THE USEFULNESS OF ECONOMETRIC MODELS WITH STOCHASTIC VOLATILITY AND LONG MEMORY: APPLICATIONS FOR MACROECONOMIC AND FINANCIAL TIME SERIES

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Abstract

This study aims to examine the usefulness of econometric models with stochastic volatility and long memory in the application of macroeconomic and financial time series. An ARFIMA-FIAPARCH process is used to estimate the two main parameters driving the degree of persistence in the US real interest rate and its uncertainty. It provides evidence that the US real interest rates exhibit dual long memory and suggests that much more attention needs to be paid to the degree of persistence and its consequences for the economic theories which are still inconsistent with the finding of either near-unit-root or long memory mean-reverting behavior.

A bivariate GARCH-type of model with/without long-memory is constructed to concern the issue of temporal ordering of inflation, output growth and their respective uncertainties as well as all the possible causal relationships among the four variables in the US/UK, allowing several lags of the conditional variances/levels used as regressors in the mean/variance equations. Notably, the findings are quite robust to changes in the specification of the model.

The applicability and out-of-sample forecasting ability of a multivariate constant conditional correlation FIAPARCH model are analysed through a multi-country study of national stock market returns. This multivariate specification is generally applicable once power, leverage and long-memory effects are taken into consideration. In addition, both the optimal fractional differencing parameter and power transformation are remarkably similar across countries.

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Declaration

Selected publications arising from the research discussed in this thesis:

- Karanasos, M., Conrad, C., and Zeng, N. 2008. Multivariate Fractionally integrated APARCH modelling of stock market volatility: a multi-country study, mimeo.
- Karanasos, M., Sekioua, S., and Zeng, N. 2006. On the order of integration of monthly US ex-ante and ex-post real interest rates: new evidence from over a century of data. *Economics Letters* 90, 163-169.
- Karanasos, M. and Zeng, N. 2007. UK inflation, output growth and their uncertainties: four variables, twelve links and many specifications, mimeo.
- Karanasos, M. and Zeng, N. 2007. The link between macroeconomic performance and uncertainty, mimeo.
- Karanasos, M. and Zeng, N. 2008. The persistence in inflation and output growth and the importance of the latter for the performance-uncertainty link, mimeo.

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Introduction

This study aims to examine the usefulness of econometric models with stochastic volatility and long memory and gain an insight into the macroeconomic and financial time series, such as interest rates, inflation, output growth and stock returns. The investigation of interest rates is motivated by the following four factors. First, the fact that an understanding of their dynamics is central to the study of prominent macroeconomic models and to the valuation of financial assets. Second, the number of economic theories which are inconsistent with the finding of nonstationarity. In particular, the long-run Fisher relationship requires the ex-ante real rate to be stationary. Third, developments of unit root tests with good size and power. Fourth, the empirical evidence to date concerning the order of integration of the US rates, which is rather mixed. For example, Sekioua (2004) suggests that they can be viewed as stationary albeit quite persistent processes, whereas Rapach and Weber (2004) conclude that they contain a unit root component. In sharp contrast, Rapach and Wohar (2004) find that the US quarterly postwar tax-adjusted real rates are consistent with either a high degree of persistence or a unit root. This evidence that the data may be generated by either an I(0) or I(1) process is at least indicative of fractional integration. Accordingly, Tsay (2000) argues that real rates do not contain a unit root but are fractionally integrated[1].

[1] However, this article has not explored the time-dependent heteroscedasticity in the second conditional moment of the real interest rate process.

The two main parameters driving the degree of persistence in the real interest rate and its uncertainty are estimated using a fractionally integrated ARMA-asymmetric power ARCH (ARFIMA-FIAPARCH) process, which is sufficiently flexible to handle the dual long memory behavior encountered in the real US rates. This study provides evidence that the US real interest rates exhibit dual long memory with orders of integration which differ significantly from zero and unity. Persistence, in the present context, is problematic not just for the Fisher hypothesis but also for the consumption based capital asset pricing model (CCAPM). The CCAPM implies that the growth rate of consumption and the real interest rate should have similar time-series characteristics. Still, the growth rate of consumption has been found to contain no unit root and does not exhibit the persistence apparent in real interest rates (Rapach and Wohar, 2004). Thus, although in finding no unit root, the results might have been seen as resolving the puzzling irregularity concerning the behavior of interest rates implied by the CCAPM, the observed persistence means that another irregularity emerges.

Another concern in this thesis is that one of contemporary debates about the inflation-growth interaction is linked to another on-going dispute, that of the existence or absence of a variance relationship.[2] As Fuhrer (1997) puts it:

> "..., it is difficult to imagine a policy that embraces targets for the level of inflation or output growth without caring about their variability around their target levels. The more concerned the monetary policy is about maintaining the level of an objective as its target, the more it will care about the variability of that objective around its target, ..." (p. 215)

[2] The terms variance, variability, uncertainty and volatility are interchangeably used in the remainder of the text.

Thus, Fuhrer focuses his attention on the trade-off between the volatility of inflation and that of output growth. The extent to which there is an interaction between them is an issue that cannot be resolved on merely theoretical grounds. To paraphrase the words of Temple (2000, p. 407): when one lists ideas about the influence of macroeconomic performance on uncertainty, it is striking that theoretical models are less common than hypotheses or conjectures.[3] Not only that, the models regarding the opposite link (the impact of uncertainty on performance) that do exist are often ambiguous in their predictions. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

The last ten years have seen an outpouring of empirical work intended to explain the links among the four variables. Many researchers who have worked on this field over the last decade or so have endorsed the GARCH model. Indeed, this model has been the driving force behind the quest to examine the interactions between the macroeconomic performance and its uncertainty.[4] Despite numerous empirical studies, there still exists controversy over the robustness of these relationships. The GARCH studies by Karanasos et al. (2004), Karanasos and Kim (2005a) and Karanasos and Schurer (2005) focus almost exclusively on the empirical linkages between any of the following three: (i) inflation and its volatility, (ii) nominal and real uncertainty and (iii) growth and its variability. It makes good sense to treat these issues together as answers to one relationship is usually relevant to the other two.

One potentially controversial aspect of nearly all bivariate GARCH processes is the way in which the conditional variance-covariance matrix is formulated. The two

[3]The term macroeconomic performance(uncertainty) is used as a shorthand for inflation(uncertainty) and output growth(uncertainty).
[4]Of course, the GARCH process is not the only possible model of the performance-uncertainty link.

most commonly used models are the constant conditional correlation (ccc) specification and the BEKK (named after Baba, Engle, Kraft, and Kroner) representation.[5] At the one extreme, the former assumes that there is no link between the two uncertainties, whereas, near the other extreme, the latter only allows for a positive variance relationship. At this point one alternative model suggests itself, that is, a formulation of the ccc model allowing for a bidirectional feedback between the two volatilities, which can be of either sign positive or negative, and therefore derives sufficient conditions for the non-negativity of the two conditional variances.

The studies by Grier and Perry (2000), Grier et al. (2004) and Shields et al. (2005) focus on the impact of uncertainty on performance (the so called in-mean effects). These studies simultaneously estimate a system of equations that allows only the current values of the two conditional variances to affect inflation and growth (see also Elder, 2004). However, any relationship where macroeconomic performance is influenced by its variability takes time to show up and cannot be fairly tested in a model that restricts the effect to be contemporaneous. In this thesis a system of equations is estimated, allowing various lags of the two variances to affect the conditional means. An empirically important issue is that it is difficult to separate the nominal uncertainty from inflation as the source of the possible negative impact of the latter on growth. This distinction is important as a policy matter, as pointed out by Judson and Orphanides (1999):

> "If inflation volatility is the sole culprit, a high but predictably stable level of inflation achieved through indexation may be preferable to a lower, but more volatile, inflation resulting from an activist disinflation strategy. If on the other hand, the level of inflation *per se*

[5]The ccc and BEKK GARCH models introduced by Bollerslev (1990) and Engle and Kroner (1995) respectively.

> negatively affects growth, an activist disinflation strategy may be the only sensible choice?" (p. 118)

Perhaps a more promising approach is to construct a model allowing for effects in opposite direction as well. There exists relatively little empirical work documenting the influence of performance on uncertainty (the so called level effects). Dotsey and Sarte (2000) point out that countries which have managed to live with relatively high levels of inflation, should exhibit greater variability in their real growth rate. Inflation breeds uncertainty in many forms. The fact that higher inflation has implications for the volatility of growth has thus far been overlooked in empirical studies. One could also imagine that when economic growth decreases, there is some uncertainty generated about the future path of monetary policy, and consequently, inflation variability increases (Brunner, 1993). Although Dotsey and Sarte's and Bruner's hypotheses are merely suggestive, their conjectures suggest the importance of devoting greater explicit attention to the effects of inflation and growth on nominal and real uncertainty.

The above considerations along with the just mentioned complexity, have led to a protracted chicken-or-egg debate about the causal relations between inflation, growth and their respective uncertainties. This study employs a ccc model with lagged inflation and growth included in the variance specifications and the conditional variances various lags of the two variables added in the mean, which is considered with the best model chosen on the basis of the minimum value[6] of the information criteria. In other words, the bidirectional causality (either direct or indirect) between the four variables examined in thesis is in contrast with the existing literature that focuses almost exclusively on the effect of uncertainty on performance.

[6] For the estimation implemented by James Davidson (2006-2008) in TSM, this should be understood as minimum absolute value.

All the interactions among the four variables of UK and US will be examined simultaneously. In doing so it is able to highlight some key behavioral features that are present across various bivariate formulations.

This thesis also draws attention to a common finding in much of the empirical finance literature, that is, although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated (see, for example, Dacorogna et al. 1993, Granger and Ding, 1995a, 1995b and Breidt et al. 1998). In particular, Ding et al. (1993) investigate the autocorrelation structure of $|s_t|^\delta$, where s_t is the daily S&P 500 stock market returns, and δ is a positive number. They found that $|s_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which they call the Asymmetric Power ARCH (APARCH). In addition, they show that this formulation comprises seven other specifications in the literature.[7] Brooks et al. (2000) analyze the applicability of the PARCH models to national stock market returns for ten countries plus a world index. Bollerslev and Mikkelsen (1996) provide strong evidence that the conditional variance for the S&P 500 composite index is best modelled as a mean-reverting fractionally integrated process. Christensen and Nielsen (2007) analyze the impulse response function for future returns with respect to a unit shock in current volatility. They show that the interaction of a positive risk-return link, long-memory in volatility, and a strong financial leverage effect, yields a perhaps surprisingly low impact of volatility shocks on asset values. McCurdy and Michaud (1996) analyze the CRSP value-weighted index using a fractionally integrated APARCH (FIAPARCH) type of model. McCurdy

[7]These models are: the ARCH (Engle, 1982), the GARCH (Bollerslev, 1986), the Taylor/Schwert GARCH in standard deviation (Taylor, 1986, and Schwert, 1990), the GJR GARCH (Glosten et al., 1993), the TARCH (Zakoian, 1994), the NARCH (Higgins and Bera, 1992) and the log-ARCH (Geweke, 1986, and Pantula, 1986).

and Michaud (1996) and Tse (1996, 1998) extend the asymmetric power formulation of the variance to incorporate fractional integration, as defined by Baillie et al. (1996).[8]

The FIAPARCH model increases the flexibility of the conditional variance specification by allowing (a) an asymmetric response of volatility to positive and negative shocks, (b) the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest, and (c) long-range volatility dependence. These three features in the volatility processes of asset returns have major implications for many paradigms in modern financial economics. Optimal portfolio decisions, the pricing of long-term options and optimal portfolio allocations must take into account all of these three findings. E.g., Giot and Laurent (2003) have shown that APARCH volatility forecasts outperform those obtained from the RiskMetrics model, which is equivalent to an integrated ARCH with prespecified autoregressive parameter values. The fractionally integrated process may lead to further improvement, if its forecasts are more accurate than those obtained from the stable specification. Another important advantage of having a FIAPARCH model is that it nests the formulation without power effects and the stable one as special cases. This provides an encompassing framework for these two broad classes of specifications and facilitates comparison between them. The main contribution of this study is to enhance the understanding of whether and to what extent this type of model improves upon its simpler counterparts.

The evidence provided by Tse (1996, 1998) suggests that the FIAPARCH model is applicable to the yen-dollar exchange rate. More recently, Degiannakis (2004) and Ñíguez (2007) applied univariate FIAPARCH specifications to stock return data. So

[8]The FIGARCH model of Baillie et al. (1996) is closely related to the long-memory GARCH process (see Karanasos et al., 2003, and Conrad and Karanasos, 2006, and the references therein). The Hyperbolic GARCH (HYGARCH) model of Davidson (2004) and the fact that Robinson (1991) was the first to consider the long-memory potential in volatility should also be mentioned.

far, multivariate versions of the framework have rarely been used in the literature. Only Dark (2004) applies a bivariate error correction FIAPARCH model to examine the relationship between stock and future markets, and Kim et al. (2005) use a bivariate FIAPARCH-in-mean process to model the volume-volatility relationship. Therefore, an interesting research issue is to explore how generally applicable this formulation is to a wide range of financial data and whether multivariate specifications can outperform their univariate counterparts. This study attempts to address this issue by estimating both univariate and multivariate versions of this framework for eight series of national stock market index returns. These countries are Canada, France, Germany, Hong Kong, Japan, Singapore, the United Kingdom and the United States. Furthermore, the ability of the FIAPARCH formulation to forecast (out-of-sample) stock volatility is assessed by a variety of forecast error statistics. By employing the tests of Diebold and Mariano (1995) and Harvey et al. (1997), it is able to verify whether the difference between the statistics from the different models is statistically significant,

The remainder of this thesis is structured as follows. Chapter 1 examines the stochastic volatility and long memory in interest rates, inflation and output growth of UK and US. Chapter 2 uses a bivariate GARCH processes to analyse the interaction among inflation, output growth and their uncertainties for UK. Chapter 3 then employs multivariate FIAPARCH for a multi-country study of stock market volatility, as well as evaluates the different specifications in terms of their out-of-sample forecast ability. Finally several remarks are concluded.

CHAPTER 1

Stochastic Volatility and Long Memory in Interest Rates, Inflation and Output Growth

1.1. Introduction

This chapter investigates the stochastic volatility and long memory in interest rates, inflation and output growth. Section 1.2 concerns the issue of that the US data is not consistent with a unit root in real interest rates, although shocks impinging upon these rates are rather persistent, using a long series of monthly US ex-post and ex-ante real interest rates spanning over 100 years. In addition, the results highlight the importance of modeling long memory not only in the conditional mean but in the power transformed conditional variance as well.

Then section 1.3 investigates the interactions between the four variables of the UK, employing a bivariate GARCH model without imposed restriction in the variance relationship. An important finding is that the significance, and even the sign, of the in-mean effects varies with the choice of the lag.

Finally section 1.4 turns to address the US inflation, output growth and their respective uncertainties, which possess significant long memory property. The two dynamics for the period 1960-2004 are modelled in a bivariate dual long-memory GARCH-type process. Findings stress a bidirectional impact of growth and nominal (real) variability.

1.2. On the Order of Integration of Real Interest Rates

1.2.1. Overview

This study of real interest rates uses an ARFIMA-FIAPARCH process, which attempts to fill a gap in the literature in a number of ways. First, since the use of data on realized inflation can produce substantial small-sample bias in estimates of the Fisher relationship, both ex-ante and ex-post real interest rates are employed. Second, several powerful tests are applied, including two unit roots developed by Elliott et al. (1996) and Ng and Perron (2001), together with the test proposed by Hansen (1999) allowing for the construction of confidence intervals for the largest roots of autoregressive processes. Third, to overcome the small-sample bias and, most importantly, to increase the power of the tests a long series of monthly data that spans over 100 years is used. Fourth, to handle the dual long memory behavior, two main parameters driving the degree of persistence in the real interest rate and its uncertainty are estimated by a ARFIMA-FIAPARCH process.

Results evidence the property of dual long memory in the US real interest rates and suggest that much more attention needs to be paid to the degree of persistence and its consequences for the economic theories which are still inconsistent with the finding of either near-unit-root or long memory mean-reverting behavior.

1.2.2. Empirical Methodology

Unit root tests

Unit root in the real interest rate is tested using recently proposed tests, the efficient generalized least squares (GLS) version of the Dickey–Fuller (DF) test due to Elliott et al. (1996) and the Ng and Perron (2001) test. While most unit root tests are only concerned with testing the null hypothesis that the largest root of an autoregressive AR (k) process is unity (H_0: $\alpha = 1$) against the alternative that it is less than one, the DF-GLS method tests the null against a specific alternative H_1: $\alpha < 1$ where α is set as local-to-unity $(1 + c/T)$ and holding c fixed as $T \longrightarrow \infty$. Further, using a sequence of tests of the null of a unit root against a set of stationary persistent alternatives, Elliott et al. (1996) showed substantial power gain from the DF-GLS method over the conventional augmented DF test (which has low power against close alternatives so that the unit root null can seldom be rejected for highly persistent variables). The unit root test of Ng and Perron (2001), which follows Elliott et al. (1996) by using local-to-unity GLS detrending, has also been shown to have good size and power properties. Nonetheless, whilst these two tests are more powerful than the traditional ADF test, rejection of the unit root hypothesis leaves little information on the actual persistence and speed of mean reversion of the real interest rate.

To remedy this, the grid bootstrap method of Hansen (1999) is used, which allows for the construction of confidence intervals for α, the largest root of the following ADF equation:

$$r_t = \alpha r_{t-1} + \sum_{t=1}^{k-1} \Delta r_{t-i} + \varepsilon_t \tag{1.1}$$

Hansen's grid bootstrap has been shown, using Monte Carlo simulations, to yield accurate confidence intervals and unbiased estimates in large samples. The lag lengths used in the aforementioned tests are chosen with the modified AIC (MAIC) of Ng and Perron (2001) as it produces the best combination of size and power. It must be stressed, however, that the long lags selected by MAIC, see table 1.1, are not surprising. MAIC is designed to select relatively long lag lengths in the presence of roots (α) near unity and shorter lags in the absence of such roots.

Finally, the KPSS test statistic (η_μ) of Kwiatkowski et al. (1992) is used to test the null hypothesis of level stationarity I(0) against a unit root alternative.

Table 1.1 Unit root tests

	η_{μ}	MZ_{α}	MZ_{t}	DF-GLS	α	95 (lower)	95 (upper)	99 (lower)	99 (upper)
Ex-post	0.770	20.404	3.177	3.064	0.977	0.968	0.988	0.965	0.991
Ex-ante	0.771	19.821	3.132	3.040	0.983	0.975	0.993	0.973	0.995

The bandwidth for the KPSS (η_{μ}) test is chosen with Newey–West using Bartlett kernel. The optimal lag lengths for the unit root tests (13 and 14 for the ex-post and ex-ante rates, respectively) are set according to the modified AIC. The 95% and 99% bootstrap confidence intervals were constructed using 1999 bootstrap replications at each of 200 grid-points. The 1% critical values are: 0.739 for η_{μ}; -13.8 and -2.58 for the Ng and Perron statistics (MZ_{α} and MZ_{t}, respectively); -2.56 for the DFGLS statistic.

Essentially, if the tests reject the unit root and stationarity hypotheses, then the US rates may potentially be fractionally integrated processes.

ARFIMA-FIAPARCH model

The traditional ARMA and ARIMA specifications are incapable of imparting the persistence to real rates, which is found in the data. Put differently, by viewing the real interest rate as an I(0) or I(1) process instead of an I(d) process, there is a downward or upward bias of estimating its persistence.

The model of ARFIMA-FIAPARCH[1] generates the long memory property in both the first and (power transformed) second conditional moments and is thus sufficiently flexible to handle the dual long memory behavior encountered in the real interest rate. In the ARFIMA($l, d_m, 0$)-FIAPARCH($1, d_v, 1$) model the mean equation is defined as:

$$(1 - \phi_1 L - ... - \phi_l L^l)(1 - L)^{d_m}(r_t - \mu) = \varepsilon_t \tag{1.2}$$

where r_t denotes the real rate and $0 \leq d_m \leq 1$; ε_t is conditionally normal with mean zero and variance h_t. That is $\varepsilon_t | \Omega'_{t-1} \sim N(0, h_t)$, where Ω'_{t-1} is the information set up to time $t-1$. The structure of the conditional variance is:

$$h_t^{\delta/2} = \omega + \left[1 - \frac{(1 - \varphi L)(1 - L)^{d_v}}{(1 - \beta L)}\right] [|\varepsilon_t| - \gamma \varepsilon_t]^{\delta} \tag{1.3}$$

[1]The properties of this model are investigated in Conrad and Karanasos (2005).

where $0 \leq d_v \leq 1$, ω, $\delta > 0$, φ, $\beta < 1$ and $-1 < \gamma < 1$ (see Tse, 1998).

The two common values of the power term (δ) imposed throughout much of the GARCH literature are the values of two and unity. The invalid imposition of a particular value for the power term may lead to suboptimal modeling and forecasting performance.

The ARFIMA-FIAPARCH model has the advantage of keeping the elegance of the ARMA-GARCH model while enhancing its dynamics. Put differently, it has at least two important implications for understanding the real rate and its uncertainty. First, it recognizes the long memory aspect of the interest rate and provides an empirical measure of real uncertainty that accounts for long memory in the power transformed conditional variance of the interest rate process. Second, it allows for a more systematic comparison of many possible models that can capture the features of the real interest rate series.

1.2.3. Empirical Results

The data is extracted from the www.globalfindata.com database and includes the monthly long-term government bond yield and the consumer price index (CPI) series for the US spanning the period from 1876 to 2000 stable over the sample[2]. Of course, there is the potential problem of structural instability when using a long span of data. However, the dynamics of the US real interest rates appear to be relatively stable over sample period of this study. Also, the expected values of inflation used to construct the ex-ante rate are obtained by means of a preliminary signal extraction procedure. Signal extraction is a procedure used to separate unobservable components, expected values in this case, from an observable variable containing noise. This is achieved through the application of the law of iterated projections by means of the Kalman filter technique. The estimated model is the following:

[2]This can be observed by reference to figure B.1 of US real interest rates in Appendix B.

$$r_t = \xi_t + v_t,\ \xi_{t+1} = \xi_t + \zeta_t$$

where ξ_t is a vector of possibly unobserved state variables and v_t and ζ_t are vectors of mean zero, Gaussian disturbances. The two expressions are the signal and state equations, respectively.

By implementing the unit root tests, from table 1.1, one can see that the DF-GLS and Ng and Perron (2001) tests reject the unit root null unequivocally at the 1% level of significance. This is an important result since it contrasts sharply with what has been reported in earlier studies on real interest rates which were essentially based on shorter samples and weaker statistical tests than those this study is using. Nevertheless, the point estimates and upper limits of the grid bootstrap intervals reveal that although the root of eq. (1.1) is not unity, it is still very close to the unit root boundary. Interestingly, it appears that the results are not critically dependent on how rates are measured, whether ex-post or exante, since α for both is close to unity. Hence, if forecast errors are to blame for the failure to detect mean-reversion in small samples due to peso problems, then the fact that there is no substantial difference between ex-post and ex-ante rates in terms of α means that these errors are likely to be much smaller over long periods than over shorter periods.

In addition, the KPSS test rejects the stationarity null hypothesis for both rates[3]. Therefore, with the two US real interest rates, evidence is against the unit root as well as the stationarity hypotheses. Although these tests are merely suggestive, the overall evidence indicates the need to go beyond the I(1)/I(0) framework. Thus, fractional integration allowing for long memory is a plausible alternative (see also Lai, 1997).

[3]The KPSS test does not perform well in the presence of Moving Average error behaviour. (Caner and Killian 2001, p. 642)

Next, eqs. (1.2) and (1.3) of the ARFIMA$(l, d_m, 0)$-FIAPARCH$(1, d_v, 1)$ are estimated, in order to take into account the serial correlation observed in the levels (and their power transformations) of the time series data, and to capture the possible long memory in the conditional mean and the power transformed conditional variance. The ARFIMA-FIAPARCH models are estimated using the quasi maximum likelihood estimation (QMLE) method as implemented by Laurent and Peters (2002) in Ox. In view of the characteristic incidence of outliers in the data, the Student's t distribution is assumed for the disturbances. Table 1.2 reports the results for the period 1876–2000.

The best fitted model is chosen according to the minimum values of the Schwarz information criterion (SIC). A FIAPARCH$(1, d_v, 1)$ specification is chosen for the power transformed conditional variances and ARFIMA models of orders $(2, d_m, 0)$ and $(3, d_m, 0)$ for the ex-post and ex-ante rates, respectively. The autoregressive parameters (ϕ_1, ϕ_2, ϕ_3) were necessary to account for the significant autocorrelation, which is evident in both series. The estimated ARCH parameters $(\widehat{\varphi}, \widehat{\beta})$ for the US rates are significant and satisfy the set of conditions sufficient to guarantee the nonnegativity of the conditional variance (see Conrad and Haag, 2004). For both series negative shocks predict higher volatility than positive shocks, since the estimated asymmetry coefficient $(\widehat{\gamma})$ is significant and positive. The estimated values of d_m for the ex-post and ex-ante rates are 0.26 and 0.40, respectively, which are significantly different from zero at the 1% level and imply some strong long-memory features. In both cases the estimates for the fractional differencing parameter $(\widehat{d_v})$ are relatively large and statistically significant. For the ex-ante rate the power term is not significantly different from two. Moreover, the hypothesis of uncorrelated standardized and squared standardized residuals is well supported.

Table 1.2 ARFIMA-FIAPARCH models

	$\widehat{\mu}$	$\widehat{d}_m$	$\widehat{\phi}_1$	$\widehat{\phi}_2^a$	$\widehat{\omega}$	$\widehat{d}_v$	$\widehat{\delta}$	$\widehat{\varphi}$	$\widehat{\beta}$	$\widehat{\gamma}$	$\widehat{t}$
Ex-post	2.48	0.26	0.79	0.13	5.68	0.83	1.74	0.10	0.88	0.26	6.34
	(2.08)	(6.13)	(16.65)	(3.52)	(2.04)	(12.23)	(10.41)	(1.85)	(32.04)	(2.30)	(4.44)
	$\widehat{Q}(12)$: 2.48 [0.01]			$\widehat{Q}^2(12)$: 4.85 [0.77]							
Ex-ante	3.61	0.40	0.93	-0.21	0.01	0.90	2^b	0.11	0.89	0.16	7.14
	(1.20)	(5.38)	(12.36)	(4.86)	(1.35)	(12.30)	-	(1.68)	(36.65)	(2.18)	(4.31)
	$\widehat{Q}(12)$: 12.15 [0.22]			$\widehat{Q}^2(12)$: 22.41 [0.01]							

For each of the two series, table 1.2 reports QMLE parameter estimates for the ARFIMA-FIAPARCH model. The numbers in (.) are absolute t-statistics and in [.] are p values.
[a]For the ex-ante rate a $\widehat{\phi}_3$ of 0.17(5.25) is estimated. [b]The estimated value of δ is 1.96 and not significantly different from 2. t are the degrees of freedom for the Student's t distribution.

To test for the persistence in the two conditional moments of the two series, the likelihood ratio (LR) tests are used for the linear constraints $d_m = 0$ ('ARMA' model), $d_v = 0$ ('APARCH' model) and $d_m = d_v = 0$ ('ARMA-APARCH' model). As seen in table 1.3 for both rates the LR statistics clearly reject the 'ARMA' and the 'APARCH' null hypotheses against the ARFIMA-FIAPARCH model. The evidence obtained from the LR tests is reinforced by the model ranking provided by the SIC model selection criterion. In both cases the criterion (not reported) favors the ARFIMA-FIAPARCH model over both the ARMA-FIAPARCH and ARFIMA-APARCH models. Hence, from the various diagnostic statistics it appears that monthly US real interest rate has long memory behavior in both its first and its (power transformed) second conditional moments.

1.2.4. Summary

This section has examined the long-term persistence of ex-ante and ex-post US real interest rates. Responses to the problem of low power of the standard unit root tests were two fold. First, employing recently developed econometric techniques greatly improves the power of these tests. Second, using a long span of monthly data covers more than a century. Estimation results show that the US real rate displays near

Table 1.3 Tests of fractional differencing parameters in the first and second conditional moments

	$\widehat{d}_m$	H_0: ARMA(d_m=0)	$\widehat{d}_v$	H_0: APARCH(d_v=0)	H_0: ARMA-APARCH(d_v=d_m=0)
		LR		LR	LR
Ex-post	0.26	27.28	0.83	8.42	34.40
	{0.04}	[0.00]	{0.07}	[0.00]	[0.00]
Ex-ante	0.40	27.95	0.90	1.85	28.52
	{0.07}	[0.00]	{0.07}	[0.17]	[0.00]

LR test: LR=2[ML_u-ML_r], where ML_u and ML_r denote the maximum log-likelihood values of unrestricted and restricted models, respectively. The numbers in [.] are *p*-values. The numbers in {.} are standard errors.

integrated behavior, precisely the type of stationary behavior that will be difficult for standard tests to detect for samples as short as the post war era which are typically used in the extant literature.

However, recognizing that the knife-edge distinction between I(0) and I(1) processes can be far too restrictive, this investigation of the long memory aspect of the US interest rates provided an empirical measure of its uncertainty that accounts for long memory in the second conditional moment of the real interest rate process. Analogous to the issues pertaining to the proper modeling of the long-run dynamics in the conditional mean of the real US rate, similar questions, therefore, become relevant in the modeling of its conditional volatility.

Finally, Rapach and Wohar (2004) find that real US consumption growth exhibits very mild persistence. These significant differences in the degree of persistence for the US real interest rate and consumption growth imply sustained violations of the Euler condition at the center of the consumption-based asset pricing model. Hence, this study reemphasises Rapach and Wohar's point: a quite persistent real interest rate, due to either near-unit-root or long memory mean reverting behavior, has important theoretical implications for the CCAPM model.

1.3. The Link between Macroeconomic Performance and Uncertainty

1.3.1. Overview

This section uses a bivariate ccc GARCH model to investigate the interactions between inflation, growth, and their respective uncertainties. Previous work on this field over the last decade or so has endorsed the GARCH model (see, for example, Grier and Perry, 2000, and Fountas et al., 2002). The distinguishing feature of this study is examining all the possible causal relationships among the four variables that are predicted by economic theory in a single empirical framework, allowing for a bidirectional feedback between the two volatilities, which can be of either sign, positive or negative, and so no restriction is imposed. This has the advantage of deriving sufficient conditions for the non-negativity of the two conditional variances.

In this section, a system of equations is estimated, which allows various lags of the two volatilities to affect the conditional means and rather than utilizing formulations that allow only the current values of the two conditional variances to affect the means involved in previous studies. The following observations, among other things, are noted about the interlinkages. One significant importance is that in all cases there is strong bidirectional feedback between inflation and growth. Another useful piece of evidence is that nominal variability has a negative but insignificant effect on real volatility. Moreover, inflation has a positive impact on macroeconomic uncertainty as predicted by Ungar and Zilberfarb (1993) and Dotsey and Sarte (2000). Whereas the link between inflation and its variability is well documented, not much attention has been paid to its effect on the variance of growth. There is also a lack of a direct influence from growth to macroeconomic variability.

Finally, the significance and even the sign of the in-mean effects vary with the choice of the lag. Thus this analysis suggests that the behavior of macroeconomic performance depends upon its uncertainty, but also that the nature of this dependence varies with time. For example, at lag one, the impact of real variability on growth is positive as predicted by Blackburn (1999), but at lag three it turns to negative as predicted by Pindyck (1991). In contrast, the negative but insignificant effect from nominal uncertainty to inflation exhibits much less sensitivity.

1.3.2. Economic Theory

The economic theory concerning the relationship between macroeconomic uncertainty and performance has been widely discussed. Some researchers find evidence that inflation negatively Granger causes growth (see Gillman and Kejak, 2005, and the references therein). Briault (1995) argues that there is a positive relationship between the two variables, at least over the short run, with the direction of causation running from higher growth (at least in relation to productive potential) to higher inflation. Fuhrer (1997) explores the nature of the long-run variance trade-off. The short-run trade-off between the two variables that exists in the models he explores implies a long-run trade-off in their volatilities. Furthermore, Ungar and Zilberfarb (1993) provide a theoretical framework in order to specify the necessary conditions for the existence of a positive impact of inflation on its uncertainty. Dotsey and Sarte (2000) present a model which suggests that as average money growth rises nominal variability increases and real growth rates become more volatile.

Moreover, one possible reason for greater nominal uncertainty to precede lower inflation is that an increase in volatility is viewed by policymakers as costly, inducing them to reduce inflation in the future (Holland, 1995). The inflation bias-producing mechanism in Cukierman and Gerlach (2003) implies a positive relationship between inflation and the variance of growth, where causality runs from the latter to the former. In his Nobel address, Friedman (1977) explains a possible positive correlation between inflation and unemployment by arguing that high inflation produces more uncertainty about future inflation. This variability then lowers economic efficiency and temporarily reduces output and increases unemployment. Finally, Pindyck (1991), among others, proposes a theory for which the negative impact of real volatility on growth relies on uncertainty through the link of investment. In another class of models the relationship between short-term variance and long-term growth is positive (see Blackburn, 1999, and the references therein).

1.3.3. Empirical Strategy

A bivariate model is used to simultaneously estimate the conditional means, variances, and covariances of inflation and growth. Let π_t denote the former and y_t the latter and define the residual vector ε_t as $\varepsilon_t = (\varepsilon_{\pi t}\varepsilon_{yt})'$. Regarding ε_t, it is assumed to be conditionally normal with mean vector 0, variance matrix H_t where $h_t = (h_{\pi t}h_{yt})'$ and covariance $h_{\pi y,t}$. That is $(\varepsilon_t|\Omega'_{t-1}) \sim N(0, H_t)$, where Ω'_{t-1} is the information set up to time $t-1$.

Note that a general bivariate vector autoregressive (BVAR) GARCH-in-mean model can be written as

$$(I - \sum_{l=1}^{p} \Phi_l L^l)(x_t - \Delta h_{t-n}) = \Phi_0 + \varepsilon_t, \quad t \in \mathbb{N}, \tag{1.4}$$

with

$$\Phi_l = \begin{bmatrix} \phi_{\pi\pi}^{(l)} & \phi_{\pi y}^{(l)} \\ \phi_{y\pi}^{(l)} & \phi_{yy}^{(l)} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \delta_{\pi\pi} & \delta_{\pi y} \\ \delta_{y\pi} & \delta_{yy} \end{bmatrix}, \quad \Phi_0 = \begin{bmatrix} \phi_{\pi 0} \\ \phi_{y0} \end{bmatrix}$$

where I is a 2×2 identity matrix, x_t is 2×1 column vectors given by $x_t = (\pi_t \ y_t)'$, $l = 0, 1, \ldots$ and $n = 0, 1, \ldots$. $\mathbf{\Delta}$ captures the in-mean effects. The ijth $(i, j = \pi, y)$ elements of the 2×2 matrices $\mathbf{\Delta}$ and Φ are denoted by δ_{ij} and ϕ_{ij} respectively and assuming all the roots of $\left|\sum_{l=1}^{p} \Phi_l L^l\right|$ lie outside the unit circle.

Following Bollerslev (1990), the ccc GARCH-level structure on the conditional covariance matrix H_t is imposed:

$$h_t = \Omega + A(L)\varepsilon_{t-1}^2 + B(L)h_{t-1} + \Gamma x_{t-1}, \tag{1.5}$$

with

$$\Omega = \begin{bmatrix} \omega_\pi \\ \omega_y \end{bmatrix}, \quad A = \begin{bmatrix} a_{\pi\pi} & a_{\pi y} \\ a_{y\pi} & a_{yy} \end{bmatrix}, \quad B = \begin{bmatrix} \beta_{\pi\pi} & \beta_{\pi y} \\ \beta_{y\pi} & \beta_{yy} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \gamma_{\pi\pi} & \gamma_{\pi y} \\ \gamma_{y\pi} & \gamma_{yy} \end{bmatrix}.$$

where $h_{\pi y,t} = \rho\sqrt{h_{\pi t}}\sqrt{h_{yt}},$ $(-1 \leq \rho \leq 1)$, Γ captures the level effects and the ijth $(i, j = \pi, y)$ elements of the 2×2 matrices are denoted by γ_{ij}. The acronym BVAR(l)-GARCH(1, 1)-ML(n,1) will be used to refer to this model.

It is worth reiterating in just a few sentences to talk about the main benefits of this model. Its greatest advantage is that it does not require making the dubious assumption that there is a positive link between the two uncertainties. That is, the coefficients that capture the volatility-relationship ($\beta_{\pi y}$, $\beta_{y\pi}$) are allowed to be negative. It has the convenience of deriving sufficient conditions for the non-negativity of the two conditional variances. These conditions can be seen as analogous to those derived by Nelson and Cao (1992) for the univariate GARCH model. Another advantage is that several lags of the conditional variances are added as regressors in the mean equation. Further, distinguishing empirically between the in-mean and level effects found in theoretical models is extremely difficult in practice so it makes sense to emphasize that both are relevant.

1.3.4. Empirical Results

Monthly UK data, obtained from the OECD Statistical Compendium, are used to provide a reasonable number of observations. The inflation and output growth series are calculated as the monthly difference in the natural log of the CPI and Industrial Production Index (IPI) respectively[4]. The data range from 1962:01 to 2004:01. Allowing for differencing this implies 504 usable observations. For the two series, based on unit root tests (see table E.1), it is able to reject the unit root hypothesis. The estimates of the various formulations were obtained by maximum likelihood estimation (MLE) as implemented by James Davidson (2006) in TSM. The best AR(GARCH) specification is chosen on the basis of LR tests and three alternative information criteria. For the conditional means[variances] of inflation and growth, AR(14)[GARCH(1,1)] and AR(12)[ARCH(1)] models are chosen respectively. Next, the results from four alternative specifications are analysed to examine the sign and the significance of the estimated coefficients, and therefore to provide some statistical evidence on the nature of the relationship between the four variables[5].

First, inflation affects growth negatively, whereas growth has a positive effect on inflation (see the coefficients). That is, there is strong evidence supporting the Gillman-Kejak theory and the Briault conjecture. Following Burke and Hunter (2005), the long-run growth-inflation relation expressed as $\overline{x}_i = \frac{\phi_0}{\alpha} + \sum \frac{\overline{x}_j}{\alpha}$ is significant as well[6]. Next, nominal uncertainty has a negative, as predicted by Fuhrer (1997), but insignificant impact on real volatility ($\beta_{y\pi} < 0$). When estimating ML models with $\beta_{\pi y} \neq 0$, the estimation routine did not converge. There is also strong evidence in favor of the Ungar-Zilberfarb theory and the Dotsey-Sarte conjecture that higher inflation has a positive impact on nominal and real uncertainty respectively ($\gamma_{\pi\pi}$, $\gamma_{y\pi} > 0$). In sharp contrast, macroeconomic variability appears to be independent of changes in growth (that is, $\gamma_{y\pi}$ and γ_{yy} are insignificant). It also

[4]Actual data is plotted in figure B.2.

[5]Note estimates on causality are conditional on the structrue of the model and may change with the number and nature of the variables (Hendry, 1995)

[6]The estimates of effect of long-run output growth on long-run inflation and that of long-run inflation on output growth are 0.04 (0.03) and -0.17 (0.10) respectively.

Table 1.4 BVAR-GARCH ML model (n = 0)

Mean	$\phi^{(5)}_{\pi y}$	$\phi^{(7)}_{\pi y}$	$\phi^{(7)}_{y\pi}$	$\phi^{(11)}_{y\pi}$	$\delta_{\pi\pi}$	$\delta_{\pi y}$	$\delta_{y\pi}$	δ_{yy}
	0.04***	0.04***	-0.20**	0.13*	-0.14	0.02*	-0.37	0.05
	{0.02}	{0.01}	{0.09}	{0.08}	{0.31}	{0.01}	{0.59}	{0.08}
Variance	-	ρ	$\beta_{\pi y}$	$\beta_{y\pi}$	$\gamma_{\pi\pi}$	$\gamma_{\pi y}$	$\gamma_{y\pi}$	γ_{yy}
		0.02	-	-0.02	0.07**	-0.002	0.53***	-0.10
		{0.06}		{0.18}	{0.03}	{0.008}	{0.09}	{0.18}
	$Q_\pi(12)$: 17.00 [0.15]			$Q^2_\pi(12)$: 16.52 [0.17]				
	$Q_y(12)$: 16.27 [0.18]			$Q^2_y(12)$: 15.94 [0.19]				

Notes: Table 1.4 reports estimates of the parameters of interest. The numbers in {.} and [.] are robust standard errors and p values respectively.
***, **, * denote significance at the 0.01, 0.05 and 0.10 levels respectively.

demonstrates the invariance of all the above findings to changes in the specification of the model (see tables 1.4-7).

One particular theoretical interest has been the relationship between growth and its variance with different analyses reaching different conclusions, which depends on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (see Blackburn and Pelloni, 2004, 2005, and the references therein). At lag one, the impact of real variability on growth is positive ($\delta_{yy} > 0$) as predicted by Blackburn but at lag three it turns to negative ($\delta_{yy} < 0$) as predicted by Pindyck (see tables 1.5-6). In addition, only at lag zero, a significantly positive causal effect from real volatility on inflation ($\delta_{\pi y} > 0$) appears, offering support for the Cukierman-Gerlach theory (see table 1.4).

The estimation results show the effect of nominal uncertainty on growth is negative ($\delta_{y\pi} < 0$) as predicted by Friedman (see table 1.7) . However, when controlling for the impact of inflation on growth, the evidence in support of the Friedman hypothesis disappears (see table 1.4). Finally, there is no direct impact of nominal variability on inflation. In contrast, the indirect effect that works via the output growth is negative. That is, the nominal volatility has a negative impact on growth, which in turn affects inflation positively.

Table 1.5 BVAR-GARCH ML model (n = 1)

Mean	$\phi_{\pi y}^{(5)}$	$\phi_{\pi y}^{(7)}$	$\phi_{y\pi}^{(7)}$	$\phi_{y\pi}^{(11)}$	$\delta_{\pi\pi}$	$\delta_{\pi y}$	$\delta_{y\pi}$	δ_{yy}
	0.04***	0.04***	-0.25*	0.11	-0.15	0.001	-0.22	0.04**
	{0.02}	{0.01}	{0.08}	{0.08}	{0.25}	{0.01}	{0.43}	{0.02}
Variance	-	ρ	$\beta_{\pi y}$	$\beta_{y\pi}$	$\gamma_{\pi\pi}$	$\gamma_{\pi y}$	$\gamma_{y\pi}$	γ_{yy}
		0.02	-	-0.09	0.07**	-0.002	0.53***	-0.15
		{0.06}		{0.22}	{0.03}	{0.009}	{0.11}	{0.20}
	$Q_\pi(12)$: 16.78 [0.16]			$Q_\pi^2(12)$: 17.55 [0.13]				
	$Q_y(12)$: 14.15 [0.29]			$Q_y^2(12)$: 14.78 [0.25]				

Notes: As in table 1.4.

Table 1.6 BVAR-GARCH ML model (n = 3)

Mean	$\phi_{\pi y}^{(5)}$	$\phi_{\pi y}^{(7)}$	$\phi_{y\pi}^{(7)}$	$\phi_{y\pi}^{(11)}$	$\delta_{\pi\pi}$	$\delta_{\pi y}$	$\delta_{y\pi}$	δ_{yy}
	0.04***	0.04***	-0.25*	0.07	-0.003	-0.01	0.53	-0.04°
	{0.02}	{0.01}	{0.08}	{0.09}	{0.15}	{0.01}	{0.54}	{0.03}
Variance	-	ρ	$\beta_{\pi y}$	$\beta_{y\pi}$	$\gamma_{\pi\pi}$	$\gamma_{\pi y}$	$\gamma_{y\pi}$	γ_{yy}
		0.02	-	-0.11	0.07**	-0.002	0.53***	-0.15
		{0.06}		{0.23}	{0.03}	{0.009}	{0.11}	{0.18}
	$Q_\pi(12)$: 17.19 [0.14]			$Q_\pi^2(12)$: 18.33 [0.11]				
	$Q_y(12)$: 14.81 [0.25]			$Q_y^2(12)$: 16.12 [0.19]				

Notes: As in table 1.4. $^{\circ}$denote significance at the .15 level.

Table 1.7 BVAR-GARCH ML model (n = 0) with restrictions

Mean	$\phi_{\pi y}^{(5)}$	$\phi_{\pi y}^{(7)}$	$\phi_{y\pi}^{(7)}$	$\phi_{y\pi}^{(11)}$	$\delta_{\pi\pi}$	$\delta_{\pi y}$	$\delta_{y\pi}$	δ_{yy}
	0.04***	0.04***	-	-	-0.11	0.02*	-0.77*	-0.03
	{0.02}	{0.01}			{0.28}	{0.01}	{0.47}	{0.08}
Variance	-	ρ	$\beta_{\pi y}$	$\beta_{y\pi}$	$\gamma_{\pi\pi}$	$\gamma_{\pi y}$	$\gamma_{y\pi}$	γ_{yy}
		0.01	-		0.07*	-		-0.15
		{0.06}			{0.03}			{0.17}
	$Q_\pi(12)$: 16.71 [0.16]			$Q_\pi^2(12)$: 17.57 [0.13]				
	$Q_y(12)$: 15.94 [0.19]			$Q_y^2(12)$: 18.89 [0.09]				

Notes: As in table 1.4. The B and Γ matrices are diagonal. The Φ matrix is upper triangular.

1.3.5. Summary

In this section, the link between UK inflation, growth and their respective uncertainties has been investigated. The variables under consideration are inextricably linked. Previous literature shows how hard it is to arrive at definitive conclusions on this topic. One of the objectives of this analysis was to consider several changes in the formulation of the bivariate model and discuss how these changes would affect the twelve interlinkages among the four variables.

One dramatic finding is that some in-mean effects are found to be quite robust to the various specifications. In particular, inflation is invariant to changes in its volatility. Some others are found to be 'fragile' in the sense that either their statistical significance disappears or their sign changes when a different formulation is used. Slight variations in the specification of the regressions appear to yield substantially different results for the influence of the two volatilities on growth. In particular, when controlling for the impact of inflation on growth the evidence for the Friedman hypothesis disappears. The interlinkage between levels of the two variables may, therefore, be an important element masking the negative effects of nominal variability on growth. Lack of robustness should often spur further investigation into causality and inter-relationships. Finding that some results are fragile could in itself be valuable information.

Moreover, inflation has a positive impact on macroeconomic uncertainty. Whereas the link between inflation and its volatility is well documented, not much attention has been paid to its effect on real variability[7]. Evidence for a positive indirect causal effect from growth on the variance of inflation has also been found. The indirect impact works through the channel of inflation. This effect has also been overlooked in the literature. There has been surprisingly little work of this kind. When examining simultaneously the direct and indirect impact of growth on the nominal uncertainty, the former disappears. In doing so, it shows that accounting for the indirect effect reduces the strength of the direct one.

[7]However, nominal effects have been observed in models of real macro variables such as growth in the past (Deaton 1977, Davidson, Hendry, Srba and Yeo 1978).

1.4. The Persistence in Inflation and Output Growth and the Importance of the Latter for the Performance-uncertainty Link

1.4.1. Overview

This section uses a bivariate dual long-memory GARCH-type of model to investigate the various interactions among inflation, output growth and their uncertainties, which can help identify the relative contributions of different influences more precisely than previous studies. A bivariate diagonal constant conditional correlation (DCCC) AR-FI-GARCH process[8] is employed to examine the four main parameters driving the degree of persistence in inflation, growth and their respective uncertainties, which provides a general and flexible framework to study complicated processes like inflation and growth. Put differently, it is sufficiently flexible to handle the dual long-memory behavior encountered in the two series. It has also the advantage of allowing one to derive sufficient conditions for the non-negativity of the two conditional variances (see, for example, Conrad and Haag, 2006, and Conrad and Karanasos, 2008a,b).

This section stresses the link between growth and nominal(real) variability, to which related research should devote greater explicit attention. As Dotsey and Sarte (2000) point out, countries which have managed to live with relatively high levels of inflation should exhibit greater variability in their real growth rate. Inflation breeds uncertainty in many forms. Brunner (1993) then suggests that some uncertainty is generated about the future path of monetary policy when economic growth decreases. Consequently, inflation volatility increases. On the other hand, the sensitivity of the in-mean effects to the exclusion of level effects is also checked.

The evidence in support of the Blackburn (1999) theory that real volatility has a positive impact on growth disappears when including the level effects. Another

[8] This section refers to a model that is fractionally integrated in both the AR and GARCH specifications as the AR-FI-GARCH process.

finding is that growth has a positive effect on inflation whereas the latter affects the former negatively. The analysis also highlights reciprocal interactions in which two or more variables influence each other, either directly or indirectly. For example, there is an indirect (that works via growth) bidirectional feedback between the two volatilities. That is, the variance of inflation has a positive impact on real variability whereas it is affected negatively by it.

It is also noteworthy that the indirect effect of nominal(real) uncertainty on inflation that works via growth is negative(positive). Moreover, the indirect effect (via the channel of growth) regarding the positive impact of inflation on its uncertainty is opposite to the negative direct effect. Ungar and Zilberfarb (1993) provide a theoretical framework in order to specify the necessary conditions for the existence of a positive or negative impact. Finally, growth has a (positive)negative (in)direct influence on its variability as predicted by Karanasos and Zeng (2007). They hypothesize that the indirect effect could work through the positive impacts of growth on inflation and of the latter on the real uncertainty.

1.4.2. Inflation Dynamics and Economic Theory

Inflation persistence

Conrad and Karanasos (2005a) summarize several empirical and theoretical studies that investigate the short-term inflation dynamics. The nature of these dynamics is a central issue in macroeconomics and one of the most fiercely debated. There is an extensive theoretical literature that attempts to develop structural models of inflation providing a good approximation to its dynamics (see, for example, Chugh, 2007, and Amano, 2007) and an equally extensive empirical literature that attempts to document the properties of inflationary shocks. For example, Pivetta and Reis (2007), using different measures and estimation procedures, find that inflation persistence has been high and approximately unchanged in the US.

Backus and Zin (1993) find that a fractional root shows up very clearly in monthly US inflation. They conjecture that the long-memory in inflation is the result of aggregation across agents with heterogeneous beliefs. Similarly, Hassler and Wolters (1995) point out, that a likely explanation of the significant persistence in the inflation rate series is the aggregation argument, which states that persistence can arise from aggregation of constituent processes, each of which has short-memory. Alternatively, Baum et al. (1999) conjecture that the long-memory property of monetary aggregates will be transmitted to inflation, given the dependence of long-run inflation on the growth rate of money.

Baillie et al. (1996) explore the time-dependent heteroscedasticity in the second conditional moment of the inflation process. They utilize the ARFIMA-GARCH model to describe its dynamics for ten countries and they emphasize that all ten series possess substantial persistence in their conditional variances. Therefore Baillie et al. (2002) have focused their attention on the topic of long-memory and persistence in terms of the nominal uncertainty. Similarly, Conrad and Karanasos (2005,a,b) find that the inflation rates for the US and many European countries display significant fractional integration in both their first and second moments.

Most importantly, Morana (2002) suggests that long-memory in inflation is due to the output growth. His model implies that the two processes must share a common long-memory component. Using a bivariate AR-FI-GARCH type of model, which allows the measurement of uncertainty about inflation and growth by the respective conditional variances, one can test for the empirical relevance of several theories that have been advanced on the relationship between the four variables.

The interactions among the four variables

As discussed in section 1.3, researchers find various evidence of the interactions among the four variables. Specifically, this section draws attention to the effect of growth on its variability. An increase in growth, given that the Briault and Dotsey-Sarte conjectures hold, pushes its variance upward. In sharp contrast, a higher growth, given that the Brunner conjecture and the Logue-Sweeney theory hold, will lower real volatility. These causal effects will be referred as the Karanasos conjecture (see, Karanasos and Zeng, 2007).

1.4.3. Methodology

The bivariate AR-FI-GARCH process

It appears from the studies of Baillie et al. (2002) and Conrad and Karanasos (2005a,b) that the apparent long-memory in the inflation rate is also present in nominal uncertainty and might be present in output growth and its variability as well (see Morana, 2002). Hence, there seems to be a need to have a joint bivariate model which incorporates long-memory in both the conditional means and variances of the two series. In other words, the time series features of inflation and growth seem to require the use of a bivariate fractionally integrated model from two different classes, namely the AR and the GARCH.

Along these lines the bivariate dual long-memory time series model for the two variables as well as its merits and properties are discussed below. The structure of the ARFI (l, d_m) GARCH in-mean equation is given by

$$\mathbf{\Lambda_m}(L)\mathbf{\Phi}(L)\left[\mathbf{x}_t - \boldsymbol{\mu} - \Delta h_{t-n}\right] = \boldsymbol{\varepsilon}_t, \quad t \in \mathbb{N}, \tag{1.6}$$

where I is a 2×2 identity matrix, $\mathbf{x}_t$ are 2×1 column vectors given by $\mathbf{x}_t = (x_{\pi t} x_{yt})'$, $\boldsymbol{\mu} = (\mu_\pi \mu_y)'$, $n = 0, 1, 2, \ldots$, and the ijth $(i, j = \pi, y)$ elements of the 2×2 matrices Φ and Δ are denoted by $\phi_{ij,t}$ and δ_{ij} respectively. $\mathbf{\Delta}$ captures the in-mean effects and $\mathbf{\Lambda}_m(L)$ captures the long-memory in the two conditional means. $\varepsilon_t = (\varepsilon_{\pi t} \varepsilon_{yt})'$, which is assumed to be conditionally normal with mean vector 0, variance matrix H_t where $h_t = (h_{\pi t} h_{yt})'$ and covariance $h_{\pi y,t} = \rho\sqrt{h_{\pi t}}\sqrt{h_{yt}}$.

Then the bivariate FIGARCH(p, d_v, q) process is defined by

$$\mathbf{B}(L)(\mathbf{h}_t - \Omega - \mathbf{\Gamma}\mathbf{x}_{t-n}) = [\mathbf{B}(L) - \mathbf{\Lambda}_v(L)\mathbf{A}(L)]\boldsymbol{\varepsilon}_t^{\hat{}2}, \tag{1.7}$$

where is Ω a $2{\times}1$ column vector given by $\Omega = (\varpi_{\pi t}\varpi_{yt})'$; ijth $(i, j = \pi, y)$ elements of the $2{\times}2$ matrices A, B and Γ are denoted by α_{ij} , β_{ij} and γ_{ij} respectively. $\mathbf{\Gamma}(L)$ captures the level effects and $\mathbf{\Lambda}_v(L)$ captures the long-memory in the two conditional variances.

Further, some more terminology and notation are established. $\mathbf{B}(L)$, $\mathbf{A}(L)$ are 2×2 diagonal polynomial matrices with diagonal elements as in equation (1.5). $\mathbf{\Lambda}_m(L)$ is a 2×2 diagonal polynomial matrix with elements $(1-L)^{d_{mi,j}}$ $(0 \leq d_{mi,j} \leq 1)$ and let also $\mathbf{d}_m = [d_{mi,j}]$. $\mathbf{\Lambda}_v(L)$ is a 2×2 diagonal polynomial matrix with elements $(1-L)^{d_{vi,j}}$ $(0 \leq d_{vi,j} \leq 1)$ and let also $\mathbf{d}_v = [d_{vi,j}]$. $\hat{}$ denotes elementwise exponentiation. $\Phi(L)$ is defined as $\Phi(L) = \mathbf{I} - \sum_{l=1}^{p} \mathbf{\Phi}_l L^l$ where $\mathbf{I}$ is an identity matrix of order 2 and assuming that all the roots of $|\Phi(L)|$ lie outside the unit circle.

This model, in particular, is capable of handling the dual long-memory behavior encountered in the two series, as well as distinguishing empirically between the in-mean and level effects [9] simultaneously. A restricted version of this formulation (with $\mathbf{d}_m = \mathbf{d}_v = 0$, and $\mathbf{\Gamma}(L) = \mathbf{\Gamma}L^k$) is applied by Karanasos et al. (2008) and Karanasos and Zeng (2007) to UK inflation and growth.

Empirical results

Monthly US data, obtained from the Datastream database, are used to provide a reasonable number of observations. The inflation and output growth series are

[9] Level effects here should be understood as the effects of the level rates of inflation and output growth on their volatilities.

calculated as the monthly difference in the natural log of the Producer Price Index (PPI) and IPI respectively [10].. The data range from 1957:01 to 2005:02. Allowing for differencing this implies 577 usable observations. The estimates of the various formulations were obtained by quasi maximum likelihood (ML) estimation as implemented by James Davidson (2008) in Time Series Modeling (TSM). The best ARFI and FIGARCH specifications are chosen on the basis of Wald (W) tests and the Akaike information criterion (AIC). For the conditional means[variances] of inflation and growth, ARFI(12)[FIGARCH(0,0)] and ARFI(3)[FIGARCH(0,0)] models are chosen respectively with $n = 3$, while the best variance specification is one with $\mathbf{B}(L) = \mathbf{A}(L) = \mathbf{I}$ and $k_{\pi i,j} = 5$, $k_{yi,j} = 3$. The Ljung–Box Q statistics at 12 lags for the levels and squares of the standardized residuals for the estimated bivariate system. The results, reported in table 1.8 and table A.1-2, show that the time-series models for the conditional means and variances adequately capture the joint distribution of the disturbances. To test for the persistence in inflation and growth, the W statistic is examined for the linear constraints $\mathbf{d}_m = 0$ (AR-FIGARCH model). As seen in table 1.8 it clearly rejects the null hypothesis against the AR-FI-GARCH specification. Thus, purely from the perspective of searching for a model that best describes the degree of persistence in the conditional means and variances of the two series, the dual long-memory process appears to be the most satisfactory representation.

From three alternative specifications, the sign and the significance of the estimated coefficients provide some statistical evidence on the nature of the relationship between the four variables. First, inflation affects growth negatively ($\phi_{y\pi} < 0$), whereas the latter has a positive effect on the former ($\phi_{\pi y} > 0$) as predicted by Gillman and Kejak, and Briault respectively (see the estimated $\Phi = [\phi_{ij}]$ matrix in

[10] Actual data is plotted in figure B.3.

Table 1.8 . Dual long-memory in-mean-level model

Mean equation

$$\mathbf{\Phi} = \begin{bmatrix} \underset{\substack{\{0.04\} \\ l_{\pi\pi}=12}}{0.11^{***}} & \underset{\substack{\{0.02\} \\ l_{\pi y}=3}}{0.03^{*}} \\ \underset{\substack{\{0.04\} \\ l_{y\pi}=12}}{-0.08} & \underset{\substack{\{0.05\} \\ l_{yy}=3}}{0.11^{***}} \end{bmatrix}, \ \Delta L^3 = \begin{bmatrix} \underset{\{0.16\}}{-0.14} & \underset{\{0.07\}}{-0.08} \\ \underset{\{0.19\}}{-0.28^{\circ}} & \underset{\{0.26\}}{0.18} \end{bmatrix} L^3, \ \mathbf{d}_m = \begin{bmatrix} \underset{\{0.04\}}{0.23^{***}} \\ \underset{\{0.05\}}{0.13^{**}} \end{bmatrix}$$

Variance equation

$$\Gamma = \begin{bmatrix} \underset{\substack{\{0.31\} \\ k_{\pi\pi}=5}}{-0.49^{\circ}} & \underset{\substack{\{0.12\} \\ k_{\pi y}=5}}{-0.15} \\ \underset{\substack{\{0.29\} \\ k_{y\pi}=3}}{0.49^{*}} & \underset{\substack{\{0.62\} \\ k_{yy}=3}}{-1.32^{**}} \end{bmatrix}, \ \mathbf{d}_v = \begin{bmatrix} \underset{\{0.08\}}{0.36^{***}} \\ \underset{\{0.08\}}{0.32^{***}} \end{bmatrix},$$

Diagnostics

$$Q(12) = \begin{bmatrix} \underset{[0.18]}{16.19} & \underset{[0.59]}{10.32} \\ \underset{[0.30]}{13.95} & \underset{[0.30]}{13.95} \end{bmatrix}, \ \begin{bmatrix} ML \\ AIC \end{bmatrix} = \begin{bmatrix} -3,290.22 \\ -3,311.22 \end{bmatrix}$$

Wald statistics

$$\begin{bmatrix} \mathbf{d}_m = 0 \\ 47.91 \\ [0.00] \end{bmatrix}, \ \begin{bmatrix} \mathbf{\Phi} \text{ is diag.} \\ 6.60 \\ [0.04] \end{bmatrix}, \ \begin{bmatrix} \mathbf{\Gamma} = 0 \\ 9.85 \\ [0.04] \end{bmatrix}, \ \begin{bmatrix} \mathbf{\Delta} = 0 \\ 4.22 \\ [0.38] \end{bmatrix}$$

Notes: Table 1.8 reports estimates of the parameters of interest.
$***$, $**$, $*$and$^{\circ}$denote significance at the 0.01, 0.05, 0.10 and 0.15 levels respectively. The numbers in {.} are robust standard errors. Q(12) and $Q^2(12)$ are the Ljung-Box statistics for 12th-order serial correlation in the standardized residuals and their squares respectively. p values are reported in [.].

table 1.8-table A.1-2). Second, higher inflation has a negative ($\gamma_{\pi\pi} < 0$) and a positive ($\gamma_{\gamma\pi} > 0$) impact on nominal and real uncertainty respectively. That is, there is evidence supporting the Ungar-Zilberfarb theory and the Dotsey-Sarte conjecture. There is also evidence in favor of the Brunner and Karanasos conjectures that higher growth has a negative impact ($\gamma_{\pi y}$, $\gamma_{yy} < 0$) on macroeconomic performance (see the estimated $\mathbf{\Gamma}$ matrices in table 1.8-table A.1).

Furthermore, the impact of real volatility on growth is positive ($\delta_{yy} > 0$) as predicted by Blackburn (1999). In addition, although there is no direct evidence supporting the Cukierman and Gerlach theory, there is an indirect positive influence of real variability on inflation through its impact on growth (the Blackburn theory and the Briault conjecture). While the effect of nominal uncertainty on growth is negative ($\delta_{y\pi} < 0$) as predicted by Friedman (see table 1.8 and table A.2). Finally, the indirect evidence via the growth channel (the Friedman hypothesis and the Briault conjecture) regarding the negative impact of nominal variability on inflation agrees well with the direct (although insignificant) negative effect.[11]

Impulse Response Analysis

The impulse response function (IRF) of the means and variances of inflation and output growth are compared for the following estimated diagonal model

$$\mathbf{\Lambda}_m(L)\mathbf{\Phi}(L)[\mathbf{x}_t - \boldsymbol{\mu}] = \boldsymbol{\varepsilon}_t,$$

and

$$(\mathbf{h}_t - \boldsymbol{\omega}) = [\mathbf{I} - \mathbf{\Lambda}_v(L)]\boldsymbol{\varepsilon}_t^{\wedge 2},$$

with estimated coefficients: $\mathbf{d}'_m = [0.24\ 0.19]'$, $\mathbf{d}'_v = [0.35,\ 0.55]'$, $\phi_{\pi\pi} = 0.12$ and $\phi_{yy} = 0.17$. The IRFs were evaluated using the formulae in Conrad and Karanasos (2006) (see also Karanasos and Kartsaklas, 2007). Figure 1.1 plots the mean IRFs for the two variables for lags up to 50. The estimated mean equation for inflation exhibits the highest long-memory parameter. As a result the impulse response weights start high, around 0.24, and decrease very slowly. Observe that the weights at lags 12, 24, and 48 are 0.16, 0.04, and 0.02 respectively. The estimated mean specification for growth exhibits lower persistence. Thus, the impulse response weights start relatively low (0.19) and decrease more rapidly. The weights at lags 12 and 24 are 0.03 and 0.02 respectively. Figure 1.1 plots the IRFs for the two conditional

[11] When we tried to estimate models with the $\mathbf{B}(L)$ matrix full the estimation routine did not converge. In the specification without in-mean and level effects and the $\mathbf{B}(L)$ matrix cross-diagonal the estimated β_{12} and β_{21} coefficients (not reported) are positive but insignificant.

Figure 1.1 IRFs for the means and variances of the bivariate AR-FI-GARCH model

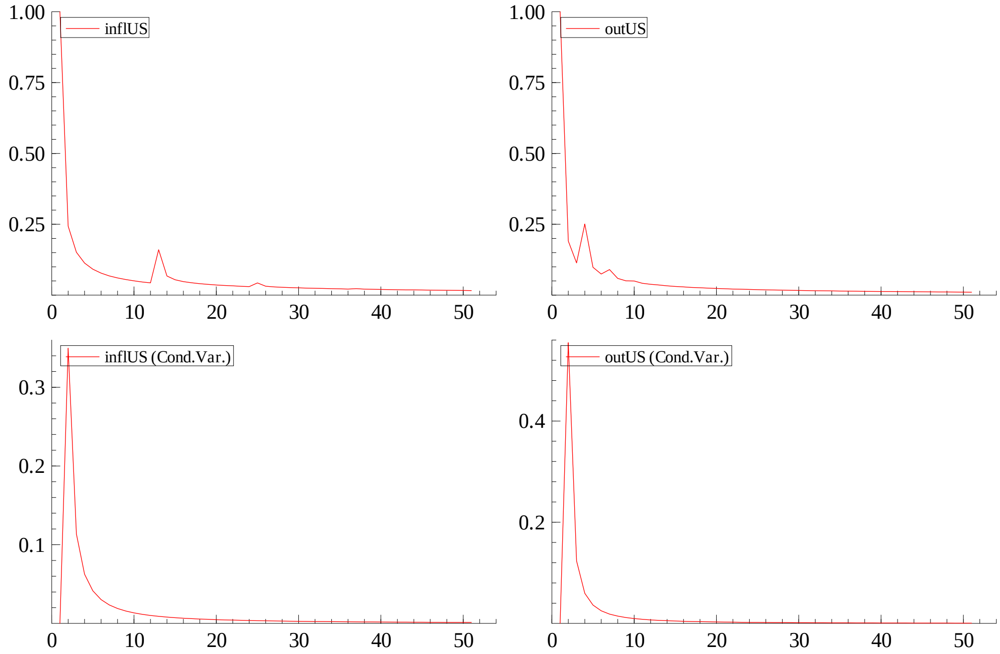

Figure 1.2

variances. The plot for the inflation uncertainty shows a very similar pattern to that for the variance of growth.

1.4.4. Summary

This section has investigated the link between US inflation, growth and their respective uncertainties simultaneously in a bivariate DCCC AR-FI-GARCH process. There are few theoretical models that come to grips with the main relationships. In addition, as a result of many econometric difficulties much of the empirical evidence is dubious. Results provide the evidence that the apparent long-memory in the inflation rate and nominal uncertainty as well as in output growth and its variability.

Early work concentrated on the impact of uncertainty on macroeconomic performance. That 'one-sidedness' of these methodologies is an important caveat and any such attempts to analyze the link between the four variables are doomed to imperfection. This study shows that not only does uncertainty affect performance but the latter influences the former as well.

The core findings of this study are: (i) growth tends to increase inflation, whereas inflation is detrimental to growth which are in line with the Briault conjecture and the Gillman-Kejak theory respectively, (ii) inflation has a negative(positive) impact on nominal(real) uncertainty thus supporting the Ungar-Zilberfarb theory(Dotsey-Sarte conjecture), (iii) there is evidence supporting Brunner's and Karanasos's conjectures that growth has a negative impact on macroeconomic performance, and (iv) real(nominal) volatility affects growth positively(negatively) as predicted by Blackburn(Friedman).

This study has also highlighted reciprocal interactions in which two or more variables influence each other, either directly or indirectly. In particular, there is an indirect positive influence of real variability on inflation through its impact on growth. In addition, the indirect evidence (via the growth channel) regarding the negative influence of nominal uncertainty on inflation agrees well with the direct effect. Finally, there is an indirect (that works via growth) bidirectional feedback between the two volatilities.

CHAPTER 2

Inflation, Output Growth and Their Uncertainties: Four Variables, Twelve Links and Many Specifications

2.1. Introduction

This chapter continues to address the links among inflation, output growth and their uncertainties in UK, which has been examined briefly in section 1.3. In this chapter, the UK inflation and output growth are estimated by various formulations, including a bivariate constant conditional correlation GARCH model as well as the BEKK representation with non-negative definiteness of the covariance matrix assured. Thus results are robust to investigate the similarities and differences between them, rather than selecting one specification as pre-eminent. Further, a VAR can be seen as a reduced form of a time series model or as is used by Harvey (2008) a structural time series model. The VAR provides a bivariate explanation of both output and inflation, and as a result does not rely on filter models to define some measure of equilibrium output. Also GARCH approach accounts for the second conditional moment of inflation and output growth series, and therefore shall be able to consider a broader range of hypothesis than would be possible in the univariate case, which is superior than other methods such as the mainly single equation approach used in the case of the New Keynesian Phillips curve (see Harvey, 2008) and the VAR when time series exhibit heteroskedasticity. If well estimated, it can help identify the relative contributions of different influences more precisely than previous studies.

Section 2.2 gives a broad overview of the economic theory concerning the link between macroeconomic performance and uncertainty and previous work. Section 2.3 describes the time series model and notation for inflation and growth. The empirical results are reported in section 2.4. Section 2.5 interprets these results and relates them to the predictions of economic theory. Section 2.6 evaluates the robustness of empirical findings and section 2.7 contains summary remarks and conclusions.

2.2. Economic Theories, Hypotheses and Conjectures

2.2.1. The Link between Inflation(Uncertainty) and Growth(Uncertainty)

The inflation-output growth relationship

Mean inflation and output growth are interrelated. Temple (2000) presents a critical review of the emerging literature which tends to discuss how inflation affects growth. Gillman and Kejak (2005) bring together for comparison several main approaches to modelling the inflation-growth effect by nesting them within a general monetary endogenous growth model with both human and physical capital. Their summary of the findings across the different formulations, establishes clearly a robust significant negative effect. Other researchers also find evidence that inflation negatively Granger causes real growth (see Gillman and Kejak, 2005, and the references therein).

Briault (1995) argues that there is a positive relationship between growth and inflation, at least over the short run, with the direction of causation running from higher growth (at least in relation to productive potential) to higher inflation. For simplicity, in what follows this positive influence will be referred as the Briault conjecture. Later, a study by Fountas et al. (2006), involving the G7, finds that growth has a significant positive impact on inflation.

The inflation-output growth variability relationship

There are some reasons to suspect a relationship between nominal uncertainty and the volatility of real growth. For example, models with a stable inflation-unemployment trade-off imply a positive relationship between the two variabilities (see, Logue and Sweeney, 1981, for details). Moreover, the discretionary equilibrium of Devereux's (1989) model predicts a close relationship between the mean rate of inflation, its volatility and the variance of output growth. Although in his model there is no direct causal link whatever from real to nominal uncertainty, for simplicity, in what follows this positive effect will be referred as the 'Devereux' hypothesis.

In contrast to the positive relationship, Fuhrer (1997) explores the nature of the long-run variance trade-off. The short-run trade-off between inflation and output growth that exists in the models he explores implies a long-run trade-off in the volatilities. Karanasos and Kim (2005a,b) discuss a number of arguments, advanced over the last 30 years, that predict a positive association between the two variables.

2.2.2. The Impact of Macroeconomic Uncertainty on Performance

Macroeconomists have placed considerable emphasis on the impact of economic uncertainty on the state of the macroeconomy. The profession seems to agree that the objectives of monetary policy are inflation and output stabilisation around some target levels.

Variability about future inflation affects the average rate of inflation. However, the direction of the effect is ambiguous from a theoretical point of view. Cukierman and Meltzer's (1986) model explains the positive association between the two variables. On the other hand, one possible reason for greater nominal variability to precede lower inflation is that an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future (Holland, 1995). This negative effect will be referred as the Holland conjecture.

The impact of nominal uncertainty on output growth has received considerable attention in the literature. However, there is no consensus among macroeconomists on the direction of this effect. Theoretically speaking, the influence is ambiguous. In his Nobel address, Friedman (1977) explains a possible positive correlation between inflation and unemployment by arguing that high inflation produces more uncertainty about future inflation. This uncertainty then lowers economic efficiency and temporarily reduces output and increases unemployment. In sharp contrast, Dotsey and Sarte (2000) employ a model where money is introduced via a cash-in-advance constraint and find that variability increases average growth through a precautionary savings motive.

Next, real variability may affect the rate of inflation. Cukierman and Gerlach (2003) using an expectations-augmented Phillips curve demonstrate that in the presence of a precautionary demand for expansions and uncertainty about the state of the economy there is an inflation bias even if policymakers target the potential level of output. Their bias-producing mechanism implies that countries with more volatile shocks to output should have, on average, higher rates of inflation. Their approach implies a positive relationship between inflation and the variance of growth where causality runs from the latter to the former.

Finally, one particular interest has been the relationship between growth and its variance with different analyses reaching different conclusions depending on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (see Blackburn and Pelloni, 2005, and the references therein). Pindyck (1991), among others, proposes a theory for which the negative impact of volatility on growth relies on uncertainty through the link of investment (see Martin and Rogers, 2000, and the references therein). In another class of models the relationship between short-term variance and long-term growth is positive (see Blackburn, 1999, and the references therein). Blackburn(1999) presents a model of imperfect competition with nominal rigidities and 'learning by doing' technology. He argues that it is possible that the additional learning during expansions more than compensates for the loss of learning during recessions so that, on average, the rate of technological progress increases when there is an increase in volatility. Under such circumstances, there is a positive relationship between growth and uncertainty. A positive correlation between the two variables does not imply a causal link. However, a positive effect from real variability to growth implies a positive correlation between the two variables. Thus, in what follows this positive influence will be referred as the 'Blackburn' theory.

2.2.3. The Influence of Macroeconomic Performance on Uncertainty

The positive relationship between inflation and its uncertainty has often been noted. According to Holland (1993) if regime changes causes unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to volatility. In addition, Ungar and Zilberfarb (1993) provide a theoretical framework in order to specify the necessary conditions for the existence of a positive or negative impact.

A number of theories have been put forward to examine the impact of inflation on real uncertainty. In a nutshell, the sign of such an effect is ambiguous. Dotsey and Sarte (2000) present a model which suggests that as average money growth rises nominal variability increases and real growth rates become more volatile. The models developed by Ball et al. (1988) assume menu costs and imply that the slope of the short-run Phillips curve should be steeper when average inflation is higher. In their New Keynesian model, nominal shocks have real effects because nominal prices change infrequently. Higher average inflation reduces the real effects of nominal disturbances and hence also lowers the variance of output.

The sign of the impact of output growth on macroeconomic volatility is also ambiguous. Consider first the influence on nominal uncertainty. As Brunner (1993) puts it: "While Friedman's hypothesis is plausible, one could also imagine that when economic activity falls off, there is some uncertainty generated about the future path of monetary policy, and consequently, about the future path of inflation". The term of 'Brunner conjecture' will be a shorthand for this negative effect. In sharp contrast, a higher growth rate will raise inflation according to the Briault conjecture, and therefore, raises/lowers its variability, as predicted by the Ungar-Zilberfarb theory. This positive/negative impact will be termed as the Karanasos conjecture (I).

Finally, consider now the effect of growth on its variability. An increase in growth, given that the Briault conjecture and Dotsey-Sarte conjecture hold, pushes its variance upward. However, if the impact of inflation on real uncertainty is negative (the Ball-Mankiw-Romer theory), the opposite conclusion applies. This causal effect will be referred as the Karanasos conjecture (II).

The causal relationships and the associated theories as well as empirical evidences found in this study are summarised in table 2.1.

2.3. Empirical Strategy

2.3.1. Model

This chapter uses a bivariate model to simultaneously estimate the conditional means, variances, and covariances of inflation and output growth as presented in 1.3.3.

Table 2.1 Theories-Hypotheses-Conjectures

Performance	Empirical evidence
Macroeconomic performance	
Inflation Granger causes growth	
Gillman-Kejak theory: $-$	Strong
Growth Granger causes inflation	
Briault conjecture: $+$	Strong
Macroeconomic uncertainty	
Inflation uncertainty Granger causes growth uncertainty	
Logue-Sweeney theory: $+$; Fuhrer theory: $-$	$+$:Strong $-$:None
Growth uncertainty Granger causes inflation uncertainty	
'Devereux' hypothesis: $+$; Fuhrer theory:$-$	None
In-Mean effects	
Inflation uncertainty Granger causes inflation	
Cukierman-Meltzer theory: $+$; Holland conjecture: $-$	$+$:Strong
Inflation uncertainty Granger causes growth	
Dotsey-Sarte theory: $+$; Friedman hypothesis: $-$	$+$:Weak $-$:Weak
Growth uncertainty Granger causes inflation	
Cukierman-Gerlach theory: $+$	None
Growth uncertainty Granger causes growth	
Pindyck (Blackburn) theory: $-(+)$	$+$:Weak $-$:Weak
Level effects	
Inflation Granger causes inflation uncertainty	
Ungar-Zilberfarb theory: $\pm$	$+$:Strong
Inflation Granger causes growth uncertainty	
Dotsey-Sarte conjecture: $+$; Ball-Mankiw-Romer theory: $-$	$+$:Strong
Growth Granger causes inflation uncertainty	
Karanasos conjecture (I): $\pm$, Brunner conjecture: $-$	$+$:Strong
Growth Granger causes growth uncertainty	
Karanasos conjecture (II): $\pm$	None

Regarding the model, it follows Zellner's (1998) 'KISS' approach, that is, 'keep it sophisticatedly simple'. It is important to notice that, despite the fact that it is simple and convenient, the model remains very general in its scope.[1] As mentioned in section 1.3.3, the main benefits of this model are that the dubious assumption of a positive link between the two uncertainties is not necessary, that is, the coefficients that capture the variance-relationship ($\beta_{\pi y}$, $\beta_{y\pi}$) are allowed to be negative[2], and

[1]And it is well known that Einstein advised in connection with theorizing in the natural sciences, "Make it as simple as possible but no simpler." (Zellner, 1998).

[2]Of course the conditional correlation ($h_{\pi y,t}/\sqrt{h_{\pi t}}\sqrt{h_{yt}},$) is constant ($\rho$). This is the price for allowing for a negative relationship between nominal and real uncertainty. The model estimated in this chapter has some more limitations. However, it is easy to see how the model might be tinkered with to overcome some of its limitations, which will be left as a task for future research.

Table 2.2 Causality effects

Twelve Links	Coefficients
Macroeconomic performance	Matrix Φ
Inflation Granger causes output growth	$\phi_{y\pi}\neq 0$
Output growth Granger causes inflation	$\phi_{\pi y}\neq 0$
Macroeconomic uncertainty	Matrix B
Inflation uncertainty Granger causes output growth uncertainty	$\beta_{y\pi}\neq 0$
Output growth uncertainty Granger causes inflation uncertainty	$\beta_{\pi y}\neq 0$
In-Mean effects	Matrix Δ
Inflation uncertainty Granger causes inflation	$\delta_{\pi\pi}\neq 0$
Inflation uncertainty Granger causes output growth	$\delta_{y\pi}\neq 0$
Output growth uncertainty Granger causes inflation	$\delta_{\pi y}\neq 0$
Output growth uncertainty Granger causes output growth	$\delta_{yy}\neq 0$
Level effects	Matrix Γ
Inflation Granger causes inflation uncertainty	$\gamma_{\pi\pi}\neq 0$
Inflation Granger causes output growth uncertainty	$\gamma_{y\pi}\neq 0$
Output growth Granger causes inflation uncertainty	$\gamma_{\pi y}\neq 0$
Output growth Granger causes output growth uncertainty	$\gamma_{yy}\neq 0$

that several lags of the conditional variances/means are added as regressors in the mean/variance equation. This approach is promising since it allows for bidirectional effects. However, there are great difficulties in drawing conclusions for the interlinkages, because the relationships between the four variables are not well understood, and theoretical models can only be used to illustrate a range of possibilities. This methodology is interesting because it tests the various theories in a variety of ways and it emphasizes that the empirical evidence is not clear-cut. The causality links and the relevant coefficients are summarised in table 2.2.

2.3.2. Notation

In order to make the analysis easier to understand, the following matrix notation is introduced. The subscripts d and f will denote diagonal and full matrices respectively, whereas the subscripts c and $u(w)$ will denote cross diagonal and upper(lower) triangular matrices respectively. For example, Φ_{ld} is a diagonal matrix: diag$\{\phi_{\pi\pi}^{(l)}, \phi_{yy}^{(l)}\}$, whereas B_d and Γ_d are diagonal matrices with $\beta_{\pi y}$, $\beta_{y\pi} = 0$ and $\gamma_{\pi y}$, $\gamma_{y\pi} = 0$ respectively. In addition, Φ_{lf}, B_f, and Γ_f are full matrices (see table 2.3).

Table 2.3 Matrix notation

Matrices	Φ	B	Γ
Diagonal	Φ_{ld} ($\phi^{(l)}_{\pi y}$, $\phi^{(l)}_{y\pi}$=0)	B_d ($\beta_{\pi y}$, $\beta_{y\pi}$=0)	Γ_d ($\gamma_{\pi y}$, $\gamma_{y\pi}$=0)
Cross Diagonal	-	-	Γ_c ($\gamma_{\pi\pi}$, γ_{yy}=0)
Upper Triangular	Φ_{lu} ($\phi^{(l)}_{y\pi}$=0)	-	-
Lower Triangular	-	B_w ($\beta_{\pi y}$=0)	-
Full	Φ_{lf} ($\phi^{(l)}_{\pi y}$, $\phi^{(l)}_{y\pi}\neq$0)	B_f ($\beta_{\pi y}$, $\beta_{y\pi}\neq$0)	Γ_f ($\gamma_{\pi y}$, $\gamma_{y\pi}\neq$0)

Φ_{ld}, B_d, and Γ_d denote diagonal matrices. Φ_{lf}, B_f, and Γ_f denote full matrices. Φ_{lu} (B_w), and Γ_c denote upper, lower triangular and cross diagonal matrices respectively.

Table 2.4 Models notation

Models	Simple	In-Mean	Level	In-Mean-Level
Matrices	$\Delta = 0, \Gamma = 0$	$\Delta \neq 0, \Gamma = 0,$	$\Delta = 0, \Gamma \neq 0$	$\Delta \neq 0, \Gamma \neq 0$
Notation $\xi(\kappa)$=d,$u(w)$,f; ζ=d,f	$\text{S}(\Phi_\xi, B_\kappa)$	$\underset{n=0,\ldots,4}{\text{M}}(\Phi_\xi, B_\kappa)$	$\text{L}(\Phi_\xi, B_\kappa, \Gamma_\zeta)$	$\underset{n=0,\ldots,4}{\text{ML}}(\Phi_\xi, B_\kappa, \Gamma_\zeta)$

S and ML refer to the simple and the in-mean-level models respectively.
M and L refer to the in-mean and level models respectively.
The d, $u(w)$ and f subscripts denote diagonal, upper(lower) triangular and full matrices respectively. n is the lag order of the in-mean effect.

To distinguish between four alternative models, the specifications with $\Delta, \Gamma = 0$ and $\Delta, \Gamma \neq 0$ are referred as the simple and the in-mean-level models respectively. Similarly, the formulations with $\Delta \neq 0, \Gamma = 0$ and $\Delta = 0, \Gamma \neq 0$ are referred as the in-mean and level models respectively. For typographical convenience the acronyms S, M, L and ML are used for reference to the simple, in-mean, level and in-mean-level models respectively (see table 2.4).

In order to simplify the description of the various models, the following notation is referred as shorthand. S (Φ_d, B_f) denotes the simple model with the Φ matrix diagonal and the B matrix full. Further, $\underset{n=0}{\text{M}}(\Phi_d,\ B_d)$ describes the in-mean model with the Φ and the B matrices diagonal and the current value of the macroeconomic uncertainty to affect performance. Moreover, L(Φ_f, B_d, Γ_d) stands for the level process with the Φ matrix full and the B and Γ matrices diagonal (see table 2.4).

Before analysing results, in order to make this analysis more concise, some specific models will be discussed. For example, in the S (Φ_f, B_f) model four out of the twelve effects are present. In particular, there is a bidirectional feedback between inflation(uncertainty) and growth(uncertainty). Moreover, in the $\underset{n=0}{\mathrm{M}}(\Phi_f, B_f)$ model eight influences are present. Specifically, in addition to the four impacts above, the four in-mean effects are also present. Further, in the $\mathrm{L}(\Phi_f, B_d, \Gamma_f)$ model six effects are present. Especially, the four level effects are present and there is also a bidirectional feedback between inflation and growth.

2.4. Data and Empirical Specifications

2.4.1. Data and Estimation Results

This section uses the same data sets as in section 1.3.4: inflation and output growth calculated as the monthly difference in the natural log of the monthly CPI and IPI with data range from 1962:01 to 2004:01. For both series, based on the Phillips-Perron (PP) and KPSS unit root tests (see table E.1), it is able to reject the unit root hypothesis.

Within the BVAR-GARCH-ML framework, the dynamic adjustments of both the conditional means and the conditional variances of UK inflation and output growth, as well as the implications of these dynamics for the direction of causality between the two variables and their respective uncertainties will be analysed. The estimates of the various formulations were obtained by maximum likelihood estimation (MLE) as implemented by James Davidson (2006) in TSM. To check for the robustness of estimates, this study used a range of starting values and hence ensured that the estimation procedure converged to a global maximum. The best model is chosen on the basis of LR tests and three alternative information criteria.

For the conditional means[variances] of inflation and growth, AR(14)[GARCH(1,1)] and AR(12)[ARCH(1)] models are chosen respectively.[3]

To select best S model, specifications with the $\Phi(B)$ matrix either diagonal or upper(lower) triangular or full are estimated. To test for the presence of an inflation-growth link the LR statistic for the linear constraints $\phi_{\pi y} = \phi_{y\pi} = 0$ is examined. To test for the existence of a variance relationship the LR test for the constraints $\beta_{\pi y} = \beta_{y\pi} = 0$ is employed. As seen in table 2.5, the LR tests clearly reject the S(Φ_f, B_d) and S(Φ_d, B_f) null hypotheses against the S(Φ_f, B_f) model. In accordance with this result, the Akaike and Hannan-Quinn Information criteria (AIC and HQIC respectively) choose the S(Φ_f, B_f) specification.[4] That is, the formulation with the simultaneous feedback between inflation(uncertainty) and growth(uncertainty). It is worth noting that the S model with the Φ matrix diagonal is not appropriate, since there is evidence (not reported) for serial correlation in the standardised residuals of inflation.

Further, for the L, M and ML models the estimation routine did not converge when the B_f matrix was used. In accordance with the results for the S models, the three criteria favor the L(Φ_f, B_d, Γ_f) specification while the L(Φ_f, B_w, Γ_f) process is ranked second. When the Φ_f and either the B_d or the B_w matrices are used all criteria favor the level model over the simple one. According to the three information criteria the optimal ML formulation is the $\underset{n=0}{\mathrm{ML}}(\Phi_f, B_d, \Gamma_f)$ while the second ranked model is the $\underset{n=0}{\mathrm{ML}}(\Phi_f, B_w, \Gamma_f)$. Finally, it is worth noting that for the specification with the Φ_f, and either the B_d or the B_w matrices the criteria favor the ML model over both the M and S ones. Thus, purely from the perspective of searching

[3]The GARCH coefficient is significant only in the conditional variance of inflation. For bivariate process the estimation shows a significant improvement in the likelihood value of the ARCH growth specification over the GARCH model. Only parameters of interest have been reported.

[4]In particular, the seventh and eleventh lags of inflation have a joint significant negative impact on growth while the fifth and seventh lags of growth affect inflation positively (see Table B.1).

Table 2.5 Information Criteria and Maximum Likelihood (MaxLik) values

Models	Information Criteria			MaxLik
	AIC	SIC	HQIC	
		Simple		
S($\Phi_d, B_f\,\vert B_w\,\vert B_d$)*	-562 | -597 | -596	$\boxed{-604}$ ‖ -632 | -630	-579 | -611 | -609	-542 | -580 | -580
S($\Phi_u, B_f\,\vert B_w\,\vert B_d$)	-559 | -580 | -576	-605 | -623 | -614	-577 | -597 | -591	-539 | -561 | -558
S($\Phi_f, B_f\,\vert B_w\,\vert B_d$)	$\boxed{-557}$ | -586 | -585	-608 | -630 | -627	$\boxed{-577}$ | -603 | -601	-533 | -565 | -565
	$Q(12)$	$Q^2(12)$		
π	27.27[0.01]	15.64[0.21]		
y	13.63[0.32]	5.39[0.94]		
		In-mean		
$\underset{n=0}{\mathrm{M}}(\Phi_u, B_w\,\vert B_d)$	-588 ‖ $\boxed{-576}$	-636 ‖ $\boxed{-622}$	-607 ‖ $\boxed{-594}$	-565 | -554
$\underset{n=0}{\mathrm{M}}(\Phi_f, B_w\,\vert B_d)$	-585 | -585	-638 | -636	-606 | -605	-560 | -561
	$Q(12)$	$Q^2(12)$		
π	20.19[0.06]	14.25[0.28]		
y	16.30[0.32]	15.96[0.19]		
		Level		
L($\Phi_u, B_w\,\vert B_d\,, \Gamma_d$)	-582 | -581	-626 | -623	-599 | -598	-561 | -561
L($\Phi_f, B_w\,\vert B_d\,, \Gamma_d$)	-579 | -579	-628 | -625	-598 | -597	-556 | -557
L($\Phi_f, B_w\,\vert B_d\,, \Gamma_f$)	-569 ‖ $\boxed{-568}$	-621 ‖ $\boxed{-618}$	-590 ‖ $\boxed{-588}$	-544 | -544
	$Q(12)$	$Q^2(12)$		
π	16.89[0.15]	18.19[0.11]		
y	14.01[0.30]	15.83[0.20]		
		In-mean-level		
$\underset{n=1}{\mathrm{ML}}(\Phi_u, B_w\,\vert B_d\,,\Gamma_d)$	-584 | -584	-636 | -634	-604 | -604	-559 | -560
$\underset{n=1}{\mathrm{ML}}(\Phi_u, B_w\,\vert B_d\,,\Gamma_f)$	-575 | -574	-631 | -629	-597 | -596	-548 | -548
$\underset{n=0}{\mathrm{ML}}(\Phi_f, B_w\,\vert B_d\,,\Gamma_d)$	-579 | -578	-636 | -633	-601 | -600	-552 | -552
$\underset{n=0}{\mathrm{ML}}(\Phi_f, B_w\,\vert B_d\,,\Gamma_f)$	-570 ‖ $\boxed{-569}$	-630 ‖ $\boxed{-627}$	-593 ‖ $\boxed{-592}$	-540 | -540
	$Q(12)$	$Q^2(12)$		
π	17.00[0.15]	16.52[0.17]		
y	16.231[0.18]	16.01[0.19]		

AIC, SIC and HQIC are the Akaike, Schwarz and Hannan-Quinn Information criteria respectively.

*The three numbers refer to the models with the B_f, B_w and B_d matrices respectively.

The numbers in $\boxed{}$ indicate the optimal type model according to the information criteria with the values of the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals eported.

The underlined numbers indicate the second ranked model.

For the L(Φ_u, $\underset{k=d,w}{B_k}$, Γ_f) models the estimation routine did not converge.

For the M, L and ML models the estimation routine did not converge when the B_f matrix was used.

for a model that best describes the link between macroeconomic performance and uncertainty, the ML model appears to be the most satisfactory representation.

2.4.2. Interconnections among the Four Variables

This section analyses the results from the various specifications and examines the sign and the significance of the estimated coefficients to provide some statistical evidence on the nature of the relationship between the four variables.

Inflation-Growth link

There is strong evidence supporting the Gillman-Kejak theory and the Briault conjecture. That is, there is strong bidirectional feedback between inflation and output growth. In particular, inflation affects growth negatively, whereas growth has a positive effect on inflation. This causal relationship is not qualitatively altered by changes in the specification of the model (see table B.1).

Variance relationship

There is evidence that nominal uncertainty has a positive impact on real volatility as predicted by Logue and Sweeney (1981). The influence is invariant to the formulation of the Φ matrix. In particular, in all three S($\underset{\xi=d,u,f}{\Phi_{\xi}}$, B_f) models the effect is significant at the 1% level (see table B.2). When trying to estimate M, L and ML models, with the B matrix full the estimation routine did not converge. In all specifications with the B matrix lower triangular (not reported) the influence disappears.

In-mean effects

The objective in the following analysis is to consider several changes in the specification of the model and to discuss how these changes affect the in-mean effects. In some cases, it has been found that by making very small changes in the formulation of the model the estimated effects vary considerably (see table B.3).

First, when the current values ($n = 0$) of the conditional variances are included in the mean equations, some very weak evidence for Friedman hypothesis (see table B.4) are found. This result is invariant to changes in the B matrix. For example, in the $\underset{n=0}{\mathrm{M}}(\Phi_d, B_d)$ and $\underset{n=0}{\mathrm{M}}(\Phi_d, B_w)$ models the effect is significant at the 18% and 20% levels respectively (see table B.4). However, controlling for the impact of inflation on growth, that is when the Φ_f matrix is used, the effect disappears (see the '$\delta_{y\pi}$' column in table B.3). On the other hand, the negative influence of nominal uncertainty on growth becomes stronger when accounting for level effects. More specifically, in the $\underset{n=0}{\mathrm{ML}}(\Phi_d, B_d, \Gamma_d)$ and $\underset{n=0}{\mathrm{ML}}(\Phi_u, B_d, \Gamma_d)$ models the in-mean coefficient becomes more significant (at the 13% and 10% levels respectively) (see table B.4).

In sharp contrast, Dotsey and Sarte (2000) argue that as inflation rises, growth begins to fall. However, as inflation continues to rise, the positive effects of higher nominal uncertainty begin to dominate and growth starts to increase. The mitigating effect of inflation variability may help partially explain why inflation might seem unrelated to growth. On the contrary, weak evidence (significant at the 14% level) for the Dotsey-Sarte theory appears at the model with the third lags of the in-mean effects and a bidirectional feedback between inflation and growth ($\underset{n=3}{\mathrm{M}}(\Phi_f, B_d)$) (see table B.3).

Second, the evidence supporting the Cukierman-Gerlach theory when either the current values ($n = 0$) or the fourth lags ($n = 4$) of the conditional variances are allowed to affect inflation and growth. When the current values are used the impact of real uncertainty on inflation is stronger (see table B.5) and is not qualitatively altered by using different versions of the Φ (diagonal or upper triangular or full) matrix (see the '$\delta_{\pi y}$' column of table B.3). However, at lag 4 the effect (not reported) disappears when the Φ_d matrix is used. Moreover, when the current values are used the impact is robust to the inclusion or exclusion of level effects and to whether the

B matrix is diagonal or lower triangular and the Γ matrix is diagonal or full. For example, when the $\underset{n=0}{\mathrm{ML}}(\Phi_f, B_w, \Gamma_d)$ and $\underset{n=0}{\mathrm{M}}(\Phi_f, B_w)$ models are estimated the effect is significant at the 4% and 7% levels respectively. However, at lag 4, the impact becomes weaker in the presence of level effects (see table B.5).

Third, there is weak evidence (significant at the 16% level) for the 'Blackburn' theory when the Φ matrix is full and the first lags of the two uncertainties are allowed to affect their means. This result is invariant to the formulation of the B matrix. When adding level effects, the impact becomes stronger. In particular, in the model with the B_w matrix, when the Γ_d matrix is used it is significant at the 11% level while when the full Γ matrix is employed it is significant at the 9% level (see table B.6). On the contrary, there is evidence for the Pindyck theory when allowing the third lags of the macroeconomic uncertainty to affect performance. However, the significance of the effect varies substantially with changes in the specification of the model. For example, in the $\underset{n=3}{\mathrm{M}}(\Phi_d, B_w)$ (not reported) and $\underset{n=3}{\mathrm{M}}(\Phi_f, B_w)$ models the effect is significant at the 19% and 12% levels respectively, whereas in the $\underset{n=3}{\mathrm{M}}(\Phi_f, B_d)$ it disappears. That is, when accounting for the bi(uni)-directional feedback between inflation (uncertainty) and growth (uncertainty) the impact is stronger. When including all four level effects the impact becomes weaker. In particular, for the $\underset{n=3}{\mathrm{ML}}(\Phi_f, B_w, \Gamma_f)$ model the effect is significant at the 15% level (see table B.6).

Level effects

There is strong evidence in favour of the Ungar-Zilberfarb theory and the Dotsey-Sarte conjecture that higher inflation has a positive impact on nominal and real uncertainty respectively. It demonstrates the invariant of these findings to changes in the specification of the model (see table B.7, columns 2 and 3). Moreover, some evidence for the Karanasos conjecture (I) regarding the positive effect of growth on

Table 2.6 Relatively robust effects

$\pi \xrightarrow{-} y$	$y \xrightarrow{+} \pi$	$h_\pi \xrightarrow{+} h_y$	$h_y \nrightarrow h_\pi$	$h_\pi \nrightarrow \pi$	$\pi \xrightarrow{+} h_\pi$	$\pi \xrightarrow{+} h_y$	$y \nrightarrow h_y$

($\nrightarrow$)$\rightarrow$(does not) Granger causes. A$+$($-$) indicates that the effect is positive(negative).

inflation variability appears at the ML model with the first lags of the two conditional variances in the mean equations, the Φ and the B matrices diagonal, and the Γ matrix cross diagonal ($\underset{n=1}{\mathrm{ML}}(\Phi_d, B_d, \Gamma_c)$) (see last row of table B.7). Finally, there is a lack (negative and insignificant) of a direct link from growth to its volatility.

2.5. Discussion

2.5.1. Summary

In general, there are three bidirectional feedbacks. There is a positive one, between inflation and real uncertainty, and two mixed ones. That is, growth has a positive direct impact on inflation and an indirect one on nominal uncertainty whereas it is affected negatively by the two variables (see tables 2.6 and 2.7). Moreover, there are two positive unidirectional feedbacks. That is, causality runs only from nominal to real uncertainty, and from inflation to its variability. Finally, there is a third unidirectional feedback. Causality runs only from real uncertainty to growth. However, the sign of the influence is altered by changes in the choice of the lag of the in-mean effect. More specifically, at lag 1 the effect is positive whereas at lag 3 switches to negative. In sharp contrast, when the current values or the second lags or the fourth lags of the conditional variances are included as regressors in the mean equations growth and its uncertainty are independent of each other.

2.5.2. Sensitivity of the In-mean Effects

Choice of the lag:

Table 2.7 Empirical evidence (summary)

Theories-Hypotheses-Conjectures	Models
Macroeconomic performance	
Gillman-Kejak theory ($\phi_{y\pi}$<0): Briault conjecture ($\phi_{\pi y}$>0)	In all models: S,L,M, and ML.
Macroeconomic Uncertainty	
Logue-Sweeney theory ($\beta_{y\pi}$>0)	$S(\underset{\xi=f,d,u}{\Phi_\xi}, B_f)$
'Devereux' hypothesis ($\beta_{\pi y}$>0)	-
In-Mean Effects	
Cukierman-Meltzer theory ($\delta_{\pi\pi}$>0)	-
Friedman hypothesis ($\delta_{y\pi}$<0)	$\underset{n=0}{M}(\underset{\varsigma=d,u}{\Phi_\varsigma}, \underset{k=d,w}{B_k})$; $\underset{n=0}{ML}(\underset{\varsigma=d,u}{\Phi_\varsigma}, \mathrm{B}_d, \Gamma_d)$
Dotsey-Sarte theory ($\delta_{y\pi}$>0)	$\underset{n=3}{M}(\Phi_f, \underset{k=d}{B_k})$
Cukierman-Gerlach theory ($\delta_{\pi y}$>0)	$\underset{n=0}{M}(\underset{\xi=f,d}{\Phi_\xi}, \underset{k=d,w}{B_k})$; $\underset{n=4}{M}(\underset{\tau=f,d}{\Phi_\tau}, \underset{k=d,w}{B_k})$ $\underset{n=0,4}{ML}(\Phi_f, \underset{k=d,w}{B_k}, \underset{\zeta=d,f}{\Gamma_\zeta})$; $\underset{n=0}{ML}(\underset{\varsigma=d,u}{\Phi_\varsigma}, B_d, \Gamma_d)$
'Blackburn' theory (δ_{yy}>0)	$\underset{n=1}{M}(\Phi_f, \underset{k=d,w}{B_k})$; $\underset{n=1}{ML}(\Phi_f, \underset{k=d,w}{B_k}, \underset{\zeta=d,f}{\Gamma_\zeta})$
Pindyck theory (δ_{yy}<0)	$\underset{n=3}{M}(\underset{\xi=d,f}{\Phi_\xi}, \mathrm{B}_w)$; $\underset{n=3}{M}(\Phi_u, \mathrm{B}_d)$; $\underset{n=3}{ML}(\underset{\varsigma=d,u}{\Phi_\varsigma}, \underset{k=d,w}{B_k}, \Gamma_d)$; $\underset{n=3}{ML}(\underset{\tau=u,f}{\Phi_\tau}, \underset{k=d,w}{B_k}, \Gamma_f)$
Level effects	
Ungar-Zilberfarb theory ($\gamma_{\pi\pi}$>0)	$L(\underset{\xi=d,u,f}{\Phi_\xi}, \underset{k=d,w}{B_k}, \underset{\zeta=d,f}{\Gamma_\zeta})$; $\underset{n=0,\ldots,4}{ML}(\Phi_f, \underset{k=d,w}{B_k}, \underset{\zeta=d,f}{\Gamma_\zeta})$ In all ML models with $\underset{\varsigma=d,u}{\Phi_\varsigma}$ that the estimation routine converge;
Dotsey-Sarte conjecture ($\gamma_{y\pi}$>0)	$L(\underset{\xi=d,u,f}{\Phi_\xi}, \underset{k=d,w}{B_k}, \Gamma_f)$; $\underset{n=0,\ldots,4}{ML}(\Phi_f, \underset{k=d,w}{B_k}, \Gamma_f)$ In all ML models with Γ_f and $\underset{\varsigma=d,u}{\Phi_\varsigma}$ that the estimation routine converge;
Karanasos conjecture (I) ($\gamma_{\pi y}$>0)	$ML(\Phi_d, B_d, \Gamma_c)$
Karanasos conjecture (II) ($\gamma_{yy}\neq$0)	-

For the $\underset{n=0}{ML}(\underset{\varsigma=d,u}{\Phi_\varsigma}, \underset{k=d,w}{B_k}, \Gamma_f)$, $\underset{n=0}{ML}(\underset{\varsigma=d,u}{\Phi_\varsigma}, B_w, \Gamma_d)$ models the estimation routine did not converge. For the L, M and ML models with the B_f matrix the estimation routine did not converge. For the $\underset{n=3}{ML}(\Phi_f, \underset{k=d,w}{B_k}, \Gamma_d)$, $\underset{n=3}{ML}(\Phi_d, \underset{k=d,w}{B_k}, \Gamma_f)$ models the estimation routine did not convergence.

When the current values of the in-mean effects are used there is evidence supporting the Friedman hypothesis and the Cukierman-Gerlach theory, whereas at lag 1 there is evidence that real uncertainty affects growth positively as predicted by Blackburn (1999). Moreover, when the third lags of the conditional variances are

Table 2.8 In-mean effects sensitive to the choice of the lag

Lags:	0	1	2	3	4		0	1	2	3	4		0	1	2	3	4
$h_\pi \rightarrow y$	$-$	0	0	$+$	0	$h_y \rightarrow \pi$	$+$	0	0	0	$+$	$h_y \rightarrow y$	0	$+$	0	$-$	0

$\rightarrow$: Granger causes. A $+(-)$ indicates that the effect is positive(negative).

allowed to affect their means there is evidence in support of the Dotsey-Sarte and Pindyck theories, whereas at lag 4 there is evidence that the variability of growth has a positive impact on inflation (see table 2.8).

Level Effects:

The changes in the specification of the model affect the in-mean effects. First, it is their sensitivity to the inclusion or exclusion of level effects. When accounting for level effects, the evidence for the Cukierman-Gerlach theory, at lag 4, becomes weaker whereas, at lag 0, it remains the same. Moreover, the evidence in support of the Friedman hypothesis and the 'Blackburn' theory becomes stronger in the presence of level effects. Further, if assuming that the two variances are independent of each other, then excluding the level effects the negative impact of real uncertainty on growth disappears. In sharp contrast, if assuming that the volatility of inflation affects real variability, then the evidence for the Pindyck theory becomes weaker when including the level effects.

Inflation-growth link:

Second, the inflation-growth link possesses the invariance of the results. The (lack of) evidence for the (Holland conjecture) Cukierman-Gerlach theory is not qualitatively altered by the presence or absence of an inflation-growth link. However, when assuming that either there is no inflation-growth link or that growth is independent of changes in inflation the evidence for the Blackburn (Pindyck) theory disappears (becomes weaker).

Variance relationship:

The Cukierman-Gerlach and 'Blackburn' theories and the Friedman hypothesis are invariant to the choice of the matrix B. Moreover, in the absence of level effects, when there is unidirectional feedback between nominal and real uncertainty there is mild evidence for the Pindyck theory, whereas when there is no variance relationship the evidence disappears. That is, the evidence for the Pindyck theory is qualitatively altered by the inclusion or exclusion of a variance relationship.

2.5.3. Direct and Indirect Events

In-mean effects:

For the purposes of this study, it helped to distinguish between direct and indirect impacts. As observed above, these kinds of interactions can be very important. Figure C. 1 presents the direct and indirect impacts for the in-mean effects. It is noteworthy that the indirect effect of nominal uncertainty on inflation that works via growth is opposite to the one that works through growth variability. In particular, the former impact is negative whereas the latter influence is positive. One possible implication of this finding is that inflation is independent of changes in its uncertainty. In essence, the offsetting indirect effects provide a partial rationale for the lack of evidence for either the Cukierman-Meltzer theory or the Holland conjecture.

Regarding the other three in-mean effects direct and indirect influences point to the same conclusion. First, the indirect negative influence of inflation variability on growth through its impact on the uncertainty about growth tells essentially the same story with the direct evidence supporting the Friedman hypothesis. Second, both types of evidence point unequivocally to a positive effect of real uncertainty on inflation. That is, the evidence supporting the Cukierman-Gerlach theory is in line with the evidence for the 'Blackburn' theory and the Briault conjecture. Finally, the indirect evidence (via the inflation channel) regarding the negative impact of real uncertainty on growth agrees well with the direct evidence supporting the Pindyck theory.

Level effects:

Figure C.2 presents the direct and indirect impacts for the level-effects. Both types of evidence point unequivocally to a positive effect of inflation on its uncertainty. That is, the evidence supporting the Friedman hypothesis is in line with the evidence for the Gillman-Kejak theory and Brunner conjecture (when including the second lag of growth as a regressor in the two variances, see section 2.6.3 below). In addition, the indirect effect (via the channel of nominal uncertainty) regarding the positive impact of inflation on the variability of growth agrees well with the direct evidence supporting the Dotsey-Sarte conjecture.

Moreover, this study hypothesizes that the effects of growth on inflation variability could work through changes in inflation. Theoretically speaking the impact is based on the interaction of two effects. A higher growth will raise inflation and, therefore, nominal uncertainty. The evidence for both these influences confirms the positive direct effect. The four variables are connected by a rich network of relationships, which may be causal (direct effects), or reflect shared causal pathways (indirect effects). Direct and indirect effects often occur together. Co-occurence depends on the strength and number of these relationships. However, in order to understand the mechanisms that are responsible for these effects sometimes it is necessary to consider them in isolation. For example, as just mentioned, the indirect impact of growth on volatility works via the channel of inflation. It is worth noting that the direct relationship is qualitatively altered by the presence of the indirect effects. That is, when including in the model the influence of growth on inflation and of inflation on its uncertainty the direct impact disappears.

Finally, the indirect positive influence of growth on its uncertainty through its (first lag) impact on the inflation variability tells essentially the same story with the indirect evidence supporting the Briault and Dotsey-Sarte conjectures. In sharp contrast, there is a lack of a direct effect. On the contrary, when including the second lag of growth as a regressor in the two variances, direct and indirect (via the channel of nominal uncertainty) evidence point to a negative impact (see section 2.6.3 below).

2.6. Robustness

2.6.1. Variance Relationship: BEKK Representation

This section reports the estimation results of a bivariate BEKK GARCH model. Following Engle and Kroner (1995), assuming that the conditional covariance matrix follows the BEKK representation. That is, H_t is parametrized as

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon_{t-1}'A' + BH_{t-1}B', \tag{2.1}$$

where A and B are defined in equation (1.5) and $\text{vech}(C) = (c_{\pi\pi}\ c_{\pi y}\ c_{yy})'$. Because of the presence of a paired transposed matrix factor for each of these three matrices non-negative definiteness of the covariance matrix is assured. Note that the two conditional variances in equation (2.1) can be expressed as

$$\begin{aligned} h_{i,t} &= c_{ii}^2 + c_{ij}^2 + \alpha_{ii}^2\varepsilon_{i,t-1}^2 + 2\alpha_{ii}\alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \alpha_{ij}^2\varepsilon_{j,t-1}^2 + \\ &\quad + \beta_{ii}^2 h_{i,t-1} + 2\beta_{ii}\beta_{ij}h_{ij,t-1} + \beta_{ij}^2 h_{j,t-1}, \quad i,j = \pi, y, \quad j \neq i. \end{aligned} \tag{2.2}$$

It is worth noting that in the BEKK model the effect of the jth variance on the ith variance is restricted to be positive (β_{ij}^2).

As seen in table B.8 the $\beta_{\pi y}$ and $\beta_{y\pi}$ coefficients are insignificant. That is, in the BEKK representation it appears that the two uncertainties are independent of each other. This result is invariant to the formulation of the Φ matrix.

Alternatively, to test for the existence of a variance relationship, the LR test is employed for the constraints $\beta_{\pi y} = \beta_{y\pi} = 0$. As seen in table B.9 the LR tests clearly accept the $\mathrm{S_B}(\Phi_f, B_d)$ null hypothesis against the $\mathrm{S_B}(\Phi_f, B_w)$ and $\mathrm{S_B}(\Phi_f, B_f)$ models. In accordance with this result, the SIC and HQIC come out in favor of the $\mathrm{S_B}(\Phi_f, B_d)$ specification.

Finally, it is worth noting that the three information criteria favor the ccc $\mathrm{S}(\Phi_f, B_f)$ model over the BEKK $\mathrm{S}(\Phi_f, B_f)$ specification (see tables 2.5 and B.9).

2.6.2. In-mean Effects: Standard Deviation

To check the sensitivity of estimation results to the form in which the time varying variance enters the specification of the mean, the conditional standard deviations are also used as regressors in the conditional means. That is, h_{t-n} in eq. (1.4) is replaced by $h_{t-n}^{(sd)}$ where $h_t^{(sd)}$ is a 2×1 column vector given by $h_t^{(sd)} = (\sqrt{h_{\pi t}}\ \sqrt{h_{yt}})'$

The picture is different to that with the conditional variances in the mean equations. At lags 3 and 4 the evidence in support of the Pindyck and the Cukierman-Gerlach theory respectively disappears. In most cases, when the current values of the in-mean effects are used, the routine did not converge. On the other hand, in the $\underset{n=0}{\mathrm{M}}(\Phi_f, B_d)$ model there is evidence in favor of the Cukierman-Gerlach theory. Moreover, there is mild evidence supporting the 'Blackburn' theory only in the $\underset{n=1}{\mathrm{ML}}(\Phi_f, B_w, \Gamma_f)$ specification (see table B.10). Overall, when the standard deviations are included as regressors in the equations of inflation and growth, the in-mean effects become weaker or disappear.

2.6.3. Level Effects: Second Lags and Squared Terms

This section checks the sensitivity of estimation results (regarding the level effects) to the linear form and the choice of the lag. It considers the ccc GARCH(1,1)-level structure eq. (1.5) with the x_{t-1} replaced by (i) $\widetilde{x}_{t-1}$, and (ii) $x_{t-1,2}$ where $\widetilde{x}_{t-1}$ and

Table 2.9 Level effects

L Models	$\gamma_{\pi\pi}$		$\gamma_{y\pi}$		$\gamma_{\pi y}$		γ_{yy}	
Panel A. Models with $\widetilde{x}_{t-1} = [(\pi_{t-1} - \overline{\pi}_{t-1})^2 \ (y_{t-1} - \overline{y}_{t-1})^2 \ ]'$								
$\widetilde{\mathrm{L}}(\Phi_f, B_w \mid B_d, \Gamma_f)^*$	0.06 [0.22]	0.06* [0.22]	**0.36** [0.00]	**0.16** [0.05]	0.00 [0.87]	0.00 [0.87]	-0.16 [0.69]	-0.19 [0.64]
$\widetilde{\mathrm{L}}(\Phi_u, B_w \mid B_d, \Gamma_f)$		0.06 [0.22]		**0.17** [0.06]		0.00 [0.87]		-0.15 [0.69]
Panel B. Models with $x_{t-1,2} = (\pi_{t-1} \ y_{t-2})'$								
$\mathrm{L}_2(\Phi_f, B_w \mid B_d, \Gamma_f)$	**0.08** [0.03]	**0.08** [0.03]	**0.55** [0.00]	**0.52** [0.00]	**-0.01** [0.01]	**-0.01** [0.01]	-0.08 [0.31]	-0.09 [0.31]
$\mathrm{L}_2(\Phi_u, B_w \mid B_d, \Gamma_f)$	**0.08** [0.03]	**0.08** [0.03]	**0.55** [0.00]	**0.49** [0.00]	**-0.01** [0.01]	**-0.01** [0.01]	**-0.11** [0.11]	**-0.11** [0.11]

*The two numbers refer to the models with the B_w and B_d matrices respectively.
The bold numbers indicate significant effects.

$x_{t-1,2}$ are 2×1 column vectors given by $\widetilde{x}_{t-1} = [(\pi_{t-1} - \overline{\pi})^2 \ (y_{t-1} - \overline{y})^2 \ ]'$ (with $\overline{\pi}$, $\overline{y}$ the two sample means) and $x_{t-1,2} = (\pi_{t-1} \ y_{t-2})'$ respectively. The estimated level parameters are reported in table 2.9.

According to Holland (1993) if regime changes causes unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to inflation uncertainty. Uncertainty about inflation regimes is a source of inflation uncertainty. As seen from panel A of table 2.9 inflation variability is independent from changes in $(\pi_{t-1} - \overline{\pi}_{t-1})^2$. In other words, on the contrary to the Holland conjecture there is a lack of a causal impact from squared inflation to the variance of inflation. Regarding the other three level effects the results from the linear causality tests and those obtained by the non-linear procedure are basically identical.

When including the second lag of growth as a regressor in the two variances the results change dramatically. That is, the impact of growth on nominal uncertainty is negative as predicted by Brunner (1993). This result is invariant to the formulation of the Φ and B matrices (see the fourth column of panel B in table 2.9). Recall, however, that the effect disappears with the first lag (see table B.7). Moreover, in the L model with the second lag of growth and the Φ matrix upper triangular growth affects its volatility negatively thus supporting the Karanasos (II) conjecture (see the last column of panel B in table 2.9). Recall that, theoretically speaking, the negative indirect impact is based on the interaction of the Brunner conjecture and the Logue-Sweeney theory. The evidence for these two effects confirm the direct negative influence of growth on its uncertainty, i.e., direct and indirect effects point to the same conclusion. However, when controlling for the impact of inflation on growth, that is when the Φ_f matrix is used, the negative influence of growth on its variance disappears.

2.7. Conclusions

This study has used a bivariate ccc GARCH model and BEKK representation to investigate the link between UK inflation, growth and their respective uncertainties. The core findings are quite robust to changes in the specification of the model, including: (i) growth tends to increase inflation, whereas inflation is detrimental to growth which are in line with the Briault conjecture and the Gillman-Kejak theory respectively (ii) inflation, under linearity, has a positive impact on macroeconomic uncertainty thus supporting the Ungar-Zilberfarb theory and the Dotsey-Sarte conjecture, and (iii) nominal variability, when allowing for both cross effects, affects real volatility positively as argued by Logue and Sweeney (1981). In addition, one significant importance is that in all specifications inflation is independent of changes in its variance, and real uncertainty does not affect inflation variability and is unaffected by the first lag of growth.

The significance and even the sign of the in-mean effects vary with the choice of the lag. Thus the analysis suggests that the behavior of macroeconomic performance depends upon its uncertainty, but also that the nature of this dependence varies with time. In particular, at lag 1, the impact of real variability on growth is positive as predicted by Blackburn (1999), but at lag 3, turns to negative. At lags 1 to 3 there is no causal effect from real volatility to inflation whereas, at lags 0 and 4 a positive impact appears offering support for the Cukierman-Gerlach theory. Also, when accounting for the level effects, it reduces the strength of the impact of real uncertainty on inflation. In sharp contrast, the evidence in support of the Friedman hypothesis and the 'Blackburn' theory becomes stronger in the presence of level effects.

In contrast, note that the lack of an effect from nominal uncertainty to inflation exhibits much less sensitivity. That is, it has been unable to verify, for the UK, the more conventional view that greater volatility in inflation either lowers or increases inflation. This astonishing result cries out for explanation. It is worth noting that the indirect effect that works via the real variability is opposite to the one that works via output growth. That is, on the one hand, nominal uncertainty has a positive impact on real volatility which in turn affects inflation positively. On the other hand, it has a negative effect on growth which in turn affects inflation positively. In essence, the offsetting indirect effects of nominal uncertainty on inflation might provide a rationale for the lack of a direct impact. This account has been fairly speculative-it is more an agenda for further research than a polished theory. In addition, when controlling for the impact of inflation on growth the evidence for Friedman hypothesis disappears. The interlinkage between levels of the two variables may, therefore, be an important element masking the negative effects of nominal volatility on growth.

CHAPTER 3

Multivariate Fractionally Integrated APARCH Modeling of Stock Market Volatility: A multi-country study

3.1. Introduction

This chapter analyzes the applicability of a multivariate ccc FIAPARCH model by estimating national stock market index returns of Canada, France, Germany, Hong Kong, Japan, Singapore, the United Kingdom and the United States in both univariate and multivariate pattern. As the general multivariate specification adopted in this chapter nests the various univariate formulations, the relative ranking of each of these models can be considered using the Wald testing procedures, with which standard information criteria can be used to provide a ranking of the specifications. In addition, this chapter also assesses the ability of the FIAPARCH formulation to forecast (out-of-sample) stock volatility. Whether the difference between the statistics from the different models is statistically significant is verified via the tests of Diebold and Mariano (1995) and Harvey et al. (1997).

Section 3.2 describes the FIAPARCH model and how various ARCH specifications are nested within it. Section 3.3 presents maximum likelihood parameter estimates for the various specifications and tests for the apparent similarity of the power and fractional differencing terms across countries. Section 3.4 evaluates the different specifications in terms of their out-of-sample forecast ability. Moreover, equal forecast accuracy of the competing models is tested by utilizing three test statistics. Section 3.5 discusses estimation results and concludes.

3.2. FIAPARCH Model

3.2.1. Univariate Process

One of the most common models in finance and economics to describe a time series s_t of stock returns is the AR(1) process

$$(1 - \zeta L)s_t = c + \varepsilon_t, \quad t \in \mathbb{N}, \tag{3.1}$$

with

$$\varepsilon_t = e_t \sqrt{h_t},$$

where $|\zeta| < 1$ and $\{e_t\}$ are independently, identically distributed $(i.i.d.)$ student-t random variables with $\mathsf{E}(e_t) = \mathsf{E}(e_t^2 - 1) = 0$. h_t is positive with probability one and is a measurable function of Σ_{t-1}, which in turn is the sigma-algebra generated by $\{s_{t-1}, s_{t-2}, \ldots\}$. That is h_t denotes the conditional variance of the returns $\{s_t\}$ and $s_t|\Sigma_{t-1} \overset{i.i.d.}{\sim} (c + \zeta s_{t-1}, h_t)$.

Tse (1998) examines the conditional heteroscedasticity of the yen-dollar exchange rate by employing the FIAPARCH$(1, d, 1)$ model. Accordingly, this chapter utilizes the following process presented in section 1.2.2:

$$h_t^{\delta/2} = \omega + \left[1 - \frac{(1 - \varphi L)(1 - L)^d}{(1 - \beta L)}\right] [|\varepsilon_t| - \gamma \varepsilon_t]^{\delta}$$

where $\omega \in (0, \infty)$, $|\varphi| < 1$, $0 \leq d \leq 1$,[1] γ is the leverage coefficient, and δ is the parameter for the power term that takes (finite) positive values.

[1] The fractional differencing operator, $(1 - L)^d$ is most conveniently expressed in terms of the hypergeometric function

$$(1 - L)^d = F(-d, 1; 1; L) = \sum_{j=0}^{\infty} \frac{\Gamma(j - d)}{\Gamma(-d)\Gamma(j + 1)} L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j,$$

where

$$F(a, b; c; z) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{z^j}{j!}$$

is the Gaussian hypergeometric series, $(b)_j$ is the shifted factorial defined as $(b)_j = \prod_{i=0}^{j-1} (b + i)$ (with $(b)_0 = 1$), and $\Gamma(\cdot)$ is the gamma function.

When $d = 0$, the process in equation (1.3) reduces to the APARCH(1,1), which nests two major classes of ARCH models. Specifically, a Taylor/ Schwert type of formulation is specified when $\delta = 1$, and a Bollerslev type is specified when $\delta = 2$. There seems to be no obvious reason why one should assume that the conditional standard deviation is a linear function of lagged absolute returns or the conditional variance a linear function of lagged squared returns. As Brooks et al. (2000) point out

> "The common use of a squared term in this role ($\delta = 2$) is most likely to be a reflection of the normality assumption traditionally invoked regarding financial data. However, if we accept that (high frequency) data are very likely to have a non-normal error distribution, then the superiority of a squared term is lost and other power transformations may be more appropriate. Indeed, for non-normal data, by squaring the returns one effectively imposes a structure on the data which may potentially furnish sub-optimal modeling and forecasting performance relative to other power terms". (p. 378)

Since its introduction by Ding et al. (1993), the APARCH formulation has been frequently applied. It is worth noting that Fornari and Mele (1997) show the usefulness of this scheme in approximating models developed in continuous time as systems of stochastic differential equations. This feature has usually been overshadowed by its well-known role as simple econometric tool providing reliable estimates of unobserved conditional variances (Fornari and Mele, 2001). Hentschel (1995) defines a parametric family of asymmetric models that nests the APARCH one.[2]

[2]For applications of the APARCH model in economics see Campos and Karanasos (2008), Campos et al. (2008a, 2008b) and Karanasos and Schurer (2008).

When $\gamma = 0$ and $\delta = 2$ the process in equation (1.3) reduces to the FIGARCH$(1, d, 1)$ specification which includes Bollerslev's (1986) model (when $d = 0$) and the integrated specification (when $d = 1$) as special cases.[3] Baillie et al. (1996) point out that a striking empirical regularity that emerges from numerous studies of high-frequency, say daily, asset pricing data with ARCH-type models, concerns the apparent widespread finding of integrated behavior. This property has been found in stock returns, exchange rates, commodity prices and interest rates (see Bollerslev et al., 1992). Yet unlike I(1) processes for the mean, there is less theoretical motivation for truly integrated behavior in the conditional variance (see Baillie et al., 1996 and the references therein).[4]

Finally, as noted by Baillie et al. (1996) for the variance, being confined to only considering the extreme cases of stable and integrated specifications can be very misleading when long-memory (but eventually mean-reverting) processes are generating the observed data. They showed that data generated from a process exhibiting long-memory volatility may be easily mistaken for integrated behavior. Andersen and Bollerslev (1997) suggest that cross-sectional aggregation of a large number of volatility components or news information arrival processes with different degrees of persistence could lead to fractional integration. Kirman and Teyssiere

[3] An excellent survey of major econometric work on long-memory processes and their applications in economics and finance is given by Baillie (1996). Karanasos et al. (2006) apply the FIAPARCH model to interest rates. For applications of the FIGARCH model to exchange rates see, among others, Conrad and Lamla (2007).

[4] In particular, the occurrence of a shock to the IGARCH volatility process will persist for an infinite prediction horizon. This extreme behavior of the IGARCH process may reduce its attractiveness for asset pricing purposes, where the IGARCH assumption could make the pricing functions for long-term contracts very sensitive to the initial conditions. This seems contrary to the perceived behavior of agents, who typically do not frequently and radically change their portfolio compositions. In addition, the IGARCH model is not compatible with the persistence observed after large shocks such as the Crash of October 1987. A further reason to doubt the empirical reasonableness of IGARCH models relates to the issue of temporal aggregation. A data generating process of IGARCH at high frequencies would also imply a properly defined weak IGARCH model at low frequencies of observation. However, this theoretical result seems at odds with reported empirical findings for most asset categories (abstracted from Baillie et al. 1996).

(2001) use a microeconomic model to link herding and swing of opinion with long-memory in volatility. According to Beltratti and Morana (2006) volatility of output growth and, to a lesser extent, the volatility of the Federal funds rate and M1 growth affect both the persistent and non-persistent components of S&P 500 volatility (see Hyung et al., 2006).

3.2.2. Multivariate Formulation

This section discusses the multivariate time series model for the stock returns and its merits and properties. Let the N-dimensional column vector of the returns $\mathbf{s}_t$ defined as $\mathbf{s}_t = [s_{it}]_{i=1,\ldots,N}$ and the corresponding residual vector $\boldsymbol{\varepsilon}_t$ as $\boldsymbol{\varepsilon}_t = [\varepsilon_{it}]_{i=1,\ldots,N}$. Regarding $\boldsymbol{\varepsilon}_t$, it is assumed to be conditionally student-t distributed with mean vector $\mathbf{0}$, variance vector $\mathbf{h}_t = [h_{it}]_{i=1,\ldots,N}$ and ccc, $\rho_{ij} = h_{ij,t}/\sqrt{h_{it}h_{jt}}$, $|\rho_{ij}| \leq 1$, $i, j = 1, \ldots, N$.

Next, the structure of the AR (1) mean equation is given by

$$\mathbf{Z}(L)\mathbf{s}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t, \tag{3.2}$$

where $\mathbf{Z}(L) = \mathbf{I}_N\boldsymbol{\zeta}(L)$ with $\mathbf{I}_N$ being the $N \times N$ identity matrix and $\boldsymbol{\zeta}(L) = [1 - \zeta_i L]_{i=1,\ldots,N}$, $|\zeta_i| < 1$.

Further, to establish terminology and notation, the multivariate FIAPARCH (M-FIAPARCH) process of order $(1, d, 1)$ is defined by

$$\mathbf{B}(L)(\mathbf{h}_t^{\wedge\frac{\delta}{2}} - \boldsymbol{\omega}) = [\mathbf{B}(L) - \boldsymbol{\Lambda}(L)\boldsymbol{\Phi}(L)][|\boldsymbol{\varepsilon}_t| + \boldsymbol{\Gamma}\boldsymbol{\varepsilon}_t]^{\wedge\delta}, \tag{3.3}$$

where $^\wedge$ denotes elementwise exponentiation and $|\boldsymbol{\varepsilon}_t|$ is the vector $\boldsymbol{\varepsilon}_t$ with elements stripped of negative values. Moreover, $\mathbf{B}(L) = \mathbf{I}_N\boldsymbol{\beta}(L)$ with $\boldsymbol{\beta}(L) = [1 - \beta_i L]_{i=1,\ldots,N}$, and $\boldsymbol{\Phi}(L) = \mathbf{I}_N\boldsymbol{\phi}(L)$ with $\boldsymbol{\varphi}(L) = [1 - \varphi_i L]_{i=1,\ldots,N}$, $|\varphi_i| < 1$. In addition,

$\boldsymbol{\omega} = [\boldsymbol{\omega}_i]_{i=1,\ldots,N}$ with $\omega_i \in (0, \infty)$ and $\boldsymbol{\Lambda}(L) = \mathbf{I}_N \mathbf{d}(L)$ with $\mathbf{d}(L) = [(1-L)^{d_i}]_{i=1,\ldots N}$, $0 \leq d_i \leq 1$. Finally, $\boldsymbol{\Gamma} = \boldsymbol{\gamma}\mathbf{I}_N$ with $\boldsymbol{\gamma} = [\gamma_i]_{i=1,\ldots,N}$.[5]

3.3. Empirical Analysis

3.3.1. Data

Daily stock price index data for eight countries were sourced from the Datastream database for the period 1st January 1988 to 22nd April 2004, giving a total of 4,255 observations. with the period 1st January 1988 to 16th July 2003 for the estimation, while producing 200 out-of-sample forecasts for the period 17th July 2003 to 22nd April 2004. The eight countries and their respective price indices are: UK: FTSE 100 (F), US: S&P 500 (SP), Germany: DAX 30 (D), France: CAC 40 (C), Japan: Nikkei 225 (N), Singapore: Straits Times (S), Hong Kong: Hang Seng (H) and Canada: TSE 300 (T). For each national index, the continuously compounded return was estimated as $s_t = 100[\log(p_t) - \log(p_{t-1})]$ where p_t is the price on day t.[6]

3.3.2. Univariate Models

Univariate estimation

This section proceeds with the estimation of the AR(1)-FIAPARCH$(1, d, 1)$ model[7] in equations (3.1) and (1.3) in order to take into account the serial correlation[8] and the GARCH effects observed in the time series data, and to capture the possible

[5] $\mathbf{Z}(L)$, $\mathbf{B}(L)$, $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\Lambda}(L)$ are $N \times N$ diagonal polynomial matrices with diagonal elements $1 - \zeta_i L$, $1 - \beta_i L$, $1 - \varphi_i L$ and $(1-L)^{d_i}$ respectively. Further, $\boldsymbol{\Gamma}$ is a $N \times N$ diagonal matrix with diagonal elements γ_i.

[6] See figure F.1-3 for actual data series.

[7] The only exceptions are the Canadian and Singaporean indices, where an AR(1)-FIAPARCH$(0, d, 1)$ model is used. For these two indices the AR(1)-FIAPARCH$(1, d, 1)$ estimates for β were insignificant and the IC came out in favor of the $(0, d, 1)$ specification. In addition, for the Hang Seng index, the criteria favor the $(1, d, 0)$ formulation.

[8] The 12th order Ljung-Box Q-statistics on the squared return series indicate high serial correlation in the second moment for all indices.

long-memory in volatility. The various specifications are estimated using the maximum likelihood estimation (MLE) method as implemented by Davidson (2008) in Time Series Modelling (TSM). The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis.[9] To accommodate the presence of such leptokurtosis, the models are estimated using student-t distributed innovations. Hence, for the univariate models, the log-likelihood to be maximized is given by

$$\begin{aligned}\log \mathcal{L} &= T\left[\log\Gamma\left(\frac{\upsilon+1}{2}\right) - \log\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\log\pi(\upsilon-2)\right] \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left\{\log h_t^2 + (\upsilon+1)\left[\log\left(1+\frac{\varepsilon_t^2}{h_t^2(\upsilon-2)}\right)\right]\right\},\end{aligned}$$

where $\Gamma(\cdot)$ denotes the gamma function. For more details, see, Davidson (2008).

Table 3.1 reports the estimation results.[10] In all countries the AR coefficient (ζ) is highly significant. The estimate for the $\varphi(\beta)$ parameter is insignificant only in one(two) out of the eight cases. In three countries the estimates of the leverage term (γ) are statistically significant, confirming the hypothesis that there is negative correlation between returns and volatility. For the other countries, the models are reestimated without an asymmetry term. For all indices the estimates of the power term (δ) and the fractional differencing parameter (d) are highly significant. Interestingly, the highest power terms are obtained for the two American indices, while the European ones are characterized by the highest degree of persistence. In all cases, the estimated degrees of freedom parameter (υ) is highly significant and leads to an estimate of the kurtosis which is different from three.[11]

[9] For all indices the Jarque-Bera statistic rejects the normality hypothesis at the 1% level. The estimated kurtosis coefficient is significantly above three for all indices but FTSE 100 and Nikkei 225.

[10] The estimates of the constants in the mean and the variance are not presented, which were significant in all cases but one.

[11] The kurtosis of a student-t distributed random variable with υ degrees of freedom is $3\frac{\upsilon-2}{\upsilon-4}$.

Table 3.1 Univariate AR-FI(A)PARCH models (ML Estimation)

	SP	T	C	D	F	H	N	S
ζ	-0.05^* (-3.28)	0.17 (10.94)	0.04 (2.34)	0.03^* (2.31)	0.04 (2.38)	0.06 (3.71)	-0.02 (-1.63)	0.15 (9.20)
β	0.54 (5.81)	–	0.66 (6.94)	0.56 (5.65)	0.59 (5.32)	0.08 (2.01)	0.51 (4.83)	–
φ	0.27 (4.11)	-0.11 (-2.95)	0.20 (3.84)	0.21 (4.82)	0.19 (3.77)	–	0.14 (2.03)	-0.07 (-2.23)
γ	–	–	–	0.46 (3.73)	–	0.69 (3.65)	–	0.76 (3.90)
δ	2.35 (23.50)	2.42 (17.28)	1.77 (12.64)	1.24 (11.46)	1.86 (14.31)	1.28 (12.80)	2.07 (18.81)	1.40 (12.73)
d	0.30 (6.00)	0.19 (6.33)	0.52 (4.33)	0.40 (4.34)	0.46 (4.60)	0.18 (4.50)	0.42 (6.00)	0.21 (5.25)
υ	5.60 (10.77)	5.38 (10.76)	8.53 (6.56)	6.83 (6.90)	10.70 (6.04)	4.56 (11.12)	5.80 (10.54)	4.86 (11.04)
Q_{12}	18.45 [0.10]	9.52 [0.66]	10.00 [0.61]	13.18 [0.36]	12.86 [0.38]	22.85 [0.03]	10.59 [0.56]	18.50 [0.10]
Q_{12}^2	5.12 [0.95]	19.47 [0.08]	11.74 [0.47]	8.13 [0.77]	18.00 [0.12]	33.24 [0.00]	20.90 [0.05]	2.20 [1.00]

Notes: For each of the eight indices, Table 3.1 reports ML parameter estimates for the AR(1)-FI(A)PARCH model. The numbers in parentheses are t-statistics. * The S&P 500 and Dax 30 indices are estimated by AR(3) and AR(4) models respectively. Q_{12} and Q_{12}^2 are the 12th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals respectively. The numbers in brackets are p-values.

In all cases, the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variance (see Conrad and Haag, 2006). According to the values of the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals there is no statistically significant evidence of misspecification.

Tests of fractional differencing and power term parameters

A large number of studies have documented the persistence of volatility in stock returns, see, e.g., Ding et al. (1993), Ding and Granger (1996), Engle and Lee (2000). Using daily data many of these studies have concluded that the volatility process is very persistent and appears to be well approximated by an IGARCH process. For the stable APARCH(1,1) model[12] the condition for the existence *of* the $\delta/2$ th

[12]Restricting d to be 0 in equation (1.3) leads to an APARCH(1,1) model with parameters β and $\varphi - \beta$.

moment of the conditional variance is $\mathsf{V} = \alpha\mathsf{E}(|e| - \gamma e)^{\delta} + \beta < 1$ which depends on the density of e. For a student-t distributed innovation with v degrees of freedom, it is $\frac{\mathsf{V}-\beta}{a} = \frac{(1+\frac{\gamma}{2})}{\sqrt{\pi}}(v-2)^{\frac{\delta}{2}}\frac{\Gamma(\frac{\delta+1}{2})\Gamma(\frac{v-\delta}{2})}{\Gamma(\frac{v}{2})}$. Notic that if $\gamma = 0$ the expression for the $\frac{\mathsf{V}-\beta}{\alpha}$ is the one for the symmetric PARCH model (see Paolella, 1997 and Karanasos and Kim, 2006). In addition, if $\gamma = 0$, $\delta = 2$, $\mathsf{V} = \alpha + \beta < 1$ reduces to the usual stationarity condition of the GARCH(1,1) model.

Thus, estimating a V which is close to one is suggestive of integrated APARCH behavior. Table 3.2 presents the estimates for V from the AR-APARCH$(1,1)$ model with student-t distributed innovations. For all indices V is close to 1, indicating that $h_t^{\frac{\delta}{2}}$ may be integrated.[13]

Table 3.2 Estimates of V for AR-APARCH$(1,1)$ models

	SP	T	C	D	F	H	N	S
V	0.998	0.991	1.000	0.985	0.985	0.963	1.013	0.946

However, from the FI(A)PARCH estimates (reported in table 3.1), it appears that the long-run dynamics are better modeled by the fractional differencing parameter. To test for the persistence of the conditional heteroscedasticity models, the Wald statistics are examined for the linear constraints $d = 0$ (stable APARCH) and $d = 1$ (IAPARCH).[14] As seen in table 3.3 the W tests clearly reject both the stable and integrated null hypotheses against the FIAPARCH one.[15] Clearly, the results which emerged from table 3.2 were misleading, i.e. imposing the restriction $d = 0$ leads to parameter estimates which falsely suggest integrated behavior. Thus,

[13]The estimated AR-APARCH$(1,1)$ coefficients are reported in table D.1.
[14]Restricting d to be one leads to an IAPARCH(1,2) model with parameters β, $1+\varphi-\beta$ and $-\varphi$ (see equation (1.3)).
[15]Various tests for long-memory in volatility have been proposed in the literature (see, for details, Karanasos and Kartsaklas, 2008).

purely from the perspective of searching for a model that best describes the volatility in the stock return series, the fractionally integrated one appears to be the most satisfactory representation.[16]

This result is an important finding because the time series behavior of volatility affects asset prices through the risk premium. Christensen and Nielsen (2007) establish theoretically and empirically the consequences of long-memory in volatility for asset prices. Using a model for expected returns to discount streams of expected future cash flows, they calculate asset prices. Within this context the risk-return trade-off and the serial correlation in volatility are the two most important determinants of asset values. Christensen and Nielsen (2007) derive the way in which these two ingredients jointly determine the level of stock prices. They also investigate the quantitative economic consequences of these changes in asset price elasticities.

Table 3.3 Tests for restrictions on fractional differencing and power term parameters

H_0:		$d=0$	$d=1$		$\delta=1$	$\delta=2$
	d	W	W	δ	W	W
S&P 500	0.30{0.05}	33[0.00]	173[0.00]	2.35{0.10}	178[0.00]	9[0.00]
TSE 300	0.19{0.03}	28[0.00]	522[0.00]	2.42{0.14}	102[0.00]	10[0.00]
CAC 40	0.52{0.12}	18[0.00]	15[0.00]	1.77{0.14}	31[0.00]	3[0.09]
DAX 30	0.40{0.09}	18[0.00]	39[0.00]	1.24{0.11}	15[0.00]	52[0.00]
FTSE 100	0.46{0.10}	21[0.00]	29[0.00]	1.86{0.13}	37[0.00]	1[0.30]
Hang Seng	0.18{0.04}	16 [0.00]	322[0.00]	1.28{0.10}	8[0.00]	72[0.00]
Nikkei 225	0.42{0.07}	35[0.00]	67[0.00]	2.07{0.11}	114[0.00]	0.50[0.54]
Straits Times	0.21{0.04}	32[0.00]	444[0.00]	1.40{0.11}	16[0.00]	36[0.00]

Notes: For each of the eight indices, table 3.3 reports the value of the Wald (W) statistics for the unrestricted FI(A)PARCH and restricted ($d=0,1;\delta=1,2$) models respectively. The numbers in $\{\cdot\}$ are standard errors. The numbers in $[\cdot]$ are p values.

Following the work of Ding et al. (1993), Hentschel (1995), Tse (1998) and Brooks et al. (2000) among others, the Wald test can be used for model selection. Alternatively, the Akaike, Schwarz, Hannan-Quinn or Shibata information criteria

[16] It is worth mentioning the empirical results in Granger and Hyung (2004). They suggest that there is a possibility that, at least part of the long-memory may be caused by the presence of neglected breaks in the series. Future work may clarify this out.

(AIC, SIC, HQIC, SHIC respectively) can be applied to rank the various ARCH type of models.[17] These model selection criteria check the robustness of the Wald testing results discussed above.[18] Specifically, according to the AIC, HQIC and SHIC, the optimal specification (i.e., FIAPARCH, APARCH or IAPARCH) for all indices was the FIAPARCH one.[19] The SIC results largely concur with the AIC, HQIC or SHIC results.[20]

Next, recall that the two common values of the power term imposed throughout much of the GARCH literature are the values of two (Bollerslev's model) and unity (the Taylor/Schwert specification). The invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance (Brooks et al., 2000). Accordingly, this study tests whether the estimated power terms are significantly different from unity or two using Wald tests. As reported in table 3.3, all eight estimated power coefficients are significantly different from unity (see column six). Further, with the exception of the CAC 40, FTSE 100 and Nikkei 225 indices, each of the power terms are significantly different from two (see the last column of table 3.3). Hence, on the basis of these results, in the majority of cases support is found for the (asymmetric) power fractionally integrated model, which allows an optimal power transformation term to be estimated. The evidence obtained from the Wald tests is reinforced by the model ranking provided by the four model selection criteria. This is a noteworthy result since He and Teräsvirta (1998) emphasized that if the standard Bollerlsev type of model is augmented by the 'heteroscedasticity' parameter, the estimates of the ARCH and GARCH coefficients

[17]As a general rule, the information criteria approaches suggest selecting the model which produces the lowest AIC, SIC, HQIC or SHIC values.

[18]The use of the information criteria techniques for comparing models has the advantage of being relatively less onerous compared to Wald testing procedures, which only allow formal pairwise testing of nested models (Brooks et al., 2000).

[19]Caporin (2003) performs a Monte Carlo simulation study and verifies that information criteria clearly distinguish the presence of long-memory in volatility.

[20]The AIC, SIC, HQIC or SHIC values are not reported.

almost certainly change. More importantly, Karanasos and Schurer (2008) show that in the univariate GARCH-in-mean level formulation the significance of the in-mean effect is sensitive to the choice of the power term.

3.3.3. Multivariate Models

The analysis above suggests that the FIAPARCH formulation describes the conditional variances of the eight stock indices well. However, financial volatilities move together over time across assets and markets. Recognizing this commonality through a multivariate modeling framework can lead to obvious gains in efficiency and to more relevant financial decision making than can be obtained when working with separate univariate specifications (Bauwens and Laurent, 2005). Therefore, multivariate GARCH models are essential for enhancing the understanding of the relationships between the (co)volatilities of economic and financial time series. For recent surveys on multivariate specifications and their practical importance in various areas such as asset pricing, portfolio selection and risk management, see e.g., Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2007). Thus this section, within the framework of the multivariate ccc model, will analyze the dynamic adjustments of the variances for the various indices. Overall seven bivariate specifications are estimated; three for the European countries: CAC 40-DAX 30 (C-D), CAC 40-FTSE 100 (C-F) and DAX 30-FTSE 100 (D-F); three for the Asian countries: Hang Seng-Nikkei 225 (H-N), Hang Seng-Straits Times (H-S) and Nikkei 225-Straits Times (N-S); one for the S&P 500 and TSE 300 indices (SP-T). Moreover, two trivariate models are estimated as well: one for the three European countries (C-D-F) and one for the three Asian countries (H-N-S).

For the multivariate models, the log-likelihood to be maximized is given by

$$\begin{aligned}\log \mathcal{L} &= T\left[\log\Gamma\left(\frac{\upsilon+1}{2}\right)-\log\Gamma\left(\frac{\upsilon}{2}\right)-\frac{1}{2}\log\pi(\upsilon-2)\right] \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left\{\log\det\mathbf{H}_t+\log\det\boldsymbol{\rho}+(\upsilon+1)\left[\log\left(1+\frac{\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1/2}\boldsymbol{\rho}^{-1}\mathbf{H}_t^{-1/2}\boldsymbol{\varepsilon}_t}{(\upsilon-2)}\right)\right]\right\},\end{aligned}$$

where $\Gamma(\cdot)$ denotes again the gamma function, $H_t = diag(h_t)$ and ρ is the 2×2 (3×3) correlation matrix with unit diagonal elements and off-diagonal entries ρ_{ij}. Note, that the degrees of freedom are constrained to be equal for all equations. For more details, see, Davidson (2008).

Bivariate processes

The best fitting bivariate specification is chosen according to LR test results and the minimum value of the information criteria (not reported). In the majority of the models the AR coefficients are significant at the 5% level or better. In almost all cases a $(1, d, 1)$ order is chosen for the FIAPARCH formulation. Only the H-S and N-S models are $(0, d, 1)$ order for the Straits Times index, and $(1, d, 0)$ order for the Hang Seng index. Note that this is in line with the findings for the univariate models where the β parameter was insignificant for Straits Times, while the φ parameter was insignificant for Hang Seng. In six out of the fourteen models the leverage term (γ) is significant.

As in the univariate case, it is significant in both indices for the H-S case and in the DAX 30 index for the D-F case. In addition, in the bivariate case it is also significant in the Tse 300 index for the SP-T model and in the Nikkei 225 for the N-S one. In almost all cases the power term (δ) and the fractional differencing parameter (d) are highly significant. In the D-F, H-S and N-S models the two countries generated very similar power terms: (1.28, 1.36), (1.42, 1.47) and (1.70, 1.62) respectively. In four out of the seven bivariate formulations the two countries generated very similar fractional parameters. These are the SP-T, the C-F, the H-N and the H-S models. The corresponding pairs of values are: (0.22, 0.21), (0.24, 0.29), (0.36, 0.35) and (0.16, 0.13). Interestingly, in the majority of the cases the estimated power and fractional differencing parameters of the bivariate models take lower values than those of the corresponding univariate models. In all cases the estimated ccc (ρ) is highly significant. Interestingly, it is rather high among the American and European indices, and rather low among the Asian indices. Finally, the degrees of freedom (υ) parameters are highly significant and the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variances (see, Conrad and Haag, 2006). In the majority of the cases the hypothesis of uncorrelated standardized and squared standardized residuals is well supported (see the last two columns of table 3.4).

Next the Wald statistics are examined for the linear constraints $d = 0$ (stable APARCH) and $d = 1$ (IAPARCH). As seen in table 3.5 the W tests clearly reject both the stable and integrated null hypotheses against the FIAPARCH one. In the presence of long-memory in volatility, Christensen and Nielsen (2007) reassess the relation between the risk-return trade-off, serial dependence in volatility, and the elasticity of asset values with respect to volatility. They show that the elasticity is smaller in magnitude than earlier estimates, and much more stable under variations in the long-memory parameter than in the short-memory case. Thus, they point out that the high elasticities reported earlier should be interpreted with considerable caution. They also highlight the fact that the way in which volatility enters in the asset evaluation model is crucial and should be considered carefully. This is due to the fact that the memory properties of the volatility process carry over to the stock return process through the risk premium link.

Table 3.4 Bivariate AR-FI(A)PARCH models (ML Estimation)

		ζ_i	β_i	φ_i	γ_i	δ_i	d_i	ρ	υ	Q_{12}	Q_{12}^2
SP-T	SP	-0.05^*	0.46	0.26	–	1.85	0.22	0.65	13.69	18.08	2.77
		(−4.51)	(4.78)	(3.73)		(8.81)	(5.50)	(21.33)	(9.85)	[0.11]	[0.99]
	T	0.17	0.33	0.18	0.34	1.59	0.21			10.74	2.81
		(13.86)	(2.27)	(1.52)	(2.46)	(8.37)	(5.25)			[0.55]	[0.99]
C-D	C	−0.03	0.50	0.26	–	1.55	0.30	0.65	16.69	34.92	20.28
		(−2.63)	(3.94)	(4.30)		(9.12)	(3.00)	(20.54)	(6.76)	[0.00]	[0.06]
	D	0.02^*	0.62	0.24	–	1.23	0.44			10.17	5.17
		(1.53)	(9.00)	(5.60)		(9.84)	(6.28)			[0.60]	[0.95]
C-F	C	0.05	0.35	0.16	–	1.76	0.24	0.67	18.96	10.33	24.51
		(3.88)	(1.55)	(1.24)		(7.65)	(2.18)	(20.90)	(6.94)	[0.59]	[0.02]
	F	0.04	0.45	0.20	–	1.55	0.29			15.80	40.18
		(2.70)	(1.48)	(1.48)		(5.54)	(1.61)			[0.20]	[0.00]
D-F	D	0.01^*	0.55	0.20	0.14	1.28	0.40	0.54	18.13	12.48	3.31
		(0.37)	(5.51)	(4.96)	(1.68)	(11.64)	(4.44)	(19.48)	(6.06)	[0.41]	[0.99]
	F	−0.03	0.42	0.17	–	1.36	0.28			36.27	17.44
		(−2.14)	(1.93)	(1.74)		(8.00)	(2.15)			[0.00]	[0.13]
H-N	H	0.05	0.57	0.33	–	1.49	0.36	0.33	12.62	22.30	35.79
		(3.44)	(3.70)	(3.94)		(17.71)	(3.18)	(11.03)	(11.03)	[0.03]	[0.00]
	N	−0.02	0.46	0.15	0.10	1.69	0.35			9.78	54.52
		(−1.07)	(3.82)	(2.11)	(1.74)	(13.75)	(5.04)			[0.63]	[0.00]
H-S	H	0.03	0.08	–	0.11	1.42	0.16	0.43	11.31	39.18	108.29
		(1.79)	(2.85)		(1.73)	(12.07)	(7.58)	(17.02)	(11.44)	[0.00]	[0.00]
	S	0.14	–	−0.02	0.47	1.47	0.13			20.69	5.03
		(9.16)		(0.87)	(3.16)	(12.01)	(5.79)			[0.06]	[0.96]
N-S	N	−0.03	0.43	0.14	0.11	1.70	0.33	0.26	12.42	12.93	58.83
		(−2.08)	(3.41)	(1.76)	(2.02)	(13.75)	(5.15)	(12.32)	(10.47)	[0.37]	[0.00]
	S	0.15	–	−0.07	–	1.62	0.23			16.36	1.50
		(9.33)		(1.78)		(15.71)	(6.64)			[0.17]	[1.00]

For each of the seven pairs of indices, table 3.4 reports ML parameter estimates for the bivariate AR-FI(A)PARCH model. SP-T denotes the bivariate process for the S&P 500 and TSE 300 indices. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stand for the three bivariate specifications for the Asian indices. *For the S&P 500 and DAX 30 indices, AR models of order 3 and 4 are estimated respectively. The numbers in parentheses are t-statistics. Q_{12} and Q_{12}^2 are the 12th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals respectively. The numbers in brackets are p-values.

Also, this study tests whether the estimated power terms are significantly different from unity or two using Wald tests. The eight estimated power coefficients are significantly different from either unity or two (see the last two columns of table 3.5).

Trivariate specifications

Table 3.6 reports the parameters of interest for the two trivariate FI(A)PARCH(1,1) models of Asian and European indices. In two out of the three Asian countries the leverage term (γ) is weakly significant. In all cases the power term (δ) and the fractional differencing parameter (d) are highly significant. Similarly, in all cases the

Table 3.5 Tests for restrictions on fractional differencing and power term parameters

H_0:		d's= 0	d's= 1		δ's= 1	δ's= 2
	d's	W	W	δ's	W	W
SP-T	0.22 {0.04}-0.21 {0.04}	37[0.00]	432[0.00]	1.85 {0.21}-1.59 {0.19}	141[0.00]	141[0.00]
C-D	0.30 {0.10}-0.44 {0.07}	39[0.00]	241[0.00]	1.55 {0.17}-1.23 {0.12}	97[0.00]	124[0.00]
C-F	0.24 {0.11}-0.29 {0.18}	5[0.10]	112[0.00]	1.76 {0.23}-1.55 {0.28}	30[0.00]	81[0.00]
D-F	0.40 {0.09}-0.28 {0.13}	25[0.00]	279[0.00]	1.29 {0.11}-1.36 {0.17}	130[0.00]	155[0.00]
H-N	0.36 {0.11}-0.35 {0.07}	36[0.00]	65[0.00]	1.49 {0.08}-1.69 {0.12}	319[0.00]	318[0.00]
H-S	0.16 {0.02}-0.13 {0.02}	33[0.00]	255[0.00]	1.42 {0.12}-1.47 {0.12}	228[0.00]	247[0.00]
N-S	0.33 {0.06}-0.23 {0.03}	77[0.00]	158[0.00]	1.70 {0.12}-1.62 {0.10}	341[0.00]	284[0.00]

Notes: For each of the seven pairs of indices, table 3.5 reports the values of the Wald (W) statistics of the unrestricted bivariate FI(A)PARCH and restricted (d's=0,1; δ's=1,2) models respectively. SP-T denotes the bivariate model for the S&P 500 and TSE 300 indices. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stand for the three bivariate models for the Asian indices. The numbers in {.} are standard errors. The numbers in [.] are p values.

estimated ccc (ρ) and degrees of freedom (υ) parameters are highly significant and the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variances (see, Conrad and Haag, 2006). In particular, the estimates of ρ confirm the results from the bivariate models, i.e. the conditional correlation between the European indices is considerably stronger than between the Asian indices.

3.3.4. On the Similarity of the Fractional/Power Parameters

The apparent similarity of the optimal fractional differencing and power term parameters for each of the eight country indices are tested using pairwise Wald tests:

$$\mathrm{W}_d = \frac{(d_1 - d_2)^2}{\mathsf{Var}(d_1) + \mathsf{Var}(d_2) - 2\mathsf{Cov}(d_1, d_2)}, \quad \mathrm{W}_\delta = \frac{(\delta_1 - \delta_2)^2}{\mathsf{Var}(\delta_1) + \mathsf{Var}(\delta_2) - 2\mathsf{Cov}(\delta_1, \delta_2)},$$

where d_i (δ_i), $i = 1, 2$, is the fractional differencing (power term) parameter from the bivariate FIAPARCH model estimated for the national stock market index for country i, $\mathsf{Var}(d_i)$, $\mathsf{Var}(\delta_i)$ are the corresponding variances, and $\mathsf{Cov}(d_1, d_2)$, $\mathsf{Cov}(\delta_1, \delta_2)$

Table 3.6 Trivariate AR-FI(A)PARCH$(1, d, 1)$ models (ML Estimation)

	C-D-F			H-N-S*		
	C	D	F	H	N	S
β_i	0.19 (1.40)	0.43 (4.61)	0.22 (1.15)	0.39 (1.92)	0.38 (2.50)	0.78 (18.85)
φ_i	0.11 (0.90)	0.22 (3.35)	0.09 (0.61)	0.28 (1.56)	0.15 (1.53)	0.81 (22.08)
γ_i	-	-	-	0.02 (1.46)	0.07 (1.60)	–
δ_i	1.83 (10.95)	1.25 (9.52)	1.56 (7.12)	1.47 (13.36)	1.53 (10.20)	1.88 (11.75)
d_i	0.11 (4.16)	0.25 (5.43)	0.15 (3.27)	0.18 (4.50)	0.26 (0.07)	0.08 (4.39)
	C-D	D-F	C-F	H-N	N-S	H-S
ρ	0.66 (21.07)	0.56 (19.86)	0.68 (21.70)	0.32 (14.84)	0.25 (12.19)	0.43 (16.92)
υ		9.60 (17.36)			8.42 (20.54)	
Q_{12}	28.77 [0.004]	33.79 [0.001]	17.19 [0.14]	47.95 [0.00]	14.42 [0.28]	21.51 [0.04]
Q^2_{12}	68.72 [0.00]	70.88 [0.00]	7.16 [0.85]	95.54 [0.00]	163.52 [0.00]	0.71 [1.00]

Notes: Table 3.6 reports ML parameter estimates for the two trivariate (white noise) FI(A)PARCH$(1, d, 1)$ models. C-D-F and H-N-S denote the models for the European and Asian countries respectively. *For the Nikkei 225 and Straits Times indices AR(1) models are estimated. The numbers in parentheses are t-statistics.

are the corresponding covariances. The above Wald statistics test whether the fractional differencing (power term) parameters of the two countries are equal $d_1 = d_2$ $(\delta_1 = \delta_2)$, and are distributed as $\chi^2_{(1)}$.

The following table presents the results of this pairwise testing procedure for the various bivariate models.[21] Several findings emerge from this table. The estimated long-memory parameters for the various (a)symmetric specifications are in the range $0.20(0.13) \leq d \leq 0.48(0.36)$ while the estimated power terms are in the range $1.19(1.18) \leq \delta \leq 2.00(1.86)$. In all cases for the American and Asian indices (and in the majority of the cases for the European countries) the values of the two coefficients $(d_i,\ \delta_i)$ for the asymmetric models are lower than the corresponding values for the

[21]For reasons of comparability, all the various bivariate models for both indices are estimated in AR(1)-FI(A)PARCH(1,1) processes. That is, the parameter values for d and δ presented in table 3.7 are not necessarily the same as the ones in table 3.4.

symmetric formulations. The values of the Wald tests in table 3.7 support the null hypothesis that the two estimated fractional parameters and the two power term coefficients are not significantly different from one another.

Table 3.7 Tests for similarity of fractional and power terms (Bivariate Models)

	Symmetric Models							Asymmetric Models						
	SP-T	C-D	C-F	D-F	H-N	H-S	N-S	SP-T	C-D	C-F	D-F	H-N	H-S	N-S
							d							
d_1	0.24	0.30	0.24	0.48	0.38	0.26	0.34	0.22	0.19	0.26	0.36	0.36	0.16	0.32
d_2	0.27	0.45	0.29	0.41	0.36	0.20	0.24	0.23	0.29	0.33	0.23	0.35	0.13	0.22
W	0.25	4.16	0.26	0.75	0.04	0.85	1.46	0.04	1.24	0.05	6.00	0.02	1.61	1.62
							δ							
δ_1	2.00	1.55	1.76	1.35	1.50	1.49	1.80	1.86	1.59	1.74	1.27	1.49	1.42	1.66
δ_2	1.68	1.19	1.55	1.40	1.79	1.68	1.68	1.51	1.18	1.51	1.39	1.70	1.47	1.58
W	2.38	3.85	1.08	0.10	4.43	1.59	0.60	2.59	6.57	1.55	0.24	1.66	0.10	0.19

Notes: SP-T denotes the bivariate model for the S&P 500 and TSE 300 indices respectively. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stand for the three bivariate models for the Asian indices. The W rows report the corresponding Wald statistics. The 5% and 1% critical values are 3.84 and 6.63 respectively.

All specifications generated very similar long-memory coefficients between countries. For example, in the asymmetric SP-T and H-N models, which generated very similar fractional parameters $(0.22, 0.23$ and $0.36, 0.35$ respectively), the two coefficients were, as expected, not significantly different ($W = 0.04, 0.02$ respectively). The null hypothesis of equal long-memory coefficients is rejected at the 5% level only for the symmetric C-D and the asymmetric D-F models. Both include the DAX 30 index with a relatively high persistence parameter. As regards the power term, the two models for CAC 40 and DAX 30 indices are those with the highest differences: $1.59 - 1.18 = 0.41$ and $1.55 - 1.19 = 0.36$ respectively. For these two cases the values of the Wald tests ($W = 6.57$, 3.85 respectively) are significant at the 5% level. For all other models, but one, the equality of the power terms cannot be rejected. For example, in models which generated very similar power terms, such as the symmetric D-F one $(1.35, 1.40)$ or the asymmetric H-S $(1.42, 1.47)$ the two coefficients were, as expected, not significantly different ($W = 0.10$ in both cases).

3.4. Forecasting Methodology

3.4.1. Evaluation Criteria

Financial market volatility is one of the most important attributes that affect the day-to-day operation of the Finance industry. It is a key driver in investment analysis and risk management. More recently, there is an increasing interest in trading on volatility itself as evidence by the volatility option contracts launched by the CBOE (Chicago Board of Option Exchange) in March 2006 (Hyung, Poon and Granger, 2006).

As Poon and Granger (2003) point out volatility forecasting is an important task in financial markets, and it has held the attention of academics and practitioners over the last two decades.[22] Elliot and Timmermann (2008) review various issues concerning economic forecasts. Since the publication of Ding et. al. (1993) there has been a lot of research investigating if the fractional integrated models could help to make better volatility forecasts. Hyung et al. (2006) compare the out-of-sample forecasting performance of various short and long-memory volatility models. They find that for volatility forecasts of 10 days and beyond, the FIGARCH specification is the dominant one. This section examines the ability of the various univariate/multivariate fractionally integrated and power asymmetric ARCH models to forecast stock return volatility.[23]

The full sample consists of 4,255 trading days and each model is estimated over the first 4,055 observations of the full sample, i.e. over the period 1st January 1988 to 16th July 2003. As a result the out-of-sample period is from 17th July 2003 to 22nd

[22]Several empirical studies examine the forecast performance of various GARCH models. The survey by Poon and Granger (2003) provides, among other things, an interesting and extensive synopsis of them.

[23]For the literature in the forecasting performance of univariate fractionally integrated and power ARCH models see, among others, Degiannakis (2004), Hansen and Lunde (2006) and Ñíguez (2007). In addition, Angelidis and Degiannakis (2005) examine whether a simple GARCH specification or a complex FIAPARCH model generates the most accurate forecasts in three areas: option pricing, risk management and volatility forecasting.

April 2004 providing 200 daily observations. The parameter estimates obtained with the data from the in-sample period are inserted in the relevant forecasting formulas and volatility forecasts $\widehat{h}_{t+1}$ calculated given the information available at time $t = T(= 4,055), \ldots, T + 199(= 4,254)$, i.e. 200 one-step ahead forecasts are calculated.

In order to evaluate the forecast performance of the different model specifications, one needs (a) to obtain a valid proxy for the true but unobservable underlying volatility and (b) to specify certain loss functions.[24] A natural candidate for the proxy are the squared returns which are an unbiased estimator for the unobserved conditional variance. However, compared to realized volatility the squared returns are a noise proxy and as shown in Patton (2007) distortions in the rankings of competing forecasts can arise when using noisy proxies. Whether such distortions arise depends on the choice of the loss function. Patton (2007) provides necessary and sufficient conditions on the functional form of the loss function to ensure that the ranking is the same whether it is based on the true conditional variance or some conditionally unbiased volatility proxy. Two loss functions which satisfy these condition are the mean square error (MSE) statistic and the QLIKE statistic.[25] Consequently, the MSE is employed which is, of course, one of the most commonly employed criteria in the existing literature (see, e.g., Andersen et al., 1999). In addition, the QLIKE statistic is employed, which corresponds to the loss implied by a Gaussian likelihood, is extensively discussed in Bollerslev et al. (1994) and

[24]As Andersen et al. (1999) point out, it is generally impossible to specify a forecast evaluation criterion that is universally acceptable (see also, e.g., Diebold et al., 1998). This problem is particularly acute in the context of nonlinear volatility forecasting. Accordingly, there is a wide range of evaluation criteria used in the literature. Following Andersen et al. (1999) this study will not use any of the complex economically motivated criteria but instead will report summary statistics based directly on the deviation between forecasts and realizations. Three out-of-sample forecast performance measures will be used to evaluate and compare the various models.

[25]Similarly, Awartani and Corradi (2005) point out that in comparing the relative predictive accuracy of various models, if the loss function is quadratic, the use of squared returns ensures the correct ranking of models actually obtained.

applied in, e.g., Hansen and Lunde (2005). Finally, in addition to those robust loss functions, an error statistic applied by Peters (2001) is used. This is the adjusted mean absolute percentage error (AMAPE) (see table 3.8 below). In contrast to the simple mean absolute percentage error the AMAPE corrects for the problem of asymmetry between the actual and forecast values.

Table 3.8 Forecast evaluation criteria

MSE:	$k^{-1} \sum\limits_{t=T+1}^{T+k} (\widehat{h}_t - s_t^2)^2$
QLIKE:	$k^{-1} \sum\limits_{t=T+1}^{T+k} [\ln(\widehat{h}_t) + s_t^2/\widehat{h}_t]$
AMAPE:	$k^{-1} \sum_{t=T+1}^{T+k} \left\vert (\widehat{h}_t - s_t^2)/(\widehat{h}_t + s_t^2) \right\vert$

Notes: k is the number of steps ahead, T is the sample size, $\widehat{h}_t$ is the forecasted variance and s_t^2 are the squared returns.

On the basis of several model selection techniques the superior fitting specification was the FIAPARCH one (see section 3.3.3). While such model fitting investigations provide useful insights into volatility, the specifications are usually selected on the basis of full sample information. For practical forecasting purposes, the predictive ability of these models needs to be examined out-of-sample. The aim of this section is to examine the relative ability of the various long-memory and power formulations to forecast daily stock return volatility. For each index the three forecast error statistics are calculated for the specifications of APARCH, IAPARCH, FIAPARCH($\delta = 1$), FIAPARCH($\delta = 2$) and FIAPARCH in the univariate, bivariate and (where possible) trivariate version. Hence, overall fifteen values of each forecast error statistic are available for each index. Instead of presenting all the figures, table 3.9 present only the best and the worst specification for each index as identified by the forecast error statistic. In addition, whether the values of the forecast error statistics from the best and the worst model are statistically significant are tested using the Diebold and Mariano (1995) test. Table 3.9 contains the corresponding p-values.

Table 3.9 Best versus worst ranked models

	MSE	QLIKE	AMAPE
S&P 500	B-FIAP vs. U-FIAP	B-IAP vs. U-FIAP	B-AP vs. U-FIAP
	[0.00]	[0.03]	[0.02]
TSE 300	B-FIAP vs. U-IAP	U-FIP vs. U-IAP	B-AP vs. U-IAP
	[0.14]	[0.00]	[0.00]
CAC 40	T-P vs. B_F-FIA(δ=2)	T-IP vs. B_F-FIA(δ=2)	T-IP vs. B_F-FIA(δ=2)
	[0.00]	[0.15]	[0.00]
DAX 30	B_F-AP vs. U-FIAP	U-FIA($\delta = 1$) vs. B_C-FIA($\delta = 2$)	B_F-AP vs. B_F-FIA(δ=2)
	[0.00]	[0.08]	[0.17]
FTSE 100	T-P vs. B_C-FIA(δ=2)	T-P vs. B_C-FIA(δ=2)	B_D-AP vs. B_C-FIA(δ=2)
	[0.00]	[0.01]	[0.00]
Hang Seng	B_S-FIA vs. U-AP	B_N-AP vs. T-FIAP	T-FIA(δ=2) vs. U-FIA(δ=2)
	[0.00]	[0.02]	[0.26]
Nikkei 225	B_S-FIA(δ=1) vs. U-FIAP	U-FI(δ=1) vs. T-AP	T-FIA(δ=2) vs. U-AP
	[0.12]	[0.03]	[0.67]
Straits Times	B_H-FIAP vs. B_N-IAP	B_H-FIA(δ=2) vs. U-AP	T-FIAP vs. U-AP
	[0.00]	[0.01]	[0.00]

Notes: U, B and T stand for univariate, bivariate and trivariate specifications respectively. (F)I, A and P indicate (fractionally) integrated, asymmetric and power models respectively. The subscripts refer to the jointly estimated index of the bivariate model, e.g., the subscript F indicates that the bivariate model is estimated with the FTSE 100 index. The numbers in brackets are the p-values from the Diebold and Mariano (1995) test.

An examination of table 3.9 reveals that either a multivariate or a fractionally integrated (FI) or a power (P) or an asymmetric (A) process is clearly superior. That is, there is strong evidence that the restrictive univariate (U), stable, symmetric Bollerlsev's type of process is inferior to one of the more flexible specifications. The results can be summarized as follows. Only in three cases is the best ranked model, as assessed by the forecasting criteria, the univariate one. Both MSE and AMAPE loss functions uniformly favor either bivariate or trivariate specifications (see the second and fourth column of table 3.9). For the two American indices in five out of the six cases a bivariate model is selected as being best (see the first two rows of table 3.9). The results for the European countries show the close connection between the three volatilities. In five cases a trivariate specification is the best performing model and in three cases a bivariate one. Similarly, for the Asian indices in only one case do the statistics rank the univariate formulation first (see the last three rows of table 3.9). Overall, the multivariate formulation has the best statistics for twenty one out of the twenty four cases.

Moreover, in the Asian countries the (fractionally) integrated model is favored in all but one case. Similarly, for the S&P 500 and the TSE 300 indices the statistics indicate the superiority of the fractionally integrated specification. The power formulation is the dominant one in the European and American countries. In particular, for the European indices the restriction that $\delta = 2$ characterizes with one exception the worst performing specification. In summary, the best formulations as ranked by the forecast error statistics are multivariate models. For the American and Asian indices the long-memory property appears to be important for the forecast performance, while for the European and American indices power specifications are dominant.

3.4.2. Tests of Equal Forecast Accuracy

In the previous section in some cases the statistics do not allow for a clear distinction between the ranking models, which is evidenced by the marginal difference in relative accuracy which separates the three models (results not reported).[26] Thus next moves to the pairwise comparison of the best and the worst specifications.

This section utilizes the tests proposed by Diebold and Mariano (1995) and Harvey et al. (1997). Before moving to the two tests some notation is needed. First, let $L_{bt}^{(i)}(s_t^2, \widehat{h}_{bt})$ and $L_{wt}^{(i)}(s_t^2, \widehat{h}_{wt})$ $(t = T+1, \ldots, T+k)$ denote the 1-step ahead loss functions for the best and worst models , where $i \in \{\text{MSE, QLIKE, AMAPE}\}$, respectively. Forecasts of the squared returns are generated using the fixed forecasting

[26]In addition, in some cases the ranking of the models varies depending upon the choice of the error statistic. Hence, as Brailsford and Faff (1996) point out, caution should be exercised in the interpretation of the obtained rankings.

scheme (described in West and McCracken, 1998, p. 819). Next, let $\Delta_t = L_{bt}^{(i)} - L_{wt}^{(i)}$ and $\overline{\Delta}$ denote its sample mean, i.e. $\overline{\Delta} = k^{-1} \sum_{t=T+1}^{T+k} \Delta_t$. The test proposed by Diebold and Mariano (1995) is formed as

$$S = [\widehat{\mathsf{Var}}(\overline{\Delta})]^{-1/2}\overline{\Delta},$$

with

$$\widehat{\mathsf{Var}}(\overline{\Delta}) = \frac{2\pi \widehat{f}_{\Delta}(0)}{k},$$

where $\widehat{f}_{\Delta}(0)$ is a consistent estimate of the spectral density function of Δ at frequency zero. Under the null hypothesis S has an asymptotic standard normal distribution.[27]

As seen in table 3.9 the evidence obtained from the loss functions is reinforced by the Diebold-Mariano test. Clearly the test discriminates between the best and the worst model. That is, in the majority of the cases (eighteen out of twenty four) the test indicates the superiority of the best formulation over the worst one. In particular, for the USA and Canada, in four out of the five cases the worst model (univariate) is rejected in favor of the best (multivariate) one. For the Asian indices, the Diebold-Mariano test indicates the superiority of the best (fractionally integrated) specification over the worst (stable) one in four out of the five cases. The long-memory characteristic has important implications for volatility forecasting and option pricing. Option pricing in a stochastic volatility setting requires a risk premium for the unhedgeable volatility risk. The fractionally integrated series lead to volatility forecasts larger than those from short-memory models which immediately translates into higher option prices. This could be an explanation for the better pricing performance of FIGARCH in this case (Hyung et al., 2006).

[27]Harvey et al. (1997) proposed a small sample correction for the Diebold and Mariano (1995) statistic. Their modified test statistic is t-distributed with $k-1$ degrees of freedom. The results from this statistic are qualitatively similar to the original Diebold and Mariano (1995) statistic and, hence, are not reported.

Further, for the European countries, in five out of the seven cases the power (best) formulation outperforms the Bollerslev (worst) one. Finally, it is noteworthy that in the majority of the cases both the best and the worst formulation is an asymmetric one.[28]

3.5. Discussion and Conclusion

3.5.1. The Empirical Evidence

Brooks et al. (2000) analyzed the applicability of the stable APARCH model to national stock market returns for various industrialized countries. However, as in all cases the estimated values of the persistence coefficients were quite close to one, there was a need to examine closely the possibility of long-memory persistence in the conditional volatility.

In this study, strong evidence has been put forward suggesting that the conditional volatility for eight national stock indices is best modeled as a FIAPARCH process. On the basis of Wald tests and information criteria the fractionally integrated model provides statistically significant improvement over its integrated counterpart. One can also reject the more restrictive stable process, and consequently all the existing specifications (see Ding et al. 1993) nested by it in favor of the fractionally integrated parameterization. Hence, the analysis has shown that the FIAPARCH formulation is preferred to both the stable and the integrated ones. In other words, the fractionally integrated process appeared to have superior ability to differentiate between stable specifications and their integrated alternatives.

The Bollerslev formulation is nested within the power specification. Brooks et al. (2000) applied the LR test to this nested pair. The results of this test

[28]Two encompassing tests proposed by Ericsson (1992) and Harvey et al. (1998) are also utilized, of which results are not reported. For example, for the FTSE 100 index, in the univariate and bivariate F-C models, the FIAPARCH formulation outperforms the restricted Taylor/Schwert and Bollerlsev specifications, and the stable/integrated ones as well.

were mixed as far as supporting the presence of power effects is concerned. For the German and French indices there was strong evidence of power effects. For a further two countries (US and Japan) there was mild evidence and for Hong Kong there was only weak evidence in support of the power specification. In contrast, United Kingdom, Canada and Singapore show no evidence of power effects as the Bollerslev formulation could not be rejected in favor of the power one.

Moreover, the Taylor/Schwert specification is nested within the power model. For all countries tested, with the exceptions of Hong Kong and Singapore, the test statistics indicated a preference for the Taylor/Schwert formulation over the power specification. Accordingly, Brooks et al. (2000) concluded that allowing the power term to take on values other than unity did not significantly enhance the model. In other words there was a lack of evidence to suggest the need for power effects in the absence of long-range volatility dependence, as the LR tests produced insignificant calculated values, indicating an inability to reject the Taylor/Schwert formulation over the power specification for eight of the national indices tested.

The results for the more general FIAPARCH model are in stark contrast. According to the analysis all eight countries show strong evidence (both the LR and Wald tests produce significant calculated values) of power effects when long-memory persistence in the conditional volatility has been taken into account, as both the Bollerslev and Taylor/Schwert specifications were rejected in favor of the power formulation. Further, by comparing the pairwise testing results of the log-likelihood procedures to the relative model rankings provided by the four alternative criteria, this study observed the findings were generally robust. That is, where the log-likelihood results provided unanimous support for the FIAPARCH specification over either the Bollerslev or Taylor/Schwert (asymmetric) FIGARCH formulations, the model selection criteria concurred without exception. Thus, the inclusion of a power term and a fractional unit root in the conditional variance equation appear to augment the model in a worthwhile fashion.

Finally, this study emphasizes that the above results were robust to the dimension of the process. That is, the evidence obtained from the univariate models on the superiority of the FIAPARCH specification was reinforced by the multivariate processes. It is noteworthy that the results are not qualitatively altered by changes in the dimension of the model.

3.5.2. Possible Extensions

The main goal of this study was to explore the issue of how generally applicable the ccc M-FIAPARCH formulation is to a wide range of national stock market returns. Possible extensions of this study can go in different directions. Kim et al. (2005) use a bivariate ccc FIAPARCH-in-mean process to model the volume-volatility relationship. In the context of the analysis in this study, incorporating volumes either in the mean or in the variance specification or in both could be at work. Future work may clarify this out. He and Teräsvirta (1999) emphasize that if the standard Bollerslev type of model is augmented by the power term, the estimates of the other variance coefficients almost certainly change. More importantly, Karanasos and Schurer (2008) find that the relationship between the level of the process and its conditional variance, as captured by the in-mean parameter, is sensitive to changes in the values of the power term (see also Conrad and Karanasos, 2008b). Therefore, one promising avenue would be to adapt the multivariate model in a way that incorporates in-mean effects.

Moreover, Conrad and Karanasos (2008a) consider a formulation of the extended constant or time varying conditional correlation M-GARCH specification which allows for volatility feedback of either sign, i.e., positive or negative. Future research will be able to deal with the unrestricted extended (and/or time varying conditional correlation) version of the M-FIAPARCH model. Also an emphasis should be on that the most commonly used measures of stock volatility apart from the conditional variance from an ARCH type of process is the realized volatility (see Andersen et al., 2003, and Conrad and Lamla, 2007) and the range-based intraday estimator (see Karanasos and Kartsaklas, 2008). In addition, Bai and Chen (2008) consider testing distributional assumptions in M-GARCH formulations based on empirical processes. To highlight the importance of using alternative measures of volatility and multivariate distributions in order to model the national stock market returns (and forecast their variances) more study should have to go into greater detail.

In addition, one can estimate multivariate versions of the Hyperbolic APARCH and Hyberbolic FIAPARCH models (see, Schoffer, 2003 and Conrad, 2007 and the references therein). Further, Baillie and Morana (2007) introduce a new long-memory volatility specification, denoted by Adaptive FIGARCH, which is designed to account for both long-memory and structural change in the conditional variance process. One could provide an enrichment of the M-FIAPARCH by allowing the intercepts of the two means and variances to follow a slowly varying function as in Baillie and Morana (2007). This is undoubtedly a challenging yet worthwhile task. Finally, Pesaran and Timmermann (2002) suggest an estimation strategy that takes into account breaks and provides gains in forecasting ability. Pesaran et al. (2006) provide a new approach to forecasting time series that are subject to discrete structural breaks. Their results suggest several avenues for further research.

Concluding Remarks

The purpose of this thesis was to examine the usefulness of econometric models with stochastic volatility and long memory in the application of macroeconomic and financial time series, and therefore provide contemporary evidence to test some economic and financial theories/theoretical models. First, the investigation of the long-term persistence of ex-ante and ex-post US real interest rates has employed an ARFIMA-FIAPARCH process and recently developed econometric techniques greatly improves the power of these tests. Estimation results show that the US real rate displays near integrated behavior, precisely the type of stationary behavior that will be difficult for standard tests to detect for samples as short as the post war era which are typically used in the extant literature. This study provided an empirical measure of its uncertainty that accounts for long memory in the second conditional moment of the real interest rate process. Analogous to the issues pertaining to the proper modeling of the long-run dynamics in the conditional mean of the real US rate, similar questions, therefore, become relevant in the modeling of its conditional volatility. Moreover, as the CCAPM model implies that the growth rate of consumption and the real interest rate should have similar time-series characteristics, the US data exhibiting significant differences in the degree of persistence nevertheless indicate this condition factually invalid. Thus, although in finding no unit root, the results might have been seen as resolving the puzzling irregularity concerning the behavior of interest rates implied by the CCAPM, the observed persistence of interest rates might be seen as being inconsistent with the simplification that gives rise to a consumption based asset pricing model. It must be noted that a discount rate rather than an observed interest rate is used in the solution to the CCAPM model, which provides a get out to the pure theorist, but provides further support to the observation that CCAPM has not worked well in practice (see the recent discussion in Gregoriou, Hunter and Wu, 2009).

There has also been some discussion of persistence in the literature on New Keynesian Phillips Curves (Harvey, 2008). Harvey observes that unit roots in inflation would seem not to be consistent with the underlying theory and the evidence of persistence found in this thesis would not go against such an argument. This is further complicated by the finding here that interest rates are persistent and the observation of both these types of persistence might suggest that such Philips Curves have no forward looking interpretation.

This thesis has also investigated the link between inflation, growth and their respective uncertainties in a bivariate GARCH type model, which has considered several changes in the specification of the bivariate model, discussed how these changes would affect the twelve interlinkages among the four variables and provided robust results including: (i) growth tends to increase inflation, whereas inflation is detrimental to growth which are in line with the Briault conjecture and the Gillman-Kejak theory respectively (ii) inflation, under linearity, has a positive impact on macroeconomic uncertainty thus supporting the Ungar-Zilberfarb theory and the Dotsey-Sarte conjecture, and (iii) nominal variability, when allowing for both cross effects, affects real volatility positively as argued by Logue and Sweeney (1981). In addition, one significant importance is that in all specifications inflation is independent of changes in its variance, and real uncertainty does not affect inflation variability and is unaffected by the first lag of growth. The significance and even the sign of the in-mean effects vary with the choice of the lag, suggesting that the behavior of macroeconomic performance depends upon its uncertainty, but also that the nature of this dependence varies with time. The attendant danger is that one might see technical sophistication as an end in itself, and lose sight of the reasons for interest in the various relationships. Be that as it may, one of the contributions of this work was to clarify the kinds of mechanisms that may be at play. Some of the conclusions this study has reached are fairly speculative. In these circumstances, this study focuses on explaining the general principles rather than the detailing them, which may have to be amended as more evidence becomes available. However, the ideas about the mechanism linking performance to uncertainty at least offer plenty of opportunities for further research. It seems likely that many more of these kind of relationships between the four variables will be uncovered in the future.

Finally, the multi-country study of stock market volatility was to consider the applicability of the multivariate fractionally integrated asymmetric power ARCH model, and to evaluate the different specifications in terms of their out-of-sample forecast ability, to the national stock market returns for eight countries. This study has found that the M-FIAPARCH formulation (both bivariate and trivariate) captures the temporal pattern of volatility for observable returns better than previous parameterizations. It also improves forecasts for volatility and thus is useful for financial decisions which utilize such forecasts. It has provided an interesting comparison to the stable and integrated specifications. The results reject both the stable and integrated null hypotheses, which is consistent with the conditional volatility profiles in Gallant et al. (1993), suggesting that shocks to the variance are very slowly damped, but do die out. In particular, the trivariate FIAPARCH results show that the conditional correlation between the European indices is considerably stronger than between the Asian indices. Moreover, all eight countries show strong evidence of power effects when asymmetries and/or long-memory persistence in the conditional volatility have been taken into account, as both the Bollerslev and Taylor/Schwert formulations were rejected in favor of the power specification. As convincingly argued by Brooks et al. (2000), for high frequency data which has a non-normal error distribution the presumption of an obvious superiority of a squared power term is lost. Other power transformations are more appropriate.

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APPENDIX A

Tables for Chapter 1

Table A.1 Dual long-memory level model

Mean equation

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{\pi\pi} & \phi_{\pi y} \\ \phi_{y\pi} & \phi_{yy} \end{bmatrix} = \begin{bmatrix} \underset{\substack{\{0.04\} \\ l_{\pi\pi}=12}}{0.11^{***}} & \underset{\substack{\{0.02\} \\ l_{\pi y}=3}}{0.04^{**}} \\ \underset{\substack{\{0.04\} \\ l_{y\pi}=12}}{-0.08^{*}} & \underset{\substack{\{0.05\} \\ l_{yy}=3}}{0.15^{***}} \end{bmatrix}, \ \mathbf{d}_m = \begin{bmatrix} d_{m\pi} \\ d_{my} \end{bmatrix} = \begin{bmatrix} \underset{\{0.04\}}{0.23^{***}} \\ \underset{\{0.05\}}{0.15^{***}} \end{bmatrix}$$

Variance equation

$$\Gamma = \begin{bmatrix} \gamma_{\pi\pi} & \gamma_{\pi y} \\ \gamma_{y\pi} & \gamma_{yy} \end{bmatrix} = \begin{bmatrix} \underset{\substack{\{0.31\} \\ k_{\pi\pi}=5}}{-0.48^{\circ}} & \underset{\substack{\{0.09\} \\ k_{\pi y}=5}}{-0.14^{\circ}} \\ \underset{\substack{\{0.29\} \\ k_{y\pi}=3}}{0.61^{***}} & \underset{\substack{\{0.54\} \\ k_{yy}=3}}{-1.50^{**}} \end{bmatrix}, \ \mathbf{d}_v = \begin{bmatrix} d_{v\pi} \\ d_{vy} \end{bmatrix} = \begin{bmatrix} \underset{\{0.07\}}{0.34^{***}} \\ \underset{\{0.08\}}{0.33^{***}} \end{bmatrix}$$

Diagnostics

$$\mathbf{Q}(12) = \begin{bmatrix} Q & Q^2 \\ Q & Q^2 \end{bmatrix} \begin{bmatrix} \underset{[0.20]}{15.76} & \underset{[0.60]}{10.22} \\ \underset{[0.41]}{12.45} & \underset{[0.71]}{8.91} \end{bmatrix}, \ \begin{bmatrix} ML \\ AIC \end{bmatrix} = \begin{bmatrix} -3,292.95 \\ -3,309.95 \end{bmatrix}$$

Notes: Table A.1 reports estimates of the parameters of interest.
***, **, *and$^{\circ}$denote significance at the 0.01, 0.05, 0.10 and 0.15 levels respectively. The numbers in {.} are robust standard errors. Q(12) and $Q^2(12)$ are the Ljung-Box statistics for 12th-order serial correlation in the standardized residuals and their squares respectively. p values are reported in [.].

Table A.2 Dual long-memory in-mean model

Mean equation

$$\mathbf{\Phi} = \begin{bmatrix} \underset{\substack{\{0.04\} \\ l_{\pi\pi}=12}}{0.11^{***}} & \underset{\substack{\{0.02\} \\ l_{\pi y}=3}}{0.03^{*}} \\ \underset{\substack{\{0.04\} \\ l_{y\pi}=12}}{-0.07^{\circ}} & \underset{\substack{\{0.05\} \\ l_{yy}=3}}{0.10^{***}} \end{bmatrix}, \ \Delta L^3 = \begin{bmatrix} \underset{\{0.16\}}{-0.18} & \underset{\{0.08\}}{-0.06} \\ \underset{\{0.25\}}{-0.43^{*}} & \underset{\{0.38\}}{0.55^{\circ}} \end{bmatrix} L^3, \ \mathbf{d}_m = \begin{bmatrix} \underset{\{0.03\}}{0.23^{***}} \\ \underset{\{0.05\}}{0.12^{**}} \end{bmatrix}$$

Variance equation

$$\mathbf{d}_v' = \begin{bmatrix} \underset{(0.09)}{0.37^{***}} & \underset{(0.07)}{0.30^{***}} \end{bmatrix}'$$

Diagnostics

$$Q(12) = \begin{bmatrix} \underset{[0.18]}{16.17} & \underset{[0.71]}{8.93} \\ \underset{[0.19]}{16.06} & \underset{[0.87]}{6.74} \end{bmatrix}, \ \begin{bmatrix} ML \\ AIC \end{bmatrix} = \begin{bmatrix} -3,297.63 \\ -3,314.63 \end{bmatrix}$$

Notes. As in table A.1.

APPENDIX B

Figures for Chapter 1

Figure B.1 US real interest rates

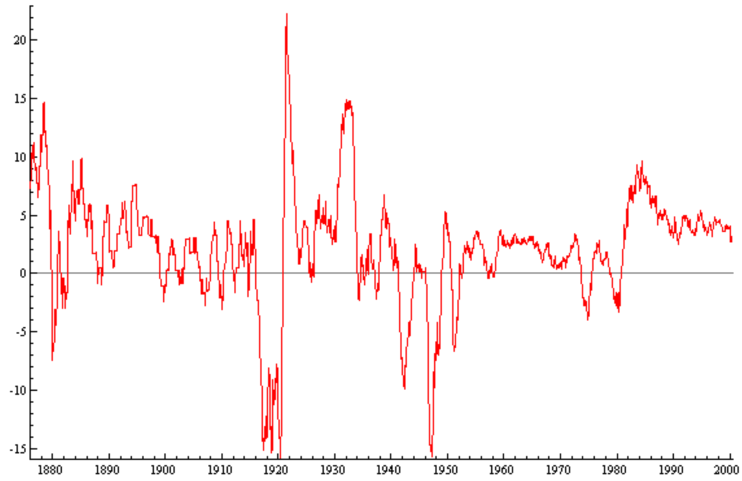


Figure B.2 UK inflation rates and output growth

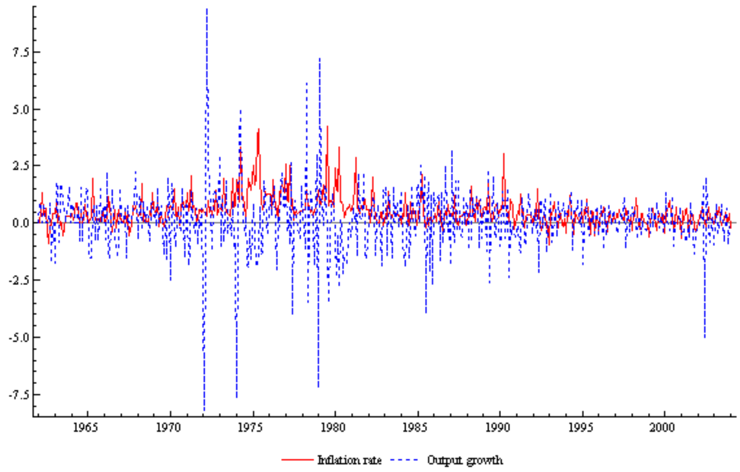
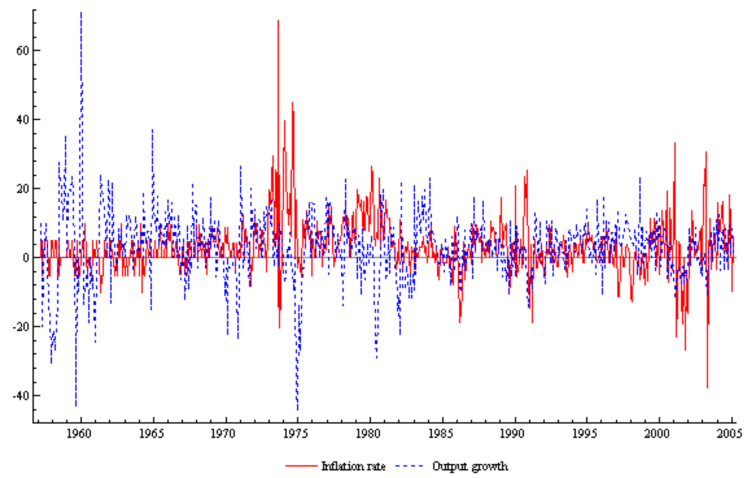


Figure B.3 US inflation rates and output growth



APPENDIX C

Tables for Chapter 2

Table C.1 Inflation-Growth Link

Models	$S(\Phi_f, B_f \mid B_w \mid B_d)$*			$M_{n=0}(\Phi_f, B_w \mid B_d)$		$L(\Phi_f, B_w \mid B_d, \Gamma_f)$		$ML_{n=0}(\Phi_d, B_l \mid B_d, \Gamma_f)$	
The effect of growth on inflation									
$\phi^5_{\pi y}$	**0.04** [0.01]	**0.04** [0.02]	**0.05*** [0.02]	**0.04** [0.02]	**0.04** [0.02]	**0.04** [0.01]	**0.04** [0.01]	**0.04** [0.01]	**0.04** [0.01]
$\phi^7_{\pi y}$	**0.03** [0.02]	**0.03** [0.05]	**0.03** [0.05]	**0.03** [0.07]	**0.03** [0.06]	**0.04** [0.01]	**0.04** [0.01]	**0.03** [0.01]	**0.03** [0.01]
The impact of inflation on growth									
$\phi^7_{y\pi}$	**-0.19** [0.01]	**-0.19** [0.02]	**-0.19** [0.02]	**-0.18** [0.15]	**-0.16** [0.13]	**-0.24** [0.00]	**-0.24** [0.00]	**-0.20** [0.02]	**-0.20** [0.02]
$\phi^{11}_{y\pi}$	0.08 [0.40]	**0.12** [0.15]	**0.12** [0.14]	**0.15** [0.10]	**0.15** [0.08]	0.10 [0.20]	0.10 [0.19]	**0.13** [0.10]	**0.13** [0.09]

*The three numbers refer to the models with the B_f, B_w and B_d matrices respectively. The bold numbers indicate significant effects. The numbers in brackets are p-values.

Table C.2 Variance relationship

Models	$\beta_{\pi y}$			$\beta_{y\pi}$		
$S(\Phi_f \mid \Phi_u \mid \Phi_d, B_f)$*	0.01 [0.26]	0.00 [0.85]	0.01* [0.35]	**2.96** [0.00]	**2.96** [0.00]	**2.95** [0.00]

*The three numbers refer to the models with the Φ_f, Φ_u and Φ_d matrices respectively. The bold numbers indicate significant effects. The numbers in [·] are p-values. For the L, M and ML models the estimation routine did not converge when the B_f matrix was used.

Table C.3 In-mean effects

$\delta_{y\pi}$	$\delta_{\pi y}$	δ_{yy}
	$\underset{\text{n=0;1;3;4}}{\text{M}}(\Phi_f, B_w)^*$	
−0.31; −0.05; 0.56; 0.17 [0.59] [0.90] [0.18] [0.70]	**0.02**; 0.01; −0.01; **0.02** [0.07] [0.51] [0.35] [0.08]	−0.07; 0.04; **−0.03**; −0.02 [0.47] [0.17] [0.12] [0.59]
	$\underset{\text{n=0;1;3;4}}{\text{M}}(\Phi_f, B_d)$	
−0.51; −0.05; **0.48**; 0.20 [0.41] [0.90] [0.14] [0.66]	**0.02**; 0.01; −0.03; **0.02** [0.08] [0.52] [0.42] [0.08]	−0.04; 0.04; 0.03; −0.02 [0.61] [0.16] [0.59] [0.53]
	$\underset{\text{n=0;1;3;4}}{\text{ML}}(\Phi_f, B_w, \Gamma_f)$	
−0.37; −0.21; 0.53; 0.15 [0.53] [0.62] [0.33] [0.78]	**0.02**; 0.00; −0.01; **0.02** [0.09] [0.94] [0.44] [0.12]	−0.05; **0.04**; **−0.04**; −0.02 [0.49] [0.09] [0.15] [0.44]
	$\underset{\text{n=0;1;4}}{\text{ML}}(\Phi_f, B_w, \Gamma_d)$	
−0.40; −0.07; 0.25 [0.37] [0.87] [0.60]	**0.02**; 0.00; **0.01** [0.04] [0.76] [0.15]	−0.08; **0.04**; −0.02 [0.21] [0.11] [0.52]
	$\underset{\text{n=0}}{\text{M}}(\Phi_d, B_w \mid B_d)^{\circ}$	
−0.62 [0.20] | −0.67 [0.18]	**0.02** [0.08] | **0.02** [0.08]	−0.01 [0.91] | 0.00 [0.99]
	$\underset{\text{n=0}}{\text{ML}}(\Phi_u \mid \Phi_d, B_d, \Gamma_d)$	
−0.77 [0.10] | **−0.78** [0.13]	**0.02** [0.08] | **0.02** [0.06]	−0.03 [0.66] | −0.02 [0.75]

*The four numbers refer to the models with the n=0,1,3,4 respectively.

The bold numbers indicate significant effects. The numbers in [·] are p-values.

For the $\underset{\text{n=3}}{\text{ML}}(\Phi_f, \underset{k=d,l}{B_k}, \Gamma_d)$ model the estimation routine did not converge.

°The two numbers refer to the models with the B_w and B_d matrices respectively.

For the M(L) models the estimation routine did not converge when the B_f matrix was used.

The $\delta_{\pi\pi}$ coefficients (not reported) are insignificant in all models.

Table C.4 Estimated coefficients for Friedman hypothesis

$\delta_{y\pi}$					
Φ_d, $n=0$			Φ_u, $n=0$		
M (B_d)	ML (B_d,Γ_d)	M (B_w)	M (B_d)	ML (B_d,Γ_d)	M (B_w)
−0.67 [0.18]	**−0.78** [0.13]	−0.62 [0.20]	−0.68 [0.19]	**−0.77** [0.10]	−0.59 [0.20]

p values are reported in [·]. For the $\underset{n=0}{\text{ML}}(\underset{\varsigma=d,u}{\Phi_\varsigma}, B_d, \Gamma_f)$ and $\underset{n=0}{\text{ML}}(\underset{\varsigma=d,u}{\Phi_\varsigma}, B_w, \underset{\zeta=d,f}{\Gamma_\zeta})$ the estimation routine did not converge.

Table C.5 Estimated coefficients for Cukierman-Gerlach theory

$\delta_{\pi y}$					
Φ_f, n=0					
M (B_d)	ML (B_d,Γ_d)	ML (B_d,Γ_f)	M (B_w)	ML (B_w,Γ_d)	ML (B_w,Γ_f)
0.02 [0.08]	**0.02** [0.08]	**0.02** [0.09]	**0.02** [0.07]	**0.02** [0.04]	**0.02** [0.09]
Φ_f, n=4					
M (B_d)	ML (B_d,Γ_d)	ML (B_d,Γ_f)	M (B_w)	ML (B_w,Γ_d)	ML (B_w,Γ_f)
0.02 [0.08]	**0.01** [0.15]	**0.02** [0.12]	**0.02** [0.08]	**0.01** [0.15]	**0.02** [0.12]

p values are reported in [.]. The estimation routine did not converge when the B_f matrix was used.

Table C.6 Estimated coefficients for 'Blackburn'/Pindyck theories

δ_{yy}									
'Blackburn' theory; Φ_f, $n = 1$						Pindyck theory; Φ_f, $n = 3$			
M (B_d)	ML (B_d,Γ_d)	ML (B_d,Γ_f)	M (B_w)	ML (B_w,Γ_d)	ML (B_w,Γ_f)	M (B_d)	ML (B_d,Γ_f)	M (B_w)	ML (B_w,Γ_f)
0.04 [0.16]	**0.04** [0.10]	**0.04** [0.09]	**0.04** [0.14]	**0.04** [0.11]	**0.04** [0.09]	0.03 [0.58]	**−0.04** [0.14]	**−0.03** [0.12]	**−0.04** [0.15]

p values are reported in [·]. For the $\underset{n=3}{\mathrm{ML}}(\Phi_f, \underset{k=d,w}{B_k}, \Gamma_d)$ the estimation routine did not converge.

Table C.7 Level effects

(M)L Models	$\gamma_{\pi\pi}$		$\gamma_{y\pi}$		$\gamma_{\pi y}$		γ_{yy}	
$\mathrm{L}(\Phi_f, B_w \mid B_d, \Gamma_f)^*$	**0.07** [0.03]	**0.07** [0.03]	**0.54** [0.00]	**0.51** [0.00]	0.00 [0.88]	0.00 [0.88]	−0.11 [0.55]	−0.12 [0.53]
$\underset{n=0}{\mathrm{ML}}(\Phi_f, B_w \mid B_d, \Gamma_f)$	**0.07** [0.02]	**0.07** [0.02]	**0.53** [0.00]	**0.53** [0.00]	0.00 [0.84]	0.00 [0.83]	−0.10 [0.57]	−0.10 [0.57]
$\underset{n=1}{\mathrm{ML}}(\Phi_u, B_w \mid B_d, \Gamma_f)$	**0.08** [0.01]	**0.07** [0.01]	**0.56** [0.00]	**0.49** [0.00]	0.00 [0.81]	0.00 [0.80]	−0.12 [0.52]	−0.13 [0.49]
$\underset{n=1}{\mathrm{ML}}(\Phi_d, B_d, \Gamma_f)$	**0.08** [0.04]		**0.49** [0.00]		0.00 [0.95]		−0.13 [0.49]	
$\underset{n=1}{\mathrm{ML}}(\Phi_d, B_d, \Gamma_c)$	-		**0.48** [0.00]		**0.01** [0.03]		-	

*The two numbers refer to the models with the B_w and B_d matrices respectively. The bold numbers indicate significant effects. The numbers in [.] are p-values. For the $\mathrm{L}(\Phi_u, \underset{\kappa=d,w,f}{B_\kappa}, \Gamma_f)$ and $\underset{n=1}{\mathrm{ML}}(\Phi_d, B_w, \Gamma_f)$ models the estimation routine did not converge. For the (M)L models the estimation routine did not converge when the B_f matrix was used.

Table C.8 Variance relationship in the BEKK model

BEKK Models	$\beta_{\pi y}$				$\beta_{y\pi}$	
$S_B(\Phi_f \mid \Phi_u \mid \Phi_d \ , B_f)^*$	-0.04 [0.25]	0.03 [0.33]	-0.04 [0.25]	0.16 [0.55]	-0.22 [0.46]	0.25 [0.38]

The subscript B denotes the BEKK model. *The three numbers refer to the models with the Φ_f, Φ_u and Φ_d matrices respectively.
The numbers in [·] are p-values.

Table C.9 Information Criteria and Maximum Likelihood (MaxLik) values for the BEKK model

BEKK Models $S_B(\Phi_f, B_f \mid B_w \mid B_d)^*$			
Information Criteria			MaxLik
AIC	SIC	HQIC	
$\boxed{-568}$ | -596 | -569	-619 | -640 | $\boxed{-615}$	-588 | -613 | $\boxed{-587}$	-544 | | -547

*The three numbers refer to the models with the B_f, B_w and B_d matrices respectively.
The numbers in $\boxed{}$ indicate the optimal type model according to the information criteria.

Table C.10 . In-mean effects

$\delta_{\pi\pi}$	$\delta_{y\pi}$	$\delta_{\pi y}$	δ_{yy}
$\underset{n=1;3;4}{M}(\Phi_f, B_w)$			
-0.12 ; -0.03 ; -0.13* [0.47] [0.85] [0.49]	0.16 ; 0.72 ; 0.38 [0.69] [0.27] [0.48]	0.05 ; -0.04 ; 0.04 [0.19] [0.39] [0.36]	0.10 ; -0.13 ; -0.06 [0.37] [0.41] [0.64]
$\underset{n=0;1;3;4}{M}(\Phi_f, B_d)$			
-0.11 ; -0.12 ; -0.04 ; -0.13 [0.62] [0.48] [0.84] [0.50]	0.01 ; 0.17 ; 0.67 ; 0.43 [0.99] [0.67] [0.28] [0.43]	**0.11** ; 0.05 ; -0.04 ; 0.04 [0.04] [0.19] [0.30] [0.37]	-0.23 ; 0.11 ; -0.10 ; -0.08 [0.40] [0.34] [0.51] [0.52]
$\underset{n=1;3;4}{ML}(\Phi_f, B_w, \Gamma_f)$			
-0.26 ; 0.01 ; -0.09 [0.40] [0.95] [0.61]	-0.18 ; 0.55 ; 0.34 [0.68] [0.61] [0.66]	0.03 ; -0.03 ; 0.03 [0.46] [0.46] [0.46]	**0.16** ; -0.16 ; -0.07 [0.12] [0.47] [0.62]
$\underset{n=1;4}{ML}(\Phi_f, B_w, \Gamma_d)$			
-0.23 ; -0.06 [0.37] [0.73]	0.07 ; 0.43 [0.88] [0.44]	0.03 ; 0.02 [0.32] [0.56]	0.13 ; -0.06 [0.24] [0.62]

The bold numbers indicate significant effects. The numbers in [.] are p-values.
*For the $\underset{n=0}{M}(\Phi_f, B_w)$ $\underset{n=0}{ML}(\Phi_f, B_w, \Gamma_f)$ and $\underset{n=0,3}{ML}(\Phi_f, B_w, \Gamma_d)$ models the estimation routine did not converge. For the M(L) models the estimation routine did not converge when the B_f matrix was used.

APPENDIX D

Figures for Chapter 2

Figure D.1 In-mean effects

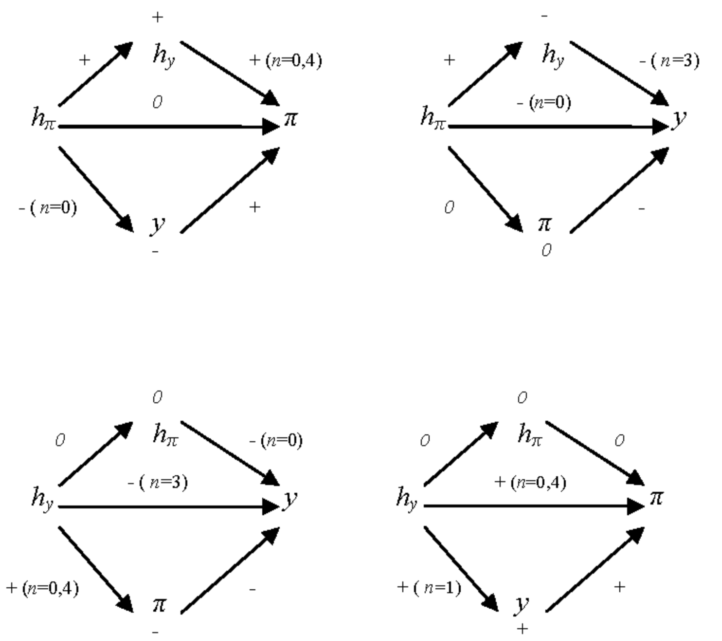


Figure D.2 Level effects

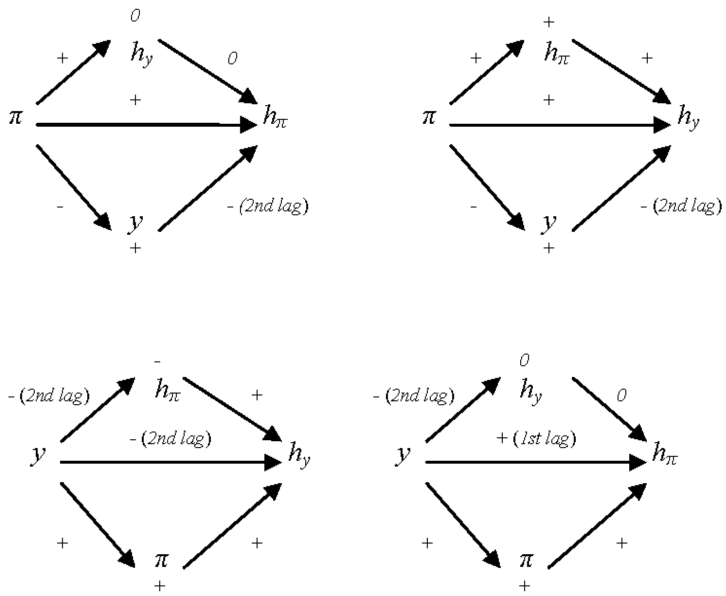


Figure D.3 Inflation-Growth link

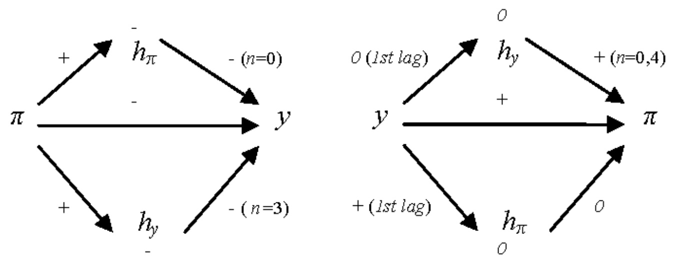
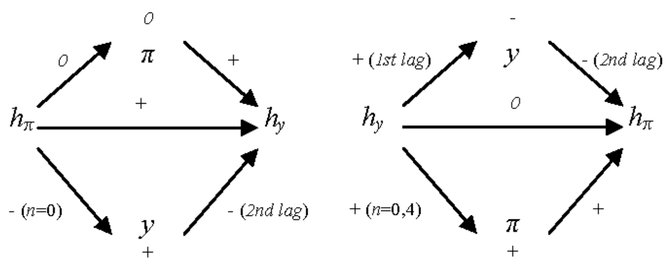


Figure D.4 Variance relationship



APPENDIX E

Tables for Chapter 3

Table E.1 Univariate AR-(A)PARCH models (ML Estimation)

	SP	T	C	D	F	H	N	S
ζ	-0.05^* (−3.25)	0.18 (9.09)	0.04 (2.39)	0.03^* (1.91)	0.04 (2.39)	0.05 (2.99)	−0.02 (−1.64)	0.15 (7.78)
β	0.96 (116.00)	−	0.91 (78.69)	0.92 (84.65)	0.91 (75.20)	0.91 (50.23)	0.92 (85.85)	−
α	0.05 (5.24)	0.46 (7.70)	0.08 (8.075)	0.06 (7.16)	0.07 (6.854)	0.09 (6.00)	0.09 (7.55)	0.36 (7.21)
γ	−	−	−	0.49 (2.01)	−	−	−	0.50 (2.06)
δ	1.56 (8.67)	1.79 (5.97)	1.76 (9.78)	1.15 (6.76)	2.08 (8.67)	1.44 (9.60)	1.64 (9.65)	1.65 (5.32)
υ	5.39 (10.78)	3.20 (17.78)	8.77 (6.40)	7.56 (6.87)	11.31 (6.02)	4.56 (11.4)	5.59 (10.75)	3.72 (14.88)
Q_{12}	17.73 [0.12]	15.37 [0.22]	10.29 [0.59]	12.00 [0.45]	10.91 [0.54]	22.64 [0.02]	9.13 [0.69]	9.80 [0.63]
Q_{12}^2	4.92 [0.96]	308.08 [0.00]	6.41 [0.89]	1.92 [1.00]	8.08 [0.78]	42.39 [0.00]	19.85 [0.07]	41.78 [0.00]

Notes: For each of the eight indices, Table D.1 reports ML parameter estimates for the AR(1)-(A)PARCH model. The numbers in parentheses are t-statistics. * The S&P 500 and Dax 30 indices are estimated by AR(3) and AR(4) models respectively. Q_{12} and Q_{12}^2 are the 12th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals respectively. The numbers in brackets are p-values.

Table E.2 Normality test

	SP	T	C	D	F	H	N	S
$\chi^2(2)$	1194.8	2517.7	706.54	1826.7	731.04	7066.7	1109.8	4618.8
p value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

APPENDIX F

Figures for Chapter 3

Figure F.1 North American stock returns

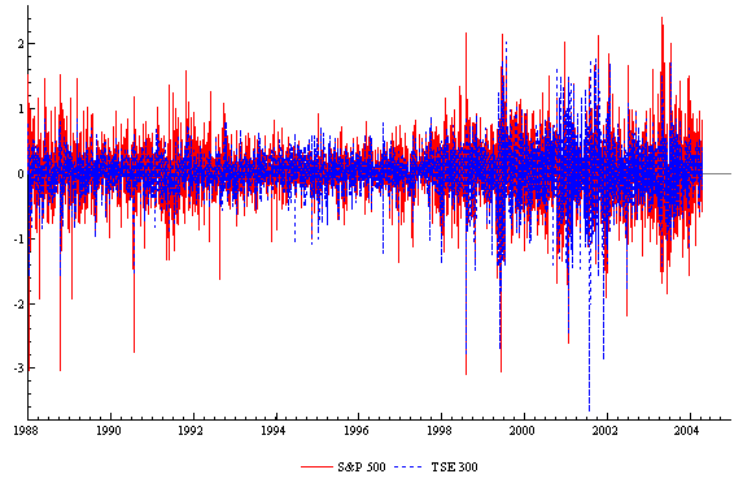


Figure F.2 European stock returns

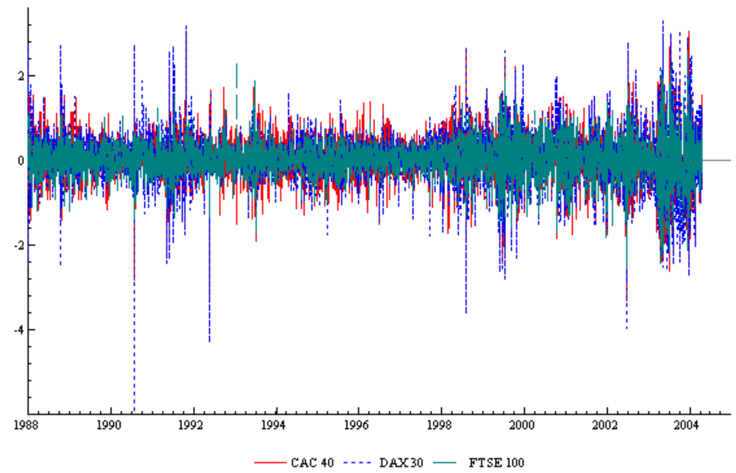
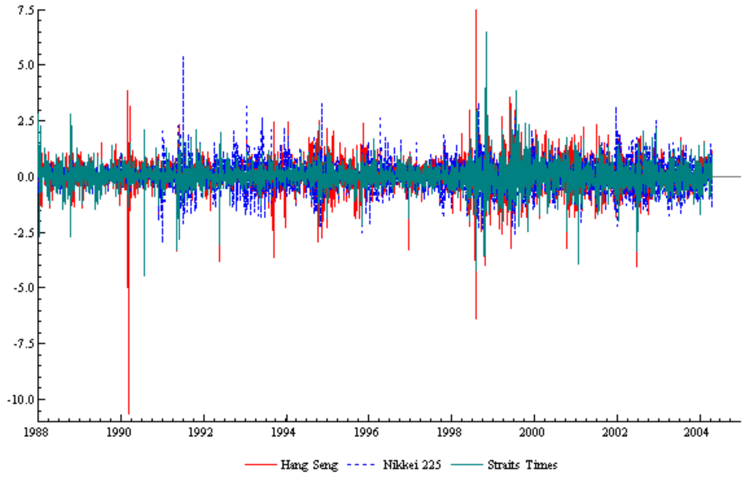


Figure F.3 Asian stock returns



APPENDIX G

Unit Root Tests

Table G.1 Unit root tests for inflation and growth of the UK and US

	PP H_0: I(1)		KPSS H_0: I(0)	
	$Z(t_{\widehat{\alpha}})$	$Z(t_{\widehat{\alpha}^*})$	η_μ	η_t
		UK(1962:01-2004:01)		
Inflation	-15.586***	-15.903***	0.424*	0.215**
Growth	-26.984***	-26.843***	0.102*	0.031*
		US(1957:01-2005:02)		
Inflation	-21.669***	-21.657***	0.535**	0.509***
Growth	-15.754***	-15.753***	0.332*	0.181**

$Z(t_{\widehat{\alpha}})$ and $Z(t_{\widehat{\alpha}^*})$ are Phillips-Perron adjusted statistic with intercept only, and intercept and time trend respectively, using Bartlett Kernel estimation method with Newey-West Bandwidth.

η_μ and η_t are LM statistic with intercept only, and intercept and time trend respectively, using Bartlett Kernel estimation method with Newey-West Bandwidth, with fixed Bandwidth at 36 and with Andrews Bandwidth for UK and US inflation respectively; using Spectral OLS AR based on SIC estimation method, with maximum lags =18 for US growth. ***, ** and* denote significance at the 0.01, 0.05 and 0.10 levels respectively.