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APPLICATION OF DAMAGE MECHANICS IN PERCUSSIVE DRILLING MODELLING

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ABSTRACT

A stationary-periodic quasi-static model of rock percussive deep drilling is introduced. First, an auxiliary problem of stationary indentation of a rigid indentor is considered. The rock is modelled by an infinite elastic medium with damage-induced material softening. The stationarity of the problem allows to reduce the damage history in a material point to the damage distribution down in space. The bore-hole is a semi-infinite cylinder with a curvilinear bottom. It is assumed the indentation is produced by a stationary motion of the rupture front at which an appropriate rock strength condition is violated. The bore-hole boundary is not known in advance and consists of four parts: a free of traction non-rupturing part, a contact non-rupturing part, a free of traction part of the rupture front, and a contact part of the rupture front. Thus the problem is formulated as a non-local non-linear free-boundary contact problem and algorithms of its numerical solution are discussed. The problem solution provides axial force necessary for the drill bit progression through the rock. Then the stationary-periodic percussive drilling problem is reduced to the stationary problem on the rupture progression stage of the cycle and to the classical contact problem on the reverse and progression-before-rupture stages of the cycle. As a result, this provides a nonlinear progression-force diagram.

1 INTRODUCTION

The progression in the percussive drilling is caused by a material rupture under the action of a drilling bit applied at the bore-hole boundary points $y(t)$ changing in time t due to rupture. This boundary loading generates the stress process $\sigma_{ij}(y,t)$ and the strain process $\varepsilon_{ij}(y,t)$ at all material points y. Let a material point y has Cartesian coordinates (y_1, y_2, y_3) in the non-deformed state. The radius-vector of the same material point y in a deformed state at a time t is $\tilde{y}(y,t) = y + u(y,t)$, where $u(y,t)$ is the displacement vector. We will use all equations in terms the non-deformed (reference) coordinates y and refer the boundary conditions to the non-deformed boundary surfaces (Lagrange approach).

Let us consider stationary-periodic percussive drilling of a half-infinite bore-hole, $\Omega_H(t)$, spreading from $y_3 = \infty$ in an infinite elastic space. Let y_3-axis of the coordinate system coincide with the bore-hole axes, and the drill bit progressive-periodic motion occurs only in the y_3 direction. Let $\Omega(t) = \mathbb{R}^3 \setminus \Omega_H(t)$ be the domain occupied by the material (i.e. the infinite space with drilled bore-hole) and $\partial\Omega(t)$ be the bore-hole surface in the non-deformed state (or, the same, in the reference coordinates y), while $\tilde{\Omega}_H(t)$, $\tilde{\Omega}(t) = \mathbb{R}^3 \setminus \tilde{\Omega}_H(t)$ and $\partial\tilde{\Omega}(t)$ be their counterparts in the deformed state. If the rupture front $\partial_F\Omega(t)$ constitutes only a finite part of the boundary $\partial\Omega(t)$, then the borehole is a semi–infinite (not necessarily circular) cylinder with a curvilinear bottom being the rupture front $\partial_F\Omega(t)$. Otherwise, the bore-hole has a monotonously widening shape. If the bit is axially-symmetric then the bore-hole is axially symmetric as well. Let $B(t)$ be the domain occupied by the bit at the instant t, and $\partial B(t)$ be its surface.

The Hook law for an elastic anisotropic damaging material can be written in the form

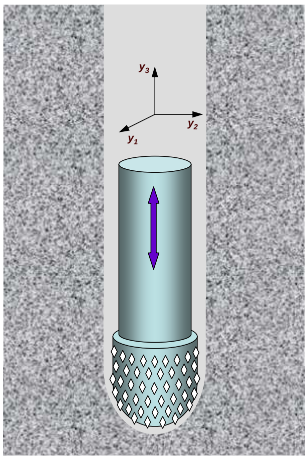

Figure 1: Stationary-periodic percussive drilling

(see e.g. Krajcinovic [1]),

$$\sigma_{ij}(y,t) = C_{ijkl}(y,t)\varepsilon_{kl}(y,t).$$

Under the damage softening we will mean a decrease of the secant elastic stiffness tensor $C_{ijkl}(y,t)$ at a point y caused by the strain tensor history $\varepsilon_{qp}(y,\tau)$ at that point during all preceding time instants, $\tau \le t$. In this terminology, the softening does not necessarily mean the decreasing part of $\sigma - \varepsilon$ diagram with negative tangent moduli. The stiffness tensor of the virgin material (before loading) at a point y is denoted by $C^0_{qpkl}(y)$.

Let the stiffness evolution equation be presented as follows,

$$\dot{C}_{ijkl}(y,t) = -\hat{C}_{ijkl}(\{\varepsilon(y)\}(t), \varepsilon(y,t)) \left\langle \left. \frac{\partial F(\{\varepsilon(y)\}(t), \varepsilon)}{\partial \varepsilon_{pq}} \right|_{\varepsilon=\varepsilon(y,t)} \dot{\varepsilon}_{pq}(y,t) \right\rangle, \qquad (1)$$

where $\hat{C}_{ijkl}(\{\varepsilon\}(t),\varepsilon)$ and $F(\{\varepsilon\}(t),\varepsilon)$ are known functionals of the strain history $\{\varepsilon\}(t) = \{\varepsilon(\tau)\}^t_{\tau=-\infty}$, and functions of the current strain ε; $F(\{\varepsilon\}(t),\varepsilon) = 1$ is the currant damage surface in the strain space ε_{ij}, and $\hat{C}_{ijkl}(\{\varepsilon\}(t),\varepsilon) = 0$ if $F(\{\varepsilon\}(t),\varepsilon) < 1$, that is if ε is inside the currant damage surface (e.g. during initial loading stage or unloading); the angular McAuley brackets are defined as $\langle a\rangle := (a + |a|)/2$. Note that (1) comprises damage rules, which may be not associated with the damage surface, as well as the strain tensor decomposition on the positive and negative parts, c.f. Lubarda and Krajcinovic [2], Krajcinovic [1].

We will suppose the functionals $\hat{C}_{ijkl}(\{\varepsilon\}(t),\cdot)$ and $F(\{\varepsilon\}(t),\cdot)$ depend on the strain history as a sequence of events only, i.e. are independent explicitly of time or the strain rates. Then the same will be true also for the stiffness tensor C_{ijkl}.

Let $\varepsilon'(t) := \varepsilon(t)$ for $\varepsilon(t)$ on the damage surface $F(\{\varepsilon\}(t),\varepsilon(t)) = 1$, and $\varepsilon'(t) := 0$ otherwise. Then $\{\varepsilon'\}(t) := \{\varepsilon'(\tau)\}^t_{\tau=-\infty}$ defines the corresponding *damage strain* history. Suppose the functionals $\hat{C}_{ijkl}$ and F depend only on the damage strain history, i.e., $\hat{C}_{ijkl}(\{\varepsilon\}(t),\cdot) = \hat{C}_{ijkl}(\{\varepsilon'\}(t),\cdot)$, $F(\{\varepsilon\}(t),\cdot) = F(\{\varepsilon'\}(t),\cdot)$, which means they do not change before the initial damage surface $F(0,\varepsilon(t)) = 1$ is reached, and then during unloading.

To describe the material strength for a point y, we will use an instant strength condition at a point y at an instant t written as

$$\Lambda(\varepsilon(y,t)) < 1, \quad y \in \Omega(t), \tag{2}$$

where the function $\Lambda(\varepsilon)$ is associated with the von Mises, Coulomb–Mohr, Drucker–Prager or another appropriate strength condition. Generally, the function $\Lambda(\varepsilon)$ may be not directly connected with the damage softening.

We suppose that the rupture appears in the form of a rupture front $\partial_F\Omega(t)$, c.f. Kachanov [3] (see also Krajcinovic [1] for discussion about damage and rupture without macro-crack nucleation at multiaxial compression). The rupture front is a part of the bore-hole boundary $\partial\Omega(t)$. The rupture front equation can be taken as

$$\Lambda(\varepsilon(y),t) = 1, \quad y \in \partial_F\Omega(t). \tag{3}$$

To formulate the simplest model, let us make the following

Model assumptions:

(i) The deformation gradient is small.

(ii) The material is linearly elastic with damage softening.

(iii) The material is homogeneous, i.e. its initial elastic moduli $C^0_{ijkl} = const$.

(iv) The bit is rigid.

(v) The borehole surface is loaded by the bit at the contact surfaces and is free of tractions at all other points.

(vi) The bit action can be reduced to the pressure $p(x)$ only.

(vii) The ruptured material, for which strength condition (2) is not satisfied, disappears (is washed away) thus leaving the fresh rupture front (the bottom of the bore-hole) either free of tractions or in contact with the bit bottom.

Under the model assumptions, the bore-hole boundary $\partial\Omega$ generally consists of four non-overlapping parts: a free of traction non-rupturing part $\partial_{00}\Omega$, a contact non-rupturing part $\partial_{c0}\Omega$, a free of traction part of the rupture front $\partial_{0F}\Omega$, and a contact part of the rupture front $\partial_{cF}\Omega$.

2 STATIONARY INDENTATION MODEL WITH DAMAGE SOFTENING

Let us consider a in this section an auxiliary problem of stationary indentation of an infinite elastic space by a rigid indentor with a constant progression rate $\dot{h}_3 < 0$ in the y_3 direction. Then $\dot{h} = (0, 0, \dot{h}_3)$ is the progression rate vector. The space is originally homogeneous (at least in y_3 direction) but becomes inhomogeneous due to damage softening caused by the inhomogeneous strain field.

In the stationary problem, the displacements, strains, and stresses are the same at the corresponding points at the corresponding instants,

$$u_i(y,t) = u_i(y - t\dot{h}, 0),\ \varepsilon_{ij}(y,t) = \varepsilon_{ij}(y - t\dot{h}, 0),\ \sigma_{ij}(y,t) = \sigma_{ij}(y - t\dot{h}, 0),\ y \in \Omega(t). \tag{4}$$

This implies

$$\dot{u}_i(y,t) = -\dot{h}_3 u_{i,3}(y,t),\ \dot{\varepsilon}_{ij}(y,t) = -\dot{h}_3\varepsilon_{ij,3}(y,t),\ \dot{\sigma}_{ij}(y,t) = -\dot{h}_3\sigma_{ij,3}(y,t), \tag{5}$$

$$\Lambda(\varepsilon(y,t)) = \Lambda(\varepsilon(y - t\dot{h}, 0)),\ y \in \Omega(t). \tag{6}$$

The dot over $u(y,t)$, $\varepsilon(y,t)$ and $\sigma(y,t)$ means partial derivative with respect to t, which for the chosen Lagrange approach coincides with the material derivative.

The boundary $\partial\Omega$ moves with the velocity $\dot{h}_3$ in the y_3 direction, the corresponding moving boundary points are related as

$$y(t) = y(0) + t\dot{h}, \quad y(t) \in \partial\Omega(t), \quad y(0) \in \partial\Omega(0).$$

This prescribes the normal velocities of the material points on the non-rupturing part of the boundary,

$$\dot{u}_i(y,t)\tilde{\eta}_i(y) = \dot{h}_3\tilde{\eta}_3(y), \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \tag{7}$$

where $\tilde{\eta}_j(y)$ is a unit outward (i.e. directed inward the bore-hole) normal vector to the boundary $\partial\tilde{\Omega}(t)$ in the *deformed* state.

On the other hand, the normal movements of the material points on the rupture front should be slower than the normal movement of the front surface itself,

$$\dot{u}_i(y,t)\tilde{\eta}_i(y) > \dot{h}_3\tilde{\eta}_3, \quad y \in \partial_{0F}\Omega(t) \cup \partial_{cF}\Omega(t), \tag{8}$$

since otherwise there will be creation of material instead of its rupture there, to fill the gap.

Taking in mind the first relation in (5), the time derivative can be replaced with the space derivative, reducing (7) and (8) to the form independent of $\dot{h}_3$,

$$u_{i,3}(y,t)\tilde{\eta}_i(y) = -\tilde{\eta}_3, \qquad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \tag{9}$$

$$u_{i,3}(y,t)\tilde{\eta}_i(y) > -\tilde{\eta}_3, \qquad y \in \partial_{0F}\Omega(t) \cup \partial_{cF}\Omega(t). \tag{10}$$

Relation (9) can be rewritten as

$$\tilde{y}_{i,3}(y,t)\tilde{\eta}_i(y) = 0, \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t). \tag{11}$$

Equation (11) is satisfied if y belongs to a cylindrical surface parallel to the y_3 axis, since the vector $\tilde{y}_{i,3}(y,t)$ is then tangent to the deformed surface which $\tilde{y}_i$ belongs to. This implies that condition (10) can not be satisfied on any cylindrical part of $\partial\Omega$, that is, *there is no rupture on the cylindrical part of* $\partial\Omega$. On the other hand, this means that we may replace condition (9) by the condition of the cylindrical non-rupturing surface,

$$\eta_3(y) = 0, \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \tag{12}$$

where $\eta_j(y)$ is a unit outward boundary normal vector to the non-deformed boundary $\partial\Omega(t)$.

On the contact surfaces we have generally the boundary inclusion $y + u(y) \in \partial B$, $y \in \partial_{c0}\Omega \cup \partial_{cF}\Omega$. Assuming the displacements $u(y)$ are small, the boundary condition can be linearized as

$$u_i(y)\eta_i(y) = d(y), \quad y \in \partial_{c0}\Omega \cup \partial_{cF}\Omega,$$

where $d(y)$ is the distance between the point y and the bit boundary ∂B in the $\eta(y)$ direction in the non-deformed state, and it is known if the contact surface is known and is to be determined otherwise.

From the second of relations (5) we have,

$$\dot{C}_{ijkl}(y,t) \; = - \; \dot{h}_3 C_{ijkl,3}(y,t) \tag{13}$$

$$\{\varepsilon(y)\}(t) \; := \; \{\varepsilon(y,\tau)\}_{\tau=-\infty}^{t} = \{\varepsilon(y-\tau\dot{h},0)\}_{\tau=-\infty}^{t} = [\![\varepsilon]\!](y-t\dot{h}) \tag{14}$$

$$[\![\varepsilon]\!](y-t\dot{h}) \; := \; \{\varepsilon(z,0)\}_{z=\{y_1,y_2,-\infty\}}^{z=\{y-t\dot{h}\}} \tag{15}$$

Thus the temporal history $\{\varepsilon(y)\}(t)$ at a point y is equivalent to $[\![\varepsilon]\!](y - t\dot{h})$, the strain distribution on the space interval $(-\infty, y - t\dot{h})$, since, similar to pure plasticity, the damage evolution laws considered here depend on the strain history as a sequence of events but not on time explicitly. Then (1) can be rewritten for $t = 0$ in the form

$$C_{ijkl,3}(y,0) = -\hat{C}_{ijkl}([\![\varepsilon]\!](y), \varepsilon(y,0)) \left\langle \left. \frac{\partial F([\![\varepsilon]\!](y), \varepsilon)}{\partial \varepsilon_{pq}} \right|_{\varepsilon=\varepsilon(y,0)} \varepsilon_{pq,3}(y,0) \right\rangle. \tag{16}$$

To solve the stationary indentation problem, it is sufficient to consider it only for $t = 0$. Thus, taking into account relations (4)-(16) and dropping the argument $t = 0$ for brevity, we arrive at the following non-classical non-linear functional-integro-differential free boundary problem,

$$\sigma_{ij,j}(y) = 0, \qquad y \in \Omega; \tag{17}$$

$$\sigma_{ij}(y)\eta_j(y)\xi_i(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\zeta_i(y) = 0, \quad u_i(y)\eta_i(y) = d(y), \qquad y \in \partial_{c0}\Omega \cup \partial_{cF}\Omega; \tag{18}$$

$$\sigma_{ij}(y)\eta_j(y) = 0, \qquad y \in \partial_{00}\Omega \cup \partial_{0F}\Omega; \tag{19}$$

$$u_i(y) = 0, \qquad y = \infty; \tag{20}$$

$$\Lambda(\varepsilon(y)) < 1, \qquad y \in \Omega; \tag{21}$$

$$\eta_3(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\eta_i(y) < 0, \quad \Lambda(\varepsilon(y)) < 1, \qquad y \in \partial_{c0}\Omega; \tag{22}$$

$$\Lambda(\varepsilon(y)) = 1, \quad \sigma_{ij}(y)\eta_j(y)\eta_i(y) < 0, \quad u_{i,3}(y)\tilde{\eta}_i(y) > -\tilde{\eta}_3(y), \qquad y \in \partial_{cF}\Omega; \tag{23}$$

$$\eta_3(y) = 0, \quad u_i(y)\eta_i(y) < d(y), \quad \Lambda(\varepsilon(y)) < 1, \qquad y \in \partial_{00}\Omega; \tag{24}$$

$$\Lambda(\varepsilon(y)) = 1, \quad u_i(y)\eta_i(y) < d(y), \quad u_{i,3}(y)\tilde{\eta}_i(y) > -\tilde{\eta}_3(y), \qquad y \in \partial_{0F}\Omega; \tag{25}$$

where

$$\sigma_{ij}(y) = C_{ijkl}(y)\varepsilon_{kl}(y), \quad \varepsilon_{kl}(y) = (u_{k,l} + u_{l,k})/2, \tag{26}$$

$C_{ijkl}(y,t)$ satisfies the functional-integral equation

$$\begin{aligned} C_{ijkl}(y) \quad = \quad & C^0_{ijkl} - \int_{-\infty}^{y_3} \hat{C}_{ijkl}([\![\varepsilon]\!](y_1,y_2,x_3), \varepsilon(y_1,y_2,x_3)) \times \\ & \left\langle \left. \frac{\partial F([\![\varepsilon]\!](y_1,y_2,x_3), \varepsilon)}{\partial \varepsilon_{pq}} \right|_{\varepsilon=\varepsilon(y_1,y_2,x_3,0)} \varepsilon_{pq,3}(y_1,y_2,x_3) \right\rangle dx_3, \quad y \in \Omega \end{aligned} \tag{27}$$

and the initial stiffness tensor C^0_{ijkl} is know; $\xi_j(y)$, $\zeta_j(y)$ are unit vectors orthogonal to $\eta_j(y)$ and to each other.

All the four boundary parts $\partial_{00}\Omega$, $\partial_{0F}\Omega$, $\partial_{c0}\Omega$, $\partial_{cF}\Omega$, and consequently $d(y)$, are generally unknown in this setting and the corresponding "excessive" boundary equalities and inequalities are provided in (22) and (25) to allow their determination.

Note that the strains decrease with the distance from the contact surface on $\partial_{cF}\Omega \cap \partial_{c0}\Omega$ in the elastic space. Thus the integrand in (27) equals to zero at sufficiently small and sufficiently large x_3 since the strains there are inside the damage surface, where $\hat{C}_{ijkl}([\![\varepsilon]\!](y_1,y_2,x_3), \varepsilon(y_1,y_2,x_3,0)) = 0$. This means the stiffness tensor $C_{ijkl}(y,t)$ will be equal to the initial one, C^0_{ijkl}, outside some neighborhood of the bore-hole, and will be independent of y_3 at some distance of the bore-hole bottom in this neighborhood.

Different strategies can be chosen to solve this problem. One of the possibilities is the multi-level iteration algorithm described below. It consists of *global iterations*, each solving a nonlinear mixed boundary value *functional-integro-differential* problem (17)-(20), (26)-(27) with some *fixed* boundaries, $\partial_{00}\Omega$, $\partial_{0F}\Omega$, $\partial_{c0}\Omega$, $\partial_{cF}\Omega$, and consequently $d(y)$. Then conditions (21)-(25) are checked and the boundaries are changed to decrease the discrepancies and the next global iteration starts.

On the first global iteration one can reasonably assume that the rupture front coincides with the contact part of the bit, $\partial_c B$, which in turn coincides with the bit bottom, $\partial_b B$, (consisting of the bit surface points with algebraically smallest y_3 coordinate, over the points with the same (y_1, y_2) coordinates), i.e. $\partial_{cF}\Omega = \partial_c B = \partial_b B$, $\partial_{0F}\Omega = \emptyset$, and there is no contact without rupture, i.e. $\partial_{c0}\Omega = \emptyset$. Those assumptions imply that the borehole free boundary $\partial_0\Omega$ is the semi-infinite cylindrical surface ended by the bit bottom, on the first iteration.

Solution of each global iteration problem can be achieved using *sub-iterations*. On each sub-iteration the *linear* mixed boundary value problem (17)-(20), (26) of inhomogeneous elasticity with *fixed boundaries* and some *fixed elastic coefficients* $C_{ijkl}(y)$ is solved. Then updated coefficients $C_{ijkl}(y)$ are obtained after substituting the solution into the right hand side of (27), and the next sub-iteration starts. On the first iteration, the stiffness coefficients $C_{ijkl}(y)$ can be taken equal to the initial one, $C^0_{ijkl}(y)$, or from the previous global iteration, where available. The iteration process should proceed before the difference between stiffnesses on neighboring iterations becomes negligible.

After the global iterations converge, the integration of the component $\sigma_{3j}(y)\eta_j(y)$ of the contact traction gives the total axial force $P(t)$ applied to the bit during the progression,

$$\mathcal{P}(t) = \int_{\partial_c\Omega} \sigma_{3j}(y)\eta_j(y)\, dS(y_c). \tag{28}$$

In the case when condition (iv) is not satisfied, the fracture front, the contact surface and the pressure are unknown and we need to take into account elasticity of the bit. If condition (vi) is violated, that is, the bit interacts with the material not only by pressure, one has to introduce some friction contact, describing it e.g. by the Coulomb–More law.

Note that in all the cases, the obtained solution and particularly the total force $\mathcal{P}$ is independent of the progression rate $\dot{h}_3$ or the progression itself.

3 STATIONARY-PERIODIC INDENTATION MODEL WITH DAMAGE SOFTENING

Let the instant bit progression $h_3(t)$ be the lowest y_3 coordinate of the bit boundary. Let T be the cycle period, $dh_3(t)/dt = \dot{h}_3(t) = -|\dot{h}(t)| \le 0$ be instant progression rate and $\dot{h}_{T3} = [h_3(t) - h_3(t-T)]/T = -|\dot{h}_T| \le 0$ be average progression rate over cycle, in the y_3 direction, where $\dot{h} = (0, 0, \dot{h}_3)$ and $\dot{h}_T = (0, 0, \dot{h}_{T3})$ are the instant and average progression rate vectors, respectively. In the stationary-periodic problems, $\dot{h}_T$ does not depend on t, the strains, stresses and boundary tractions are independent of the cycle number m in the corresponding points at the corresponding instants,

$$\varepsilon_{ij}(y + mT\dot{h}_T, t + mT) = \varepsilon_{ij}(y, t), \quad y \in \Omega(t); \tag{29}$$

$$\sigma_{ij}(y + mT\dot{h}_T, t + mT) = \sigma_{ij}(y, t), \quad y \in \Omega(t) \tag{30}$$

for any integer $m > -\infty$.

Rupture with the chosen material damage softening does not in fact depend on the natural time t but depends only on the loading history as a sequence of strain process history and on the actual state in the considered material point.

In addition to the Model assumptions (i)-(vii), let as make the following *Reverse assumption*

(viii) The damage softening and the rupture do not proceed during the reverse stage of the bit motion, i.e. the stiffness tensor and the (non-deformed) borehole boundary do not change before the load reaches the extremum value during the next cycle.

Due to assumption (viii), the stress and strain return to the same states during the reverse and the following progressive stages of the bit motion up to the rupture restarts. This implies the reverse and the following progressive stages can be considered as some interruptions of the stationary progression process, analyzed in the previous section, and moreover, the interruptions do not influence the material rupture. This means the relation between the total force and the bit progression looks as on Fig. 2, that is the elastic loading stage is followed by the rupture stage followed by the elastic unloading stage. Generally the elastic stages are non-linear on the loading and unloading stages due to the changing contact surface between the bit and the drilled material, and the force is constant on the rupture stage. The extremum force and strains in the process coincide with those obtained in the stationary indentation analysis in the previous section.

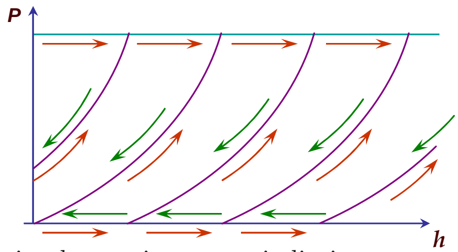

Figure 2: $h - P$ diagram in the stationary-periodic instant rupture model with damage softening.

Then the solution of the stationary indentation problem from the previous section equation and the integral (28) give the extremum values of the contact distribution $p(y, t^{ex})$ and the maximum force $\mathcal{P}^{ex} = \mathcal{P}(t^{ex})$ on the bit, as well as fixes the bore-hole boundary for the reverse stage.

To predict the curvilinear part of the $h - P$ diagram, one has to solve the classical linearly elastic contact problem with the fixed material (non-deformed) boundary $\partial\Omega$ and fixed elastic coefficients $C_{ijkl}(y)$ for the progressive (before rupture restart) or, the same, reverse stages of the cycle. The problem consists of equations

$$\sigma_{ij,j}(y) = 0, \qquad y \in \Omega; \tag{31}$$

$$\sigma_{ij}(y)\eta_j(y)\xi_i(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\zeta_i(y) = 0, \quad u_i(y)\eta_i(y) = d(y, h_3), \qquad y \in \partial_c\Omega(h_3); \tag{32}$$

$$\sigma_{ij}(y)\eta_j(y) = 0, \qquad y \in \partial_0\Omega(h_3); \tag{33}$$

$$u_i(y) = 0, \qquad y = \infty; \tag{34}$$

$$\sigma_{ij}(y)\eta_j(y)\eta_i(y) < 0, \qquad y \in \partial_c\Omega(h_3); \tag{35}$$

$$u_i(y)\eta_i(y) < d(y, h_3), \qquad y \in \partial_0\Omega(h_3); \tag{36}$$

and the stress-strain and strain-displacement relations (26). The overall boundary $\partial\Omega$ here is known from the end of the previous progression-rupture stage, although the boundary partition into the free and contact parts is to be determined for each h_3.

Classical conforming contact problem (31)-(36), (26) can be solved by any of the well known methods, see e.g. Johnson [4]. Particularly, one can use the algorithm similar to the total iteration algorithm of the progression-rupture stage described above, that is, to choose some reasonable partition of $\partial\Omega$ onto $\partial_0\Omega$ and $\partial_c\Omega$, solve mixed elasticity problem (31)-(34), (26), modify the partition of $\partial\Omega$ to alleviate the discrepancies in (35)-(36) and start the next iteration.

CONCLUSIONS

A stationary-periodic quasi-static model of percussive drilling is obtained with account of damage softening. The cycles of the bit progression – force diagram consist of three stages: elastic loading, constant-force rupture progression, and elastic unloading parallel to the loading. The problem is split into a stationary free-boundary non-linear non-local problem for the rupture stage of the cycle, and a classical contact problem for the rest of the cycle. Some iteration algorithms are described reducing the solution to a sequence of linear problems of inhomogeneous elasticity. Those problems are to be solved by a general numerical method, e.g. the FEM or the Localized Boundary-Domain Integral Equation Method, see Mikhailov [5]. As a result, this provides a nonlinear progression-force diagram, which is to be used in the bit dynamic motion prediction.

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