A Mixed-Game Agent-Based Model of Financial Contagion

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Abstract— **Over the past two decades, financial market crises with similar features have occurred in different regions of the world. Unstable cross-market linkages during financial crises are referred to as financial contagion. We simulate the transmission of financial crises in the context of a model of market participants adopting various strategies; this allows testing for financial contagion under alternative scenarios. Using a minority game approach, we develop an agent-based multinational model and investigate the reasons for contagion. Although contagion has been extensively investigated in the financial literature, it has not been studied yet through computational intelligence techniques. Our simulations shed light on parameter values and characteristics which can be exploited to detect contagion at an earlier stage, hence recognising financial crises with the potential to destabilise cross-market linkages. In the real world, such information would be extremely valuable to develop appropriate risk management strategies.**

I. INTRODUCTION

The financial crises which occurred in Mexico in 1987, in Asia in 1997, and in Russia in 1998, all spread from one country to neighbouring economies in the region - a phenomenon referred to as 'financial contagion'. Forbes and Rigobon [1] provide a more precise definition, describing contagion as a significant increase in cross-market linkages after a shock to a group of countries. "Interdependence" instead applies to a situation when two markets exhibit a high degree of co-movement during both stability periods and crisis periods [2]. Many studies using simulation and forecasting techniques or developing early warning systems focus on the origins of financial crisis [4]-[6] rather than financial contagion. More recent works [1]-[3] have instead analysed either contagion or interdependence. In this paper, we simulate the transmission of financial crises modelling the behaviour of market participants and their various strategies using a mixed-game agent-based approach. Ours is a multinational framework which is suitable for the analysis of financial contagion.

Herd behaviour has been identified as a major factor behind contagion. This arises because of irrational investment choices made by noise traders [7][8]. Agent-based models are an effective tool to describe and simulate such behaviour [9]. Experiments show that the presence of noise traders increases price fluctuations. Agent-based models may be further employed to simulate linked markets rather than a single market. For example, international financial crises are studied in Kaizoji [10] investigating correlated markets. Each market has a certain number of participants, where the proportion of participants with an attitude to buy/sell in each market is determined by the average domestic attitudes, average foreign attitudes and global fundamentals. Thus the model is able to simulate interdependence, when a shock to one of the markets is followed by a fall in the related market. By introducing noise traders as well, it is possible to simulated contagion during a financial crisis [11].

An effective framework for studying more realistic and complex markets is provided by Game Theory, the Minority Game approach being particularly influential [12]. Jefferies et al. [13] build on it and develop Grand Canonical games, using that approach in a multi-agent game model to predict future movements in financial time-series [14]. Improved forecasting accuracy is achieved by adding majority game agents who play together with minority game agents in a mixed-game model [15]. It appears that using a mixed-game approach improves forecasting accuracy by at least 3% compared to a minority game approach. In this paper, we further develop the mixed-game model into a mixed-game multinational model, to simulate contagion between linked markets in the presence of noise traders. Further, we provide evidence on financial contagion using real data for the Asian countries.

The paper is organised as follows. Section II describes the framework for the mixed-game multinational model. In Section III, we introduce an approach to parameter estimation. The simulations results are summarised in Section IV. Finally, in Section V we provide some concluding remarks.

II. MIXED-GAME MULTINATIONAL MODEL

A. Minority Game

The mixed-game model [15][16] and the mixed-game multinational model [11] are both extensions of the minority game model [12]. Challet and Zhang [12] used minority game to solve initially the El-Farol bar problem [17], and then extended the application to modelling market behaviour of heterogeneous agents. Currently, minority game is becoming a very influential approach in behavioural studies [14][18].

The basic minority game structure involves an odd number of players N, each of whom has to choose an action, buy or sell, in every time period. The actions are denoted as $+1$ for buy and -1 for sell. The information available to an agent at any time t, is the string $\mu = (\chi_{t-1}, ..., \chi_{t-M})$ of the last M actions on the winning side. The winning side is the minority side, $\chi \in \{-1, +1\}$, and M is an agent's memory size. There are 2^M possible strings $\mu_t = \mu_{t,1}...\mu_{t,2^M}$, and

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the string corresponding to the last M actions on the winning side is called present history. For each possible history μ_t, a strategy recommends a fixed action $\alpha_{\pm,i}^{\mu_t}$. Each agent has a decision table of K strategies, therefore all strategies on the market can be denoted with $S_{j,i}, j = 1...K, i = 1...N$. Table I illustrates an agent's decision table for $M = 3, K = 2$.

μ_t	$S_{1,i}$	$S_{2,i}$
$-1, -1, -1$	-1	$+1$
$-1, -1, +1$	$+1$	-1
$-1, +1, -1$	-1	$+1$
$-1, +1, +1$	$+1$	$+1$
$+1, -1, -1$	-1	-1
$+1, -1, +1$	-1	$+1$
$+1, +1, -1$	-1	$+1$
$+1, +1, +1$	$+1$	-1

Each strategy can collect a virtual point if its predicted action is on the winning side. The number of virtual points of strategy $S_{j,i}$ over a horizon till period t is denoted as U_{it}^j. The sum of virtual points for all strategies of agent i predicting action buy at t is $U_{i,t}^{(+)}$, and by analogy for sell and $U_{i,t}^{(-)}$. Therefore, the difference of virtual points for the two actions of agent i at time t is $\Delta_{i,t} \equiv U_{i,t}^{(+)} - U_{i,t}^{(-)}$. His choice is given by a strategy $S_{j,i}, j = 1...K$, with the highest virtual points among the strategies matching the sign of $sgn(\Delta_{i,t})$. That will be the strategy of agent i at time t, and we will denote it as S_i^t. Summarizing the actions of the population of agents choosing the $+$ sign, buy, and the $-$ sign, sell, at time t, we get the excess demand $A_t = (\alpha_1^\mu + \alpha_2^\mu + ... + \alpha_N^\mu)$. The minority side wins the game, therefore the sign chosen by the minority at time t is $\chi_t = -sgn(A_t)$. Thus the new history string $\mu_{t+1} = (\chi_t, ..., \chi_{t-M+1})$ is built up.

B. Mixed-game and Mixed-game Multinational Model

The standard minority game is further modified by Gou [15][16] to develop a mixed-game model that better simulates financial markets. In a mixed-game model, odd number of participants N are divided into two groups playing a majority game and a minority game, respectively. $N1$ is the number of agents playing the majority game, and $N2$ is the number of agents playing the minority game. Let $M1$ and $M2$ denote the memory size of majority and minority game players, accordingly; while $K1$ and $K2$ denote the size of the strategy table for majority and minority game players. A player in the majority game has approaches to taking actions analogous to a player in the minority game. However, a majority player wins if his action is on the majority side. Therefore, the sign chosen by a majority player at time t is $\chi_t = sgn(A_t)$. Furthermore, agents collect virtual points for their strategies over some time windows, $T1$ for majority players and $T2$ for minority players. The price P_t in period t depends on the excess demand, as in (1)

$$P_t = \delta \sum_{i=1}^{N} \alpha_{S_i}^{\mu_t} + P_{t-1} \qquad (1)$$

where parameter δ is a scale factor.

We extend the mixed-game model to work with two or more markets. This is referred to as a mixed-game multinational model. In this advanced model, there are two markets A and B, players in market A only invest in market A, but take actions taking into account both markets A and B. Vice versa for players in market B only invest in market B, but take actions based on both markets B and A. Thus for players in market A, the actions depending on the historical string for market A, μ_t^A at time t, are denoted with $\alpha_{\pm,i}^{\mu_t^A}, i = 1...N_A$, and the actions depending on the historical string for market B, μ_t^B, are $\alpha_{\pm,i}^{\mu_t^B}, i = 1...N_A$. Analogous notations can be introduced for the actions of players in market B. Therefore an agent faces different possible actions depending on information from corresponding different markets.

Let us consider market A. We assign each player a parameter $\omega_{i,t}^A, i = 1...N_A$, so that the probability of agent i in market A to choose in period t an action, $\pi_{i,t}^A$, based on the domestic market is in (2)

$$\pi_{i,t}^A = \frac{exp(\lambda^A \omega_{i,t}^A)}{exp(\lambda^A \omega_{i,t}^A) + exp(-\lambda^A \omega_{i,t}^A)} \qquad (2)$$

Where λ^A is a scale factor. Agents will take action based on either domestic market or foreign market. Thus the probability of an agent choosing an action based on a foreign market is $1 - \pi_{i,t}^A$. Parameter $\omega_{i,t}^A$ is updated after each turn. If the actions based on the domestic market history, μ_t^A, win the game, and the actions based on the foreign market history, μ_t^B, loose the game, then $\omega_{i,t+1}^A = \omega_{i,t}^A + 1$. If the actions based on the foreign market history μ_t^B win, and the actions based on the domestic market history μ_t^A loose, then $\omega_{i,t+1}^A = \omega_{i,t}^A - 1$. Therefore, if and only if $\alpha_{\pm,i}^{\mu_t^A} \neq \alpha_{\pm,i}^{\mu_t^B}$, the parameters ω_i^A are updated.

Players in market B behave in the same way.

C. Noise Traders

Next, we introduce a proportion of noise traders in each market. $N_{A,noise}$ and $N_{B,noise}$ denote the number of noise-trader agents, in the two markets respectively. In our model, noise traders have a tendency to follow the sign of the last price change on the markets. For example in market A, the probability $\pi_{n,t}^A$ of noise traders to take a buy action $(+1)$ is in (3):

$$\pi_{n,t}^A = \frac{exp(\zeta_{t-1}^A)}{exp(\zeta_{t-1}^A) + exp(-\zeta_{t-1}^A)} \qquad (3)$$

Where

$$\zeta_{t-1}^A = \sum_{Z=1}^{Z} \left(\frac{P_{t-1}^Z - P_{t-2}^Z}{P_{t-2}^Z} \right) \tau^A \qquad (4)$$

and $Z \in \{marketA, marketB, marketC, marketD, ...\}$ for the multinational model involving K markets. P_t^Z is the

price in market Z at time t. Parameter τ^A manages the sensitivity of noise traders in market A to changes in market Z. Finally, the probability of noise traders in market A to choose a sell action is $1 - \pi_{n,t}^A$.

Let us further analyse (2) and (3). They seem similar, and a feature of this type of equations is that in an equilibrium state $\omega_{i,t}^A = 0$ or $\zeta_{t-1}^A = 0$, e.g. at the very beginning, agents have the same probability to take one or the other of the two possible actions. If the action based on the historical string from the domestic market wins the game and the other one based on foreign market loses, then $\omega_{i,t}^A$ will be updated from $\omega_{i,t}^A = 0$ to $\omega_{i,t}^A > 0$, thus the probability of taking the action based on domestic market increases. Equally, if the sum of relative changes from all related markets are positive $\zeta_{t-1}^A > 0$, then the probability of noise traders selecting "buy" is larger than selecting "sell", and vice versa. Factors λ^A and τ^A are employed to adjust the speed of the probability getting close to 100% and 0%. In Fig. 1, we plot (2) for a variety of scale factors λ^A.
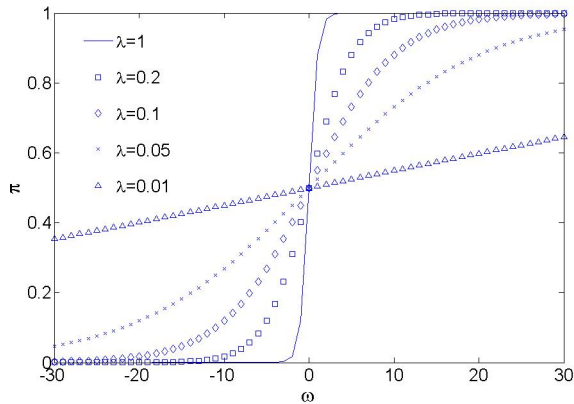

Fig. 1. Calculated probability π using formula (2) and varying the scale factor λ. The line represents $\lambda = 1$, the square corresponds to $\lambda = 0.2$, and the diamond stands for $\lambda = 0.1$, while the dot is $\lambda = 0.05$ and the triangle is $\lambda = 0.01$

It is obvious that the probability approaches 100% and 0% more slowly when the scale factor decreases. Therefore, scale factors λ^z, $Z \in \{A, B, C, D, ...\}$, can be used to describe different markets. By analogy, scale factors τ^Z, $Z \in \{A, B, C, D, ...\}$, are used to describe the sensitivity to different markets.

III. PARAMETERS ESTIMATION

The objective of the mixed-game multinational model is to simulate the effects of the crisis on the markets hit by the shock. We adopt an appropriate procedure to estimate the relevant parameters. For this purpose, the following issues need to be considered [15][16]:

1) the memory size of both types of game players must be larger than 2, and the memory size of majority game players should be less than that of minority players, while the time window for majority game players can not be larger than for minority players.

2) the plots of return distributions should look similar;
3) following the standard by Gou [15] [16], the median of the local volatilities of simulation results should be similar to the target real market;
4) co-movements with market from which the crisis originated should be look similar.

Following these restrictions, first the general parameters for market participants are determined. These parameters include $M1$, $M2$, $K1$, $K2$, $T1$, $T2$, and N (total number of participants only, not distinguishing between $N1$, $N2$ and N_{noise}, $i.e. N = N1 + N2 + N_{noise}$). We have tested for parameters robustness, and confirmed the conclusion in Gou [16] that changes in M, K, T or N contribute little to the simulation of a financial market. In the next section, we will simulate the movement of the stock markets in South Korea and Hong Kong during the Asian financial crisis of 1997.

Beside the above parameters, we need values for the remaining parameters to be able to simulated heterogeneous markets. The additional parameters include the proportions of $N1$, $N2$, N_{noise} in the total number of participants N, as well as the scale factor δ in (1) and the sensitivity factor τ in (4). In contrast to the general parameters, these specific parameters affect significantly daily return distributions in the simulated markets.

Empirical studies of price fluctuation for various financial assets have shown that the stock return distributions deviate from the Gaussian distribution and are characterised with fat tails [19][20]. Indeed, inappropriate proportions of $N1$, $N2$ and N_{noise} will lead to ridiculous return distributions. Fig. 2 plots kurtosis in grid as the result generated through simulations under different values for $N1$, $N2$, and N_{noise}. We assume an extreme scenario where noise traders sensitivity factor $\tau = 0$, i.e. noise traders have equal probability to "buy" or "sell". Indeed, noise traders are trend followers, and they will increase the kurtosis of return distributions. We observe in Fig. 2 that some proportions for $N1$, $N2$, and N_{noise} result in extremely large kurtosis. In real markets, particularly near 1000 days return, the kurtosis is under 10. For example, kurtosis of the stock market of South Korea for the relative stable period from 03/01/1994 to 01/07/1997 is 4.13. Kurtosis of the stock market of Hong Kong in the same period is 6.20 (see Fig. 3). Fig. 2 also indicates that simulations under some values are unstable. Thus, we choose appropriate values for $N1$, $N2$ and N_{noise}, such that the corresponding simulations are stable and the return distributions are plausible.

Furthermore, owing to the different index and trading volume in each market, we need to adjust the factor δ in (2) to assure that the median of local volatilities is similar to the target real market. For the purpose of more precisely simulating the movement of the target real market, it is recommended to use return distributions and median of local volatilities collected over a longer horizon.

Based on the definition of [1][2], financial contagion occurs when the correlation between the original market and the affected market increases significantly from a period
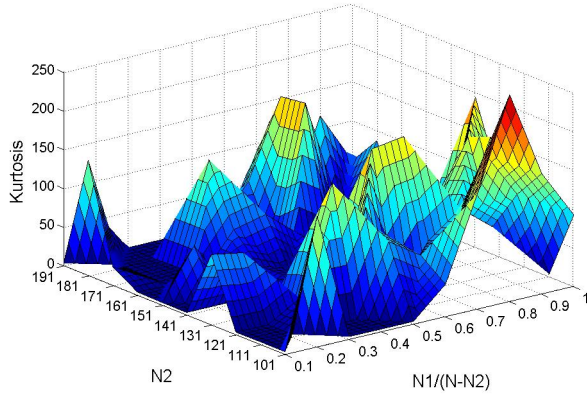
Fig. 2. Kurtosis in grid simulated under varying proportions for $N1$, $N2$ and N_{noise}. $N = 201$ is a constant and $N = N1 + N2 + N_{noise}$. Other parameter setting is $\tau = 0$ and $\delta = 2$. The results are averaged over 100 trials, where each trial has 50 repetitions of mixed-game multinational model, and each repetition simulated 912 days movement.
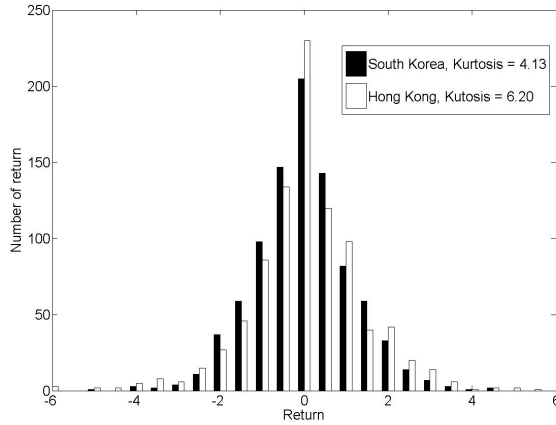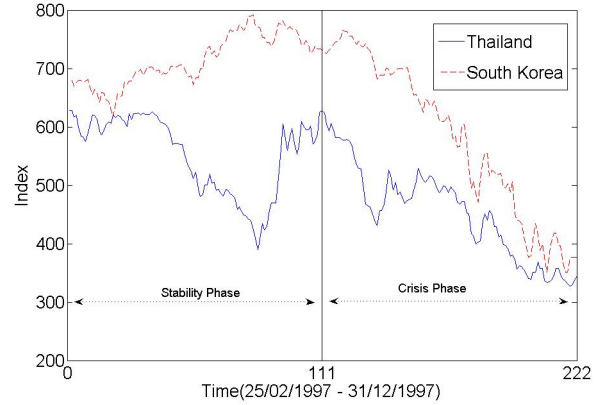

Fig. 4. Indices of Thailand's and South Korea's stock market, from 25/02/1997 to 31/12/1997. The line corresponds to the crisis origin, Thailand, and the dash represents an affected market, South Korea. The left side corresponds to the pre-crisis phase, and the right side represents the crisis phase. The correlation coefficient is -0.64 prior to the crisis, and 0.92 during the crisis phase.

Fig. 3. Plots of daily returns distributions of South Korea and Hong Kong stock markets from 03/01/1994 to 01/07/1997, 912 days. The kurtosis of South Korea is 4.13 and of Hong Kong is 6.20

of relative stability to a period of crisis. Otherwise, the phenomenon is defined as interdependence. For example, the period from 25/02/1997 to 31/12/1997 can be divided into two equal phases around the beginning of the Asian crisis. The first period corresponds to the pre-crisis phase, and the second period corresponds to the crisis phase. Let us consider the correlation coefficient between the country from which the crisis originated, i.e. Thailand, and one of the affected markets, South Korea. The correlation during the pre-crisis phase is -0.64, while during the crisis phase it is 0.92 (see Fig. 4). This confirms the expectation of a significant increase in the co-movement of markets. In order to simulate contagion, we need to identify properly the sensitivity parameter.

IV. SIMULATION AND ANALYSIS

We apply the parameters estimation procedure outlined above for the mixed-game multinational model then used

to simulate the Asian financial crisis in 1997. The crisis originated in Thailand (TH), and affected markets are South Korea, Hong Kong, Indonesia, Malaysia and other Asian countries. We simulate the markets in South Korea (SK) and Hong Kong (HK) as target real markets over 222 trading days, 25/02/1997-31/12/1997, in relation to the movement of Thailand's stock market. We collect the target daily return distributions and the median of local volatilities, using historical index movements over the period 03/01/1994-01/07/1997, corresponding to 912 trading days. The target correlation coefficients of real markets are collected from historical index movements over 25/02/1997-31/12/1997 or 222 trading days. The first 111 days correspond to the pre-crisis phase, and the last 111 days represent the crisis phase. The target real market values are listed in Table II. Based on these, the parameters for simulating target markets are listed in Table III.

TABLE II
TARGET VALUES OF REAL STOCK MARKETS

Target Value	South Korea	Hong Kong
Kurtosis of daily return distribution	4.13	6.20
Median of local volatility	63.47	9283.13
Correlation coefficient in pre-crisis phase	-0.64	-0.52
Correlation coefficient in crisis phase	0.92	0.86

Fig. 5 shows the kurtosis of daily returns and the median of local volatilities over 912 days, from a simulation under the parameters values given in Table III. Kurtosis of SK simulation is 4.6, while the corresponding value for the HK simulation is 5.6. Fig. 6 presents the SK simulation of daily returns and median of local volatilities over 912 days, under the parameter values in Table III. The simulated SK median is 59.43. Fig. 7 represents the HK simulation of daily returns and median of volatilities, where the simulated median is

SIMULATION PARAMETERS BASED ON TARGET VALUES IN TABLE II

Parameter Symbol	South Korea	Hong Kong
N	201	201
$M1$	3	3
$M2$	6	6
$K1$	4	4
$K2$	6	6
$T1$	12	12
$T2$	60	60
$N1$	25	28
$N2$	151	131
N_{noise}	25	42
δ	2.2	0.2
τ	30	40

8962.76.
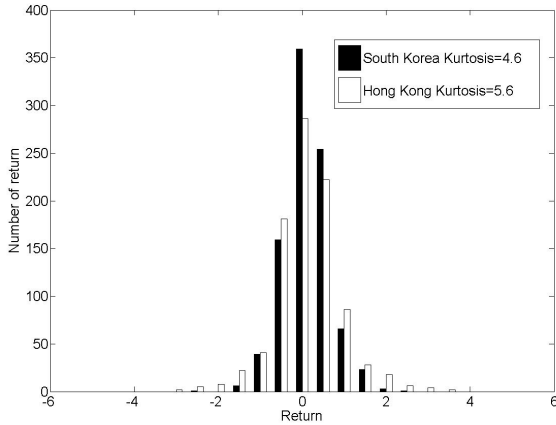
Fig. 5. Plots of daily return distributions from a simulation of SK and HK over 912 days. The simulated kurtosis for SK is 4.6, while it is 5.6 for HK. The simulation parameters are based on Table III.

Finally, Fig. 8 shows the movements in the market where the crisis originated(top), the real index in the SK affected market (middle), and the simulated SK index (bottom), from 25/02/1997 to 31/12/1997 or 222 trading days. The crisis phase is on the right-hand side, while the pre-crisis phase is on the left. The correlation coefficient of the simulated SK series and the crisis origin (TH) real series is 0.63 in pre-crisis phase, and rises to 0.94 during the crisis phase. Fig. 9 presents the corresponding information for the HK market. The correlation coefficient of the simulated HK series and the crisis origin (TH) real series is 0.48 in the pre-crisis phase, and rises to 0.96 during the crisis.

During the pre-crisis phase, Thailand suffers an initial tumble and manages to recover, while SK and HK do not follow that tumble and their markets are unaffected. In our model, SK and HK will be affected to a degree, through the noise traders. However, both the real and simulated correlation coefficients of the affected markets in the pre-crisis phase are within the range of correlation coefficients in relatively stable or non-crisis periods. Furthermore, the simulated results approximate well the target real markets
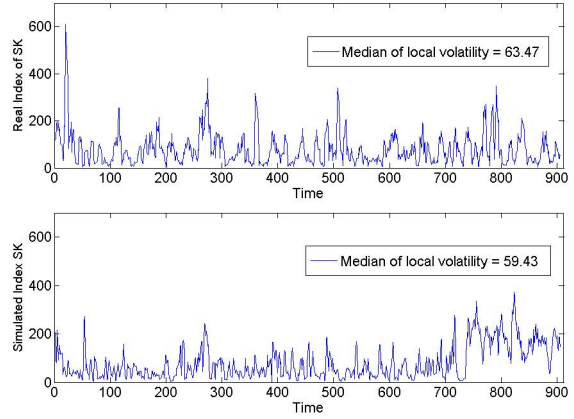
Fig. 6. The top plot corresponds to the volatilities from the real SK index. The bottom plot represents and a simulation over 912 trading days. The median of local volatilities for the real index is 63.47, and the simulated median is 59.43
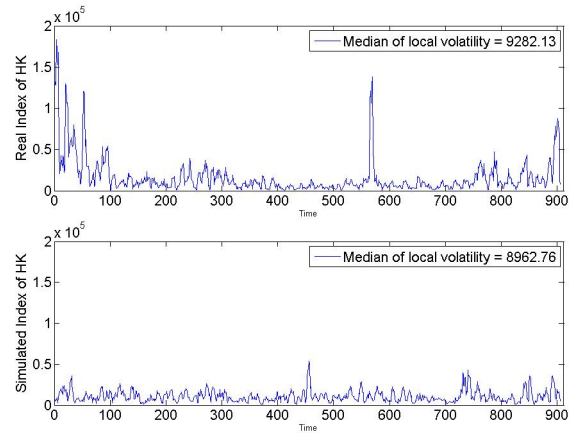
Fig. 7. The top plot corresponds to the volatilities from the real HK index. The bottom plot represents and a simulation over 912 trading days. The median of local volatilities for the real index is 9282.13, and the simulated median is 8962.76.
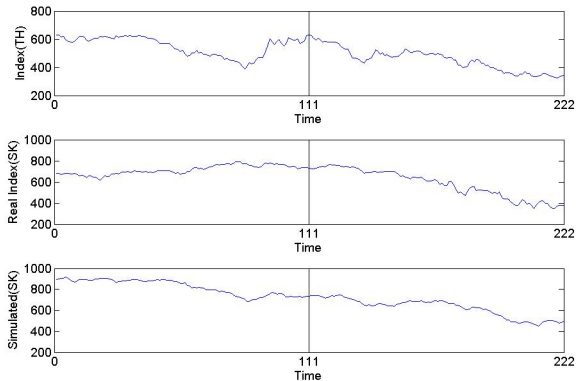
Fig. 8. Real index movement in the contagion-origin market of TH (top), real SK market (middle), and a simulated SK market (bottom) based on the parameters in Table III. Period 25/02/1997-31/12/1997.
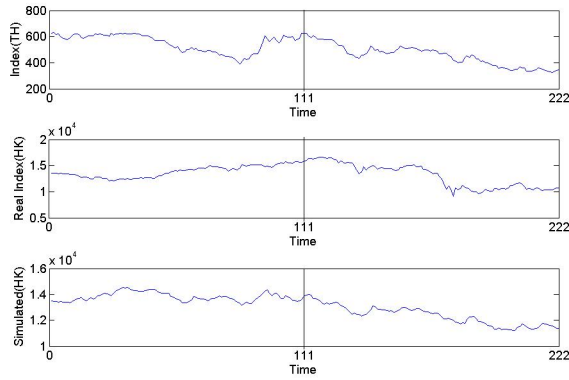
Fig. 9. Real index movement in the contagion-origin market of TH (top), real HK market (middle), and a simulated HK market (bottom) based on the parameters in Table III. Period 25/02/1997-31/12/1997.

movements and the correlation with the crisis-origin, during the crisis phase. In particular, both the real and simulated correlation coefficients rise significantly during the crisis period.

V. CONCLUSION

In this paper we have developed a mixed-game multi-national model with the purpose of simulating contagion occurring during financial crises. The aim is to capture characteristics of linked financial markets contributing to financial contagion. Using real data for Thailand, where the Asian crisis of 1997 originated, and simulating the movements of the affected markets of South Korea and Hong Kong, we detect some important features. These include fat-tail return distributions, and similarity between real and simulated volatility. We find a significant increase in the correlation and co-movement between markets during the crisis phase.

Future research will also consider evolutionary games to shed further light on financial contagion using simulation techniques. The investigation of market characteristics leading to financial contagion will contribute to recognising at an earlier stage financial crises that potentially destabilise cross-market linkages. Such information would be extremely valuable to develop appropriate risk management strategies.

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