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# **A Multifactor Consumption based Asset Pricing Model of the UK Stock Market: The US Stock Market as a Wealth Reference.**

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## **Abstract**

Here a multifactor model of UK stock returns is developed, replacing the conventional consumption habit reference by a relation that depends on US wealth. Two step Instrumental Variables and Generalized Method of Moments estimators are applied to reduce the impact of weak instruments. The standard errors are corrected for the generated regressor problem and the model is found to explain UK excess returns by UK consumption growth and expected US excess returns. Hence, controlling for nominal effects by subtracting a risk free rate and conditioning on real US excess returns provides a coherent explanation of the equity premium puzzle.

Keywords: Consumption-CAPM, Excess Returns, Generated Regressor, GMM, Habits, Wealth Reference

JEL Classification: C52, E44, G12

# 1 Introduction

Traditional Consumption-based Capital Asset Pricing Models (C-CAPM) have a record of poor performance in describing the relationship between returns and consumption growth. One important reason for the failure of C-CAPM is that consumption itself is not a good state-dependent variable. There is now a substantial body of literature that has documented alternative models that attempt to find better indicators, in particular, Campbell (1996, 2002) and Campbell and Cochrane (1999, 2000). In principle, these models try to remove the smoothness of consumption via the inclusion of habit formation or allowing for time varying risk aversion such that consumption is sufficiently volatile to effectively explain the co-variation between the real economy and financial markets. Furthermore, asset prices are often normalized by aggregate inflation, even though inflation series prior to 1980 are often viewed as non-stationary and after that would seem to exhibit long memory.

This article revisits the explanation of stock performance driven by a consumption habit reference and find that a more appropriate comparator for asset pricing is an external wealth reference. This gives rise to a multi-factor C-CAPM model that is then applied to the UK and compared with extended C-CAPMs based on internal and external consumption habits. Recent experience and empirical work suggests via globalization that stock prices are inter-related, while the dynamics of consumption behaviour is more complicated than the simple habit explanation would have it. In particular, the US market can be regarded as the good proxy for the “world” market. This would suggest that it makes both theoretical and practical sense to draw together, a consumption based and external wealth based explanation of UK asset prices. Given that world interest rates are also highly inter-dependent and a monetary environment driven by the need to control inflation, the rate of return is normalized by a measure of the risk free rate. As a result, for the period consider, the excess return can be viewed as a real rate of return and has statistical properties directly comparable with similar data for the US.

The rest of this paper is organised as follows. Section 2, contains a review of C-CAPM with consumption habit and the derivation of a generalised two-factor Consumption-CAPM driven by wealth reference. Section 3 consider the methodology

used to estimate such models with UK data, section 4 and 5 report the data descriptive statistics and results. Section 6 sets out the conclusions.

## 2 Consumption based Asset Pricing Models

The conventional C-CAPM theory introduced by Lucas (1978) and Breeden (1979) has been tested extensively on data for both the US and a wide range of other countries. However, the results associated with this research have been largely negative.<sup>1</sup> The failure of the C-CAPM has led to a range of alternative models intended to solve the problem. For example, Evan and Hasan (1998) have considered a finite-horizon C-CAPM, Gregoriou and Ioannidis (2007) have suggested the problem lies in market microstructure effects driven by transaction costs, while Smoluk and Vander-Linden (2004) extend the C-CAPM to take account of a US consumption reference.<sup>2</sup>

The various solutions to these empirical puzzles, have attempted to maintain a Constant Rates of Risk Aversion (CRRA) by incorporating habits or referencing current behaviour on past consumption. The notion that rational consumers wish to maintain their consumption position relative to some reference was first considered by Dusenberry (1949). In the context of asset pricing models these ideas were re-visited in a choice theoretic framework by Abel (1990), who has claimed that consumers are creatures of habit, and want to maintain their relative living standards, as measured by their capacity to continue to purchase a basket of consumption goods. Dusenberry describes this as a ratchet effect where by the utility of a current basket is viewed as being relative to the previous basket enjoyed by the household. Or in the aggregate, consumption today is seen relative to consumption in the past consumption. Abel (1990) calls this behaviour an “external habit” or “catching up with the Joneses”. In comparison, individual behaviour relative to current per capita consumption is called “internal habit” or “keeping up with the Joneses”.

The notion of consumption habits developed by Abel (1990) has received some degree of support. However, the appropriate reference level to be used for comparison

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<sup>1</sup> See Gregoriou, Hunter and Wu (2007) for example.

<sup>2</sup> See also the excellent survey by Mehra (2003) of earlier alternative models.

by the representative agent is still not easy to determine. More specifically, Campbell (2001) argues that the ratio of consumption relative to average per capita consumption used for habit utility in Abel (1990) can only explain constant risk aversion by an agent, because they prefer a habit function that includes the difference in consumption levels. To this purpose, Campbell and Cochrane (1999) develop a consumption-based model derived from a habit-formation economy, where the consumption-surplus ratio is defined as the extent to which the current level of consumption exceeds habit based consumption. It is this form of consumption reference that can give rise to cyclical variation in expected returns and volatility. Campbell and Cochrane (2000) use this ratio extensively to examine different forms of the CAPM and conclude that the poor performance of the C-CAPM is due to the low unconditional correlation between consumption growth and other state variables such as the price–dividend ratio.

The above results suggest that a state-dependent (conditional, reference level) C-CAPM is likely to perform better than the standard (state-independent or unconditional) C-CAPM. Campbell and Cochrane (1999) suggested that a good state-dependent variable that derives from the external habit-preference model is the log surplus consumption ratio, which is further proved by Li (2001) to perform almost as well as the finite-horizon, linear habit version of the model derived by Campbell and Cochrane (1999). Li (2001) analyses this type of model for the US, while Li and Zhong (2004, 2005) provide similar evidence for other national stock markets. Lettau and Ludvigson (2001) consider the log consumption-wealth ratio. Furthermore, Jacobs and Wang (2004) produce similar findings to Campbell and Cochrane (2000) when they add as an extra facto to the C-CAPM, cross-sectional consumption variation to capture the possibility of idiosyncratic risk.

Thus far, the external consumption reference addressed by the types of state-variables considered above is better able to capture time-varying returns, since they eliminate the effect of the representative agent's habit preferences in the model. However, they all neglect the possible inter-relatedness between major world stock markets. It is well-known that the world's major stock markets are at least partially integrated in the globalised economy and there is increasing evidence of common real dynamics

(Engsted and Tanggaard, 2004).<sup>3</sup> In particular, since the introduction of co-integration, there has been a vast literature on international co-movement of financial markets and co-movement of economic fundamentals over long sample periods.<sup>4</sup> The work on market co-movement is largely explored using data for the UK and the US, that has highlighted that the UK market is strongly affected by the US.

Excepting, for the impact of large shocks, the notion that stock prices are inter-related does not seem obvious and once one moves away from effectively functioning highly capitalized markets the evidence in support of cointegration seems thin. However, if the observation of time-varying expected returns from developed countries' equity markets is consistent, with a world, consumption based asset pricing model with habits, then it is also more likely that the utility function of consumers/investors in these countries is also time-varying where the time variation may depend on the performance of the "world" market. Hence, agent decisions on consumption and savings may in turn depend on the world and based on the level of integration of the UK in global capital markets this type of explanation would appear particularly pertinent. Here, two factors are considered that might influence a UK based C-CAPM: one is excess returns on the US stock market and the other is the aggregate habit preferences of UK consumers. The former affects the movement of the returns of economic agents in their domestic market and thus influences their decisions about consumption allocations. The latter implies that consumers/investors have to strive to close any gap in living standards in an attempt to maintain their own consumption levels for the preceding period.<sup>5</sup>

However, it may not be appropriate to treat US excess return as exogenous to UK agent behaviour. Firstly, there are unobserved traits such as shocks that might affect both UK and US series, hence the error sequences will not be independent. Otherwise, one might view US consumption growth as a more appropriate proxy for this variable (see Li and Zhong, 2004). However, Gregoriou, Hunter and Wu (2008) suggest that

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<sup>3</sup> See Dumas and Solnik (1995) and De Santis et al (1997, 1998) for some international asset pricing examples.

<sup>4</sup> For example, see Kasa (1992) and Engsted & Lund (1997) and Beltratti and Shiller (1993), among others.

<sup>5</sup> Smoluk & Vander-Linden (2004) test for an international version of C-CAPM with local consumption catching up with the Americans, and the poor performance of this model suggests that the consumptions between countries are little correlated.

although the US stock market is affected by the real domestic economy, this effect is dominated by the reverse impact of stock market windfalls on US consumption growth. Hence, US excess returns might be consumption based, but the relationship is interdependent and as a result US consumption growth is not an appropriate proxy for US wealth. Also given the timing differences in the opening of the two markets, it would appear more pertinent to explain UK excess returns by the expectation of US returns or some sort of long-term average. In what follows the C-CAPM is considered with both habit and external wealth references.

## 2.1 Consumption-CAPM

The conventional C-CAPM theory relates asset prices to the economic agent's consumption and portfolio decisions over time. The asset pricing model follows from maximizing agent utility over time:

$$E_t \left[ \sum_{k=0}^{\infty} \mathbf{b}^k U(C_{t+k}) \right]$$

subject to the intertemporal budget constraint  $W_{t+1} = (W_t - C_t) \sum_{i=0}^N x_i R_{it+1}$ . The solution

to the problem gives rise to a first-order Euler equation:

$$1 = E_t \left[ M_{t+1} (1 + r_{t+1}^e) \right], \quad (1)$$

where  $r_{t+1}^e$  is the excess returns on risky assets over risk-free rates, and

$M_{t+1} = \mathbf{b} \left[ \frac{U'(C_{t+1})}{U'(C_t)} \right]$  is a Stochastic Discount Factor (SDF) or the Intertemporal

Marginal Rate of Substitution (IMRS). It is common to make this problem operational for a single representative agent by selecting a specific utility function. A common specification in the C-CAPM literature is the power utility function,

$$U(C_t) = \frac{C_t^{1-g} - 1}{1-g}.$$

One implication of this choice is that  $g$ , defines a rate of Constant Relative Risk Aversion (CRRA). Unfortunately, the C-CAPM model with a power utility function does not seem to satisfy the data (Kocherlakota 1996, Campbell & Cochrane 2000).

### 3.2 The Utility Function with Consumption Habit Revisited

Dusenberry (1949) first suggested a reason for the observed inertia in consumption data based on a ratchet in aggregate consumption. This can be derived as a feature of optimal dynamic consumption and investment policy with extreme habit formation that prevents consumption from falling over time. This concept is what has entered the utility literature that then drives the habit based C-CAPM.

Thus far the appropriate reference has been seen as past consumption, rather than some form of external wealth. However, there is clear evidence that consumption is driven by stock market wealth.<sup>6</sup> Thus far the literature on globalization and contagion has not paid specific attention to the underlying choice problem that might give rise to home asset pricing decisions driven by external current or future values of external asset prices. Here it is suggested that this arises via an external wealth reference. This is because of the need for investors to gauge their investment performance. In fact, many investors do not participate in markets directly, they do this via fund managers, who are usually required to hedge risk and perform on average better than the market. It is increasingly the case that UK assets are traded on the US stock markets and that fund managers diversify risk by holding assets from other markets. Therefore, they use a variety of benchmark indices to gauge the performance of their funds.

In order to extend the external consumption habit model to incorporate an external stock market wealth reference, we apply a simple Cobb–Douglas power type utility function:

$$U(C_t, X_t) = \mathbf{b} \frac{C_t^{1-g_1} X_t^{g_2} - 1}{1-g_1}. \quad (2)$$

$X_t$  is the level of the habit reference usually determined externally. For example, for the consumption reference considered by Abel (1990),  $X_t = C_{t-1}$ . However, instead of using past consumption as has occurred in the literature, an external wealth reference  $W_t$  is used here. Therefore:

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<sup>6</sup> See Gregoriou, Hunter and Wu (2008) for example.



$$U(C_t, W_t) = \mathbf{a}\mathbf{b} \frac{C_t^{1-g_1} W_t^{g_2} - 1}{1-g_1}. \quad (3)$$

$\mathbf{a}$  is an implicit discount factor for the external wealth reference associated with the conventional consumers' optimisation problem and the corresponding pricing kernel is:

$$M_{t+1} = \mathbf{a}\mathbf{b} \left( \frac{C_{t+1}}{C_t} \right)^{-g_1} \left( \frac{W_{t+1}}{W_t} \right)^{-g_2}. \quad (4)$$

Due to the dominant role of the US stock market in the global stock markets,<sup>7</sup> we choose a US stock index as a proxy of this wealth reference and test whether this is the factor that drives average non-US investor optimising behaviour.

If we denote  $r_{US,t}^e$  as the excess return at the time  $t$ , the following equality can be satisfied:

$$\frac{W_{t+1}}{W_t} = 1 + r_{US,t+1}^e.$$

Therefore

$$M_{t+1} = \mathbf{a}\mathbf{b} \left( \frac{C_{t+1}}{C_t} \right)^{-g_1} (1 + \hat{r}_{US,t+1}^e)^{-g_2}. \quad (5)$$

In practice, we use  $\hat{r}_{US,t+1}^e$ , which is an estimate of  $r_{US,t+1}^e$  based on the information available at the time  $t$ . Specifically,  $r_{US,t+1}^e$  can be proxied by expected returns on the S&P500 index. It follows for (5) to be consistent with agent rationality that  $\mathbf{a}, \mathbf{g}_1, \mathbf{g}_2$  are all positive. Unfortunately, the subjective discount factor  $\mathbf{b}$  in this model is not identified (Sargan, 1983). Following, Gregoriou and Ioannidis (2007) we set  $\mathbf{b} = 0.99$ .

### 3 The Methodology

One advantage of a wealth reference over consumption habit formation is that the wealth effect proxied by the US stock market index can capture transitory innovations as well as permanent shocks (Lettau and Ludvigson, 2004). Given the extreme

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<sup>7</sup> This ratio is 44% from IMF annual Report (2006).

volatility in stock prices, consumption-smoothing households may not want to vary their consumption to react to daily, monthly, or even yearly equity price movements. Thus an external wealth reference already captures an external consumption habit.<sup>8</sup>

As US excess returns are viewed as being endogenous we require some form of systems estimator. As the focus is on UK market behaviour and the feedback is viewed as being unidirectional we have restricted ourselves to Instrumental Variables (IV) and Generalized Method of Moments (GMM) estimators. It is reasonable to consider that UK information on consumption and returns might define reasonable forcing variables for UK consumption growth, but this is unlikely to be the case for US excess returns. In the light of the weak instrument problem (Stock, Wright and Yogo, 2002) we apply a two step estimator to this part of the problem. Hence, future excess returns are estimated from the model of US excess returns developed by Gregoriou, Hunter and Wu (2008). This has the advantage that excess returns are explained by a well specified model that depends on the key relations driving the US economy. However, the two step approach, gives rise to inconsistency in the conventional estimate of the standard error that can be corrected either by the bootstrap or direct calculation of an appropriate asymptotic estimator.<sup>9</sup> Indeed, such a problem will always arise when the generated variable is correlated with the residuals ( $E(\hat{r}_{US,t}^e \mathbf{n}_1) \neq 0$ ), this may be the result of omitted explanatory variables or unobserved factors in the regression. This can also be caused by the dynamic process generating the regressors, so when the residuals are not serially correlated, nor heteroscedastic, and are normally distributed, then the bias may be small. Such generated variables are potentially useful, since good instruments are often difficult to obtain and this is particularly the case for return data. As is common in the case of IV regressions, the simple choices of different lagged values of returns may not be sufficient to describe the current behaviour of the variable. Moreover, the regressor

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<sup>8</sup> The similar three-factor model has also been tested but the coefficient of consumption habit is not statistically significant.

<sup>9</sup> The predicted variable is called “generated regressor” (see Murphy and Topel, 1985). However, bootstrap tests have to be considered carefully before applying them to two stage regression models since the impact of the residuals of the first step regression cannot be neglected. Further, caution is necessary for GMM bootstrapping, where inference can be biased, because the bootstrap estimates are based on an empirical distribution function that implements a moment condition that does not necessarily hold in the population of bootstrap samples (Hall and Horowitz, 1996). Moreover, even after the adjustment of the moment conditions, biases in the augmented GMM bootstrap is reduced, but not eliminated.

selected here is consistent with the more fundamental view that rational expectations are model generated, where the model attempts to explain the complex inter-relations associated with the inter-action between financial and real sectors of the economy.

### 3.1 Extended C-CAPM Equations

Inserting (5) into (1) results in the following orthogonality condition:

$$E_t \left[ \left(1 + r_{UK,t+1}^e\right) \mathbf{a} \mathbf{b} \left(\frac{C_{t+1}}{C_t}\right)^{-g_1} \left(1 + \hat{r}_{US,t+1}^e\right)^{-g_2} \right] = 1. \quad (6a)$$

Eq. (6a) is a nonlinear form of the generalised C-CAPM. If the error is viewed as being multiplicative or the joint distribution of consumption and returns log-normal, then taking logs of (6a) gives rise to the following model.<sup>10</sup>

$$r_{UK,t+1}^e = -\log \mathbf{a} - \log \mathbf{b} + g_1 c g_{UK,t+1} + g_2 \hat{r}_{US,t+1}^e + \mathbf{e}_{t+1}. \quad (6b)$$

It is convention for (6a) and (6b) to be evaluated using expectations based on information available at time  $t-1$  instead of time  $t$ . Therefore:

$$r_{UK,t}^e = -\log \mathbf{a} - \log \mathbf{b} + g_1 c g_{UK,t} + g_2 \hat{r}_{US,t}^e + \mathbf{e}_t. \quad (7a)$$

$$E_{t-1} \left[ \left(1 + r_{UK,t}^e\right) \mathbf{a} \mathbf{b} \left(\frac{C_t}{C_{t-1}}\right)^{-g_1} \left(1 + \hat{r}_{US,t}^e\right)^{-g_2} \right] = 1. \quad (7b)$$

The differences between linear and nonlinear models only relates to nature of the econometric methodology and the linear approximation. As the expectation is conditional on information at time  $t-1$ , then we use the expected value of the dynamic equation explaining excess returns from Gregoriou, Hunter and Wu (2008), excluding the influence of current variables such as the dummies that capture the effect of large outliers related to the stock market crash of 1987 and the Asian markets crisis.

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<sup>10</sup> Similar Log-linear Consumption-based CAPMs (LCC) have already been reported in the finance literature, for example, Chen et al (1986) and Breeden, Gibbons and Litzenberger (1989), and also been extended into time-varying models, i.e. Hodrick and Zhang (2001)

### 3.2 Correcting the Equation and Coefficient Variance

Although the generated variable may be econometrically plausible, in the sense that the innovations are white noise, the standard errors of the coefficients are not correct. Consider the variance-covariance matrix of the parameters from the IV/GMM estimator in the two-stage regression:

$$\text{v\ddot{a}r}(\tilde{\mathbf{b}}) = \tilde{s}^2 (X^* P_u X^*)^{-1}$$

where  $\tilde{s}^2 = \frac{\tilde{\mathbf{e}}' \tilde{\mathbf{e}}}{n}$  is the sum of the squared residuals  $\tilde{\mathbf{e}}$ . For linear models

$\tilde{\mathbf{e}} = Y - X^* \hat{\beta}$  and for nonlinear ones,  $\tilde{\mathbf{e}}$  is calculated by some (possibly nonlinear) orthogonal function of the parameters and a set of instrumental variables  $Z$ .  $X^*$  are explanatory variables including any generated regressors.  $P_u$  is the orthogonal projection matrix of the instrument variable set. For IV estimation  $P_u = Z(Z'Z)^{-1}Z'$ , and for linear GMM,  $P_u = ZWZ'$  where  $W$  is a weighting matrix. To obtain the optimal GMM estimator,  $W$  is required to be the inverse of the variance-covariance matrix of the moment conditions  $g(\hat{\mathbf{b}}) = \frac{1}{n} E(Z' \tilde{\mathbf{e}})$ , that is  $W = S^{-1}$  and  $S \equiv \frac{1}{n} E(Z' \tilde{\mathbf{e}} \tilde{\mathbf{e}}' Z)$ .

Consequently, the standard error of the  $i^{\text{th}}$  coefficient is

$$SE(\hat{\mathbf{b}}_i) = \tilde{\mathbf{e}} / \mathbf{I}_{ii}$$

where  $\mathbf{I}_{ii}$  is the  $i^{\text{th}}$  diagonal value of  $(X^* P_u X^*)^{-1}$ . The conventional estimate of the residual variance is calculated as

$$\text{v\ddot{a}r}(\hat{\mathbf{b}}_{IV}) = \hat{s}^2 (X' P_u X)^{-1}.$$

$\hat{s}^2 = \frac{\hat{\mathbf{e}}' \hat{\mathbf{e}}}{n}$ ,  $\hat{\mathbf{e}} = Y - f(X, \hat{\beta})$ , and  $X$  are explanatory variables including the actual values corresponding to the generated regressors. That is, to correct the bias in the standard errors, we need to calculate  $SE(\hat{\mathbf{b}}_{IV,i})_{BC}$  that are based on residuals computed using actual values of variables instead of the generated ones. Comparing the two formulae above, the standard errors are correct to a factor that relates to the

differential in the squared residuals, when  $(X^{*'} P_w X^*)^{-1}$  and  $(X' P_w X)^{-1}$  asymptotically converge to the same limit. The latter requirement is satisfied when the instruments are stationary and residuals of the first step regression have the normal as their limiting distribution. Then the corrected standard errors are given by rescaling using the factor  $\sqrt{\tilde{e}' \tilde{e} / \hat{e}' \hat{e}}$ :

$$SE(\hat{\beta}_{IV})_{BC} = SE(\hat{\beta}_{IV}) \times \sqrt{\tilde{e}' \tilde{e} / \hat{e}' \hat{e}}. \quad (8)$$

Prior to any analysis we consider the properties of the data.

## 4 Data Description

We use the same seasonally adjusted aggregate consumption expenditure data  $C_{UK,t}^N$  for the UK as Gregoriou and Ioannidis (2007), and for comparison, seasonally adjusted US personal consumption expenditure data  $C_{US,t}^N$ . The UK FTSE100 index and the 3-month UK government Treasury bill rate are used respectively as the risky asset returns  $r_{UK,t}$  and risk-free rate of return  $r_{UK,t}^f$ . US excess returns are calculated from actual returns  $r_{US,t}$  on SP500 index less the returns  $r_{US,t}^f$  on 3-month US Treasury Bills. The expected values are measured using fitted values  $(\hat{r}_{US,t}^e)$  of US excess returns generated by the system of equations estimated by Gregorious, Hunter and Wu (2008). Nominal consumption data  $C_t^N$  have been deflated by the CPI index  $p_t$ , and for this purpose, we set the  $p_t$  over the period 1980:01 as the base value. Then real consumption is denoted  $C_t$  and continuously compounded consumption growth  $cg_t$ . All the data are monthly series for the period 1980:01-1999:12, which is the extant monthly consumption data available from the Office for National Statistics (ONS), while for estimation we have used the sample 1983:01-1999:12. Table 1 reports the correlations between these variables.

It should be noted that based upon the Table 1, it would appear that volatility in UK excess returns would seem to be transmitted from the US stock market as is indicated by the strong correlation between the two markets. For instance, the correlation

**Table 1 Correlations of Excess Returns and Consumption Growth for both the UK and the US**

	$r_{UK}^e$	$r_{US}^e$	$\hat{r}_{US}^e$	$cg_{UK}$	$cg_{US}$
$r_{UK}^e$	1				
$r_{US}^e$	0.735	1			
$\hat{r}_{US}^e$	0.499	0.546	1		
$cg_{UK}$	0.044	0.054	0.016	1	
$cg_{US}$	0.054	-0.074	0.064	0.130	1

coefficient between UK excess returns and US excess returns is 0.75, and even that associated with our estimates of the conditional expectations of the mean of US excess returns is close to 0.50. Further, such volatility transmission can be readily detected through extreme observations such as those associated with the stock market crash in October 1987 and the Asian markets crisis. Consequently it would seem necessary to account for this co-movement of returns as an explanatory variable in any UK asset pricing model. This can be compared with the weak association of consumption growth between the two countries (0.130), which suggests that the representative agent from the UK might be less likely to share common consumption habit behaviour with similar agents in the US and thus base consumption and asset pricing decisions without direct reference to US consumer behaviour.<sup>11</sup>

There is an issue of stock market timing that is related to the expectations of returns on extreme observations, since these shocks are not predictable. To purge the equations of the influence of these extreme observations the expectations are calculated only using information available before time  $t-1$ .<sup>12</sup> Thus, the new fitted values of the observations associated with 1987 stock market crash and the Asian Crisis are obtained as 0.019257 and 0.026543, respectively.

<sup>11</sup> This may be evidence that the idea of catching up with American consumption as suggested in Smoluk and VanderLinden (2004) is not supported by our analysis.

<sup>12</sup> Nevertheless, shock dummies are necessary for of US asset pricing models, again since the shocks are unforeseeable and thus have to be excluded when we want rational expectations.

## 5 Empirical Results

In this section, we consider the two-factor C-CAPM models (7a) and (7b), we present results based on a range of different instruments and then turn to the correction of the standard errors using (8).

Both IV and GMM can be estimated by linear and non-linear procedures. Underlying the early treatment of IV is the notion that the error process is driven by measurement error and that this relates ostensibly to well defined structures (Sargan, 1959), but more recently this distinction between errors driven by shock and measurement error has been diluted (Arrelano, 2002). If we consider linear IV estimators, then the key criterion is that the moment matrix of the data has full rank, the moment matrix has a limit and the cross moment matrix for the right regressors and the instruments has a limit (Sargan, 1988). Hence, minimising the IV problem depends on the nature of the instrument set used, more specifically they ought to be stationary and appropriately dimensioned. Should serial correlation be an issue then this might preclude the use of certain types of lagged information. On the basis of selecting an optimal set of instruments (Sargan, 1959), an efficient estimator will yield consistent parameter estimates that are asymptotically normal and yield conventional inference on parameters and on the specification of the model (Sargan, 1988).

Sargan first moved to describe the econometric problem in terms of a set of moment conditions that might be viewed as some form of sufficient statistics for the underlying the Data Generation Process (Arrelano, 2002). Although Sargan (1959) extended the IV estimator to the non-linear form, it is now more usual to estimate non-linear models by GMM. Hansen (1982) extended this use of moment conditions to the non-linear context to make estimation robust to serial correlation and heteroscedasticity. Although, IV and GMM have removed the need to specify likelihood functions and systems of equations, the penalty associated with this emphasis on consistent estimation is often bought at a cost. In the first instance this relates to efficiency and Davidson and MacKinnon (1993) warn that it makes little sense to base inference on inefficient estimators as such inference is significantly more difficult. It should also be born in mind that this might not be rectified via application of the semi-parametric bootstrap. Also in the context of complex

expectational models, discarding structure may give rise to a fundamental loss of identification (Hunter and Ioannidis, 2000).

Therefore, we emphasise the use of GMM to estimate the non-linear first-order conditions of C-CAPM. As firstly there is no requirement for the data to be stationary. Secondly the linear representation forces the conditional covariance between returns and marginal rates of substitution to be constant through time, while non-linear GMM does not impose this restriction. Thirdly, the error process can be autocorrelated and conditionally heteroscedastic.

Removing the expectations reveals an error in variables problem that is solved either via IV (Sargan, 1958) or GMM estimation (Hansen, 1982). The chosen instrument sets are quite different between linear and nonlinear models. For the linearised models we not only include the explanatory variables, but also lagged US excess returns ( $r_{US,t}^e$ ). For the nonlinear models, instruments are gross excess returns on both markets: the expected US gross excess returns ( $\hat{r}_{UK,t}^e$ ) and the UK gross consumption growth ( $cg_{UK,t}$ ). The lag length for each instrument set is 2, 4, 6 and 12.<sup>13</sup> In order to see the predictive power of the variable  $\hat{r}_{US}^e$ , we also include it in the instrument set even when it is not a regressor in the model.<sup>14</sup> To choose the best model, economically and econometrically, we apply several tests to the residuals, namely, where appropriate, Box-Pierce tests of the autocorrelation structure in the residual correlogram, LM test for serial correlation, LM test for ARCH and the Jarque-Bera test for normality.

The normality test is quite common in econometrics and it should be addressed here as an important criterion for model selection. In particular, it can be used as a means of detecting omitted variables or unobserved variables in the regression. However, such a test has always been neglected in the C-CAPM literature that seems just to select a decent model based upon the J-test (Hansen, 1982) and t-tests. The J-test has already been demonstrated to be a weak model criterion since it is only considers

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<sup>13</sup> The similar lag length for IV estimation was suggested by Hansen and Singleton (1982), among others. In order to meet the over-identification restrictions, we here use the lag 12 instead of the lag 1.

<sup>14</sup> Applying generated instruments will not lead to inconsistency of 2SLS estimates (Wooldridge, 2002, pp. 117), provided that they are not correlated with the residuals.



whether the instruments can be accepted based on acceptance of a set of over-identifying restrictions, but this is not a direct test of the models specification. It should be noted that the expected US return series already take the major shocks into account and as a result when they are included in the UK model, they should help to generate well defined models with normally distributed residuals.<sup>15</sup>

Next we consider in Table 2 results that relate to the C-CAPM with habit preferences using a number of different instrument sets. These dominate the conventional C-CAPM without habits, but do not explain the non-normality and also the coefficient on the habit preference is not significant, similar results arise when the habit reference is external.<sup>16</sup>

**Table 2 IV and GMM estimates for the UK C-CAPM with habit preferences**

	NLAG	Constant	$g_1$	$-g_2$	DF	$C^2$	Test for Serial Correlation	Test for ARCH	Test for Normality
IV	2	-.9E-5(.99)	3.336(.45)	-1.239(.47)	2	1.98(.37)	Non at 10%	***	275(0)***
	4	-1.5E-5(.99)	3.259(.24)	-1.211(.21)	6	2.87(.82)	Non at 10%	Non at 10%	276(0)***
	6	.002(.65)	1.490(.18)	-.560(.37)	10	8.37(.59)	Non at 10%	“	515(0)***
	12	.002(.52)	.964(.11)	-.367(.48)	22	17.1(.76)	Non at 10%	“	573(0)***
GMM	2	-.004(.35)	-.276(.93)	.058(.97)	2	3.33(.19)	Non at 10%	“	560(0)***
	4	.003(.42)	1.546(.46)	-.502(.61)	6	5.37(.50)	Non at 10%	“	511(0)***
	6	.006(.06)*	1.493(.13)	-.214(.70)	10	10.8(.37)	Non at 10%	“	527(0)***
	12	.006(0)***	.986(.01)***	.202(.51)	22	17.0(.76)	Non at 10%	“	586(0)***
GMM	2	.996(0)***	.290(.93)	-.065(.96)	2	3.33(.19)	Non at 10%	“	563(0)***
	4	.997(0)***	1.694(.46)	.539(.59)	6	5.21(.52)	Non at 10%	“	470(0)***
	6	.994(0)***	1.476(.15)	.196(.71)	10	10.9(.37)	Non at 10%	“	507(0)***
	12	.994(0)***	0.974(.01)***	-.193(.52)	22	17.1(.76)	Non at 10%	“	566(0)***

Notes: Standard errors are corrected for autocorrelation and heteroskedasticity in the IV estimation, and p-values are given in parenthesis. Dynamic tests are carried out up to 12 lags for residuals, and relate both to the LM test and the Box-Ljung Q statistics for the IV estimation, and only the Box-Ljung Q statistics for the GMM estimation. The instruments are the constant and the lagged explanatory variables plus the UK excess returns up to the lag  $n=NLAG$ , and the test for the validity of overidentifying restrictions are given by Sargan's test for the IV estimation and Hansen's J-test for the GMM estimation. \*,\*\*,\*\*\*: Statistically significant at the 10% level, 5% and 1%, respectively. Number of observations used 204.

<sup>15</sup> However, normality test cannot be met for regressions with the US returns corrected for market timing.

<sup>16</sup> We also looked at other C-CAPM models for the UK, in both linear and nonlinear forms: the traditional. These models either perform as poorly as the habit preference model or worse than our final model with the US wealth reference.

The results in table 2 provide some evidence in favour of a linear version of C-CAPM models, but these results are not robust to the specification and are very sensitive to the inclusion of the habit reference that appears not to be significant for any of the cases estimated. There are also quite considerable shifts in the coefficients for the non-linear GMM models. The errors though generally uncorrelated are not normal that might call into question any of the inference on such models. Certainly the error bands are likely to be greater than those ordinarily considered.

In the next table the model based on the US wealth reference is considered.

**Table 3 IV and GMM Estimates of the C-CAPM for the UK**

Notes: The subjective discount factor is restricted to assume the value of  $\beta = 0.99$ . Standard errors are corrected for autocorrelation and heteroscedasticity in the IV estimation, and p-values are given in parenthesis.

		NLAG	$-\log \mathbf{a}$	$\mathbf{g}_1$	$\mathbf{g}_2$	DF	$\mathbf{c}^2$	Test for Serial Correlation	Test for ARCH	Test for Normality
Linear Two-factor C-CAPM <sup>†</sup>	IV	2	-.013(.45)	1.541(.21)	.943(0)***	5	9.75(.08)*	Non at 10%	Non at 10%No	.05(.98)
		4	-.012(.50)	1.237(.19)	.945(0)***	11	15.0(.18)	Non at 10%	“	.05(.97)
		6	-.012(.64)	.663(.34)	.948(0)***	17	20.6(.24)	Non at 10%	“	.11(.95)
		12	-.012(.59)	.680(.22)	.948(0)***	35	30.2(.70)	Non at 10%	“	.10(.95)
	GMM	2	-.008(.60)	.832(.25)	.696(0)***	5	7.10(.21)	Non at 10%	“	1.93(.38)
		4	-.007(.23)	.205(.70)	.629(0)***	11	11.9(.37)	Non at 10%	“*	6.20(.04)**
		6	-.009(.91)	.218(.56)	.899(0)***	17	16.6(.48)	Non at 10%	“	.54(.76)
		12	-.018(0)***	1.065(0)***	1.067(0)***	35	29.5(.73)	Non at 10%	“	.25(.88)
Nonlinear Two-factor C-CAPM <sup>††</sup>	GMM	2	1.008(0)***	.8556(.26)	.702(0)***	5	7.11(.68)	Non at 10%	“	4.98(.08)*
		4	1.007(0)***	.204(.70)	.608(0)***	11	11.9(.37)	Non at 10%	“	.17(0)***
		6	1.010(0)***	.225(.55)	.915(0)***	17	16.9(.46)	Non at 10%	“	.66(.72)
		12	1.018(0)***	1.039(0)***	1.094(0)***	35	29.6(.73)	Non at 10%	“	.61(.74)

<sup>†</sup>:For the linear C-CAPM, dynamic tests are carried out up to 12 lags for residuals, and are reported by both the LM test and the Box-Ljung Q statistics for the IV estimation, and only the Box-Ljung Q statistics for the GMM estimation. The instruments are the constant and the lagged explanatory variables plus the UK excess returns up to the lag  $n = \text{NLAG}$ , and the test for the validity of overidentifying restrictions are given by Sargan's test for the IV estimation and Hansen's J-test for the GMM estimation. \*, \*\*, \*\*\*: Statistically significant at the 10% level, 5% and 1%, respectively.

If we consider the results in Table 3, in no case can the null of normality be rejected even at the 10% level, also the alternative hypotheses of serial correlation and ARCH behaviour in the residuals cannot be accepted. Further, the coefficients on the US return habit are all significant ranging from 0.943 to 0.948 for IV estimation, and from 0.696 to 1.067 for GMM estimation. However, estimates of the constant and risk

aversion coefficients are all insignificant except for the non-linear GMM case with 12 lags in the instrument set.

According to model selection criteria, and compared with the results of other specifications, the estimates based on models with 12-lagged instruments should be chosen as the best model. All three coefficients are statistically significant at 1% level (bias unadjusted), and the null hypothesis for normality of the residuals cannot be rejected at 10% level, and there is also no sign of either autocorrelation or ARCH. The coefficient on US excess returns, (1.094) is even bigger than that of risk aversion, (1.039) suggesting that risk aversion associated with investment comparisons made relative to the US stock market cannot be neglected. If agents are engaged in keeping up with the Joneses, they live in the US or more pertinently, a rational investor ought to determine their asset allocations based on the highest returns obtainable across an international portfolio of assets.

Thus far, we have examined all the specifications of the UK C-CAPM model, and for both economic and econometric purposes, the best models appear to be the non-linear model with US expected return preference, using 12 lagged instruments. Generally, the nonlinear models perform better than corresponding linear ones, and they also improve as the order of lags included in the IV sets increases. As far as the econometric methodology is concerned, the non-linear GMM estimator would seem more powerful than linear IV estimation, but no comparison exists with respect to non-linear IV.

It is not surprising that when compared with IV, the GMM estimator performs better, since the former relies heavily on the quality of the instrument set and poor instruments can affect statistical inference. Furthermore, the GMM weights yield a minimum that is as close to zero as is possible, while the weighting matrix defined by GMM is the covariance matrix of the sample moments and this in the limit is the minimum variance estimator in the class of estimators. As both linear and non-linear GMM estimators rely on a different instrument set to IV, they are invariably over-identified, and hence larger covariances terms with respect of the orthogonality conditions associated with the instruments have smaller weights in the objective

function and this implies that the estimator is less sensitive to the selection of the instrument set.

Furthermore, the optimal linear and nonlinear estimation by GMM yields similar coefficient estimates when the same instrument set and sample are used. For example, in the linear model, 0.992, 1.065 and 1.067, compare with 1.008, 1.039 and 1.094 in the nonlinear model. Of course, we cannot decide which model is preferred on the basis of p-values or t-statistics without correcting the standard errors. For this purpose, equation (8) is used and adjusted p-values for one tail t-tests that reflect the theoretical restriction that the signs are positive are reported in Table 4.

**Table 4 IV and GMM Estimates for Optimal UK C-CAPM with Correction for Standard Errors and Market Timing**

Estimates	Constant	$g_1$	$g_3$
Linear-IV	$\log \mathbf{a} = -.0104 (.443)$	.471(.149)	.468(.01)***
Linear-GMM	$\log \mathbf{a} = -.012(.01)$ ***	2.862(0)***	.613(0)***
Nonlinear-GMM	$\mathbf{a} = 1.012(0)$ ***	2.908(0)***	.645(0)***

Note:

One-tail p-values are given in parenthesis.

Three rescaling factors ( $\sqrt{\tilde{e}'\tilde{e}/\hat{e}'\hat{e}}$ ) for standard error correction are 0.758, 0.787 and 0.786, respectively.

\*, \*\*, \*\*\*: Statistically significant level for one-tail test at the 10%, 5% and 1%, respectively.

Table 4 demonstrates that all coefficients are significant at the 1% level when a one-tailed test is applied, and the only two exceptions are the constant and the coefficient of risk aversion in the linear consumption model estimated by IV with 12 lagged instruments. This suggests that on statistical grounds preference might be given to non-linear GMM over the linear IV methodology as the enhanced t-values ought to reflect the relative efficiency of the estimator, when the residuals are heteroscedastic. The similarity of the estimates in the linear and nonlinear GMM cases, suggests that the assumption of log-normality embedded in linear GMM is satisfied and that the conditional covariance between returns and IMRS is constant. If estimating C-CAPM by a non-linear estimator has the virtue of depicting the nonlinearity, then this is not obvious for the relation between returns and IMRS for the UK.

## 6 Conclusion

Many augmented models have been developed to improve the performance of the C-CAPM. In essence, they are trying to find or construct state-dependent variables for consumption that might help remove any excess smoothness or heterogeneity. However, all but a small number of these models neglect external factors, here the co-movement across markets that has arisen with globalisation and this suggests why returns seem to be less directly dependent on consumption growth.

In this article, it is shown that C-CAPM for the UK can be fruitfully extended by replacing the consumption habit by a wealth reference that can be proxied by the US stock market. As a result, it is argued that a primary driver of UK agent behaviour is a US wealth reference. Thus, the Intertemporal Marginal Rate of Substitution depends both on domestic consumption and movements in the US market. Consequently, future US excess returns and the growth rate of the UK consumption are key factors in explaining UK excess returns controlled for the risk free rate. The empirical results suggest that this two-factor model can well explain the equilibrium between UK returns and domestic consumption, since after correction of the standard errors, both linear and nonlinear models reveal the statistically significant and substantial effect of the US market on the UK. Further, this two-factor C-CAPM suggests that it is not external habit effects or comparison with external consumption, i.e. US consumption that has driven C-CAPM models for the UK.

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