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Performance Analysis of a Low Complexity Detector for MCIK-OFDM over TWDP Fading

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Abstract—This letter analyzes the performance of a low complexity detection scheme for a multi-carrier index keying (MCIK) with orthogonal frequency division multiplexing (OFDM) system over two-wave with diffused power (TWDP) fading channels. A closed-form expression for the average pairwise error probability (PEP) over TWDP fading channels is derived. This expression is used to analyze the performance of MCIK-OFDM in moderate, severe and extreme fading conditions. The presented results provide an insight on the performance of MCIK-OFDM for wireless communication systems that operate in enclosed metallic structures such as in-vehicular device-to-device (D2D) wireless networks.

Index Terms—Multi-carrier index keying (MCIK), orthogonal frequency division multiplexing (OFDM), tow-wave with diffuse power (TWDP) fading, Device-to-device (D2D), pairwise error probability (PEP).

I. INTRODUCTION

frequency RTHOGONAL division multiplexing (OFDM) has been adopted by the majority of current wireless communication standards due to its capability of combating inter-symbol interference caused by frequency selective fading channels. Multi-carrier index keying (MCIK) has been recently proposed as a means of improving the efficiency of traditional OFDM [1]. Akin to the spatial modulation (SM) concept, MCIK-OFDM exploits both the M-ary constellations and the sub-carrier indices to transmit data. The use of the sub-carrier indices as an additional degree of freedom enables the transmission of extra information bits without any additional bandwidth and power requirements. Given that MCIK-OFDM activates only a subset of sub-carriers, it leads to a decreased number of modulators/demodulators. This in turn results to low-cost, low-complexity transceiver designs, suitable for OFDM based device-to-device (D2D) wireless systems. Such D2D systems (e.g., in-vehicle camera systems), which will require higher data rates than current D2D systems, are expected to be one of the key components of future wireless networks.

Accurate and efficient sub-carrier detection is a key aspect of MCIK-OFDM. Maximum-likelihood (ML) detection is considered as the optimal detection method and has already been used in MCIK-OFDM [2]. However, for the successful development of MCIK-OFDM D2D wireless systems, which are expected to consist of low-cost, energy constrained nodes, low complexity detection schemes are required. A greedy detector that substantially reduces the computational complexity has been proposed in [3]. Unlike the ML, which searches all possible sub-carrier index combinations and their corresponding constellation symbols, the greedy detector first estimates the active sub-carriers, based on their received power, and then applies ML to estimate the corresponding symbols. Hence, given that one of the main factors that affect the received signal power is the channel fading, the performance analysis of the greedy detector over fading channels is essential.

Compared to traditional wireless systems, D2D systems introduce a new wireless landscape, where machine type sensing devices are expected to operate within non-conventional environments such as enclosed metallic structures (e.g., aircraft, vehicles) and confined urban settings. Such environments have been found to exhibit fading conditions that are not adequately characterized by existing fading models [4]. For example, wireless sensor nodes deployed in metallic cavities have been found to exhibit frequency and spatially dependent fading whose severity exceeds those predicted by the Rayleigh fading model, which is traditionally considered as the worstcase fading scenario for wireless systems.

In this work we examine the potential of a low-complexity detection scheme for MCIK-OFDM over two-wave with diffused power (TWDP) fading channels. A low complexity greedy detector [3] is considered to obtain a simple approximate expression for the pairwise error probability (PEP). Both the exact and approximate closed-form expression for the average PEP over TWDP fading channels are derived. These expressions enable the performance analysis of MCIK-OFDM over different fading scenarios including worse than Rayleigh conditions. The obtained results provide a new insight into the performance of MCIK-OFDM for a variety of non-conventional fading environments such as those of emerging in-vehicular D2D wireless communication systems.

II. SYSTEM MODEL

A. MCIK-OFDM

Let us consider an MCIK-OFDM system with N_c subcarriers that consists of c clusters of N sub-carriers, i.e, $N_c = cN$. In every transmission, only L out of N sub-carriers per cluster are randomly activated to deliver data symbols, while N - L sub-carriers are zero padded. For each cluster, let $\mathbf{I} = \{i_1, ..., i_L\}$, where $i_l \in [1, ..., N]$ for l = 1, ..., Ldenote the indices of L active sub-carriers. The corresponding block of symbols is denoted by $\mathbf{s} = [s_{(1)}, ..., s_{(L)}], s_{(l)} \in S$. Using both \mathbf{I} and \mathbf{s} , an MCIK-OFDM block is generated as $\mathbf{x} = [x(1), ..., x(N)]^T, x(\alpha) \in \{0, S\}$ and $\alpha = 1, ..., N$, where the indices of N - L inactive sub-carriers are employed for additional data transmission.

For such an MCIK-OFDM, the total number of active subcarrier index combinations is $\binom{N}{L}$. For simplicity and efficient mapping of the data bits, $2^{\lfloor log_2 \binom{N}{L} \rfloor}$ combinations are used, where $\binom{a}{b}$ and $\lfloor . \rfloor$ denote the binomial coefficient and the floor function, respectively. In every MCIK-OFDM transmission $m_1 = \lfloor \log_2 \binom{N}{L} \rfloor$ bits are used to modulate the sub-carrier indices and $m_2 = L \log_2 M$ bits are transmitted via the corresponding M-ary signal constellation.

The input-output model of MCIK-OFDM is described as,

$$\mathbf{y} = \mathbf{x}\mathbf{H} + \mathbf{n},\tag{1}$$

where $\mathbf{y} = [y(1), ..., y(N)]$, **x** denotes an MCIK-OFDM signal block, $\mathbf{H} = diag(h(1), ..., h(N))$ where $h(\alpha)$ represents the channel fading coefficient, and $\mathbf{n} = [n(1), ..., n(N)]$ is additive white Gaussian noise (AWGN), i.e., $n(\alpha) \sim C\mathcal{N}(0, N_0)$.

B. TWDP Fading

A parametric family of probability density functions (PDF) that describes small-scale fading in the presence of two dominant multipath components has been derived in [5]. The TWDP PDF is characterized by two intuitive physical parameters of the wireless channel: 1) the ratio between the average specular and diffused power, given as $K = (V_1^2 + V_2^2)/2\sigma^2$, where V_1 and V_2 denote the magnitudes of the two specular waves, and $2\sigma^2$ is the average power of the diffused waves; 2) the relative strength of the two specular waves expressed as $\Delta = 2V_1V_2/(V_1^2 + V_2^2)$. In two special cases, the TWDP PDF reduces to the Rician PDF when $K \neq 0$ and $\Delta = 0$, and to the Rayleigh PDF when K = 0. Furthermore, for K >> 0and $\Delta = 1$ the TWDP PDF describes a fading channel that consists of two multipath components of equal weights. Under such fading conditions the envelope statistics no longer follow a Rayleigh PDF and has been experimentally validated as a worse than Rayleigh fading channel in [4].

The approximate PDF of the fading envelope, r, over a TWDP channel is given in [5] as,

$$f_r(r) = \frac{r}{\sigma^2} e^{\left(-\frac{r}{2\sigma^2} - K\right)} \sum_{i=1}^C w_i D\left(\frac{r}{\sigma}; K, w_i\right)$$
(2)

with,

$$D\left(\frac{r}{\sigma}; K, w_i\right) = \frac{1}{2}e^{(w_i K)}I_0\left(\frac{r}{\sigma}\sqrt{2K(1-w_i)}\right) + \frac{1}{2}e^{(-w_i K)}I_0\left(\frac{r}{\sigma}\sqrt{2K(1+w_i)}\right)$$
(3)

where $I_0(x) = \sum_{k=0}^{\infty} \frac{(1/4x^2)^k}{(k!)^2}$ is the zero order modified Bessel function of the first kind, C is the order of approximation of the TWDP PDF and w_i denotes the corresponding approximation coefficients as per [5, Table II].

By making a change of random variable in (2) the PDF of the instantaneous SNR γ is given by [7],

$$f_{\gamma}(\gamma) = \frac{K+1}{2\bar{\gamma}} e^{-K} \sum_{i=1}^{C} w_i \times \left[e^{w_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} Z(-w_i) + e^{-w_i K} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}} Z(w_i) \right],$$
(4)

where
$$Z(\pm w_i) = I_0\left(2\sqrt{\frac{K(K+1)(1\pm w_i)\gamma}{\bar{\gamma}}}\right)$$
 with γ and $\bar{\gamma}$ denoting the instantaneous and average SNR, respectively.

III. PERFORMANCE OVER TWDP FADING CHANNELS

A. Exact Instantaneous PEP

Let the pairwise error event (PEE) describe the case when an active sub-carrier index is incorrectly detected as the index of an inactive sub-carrier, i.e., $\alpha \to \tilde{\alpha}$ where $\alpha, \tilde{\alpha} \in \{1, ..., N\}$ with $\alpha \neq \tilde{\alpha}$. The conditional PEP can be defined as the probability of having the PEE, $P(\alpha \to \tilde{\alpha})$.

For the greedy detector, the received power of an inactive and an active sub-carrier are given by $r_{\tilde{\alpha}} = \max\{||n(\alpha)||^2\}$ and $r_{\alpha} = ||\sqrt{E_s/N_0}h(\alpha)x + n(\alpha)||^2$, respectively, whereas $\gamma_{\alpha} = E_s/N_0||h(\alpha)||^2$ denotes the instantaneous SNR of an active sub-carrier. For more information on the operation of the greedy detector the reader may refer to [3]. The probability that the greatest received power of a inactive sub-carrier is greater than the received power of an active sub-carrier is expressed as [3],

$$P(\alpha \to \tilde{\alpha}) = 1 - \int_0^\infty [1 - e^{-r_\alpha}]^{N-L} \times e^{-\gamma_\alpha - r_\alpha} I_0 \left(2\sqrt{\gamma_\alpha r_\alpha}\right) dr_\alpha.$$
(5)

For simplicity on the notations, the SNR of the active subcarrier is denoted by γ hereafter.

A closed form expression for (5) is derived in [3] as,

$$P(\alpha \to \tilde{\alpha}) = 1 - \sum_{q=0}^{N-L} {N-L \choose q} \frac{-1^q}{q+1} e^{\gamma \left(\frac{1}{q+1}-1\right)}.$$
 (6)

Note that, for the MCIK-OFDM system with L out of N active sub-carriers, the overall instantaneous PEP is expressed as the miss-detection of sub-carrier indices. Hence, by using the law of total probability, the upper bound for the overall instantaneous PEP is given by,

$$P \le \frac{L}{N} \sum_{\alpha=1}^{N} P(\alpha \to \tilde{\alpha}).$$
(7)

Inserting (6) into (7), we obtain the overall PEP in closed-form but it is not simple to see the impact of N and L on P.

B. Approximate Instantaneous PEP

By using Maclaurin series expansion on the the power term $[1 - e^{-r_{\alpha}}]^{N-L}$ in (5), the instantaneous PEP can be approximated as,

$$P(\alpha \to \tilde{\alpha}) \simeq 1 - e^{-\lambda} \left[\int_0^\infty e^{-r_\alpha} I_0 \left(2\sqrt{\lambda r_\alpha} \right) dr_\alpha - (N-L) \int_0^\infty e^{-2r_\alpha} I_0 \left(2\sqrt{\lambda r_\alpha} \right) dr_\alpha \right].$$
(8)

It can be seen that (8) involves two integrals of the same form, which can be solved by using [6, eq.(6.614-3)]. Hence,

$$P(\alpha \to \tilde{\alpha}) \simeq 1 - e^{-\gamma} \left[\frac{e^{\frac{\gamma}{2}}}{\sqrt{\gamma}} M_{-\frac{1}{2},0}(\gamma) \right] - (N-L) \left[\frac{e^{\frac{\gamma}{4}}}{\sqrt{\gamma}} M_{-\frac{1}{2},0}\left(\frac{\gamma}{2}\right) \right],$$
(9)

where $M_{a,b}(x)$ denotes the Whittaker function.

By substituting (9) into (7) and after some simplification, the overall approximate instantaneous PEP is obtained as,

$$P \le \frac{L}{N} \sum_{\alpha=1}^{N} \frac{N-L}{2} e^{-\frac{\gamma_{\alpha}}{2}}.$$
(10)

It is clearly seen from (10) that P relies on the balance between N and L. Also, it is worth pointing out that (10) is much simpler that (7) at acceptable accuracy level of 3 dB as shown in our analysis in the following section.

C. Average PEP over TWDP fading

The PEP over TWDP fading, $P_{TWDP}(\alpha \rightarrow \tilde{\alpha})$, can be obtained by averaging (6) over the TWDP fading SNR γ . Formally,

$$P_{TWDP}(\alpha \to \tilde{\alpha}) = \int_0^\infty P(\alpha \to \tilde{\alpha}) f_\gamma(\gamma) d\gamma.$$
(11)

Hence, by substituting (4) and (6) in (11),

$$P_{TWDP}(\alpha \to \tilde{\alpha}) = 1 - \sum_{q=0}^{N-L} {N-L \choose q} \frac{-1^q}{q+1}$$

$$\times \int_0^\infty e^{\gamma \left(\frac{1}{q+1}-1\right)} \frac{K+1}{2\bar{\gamma}} e^{\left(-\frac{\gamma(K+1)}{\bar{\gamma}}\right)} \qquad (12)$$

$$\times \sum_{i=1}^C w_i \left[e^{-K+w_i K} Z(-w_i) + e^{-K-w_i K} Z(w_i) \right] d\gamma.$$

Using [6, eq.(6.614-3)], a closed-form expression for the upper bound of the *overall PEP* over TWDP is derived in (13) (see footnote). The details of the derivation are shown in the Appendix. Similarly, the approximate *overall PEP* over TWDP is derived in (14) (see footnote) by using (10) and following the same approach. Notice that P_{TWDP} in (14) is concave with L due to $\frac{\partial^2(14)}{\partial^2 L} < 0$ regardless of K which means that a proper choice of L can decrease (14) for given N and K.

D. Special Cases

1) Rayleigh Fading: Consider that exponentially distributed fading as a special case of the TWDP when K = 0. Hence, by averaging (7) over the Rayleigh fading statistics,

$$P_{Ray} \le L \left[1 - \sum_{q=0}^{N-L} \binom{N-L}{q} \frac{-1^q}{q\bar{\gamma} + q + 1} \right].$$
(15)

The central limit theorem suggests that the existence of infinite uncorrelated multipath components leads to a Rayleigh fading channel. Hence, it can be observed that by setting K = 0, the generalized expression (13) is simplified to (15).

2) *Rician Fading:* Rician fading occurs in the presence of a single dominant specular component. Hence, by averaging (7) over the Rician fading statistics,

$$P_{Ric} \leq L \left[1 - \sum_{q=0}^{N-L} {N-L \choose q} \frac{-1^q (K+1)}{q (\bar{\gamma} + K + 1) + K + 1} \times e^{\frac{K(K+1)(q+1)}{q (\bar{\gamma} + K + 1) + K + 1} - K} \right].$$
(16)

It is observed that by setting $\Delta = 0$ in (13), it reduces to (16).

3) Two-ray Fading: Two-ray fading occurs in the presence of two anti-phase specular components of equal power. Hence, the average PEP over two-ray fading can be obtained by the asymptotic expression of (13), $\lim_{K\to\infty} P_{TWDP}$ for $\Delta = 1$,

$$P_{Two-ray} \le L \left[1 - \sum_{q=0}^{N-L} \binom{N-L}{q} \frac{-1^q}{q+1} \times \sum_{i=1}^{\infty} w_i \left[e^{\frac{\tilde{\gamma}q(w_i-1)}{q+1}} + e^{-\frac{\tilde{\gamma}q(1+w_i)}{q+1}} \right].$$
(17)

IV. NUMERICAL RESULTS AND DISCUSSION

We consider an MCIK-OFDM system with BPSK modulation and $N_c = 128$ sub-carriers with two different configurations $N = \{8, 4\}$ and $L = \{4, 1\}$. The performance of the greedy detector is evaluated over TWDP fading channels in terms of average PEP and effective SNR, i.e., E_s/N_0 on a cluster basis since the PEP is not affected by the number of clusters. The considered fading scenarios have been selected to represent moderate, severe and worse than Rayleigh fading conditions for D2D wireless in enclosed environments.

In Fig. 1 the average PEP over hyper-Rayleigh fading for N = 8 and L = 4 is illustrated. It is shown that as K increases the detection performance deteriorates with 7 dB difference in the required SNR between Rayleigh and TWDP fading with $\Delta = 1$ and K = 10 dB for a target PEP of 10^{-2} . This results

$$P_{TWDP} \le L \left[1 - \frac{1}{2} \sum_{q=0}^{N-L} \binom{N-L}{q} \frac{-1^q (K+1)}{q (K+\bar{\gamma}+1) + K+1} \sum_{i=1}^C w_i \left[e^{\frac{K(K+1)(1-w_i)(q+1)}{q (K+\bar{\gamma}+1) + K+1} - K + w_i K} + e^{\frac{K(K+1)(1+w_i)(q+1)}{q (K+\bar{\gamma}+1) + K+1} - K - w_i K} \right] \right]$$

$$(13)$$

$$P_{TWDP} \le L \left[\frac{N-L}{2} \frac{K+1}{2(K+1)+\bar{\gamma}} \sum_{i=1}^{C} w_i \left[e^{\frac{2K(K+1)(1-w_i)}{2(K+1)+\bar{\gamma}} - K + w_i K} + e^{\frac{2K(K+1)(1+w_i)}{2(K+1)+\bar{\gamma}} - K - w_i K} \right] \right].$$
(14)

from the cancellation of the two anti-phase specular waves and the low power of the remaining diffused component. The curve for extreme case when $K \to \infty$ and $\Delta = 1$ is provided for comparison. Furthermore, it is shown the the error curves move from the Rician, to Rayleigh and hyper-Rayleigh region as Δ increased from 0 to 1. In Fig. 2 the comparison between the exact and approximate expression is depicted for MCIK-OFDM with N = 4 and L = 1 over Rayleigh and TWDP fading with $\Delta = 1$ and K = 3 dB. An up to 3 dB difference between the results from (13) and (14) is observed, which reveals the tightness of the approximation.

V. CONCLUSION

We studied that MCIK-OFDM system with the greedy detector over TWDP fading. To evaluate the performance we derived a closed-form expression for the average PEP over TWDP fading channels. The approximate expression for the average PEP over TWPD is also derived with significantly reduced complexity. The accuracy of the approximate expressions has been validated through simulations. The derived expressions for the PEP will be useful to evaluate the performance of the MCIK-OFDM over moderate, severe and worse than Rayleigh fading conditions for the design of future wireless D2D systems.

APPENDIX A DERIVATION OF (13)

After expanding (12) it can be rewritten as,

$$P_{TWDP}(\alpha \rightarrow \tilde{\alpha}) = 1 - \sum_{q=0}^{N-L} {\binom{N-L}{q}} \frac{-1^q}{q+1} \frac{K+1}{2\bar{\gamma}}$$

$$\times \sum_{i=1}^{C} w_i \left[e^{-K+w_i K} \underbrace{\int_0^\infty e^{-\gamma \left(1 - \frac{1}{q+1} + \frac{K+1}{\bar{\gamma}}\right)} Z(-w_i) d\gamma}_{I(w_i)} + e^{-K-w_i K} \underbrace{\int_0^\infty e^{-\gamma \left(1 - \frac{1}{q+1} + \frac{K+1}{\bar{\gamma}}\right)} Z(w_i) d\gamma}_{I(-w_i)} \right].$$
(A.1)

It can be seen that (A.1) involves 2C integrals of the form $\int_0^\infty e^{-ax} I_\nu(2\sqrt{bx}dx)$. By using [6, eq. (6.614-3)] for $\nu = 0$, $a = \frac{\bar{\gamma}q + Kq + q + K + 1}{\bar{\gamma}(q+1)}$, and $b = \frac{K(K+1)(1\pm w_i)}{\bar{\gamma}}$ the integrals in (A.1) can be solved as,

$$I(\pm w_i) = e^{\frac{K(K+1)(1\pm w_i)(q+1)}{2(\bar{\gamma}q+Kq+K+q+1)} - K \pm w_i K} \times M_{-\frac{1}{2},0} \left(\frac{K(K+1)(1\pm w_i)(q+1)}{\bar{\gamma}q + Kq + K + q + 1}\right).$$
(A.2)

Taking into account that $M_{a,b}(x) = e^{-\frac{1}{2}x}x^{\frac{1}{2}+b}M(\frac{1}{2}-a+b;1+2b;x)$ and $M(a;b;x) = \sum_{s=0}^{\infty} \frac{a}{bs!}x^s$, (A.2) can be simplified to,

$$I(\pm w_i) = \frac{\bar{\gamma}(q+1)}{q(K+\bar{\gamma}+1)+K+1} e^{\frac{K(K+1)(1\pm w_i)(q+1)}{\bar{\gamma}_{q+K}q+K+q+1} - K \pm w_i K}.$$
(A 3)

By replacing (A.3) into (A.1) and using (7), the average PEP over TWDP is derived (13). Similarly, the approximate average PEP over TWDP is derived in (14).



Fig. 1. Average PEP of MCIK-OFDM with greedy detection over TWDP fading with varying K and $\Delta.$



Fig. 2. Comparison of average PEP from (13) and (14) over Rayleigh and TWDP with K = 3 dB and $\Delta = 1$.

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