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A Long-Term Economic Model for Allocation of FACTS Devices in Restructured Power Systems Integrating Wind Generation

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Abstract- This paper proposes an approach to optimally allocate multiple types of flexible AC transmission system (FACTS) devices in market-based power systems with wind generation. The main objective is to maximize profit by minimizing device investment cost, and the system's operating cost considering both normal conditions and possible contingencies. The proposed method accurately evaluates the long-term costs and benefits gained by FACTS devices installation to solve a large-scale optimization problem. The objective implies maximizing social welfare as well as minimizing compensations paid for generation re-scheduling and load shedding. Many technical operation constraints and uncertainties are included in problem formulation. The overall problem is solved using both Particle Swarm Optimizations (PSO) for attaining optimal FACTS devices allocation as main problem and optimal power flow as sub optimization problem. The effectiveness of the proposed approach is demonstrated on modified IEEE 14-bus test system and IEEE 118-bus test system.

Keywords: Optimal Allocation, FACTS Devices, Congestion Management, Wind Generation.

Nomenclature

B, C	Consumer benefit and generation cost respectively.
D ,G	Set of demands and generators, respectively.
i , j	Bus indices
k	Symbol indicating under contingency state.
Ks	Variable used to represent system losses related to the stressed loading condition.
M	Set of location candidates for TCSC
N	Set of location candidates for SVC.
r	The bilateral transaction index;

t	Denote a time interval
T	Total number of time intervals in a year
U	Set of location candidates for UPFC.
B_{SVC}	The susceptance of the SVC at the voltage of 1 p.u.
$c_{1x}, c_{1x,m}$	Installed capacity and maximum capacity of FACTS device candidate at location x.
C_{LS}	Compensation paid to demand for decreasing active power.
C_{SVC}	SVC investment cost per KVar-installed.
C_{TCSC}	TCSC investment cost per KVar-installed.
C_{UPFC}	UPFC investment cost per KVar-installed.
C_{wi}	The wind power generation cost.
C_{GD}^{up}	Compensation paid to generator for increasing active power.
C_{GD}^{down}	Compensation paid to generator for decreasing active power.
IC_{dev}	Investment cost of FACTS devices.
I_G	The set of injection buses for bilateral transaction.
J_D	The set of extraction buses for bilateral transaction.
N_g	The set of pool and multilateral generators.
N_L	The set of pool and multilateral loads.
N_W	The set of wind power generation units.
P_G	Active power generation.
P_D,Q_D	The active and reactive pool power demand, respectively.
$P_{Gr,i}$	The total real power for multilateral injections at bus *i*.
$P_{Dr,i}$	The total real power for multilateral extractions bus *i*.
P_{wi}	The power generated by wind generator at bus *i*.
$Q_{Gr,i}$	The total reactive power for multilateral injections at bus *i*.
$Q_{Dr,i}$	The total reactive power for multilateral extractions bus *i*.
$\mathrm{P}_{\mathrm{Li}}^{k}$	Real power of dispatchable load part at bus i for the k^{th} contingency
$\mathrm{Q}_{\mathrm{Li}}^{k}$	Reactive power of dispatchable load part at bus i for the k^{th} contingency
S_{SVC}	SVC capacities in MVar.
S_{TCSC}	TCSC capacities in MVar.
S_{UPFC}	UPFC capacities in MVar
X_{line}	the reactance of the transmission line between bus *i* and *j*
X_{TCSC}	The reactance contributed by TCSC
r_{TCSC}	The degree of compensation of TCSC.
ΔP_g	Generation re-scheduling vector ($\Delta Pg = 0$ at normal state).
ΔP_d	Load shedding vector ($\Delta P_d = 0$ at normal state).

ΔP_G^{up} Active power generation adjustment up.

ΔP_G^{down} Active power generation adjustment down.

ΔP_D^{down} Active power demand adjustment down.

λ Load margin ($\lambda = 0$ at current loading condition).

– Symbol indicating under stressed loading condition.

N_w The number of wind power generators.

C_w The cost of actual wind power generated by the i^{th} generator

C_o The penalty cost associated with overestimation of the available wind power (required reserve cost).

C_u The underestimation penalty cost of the available wind power (the penalty of not using all available power).

$P_{w_{act},i}$ The actual wind power from the i^{th} wind generator.

$P_{w_{sto},i}$ The scheduled (forecasted) wind power from the i^{th} wind power generator.

1. Introduction

Building of new transmission lines (TLs) is difficult for environmental and political reasons. Hence, the power transmission systems are driven closer to their limits endangering the system security [1]. When a TL becomes congested, generating units may have to be brought on one of its sides. This causes different locational marginal prices (LMPs) in the two sides. The difference in LMPs between the two ends of a congested TL is related to the extent of congestion and power losses on this line [2]. To ensure secure and economic operation, properly located and sized flexible ac transmission system (FACTS) devices offer an effective means [3]. During normal state, they can relieve congestion, increase voltage stability, increase system loadability, minimize transmission loss, minimize the compensations for generations re-scheduling, and minimize the LMPs difference. This leads to maximizing the social welfare. During contingency states, the devices are utilized to secure the system and to minimize operating cost.

FACTS devices (FD) can be connected to a TL in various ways, such as in series, shunt, or a combination of series and shunt. The static VAR compensator (SVC) and static synchronous compensator (STATCOM) are connected in shunt. The static synchronous series compensator (SSSC) and thyristor controlled series capacitor (TCSC) are connected in series. The thyristor controlled phase shifting transformer and unified power flow controller (UPFC) are connected in series and shunt combination [4].

Compensation by FD enhances the real power handling capacity of a TL [5]. FD should be located and sized properly to be effective [3]. The techniques used for optimal placement and setting of FD can be broadly classified into two methods:

i) *Index-based method:* the priority list is formed to reduce solutions space based on sensitivity indexes with respect to each line and bus [6]-[10].

ii) *Optimization-based method:* use either conventional or heuristic optimization methods such as simulated annealing, Tabu search, or particle swarm optimization (PSO) [11]-[16]. The objective function can be single or multi-objective optimizing certain technical/economic operational goals [17], [18].

References [6] and [7] have proposed optimal allocation methods for TCSC to eliminate the line overloads against contingencies, where sensitivity index called single contingency sensitivity is introduced for ranking the optimal placement. In [8], an index developed by reactive power spot price has been used for optimal allocation of SVC. Priority list method based on the LMPs is used in [9] to reduce solutions space for TCSC allocation for congestion management. Reference [10] has presented a technique to recover the investment cost of TCSC for congestion management in deregulated electricity markets. The technique evaluates the benefits of TCSC and converts them into monetary values. In [11], the FD location problem is solved by genetic algorithm to lower the cost of energy production and to improve the system loading margin. In [12], the same problem is formulated as a mixed-integer nonlinear programming problem. The optimal placement is obtained by optimizing both the investment cost in FACTS and the security in terms of the cost of operation under contingency events. Reference [13] describes an improved solution using the multi-start Benders decomposition technique to maximize the loading margin of a transmission network through the placement of SVCs. In [15], PSO technique is presented to seek the optimal places of TCSC, SVC and UPFC in power system. The objectives of optimization are minimizing the cost of FACTS installation and improving the system loadability. Economic feasibility analysis is not included in that paper. In [16], a non-dominated sorting PSO optimization has been used to find optimal locations of FD to maximize loading margin, reduce real power losses, and reduce load voltage deviation.

Ghahremani and Kamwa in [19] discuss the effects of six different types of FD on the steady-state performance of Hydro-Québec's power system. The improvement in the system loading margin and the network security are evaluated for different FD arrangements. Optimization problems are formulated and solved by genetic algorithm optimization to determine the sizes and locations of FD. In [20], the authors develop a graphical user interface toolbox based on genetic algorithm optimization to determine the optimal locations and sizing parameters of multi-type FD. The objective of the optimization problem is to maximize the system static loadability with maintaining the system security. However, in contrary to the proposed method, the last two references do not consider: the cost of FD, the value assessment of resulting benefits, the economic feasibility analysis of FD installation project, the types of energy market model, and the presence of renewable energy resources. In addition, they only consider the system operation under normal state condition and ignore the possible contingency states.

Most of the reported methods cater a single-type FD allocation problem. They do not take the compensations for generations re-scheduling into account. Also, only annual economic model is typically presumed. Furthermore, the appropriate market model is mostly missing and effect of wind power integration is not tackled. This paper proposes a new long-term economic model approach for optimal allocation of FD in restructured power system integrating wind generation. The objective is to maximize the annual profit under both normal and contingency operation maintaining system stability and security. This implies to: minimize FD investment cost, and maximize benefit due to FD installation. Variation of load and wind generation is treated by including daily load and wind generation curves. PSO is utilized for determining FD locations and capacities, while optimal power flow (OPF) is used to determine operating cost. Modified IEEE 14-bus and IEEE 118-bus systems are used to verify the efficacy of proposed method.

2. FACTS devices models

For static applications, FACTS devices (FD) can be modeled by two methods: (i) Power Injection Model (PIM), (ii) Impedance Insertion Model (IIM). The power injection model handles the FD as a device that injects a certain amount of active and reactive power to a node. The impedance insertion model represents the FD as known impedance inserted to the system in series, shunt or a combination of both according to the device type. These methods do not disturb the symmetry of the admittance matrix and allows efficient and convenient integration of FD into existing power system analytical software tools [9], [10]. This paper focuses on the optimal location and sizing of three types of FACTS, namely the SVC, TCSC, and the UPFC. They are chosen because of their fast control responses, low investment costs and ability to increase loadability as discussed in [11] and [21].

2.1Model of SVC

The SVC is a shunt compensator that may have two modes: inductive or capacitive [11]. The SVC combines a capacitor bank shunted by a thyristor-controlled reactor as shown in Fig. 1a. In this paper, the SVC is modeled as a variable admittance as in Fig. 1b.

The reactive power provided is limited as given in (1).

$$S_{SVC} = -V_i^2 \times B_{SVC} \tag{1}$$

and

$$B_{SVC\,min} \leq B_{SVC} \leq B_{SVC\,max} \tag{2}$$

2.2 Model of TCSC

The TCSC is a series compensation component which consists of a series capacitor bank shunted by a thyristor-controlled reactor as in Fig. 2a. The basic idea behind power flow control with the TCSC is to vary the overall line's effective series impedance, by adding capacitive or inductive impedance [16], [22]. The TCSC is modeled as a variable impedance as depicted in Fig. 2b. After installing TCSC, the new reactance of the line is estimated by (3).

$$X_{ij} = X_{line} + X_{TCSC} = r_{TCSC} \text{ x } X_{line} \tag{3}$$

To avoid overcompensation, X_{TCSC} is set between -0.7 X_{line} (capacitive) and 0.2 X_{line} (inductive) [22].

2.3 Model of UPFC

Basically, the UPFC consists of series and shunt voltage source inverters. These two inverters share a common DC-link. They are connected to the power system through two coupling transformers. The basic structure of UPFC is shown in Fig.3. The UPFC can control the voltage, impedance, phase angle, real and reactive power flow in a TL. The voltage drop on the line can be regulated by the shunt converter of UPFC whereas the power flow is controlled by the series converter [23].

The UPFC can have a coupled model or a decoupled model. For the coupled model, UPFC is modeled as two series combinations of a voltage source and an impedance. One of them is series connected to the TL. The second is shunt connected to the line. The two combinations are coupled through the UPFC control system. For the decoupled model, the above two voltage source-impedance combinations are independent [24]. The first model is more complex compared to the second one because modification of Jacobian matrix in coupled model is inevitable [23]. Decoupled model can be easily implemented in conventional power flow algorithms without modification of Jacobian matrix. In this paper, decoupled model is used for modeling UPFC.

3. Problem formulation

The problem is composed of two levels, the FD sizing and location sub-problem (upper level) and operation sub-problem (lower level). The upper level sub-problem is to determine locations and capacities of FD. The lower level problem is an OPF-based problem to obtain minimum operating cost incorporating FD given by the upper level. Then, the operating costs, as a component of the total annual cost, are fed back to the upper level. The iterative process is repeated until a termination criterion is satisfied.

Many restructured utilities in the world have considerable penetration levels of renewable resources, particularly wind energy. Increasing penetration of renewable resources in the electric grid is expected to have significant impact on transmission operation and planning. So, the power system is assumed to have an integrated wind generation in this analysis. The power from renewable resources is highly stochastic in nature. Wind power generation is generally treated as a negative load in power system studies. This is to indicate their capability for delivering current meanwhile their voltage is imposed by the electrical system at the connection point [25].

3.1 Load and wind power models

As an intermittent power source, wind generation power is best modeled as a random variable. In this case, many thousands of simulation runs must be executed to cover all possible wind states [26-29]. However, this approach will prohibitively elongate the solution time of the tackled multi-type FD planning problem considering the capabilities of the available commercial computers. As a proper model, a realistic time-wind power pattern depicted in Fig.4 is adopted in this study. Fig.4 provides the average daily forecasted and actual values of generated wind power recorded at 15-minutes intervals at a selected site in June [27]. Similar patterns are considered for each month.

Representing load power as a random variable will greatly slow down the solution of the optimal FD allocation problem [15], [26]. Millions of OPF runs are required to perfectly consider the random variations of both loads and wind generation. This may be necessary in energy adequacy studies. Since the optimal FD capacities are mainly focused, only peak loads levels are considered sufficient in some papers [9]. However, in this paper, typical daily load curves are used. The network loads are assumed to be categorized into three groups as revealed in Fig.5. The third load group includes loads presumably involved in multilateral contracts for energy purchases that may have a fixed demand [10].

The incomplete agreement between forecasted and actual wind power introduces an uncertainty in operation that must be included in wind generation cost function as addressed below [28].

3.2 Optimization problem

The objective function is formulated as follows:

$$Minimize \qquad Total\ cost\ = AIC_{dev}\ +\ (EB_{with\ FACTS} -\ EB_{without\ FACTS}) \tag{4}$$

Where AIC_{dev} is the annual investment cost of installed FD and calculated as in Section 3.3. The value of this cost is positive and depends on the number and capacities of installed FD.

$EB_{with\ FACTS}$ is the social welfare (annual benefit) of power system due to instating FD. For normal state, $EB_{with\ FACTS}$ is estimated as:

$$EB_{with\ FACTS} = \sum_{t=1}^{T}\left(\sum_{i=1}^{Ng} C_i\ (P_{Gi}^{o})_{\ t} + CW_{\ t}\ -\ \sum_{i=1}^{Nl} B_i\ (P_{Di}^{o})_{\ t}\ \right) \tag{5}$$

The right hand side in (5) has three terms. The first term is the generation cost of conventional power generators. The second term is the wind power generation cost. The third term is the customers' revenue. Since revenue is usually greater than generation costs, the value of $EB_{with\ FACTS}$ as expressed in (5) is with negative sign and depends on the locations and capacities of FD. So, minimizing $EB_{with\ FACTS}$ implies minimizing the generation costs meanwhile maximizing the customers' revenue. $EB_{without\ FACTS}$ is the social welfare (annual benefit) of power system without instating FD. Because the same operating conditions of the power system are compared with and without FD, $EB_{without\ FACTS}$ is a fixed value with negative sign computed before executing the FD allocation algorithm. Computation of $EB_{with\ FACTS}$ depends on OPF results.

When a contingency state occurs, corrective actions such as FD control (as a cost-free means), generation re-scheduling, and load shedding (as non-cost-free means) are utilized to avoid line overload, voltage instability, and to maintain load margin. Generation companies receive compensations for changing the output power to non-optimal value. If load shedding should be executed, demands will also be compensated for their interrupted load during contingency [30]. Therefore, under emergency state, $EB_{with\ FACTS}$ is formulated as follows:

$$EB_{with\ FACTS} =$$

$$\sum_{t=1}^{T}\left(\sum_{i=1}^{Ng} C_i \left(P_{Gi}^{k}\right)_t + CW_t - \sum_{j=1}^{Nl} B_j \left(P_{Dj}^{k}\right)_t + \sum_{i=1}^{Ng} \left(C_{GD}^{up}\, \Delta P_{Gi}^{up,k} + C_{GD}^{down}\, \Delta P_{Gi}^{down,k}\right)_t + \sum_{j=1}^{Nl} C_{LS} \left(\Delta P_{Dj}^{down,k}\right)_t \right) \quad (6)$$

The total cost of generated wind power (CW) is expressed as [26]:

$$CW = \sum_{i=1}^{N_w} C_{w,i}\, P_{w_{act},i} + \sum_{i=1}^{N_w} C_{o,i} \left(P_{w_{act},i} - P_{w_{sto},i}\right) + \sum_{i=1}^{N_w} C_{u,i} \left(P_{w_{sto},i} - P_{w_{act},i}\right) \quad (7)$$

Subject to:

Bus power balance, line flow, and voltage constrains:

$$p(V_i, \theta_i, C) = P_{Gi} + P_{Wi} - P_{Li} + P_{Gr,i} - P_{Dr,i} \quad (8)$$

$$q(V_i, \theta_i, C) = Q_{Gi} + Q_{Wi} - Q_{Li} + Q_{Gr,i} - Q_{Dr,i} \quad (9)$$

$$Q_{Li} = P_{Li} \times \tan\theta \quad (10)$$

$$P_{Gi\,min} \le P_{Gi} \le P_{Gi\,max}\ ,\ Q_{Gi\,min} \le Q_{Gi} \le Q_{Gi\,max} \quad (11)$$

$$\mathrm{P_{Li\,min}} \le \mathrm{P_{Li}^{k}} \le \mathrm{P_{Li\,max}}\ ,\ \mathrm{Q_{Li\,min}} \le \mathrm{Q_{Li}^{k}} \le \mathrm{Q_{Li\,max}} \quad (12)$$

$$\mathrm{V_{i\,min}} \le V_i \le \mathrm{V_{i\,max}}\ ,\quad \mathrm{MVA_{ij}} \le \mathrm{MVA_{ij\,max}} \quad (13)$$

bilateral/multilateral power balance:

$$\sum_{i \in I_G} P_{Gr,i} = \sum_{j \in J_G} P_{Dr,i} \tag{14}$$

constraints for generation re-scheduling and load shedding in contingencies states:

$$P_{Gi}^{k} = P_{Gi}^{0} + \Delta P_{Gi}^{up,k} - \Delta P_{Gi}^{down,k} \text{ , } P_{Dj}^{k} = P_{Dj}^{0} - \Delta P_{Dj}^{down,k} \tag{15}$$

$$\Delta P_{Gi}^{up,k} \text{ , } \Delta P_{Gi}^{down,k} \text{ , } \Delta P_{Dj}^{down,k} \geq 0 \tag{16}$$

Under stressed loading conditions, denoted below by "—", there should be a minimum loading margin:

$$\overline{\lambda} \geq \lambda_{\min} \tag{17}$$

Also, demand and generation are updated as:

$$\overline{P}_{Gi} = P_{Gi} \left(1 + \overline{\lambda} + K_s \right)$$

$$\overline{P}_{Di} = P_{Di} \left(1 + \overline{\lambda} \right)$$

$$\overline{Q}_{Di} = Q_{Di} \left(1 + \overline{\lambda} \right) \tag{18}$$

Constraints in (18) correlate normal and contingency states. Also, it is a way to ensure that compensations are always positive values.

3.3 FACTS devices investment cost

The range of cost of major FD is presented in Siemens AG Database [21]. A polynomial cost function of FD is derived and used for FACTS allocation study as in [3], [11]. The investment costs of TCSC, SVC and UPFC can be formulated as follows:

$$C_{TCSC} = 0.0015\, S_{TCSC}^{2} - 0.713\, S_{TCSC} + 153.7 \tag{19}$$

$$C_{SVC} = 0.0003\, S_{SVC}^{2} - 0.3051\, S_{SVC} + 127.38 \tag{20}$$

$$C_{UPFC} = 0.0003\, S_{UPFC}^{2} - 0.2691\, S_{UPFC} + 188.22 \tag{21}$$

$$IC_{dev} = \sum_{m \epsilon M} S_{TCSC,m} \times C_{TCSC,m} + \sum_{n \epsilon N} S_{SVC,n} \times C_{SVC,n} + \sum_{u \epsilon U} S_{UPFC,u} \times C_{UPFC,u} \tag{22}$$

Constraint of FD is given as follows:

$$0 \leq c_{Ix} \leq c_{Ix,max} \tag{23}$$

Then, the following expression is used to convert the investment cost into annual term:

$$AIC_{dev} = IC_{dev} \times \frac{ir(1+ir)^{LT}}{(1+ir)^{LT} - 1} \tag{24}$$

where ir is interest rate and LT is lifetime of FACTS device.

3.4 Market model

In this study, a hybrid market model is considered. A voluntary central pool is the most likely arrangement that will emerge in practical restructured power system [10]. This pool will set the price of bilateral and/or multilateral transactions [31]. The generation companies (GENCOs) submit a bid curve (supply bid) to independent system operator (ISO) and distribution companies (DISCOs) has the flexibility to submit either price-elastic demand (with benefit bid curve) or fixed demand. The bilateral/multilateral transaction holders request transaction of power specifying the points of injection and points of extraction. They pay the energy charge based on the difference in LMP at the points of injection and extraction. Based on the submitted bids by GENCO and DISCO, and considering the bilateral/multilateral transactions, the ISO solves the security-constrained OPF to find the optimum dispatch [32].

4. Solution algorithm

The overall problem is formulated as a two-level mixed integer nonlinear programming problem solved by hybrid PSO-sequential quadratic programming (SQP) method. The upper level is solved using standard PSO [33], [34]. Locating FD is a discrete problem. Determining devices capacities is a continuous problem. The outcomes of the upper level is passed to the lower level (operation sub-problem). It is formulated as an OPF problem solved by SQP. Matpower version 4.1 [35] is used to solve the operation sub-problem for normal and contingency states of the power system. The lower level will provide the upper level with $EB_{with\ FACTS}$ component of the objective function.

The proposed solution algorithm is described below.

Step 1: For a given year number starting from year 1, define line and bus data of the power system for a given system state (normal or contingency), all operational constraints, and PSO parameters.

Step 2: Generate an initial population of PSO particles with random positions and velocities representing location and sizes of FD. Set iteration index ite =0.

Step 3: For a given particle, update bus data (for SVC and shunt part of UPFC) and line data (for TCSC and series part of UPFC) based on its locations and size values. Initiate the time interval counter.

Step 4: Determine the average bus load level and wind generation output power according to Figs.4 and 5. Compute the system generation cost, customer benefit, and hence $\mathrm{EB}_{\text{with FACTS}}$ using (5) or (6).

Step 5: For the next 15 minutes time interval, update the average bus load level and wind generation output power for the next time interval. If all time intervals are not done, then go to step 4.

Step 6: Calculate FD investment cost using (22). Evaluate the value of the fitness function as given in (4). Check all the constraints. If any of the constraints is violated, a penalty term is applied. The calculated value of the fitness function including the added penalty terms (if any) serves as a fitness value of a particle. Consider the next particle. If all particles are not done, go to step 3.

Step 7: Compare the fitness value of each particle with the personal best, Pbest. If the fitness value is lower than Pbest, set this value as the current Pbest, and save the particle position corresponding to this Pbest value.

Step 8: Select the minimum value of Pbest from all particles to be the current global best, Gbest, and record the particle position corresponding to this Gbest value. Update the velocity and position of all particles.

Step 9: Set ite = ite +1. If maximum iterations are not exceeded, go to Step 2. Otherwise, the particle associated with the current Gbest is the optimal solution. Print and save the results.

Step 10: Fix the determined FD in the system. Consider the next year in the planning period. If all years are not done, then consider load and wind generation growth, configuration changes and go to step 1.

The above FD allocation algorithm is run first for the normal state. Then, it is run for contingency states allowing FD only at the same locations determined for normal state. In this case, the algorithm only identifies the FD capacities under contingencies. Fig.6 depicts the flowchart of the solution algorithm.

The overall size of one of FD (CFD_i) that suits both normal and contingency states is estimated as:

$$CFD_i = \sum_{m=1}^{Ns} CFD_{i,m} \times \frac{T_m}{8760} \qquad (25)$$

where, Ns is the number of considered states (normal and contingencies). T_m is the expected time duration of the m^{th} state in hours/year.

5. Case studies and results

The proposed solution algorithm is coded as one entity in MATLAB environment. It is applied to the IEEE 14- bus and 118-bus test systems. The load and wind generator output power are assumed to grow by 5% yearly. The planning period is taken as 10 years.

Nonetheless, to approach the evolving discretized commercial FD capacities, the obtained optimal FD capacities are rounded-up. Considering the maximum FD capacities assumed in this work, the obtained SVC capacity (and shunt UPFC element) is rounded-up to the nearest 1MVA. The obtained TCSR capacity (and series UPFC element) is rounded-up to the nearest 0.1 MVA. Due to this rounding-up in FD capacities, all performance results slightly change. Optimal results are compared to those obtained after FD capacities rounding.

5.1 IEEE 14-Bus system

The Modified IEEE 14-bus system is used to evaluate the proposed approach. Detailed data of generators, demand, and lines limits are given in [34]. The system includes a 20MW wind generator at bus8. According to Fig.5, load group 1 includes loads at buses 5, 10 and 12. Load group 2 includes loads at buses 4, 11 and 13. Loads at buses 9 and 14 are included in load group 3. There is a multilateral transaction of 35MW between the seller at bus6 and two buyers at bus9 and bus14. This transaction holder has requested ISO to provide transmission access to transmit power from bus6 to bus9 and bus14. The details of this transaction are given in the appendix.

5.1.1 Normal state

The determined optimal locations and capacities of FD under normal operating conditions are presented in Table 1. New FD are added every year in mostly new locations. By the end of the 10^{th} year, 13 SVC, 14

TCSC, and one UPFC are installed at 15 buses and lines. Table 2 shows the annual cost and benefit due to FD installation. The aggregated cost and benefit due to FD assuming zero interest rate is given in Table 3. Adding FD obviously increases social welfare because the system loadability and power losses are much improved. FD can greatly mitigate the risks of voltage violations and line congestion that enable supplying higher loads. Fig.7 depicts the actually-supplied yearly average load with and without FD. The improvement in the supplied load due to FD is evident especially for the late years of planning period with much growth in the customer loads. Besides, the average annual interruptible and uninterruptible loads at each bus are revealed in Fig.8. The increase in the uninterruptible load and the reduction in interruptible load parts due to FD are noted. Minimum bus voltage at the 10th year is shown in Fig.9 whereas maximum line power flow is displayed in Fig.10. Since the system is supported by FD, the bus voltage is improved and kept well above 0.95 p.u even under the highest load of the 10th year. Also, no congestion occurs in any line under the heaviest loading conditions. The power flow in many transmission lines securely increases after installing FD allowing increased system loadability. The system yearly average active power loss is reduced significantly after FD installation as depicted in Fig.11.

Due to the rounding-up of FD capacities, all performance results slightly change. This is noticed in Tables 1-3 and Figs. 7, 9, 10 and 11. Nonetheless, it is obvious that capacity rounding-up does not prevent FD from supporting the power system stability and security as well as improving social welfare.

5.1.2 Contingency state

It is practical to assume that FD locations for the contingency states are the same as for normal state. The optimal FD capacities should be searched for each contingency state. The determined optimal FD capacities under selected contingency states are shown in Table 4. Also, it is observed that the total required optimal FD size varies from one contingency to another. Table 5 provides the average operating cost components under the selected contingency states. Costs of generation re-scheduling and load shedding are much reduced for all contingencies due to optimal FD. Meanwhile, social welfare is markedly improved for all examined contingencies due to FD.

5.2 IEEE 118-bus system

The IEEE118 bus test system consists of 54 generator buses, 99 loads and 186 branches (TLs plus transformers). The bus data and line data values are taken from [34]. The system contains two 20MW wind generators at buses 37 and 38 with the power production pattern shown in Fig.4. The loads are grouped into three groups as shown in Fig.5. Group 1 includes loads at buses 1-40. Group 2 includes loads at buses 41-80. Group 3 includes loads at buses 81-118. The system contains two multilateral contracts. The first has loads at buses 107 and 110 as buyers and generator at bus100 as seller. The second multilateral contract has loads at bus 116 as a buyer and generators at buses 89 and 111 as sellers.

Simulations are carried out for optimal locations and capacities for mixed-type FD. It is assumed that there are 15 locations available for installing FD every year. This helps to limit the total number of FD and allows enough space to reach the solution of the large-dimension optimization problem at the same time. Table 6 shows the optimal locations and sizes of multi-type FD under normal state at selected years. The required FD are located at 50 buses and lines for the 10 years planning period. The average annual cost of FD is 13.6 M$/year. The average annual increase in social welfare due to FD is 20.1M$/year. However, due to FD capacity rounding-up, the average annual cost of FD becomes 15.37 M$/year. The average annual increase in social welfare due to FD becomes 18.27 M$/year.

The PC used in simulation has an AMD FX 4100 Quad Core, 3.60 GHz CPU, and 4 GB of RAM. The simulation time for the IEEE 14-bus test system under normal state is about 21 hours for 100 iterations and 50 PSO particles. For the IEEE 118-bus system, the simulation time is about 23 hours for 30 iterations and 30 PSO particles. It should be kept in mind that the maximum number of FD that can be installed in a year is limited to 15 for the IEEE 118-bus system. Whereas, the FD search for the IEEE 14-bus system was unrestricted.

5.3 Comparative evaluation

The results obtained for the IEEE 14-bus test system are compared to the results reported in [3] as given in Table 7. SVC and TCSC FD types only are used in [3]. It is noted that the FD locations reported in [3] are compatible with the results obtained in this work. But the FD sizes and number in [3] tend to be less than their counterparts in this work. Therefore, the FD cost is higher and the annual increase of social welfare is greater than that reported in [3]. However, the benefit-cost ratio is around 2.2 in both studies.

Moreover, the results obtained for the IEEE 118-bus test system are compared to the results reported in [11] and [15]. Genetic algorithm is used in [11] and PSO is used in [15]. The FD locations obtained in [11] for a total number of 15 multi-type FD much coincide with the FD locations obtained for the first year in Table 6. This number of multi-type FD used in [11] raises maximum system loadability to 140%. The cost of installing these FD and annual saving are not considered in [11]. Besides, the number of multi-type FD determined in [15] is 32 with installing cost of 21.1 M$ for 1 year planning period. These FD enable maximum system loadability of 136%. The locations of these FD are not reported. The total number of FD identified in this study is 50 for the 10 years planning period. FD improve maximum loadability to 145%.

6. Conclusion

This paper presents an approach to optimally allocate multiple FD in deregulated electricity market environment. The proposed approach is based on a comprehensive cost model that considers the annual cost of FD, operation cost, and customer benefit. The effect of wind generation and load growth are addressed. The task is formulated as a two-level mixed-integer nonlinear optimization problem. The annual net cost is taken as the objective function. Bus voltage limits, line flow limits, generator capacity limits are the main constraints. Hybrid Particle-swarm and sequential quadratic programming-based OPF are employed to solve the optimization problem. The impact of the optimally allocated FD includes increasing social welfare and reducing the compensation paid to market participants due to generation re-scheduling and load shedding.

7. References

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Appendix

Parameters and constants used in simulation are as follows:

1) The MVA limits of transmission lines are three times of base case line flow. The voltage limits are 0.95 and 1.1 p.u. All loads have constant power factor of 0.9 lagging.

2) Parameters c_1, c_2, ω_{max} and ω_{min} used in PSO are 1, 1, 0.9, and 0.4, respectively.

3) Maximum equivalent reactance of TCSC is assumed between -0.7 X_{line} (capacitive) and 0.2 X_{line} (inductive), while maximum installed capacity of SVC is 0.3 pu. The capacity range for UPFC is the same as for TCSC and SVC.

4) Interest rate and life time of devices are assumed to be 0.04 and 15 years, respectively.

5) C_{GD}^{up} and C_{GD}^{down} are 0.4 of power price in normal state. Meanwhile, C_{LS} is $10838 per MWh-curtailed load [3].

6) The total duration of contingencies is 240 hours per year.

7) The cost coefficient C_{wi} is $ 20 per MW/h of peak output power.

Data of multilateral contract for the IEEE 14-Bus system is given in Table A1.

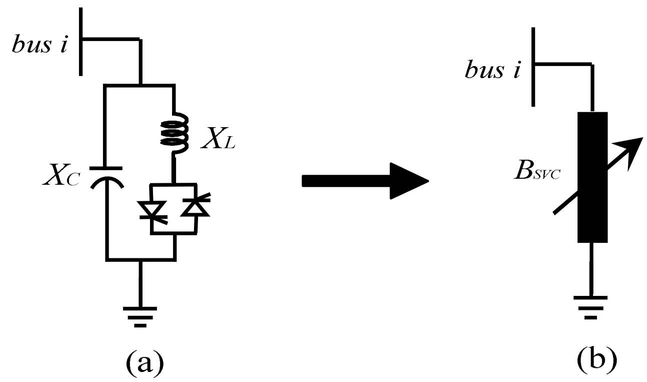

Fig. 1 Static var compensator (a) basic structure, (b) model

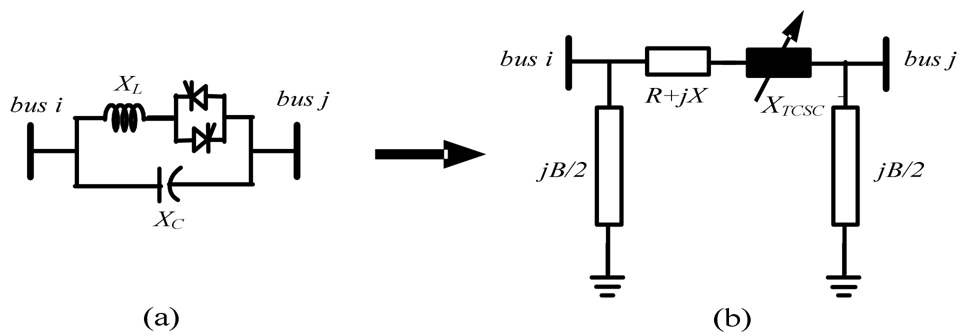

Fig. 2 Thyristor controlled series compensator (a) basic structure, (b) model

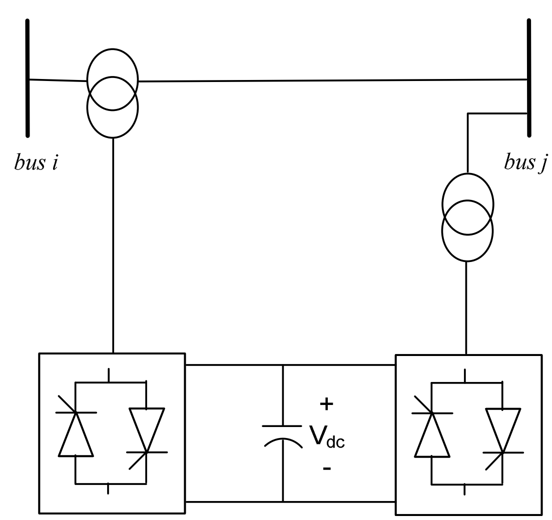

Fig.3 Basic structure of UPFC

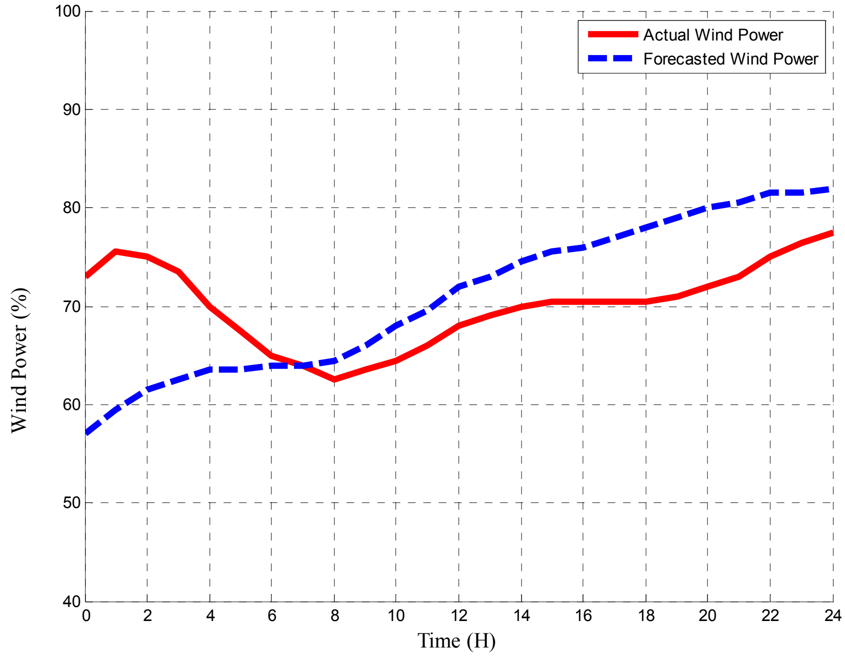

Fig.4 The actual and forecasted wind power pattern

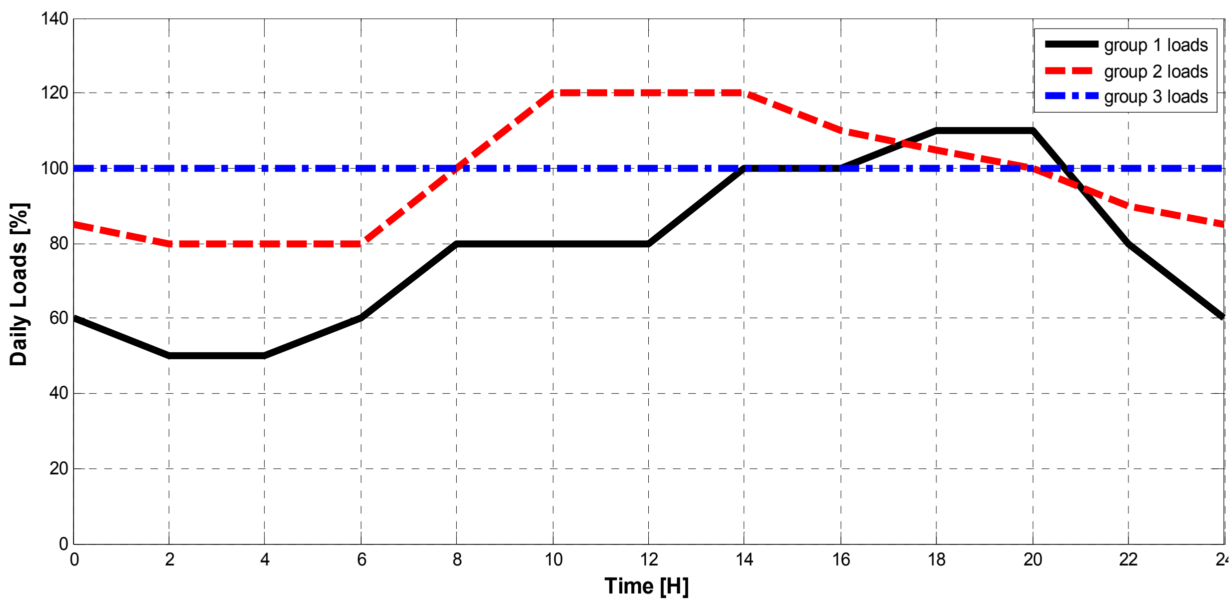

Fig. 5 The daily load curve

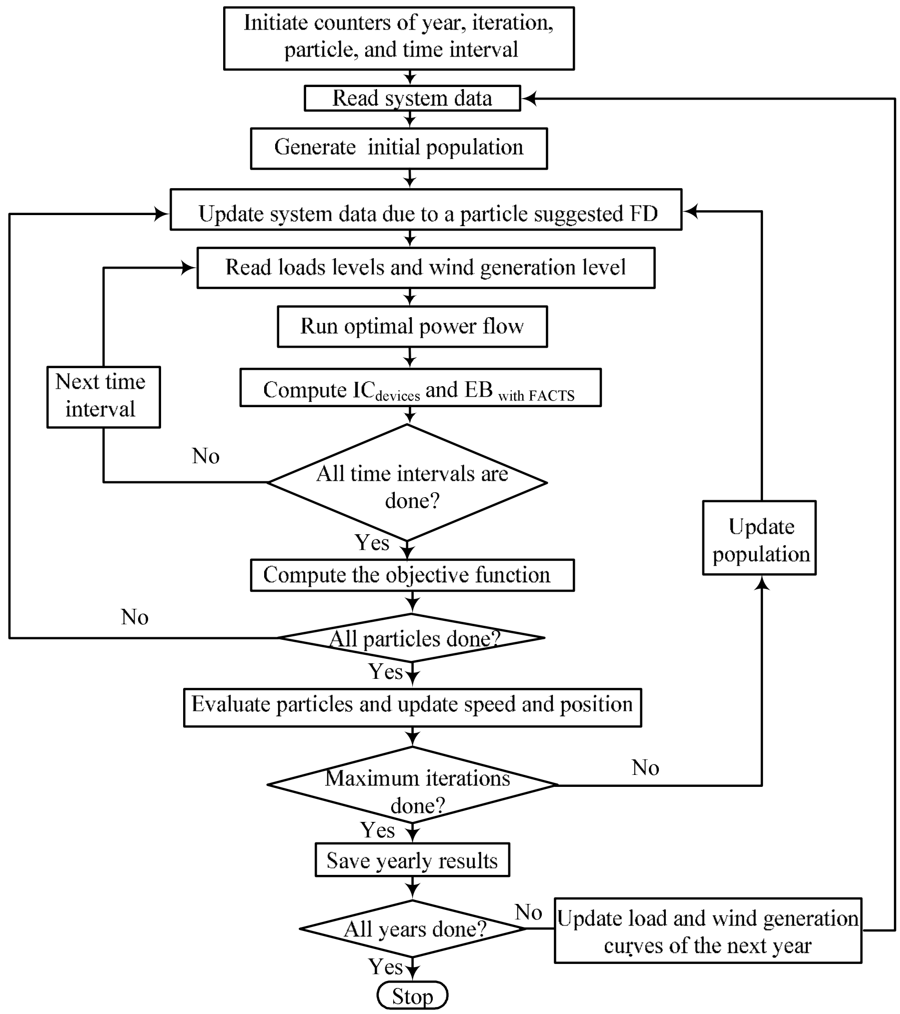

Fig.6 Flowchart of the solution algorithm

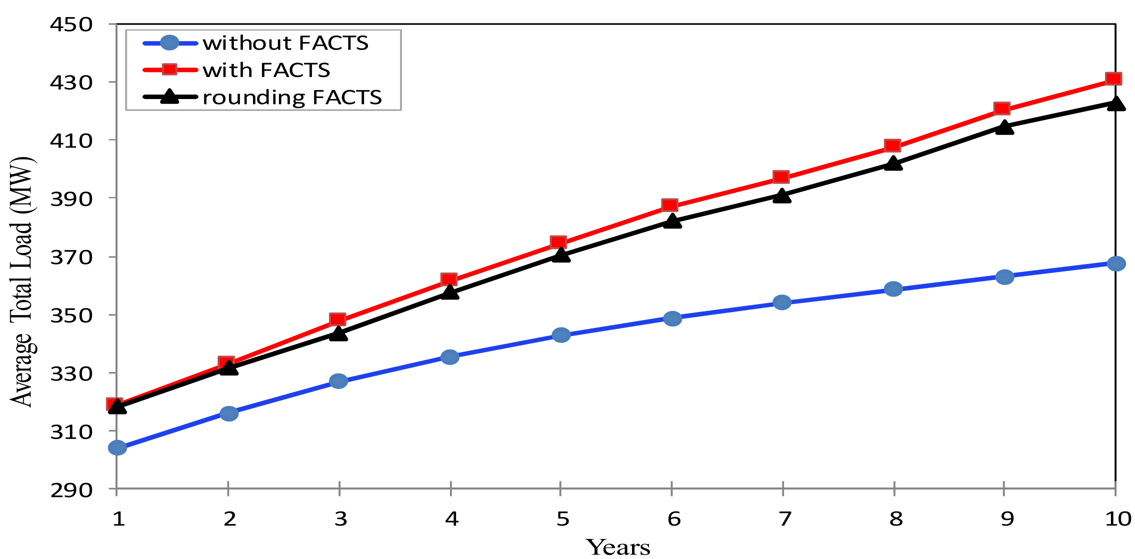

Fig. 7 Yearly average load for the 14-bus system

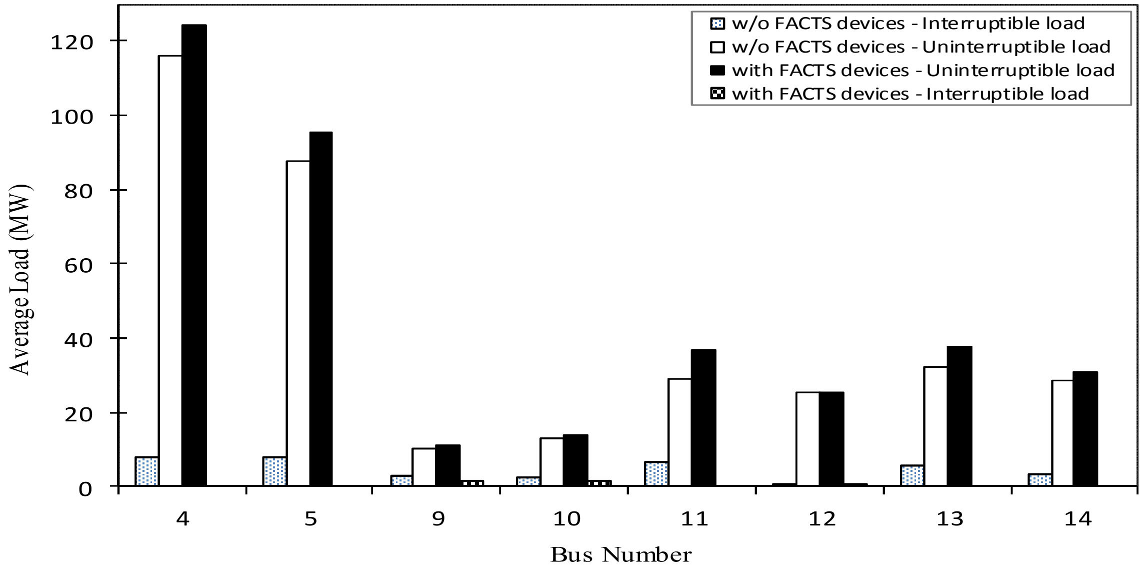

Fig.8 Average annual interruptible and uninterruptible load at each bus

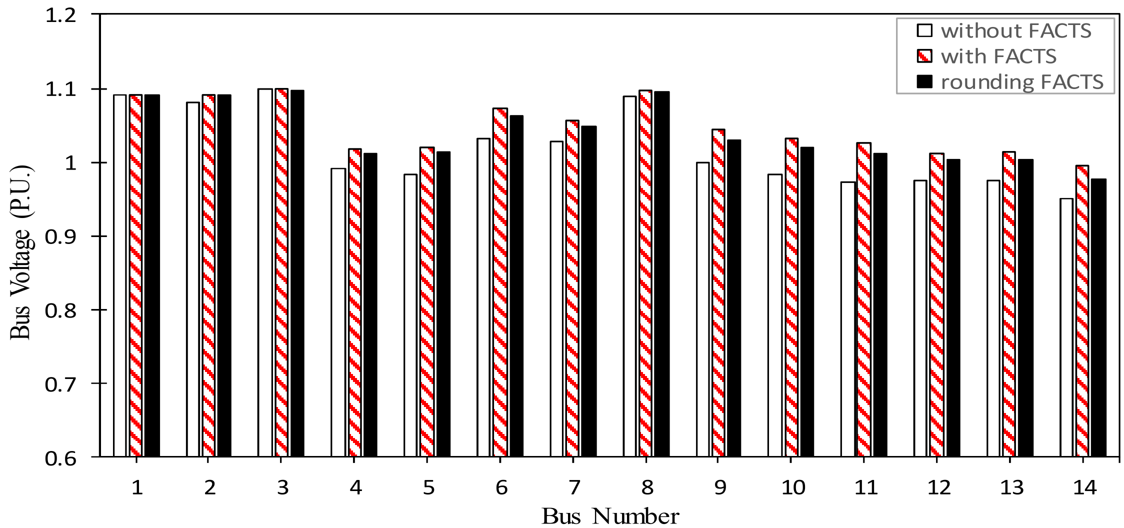

Fig.9 Minimum bus voltage at the 10th year

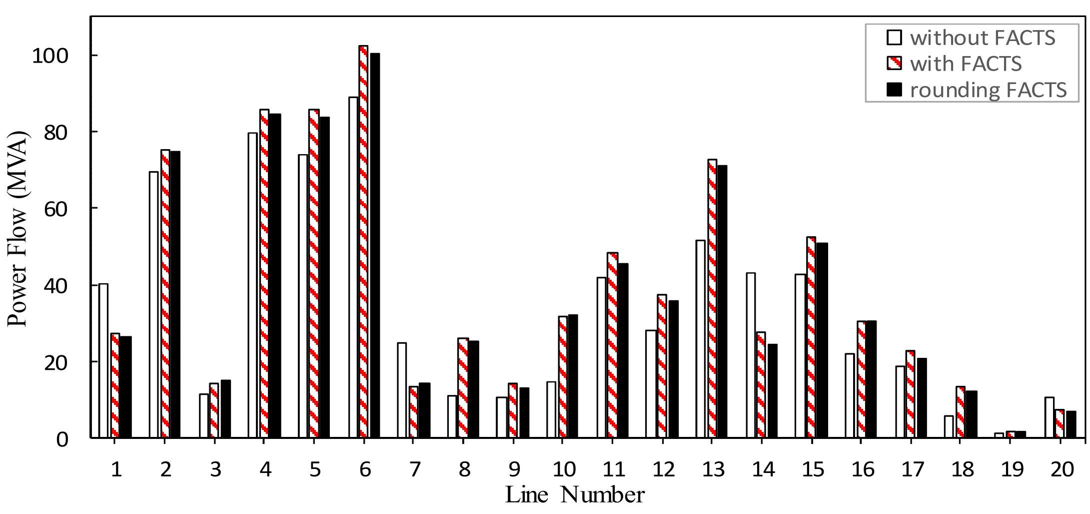

Fig.10 Maximum line power flow

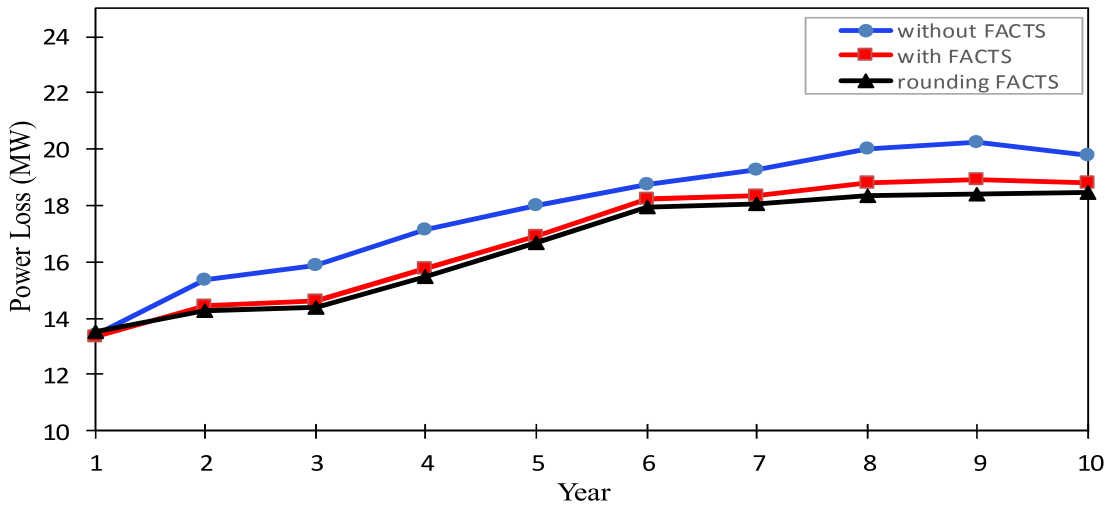

Fig.11 Yearly average power loss

Table 1 Optimal locations and capacities of FD for the IEEE 14-bus system under normal state

Year \ Item	FACTS type	FD Locations (Bus/Line)	FD capacities (MVar)	
			actual	rounded-up
1	SVC	14	0.08163	0.0
	TCSC	4 - 5	0.00454	0.0
		9 - 14	0.88778	0.90
		13 - 14	0.17533	0.20
2	SVC	14	0.395674	0.0
	TCSC	1 - 2	0.12807	0.10
3	SVC	11	14.84195	15.0
		10	29.16916	29.0
4	SVC	9	29.33764	29.0
	TCSC	2 - 3	0.02633	0.0
		10 - 11	0.01889	0.0
5	SVC	4	11.39267	11.0
	TCSC	1 - 2	0.12128	0.10
		6 - 11	1.55983	1.60
6	SVC	11	29.47755	29.0
	TCSC	1 - 2	0.46363	0.5
7	SVC	14	13.27538	13.0
		5	23.19742	23.0
	TCSC	1 - 2	0.0742	0.10
8	SVC	4	8.232627	8.0
	TCSC	1 - 5	1.47049	1.50
		4 - 7	0.37804	0.40
9	SVC	4	28.1699	28.0
		5	28.25378	28.0
	TCSC	3 - 4	4.8276	4.80
10	SVC	4	28.8532	29.0
	TCSC	4 - 7	0.83004	0.80
	UPFC	5	28.96912	29.0
		1 - 5	3.21967	3.20

Table 2 Annual FD cost and benefit for the IEEE 14-bus system under normal state

Year	FD Cost ($/Year)	Social Welfare (M$/Year)		Increase in social welfare due to FD (M$/Year)		Net benefit due to FD (M$/Year)	
		with FD	without FD	without FD capacity rounding	with FD capacity rounding	without FD capacity rounding	with FD capacity rounding
1	26582.88	-15.32	-13.99	1.33	1.05	1.30	1.03
2	11397.43	-21.62	-20.03	1.58	1.12	1.57	1.11
3	820356.87	-31.86	-29.25	2.61	2.00	1.79	1.18
4	567811.56	-37.57	-34.28	3.29	2.67	2.72	2.11
5	271541.48	-44.28	-40.11	4.17	3.48	3.89	3.22
6	580765.92	-49.75	-45.02	4.72	3.61	4.14	2.99
7	694272.06	-56.01	-49.95	6.06	4.86	5.36	4.13
8	213187.60	-62.56	-55.12	7.43	5.53	7.22	5.32
9	1138502.87	-73.34	-63.44	9.89	7.92	8.75	6.79
10	1520923.61	-83.27	-70.76	12.50	10.20	10.98	8.67

Table 3 Total FD cost and benefit for the IEEE 14-bus system under normal state

Items	Amount (M$)	
	without FD capacity rounding	with FD capacity rounding
Total social welfare with FD	-475.62	-464.50
Total social welfare without FD	-422.00	-422.00
Increase in social welfare due to FD	53.62	42.50
Total cost of FD	22.65	22.79

Table 4 Optimal FD locations and capacities for the IEEE 14-bus system under contingency states

Item / Year	FACTS type	FD Locations (Bus/Line)	Open line 1-2	Open line 2-3	Open line 4-5	Open line 4-7	Open line 10-11	Open line 12-13
			FD capacities (MVar)	FD capacities (MVar)	FD capacities (MVar)	FD capacities (MVar)	FD capacities (MVar)	FD capacities (MVar)
1	SVC	14	9.8269	11.9942	16.3915	3.9828	10.5916	28.7685
	TCSC	4 - 5	0.0	0.0180	0.0	0.0	0.0124	0.0206
		9 - 14	0.1423	0.4971	0.7546	0.30	0.2206	0.5929
		13 - 14	0.2954	0.1712	0.0308	0.0417	0.0669	0.2428
2	SVC	14	1.9373	0.0	0.0	0.8942	0.6595	4.4191
	TCSC	1 - 2	0.0	0.6676	0.4074	0.5840	0.4507	0.2706
3	SVC	11	29.7821	23.80	29.8819	24.4472	26.9141	28.6217
		10	24.4956	26.9036	25.9089	29.8841	29.7556	25.8116
4	SVC	9	14.0866	14.4717	3.0991	0.0	2.2911	9.2819
	TCSC	2 - 3	0.0	0.0	0.0974	0.0606	0.1227	0.0958
		10 - 11	0.0	0.0125	0.0	0.0	0.0	0.0
5	SVC	4	1.2048	1.6558	0.0	5.3042	6.0657	0.0
	TCSC	1 - 2	0.0	0.0	0.4108	0.2502	0.0248	0.4418
		6 - 11	0.7878	1.8788	1.4372	0.0	1.5418	0.4883
6	SVC	11	0.0	3.0172	0.0	0.0	0.0	0.0
	TCSC	1 - 2	0.0	0.0	0.1146	0.0	0.0	0.1230
7	SVC	14	1.0552	0.9051	7.6780	13.390	8.4514	0.0
		5	27.5097	27.6135	28.0448	0.0	27.6848	13.1858
	TCSC	1 - 2	0.0	0.0	0.0	0.0661	0.0286	0.0420
8	SVC	4	6.1655	0.0	24.6102	0.0	0.0	14.8242
	TCSC	1-5	16.1109	1.6891	15.2390	5.4799	0.0	7.2148
		4-7	0.0	0.0	0.1598	0.0	0.0394	0.1858
9	SVC	4	0.0	28.0403	25.5308	2.7663	0.6124	27.7840
		5	17.1502	20.5186	28.3368	3.2046	23.0220	24.8386
	TCSC	3-4	5.6742	9.1404	0.0	0.0	0.0	0.0
10	SVC	4	27.9722	28.3861	18.1736	27.8899	28.0035	27.3337
	TCSC	4-7	0.0	0.3405	2.0451	0.0	0.0888	0.0
	UPFC	5	0.0	25.6658	28.8163	27.8730	22.1643	28.5541
		1 - 5	0.0	7.6297	2.2498	0.6503	1.0372	0.2579
Total FD sizes (MVar)			184.2	235	259	147	190	243

Table 5 Average operating cost under contingency states

Open line	Social welfare ($/h)		Generation re-scheduling cost ($/h)		Load shedding cost ($/h)	
	without FD	with FD	without FD	with FD	without FD	with FD
1-2	-3775.2	-5208.7	429.24	90.10	163080	44820
2-3	-4486.4	-5276.8	319.95	94.96	124350	49849
4-5	-4799.9	-5329.3	98.35	11.49	74523	25217
4-7	-4630.4	-5297.1	103.93	82.84	85561	32474
10-11	-4741.8	-5266.5	45.97	9.36	45343	16887
12-13	-4823.1	-5346.2	20.19	10.5	28832	2204

Table 6 Optimal locations and capacities of FD for IEEE 118-bus system under normal state

Item / Year	FACTS type	FD Locations (Bus/Line)	FD capacities (MVar)	
			actual	rounded-up
1	SVC	81	25.2106	25.0
		5	18.08828	18.0
		59	25.6029	26.0
		12	18.86372	19.0
	TCSC	23 - 32	4.84881	4.80
		8 - 5	2.01	2.0
		38 - 65	13.88	13.90
		100 - 104	3.9188	3.90
	UPFC	50	20.53099	21.0
		49 - 50	0.93944	0.90
		89	27.65446	28.0
		88- 89	5.35643	5.40
		95	15.83572	16.0
		94 - 95	0.27912	0.30
5	SVC	72	19.49145	19.0
		89	28.81271	29.0
	TCSC	35 - 37	0.34113	0.30
		46 - 48	0.89744	0.90
		89 - 90	3.89923	3.90
	UPFC	68	8.706254	9.0
		65 - 68	1.41892	1.40
		46	11.03571	11.0
		45 - 46	0.75659	0.80
10	SVC	72	12.61177	13.0
		107	16.70158	17.0
		20	15.8661	16.0
		62	16.95905	17.0
		10	12.05453	12.0
	TCSC	5 - 11	0.70792	0.70
		37 - 40	3.57687	3.60
		23 - 25	0.538445	0.50
		34 - 37	0.39015	0.40
	UPFC	97	24.46226	24.0
		96 - 97	0.87599	0.90

Table 7 Comparison of results for the IEEE 14-bus test system

		Results reported in [3]	Results obtained in this work
Shunt FD	Bus	10, 12, 13, and 14	14, 11, 10, 9, 4, and 5
	Capacity (MVar)	22.5, 7.89, 10, 42, 0	13.7, 44.3, 29.2, 29.3, 76.6, 80.4
Series FD	Line	(6–11), (2–4), (3–4), (9–14), and (7–9)	(4–5),(9–14), (13–14), (1–2), (2–3), (10–11), (6–11), (1–5), (4–7), and (3–4)
	Capacity (MVar)	1.06, 0, 0, 0.52, 0.48	0.0045, 0.89, 0.18, 0.79, 0.03, 1.6, 4.7, 1.2, 4.8, 0.83
Average annual cost of FD		1.2 M$/year	2.3 M$/year
Average annual increase in social welfare due to FD		2.6 M$/year	5.4 M$/year

Table A1 Multilateral contract data

No.	Bus No.	Type	Minimum Power (MW)	Cost coefficient		
				C_2	C_1	C_o
1	6	seller	35	-0.15	100	0.0
2	9	Buyer 1	10	-0.15	100	0.0
3	14	Buyer 2	25	-0.15	100	0.0