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# Compressive Sensing based Detector for Sparse Signal Modulation in Precoded OFDM

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**Abstract**—In this paper, we propose a sparse signal modulation (SSM) method for precoded orthogonal frequency division multiplexing (OFDM) systems and study the signal detection. Although a receiver is able to exploit a path diversity gain with random precoding in OFDM, the complexity of the receiver is usually high as the orthogonality is not retained due to precoding. However, with SSM, we can derive a low-complexity detector that can provide reasonably good performances with a low sparsity ratio based on the notion of compressive sensing (CS). An important feature of a CS detector is that it can estimate SSM signals with a small fraction of the received signals over sub-carriers. This feature can allow us to build a low cost receiver with a small number of demodulators.

**Index Terms**—compressive sensing, sparse signal modulation

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been extensively studied and employed for wireless standards [1], [2] due to various advantages over other schemes. In particular, OFDM effectively mitigates inter-symbol interference (ISI) and allows to use low-complexity a one-tap equalizer for signal detection. However, OFDM cannot exploit path diversity at the expense of no ISI [3].

To exploit path diversity, precoding can be used in OFDM [4]. Unfortunately, precoding can offset one of the advantages of OFDM systems, which is the orthogonality. Since the orthogonality cannot be retained due to precoding, one-tap equalizers cannot be used and more complicated equalizers or detectors are to be used to mitigate ISI. There can be certain precoding schemes for the trade-off between performance and complexity of detectors. To this end, low density spreading is devised in [5] and extended for multiple access in [6] to exploit path diversity by receivers of reasonably low complexity.

In [7], spatial modulation (SM) is proposed to transmit signals over multiple input multiple output (MIMO) channels with a single transmit antenna at a time. In SM, the index of the active transmit antenna is to bear information bits. Thus, if there are 4 transmit antennas, SM can transmit 2 bits per channel use. Furthermore, a modulated symbol can be transmitted by the active transmit antenna. SM can be generalized by activating more than one transmit antennas.

Thus, the number of bits that can be transmitted by the indices of the active transmit antennas becomes  $\lfloor \log_2 \binom{N_{\text{TX}}}{K} \rfloor$ , where  $N_{\text{TX}}$  is the number of transmit antennas and  $K$  is the number of active antennas at a time. Since a fraction of  $N_{\text{TX}}$  antennas are activated, SM can be energy efficient, and cost-effective as only  $K$  radio frequency (RF) chains are required. Furthermore, if  $K = 1$ , there is no interference from the other transmit antennas, which allows to use low-complexity detectors [7], [8]. In order to improve the performance of SM, the notion of channel coding can also be employed [9], [10].

The notion of SM can be applied to OFDM, which results in sub-carrier-index modulation (SIM) OFDM [11]. In SIM OFDM, a subset of sub-carriers are activated and their indices are used to transmit information bits. While SIM OFDM is energy efficient as SM, it cannot exploit path diversity, which is the same as OFDM. For a path diversity gain, precoding can be used in SIM OFDM. In this case, however, the complexity of detection can be high as the orthogonality is not retained.

In this paper, we study SIM for precoded OFDM with a small number of activated sub-carriers. Since only a fraction of sub-carriers are activated, the resulting modulation is referred to as sparse signal modulation (SSM). Due to the sparsity, the notion of compressive sensing (CS) [12], [13] can be employed to derive a low-complexity detector in this paper. CS is to recover sparse signals with a considerably low sampling rate compared to the bandwidth of observed signals. There have been various CS algorithms for the estimation of sparse signals [14]–[16]. The main advantage of a CS detector is that it can estimate SSM signals by using only a small fraction of the received signals over sub-carriers. Since a small number of demodulators would be required, we can see that the cost to build a CS detector can be low.

*Notation:* Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscripts  $\text{T}$  and  $\text{H}$  denote the transpose and complex conjugate, respectively. The  $p$ -norm of a vector  $\mathbf{a}$  is denoted by  $\|\mathbf{a}\|_p$ .  $\mathbb{E}[\cdot]$  denotes the statistical expectation.  $\mathcal{CN}(\mathbf{a}, \mathbf{R})$  ( $\mathcal{N}(\mathbf{a}, \mathbf{R})$ ) represents the distribution of circularly symmetric complex Gaussian (CSCG) (resp., real-valued Gaussian) random vectors with mean vector  $\mathbf{a}$  and covariance matrix  $\mathbf{R}$ .

## II. SYSTEM MODEL

Consider an OFDM system with  $L$  sub-carriers. The signal vector to be transmitted is denoted by  $\mathbf{s}$ . For convenience,  $\mathbf{s}$  is referred to as an OFDM signal block, while its elements are referred to as data symbols. The received signal is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where  $\mathbf{n} \sim \mathcal{CN}(0, N_o\mathbf{I})$  is the background noise vector and  $\mathbf{H} = \text{diag}(H_0, \dots, H_{L-1})$  is a diagonal channel matrix and

$$H_l = \sum_{p=0}^{P-1} h_p e^{-j2\pi \frac{pl}{L}}.$$

Here,  $\{h_p\}$  is the channel impulse response (CIR) and  $P$  is the length of CIR.

In the conventional OFDM system, each data symbol is an element of a given signal constellation. That is,  $s_l \in \mathcal{S}$ , where  $s_l$  is the  $l$ th element of  $\mathbf{s}$  and  $\mathcal{S}$  represents the signal constellation. Since  $\mathbf{H}$  is diagonal,  $s_l$  can be detected from the  $l$ th element of  $\mathbf{r}$ , i.e.,  $r_l$ , as

$$\hat{s}_l = \underset{s_l \in \mathcal{S}}{\text{argmin}} |r_l - H_l s_l|^2.$$

Thus, the performance depends on each sub-carrier's channel coefficient,  $H_l$ , and no path diversity can be exploited.

## III. SSM FOR PRECODED OFDM

Denote by  $\mathbf{P}$  the precoding matrix. Then, the received signal becomes

$$\begin{aligned} \mathbf{r} &= \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n} \\ &= \mathbf{G}\mathbf{s} + \mathbf{n}, \end{aligned} \quad (2)$$

where  $\mathbf{G} = \mathbf{H}\mathbf{P}$ . In SSM, most  $s_l$ 's are zero, while information bits can be delivered through non-zero data symbols as well as their indices. Assume that there are  $K$  non-zero symbols within  $\mathbf{s}$ , i.e.,  $\mathbf{s}$  is a  $K$ -sparse signal. The number of bits that can be sent through indices is

$$N_I = \lfloor \log_2 \binom{L}{K} \rfloor,$$

while the number of bits that can be sent by non-zero data symbols is  $K \lfloor \log_2 |\mathcal{S}| \rfloor$ . For example, if  $L = 64$  and  $K = 6$ , we have  $N_I = 26$  bits. Since each non-zero symbol can represent  $\log_2 |\mathcal{S}|$  bits, the total number of bits becomes

$$N = \lfloor \log_2 \binom{L}{K} \rfloor + K \log_2 |\mathcal{S}|,$$

if  $|\mathcal{S}|$  is a power of 2.

If  $K \ll L$ , from [17], we can show that

$$N \approx L\psi\left(\frac{K}{L}\right) + K \log_2 |\mathcal{S}|. \quad (3)$$

where  $\psi(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ ,  $0 \leq x \leq 1$ , which is the entropy of a binary random variable. Furthermore, using the Taylor series, it can be shown that

$$\psi(x) = x \log_2 \frac{1-x}{x} + O(x^2).$$

Letting  $\alpha = \frac{K}{L}$ , it can be shown that

$$\begin{aligned} N &\approx L \frac{K}{L} \beta + K \log_2 |\mathcal{S}| \\ &= K (\log_2 |\mathcal{S}| + \beta) \\ &= L \alpha (\log_2 |\mathcal{S}| + \beta), \end{aligned} \quad (4)$$

where  $\beta = \log_2 \frac{L-K}{K} = \log_2 \left(\frac{1}{\alpha} - 1\right)$ . This shows that the number of bits,  $N$ , grows linearly with  $L$  as long as  $\alpha \ll 1$  is fixed. For convenience,  $\alpha$  is referred to as the sparsity ratio.

On the other hand, for a given large  $L$ ,  $N$  can grow quickly with  $K$ . In particular,  $N_I$ , which is the number of bits that can be transmitted by the indices of activated sub-carriers, can be maximized as  $K$  approaches  $L/2$ . Thus, the maximum  $N_I$  is

$$\max N_I = \lfloor \log_2 \binom{L}{\lfloor L/2 \rfloor} \rfloor.$$

However, the number of bits that can be reliably detected at a receiver would be lower than the maximum  $N_I$  under a finite signal-to-noise (SNR). Furthermore, the performance of the detector employed affects the number of bits that can be reliably detected. In the next section, we propose a CS based detector and study a bound on  $N_I$  for reliable detection.

## IV. COMPRESSIVE SENSING BASED DETECTION

To derive a low-complexity detector, we consider CS approach under the assumption that  $K$  is sufficiently smaller than  $L$ . The proposed approach consists of two steps and the second step is optional.

### A. Two-Step Approach for Detection

The maximum likelihood (ML) detection can be considered to detect  $\mathbf{s}$  when SSM is used. Let  $\tilde{\mathcal{S}} = \mathcal{S} \cup \{0\}$ , which is the extended signal constellation of  $\mathcal{S}$  including zero. Then, the ML detection is given by

$$\begin{aligned} \hat{\mathbf{s}} &= \underset{\mathbf{s}}{\text{argmin}} \|\mathbf{r} - \mathbf{G}\mathbf{s}\|^2 \\ &\text{subject to } s_l \in \tilde{\mathcal{S}} \text{ and } \mathbf{s} \in \Sigma_K, \end{aligned} \quad (5)$$

where  $\Sigma_K$  is the set of  $K$ -sparse signals, which is defined as

$$\Sigma_K = \{\mathbf{x} \mid \|\mathbf{x}\|_0 = K\}.$$

Certainly, an exhaustive search for the ML detection in (5) is infeasible due to a high computational complexity.

Prior to deriving a CS detector in this section, we briefly discuss its complexity. In general, CS algorithms do not require all the samples to estimate sparse signals [12], [13]. In our context, it implies that all the received signals from  $L$  sub-carriers are not required in a CS detector, while the ML detector should have all the signals. Denote by  $M$  the number of samples. In a CS detector,  $M$  is the number of the sub-carriers that are used to detect SSM signals. Usually,  $M$  is greater than  $K$ , but less than  $L$ . This implies that a CS detector does not need to have all the received signals from  $L$  sub-carriers. Thus, the complexity of a CS detector can be much lower as it only needs received signals from  $M$  out of  $L$  sub-carriers. Furthermore, the cost for building a CS detector

can be lower than that for building an ML detector, since only  $M$  demodulators would be required.

For practical detection schemes, we can consider two-step approaches with CS. In the first step, we can solve the following problem:

$$\tilde{\mathbf{s}} = \underset{\mathbf{s}}{\operatorname{argmin}} \|\mathbf{r} - \mathbf{G}\mathbf{s}\|_2^2 + \lambda \|\mathbf{s}\|_1, \quad (6)$$

where  $\lambda$  is a design parameter. From (5),  $\lambda$  can be considered as the Lagrange multiplier. As  $\lambda$  increases, the sparsity constraint is more emphasized. The problem in (6) is a typical CS problem that has a number of algorithms to solve it (e.g., [15]). Alternatively, we can use the orthogonal matching pursuit (OMP) algorithm [14] to estimate  $K$ -sparse signals.

The second step is optional. From the estimated  $K$ -sparse signals, the detection of non-zero symbols can be carried out using the ML approach with the estimated indices of the active sub-carriers. Let  $\mathcal{I}$  denote the (estimated) index set of non-zero elements. In addition, let

$$\begin{aligned} \bar{\mathbf{r}} &= [\mathbf{r}]_{\mathcal{I}} \\ \bar{\mathbf{s}} &= [\mathbf{s}]_{\mathcal{I}} \\ \bar{\mathbf{G}} &= [\mathbf{G}]_{\mathcal{I}}. \end{aligned} \quad (7)$$

Then, the values of the non-zero symbols in  $\mathbf{s}$  can be detected as

$$\hat{\mathbf{s}} = \underset{\bar{\mathbf{s}} \in S^K}{\operatorname{argmin}} \|\bar{\mathbf{r}} - \bar{\mathbf{G}}\bar{\mathbf{s}}\|^2. \quad (8)$$

It is also possible to employ the minimum mean squared error (MMSE) detector at the expense of degraded performance.

### B. An Upper-Bound on Throughput

The performance of the first step in the CS based detection mainly depends on  $\alpha$ , not the SNR, while the second step depends on the SNR. Thus, at high SNR, the performance of the overall CS based detection mainly depends on  $\alpha$ .

Suppose that the elements of the precoding matrix  $\mathbf{P}$  are independent identically distributed (iid). Furthermore, the  $\mathbf{H}_l$ 's are also iid. Then, from [13], it is known that the number of samples,  $M$ , to detect  $K$ -sparse signals is bounded as

$$M \geq cK \ln \frac{L}{K}, \quad (9)$$

where  $c$  is a positive constant. This inequality is valid with a sufficiently high probability. In most CS applications, it is desirable to find a minimum  $M$ . However, we are interested in finding a maximum  $K$ . Suppose that the receiver uses all the received signals for signal detection, i.e.,  $M$  becomes  $L$ . In this case, we have

$$\frac{L}{K} \geq c' \log_2 \frac{L}{K},$$

where  $c' = c \ln 2$ . Since  $\alpha = \frac{K}{L}$ , we have

$$\frac{1}{\alpha} \geq c' \log_2 \frac{1}{\alpha} \geq c' \log_2 \left( \frac{1}{\alpha} - 1 \right). \quad (10)$$

For a sufficiently small  $\alpha$ , from (4) and (10), it can be shown that

$$\begin{aligned} N_I &\approx L\alpha \log_2 \left( \frac{1}{\alpha} - 1 \right) \\ &\leq L\alpha \frac{1}{c'\alpha} \\ &= \frac{L}{c'}. \end{aligned} \quad (11)$$

This shows that the number of bits to be reliably transmitted by indices is bounded for a given  $L$  and its growth rate cannot be faster than linear when  $L$  increases for reliable  $K$ -sparse signal detection. Therefore, in SSM,  $\alpha$  is to be fixed or  $K$  is to be proportional to  $L$  for reliable  $K$ -sparse signal detection.

## V. SIMULATION RESULTS

We consider the SSM for a precoded OFDM system of  $L = 128$  sub-carriers with various configurations:  $K \in \{1, 2, 4, 8, 16, 32\}$  and  $M \in \{10, \dots, 100\}$  (per-cent) measured samples. The average bit error rates (BERs) are obtained by simulating the compressive sensing detector for the proposed scheme. For convenience, it is assumed that the  $H_l$ 's are independent CSCG random variables, i.e., an independent Rayleigh fading channel per sub-carrier is assumed. For random precoding, we assume a random matrix with iid  $[\mathbf{P}]_{l,m} \sim \mathcal{N}(0, 1)$ .

At the receiver, we employ the OMP algorithm to estimate  $K$ -sparse signals. The number of samples,  $M$ , is considered as a design parameter for the receiver. In implementing a receiver based on the OMP algorithm (or other CS algorithms), since we only need to demodulate the received signals from  $M$  out of  $L$  sub-carriers, we can claim that the complexity of the receiver grows linearly with  $M$ .

Fig. 1 depicts the average BER of the SSM with  $K \in \{1, 2, 4, 8, 16, 32\}$ ,  $E_s/N_o = 15$  (dB) and  $M = 50$  per-cent measured samples for a given  $L = 128$ . Binary phase shift keying (BPSK) is employed for modulation of non-zero signals. This figure illustrates that the average BER increases with  $\alpha = \frac{K}{L}$ . For a small  $\alpha$ , intuitively, the CS detector with only a subset of measured samples is capable of decreasing the average BER at the cost of the data rate, and it can be seen in this figure that the average BER performance relies on  $\alpha$  at the properly chosen number  $M$  of measured samples. Overall, since the transmission rate increases with  $K$ , we can observe a trade-off between the transmission rate and performance in Fig. 1.

In order to capture the relationship between the average BER and the number  $M$  of samples, simulations are carried out with various values of  $M$  and results are illustrated in Fig. 2 with the average BERs for fixed  $\alpha = \frac{K}{L}$  and SNR. In this figure, the impact of  $M$  on the average BER can be observed at different SNRs. Particularly, it can be seen in this figure that for given SNR and  $\alpha$ , the average BER decreases with  $M$  which validates our analysis in (9). Also, its decreasing rate with  $M$  becomes faster at a higher SNR. For example, to achieve an average BER of  $10^{-2}$ , the required

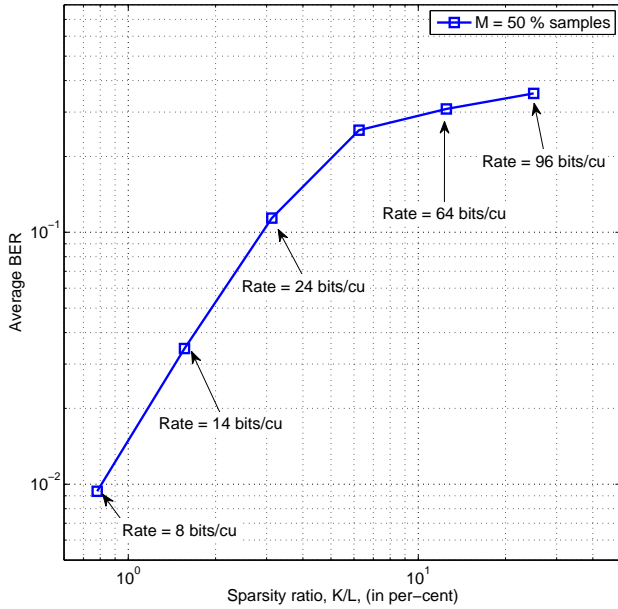


Fig. 1. Average BER performance versus the sparsity ratio  $\alpha = \frac{K}{L}$  from the compressive sensing detector over an independent Rayleigh fading channel per sub-carrier when  $K \in \{1, 2, 4, 8, 16, 32\}$ , BPSK,  $L = 128$ ,  $M = 50$  (per-cent) samples are used at  $E_s/N_o = 15$  dB.

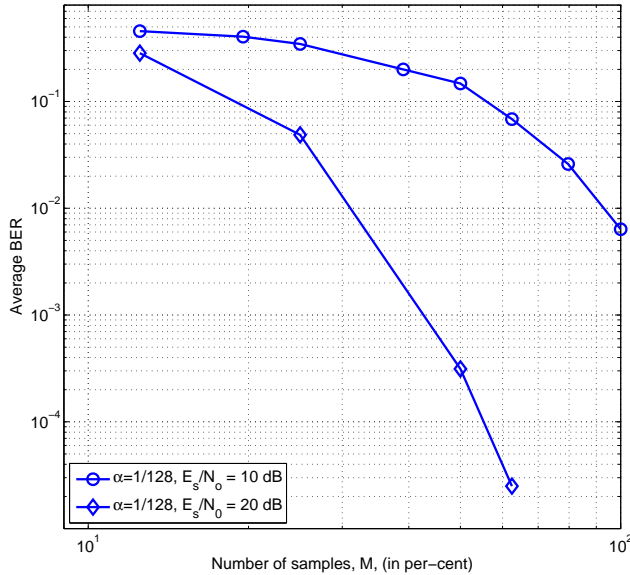


Fig. 2. The impact of the number  $M$  of samples on the average BER performance with the presence of compressive sensing detector over the Rayleigh fading per sub-carrier, when a fixed  $\alpha = 1/128$ ,  $L = 128$ ,  $E_s/N_o \in \{10, 20\}$  dB and BPSK are used.

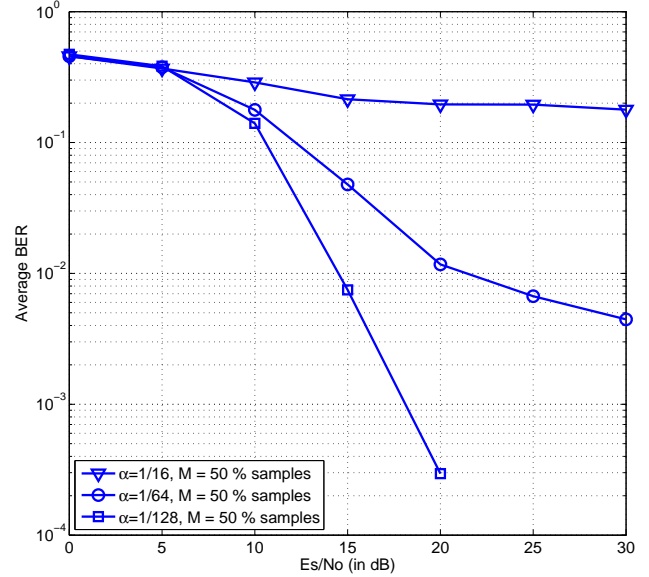


Fig. 3. Average BER performance versus the SNR with a CS detector over an independent Rayleigh fading channel per sub-carrier at various values of  $\alpha = \frac{K}{L} \in \{1/16, 1/64, 1/128\}$ . We have assumed that  $L = 128$ , BPSK, and  $M = 50$  per-cent.

minimum number of samples,  $M$ , at  $E_s/N_o = 20$  dB is 30 per-cent (i.e.,  $M = 30$  per-cent), which increases up to  $M \geq 90$  per-cent at a low SNR (i.e.,  $E_s/N_o = 15$  dB). This leads us to that at a high SNR, the required number of samples,  $M$ , to obtain the desired average BER can be small. This implies that the complexity of the receiver can be low when the SNR is high as  $M$  can be small.

Similarly, in Fig. 3, the average BER is illustrated with respect to  $E_s/N_o$ . For the illustrations, it is assumed that BPSK,  $\alpha \in \{1/16, 1/64, 1/128\}$ , and  $M = 50$  per-cent measured samples. We can observe from this figure that the average BER decreases with the SNR for a given  $\alpha$  and its decreasing rate relies on the values of  $\alpha$ , at the cost of the data rate. In particular, considering  $\alpha = 1/64$ , the average BER decreases with the SNR slowly at high SNRs (i.e.,  $E_s/N_o \geq 20$  dB). The intuition behind this behavior is that the accuracy level of the CS detector can be limited by the number of samples,  $M$ .

## VI. CONCLUSIONS

In this paper, we proposed the SSM for precoded OFDM that can exploit a path diversity gain with a low-complexity CS detector. Since a CS detector requires a small fraction of the received signals over sub-carriers, the number of demodulators can be small, which could allow us to build it at a low cost. In addition, another important feature of the derived CS detector was that it can exploit the trade-off between the SNR, complexity, and performances (in terms of BER and transmission rates). For example, the complexity of the CS detector could be lower as the SNR increases with a reasonable

SNR.

While we only considered the OMP algorithm for the CS detector in this paper, other CS algorithms can be employed. Furthermore, we did not address bit allocation problems (over sub-carrier indices as well as non-zero symbols) to optimize performances. Other CS algorithms as well as bit allocation problems will be studied for further work in the future.

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