Simms, Victoria and Clayton, Sarah and Cragg, Lucy and Gilmore, Camilla and Johnson, Samantha (2016) Explaining the relationship between number line estimation and mathematical achievement: the role of visuomotor integration and visuospatial skills. Journal of Experimental Child Psychology, 145 . pp. 22-33. ISSN 1096-0457

RUNNING HEAD: NUMBER LINE ESTIMATION & MATHEMATICAL ACHIEVEMENT: VSS

Explaining the relationship between number line estimation and mathematical achievement: The role of visuo-motor integration and visuo-spatial skills

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This study was funded by a project grant (SP4575) from Action Medical Research (AMR), UK. C.G. is supported by a UK Royal Society Dorothy Hodgkin Fellowship.

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Word count: 4,832

Abstract

Performance on number line tasks, typically used as a measure of numerical representations, are reliably related to children's mathematical achievement. However, recent debate has questioned what precisely performance on the number line estimation task measures. Specifically, there has been a suggestion that this task may not only measure numerical representations but also proportional judgement skills; if this is the case, then individual differences in visuo-spatial skills, not just the precision of numerical representations, may explain the relationship between number line estimation and mathematical achievement. The current study investigated the relationships among visuo-spatial skills, visuo-motor integration, number line estimation and mathematical achievement. Seventy-seven children were assessed using a number line estimation task, a standardised measure of mathematical achievement and tests of visuo-spatial skills and visuo-motor integration. The majority of measures were significantly correlated. Additionally, the relationship between one metric from the number line estimation task (R^2_{LIN}) and mathematical achievement was fully explained by visuo-motor integration and visuo-spatial skill competency. These results have important implications for understanding what the number line task measures as well as the choice of number line metric for research purposes.

Explaining the relationship between number line estimation and mathematical achievement: The role of visuo-motor integration and visuo-spatial skills

Achievement in mathematics is not only important for general academic achievement but also for future health and life chances (Williams, Clements, Oleinikova & Tarvin, 2003) (Every Child a Chance Trust, 2008). A large body of research has identified individual differences in mathematical achievement and researchers have begun to explore the range of factors that are important for success in mathematics. Of the cognitive factors studied, individuals' internal numerical representations have frequently been identified as a potential contributing factor to individual differences in mathematical performance (e.g., Muldoon et al., 2013; Siegler & Opfer, 2008).

Representations of number are believed to be stored along a mental “number line” (Dehaene, 1997). A number of tasks have been used to provide an indication of the precision of individuals' numerical representations, of which the number line estimation task is a popular measure (Geary, Hoard, Nugent, & Byrd-Craven, 2008, Siegler & Opfer, 2003, 2008; Simms, Muldoon & Towse, 2013; van den Bos et al, 2015). In a number line estimation task, participants are generally presented with number lines which include the value of the start and end point of the scale (e.g. 0 and 10) and are asked to position a series of numbers on the line. As these tasks may be confounded with number knowledge, a variety of age appropriate scales have been used (i.e. 0-10, 0-20, 0-100, Muldoon, Towse, Simms, Perra & Menzies, 2013) with larger scales being used with older children.

Two metrics can be calculated from participants' responses on the number line task to try and capture developmental change in performance. Curve estimation, using the estimated position as the dependent variable and the actual position as the independent variable, produces R^2 values for both linear (R^2_{LIN}) and logarithmic (R^2_{LOG}) functions that fit the data points. However, it is important to note that even though a participant has a R^2_{LIN} value approaching 1, this does not *necessarily* mean that their responses are highly accurate; instead this indicates that their estimates are linearly spread across the number line. In contrast, percent absolute error (PAE) quantifies the difference between an individual's estimate and the actual position of the number in relation to the scale of the line, thus providing a metric of accuracy of the positioning of estimates. These two metrics are typically presented concurrently (e.g. Muldoon et al., 2012, Opfer & Siegler, 2007, Siegler & Booth, 2004, Simms et al., 2013) with many researchers acknowledging that these metrics may provide distinct information on numerical representations. However, the literature to date currently lacks an in depth discussion as to how and why these metrics differ.

Clear developmental changes in number line task performance have been noted, with young children producing estimates that are best explained by a logarithmic function, with small numbers spread out over the lower end of the number line and larger numbers squashed together at the top of the number line. With development, children's estimates become more evenly spread across the number line and are thus best explained by a linear function (Siegler & Opfer, 2003). However, this change is gradual and children have been observed to concurrently hold linear representations for some scales and logarithmic representations for other scales (Muldoon et al., 2013), consistent with an overlapping waves model of cognition (Siegler, 1996).

Performance on the number line estimation task has been related to children's mathematical achievement in a number of studies (e.g., Booth & Siegler, 2008; Muldoon et al., 2013; Siegler & Opfer, 2008; van den Bos et al, 2015) in which participants with more linear and accurate performance demonstrate better mathematical achievement. Moreover, interventions that have focused on improving numerical representations through game-based tasks such as a number line board games or repetitive physical movements along a large-scale number line have noted transfer to arithmetic learning and mathematical performance

(Siegler & Ramani, 2008; Fischer, Moeller, Bientzle, Cress & Nuerk, 2011; Link, Moeller, Huber, Fischer & Nuerk, 2013). These findings provide evidence to support a causal relationship between accuracy of performance on number line estimation tasks and mathematical achievement more generally (Moeller, Fischer, Nuerk & Cress, 2015). However, there has been recent debate as to what precisely the number line estimation task measures. Numerous studies refer to metrics of number line estimation as indicators of numerical representations (Booth & Siegler, 2006, 2008; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Siegler & Ramani, 2009). However, a more complex interpretation of these studies may be necessary; for example, some researchers have suggested that performance on number line estimation task measures may be highly confounded by an individual's general cognitive skills (e.g. LeFevre et al., 2013).

Specifically, it has been suggested that performance on the number line task is highly reliant on proportional reasoning skills, or the ability to accurately divide up space and/or number (Ashcraft & Moore, 2012; Barth & Paladino, 2011; Berteletti et al., 2010; Rouder & Geary, 2014). This explanation suggests that children primarily require sufficient number familiarity in order to make a proportional judgement based on the scale of the line (e.g. Ebersbach, Luwel, Frick, Onghena & Verschaffel, 2015). In addition, the proportional reasoning account implies that children will use sophisticated strategies to complete the task, such as using potential anchor points (e.g. end, quartile and mid-points) to produce accurate estimates of numerical positions. It is important to note that very few studies have *directly* measured strategy use during this task, but those that have report that systematic strategies are often used (Heine et al., 2010; Pettito, 1990). However, the proportional judgement explanation of number line estimation performance implies that in order to employ successful strategies on the number line task, children must have sufficient visuo-spatial skills to be able to judge the scale of the line and to parse the space into segments. Moreover, this suggests that the relationship between performance on number line estimation tasks and mathematics achievement may in fact be driven by an individual's visuo-spatial skills.

More generally, visuo-spatial skills have also been found to predict mathematical achievement (Assel et al., 2003; Carlson, Rowe & Curby, 2013; Wai et al., 2009; Mix & Cheng, 2012). However, it is important to recognise that these skills are not unitary. For example, Mix and Cheng (2012) discuss five subsets of visuo-spatial skills including visuo-spatial working memory, mental rotation, spatial visualisation (mental transformations of 2D and 3D objects), perspective taking and disembedding (identifying targets in scenes that contain distracting information) which were identified by Uttal et al. (2013) through a large-scale meta-analysis. Mix and Cheng (2012) note the importance of identifying and assessing more precise relationships between specific components of visuo-spatial and mathematics skills in future research.

Addressing this gap in the literature, Thompson, Nuerk, Moeller and Cohen Kadosh (2013) investigated the specific relationship between mental rotation skill and performance on a variety of numerical representations tasks in an adult sample. In particular, this study indicated individuals who had better mental rotation skills produced less erroneous estimates on a number line estimation task. Currently, only three studies have investigated the relationship between visuo-spatial skills and numerical estimation performance in children, and only two of these have investigated whether visuo-spatial skills can account for the relationship between performance on number line tasks and mathematics achievement.

In a recent study, Crollen and Noël (2015) assessed the impact of visuo-spatial skills on a variety of basic numerical tasks in a sample of 9½ year-olds. Children were defined as either having good or poor visuo-spatial processing based on a composite score of two standardised picture copying tasks and a parent report of visuo-spatial skills. Children with poor visuo-spatial skills had less accurate number line estimation performance, number line bisection skills and symbolic comparison skills than children with good visuo-spatial skills. This suggests that visuo-spatial skills play a role in successful performance on the number line task. However, there are two limitations with this study. Firstly, the measure of visuo-spatial processing used in the study included an assessment of visuo-motor integration, a combination of visuo-spatial processing and fine-motor control. Thus, performance on the visuo-spatial tasks may have been confounded by children's fine motor proficiency. However, visuo-motor integration may be a particularly important skill for number line estimation tasks where the ability to translate visuo-spatial representations into physical space using a motoric response is also required. Secondly, it is possible that this study may have underestimated the importance of visuo-spatial skills for number line estimation as a 0-100 number line estimation task was used. It has generally been observed that 7-8 year-old children perform well on the 0-100 scale and produce strong linear representations (Siegler & Booth, 2004). This may have reduced the individual differences in performance on this task and therefore the strength of the relationship between visuo-spatial skills and task performance.

LeFevre et al. (2013) assessed the relationship between visuo-spatial skills, number line estimation and mathematical achievement in a longitudinal study of 5-9-year-olds. Visuo-spatial processing was assessed using the Analogy subset of the Cognitive Intelligence Test Nonverbal (Gardner, 2000) which measures mental rotation, analogical reasoning and general spatial processing. In addition, visuo-spatial working memory was assessed using a computerised task that required children to indicate on a computer screen the position of a frog on a series of lily-pads, with increasing numbers of pads across trials. These two measures were combined into one composite spatial ability measure. It was observed that there were moderate significant relationships between visuo-spatial processing, number line estimation and mathematical achievement and that growth in number line estimation performance was predicted by visuo-spatial skills. After controlling for visuo-spatial skills, number line estimation task score (as quantified by linear slope values between the to-be-estimated and actual estimates) remained a significant independent predictor of calculation performance, but not of number system knowledge. The authors suggested that future studies should include more refined measurement of visuo-spatial skills using a wider battery of tasks in order to further investigate these relationships.

Gunderson, Ramirez, Beilock and Levine (2012) observed that 7 year-olds with good mental rotation skills displayed greater gains in number line estimation over time than children with poor mental rotation skills. In addition, mental rotation skill was a significant predictor of later number line performance after controlling for previous estimation scores and prior mathematical achievement, thus emphasising the importance of visuo-spatial skills for growth in number line estimation performance. In a second study, Gunderson et al. (2012) tracked children between the ages of 5 and 8 years-old and measured visuo-spatial skills using both a mental rotation and a mental transformation task (i.e. children were asked to indicate which shapes were made from two component pieces joined together). The relationship between visuo-spatial skills at 5 years-old and approximate symbolic calculation skills at 8 years-old was fully mediated by performance on the 0-100 number line estimation task at 6 years-old. Therefore, using a limited task battery of visuo-spatial measures, these studies emphasised the relationship between mental rotation, mental transformation, number line estimation and calculation skills. However, more in depth assessment of additional specific visuo-spatial processing skills, such as those discussed by Mix and Cheng (2012), is still required. In addition, the number line estimation tasks in both studies utilised an exceptionally low number of estimation trials in contrast to comparable studies (Study 1: 6 estimation points; Study 2: 10 estimation points), therefore the validity of these metrics may be questioned.

In summary, previous research has suggested that performance on the number line estimation task is a predictor of general mathematical achievement (Booth & Siegler, 2008; Muldoon et al., 2013). In addition, visuo-spatial skills and visuo-motor integration have been observed to be important for mathematical achievement (Assel et al., 2003; Carlson, Rowe & Curby, 2013; Mix & Cheng, 2012; Wai et al., 2009). Accounting for visuo-spatial skills has diminished the importance of number line estimation performance (Crollen and Noël, 2015; LeFevre et al., 2013); however, there are methodological problems with these previous studies. Therefore, the current study aimed to assess if the previously observed relationship between number line estimation and mathematical achievement can be explained by individual differences in visuo-motor integration and visuo-spatial skills.

Method

Participants

Seventy-seven 8-10 year-old children (mean age = 9.5 years; SD= 0.6 years months; males= 40) were recruited as control children for the Premature Infants' Skills in Mathematics (PRISM) study. These children were recruited from primary schools in the greater London and Leicestershire areas. Using the Index of Multiple Deprivation score based on participants' postcodes (Lad, 2011), 26%, 23% and 48% of children resided in areas of low, medium and high deprivation respectively. Ethical approval was granted from Derbyshire National Health Service Research Ethics Committee and informed parental consent was gained for all participating children.

Measures

Number line estimation. In order to utilise an age-appropriate number line scale in which sufficient variation in performance would be observed (Crollen & Noël, 2013), a 0-1000 scale was used. The task was administered following the procedure in Siegler and Opfer

(2003). Children completed a series of 0-1000 number line estimation tasks in which they were required to position numbers on blank number lines. The experimenter introduced the task to the child and gave an example of how to complete a trial stating: “This number line goes from 0 at this end to 1000 at this end. If this is 0 and this is 1000 where would you put 500?”. Children indicated the position of the number on the line using a pencil. Children received feedback on the practice trial in order to ensure that they understood the task demands. The experimenter pointed to the 0 and 1000 end points for each trial, labelled them verbally and then stated the specific number that the child should position. The presentation order of the twenty-two numbers that were to be estimated (2,5,18,34,56,78,100,122,147,150,163,179,245,366,486,606,722,725,738,754,818 and 938) was randomised. PAE and R^2_{LIN} scores were calculated for each participant.

Visuo-motor integration. Following from the work of Carlson et al. (2013), visuo-motor integration was measured using the Developmental Neuropsychology Test (NEPSY-II) Design Copying task. This task requires children to copy 21 increasingly complex images using paper and pencil. Separable measures of motor, global and local visuo-motor integration are generated. The global score indicates the accuracy to which participants can replicate the overall composition of the stimuli in the design copying task; in contrast the local score reflects the accuracy of the child’s reproduction of specific details contained within these images. The motor score reflects the level of fine motor control demonstrated in reproducing the images. All three raw scores were converted to scaled scores (Mean= 10, SD= 3). The reliability of these scores is relatively low for the motor (α=.65) global (α=.64) and local (α=.48) measures of visuo-motor integration.

Visuo-spatial skills. As disembedding was identified as a pertinent skill for general mathematical achievement (Mix & Cheng, 2012), the Arrows subtest from the NEPSY-II was used to assess children’s disembedding skills. This test has acceptable reliability (α=.76). The task requires children to identify arrows that are aligned to a centre of a target that are presented among distractor arrows. The raw score was converted to a scaled score (Mean= 10, SD= 3).

In addition, visuo-spatial working memory (Szucs, Devine, Soltesz, Nobes & Gabriel, 2013) and mental rotation (Mix & Cheng, 2012) are consistently observed to be significant predictors of mathematical achievement. These were assessed using the Mr X task from the Automated Working Memory Assessment (AWMA, Alloway, 2007). This task requires children to make judgements about a series of rotating images whilst remembering the position of those images. Children are then asked to recall the positions by tapping the screen. The series of positions increases in length throughout the task. Standardised working memory and mental rotation scores were calculated from the raw score for analyses (Mean= 100, SD= 15). The test-retest reliability of both the working memory (α=.84) and mental rotation (α=.81) scores was good.

Mathematical achievement. Mathematical achievement was measured using the combined score of the Weschler Individual Achievement Test-II (WIAT-II) Mathematical Reasoning and Numerical Operations sub-scales. A standardised score, which has excellent reliability (α=.94), was calculated from the raw combined scores (Mean= 100, SD =15).

Results

The two metrics from the number line task were first calculated for each participant. The PAE was calculated as the absolute average distance between the actual and estimated positions of numbers on the line, divided by the scale of the line multiplied by 100 (following Siegler & Booth, 2004), The linear R^2 function (R^2_{LIN}) was calculated by running a curve estimation per participant (independent variable = actual number positions; dependent variable = estimated points). Overall, 86% of participants’ distributions of estimates were best explained by a linear function. Table 1 displays the descriptive statistics and Table 2 displays the first order correlations for all measures. Mathematical achievement was significantly correlated with both R^2_{LIN} ($r = .257$) and PAE ($r = -.574$) measures of number line task performance.

Creation of component variables from visuo-motor integration and visuo-spatial skills measures

Principal Component analysis with direct oblimin rotation was conducted using the six visuo-spatial and visuo-motor integration measures in order to create component variables to aid further analyses and to identify commonalities across variables without assuming that these are distinct underlying constructs. Kaiser-Meyer-Olkin (KMO) value of .640 indicates that the sample was adequate for principal component analysis. The Bartlett’s test of sphericity was also significant $\chi 2$ (15) = 287.7, $p < .001$. Table 3 displays the results of the principal components analysis in which two component variables were created when loading values of <.3 were suppressed. These were labelled Visuo-spatial skills (Component Variable 1) and Visuo-motor integration (Component Variable 2). These two component variable scores were used in subsequent analyses.

The relationship between the two component variable scores, number line performance and mathematics achievement can be observed in Table 2. Both the visuo-spatial component variable and the visuo-motor integration component variable were significantly correlated with each other and also with the number line performance measures and mathematical achievement.

Predicting mathematical achievement: Assessing the relationship with number line estimation after controlling for visuo-spatial and motor skills

Two separate linear regressions were conducted predicting mathematical achievement, one model utilised R^2_{LIN} score as the metric of number line estimation, the other utilised PAE. In Step 1 of the model, the visuo-spatial and visuo-motor integration component variable scores were entered as independent variables, and at Step 2 either R^2_{LIN} or PAE was also entered as an independent variable (Table 4). The model including R^2_{LIN} score explained 33.1% of the variance in mathematical achievement. Both the visuo-spatial and the visuo-motor integration component variable, but not the R^2_{LIN} values, were unique significant predictors. The model including PAE score explained 42.0% of variance in mathematical achievement. The visuo-spatial component variable, visuo-motor integration component variable and PAE scores were all significant unique predictors. Importantly, the addition of PAE added significantly to the model over and above visuo-spatial and visuo-motor integration component variables in explaining variance in mathematical achievement. However, in a separate model, the addition of R^2_{LIN} . did not significantly increase the amount of variance explained in mathematical achievement over visuo-spatial skills and visuo-motor integration component variables. Overall, Fishers-r-to-z transformation revealed that the amount of variance explained for mathematical achievement by both of the models (i.e either containing PAE or R^2_{LIN}) did not differ significantly ($z= -0.63$, $p= .529$).

Reversing the order by which variables were entered into the hierarchical regression (i.e. by first entering the number line estimation metrics and then entering the visuo-spatial and visuo-motor integration component variable scores; Table 5) indicated that the variance in mathematical achievement accounted for by the R^2_{LIN} measure was shared with the visuo-spatial and visuo-motor integration component variables, that is, R^2_{LIN} did not account for any unique variance in mathematical achievement. In contrast, the model using the PAE measure indicated that PAE, visuo-spatial and visuo-motor integration component variables contributed both shared and unique variance to mathematical achievement.

Discussion

This study assessed whether the relationship between number line estimation and mathematical achievement could be explained by individual differences in visuo-spatial skills and visuo-motor integration. A significant relationship was found between number line performance and mathematical achievement, replicating the large body of research showing that children who have more linear and accurate performance on number line tasks demonstrate better mathematical achievement (Booth & Siegler, 2008; Muldoon et al., 2013; Siegler & Opfer, 2008; van den Bos et al, 2015). It was also observed that there were medium to strong significant correlations between all visuo-spatial and visuo-motor integration measures and number line estimation, barring between R^2_{LIN} and both visuo-spatial working memory and mental rotation. Therefore, these data highlight the important relationship between visuo-spatial processing, visuo-motor integration skills and number line estimation and replicate previous research by both LeFevre et al. (2013) and Crollen and Noël (2015). These data also emphasise that results from studies using the number line estimation task as a measure of numerical representations should be interpreted with caution due to the identification of general cognitive skills that are important for successful task completion.

Importantly, in contrast to Crollen and Noël (2015), the current study utilised a number line that was appropriate for the sample age group and still found a relationship between visuo-spatial skills, visuo-motor integration and number line estimation. Additionally, in response to the limitations of LeFevre et al. (2013), this study administered a wide range of visuo-spatial and visuo-motor integration measures. Principal Component analysis was utilised to create two component variables to aid analyses and interpretation, the first of which comprised measures of disembedding (NEPSY-II Arrows), visuo-spatial working memory and mental rotation (AWMA Mr X task) that were identified as contributing to a visuo-spatial skills component variable. Visuo-motor tasks (NEPSY-II

Design Copying Motor, Global and Local scores) were identified as forming a visuo-motor integration component variable. The divergence of these component variables is logically explained due to the necessity of motor skills for the completion of measures that were identified as contributing to the visuo-motor integration component variable. Uttal et al. (2013) previously established that visuo-spatial working memory, mental rotation and disembedding were not statistically separable visuo-spatial factors through meta-analyses. Our analyses support these previous findings as these skills also were identified as contributing to the same component variable in the present study.

Number line estimation was significantly related to mathematical achievement, replicating previous findings (e.g. Booth & Siegler, 2008; Muldoon et al., 2013). However, hierarchical regressions indicated that PAE, but not R^2_{LIN}, was a significant predictor of mathematical achievement over and above visuo-spatial and visuo-motor integration scores. LeFevre et al. (2013) observed that number line estimation performance (albeit as measured by linear slope values between to-be-estimated and actual estimates) was not an independent predictor of mathematical achievement after accounting for visuo-spatial skills, and the current study has replicated this finding using R^2_{LIN} values.

However, it is important to emphasise that number line estimation as measured by PAE remained a significant predictor of mathematical achievement after adjustment for visuo-motor integration and visuo-spatial skills in the present study. These results indicate that the choice of metric for number line estimation is important as the relationship between R^2_{LIN} and mathematical achievement is driven by visuo-motor integration and visuo-spatial skills more so than for PAE. R^2_{LIN} is a substantially more skewed variable than PAE (2.37 vs 1.96 respectively) indicating that these metrics are potentially measuring different constructs. However, it should also be noted that there is substantial variance in the performance on the number line task performance when using either metric (Table 1).

These results are important for future research as to date as the choice of metric to be reported has not always been fully justified, however, as shown here, these values potentially represent different processes. Instead we would suggest that decisions about which metric to use should be made with caution and that PAE may provide a measure of numerical representations that is less confounded with visuo-spatial skills and visuo-motor integration.R^2_{LIN} values indicate the relative positioning of estimated marks on the number line, in contrast PAE indicates the accuracy of absolute positions of estimated numbers. R^2_{LIN} values was particularly correlated with global visuo-motor integration scores, which assesses ability to produce an overall representation of target stimuli. This is an important skill when attempting to use proportional skills to evenly spread estimated points across the number line and may therefore explain why controlling for visuo-motor integration skills negated the relationship between R^2_{LIN} and mathematical achievement.

Additionally, for both the model including PAE and the model including R^2_{LIN} values visuo-spatial and visuo-motor integration skills component variables explained a similar amount of variance to each other in mathematical achievement *within* the models. Thus our data suggest that both of these skills are equally important for mathematical achievement. Although these data emphasise that the relationship between number line estimation and mathematical achievement can at least be partially accounted for by visuo-spatial skills and visuo-motor integration, there may be unmeasured factors that explain additional variance, such as working memory or processing speed.

More generally, visuo-spatial and visuo-motor integration skills are pertinent for many mathematical tasks (Assel et al., 2003; Carlson, Rowe & Curby, 2013; Wai et al., 2009; Mix & Cheng, 2012) but we suggest they are particularly important for number line estimation due to the high spatial component included in the task and the need for motor precision. Therefore, in addition, the widely reported relationship between number line estimation and other specific mathematical skills (e.g. simple addition, Booth & Siegler, 2008; knowledge of numerical order, Berteletti et al., 2010; identifying ordinal relations, Muldoon, Simms, Towse, Menzies & Yue, 2011) may at least be partially explained by their shared reliance on visuo-spatial and/or visuo-motor integration skills.

These results also have ramifications for the future use of the number line estimation task in education. It essential that teachers recognise that the number line task does not just provide an indication of children's understanding of the number system, but also their visuo-motor integration and visuo-spatial processing skills. Therefore, although this task is a useful teaching tool it should be recognised that it may be more challenging for use with children with problems in visuo-motor integration and poor visuo-spatial skills. Children's performance on the number line estimation task is therefore not only constrained by their knowledge of the number system, but also general cognitive skills that enable them to approach and complete the task successfully.

A strength of this study is the wide range of components of visuo-motor integration and visuo-spatial skills assessed. Together, these measures provide a novel picture of the factors that contribute to number line estimation. Of course numerous additional factors may contribute to number line estimation performance, such as number knowledge (Muldoon et al., 2012), attention and specific strategy use (Heine et al., 2010). Although this study makes a contribution to the understanding of the complexities of number line estimation further research should expand on the breadth of measures utilised. These data will contribute to the debate on precisely *what* this task measures. As these data were cross-sectional, it would beneficial to collect further data longitudinally to establish if there are changes in the nature of the relationship between number line estimation, mathematics attainment, visuo-motor integration and visuo-spatial processing skills as these may change over time. As such, the utility of the number line task may as a measure of numerical representations may vary with age.

This study provides additional evidence on the relationship between visuo-spatial skills, number line estimation and mathematical achievement in 8-10 year-old children. These data emphasise the importance of understanding the complexities of the general cognitive skills that contribute to performance on this task. In addition the study highlights that R^2_{LIN} and PAE values should not be used interchangeably in numerical cognition research. Finally, we suggest that researchers may wish to reconceptualise their interpretation of precisely what this task measures and adopt a more cautious approach if describing the number line task as one which solely measures numerical representations.

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Table 1: Descriptive statistics for all measures

Measure	**Construct**	**Mean**	**S.D.**	**Min**	**Max**
WIAT-II Composite	Mathematical achievement	103.6	20.7	53.0	151.0
NEPSY-II Arrows scaled score	Visuo-spatial skills: Disembedding	10.5	3.1	1.0	18.0
NEPSY-II Design Copying Motor scaled score	Visuo-motor integration: Motor control	14.1	2.5	5.0	19.0
NEPSY-II Design Copying Global scaled score	Visuo-motor integration: Global processing	13.2	2.9	3.0	19.0
NEPSY-II Design Copying Local scaled score	Visuo-motor integration: Local processing	14.1	3.0	4.0	19.0
Mr X task (AWMA)	Visuo-spatial skills: Working memory	108.9	15.9	75.0	146.0
Mr X task (AWMA)	Visuo-spatial skills: Mental rotation	108.5	16.0	77.0	135.0
Number line R^2_{LIN}	Linearity on number line task	0.9	0.2	0.2	1.0
Number line PAE	Accuracy on number line task	8.7	6.8	2.4	35.1

Note: WIAT-II= Weschler Individual Achievement Test- II; NEPSY-II= A Developmental Neuropsychological Assessment-II; AWMA= Automated Working Memory Assessment; R^2_{LIN}= R^2 value of the linear function; PAE= Percent absolute error

Table 2: First order correlations between all measures

	Arrows	**DC Motor**	**DC Global**	**DC Local**	**VS working memory**	**Mental rotation**	**R^2_{LIN}**	**PAE**	**VSS component variable**	**VMI component variable**
WIAT-II Mathematics	.283*	.326*	.470**	.421**	.453**	.436***	.257*	-.574**	.464***	.478***
Arrows	-	.131	.347*	.228*	.438**	.446***	.306*	-.438**	.660***	.285*
DC Motor	-	-	.488**	.496**	.218	.130	.322*	-.328*	.150	.829***
DC Global	-	-	-	.547**	.371*	.361**	.539**	-.446**	.446***	.806***
DC Local	-	-	-	-	.315*	.312**	.389**	-.397**	.346**	.823***
VS working memory	-	-	-	-	-	.962***	.198	-.407**	.950***	.328**
Mental rotation	-	-	-	-	-	-	.199	-.413***	.959***	.283*
R^2_{LIN}	-	-	-	-	-	-	-	-.644**	.272*	.502***
PAE	-	-	-	-	-	-	-	-	-.478***	-.463***
VSS component variable	-	-	-	-	-	-	-	-	-	.345**

Note: DC= Design Copying; VS= Visuo-spatial; PAE= Percent Absolute Error; VSS= visuo-spatial skills; VMI= visuo-motor integration; *** p< .001; ** p< .01; * p< .05

Table 3: Results of Principal Component Analysis

Measure	**Component variable 1 VS skills**	**Component variable 2 VMI**
Arrows	0.638	0.065
DC Motor	-0.154	0.882
DC Global	0.191	0.740
DC Local	0.071	0.798
VS working memory	0.950	0.001
Mental rotation	0.977	-0.054

Note: DC= Design Copying; VS= Visuo-spatial; VMI= Visuo-motor integration; Values < .3 suppressed.

Table 4: Linear regressions predicting mathematical achievement

		R^2_{LIN} model		PAE model	
		ß	**R^2 change**	**ß**	**R^2 change**
Step 1	**VS component variable**	0.340**	.330***	0.339**	.330***
	VMI component variable	0.361**		0.360**	
Step 2	**VS component variable**	0.342**	.000	0.208*	.089**
	VMI component variable	0.371**		0.237*	
	Number line	-0.022		-0.365**	

Note: VS= visuo-spatial; VMI= visuo-motor integration; *** p< .001; ** p< .01; * p< .05

Table 5: Linear regressions predicting mathematical achievement

		R^2_{LIN} model		PAE model	
		ß	**R^2 change**	**ß**	**R^2 change**
Step 1	**Number line**	.257*	.066*	-.574***	.329***
Step 2	**Number line**	-.022	.264**	-.365**	.09**
	VS component variable	.342**		.208*	
	VMI component variable	.371**		.237**	

Note: VS= visuo-spatial; VMI= visuo-motor integration; *** p< .001; ** p< .01; * p< .05