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The effective mathematics department: adding value *and* increasing participation?

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Abstract

Given the commonly accepted view that having a mathematically well-educated populace is strategically important, there is considerable international interest in raising attainment, and increasing participation, in post-compulsory mathematics education. In this article I develop multi-level models using datasets from the UK Department for Education's National Pupil Database (NPD) in order to explore 1) school effects upon student progress in mathematics from age 11-16 in England, and 2) student participation in advanced level mathematics over the following two years. These analyses highlight between-school variation in the difference between mathematical and general academic progress. Furthermore, the between–school differences in post-compulsory mathematics participation are large. Importantly, there is no evidence to suggest that schools/departments with higher 'contextual value added' from 11-16, a key measure in government accountability processes in England, are also more effective in recruiting and retaining students in post-16 advanced mathematics courses.

Keywords: mathematics, attainment, participation, multi-level modeling

Introduction

School mathematics is of central importance in school curricula across the world. Its inclusion in major international comparison studies such as the OECD's Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Survey (TIMSS) have resulted in successive UK governments using mathematics as a barometer for judging the efficacy of the education system as a whole. As a result, the teaching and learning of mathematics receives particularly close scrutiny. In turn, policy development has been predicated upon the belief that these international comparisons have validity in predicting future economic productivity and fiscal security.

For some years there has been a concern amongst policymakers and stakeholders that the supply of STEM (science, technology, engineering and mathematics) academics, professionals and technicians needs to be increased. Such concerns are heard in the UK (Roberts, 2002), Europe (Gago, 2004) and elsewhere in the developed world (e.g. in the US, National Academies, 2007). There is a strong utilitarian current in policymaking that aims to increase the level of mathematical skills to ensure a continued strong position in the changing world economy. For example, there has been a recent drive to introduce 'functional mathematics' (Roper, Threlfall, et al., 2006) into schools and colleges in England in order to placate employers who repeatedly complain about the skills of new employees (e.g. CBI, 2010). Although these debates are echoed around the developed world they are inflected locally, resulting in different development trajectories in education systems generally and in mathematics education in particular. The current UK coalition government has recently introduced changes to the Programme of Study for 14-16 year olds as well as a new national mathematics qualification, but at the same time it is conducting yet another full curriculum review. However, research suggests that enacted curriculum and pedagogy change little over time (Galton & Hargreaves, 2002). The same is true of deeply embedded societal attitudes towards mathematics, which, in England, contribute to the vast majority of young people

happily ceasing their formal study of the subject at age 16. The sentiment of the student who reported that they "would rather die" (Brown, Brown, et al., 2008) than continue to A level mathematics is not uncommon.

A recent report (Hodgen, Pepper, et al., 2010) has highlighted England's position as an international outlier in terms of post-16 mathematical participation. Such concerns are well documented (Mendick, 2005; Noyes, 2009; Royal Society, 2008; Wiliam, Brown, et al., 1999) but there is currently little idea of how to tackle this problem. One of the causes of this general 'quiet disengagement' (Nardi & Steward, 2003) with high school mathematics is the increasingly *performative* (Ball, 2003) nature of schooling, with teachers working under the panoptical gaze of performance tables and the schools inspectorate. For the last four years mathematics has been included in the published school performance measure of five or more higher grade passes (A*-C) for 16 year olds and this has further embedded atomised, test-oriented curricula and pedagogy (Ofsted, 2008).

In England, young people complete their compulsory schooling at age 16 (Year 11) with the General Certificate of Secondary Education (GCSE) qualifications. Obtaining five or more higher grades (A*-C) allows students access to a wide range of further educational opportunities. The majority of those achieving this level at GCSE proceed to the traditional academic track of Advanced level awards (General Certificate of Education or GCE). These are the standard university-entrance qualifications and most students would study three or four subjects over the following two years, up to the age of 18 (Year 13). Sometimes a student might complete half of one of these two-year, modular A level courses and so receive an Advanced Supplementary (AS) award. Most proceed to the second year of study to complete the full Advanced level qualification (A2). Advanced level Mathematics is a pre-requisite for most Science, Technology, Engineering and Mathematics (STEM) courses in higher education. Around 10-15% of each cohort of 16-year-olds chose to continue with their study of mathematics, a proportion which is unusually low amongst developed countries.

A number of school effectiveness studies focus either partly or exclusively on school mathematics (for example, Cervini, 2009; Opdenakker, Van Damme, et al., 2002; Teodorović, 2011). However there is a paucity of research in England indicating a) whether there is a significantly different uptake of advanced level mathematics in different schools, and b) what might cause such differences. Understanding complex school environments in order to better inform policies and strategies designed to increase participation in mathematics (and science, e.g. Smyth and Hannan (2006)) are therefore of the utmost importance. The broader study from which this article arises is a longitudinal, multi-scale (Noyes, 2012, in press), mixed-methods project exploring regional patterns of mathematics attainment and participation and the roles of families, peers, teachers and schools in creating these patterns. In England, official data is reported at the level of schools in what are commonly termed *league tables*. So, whilst considering how schools effect students' progress in mathematics my real interest is with the mathematics 'department'. Analysis of fieldwork and survey data shows that departments do not always reflect the qualities of the school. For example, some strong mathematics departments seem to have a much greater positive impact upon student progress than other departments in the school. As a consequence, school-level results can hide considerable variation for particular departments, an issue explored by Sammons et al. (1997). Nearly fifteen years ago Sammons et al. suggested moving away from school league tables in order to look more closely at departments. This hasn't happened, at least in the public domain.

What difference does a department make? The answer to this question depends upon what one is interested in exploring: attainment, participation, learner self-efficacy, engagement, interest, etc. Perhaps more importantly we might ask which of these measures might be necessary to describe a 'good' department. The problem here is one of values – what does one mean by good? This paper is ultimately interested in exploring a particular kind of *good*, namely the level of participation in post-16 mathematics education, but I am also concerned with the progress made by learners and whether they are significantly more likely to attain that all-important GCSE grade C in one school over another. The attainment of a GCSE grade C or above, or

participation in advanced level mathematics (completed with a *good* grade), are both cultural 'goods' with particular exchange value. For example, Wolf (2002) points out that mathematics is the only A level that increases likely future earnings. This 'fact' about the economic return on A level mathematics, questionable as it is due to changing demographics, work and the shifting qualifications frameworks, is well known by teachers who exploit the claim in their drive to recruit students to courses.

Notwithstanding the criticisms of school effectiveness research (see Luyten, Visscher, et al., 2005, for a recent discussion), this paper reports multilevel models to explore the extent to which mathematics departments impact pupil progress from 11-16. Such differences will have a knock-on effect on the likelihood of further participation in mathematical study (Noyes, 2009). That said, it is clear from other data from this project that the differences between classes in a department are greater than the aggregated differences between departments (Noyes, 2011, under review). Studies of school and teacher effects have also suggested this (Opdenakker, et al., 2002) and even that such differences might be greater in mathematics than in English, for example (Nye, Konstantopoulos, et al., 2004). The data used in this study is taken from the Department for Education's National Pupil Database (NPD). This database has a comprehensive record for every student in the country. These records are not organised into classes but include Unique Pupil Reference Numbers (UPRN), school identifiers, a range of social background variables and attainment measures from various Key Stages of the education system. The NPD consists of a range of datasets which can be matched through the UPRN. In the following analysis I want to ascertain whether it matters which school a child attends in terms of their mathematical attainment and progression. That is not to say that one could choose a better school or mathematics department as Leckie and Goldstein (2009) have shown that, in contrast to the claims made for them, typical school effectiveness models are not good predictors of future performance. My interest here is more exploratory and explanatory rather than predictive.

The second part of the paper then considers a different issue about departmental effects which is concerned with their impact on recruitment and retention of students in post-16 mathematics. This is a particular policy concern in England as outlined above. Due to constraints on the data this second analysis draws on a different, but intersecting, dataset. These two sets of models are brought together in this paper in order to consider whether the same mathematics departments are equally strong in these two areas, and indeed whether there is a correlation at all. Or, have performative cultures in schools led to some departments being very effective in raising attainment at 16 but in ways which negatively impact ongoing participation?

Modelling progress from 11-16

The hierarchical data structure of the NPD (e.g. students within schools) allows researchers to construct multilevel models which partition variance in student outcomes and progress at different levels. These might include classes, years groups, regions, etc. and various studies model different data structures, depending upon what is available or easily collectable (e.g. Cervini, 2009; Opdenakker & Van Damme, 2000). There is a great deal of technical discussion in the literature, for example regarding sample sizes for multilevel modelling (Cools, Fraine, et al., 2009), but this study does not get too far into such technical matters due to the space required to develop the two distinctive models.

The following bivariate analysis considers Key Stage 2-4 (i.e. aged 11-16) mathematics contextual value added (CVA) models against those for all GCSE (excluding mathematics) for 130 state funded schools in four Local Authorities of the Midlands of England from summer 2004-2008. This sample of the whole national dataset is considered to be representative and sufficient for exploring potential between-school variation. This dataset includes five consecutive year cohorts for each school. The model aims to identify whether there exist schools in which significantly more or less progress is made in mathematics, and where this mathematical progress differs from progress more generally. In other words, is it possible to identify particularly effective or ineffective departments? This is important for the broader questions

about the ways in which departmental effectiveness from 11-16 might relate (or not) to participation in post-16 mathematics. For this reason the analysis falls into the grey area between what can be considered 'the school' and 'the mathematics department'. In the models I include the mathematical outcome variable alongside a 'mean GCSE' variable. Although these might appear to be independent, there is of course an interdependence between what happens in the department and what happens in the school more generally. That said, the models do show that there can be quite marked differences between progress in mathematics and progress more generally (in 'not mathematics').

The original NPD dataset for students in state-funded secondary schools had a small amount of missing data. Running models without the cases for whom data is missing tends to understate the significance of estimates. So whilst the decision has been made to only work with students in state schools (which excludes a sizable group of privately educated student) students with missing GCSE and prior attainment results are retained in the dataset. By running a multiple imputation[i] process, estimates can predict more faithfully those of the full population (i.e. assuming no missing data/cases). In the current dataset there are 118462 students in 131 schools[ii] over a five year period and 7% of these students have some missing data. Not all schools have a cohort in all of the five years, for example where schools have closed or opened during this period but all schools are retained in the dataset. These are state-funded secondary schools so the dataset does not include special or selective schools. There were no schools removed from the dataset. In constructing a five year dataset like this I am assuming that the distribution of prior attainment (Key Stage 2 scores) and outcomes (GCSE scores) are similarly distributed over time. Although year on year trends for schools vary slightly the following analysis assumes that there is an underlying school effect, i.e. there are general school characteristics that effect pupil progress and that these do not change that quickly. With this level of missing data the imputed model makes very small reductions in some of standard errors for estimates that were already highly significant.

In order to conduct this analysis new variables are constructed for the mean GCSE attainment excluding mathematics. The two outcomes in which I am interested are GCSE mathematics grade and mean GCSE (not including mathematics) grade. For simplicity I will refer to these as Maths and GCSE from now on. I treat the student as level 2 in a bivariate multilevel model and these two 'within-student' GCSE outcomes are the level 1 measurements. Level 3 of the model is the within-school year group and level 4 is the school. So student outcomes (*i*) are nested within students (*j*), within cohorts (*k*) within schools (*l*). The basic model for student scores (y_{ijkl}) *i*s as follows:

$$y_{ijkl} = \beta_{1jkl}x_{1ijkl} + \beta_{2jkl}x_{2ijkl}$$

$$x_{1ijkl} = \begin{cases} 1 \text{ if maths} \\ 0 \text{ if GCSE} \end{cases}, \qquad x_{2ijkl} = 1 - x_{1ijkl}$$

$$\beta_{1jkl} = \beta_1 + f_{1l} + v_{1kl} + u_{1jkl}\,, \quad \beta_{2jkl} = \beta_2 + f_{2l} + v_{2kl} + u_{2jkl}$$

$$var\,(f_{1l}) = \sigma^2_{f1}\,, \quad var\,(f_{2l}) = \sigma^2_{f2}\,, \quad cov(f_{1l}, f_{2l}) = \sigma_{f12}$$

$$var\,(v_{1kl}) = \sigma^2_{v1}\,, \quad var\,(v_{2kl}) = \sigma^2_{v2}\,, \quad cov(v_{1kl}, v_{2kl}) = \sigma_{v12}$$

$$var\left(u_{1jkl}\right) = \sigma^2_{u1}\,, \quad var\left(u_{2jkl}\right) = \sigma^2_{u2}\,, \quad cov\left(u_{1jkl}, u_{2jkl}\right) = \sigma_{u12}$$

There is no variation at level one (*i*) as this exists to create the bivariate structure. Variance is partitioned between students (*u*), cohorts (*v*) and schools (*f*) and the modelling process begins by specifying the empty model in order to explore these initial variances. Further models are then specified with the inclusion of a range of predictors and estimates calculated separately for the two response variables. The following analyses are conducted in MLwiN. Non-categorical explanatory variables and the GCSE outcomes have been normalised.

	Empty model							
	Math					GCSE		
Fixed Part	estimate	s.e.	VPC			Estimate	s.e.	VPC
Constant	-0.071	(0.034)				-0.057	(0.038)	
Variance				Maths.GCSE covariance				
School	0.146	(0.019)	0.153	0.156	(0.02)	0.180	(0.023)	0.177
Cohort	0.016	(0.001)	0.017	0.011	(0.001)	0.013	(0.001)	0.013
Student	0.792	(0.003)	0.830	0.071	(0.003)	0.826	(0.003)	0.810
deviance	441850							
Schools	131 (116744 students)							

Table 1: Empty bivariate model (A) for attainment at GCSE (standard errors are reported in parentheses). Variance participation coefficients (VPC) are also included.

	Model A							
	Math					GCSE		
Fixed Part	estimate	s.e.	VPC			estimate	s.e.	VPC
Constant	-0.038	(0.019)				-0.022	(0.020)	
KS2 ave points	0.245	(0.008)				0.360	(0.008)	
KS2 English	0.084	(0.004)				0.273	(0.005)	
KS2 maths	0.422	(0.005)				0.116	(0.005)	
Variance				Math.GCSE covariance				
School	0.043	(0.006)	0.116	0.040	(0.006)	0.051	(0.007)	0.121
Cohort	0.009	(0.001)	0.024	0.005	(0.001)	0.007	(0.001)	0.016
Student	0.319	(0.001)	0.860	0.259	(0.001)	0.365	(0.002)	0.863
deviance	299194							
schools	130 (111305 students)							

Table 2: Basic prior attainment bivariate model (A) for attainment at GCSE.

The empty model (Table 1) provides a baseline from which to compare later models. Students make slightly less progress in mathematics than they do generally (or GCSE mathematics is slightly harder than other subjects generally). The model suggests that, without any attempt to explain away any variation in progress, around 15% is attributable to schools and over 80% to the student. Only a very small amount of the variance (less than 2%) is attributable to the cohort. This cohort measure is not simply a measure of the cohort of students but also the group of teachers that have worked with them. It might also reflect school changes that contribute to longer term trends in increased/decreased GCSE attainment.

When prior attainment measures (which are standardised normal scores) are included in model A (Table 2), prior attainment in mathematics has a significant effect on progress, much more so than English prior attainment has on general progress. Being 1 standard deviation higher in mathematics score at age 11 yields nearly a whole GCSE grade at age 16. English attainment at age 11 has a small but significant role in predicting GCSE mathematics. What is also clear is that the inclusion of these prior attainment measures explains quite a lot of the variance in attainment. Mathematics attainment variance is reduced by 60% and of the remaining residual variance slightly more is attributable to the student (86%) than in the empty model and now 12% and 2% to the school and cohort levels of the model. For the GCSE attainment a similar amount of the variance is attributable at each level. At this stage this middle *level* – the cohort – is merely an exploratory component in the model. It does however indicate some small variations over time and this stability of school effectiveness is an important issues that there is not space to explore herein (Creemers & Kyriakides, 2010). Figure 1 indicates that the assumption of normality underpinning this model is justified.

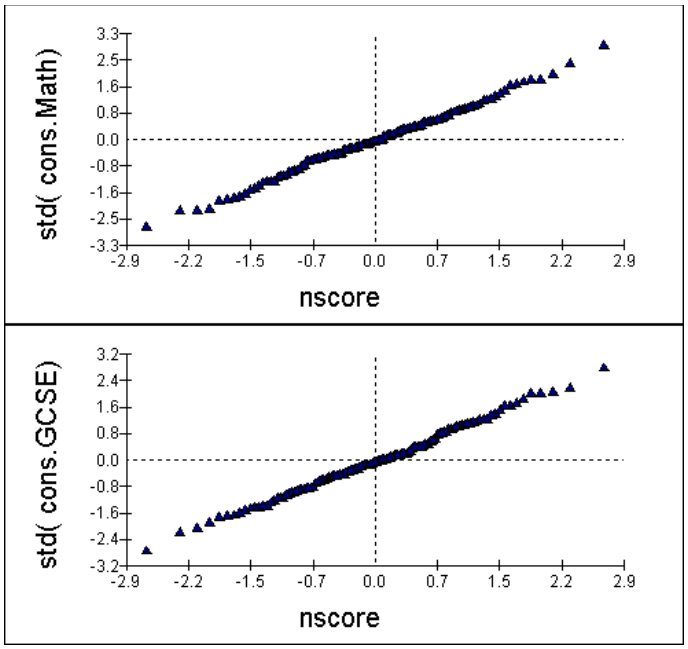

Figure 1: normal score plots indicated the appropriateness of the model.

In order to specify the model more fully (model B, Table 3) a range of explanatory variables are included and experimented with to improve the model fit (as indicated by the reduced deviance). Most of these are at the individual level (Table 3). Cohort level compositional variables are generally not significant and have only a marginal effect on model fit. The mean cohort prior average attainment at age 11 is the exception so this is retained.

	Mathematics					GCSE		
Fixed Part	Est.	s.e.				Est.	s.e	
Constant	-0.019	0.015				-0.062	0.014	
KS2 ave	0.251	0.008				0.382	0.008	
KS2 Eng	0.061	0.005				0.211	0.005	
KS2 mat	0.421	0.005				0.131	0.005	
Female	0.044	0.004				0.181	0.004	
FSM	-0.128	0.006				-0.171	0.006	
IDACI	0.020	0.002				0.026	0.002	
SEN								
school action	-0.081	0.006				-0.089	0.006	
school action plus	-0.136	0.009				-0.152	0.01	
statement	-0.005	0.012				0.013	0.012	
Ethnicity								
Chinese	0.313	0.039				0.285	0.041	
Pakistani	0.148	0.018				0.169	0.019	
Bangladeshi	0.140	0.039				0.141	0.041	
Indian	0.089	0.016				0.103	0.017	
African	0.080	0.041				0.071	0.043	
Any Other Asian Background	0.074	0.035				0.101	0.037	
Any Other Ethnic Group	0.067	0.037				0.075	0.039	
White and Asian	0.046	0.03				0.044	0.032	
Any Other White Background	0.033	0.017				0.051	0.018	
Any Other Mixed Background	0.008	0.024				0	0.026	
Caribbean	0.007	0.018				0.030	0.019	
White and Black Caribbean	-0.038	0.017				-0.033	0.018	
White and Black African	-0.078	0.049				-0.120	0.052	
Any Other Black Background	-0.081	0.029				-0.027	0.031	
Cohort Average KS2 pts	0.107	(0.014)				0.138	0.013	
Variance			vpc	Math.GCSE covariance				vpc
School	0.026	(0.004)	0.076	0.019	0.003	0.024	(0.003)	0.063
Cohort	0.001	(0.001)	0.003	0.005	0.001	0.008	(0.001)	0.021
Student	0.313	(0.001)	0.921	0.249	0.001	0.348	(0.001)	0.916
Variance				291168				
schools				130 (110747 students)				

Table 3: Bivariate model B including a range of explanatory variables.

Firstly, consider how this fully specified model has accounted for more of the initial variation in student attainment. Compared to the empty model there remains 38% and 41% of the total variance unexplained for maths and GCSE. Interestingly, the inclusion of these explanatory variables now partitions the unexplained variance for school, cohort and student as 8, <1 and 92% for mathematics and 6, 2 and 92% for GCSE. This gives us a sense of the year on year variation in student performance which appears to be greater for general attainment than for mathematics.

I now consider some of the estimates for the explanatory variables. Girls make more progress than boys in both mathematics and GCSE but the difference is less in mathematics. As strong as the effect of being female is positive, the impact of being eligible for free school meals (FSM) is negative, although this is slightly less in mathematics than for GCSE generally. The IDACI (Income Deprivation Affecting Children Index) score suggests a small positive effect for those living in more affluent areas but this is much smaller in size than the FSM effect. The IDACI measure is not particular to the individual child (like FSM) but is derived from census data and therefore related to the neighbourhood where the student lives[iii]. As might be expected, students on the special educational needs (SEN) register make less progress than their peers.

The ethnicity categories in Table 3 have been ordered (for mathematics) and show that, compared to the White British base category, Asian, Chinese and African students all make better progress in mathematics and GCSEs generally. Chinese students gain over half of a GCSE grade on their White British peers. Table 3 also shows that there is a compositional effect upon learner progress whereby students in schools with a higher mean score at age 11 make more progress in both mathematics and GCSE generally.

The familiar caterpillar plots in Figure 2 have been plotted on the same scales and indicate the school level residuals. The error bars (1.96 x s.e.) are shorter for GCSE than mathematics. This is due to the effect of using the 'mean GCSE' score across a range of subjects which reduces the variance. In the top ranked thirty or so schools, students make significantly better progress than in the similar number of schools at the bottom of this 'contextual value added' (CVA) ranking, for both mathematics and GCSE. Despite the apparently fine grained differences between schools we can only be confident of these rather broad differences between groups of schools (van de Grift, 2009), a point that often goes unrecognised in schools when they receive such plots. If the outcomes are plotted as grades the few schools at the extreme of this ranking add (or subtract) around half of a GCSE grade per student on average, all other things being equal. This is an important difference. However, what we do not know from these two plots is the relationship between progress in mathematics and progress generally. For example, can a particular school appear at a very different place in each of these two plots and what would that tell us? The pairwise plot in Figure 3 below gives an indication of this relationship.

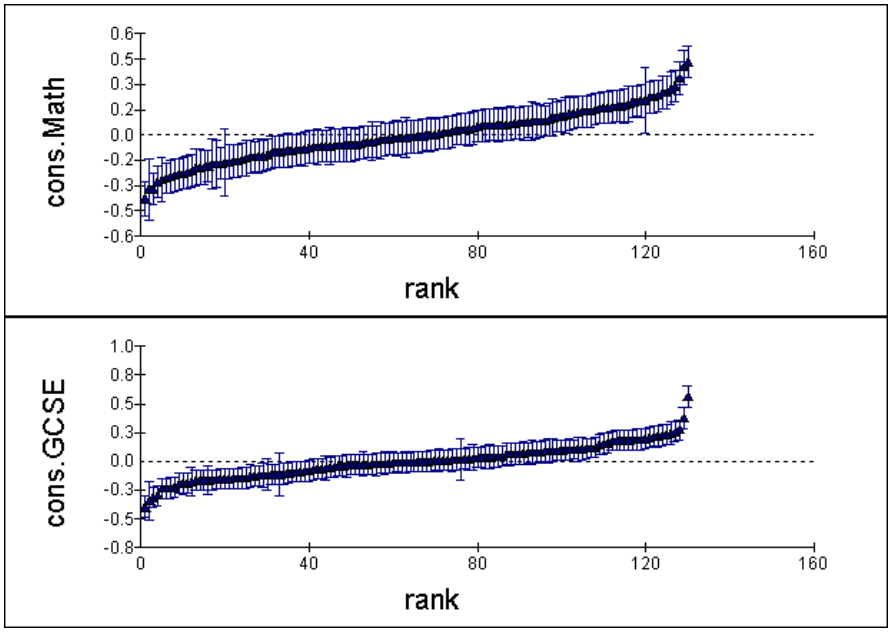

Figure 2: school level residual plots for CVA in math and GCSE respectively

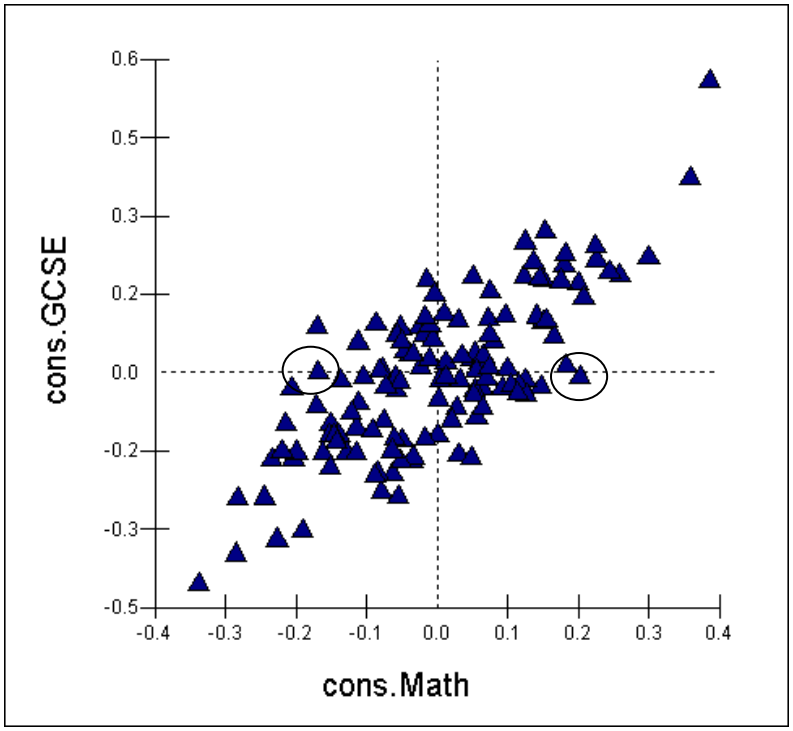

Figure 3: Pairwise plot of school level residuals

Although there is some correlation between CVA in mathematics and more generally, there is also a considerable degree of variation with some departments performing quite differently from the school as a whole. Of particular interest are those schools that are off the y=x diagonal (i.e. where mathematics CVA is different from the GCSE CVA). Those further off the diagonal are particularly interesting as this signals that there might be something peculiar occurring in the mathematics department and that this might have some impact upon future participation which can be connected to models of post-16 completion. Perhaps participation in A level might be related to the distance from the y=x line, i.e. not raw maths CVA but the relative difference between the two measures. It seems from models of post-16 participation (see below) that the difference between attainment in mathematics and generally (and mathematics and English) are small but significant predictors of A level participation. The best and worst measures of value added suggest over a grade difference in mathematical progress from 11-16 between schools. Even for those in the middle of the plot the implications of a more modest shift in attainment, particularly around the C/D borderline are significant.

Figure 3 raises the question of how the maths CVA measures should be interpreted and indeed what the value of the published CVA scores are (which are used for ranking in the school performance tables) when interested in a single subject such as mathematics. Consider the right-hand outlier of the two schools circled in Figure 3. This school is typical in terms of progress made in GCSEs generally but is in the top 10 for mathematics value added and so we might expect to see something in that department which might explain such difference. Similarly schools at approximately (-0.5, 0) do similarly well with GCSE generally but are in the bottom 10% for mathematics value added. These schools might appear very similar generally but make nearly a whole grade difference in pupil progress in mathematics. This is highly significant given the exchange value of mathematics and is a particular issue for those students around the C/D borderline (C and above are all-important 'higher grade' passes). It would be a profitable line of inquiry to take two such schools and research what is different about the mathematics departments (e.g. staffing, teaching and learning, etc.) and how this might be related to differences in progress. The cluster of schools in the upper right quadrant achieve well in mathematics but this is not much different from what happens in the school generally.

Modelling participation 16-18

We now move on to a second modelling context, that of participation in advanced level mathematics. The NPD dataset used here is the 2005 cohort of 16 year olds completing their GCSEs in the East and West Midlands (Government Office Regions) of England who then completed any advanced level qualification (in any subject) over the following two years (36696 students). This dataset covers a larger geographic region than that used in the previous section but only focuses on the GCSE cohort from 2005, who completed A levels in 2007. Admittedly, there is not a neat connection between the two datasets but they do include the same schools, and intersecting sets of students and teachers.

Several important analytical decisions have been made in preparing this data for multilevel modelling and as Gorard (2008) explains, it is important to bear these in mind throughout the analysis. These kinds of processes are explained elsewhere in more detail (Noyes, 2009) but the key points for this analysis are:

- Only students completing one or more A level courses are included in the dataset, i.e. we are concerned only with those students who have chosen some A levels, and might have included mathematics amongst these;

- Only students who obtained a GCSE grade C in mathematics have been included as this is the official eligibility criteria for entry to A Level mathematics. However, this presents a significant problem since entrance criteria vary between schools;

- Only those students from mainstream state secondary schools are included here (around 90% of the cohort)

Learner trajectories do not all fit into this two year cycle (i.e. 2005-7) but it is generally applicable. This analysis accounts for student qualifications in the two years following GCSE awards in 2005. When modelling 'completion', we are unable to tell from the dates of awards in the NPD whether an AS in 2007 took one or two years to complete. The model considers whether a student has gained at least this AS (Advanced Supplementary) qualification.

Another limitation of using the NPD data is that it only reports results (and therefore entries) and so doesn't give the full picture about participation and attrition. Survey data from another strand of the larger project (Noyes & Sealey, 2010) indicates that approximately 10% of 16/17-year-olds who start mathematics do not complete. This is one of the highest attrition rates for A level subjects and a different methodology is required to explore that aspect of participation.

The modelling in this analysis consists of three level, cross-classified binary response models. Students (level 1) are nested within schools when aged 14-16 (level 2) and either the same or a different school when aged 16-18 (level 3). The majority of these students (58%) stay in the same school but since there is movement at 16 both into and out of many schools, levels 2 and 3 of the model are cross-classified. A dummy variable is included to account for changing schools at 16. Models are run initially using predictive quasi-likelihood (PQL) estimation and these coefficients then act as prior estimates for the Markov Chain Monte Carlo (MCMC) estimation which a) gives more reliable estimates of the size of effect attributable to a range of factors and b) is required due to the cross-classified data structure.

The modelling is developed from a single level logistic regression model in which the binary response (0,1) (whether or not they completed any A level mathematics between 2005-7) for the *i*th student with prior attainment x_i is y_i. Denoting as π_i the probability that $y_i = 1$ gives the general model:

$$f(\pi_i) = \beta_0 + \beta_1 x_i + e_i$$

There are a number of possible link functions $f(\pi_i)$ which can be used in such logistic regression

models but here I adopt the logit link function (Rasbash, Steele, et al., 2005) where $f(\pi_i)$ = log $(\pi_i /(1- \pi_i))$. The following model is developed for the *i*th student in the *j*th school for GCSE (up to 16) and the *k*th school for A level mathematics (post-16):

$$\text{logit}\,(\pi_{ijk}) = \beta_{0jk} + \beta_1 x_{ijk} + e_{ijk}$$

$$\beta_{0jk} = \beta_0 + v_{0k} + u_{0j}\ ,\ \ v_{0k} \sim \text{N}(0, \sigma_v^2)\ ,\ u_{0j} \sim \text{N}(0, \sigma_u^2)\ ,\ e_{ijk} \sim \text{N}(0, \sigma_e^2)$$

As before the models were run in MLwiN. Due to the size and complexity of the model a burn in period of 5000 with 200000 iterations of the model was used in order for the effective sample size to be sufficiently high (>1000). The resulting parameter estimates are shown in Table 4.

Fixed Part	
Constant	-5.764 (0.155)
GCSE mathematics grade (ref. grade C)	
Grade B	1.755 (0.067)
Grade A	3.432 (0.074)
Grade A*	4.630 (0.096)
Female	-0.824 (0.037)
Difference of GCSE mathematics and English grades	0.486 (0.027)
Difference of GCSE mathematics and average grade	0.283 (0.041)
Number of A level entries	0.658 (0.036)
IDACI score	0.654 (0.150)
Ethnicity (ref. White British. Only statistically significant categories included here)	
Any Other Asian Background	0.950 (0.191)
Indian	0.946 (0.075)
Pakistani	0.802 (0.119)
African	1.151 (0.233)
Bangladeshi	0.691 (0.224)
Chinese	1.167 (0.193)
Post_16 School s.d. of number of A level entries	-0.128 (0.042)
Random Part	
Post-16 between-school variance	0.569 (0.075)
Pre-16 between-school variance	0.252 (0.038)
Number of post-16 centres	509
Number of pre-16 centres	634

Table 4 Parameter estimates for the three-level, cross-classified model of Advanced level mathematics completion 2005-7

A number of things are worth pointing out from the above model. Firstly, consider the between-school variance in completion of some A level mathematics. The variance participation coefficient (Goldstein, Browne, et al., 2002) is the total amount of residual variance attributable to levels 2 and 3 in the model and can be estimated in more than one way. Here I use the following linear threshold model:

$$\text{VPC} = \sigma_u^2/(\ \sigma_u^2 + 3.29)$$

Using this model, estimates for the variances can be calculated as 0.569/(0.569+3.29) = 0.147 at level 3, i.e. the A level centres, and 0.252/(0.252+3.29) = 0.071 at level 2; the GCSE centres. So around 15% of the residual variance in completion of any Advanced level mathematics is attributable to the school or college attended after 16. Schools attended for GCSE (age 14-16) contribute half as much variation again. Together, the schools attended account for over 20% of the variation of completion of some advanced mathematics, after accounting for prior attainment, social background and school mix. This is substantial and much greater than the typical between-school/department variances (8-10%) of secondary school CVA modelling as shown in the first analysis above.

The most significant predictor of completion of A level mathematics is, unsurprisingly, prior attainment. Also, a positive difference between GCSE mathematics grade and students English and mean GCSE[iv] grades increases the likelihood of them completing some A level mathematics. It is reasonable that completing a greater number of A levels increases the chances of having some mathematics included in one's portfolio of qualifications. From interviews with students and teachers it is clear that different schools and colleges have different policies on admission to A level (see also Matthews & Pepper, 2007). Having explored the potential significance of this by including school level measures (mean and standard deviation of the number of subjects awarded) only one measure was significant. The negative influence of 'standard deviation of number of Advanced level entries' suggests that a more heterogenous post-16 cohort has some small detrimental effect upon likely completion of some mathematics. However, caution needs to be exercised here as we don't know the true mix of the centres from this data as we have only included students on A level pathways and not those following vocational pathways. That said, if this measure of heterogeneity were important then it would only become more so if the full range of college students were included in the model.

Turning to the social variables it can be seen, as anticipated from the research literature, that gender has a significant impact on participation with girls being less likely to complete some A level mathematics. The IDACI score shows that students from more deprived backgrounds are actually more likely to study some mathematics, when all other factors have been taken into account. I have shown elsewhere (Noyes, 2009) that GCSE mathematics performance is associated with social class. So any 'classed' pattern of post-compulsory mathematics participation was shaped earlier in the education system. It should also not be a surprise that the impact of ethnicity is very variable with Chinese/Indian/Pakistani/African students having a much increased predicted probability of completing some mathematics compared to the White British base category.

Having looked at the effect of these background variables, probability estimates for different types of students can be made. For example, consider students with a grade A in GCSE mathematics taking 3 A levels, remaining in the same school for A levels, with a very low (i.e. 0, affluent background) IDACI score:

	White British	Chinese
Male	0.41	0.69
Female	0.23	0.50

Table 5: Predicted probabilities of completing pre-college mathematics course

The differences here are striking and reflect a far more complex patterning of participation than that which can be explored using only GCSE maths grades or gender, which are the typical units of analysis in England. And these differences are in addition to any earlier school effect that results in higher GCSE attainment, for Chinese students, for example.

Concluding Comments

So, which departments are most effective? From the first analysis it is clear that one needs to distinguish between general school effects (as measured by mean GCSE attainment/progress)

and department effects (i.e. mathematics attainment/progress) but this is not straightforward. It is also important to return to the question of values raised at the outset of the paper. GCSE attainment, both absolute and relative, is critically important in shaping the likelihood of young people's progressing to study A level mathematics as shown in the second analysis. That model pointed to the high between-school variation in completion of A level mathematics. Any policy action aimed at increasing participation in A level mathematics could start by considering how to get those with low participation to recruit and retain students as successfully as those with higher rates of participation and retention. What this analysis does not do is identify particular cultural, curricular or pedagogic influences upon these between-school variations and there is not sufficient space here to explore the qualitative research in the project that was designed to explore these differences. However, it is important to understand how the results of these two modelling processes shed light on this important issues of mathematical participation beyond the age of 16 in England. Is a department that ranks highly in CVA from 11-16 also one which also encourages future participation? Indeed, there are important implications here for other education systems that include similar transition points at which students can opt into or out of mathematical pathways.

In order to bring these two analyses together the school level residuals from each model (11-16 mathematics CVA residuals from 2005 and 16-18 mathematics participation 2005-7) were compared, bearing in mind that they are different types of model using different datasets. That said, the binomial participation model includes students in the 131 schools in the 11-16 CVA model during the same period of time, with largely the same teachers, school and departmental culture. Comparison between the two sets of school level residuals shows that there is a small, negative but statistically insignificant correlation between these two sets of residuals for the GCSE year 2005. This suggests that modelling departmental effectiveness in mathematics from 11 – 16 years of age tells us very little about which schools are likely to recruit and retain more A level mathematics students. There are of course some difficulties with this approach as the participation model takes account of prior attainment and the CVA model includes students across the full attainment range and not just those who are likely to progress to advanced study. However, this is an important insight in the current performative context of schooling in England. Schools that appear strong in terms of whole-school contextual value added might not be those with mathematics departments that can add significant value to student progress. That is, mathematics departments that 'add value' between 11 and 16 years of age are not more likely to have better recruitment and retention in post-16 advanced mathematics courses. This returns us to the question of 'goods' and what it is that is required from a school mathematics education. Is increased success at age 16 sufficient if it bears no relation to the levels of motivation for further study. Moreover, is it right to laud the 'effectiveness' of departments that do particularly well in enhancing student progress from 11-16 but who cannot motivate those students to continue mathematical study beyond the age of 16?

The main aim of conducting these analyses was to explore between-school variation in mathematical progress from age 11-16 and any relationship to to the recruitment and retention of A level mathematic students. This KS2-4 CVA modelling discussed above identifies a range of school effects, a small part of which is year on year variation due to the cohort effect. Put together, these variances are similar to those reported in other CVA models of school effects. The bivariate modelling of 11-16 progress suggests that mathematics departments can have significantly different impacts on pupil progress than the rest of the school in general. Explanations for these differences would require further curricular and pedagogic data, the likes of which are not included in the NPD. The performative culture of English schools is well documented and what we are probably seeing here is that the *effects* measured in the KS2-4 contextual value added models are schools' capacity to prepare students for high stakes tests. Other *values* would need to be *added* to the student experience in order for there to be increased uptake of A level mathematics. That said, there is some suggestion that where students do better in mathematics relative to their other subjects they are more likely to proceed to advanced level study.

What is more interesting is the amount of unexplained variance in completion of A level mathematics that might be attributed to the school or department (over 20%). The school attended seems to have a very real impact on one's likelihood of completing some advanced level mathematics. So, taken together, schools have a very real effect upon progress to 16 and likely participation post-16 in mathematics, but the evidence suggests that there is little correlation between these two effects. At a time where there continues to be considerable political interest in the levels of participation in post-compulsory mathematics education in England, policymakers would do well to attend to this variation between schools. In addition, further research studies that develop our understanding of these between-school variations would be invaluable.

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[i] Multiple imputation procedures are increasingly being used in multilevel modelling to account for missing data and produce increasingly reliable parameter estimates. In this case, REALCOM was used to conduct these imputation processes. See (Goldstein, 2011)

[ii] All of the cells in this school/year matrix are over 70.

[iii] The IDACI measure is based upon Lower Level Super Output Areas. It assumes a relatively homogenous type of household. There will be some variability however, so the IDACI score can only ever be an approximation.

[iv] The 'mean GCSE' grade is calculated as the mean of all GCSE grades (A*=8…G=1) with the exception of Mathematics. Students typically have 8-10 GCSE 'scores'