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Interval Type-2 Fuzzy Sets in Supplier Selection

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***Abstract*—Selection of an appropriate supplier is a crucial and challenging task in the effective management of a supply chain. This study introduces a model for solving the supplier selection problem using interval type-2 fuzzy sets. Moreover, the influence of the membership function shape on the results obtained from the model has been investigated on a real-world problem instance tackled by Ordoobadi.**

I. INTRODUCTION

Supplier Chain Management (SCM) is the management of material flows from the procurement of basic raw materials to final product delivery considering information flows among whole processes of supply chains (SCs), material flows and long-term relations between customers and suppliers [14]. Therefore, it is very important for all companies to have a small number of reliable suppliers for sourcing a product efficiently. Poorly managed suppliers also result in low quality products with high raw material cost.

In a competitive environment, without considering selection and evaluation of suppliers successfully, it is extremely hard to manage any production process with high quality and low cost. Thus, supplier evaluation and selection problems have been addressed by using different approaches such as; data envelopment analysis (DEA), analytic hierarchy process (AHP) and fuzzy set theory [8]. Ho et al. [8] reviewed the literature considering 78 articles between 2000 and 2008 in order to summarise which approaches were applied, which evaluating criteria were emphasised and the adequacy of the approaches. Based on this literature review, DEA, mathematical programming, AHP, case-based reasoning, analytic network process, fuzzy set theory, simple multi-attribute rating technique and genetic algorithm have been mostly used. The most popular five evaluation criteria also have been found as quality, delivery, price, manufacturing capacity, service.

In addition, supplier selection problems are a kind of multiple attribute decision making (MADM) problem which makes decisions over a finite number of alternatives in terms of the decision-maker's constraints and preference priorities [1]. Wu and Chen [15] addressed group multiple attribute decision making problems related to uncertainties in a linguistic environment by a new method which deals with given decision information to get the overall preference value of each criterion weighting them with linguistic weighted arithmetic averaging approach.

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Chen and Lee [3] presented a method for MADM problems on the basis of ranking values and the arithmetic operations of fuzzy logic in type-2 fuzzy sets. Chen and Lee [2] developed their method using type-2 TOPSIS approach. Chen et al. [4] pointed out ranking interval type-2 fuzzy sets in order to deal with fuzzy multiple attributes group decision making (FMAGDM) problems. Gong [6] presented an approach to handle the FMAGDM problems under interval type-2 (IT2) fuzzy environment where the criteria weights are unknown. This research shows that type-2 fuzzy models are able to handle FMAGDM problems.

Ordoobadi [13] described an approach in order to get a fuzzy score for each supplier considering selection criteria and evaluation of the potential suppliers with respect to determined criteria in type-1 fuzzy sets. Then, these fuzzy scores were converted to crisp values through the Center of Area (COA) defuzzification approach in order to make the ranking of the suppliers. The aim of the work was to select the supplier with the highest ranking and provide more information about suppliers to the decision maker.

In this paper, the same problem described by Ordoobadi [13] is used to see the influence of the shape of the membership functions on the rank of suppliers. Three different methods are adapted in order to create membership functions for both importance of criteria and performance of suppliers. In addition, in order to minimise the impact of uncertainties revealed into the linguistic evaluation of suppliers, these approaches have been extended by using interval type-2 fuzzy sets.

The rest of paper is organised as follows. In section II, some concepts of type-1 fuzzy sets and interval type-2 fuzzy sets are reviewed briefly. In section III, the definition of the problem and development of the methodology using the interval type-2 fuzzy model with regard to three methods are examined. In section IV, the conclusion and contribution of the studying is discussed.

II. TYPE-2 FUZZY SETS

In order to deal with uncertainties, Zadeh [16] introduced the fuzzy set theory which allows varying degrees of membership values in a given set. Mendel and John [11] stated that there are several sources of uncertainties in type-1 fuzzy logic systems due to the following reasons:

1) Meanings of the words can be varied from different people to people. Thus, vagueness can be revealed in

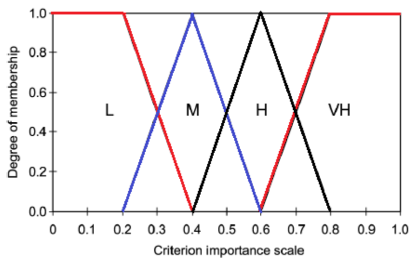

Fig. 1. The membership functions of the linguistic importance weight [13].

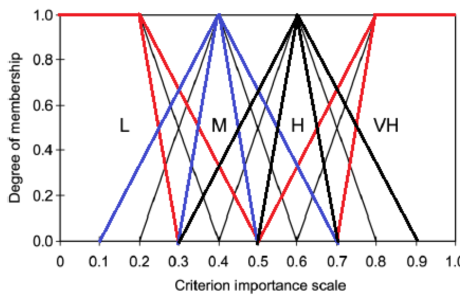

Fig. 2. The interval type-2 membership functions of the linguistic importance weight.

provided rules.

2) When a group of experts have different ideas to define consequents of the same problem linguistically, uncertainties could arise.
3) The use of noisy data from measurements activating type-1 fuzzy logic sets.
4) The use of noisy data to tune the parameters of type-1 fuzzy sets.

Zadeh [16] developed the ideas in type-2 fuzzy logic which is able to handle more uncertainty. In type-2 fuzzy sets, using three-dimensional membership function minimises effects of uncertainty [11]. On the other hand, type-2 fuzzy sets are difficult to apply and understand. Especially, type-reduction and defuzzification processes have been challenging.

Because of the the computational complexity of type-2 fuzzy sets, Greenfield [7] stated that the most researchers have concentrated on interval type-2 fuzzy models. In addition, according to Mendel et al. [12], interval type-2 fuzzy sets are quite practical due to the their manageable computational complexity. Therefore, in this work, interval type-2 fuzzy sets with three different approaches are used to cope with uncertainties in the system that requires linguistic descriptions covered with type-1 fuzzy logic by Ordoobadi [13].

A. Fuzzy Membership Functions

Based on the work proposed by Ordoobadi [13], to evaluate suppliers, decision maker considered two different attributes as importance weight of the selection criteria and performance rating of the suppliers with considering trapezoidal representations of each set.

1) Importance weight of attributes: In Ordoobadi's work, the importance of each criterion used for the evaluation of supplier selection addressed by a question with an answer set of the linguistic weights named as 'low importance', 'moderate importance', 'high importance', 'very high importance'. The numeric scale defined between 0 and 1 corresponded the fuzzy numbers of each criterion value and Figure 1 demonstrates four membership functions [13].

TABLE I. Linguistic weights of the attributes represented by type-1 fuzzy sets

Linguistic terms (LT)	Type-1 fuzzy sets	Fuzzy sets in Approach 1
Low importance(L)	(0.0,0.0,0.2,0.4)	(0.0,0.0,0.25,0.5)
Moderate Importance(M)	(0.2,0.4,0.4,0.6)	(0.25,0.5,0.5,0.75)
High Importance(H)	(0.4,0.6,0.6,0.8)	(0.5,0.75,0.75,1.0)
Very High Importance(VH)	(0.6,0.8,1.0,1.0)	(0.75,1.0,1.0,1.0)
Linguistic terms (LT)	**Fuzzy sets in Approach 2**	**Fuzzy sets in Approach 3**
Low importance(L)	(0.0,0.0,0.2,0.5)	(0.0,0.0,0.15,0.5)
Moderate Importance(M)	(0.2,0.5,0.5,0.8)	(0.15,0.5,0.5,0.85)
High Importance(H)	(0.5,0.8,0.8,1.0)	(0.5,0.85,0.85,1.0)
Very High Importance(VH)	(0.8,0.1,1.0,1.0)	(0.85,1.0,1.0,1.0)

TABLE II. Linguistic weights of the attributes represented by interval type-2 fuzzy sets

LT	Type-1 fuzzy sets	Fuzzy sets in Approach 1
L	((0.0,0.0,0.2,0.3),(0.0,0.0,0.2,0.5))	((0.0,0.0,0.25,0.4),(0.0,0.0,0.25,0.6))
M	((0.3,0.4,0.4,0.5),(0.1,0.4,0.4,0.7))	((0.35,0.5,0.5,0.65),(0.15,0.5,0.5,0.85))
H	((0.5,0.6,0.6,0.7),(0.3,0.6,0.6,0.9))	((0.6,0.75,0.75,0.9),(0.4,0.75,0.75,1.0))
VH	((0.7,0.8,1.0,1.0),(0.5,0.8,1.0,1.0))	((0.85,1.0,1.0,1.0),(0.65,1.0,1.0,1.0))
LT	**Fuzzy sets in Approach 2**	**Fuzzy sets in Approach 3**
L	((0.0,0.0,0.2,0.4),(0.0,0.0,0.2,0.6))	((0.0,0.0,0.15,0.4),(0.0,0.0,0.15,0.6))
M	((0.3,0.5,0.5,0.7),(0.1,0.5,0.5,0.9))	((0.25,0.5,0.5,0.75),(0.05,0.5,0.5,0.95))
H	((0.6,0.8,0.8,1.0),(0.4,0.8,0.8,1.0))	((0.6,0.85,0.85,1.0),(0.4,0.85,0.85,1.0))
VH	((0.9,1.0,1.0,1.0),(0.7,1.0,1.0,1.0))	((0.95,1.0,1.0,1.0),(0.75,1.0,1.0,1.0))

In this study, the uncertainty is analysed by using three different shapes of type-1 membership functions associated to linguistic terms and assume that there are interval type-2 fuzzy sets formed from above three different type-1 fuzzy sets with the same hight. Figure 2 is an example to illustrate one of these interval type-2 sets obtained blurring type-1 sets from left to right in the same amount shifting for each set. In addition, linguistic scales for these membership functions are shown in Table I for type-1 fuzzy sets and in Table II for interval type-2 fuzzy sets.

Table I demonstrates values of trapezoidal type-1 fuzzy sets. Trapezoidal type-1 fuzzy set can be defined by four parameters as a, b, c, d where $a < b < c < d$. In Table I, second column demonstrates a, b, c, d values in sequence for each membership set for type-1 fuzzy sets where the x axis represents the universe of discourse defined the importance scale 0-1 shown in Figure 1 and the y axis represents the importance degrees of membership in the $[0, 1]$ interval shown in Figure 1. The rest of columns show values of fuzzy sets for each approach with the same sequence (explanations of each approach are detailed in section III).

In addition, in Table II, their corresponding interval type-2 fuzzy sets are indicated with respect to values of both upper and lower membership functions in the same way explained above for type-1 fuzzy sets.

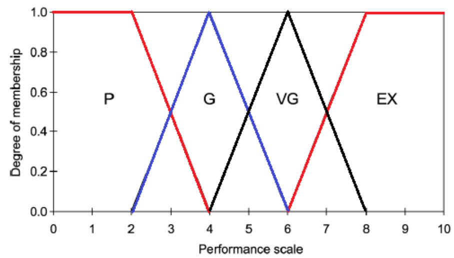

Fig. 3. The membership functions of the linguistic performance rate [].

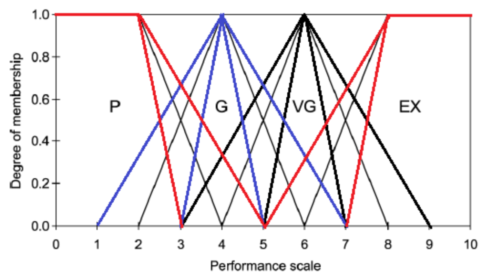

Fig. 4. The interval type-2 membership functions of the linguistic performance rate.

TABLE III. LINGUISTIC PERFORMANCE RATES REPRESENTED BY TYPE-1 FUZZY SETS

Linguistic terms (LT)	Type-1 fuzzy sets	Fuzzy sets in Approach 1
Poor (P)	(0,0,2,4)	(0,0,2.5,5)
Good (G)	(2,4,4,6)	(2.5,5,5,7.5)
Very Good (VG)	(4,6,6,8)	(5,7.5,7.5,10)
Excellent (E)	(6.8,10,10)	(7.5,10,10,10)
Linguistic terms (LT)	**Fuzzy sets in Approach 2**	**Fuzzy sets in Approach 3**
Poor (P)	(0,0,2,5)	(0,0,1.5,5)
Good (G)	(2,5,5,8)	(1.5,5,5,8.5)
Very Good (VG)	(5,8,8,10)	(5,8.5,8.5,10)
Excellent (E)	(8,10,10,10)	(8.5,10,10,10)

2) Performance Rating of Suppliers: Based on the work of Ordoobadi, the performance of a supplier with considering each criterion was dealt with a question with the answer set named as ‘excellent’, ‘very good’, ‘good’, ‘poor’. The numeric scale defined between 0 and 10 corresponded the fuzzy numbers of each criterion value and Figure 3 demonstrates four membership functions [13].

In this work, both type-1 fuzzy sets and their corresponding interval type-2 sets are created in the same manner as explained in the subsection of the importance weight of attributes and their values are illustrated in Table III and Table IV. Figure 4 shows one of interval type-2 sets for performance rates of suppliers generated by shifting original type-1 sets in the same manner as mentioned in the importance weight of attributes.

B. Basic Concepts of IT2 FS

According to Zadeh [17], fuzzy logic systems(FLSs) have been becoming essential in the environment of ‘Computing with Words’. Thus, there has been much work carried out on the theoretical side of type-2 FLSs [10]. Figure 5 demonstrates the structure of a type-2 FLS. As seen in Figure 5, a type-2 FLS comprise fuzzifier, inference, rules and output processing. In other words, a type-2 FLS is very similar to a type-1 FLS, the only difference is that the defuzzifier process of a type-1 FLS is replaced by the output processing consists of type-reduction followed by defuzzification [9]. In this section, rather than explaining each of blocks shown in Figure 5, only selected relevant definitions and operations of type-2 FS are briefly introduced from Mendel et al. [12].

TABLE IV. LINGUISTIC PERFORMANCE RATES REPRESENTED INTERVAL TYPE-2 FUZZY SETS

LT	Type-1 fuzzy sets	Fuzzy sets in Approach 1
P	((0,0,2,3),(0,0,2,5))	((0,0,2.5,4),(0,0,2.5,6))
G	((3,4,4,5),(1,4,4,7))	((3.5,5,5,6.5),(1.5,5,5,8.5))
VG	((5,6,6,7),(3,6,6,9))	((6,7.5,7.5,9),(4,7.5,7.5,10))
E	((7,8,10,10),(5,8,10,10))	((8.5,10,10,10),(6.5,10,10,10))
LT	**Fuzzy sets in Approach 2**	**Fuzzy sets in Approach 3**
P	((0,0,2,4),(0,0,2,6))	((0,0,1.5,4),(0,0,1.5,6))
G	((3,5,5,7),(1,5,5,9))	((2.5,5,5,7.5),(0.5,5,5,9.5))
VG	((6,8,8,10),(4,8,8,10))	((6,8.5,8.5,10),(4,8.5,8.5,10))
E	((9,10,10,10),(7,10,10,10))	((9.5,10,10,10),(7.5,10,10,10))

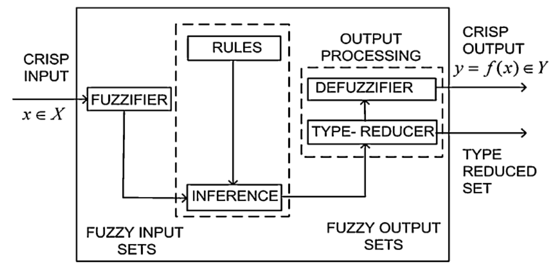

Fig. 5. Type-2 fuzzy logic system [9].

Definition II.1. A type-2 fuzzy set $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$ shown as follows:

$$\tilde{A} = ((x,u), \mu_{\tilde{A}}(x,u))|\ \forall x \in X, \forall u \in J_x \subseteq [0,1] \quad (1)$$

where $x \in X$ and $u \in J_x \subseteq [0,1]$ in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. As the primary membership function is between 0 and 1, thus can be expressed as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u)\ \ J_x \subseteq [0,1] \quad (2)$$

where $\int\int$ denotes a union over all admissible x and u [12].

Definition II.2. Let $\tilde{A}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{A}}$. If all $\mu_{\tilde{A}}(x,u) = 1$ for $\forall x \in X$ and $u \in J_x \subseteq [0,1]$, then $\tilde{A}$ is called an interval type-2 fuzzy set, shown as followings:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u)\ \ J_x \subseteq [0,1] \quad (3)$$

where $J_x \subseteq [0,1]$, i.e. [12].

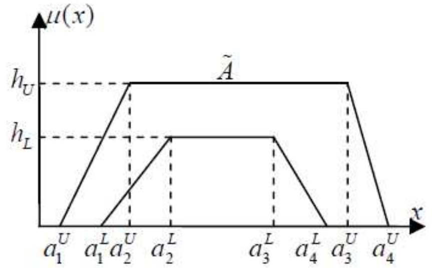

Fig. 6. The upper trapezoidal membership function $\tilde{A}^U$, the lower trapezoidal membership function $\tilde{A}^U$ of the IT2FS $\tilde{A}$ [6].

Definition II.3. The upper membership function and the lower membership function of an interval type-2 fuzzy set are both type-1 membership functions, respectively [12].

A trapezoidal IT2 FS $\tilde{A}_i$ in the universe of discourse X represented by;

$$\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u; h_1(\tilde{A}_i^U), h_2(\tilde{A}_i^U)), (a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l; h_1(\tilde{A}_i^L), h_2(\tilde{A}_i^L)) \quad (4)$$

where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u, a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l$ are the reference points of the interval type-2 fuzzy set $\tilde{A}_i$, $h_j(\tilde{A}_i^U)$ denotes the membership value of the element $a_{i(j+1)}^u$ in the upper trapezoidal membership function $\tilde{A}_i^U$ while $1 \leq j \leq 2$, $h_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{i(j+1)}^l$ in the lower trapezoidal membership function $\tilde{A}_i^L$ while $1 \leq j \leq 2$, $h_1(\tilde{A}_i^U) \in [0,1]$, $h_2(\tilde{A}_i^U) \in [0,1]$, $h_1(\tilde{A}_i^L) \in [0,1]$, $h_2(\tilde{A}_i^L) \in [0,1]$, $1 \leq i \leq n$.

In Figure 6 as an instance, it is shown that $h_1(\tilde{A}_i^L)$ is equal to $h_2(\tilde{A}_i^L)$, defined with the label h_L and $h_1(\tilde{A}_i^U)$ equals $h_2(\tilde{A}_i^U)$, defined with the label h_U and $a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u, a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l$ are illustrated as; $a_1^U, a_2^U, a_3^U, a_4^U, a_1^L, a_2^L, a_3^L, a_4^L$.

Definition II.4. In this work, algebraic operations used are addition, multiplication defined as follows for the trapezoidal interval type-2 fuzzy sets $\tilde{A}_1$ and $\tilde{A}_2$ [12];

$$\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^u, a_{12}^u, a_{13}^u, a_{14}^u; h_1(\tilde{A}_1^U), h_2(\tilde{A}_1^U)), (a_{11}^l, a_{12}^l, a_{13}^l, a_{14}^l; h_1(\tilde{A}_1^L), h_2(\tilde{A}_1^L)) \quad (5)$$

and

$$\tilde{A}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^u, a_{22}^u, a_{23}^u, a_{24}^u; h_1(\tilde{A}_2^U), h_2(\tilde{A}_2^U)), (a_{21}^l, a_{22}^l, a_{23}^l, a_{24}^l; h_1(\tilde{A}_2^L), h_2(\tilde{A}_2^L)) \quad (6)$$

The addition operation is:

$$\begin{aligned}\tilde{A}_1 \oplus \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^u + a_{21}^u, a_{12}^u + a_{22}^u, a_{13}^u + a_{23}^u, a_{14}^u + a_{24}^u; \\ &\quad min(h_1(\tilde{A}_1^U), h_1(\tilde{A}_2^U)), min(h_2(\tilde{A}_1^U), h_2(\tilde{A}_1^U))), \\ &\quad (a_{11}^l + a_{21}^l, a_{12}^l + a_{22}^l, a_{13}^l + a_{23}^l, a_{14}^l + a_{24}^l; \\ &\quad min(h_1(\tilde{A}_1^L), h_1(\tilde{A}_2^L)), (h_2(\tilde{A}_1^L), h_2(\tilde{A}_2^L))) \end{aligned} \quad (7)$$

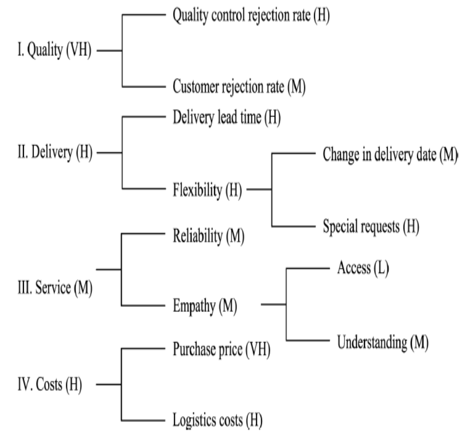

Fig. 7. The criteria and sub-criteria used for selection of suppliers [13].

The multiplication operation is:

$$\begin{aligned}\tilde{A}_1 \otimes \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^u \times a_{21}^u, a_{12}^u \times a_{22}^u, a_{13}^u \times a_{23}^u, a_{14}^u \times a_{24}^u; \\ &\quad min(h_1(\tilde{A}_1^U), h_1(\tilde{A}_2^U)), min(h_2(\tilde{A}_1^U), h_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^l \times a_{21}^l, a_{12}^l \times a_{22}^l, a_{13}^l \times a_{23}^l, a_{14}^l \times a_{24}^l; \\ &\quad min(h_1(\tilde{A}_1^L), h_1(\tilde{A}_2^L)), (h_2(\tilde{A}_1^L), h_2(\tilde{A}_2^L))) \end{aligned} \quad (8)$$

Multiplying a fuzzy set by a constant k is defined as follows:

$$\begin{aligned} k\tilde{A}_1 &= k(\tilde{A}_1^U, \tilde{A}_1^L) \\ &= ((ka_{11}^u, ka_{12}^u, ka_{13}^u, ka_{14}^u; h_1(\tilde{A}_1^U), h_2(\tilde{A}_1^U)), \\ &\quad (ka_{11}^l, ka_{12}^l, ka_{13}^l, ka_{14}^l; h_1(\tilde{A}_1^L), h_2(\tilde{A}_1^L)) \end{aligned} \quad (9)$$

In this study, it is assumed that all height of each fuzzy set is equal to one. For this reason, they are not considered in the problem.

III. Problem Definition and the Development of the Methodology

The study proposed by Ordoobadi is summarised in this section. Ordoobadi [13] provided decision makers an appropriate way to select a supplier by identifying the criteria which have played an essential role in supplier selection, evaluating supplier performance with respect to the selection criteria and developing the methodology for ranking of the suppliers.

Based on the work provided by Ordoobadi [13], to evaluate suppliers, a decision maker considered two different attributes as importance weight of the selection criteria and performance rating of the suppliers.

Briefly, in the work of Ordoobadi [13], the decision maker selected the criteria relevant to the circumstance at hand from a list of criteria. After careful review of the selected criteria, five main criteria and several sub-criteria were defined as shown in Figure 7. Then, they were evaluated in a linguistic way such as 'low', 'moderate', 'high', 'very high' for the importance of each criterion based on thoughts of the decision maker. The numeric scale defined between 0 and 1 corresponded the fuzzy

TABLE V. CRISP SCORES AND SUPPLIER RANKINGS [13]

Suppliers	Crisp Scores	Rank
Supplier A	13.50	3
Supplier B	15.78	1
Supplier C	14.44	2

TABLE VI. RANKING OF SUPPLIERS RESULTS FOR THREE APROACHES

Suppliers	Approach 1	Rank	Approach 2	Rank	Approach 3	Rank
Supplier A	25.41	3	27.20	3	29.11	3
Supplier B	27.61	1	29.37	1	31.18	2
Supplier C	27.27	2	29.21	2	31.36	1

numbers of each criterion value and Figure 1 demonstrates four fuzzy sets of the linguistic importance weight of criteria [13].

Following this, the linguistic terms were converted into fuzzy weights using fuzzy membership functions. For instance, if the importance of criterion is 'low' then the fuzzy importance weight is assigned as (0.0,0.0,0.2,0.4) shown in Table I. In Table I, each number represents each parameter of the trapezoidal fuzzy set in sequence for each row (generally a trapezoidal fuzzy set can be defined by four parameters as a, b, c, d where $a < b < c < d$ explained in Section II).

Then, the suppliers were identified to consider for selection by the decision maker and their performances were determined in the same manner of the criteria using linguistic terms such as 'excellent', 'very good', 'good', 'poor'. The numeric scale defined between 0 and 10 corresponded the fuzzy numbers of each criterion value and Figure 3 demonstrates four fuzzy sets and Table I illustrates parameters of each trapezoidal fuzzy sets in the same manner as mentioned in the importance weight of attributes. Following this, the linguistic terms were converted into fuzzy performance ratings using fuzzy membership functions.

In the subsequent stage, the aggregate fuzzy score for each supplier was calculated by aggregating fuzzy score all the pertinent criteria and converted into crisp scores using the Center of Area (COA) defuzzification method. Finally, the suppliers were ranked according to their their crisp scores illustrated in Table XII.

In conclusion, the proposed approach by Ordoobadi [13], was applied in a case study comprised three suppliers and ten selected criteria and sub-criteria as illustrated in Figure 7.

In this study, firstly, shapes of membership functions of Ordoobadi's work is investigated and the influence of the membership functions' shapes on the result is evaluated by using three different modified fuzzy sets and then a new method for ranking suppliers is proposed using the interval type-2 fuzzy sets with regard to these three approaches.

A. Effect of the fuzzy membership functions shape on the result

Garibaldi and John [5] point out that choosing membership functions of linguistic terms has played an essential role in fuzzy systems to capture the opportunities to the best represent human knowledge. For this reason, this work is carried out on investigation of shapes of membership functions in order to see whether there is any change on the ranking of suppliers. This task is done in a two-step process:

1) In order to demonstrate the effect of uncertainty on fuzzy sets in the linguistic terms, ranking is done for three different membership functions. Each values of membership functions are determined based on original type-1 fuzzy sets of the work of Ordoobadi. In the first approach, membership functions are created by shifting the centre point of the original type-1 membership functions from 0.4 to 0.5 for the criteria importance and from 4 to 5 for performance of suppliers and values of each set are assigned as in Table I and III. In the second and third approaches, membership functions are created by varying the parameters of membership functions determined in the first approach and changes in the widths of membership functions are increased gradually.
2) For these approaches, the suppliers are ranked by using the same way explained in the study of Ordoobadi [13] briefly mentioned in this section.

Results of experiments for analysing the effect of uncertainty in membership functions on variation in ranking of suppliers is illustrated in Table VI. In comparison to crisp scores of suppliers using original type-1 sets shown in Table XII and results of these three approaches, there are significant raises in fuzzy scores of each supplier because of shifting the centre points from 0.4 to 0.5.

Nevertheless, in the first and second approaches, the rank of suppliers is found same as the result in the work of Ordoobadi, in the third approach, supplier C became the best among three suppliers. In Table VI, it is also clearly seen that the difference between the crisp score of supplier B and supplier C becomes smaller when the widths of fuzzy sets are increased regardless of the fact that there is no important change on Supplier A. Based on results, it is observed that shapes of membership functions have a significant role in the rank of suppliers. Thus, these three approaches are carried out on the next stage in order to develop them using interval type-2 fuzzy sets.

B. Developing approaches with IT2 FS

The type-1 FLSs are extended by using interval type-2 sets. The process for each approach is performed by going through the following steps:

Step 1: The original type-1 FLS is used to generate trapezoidal interval type-2 fuzzy sets shown in Figure 2 as the first experiments of the study. According to Table I, the rest fuzzy sets are obtained in the same manner for three approaches. Let w_i demonstrates the fuzzy importance weight of criterion i where $i = 1, 2, ..., 10$. As an example, if the first criterion's importance weight is decided as 'high' by the decision-maker then w_i is defined as $((0.5, 0.6, 0.6, 0.7), (0.3, 0.6, 0.6, 0.9))$ using the linguistic scales of Table II.

Step 2: The weighted trapezoidal interval type-2 sets of end nodes of each branch are calculated by multiplying all nodes on the same branch illustrated in Figure 7. For instance, w_1 is computed by multiplying the importance weight of the quality

TABLE VII. THE WEIGHTED TRAPEZOIDAL INTERVAL TYPE-2 SETS FOR CRITERIA

Criterion Weight	Interval type-2 fuzzy sets
w1	((0.35,0.48,0.60,0.70),(0.15,0.48,0.60,0.90))
w2	((0.21,0.32,0.40,0.50),(0.05,0.32,0.40,0.70))
w3	((0.25,0.36,0.36,0.49),(0.09,0.36,0.36,0.81))
w4	((0.08,0.14,0.14,0.24),(0.01,0.14,0.14,0.57))
w5	((0.13,0.22,0.22,0.34),(0.03,0.22,0.22,0.73))
w6	((0.09,0.16,0.16,0.25),(0.01,0.16,0.16,0.49))
w7	((0.00,0.00,0.03,0.08),(0.00,0.00,0.03,0.24))
w8	((0.03,0.06,0.06,0.13),(0.00,0.06,0.06,0.34))
w9	((0.35,0.48,0.60,0.70),(0.15,0.48,0.60,0.90))
w10	((0.25,0.36,0.36,0.49),(0.09,0.36,0.36,0.81))

TABLE VIII. SUPPLIERS' PERFORMANCE RATINGS WITH RESPECT TO THE SELECTION CRITERIA-I

Suppliers	w1	w2	w3
A	((3,4,4,5),(1,4,4,7))	((5,6,6,7),(3,6,6,9))	((0,0,2,3),(0,0,2,5))
B	((7,8,10,10),(5,8,10,10))	((5,6,6,7),(3,6,6,9))	((3,4,4,5),(1,4,4,7))
C	((5,6,6,7),(3,6,6,9))	((3,4,4,5),(1,4,4,7))	((0,0,2,3),(0,0,2,5))

TABLE IX. SUPPLIERS' PERFORMANCE RATINGS WITH RESPECT TO THE SELECTION CRITERIA-II

	w4	w5	w6
A	((7,8,10,10),(5,8,10,10))	((3,4,4,5),(1,4,4,7))	((0,0,2,3),(0,0,2,5))
B	((0,0,2,3),(0,0,2,5)))	((0,0,2,3),(0,0,2,5))	((3,4,4,5),(1,4,4,7))
C	((5,6,6,7),(3,6,6,9))	((5,6,6,7),(3,6,6,9))	((7,8,10,10),(5,8,10,10))

by importance weight of quality control rejection rate as;

$$w_1 = ((0.7, 0.8, 1.0, 1.0), (0.5, 0.8, 1.0, 1.0))\\ ((0.5, 0.6, 0.6, 0.7), (0.3, 0.6, 0.6, 0.9))\\ = ((0.35, 0.48, 0.60, 0.70),\\ (0.15, 0.48, 0.60, 0.90)) \quad (10)$$

The rest of the weights are computed in the same way and outcome of the first experiment derived from the original type-1 fuzzy sets are shown in Table VII.

Step 3: The linguistic terms of suppliers' performance are converted into interval type-2 fuzzy performance ratings in the same manner as explained in the step 1 and results of the first experiment are shown in Table VIII, IX, X, XI.

Step 4: The aggregate fuzzy sets for each supplier is calculated by multiplying fuzzy performance rates matrix indicated in Table VIII, IX, X, XI by fuzzy importance weights of Table VII. The rest approaches are performed in the same manner.

Step 5: In order to convert the fuzzy set values for each supplier into crisp values, Centroid type-reduction and defuzzification methods are applied and the output of each approach is illustrated in Table XII and Table XIII.

Based on crisp values that we obtained, the suppliers are ranked as shown in Table XII and Table XIII. Table XII-XIII demonstrate crisp scores of suppliers over results of experiments. In Table XII, the crisp score of Ordoobadi's work using type-1 fuzzy sets and the output of our study using

TABLE X. SUPPLIERS' PERFORMANCE RATINGS WITH RESPECT TO THE SELECTION CRITERIA-III

Suppliers	w7	w8	w9
A	((5,6,6,7),(3,6,6,9))	((3,4,4,5),(1,4,4,7))	((0,0,2,3),(0,0,2,5))
B	((7,8,10,10),(5,8,10,10))	((0,0,2,3),(0,0,2,5))	((3,4,4,5),(1,4,4,7))
C	((0,0,2,3),(0,0,2,5))	((3,4,4,5),(1,4,4,7))	((3,4,4,5),(1,4,4,7))

TABLE XI. SUPPLIERS' PERFORMANCE RATINGS WITH RESPECT TO THE SELECTION CRITERIA-IV

Suppliers	w10
Supplier A	((7,8,10,10),(5,8,10,10))
Supplier B	((3,4,4,5),(1,4,4,7))
Supplier C	((0,0,2,3),(0,0,2,5)))

TABLE XII. FOUND SCORES FOR SUPPLIERS BY ORDOOBADO'S WORK

Suppliers	Score of T1FS		Score of IT2FS	
	Crisp Scores	Rank	Crisp Scores	Rank
Supplier A	13.50	3	16.72	3
Supplier B	15.78	1	18.56	1
Supplier C	14.44	2	17.79	2

TABLE XIII. RANKING OF SUPPLIERS RESULTS FOR THREE APROACHES

Suppliers	Approach 1	Rank	Approach 2	Rank	Approach 3	Rank
Supplier A	27.41	3	30.03	3	32.03	3
Supplier B	29.74	1	32.12	1	34.00	2
Supplier C	29.32	2	32.00	2	34.06	1

interval type-2 fuzzy sets without shifting the centre points are illustrated. In comparison to the effect of using type-1 and interval type-2 fuzzy sets, the rank of suppliers are found the same, despite that there is an increase in the score of each supplier (Table XIII).

In comparison to interval type-2 membership functions created with shifting the center points and changing the widths of this membership functions, the difference in scores of supplier B and C becomes small and in the third approach, the crisp value of supplier C becomes greater than the output of supplier B.

During the experiments, we have observed that there is a relationship between the shape of membership functions reflecting the uncertainty in linguistic terms. As this uncertainty is increased by enlarging the widths of the fuzzy sets, there is a rise in the fuzzy scores of suppliers.

Another important observed feature of this work is that in the third approach, the crisp score of supplier C is greater than crisp score of supplier B in type-1 fuzzy logic system, however, in interval type-2 fuzzy logic system, the output of supplier B is almost same as the result of supplier C. Thus, it could show that interval type-2 FLSs are able to deal with linguistic uncertainty better than type-1 FLSs.

IV. CONCLUSIONS

Although fuzzy logic has been applied in many fields successfully, this success has not been carried over to dealing with human reasoning [17]. In this paper, the level of uncertainty having influence on variation of ranking of suppliers is analysed by changing the width of fuzzy sets in both type-1 and interval type-2 membership functions. The interval type-2 fuzzy sets are capable of handling linguistic uncertainty better than type-1 fuzzy sets. Simply because, type-2 fuzzy sets are more effective in capturing the linguistic assessment and considering them as an input of the main problem [9]. Hence, we resolved the same issue while handling a supplier selection problem by considering different membership functions under an interval type-2 fuzzy environment. It has been observed that the shape of membership functions influences the ranking of suppliers. Looking at the the scores of suppliers obtained using type-I and type II fuzzy models, clearly, the same objective is achieved when the interval type-2 FLS without shifting the centre points is used, as pointed out in the work of Ordoobadi [13]. However, when there is a significant change in the widths of the fuzzy sets, the ranking of suppliers changes both in type-1 and interval type-2 fuzzy sets.

The research on understanding variation in human decision making is ongoing. In this work, trapezoidal upper membership functions and lower membership functions with the same heights are considered in interval type-2 fuzzy sets. For future research, different shapes, particularly, trapezoidal upper and lower membership functions with different heights will be taken into consideration.

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