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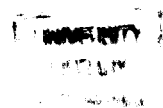
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# **THE STABILITY OF ARTICULATED TIPPING TRAILER UNITS**

**by Simon G Pickering, BEng**

**Thesis submitted to the University of Nottingham  
for the degree of Doctor of Philosophy  
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## **ABSTRACT**

When an articulated tipper unit is being loaded or is tipping, it is unlikely to be standing on perfectly level ground. Also, the centre of gravity of the load is unlikely to be in the centre of the body. Hence the loads carried by the suspension and tyres on one side of the tipper will be greater than those on the other side. This uneven loading will cause the tyres and suspension on one side of the tipper unit to deform more than those on the other side. It will also cause the chassis to deform; the twisting about its longitudinal axis being the most significant mode of deformation. As a result of these deformations caused by the uneven loading, the position of the centre of gravity will be shifted even further towards the more heavily loaded side. This will cause even more uneven loading and further deformations.

Under stable conditions a situation will exist at which the position of the centre of gravity, the deformations and the forces transmitted through the system are compatible. Instability, resulting in roll-over would occur if the overall centre of gravity of the load, body, chassis etc. were to fall outside the area bounded by the contact of the wheel with the ground, before a stable condition was reached.

Many factors influence the roll stability. To increase stability, an understanding of the influence of components of the lorry on the stability is required. In order to achieve this, a theoretical model of an articulated tipper was developed which will allow roll-over predictions to be made for a given lorry in likely attitudes. In this model dimensions and stiffness of the lorry components can be altered to assess their influence on roll stability.

The previous theoretical roll-over models were based on lumped mass systems, representing various parts of the lorry inter-connected by compliant elements. Certain flexibilities such as the tyres, suspension units, etc. could be obtained from the respective components manufacturers but the tractor and trailer chassis flexibilities are unknown. To overcome this problem the flexibilities were obtained from full scale static tilt tests. This is a very expensive undertaking, providing a limited means in which to assess those elements of trailer design which are important in improving stability, without further recourse to more tilt tests. It was decided that the finite element method should be used to model the tractor and trailer, in order to determine the important deformations. Once the finite element model is created it is relatively straight forward to make changes to the structure. Hence an assessment of component contribution to roll stability can be undertaken relatively inexpensively.

Whilst a vehicle operator should always endeavour to discharge the payload with the vehicle standing on level ground, practical situations arise where this is not possible. This may be due to the absence of level ground or poor judgement by the operator, which may result in the vehicle being tipped on a lateral ground slope. As a result of this, the maximum ground slope angle considered for the theoretical model is limited to eight degrees, as this position is at least twice the severity of ground slope on which a vehicle should normally be tipped.

For each trailer design, the magnitude of the load, position of the load, ram length and ground slope can be varied in any combination. Four payloads and up to nine payload positions are considered, varying the ground slope from 0 to 8 degrees and varying the ram length from 2 to 8 meters. Also, three further chassis configurations, based on the reference chassis were modelled to investigate the contribution of important component flexibilities on roll stability.

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## NOTATION

### Forces

$A_L, A_R$	left and right airbag forces, see Fig. 3.5
$A_{R1}$ and $A_{R2}$	forces at the top and bottom of the right hand airbag of the 1st suspension, see Fig. 3.5
$A_{R3}$ and $A_{R4}$	forces at the top and bottom of the right hand airbag of the 2nd suspension, see Fig. 3.5
$A_{R5}$ and $A_{R6}$	forces at the top and bottom of the right hand airbag of the 3rd suspension, see Fig. 3.5
$\underline{A}_R$ ( $A_{R1}, A_{R2}, A_{R3}, A_{R4}, A_{R5}, A_{R6}$ )	column of forces applied to the right hand airbags
$\underline{B}$ ( $B_y, B_z$ )	body force vector
$\underline{F}, \underline{F}', \underline{F}''$	column matrices of all important chassis forces
$\underline{G}$ ( $G_{L1}, G_{L2}, G_{L3}, G_{R1}, G_{R2}, G_{R3}$ )	column matrix containing the left and right vertical tyre forces
$\underline{H}_L$ ( $H_{Lx}, H_{Ly}, H_{Lz}$ )	left hinge force vector
$\underline{H}_R$ ( $H_{Rx}, H_{Ry}$ )	right hinge force vector
$L$	force applied to cantilever
$\underline{P}$ ( $P_x, P_y, P_z$ )	payload force vector
$\underline{R}$ ( $R_x, R_y, R_z$ )	ram force vector
$\underline{T}$ ( $T_L, T_R$ )	column matrix containing the left and right tie bar forces
$\underline{W}$ ( $W_x, W_y, W_z$ )	wind force vector
$\underline{W}_T$ ( $W_{Ty}, W_{Tz}$ )	tractor self-weight vector
$\underline{W}_c$ ( $W_{cy}, W_{cz}$ )	chassis self-weight vector

Position Vectors (relative to origin of X, Y, Z coordinate system)

$\underline{b}$ ( $b_x, b_y, b_z$ )	position of the centre of gravity of body
$\underline{p}$ ( $p_x, p_y, p_z$ )	position of the centre of gravity of payload
$\underline{h}_l$ ( $h_{lx}, h_{ly}, h_{lz}$ )	position of the left hinge
$\underline{h}_r$ ( $h_{rx}, h_{ry}, h_{rz}$ )	position of the right hinge
$\underline{r}_b$ ( $r_{bx}, r_{by}, r_{bz}$ )	position of the bottom of ram
$\underline{r}_t$ ( $r_{tx}, r_{ty}, r_{tz}$ )	position of the top of ram
$\underline{w}$ ( $w_x, w_y, w_z$ )	position of the centroid of wind pressure
$\underline{r}_{tb}$	position of the top of the ram relative to the bottom of the ram

Displacements

$\underline{u}, \underline{u}^T$	column matrix of all important chassis displacements
$\underline{u}_{hl}$ ( $u_{hl}, v_{hl}, w_{hl}$ )	left hinge displacement vector
$\underline{u}_{hr}$ ( $u_{hr}, v_{hr}, w_{hr}$ )	right hinge displacement vector
$\underline{u}_{rb}$ ( $u_{rb}, v_{rb}, w_{rb}$ )	bottom of ram displacement vector
$v_{a1}$ and $v_{a2}$	y components of displacement of the upper and lower, right hand airbag ends of 1st suspension, see Fig. 3.5
$v_{a3}$ and $v_{a4}$	y components of displacement of the upper and lower, right hand ends of the 2nd suspension, see Fig. 3.5
$v_{a5}$ and $v_{a6}$	y components of displacement of the upper and lower, right hand ends of the 3rd suspension, see Fig. 3.5
$\underline{v}_a$ ( $v_{a1}, v_{a2}, v_{a3}, v_{a4}, v_{a5}, v_{a6}$ )	column matrix of the y components of displacements of the right hand airbags
$\Delta^{(0)}$	vertical displacement of cantilever beam tip
$\phi^{(0)}$	rotation of cantilever beam tip
$\theta$	hinge bar rotation about X axis, see Fig. 3.1
$\theta_p$	slope of the ground, see Fig. 3.1
$\psi$	hinge bar rotation about Y axis, see Fig. 3.1

## Others

a, b	dimensions of the rigid frame attached to the tip of the cantilever, see Fig. 3.2
E	Young's modulus
I	second moment of area
$[I^{(a)}], [I^{(b)}], [I^{(c)}]$	influence matrices relating the important chassis displacements to the important chassis forces for different airbag conditions
$[I^{(a)}_T], [I^{(b)}_T], [I^{(c)}_T]$	influence matrices relating the tie bar forces to the important chassis forces for different airbag conditions
$[I^{(a)}_G], [I^{(b)}_G], [I^{(c)}_G] [I^{(d)}_G]$	influences matrices relating the ground/tyre reaction forces to the important chassis forces (or displacement) for different airbag conditions
$[I^{(b)}]$	influence matrix relating the right hand airbag displacements to the important chassis forces
[K]	stiffness matrix relating the important chassis forces to the important chassis displacements
l	length of cantilever, see Fig. 3.2
$l_0$	length of gas contained in the airbag cylinders at the limiting pressure
n	exponent in the equation for polytropic compression of a gas
Q	constant in Equation 3.17
X, Y, Z	co-ordinate system with its origin at the centre of the hinge bar
$\beta$	inclination of the body relative to the x axis
$\gamma$	angle between the vectors $\underline{r}_t$ and $\underline{r}_b$
$\delta$	angle between the vectors $\underline{r}_t$ and $\underline{p}$
$\wedge$	vector product
$\cdot$	scalar product
+	vector addition
-	vector subtraction

# CHAPTER ONE

## INTRODUCTION

### 1.1 Size of the problem

Articulated tipping units are used in many different industries for transporting many different types of payloads. In 1984 Keen et al [1] estimated that the number of vehicles having tipping facilities (articulated and rigid) was 120,000 with articulated tippers accounting for 15%. Following the amendments to the Construction and Use Regulations, which came into effect on 1<sup>st</sup> May 1983, the maximum gross vehicle weight of articulated tipper units was increased from 32 to 38 tonnes. Due to the increase in payload capacity, articulated tipping trailers have become more popular.

Information from official sources on the extent of tipper lorry rollovers during discharge of loads is limited to those where fatalities and or serious injuries result, as notification of non injury type incidents are not mandatory. A survey of hauliers by Keen et al [1], with a combined fleet of 600 tipper, found that the number of static rollovers accounted for 0.75% of the total fleet. Scaling up suggested an annual occurrence of 900 static rollovers for the UK.!

Many factors influence the roll stability of articulated tippers, while they are discharging their payloads. To increase stability, an understanding of the influence of components of the lorry on stability is required. This work describes a theoretical model of an articulated tipper which will allow rollover prediction to be made for a



given lorry in likely attitudes. In this model dimensions and stiffnesses of the lorry components, payload magnitude and position body attitude and transverse ground slope can be altered to assess their influence on roll stability.

## **1.2 Trailer construction**

There are two main categories into which tipper lorries can be grouped, rigid and articulated tipper units. The rigid tipper has a maximum gross vehicle weight of 30 tonnes, with the payload body and cab supported on one chassis, mounted on two steering wheels and four driving wheels. The articulated tipper has a maximum gross vehicle weight of 38 tonnes, with the payload body supported by the trailer chassis, mounted on six wheels and the driver's cab supported by the tractor chassis mounted on two steering wheels and two driving wheels. The tractor and trailer unit are detachable and are connected by a coupling known as the "fifth wheel".

This work is only concerned with 38 tonne articulated tipper units. A typical tractor and trailer unit is shown in Fig. 1.1.

### **1.2.1 Trailer design**

The design of articulated tippers has been concerned more with the static strength of the vehicle, than its stability characteristics in either dynamic or tipping situations. The object of this work is to study the static tipping of articulated tipper units and develop a mathematical model which predicts the stability characteristics for a given lorry

configuration. The mathematical model incorporates all of the important tractor and trailer flexibilities which can be changed to enable an assessment of their contribution to stability. This will allow designs to be produced based on stability criteria as well as strength criteria.

### 1.2.2 Trailer structures

Lorry chassis structures may be comprised of all open section channel or two longitudinal I beams inter-connected by cross members to form a ladder type structure. Open section channel chassis are used for rigid vehicles to form a ladder type structure. Two longitudinal I beams inter-connected by cross members forming a ladder type structure are used for articulated tipper units.

Mild steel is widely used in the construction of the ladder type structures for articulated tippers. The cross members are constructed from closed sections (rectangular, circular etc.) as the section is efficient in resisting bending and torsional forces. The longitudinal members are constructed from I beams as the section is efficient in resisting bending forces due to vertical loads.

The payload is transported in the "body" which is usually constructed from an aluminium alloy. The body is supported at three points, during tipping, by two hinges at the rear of the chassis and by the lifting ram at the front of the chassis. The chassis has to predominantly resist twisting about the longitudinal axis and bending about the transverse axis during tipping.

The forces generated by the payload the self weight of the vehicle are transmitted through the suspension system, which are more commonly air suspension, and tyres to ground reactions.

Chapter 3 describes the method of modelling the air suspension in the theoretical model.

### **1.3 Theoretical model**

No published material has been found relating to the stability of articulated tipping trailer units. Previous work has been undertaken on the dynamic stability of non tipping trailer negotiating corners. Theoretical models developed to predict vehicle stability in such conditions relies upon full scale experimental tests to provide flexibility data. This is not only an expensive undertaking, but makes further design investigations difficult to undertake without further experimental testing.

This work is only concerned with the static stability of articulated tipping trailer units.

The theoretical model developed to predict vehicle stability incorporates the following flexibilities.

- (i) Tractor and trailer chassis structure
- (ii) Tractor and trailer axles
- (iii) Tractor leaf spring suspensions

- (iv) Trailer air suspension and height control mechanism
- (v) Tractor and trailer tyres

The important flexibilities (i), (ii) and (iii) are determined using the finite element method. General flexibility matrices are developed using the finite element results, to enable stability characteristics to be determined, for a given lorry configuration, for any payload magnitude and position and body attitude.

Chapter 4 describes the finite element model.

The flexibilities (iv) and (v) are determined experimentally as described in Appendices A and B and are included in the F.E. model.

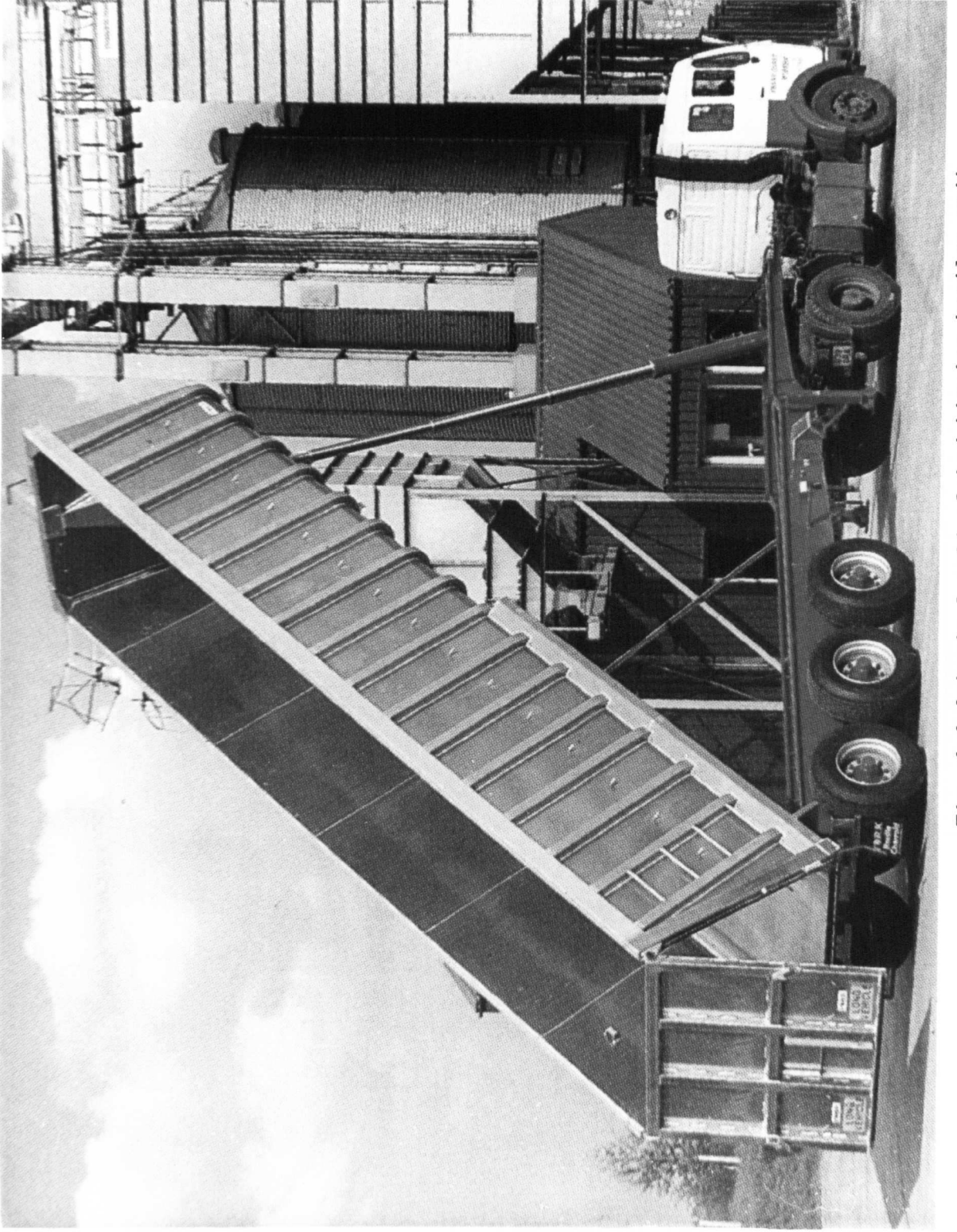


Fig. 1.1 A typical articulated tipping trailer unit

## **CHAPTER 2**

### **GENERAL BACKGROUND**

#### **2.1 General description of tipping trailer units and operation**

A schematic of a typical tipping trailer unit is shown in Fig. 2.1, it consists of a tractor unit connected to a trailer unit by a fifth wheel coupling. The tractor unit provides the motive force to pull the trailer and in most cases; provides the hydraulic pressure to raise the trailer's body by means of a telescopic ram. A tractor unit has a flexible chassis with a pair of single steering wheels and a pair of dual driving wheels connected to the chassis by leaf springs, as shown in Fig. 2.2.

A trailer unit is comprised of a flexible chassis, suspension and tyres supporting a body used for transporting the payload, as shown in Fig. 2.3. The longitudinal members (usually I sections) with circular, rectangular, hexagonal or similar hollow cross-members. The cross-members are welded to the longitudinal members with the joints often stiffened by local reinforcing, as indicated in Fig. 2.4. Steels or aluminium alloys are used in the manufacture of the chassis.

There are two main types of suspension used on articulated tippers, ie. leaf springs and air suspension. Because the modern trend is towards the use of air-bag suspension systems, only this type of suspension is described. Typically there are three air-bags on each side of the trailer, connected to three pairs of wheels, as indicated in Fig. 2.5. Each air-bag is connected to a wheel by a "trailing arm",

which rotates about the pivot mounting and each wheel is attached to an axle tube, which in turn is attached to a pivot arm on the left and right side of the trailer, shown in Fig. 2.5. The distance between the middle axle and chassis frame, known as the 'ride height' is automatically maintained by the airbags; each set of three airbags, on the left and right hand sides of the chassis, are pressurised independently. Because there is a maximum pressure limit, during tipping, the ride height may not be maintained due to excessive loading on one side of the trailer.

The tyres now predominantly used in modern tipping trailer units are so called "super singles", which, as the name suggests, are one large tyre instead of two smaller tyres used side by side. Flexibility data exists so that these tyres can be modelled mathematically using three mutually perpendicular springs.

The telescopic ram used to raise and lower the body usually have four or five stages. These rams can be used to tip the body to between 40 and 50 degrees, relative to the horizontal. Since this type of ram has a relatively low resistance to transverse deformation, it can be regarded as a strut, with ball joints at the ram to body and ram to chassis connecting points, for modelling purposes.

The fifth wheel and king pin unit which connects the tractor unit to the trailer allows relative rotation between the tractor and trailer about the vertical and transverse axis. However, only limited relative rotation of the tractor and trailer can occur about the longitudinal axis. The relative longitudinal rotation, which is not intended for design purposes, is due to the inevitable relative vertical displacement which must exist for

practical purposes. This limited, relative vertical movement has a non-linear effect on the roll stability. However, since this "play" can only contribute to the behaviour when roll-over is in progress it has not been taken into account in the modelling presented in this thesis.

The body, in which the payload is transported, is usually made from an aluminium alloy to minimise the weight of the trailer. It is attached to the chassis by two hinges at the rear and by the ram at the front. For the purpose of calculation, in this thesis it is assumed that the body is rigid and that during tipping operation, the body is only connected to the chassis via the hinges and the ram.

Standing on a firm, level site is the ideal situation in which to tip the body, in order to discharge the payload. However, tipping takes place under a wide range of conditions on sites such as building sites, landfill sites etc. and the vehicle operator may not always be able to find a firm and level place to tip. Hence, in practice some vehicles are tipped on uneven, inclined and soft ground and in these adverse conditions, the possibility of roll over exists. Although, with experience, an operator may be able to judge, by eye, whether a site is reasonably level. The operator cannot be sure that there isn't soft ground under one side of the trailer and that the payload will not discharge in a non-uniform manner, moving the payload centre of gravity from the longitudinal centre line of the body; a combination of these events may lead to vehicle roll over.



If the payload sticks in the body of a trailer at maximum tip angle, with the body fully tipped an operator may drive the vehicle, and then apply the brakes sharply, in an attempt to free the stuck payload. It has been suggested by Keen et al [1] that if a vehicle is driven over irregular terrain, under these conditions, a pendulum motion can occur and that this could lead to roll over; this problem is not specifically addressed in this thesis.

## **2.2 Loading modes**

When stationary, the external forces experienced by a tipping trailer unit are due to the gravitational load, wind loads and tyre reaction forces.

When standing on horizontal ground the gravitational forces only act normal to the ground and therefore there are no resultant transverse or longitudinal forces. However, when standing on a slope, components of the gravitational force may be experienced in the transverse and longitudinal directions.

The unsprung masses of a tipping trailer unit are defined as those masses which are not directly supported by the vehicles suspension. These masses on the trailer consist of the wheels, hubs and axles. For modelling purposes, the unsprung mass of each axle may be divided into two lumped masses located at the centre of each tyre. No detailed data, relating to the mass distribution, was obtainable for the tractor. Only the gross vehicle weight was obtained, and therefore the mass was taken to be entirely sprung for modelling purposes.

The trailer sprung masses, which are those supported by the trailer suspension, are the chassis mass and all of the accessories which are located on, or attached to, the chassis. The PAFEC finite element package [2] was used to obtain some of the information used in the model developed to assess the stability of a tipping trailer unit, part of the sprung mass is determined using the density and the volume of the material used for constructing the chassis. The remaining sprung masses were modelled by locating a lumped mass at the centre of gravity determined using the finite element package. The tractor's sprung mass is comprised of the chassis, cab and engine, together with all the associated accessories.

The tractor's gross weight and the position of its centre of gravity were obtained from the information provided by the manufacturer. As no information was available relating to the distribution of the tractor mass, it was modelled using a single lumped mass, positioned at the centre of gravity of the tractor unit.

The present work is concerned with the stability of a vehicle tipping in a static position. Although there is no published information relating to this problem, work has been undertaken relating the stability of vehicles travelling round corners Sweatman et al [3], Kemp et al [4], Miller et al [5] and Isermann [6]. Where a vehicle travels around a corner of radius  $r$ , speed  $v$  and mass  $m$ , a transverse force, acting in the radial direction, of magnitude  $mv^2/r$  is induced. If this force should become large enough, the vehicle can roll over. This previous work has been concerned with predicting roll over under cornering conditions, by comparing results obtained from static tilt tests, where the concerning force  $mv^2/r$  was replaced by a

component of the gravitational force, ie.  $mg\cos\theta$ , where  $\theta$  is the transverse tilt angle and  $g$  is the acceleration due to gravity.

In the present investigation the body and payload are assumed to be rigid. A single force, representing the body self weight is taken to act along its centre line, whereas a single force, representing the payload, may be taken to act at a suitable position, which can be defined by the use of the theoretical model. During the analysis, the position of the payload relative to the body remains unchanged, although the position of the body can change as the body/chassis contact points move due to deformation of the chassis, tyres, suspension etc.

Wind loading can produce significant forces, especially when the vehicle is at the maximum tip angle. A transverse wind force has been included in the theoretical model. However, the method for obtaining the magnitude and line of action of wind forces was outside the scope of the present work. Previous work by Hollis [7] undertaken on determining wind forces on crane structures, up to 20 m high, showed that the measured values could be as much as 100 to 150% greater than those predicted, shows that this problem is relevant to the tipping trailer situation.

Conventional leaf springs suspension exert forces on the chassis which are proportional to the suspension deformation. However, with air suspension the airbag exerts a combined force which is related to the sprung mass (including payload and body weight). Under normal conditions, the pressures on the left hand and right hand sets of airbags are increased or decreased independently to maintain a fixed "ride height" for the centre axle. However, once a maximum allowed pressure is reached

on one side, a valve closes and the set of three airbags compress under constant mass of gas conditions; the behaviour is taken to be adiabatic compression of the gas for modelling purposes, the airbag forces are treated as externally applied loads.

### **2.3 Design features**

Although no research work, related to the stability of tipping trailer units, operating under tipping conditions, has been published, a number of experimental [3, 4, 5, 6, 8, 9, 10] and theoretical [3, 4, 5, 6, 10, 11, 12, 13] investigations on stability under cornering conditions have been published.

A survey undertaken by Miller and Barter [5] found that operators were surprised at the lack of warning they had obtained before roll over occurred. This was put down to a lack of "feel" in the drivers cab for the trailer roll behaviour. Kemp, Chinn and Brock [4] indicated that design changes could improve roll stability performance. Although the greatest reduction in roll over accidents might be achieved by operators accepting that roll over is caused by travelling at too high a speed for a particular radius of turn, it is clear that designers have a significant role to play in minimising the possibility of roll over instability.

Some of the major details over which the design has control include:

- (i) the tyre and suspension type and operating conditions.

- (ii) dimensions and materials, which may be varied in order to increase the chassis stiffness, particularly the torsional stiffness about the longitudinal axis.
- (iii) the body materials and construction, which can be altered to minimise weight and increase rigidity.

The tyre and suspension stiffness were found by Laird [10] to significantly effect roll stability. It was found that the tractor drive suspension and trailer suspension could significantly affect roll stability, while the tractor front suspension only has a moderate influence.

Encouraged by legislative procedures, air suspension systems have become more popular than conventional leaf springs. However, the design of air suspension systems is still relatively new and further significant improvements are likely to occur. Current suspension systems predominantly use a combination of traditional leaf springs and air suspension bags, as indicated in Fig. 2.5. Current air bag designs, only accommodate loading along their longitudinal axis. Therefore leaf springs have been used to resist the large multi-directional forces resulting from side scrub between tyres and road, bumping into curbs or verges etc. It has been estimated by Dickson-Simpson [14] that these forces can be more than three times the nominal axle load. The air suspension system, modelled in this thesis, see Fig. 2.5, consists of elliptic leaf springs, each hinged to the chassis at one end, and supported at the other end by an air bag. Each axle tube is then rigidly clamped to the middle of the left and right leaf springs. Simultaneous displacements of the left and right tyres are primarily accommodated by the air suspension bags. However, when

uneven loading on the tyres occurs (when cornering or tipping on uneven ground, etc.) the leaf springs and axle tube, which in effect form an anti-roll bar system, accommodate the uneven displacements.

Due to the trend towards the use of air suspension systems on commercial vehicles, height control valves have been developed to automatically govern the vehicles design ride height. This is accomplished by adjusting the pressure in the air bags in response to changes in vehicle loading which force the valve lever to move up or down as the chassis to axle spacing changes. A consistent design height is one advantage of air suspension over mechanical suspension. To maintain the design height while using a minimum amount of air, a time delay is incorporated into the design. The time delay feature prevents the valve from allowing air flow to or from the airbags during momentary changes in the chassis to axle spacing. In addition to the time delay, the valve may be biased to the centre position, so that it is slow to open but quick to close, thus eliminating the tendency for the valve to oscillate, when repeated small variations in load occur.

A typical control valve operation, described by Hillebrand [15], is shown in Fig. 2.6. The valve is designed with a minimal "dead band" shown in Fig. 2.6a. This is the neutral zone where the lever arm attached to the axle can move without allowing air to enter or leave the air bag. The "dead band" is kept to a minimum to maintain an accurate spacing between the axle and chassis and minimise air usage.

The valve is attached to the vehicle frame and the lever arm is connected to the middle axle only through a linkage. On tipping trailer units there are two height control valves for left and right air bags. When tipping it is assumed that the left and right airbags operate independently. As the valve is opened the air flow into the airbag is shown in Fig. 2.6b. As the valve closes the air flow into the air as is shown in Fig. 2.6c. This combination of delayed opening and rapid closing provides a biasing tendency towards the centre closed position.

An increase in the torsional rigidity of a typical trailer chassis about this longitudinal axis, was shown by Holmes [8] to improve its stability and handling. Also, by eliminating fifth wheel play, it was shown by Sweatman and Mai [12] that the transverse acceleration could be increased by 4% before first wheel lift occurred.

The most important design parameters, effecting roll stability is obviously the height of the centre of gravity of the trailer. Sweatman and Mai [12] showed that for a typical trailer design, an increase in the height of the centre of gravity of 26% (1.95 m to 2.35 m) caused the first wheel lift acceleration to reduce by 26%. Also, they showed that increasing the trailer axle track width by 10% resulted in an increase in first wheel lift acceleration of 10%.

In experiments conducted by MIRA, [16], a fifth wheel, which was constructed with a load cell, to measure vertical, longitudinal and transverse loads was used to determine fifth wheel dynamic loads during driving conditions. These loads were required in order to perform realistic long term fatigue tests under laboratory conditions. The typical and maximum forces measured are given in Table 2.1.

Since 1978, some trailer designs have been constructed using extruded sections welded to aluminium plates, to form a strong light weight box, Brock [17]. There are two basic types of box design which are now commonly used. The first design is constructed from extruded "U" shaped channels welded to a large rail forming the top rail of the body. Aluminum plate forms the skin of the body to contain the payload. The other design is constructed from extruded aluminium channels which form the side and floor supports, as shown in Fig. 2.7. These are welded to a large rail which forms the floor corners of the body. Aluminium plate is again used for the skin of the body. Finite element analyses [17] indicated that, for the latter design, the bottom pillar joints, shown in Fig. 2.7, were the most highly stressed joints.

The stiffness of the bottom rail and the way in which it is connected to the floor sections is intrinsic to construction strength. Fabrication procedures and the adoption of MIG welding procedures have enabled improved quality and enhance fatigue life performance.

#### **2.4 Roll over models**

A thorough literature survey and discussions with manufacturers of tipping trailer units has shown that all previous work on articulated roll stability has been directed towards non-tipping trailers, negotiating corners. The theoretical models, developed to predict the cornering roll stability, have all been based on unsprung and sprung masses, connected by compliant elements under the influence of gravitational and transverse forces. In 1970 <sup>a</sup> theoretical <sup>model</sup> was developed by Isermann [6] which



included the flexibilities of the tyres, the suspension and tractor unit chassis with a rigid trailer chassis. This was later refined by Kemp et al [4] and Sweatman et al [5] to incorporate a torsional stiffness associated with the trailer. One of the major drawbacks with these models is that a theoretical estimation of the various flexibilities is required or, alternatively, a vehicle has to be experimentally tested under static rolling conditions so that measurements of various displacements can be obtained in order to give an indication of the trailers torsional stiffness etc. In 1984 Sweatman [12] developed a further model which reduced the number of variables to be measured experimentally to three, namely the percentage of wheel load transferred laterally in the trailer and the drive and steering axle groups of the tractor unit during a tilt test. Again the model is based on a set of constant masses interconnected by massless compliant elements.

The above models have been used to assess a number of design parameters in order to determine their effect on vehicle roll stability. Structural changes to an actual chassis would require expensive tilt tests to be undertaken, to enable accurate assessment of their effect on roll stability. Therefore, in order to make an initial assessment of possible design changes, these models have been found to be useful.

## **2.5 General discussions and conclusions**

No information, which relates directly to the roll stability of articulated tipping trailer units in operation, has been published in the open literature. Although some of the results of work related to cornering roll stability is applicable in a general sense, to

static tipping roll stability, little or no information exists for the types of forces generated by a tipped vehicle and on the effect of vehicle design parameters on roll stability during this mode of operation.

Published theoretical models, relating to roll stability are all based on rigid masses interconnected by compliant elements. Application of these methods requires full scale experimental tilt testing, in order to determine important compliances. In the present work, the finite element method is introduced to eliminate the need for static tilt tests. Also air bag suspension systems are modelled so that the forces generated by the tipped body, on the chassis, can be properly determined, in order to accurately model the roll stability of an articulated tipper unit. All of the important design parameters can be changed in order to assess the influence of them on the roll stability, without the need for static tilt tests. Also, although the required finite element analyses are simple small deformation, linear elastic analyses, the computer program written to perform the stability calculations is iterative and can cope with the large movements of the payload.

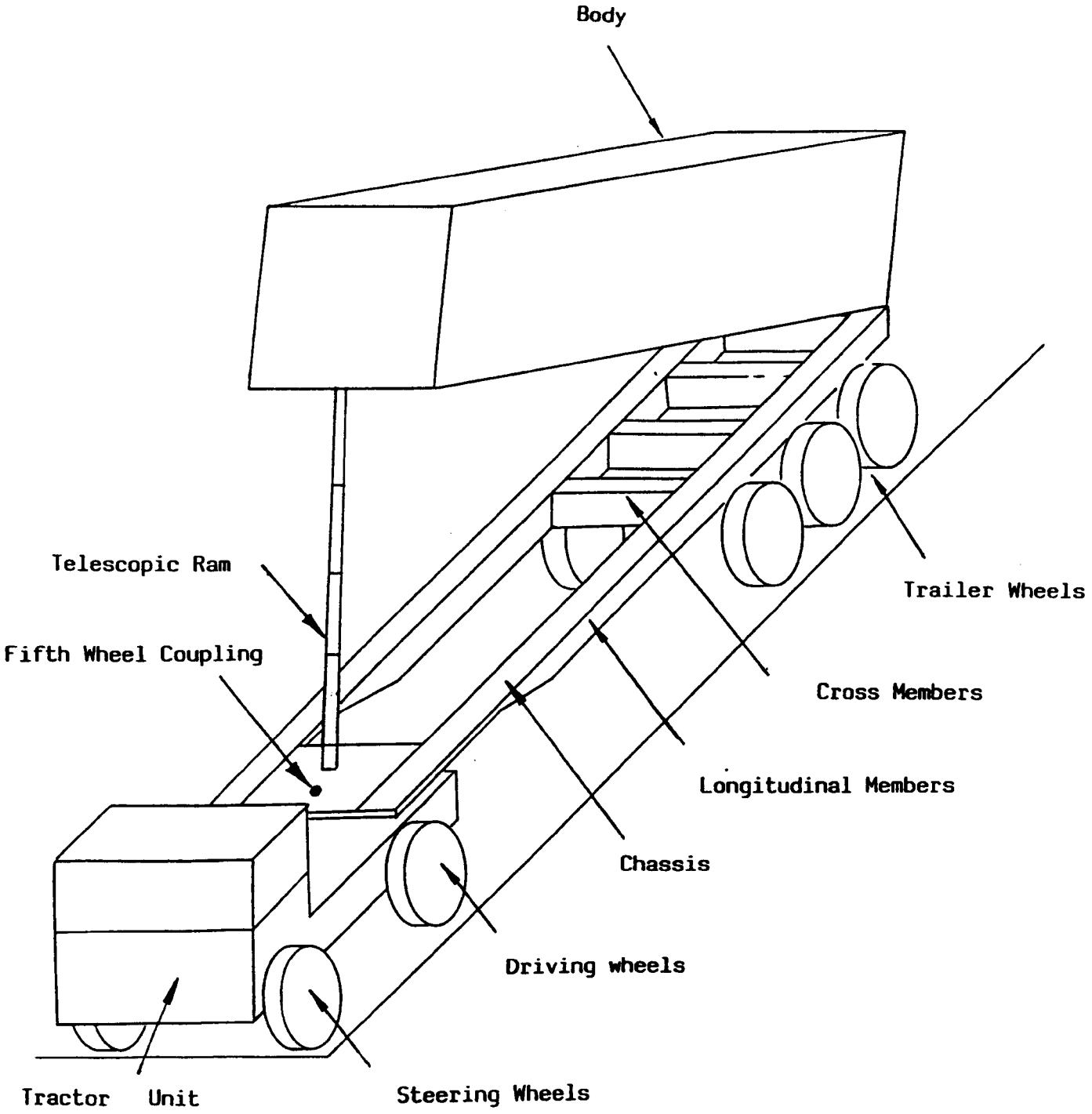
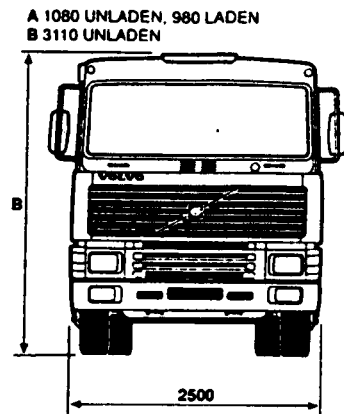
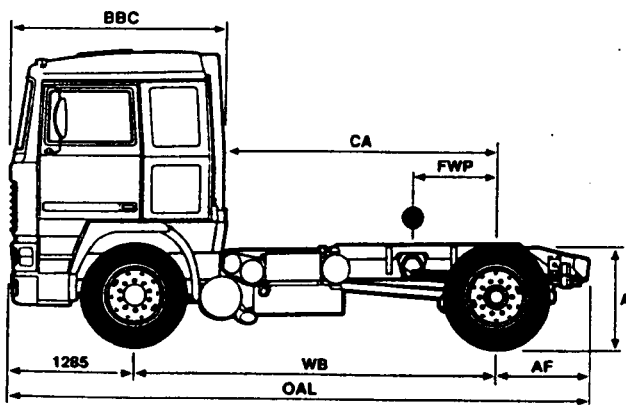


Fig. 2.1 A schematic diagram of a tipping trailer unit

Plated Weights Kg	Design	U.K. Legal
GVW	19500	17000**
Front Axle	6700	6700*
Rear Axle	13000	10500
GCW/GTW	44000	38000



### Chassis Dimensions mm

WB	Wheelbase	3400	3700
OAL	Overall Length	5619	5919
AF	Rear overhang	934	934
BBC#	Bumper to back of air intake	2220	2220
CA@	Back of air intake to axle centre	2465	2765
Turning Circle Diameter(Kerb to Kerb)		12100	13000
FWP	Fifth Wheel Position for 16.5m OAL	350	650

F.W.P. For accurate fifth wheel positioning, in terms of axle weights and dimensions, consult SSS program

Note 1) Height measurements calculated on 295/80R 22.5 tyres.

2) Do not use this drawing for 5th. wheel mounting, refer to F superstructures manual and coachbuilder drawing.

# Day cab -320. @ Day cab +320

### Chassis Kerb Weights Kg (Tolerance $\pm 2\%$ )

Front Axle	4560	4570
Rear Axle	1930	1940
Total Kerb Weight	6490	6510

Note: Weights for vehicles to standard specification i.e. sleeper cab, R1400 gearbox, twin plate clutch, 550 litre fuel tank(full), 295/80R22.5 tyres, water and tools, but excluding spare wheel and carrier.

Fig. 2.2 A Typical tractor unit

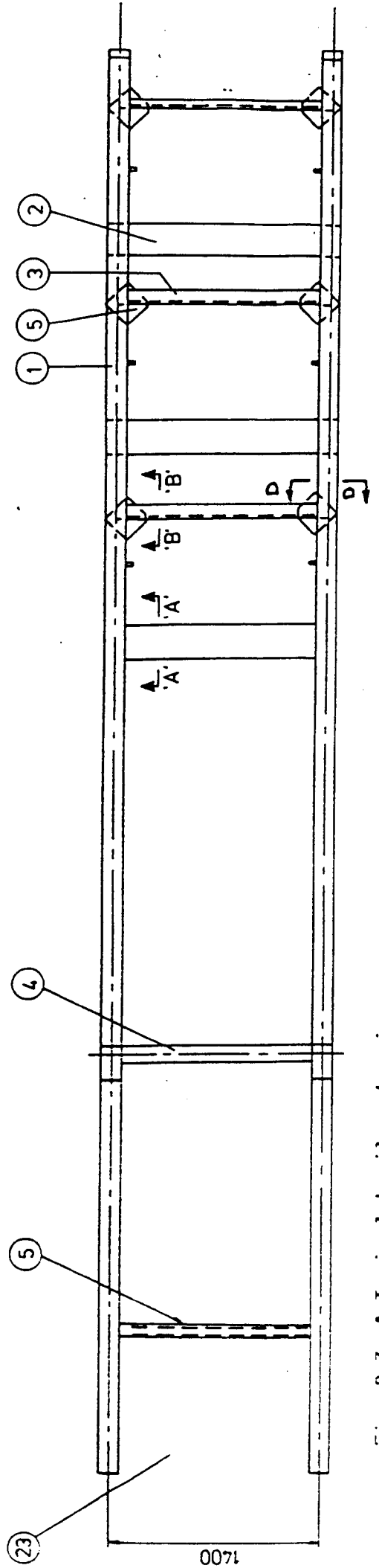
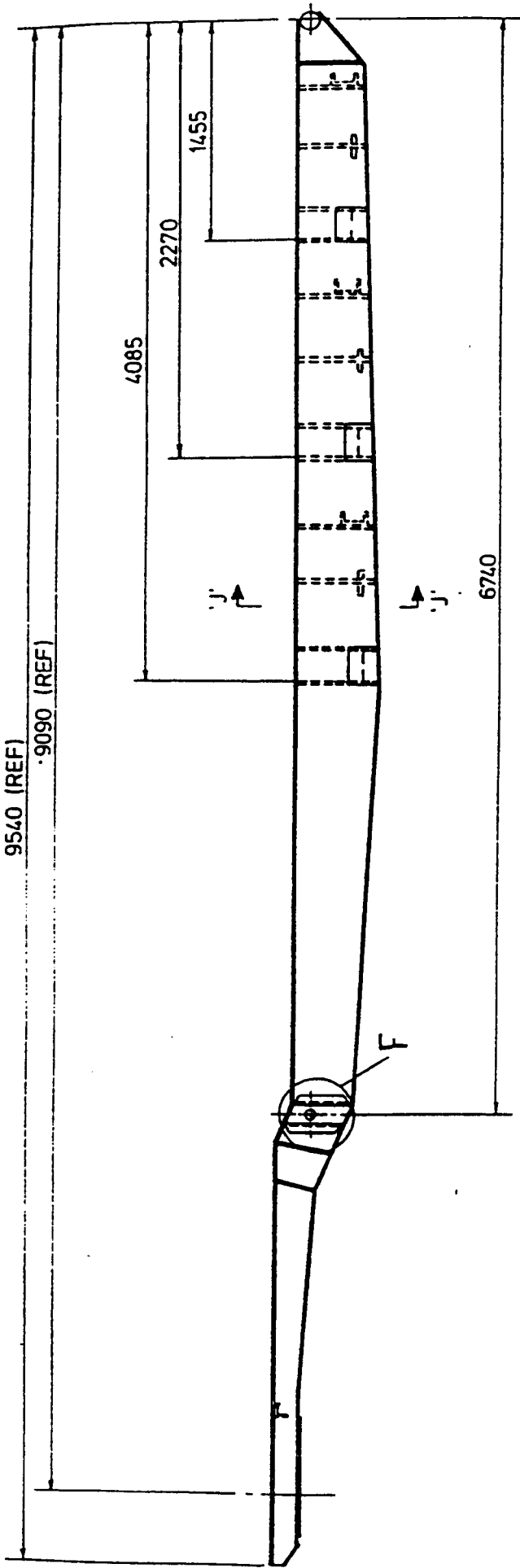


Fig. 2.3a A Typical trailer chassis

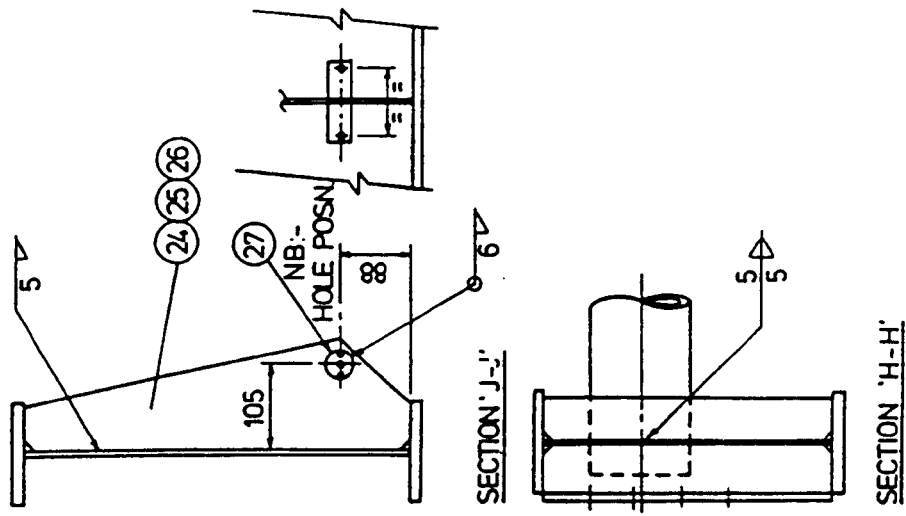
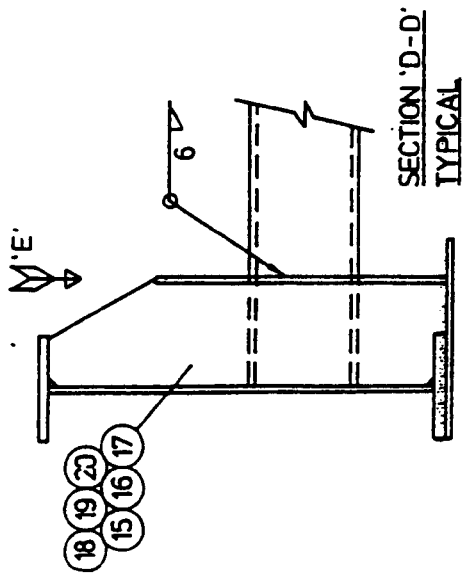
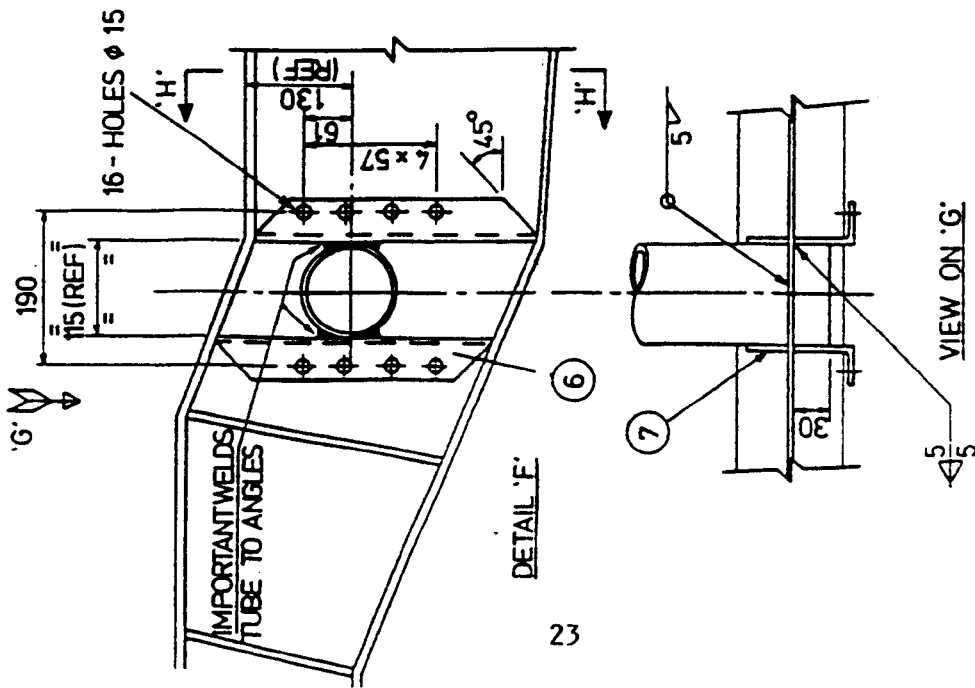


Fig. 2.3b Typical trailer chassis details

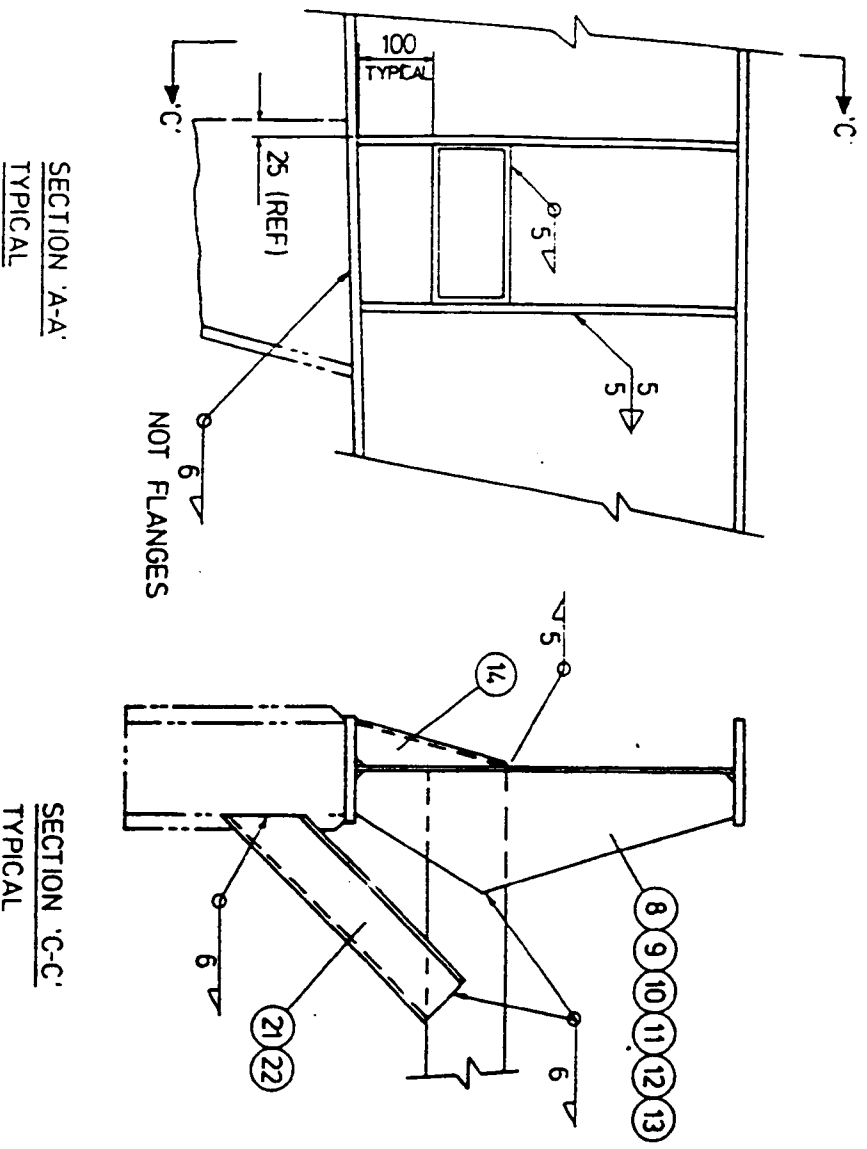


Fig. 2.3c Typical trailer chassis details

ITEM	PART NUMBER	QTY	DESCRIPTION
1	RD 891112	2	MAINRAIL
2		3	CROSSMEMBER RHS 200x100x5x3951G
3		3	CROSSMEMBER
4		1	TUBE $\phi$ 114.3x5 WTx1460 LG
5		1	CROSSMEMBER BS4360-GR50B
6		4	RSA 50x75x6x360 LG
7		4	STIFFENER
8		2	"
9		2	"
10		2	"
11		2	"
12		2	"
13		2	STIFFENER
14		6	GUSSET - FOLDED
15		1	" RH
16		1	" RH
17		1	STIFFENER RH
18		1	" LH
19		1	" LH
20		1	STIFFENER LH
21	896776	2	HANGER BRACE RH
22	896775	2	HANGER BRACE LH
23	RD 891113	1	PUP ASS.
24		2	"
25		2	"
26		2	AXLE RESTRAINT RIB
27		6	PIN
28			
29			
30			

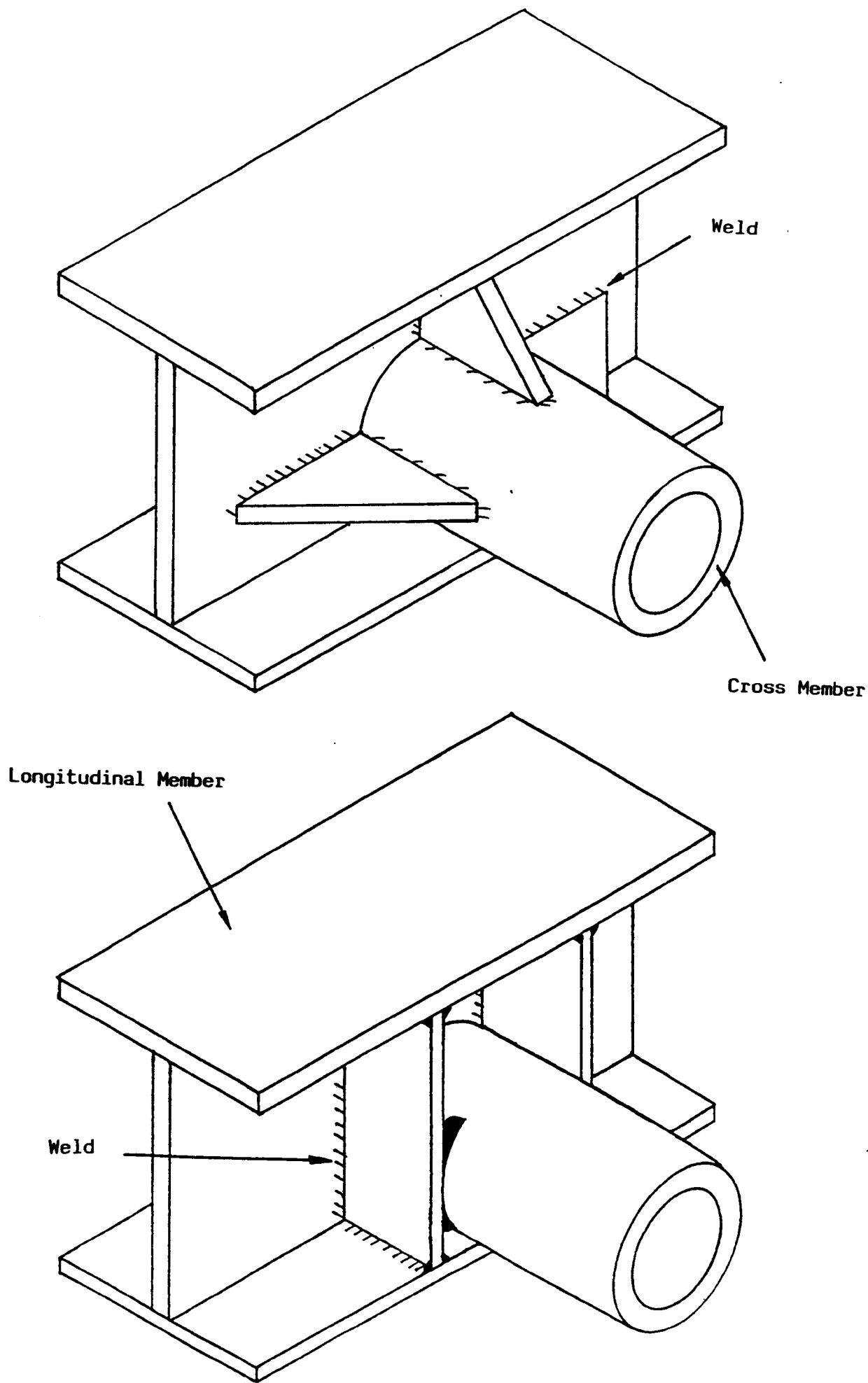


Fig. 2.4 Connection of chassis cross-members to longitudinal members



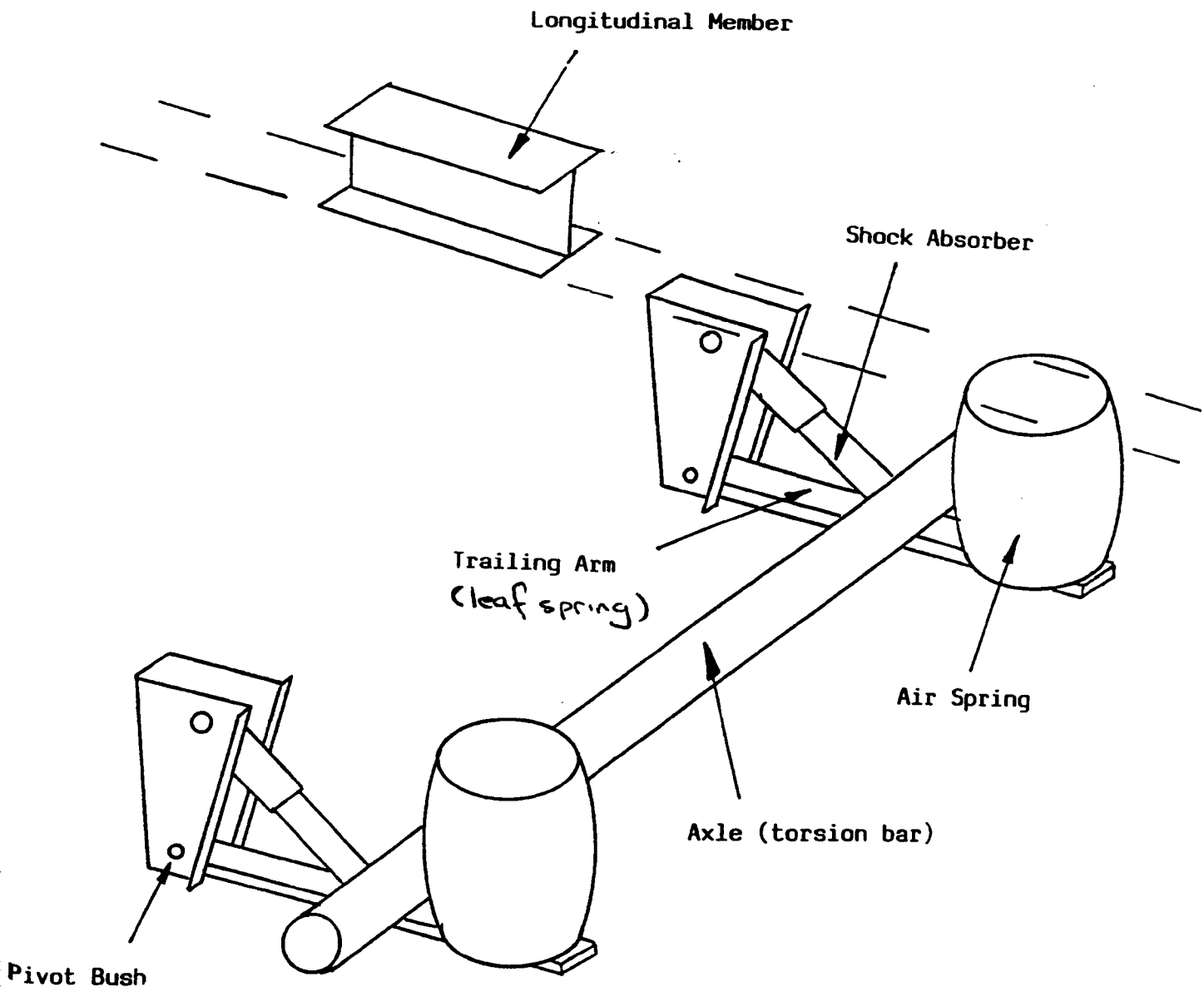


Fig. 2.5 Airbag suspension system

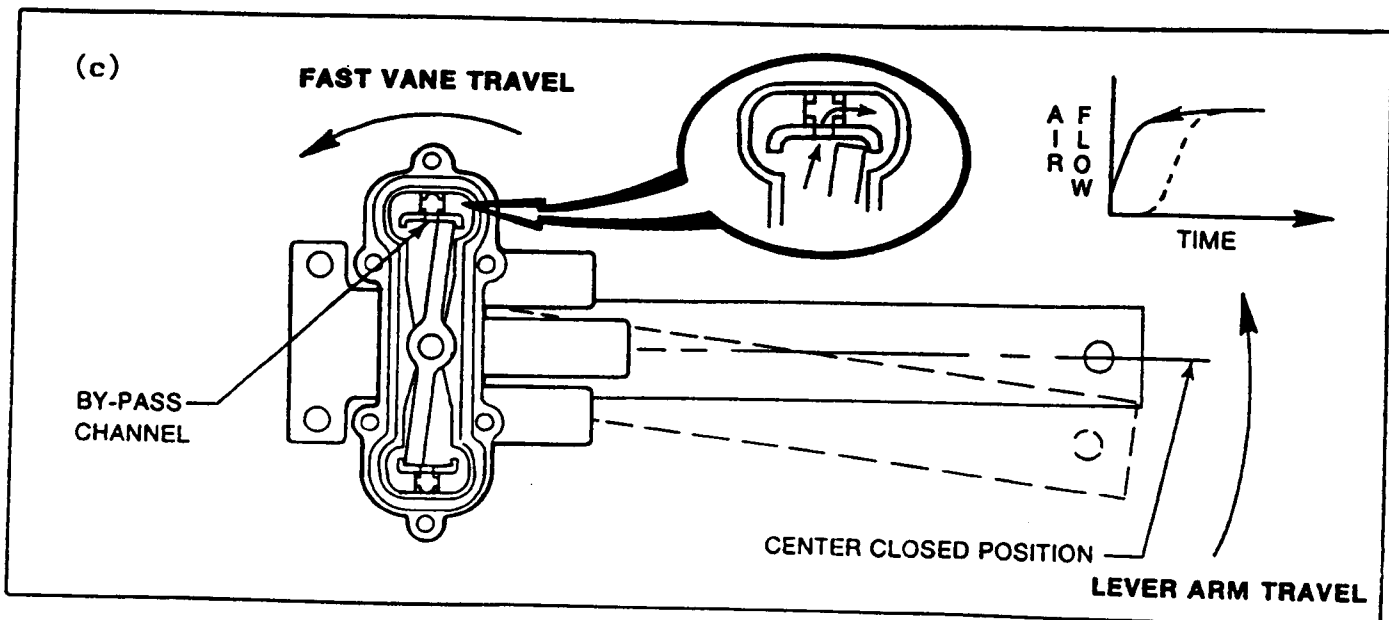
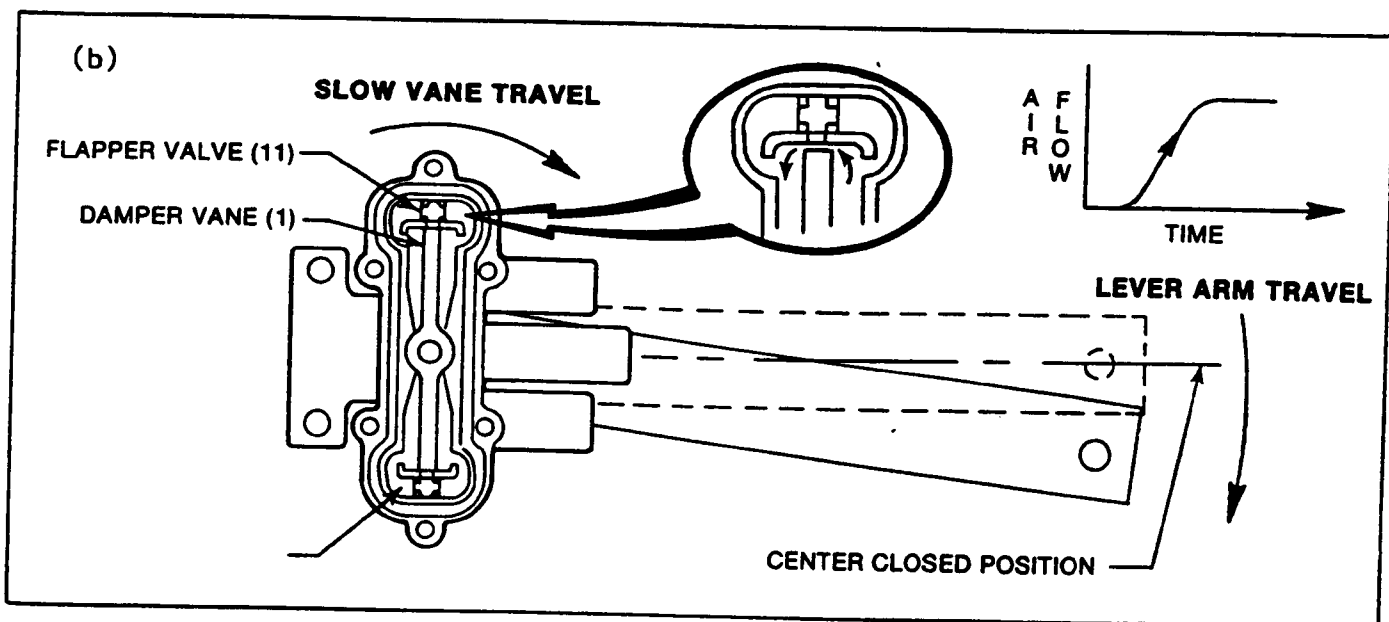
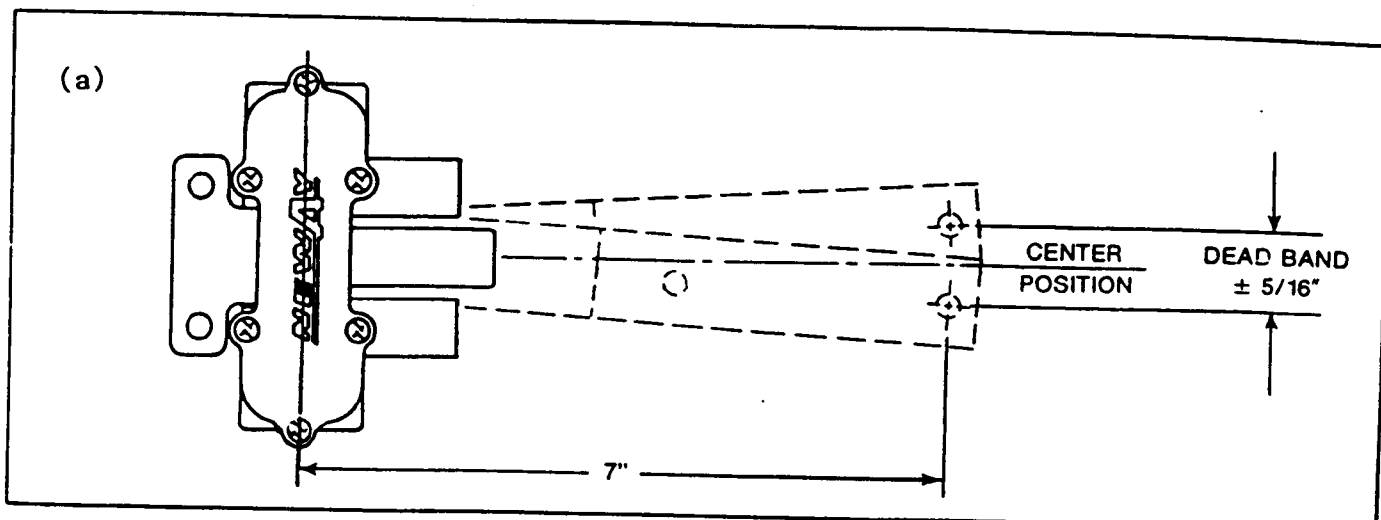


Fig 2.6 Typical suspension height control valve operating modes

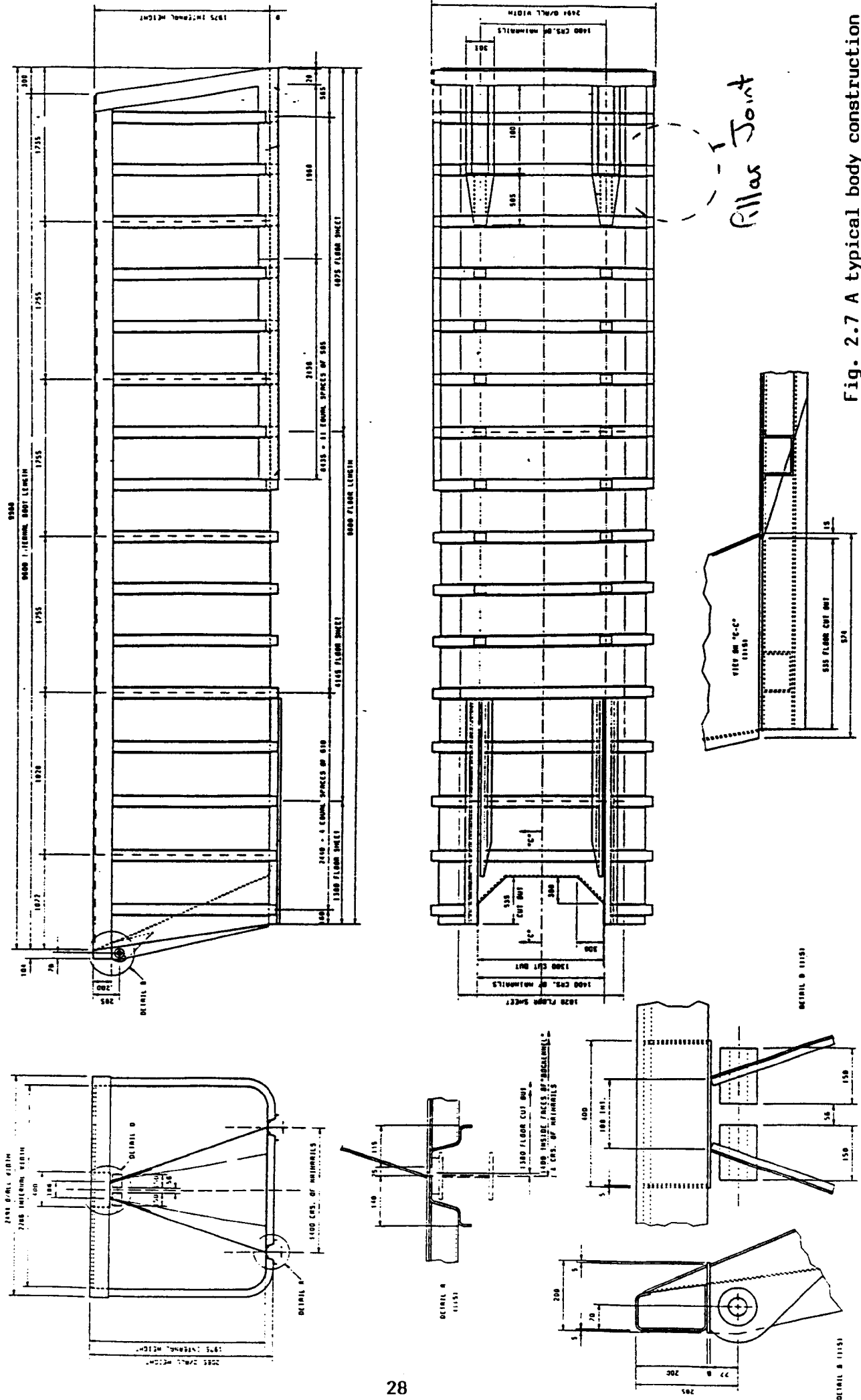


Fig. 2.7 A typical body construction

**Table 2.1 - Dynamic fifth loads occurring once per km  
for a 32 tonne fully laden articulated lorry**

LOADING MODE	DYNAMIC LOADS	
	Average	Maximum
LONGITUDINAL LOADING (+ve load, trailer pushing tractor)	+20kN to -40 kN	+120 kN
VERTICAL LOADING (+ve load downwards)	±7 kN	±24 kN
LATERAL LOADING (+ve loading applied from Kerbside)	±20 kN	+33 kN

# CHAPTER 3

## BASIS OF THE THEORETICAL MODEL

### 3.1 Introduction

When an articulated tipper unit is being loaded or is tipping, it is unlikely to be standing on perfectly level ground. Also, the centre of gravity of the load is unlikely to be in the centre of the body. Hence the loads carried by the suspension and tyres on one side of the tipper will be greater than those on the other side. This uneven loading will cause the tyres and suspension on one side of the tipper unit to deform more than those on the other side. It will also cause the chassis to deform; the twisting about its longitudinal axis is the most significant mode of deformation. As a result of these deformations, caused by the uneven loading, the position of the centre of gravity moves even further towards the more heavily loaded side. This causes even more uneven loading and further deformations.

Under stable conditions, a situation will exist at which the position of the centre of gravity, the deformations and the forces transmitted through the system are compatible. Instability, resulting in roll-over would occur if the overall centre of gravity of the load, body, chassis etc. were to fall outside the area bounded by the contact of the wheels with the ground.

Although the stresses within the chassis, suspension, tyres etc. may be well within the linear elastic limits for the materials used, the position of the centre of gravity of the

load can be significantly altered by the elastic deformations. The movement of the centre of gravity is largest when tipping is in progress. In order to analyse this type of non-linear, elastic system, an iterative procedure must be adopted. The basic procedure is illustrated in Section 3.4.1 using a simple example.

In some articulated tipper units, a further non-linearity arises from the use of height-controlled, pneumatically pressurised, airbag suspension units. In such systems, the distance between the axle of the centre pair of wheels and the chassis, is maintained constant on both, by increasing or decreasing the pressure in the airbags on each side independently. Under normal driving conditions this reduces the significant effect that the suspension deformations have on the overall movement of the centre of gravity. However, other deformations, such as the tyre compression, chassis twisting etc. still cause significant movements of the centre of gravity.

Two further non-linear situations can result from this type of airbag suspension. Firstly, if the pressure to one side should become zero (gauge), no further control is exerted, ie. the airbag is effectively removed from that side. Secondly, above a maximum allowable pressure, a cut-off valve operates and the compression of the fixed volume of air trapped in the system at this stage controls the deformation of the suspension. Under these conditions, an effectively polytropic compression of the gas occurs.

Other non-linear effects, such as clearance in the hinges connecting the body to the chassis and separation of the fifth wheel plate and the king pin have not been

included. However, unless wear is very great and/or the tipper unit is in an unstable condition, these effects are likely to be small compared to those described above.

The theoretical model developed for predicting the behaviour of a tipping trailer unit, under tipping conditions, requires only one set of finite element analyses to calculate the flexibility matrix for a particular chassis and tractor. A computer program was developed based on the theoretical model. It requires the use of the chassis finite element results, tyre stiffnesses, air-bag characteristics, etc. It can be used to assess the stability of the particular tipping trailer unit, for any combination of ground slope, payload and tipping angle.

### **3.2 Co-ordinate system**

The theoretical model requires a system to define the initial undeformed and subsequent deformed positions of the tipping trailer unit. A trailer datum, coordinate axis, ground slope and ram length are required.

- (i) The trailer datum and coordinate axis are used to describe the position of the elements in the finite element model in an undeformed and deformed position. The coordinate system is also used to describe the position of the ram to body contact when tipped and the body and payload centre of gravity in an undeformed or deformed position. The datum position is at the centre of the chassis/body hinges. The +X direction is defined from the datum along a line parallel to the ground in the forward direction. The +Z direction is defined from the datum along a line passing through the right body chassis hinge in

a undeformed state when viewed from the rear of the trailer. The +Y direction is upwards (perpendicular to the XZ plane), they form a right handed system of axes, as shown in Fig. 3.1.

- (ii) The ground slope is used to describe the trailer unit's position (XZ plane) relative to the horizontal, and is a rotation about the X axis only. The ground slope,  $\theta_p$ , is shown in Fig. 3.1. Only a positive ground slope (clockwise rotation about the X axis) is considered.
- (iii) The ram length is used to describe the tipped position of the body, and varies between 2 and 8 meters.

### 3.3 Assumptions

The main assumptions made in this work are as follows:

#### 3.3.1 The body

The body discussed in Section 2.1 is assumed to be rigid and to be connected to the chassis by a hinge bar at the back and a ram at the front. The connection at the hinge bar is assumed to have no clearance in it. It is further assumed that there is negligible friction in this hinge and that the force acting along the hinge bar, due to its contact with the chassis, ie. in the transverse direction of the tipper unit, occurs at the left hinge. The connection between the ram and the body is assumed to be a pin-joint with negligible friction.



### 3.3.2 The chassis (Trailer & Tractor)

The mainrails and cross-members of the trailer and tractor chassis described in Section 2.1 are assumed to stretch, compress, bend and twist according to beam theory. That is, local deformations at the connections between cross-members and mainrails, and at load application points are assumed to be negligible. The components welded to the underside of the chassis, containing the pivot bush (hinge), which connects the trailing arm to the chassis, is also represented by a cantilever beam.

The trailer chassis is connected to the body by a hinge bar at the back and a ram at the front. It is assumed that there is negligible friction in the hinge and that the transverse force acting on the chassis, at the hinge bar, occurs on the upper side of the chassis. The connection between the ram and the chassis is assumed to be a pin-joint with negligible friction.

The fifth wheel and king pin unit, described in Section 2.1, connects the chassis to the fifth wheel plate on the tractor unit. It is assumed to allow relative rotation of the fifth wheel plate and the chassis plate, about an axis perpendicular to these plates, through the king pin, and about a transverse axis in the Z direction through the king pin only. Other displacements of the tractor and chassis, at the point of contact between the king pin and the fifth wheel plate, are assumed to be the same for the tractor and chassis. That is, it is assumed that there is no clearance in the connection between the king pin and fifth wheel and that there is no separation between the fifth

wheel plate and chassis plate. It is also assumed that friction effects in the region of contact between the tractor and chassis is negligible.

The trailing arms on the suspension, which connect the axle to the chassis, described in Section 2.1, are assumed to be connected by hinges. That is, free (frictionless) relative rotation of the chassis and trailing arms is allowed about an axis through the point of contact which runs transverse relative to the chassis. Other deflections of the chassis and trailing arms at the points of contact between them, are assumed to be the same, ie. it is assumed that there is no clearance in the hinges.

Each airbag is assumed to apply a force to the chassis at the point on the chassis which is on the centre-line of the airbag. It is further assumed that the force acts along the axis of the airbag, ie. the resistance to motion, of one end of the airbag relative to the other, perpendicular to its axis, is assumed to be negligible.

### 3.3.3 The suspension

The suspension systems, described in Section 2.1, are comprised of trailing arms and torsion beams and are assumed to stretch, compress, bend and twist according to beam theory. That is, local deformations at the connections between these components and the mainrails and at load application points are assumed to be negligible. Also, it is assumed that the trailing arm seats rigidly attach the trailing arms to the torsion beams. The shock absorbers, which are intended to damp out vibrations when the trailer is in motion have little effect when the trailer is stationary. The shock absorbers are therefore assumed to be inactive in the present analysis.

Each airbag is assumed to apply a force to a trailing arm at the point on the trailing arm which is on the axis of the airbag. It is further assumed that the force acts along the axis of the airbag, ie. the resistance to transverse motion of one end of an airbag relative to the other, perpendicular to its axis, is assumed to be negligible.

The forces exerted by the airbags, when operating under pressure control, are assumed to be proportional to the pressure. This has been verified experimentally; details are given in Appendix A. Once the maximum pressure is achieved in the airbags on one side, and the cut-off valve operates, the force,  $F$ , in the three bags on that side and the sum of the lengths of the three airbags,  $\Sigma l$ , are assumed to be governed by a polytropic gas law, ie.  $F (\Sigma l)^n = \text{a constant}$ . This has been verified experimentally and the value of  $n$  has been determined; details are given in Appendix A.

Each of the trailer wheel/tyre units, which is connected to the end of a torsion bar, is assumed to be represented by three springs. Each of these three springs has a stiffness which represents the relationship between the force and displacements of the axis of the wheel, in the radial, rolling and transverse directions, assuming no slip between the tyre and the ground.

The vertical spring stiffness was obtained experimentally; details are given in Appendix B. The fore/aft and transverse spring stiffnesses were also determined experimentally, however the manufacturers data was used, as the experimental procedure over estimated them, as explained in Appendix B.

Each of the tractor wheel/tyre units is assumed to be represented by three mutually perpendicular springs. The stiffnesses were obtained from manufacturers information.

#### 3.3.4 Ground conditions

All contact areas between wheels and the ground (tractor and trailer) are assumed to lie on a plane with all lower wheels at the same elevation and all upper wheels at the same elevation. The ground is assumed to be rigid, ie. the axles only move due to compression of the tyres, no compression of soft or loose ground is included because this would be difficult to define.

### 3.4 Theoretical basis

#### 3.4.1 Simple illustration of principle

For the tipping trailer unit, the body containing the payload is assumed to be rigid and the rest of the system, comprising the tractor unit, the trailer chassis, suspension and tyres, is assumed to undergo linear elastic deformations. A tipping trailer unit is similar to a number of other engineering systems in the sense that small, linear elastic deformations of some parts of the system can cause large displacements of the positions of the loads (eg. the centre of gravity of the payload, the top of the ram and the line of action of the wind load).

The simple cantilever beam, of length  $l$ , made of a material with a Young's modulus of  $E$  and with a cross-section having a second moment of area about the horizontal axis of  $I$ , loaded by a constant, relatively small load,  $L$ , carried on a rigid frame, as shown in Fig. 3.2a, illustrates the type of problem for which this is the case. In the undeformed position, the end of the beam is subjected to a force  $L$  and a moment of magnitude  $La$ , as shown in Fig. 3.2b. However, these cause displacement  $\Delta^{(1)}$  and rotation  $\phi^{(1)}$  of the tip of the cantilever, where

$$\Delta^{(1)} = \frac{Ll^3}{3EI} + \frac{Lal^2}{2EI} \quad (3.1a)$$

and

$$\phi^{(1)} = \frac{Ll^2}{2EI} + \frac{Lal}{EI} \quad (3.1b)$$

resulting in a movement of the rigid frame, and hence a change in the load position, as shown in Fig. 3.2c. In this deformed position, the load on the beam is still  $L$ , but the moment becomes  $L(a+b\theta)$ , assuming that  $\theta$  is small enough for  $\sin \phi \approx \phi$  and  $\cos \phi \approx 1$ .

Hence, using the updated load and moment, the displacement and rotation of the tip of the beam,  $\Delta^{(2)}$  and  $\phi^{(2)}$ , can be obtained, ie.,

$$\Delta^{(2)} = \frac{Ll^3}{3EI} + \frac{L(a+b\phi^{(1)})l^2}{2EI} \quad (3.2a)$$

and

$$\phi^{(2)} = \frac{Ll^2}{2EI} + \frac{L(a+b\phi^{(1)})l}{EI} \quad (3.2b)$$

Although, in this example a further iteration is not necessary.

If the process is repeated  $i$  times, the displacement,  $\Delta^{(i)}$ , and rotation,  $\phi^{(i)}$ , of the tip of the cantilever are given by

$$\Delta^{(i)} = \frac{Ll^3}{3EI} + \frac{L(a+b\phi^{(i-1)})l^2}{2EI} \quad (3.3a)$$

and

$$\phi^{(i)} = \frac{Ll^2}{2EI} + \frac{L(a+b\phi^{(i-1)})l}{EI} \quad (3.3b).$$

By continuing the iterative procedure until the differences between  $\Delta^{(i)}$  and  $\Delta^{(i-1)}$  and between  $\phi^{(i)}$  and  $\phi^{(i-1)}$  are negligible, the actual solution is obtained. At this stage, the load and moment transmitted to the beam, through the displaced rigid frame, are compatible with the displacement and rotation of the tip of the beam calculated using these forces and moments.

### 3.4.2 Principle applied to tipping trailer

The whole of the following description and calculation applies to one initial position of the body, ie. one fixed ram length. It has to be repeated for other ram lengths to analyse the complete tipping operation of a particular truck, unloading in a particular position. The body (assumed to be rigid) of the tipping trailer corresponds to the rigid frame in Fig. 3.2a. It is loaded by the payload  $\underline{P}$ , and wind load  $\underline{W}$ . As shown in Fig. 3.3, the equilibrium of the body is maintained by forces  $\underline{H}_L$  at the left hand hinge,  $\underline{H}_R$  at the right hand hinge and  $\underline{R}$  at the bottom of the ram (which is considered as part of the body). Just like the tip of the cantilever in Fig. 3.2a, the positions of these reactions change under load, because  $\underline{H}_L$ ,  $\underline{H}_R$  and  $\underline{R}$  are supported

by the flexible chassis. Instead of Equ. 3.2, a stiffness matrix for the flexible parts of the vehicle (chassis, tyres and suspension etc.) has to be used to obtain the displacements of the connecting 'points' between the body and chassis (see Fig. 3.4). A similar iterative procedure to that used in the simple cantilever example allows the true displacements to be determined.

There are two other important differences between the simple illustration and the tipping trailer. The distributed weight of the chassis, wheels and suspension of the trailer,  $\underline{W}_C$ , and the weight of the tractor,  $\underline{W}_T$ , are significant loads on the chassis, which have to be included in the calculation of the displacements. Also, the airbags exert forces which have to be determined.

The three airbags on each side are connected so that they exert six identical forces,  $A_L$ , on the left and six other identical forces,  $A_R$ , on the right, as shown in Fig. 3.5. However, there are four possible operating conditions which exist for the airbag suspension system. Under normal conditions, the airbag pressure is continuously changed to maintain 'height control' (called 'H'), ie., the distance between each side of the central wheel axle and the chassis is maintained at a constant value. If the pressure in one of the sets of airbags reaches a maximum value, a cut-off valve operates and polytropic compression of the fixed mass of gas (called 'C') controls the airbag force and hence the deformations of the suspension on this side, while the other side can still be height controlled. Also, if the gauge pressure becomes zero for one of the sets of airbags, no further control is exerted on that set of airbags (called 'F'). Hence the airbags will be effectively removed from one side while the

other side may be either height controlled, or controlled by the polytropic compression of a fixed mass of gas. In summary, for rolling to the right, the four possible airbag conditions are:

- (a) the left and right side airbag pressures are both adjusted to give height control on both sides (ie., LH/RH),
- (b) the left airbag pressure is adjusted to give height control on this side while the right side is controlled by the polytropic compression of a fixed mass of gas (ie., LH/RC),
- (c) the left airbag is inoperative (gauge pressure of zero) while the right airbag pressure is adjusted to give height control on this side (ie., LF/RH) and
- (d) the left airbag is removed while the right side is controlled by the polytropic compression of a fixed mass of gas (ie., LF/RC).

### 3.4.3 Basis of the tipping trailer analysis

The forces on the rigid body, and hence on the elastic parts of the system, (in a deformed position) are given in the free body diagrams shown in Fig. 3.3. The origin of the orthogonal, cartesian, co-ordinate system (X, Y, Z) is in the middle of the axis of the hinges which allow the body to be tipped.  $\underline{H}_L$  and  $\underline{H}_R$  are the reactions at the centres of the left and right hinge,  $\underline{B}$  is weight of the body,  $\underline{P}$  the magnitude of the payload and  $\underline{W}$  is the resultant of the wind forces acting on the body.  $\underline{R}$  is the force in the ram which lifts the front end of the body during tipping. In the analysis these forces are split into their cartesian components.



The components of rotation about the X and Y directions of the hinge-bar, which connects the body to the trailer chassis, are defined by  $\theta$  and  $\psi$ , as indicated in Fig. 3.1 and Fig. 3.4.

The points on the chassis at which the deformations control the displaced position of the body and payload are the connection points with the hinge bar (the left, L, and right, R, rear positions on the chassis, see Fig. 3.3) and the connection point with the bottom of the ram. Although analytical solutions, such as those applicable to the deflection and rotation of the cantilever (ie. Equation 3.1), cannot be obtained to relate the deformations of the important points on the chassis to the loads applied to the chassis, the finite element method can be used to obtain the required relationships for each of the four possible suspension conditions.

Unlike the force,  $L$ , in the simple illustration in Fig. 3.2, the forces  $\underline{H}_L$ ,  $\underline{H}_R$  and  $\underline{R}$  vary during the iteration. For each of the four suspension conditions (a, b, c, d in Section 3.4.2) the finite element program was run for one load at a time, making this load unity. The required displacements, due to each unit load, were stored and later multiplied by the  $i$  th values of the loads to calculate the  $(i+1)$  th values of deformation.

The 14 unit loads were:

- a) the cartesian components of the body contact forces,  $H_{Lx}$ ,  $H_{Ly}$ ,  $H_{Lz}$ ,  $H_{Rx}$ ,  $H_{Ry}$ ,  $R_x$ ,  $R_y$ , and  $R_z$ . (NB.  $H_{Rz} = 0$  because there must be axial clearance between the body and chassis at the hinge bar);

- b) the distributed weight of the tractor,  $W_{Ty}$  and  $W_{Tz}$ ;
- c) the distributed weight of the chassis, suspension and wheels,  $W_{Cy}$  and  $W_{Cz}$ ;
- d) the airbag forces,  $A_L$  and  $A_R$ , each acting in six positions, as shown in Fig. 3.5.

The reactions to each of these loads were taken where the ten wheels touch the ground (see Fig. 3.3).

The 15 important cartesian components of displacement were:

- a) at the connections to the body,  $u_{hl}$ ,  $v_{hl}$ ,  $w_{hl}$ ,  $u_{hr}$ ,  $v_{hr}$ ,  $w_{hr}$ ,  $u_{rb}$ ,  $v_{rb}$  and  $w_{rb}$ ;  
(suffices refer to hinges at left and right and the ram bottom end);
- b) at the upper ends of the airbags  $v_{a1}$ ,  $v_{a3}$  and  $v_{a5}$ ;
- c) at the lower ends of the airbags,  $v_{a2}$ ,  $v_{a4}$ ,  $v_{a6}$ ;

Suffices 1 and 2 refer to the 1st airbag, 3 and 4 to the 2nd airbag and 5 and 6 to the 3rd airbag, see Fig. 3.5.

#### 3.4.3a Suspension on both sides operating under height control (LH/RH)

In the finite element analysis, in order to model the situation when both sides are height controlled, tie bar elements were placed between E and B and between C and D (see Fig. 3.5) to keep the distances EB and CD constant. In the truck, the tie bars do not exist, there are only height sensing devices, which do not transmit any forces between EB or CD. Therefore the forces exerted by the airbags must be adjusted during each iteration to make the tie bar forces zero. Hence, an influence matrix,

[I<sup>a</sup>], relating the important displacements to the forces can be obtained, such that

$$\underline{\underline{u}} = [I^{(a)}] \underline{\underline{F}} \quad (3.4a)$$

where [I<sup>a</sup>] is the flexibility matrix obtained from the finite element work,

$$\underline{\underline{u}}^t = [u_{rb}, v_{rb}, w_{rb}, u_{hl}, v_{hl}, w_{hl}, u_{hr}, v_{hr}, w_{hr}, v_{a1}, v_{a2}, v_{a3}, v_{a4}, v_{a5}, v_{a6}] \quad (3.4b)$$

and

$$\underline{\underline{F}}^t = [R_x, R_y, R_z, H_{Lx}, H_{Ly}, H_{Lz}, H_{Rx}, H_{Ry}, W_{Ty}, W_{Tz}, W_{Cy}, W_{Cz}, A_L, A_R] \quad (3.4c).$$

[N.B. The superscript, t, used in equations 4b and 4c, and in subsequent equations, refers to the transpose of the matrix].

Equations 3.4 are the equivalent, for the tractor and trailer system, of Equations 3.1 for the simple cantilever problem used to illustrate the approach.

As well as determining the important displacements, the components of the forces in the tie bars can be obtained from the finite element results for each unit load applied to the finite element mesh; hence, it is possible to determine an influence matrix, [I<sub>T</sub><sup>(a)</sup>], which relates the tie bar forces,  $\underline{\underline{T}}$ , to the force vector,  $\underline{\underline{F}}$ , ie.

$$\underline{\underline{T}} = [I_T^{(a)}] \underline{\underline{F}} \quad (3.5a)$$

where

$$\underline{\underline{T}}^t = [T_L, T_R] \quad (3.5b)$$

The relationships between  $A_L$  and  $A_R$  and the other forces can be obtained from equation 3.5a by imposing the condition  $\underline{T} = \underline{0}$ .

From the finite element analyses, the relationship between the tyre forces transmitted to ground,  $\underline{G}$ , and the other forces,  $\underline{F}$ , can be obtained, ie.

$$\underline{G} = \left[ I_G^{(a)} \right] \underline{F} \quad (3.6)$$

The initial components of the force vector,  $\underline{F}$ , at the connection points between the body and chassis, related to the undeformed position of the system can be obtained by considering the equilibrium of the body and payload in the undeformed position. In the undeformed position,  $\theta$  and  $\psi$ , indicated in Fig. 3.4, are both zero. The force and moment equilibrium equations for the body and payload, in vector form, are as follows:

$$\underline{\tilde{H}}_L + \underline{\tilde{H}}_R + \underline{\tilde{R}} + \underline{\tilde{B}} + \underline{\tilde{P}} + \underline{\tilde{W}} = \underline{0} \quad (3.7a)$$

and

$$\underline{\tilde{H}}_L \wedge \underline{h}_l + \underline{\tilde{H}}_R \wedge \underline{h}_r + \underline{\tilde{R}} \wedge \underline{r}_t + \underline{\tilde{B}} \wedge \underline{b} + \underline{\tilde{P}} \wedge \underline{p} + \underline{\tilde{W}} \wedge \underline{w} = \underline{0} \quad (3.7b)$$

When expanded, equations 3.7 produce six linear equations with eight unknown forces, shown in Fig. 3.6, ie.,  $R_x$ ,  $R_y$ ,  $R_z$ ,  $H_{Lx}$ ,  $H_{Ly}$ ,  $H_{Lz}$ ,  $H_{Rx}$  and  $H_{Ry}$ . By assuming that the ram is pin-jointed at both ends, the ram load components ( $R_x$ ,  $R_y$  and  $R_z$ ) can be related to the resultant ram force  $\underline{\tilde{R}}$ , which must act along the axis of the ram. In the initial undeformed position, the co-ordinates of the top and bottom of the ram

are known from the dimensions of the vehicle. Therefore, the vector,  $\underline{r}_{tb}$ , which defines the position of the top of the ram relative to the bottom of the ram, which in turn defines the line of action of the ram force,  $\underline{R}$ , can be obtained, ie.,

$$\underline{r}_{tb} = \underline{r}_t - \underline{r}_b \quad (3.8)$$

Hence equations 3.4 can be solved to obtain the components of the chassis forces,  $\underline{F}$ , at the connection points between the chassis and body, related to the undeformed position of the body and payload. Using these forces together with the tractor and chassis self weight forces, equations 3.5a can be used, with  $\underline{T} = \underline{0}$ , to obtain the suspension forces,  $A_L$  and  $A_R$ . Hence, the complete force vector,  $\underline{F}^{(1)}$ , related to the undeformed position can be determined. Substituting  $\underline{F}^{(1)}$  into equations 3.4 and 3.6 allows a first estimates for the important chassis displacements,  $\underline{u}^{(1)}$ , and the left, rear tyre force,  $G_{L3}^{(1)}$ , to be determined.

Having determined  $\underline{u}^{(1)}$  values, the updated loads,  $\underline{F}^{(2)}$ , can be obtained using the equilibrium equations 3.7 with the updated position vectors  $\underline{h}^{(1)}$ ,  $\underline{h}^{(1)} \sigma \underline{r}_t^{(1)}$ ,  $\underline{b}^{(1)}$ ,  $\underline{p}^{(1)}$  and  $\underline{w}^{(1)}$ . The facts that the body is assumed to be rigid, the ram length is constant during the iteration process and the small changes in distance between the centre of the hinge bar and the bottom of the ram during the deformation process are insignificant (further explanation is given later) allows the updated, position vectors to be related to the initial position vectors.

The vector defining the position of the top of the ram,  $\underline{r}_t$  (or  $\underline{r}_t^{(1)}$ ), is at right angles to the vector defining the position of the left hinge,  $\underline{h}_l$  (or  $\underline{h}_l^{(1)}$ ); displacement of the

body does not affect this, because the body is assumed to be rigid. Therefore, the scalar product of the vectors  $\underline{r}_t$  (or  $\underline{r}_t^{(1)}$ ) and  $\underline{h}_t$  (or  $\underline{h}_t^{(1)}$ ) is zero, ie.,

$$\underline{r}_t^{(1)} \cdot \underline{h}_t^{(1)} = 0 \quad (3.9)$$

Also, if the distance between the centre line of the hinge bar and the bottom of the ram,  $r_b$ , is constant (finite element analysis showed the changes in this distance to be negligible) and the ram length remains constant, then in the displaced position, the angle between the vectors  $\underline{r}_t$  (or  $\underline{r}_t^{(1)}$ ) and  $\underline{r}_b$  (or  $\underline{r}_b^{(1)}$ ), defined as  $\gamma$ , would remain constant. Hence,

$$\underline{r}_t^{(1)} \cdot \underline{r}_b^{(1)} = r_t r_b \cos \gamma \quad (3.10)$$

and

$$\begin{aligned} & \left( r_{tx}^{(1)} - r_{bx}^{(1)} \right)^2 + \left( r_{ty}^{(1)} - r_{by}^{(1)} \right)^2 + \left( r_{tz}^{(1)} - r_{bz}^{(1)} \right)^2 \\ & = \left( r_{tx} - r_{bx} \right)^2 + \left( r_{ty} - r_{by} \right)^2 + \left( r_{tz} - r_{bz} \right)^2 \end{aligned} \quad (3.11)$$

From equations 3.9, 3.10 and 3.11,  $\underline{r}_t^{(1)}$  ( $r_{tx}^{(1)}$ ,  $r_{ty}^{(1)}$ ,  $r_{tz}^{(1)}$ ) can be determined. Having obtained  $\underline{r}_t^{(1)}$  the other unknown position vectors, ie.  $\underline{p}^{(1)}$ ,  $\underline{b}^{(1)}$  and  $\underline{w}^{(1)}$  are all obtained in a similar manner. For example, the triangle defined by the centre of the hinge bar, the top of the ram and the centre of gravity of the payload remains fixed in shape, because the body/payload is assumed to be rigid. If the angle between the vectors  $\underline{r}_t$  and  $\underline{p}$  is taken to be  $\delta$  (say), this angle can be obtained on the basis of the undeformed position of the chassis, ie.,

$$\underline{r}_t \cdot \underline{p} = r_t p \cos \delta \quad (3.12).$$

In the displaced position,  $\delta$  remains constant, therefore

$$\underline{r}_t^{(1)} \cdot \underline{p}^{(1)} = r_t p \cos \delta = r_t \cdot \underline{p} \quad (3.13)$$

Also, and

$$\underline{p}^{(1)} \cdot \underline{h}_l^{(1)} = \underline{p} \cdot \underline{h}_l \quad (3.14)$$

$$\left(\underline{p}_x^{(1)}\right)^2 + \left(\underline{p}_y^{(1)}\right)^2 + \left(\underline{p}_z^{(1)}\right)^2 = p^2 \quad (3.15).$$

The solution of equations 13, 14 and 15 allow  $\underline{p}^{(1)}$  ( $p_x$ ,  $p_y$ ,  $p_z$ ) to be determined. By replacing  $\underline{p}^{(1)}$  by  $\underline{b}^{(1)}$  or  $\underline{w}^{(1)}$ , then  $\underline{b}^{(1)}$  and  $\underline{w}^{(1)}$  can also be obtained.

With the updated position vectors, ie.,  $\underline{h}_l^{(1)}$ ,  $\underline{h}_r^{(1)}$ ,  $\underline{r}_t^{(1)}$ ,  $\underline{b}^{(1)}$ ,  $\underline{p}^{(1)}$  and  $\underline{w}^{(1)}$ , equations 3.7 can be solved to obtain the updated components of the forces between the body and chassis. These forces, together with the tractor and chassis self-weight forces can be used in equation 3.5, with  $\underline{T} = \underline{0}$ , to obtain updated suspension forces. Hence, an updated total force vector,  $\underline{F}^{(2)}$ , can be obtained. An improved set of displacements,  $\underline{u}^{(2)}$ , and rear, left-hand tyre force,  $G_{L3}^{(2)}$ , can be obtained using equations 3.4 and 3.6, respectively. These updated displacements can then be used to obtain the updated position of the top of the ram  $\underline{r}_t^{(2)}$ , as indicated by equations 3.9, 3.10 and 3.11. Then, the updated positions of the other important points can also be obtained and used to obtain a further update of the chassis load vector. This process is repeated until either the changes in displacement are negligible or an unstable solution, indicating a rollover condition, is obtained. The onset of instability is taken to be when the rear, left-hand tyre reaction force  $G_{L3}$  becomes zero.

A similar approach can be used to assess stability with the other three suspension conditions. The way in which these analyses differ from the one described above are outlined below.

**3.4.3b Left suspension height controlled and right suspension governed by polytropic compression of a fixed mass of gas (LH/RC)**

In this case, a tie bar is placed between C and D (see Fig. 3.5), as in the previous case. However, the tie bar is removed from the right hand side because the distance between E and B can decrease, governed by the polytropic compression of a fixed mass of gas in the three, connected, right hand airbags. Since the force in the right hand suspension,  $A_R$ , is related to the deformation of the system, an iterative procedure is required in this case. Again, unit forces  $R_x = 1 \dots H_{Ry} = 1, \dots W_{cz} = 1, A_L = 1$  and  $A_R = 1$ , are applied in turn to the finite element mesh. For these units forces the left hand tie bar force,  $T_L$ , and the important chassis displacements, at points where the body and ram are attached, ie.,  $\underline{u}$  are determined. As before,  $\underline{u}$  can be related to  $\underline{F}$ , ie.,

$$\underline{u} = [I^{(b)}] \underline{F} \quad (3.16a),$$

where a subset of matrix equation 16a, related to the right hand side air-bag displacement, is

$$\underline{v}_a = [I_a^{(b)}] \underline{F} \quad (3.16b)$$



Also, as before, the tie-bar force,  $T_L$ , can also be related to  $\underline{F}$ , ie.

$$T_L = [I_T^{(b)}] \underline{F} \quad (3.16c).$$

and the tyre forces,  $\underline{G}$ , can be related to the  $\underline{F}$ , ie.

$$\underline{G} = [I_G^{(b)}] \underline{F} \quad (3.16d).$$

In addition to the above, the polytropic compression of the fixed mass of gas dictates that

$$A_R \left( 3l_o + \sum_{i=1}^6 v_{ai} \right)^n = \text{constant} (= Q) \quad (3.17)$$

where  $l_o$  is the length of gas contained in the constant cross-sectional area cylinders, at the limiting pressure, ie., when the pressure supply is cut off, and  $\Sigma v_{ai}$  is the sum of the changes of length of the three right hand airbags. The constant is obtained by substituting the instantaneous values of  $A_R$  and  $v_{ai}$  when  $A_R$  (which is proportional to the pressure) reaches the limiting value. The values of  $n$  and  $Q$  (for a given mass of gas) have been determined by performing experiments on airbags.

In order to obtain the solution to equations 3.16 and 3.17, an iterative procedure is adopted. A first estimate,  $A_R^{(1)}$  of the force in the right hand airbags is substituted into equation 3.16c with  $T_L = 0$ , to obtain a first estimate,  $A_L^{(1)}$ , of the force in the left hand airbags. These first estimates for the airbag forces are then substituted into equation 3.16b to obtain a first estimate,  $\underline{v}_a^{(1)}$ , for the displacements of the ends of the right hand airbags. The  $\underline{v}_a^{(1)}$  values are then substituted into equation 3.17 to obtain an updated estimate,  $A_R^{(2)}$ , for the force in the right hand airbags. This

iterative procedure is continued until negligible change in the forces and displacements occur between one iteration and the next. At this stage, the  $\underline{F}$  matrix is substituted into equation 3.16a to obtain the updated displacements,  $\underline{u}^{(2)}$ , which are used to obtain the new displaced position of the body and payload. As for the case described in Section 3.4a, the procedure is repeated, iteratively, until the changes in displacement and forces, including the right-hand airbag values are negligible.

### 3.4.2c Left suspension airbags inoperative and right suspension height controlled (LF/RH)

When the gauge pressure falls to zero in the left hand suspension, the airbags become inoperative and the forces  $A_L$  no longer act. For this condition, it is therefore only necessary to place a tie bar between E and B (Fig. 3.5) and to leave the left suspension free. By obtaining the displacements,  $\underline{u}$ , and right-handed tie bar force,  $T_R$ , from the finite element results for each unit load case  $R_x = 1, \dots W_{cz} = 1$  and  $A_R = 1$ , the influence matrices  $[I^{(c)}]$ ,  $[I_T^{(c)}]$  and  $[I_G^{(c)}]$  can be determined, such that

$$\underline{u} = [I^{(c)}] \underline{F} \quad (3.18a),$$

$$T_R = [I_T^{(c)}] \underline{F} \quad (3.18b).$$

and

$$\underline{G} = [I_G^{(c)}] \underline{F} \quad (3.18c)$$

Note, in this case, the force vector,  $\underline{F}'$ , does not contain left-hand airbag force, ie.  $A_L = 0$ , and the influence matrices  $[I^{(c)}]$  and  $[I_T^{(c)}]$  are smaller than the corresponding  $[I^{(a)}]$ ,  $[I^{(b)}]$ ,  $[I_T^{(a)}]$  or  $[I_T^{(b)}]$  matrices.

In operation, with height control between E and B,  $T_R = 0$  would be applicable. Therefore, for a given set of forces, at the connections between the body and chassis, and using tractor and chassis components of self-weight, the right hand airbag force,  $A_R$ , can be obtained by setting  $T_R = 0$  in equation 3.18b. This value of  $A_R$  is then substituted into equation 3.18a, along with the other forces to obtain the displacement,  $\underline{u}$ , which define the updated displaced position of the body and payload. An iterative procedure is again adopted until the changes in displacements and forces are negligible.

#### 3.4.2d Left suspension airbags inoperative and right suspension governed by polytropic compression of a fixed mass of gas (LF/RC)

In this case,  $A_L = 0$  and the left hand tie bar is removed, because the left hand suspension system is inoperative, ie., no tie bar is present between C and D, as well as  $A_L = 0$ . Because the maximum pressure, and hence the maximum allowed value of  $A_R$ , has been reached, no tie bar is included between E and B either (see Fig. 3.5). Therefore, the method of applying unit loads in order to determine the influence matrices cannot be applied in this situation, because the finite element mesh would act as a mechanism.

By applying unit displacements to each of the important positions on the chassis in turn (ie.,  $u_{tb} = 1$ , or  $v_{tb} = 1$  or  $w_{tb} = 1$  or  $u_{hl} = 1$  or  $v_{hl} = 1$  or  $w_{hl} = 1$  or  $u_{hr} = 1$  or  $v_{hr} = 1$ ) and to the six right-hand airbag connection points in turn (ie.,  $v_{a1} = 1$  or  $v_{a2} = 1$  ... or  $v_{a6} = 1$ ), while restraining each of the other points, the reaction forces at the restrained positions can be obtained. These reaction forces form components in a stiffness matrix, such that,

$$[K] \underline{u} = \begin{Bmatrix} \underline{F''} \\ \underline{A_R} \end{Bmatrix} \quad (3.19a)$$

where  $\underline{F''}$  is the same as  $\underline{F}$ , but with the terms  $A_L$  and  $A_R$  omitted and

$$\underline{A_R} = (A_{R1}, A_{R2}, A_{R3}, A_{R4}, A_{R5}, A_{R6}) \quad (3.19b)$$

which are the forces corresponding to the right-hand airbag displacement,  $\underline{v}_a$ . In reality, all of the  $A_{Ri}$  values ( $i = 1$  to  $6$ ) must be the same, because the airbags are interconnected.

Matrix equation 3.19a can be rewritten as

$$\begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \underline{u''} \\ \underline{v}_a \end{Bmatrix} = \begin{Bmatrix} \underline{F''} \\ \underline{A_R} \end{Bmatrix} \quad (3.20).$$

By eliminating  $\underline{u''}$ , which is the same as  $\underline{u}$  but with the  $\underline{v}_a$  terms omitted, from matrix equation 3.20 and rearranging, an expression for  $\underline{v}_a$  can be obtained, ie.,

$$\underline{v}_a = \left[ [K_{22}] - [K_{21}] [K_{11}]^{-1} [K_{12}] \right]^{-1} \left\{ \underline{A_R} - [K_{21}] [K_{11}]^{-1} \underline{F''} \right\} \quad (3.21).$$

In addition to the above, the right hand side airbag forces,  $A_R$ , are related to the airbag displacement,  $\underline{v}_a$ , via the gas equation which governs the polytropic compression of the fixed mass of gas, ie., equation 3.17 is applicable.

The solution, in this case, also requires an iterative approach. A first estimate,  $\underline{A}_R^{(1)}$ , of the forces in the right hand airbags (each component in  $\underline{A}_R$  is taken to be the same) is substituted into equation 3.21 to obtain the first estimate for the right hand side airbag displacements,  $\underline{v}_a^{(1)}$ . These airbag displacements can then be used in equation 3.17 to obtain an updated airbag force,  $\underline{A}_R^{(2)}$ . This iteration, between equations 3.21 and 3.17, is continued until negligible changes in the airbag forces and displacements occur between one iteration and the next. Having obtained the airbag forces and displacement, equation 3.20 can be rearranged to allow the other chassis displacements,  $\underline{u}^*$ , to be determined, ie.,

$$\underline{u}^* = [K_{11}]^{-1} \{ \underline{F}^* - [K_{12}] \underline{v}_a \} \quad (3.22a)$$

These updated chassis displacements,  $\underline{u}^*$ , can now be used to obtain the updated, displaced position of the body and payload. Also, the forces at the contact points between the tyres and the ground can be related to  $\underline{u}$ , ie.,

$$\underline{G} = [I_G^{(d)}] \underline{u} \quad (3.22b)$$

An iterative procedure is again adopted until the changes in displacement and forces are negligible.

### 3.5. Outline of the overall solution procedure

The theoretical model, which enables the positions of the important points on the body, body/chassis contact point reactions, suspension forces, chassis displacements and tyre forces to be determined, requires a method to govern the sequence of operations. The theoretical model involves the solution of complicated equations, repetitive iterations and the multiplication of many matrices. This type of problem lends itself to solution by digital computer. Hence a program was written for an Akhter 286 PC using Turbo Basic; with this language, modifications are relatively easy and separate sub-routines can be written.

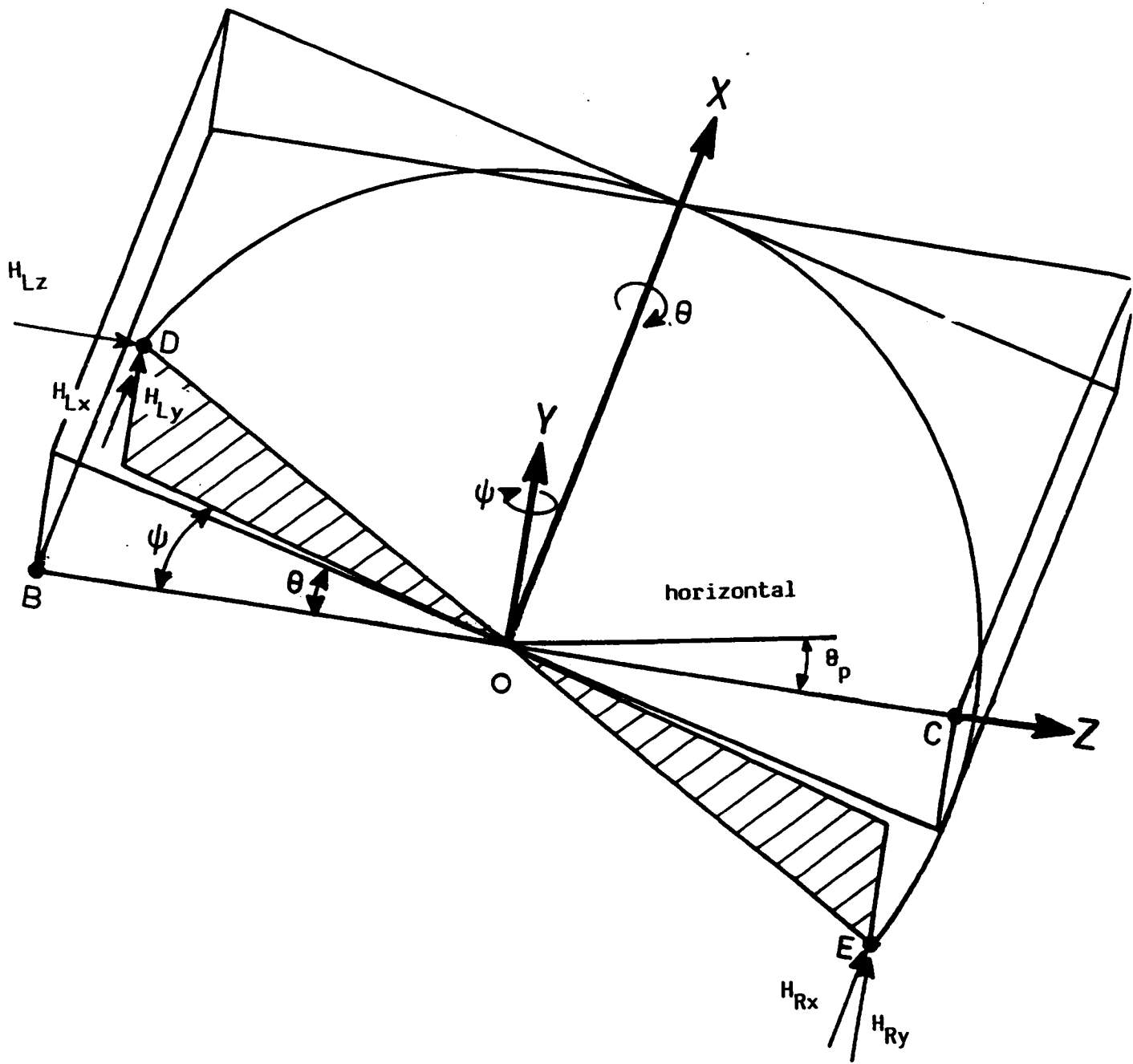
For each ground slope, ram length, wind loading and payload, the iterative calculation procedure used to establish the airbag operating conditions, deformed position, tyre force, etc., is outlined in the flow diagram shown in Fig. 3.7. A separate loop is applicable for each of the four possible airbag conditions. Within each of the four loops, there are some common features. The important position vectors (ie.  $\underline{h}_l$ ,  $\underline{h}_r$ ,  $\underline{r}_l$ ,  $\underline{b}$ ,  $\underline{p}$  and  $\underline{w}$ ) are determined using equations 3.9 to 3.15 and the direction of the ram is obtained using equation 3.8. With these position and direction vectors, equations 3.7a and 3.7b are solved to determine the forces on the body (ie.  $\underline{R}$ ,  $\underline{H}_L$  and  $\underline{H}_R$ ). Hence, the equal and opposite forces applied to the chassis by the hinge bar, connecting the chassis to the body, and the bottom of the ram are obtained. Using the appropriate airbag conditions, the airbag forces can be determined and hence the total force vector,  $\underline{F}$ , applied to the chassis can be obtained. This force vector,  $\underline{F}$ , is then used with the influence matrices (for the

appropriate airbag conditions) to obtain the updated displacements. The procedure is continued until the changes in the predicted displacements are negligible. The tyre forces are then calculated.

The program's initial execution of each iteration is governed by the undeformed position of the tractor and trailer and body. For the first iteration, the transverse ground slope is taken to be zero and the body is considered in the untipped position. For this ram length, after each iteration of the ground slope is increased in  $0.1^\circ$  interval until a ground slope of  $10^\circ$  is reached or until instability occurs, ie. the rear left wheel force becomes zero. The ground slope is then zeroed and the ram length is increased by 0.5 m. Further iterations are again undertaken increasing the ground slope up to  $10^\circ$  or until instability occurs. The program's execution is terminated when the ram length reaches 8 m and instability occurs for the given ground slope.

Appendix C, "Computer Program and Flow Chart", describes the program subroutines and how they operate in a flow diagram and the program coding.

Appendix D, "Program User Guide" gives the necessary information to enable the user to run the program, use the results and change positional, loading and flexibility/stiffness matrices variables.



BC - underformed hinge bar

DE - deformed hinge bar

Fig. 3.1 Coordinate system and angular movement of hinge bar



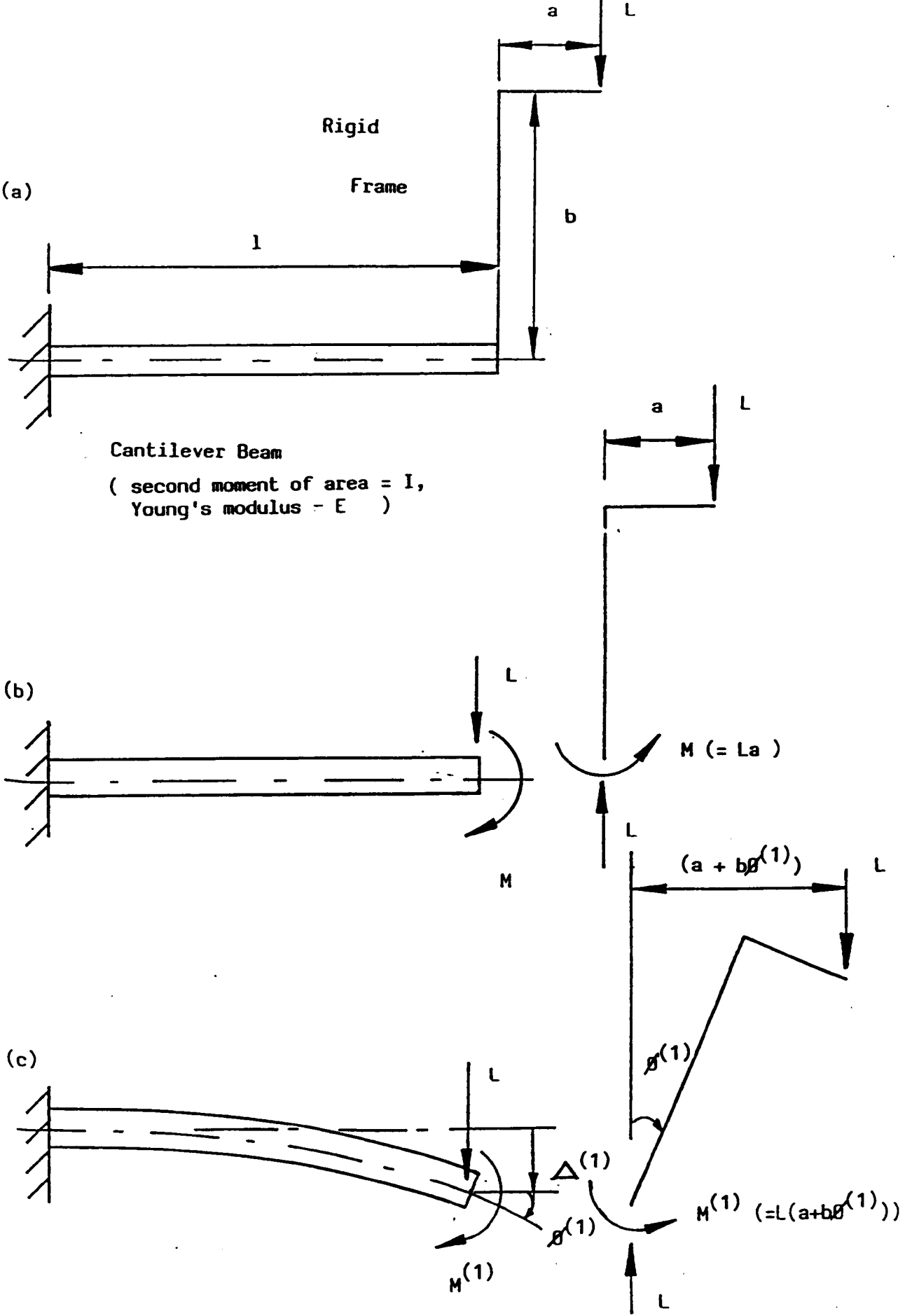


Fig. 3.2 Simple, cantilever example, to illustrate the interaction between small, elastic deformation and load position

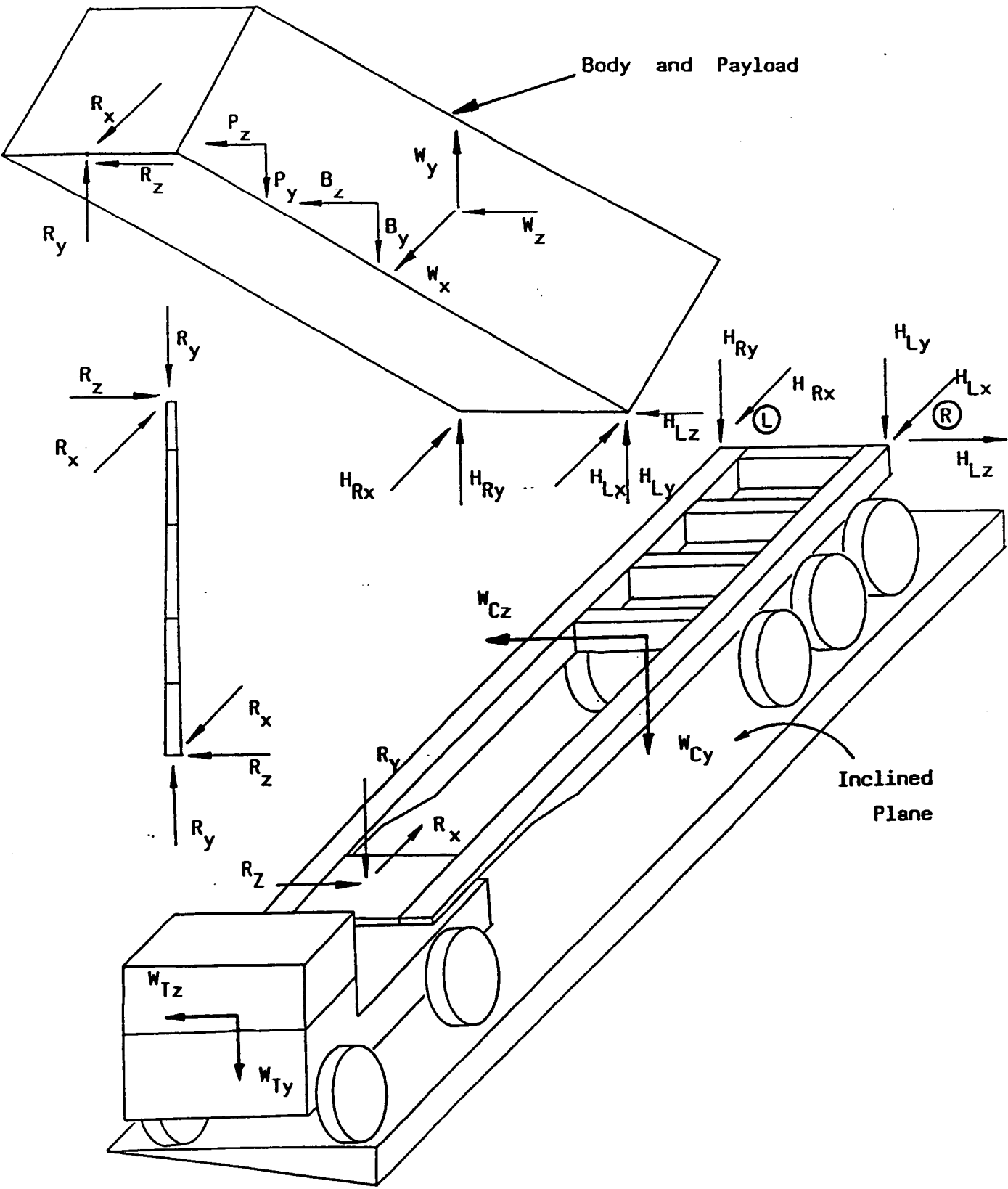
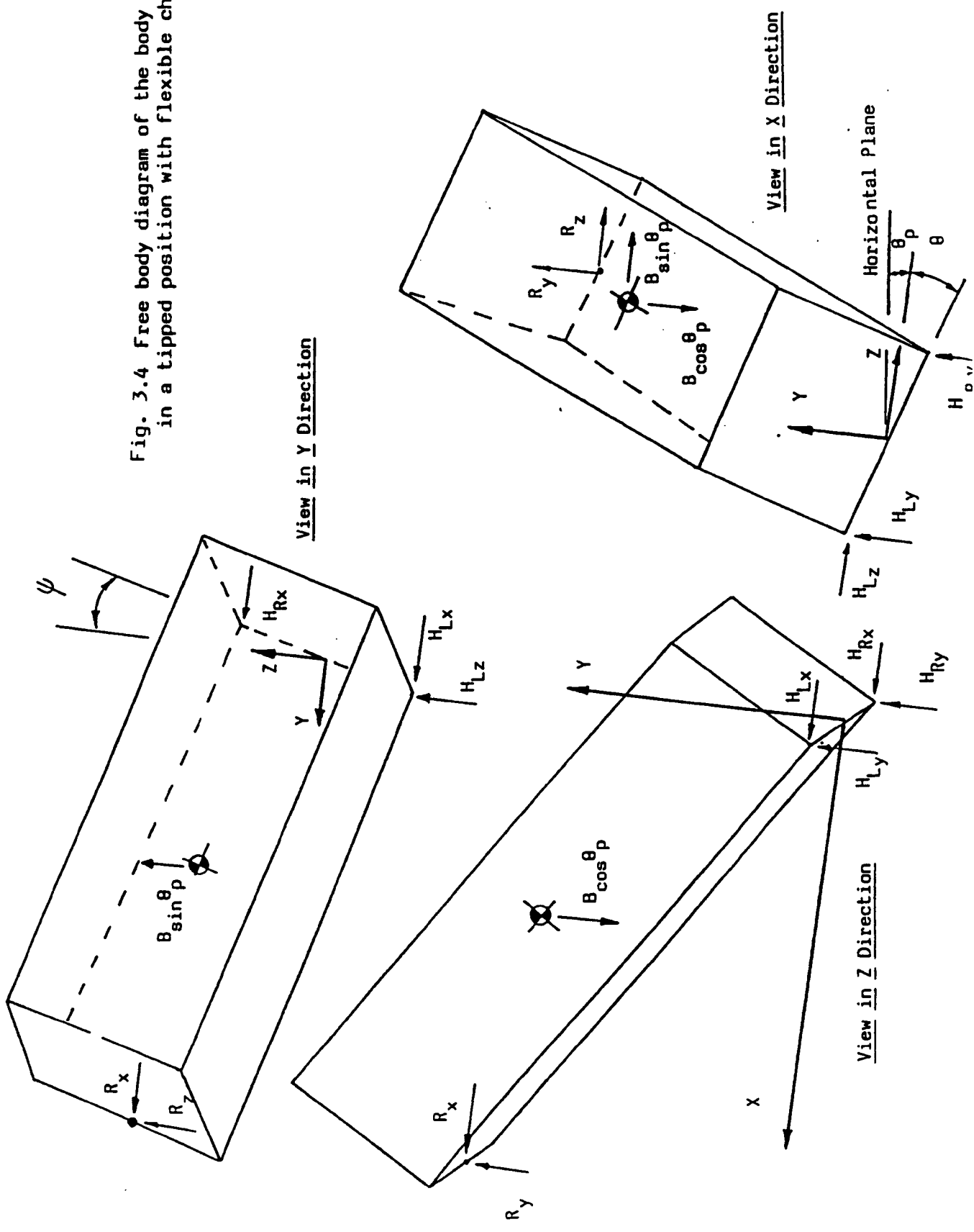


Fig. 3.3 Free body diagrams of

- (i) the body and payload in a displaced position,
- (ii) the ram in a displaced position, and
- (ii) the tractor unit, trailer chassis, suspension and tyres combination in an undeformed position.

Fig. 3.4 Free body diagram of the body and payload in a tipped position with flexible chassis



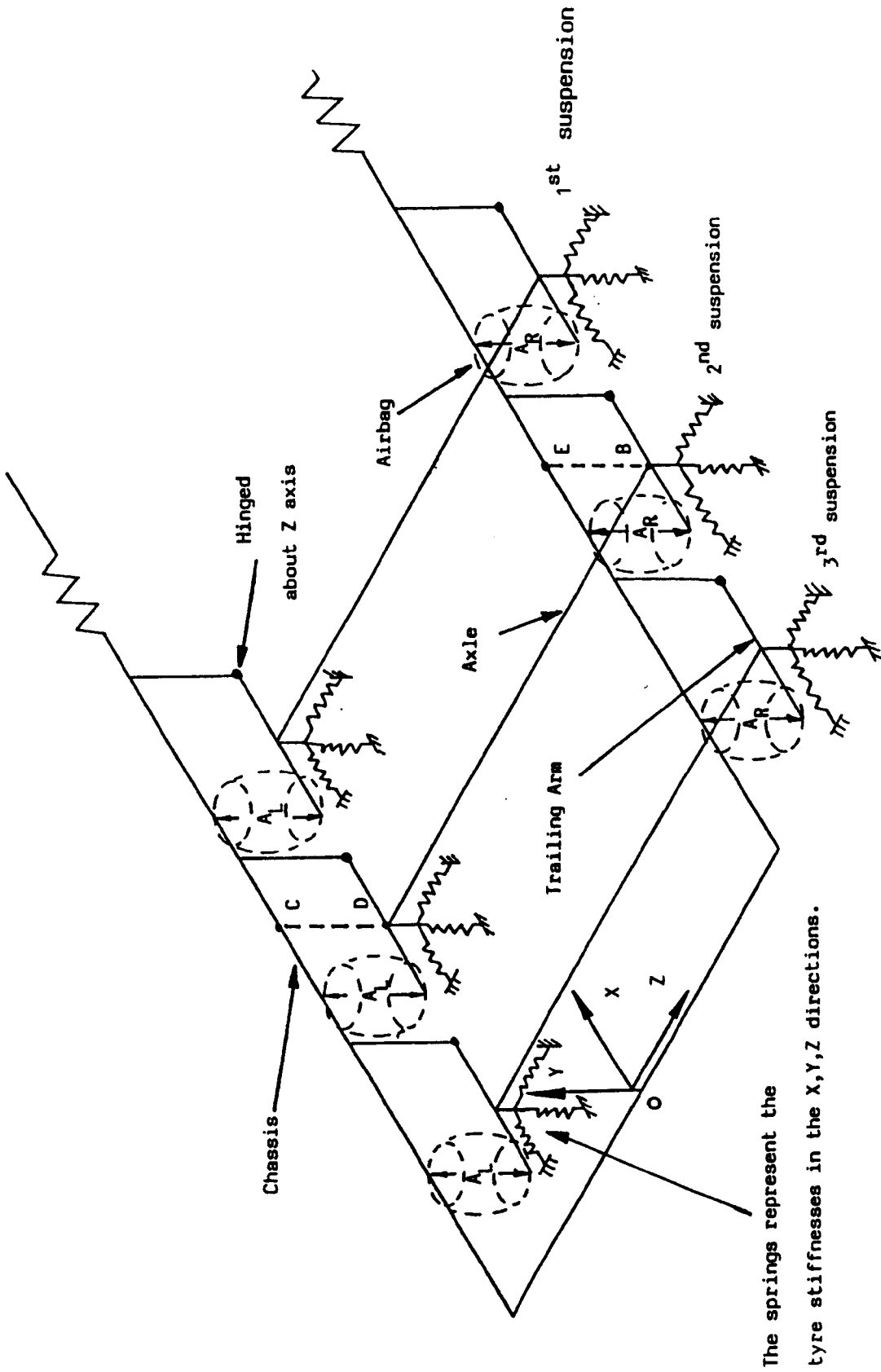


Fig. 3.5 Schematic diagram representing the rear part of the chassis, the suspension and tyres

The forces  $A_L$  and  $A_R$  represent the forces applied by the left and right airbags. EB and CD are the height control positions.

Fig. 3.6 Force and moment equilibrium equations expanded into matrix form

$$\begin{bmatrix}
 -W_x \\
 -W_z(w_x) \\
 +W_z(w_z) \\
 -B_x(b_x) \\
 -P_z(p_x)
 \end{bmatrix}
 =
 \begin{bmatrix}
 H_{Lx} & H_{Rx} & H_{Ly} & H_{Ry} & H_{Lz} & R
 \end{bmatrix}
 \times
 \begin{bmatrix}
 nx & -nx(r_{tz}) + nz(r_{tz}) & -ny(r_{tz}) + nz(r_{ty}) & ny & nz & -nx(r_{ty}) + ny(r_{tx}) \\
 0 & h_{lx} & h_{ly} & 0 & 1 & 0 \\
 0 & 0 & -h_{rz} & 1 & 0 & h_{rx} \\
 0 & 0 & -h_{lz} & 1 & 0 & h_{lx} \\
 1 & -h_{rz} & 0 & 0 & 0 & -h_{ry} \\
 -h_{lz} & 0 & 0 & 0 & 0 & -h_{ly}
 \end{bmatrix}$$

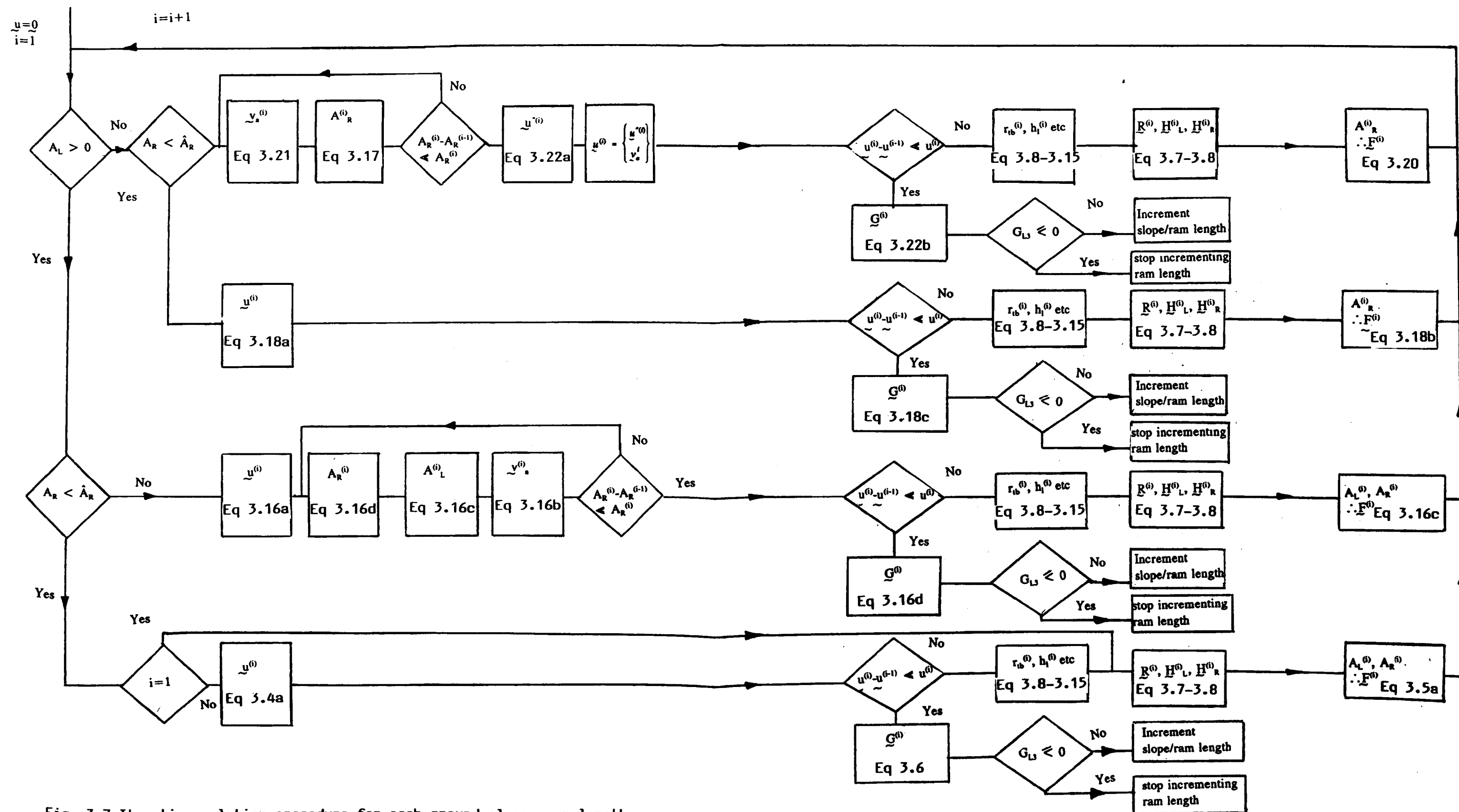


Fig. 3.7 Iterative solution procedure for each ground slope, ram length and payload.

# CHAPTER FOUR

## FINITE ELEMENT ANALYSES

### 4.1 Introduction

The previous theoretical roll-over models were based on lumped mass systems, representing various parts of the lorry inter-connected by compliant elements. Certain flexibilities such as the tyres, suspension units etc. could be obtained from the respective component manufacturers but the tractor and trailer chassis flexibilities are unknown. To overcome this problem the flexibilities of these components were obtained from full scale tests. This is a very expensive undertaking, providing a limited means in which to assess those elements of trailer design which are important in improving stability, without further recourse to more tilt tests.

It was decided that the finite element method should be used to model the tractor and trailer, in order to determine the important deformations. Once the finite element model is created it is relatively straight forward to make changes to the structure. Hence an assessment of component contribution to roll stability can be undertaken relatively inexpensively.

#### 4.1.1 Flexible elements

Finite element analysis allows a structure to be simulated by dividing it into a number of elements which model its flexibility. This enables a relationship to be established between the structures applied loads and its displacement.

The tractor's components which are considered to be flexible are the chassis, suspension and tyres. The driver's cab is not considered as it is connected to the chassis by very flexible springs.

The trailer's components which are considered to be flexible are the chassis, suspension, tyres and axial tubes. The trailer's body is considered to be rigid as its flexibility is thought to have a second order effect on roll over stability. The fifth wheel model enables rotation between the tractor and trailer about the vertical (Y axis) and transverse (Z axis) axes only.

The finite element line diagram shown in Figs. 4.1 and 4.2 represent the Tractor and Trailer structure respectively, with node numbers denoted by a prefix "N" and the remaining numbers referring to element numbers. The element numbers can be used in conjunction with Table 4.1 to obtain the element property number. This can then be used in conjunction with Table 4.2 to obtain the element's mechanical properties (ie. second moment of area, stiffness, cross sectional area etc.) depending on the type of element.

#### 4.1.2 Suspension operating modes

The suspension operating modes discussed in Section 3.4.2 affect the loading and boundary/loading conditions for the finite element model. For the three suspension operating modes (a, b, c) where at least one side is height controlled, the displacement and forces are derived from unit load flexibility and influence matrices.



As the air suspensions are represented by only forces, the finite element model of the suspension forms a mechanism. To overcome the problem of loading a mechanism, tie bars are inserted in the finite element model to maintain the required ride height. A tie bar is only inserted in the left or right suspension if it is height controlled.

For the remaining suspension operating mode (d) where neither side is height controlled, a tie bar cannot be inserted in the finite element model to overcome the problem of loading a mechanism. It is therefore not possible for the displacement and forces of such a structure to be obtained from unit load, flexibility matrices. Instead a unit displacement stiffness matrix was generated to determine the chassis displacements and suspension forces. A module within the PAFEC finite element package allows prescribed displacements to be imposed on the structure. The nodes of interest, where forces or displacements are required, are restrained in the appropriate directions.

#### **4.2 Finite element meshes**

The finite element mesh is constructed from six different elements and modules. These are used to model component flexibility, fifth wheel and suspension hinges and tyre/ground contacts.

#### 4.2.1 Elements and modules

- (i) **Simple Beam Element:** A straight simple beam element with a node at each end of the shear centre. The element is connected to the remainder of the structure by these nodes or by offset nodes. The element formulation includes bending in two directions, axial force and twisting. The main application of this element is for frame structures and stiffened plates.
- (ii) **Eight Noded Facet Shell Element:** A flat thin shell element which can carry bending and membrane loads. The main applications are for shell problems in which either or both in-plane and out-of-plane effects are important.
- (iii) **Spring Element:** A spring element which connects any pair of nodes with translatory stiffness in the  $x, y, z$  directions and with rotational stiffness about the  $X, Y, Z$  axes. The element is used whenever a discrete flexibility or stiffness needs to be introduced.
- (iv) **Mass Element:** The mass element is used to model concentrated masses with a stiffness greater than those of the surrounding structure.
- (v) **Repeated Freedoms:** The repeated freedom module is used to join two nodes which are to have one or more identical freedoms. The module can be used to provide relative movement between coincident nodes in order to model a hinge or slide connection.

- (vi) **Restraints:** The restraints module is used to restrain the freedom of a node. The node can have all or just one of its six freedoms restrained.
  
- (vii) **Gravity:** The gravity module is used to simulate the self weight of a structure by applying an acceleration of  $9.81 \text{ m/s}^2$ , or whatever the user defines.

#### 4.2.2 Tractor chassis

The tractor unit described in Section 2.1, models the flexibilities of the chassis, suspension, tyres and fifth wheel. The finite element line diagram of the tractor unit, shown in Fig. 4.1 represents these details. The element numbers are given and the corresponding property values (depending on which type of element is being considered) can be obtained from Table 4.1 and 4.2.

The tractor chassis is divided into a series of beam elements to represent the ladder structure, with dimensions being obtained from detail drawings and the measurement of an actual vehicle. The chassis beams are channel sections welded together forming a ladder type structure, with the intersection of cross members and longitudinal members being treated as rigid joints. Beam elements 175 to 200 describe the chassis.

The leaf spring suspensions are modelled by translation springs, with their stiffnesses being obtained from manufacturers information. The front and rear suspension

springs are connected between the chassis beams and the axle beam and are described by elements 213, 214 and 215, 216 respectively. In order to model the leaf springs the ends of each spring are made to have the same  $U_x$  and  $U_z$  displacement, with the  $U_y$  displacement being independent of each other, hence achieving a flexible connection in the y direction only.

The tractor has front and rear axles connected to the chassis by leaf springs and roll bars with single steering wheels attached to the front axle and dual driving wheels attached to the rear axle. The front and rear roll bars are described by elements 227 to 238, the front and rear axles are described by elements 219 to 226 and the tyres are described by elements 210 to 121. The tyre model is described in Section 4.2.5.

The tractor's mass is modelled by applying unit loads to the centre of gravity in the Y and Z directions. From gross vehicle axle weights the position of the centre of gravity could be determined in the fore/aft direction but the vertical position was obtained from an estimation made by the manufacturer.

#### 4.2.3 Fifth wheel connection

The fifth wheel coupling described in Section 2.1 is modelled using the repeated freedoms module between nodes on the tractor unit and trailer unit. This allows rotation between the tractor and trailer unit about the Y and Z axes. The play between components which allows limited rotation about the X axis between the tractor and trailer is not modelled. Three nodes (292, 293, 294) shown in Fig. 4.2

on the tractor and three corresponding nodes (43, 40, 47) shown in Fig. 4.2 on the trailer were constrained to have certain displacements the same. The nodes 292, 43 and 294, 47 were constrained to have the same  $U_y$  displacements, while the nodes 293, 40 were constrained to have the same  $U_x$  and  $U_z$  displacements.

#### 4.2.4 Trailer chassis

The trailer unit described in Section 2.1 models the flexibilities of the chassis suspension, tyres and king pin. The finite element line diagram shown in Fig. 4.2 represents these details. The element numbers are given and the corresponding property values (depending on which type of element is being considered) can be obtained from Table 4.1 and 4.2.

The chassis is divided into a series of beam elements with appropriate second moments of area and plate elements to represent the ladder structure, with dimensions being obtained from detail drawings. As the chassis beams vary in cross-section along their length, the average value of the second moment of area at the ends of each beam are used. The intersection of cross members and longitudinal members have been treated as rigid joints. The fifth wheel plate is represented by plate elements interconnected with beam elements. Plate elements 1 to 20 and beam elements 21 to 127 describe the chassis, see Fig. 4.2.

#### 4.2.5 Trailer suspension

Each suspension unit consists of two airbags. The middle suspension system is described by nodes 180-211, 199-220, an axle tube described by beam elements 179, 252, 253, 173 two suspension supports described by beam elements 133, 148, two trailing arms described by beam elements 134, 135, 149, 150 and two tyres described by spring elements 137, 163, 152, 166 as shown in Fig. 4.2. The trailing arms on either side pivot about nodes 226 and 229 as the airbag height changes. In order to model the pivot the suspension support beam 133 and trailing arm beam 134 and 135 connection is made to have the same  $U_x$ ,  $U_y$ ,  $U_z$  displacements rotation. The  $z$  rotation is made independent of each other, hence achieving a pivot connection. This arrangement is repeated for the opposite side.

The tyres on the tractor and trailer are modelled using a vertical spring and two horizontal springs connected to the vertical spring at a distance of a tyre radius below the tyre centre. The base of the vertical spring is restrained in the vertical direction, with the fore/aft and lateral freedoms being restrained by the two horizontal springs. The other ends of these springs are completely restrained as shown in Fig. 4.3. The tyres on the trailer are described by the elements 132, 137, 142, 147, 152 and 157 as shown in Fig. 4.2. The tyres on the tractor are described by 201, 203, 205, 207, 209, and 211 as shown in Fig. 4.1.

### 4.3 Design investigations

Individual lorry components can be altered in the finite element model allowing an assessment of their contribution to roll stability to be made. Following discussions with the trailer manufacturer, three areas of chassis design were changed individually to assess their effect on roll stability.

- (i) A 3 mm plate was attached between the left and right longitudinal chassis I beams, being located along their neutral axes.. The plate is attached from the hinge bar, nodes 186 and 205, to the third torsion box cross member, nodes 174 and 193 as shown in Fig. 4.1. It is thought that this would stiffen the rear of the chassis when subjected to fore/aft loading.
- (ii) The three large torsion box cross members were doubled in size, from 100 x 200 x 5 mm to 200 x 200 x 5 mm. These elements are attached between nodes 182 to 201, 178 to 197 and 174 and 193 as shown in Fig. 4.2. It is thought that this would increase the torsional stiffness of the chassis as a whole.
- (iii) The six trailing arm leaf springs were reduced in size to double their bending flexibility. The leaf springs are connected between nodes 214 to 212, (211 to 209), 208 to 206, 223 to 221, (220 to 218) and 217 to 215, as shown in Fig. 4.3.

#### **4.4 Loading of F.E. meshes**

The loads applied to the vehicle, described in Section 2.2, are used to obtain the displacements from the finite element analyses. The iterative procedure adopted in Chapter 3 to analyse this type of non-linear, elastic system, necessitates the need for a general procedure for determining the displacement. There are two solutions required to determine the F.E. displacement, depending upon the suspension operating conditions.

For the three suspension operating conditions a, b or c, described in Section 3.4.2, in which at least one suspension side is height controlled, unit loads (one newton) are individually applied to the F.E. model, at nodes where forces are applied on the real structure. The forces are always applied to the F.E. model in the positive X, Y, Z directions (except for the suspension forces), representing the body/chassis contact forces at nodes 186, 205, 314, the tractor self weight at node 313, the chassis self weight using the gravity module, the left suspension forces at nodes 176, 208, 180, 211, 184, 214 and the right suspension forces at nodes 195, 217, 199, 220, 203, 233. The body/chassis contact forces are modelled by applying individual unit forces to the right hinge in the X, Y, Z direction, to the right hinge in the X, Y directions and to the ram/chassis contact point in the X, Y, Z direction. The tractor unit self weight in the vertical and lateral directions is modelled by applying individual unit forces in the Y, Z direction. The unsprung and sprung mass of the actual trailer and the finite element model are different as only the masses of the flexible components are taken into account. Therefore, the trailers actual sprung and unsprung mass is modelled by



locating an additional mass at the trailers centre of gravity and at the ends of each axle and applying an individual unit acceleration in the Y and Z direction using the gravity module of the finite element package. The suspension forces are modelled by applying three pairs of forces to each side, with each pairs of forces representing the expansion forces of each airbag. The six forces on one side are applied simultaneously and in the direction of the actual forces. The forces are applied simultaneously as the suspension forces on each side are all equal.

For the remaining suspension operating conditions, (d) in which neither side is height controlled, unit displacements (one mm) are individually applied to the F.E. model at nodes where forces are applied to the structure. The displacements are always applied to the F.E. model in the positive X, Y, Z directions, (except for the suspension displacements), to the body/chassis contact points at nodes 186, 205, 314, the tractor self weight at node 313, the chassis sprung weight at node 315, the chassis unsprung weight at nodes 316, 317, 318 and the right suspension forces at nodes 195, 217, 199, 220, 203, 223.

The body/chassis contact point forces are modelled by applying individual unit displacements to the left hinge in the X, Y, Z directions to the right hinge in the X, Y direction and to the ram/chassis contact point in the X, Y, Z directions. The tractor self weight is modelled by applying individual unit displacements to the centre of gravity in the Y, Z directions. The chassis unsprung weight is modelled by applying individual unit displacements to the middle of each axle in the Y, Z directions. The right suspension forces are modelled by applying individual unit

displacements in the Y direction representing the expansion forces of each airbag. Although the displacements are applied individually it is still assumed that the right suspension forces are all equal.

To obtain the tyre forces in the Y direction the restraint in the Y direction at the tyre/ground contact node is released and unit displacements are individually applied in the y direction to the nodes 231 to 233.

#### **4.5 Flexibility, influence and stiffness matrices**

For the three suspension operating conditions, (a, b, c described in Section 3.4.2) in which at least one suspension side is height controlled, the unit load finite element results enable flexibility and influence matrices to be generated. From these matrices, chassis displacements, suspension displacements, suspension forces and tyre forces can be determined for any given loading condition. For each flexibility or influence matrix described there are three types which are directly related to the suspension operating condition. The matrices will be discussed in general with the specific matrices given at the ends of this section for the original chassis design.

For the remaining suspension operating condition, (described in Section 3.4.2), in which neither suspension is height controlled, unit displacement finite element results enable a stiffness matrix to be generated. From this single matrix chassis displacement, suspension displacement, suspension forces and tyre forces can be determined for any given loading condition. The matrix will be discussed in general with the specific matrix given at the end of this section for the original chassis design.

#### **4.5.1 Chassis displacement flexibility matrix**

To determine the chassis displacements at the body/chassis contact points for any given loading condition, a unit load flexibility matrix was generated using the results from the finite element model. At the nodes of interest, the displacement at these points, resulting from the individually applied unit loads to the finite element model, form the coefficients of the unit load flexibility matrix. These results are used to form the unit load flexibility matrix which gives a relationship between the body/chassis contact forces, the tractor self weight, the chassis self weight, the suspension forces and the chassis displacements at the body/chassis contact point. The unit load flexibility matrices relating to the suspension operating conditions a, b, and c are given in Tables 4.3, 4.4 and 4.5 respectively.

#### **4.5.2 Suspension displacement flexibility matrix**

To calculate the compression of the left airbags, the displacement of the two ends of each airbag are required. To determine these displacements for any given loading condition a unit load flexibility matrix was generated using the results from the finite element model in the same manner as the unit load chassis, displacement flexibility matrix. These results are used to form the unit load flexibility matrix, which gives a relationship between the body/chassis contact forces, the tractor self weight, the chassis self weight, the suspension forces and the suspension displacements at the ends of each airbag. The suspension displacements form the coefficients of the unit load suspension displacement flexibility matrix. The unit load flexibility matrices

relating to the suspension operating conditions a, b, c are given in Tables 4.6, 4.7 and 4.8 respectively.

#### 4.5.3 Suspension force influence matrix

To determine the left and right suspension forces for any given loading condition, a unit load influence matrix was generated using the results from the finite element model. The tie bar forces resulting from the individually applied unit loads to the finite element model form the coefficients of the unit load influence matrix. From these results the unit load influence matrix can be generated which gives a relationship between the body/chassis contact forces, unit tractor self weight, the chassis self weight and the suspension forces. The unit load influence matrices relating to the suspension operating conditions a, b and c are given in Tables 4.9, 4.10 and 4.11 respectively.

#### 4.5.4 Tyre force influence matrix

To determine the vertical tyre forces in the trailer wheels for any given loading condition, a unit load influence matrix was generated using the results from the finite element model. The forces in the tyre springs resulting from the individually applied unit loads to the finite element model form the coefficients of the unit load influence matrix. These results are used to form the unit load influence matrix which gives a relationship between the body/chassis contact forces the tractor self weight, the chassis self weight, the suspension forces and the vertical tyre forces. The unit load

influence matrices relating to the suspension operating conditions a, b and c are given in Tables 4.12, 4.13 and 4.14 respectively.

#### 4.5.5 Stiffness matrix

For the remaining suspension operating condition, d, described in Section 3.4.2, in which neither suspension is height controlled, unit displacements are applied to the finite element model instead of unit forces, with the nodes of interest being restrained in the desired directions. The reaction forces resulting from the individually applied unit displacements to the finite element model form the coefficient of the unit displacement stiffness matrix. The finite element results enable a unit displacement stiffness matrix, shown in Table 4.15, to be generated from which the chassis displacements, the suspension displacements, the suspension forces and the tyre forces can be determined for any given loading condition. The method of calculating the unknown displacement and forces is fully described in Section 3.4.3.

#### 4.6 Discussion

Two different methods have been devised for determining chassis displacements, suspension displacements, suspension forces and tyre forces using the finite element method. Depending upon the suspension operating condition, unit forces are used to generate the flexibility and influence matrices where at least one suspension is height controlled, with unit displacements being used to generate the stiffness matrix where neither suspension is height controlled.

The validity of each method for determining displacements and forces is confirmed by cross checking the forces and displacements obtained at the point at which the analysis changes the suspension operating condition. The four different methods can all be cross checked in a similar manner to confirm their validity, as the analysis changes from one suspension operating condition to another.

The continuity of the graphs shown in Figs. 5.1 to 5.4, show that good agreement has been obtained between the forces and displacements determined by either suspension operating condition at the point of cross over.

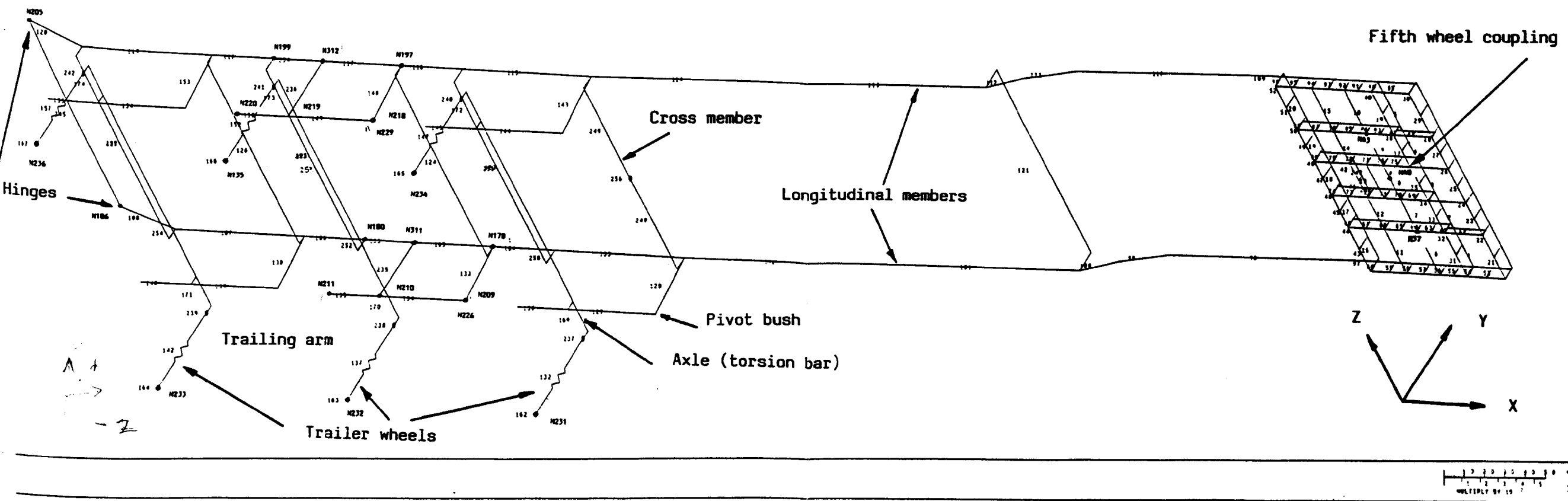
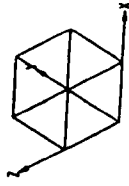


Fig. 4.1 Trailer finite element line diagram

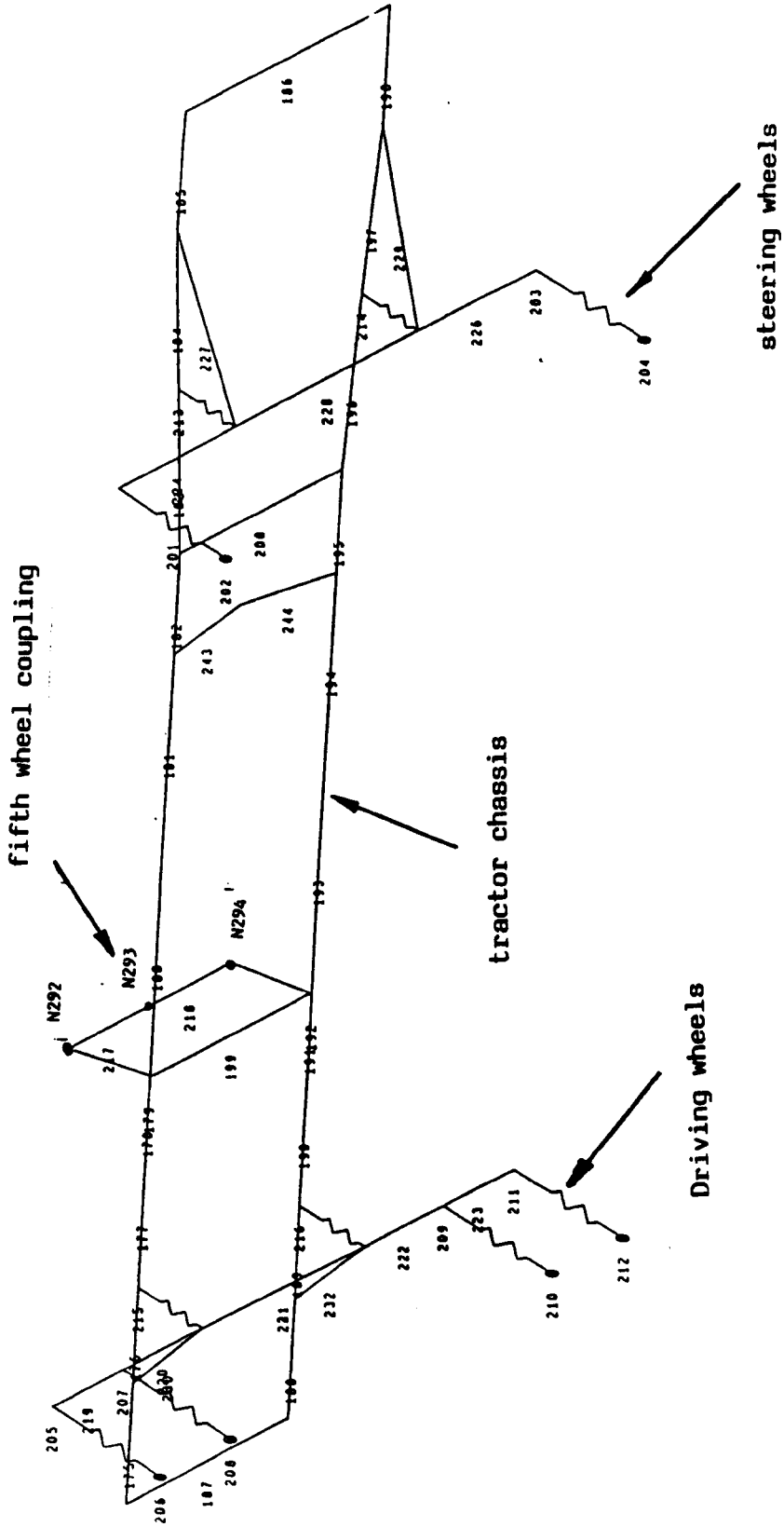
PAFEC

VIEW FROM X = 1000  
Y = 1000  
Z = 1000



ELEMENT GROUPS = 3

DRAWING NO. 1  
SCALE = 0.5250 E 4  
DRAWING TYPE = 3



MULTIPLY BY 10<sup>3</sup>

0 2 4 6 8 10 12 14 16 18 20 MM.  
STRUCTURAL  
UNITS

Fig. 4.2 Tractor finite element line diagram



**Table 4.1a - Element Property Numbers**

<b>Element Numbers</b>	<b>Property Numbers</b>	<b>Element Numbers</b>	<b>Property Numbers</b>
1-20	28	112	13
21-30	7	113	14
31-40	2	114	15
41-42	3	115	16
43-52	6	116	17
53-60	1	117	18
61-62	3	118	19
63-68	4	119	20
69-80	5	120	21
81-82	3	121	10
83-88	4	123	9
89-97	1	124	8
98	11	125	9
99	12	126	8
100	13	127	9
101	14	128	22
102	15	129	24
103	16	130	31
104	17	132	26
105	18	133	22
106	19	134	24
107	20	135	31
108	21	137	26
109	1	138	22
110	11	139	24
111	12	140	31

**Table 4.1b - Element Property Numbers**

<b>Element Numbers</b>	<b>Property Numbers</b>	<b>Element Numbers</b>	<b>Property Numbers</b>
142	26	210	34
143	22	211	35
144	24	212	34
145	31	213, 214	33
147	26	215, 216	32
148	22	217, 218	36
149	24	219 - 226	37
150	31	227 - 232	38
152	26	233, 234	18
153	22	235, 236	40
154	24	237 - 242	41
155	31	243, 244	18
157	26	245	42
162 - 167	29	246	17
169 - 174	23	247	17
175 - 200	36	248, 249	8
210	35	250 - 255	23
202	34	256	43
203	35		
204	34		
205	35		
206	34		
207	35		
208	34		
209	35		

**Table 4.2a - Property Number Values**

Beam Property Number	$I_{yy} \text{ (mm)}^4 \times 10^6$	$I_{zz} \text{ (mm)}^4 \times 10^6$	$K \text{ (mm)}^4 \times 10^3$	Area (mm) <sup>2</sup>
1	3.663	13.667	92.5	3250
2	1.464	9.193	93.333	2800
3	2.46	0.393	60.0	1800
4	0.571	5.343	77.666	6570
5	0.189	6.351	755.208	3625
6	0.813	2.686	10.208	1225
7	0.388	7.801	43.52	2040
8	15.224	5.124	11834	2900
9	0.362	1.298	8125	1000
10	2.569	2.569	5138	1717
11	3.664	25.025	0.0942	3458
12	3.665	69.249	0.0988	4008
13	3.665	105.286	0.1019	4375
14	3.664	125.099	0.0952	4162
15	3.664	170.164	0.0965	4406
16	3.664	186.195	0.09695	4488
17	3.664	174.025	0.0966	4430
18	3.664	162.496	0.0964	4372
19	3.664	151.253	0.0961	4314
20	3.664	140.986	0.0957	4258
21	3.664	140.986	0.0957	4258
22	8.919	17.756	0.0241	2875
23	80.55	80.55	16109	49700
24	2.084	0.777	1926	3908
31	1.042	0.389	963	1954
36	1.585	27.551	30.744	2598
37	100000	10000	100	1000
38	0.134	0.134	267.803	1297.2
39	0.397	0.397	794.716	2234.6
40	0	0	0	900
42	1.113	1.113	2226.26	1995

Cross  
members

longitudinal  
beams

**Table 4.2b - Property Number Values**

<b>Spring Property Number</b>	<b>Spring Stiffness</b>	<b>Spring Stiffness</b>	<b>Spring Stiffness</b>
	<b>K<sub>x</sub> (N/mm)</b>	<b>K<sub>y</sub> (N/mm)</b>	<b>K<sub>z</sub> (N/mm)</b>
26	-	1070	-
29	749	-	412
32	-	872	-
33	-	403	-
34	755	-	350
35	-	981	-
<b>Mass property</b>	<b>Mass (kg)</b>		
41	17.666		
43	1220		
<b>Plate Property Number</b>	<b>Plate Thickness (mm)</b>		
28	8		

**Table 4.3 - Displacement flexibility matrix, original chassis, case 1**

Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N186-X	0.1562	0.0919	0.0839	0.0505	-0.1082	0.1163	-0.0135	0.0408	-0.0122	0.0286	0.028	-0.0043	0.0926	0.0693
N186-Y	0.0839	0.0505	1.3528	0.7072	0.1345	0.0442	-0.799	0.0088	-0.0777	0.0015	0.2506	0.0145	2.2495	1.7450
N186-Z	-0.1082	0.1082	0.1345	-0.1345	1.2801	0	0	-0.0836	0	-0.0630	0	0.3613	0.1051	-0.1050
N250-X	0.0919	0.1562	0.0505	0.0839	0.1802	0.1163	-0.0135	-0.0408	-0.0122	-0.0286	0	0.0043	0.0693	0.0296
N250-Y	0.0505	0.0839	0.7072	1.3528	-0.1345	0.0442	-0.799	0.0088	-0.0777	-0.0012	0.2056	-0.0145	1.7450	2.2495
N250-Z	-0.1082	0.1082	0.1345	-0.1345	1.2785	0	0	-0.836	0	-0.0630	0	0.3613	1.051	-0.1050
N314-X	0.1163	0.1163	0.0442	0.0442	0	0.1185	-0.0126	0.0150	-0.0112	0	0.0043	0	0.0667	0.0667
N314-Y	-0.0135	0.0135	0.0800	-0.0800	0	-0.0126	0.2826	0.004	0.2639	0	0.0579	0	0.0316	0.0316
N314-Z	0.0408	-0.0408	0.0088	-0.0088	-0.0836	0	0	0.3203	0	0.4085	0	0.1219	0.069	-0.0070

**Table 4.4 - Displacement flexibility matrix, original chassis, case 2**

Unit forces / Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>
N186-X	0.1567	0.0922	0.1011	0.0511	-0.1046	0.1166	-0.0133	0.0411	-0.0122	0.0287	0.056	-0.0040	0.0954
N186-Y	0.1011	0.0615	1.965	0.8703	0.2617	0.0572	-0.0736	0.0167	-0.0763	0.0043	0.3481	0.0263	1.6727
N186-Z	-0.1047	0.1105	0.2617	-0.1006	1.3065	0.0027	0.0013	-0.0820	0.0003	-0.0624	0.0203	0.3638	0.0877
N250-X	0.0922	0.1564	0.0615	0.0868	0.1105	0.1165	-0.0134	-0.0406	-0.0122	-0.0285	0.0046	0.0045	0.1092
N250-Y	0.0551	0.0868	0.8703	1.3963	-0.1006	0.0477	-0.0782	-0.0067	-0.0773	-0.0003	0.2766	-0.0113	2.4966
N250-Z	-0.1047	0.1105	0.2617	-0.1006	1.3049	0.0027	0.0013	-0.0820	0.0003	-0.0624	0.0203	0.3638	0.0877
N314-X	0.1167	0.1165	0.0572	0.0477	0.0027	0.1187	-0.0124	0.0002	-0.0112	0.0007	0.0064	0.0003	0.0863
N314-Y	-0.0133	-0.0134	-0.0736	-0.0783	0.0013	-0.0124	0.2827	0.0001	-0.2640	0	0.0589	0.0001	0.0413
N314-Z	0.0411	-0.0407	0.0167	-0.0067	-0.0820	0.002	0.0001	0.6212	0	0.4086	0.0013	0.1220	0.050

**Table 4.5 - Displacement flexibility matrix, original chassis, case 4**

Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N186-X	0.1564	0.0922	0.0868	0.0615	-0.1105	0.1165	-0.0134	0.0407	-0.0122	0.0285	0.0045	-0.0045	0.1092	0.1035
N186-Y	0.0868	0.0551	1.3963	0.8703	0.1007	0.0477	-0.0782	0.0067	-0.0773	0.0003	0.2766	0.0113	2.4966	2.2539
N186-Z	-0.1105	0.1047	0.1007	0.2617	1.3065	-0.0027	-0.0013	-0.0820	-0.0003	-0.0624	-0.0203	0.3638	-0.0876	-0.5017
N250-X	0.0922	0.1567	0.0551	0.1012	0.1046	0.1166	-0.0133	-0.0410	-0.0122	-0.0286	0.0055	0.0040	0.0954	0.1464
N250-Y	0.0615	0.1012	0.8703	1.9650	-0.2616	0.0572	-0.0736	-0.0167	-0.0763	-0.0043	0.2481	-0.0263	2.6727	4.1598
N250-Z	-0.1105	0.1047	0.1007	-0.2617	1.3049	-0.0027	-0.0013	-0.0820	-0.003	-0.0624	-0.0202	0.3638	-0.0876	-0.5018
N314-X	0.1165	0.1167	0.0477	0.0572	-0.0027	0.1187	-0.0124	-0.0002	-0.0001	-0.0001	0.0064	-0.0002	0.0863	0.1071
N314-Y	-0.0133	-0.0133	-0.0783	-0.0736	-0.0013	-0.0214	0.2827	-0.0001	-0.2640	0	0.0589	-0.0001	0.0413	0.0515
N314-Z	0.0407	-0.0411	0.0067	-0.0167	-0.0820	-0.0002	-0.0001	0.06212	0	0.4086	-0.0013	0.1220	-0.0050	-0.0315

**Table 4.6 - Suspension displacement flexibility matrix, original chassis, case 1**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N195-Y	0.0148	-0.0023	0.3534	0.5399	-0.1094	0.0012	0.0511	-0.0065	0.00386	-0.0075	0.2202	-0.0176	1.1810	1.3474
N217-Y	0.0103	-0.0041	-0.2113	-0.5139	-0.0614	-0.0010	-0.0528	-0.0042	-0.0430	-0.0064	0.0013	-0.0127	-1.7704	-2.1217
N199-Y	0.0287	0.0279	0.4941	0.8413	-0.01186	0.0238	-0.0047	-0.0072	-0.0109	-0.0043	0.2329	-0.0165	1.4093	1.6999
N220-Y	0.0304	0.0248	0.5108	0.8104	-0.1158	0.00236	-0.0047	-0.0069	-0.0109	-0.0049	0.2322	-0.0168	1.3457	1.4240
N203-Y	0.0447	0.0679	0.6508	1.2129	-0.1301	0.0388	-0.0606	-0.0084	-0.0603	-0.0019	0.2549	-0.0150	1.6568	2.1038
N223-Y	-0.0219	-0.0199	-0.5461	-0.3118	-0.0760	-0.0165	0.0119	-0.0040	0.0145	-0.0018	-0.0148	-0.0117	-2.1747	-2.3856



**Table 4.7 - Suspension displacement flexibility matrix, original chassis, case 2**

Unit forces / Unit force displacements	$H_{Lx}$	$H_{Rx}$	$H_{Ly}$	$H_{Ry}$	$H_{Lz}$	$R_x$	$R_y$	$R_z$	$W_{Ty}$	$W_{Tz}$	$W_{xy}$	$W_{cz}$	$A_R$
N195-Y	0.01878	0.0002	0.4946	0.5575	-0.0801	0.0131	0.0526	-0.0047	0.0389	-0.0068	0.2426	-0.0149	1.5613
N217-Y	0.0054	-0.0072	-0.3871	-0.2008	-0.0979	-0.0047	-0.0546	-0.0064	-0.0438	-0.0073	-0.0266	-0.0161	-2.3880
N199-Y	0.0328	0.0305	0.6413	0.8805	-0.0881	0.0269	-0.0031	-0.0053	-0.0106	-0.0035	0.2563	-0.0137	1.9229
N220-Y	0.0301	0.0246	0.5002	0.8076	-0.1180	0.0234	-0.0048	-0.0071	-0.0110	-0.0030	0.2306	-0.0170	1.4081
N203-Y	0.0492	0.0708	0.8093	1.2552	-0.0971	0.0422	-0.0587	-0.0063	-0.0599	-0.0011	0.2712	-0.0120	2.3441
N223-Y	-0.0302	-0.0252	-0.8407	-0.3903	-0.1372	-0.0227	-0.0088	-0.0078	0.0139	-0.0033	-0.0617	-0.0174	-2.8321

**Table 4.8 - Suspension displacement flexibility matrix, original chassis, case 4**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>Ty</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N195-Y	0.0215	0.0082	0.04532	0.9145	-0.1873	0.0180	0.0550	-0.0114	0.0394	-0.0094	0.2800	-0.0249	1.7487	2.5165
N217-Y	0.0849	-0.0062	-0.02330	-0.2314	-0.0453	-0.0027	-0.0536	-0.0032	-0.0432	-0.0060	-0.0110	-0.0112	-1.8879	-2.3635
N199-Y	0.0372	0.0413	0.6218	1.3204	-0.2182	0.0340	0.0003	-0.0103	-0.0099	-0.0068	0.3092	-0.0258	2.1354	3.1951
N220-Y	0.0248	0.0160	0.4270	0.4961	-0.0505	0.0170	-0.0080	-0.0029	-0.0117	-0.0034	0.1822	-0.0108	0.8695	0.4434
N203-Y	0.0551	0.0842	0.8047	1.7908	-0.2501	0.0511	-0.0543	-0.0158	-0.5090	-0.0049	0.3380	-0.0262	1.5236	3.9072
N223-Y	-0.0234	-0.0233	-0.5692	-0.3987	-0.0579	-0.0183	0.0110	-0.0029	0.0143	-0.0014	-0.0287	-0.0100	-2.3064	-2.6568

**Table 4.9 - Suspension force influence matrix, original chassis, case 1**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N311-Y	-0.0286	-0.0182	-1.0618	-0.2709	-0.2113	-0.0215	-0.0106	-0.0131	-0.0022	-0.0052	-0.1619	-0.196	-3.1736	-1.5410
N312-Y	-0.0182	-0.0286	-0.2708	-1.1068	0.2112	-0.0215	-0.0106	0.0131	-0.0022	0.0052	-0.01619	0.0196	-1.5413	-3.1733

**Table 4.10 - Suspension force influence matrix, original chassis, case 2**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>
N312-Y	-0.0457	-0.0468	-1.2827	-1.2864	0.0010	-0.0429	-0.0211	0.001	-0.0044	0	-0.3232	0	4.7075

**Table 4.11 - Suspension force influence matrix, original chassis, case 4**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N311-Y	-0.0467	-0.0467	-1.2866	-1.2832	-0.0011	-0.0429	-0.0212	-0.001	-0.004	0	-0.3231	-0.0001	-4.7075	-4.6992

**Table 4.12 - Tyre force influence matrix, original chassis, case 1**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N231-Y	0.0105	-0.0102	-0.0413	0.0412	-0.088	0.0001	0	-0.0059	0.0018	-0.0091	-0.0826	-0.0181	0.911	0.669
N232-Y	-0.0192	-0.0274	-0.854	-0.434	-0.0163	-0.0214	-0.0106	-0.0098	-0.0022	-0.0071	-0.2446	-0.0238	-1.74	-1.39
N233-Y	-0.0014	0.0017	-0.168	0.168	-0.109	0.001	0	-0.0058	0	-0.0027	-0.0826	-0.0167	0.810	0.770
N234-Y	-0.0102	0.0105	0.0412	-0.0413	0.088	0.0001	0	0.0059	0	0.0091	-0.0826	0.0181	0.669	0.911
N235-Y	-0.0274	-0.0192	-0.434	-0.854	0.163	-0.0214	-0.0106	0.0098	-0.0022	0.0071	-0.2446	0.0238	-1.39	-1.74
N236-Y	0.0017	-0.0014	0.168	-0.168	0.109	0.0001	0	0.0058	0	0.0027	-0.0826	0.0167	0.770	0.810

**Table 4.13 - Tyre force influence matrix, original chassis, case 2**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>Ty</sub>	W <sub>cz</sub>	A <sub>R</sub>
N231-Y	0.085	-0.0115	-0.112	0.0224	-0.103	-0.0014	-0.0007	-0.0068	-0.0001	-0.0095	-0.0938	-0.0194	-0.5620
N232-Y	-0.0134	-0.0237	-0.647	-0.379	-0.120	-0.017	-0.0084	-0.0071	-0.017	-0.0060	-0.2116	-0.0198	-1.080
N233-Y	-0.0056	-0.0010	-0.318	-0.129	-0.140	-0.003	-0.0015	-0.0678	-0.0003	-0.0034	-0.1062	-0.0195	0.544
N234-Y	-0.0082	-0.0118	0.112	-0.0225	0.103	0.0016	0.0008	0.0068	0.0002	0.0095	-0.0713	0.0194	1.02
N235-Y-	-0.0331	-0.0228	-0.636	-0.908	-0.121	-0.0256	-0.0127	0.0072	-0.0026	0.0061	-0.2768	0.0199	-2.05
N236-Y	0.0059	0.0013	0.318	-0.129	0.140	0.0033	0.0016	0.0077	0.0003	0.0034	-0.059	0.0195	1.04

**Table 4.14 - Tyre force influence matrix, original chassis, case 4**

Unit forces Unit force displacements	H <sub>Lx</sub>	H <sub>Rx</sub>	H <sub>Ly</sub>	H <sub>Ry</sub>	H <sub>Lz</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>Ty</sub>	W <sub>Tz</sub>	W <sub>xy</sub>	W <sub>cz</sub>	A <sub>R</sub>	A <sub>L</sub>
N231-Y	0.0117	-0.0082	-0.0225	0.112	-0.103	0.0016	0.0008	-0.0068	0.0002	-0.0095	-0.7125	-0.01944	1.0	0.890
N232-Y	-0.0228	-0.0331	-0.908	-0.636	-0.121	-0.0256	-0.0127	-0.0072	-0.0026	-0.0061	-0.2768	-0.199	-2.05	-2.02
N233-Y	0.0013	0.0059	-0.129	0.318	-0.140	0.0044	0.0016	-0.0078	0.0003	-0.0034	-0.0588	-0.0195	1.04	1.24
N234-Y	-0.0115	0.0085	0.0224	-0.112	0.103	-0.0014	-0.0007	-0.0068	-0.0001	0.0095	-0.938	0.0194	0.562	0.690
N235-Y-	-0.0237	-0.0134	-0.379	-0.637	0.120	-0.0170	-0.0008	0.0071	-0.0018	0.0060	-0.2116	0.0198	-1.08	-1.10
N236-Y	-0.0010	-0.0056	0.129	-0.318	0.140	-0.0030	-0.0015	-0.0078	-0.0003	0.0034	-0.1062	0.0195	0.544	0.344

Unit displacement forces	H <sub>Lx</sub>	H <sub>Ly</sub>	H <sub>Lz</sub>	H <sub>Rx</sub>	H <sub>Ry</sub>	R <sub>x</sub>	R <sub>y</sub>	R <sub>z</sub>	W <sub>xy</sub>	W <sub>xz</sub>	W <sub>yz</sub>	W <sub>mx</sub>	W <sub>my</sub>	W <sub>mz</sub>	W <sub>ny</sub>	W <sub>nz</sub>	W <sub>tz</sub>	W <sub>ty</sub>	W <sub>tz</sub>	A <sub>xx</sub>	A <sub>yy</sub>	A <sub>zz</sub>	A <sub>xy</sub>	A <sub>xz</sub>	A <sub>yz</sub>	
Unit displacements																										
N186-X	49187	-2497	2785	10570	-2948	-57919	-1715	-839	4599	-6917	960	2205	1156	1161	929	1743	232	-52	-5088	-617	4363	-739	2245	-594		
N186-Y	-2497	1654	-721	63	-213	2575	519	37	-865	454	-151	-280	-721	-10	-1489	493	-41	18	739	97	-778	461	-113	952		
N186-Z	2785	-721	2745	-2550	836	-228	-6	7	-329	-339	-66	48	-71	-469	-39	-1992	2	-0.1	327	43	414	45	-459	25		
N205-X	10569	63	-2550	5330	-16054	-61830	-1891	914	-768	8539	-113	-3120	81	-1602	155	-2317	275	59	3176	74	1436	-52	13563	-98		
N205-Y	-2948	-213	836	-16051	258220	18491	951	-273	1271	-3219	-83	1506	-71	617	-32	494	-116	-38	-28689	52	139440	45	-37804	20		
N314-X	-57919	2575	-288	-61829	18491	124720	3673	-78	-3472	-1554	-787	869	-1194	423	-1058	565	-508	-6	2147	506	-5941	764	-15556	667		
N314-Y	-1705	516	-6	-1891	943	3674	9520	-9	-1736	-198	-234	172	-54	56	3	-20	-5626	-5	-1674	152	2309	37	-1367	-2		
N314-Z	-841	38	7	918	-279	-71	1	2535	-74	-514	-15	49	-19	66	-16	36	0.	-1005	67	10	10	12	256	10		
N315-Y	4598	-865	-329	-768	1271	-3471	-1073	-72	11796	-3193	-2624	2286	-1892	958	-990	367	195	-39	-11540	1670	4994	1210	-2218	634		
N315-Z	-6917	454	-339	8539	-3219	-1555	-167	-513	-3193	13055	-632	-8455	-529	-2947	-320	-771	29	-33	6644	406	-3773	339	3670	205		
N316-Y	960	-151	-66	-113	-83	-787	-236	-15	-2624	-632	8260	440	-376	174	-194	69	30	-13	1655	-3935	-1007	240	328	124		
N316-Z	2205	-280	48	-3121	1506	869	143	48	2286	-8455	442	9424	363	-146	184	-83	-22	28	-5414	-285	3672	-232	-2246	-117		
N317-Y	1155	-721	-71	82	-71	-1194	-53	-19	-1891	-529	-376	362	8378	88	-261	134	14	-4	1115	242	741	-4009	252	167		
N317-Z	1161	-10	-469	-1603	617	422	54	66	957	-2497	174	-146	92	5698	106	-1645	-9	5	-2066	-112	1240	-60	-919	-68		
N318-Y	929	-1489	-39	155	-32	-1057	3	-16	-990	-320	-194	184	-261	106	8545	48	6	-1	565	125	-413	167	126	-4119		
N318-Z	1743	492	-1992	-2318	494	565	-20	36	367	-771	69	-83	134	-1645	51	5312	-1	1	397	-44	-1472	-85	-353	-33		
N313-Y	232	-41	2	275	-116	-508	-5626	1	194	28	30	-22	14	-9	6	-1.2	5910	0.2	172	-19	-186	-9	149	-4		
N313-Z	-52	19	-0.1	59	-38	-6	-3	-1005	-39	-33	-13	28	-4	5	-1	1	-0.2	1547	128	8	-124	3	63	1		
N195-Y	-5088	739	327	3175	-28689	2145	-1690	66	-11540	6644	1656	-5412	1114	-2062	565	397	171	128	53949	-1051	-80065	-712	65800	-362		
N217-Y	-617	97	43	74	52	506	152	9	1670	406	-3934	-283	242	-112	125	-44	-19	8	-1051	2515	641	-155	-208	-80		
N199-Y	4363	-778	414	1437	139440	-5940	2332	11	4994	-3773	-1008	3670	-741	1239	-412	-1470	-184	-123	-80065	641	183970	474	-25804	264		
N220-Y	-738	461	45	-52	45	764	34	12	1210	339	240	-232	-4010	-56	167	-85	-9	3	-712	-155	474	2565	-161	-107		
N203-Y	2245	-113	-459	13560	-37804	-15556	-1403	250	-2218	3670	328	-2246	252	-919	126	-353	148	64	65800	-208	-25804	-161	556980	-81		
N223-Y	-594	952	25	-98	20	677	-2	10	634	205	124	-117	167	-68	-4118	-31	-4	1	-362	-980	264	-107	-81	2634		
N231-Y	93	-13	-6	-0.05	-62	-90	-4	-1	-155	-50	-1740	32	-33	13	-17	6	1	-2	761	440	-410	21	153	11		
N232-Y	77	-52	-4	11	19	-85	-6	-2	-143	-29	-27	14	-1722	-10	-19	12	1	-0.2	372	17	114	428	-40	12		
N233-Y	63	-109	1	16	-82	-78	1	-1	-71	-22	-14	14	-19	12	-1710	-11	0.4	-0.1	7	9	311	12	203	421		
N234-Y	-77	10	5	-2	63	77	-0.2	1	141	40	-303	-25	27	-10	14	-5	-0.5	2	-756	-479	405	-17	-152	-9		
N235-Y	-65	44	4	-10	-20	72	5	1	123	23	23	-20	317	11	16	-11	-1	0.2	-361	-15	-122	-471	43	-10		
N236-Y	-54	93	-1	-14	81	67	-0.8	1	61	19	12	-12	17	-10	-327	12	0.3	0.1	3	-8	-315	-11	-201	-464		

Table 4.15 - Unit displacement stiffness matrix, original chassis, case d

## **CHAPTER FIVE**

### **DESIGN INVESTIGATIONS**

#### **5.1 Introduction**

Whilst a vehicle operator should always endeavour to discharge the payload with the vehicle standing on level ground, practical situations arise where this is not possible. This may be due to the absence of level ground or poor judgement by the operator, which may result in the vehicle being tipped on a lateral ground slope. As a result of this, the maximum ground slope angle considered for the theoretical model is limited to ten degrees, as this position is at least twice the severity of ground slope on which a vehicle should normally be tipped.

For each trailer design the magnitude of the load, position of the load, ram length and ground slope can be varied in any combination. Four payload and up to nine payload positions are considered, varying the ground slope from 0 to 8 degrees and varying the ram length from 2 to 8 meters. Also, three further chassis configurations, described in Section 4.3, based on the reference chassis were modelled to investigate the contribution of important component flexibilities to stability.

From the theoretical results, three types of graphs are obtained giving details of the stability of the vehicle. The first set of graphs give details of how the rear left trailer tyre force varies with ground slope and ram length for a given payload magnitude and position. The ground slope varies between 0 to 10 degrees in steps of 1 degree for



each graph. The graphs for the four chassis configurations are shown on one page for each payload magnitude and position, to enable a direct comparison of their stability characteristics and are shown in Fig. 5.1 to Fig. 5.4. The graph also shows what condition the suspension is operating in. The data points are only marked if the suspension is operating using condition, b, c or d <sup>as discussed in section 3.4.2.</sup>. The data points are labelled only where the suspension operating condition changes (eg. a to b or b to c etc). The data points following a labelled data point are the same (ie. no change in the suspension operating condition).

The second set of graphs give details of how the rear left trailer tyre force varies with payload magnitude and horizontal and vertical payload position for a 4 degree ground slope and a 4 meter ram length and are shown in Fig. 5.5 to Fig. 5.10.

The third set of graphs give details of a stability envelope for each chassis configuration for a given payload magnitude and position. The graphs give details of what ram length and ground slope give rise to a zero rear left trailer tyre force for a given payload magnitude and position and are shown in Fig. 5.11 to Fig. 5.13.

## **5.2 Reference chassis**

The reference chassis is the tipper chassis in production at the moment, and is described fully in Section 2.1. The stability criterion is quantified by the rear left trailer tyre force for a given payload magnitude and position. For the present analysis the vehicle is considered to be in a stable position as long as the rear left tyre force is greater than zero.

For stability comparisons between different chassis configurations, where the tyre force is greater than zero, the larger the tyre force the more stable the vehicle is for the given situation. Where the tyre force has reached zero the larger the ram length the more stable the vehicle is for the given situation.

Nine payload positions are considered for the present analyses to demonstrate how the initial payload position within the body effects the vehicles stability. As the body is assumed to be rigid the payload remains the same position relative to the body throughout the analysis. The positions are described with the vehicle standing on horizontal ground with the body untipped. The positions lie in the X-Y plane (ie.  $z=0$ ) and are spaced symmetrically about the centroid of the body. The payload of centre of gravity positions and their co-ordinates are shown in Fig. 5.14.

### 5.2.1 General tipping behaviour

The suspension's operating mode, discussed in Section 3.4.2, is dependent upon the payload magnitude and position within the body, ground slope and ram length for a given chassis. From the stability graphs in Fig. 5.1 to 5.4 which show how the rear left tyre force varies with ram length and ground slope for a given payload magnitude and position, described in the previous section, it can also be seen how the suspension operating condition changes.

For smallest payload of 2500 kg the suspension operates using predominantly mode a with mode b being used in the more extreme body attitudes greater than  $\theta_p = 5^\circ$  and

RL = 6m when considering payload position nine, as shown in Fig. 5.1i. The position of the payload in the fore/aft direction increases the occurrence of suspension mode b. However for each of the fore/aft positions considered, the vertical position does not significantly change the suspension mode.

As the payload increases to 10000 kg the suspension still operates using mode a with mode b being used more frequently in body attitudes greater than the  $\theta_p = 3^\circ$  and RL = 5m when considering payload position nine, as shown in Fig. 5.2i. The position of the payload in the fore/aft direction increases the occurrence of suspension b, mode b with the vertical position beginning to influence the occurrence of mode b.

For the 17500 kg payload the suspension operates under all four operating modes. Where the payload is positioned at the rear of the body the suspension operates using mode a with mode c being used in body attitudes greater than  $\theta_p = 2^\circ$  and RL = 7 m, when considering payload position seven as shown in Fig. 5.3g. With the payload positioned mid-way along the body the suspension operates under mode a with modes b, c, d occurring in body attitudes greater than  $\theta_p = 2^\circ$  and RL = 7m, when considering payload position eight as shown in Fig. 5.3h. When the payload is positioned in the most forward position the suspension operates under mode a and b with modes c and d only occurring in extreme body attitudes. Suspension mode b and d occurs in body attitudes greater than  $\theta_p = 2^\circ$  and RL = 6m when considering payload position nine as shown in Fig. 5.3i.

With the maximum payload of 25000 kg the suspension operates under modes a and c, with the latter being most prevalent. It should be noted that the most forward payload positions three, six and nine are not considered for this payload as they are impractical positions for this payload. The suspension operates using mode a with mode c being used in body attitudes greater than  $\theta_p = 1^\circ$  and  $RI = 4m$ , when considering payload position seven, as shown in Fig. 5.4e. For each of the fore/aft positions, considered, the vertical position does not significantly influence the suspension mode.

The payload magnitudes and the position of the payload centre of gravity's effect on the rear left tyre force can be conveniently demonstrated from the second set of stability graphs, by comparing the rear left tyre force for all nine payload positions with the vehicle standing on a ground slope of  $4^\circ$  at a 4m ram length. From Figs. 5.5 to Fig. 5.10 it can be seen that the tyre force is not proportional to load, for loads above 10000 kg. The graphs in Fig. 5.5 to Fig. 5.7 show how the rear left tyre force varies with the fore/aft position and payload magnitude of three vertical payload heights. For loads of above 10000 kg <sup>this</sup> tyre force is not proportional to the change in the fore/aft payload position and is made worse with increasing the vertical height. The graphs in Fig. 5.8 to Fig. 5.10 show how the tyre force varies with vertical height and payload magnitude for three fore/aft payload positions. For loads above 17500 kg the tyre force is not proportional to the change in vertical height and is made worse with increasing the fore/aft position.

### **5.2.2 Rollover condition**

When the trailer's rear left tyre force approaches or becomes zero it is in an unstable position and rollover is imminent. From the first set of stability graphs in Fig. 5.1 to Fig. 5.4 the stability characteristics of a trailer can be determined for a range of payload magnitudes and positions. To obtain a more general view of the rollover characteristics the ram length and ground slope data at which the graphs cross the X-axis (ie. zero tyre force) can be plotted against each other to produce the graphs in Fig. 5.11 to Fig. 5.13. From these results a stability envelope is created for a payload magnitude and position, which show that the vehicle is in a stable position below the graph line and in an unstable position above the graph line. As some payload position and magnitude combinations do not cause the vehicle to become unstable ie. rollover, not all payload positions are shown. Each payload magnitude and its unstable payload positions are shown on one graph. It should be noted that the 2500 kg payload does not have a stability envelope graph due to the tyre force remaining positive for all of the payload positions considered.

### **5.3 The effect of design changes on stability**

To quantify the increase or decrease in stability resulting from design changes, the rear left tyre force, for a given body attitude, ground slope, payload and chassis configuration, is compared to the reference chassis.

### 5.3.1 The effect of introducing a sheardeck on stability

The sheardeck, described in Section 4.3 made a reasonable improvement to the stability characteristics.

For the 2500 kg payload the chassis change hardly effected the stability characteristic. Considering payload position 9, shown in Fig. 5.1i,  $\theta_p = 8^\circ$  and RL = 8m, the rear left tyre force is increased from 3047 N to 3509 N, a 15% increase.

For the 10000 kg payload the chassis change has a more noticeable effect on the stability characteristics considering payload position 9, shown in Fig. 5.2i,  $\theta_p = 4^\circ$  and RL = 8m, the rear left tyre force is increased from 2795 N to 4005 N, a 43% increase.

For the 17500 kg payload the chassis change makes a noticeable effect on the stability characteristics. Considering payload position 9, shown in Fig. 5.3i,  $\theta_p = 2^\circ$  and RL = 8m, the rear left tyre force is increased from 6188 N to 8531 N, a 38% increase.

For the 25000 kg payload the chassis change has a noticeable effect on the stability characteristics. Considering payload position 5, shown in Fig. 5.4d,  $\theta_p = 1^\circ$  and RL = 8m, the rear left tyre force increases from 21511 N to 24714 N, a 15% increase.

#### **5.4 The effect of increasing the torsional stiffness of the cross members**

The torsion chassis, described in Section 4.3, made a significant improvement to the stability characteristics. The improvement to stability is quantified in the same manner as described in Section 5.3.

For the 2500 kg payload the chassis change has made a reasonable effect upon the stability characteristics. Considering payload position 9, shown in Fig. 5.1i,  $\theta_p = 8^\circ$  and  $RL = 8$ , the rear left tyre force is increased from 3047 N to 4203 N, a 38% increase.

For the 10000 kg payload the chassis change has a large effect on the stability characteristics. Considering payload position 9, shown in Fig. 5.2i,  $\theta_p = 4^\circ$  and  $RL = 8m$ , the rear left tyre force is increased from 2795 N to 6714 N, a 140% increase.

For the 17500 kg payload the chassis change has a large effect on the stability characteristic. Considering payload position 9, shown in Fig. 5.3i,  $\theta_p = 2^\circ$  and  $RL = 8m$ , the rear left tyre force is increased from 6188 N to 14703 N, a 138% increase.

For the 25000 kg payload the chassis change has a noticeable effect on the stability characteristic. Considering payload position 5, shown in Fig. 5.4d, the  $\theta_p = 1^\circ$  and  $RL = 8m$ , the rear left tyre force increases from 21511 N to 32017 N a 49% increase.

## 5.5 The effect of reducing the trailing arm stiffness

The trailing arm chassis, described in Section 4.3, made little improvement to the stability characteristics. The improvement to stability is quantified in the same manner as described in Section 5.3.

For the 2500 kg payload the chassis change hardly effected the stability characteristics. Considering payload position 9, shown in Fig. 5.1i,  $\theta_p = 8^\circ$  and  $RL = 8\text{m}$ , the rear left tyre force is increased from 3047 N to 3436 N, a 13% increase.

For the 10000 kg payload the chassis change had a slightly adverse effect on the stability characteristic. Considering payload position 9, shown in Fig. 5.2i,  $\theta_p = 4^\circ$  and  $RL = 8\text{m}$ , the rear left tyre force is reduced from 2795 N to 2500 N, an 11% decrease.

For the 17500 kg payload the chassis change had a slightly adverse effect on the stability characteristic. Considering payload position 8, shown in Fig. 5.3i,  $\theta_p = 2^\circ$  and  $RL = 8\text{m}$ , the rear left tyre force is reduced from 11122 to 10549 N, a 5% decrease.

For the 25000 kg payload the chassis change had a noticeable adverse effect on the stability characteristic. Considering payload position 5, shown in Fig. 5.4d,  $\theta_p = 1^\circ$  and  $RL = 8\text{m}$  the rear left tyre force decreases from 21511 N to 11479 N, a 47% decrease.



## 5.6 Discussion

From the design investigations it can be seen, for a given chassis configuration, how the rear left tyre force varies with payload magnitude and position, ram length and ground slope. From the results it was determined that the rear left tyre force does not vary linearly with these variables and that the non-linear model developed is necessary to determine the stability of a tipping trailer.

The results enable a stability envelope to be generated which gives detail of what ground slope and ram length give rise to a stable/unstable vehicle for a given payload magnitude and position. Future work may be able to utilise these results to develop a system which would not allow the vehicle operator to tip in unstable conditions.

The design investigation demonstrates how the stability characteristics can be assessed for a given chassis configuration and how subsequent chassis modifications can be assessed in improving vehicle stability. The typical stability envelope for all four chassis configurations, is shown in Fig. 5.15 for a 25000 kg payload, in payload position 8. From the results the chassis modifications showed that the torsional stiffness of the cross-members make an important contribution to the vehicle's stability. The introduction of a sheardeck to the chassis made a small but significant increase in the vehicle stability, with the trailing stiffness being found to make little contribution to the vehicle's stability, and in some case an adverse effect resulted.

The program could also be used to investigate the effect of other design parameters on stability

- (i) tyre and suspension flexibilities
- (ii) suspension operating conditions
- (iii) different chassis design (dimensions and materials)
- (iv) the importance of matching a tractor and trailer flexibilities (eg. a stiff tractor suspension combined with a stiff trailer suspension etc.)
- (v) wind loading

With some modifications to the program the effect of other design parameters on stability could also be investigated

- (vi) a flowing payload during tipping
- (vii) trailer body flexibility (currently assumed rigid).

Unit load or unit displacement stress matrices can be generated from the finite element analyses undertaken, these results would indicate which parts of the chassis are under or over stressed, during tipping highlighting those areas that would benefit from design changes.

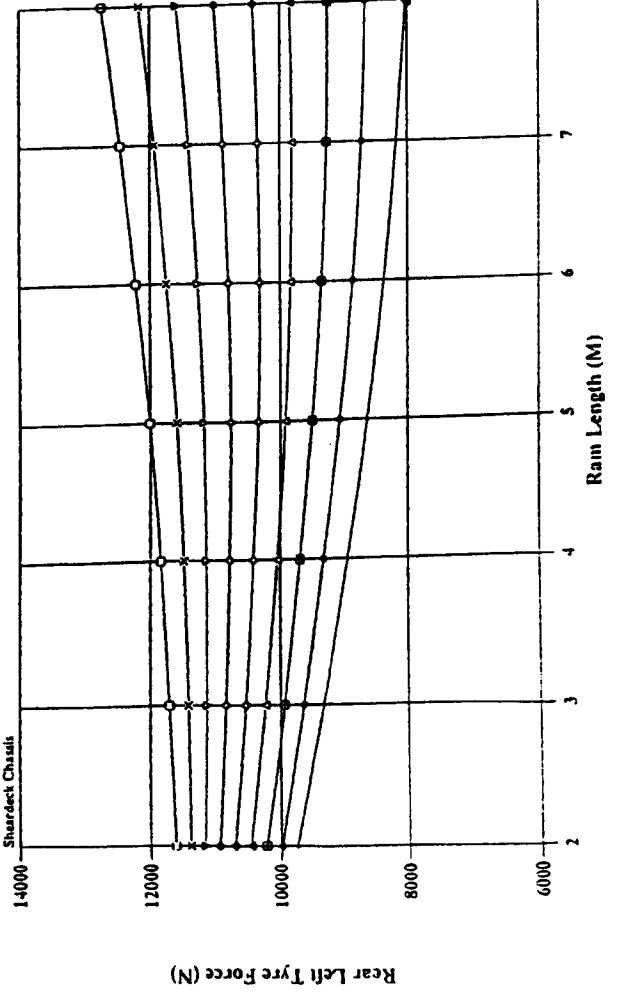
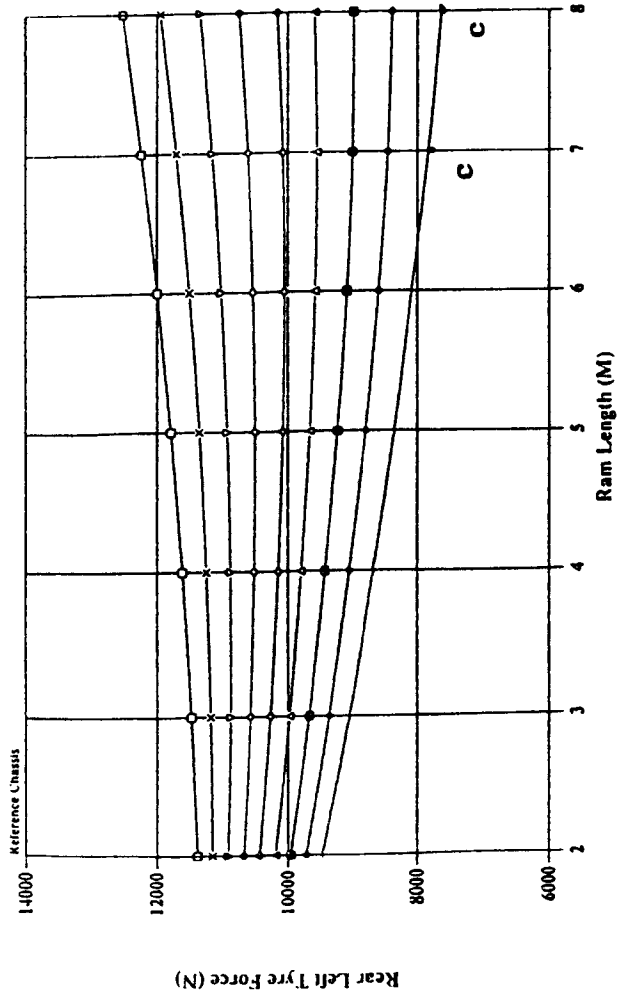
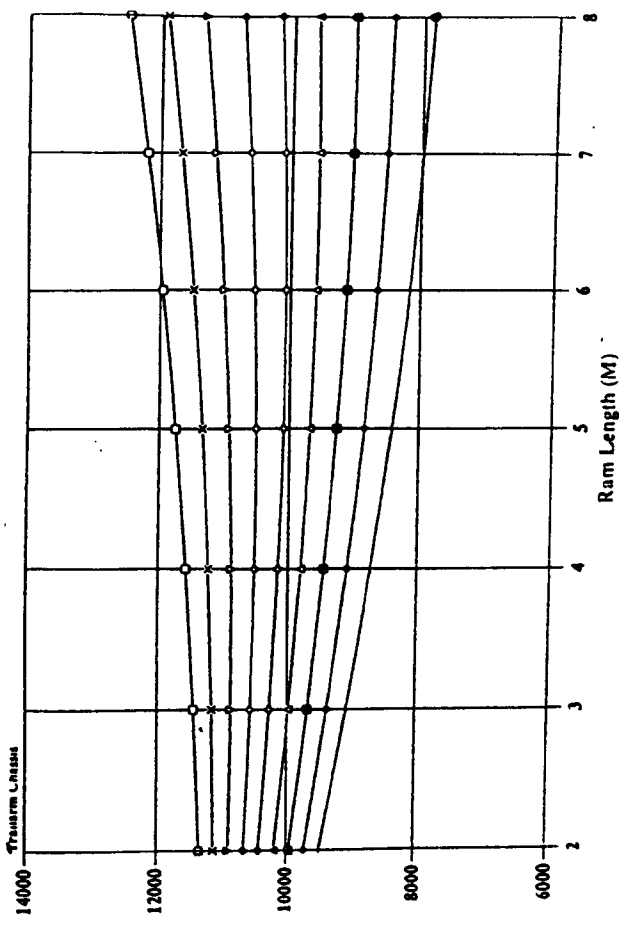
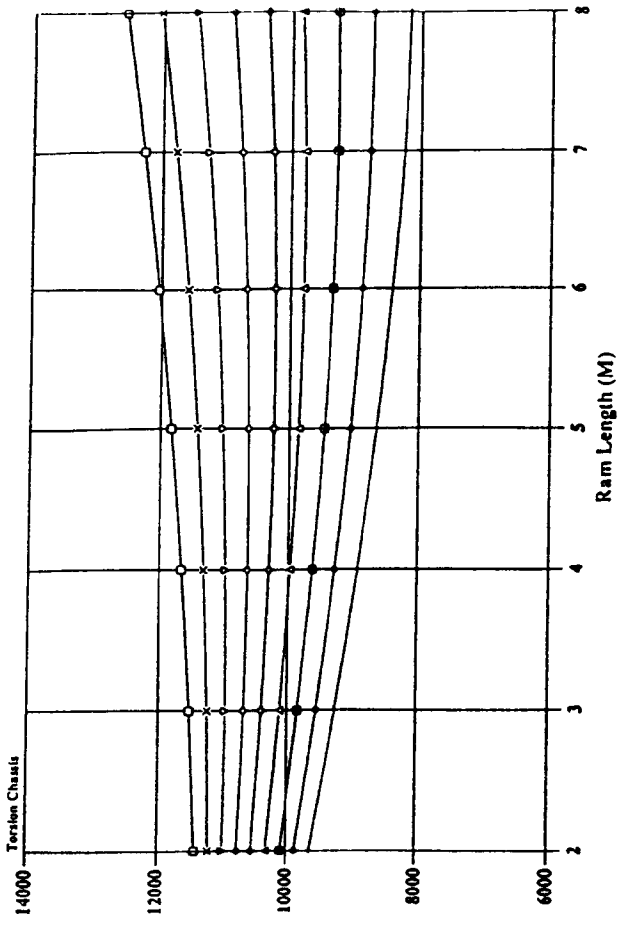
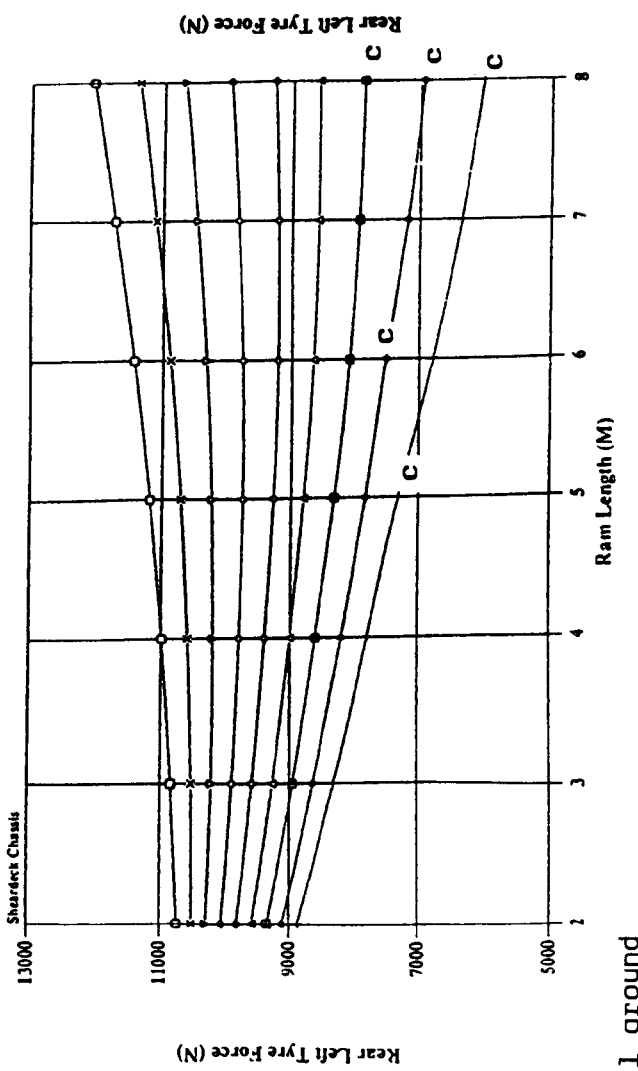
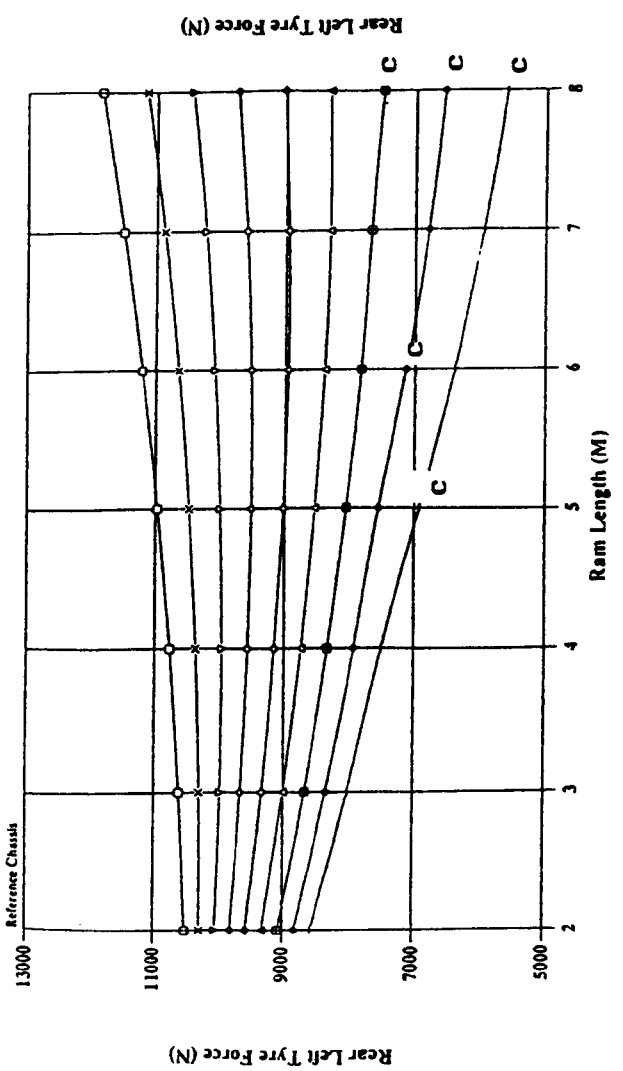
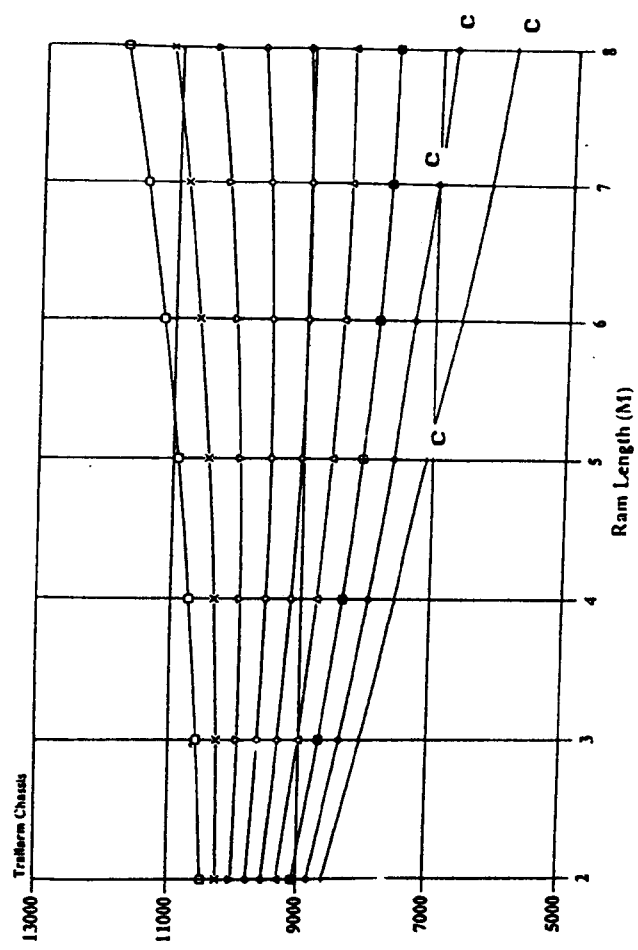
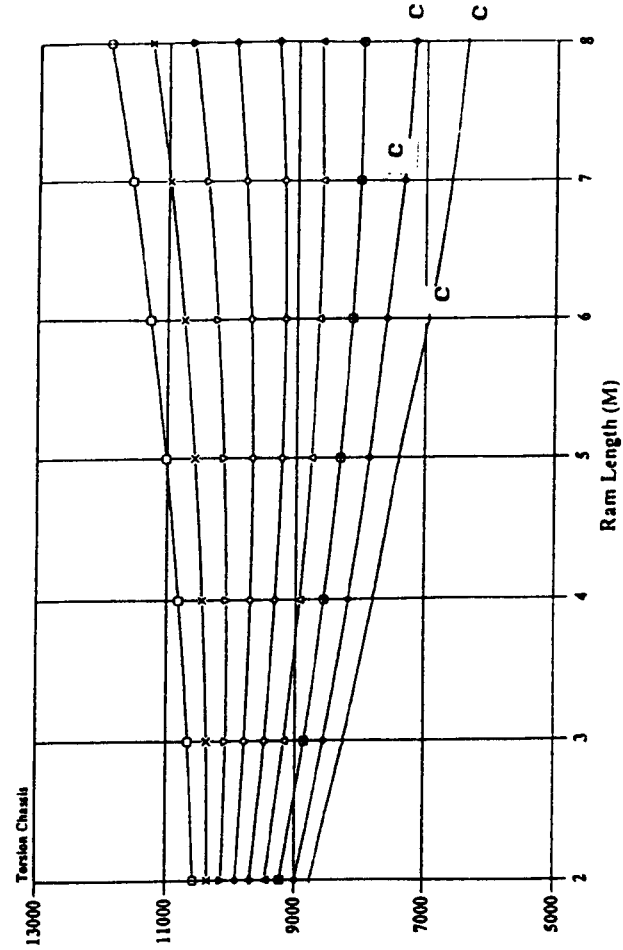


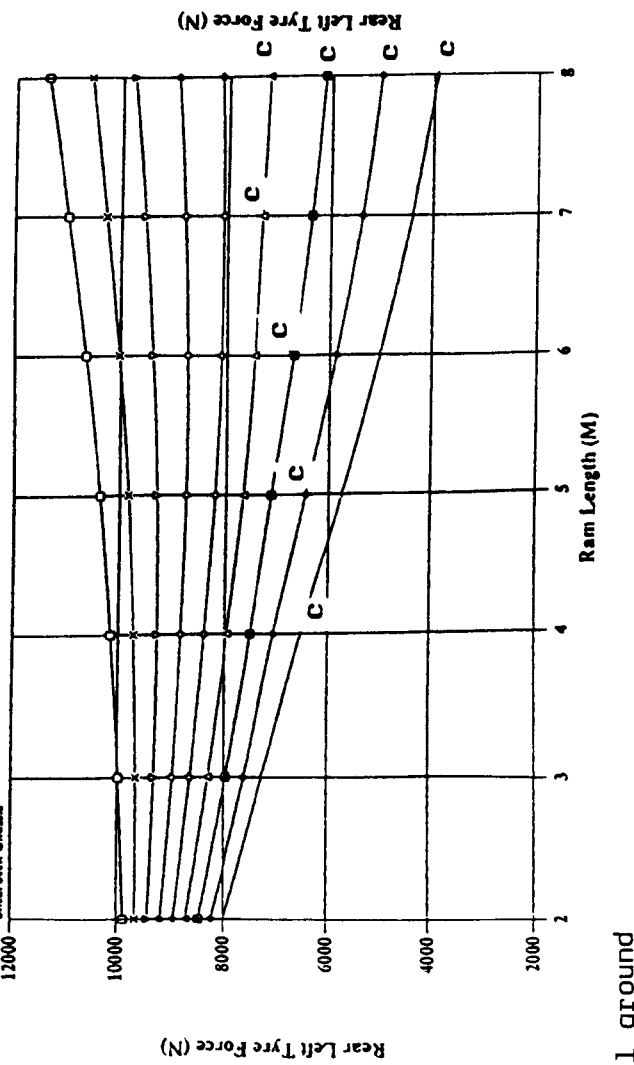
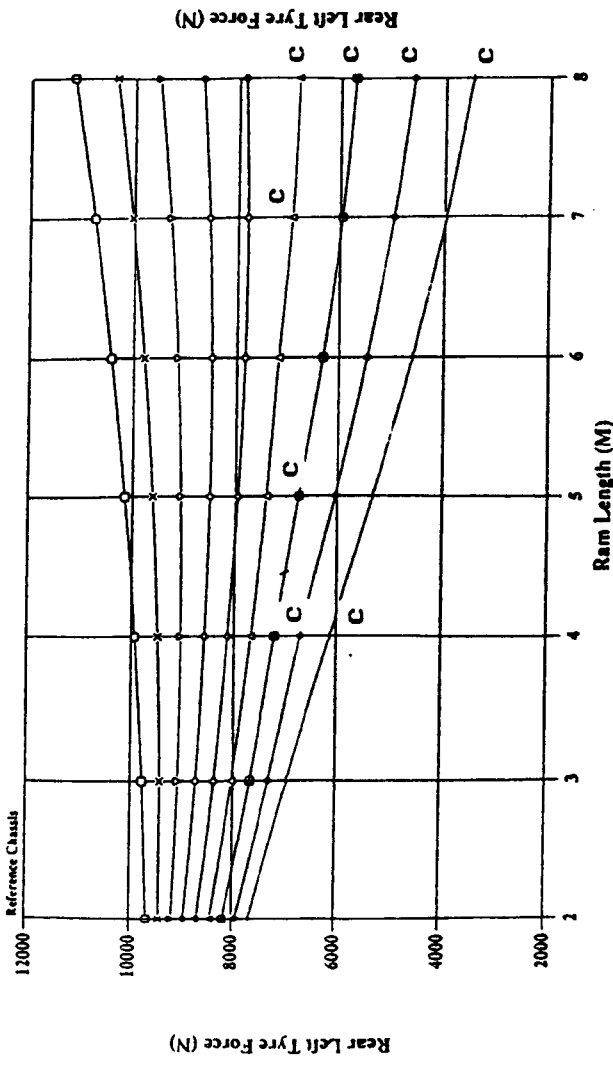
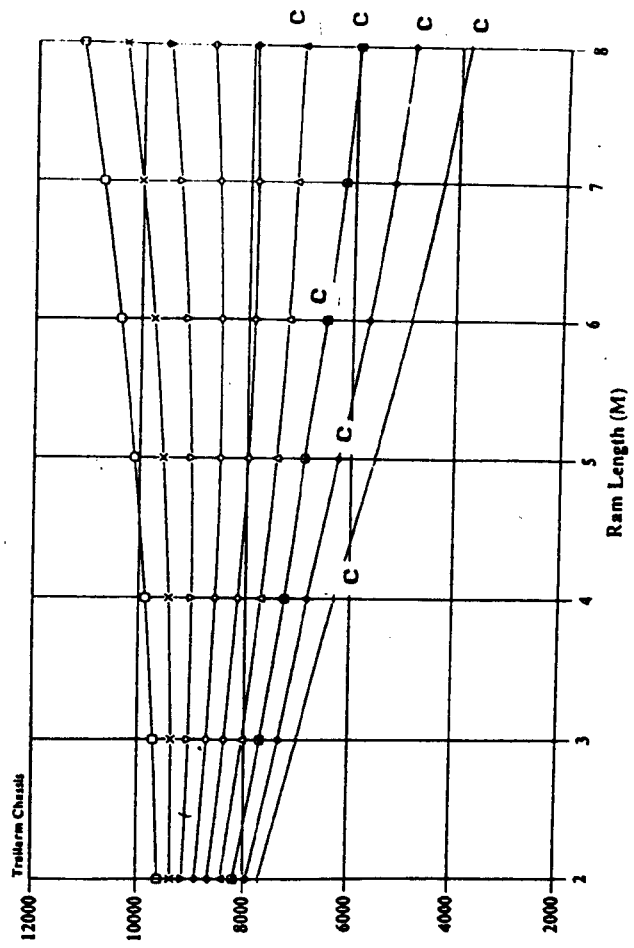
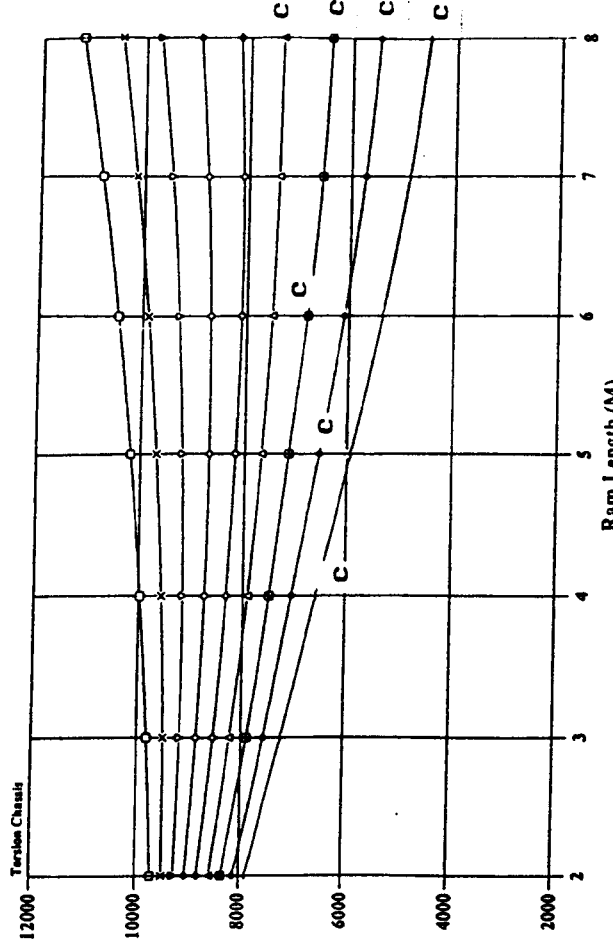
Fig. 5.1a Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 1 .

- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope



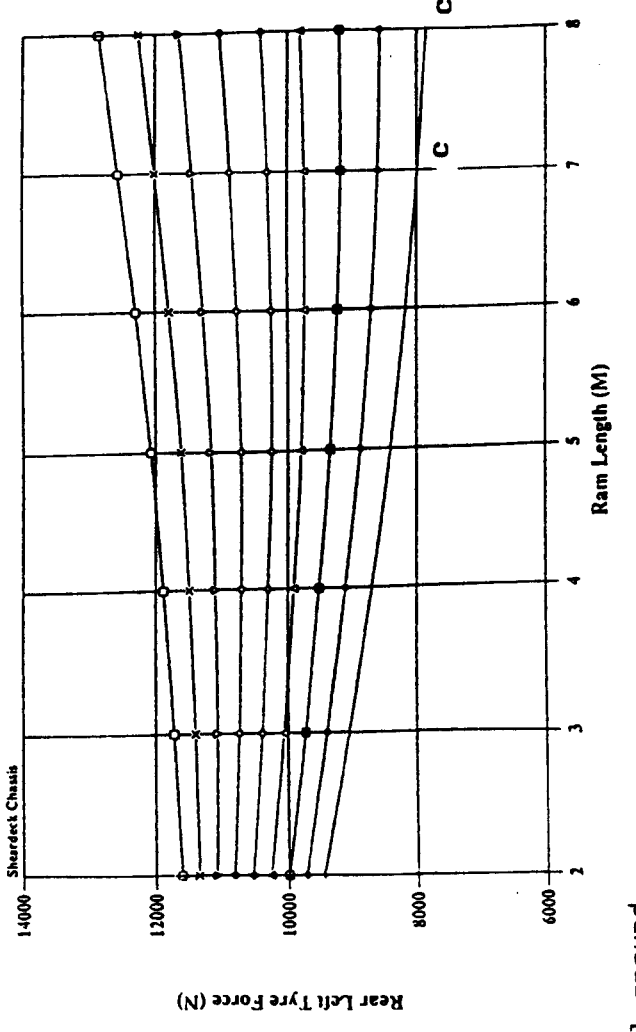
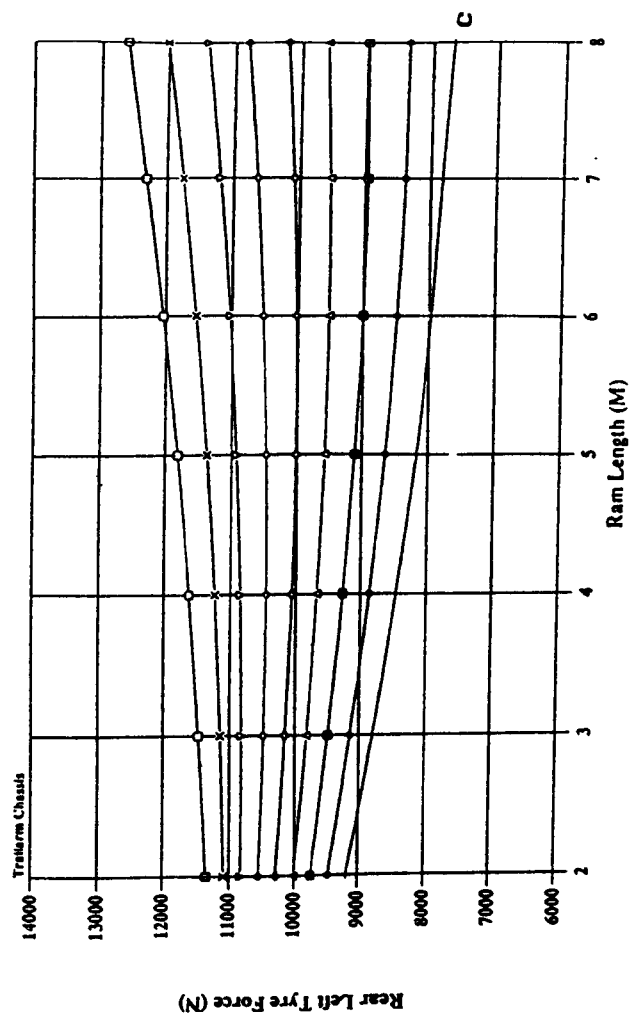
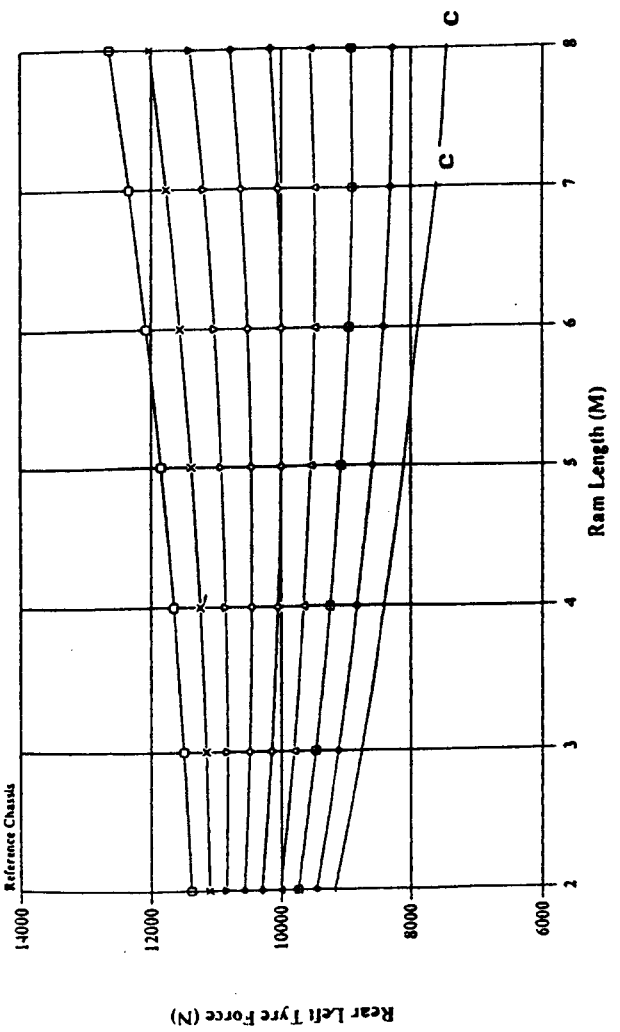
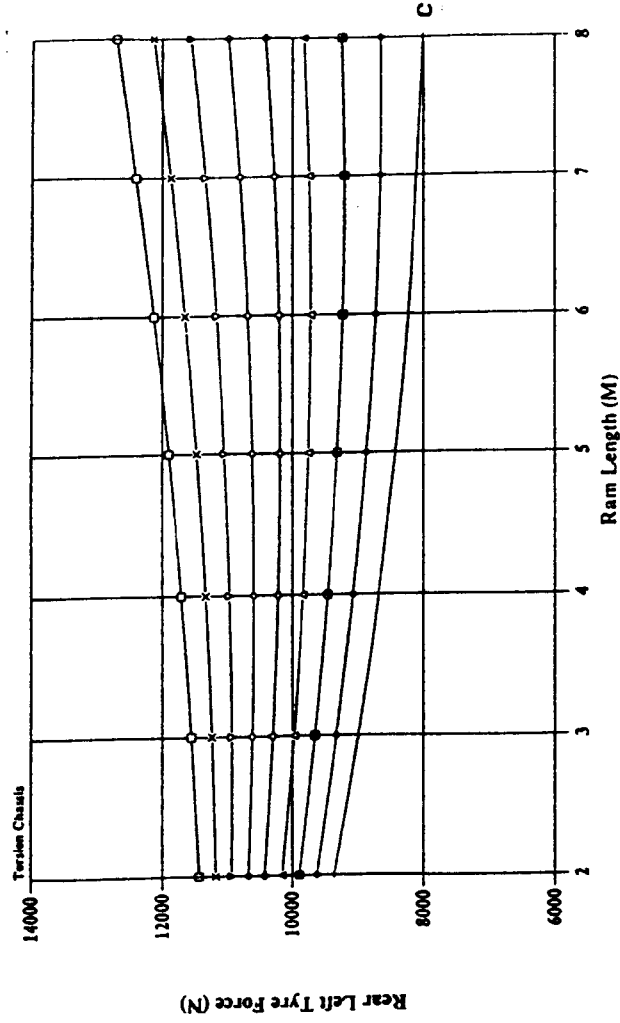
- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- = 6° ground slope
- = 8° ground slope

Fig. 5.1b Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 2 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ◊ = 6° ground slope
- = 8° ground slope

Fig. 5.1c Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 3.



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ◻ = 6° ground slope
- ∠ = 8° ground slope

Fig. 5.1d Rear left tyre force versus Ram Length and ground slope combinations for a 2500 Kg payload in body position 4 .

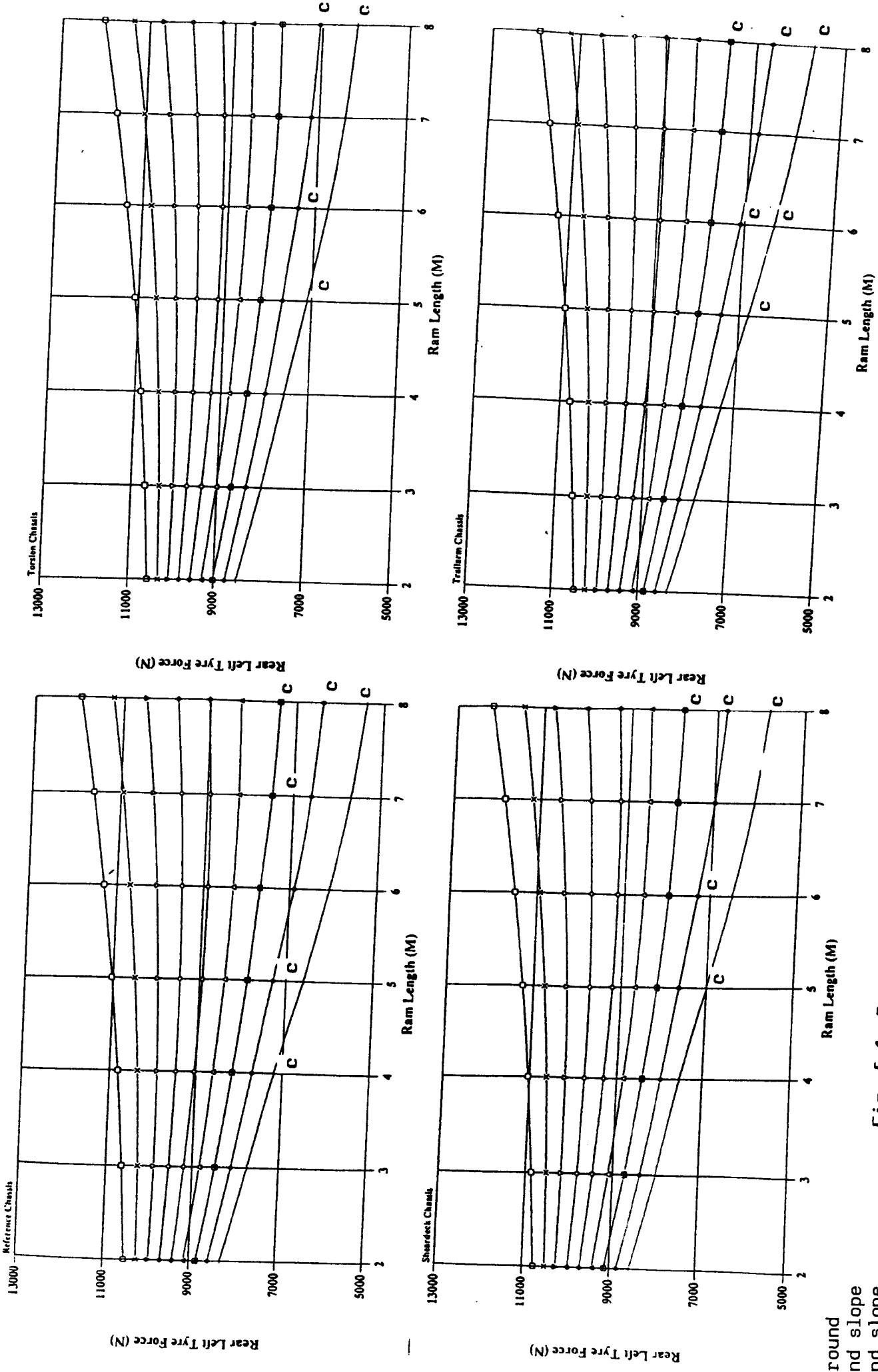
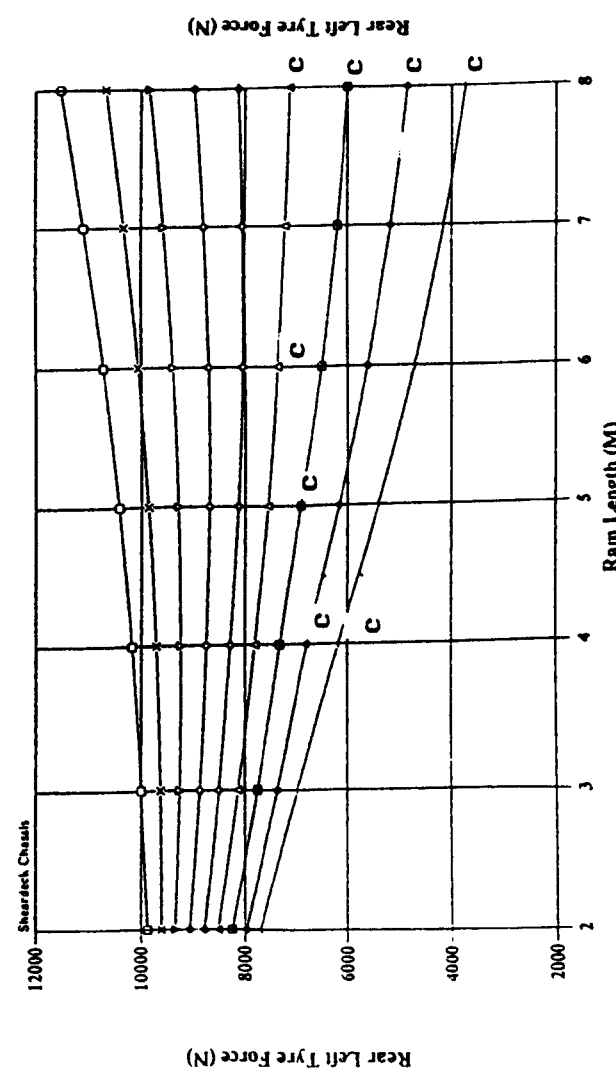
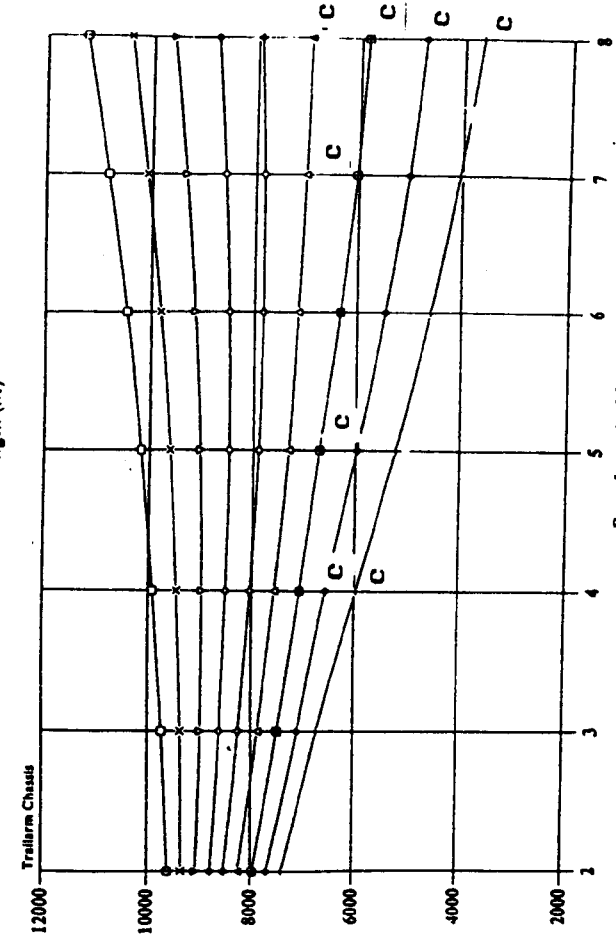
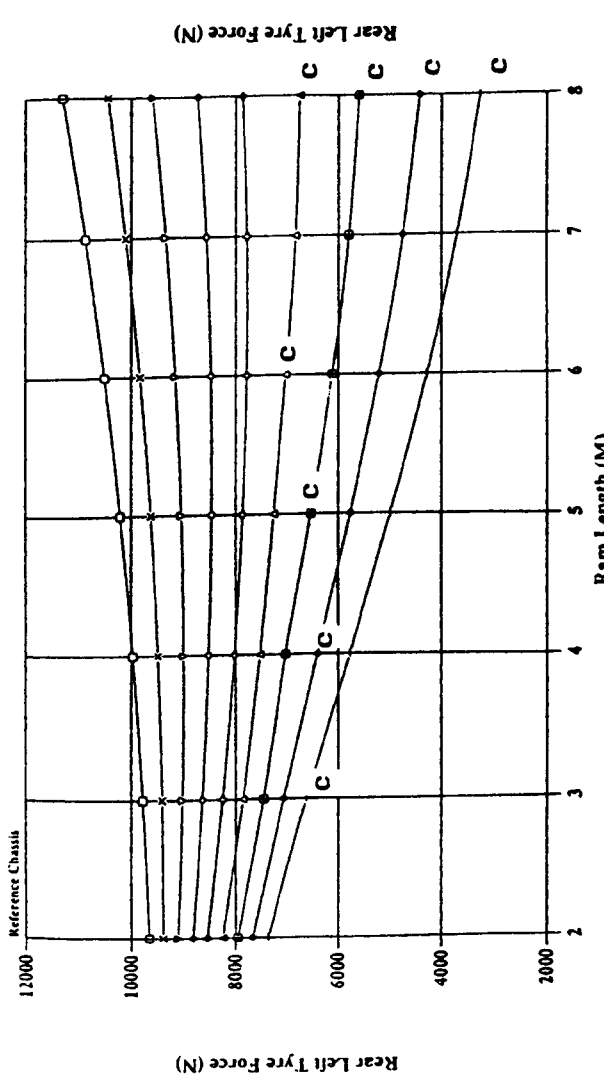
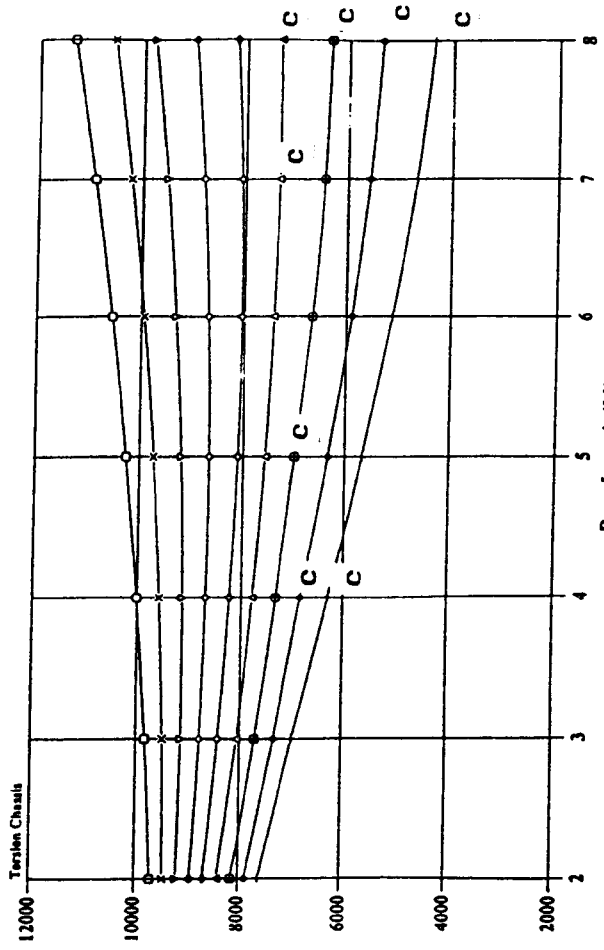


Fig. 5.1e Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 5.

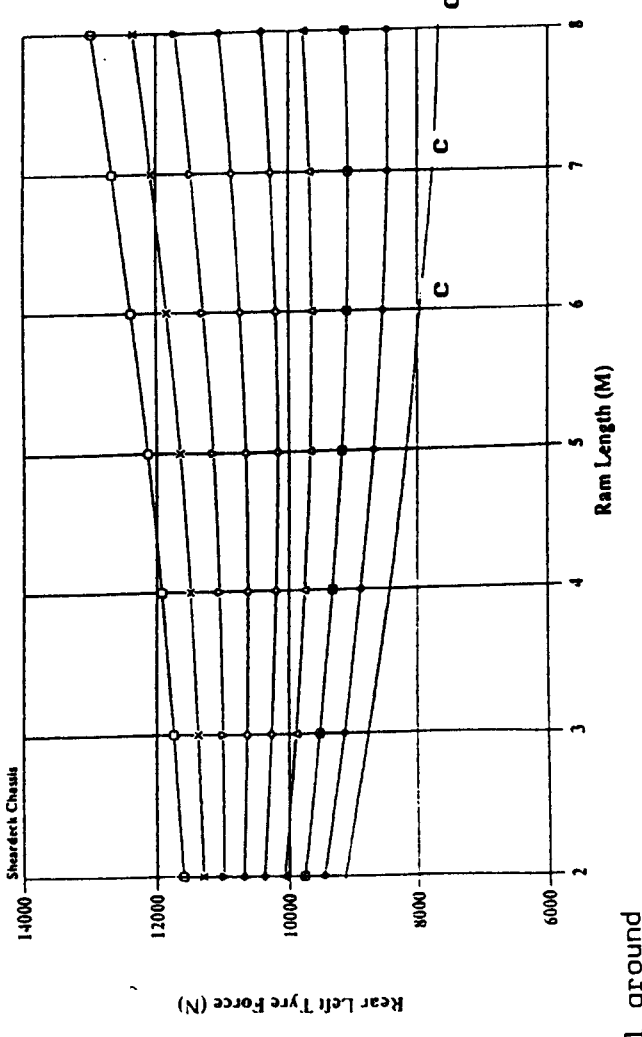
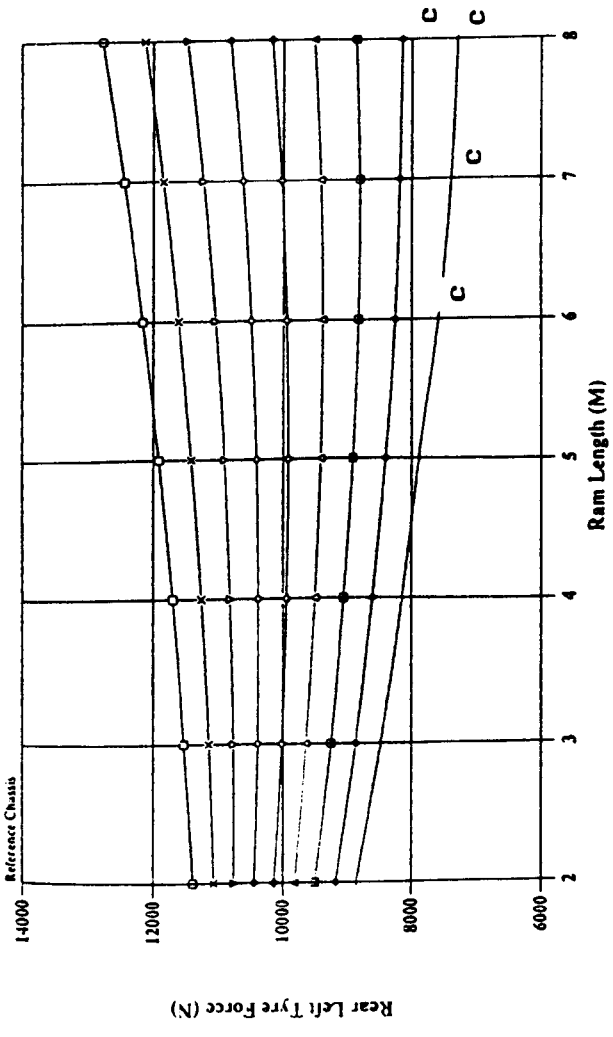
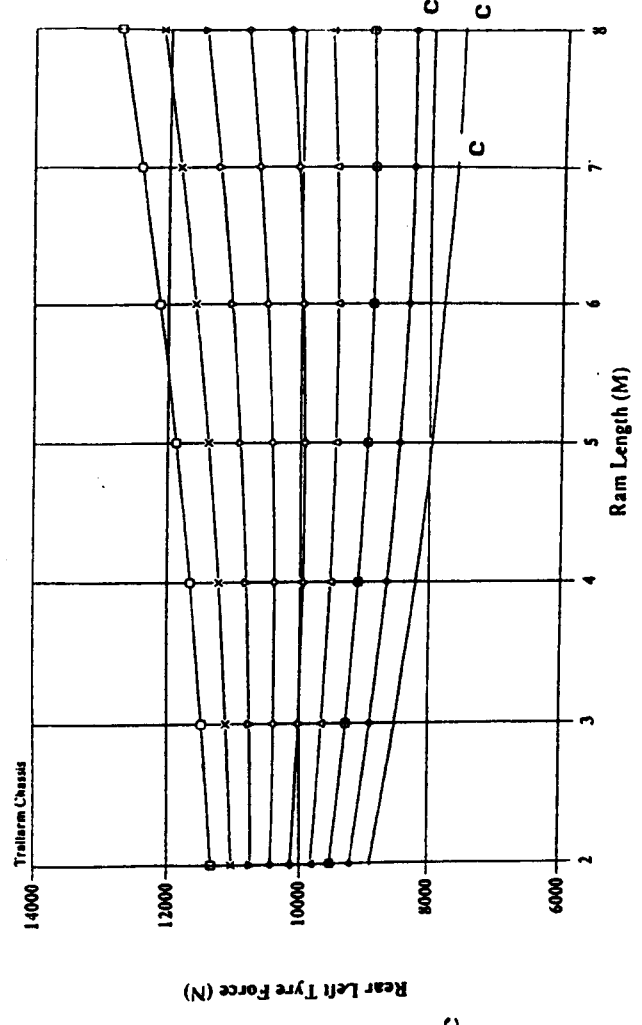
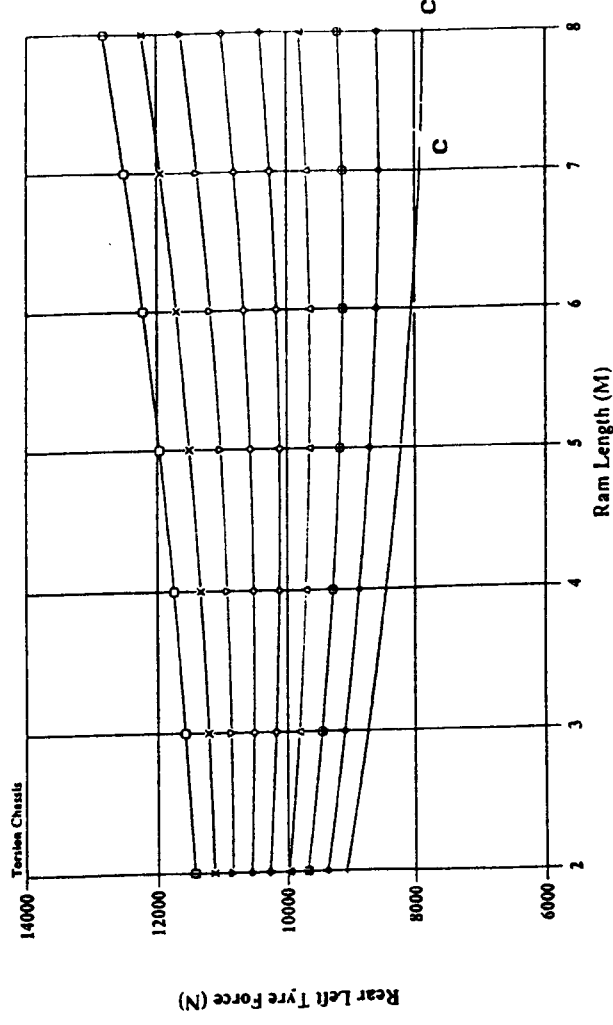
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊗ = 6° ground slope
- = 8° ground slope



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊕ = 6° ground slope
- = 8° ground slope

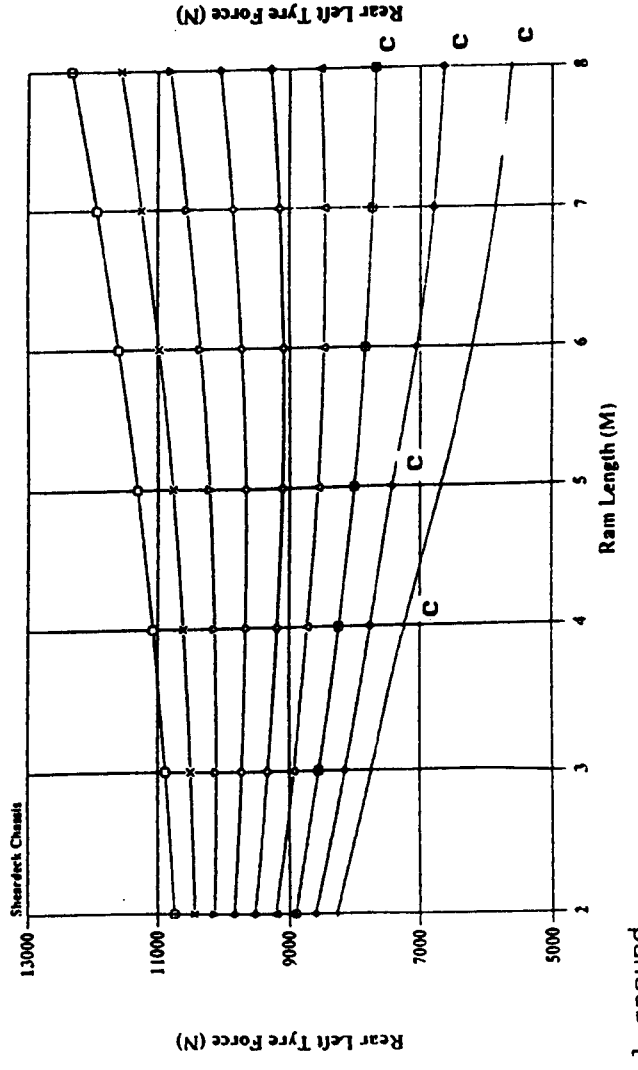
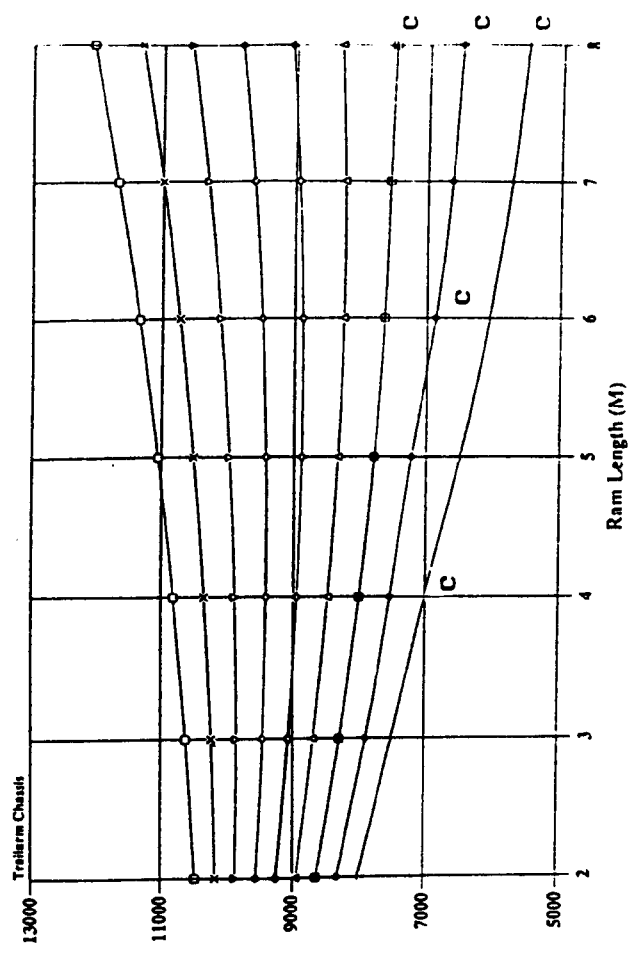
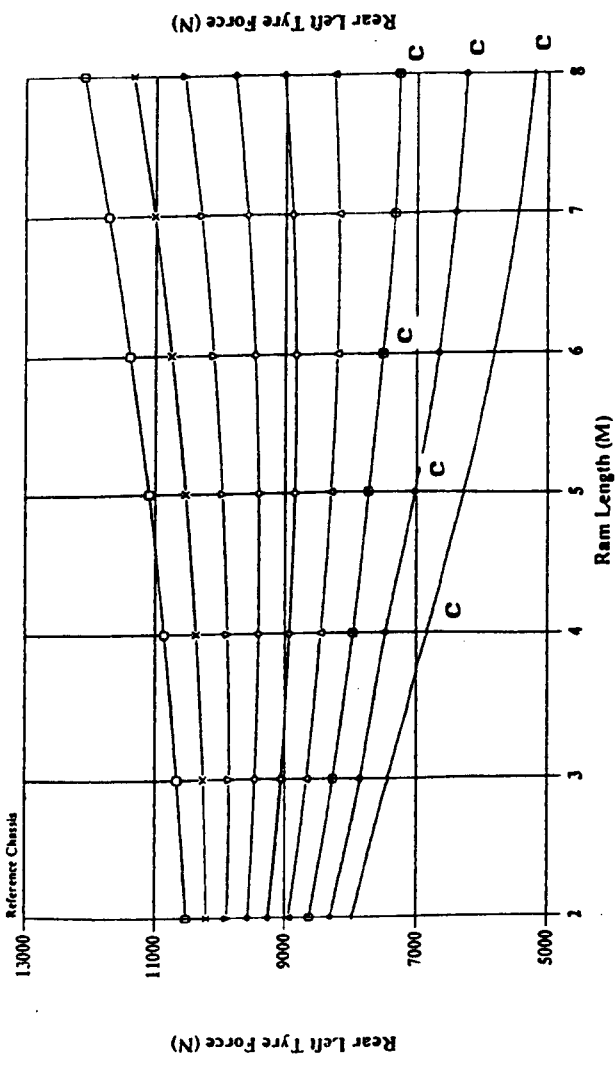
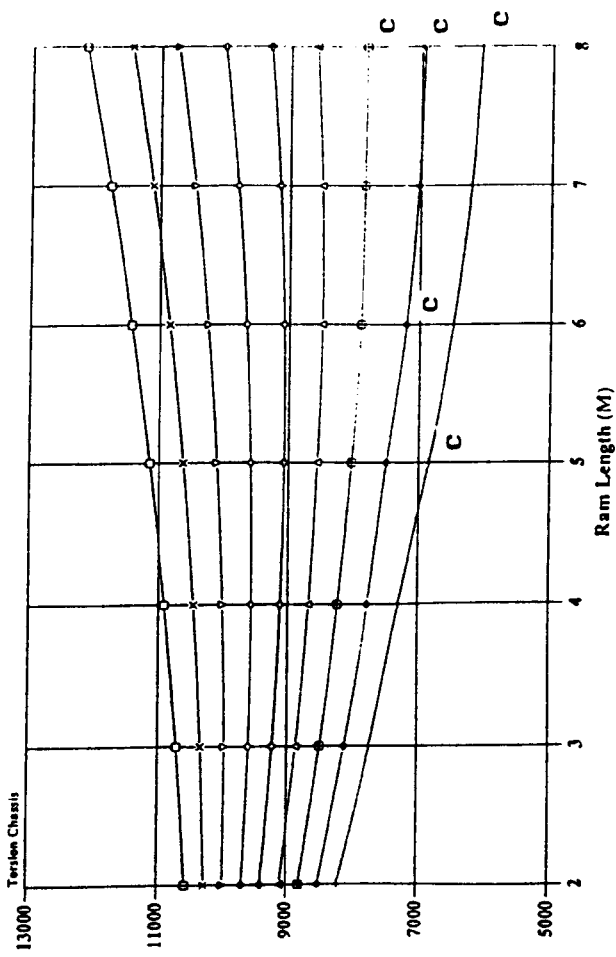
Fig. 5.1f Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 6 .





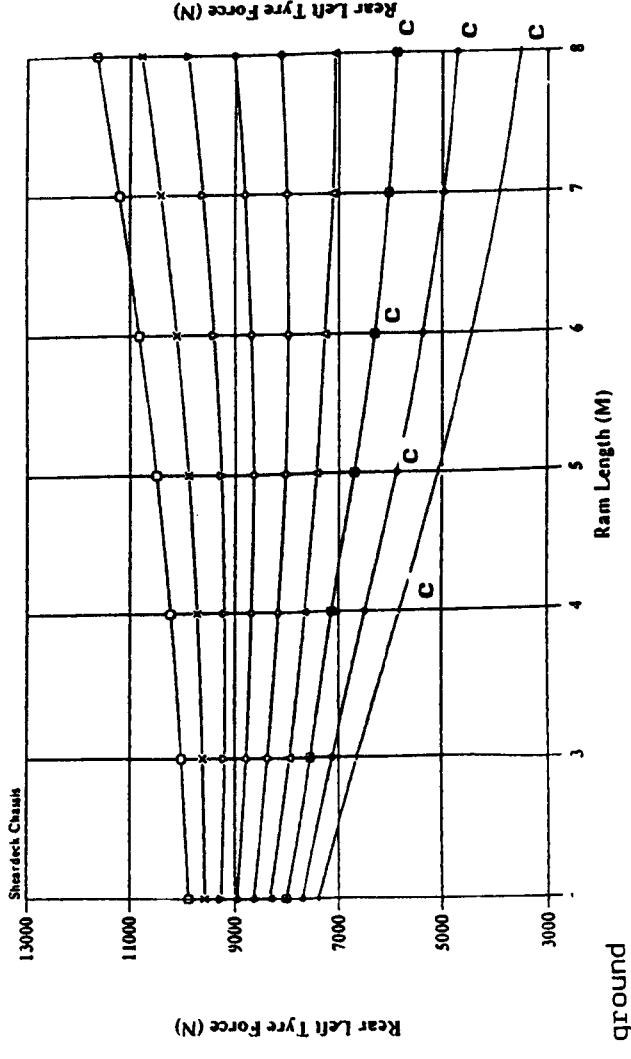
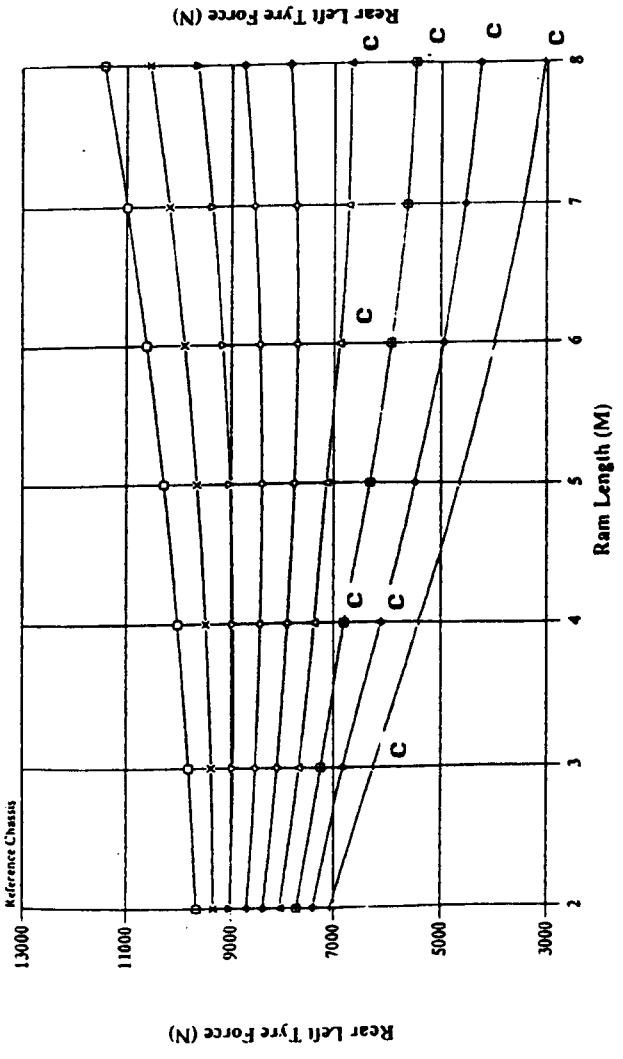
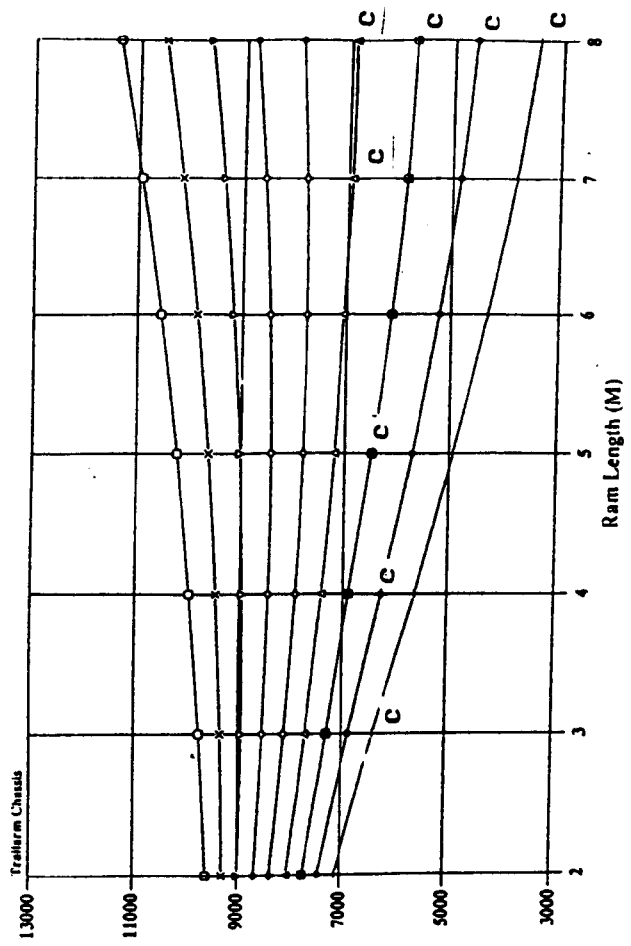
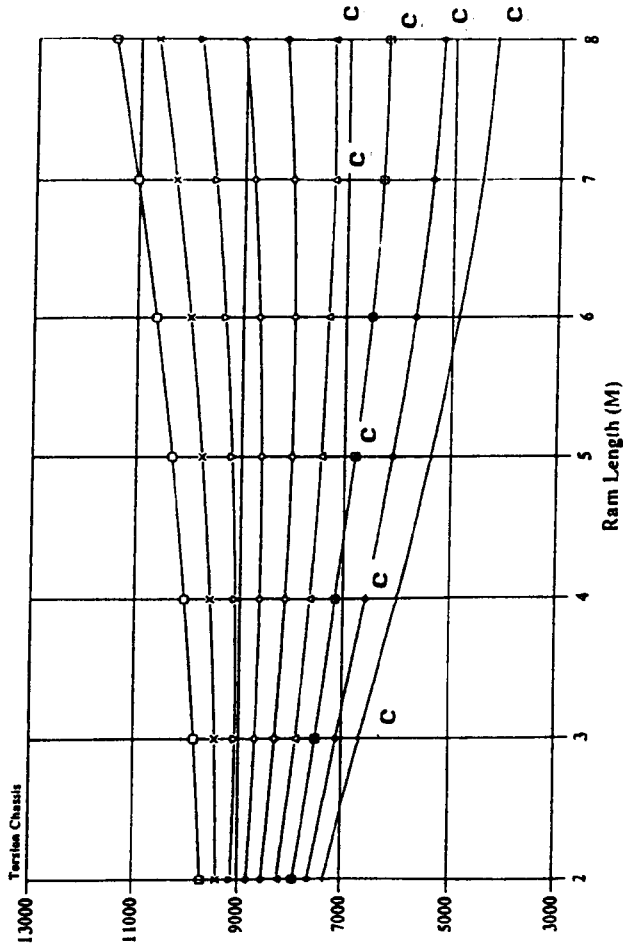
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

Fig. 5.1g Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 7 .



- = level ground
- △ = 2° ground slope
- ▤ = 4° ground slope
- ▥ = 6° ground slope
- ▧ = 8° ground slope

Fig. 5.1h Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 8 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

Fig. 5.1i Rear left tyre force versus Ram length and ground slope combinations for a 2500 Kg payload in body position 9 .

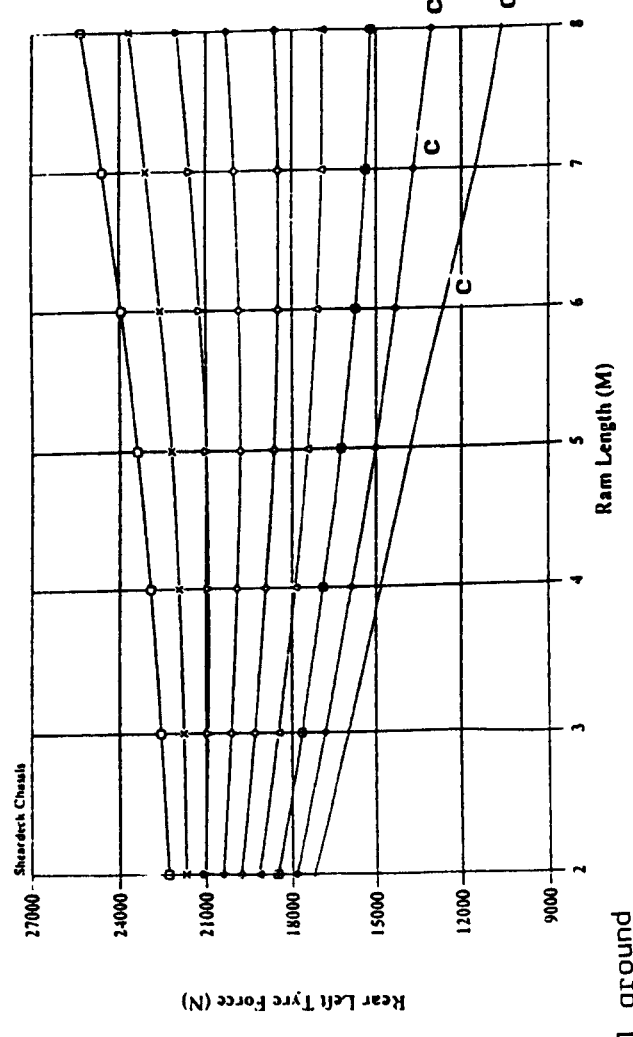
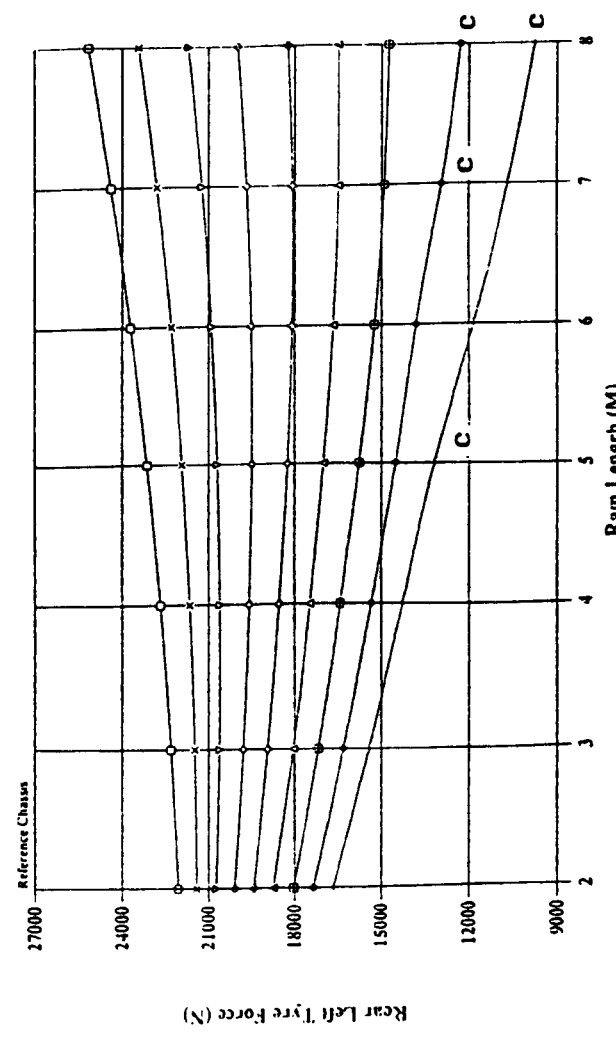
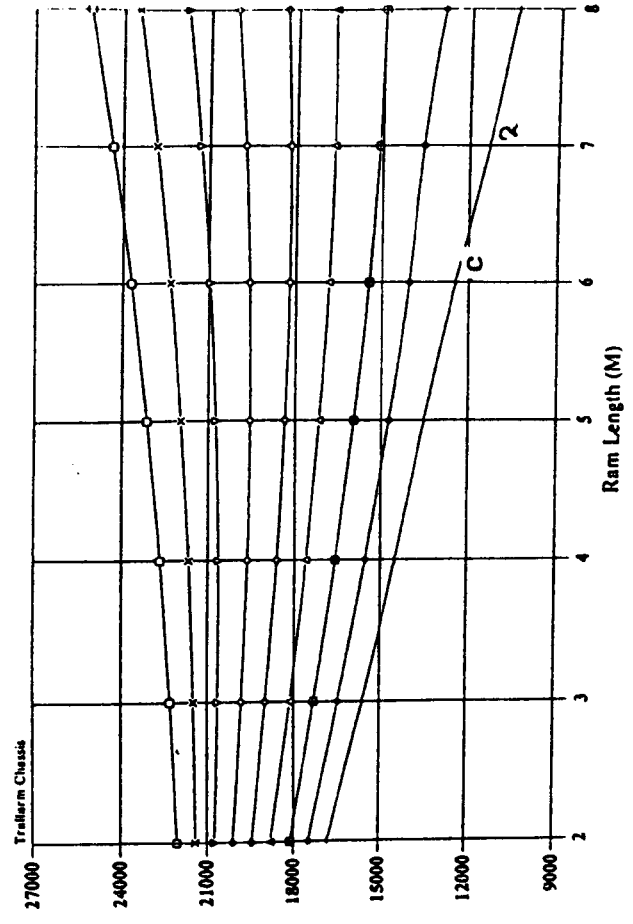
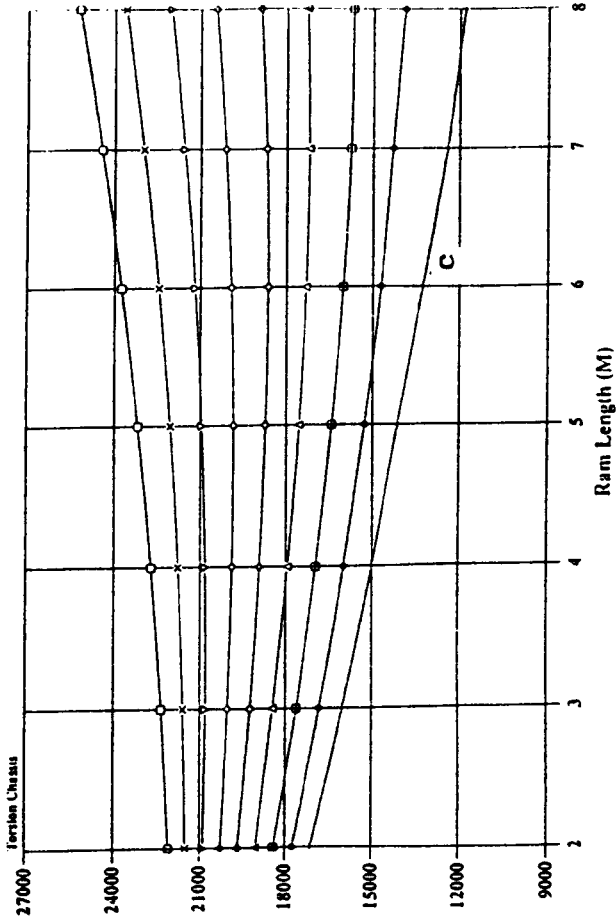
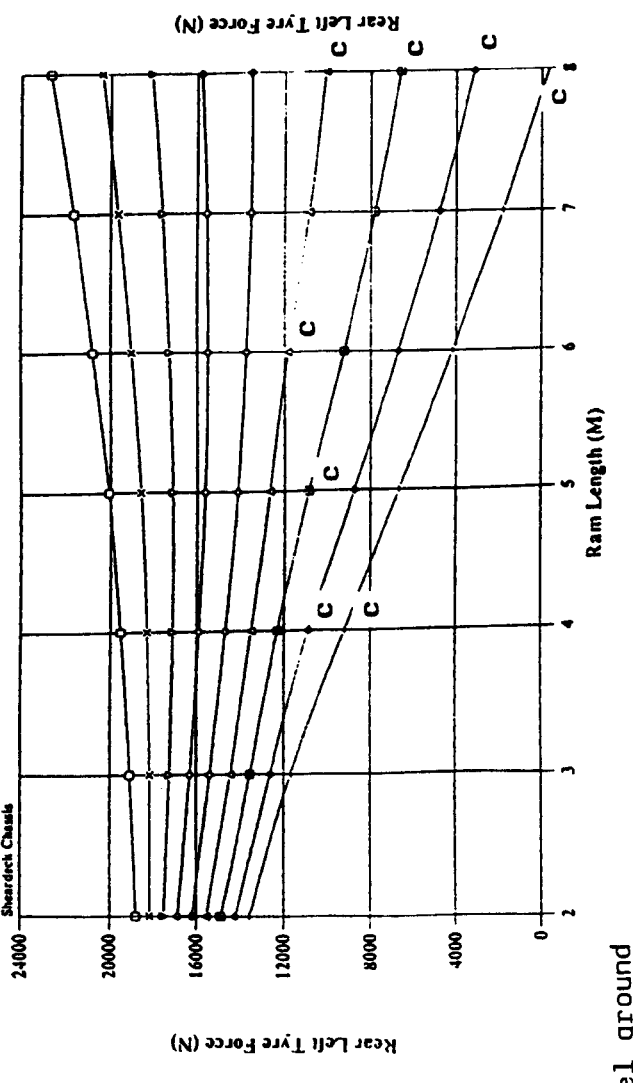
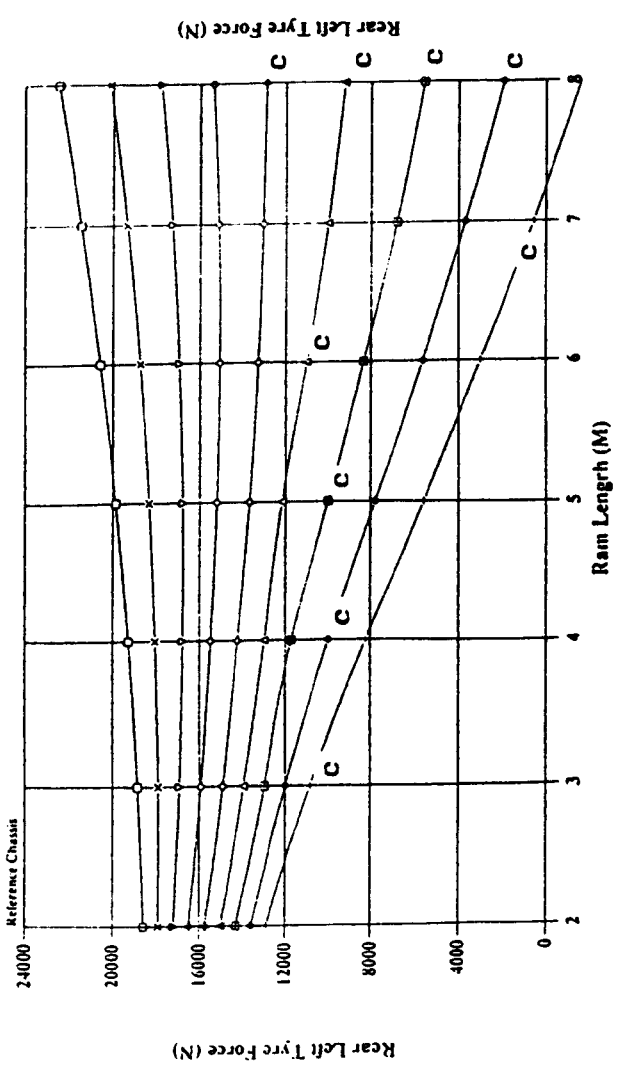
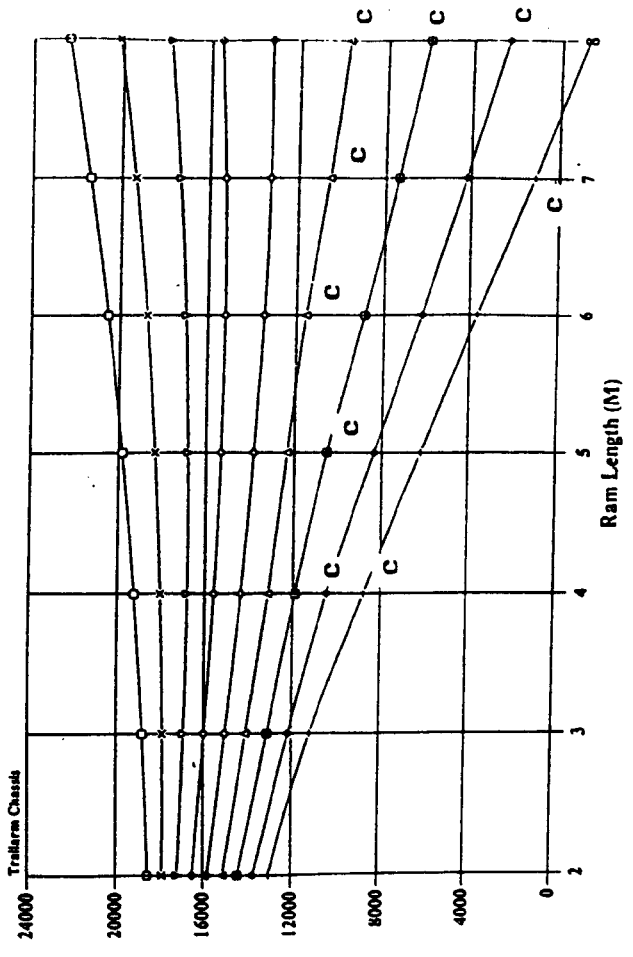
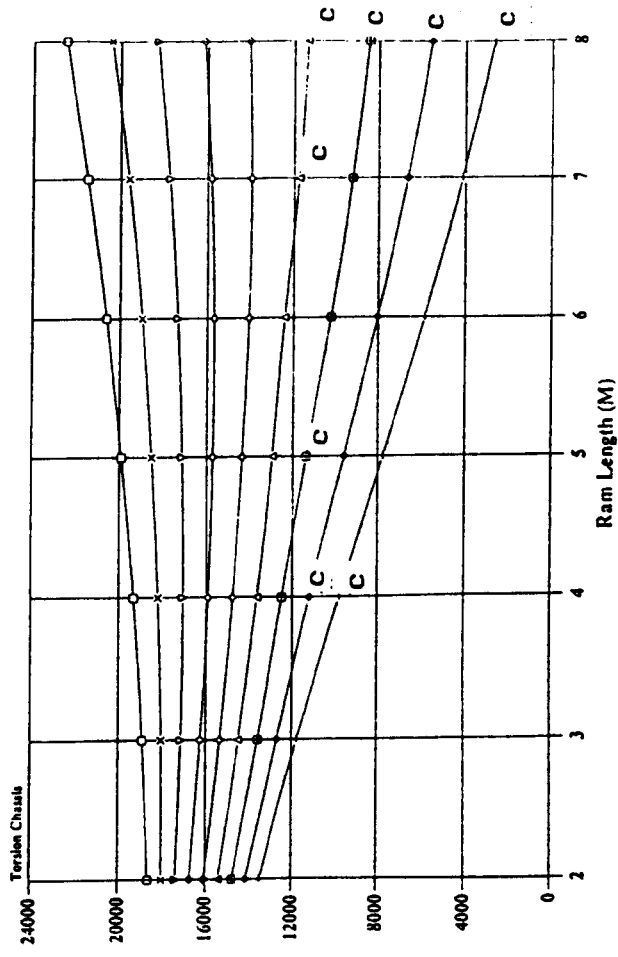


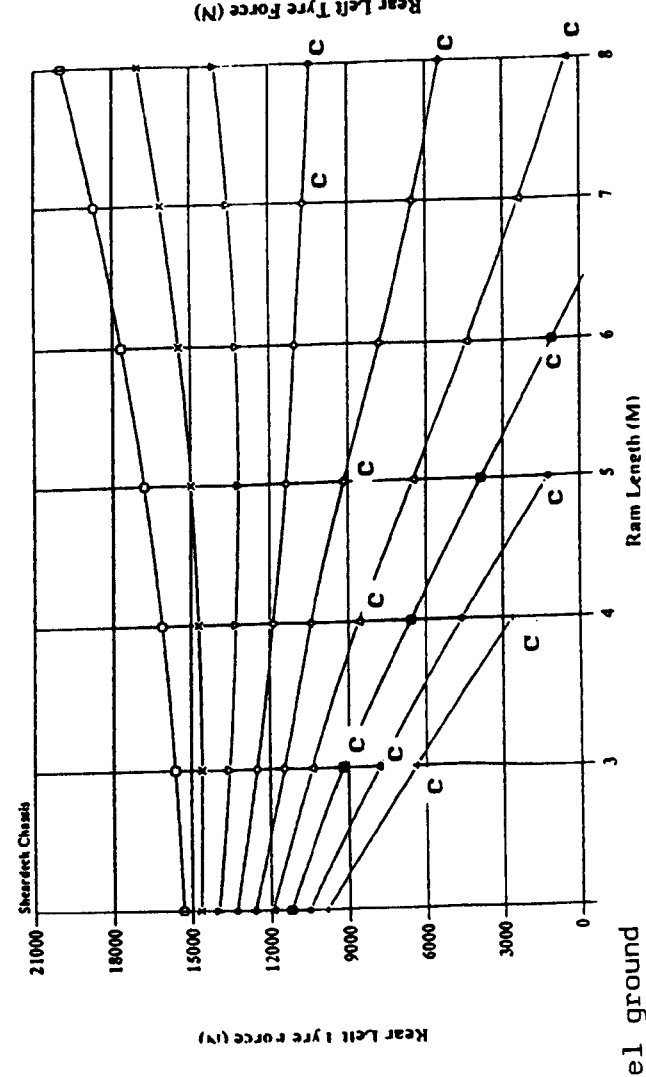
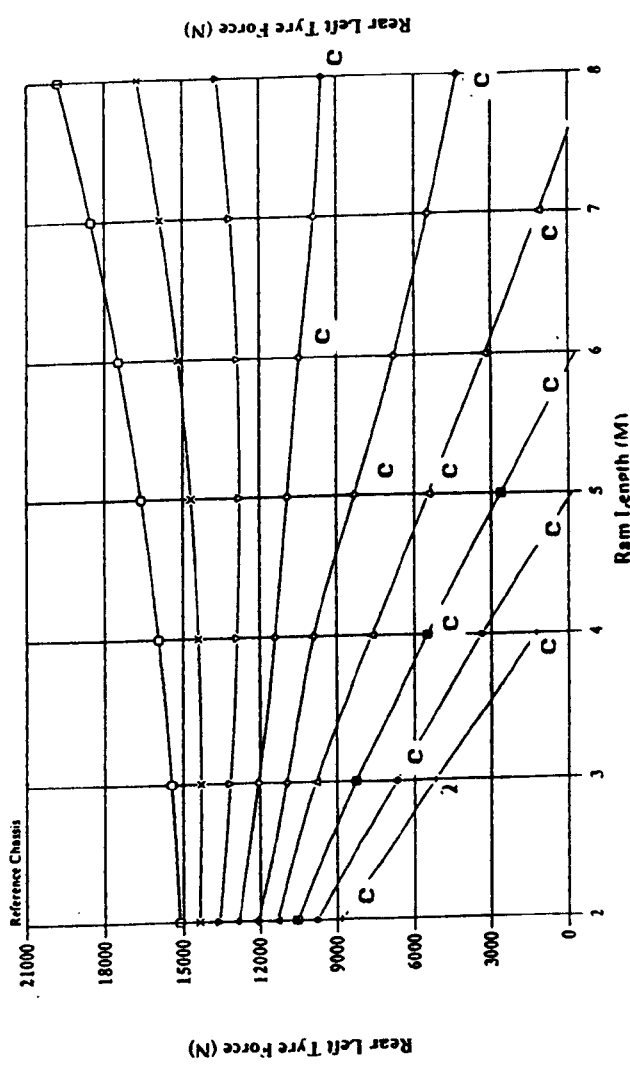
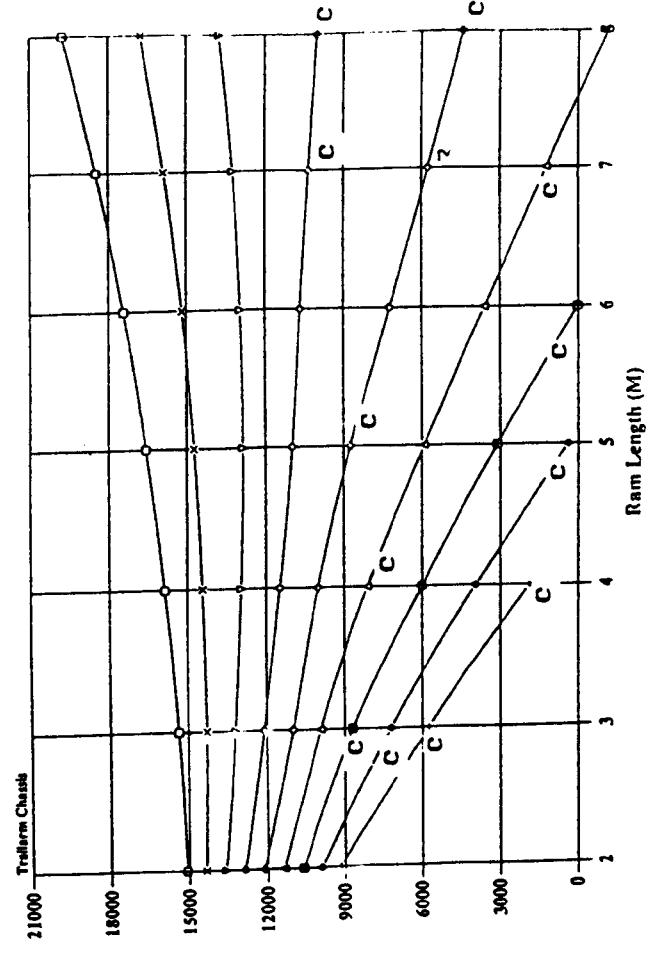
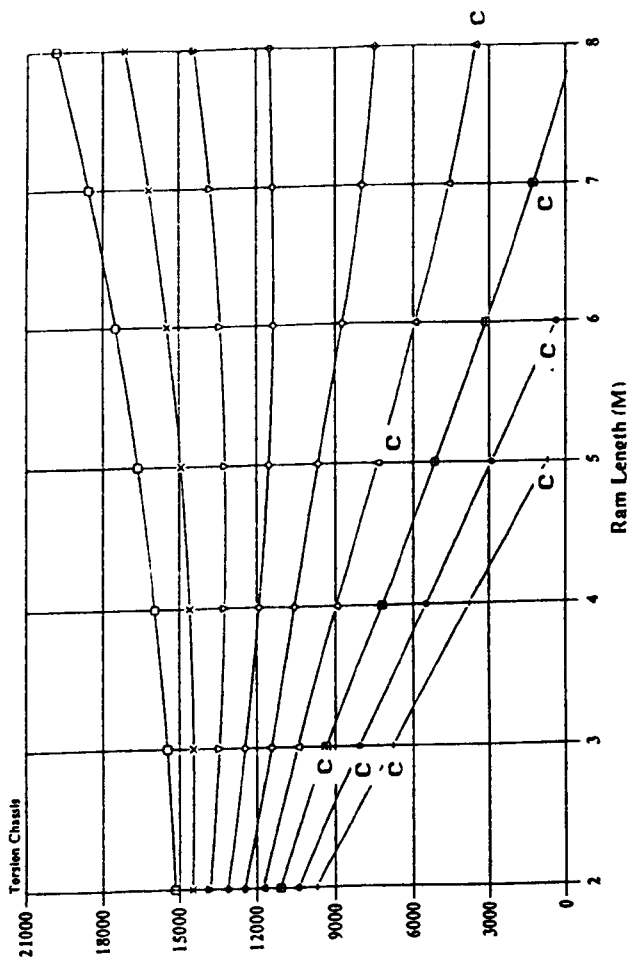
Fig. 5.2a Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 1 .

- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ▣ = 6° ground slope
- = 8° ground slope



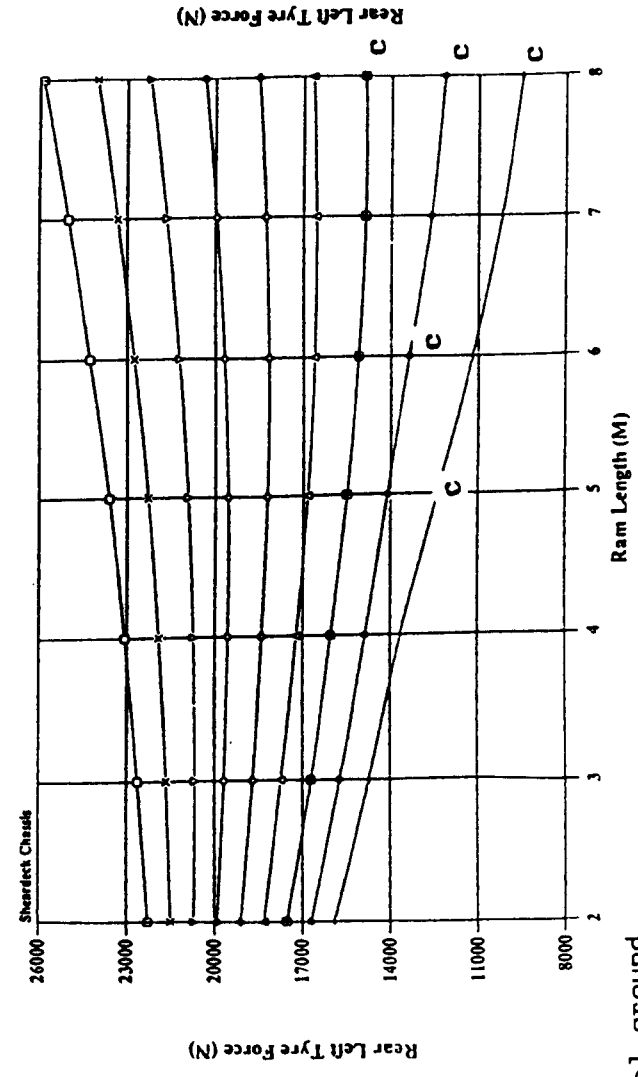
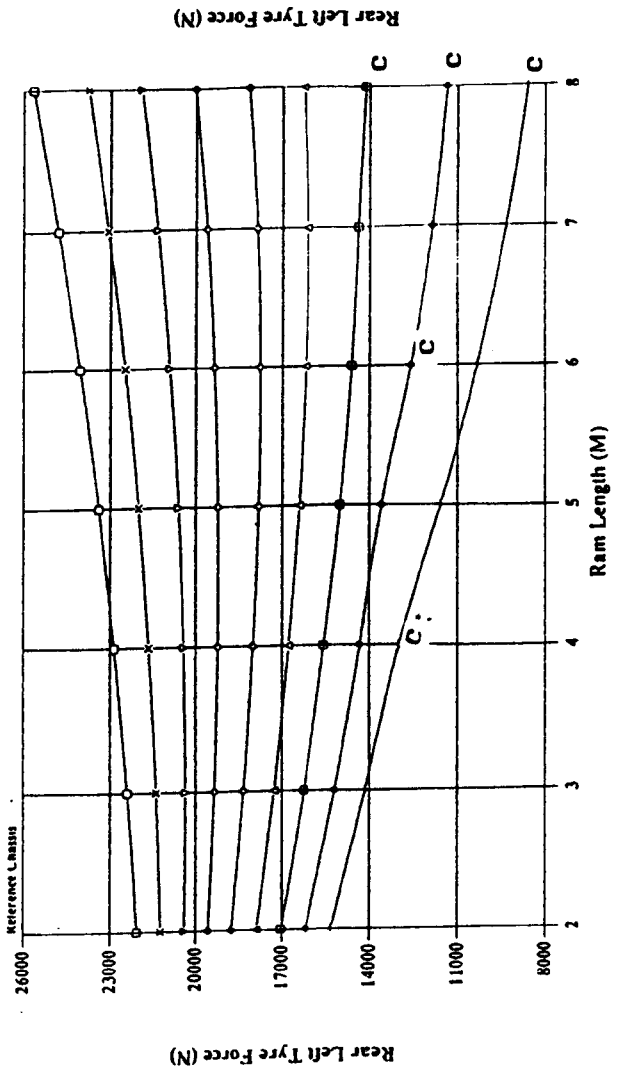
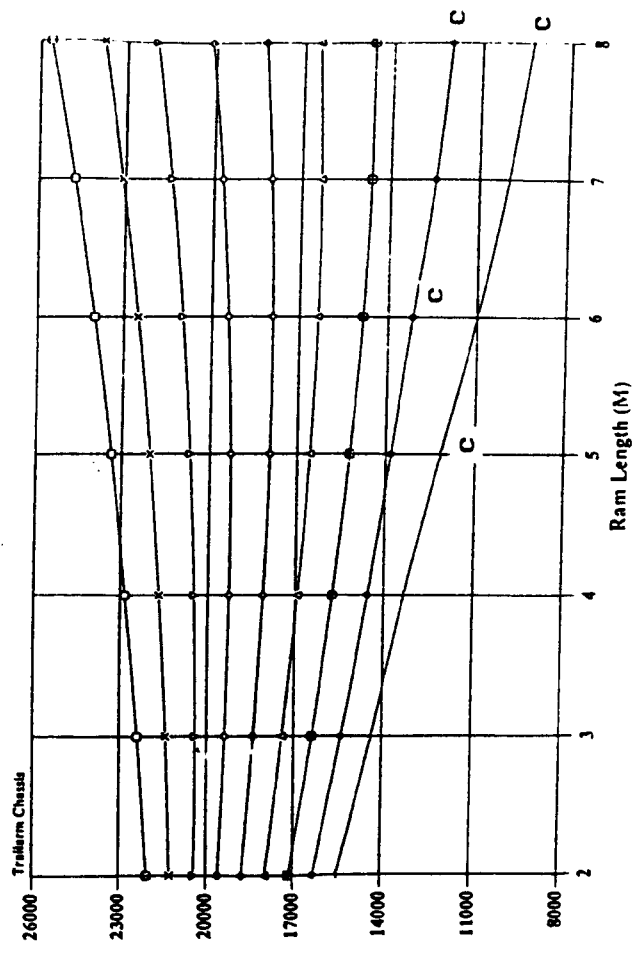
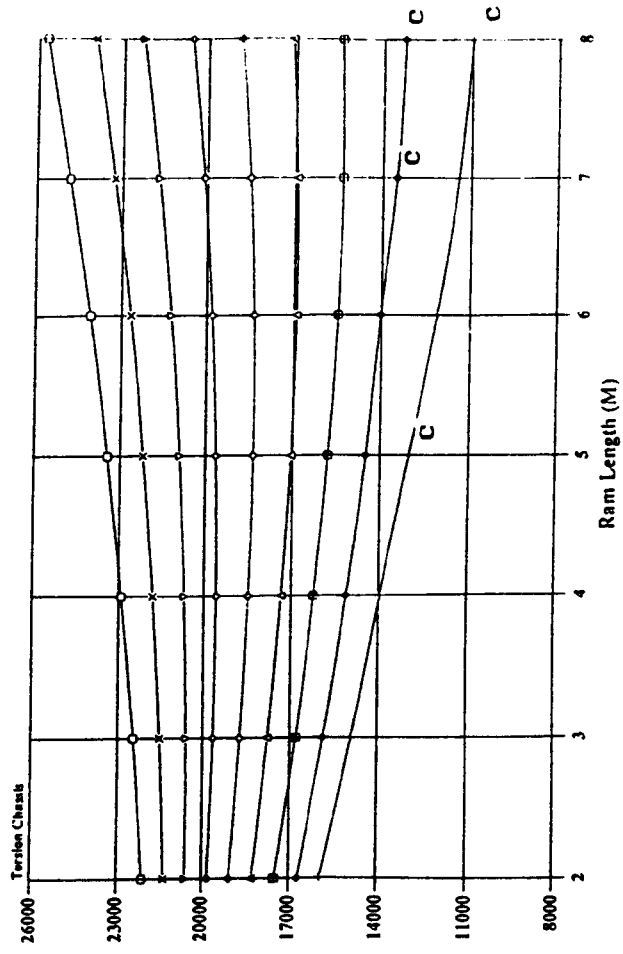
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ▣ = 6° ground slope
- ▤ = 8° ground slope

Fig. 5.2b Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 2 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

Fig. 5.2c Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 3 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- = 6° ground slope
- = 8° ground slope

Fig. 5.2d Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 4 .

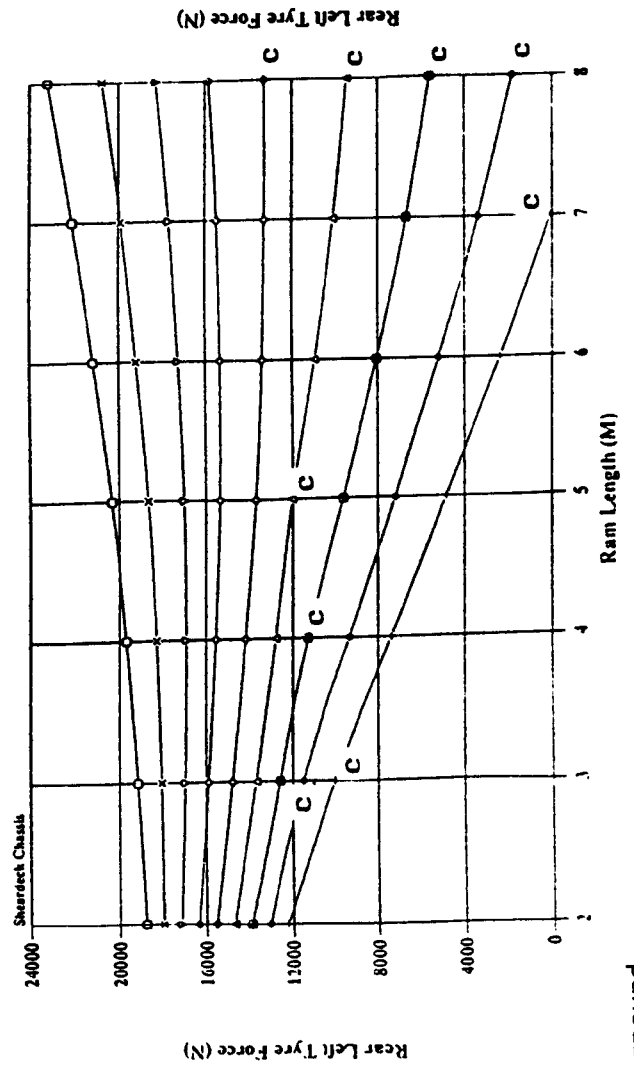
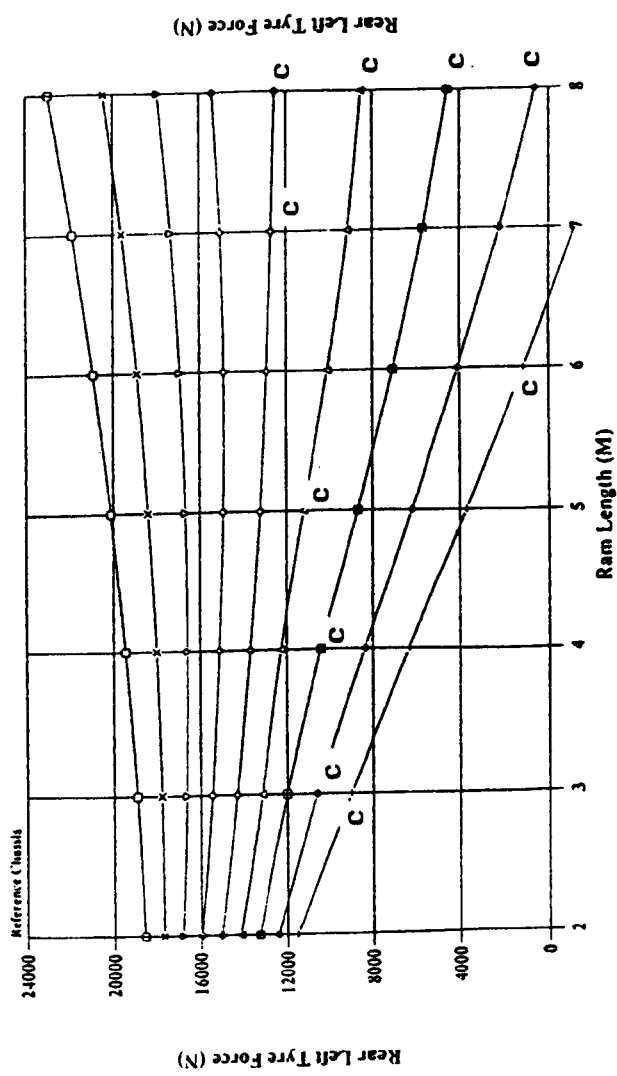
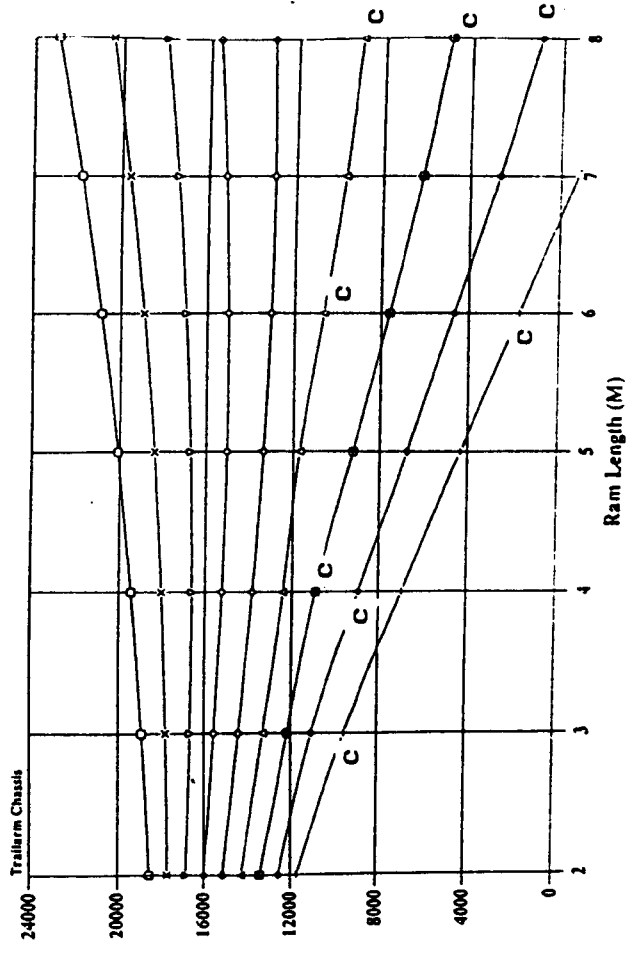
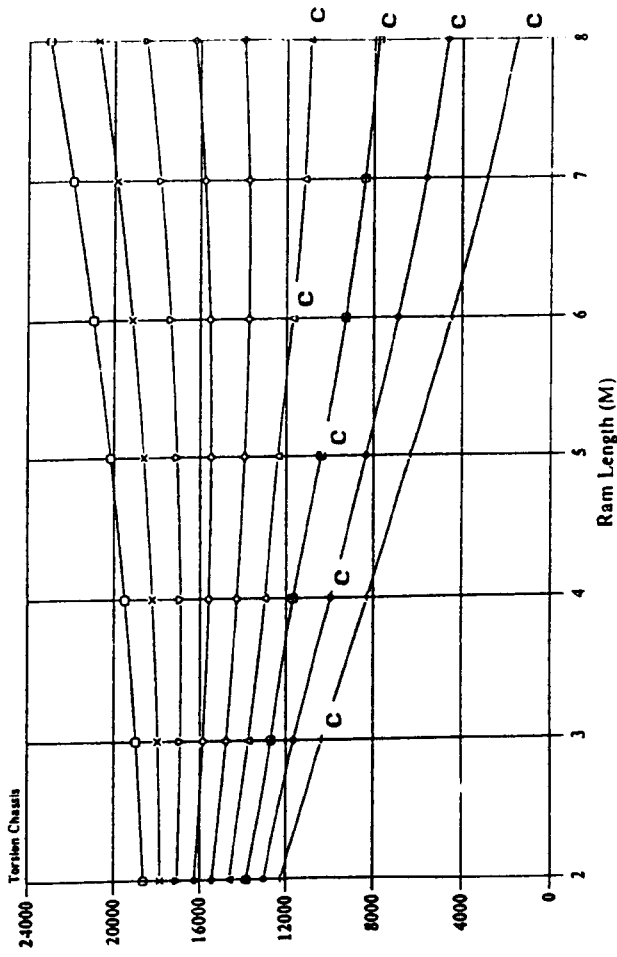
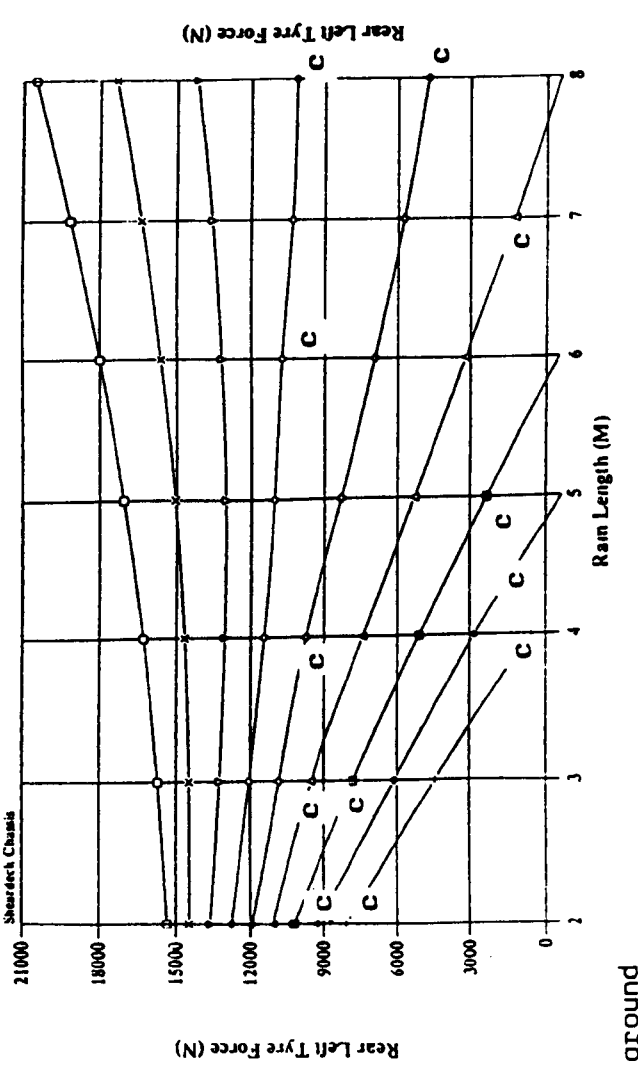
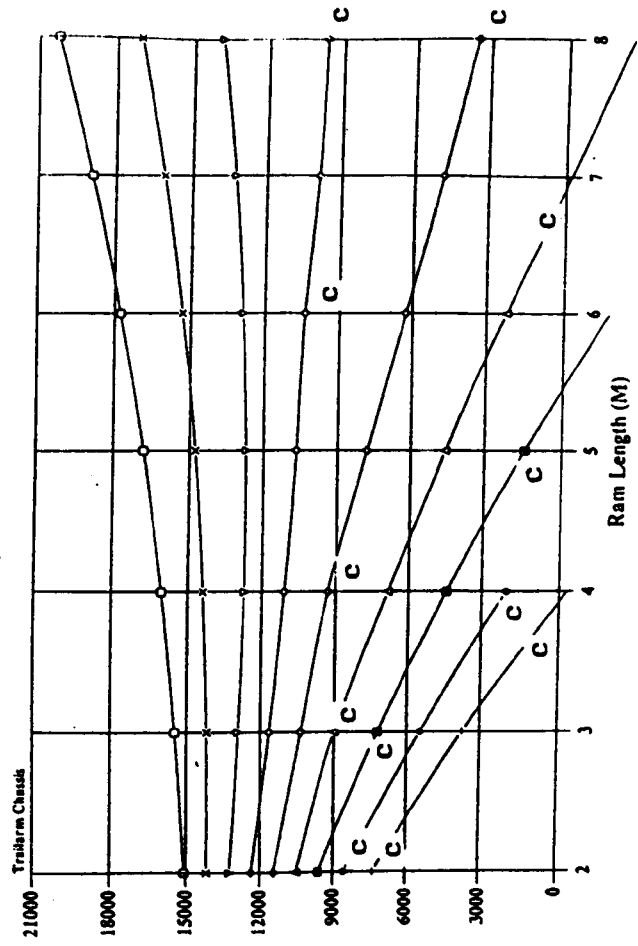
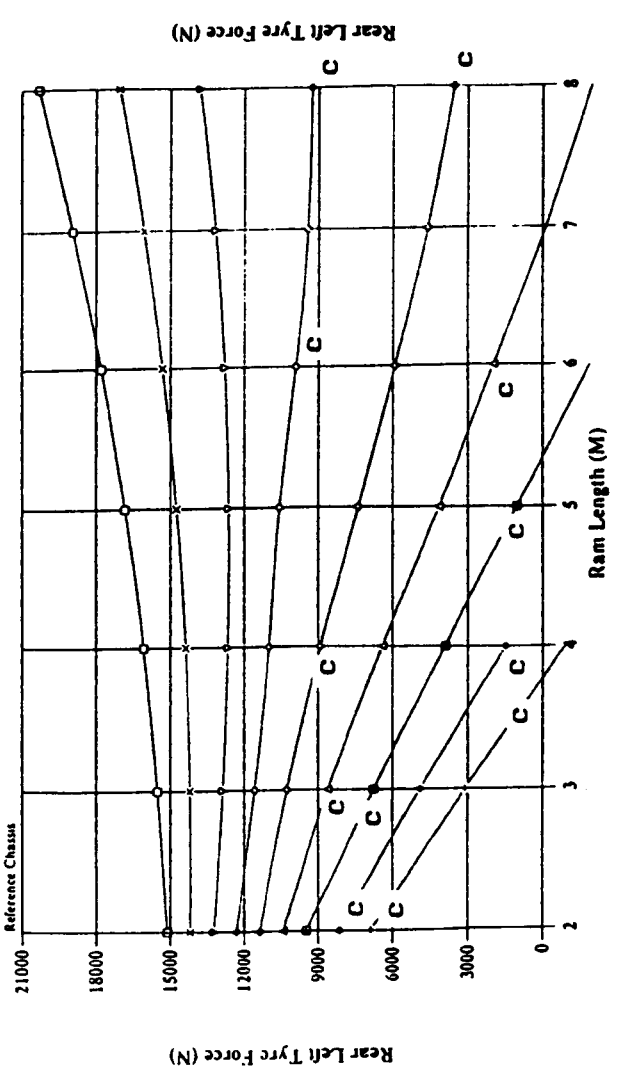
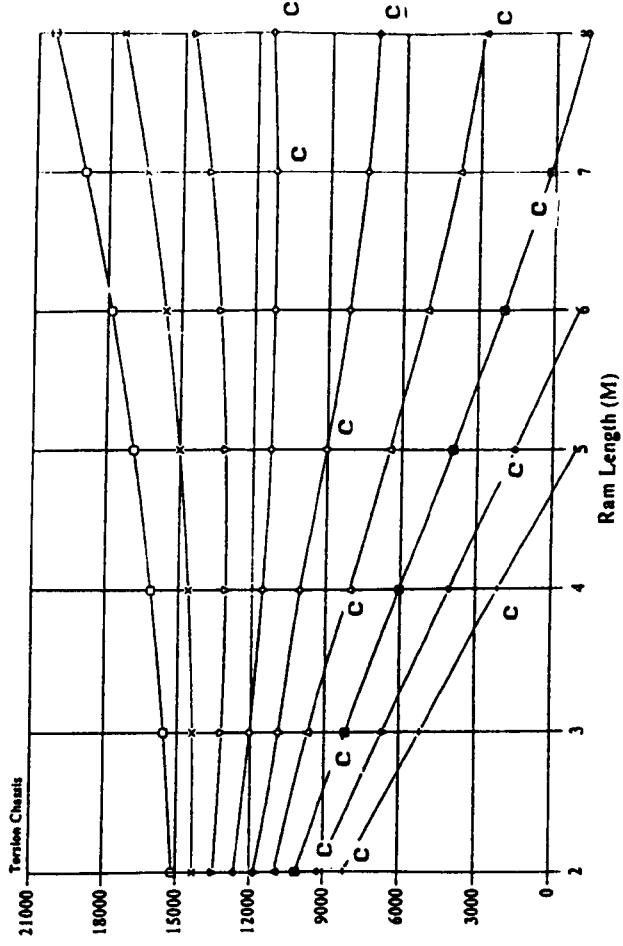


Fig. 5.2c Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 5 .

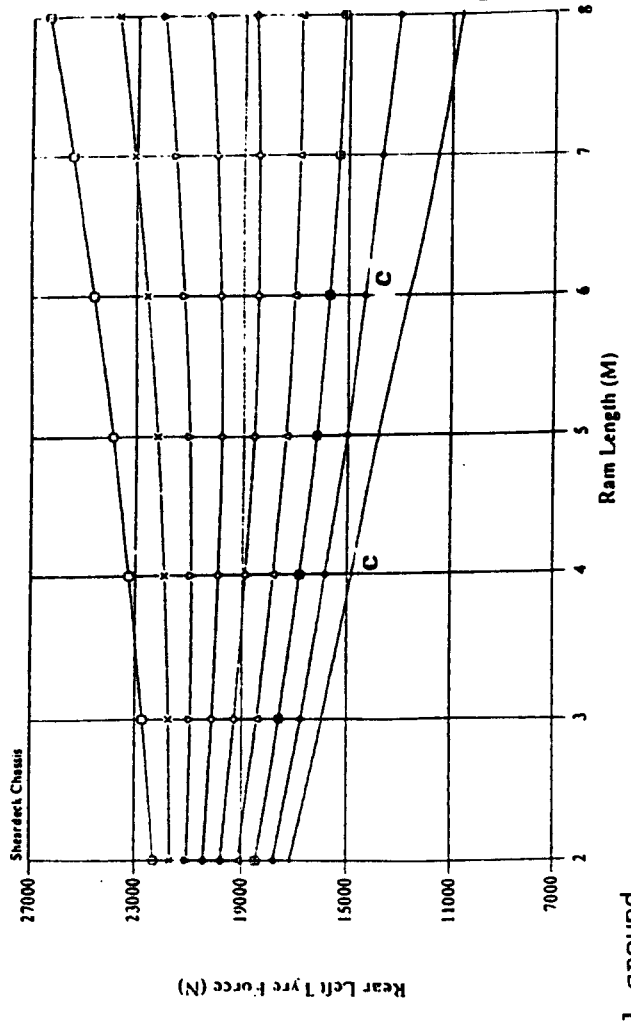
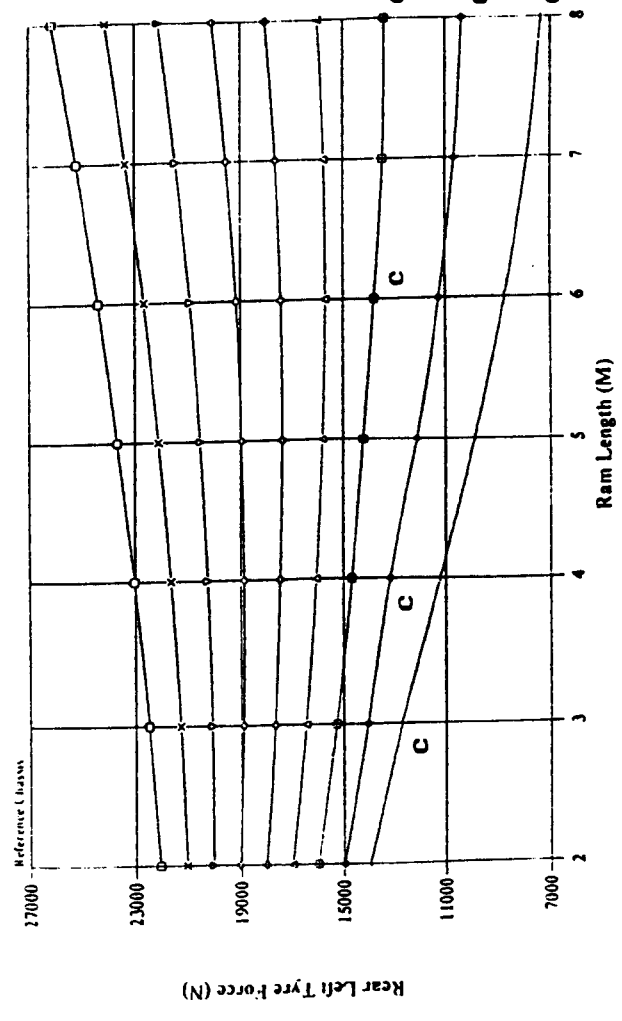
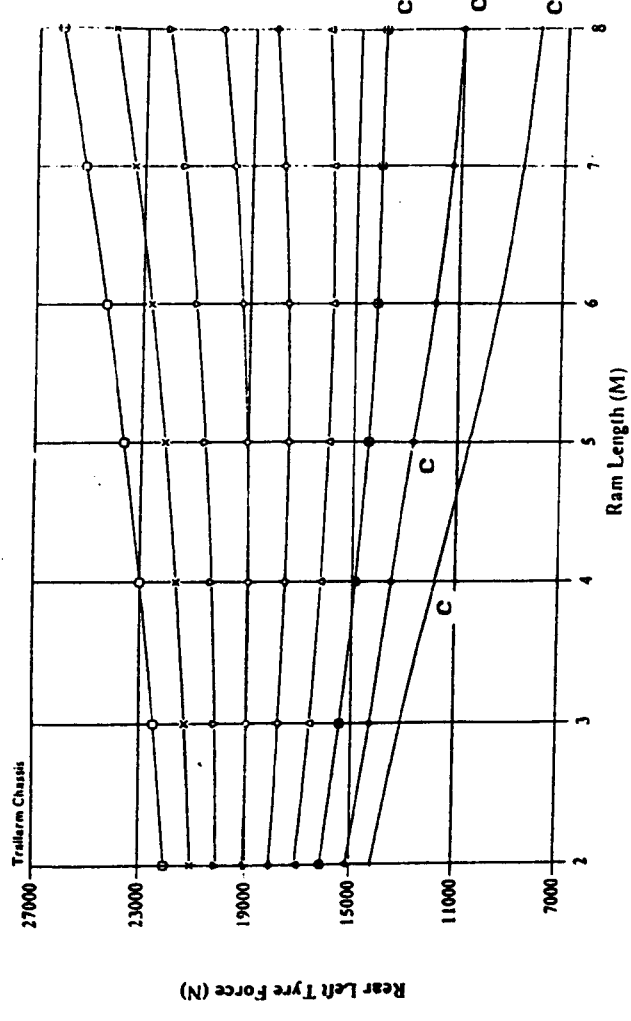
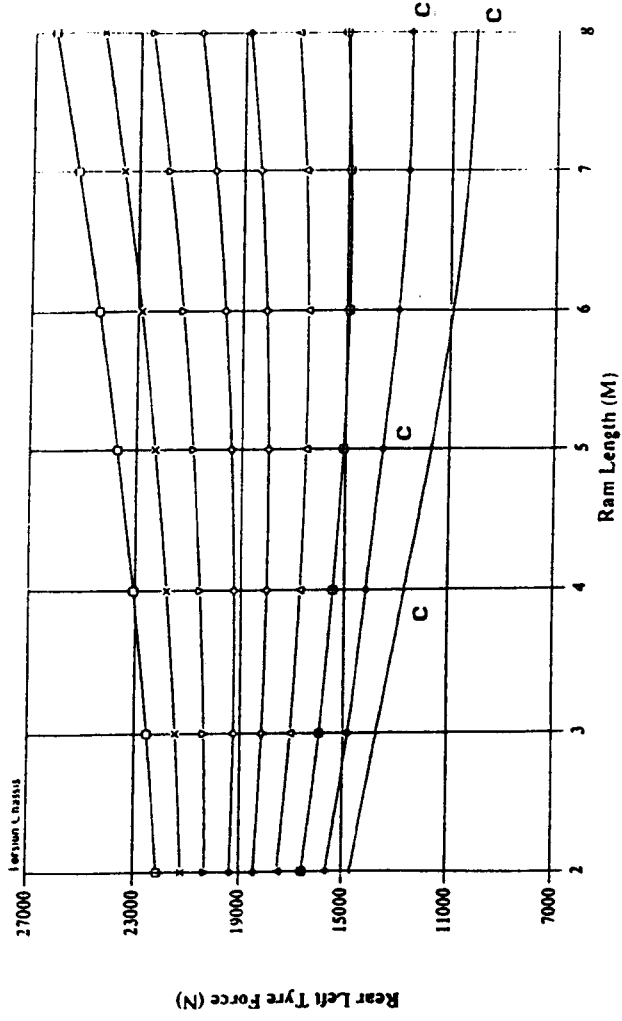
- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope





- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- ∗ = 8° ground slope

Fig. 5.2f Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 6 .



- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

Fig. 5.2g Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 7 .

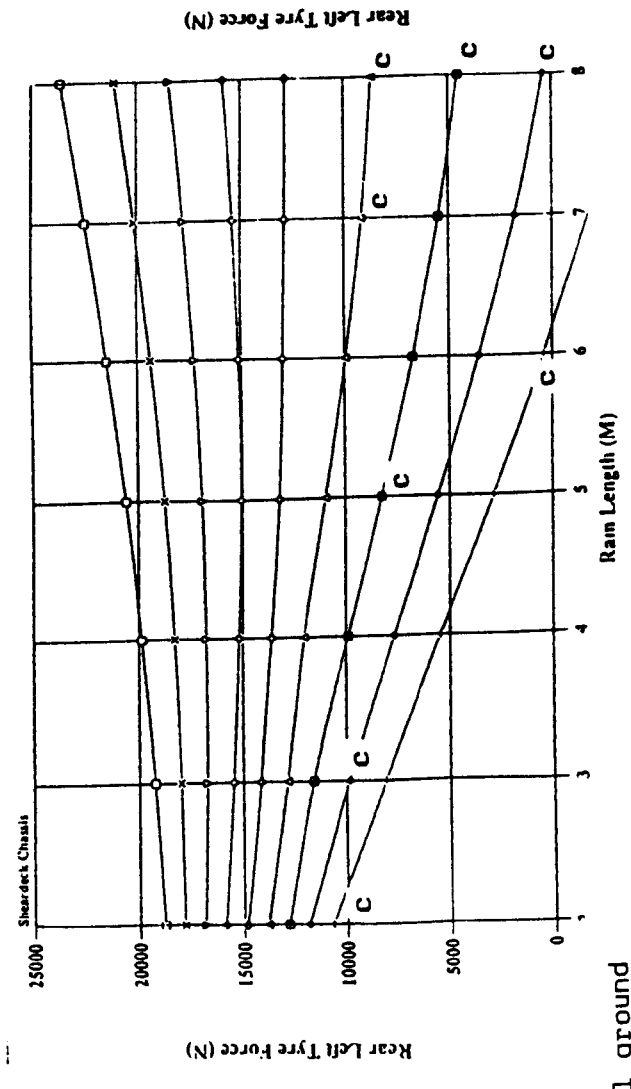
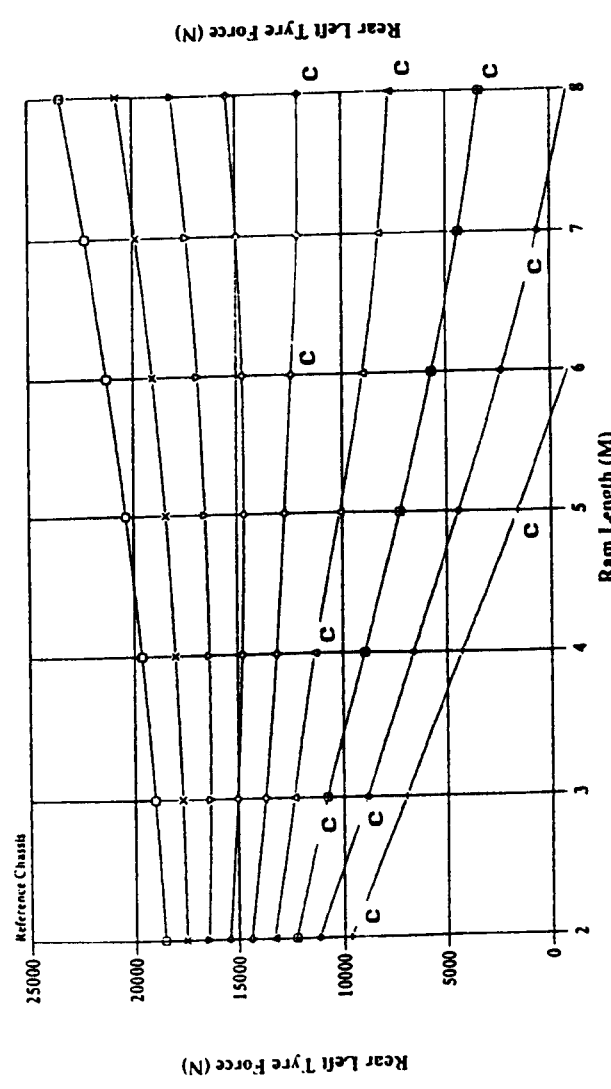
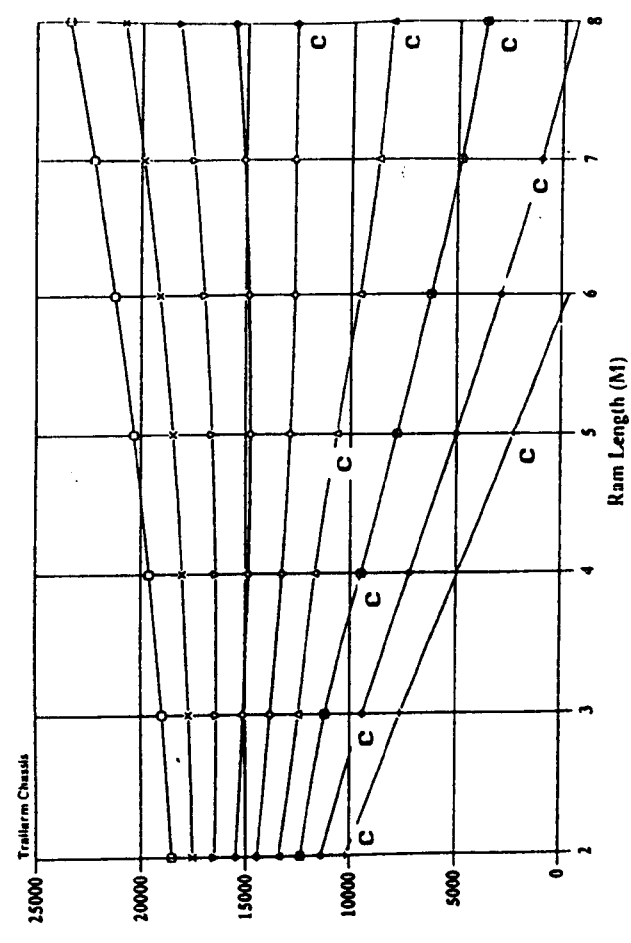
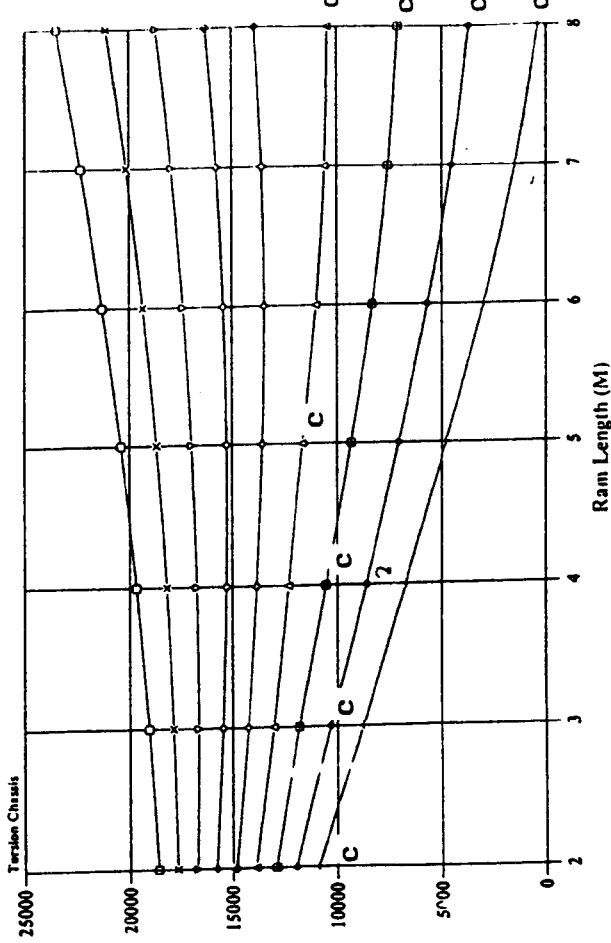


Fig. 5.2h Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 8 .

□ = level ground  
 ▽ = 2° ground slope  
 ◇ = 4° ground slope  
 ⊠ = 6° ground slope  
 — = 8° ground slope

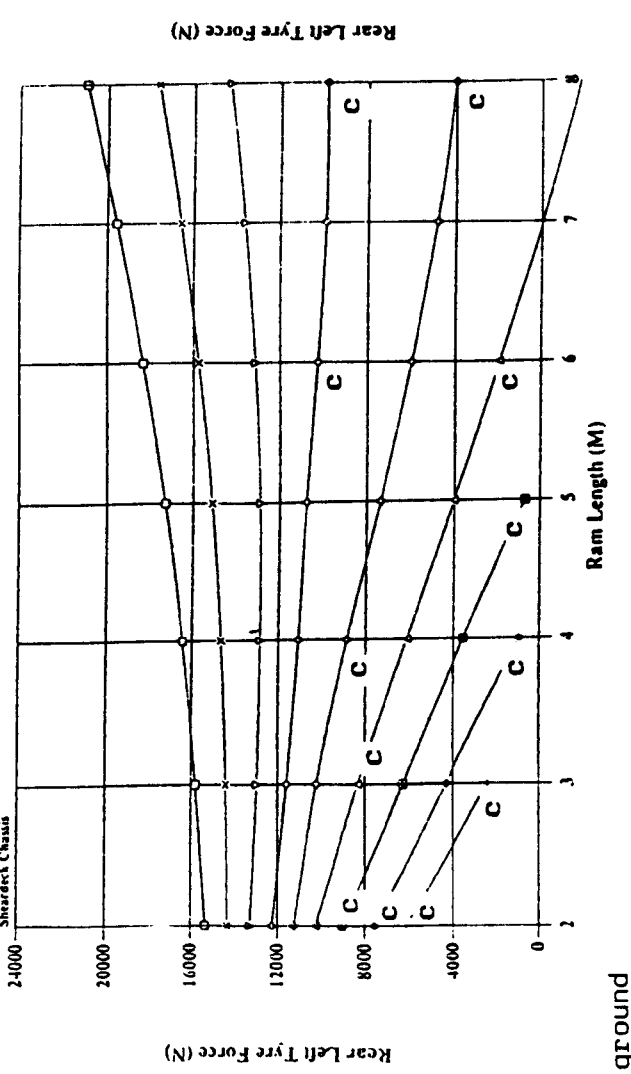
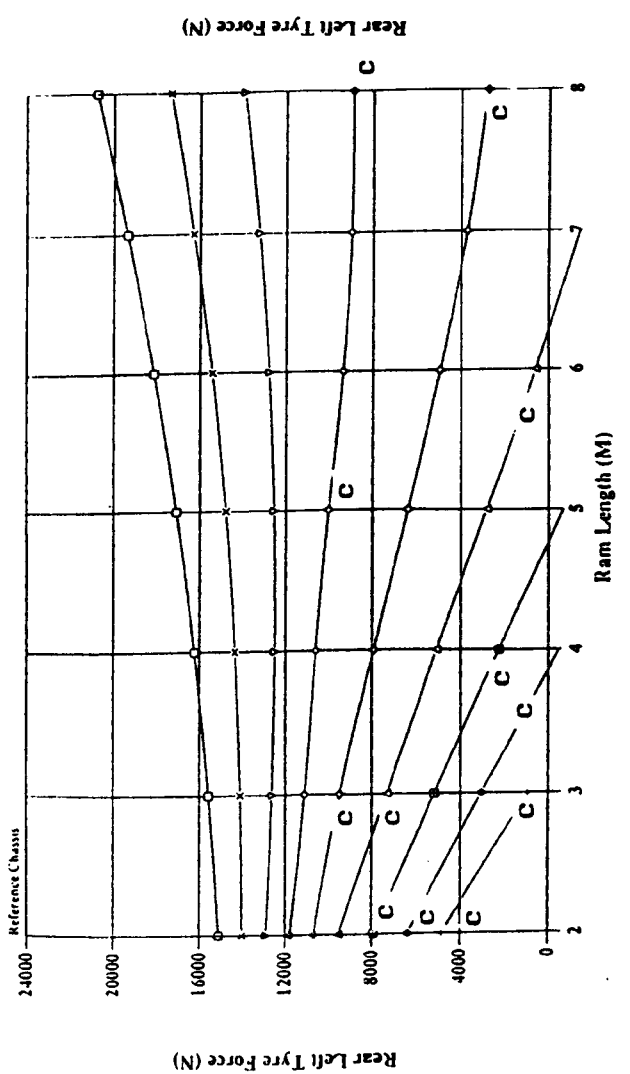
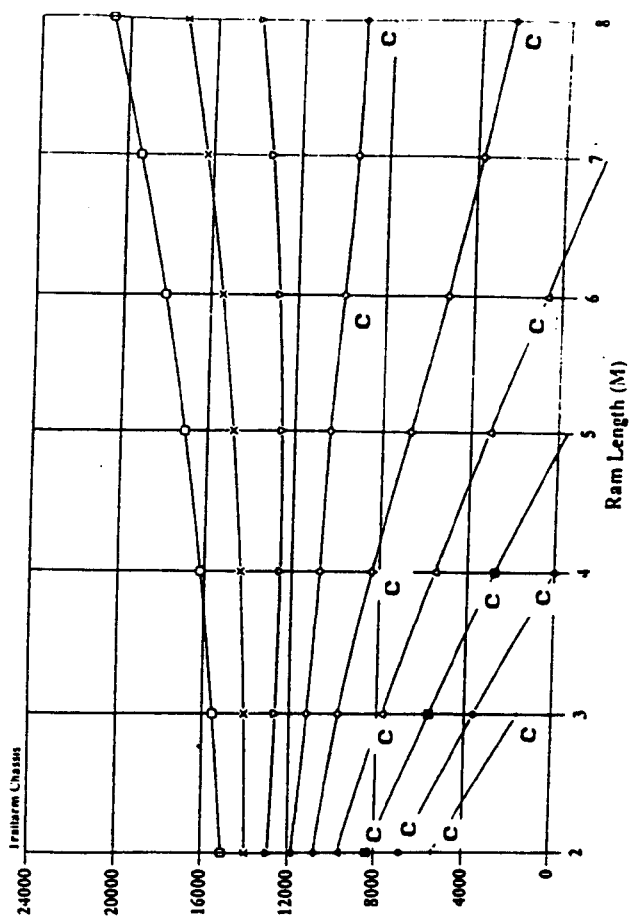
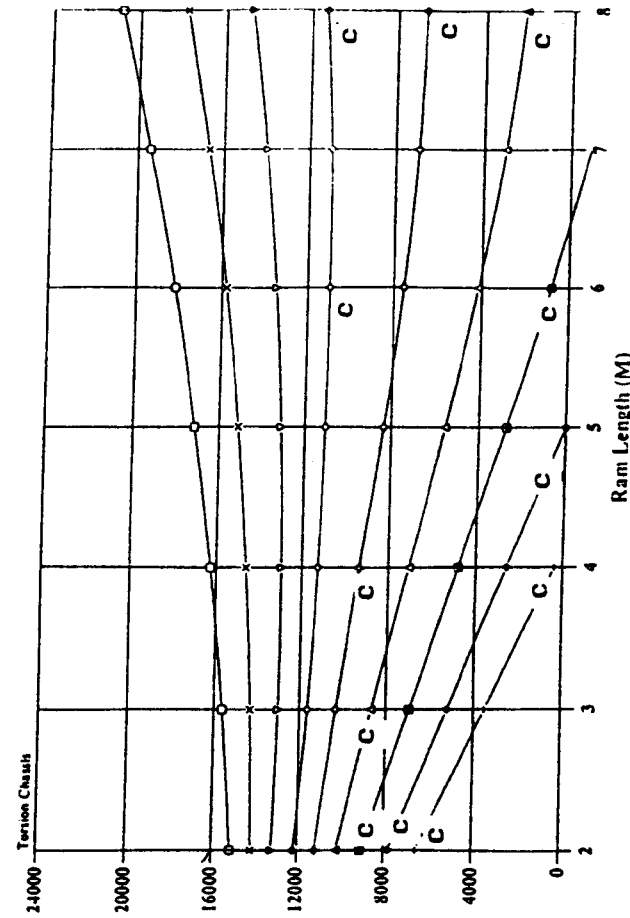
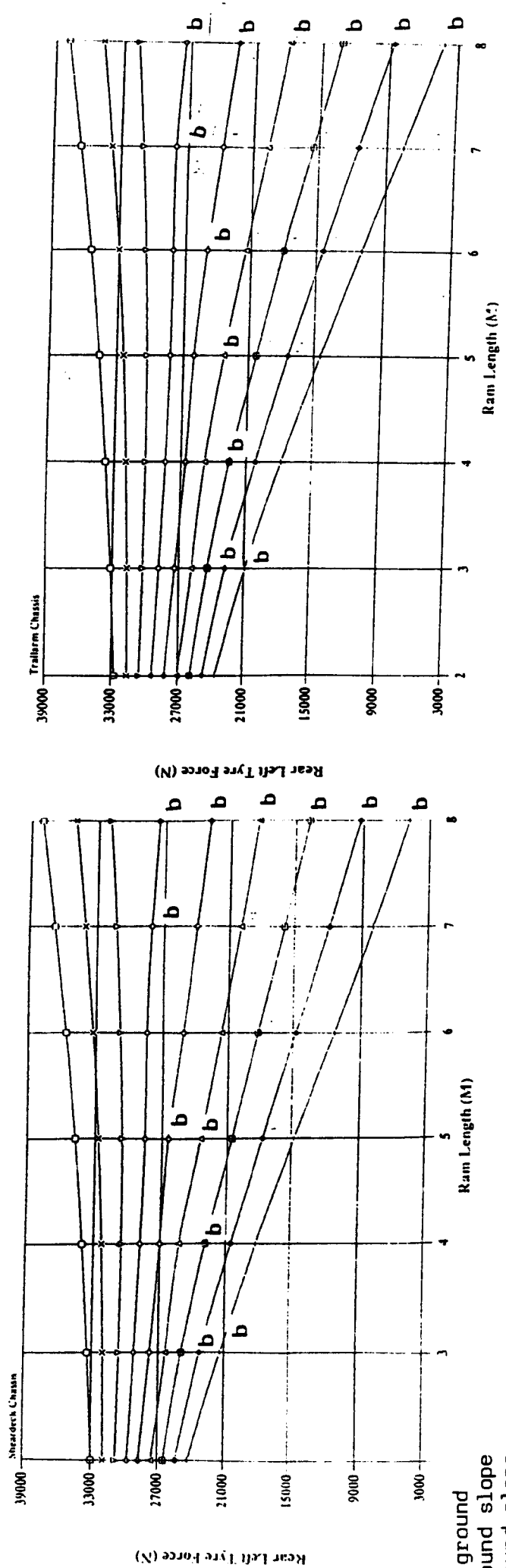
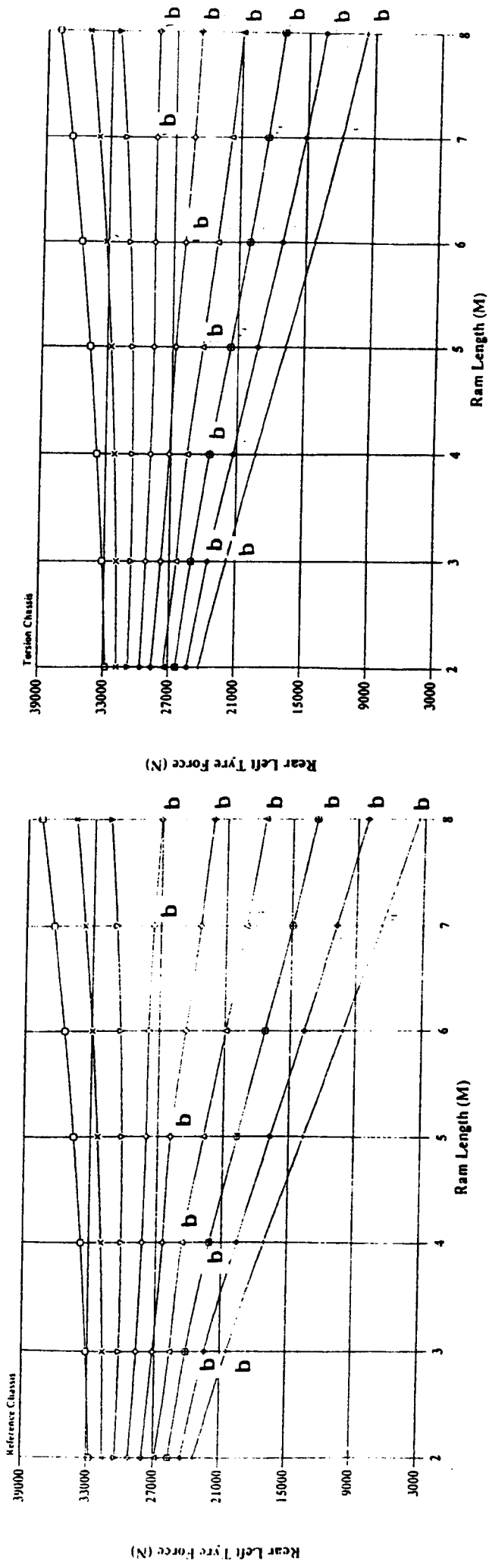


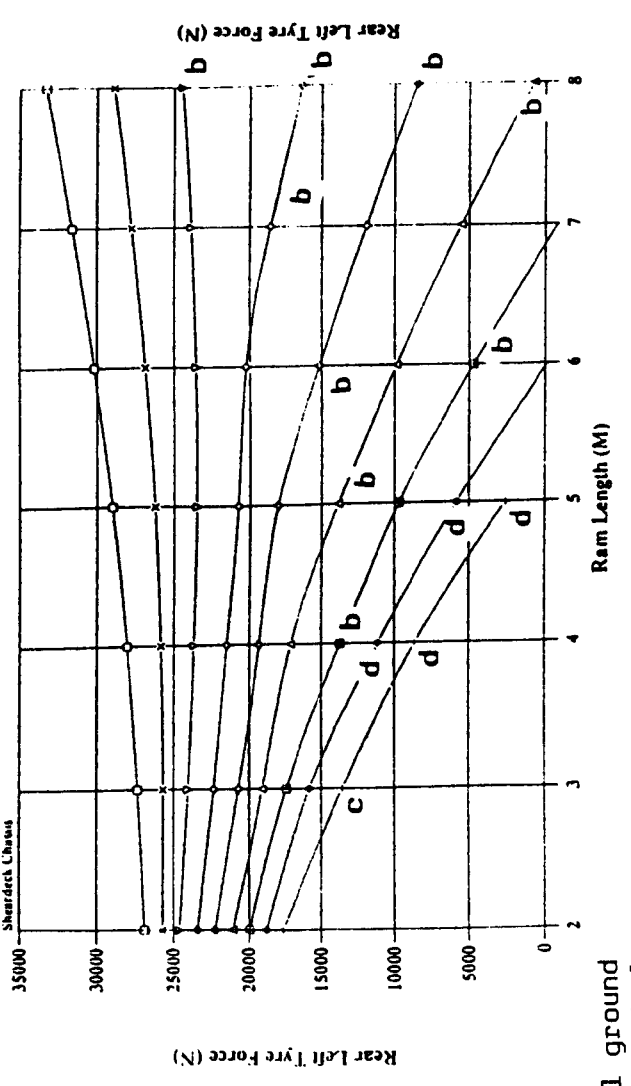
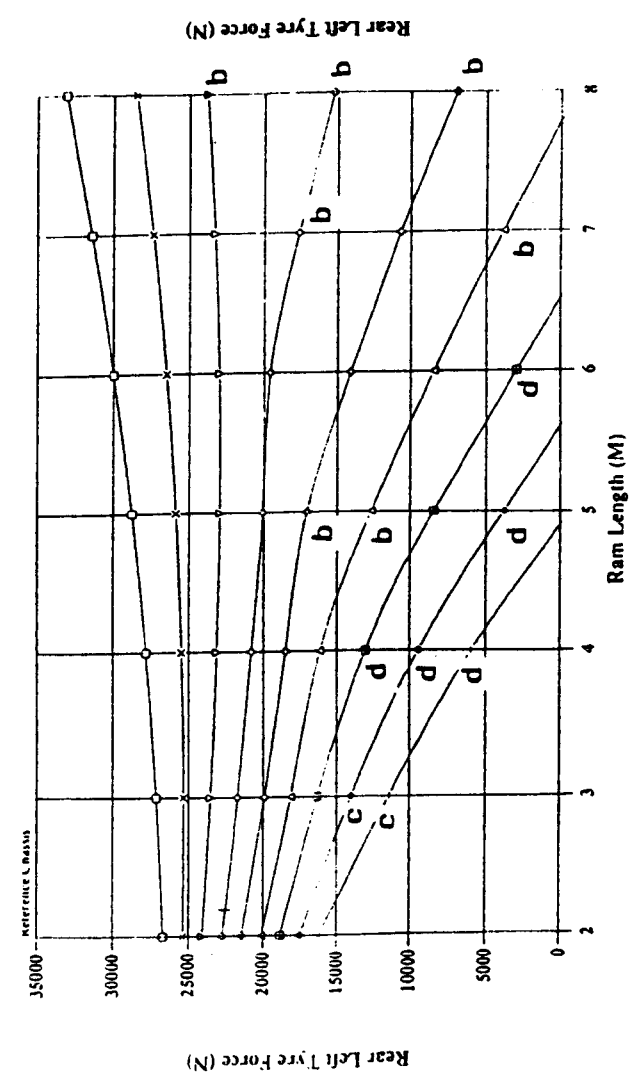
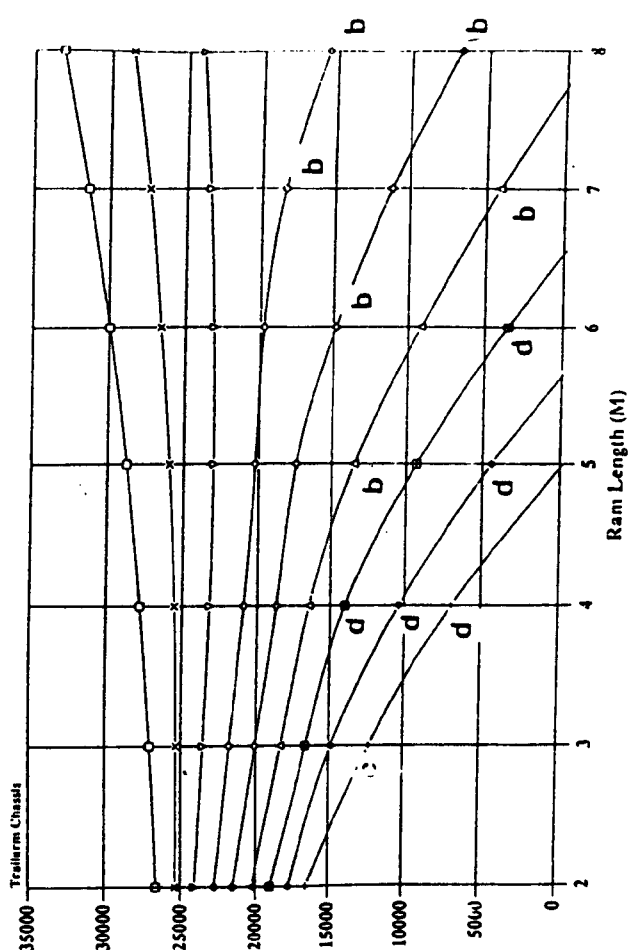
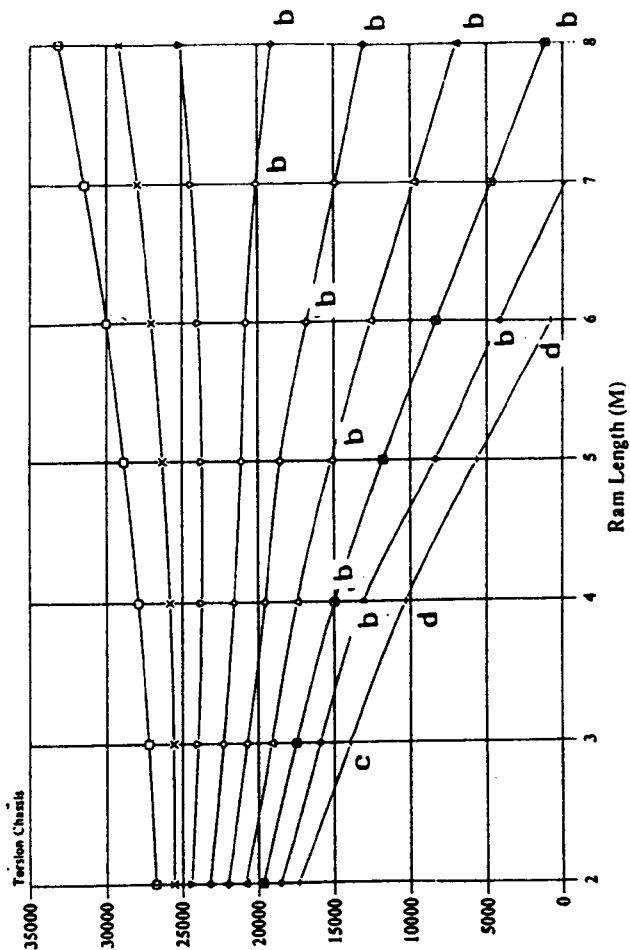
Fig. 5.2i Rear left tyre force versus Ram length and ground slope combinations for a 10000 Kg payload in body position 9 .

- = level ground
- △ = 2° ground slope
- = 4° ground slope
- ◇ = 6° ground slope
- × = 8° ground slope



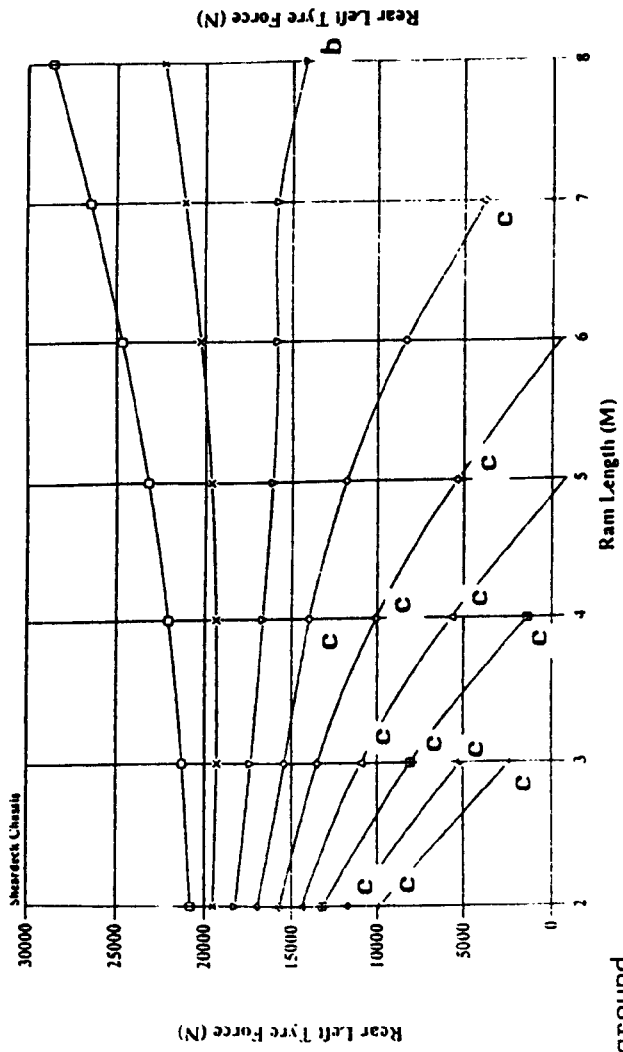
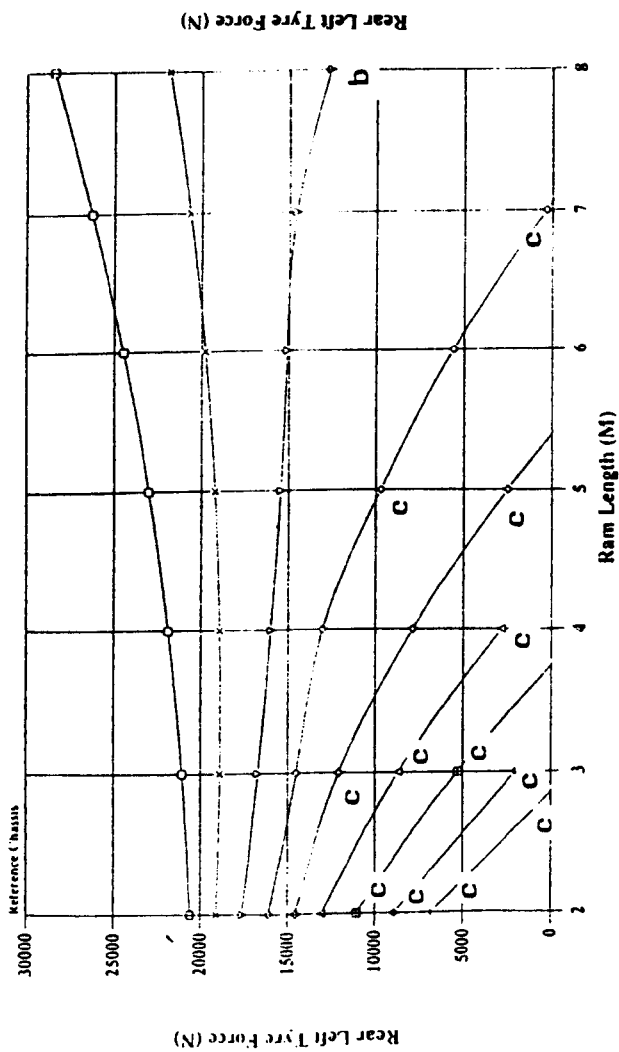
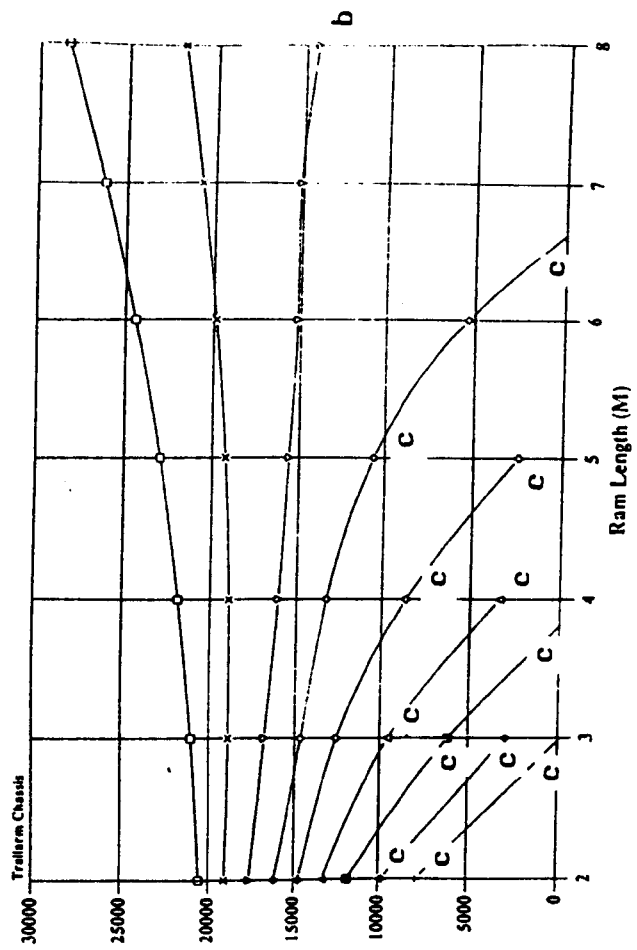
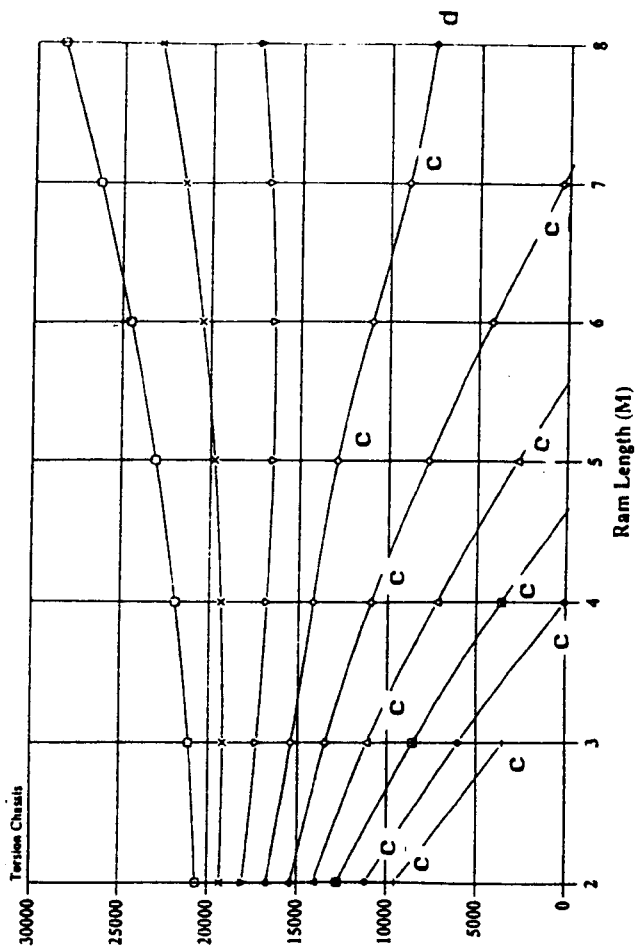
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊗ = 6° ground slope
- = 8° ground slope

Fig. 5.3a Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 1.



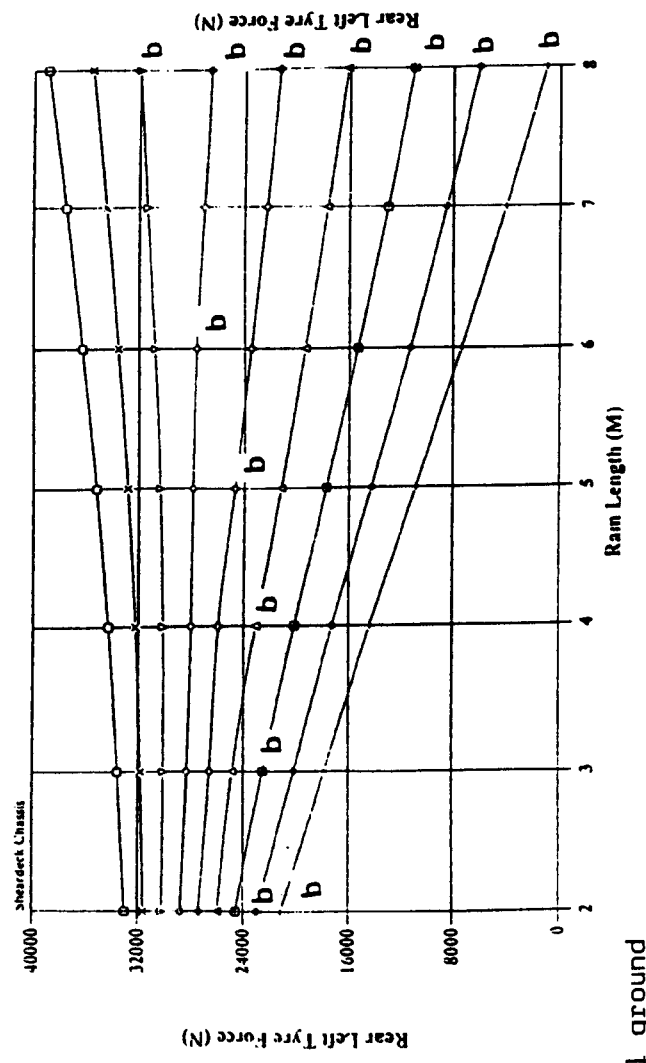
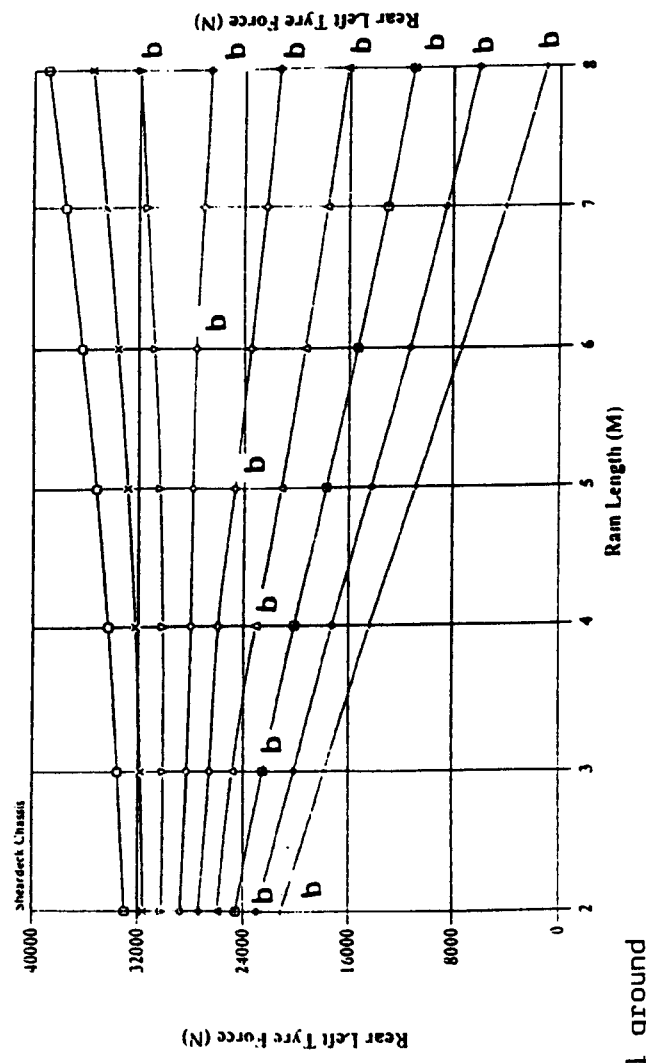
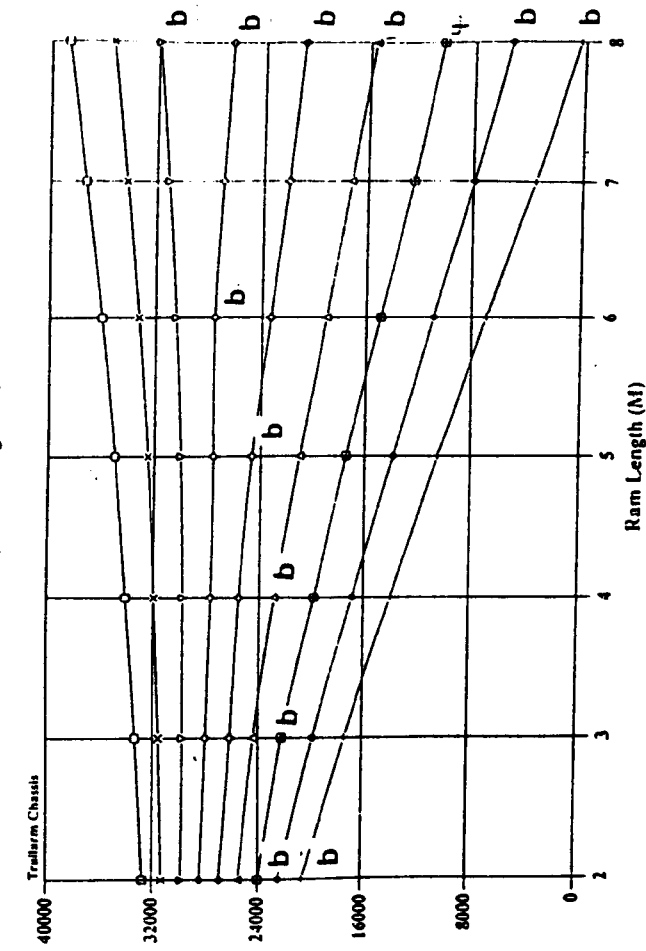
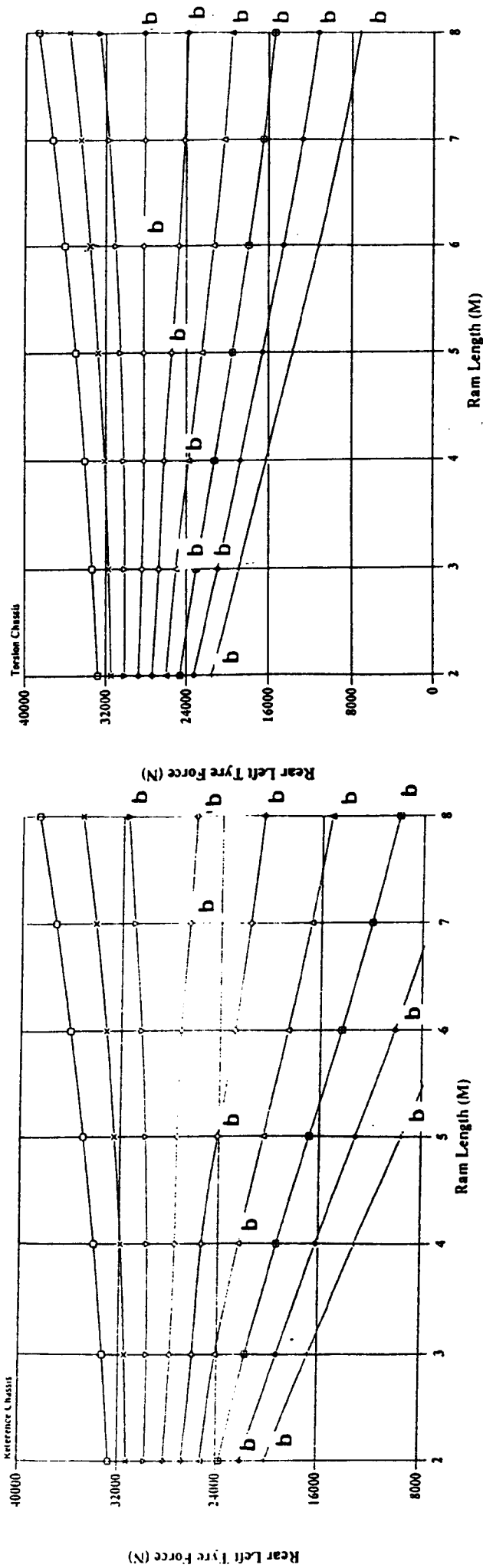
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ◻ = 6° ground slope
- = 8° ground slope

Fig. 5.3b Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 2 .



- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- ▣ = 6° ground slope
- = 8° ground slope

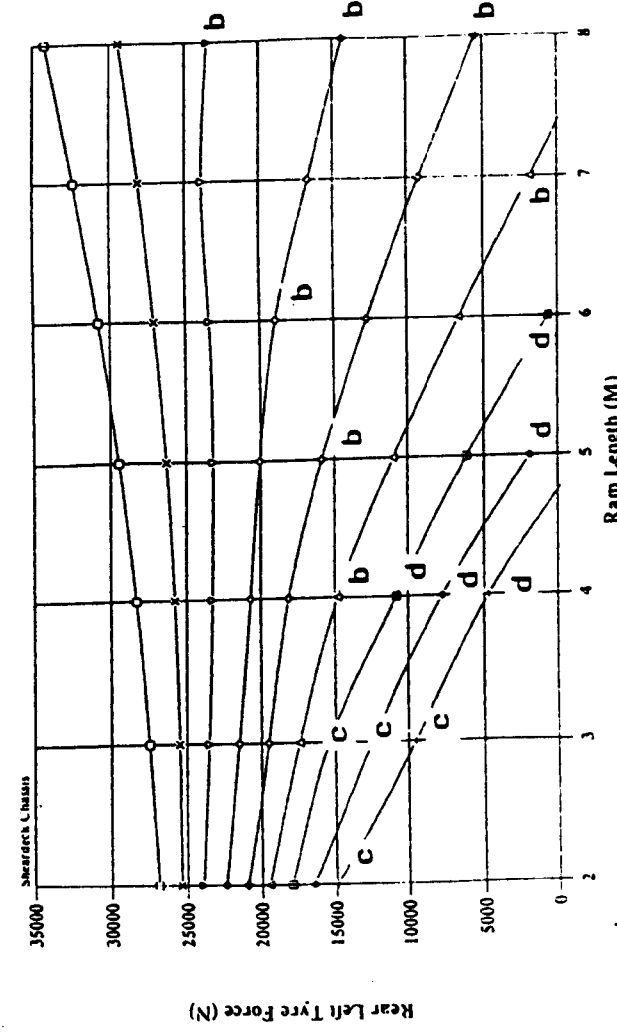
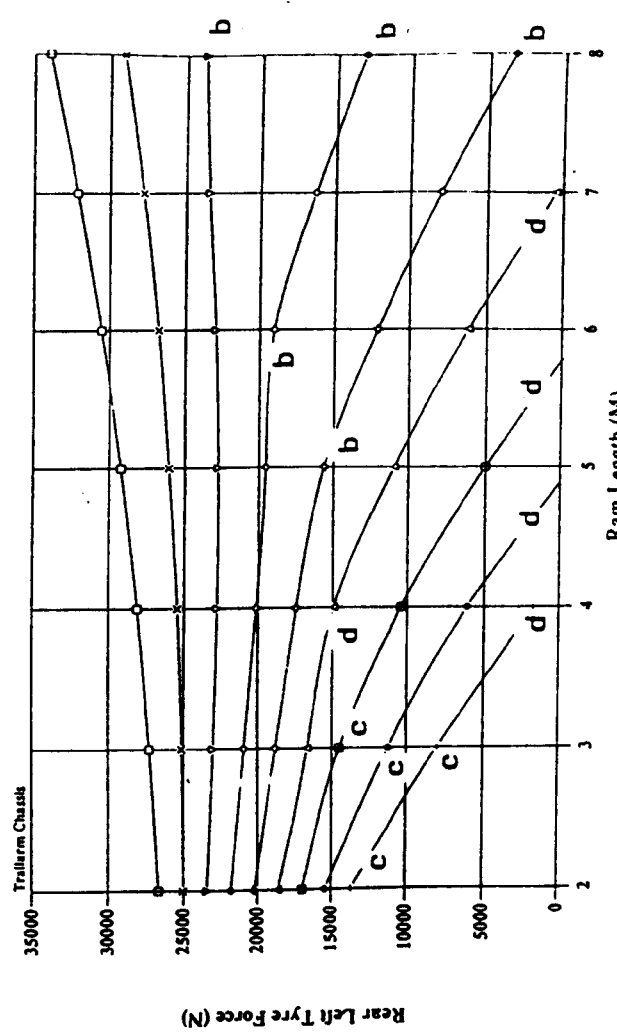
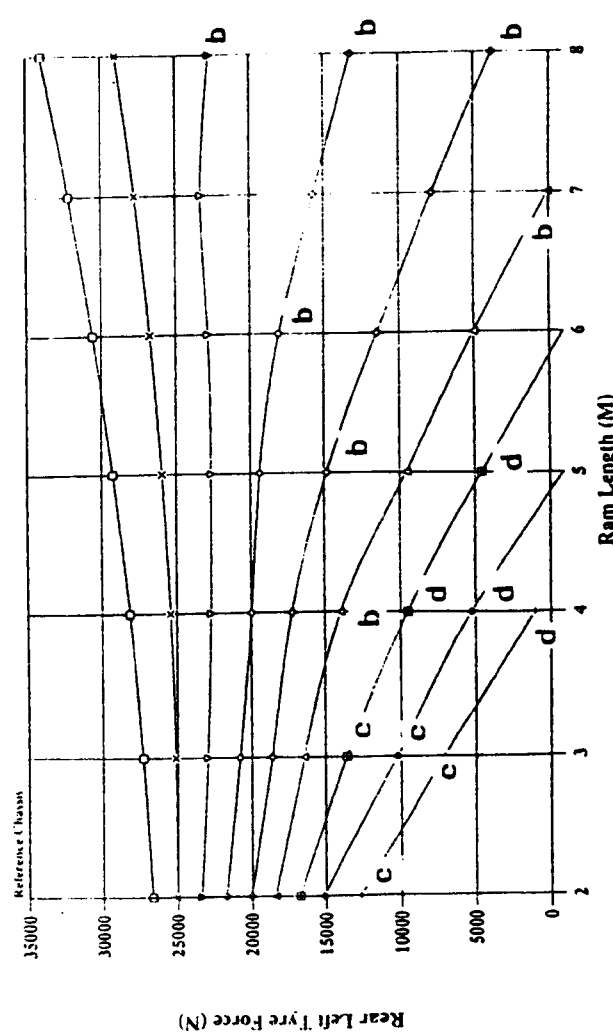
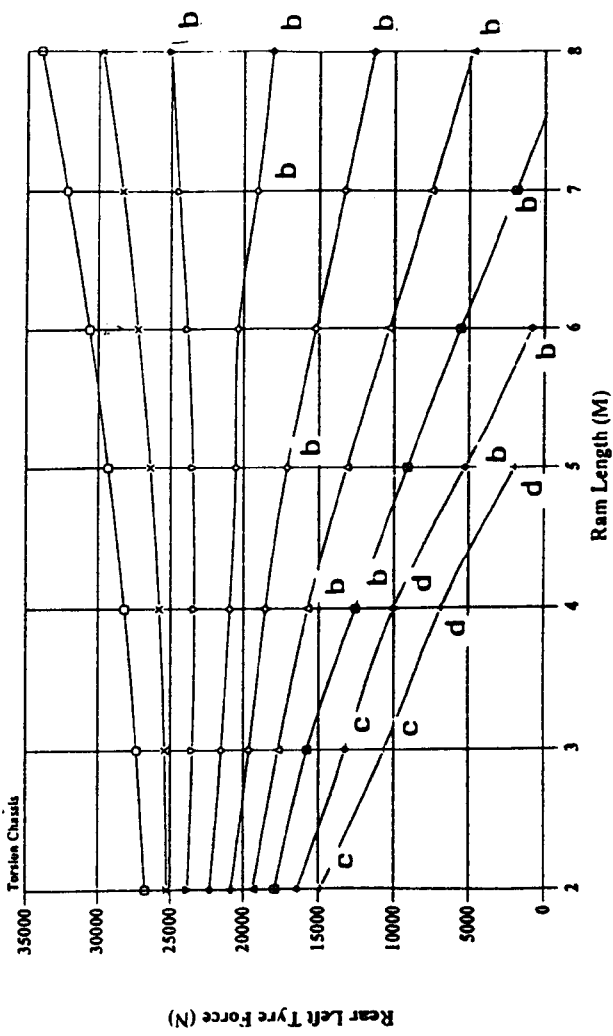
Fig. 5.3c Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 3 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- = 6° ground slope
- × = 8° ground slope

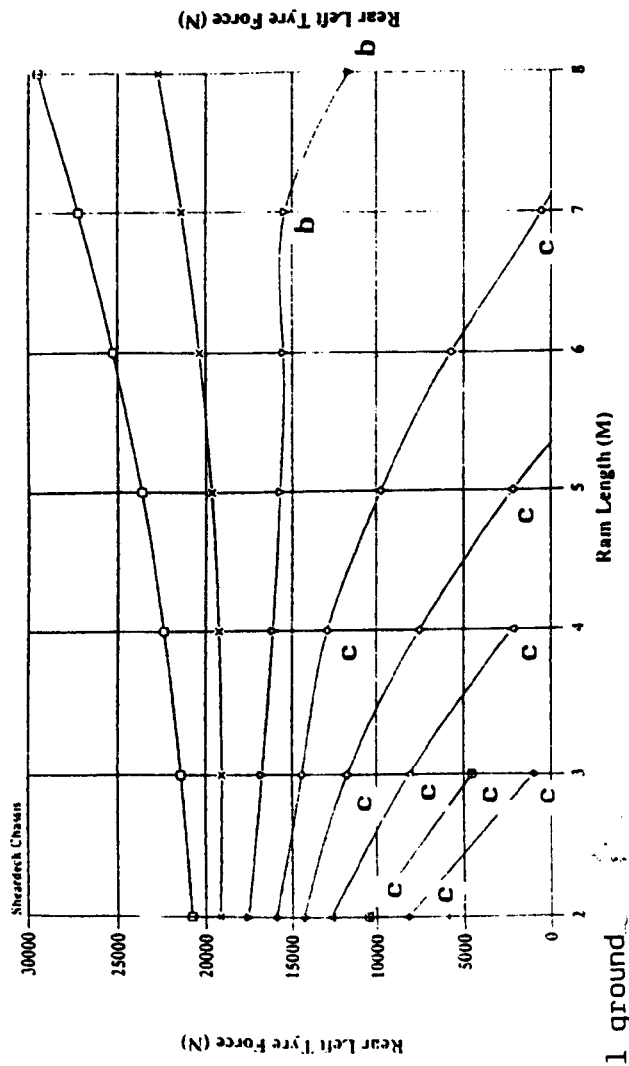
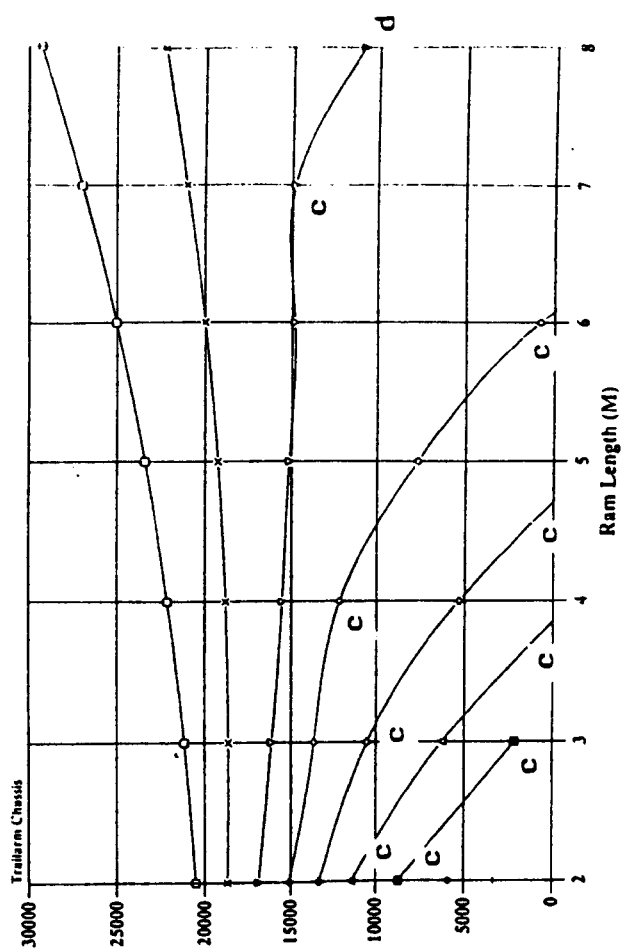
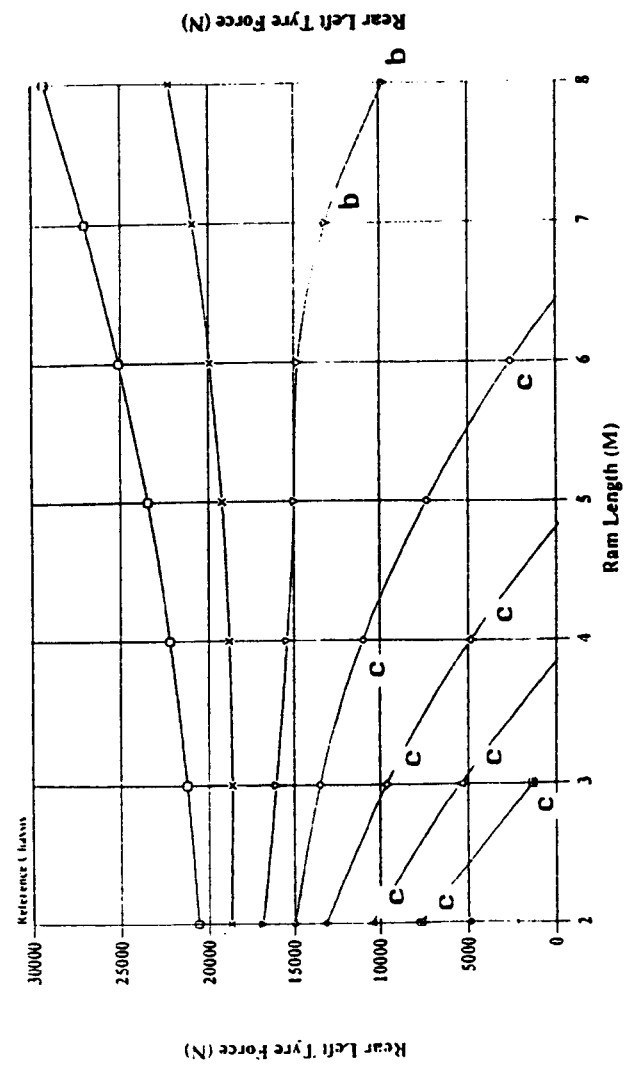
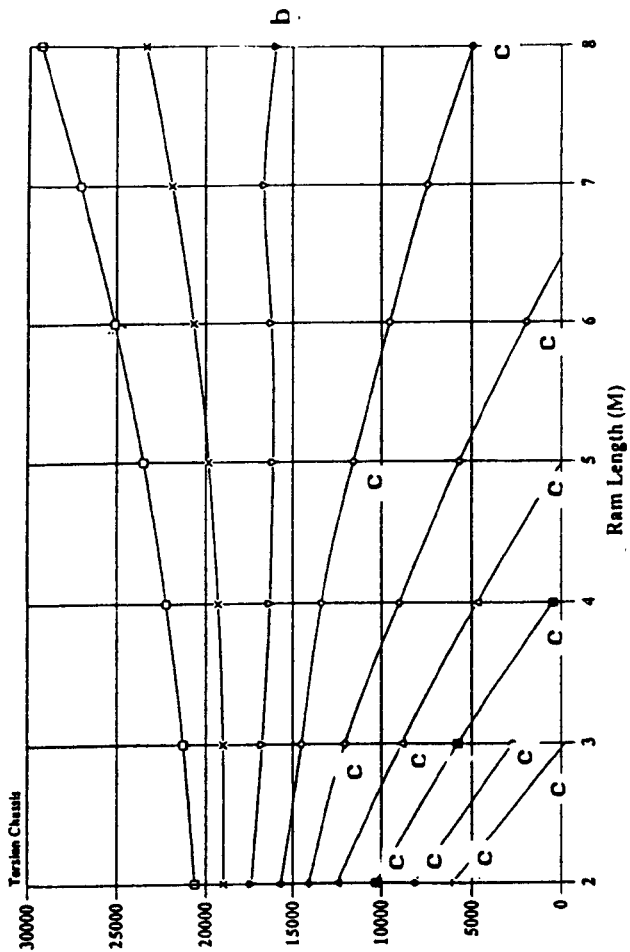
Fig. 5.3d Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 4 .





- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

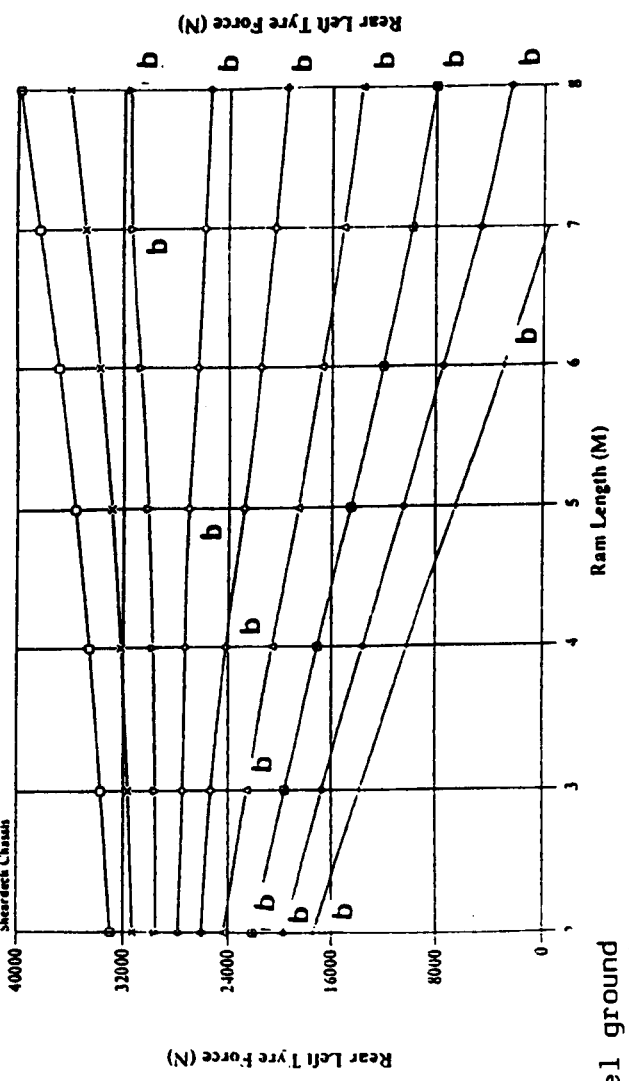
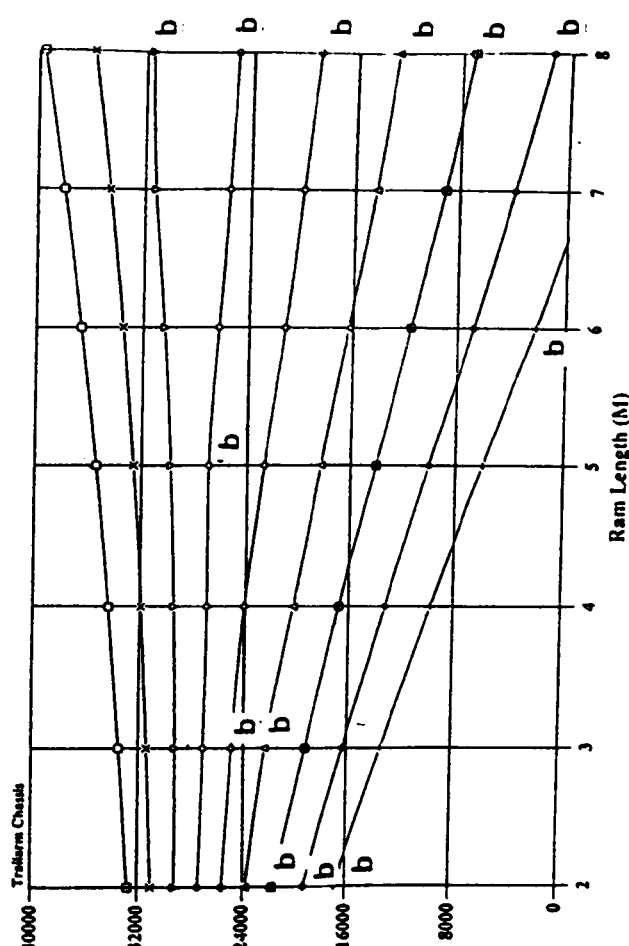
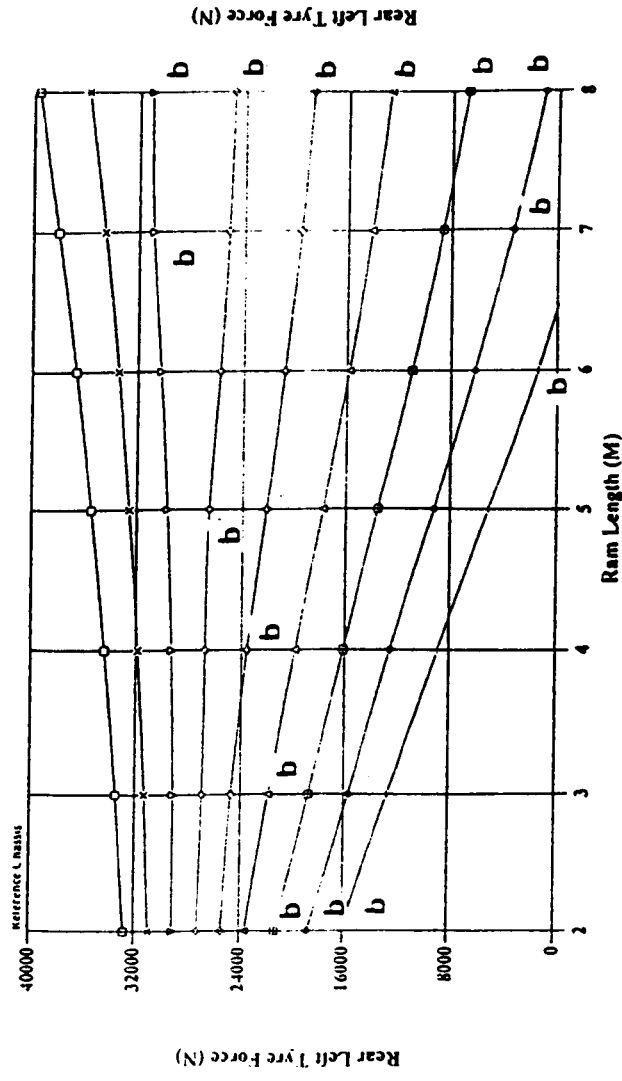
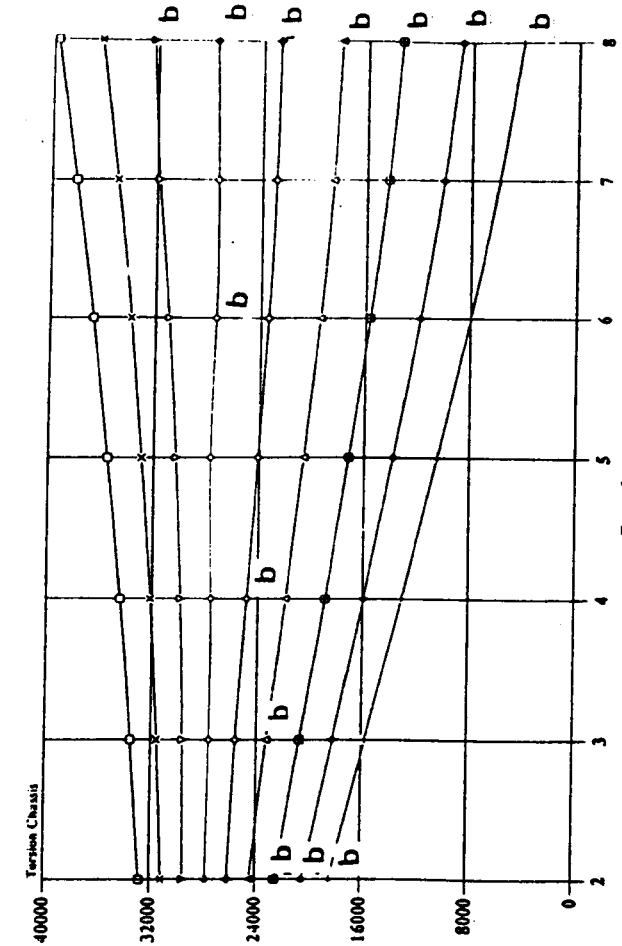
Fig. 5.3e Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 5 .



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

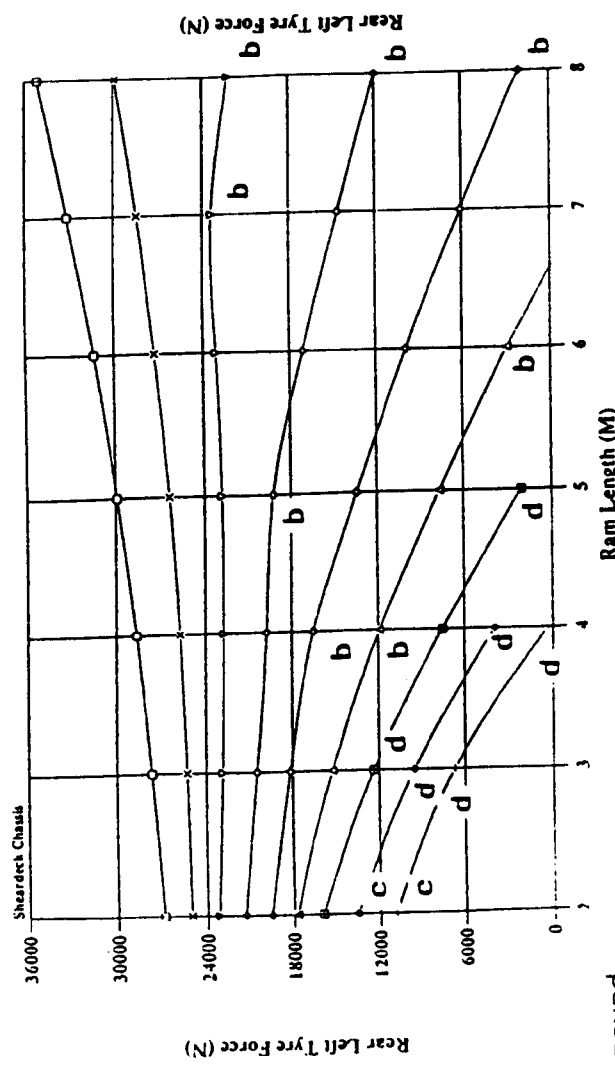
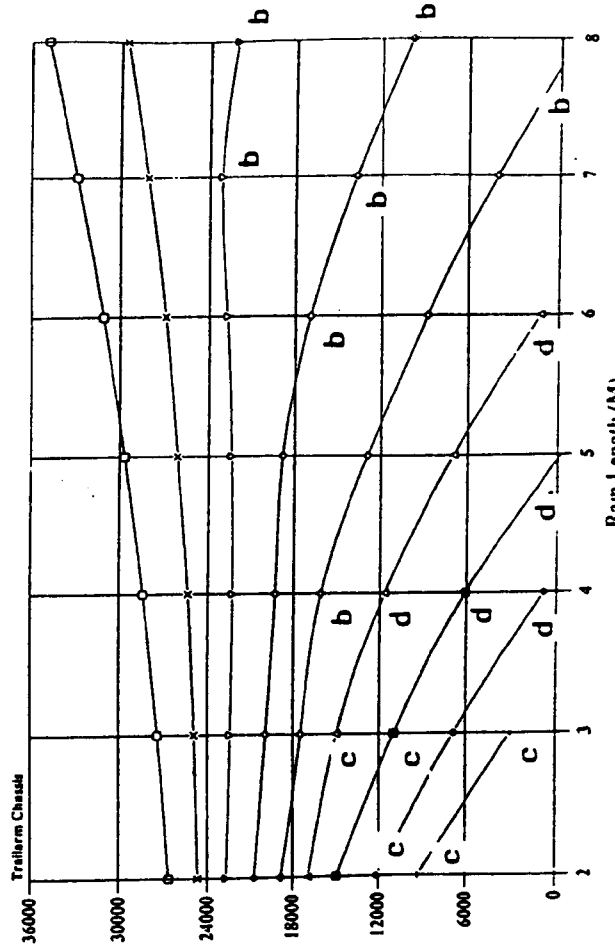
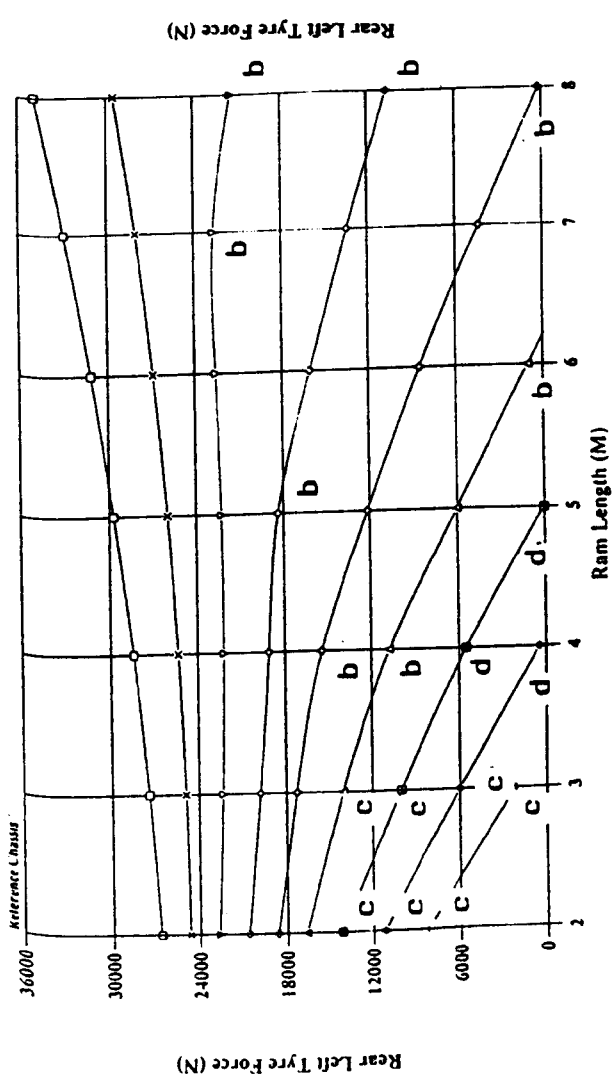
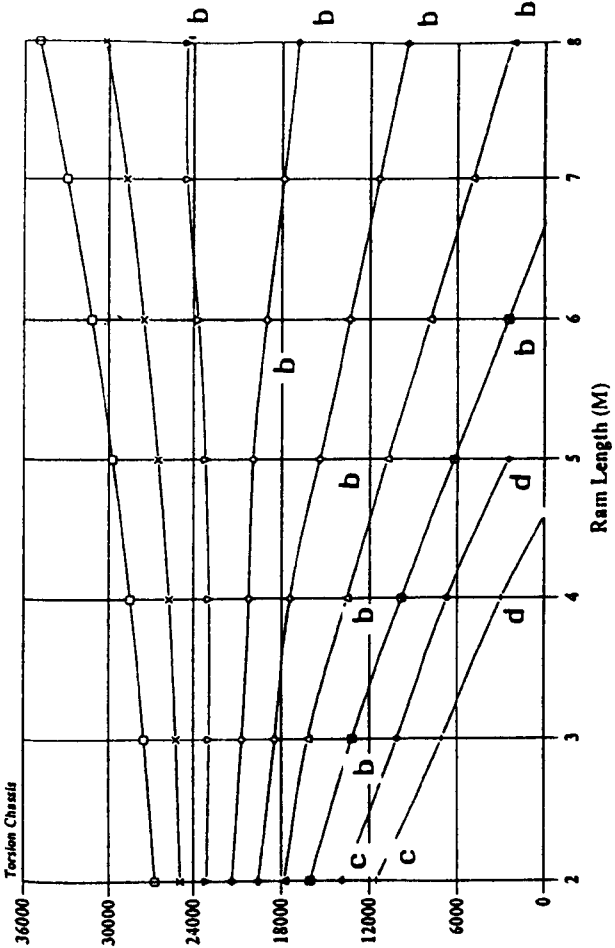
Fig. 5.3f Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 6 .

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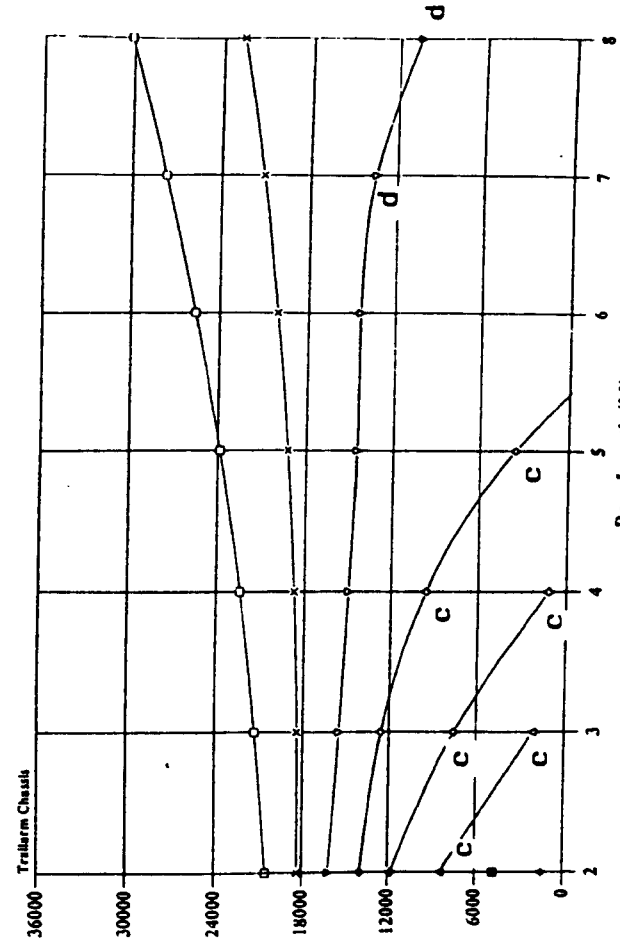
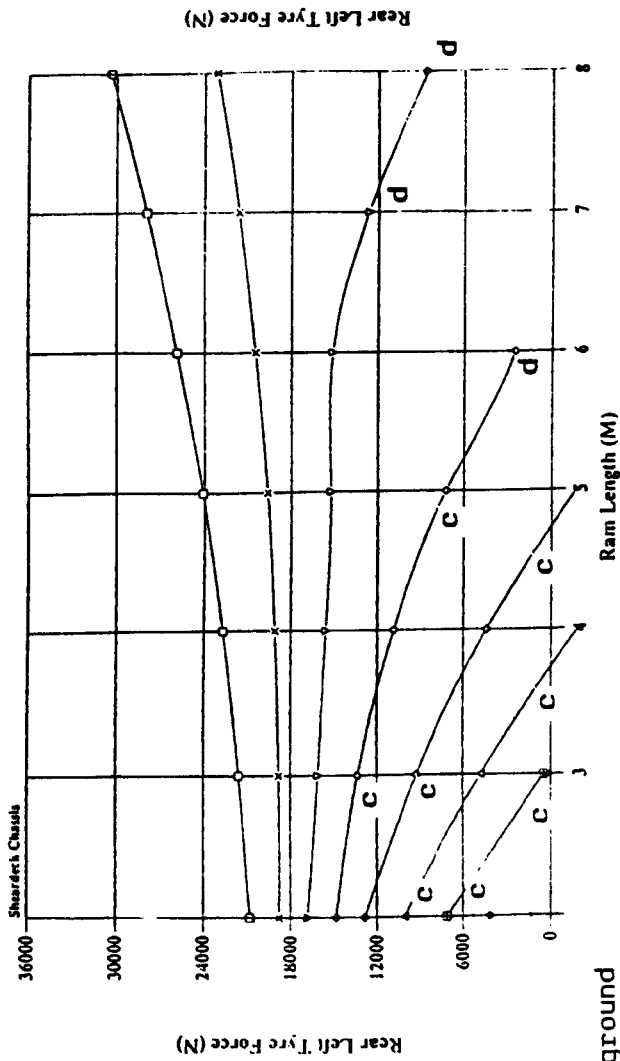
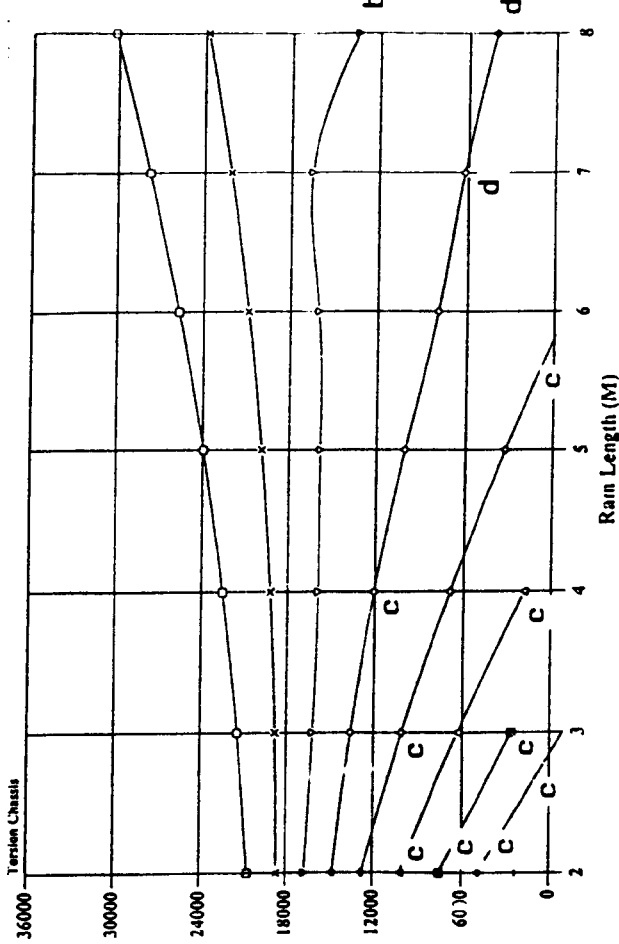
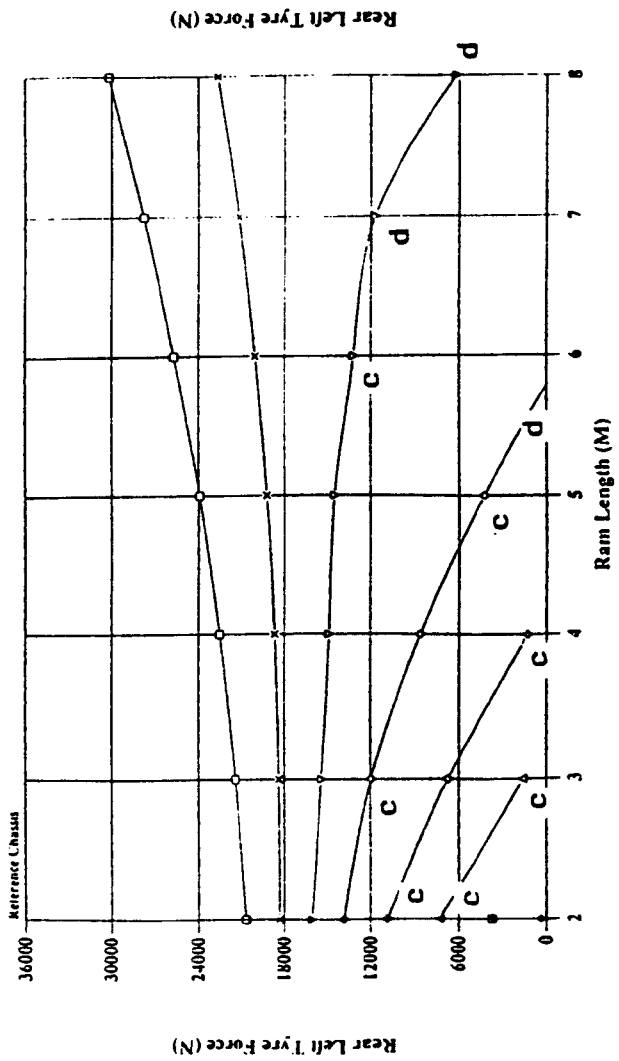
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- = 8° ground slope

Fig. 5.3g Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 7 .



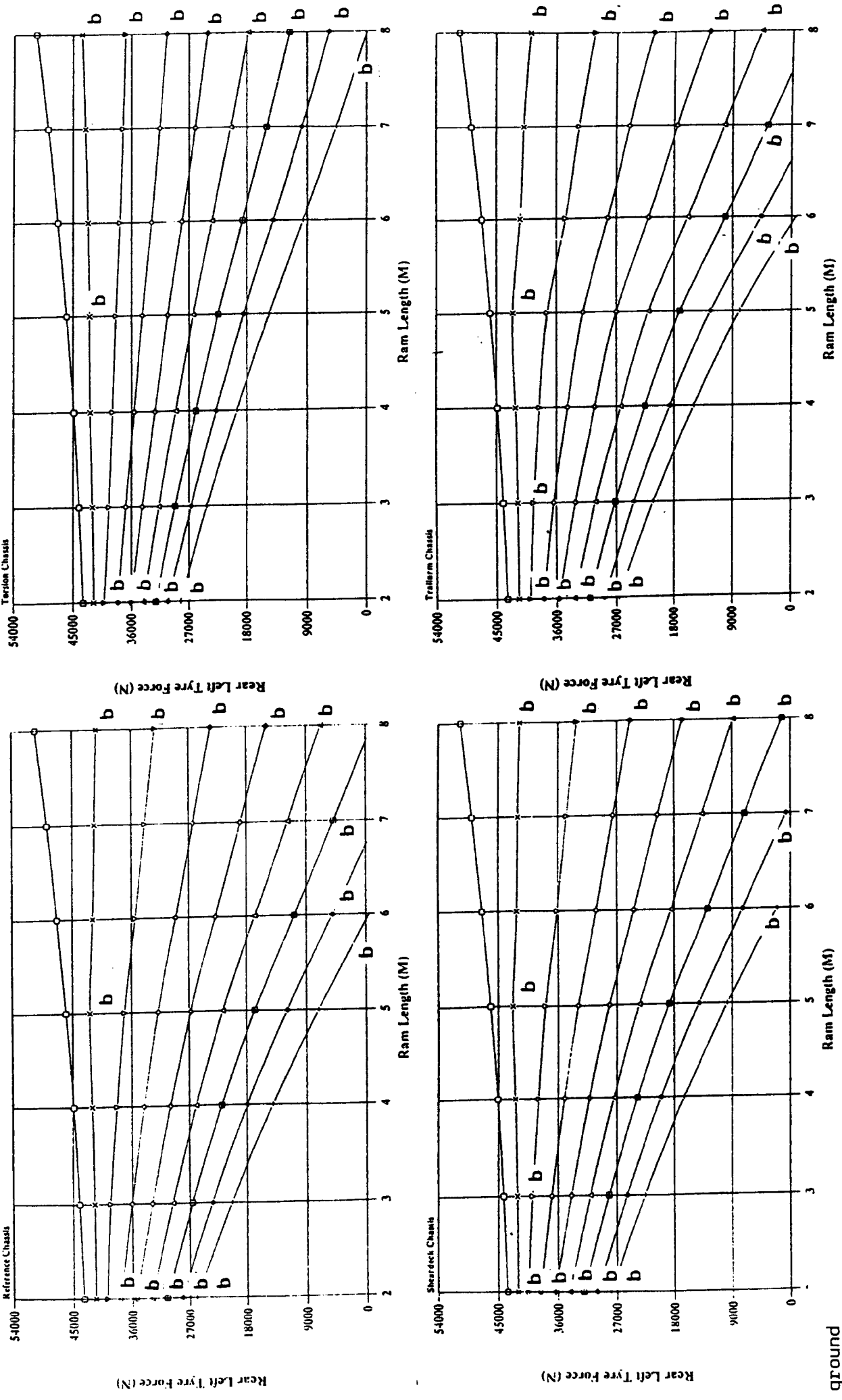
- = level ground
- △ = 2° ground slope
- ▣ = 4° ground slope
- ▤ = 6° ground slope
- ▥ = 8° ground slope

Fig. 5.3h Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 8 .



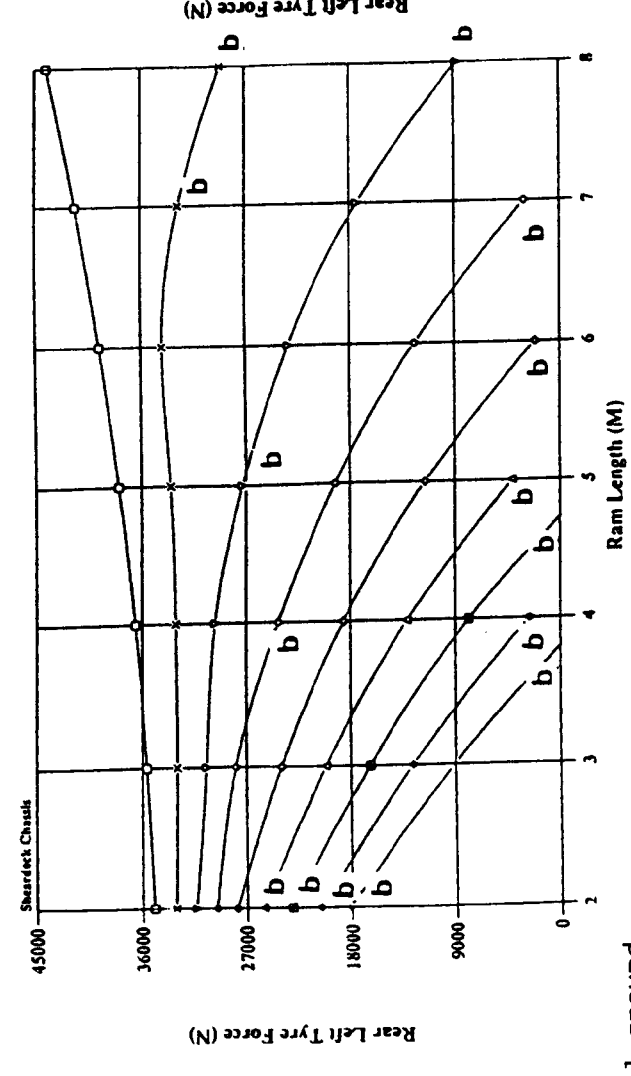
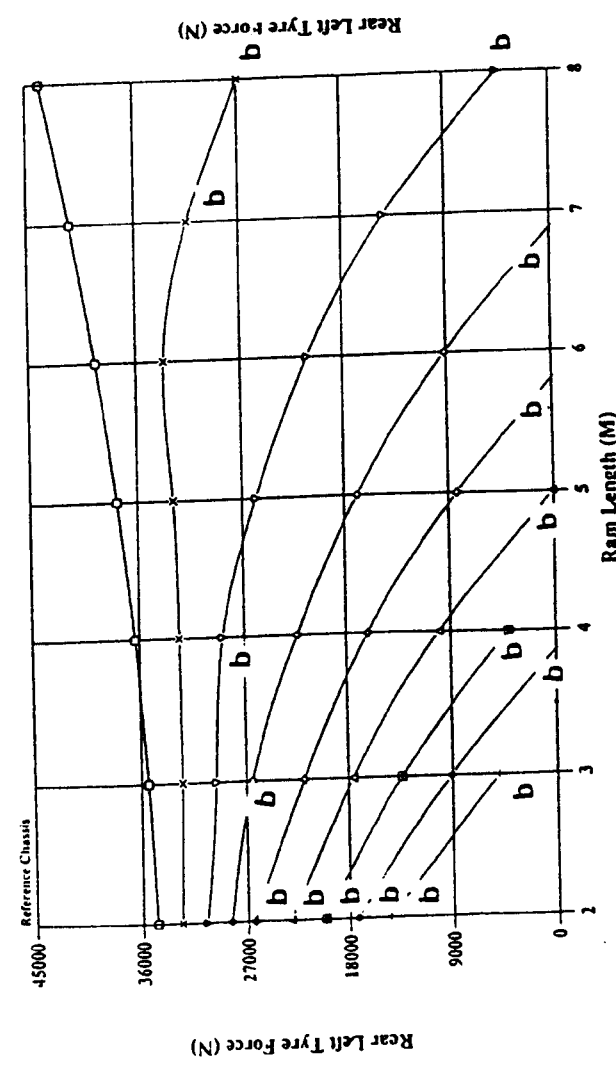
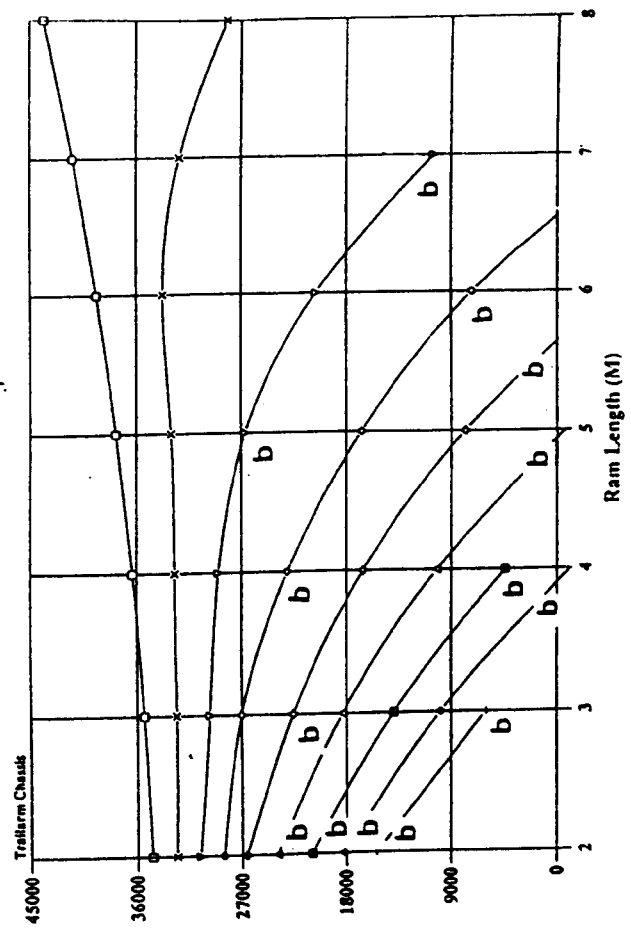
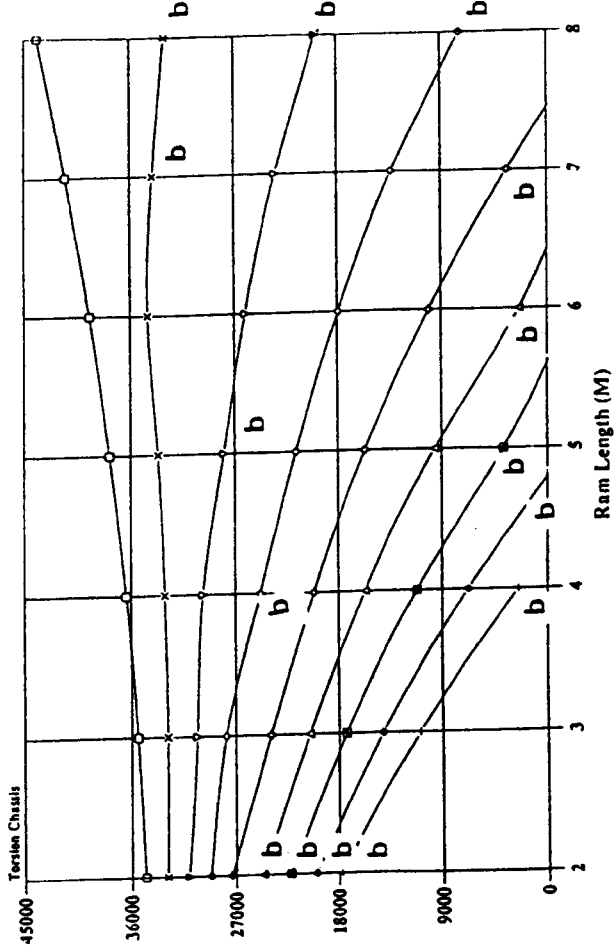
- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- ⊞ = 8° ground slope

Fig. 5.3i Rear left tyre force versus Ram length and ground slope combinations for a 17500 Kg payload in body position 9 .



- = level ground
- ▽ = 2° ground slope
- ◇ = 4° ground slope
- ⊗ = 6° ground slope
- \* = 8° ground slope

Fig. 5.4a Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 1.



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊗ = 6° ground slope
- + = 8° ground slope

Fig. 5.4b Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 2 .



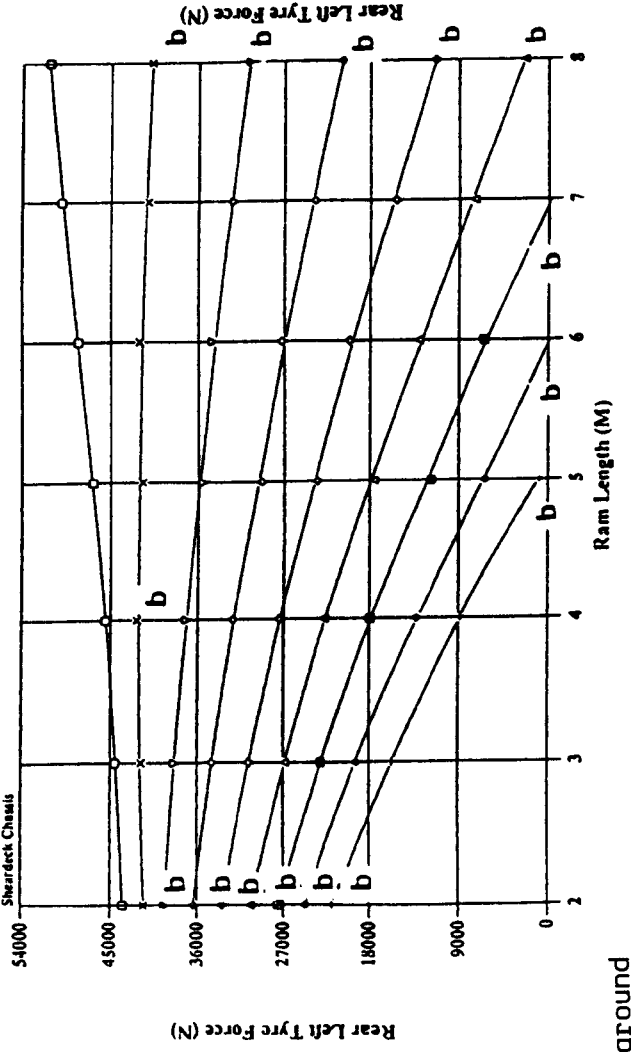
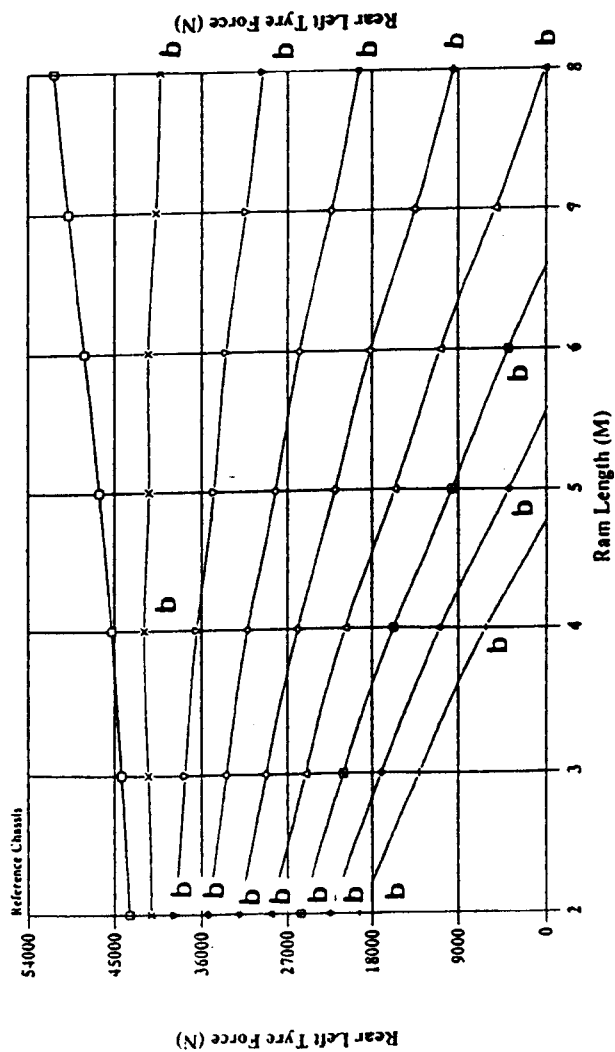
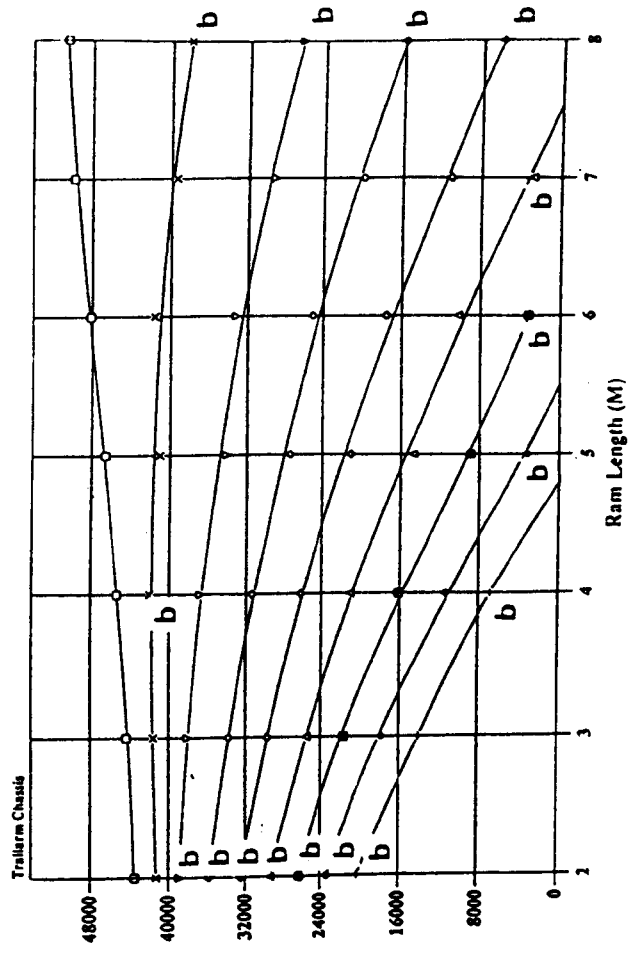
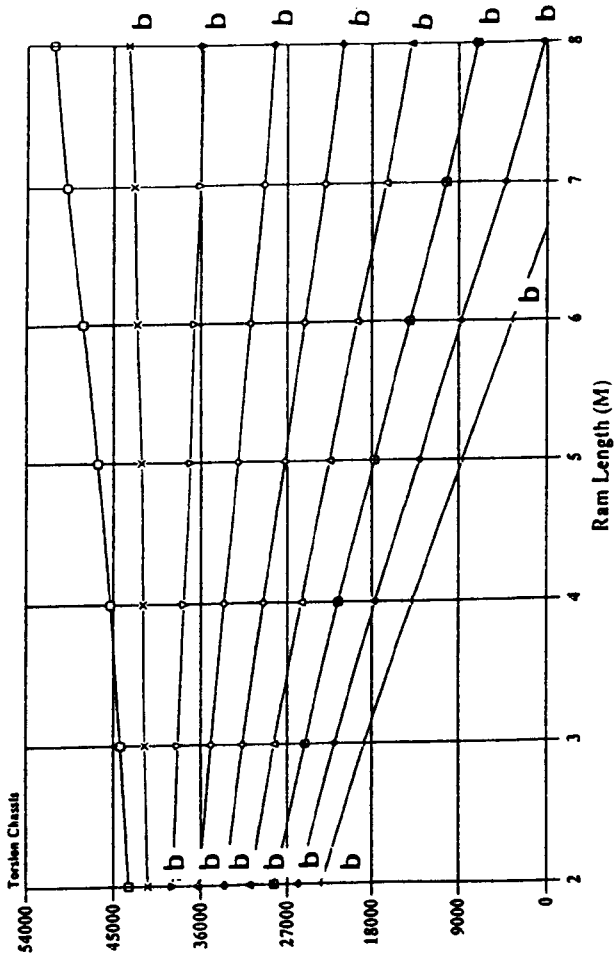
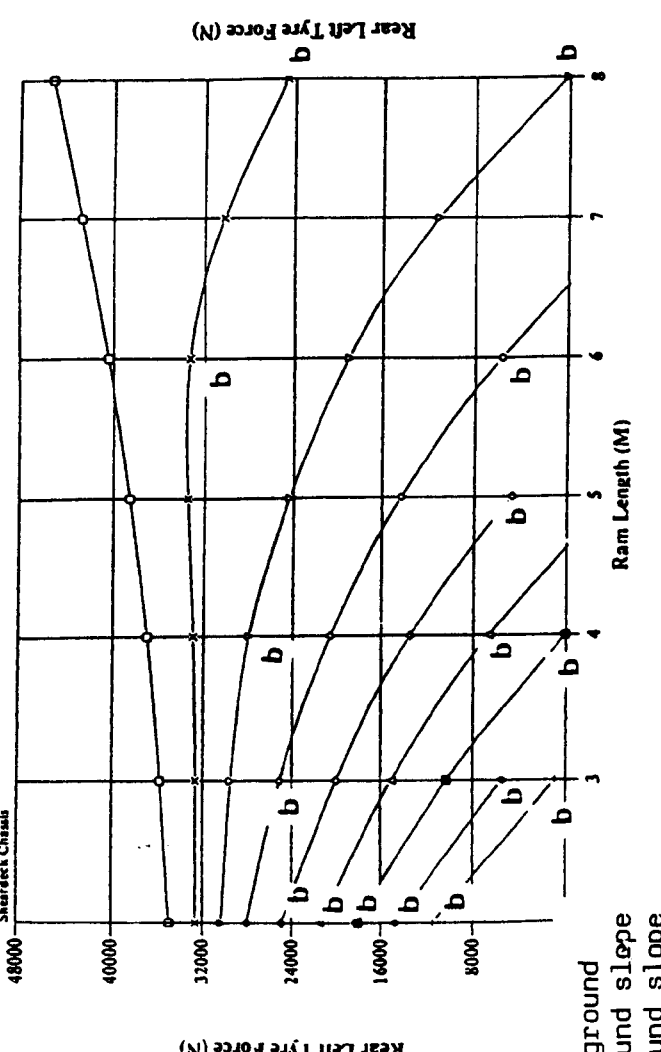
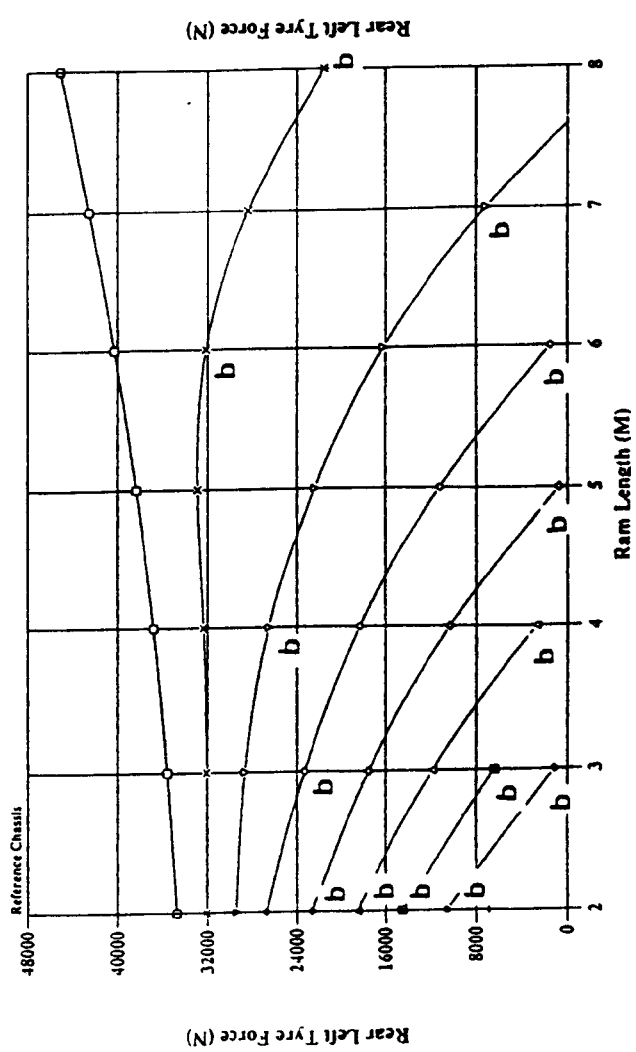
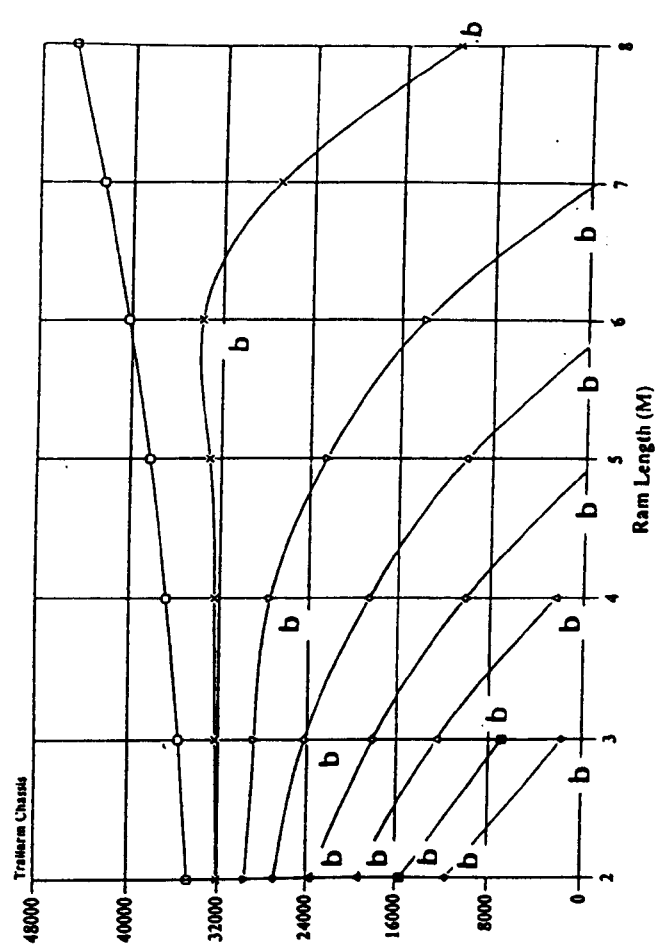
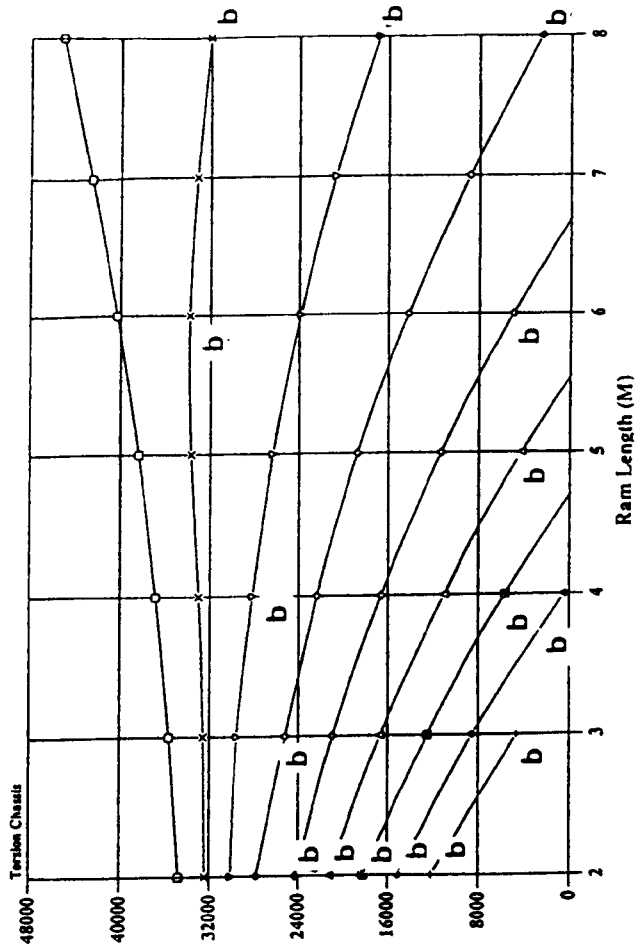


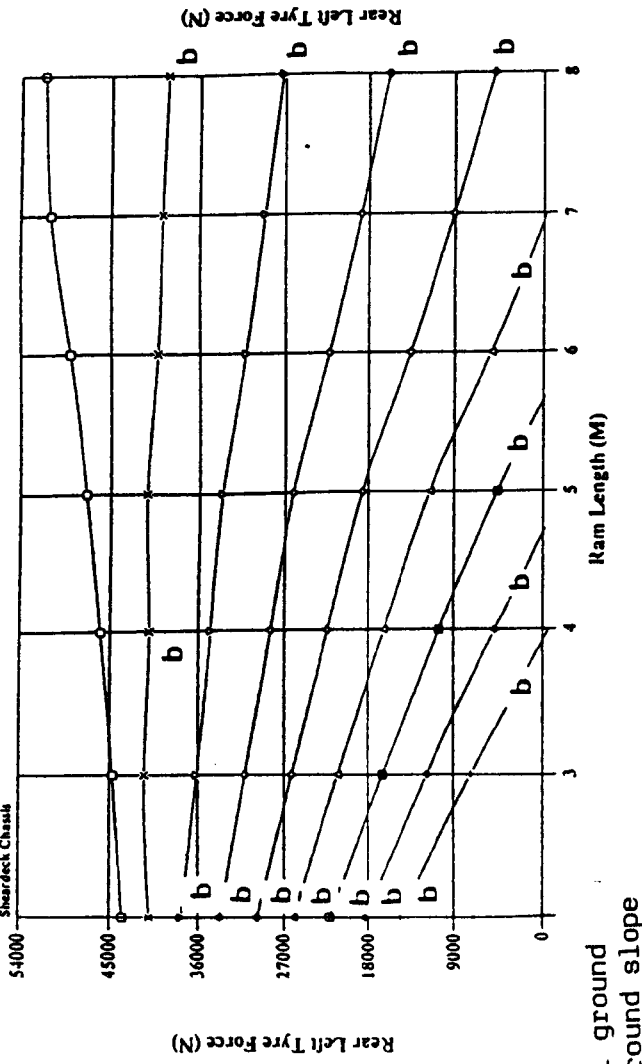
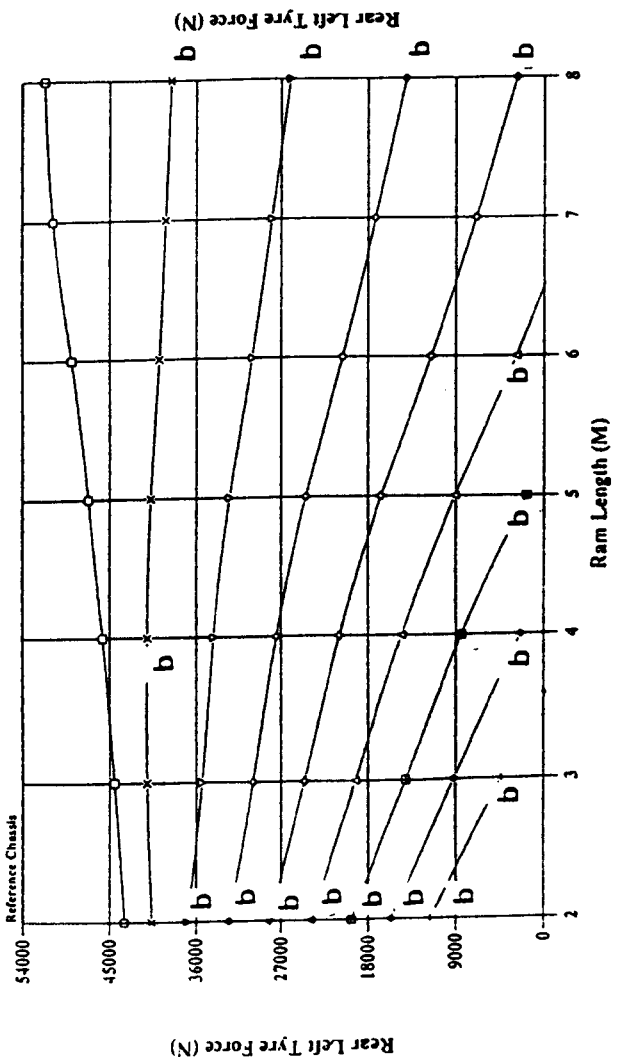
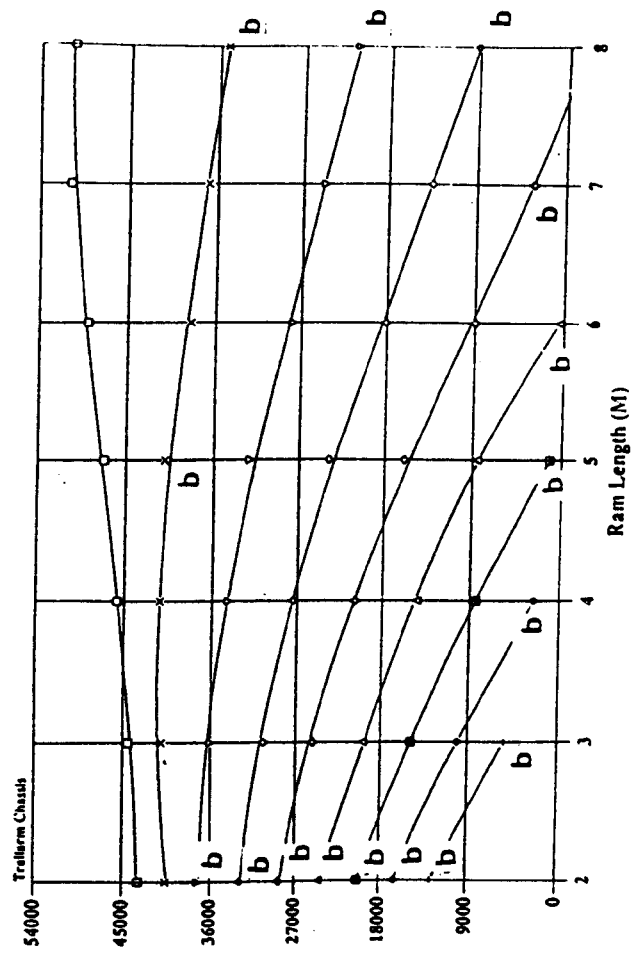
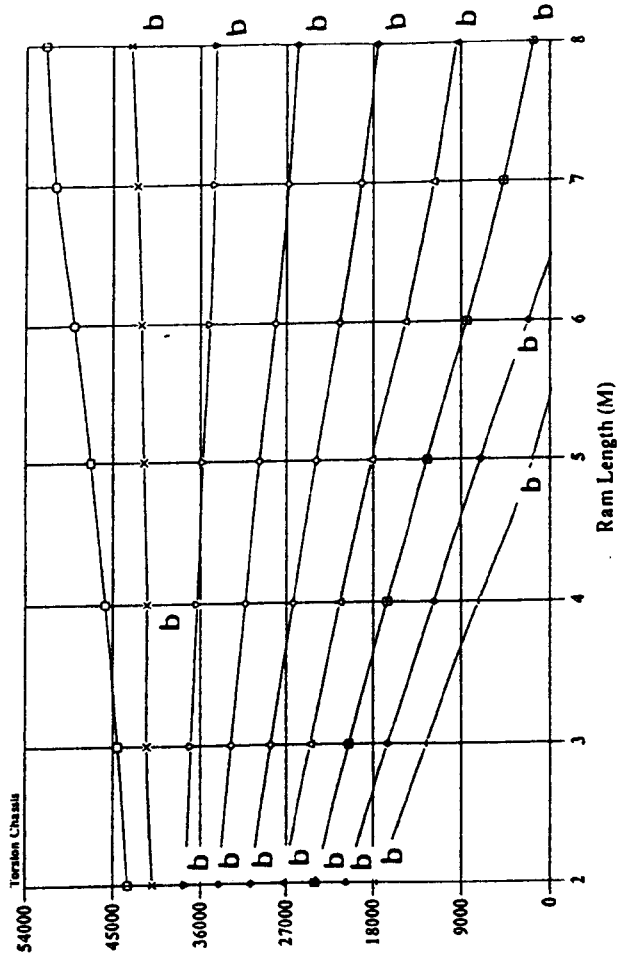
Fig. 5.4c Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 4 .

- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- = 6° ground slope
- = 8° ground slope



- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ◊ = 6° ground slope
- ▽ = 8° ground slope

Fig. 5.4d Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 5.



- = level ground
- = 2° ground slope
- △ = 4° ground slope
- ◇ = 6° ground slope
- ⊠ = 8° ground slope

Fig. 5.4e Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 7 .

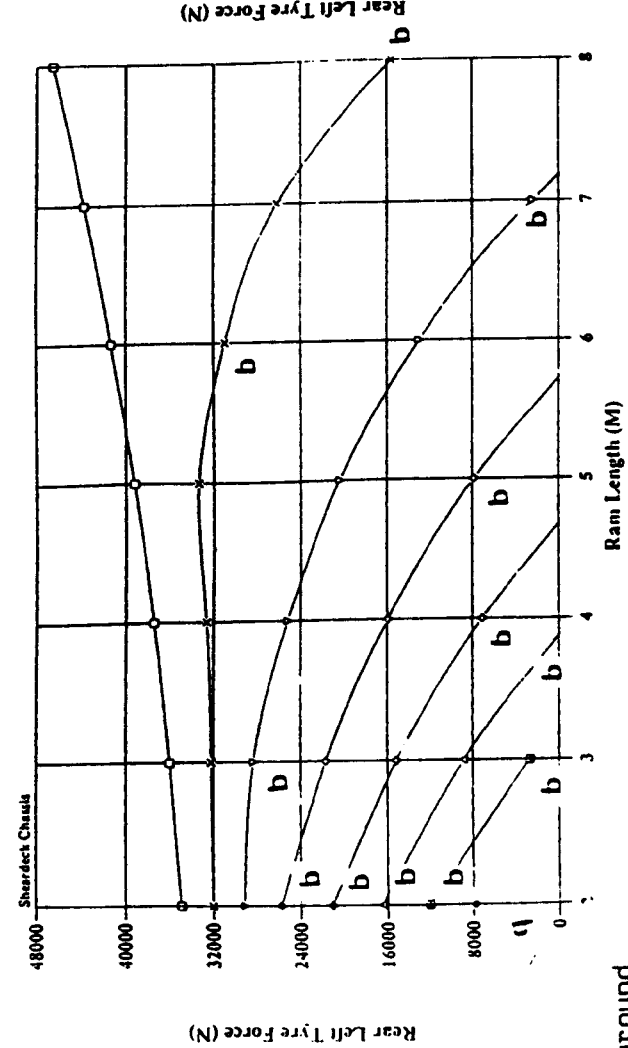
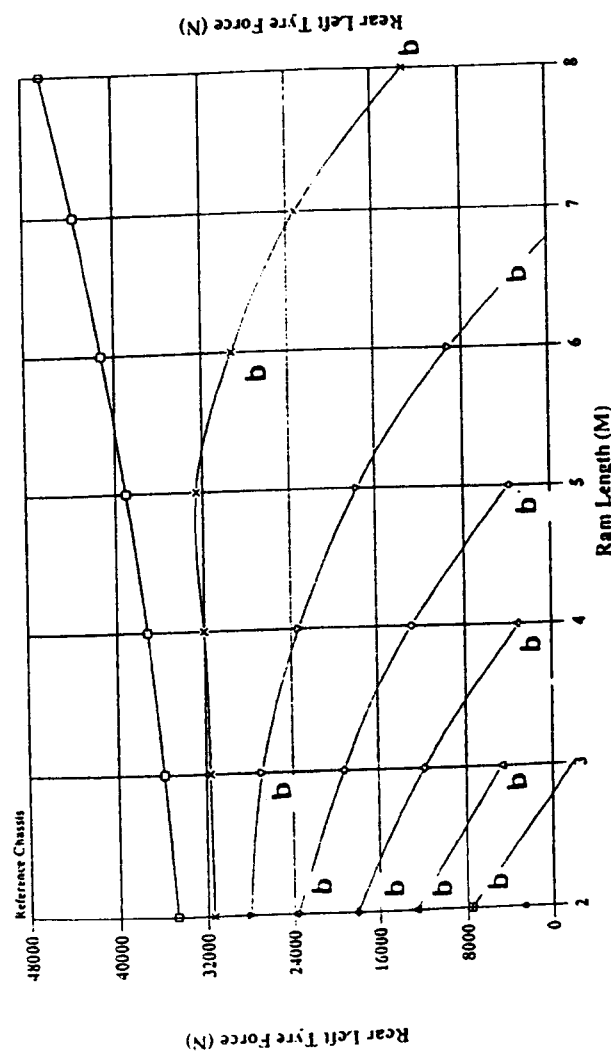
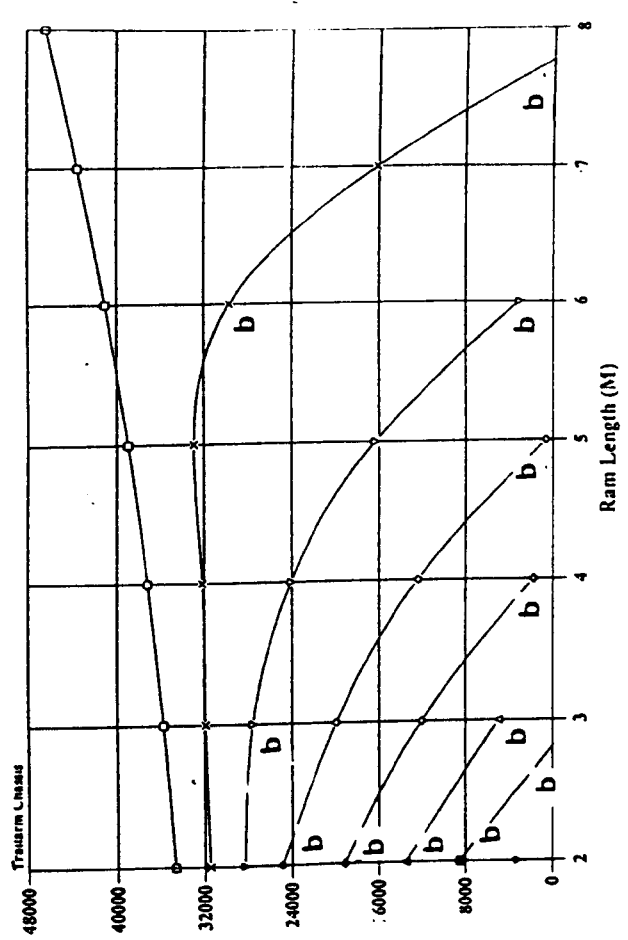
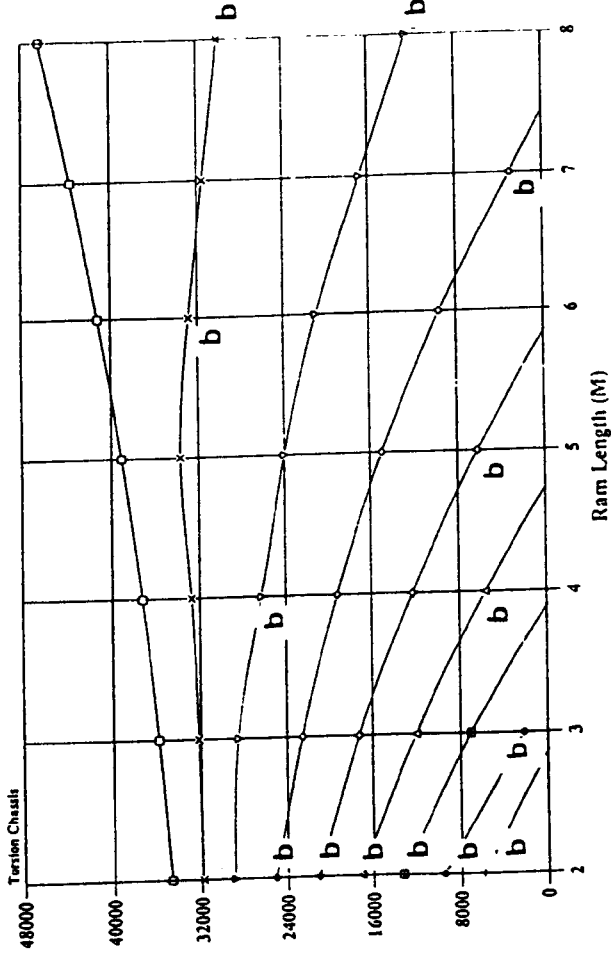


Fig. 5.4f Rear left tyre force versus Ram length and ground slope combinations for a 25000 Kg payload in body position 8 .

- = level ground
- △ = 2° ground slope
- ◇ = 4° ground slope
- ⊠ = 6° ground slope
- ⊞ = 8° ground slope

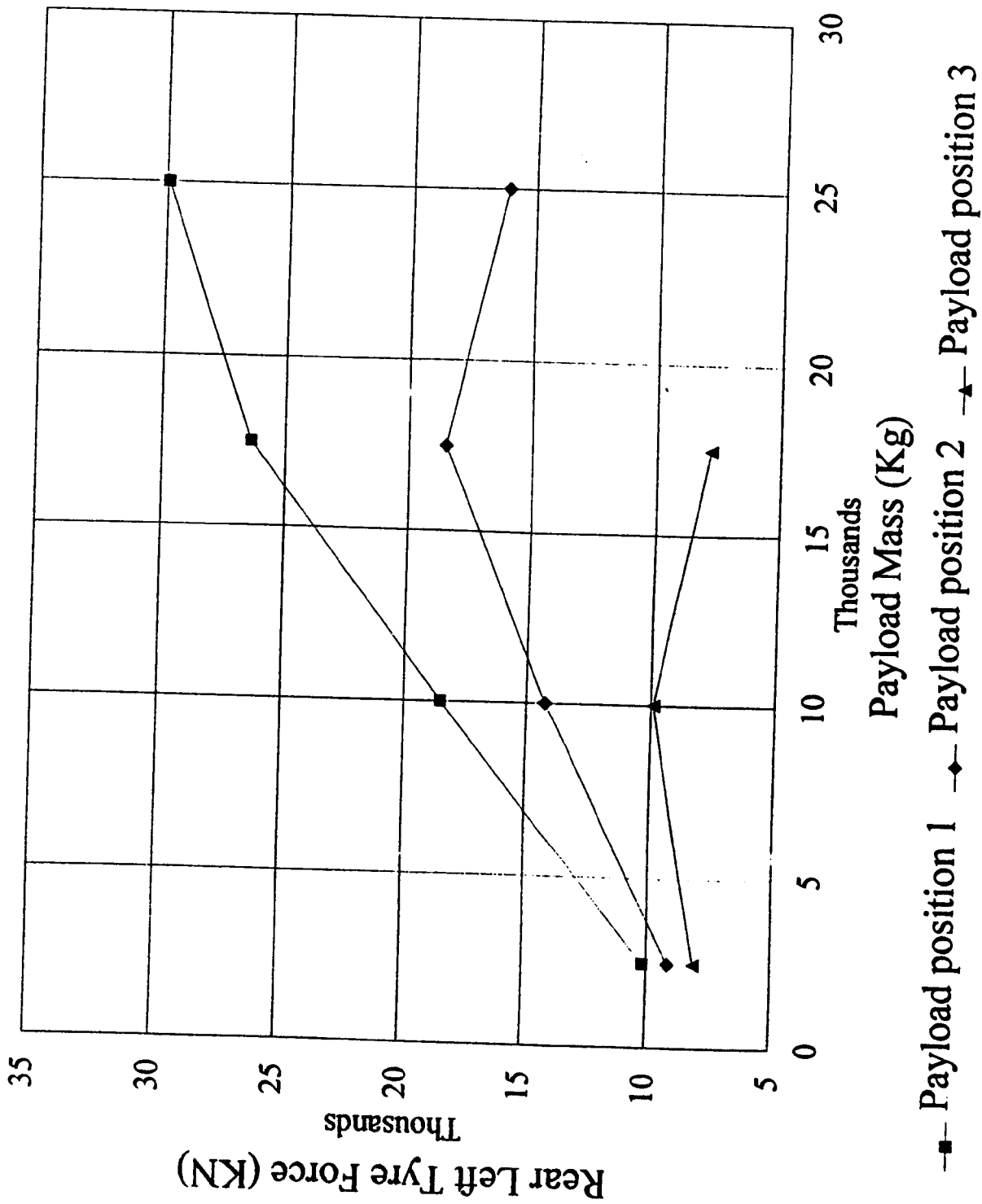
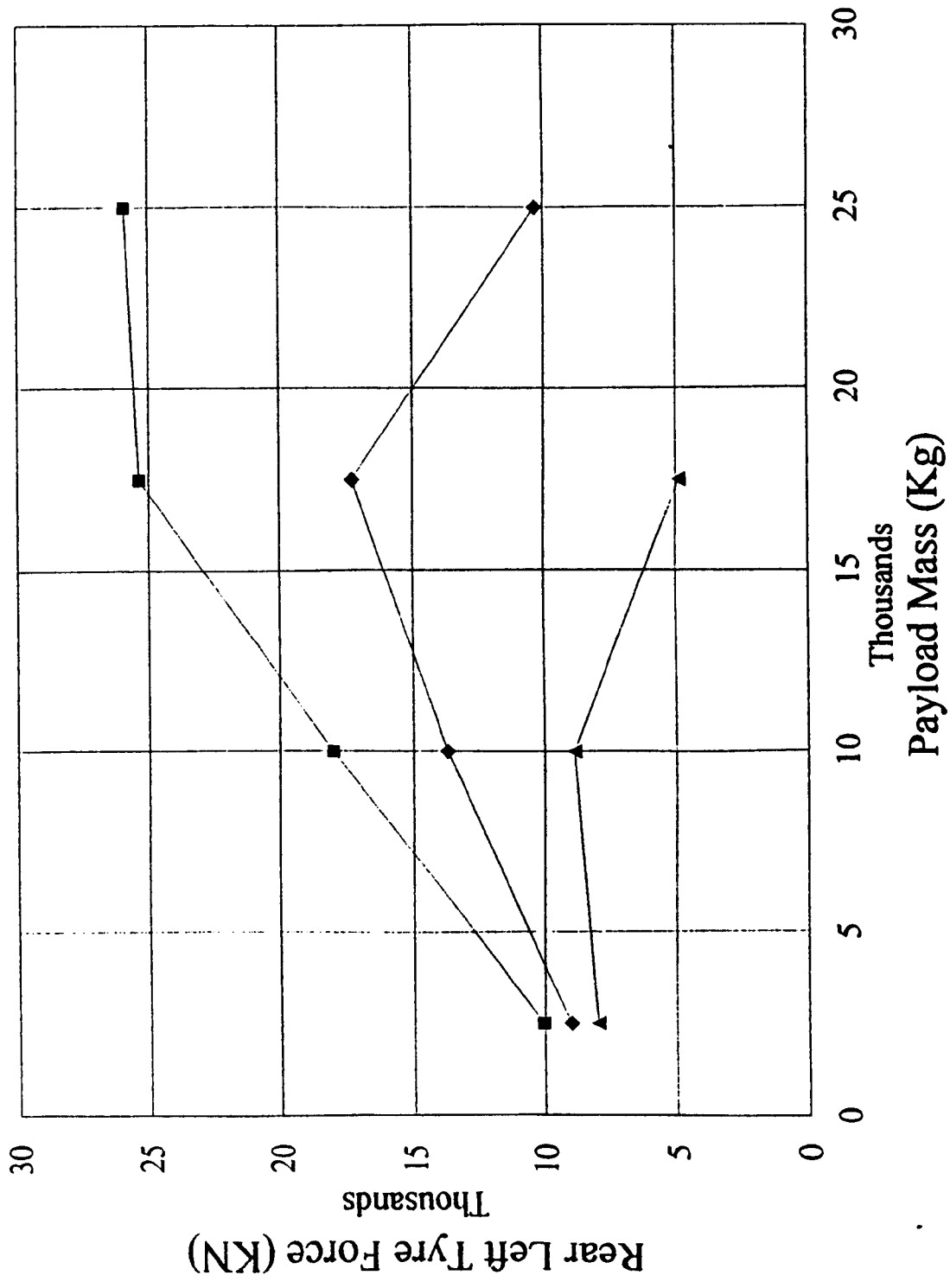
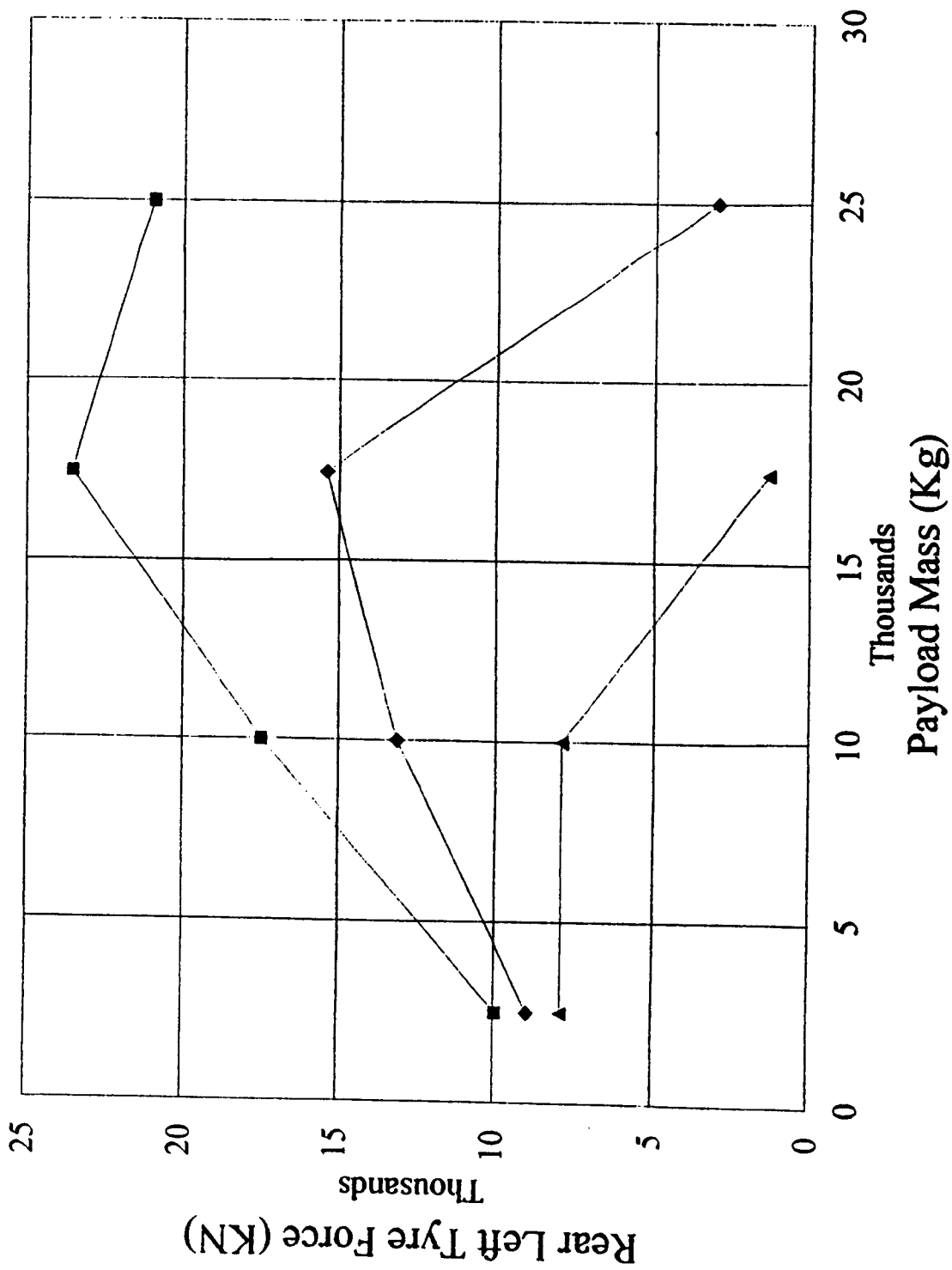


Fig 5.5 Rear left Tyre force versus payload magnitude in fore/aft payload positions 1,2 and 3.



-■- Payload position 4    -◆- Payload position 5    -▲- Payload position 6  
 Payload Mass (Kg)

Fig 5.6 Rear left Tyre force versus payload magnitude in fore/aft payload positions 4,5 and 6.



■ Payload position 7    ◆ Payload position 8    ▲ Payload position 9

Fig 5.7 Rear left Tyre force versus payload magnitude in fore/aft payload positions 7,8 and 9.

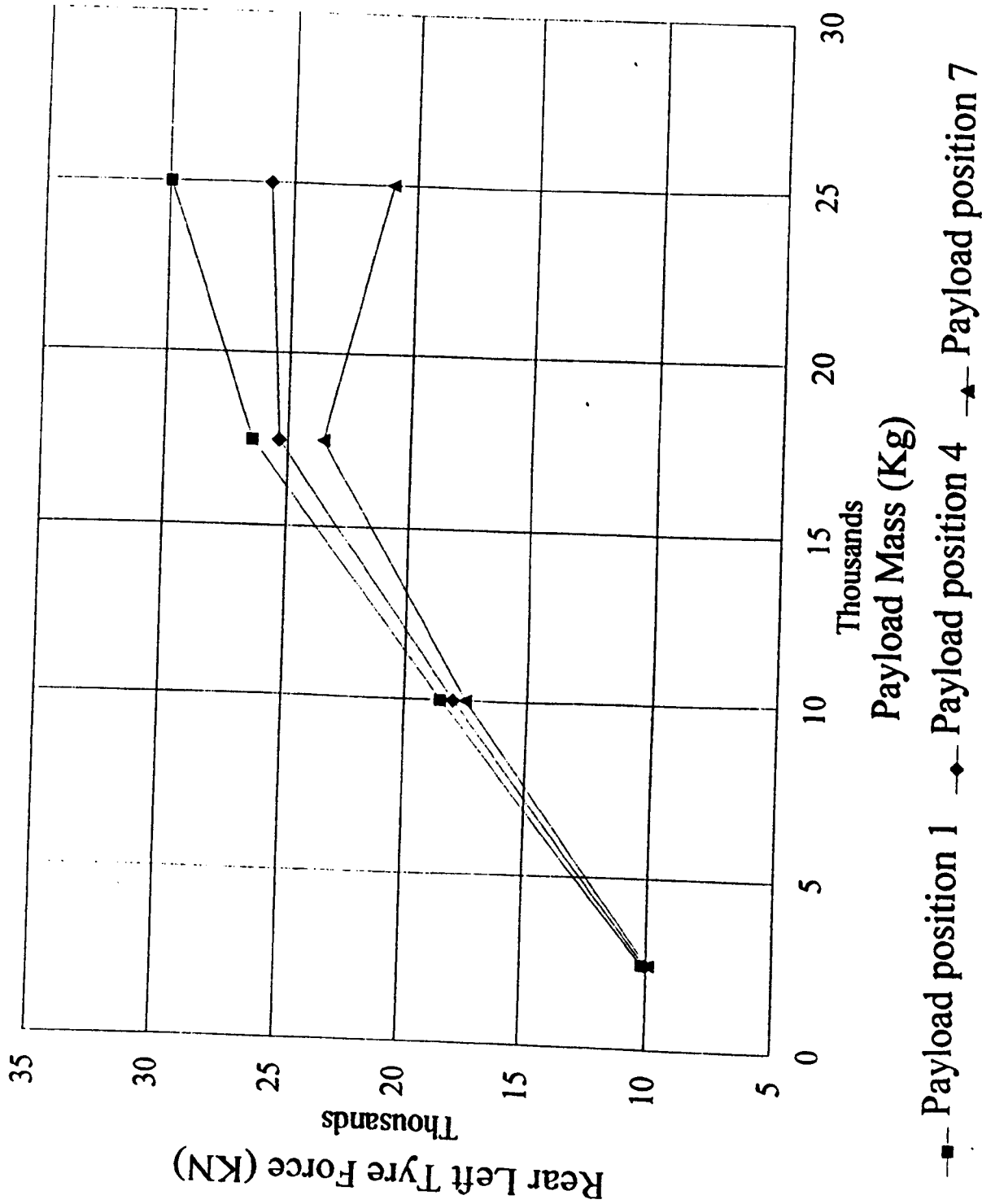
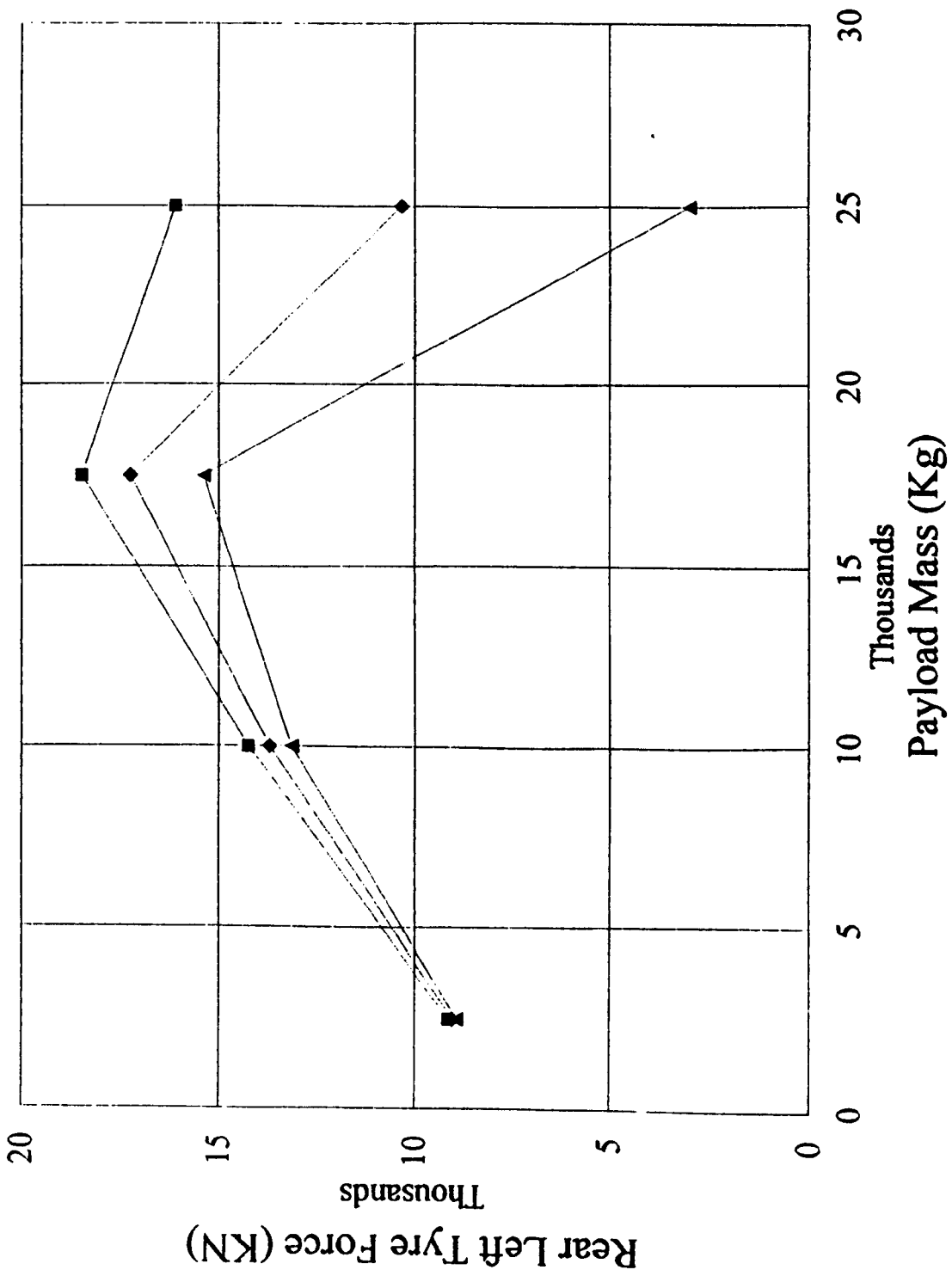


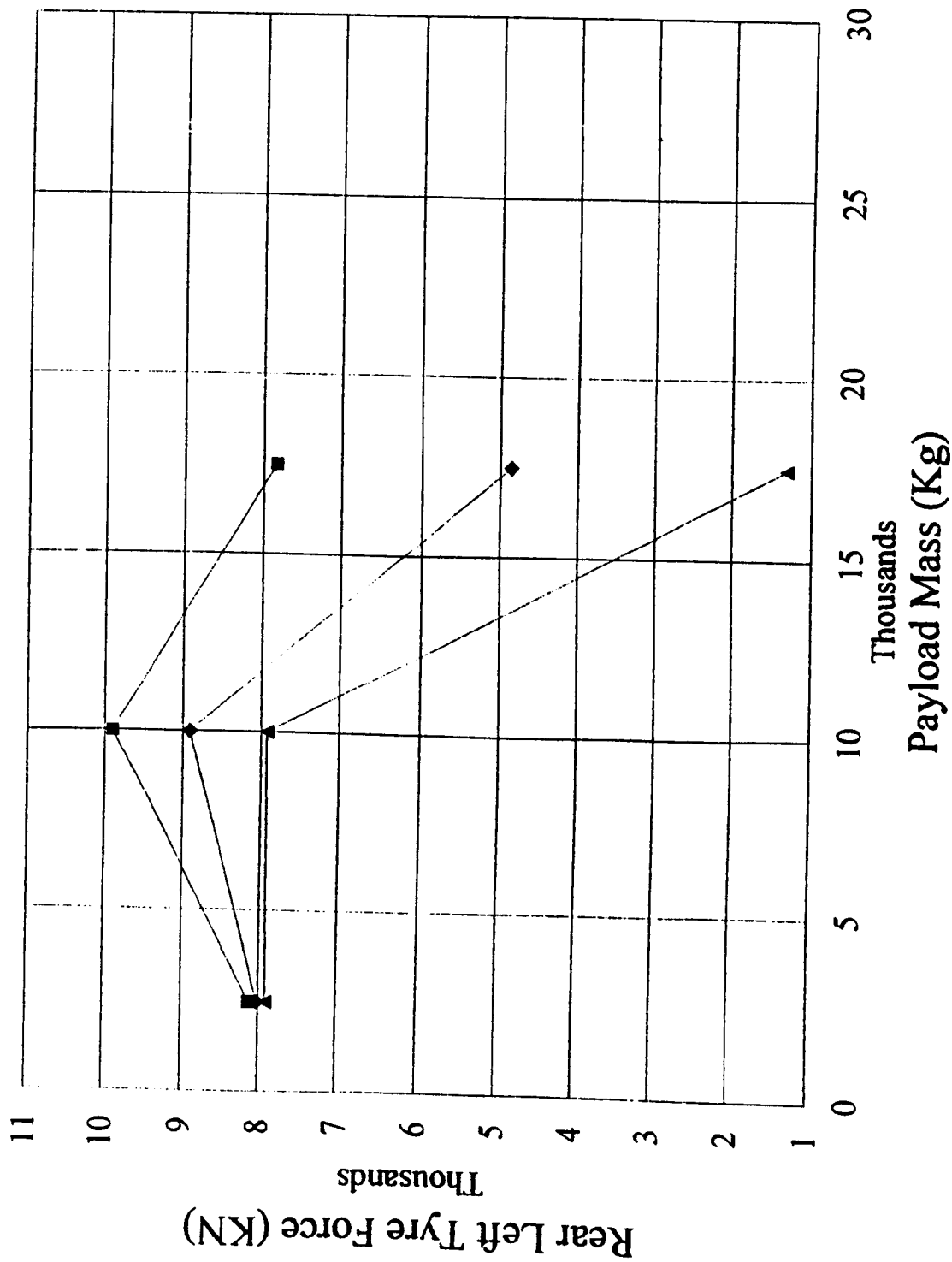
Fig 5.8 Rear left Tyre force versus payload magnitude in vertical payload positions 1,4 and 7.





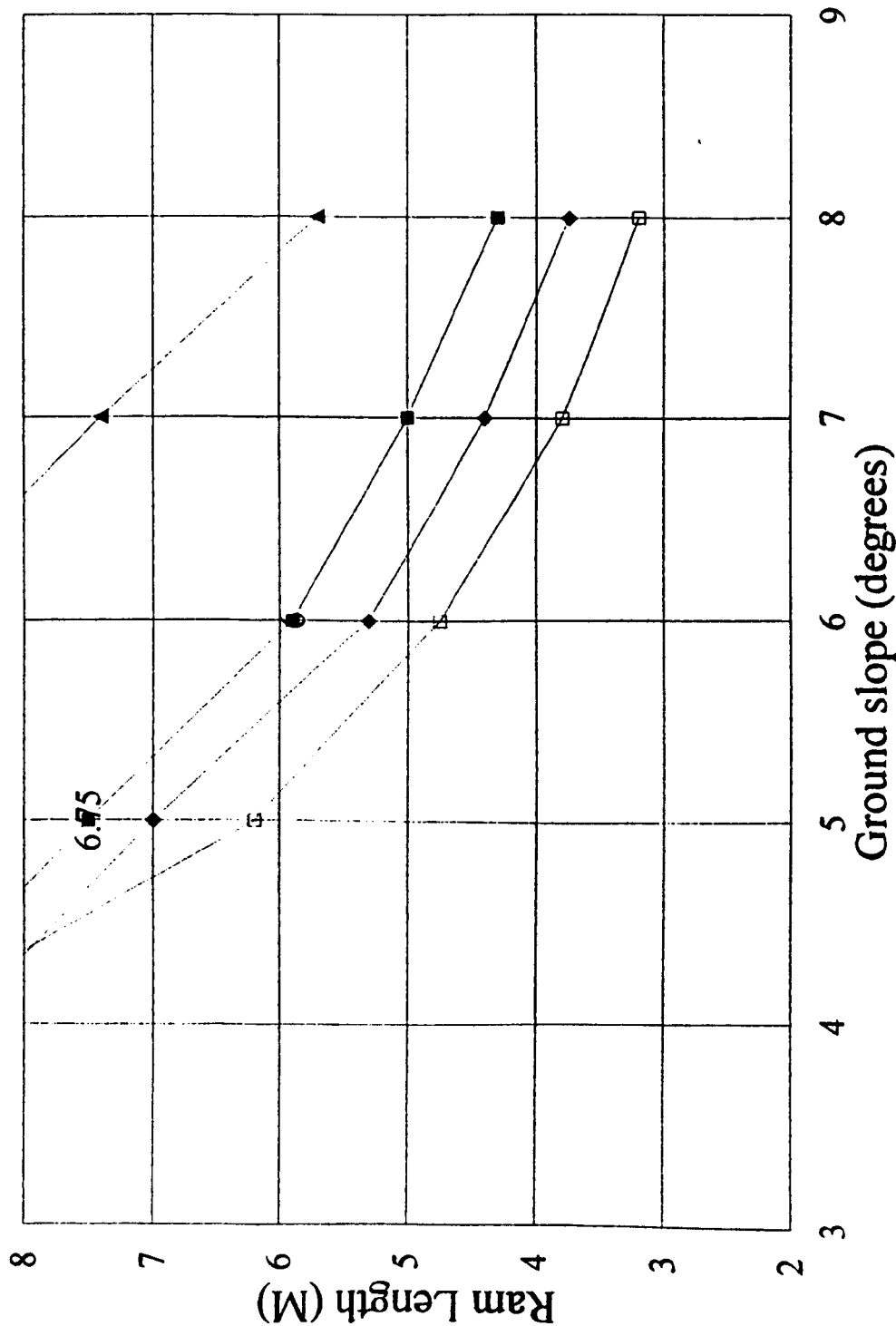
■ Payload position 2    ◆ Payload position 5    ▲ Payload position 8

Fig 5.9 Rear left Tyre force versus payload magnitude in vertical payload positions 2, 5 and 8.



• Payload position 3    ◆ Payload position 6    ▲ Payload position 9

Fig 5.10 Rear left tyre force versus payload magnitude in vertical payload positions 3,6 and 9.



17500 Kg payload

—■— payload position 3 —◆— payload position 6  
 —▲— payload position 8 —◻— payload position 9

Fig 5.11 Stability envelopes for a 10000 Kg payload in payload positions 3,6,8, and 9.(based on maximum ram length/zero rear left tyre forces)

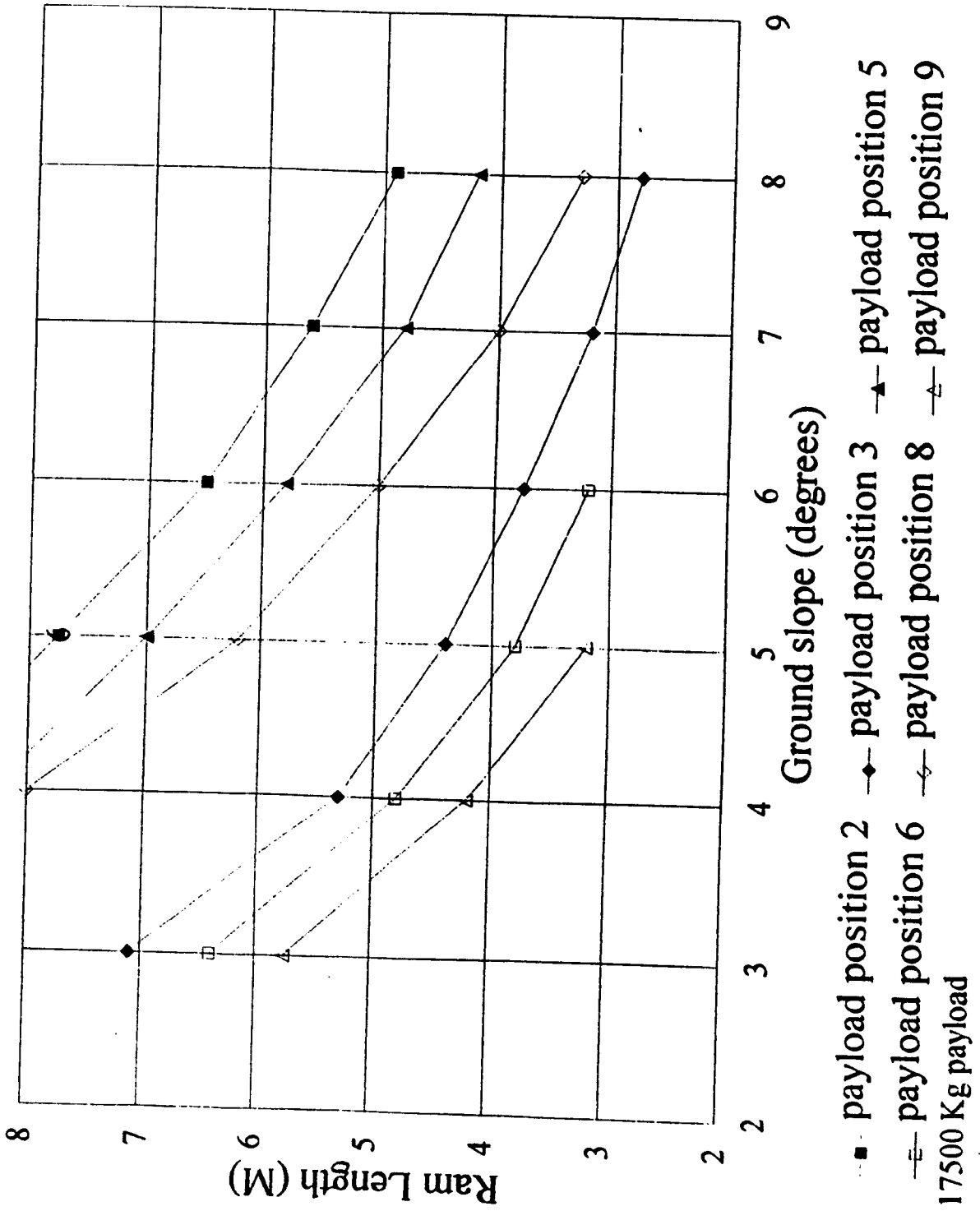
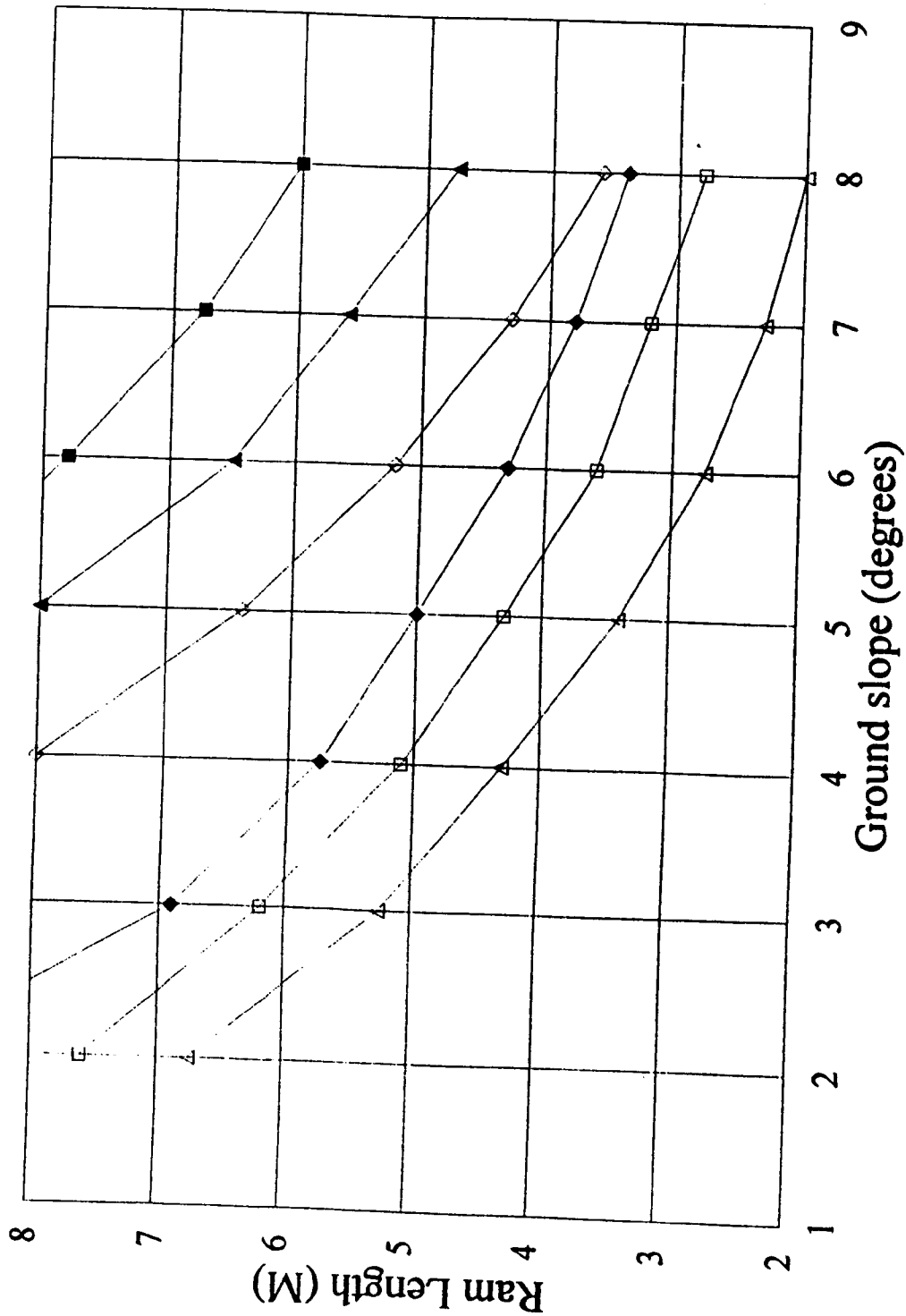
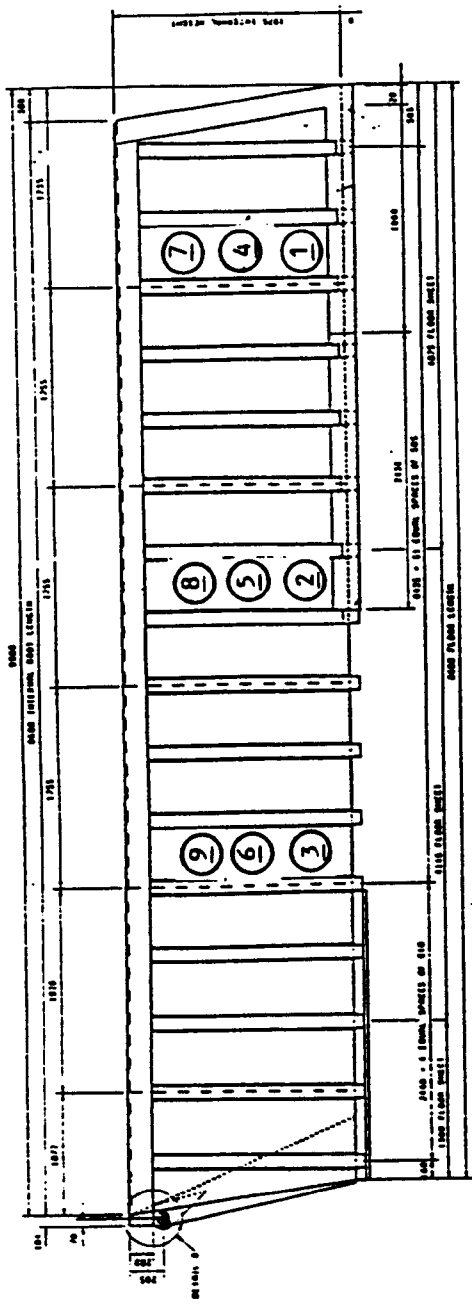


Fig 5.12 Stability envelopes for a 17500 Kg payload in payload positions 2,3,5, 6,8 and 9.(based on maximum ram length/zero rear left tyre forces)



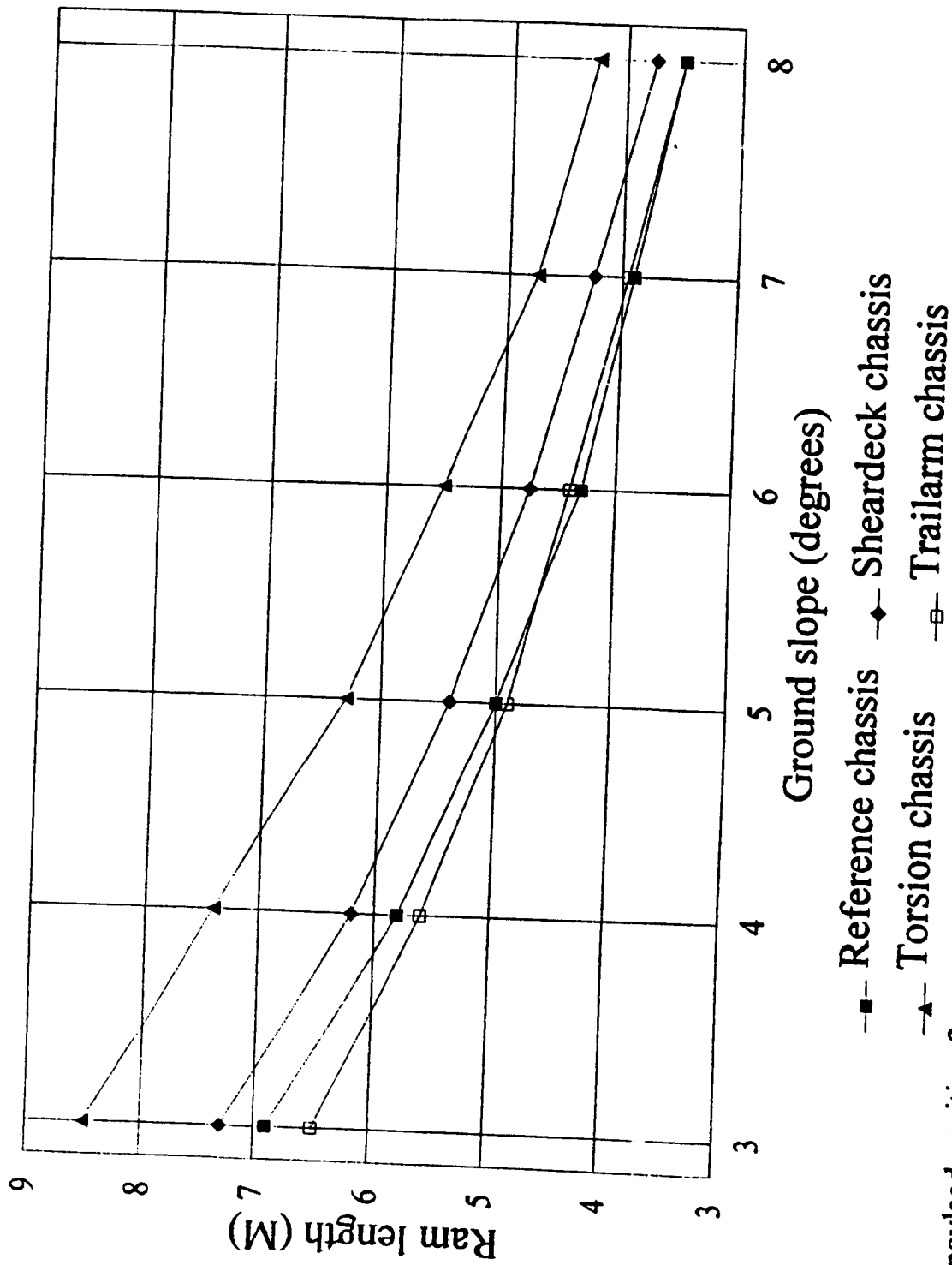
-■- payload position 1    -◆- payload position 2    -▲- payload position 3  
 -□- payload position 4    -◇- payload position 5    -▽- payload position 6  
 -○- payload position 7    -△- payload position 8  
 25000 Kg payload

Fig 5.13 Stability envelopes for a 25000 Kg payload in payload positions 1,2,4  
 5,7 and 8.(based on maximum ram length/zero rear left tyre forces)



Payload position	Payload coordinates (mm)		
	X	Y	Z
1	2992	676	0
2	4488	676	0
3	5984	676	0
4	2992	1014	0
5	4488	1014	0
6	5984	1014	0
7	2992	1350	0
8	4488	1350	0
9	5984	1350	0

Fig. 5.14 Body payload positional coordinates



payload position 2  
25000 Kg payload

Fig 5.15 Stability envelopes for a 25000 Kg payload in payload position 2 for the four different chassis configurations. (based on maximum ram length/zero rear left tyre forces)

## CHAPTER SIX

### GENERAL DISCUSSION AND CONCLUSION

#### 6.1 Technique

Theoretical models for predicting the stability behaviour of non-tipping trailers negotiating corners have been established by a number of people. However, no specific work has been found, in published material, relating to tipping trailer stability. The fundamental ideas associated with the dynamic non-tipping trailer models can be used in the static tipping trailer situation. The main criticism of the past models are that they require full scale experimental tests to determine important flexibilities (eg. chassis torsional stiffness and that the contribution of important flexibilities towards lorry stability cannot be assessed very easily. Also, air suspensions have not been considered. At the start of this project in January 1989, it was decided that the model developed for assessing the tipping stability of a trailer, should not require experimental tests to determine important flexibilities, and that design investigations should be relatively easy and cheap to establish the effect of component flexibilities on stability.

#### 6.2 Theoretical model

It was decided that the finite element method, described in Section 1, would be used to model the important flexibilities of the lorry. This enabled the important flexibilities of the lorry to be determined, without the requirement of full scale tilt



tests. It also enables design investigations to be undertaken relatively easily and quickly. However, not all flexibilities can be determined theoretically. Experimental investigations, described in Appendix A and B had to be undertaken in order to determine the flexibilities of the airbags and tyres.

### 6.2.1 Finite element model

Although large movements of the body and payload occur during tipping, these result from small linear elastic deformations of the body to chassis connection points, tyres and suspensions. The finite element model therefore uses beam plate and spring elements which are capable of dealing with small linear elastic deformations. Only the basic beam elements were used to model the chassis structure. However, brick elements could be used instead of the beam elements, which may result in a more accurate model but would significantly complicate the model, and subsequently increasing the cost of running the finite element model design investigations.

The tyres were modelled accurately using linear springs, with the flexibilities obtained from manufacturers information. It was thought at first that they might be incorrect. The vertical flexibility was confirmed to be correct through experimental work described in Appendix B, due to difficulties in the experiment the transverse and fore/aft flexibilities were not confirmed.

The relatively recent developments in suspension systems has seen the proliferation of air bag suspensions, mainly due to the airbag providing a flexible support

irrespective of the loading carried, as described in Appendix B. These supports are comprised of a rubber airbag inflated by air, with a mechanical high control device. The ability for the air suspension to increase or decrease the pressure in the airbag, depending on the load carried, results in a non linear system, described in Section 3.4.2. Experimental investigations into the load/deflection characteristics of an airbag, described in Appendix B, enabled a relationship to be determined of the form

$$FL^n = Const$$

Whereas leaf springs could be modelled using a simple linear spring, the operation of airbags and the non linear characteristics prevented this, and a novel approach had to be developed. The novel method of applying non linear loadings to a linear elastic structure, modelled by the finite element method, was successfully achieved and is described in Chapter 3.

Although the accuracy of the finite element model may be improved by the use of beam element with local flexibilities or by the use of brick elements, the method of solution is still good. The method of solution can still be used to highlight areas in the tipping trailer units that influence the stability characteristics, even if the flexibility matrices could be modelled more accurately. Also, the computer program written to execute the many matrix multiplications and iteration procedures has been shown to produce converged solutions with continuity as the suspension changes between the different operating conditions a, b, c, and d, as described in Section 4.6.

### 6.2.2 Solution procedure

The idea of using the results from finite element model to predict the flexibility of the tractor and trailer unit chassis and the realisation that the finite element package on its own could not be readily used to analyse the non linear loading and combination of payload magnitudes and positions, led to the development of general matrices for the calculation of forces and displacements. This required large unit load flexibility and influence matrices or unit displacement matrices described in Section 4.5 to be generated from the finite element results. This optimised the number of finite element runs required so that the stability characteristics of a lorry could be determined for any combination of payload magnitude and position and ground slope without further finite element runs.

Extensive processing of the finite element results, due to the non linearity of the problem is required by the PC program to determine the required trailer tyre forces. The continuity of the stability graphs, discussed in Chapter 5, shows that the flexibility matrices are accurate, relative to each other.

### 6.2.3 Configuration of flexibility data

The general force and displacement matrices derived from the finite element results are incorporated into a PC based program. The data for each matrix is inputted by hand. This is a laborious task and would benefit from a more sophisticated data handling system, between the finite element package and the PC program. Any

obvious mis-readings were later corrected, by editing the stored data files. Errors can be identified using a number of methods. Firstly there is a degree of symmetry within each matrix and this can be used to cross-check data values. Secondly the program can be run with a symmetrical payload load (ie.  $Z = 0$ ) and a zero ground slope, resulting in symmetrical vertical and horizontal displacement of the left and right hinges and zero transverse hinge displacement. The vertical tyre forces should be the same for corresponding left and right tyres and the transverse and fore/aft tyre forces should be zero. Thirdly the continuity of forces and displacements can be cross-checked before and after the suspension operating condition changes.

### **6.3 Implications**

The design changes have been investigated in order to assess the influence of the respective components on roll stability, and to demonstrate how design changes can be analysed.

The design changes investigated in Section 5.3, highlight that there is scope for improving the roll stability of articulated tipping vehicles.

The effect of the changes on roll stability compared to the reference chassis can be seen in Fig. 5.15. The graph gives details for a payload magnitude of 2500 kg and payload position of  $x = 4465$ ,  $y = 1533$ , 20, stability envelope for each chassis configuration. The vehicle is in a stable position below the line and in an unstable position above the line, for each respective chassis configuration. The reference

chassis enables a maximum ram length of 5.4 m to be reached, for  $\theta_p = 3^\circ$ , before the onset of instability. The trailing arm chassis enables a ram length of 5.0 m. The sheardeck chassis enabled a ram length of 5.7 m, and the torsion chassis which made the greatest improvement, enabling a ram length of 7.4 m to be reached, for the same body attitude before the onset of instability was reached.

Also, that the traditional chassis configuration comprising of two I-beams connected by cross members, may not be the optimum design and further design investigations could lead to alternative chassis designs.

#### **6.4 Requirements for further investigations**

The current model allows dimensions and flexibilities of lorry components to be changed in order to assess their influence on roll stability. The solution procedure of the theoretical model is correct, however the flexibilities derived from the current finite element model have not been verified. In order to have complete confidence in the flexibilities coefficients, the results from a full scale test should be compared against the F.E. results. While the current flexibility matrices may contain inaccuracies, they can still be used to assess design changes.

There are a number of areas in which the model accuracy could be improved. Firstly the tapered I beams have been modelled as uniform I beams, using average beam properties based on the two ends of the beam due to limitations of the finite element program.

Secondly inter-connections between cross members and the main I beams have been treated as rigid joints. Incorporating local flexibilities would enable simple beam plate elements to model the chassis more accurately. as shown by Baermann [18] and Mayson [19]

Otherwise brick elements could be used instead of the plate and beam elements to model the chassis enabling tapered sections and local flexibilities to be modelled. The accuracy achieved through the use of brick elements in the finite element model has to be considered against the increased complication producing the F.E. model, the increase in computing time to run the finite element program and the increase in difficulty in making design changes.

Currently, fifth wheel separation has not been taken into account and that the rotation of the tractor and trailer at the fifth wheel are the same. It has been shown in the dynamic roll-over situation by Sweatman [3] that fifth wheel separation reduces roll stability. Introducing fifth wheel separation capability would further increase the model accuracy.

A useful inclusion in the computer program would be the facility to calculate stresses within the beam elements. Unit load or unit displacement stress matrices could be generated in the same manner as the flexibility matrices. This would enable the design engineers to identify those areas that are over or understressed, and that would benefit from design investigations.

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# APPENDIX A

## AIRBAG STIFFNESS

The aim of this experimental work was to determine the force/deflection characteristics of an airbag for given initial air pressures while operating at the normal ride height of 381 mm.

The actual loading and support conditions were found to be difficult to apply with existing equipment. Therefore, simpler, reasonably representative loading and support conditions were imposed. Although these loading and support conditions were incorrect, it was considered that the results obtained could be used as an approximate check on the manufacturers data.

### A1 Introduction

An airbag is a rubber/fabric bellow which contains a column of compressed air. The rubber below itself does not significantly contribute to load support, this is mainly due to the column of compressed air. The air spring can only support axial loads with the existence of a suitable mechanism to eliminate sideways movement. When considering axial loads for an airbag, an initial ride height and pressure must be selected before loading can take place.

### A2 Theoretical prediction of airbag force/height relationship

A typical curve of force versus height is shown in Fig. A1(a). It was assumed that the force displacement graph can be represented by an equation of the form

$$FL^n = C \quad (A1)$$

The constant  $n$  and  $C$  are determined from the intercept and slopes of log-log graphs of  $F$  and  $L$  and are equivalent to those of an adiabatic compression of a gas.

### **A3 Experimental procedure**

Because the airbags have a normal operating height of 381 mm, the first set of tests were set up using this height, and varying the initial pressure only. Five different initial pressures of 25, 40, 70 and 80 psi were used to obtain the results shown by Fig. A1(a)-A1(e). The airbag was placed in an Instron universal testing machine and allowed to expand to a height of 381 mm. Following pressurisation to 25, 40, 55 70 or 80 psi the air supply was closed. It was then unloaded to a 500 mm height and then compressed to a 250 mm height or up to a 100 psi pressure, whichever came first, and then unloaded back to 381 mm height.

A second set of tests were setup using a constant airbag height and varying the air supply pressure only. Four different airbag heights of 300, 325, 375, 400 and 425 mm were used to obtain the results in Fig. A2. The airbag was placed in an Instron universal testing machine and allowed to expand to one of the predetermined height previously mentioned. It was then pressurised to 90 psi in steps of 10 psi and the corresponding axial force was recorded.

### **Results**

From the experimental work three important results were obtained which enabled the airbag to be accurately modelled. Firstly it was found that when the airbag was compressed with a constant mass of air, equation A1 could predict the force/deflection curve accurately, using suitable values for the constants. The

theoretical and experimental force/deflection curves are shown in Fig. A1(a)-A1(e). The constants  $\gamma$  and C, for equation A1, are shown in Table A1, for the five different initial pressures considered.

Secondly, it was found that the airbag axial force was not influenced by its height and was directly related to air pressure only, where the mass of air within the airbag is not constant. The experimental results showing the airbag force/pressure relationship for four different fixed heights are shown in Fig. A2.

Thirdly, it can be seen from Fig. A2 that the maximum airbag force of 30 kN can be used to indicate when the airbag air supply reaches its maximum pressure of 80 psi. Thus preventing the airbags from maintaining height control and allowing the airbags to compress according to equation A1.

From these results the airbags on one side have the same axial force as they are always at the same pressure due to being interconnected. When the suspensions are operating below their maximum working pressure of 80 psi (ie. the mass of air within the airbags is not constant), the airbag force is proportional to airbag pressure. When the suspensions are operating above their maximum working pressure (ie. the mass of air within the airbags is constant), the airbag force is determined from equation A1.

The value of  $n = 1.57$  is constant for all airbag operating condition as they change from height control to compression of a fixed mass of gas always occurs when the air pressure reaches 80 psi. It is assumed that the force/deflection characteristics of three

airbags side by side, as in the case of the actual trailer air bags, can be represented by a modified form of equation A1. Where  $F$  is the airbag force,  $l$  is the combined length of the three displaced airbags and  $C$  the constant in equation A1.  $C$  is unknown as no experimental data exists for the compression of three airbags. Instead,  $C$  is calculated using the previous height controlled airbag forces and lengths substituted into equation A1.

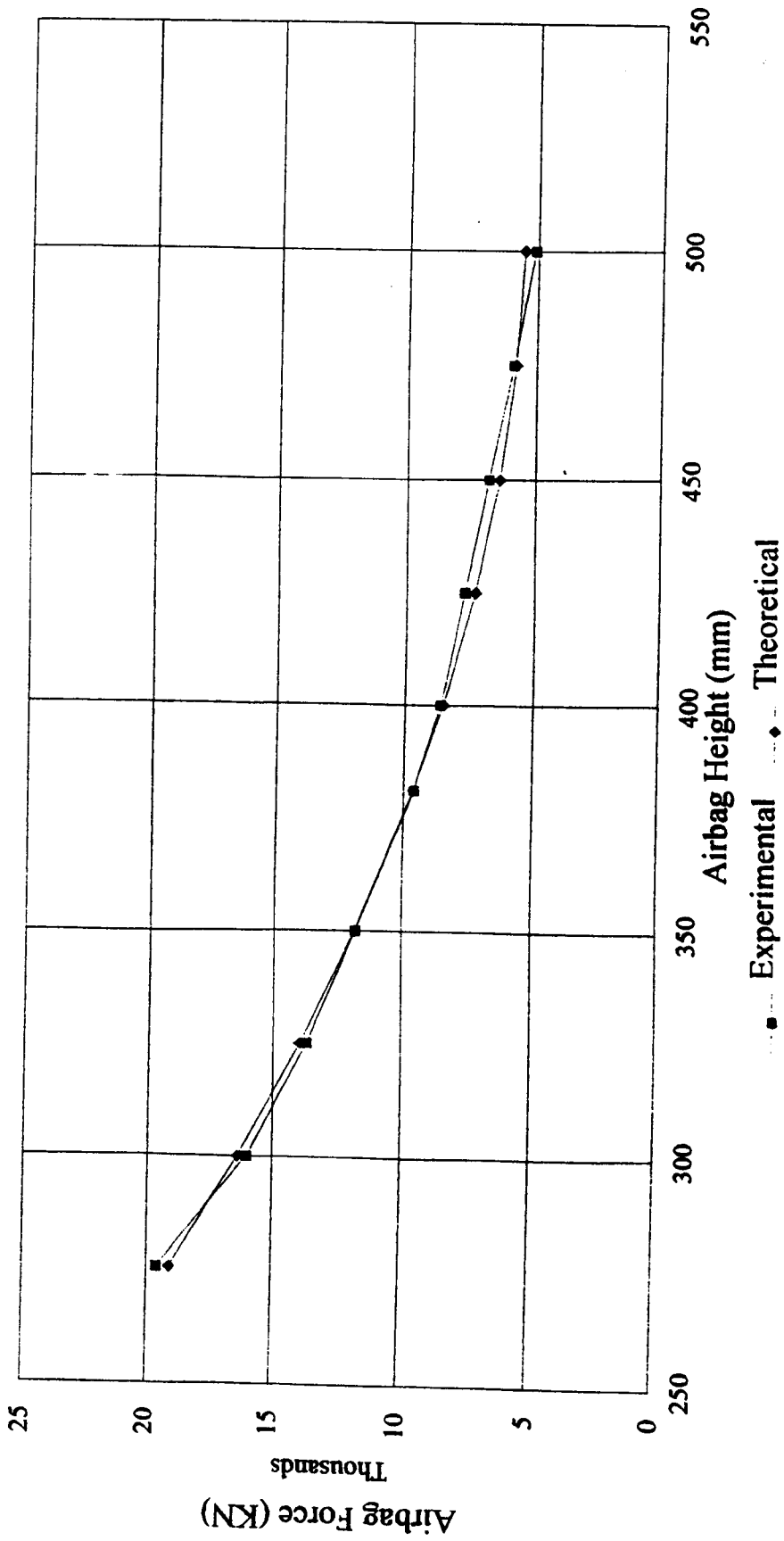


Fig. A1 Variation of airbag force with airbag height for an initial air pressure of 25 psi at an airbag height of 381 mm.

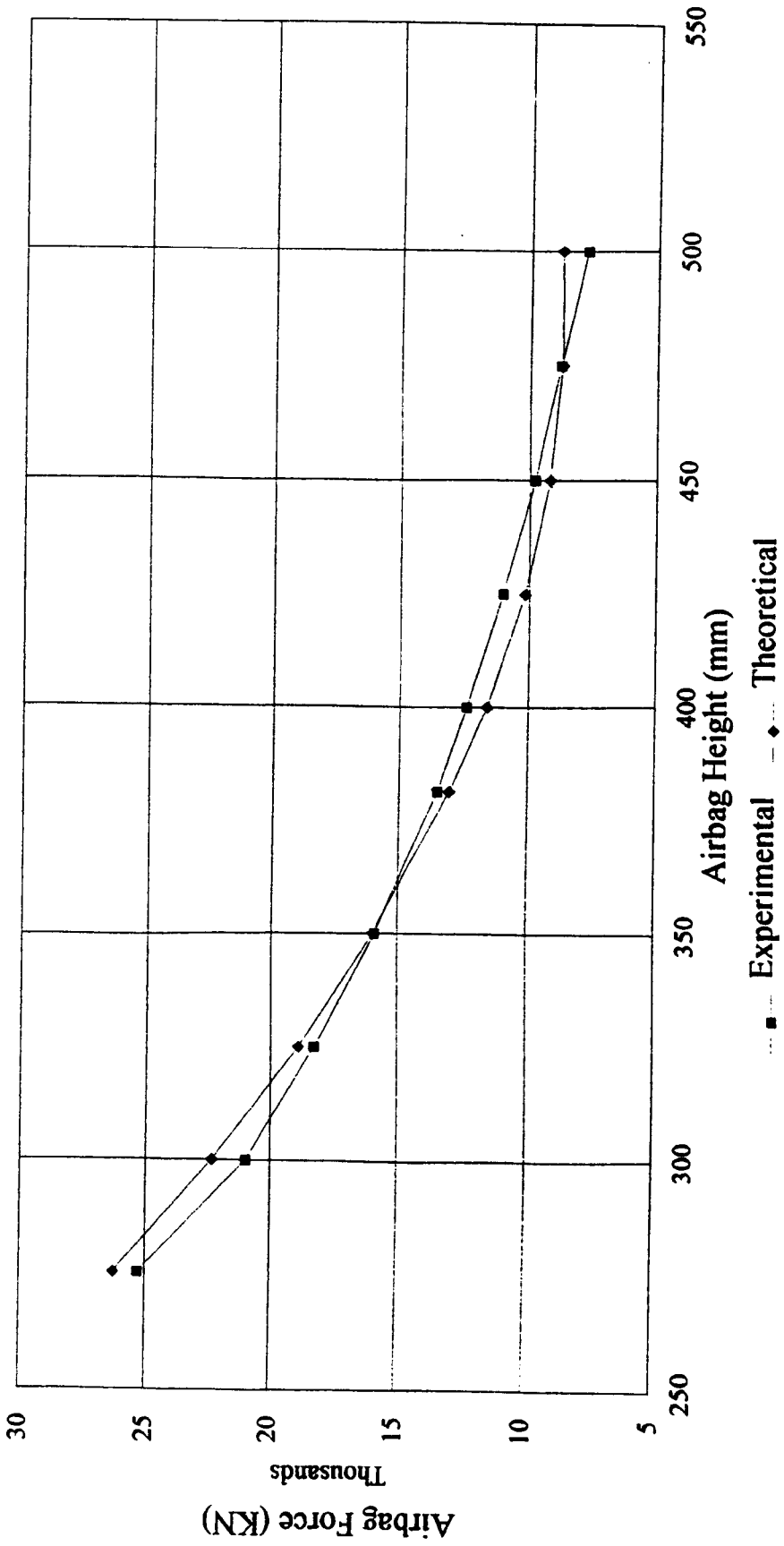


Fig. A2 Variation of airbag force with airbag height for an initial air pressure of 40 psi at an airbag height of 381 mm.

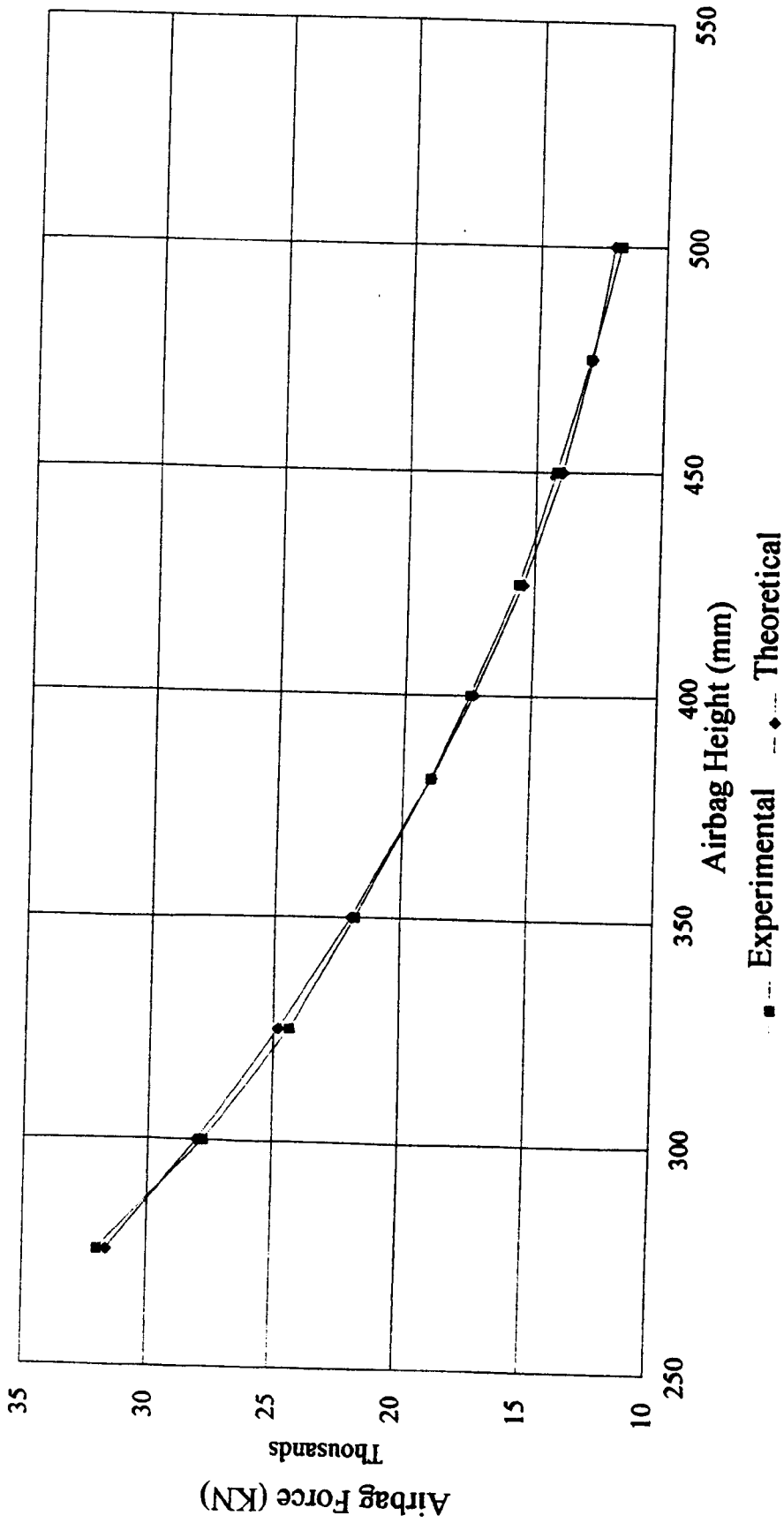


Fig A3 Variation of airbag force with airbag height for an initial air pressure of 55 psi at an airbag height of 381 mm.



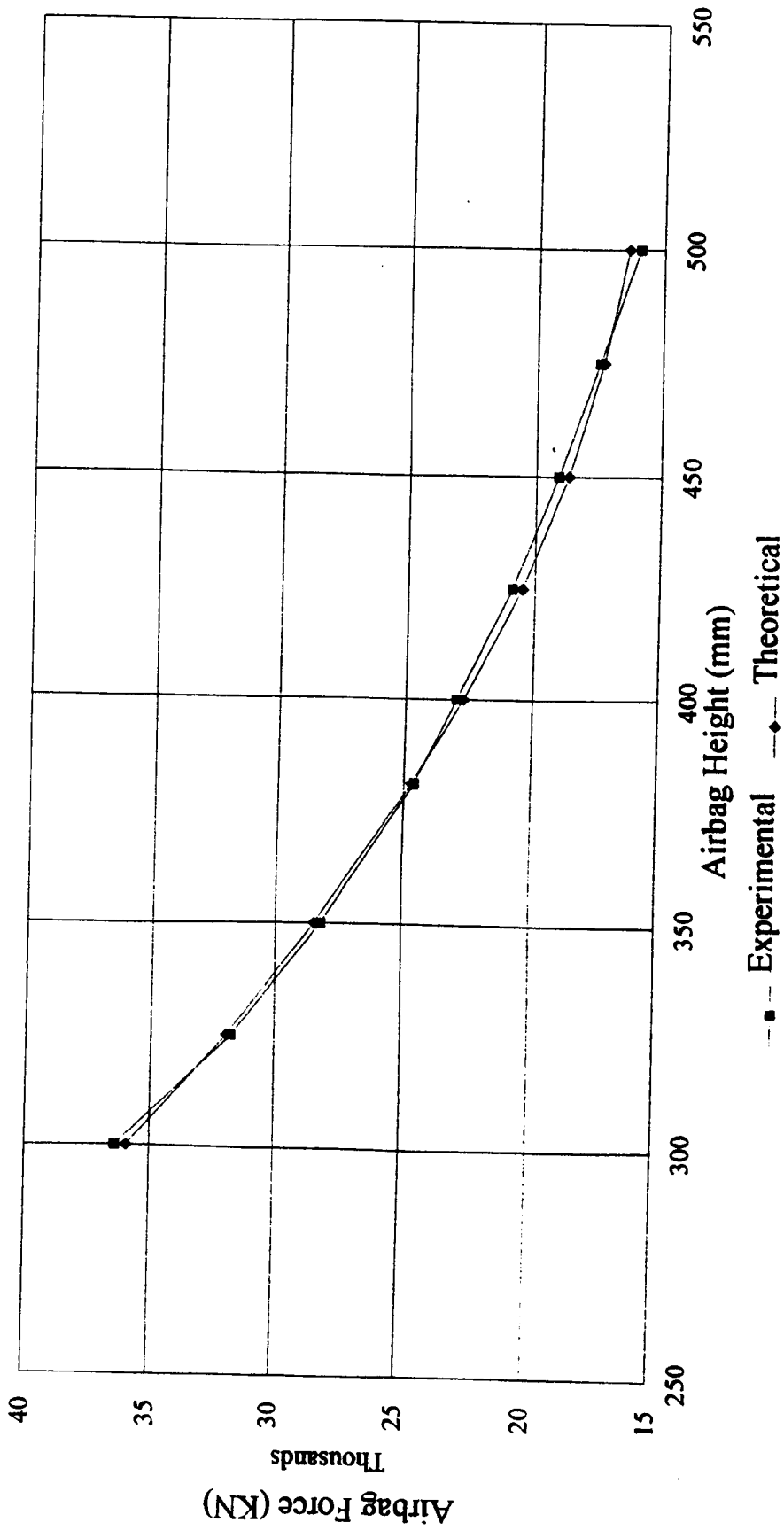


Fig. A4 Variation of airbag force with airbag height for an initial air pressure of 70 psi at an airbag height of 381 mm.

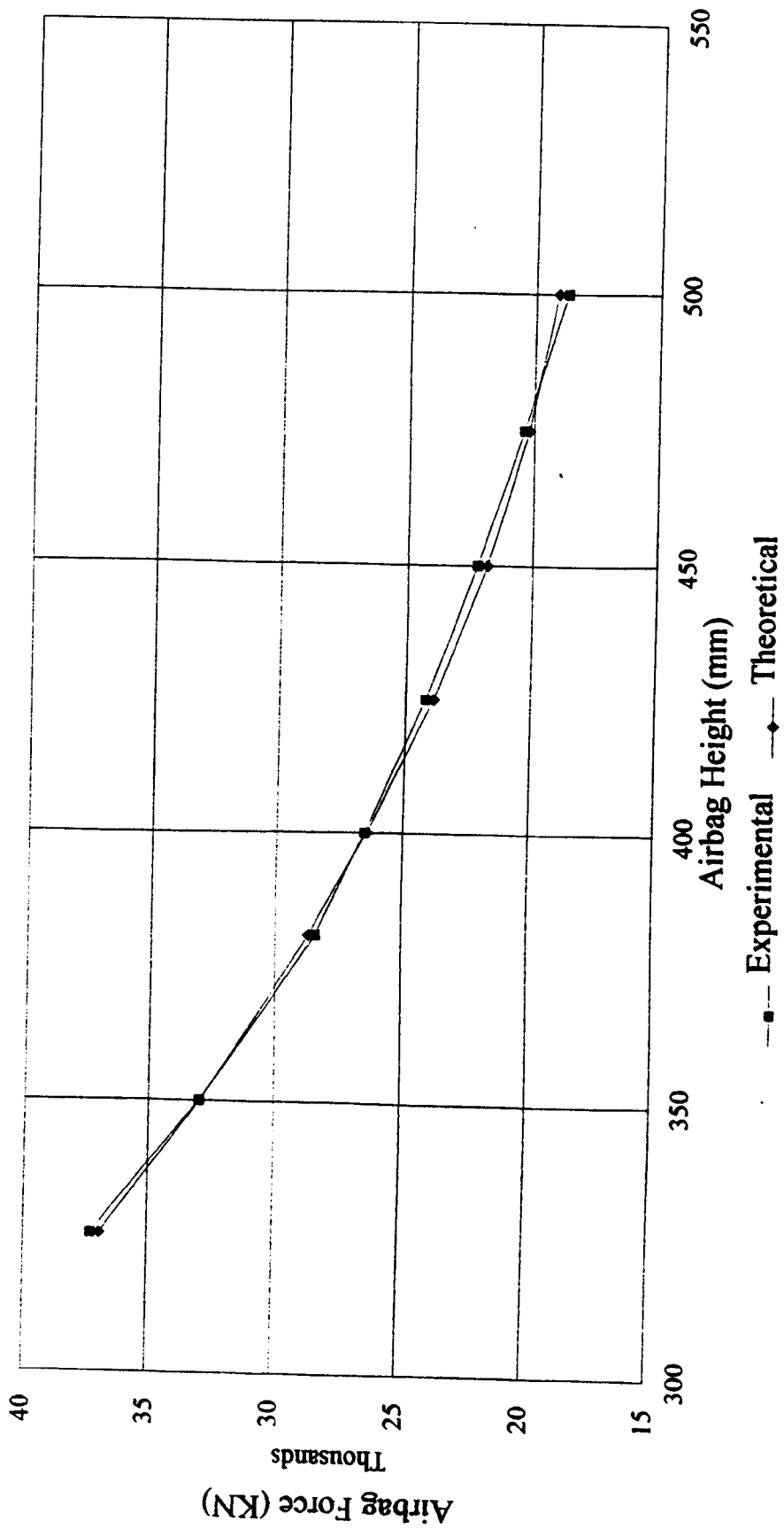


Fig. A5 Variation of airbag force with airbag height for an initial air pressure of 80 psi at an airbag height of 381 mm.

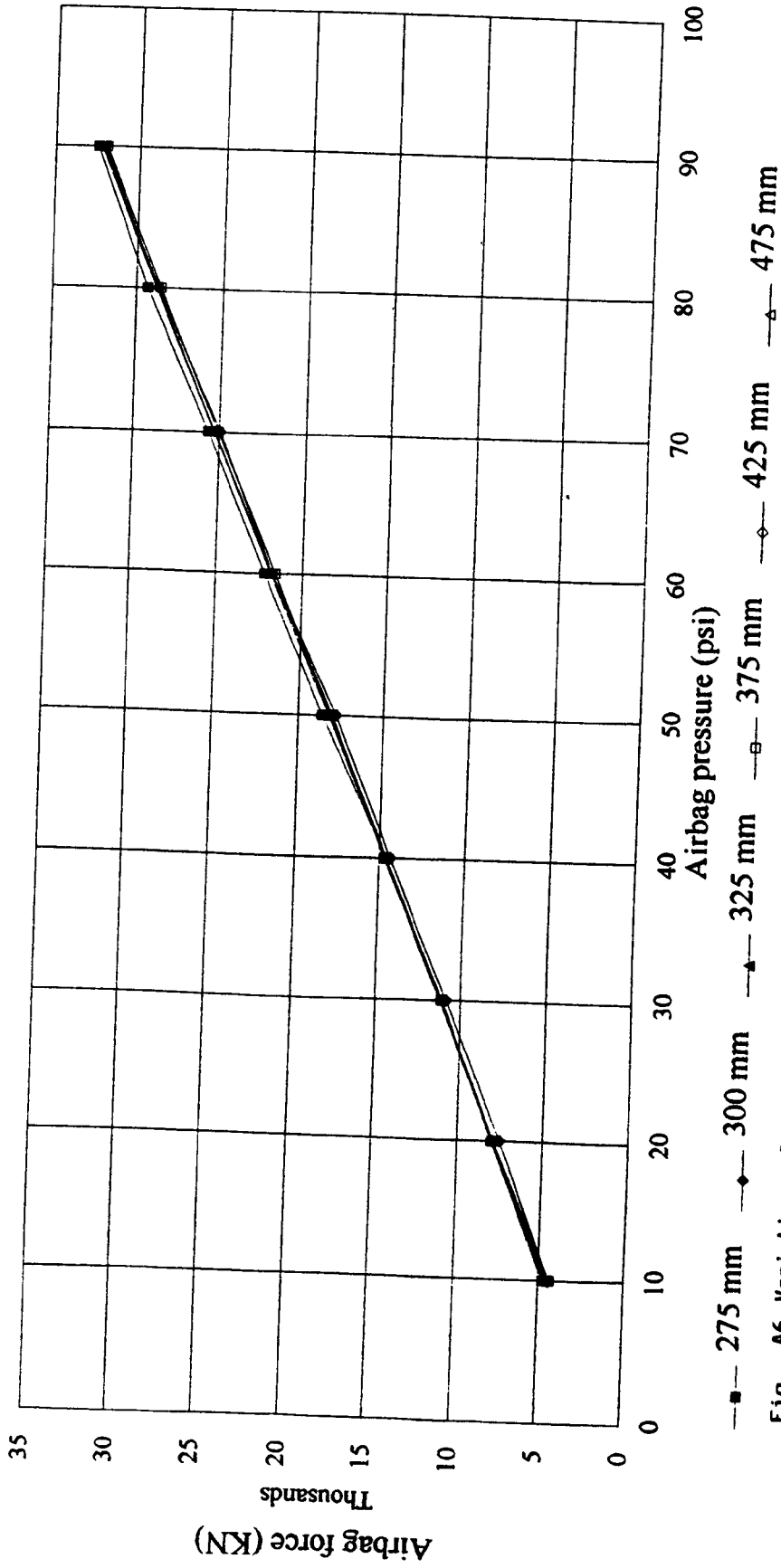


Fig. A6 Variation of airbag force with airbag pressure for six different airbag heights.

## APPENDIX B

### TYRE STIFFNESS

The aim of this experimental work is to determine the flexibility of a stationary, loaded "super single" lorry wheel in the direction of motion, lateral direction and vertical direction.

#### **B1 Theory**

##### **B1.1 In-plane horizontal forces**

When considering the deformation of the wheel in the direction of motion during roll over, shown in Fig. B1, the axle is prevented from rotating by the action of the brake. Hence, a movement  $M_i$ , applied at the axle results in a pair of reaction forces,  $F_i$ , at the axle and the tyre/ground contact. When determining the stiffness  $K_x = F_i/\delta_i$  of a tyre when not connected to a trailer, it is more convenient to restrain the top and bottom of the tyre to determine  $K_x$ , shown in Fig. B2. The stiffness of the tyre when connected to the trailer will be the same as the stiffness  $K_x$  determined experimentally.

The gradient obtained from the graph of  $\delta_i$  versus  $F_i$  shown schematically in Fig. B3, is used to determine the flexibility  $h_i$ .

$$\text{Tyre deflection} = h_i * F_i.$$

Therefore, the in-plane horizontal stiffness of the tyre  $K_x$  is given by

$$K_{xc} = \frac{1}{h_i} \quad (B1)$$

### B1.2 Out-of-plane horizontal forces

When considering the deformation of the wheel when loaded in the lateral, horizontal direction to motion during roll over, shown in Fig. B4, the axle is prevented from rotating by the axle to which it is connected. Hence, a horizontal force,  $F_o$  applied at the axle results in a reaction  $F_o$ , at the tyre/ground contact and a moment  $M_o$  when determining the stiffness,  $K_z = F_o/\delta_o$ , of a tyre when not connected to a trailer, it is more convenient to restrain the top and bottom of the tyre to determine  $K_z$ , shown in Fig. B5. The actual stiffness of the tyre, when connected to the trailer, will be the same as the stiffness  $K_z$ , determined experimentally.

The gradient obtained from the graph of  $\delta_o$  versus  $F_o$ , shown in Fig. B6, is used to determine the flexibility  $h_o$

$$\text{Tyre deflection} = h_o * F_o$$

Therefore, the out-of-plane horizontal stiffness of the tyre  $K_z$  is given by

$$K_z = \frac{1}{h_o} \quad (B2)$$

### **B1.3 Vertical forces**

The vertical load,  $F_v$ , is applied to the wheel through its axle, as shown in Fig. B7. However, it is experimentally more convenient to apply a diameter load to the whole wheel, as shown in Fig. B8. The gradient of a graph of  $\delta_v$  versus  $F_v$ , shown in Fig. B9, is the same for axle loading as for diametral loading, due to the measuring of displacement at the axle during diametral loading. The gradient,  $h_v$ , of the graph of  $\delta_v$  is the flexibility of the wheel in the vertical direction.

The stiffness of the tyre,  $K_y$ , in the vertical direction is given by

$$K_y = 1/h_v$$

(B3)

### **B2 Experimental equipment and procedures**

The tyre and rig used to apply a vertical compressive load are shown in Fig. B10. In the position shown an in-plane horizontal force can be applied to the hub of the tyre. In order to apply an out-of-plane horizontal force the tyre is rotated 90° about the vertical axis. To avoid the possibility of slip (due to limiting friction) at the contact between the tyre and the flow or load cell, two Araldite moulds of the vertically loaded tyre tread, under a load of 40 kN, were made. The tyre was placed in the grooves in the Araldite mould during testing. The Araldite moulds at the top and bottom of the tyre were held in the X and Z directions to prevent movement under loading. The effect of the depth of penetration of the Araldite moulds into the tyre was investigated by carrying out full tread penetration and half tread penetration (ie. 6 mm and 12 mm penetrations).

Equations B1, B2, B3, can be used together with experimental measurements to calculate the tyre flexibilities  $K_x$ ,  $K_y$ , and  $K_z$ . In particular the gradients of the graphs of  $\delta_i$  versus  $F_i$ ,  $\delta_o$  versus  $F_o$  and  $\delta_v$  versus  $F_v$  are required.

### **B2.1 In-plane horizontal tyre loading (to obtain $K_x$ )**

The tyre was placed in a framework securely bolted to the floor and loaded vertically by a hydraulic ram to 40 kN, which is the approximate load on each tyre under operating conditions. The horizontal load was applied using dead weights, with a wire attached to the hub of the tyre and a pulley attached to the framework to convert the vertical load into the required horizontal load. The horizontal displacement was measured using two dial gauges, at a distance of 330 mm above and below the tyre centre, with the average of these displacements taken for the horizontal displacement of the tyre.

### **B2.2 Out-of-plane horizontal tyre loading (to obtain $K_y$ )**

The out-of-plane horizontal tyre stiffness was obtained in the same manner as the in-plane horizontal tyre stiffness, except that the tyre was placed in the framework when rotated 90° about the vertical. The tyre was loaded in the vertical and horizontal direction in the same manner as for the in-plane tyre test. The horizontal displacement was also measured and used in the same manner as previously described in relation to the in-plane loading.

### **B2.3 Vertical tyre loading (to obtain $K_y$ )**

The tyre was placed in the framework and loaded in increments of 5 kN from 0 to 45 kN. The vertical displacement was measured using two dial gauges, at a distance of 330 mm either side of the tyre centre, with the average of these displacements taken for the vertical displacement of the tyre hub.

### **B3 Results**

The experimental results, for a 6 mm tyre tread depth in the Araldite mould, are shown in Figs. B11, B12, and B13, with only the final result for the 12 mm tread given. The gradients for these graphs were obtained using a linear regression analysis. The gradients and corresponding tyre stiffnesses for the in-plane, out-of-plane and vertical loadings are given in Tables B1 , B 2, and B 3, respectively. The tyre stiffnesses are compared with the manufacturers' stiffnesses in Table B. 4.

The different tread depths used in the Araldite mould did not make any significant difference to the stiffnesses obtained. The  $K_x$  and  $K_z$  stiffnesses obtained are greater than the manufacturers' supplied stiffness. The  $K_y$  stiffness agreed well with the manufacturers' stiffness.

The present results were found to be within 67%, 8% and 32% of the manufacturers' results for the  $k_x$ ,  $k_y$  and  $k_z$  stiffnesses, respectively. The level of agreement, bearing in mind that the loading and support conditions were not correct, was considered to be good enough to allow the manufacturers' data to be confidently used.





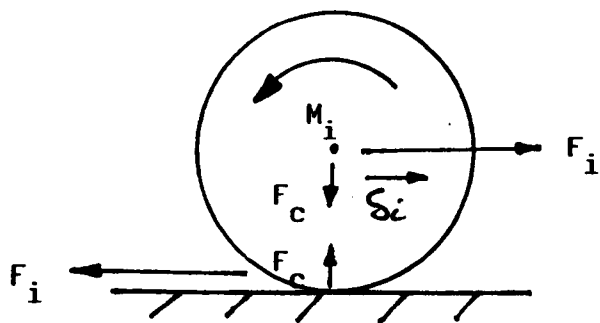


Fig. B1 Free body diagram for in-plane horizontal loading of the wheel

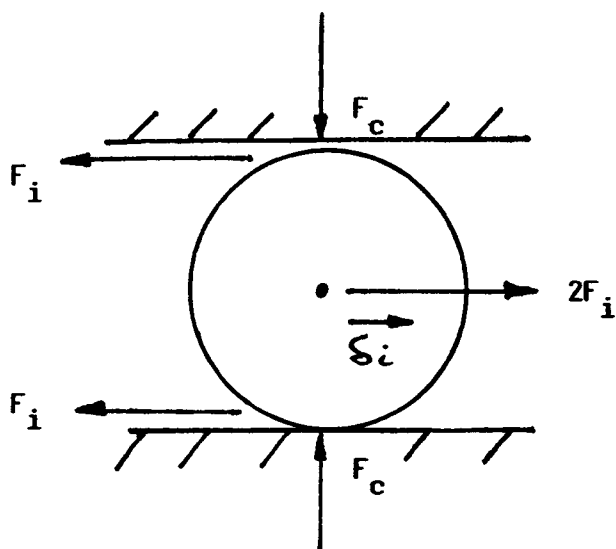


Fig. B2 Free body diagram for applying an inplane horizontal force  $2F_i$  to the wheel

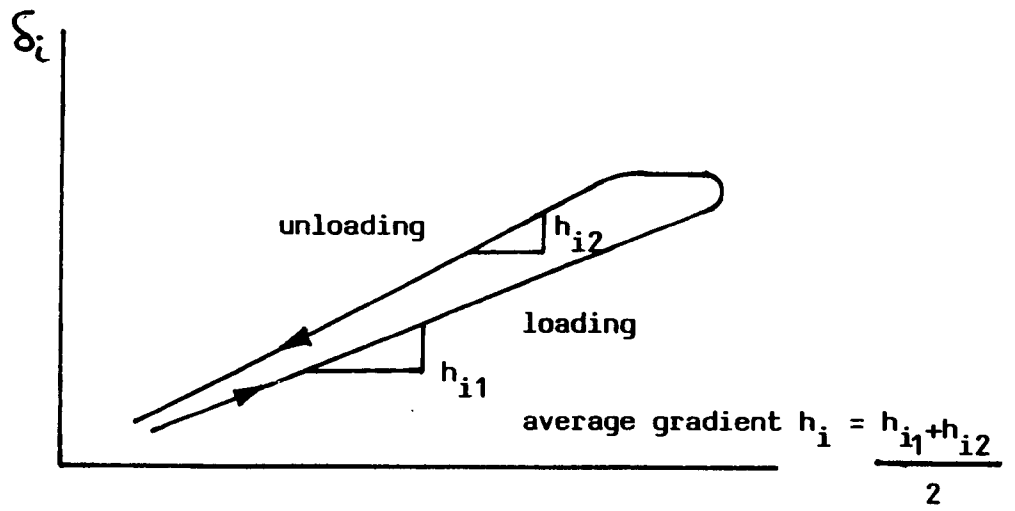


Fig. B3 Schematic diagram of  $\delta_i$  versus  $F_i$

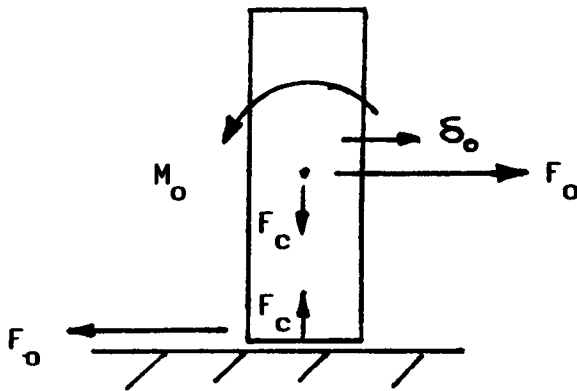


Fig. B4 Free body diagram for out of plane loading of the wheel

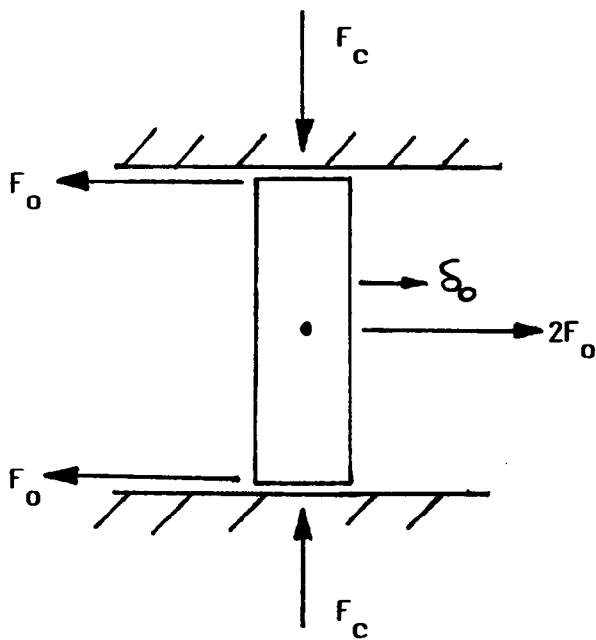


Fig. B5 Free body diagram for applying an out of plane horizontal force,  $2F_o$ , to the wheel

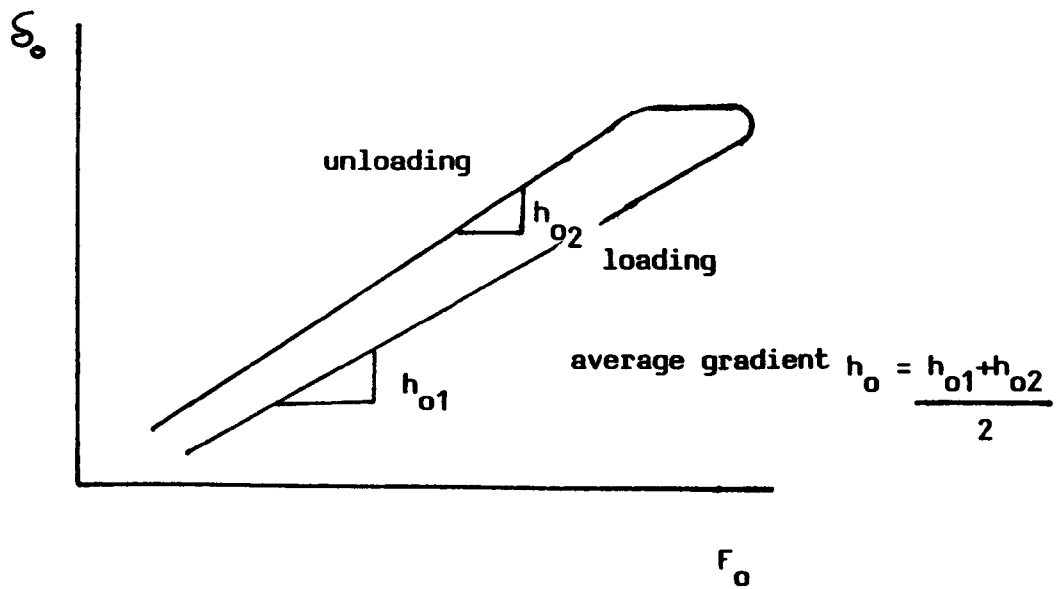


Fig. B6 Schematic diagram of  $\delta_o$  versus  $F_o$

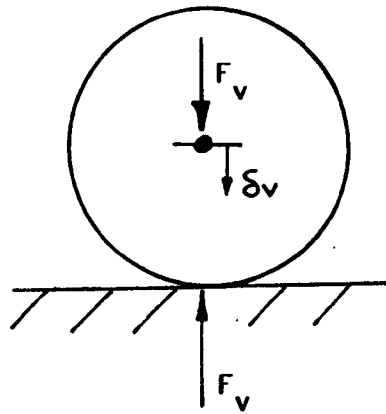


Fig. B7 Free body diagram for vertical wheel loading

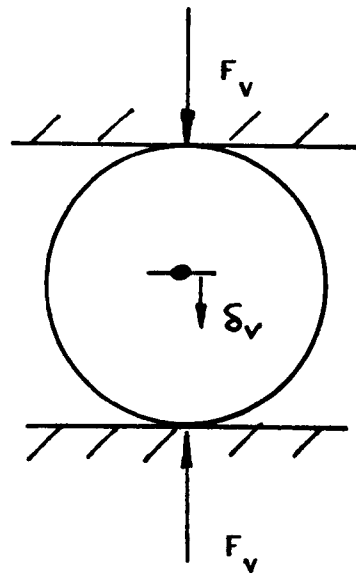


Fig. B8 Free body diagram for applying a vertical force,  $F_v$ , to the wheel.

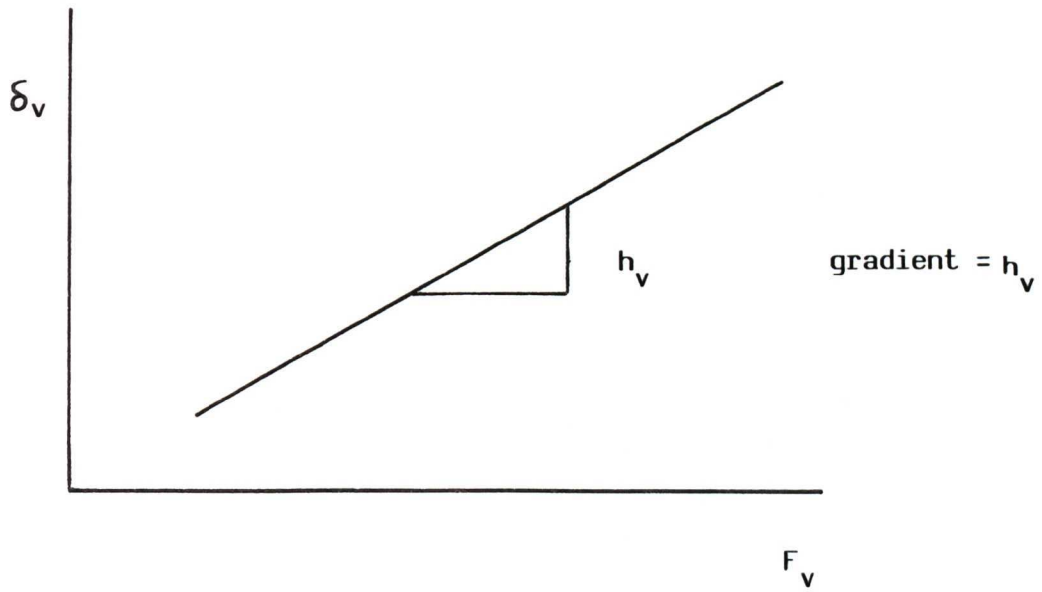


Fig. B9 Schematic diagram of  $\delta_v$  versus  $F_v$

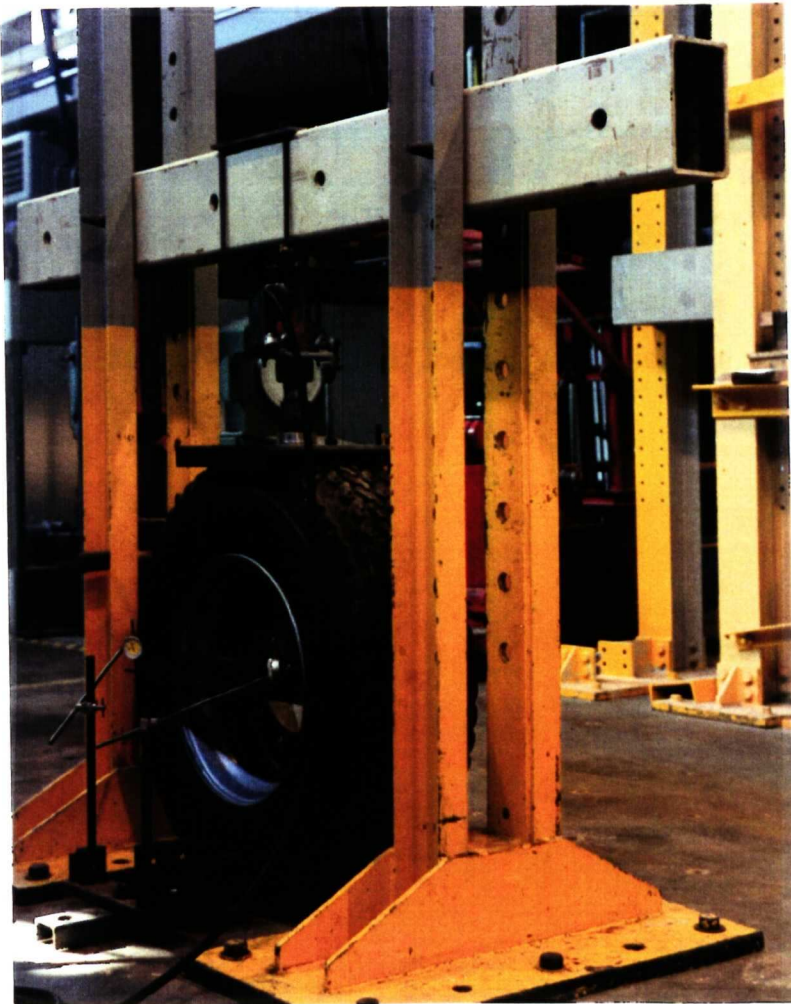


Fig. B10 Loading rig used to apply a vertical force  $F_v$  to the tyre

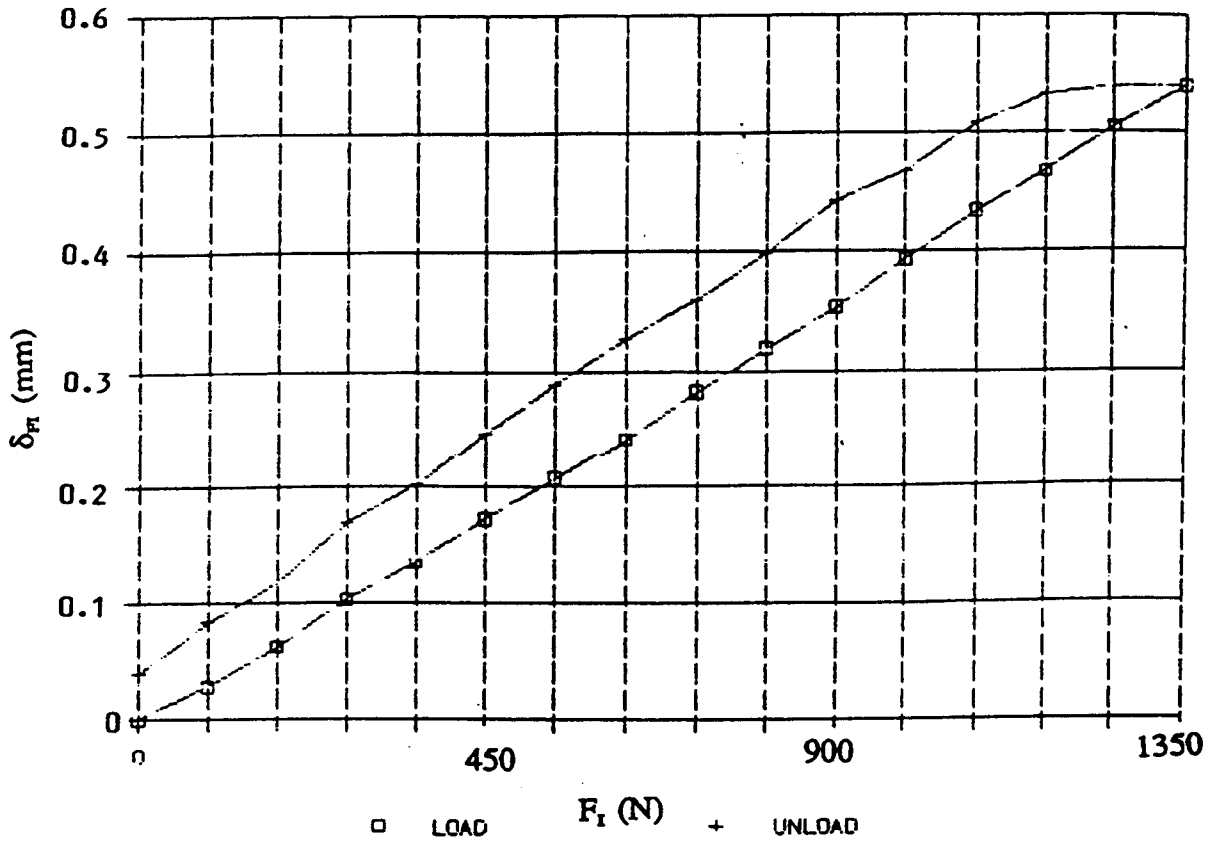


Fig. B11 Variation of  $\delta_{F1}$  with  $f_1$ , for loading and unloading, with a 6 mm tread depth in the Araldite mould.

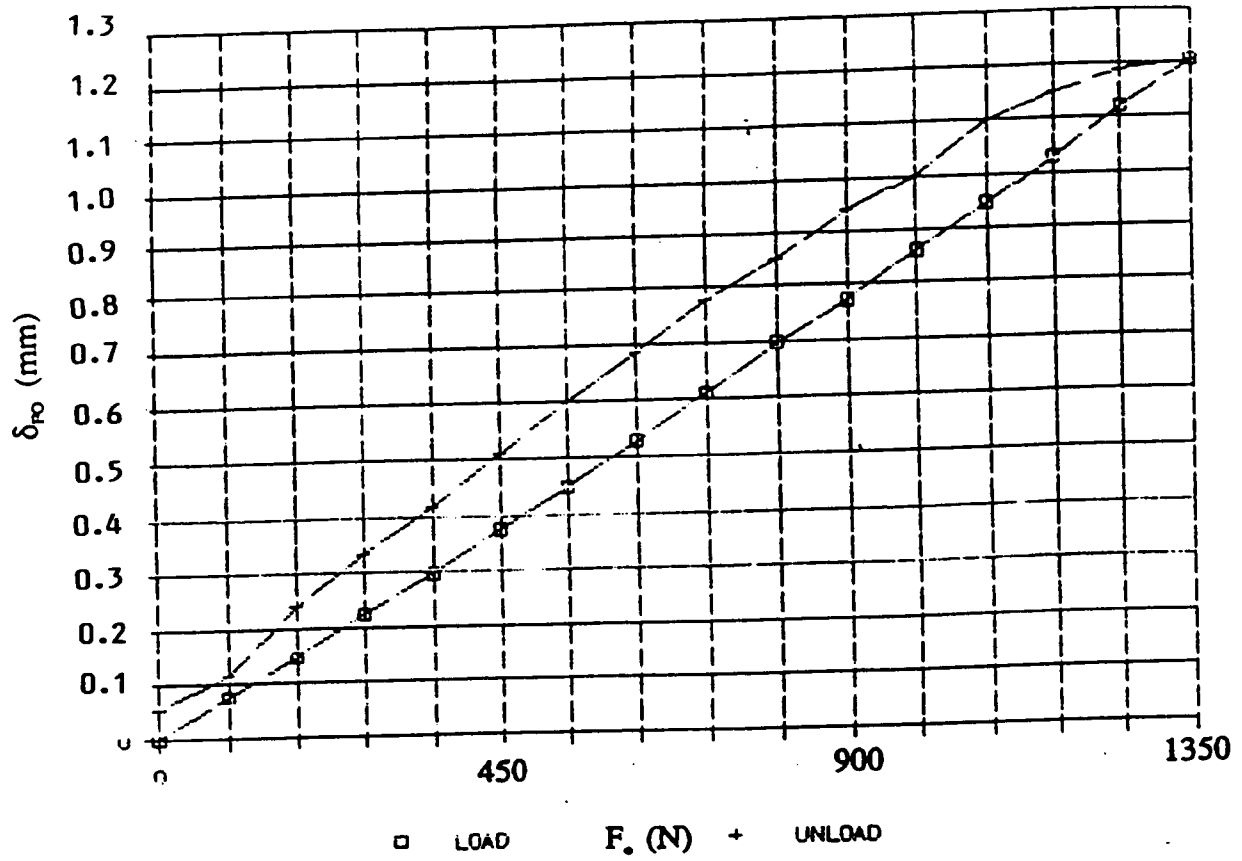


Fig. B12 Variation of  $\delta_{FPO}$  with  $F_o$ , for loading and unloading, with a 6 mm tread depth in the Araldite mould.



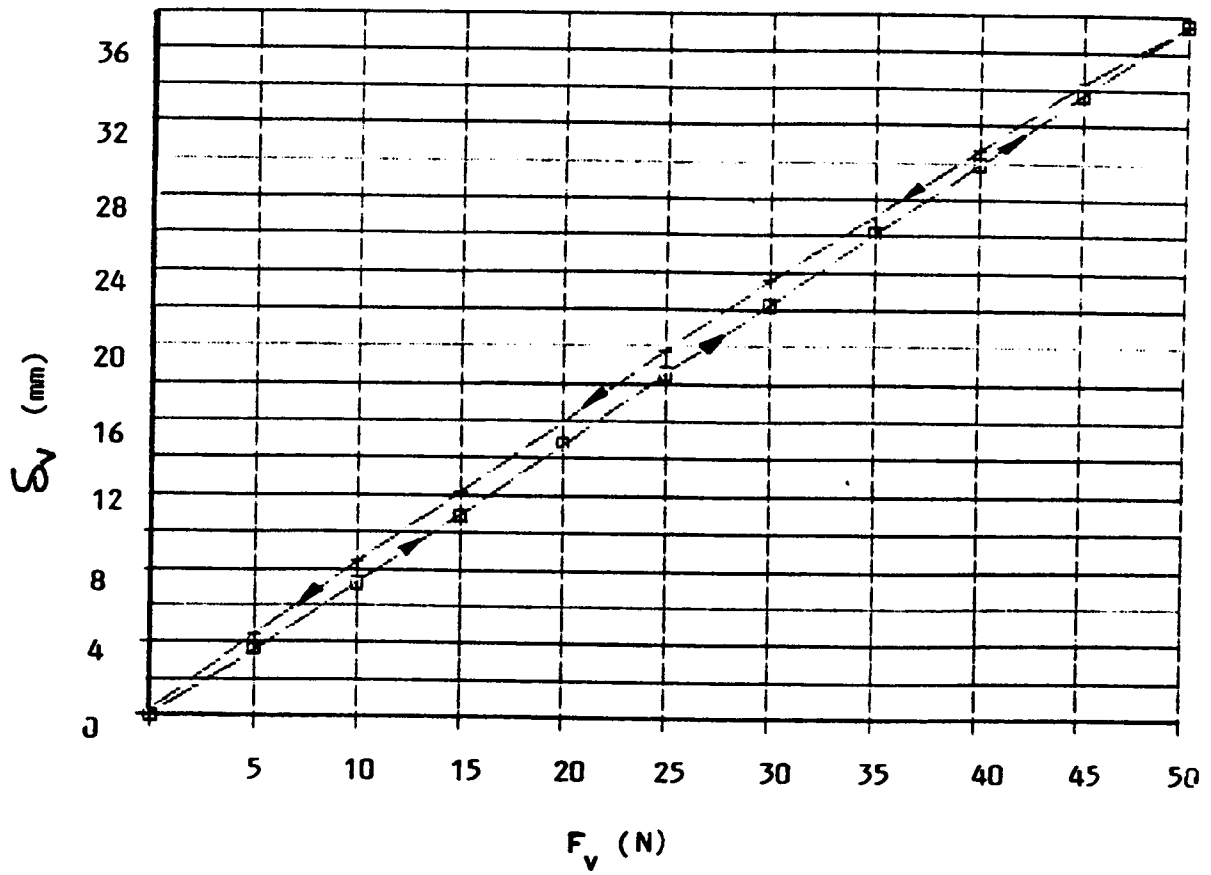


Fig. B13 Variation of  $\delta_V$  with  $F_V$ , for loading and unloading, with a 12 mm araldite mould.

**Table B1: In-plane, Horizontal Loading Results**

Depth of mould tread (mm)	$h_{i1}$ (mm/N)	$h_{i2}$ (mm/N)	$h_i$ (mm/N)	$k_x$ (N/mm)
6	$4.068 \times 10^{-4}$	$4.363 \times 10^{-4}$	$4.216 \times 10^{-4}$	1186
12	$4.200 \times 10^{-4}$	$4.575 \times 10^{-4}$	$4.388 \times 10^{-4}$	1140

**Table B2: Out-of-plane, Horizontal Loading Results**

Depth of mould tread (mm)	$h_{o1}$ (mm/N)	$h_{o2}$ (mm/N)	$h_o$ (mm/N)	$k_z$ (N/mm)
6	$8.968 \times 10^{-4}$	$9.633 \times 10^{-4}$	$9.301 \times 10^{-4}$	538
12	$8.843 \times 10^{-4}$	$9.599 \times 10^{-4}$	$9.221 \times 10^{-4}$	542

**Table B3: Vertical Loading Results**

Depth of mould tread, (mm)	$h_v$ (mm/N)	$k_y$ (N/mm)
12	$8.666 \times 10^{-4}$	1154

**Table B4: Comparison of Manufacturers Tyre Stiffnesses with Experimental Results**

Depth of mould tread (mm)	$k_x$ (N/mm)	$k_y$ (N/mm)	$k_z$ (N/mm)
6	1186	-	538
12	1140	1154	542
Manufacturers' value	712	1070	412

## **APPENDIX C**

### **COMPUTER PROGRAM AND FLOW CHART**

The program was written on an Akhter 286 PC using Turbo Basic. This language was chosen as a novice programmer would be able to modify the program relatively easily, while the language offered a more sophisticated program construction compared to ordinary Basic. The main advantage is that completely separate subroutines can be developed independently, enabling a large program to be split into manageable sections. The program subroutines are described in conjunction with the program flow chart, shown in Fig. C1(a) to Fig. C1(c) and the corresponding Turbo Basic program(s) shown in Fig. C2 to C28.

#### **C1 Start**

The program is controlled by the main program TIPPER.BAS, shown in Fig. C2 which contains the subroutine described next. The subroutines are linked together through the main program, allowing variables to be passed between the subroutines. The subroutines are executed in a sequence described by the routine CONTROL.BAS, shown in Fig. C3.

#### **C2 Input routines**

- (i) The first input routine contains the undeformed co-ordinates of the hinges, ram/chassis contact, ram/body contact and body centre of gravity, and the

beam mass. These variables are fixed for each trailer under consideration with the coding described by the routine INPUTR.BAS. shown in Fig. C4.

- (ii) The second input routine contains the undeformed co-ordinates of the payload centre of gravity, the payload mass, the initial ground slope and the ram length, all of which can be determined by the operator. The program is currently set to automatically consider four payload masses in the nine payload positions. The ram length is varied from 2 to 8 in steps of 1 with the ground slope varying from  $0^\circ$  to  $10^\circ$  in steps of  $0.1^\circ$ , for each ram length considered. The coding describing the subroutine is given in VARIAB.BAS, shown in Fig. C5.

### **C3 Initialise variables routine**

The variables used during the program operation are set to zero at the beginning of each new payload magnitude, or payload position, or ground slope, or ram length considered. The coding describing this routine is given in ZERO.BAS, shown in Fig. C6.

### **C4 Vector positional routine**

The co-ordinates of the top of the ram, payload centre of gravity and body mass centre of gravity are determined using the vector analysis described in Section 3.4.1. For the first iteration for each new body or chassis position considered, their co-

ordinates are determined for a rigid chassis, with subsequent iterations taking into account chassis deformations. The coding describing this routine is given in VECTOR1.BAS and VECTOR.BAS, shown in Fig. C7 and Fig. C8.

#### **C5 Ram direction routine**

The ram direction is described by three cosine resolving functions which independently relate the direction of the ram to the X, Y, or Z axis. These functions are used in the body equilibrium analysis described in Section 3.4.2. The coding describing this routine is given in RAM.BAS, shown in Fig. C9.

#### **C6 Body equilibrium matrix routine**

The body equilibrium matrix, EQM(6,6) is formed using the co-ordinates of the hinges and the ram/body contact, and the ram resolving functions described in Section 3.4.2. The coding describing this routine is given in EQUILB.BAS, shown in Fig. C10.

#### **C7 Body equilibrium matrix inversion routine**

The body equilibrium matrix EQM(6,6), described in Section 3.4.2, is inverted to form EQMI(6,6), in order to calculate the body reactions. The coding describing this routine is given in INVERT1.BAS, shown in Fig. C11.

## **C8 Loading matrix routine**

The co-ordinates of the payload and body mass centres of gravity, the payload mass and the body mass, are used to form the force and moment coefficients of the loading matrix LM(6), described in Section 3.4.2. The coding describing this routine is given in BODYF.BAS, shown in Fig. C12.

## **C9 Body reactions routine**

The inverted body equilibrium matrix EQM1(6,6) is multiplied by the loading matrix LM(6) to give the body/chassis contact point reaction forces described in Section 3.4.2. The coding describing this routine is given in BODYF.BAS, shown in Fig. C12.

## **C10 Suspension routine**

This routine chooses the appropriate suspension forces and suspension operating condition, which is compatible with the chassis forces based on one of the four suspension operating conditions described in Section 3.4.3. There are four different matrices corresponding to the four suspension operating conditions, which enable the suspension forces to be calculated. The coding describing this routine is given in SUSP.BAS, CASEa.BAS, CASEb.BAS, CASEc.BAS and CASEd.BAS, show in Fig. C13 to Fig. C17.

### **C11 Chassis flexibility routine**

There are four different matrices for calculating chassis deformations, which correspond to the four suspension operating conditions described in Section 3.4.3. Depending upon the suspension condition, the appropriate matrix is chosen to calculate the chassis deformations, described in Section 3.4.3. The coding describing this routine is given in FLEXI.BAS, FLEXI1.BAS, FLEXI2.BAS, FLEXI3.BAS, FLEXI4.BAS and DISP.BAS, shown in Fig. C18 to Fig. C23.

### **C12 Tyre force routine**

There are four different matrices for calculating tyre forces, which correspond to the four suspension operating conditions described in Section 3.4.3. Depending upon the suspension condition, the appropriate matrix is chosen to calculate the tyre forces, described in Section 3.4.3. The coding describing this routine is given in TYRE.BAS, TYRE1.BAS, TYRE2.BAS, TYRE3.BAS and TYRE4.BAS, shown in Fig. C24 to Fig. C27.

### **C13 Converge routine**

At the end of each iteration except the first, a check is made to see if the procedure has converged or has become unstable. The displacement of the top of the ram is compared with the previous displacement and if the change in this is within the specified tolerance of 1 mm, the solution is deemed to have converged, else a further



iteration is undertaken. After convergence the tyre forces are examined and as long as the forces are positive (indicating a downwards force) the trailer is in a stable position. The ram length, tyre force, payload position and payload magnitude are then written to a file. If the trailer is in a stable position, this process is repeated, increasing the ground slope after every converged iteration, so long as the trailer remains stable, up to a maximum ground slope of  $10^\circ$ .

If the tyre force becomes negative (indicating an unstable trailer position) or when the ground slope is equal to  $10^\circ$ , the ram length is increased by 1 m and the ground slope is re-set to zero. The program is then run again for this new body position. When the ram length is 8 m and the ground slope is  $10^\circ$ , the program will have considered the current payload mass and position in 147 body attitudes and ground slopes. At this point, the program can choose a new payload mass and position which is dependent upon the operators supplied data. The program is initiated again until all payload mass and positions have been considered. The program is then terminated. The coding describing this routine is given in CONVERGE.BAS, shown in Fig. C28.

Fig. C1(a) Program flow chart

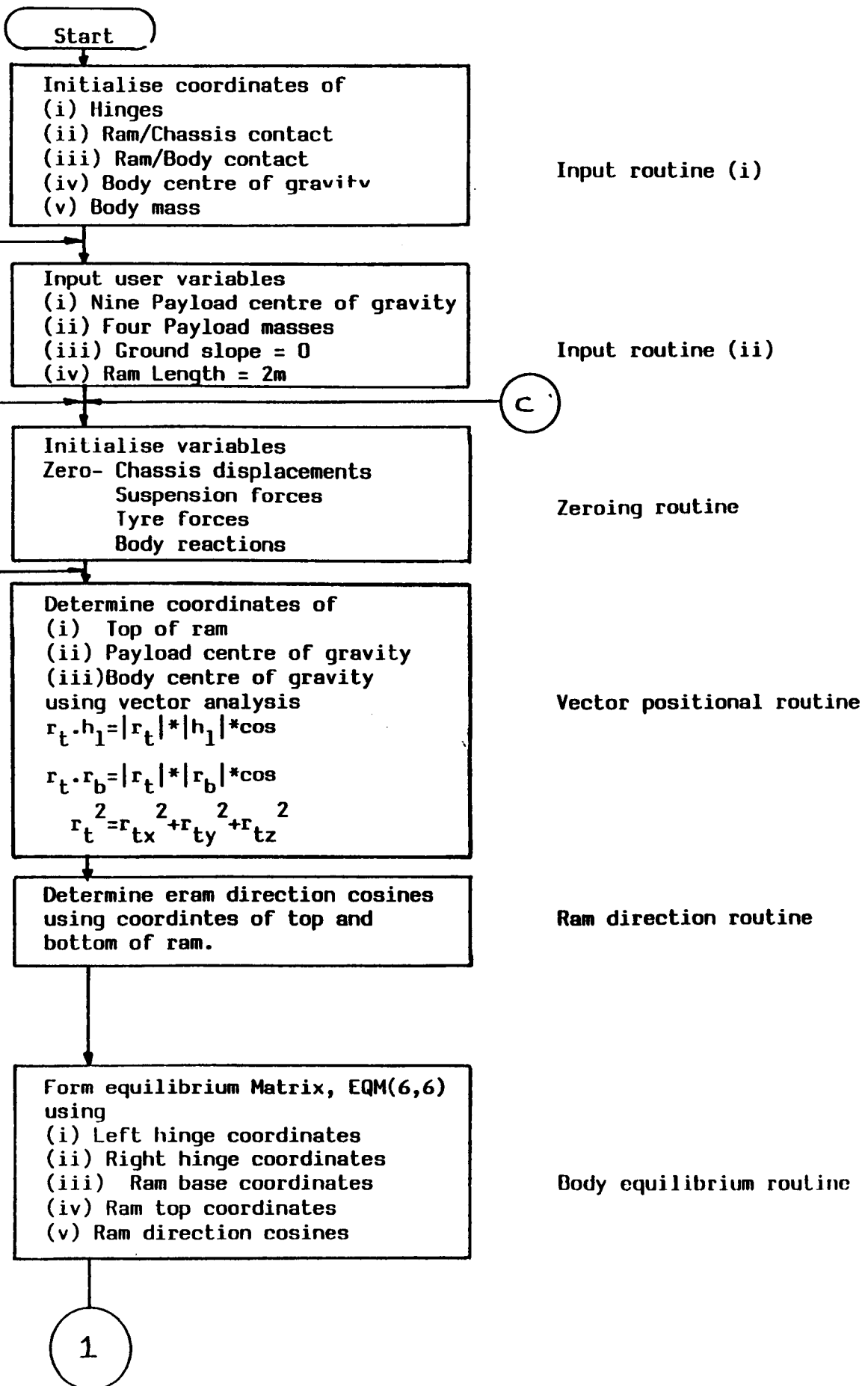


Fig. C1(b) Program flow chart

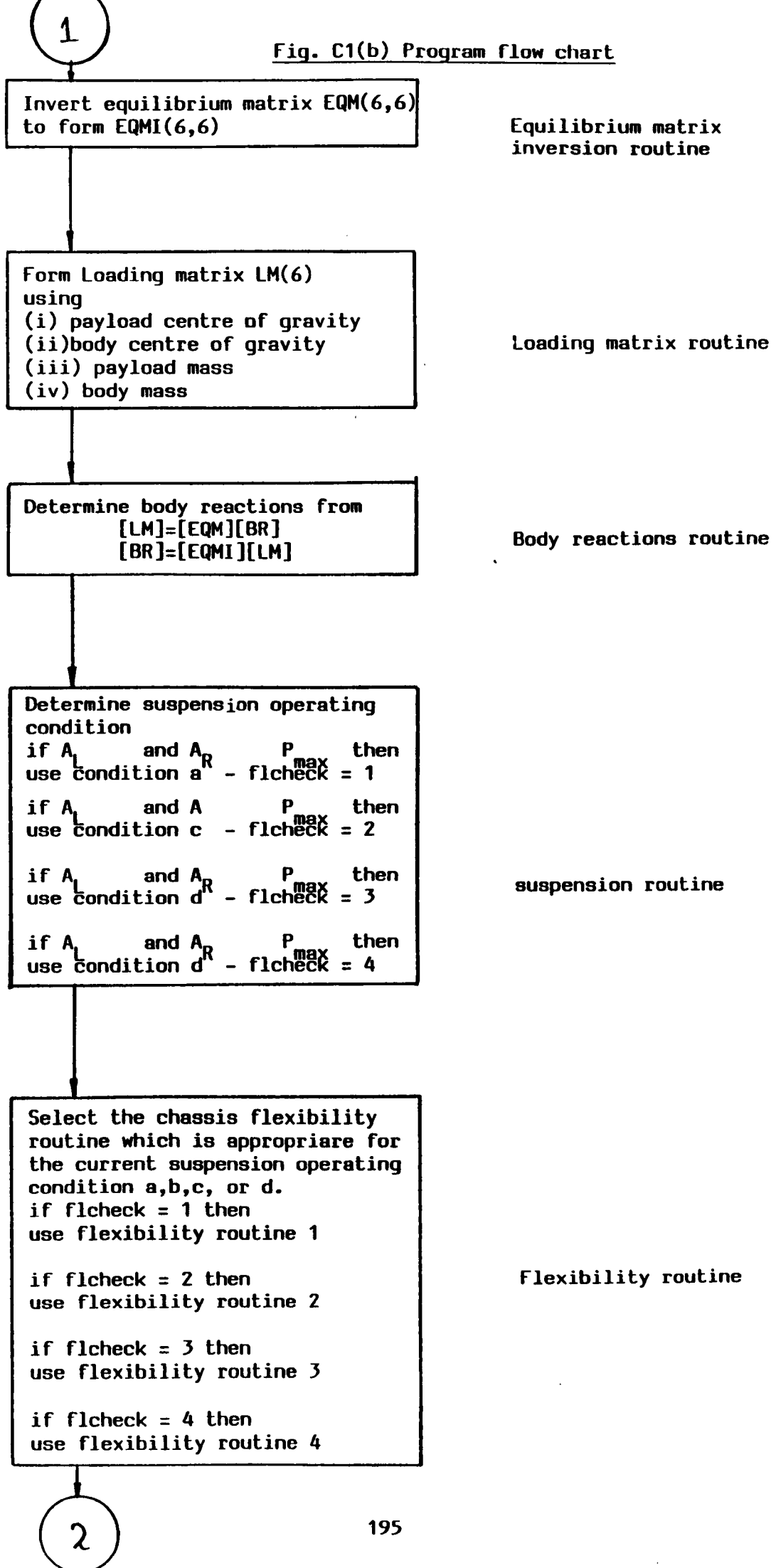
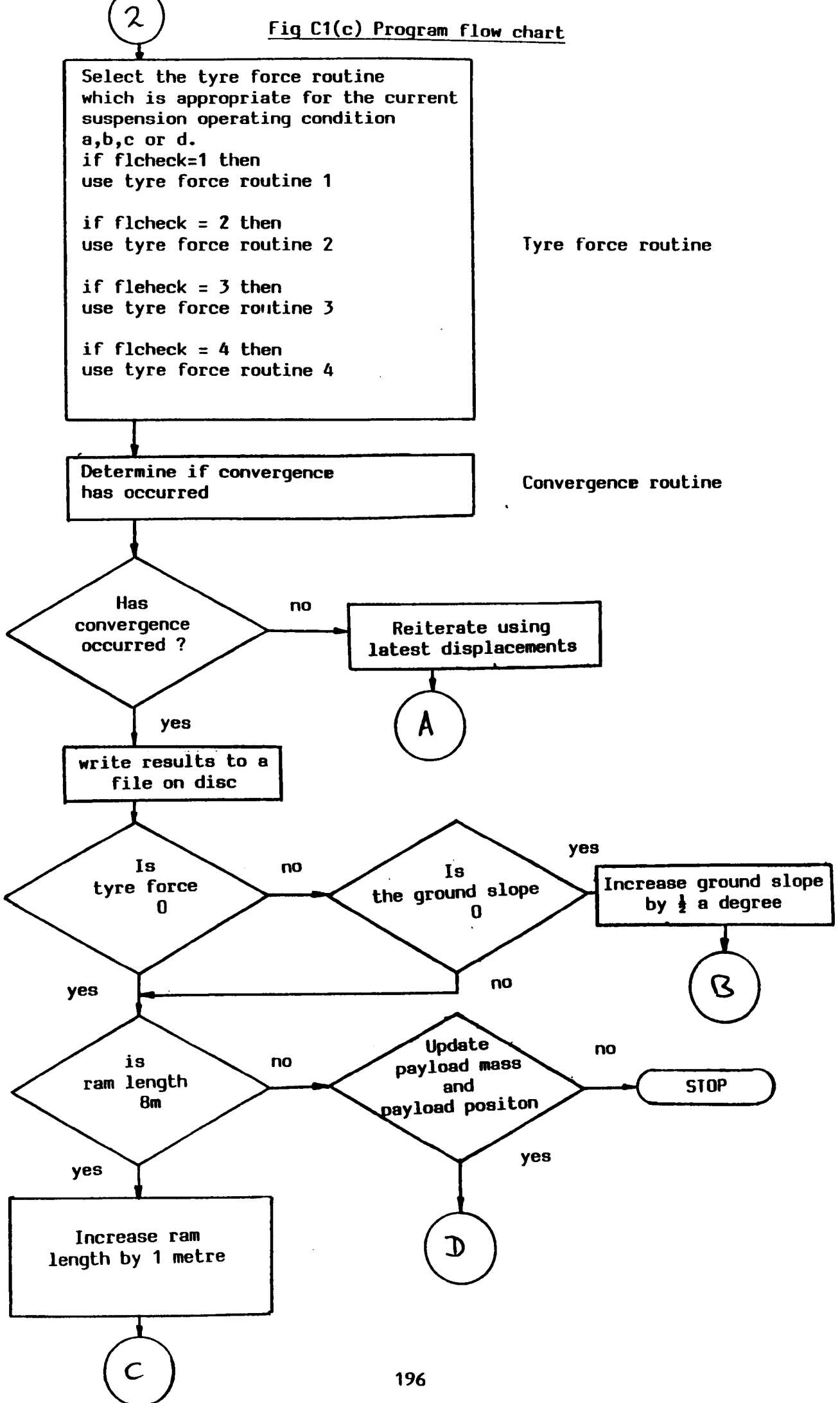


Fig C1(c) Program flow chart



' S.G.PICKERING

' \*\*\*\*\*  
 \*\*\*\*\*

' This is the main program described in Appendix C1 which links  
 the  
 ' subroutines together.

' \*\*\*\*\*  
 \*\*\*\*\*

cls

```

DIM DYNAMIC lhc#(2,35),rhc#(2,35),rb#(2,35),rt#(2,35),wc#(2,35)
DIM DYNAMIC bcg#(2,35),pcg#(2,35),BF#(5,35),BR#(7,35)
DIM DYNAMIC CHL#(15),flex#(8,15),uchassis#(8,35)
DIM DYNAMIC SUSPR#(35),SUSPL#(35),brconst#(35)
DIM DYNAMIC eqm#(5,5),eqmi#(5,5)
DIM DYNAMIC k11#(17,17) , k11i#(17,17) , k12#(17,5) , k21#(5,17)
, k22#(5,5)
DIM DYNAMIC prescribe#(29,23) , k21k11i#(5,17) ,
k21k11ik12#(5,5)
DIM DYNAMIC k22k21k11ik12#(5,5) , k22k21k11ik12i#(5,5)
DIM DYNAMIC a#(5) , fab#(5) , fch#(5) , fch1#(5,17) ,
FCH2#(23),uab#(5,35)
DIM DYNAMIC TYREF#(5,35),uch#(23,35),FLCHECK(35)
D I M D Y N A M I C
POSI$(4,8),PCGX$(8),PCGY$(8),PCGZ$(8),PM$(4),PM$(4,8)

```

CALL VARIABS

for y%=1 to 4

for x%=1 to 8

```

if POSI$(y%,x%)=1 then
PM#=PM$(y%)*9.81
pcg#(0,0)=PCGX$(x%)
pcg#(1,0)=PCGY$(x%)
pcg#(2,0)=PCGZ$(x%)
B$=STR$(x%)
FILEN$=PM$(y%,x%)
A$="PAYLOAD POSITION="+B$

```

ITER=0

r1#=2000

focount=0

FLAG3=0

PRINT FILEN\$,A\$

OPEN "A",#1,FILEN\$

WRITE#1,A\$

CLOSE#1

```

DO UNTIL rl# > 8000
print "ram length=";rl#;pm#;pcg#(0,0);pcg#(1,0);pcg#(2,0)
OPEN "A",#1,FILENS
WRITE#1,rl#
CLOSE#1

concount=0
FLAGC=0
thetap#=0
thetactr=0
tyre$="go"

DO UNTIL tyre$="stop" or thetactr>20.1
print "thetap=";thetactr
CALL CONTROL
CALL ZERO
thetactr=thetactr+1/2
thetap#=thetap#+3.1415927/(180*2) 'increment by 1/2 degree
in rads.
ITER=1
n=0
l=0
m=0
pcg#(0,0)=PCGX#(x%)
pcg#(1,0)=PCGY#(x%)
pcg#(2,0)=PCGZ#(x%)

LOOP

rl#=rl#+1000
focount=focount+1

LOOP
end if
A$=""
B$=""
C$=""
FILENS=""
D$=""
E$=""
next x%
next y%
stop

$INCLUDE "VARIABS.BAS"
$INCLUDE "ZERO.BAS"
$INCLUDE "CONTROL.BAS"
$INCLUDE "INPUTR.BAS"
$INCLUDE "VECTOR1.BAS"
$INCLUDE "VECTOR.BAS"
$INCLUDE "RAM.BAS"
$INCLUDE "EQUILIB.BAS"
$INCLUDE "BODYF.BAS"

$SEGMENT

$INCLUDE "SUSP.BAS"

```

```
$INCLUDE "CASE1.BAS"  
$INCLUDE "CASE2.BAS"  
$SEGMENT  
$INCLUDE "CASE3.BAS"  
  
$SEGMENT  
  
$INCLUDE "CASE4.BAS"  
$INCLUDE "INVERT1.BAS"  
$INCLUDE "FLEXI.BAS"  
$INCLUDE "FLEXI1.BAS"  
$INCLUDE "FLEXI2.BAS"  
$INCLUDE "FLEXI3.BAS"  
$INCLUDE "FLEXI4.BAS"  
  
$SEGMENT  
  
$INCLUDE "DISP.BAS"  
$INCLUDE "TYRE.BAS"  
$INCLUDE "TYRE1.BAS"  
$INCLUDE "TYRE2.BAS"  
$INCLUDE "TYRE3.BAS"  
$INCLUDE "TYRE4.BAS"  
$INCLUDE "CONVERGE.BAS"  
$INCLUDE "OUTP.BAS"
```

end

```
SUB CONTROL
'*****
' This subroutine controls the execution of the subroutines
' listed below.
'*****
```

```
SHARED CONVERGED%,n,ITER
```

```
IF n = 0 AND ITER =0 THEN
  CALL INPUTR
END IF
```

```
  LABEL.LOOPC:
  CALL VECTOR1
```

```
  CALL RAM
```

```
  CALL EQUILIB
```

```
  CALL INVERT1
```

```
  CALL BODYF
```

```
  CALL SUSP
```

```
  CALL FLEXI
```

```
  CALL DISP
```

```
  CALL TYRE
```

```
  CALL CONVERGE
IF CONVERGED%=0 THEN
  n=n+1
  GOTO LABEL.LOOPC
```

```
END IF
```

```
END SUB
```



```

SUB INPUTR
SHARED rt#(),rb#(),wc#(),bcg#()
SHARED pcg#(),lhc#(),rhc#(),l,m,n
SHARED rl#,thetap#,BM#,PM#,WX#,WY#,WZ#

'*****
'*****
'   This subroutine contains the undeformed coordinates of
'   the trailer/body hinges, ram/chassis contact, ram/body
contact,
'   body c. of g. and the body mass
'
'
'           lhc#(0) - LEFT HINGE X CO-ORD.
'   *
'           lhc#(1) - LEFT HINGE Y CO-ORD.
'   *
'           lhc#(2) - LEFT HINGE Z CO-ORD.
'   *
'
'   - THE BODY ATTITUDE IS SPECIFIED BY DEFINING THE
'   *
'   THE GROUND SLOPE ANGLE, thetap#, AND THE LENGTH
'   *
'   OF THE RAM , rl#.
'   *
'
'   - THE APPLIED LOADS TO THE BODY AND THEIR RESPECTIVE
'   *
'   POSITIONS ARE GIVEN .
'   *

'*****
'****
l=m=n=0
rt#(0,0)=8977.2 : rt#(1,0)=2029.5 : rt#(2,0)=1E-06
rb#(0,0)=8977.2 : rb#(1,0)=0.0 : rb#(2,0)=1E-06
wc#(0,0)=4488.6 : wc#(1,0)=1014.75 : wc#(2,0)=-1247.0
bcg#(0,0)=4488.6 : bcg#(1,0)=1014.75 : bcg#(2,0)=1E-06
lhc#(0,0)=1E-06 : lhc#(1,0)=1E-06 : lhc#(2,0)=-700
rhc#(0,0)=1E-06 : rhc#(1,0)=1E-06 : rhc#(2,0)=700
BM#=1250*9.81
end sub

```

Fig. C5 Variabs.bas

SUB VARIABS

SHARED POSI%( ), PCGX%( ), PCGY%( ), PCGZ%( ), PM\$( )

\*\*\*\*\*  
\*\*\*\*\*

' This routine contains user input variables, which are the undeformed

' coordinates of the payload centre of gravity, payload mass and the

' initial ground slope and ram length.

\*\*\*\*\*  
\*\*\*\*\*

'Line 12

'PAYLOAD CENTRE OF GRAVITY X COORDINATES STORED IN PCGX%(8) IN mm

'-----  
-----

PCGX%(0)=2992  
PCGX%(1)=4488  
PCGX%(2)=5984  
PCGX%(3)=2992  
PCGX%(4)=4488  
PCGX%(5)=5984  
PCGX%(6)=2992  
PCGX%(7)=4488  
PCGX%(8)=5984

'Line 28

'PAYLOAD CENTRE OF GRAVITY Y COORDINATES STORED IN PCGY%(8) IN mm

'-----  
-----

PCGY%(0)=676  
PCGY%(1)=676  
PCGY%(2)=676  
PCGY%(3)=1014  
PCGY%(4)=1014  
PCGY%(5)=1014  
PCGY%(6)=1350  
PCGY%(7)=1350  
PCGY%(8)=1350

'Line 45

'PAYLOAD CENTRE OF GRAVITY Z COORDINATES STORED IN PCGZ%(8) IN mm

'-----  
-----

PCGZ#(0)=1E-06  
PCGZ#(1)=1E-06  
PCGZ#(2)=1E-06  
PCGZ#(3)=1E-06  
PCGZ#(4)=1E-06  
PCGZ#(5)=1E-06  
PCGZ#(6)=1E-06  
PCGZ#(7)=1E-06  
PCGZ#(8)=1E-06

'Line 60  
'PAYLOAD MASSES STORED IN PM£(3) IN KG

'-----  
-----

PM#(0)=2500  
PM#(1)=10000  
PM#(2)=17500  
PM#(3)=25000

'Line 72  
'OUTPUT FILENAMES FOR 2500 KG PAYLOAD (9 FILES)

'-----  
-----

PM\$(0,0)="A:\P2500-1.ROL"  
PM\$(0,1)="A:\P2500-2.ROL"  
PM\$(0,2)="A:\P2500-3.ROL"  
PM\$(0,3)="A:\P2500-4.ROL"  
PM\$(0,4)="A:\P2500-5.ROL"  
PM\$(0,5)="A:\P2500-6.ROL"  
PM\$(0,6)="A:\P2500-7.ROL"  
PM\$(0,7)="A:\P2500-8.ROL"  
PM\$(0,8)="A:\P2500-9.ROL"

'Line 90  
'OUTPUT FILENAMES FOR 10000 KG PAYLOAD (9 FILES)

'-----  
-----

PM\$(1,0)="A:\P10000-1.ROL"  
PM\$(1,1)="A:\P10000-2.ROL"  
PM\$(1,2)="A:\P10000-3.ROL"  
PM\$(1,3)="A:\P10000-4.ROL"  
PM\$(1,4)="A:\P10000-5.ROL"  
PM\$(1,5)="A:\P10000-6.ROL"  
PM\$(1,6)="A:\P10000-7.ROL"  
PM\$(1,7)="A:\P10000-8.ROL"  
PM\$(1,8)="A:\P10000-9.ROL"

'Line 106

'OUTPUT FILENAMES FOR 17500 KG PAYLOAD (9 FILES)

'-----  
-----

PMS(2,0)="A:\P17500-1.ROL"  
PMS(2,1)="A:\P17500-2.ROL"  
PMS(2,2)="A:\P17500-3.ROL"  
PMS(2,3)="A:\P17500-4.ROL"  
PMS(2,4)="A:\P17500-5.ROL"  
PMS(2,5)="A:\P17500-6.ROL"  
PMS(2,6)="A:\P17500-7.ROL"  
PMS(2,7)="A:\P17500-8.ROL"  
PMS(2,8)="A:\P17500-9.ROL"

PMS(3,0)="A:\P25000-1.ROL"  
PMS(3,1)="A:\P25000-2.ROL"  
PMS(3,2)="A:\P25000-3.ROL"  
PMS(3,3)="A:\P25000-4.ROL"  
PMS(3,4)="A:\P25000-5.ROL"  
PMS(3,5)="A:\P25000-6.ROL"  
PMS(3,6)="A:\P25000-7.ROL"  
PMS(3,7)="A:\P25000-8.ROL"  
PMS(3,8)="A:\P25000-9.ROL"

END

## SUB ZERO

```

'*****
'*****
' This subroutine sets the program's variables to zero at the
' beginning of each new payload magnitude or payload mass or
ground
' slope or ram length considered.
'*****
'*****

```

```

SHARED lhc#(),rhc#(),rb#(),rt#(),wc#()
SHARED bcg#(),pcg#(),BF#(),BR#()
SHARED CHL#(),flex#(),uchassis#()
SHARED SUSPR#(),SUSPL#(),brconst#()
SHARED a#(), fab#(), fch#(), fchl#(), FCH2#(),uab#()
SHARED TYREF#(),uch#(),FLCHECK()
FOR x=0 TO 2
  FOR y=1 TO 20
    lhc#(x,y)=0
    rhc#(x,y)=0
    rb#(x,y)=0
    rt#(x,y)=0
    wc#(x,y)=0
    bcg#(x,y)=0
    pcg#(x,y)=0
  NEXT y
NEXT x
  ERASE BF#,BR#
  ERASE CHL#,flex#,uchassis#
  ERASE SUSPR#,SUSPL#,brconst#
  ERASE fch# , FCH2#,uab#
  ERASE TYREF#,uch#,FLCHECK

DIM DYNAMIC BF#(5,35),BR#(7,35)
DIM DYNAMIC CHL#(15),flex#(8,15),uchassis#(8,35)
DIM DYNAMIC SUSPR#(35),SUSPL#(35),brconst#(35)
DIM DYNAMIC fch#(5) , FCH2#(23),uab#(5,35)
DIM DYNAMIC TYREF#(5,35),uch#(23,35),FLCHECK(35)
END SUB

```

```
SUB VECTOR1
' VECTOR ANALYSIS BY S.G.PICKERING 28/1/91
'*****
'*****
'This subroutine calls the routine VECTOR.BAS for the payload c.
of g.,
'the body c. of g. and ram/body contact to determine their
positional
' coordinates using vector analysis
'*****
'*****

  SHARED rt#(),rb#(),wc#(),bcg#()
  SHARED pcg#(),lhc#(),rhc#(),rl#,l,m,n
  CALL VECTOR(rt#(),rb#(),lhc#(),0,l,m,n)
  m=m+1
  CALL VECTOR(bcg#(),rt#(),lhc#(),1,l,m,n)
  CALL VECTOR(pcg#(),rt#(),lhc#(),1,l,m,n)

l=l+1
if n=0 then
  n=n+1
  '*****
  ' hinge and ram base co-ords are modified by subtracting gravity
  load disp-
  ' lacements as these have been taken into account in the chassis
  FE mesh.
  '*****

  lhc#(0,n)=lhc#(0,n-1)+0.2714-0.778
  lhc#(1,n)=lhc#(1,n-1)+10.176-4.946
  lhc#(2,n)=lhc#(2,n-1)
  rhc#(0,n)=rhc#(0,n-1)+0.2714-0.778
  rhc#(1,n)=rhc#(1,n-1)+10.176-4.946
  rhc#(2,n)=rhc#(2,n-1)
  rb#(0,n)=rb#(0,n-1)+0.2542-0.706
  rb#(1,n)=rb#(1,n-1)+1.667+16.809
  rb#(2,n)=rb#(2,n-1)
end if
end sub
```

```

231  SUB VECTOR(a#(2),b#(2),c#(2),flag,l,m,n)
'*****
'*****
'This routine contains the coding for multiplying vectors to
determine
'the positional coordinates of the important points on the body
'*****
'*****

240  SHARED rl#
250  'N.B a#(,)-uses l subscript
260  '      b#(,)-uses m subscript
270  '      c#(,)-uses n subscript
280  if ABS(a#(0,1)) <1e-06 then
290  a#(0,1)=1e-06
300  end if
310  if ABS(a#(1,1)) < 1e-06 then
320  a#(1,1)=1e-06
330  end if
340  if ABS(a#(2,1)) < 1e-06 then
350  a#(2,1)=1e-06
360  end if
370
380  if ABS(b#(0,m)) <1e-06 then
390  b#(0,m)=1e-06
400  end if
410  if ABS(b#(1,m)) < 1e-06 then
420  b#(1,m)=1e-06
430  end if
440  if ABS(b#(2,m)) < 1e-06 then
450  b#(2,m)=1e-06
460  end if
470  if ABS(c#(0,n)) < 1e-06 then
480  c#(0,n)=1e-06
500  end if
510  if ABS(c#(1,n)) < 1e-06 then
520  c#(1,n)=1e-06
530  end if
540  if ABS(c#(2,n)) < 1e-06 then
550  c#(2,n)=1e-06
560  end if
570
580  if flag=0 then
590  'CALC a# VECTOR LENGTH
600  a#=SQR(a#(0,0)^2+a#(1,0)^2+a#(2,0)^2)
610  'CALC b# vector length
620  b#=SQR(b#(0,m)^2+b#(1,m)^2+b#(2,m)^2)
630  'c# vector length
640  c#=700.0
650  ab#=rl#
6      6      0
ac#=SQR((a#(0,0)-c#(0,0))^2+(a#(1,0)-c#(1,0))^2+(a#(2,0)-c#(2,
0))^2)
670  end if
680

```

```

690 if flag=1 then
700 'CALC a# VECTOR LENGTH
710 a#=SQR(a#(0,0)^2+a#(1,0)^2+a#(2,0)^2)
720 'CALC b# vector length
730 b#=SQR(b#(0,0)^2+b#(1,0)^2+b#(2,0)^2)
740 'c# vector length
750 c#=700.0
760 'calc ab# vector lngth
7          7          0
ab#=SQR((a#(0,0)-b#(0,0))^2+(a#(1,0)-b#(1,0))^2+(a#(2,0)-b#(2,
0))^2)
7          8          0
ac#=SQR((a#(0,0)-c#(0,0))^2+(a#(1,0)-c#(1,0))^2+(a#(2,0)-c#(2,
0))^2)
790 end if

810 'calc cos(gamma) ie angle between vectors a# & b#
820 g#=(a#^2+b#^2-ab#^2)/(2*a#*b#)
830 'calc cos(delta) ie angle between vectors a# & c#
840 d#=(a#^2+c#^2-ac#^2)/(2*a#*c#)
850 'calc ak#=dj#+ej#(aj#) ie ak# in terms of aj#
860 'where dj#=
8          8          0
dj#=(a#*c#*d#*b#(0,m)-a#*b#*g#*c#(0,n))/(c#(2,n)*b#(0,m)-L#(2,
m)*c#(0,n))
900 'and
9          1          0
ej#=(b#(1,m)*c#(0,n)-c#(1,n)*b#(0,m))/(c#(2,n)*b#(0,m)-b#(2,m)
*c#(0,n))
930 'calc ai#=fi#+gi#(aj#) ie ai# in terms of aj#
940 'where fi
950 fi#=(a#*b#*g#-dj#*b#(2,m))/b#(0,m)
970 'and gi=
980 gi#=(-ej#*b#(2,m)-b#(1,m))/b#(0,m)
' sub ai# & aj# into
1010 'ai#^2+aj#^2+ak#^2=a#^2 to give
1          0          2          0
'ak#^2(gi#^2+ej#^2+1)+ak#(2*fi#*gi#+2*dj#*ej#)+fi#^2+dj#^2-a#^2=0
1030
1040 ae#=gi#^2+ej#^2+1
1050 be#=-2*fi#*gi#+2*dj#*ej#
1060 ce#=fi#^2+dj#^2-a#^2
1080 ' CALC ROOTS
1090 disc#=be#*be#-4.0*ae#*ce#
1100 if ABS(disc#/be#) < 1E-04 THEN
1120 disc#=0.0
1130 'print "suspected error in quadratic ??????????????????"
1140 end if
1150 if ae#=0 then
1160 xj0#=-ce#/be#
1170 check=1
1180 elseif disc# >= 0 then
1190 xj1#=(-be#+SQR(disc#))/(2*ae#)
1200 xj2#=(-be#-SQR(disc#))/(2*ae#)
1210 check=2
1220 else
1230 print "STOP error in vector quadratic "
1250 end

```



```

1260   end if
1270
1280  if check=1 then
1290   a#(1,(l+1))=xj0#
1300   a#(2,(l+1))=dj#+ej#*xj0#
1310   a#(0,(l+1))=fi#+gi#*xj0#
1320  elseif check=2 then
1330   'calc original cross product of a & b for k term only
1340   acrossb1=a#(0,0)*b#(1,0)-a#(1,0)*b#(0,0)
1350
1360   'calc current cross product of a & b for k term only using
xj1#
1370   xil#=fi#+gi#*xj1#
1380   xk1#=dj#+ej#*xj1#
1390   acrossb2=xil#*b#(1,m)-xj1#*b#(0,m)
1410   'calc current cross product of a & b for k term only using
xj2#
1420   xi2#=fi#+gi#*xj2#
1430   xk2#=dj#+ej#*xj2#
1440   acrossb3=xi2#*b#(1,m)-xj2#*b#(0,m)
1455   if acrossb2=0 and acrossb3=0 then
1456   check1=0
1457   goto 1480
1458  end if
1460  check1=acrossb1/acrossb2   'using xj1# co-ord
1470  check2=acrossb1/acrossb3   'using xj2# co-ord
1480   if check1 >=0 then
1490     a#(0,l+1)=xil#
1500     a#(1,l+1)=xj1#
1510     a#(2,(l+1))=xk1#
1520   elseif check2 >=0 then
1530     a#(0,l+1)=xi2#
1540     a#(1,l+1)=xj2#
1550     a#(2,(l+1))=xk2#
1560   else
1570     print "error in choosing y value on body?????????"
1580     stop
1590     end
1600   end if
1610  end if
1930  end sub

```

Fig. C9 Ram.bas

```
SUB RAM
SHARED rt#(),rb#(),n
SHARED rl#,nx#,ny#,nz#
'*****
*****
'This routine determines the ram resolving cosines functions
nx#,ny#,nz#
'to enable X,Y,Z components to be calculated.
'*****
*****

'bottom of ram co-ords is stored in rb#(2,20)
'top of ram co-ords stored in rt#(2,20)
'ram direction co-sines stored in nx,ny,nz

nx#=(rt#(0,n)-rb#(0,n))/(rl#)
ny#=(rt#(1,n)-rb#(1,n))/(rl#)
nz#=(rt#(2,n)-rb#(2,n))/(rl#)
end sub
```

Fig. C10 Equilib.bas

```
SUB EQUILIB
shared eqm#(), thetap#, nx#, ny#, nz#, WX#, WY#, WZ#, BM#, PM#, alpha#
shared rt#(), rb#(), wc#(), bcg#(), pcg#(), lhc#(), rhc#(), BF#(), n
'*****
' EQUILIB.MAT - VALUES ARE ASSIGNED TO THE EQUILIBRIUM MATRIX
'
'          eqm() AND THE LOADING MATRIX BF() .
'
'*****
'alpha=1 ASSUMES LEFT HINGE TAKES ALL LATERAL LOADING
alpha#=1
for r=0 to 5
for c=0 to 5
eqm#(r,c)=0.0
next c,r
eqm#(0,0)=1.0
eqm#(0,1)=1.0
eqm#(0,2)=0.0
eqm#(0,3)=0.0
eqm#(0,4)=0.0
eqm#(0,5)=nx#
eqm#(1,0)=-lhc#(2,n)
eqm#(1,1)=-rhc#(2,n)
eqm#(1,2)=0.0
eqm#(1,3)=0.0
eqm#(1,4)=alpha#*lhc#(0,n)+(1-alpha#)*rhc#(0,n)
eqm#(1,5)=-nx#*rt#(2,n)+nz#*rt#(0,n)
eqm#(2,0)=0.0
eqm#(2,1)=0.0
eqm#(2,2)=-lhc#(2,n)
eqm#(2,3)=-rhc#(2,n)
eqm#(2,4)=alpha#*lhc#(1,n)+(1-alpha#)*rhc#(1,n)
eqm#(2,5)=-ny#*rt#(2,n)+nz#*rt#(1,n)
eqm#(3,0)=0.0
eqm#(3,1)=0.0
eqm#(3,2)=1.0
eqm#(3,3)=1.0
eqm#(3,4)=0.0
eqm#(3,5)=ny#
eqm#(4,0)=0.0
eqm#(4,1)=0.0
eqm#(4,2)=0.0
eqm#(4,3)=0.0
eqm#(4,4)=1.0
eqm#(4,5)=nz#
eqm#(5,0)=-lhc#(1,n)
eqm#(5,1)=-rhc#(1,n)
eqm#(5,2)=lhc#(0,n)
eqm#(5,3)=rhc#(0,n)
eqm#(5,4)=0.0
eqm#(5,5)=-nx#*rt#(1,n)+ny#*rt#(0,n)

END SUB
```

Fig. C11 Invert1.bas

```
SUB INVERT1
shared eqm#(),eqmi#()

'*****
' INVERT - THE EQUILIBRIUM MATRIX IS STORED IM EQM() . TO SOLVE
' *
'           FOR BODY REACTIONSN THE EQUILIBRIUM HAS TO BE
INVERTED.*
'           THIS ROUTINE INVERTS EQM() AND STORES THE INVERTED
' *
'           MATRIX IN EQMI().
' *
'*****
*****
for r=0 to 5
  for c=0 to 5
    eqmi#(r,c)=0
    test#(r,c)=eqm#(r,c)
next c,r

for x=0 to 5
  eqmi#(x,x)=1
next x

for x=0 to 5

  for y=x to 5

    d#=eqm#(x,y) : if d#=0 or d#=1 goto LABEL.ONE
    for k=x to 5
      eqm#(k,y)=eqm#(k,y)/d#
    next k
    for k=0 to 5
      eqmi#(k,y)=eqmi#(k,y)/d#
    next k
LABEL.ONE:

  next y

  if x=5 goto LABEL.TWO

  for y=x+1 to 5

  if eqm#(x,y)=0 goto LABEL.THREE

    for k=x to 5
      eqm#(k,y)=eqm#(k,y)-eqm#(k,x)
    next k
    for k=0 to 5
      eqmi#(k,y)=eqmi#(k,y)-eqmi#(k,x)
    next k
LABEL.THREE:

  next y

next x
```

LABEL.TWO:

for x=0 to 5

if eqm#(x,x)=1 then next x

if x<>6 then print "\*\*\*\*\*NOT INV\*\*\*\*\*equilib matrix": end

for x=5 to 1 step -1

for y=x-1 to 0 step -1

d#=eqm#(x,y)

for k=x to 5

eqm#(k,y)=eqm#(k,y)-eqm#(k,x)\*d#

next k

for k=0 to 5

eqmi#(k,y)=eqmi#(k,y)-eqmi#(k,x)\*d#

next k

next y

next x

end sub

Fig. C12 Bodyf.bas

```

SUB BODYF
s          h          a          r          e          d
eqm#( ), thetap#, psi#, ja#, nx#, ny#, nz#, WX#, WY#, WZ#, BM#, PM#, alpha#
s          h          a          r          e          d
eqmi#( ), rt#( ), rb#( ), wc#( ), bcg#( ), pcg#( ), lhc#( ), rhc#( ), BR#( ), n
shared brconst#( ), BF#( )

'*****
'*****
'  VALUES ARE ASSIGNED TO THE BODY LOADING MATRIX BF#(5)
'*****
'*****

BF#(0, n)=-WX#

BF1a#=-WZ#*wc#(0, n)+WX#*wc#(2, n)-BM#*SIN(thetap#)*bcg#(0, n)
BF1b#=-PM#*SIN(thetap#)*pcg#(0, n)
BF#(1, n)=BF1a#+BF1b#

BF2a#=-BM#*(SIN(thetap#)*bcg#(1, n)+COS(thetap#)*bcg#(2, n))
BF2b#=-PM#*(SIN(thetap#)*pcg#(1, n)+COS(thetap#)*pcg#(2, n))
BF2c#=-WZ#*wc#(1, n)+WY#*wc#(2, n)
BF#(2, n)=BF2a#+BF2b#+BF2c#

BF#(3, n)=COS(thetap#)*(PM#+BM#)-WY#

BF#(4, n)=-SIN(thetap#)*(PM#+BM#)-WZ#

BF#(5, n)=COS(thetap#)*(BM#*bcg#(0, n)+PM#*pcg#(0, n))-WY#*wc#(0,
n)+WX#*wc#(1, n)

'*****
'*****
'      OBTAIN BODY REACTIONS BY MULTIPLYING INVERTED EQUILIBRIUM
'
'      MATRIX ,eqmi#( ), BY APPLIED BODY LOADS MATRIX BF#( ).
'
'
'*****
'*****

for x=0 to 5
BR#(x, n)=0
  for y=0 to 5
    BR#(x, n)=BR#(x, n)+eqmi#(x, y)*BF#(y, n)
  next y
next x

'**** change sign of body reactions so they can be applied
'**** directly to chassis
for x=0 to 5
  BR#(x, n)=-BR#(x, n)
next x
brconst#(n)=BR#(5, n)
BR#(5, n)=brconst#(n)*nx#
BR#(6, n)=brconst#(n)*ny#
BR#(7, n)=brconst#(n)*nz#

```

Fig. C13 Susp.bas

```
SUB SUSP
SHARED SUSPL#(), SUSPR#(), n, FLCHECK(), thetactr

' *****
' *****
' This routine determines the appropriate suspension operating
' condition and calls the appropriate suspension force matrix
' *****
' *****

PMAX=30000
FLCHECK(n)=0

IF thetactr > 0 THEN

IF FLCHECK(n-1)=3 THEN
CALL CASE3
FLCHECK(n)=3

ELSE

IF FLCHECK(n-1)=4 THEN
CALL CASE4
FLCHECK(n)=4
IF SUSPL#(n) < 0 THEN
CALL CASE3
FLCHECK(n)=3
END IF
END IF
END IF
END IF

IF FLCHECK(n) = 0 THEN
CALL CASE1
FLCHECK(n)=1

IF SUSPL#(n) < 0 THEN '(1)

IF SUSPR#(n) > PMAX THEN '(2)

CALL CASE4
FLCHECK(n)=4

if SUSPL#(n) < 0 then '(3)

SUSPL#(n)=0
SUSPR#(n)=0
CALL CASE2
FLCHECK(n)=2

if SUSPR#(n) > PMAX THEN '(4)

SUSPL#(n)=0
SUSPR#(n)=0
CALL CASE3
FLCHECK(n)=3
end if '(4)
```

```
        end if          '(3)
ELSE          '(2)
SUSPL#(n)=0
SUSPR#(n)=0
CALL CASE2
FLCHECK(n)=2
END IF      '(2)

ELSE '(1)
    IF SUSPR#(n) >= PMAX THEN
        CALL CASE4
        FLCHECK(n)=4
    END IF

END IF

END IF
END SUB
```



```

SUB CASE1
SHARED BR#( ),n,thetap#,CHL#( ),SUSPR#( ),SUSPL#( )
SHARED uchassis#( )

dim suspml#(1,13)
'*****
' This routine contains the suspension force matrix for
suspension
' operating condition condition a.
' Suspension flexibility matrix is formed (suspml#( )).The two
equations *
' are solved to give right and left suspension forces, using
*
' the condition of zero relative displacement between the middle
axle *
' and chassis. This is determined by placing two tie bars between
the *
' middle axle and chassis and deriving a flexibility matrix
*
' which relates tie bar forces to body reactions,chassis and
tractor
' self weight and suspension forces. Setting the tie bar forces
to zero
' simulates the desired condition enabling suspension forces to
be obtained.
'*****
suspml#(0,0)=-0.28627e-01
suspml#(0,1)=-0.18198e-01
suspml#(0,2)=-1.0168
suspml#(0,3)=-0.27087
suspml#(0,4)=-0.21127
suspml#(0,5)=-0.21508e-01
suspml#(0,6)=-0.10598e-01
suspml#(0,7)=-0.13069e-01
suspml#(0,8)=-0.22033e-02
suspml#(0,9)=-0.52137e-02
suspml#(0,10)=-0.16194
suspml#(0,11)=-0.01963
suspml#(0,12)=-3.1736
suspml#(0,13)=-1.5410

suspml#(1,0)=-0.18186e-01
suspml#(1,1)=-0.28627e-01
suspml#(1,2)=-0.2708
suspml#(1,3)=-1.0168
suspml#(1,4)=0.21124
suspml#(1,5)=-0.21502e-01
suspml#(1,6)=-0.10605e-01
suspml#(1,7)=0.13083e-01
suspml#(1,8)=-0.22123e-02
suspml#(1,9)=0.52242e-02
suspml#(1,10)=-0.16192
suspml#(1,11)=0.01963
suspml#(1,12)=-1.5413
suspml#(1,13)=-3.1733

```

**PAGE  
NUMBERING  
AS  
ORIGINAL**

'\*\*\*\*\* body reactions, tractor weight, chassis weight and suspension

'\*\*\*\*\* forces (not known yet) are stored in chl(14).

```
CHL#(0)=BR#(0,n)
CHL#(1)=BR#(1,n)
CHL#(2)=BR#(2,n)
CHL#(3)=BR#(3,n)
CHL#(4)=BR#(4,n)
CHL#(5)=BR#(5,n)
CHL#(6)=BR#(6,n)
CHL#(7)=BR#(7,n)
CHL#(8)=-63666.9*cos(thetap#)
CHL#(9)=63666.9*sin(thetap#)
CHL#(10)=-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
' CHL#(12)-unknown yet (SUSPL)
' CHL#(13)-unknown yet (SUSPR)
A#=0
B#=0
C#=0
D#=0
'PRINT A#,B#,C#,D#
for x=0 to 11
  A#=A#-suspml#(1,x)*CHL#(x)/suspml#(1,13)
next x

  B#=-suspml#(1,12)/suspml#(1,13)

for x=0 to 11
  C#=C#-suspml#(0,x)*CHL#(x)/suspml#(0,12)
next x

  D#=-suspml#(0,13)/suspml#(0,12)

SUSPR#(n)=(A#+B#*C#)/(1-B#*D#)
SUSPL#(n)=C#+D#*SUSPR#(n)

CHL#(12)=SUSPL#(n)
CHL#(13)=SUSPR#(n)

END SUB
```

```

SUB CASE2
SHARED BR#(),n,thetap#,CHL#(),SUSPR#(),SUSPL#()
shared uchassis#()

DIM suspm2#(12)
'*****
' This routine contains the suspension force matrix for
suspension
' operating condition b.
' Suspension flexibility matrix is formed (suspm#()).The
equation *
' is solved to give the right hand suspension force only, using
*
' the condition of zero relative displacement between the middle
axle *
' and chassis. This is determined by placing a tie bar between
the *
' middle axle and chassis and deriving a flexibility matrix
*
' which relates tie bar force to body reactions,chassis and
tractor
' self weight and suspension forces. Setting the tie bar force
to zero
' simulates the desired condition enabling suspension force to
be determined
' for the RIGHT SUSPENSION ONLY. LEFT SUSPENSION FORCE=0
'*****
suspm2#(0)=-0.04669
suspm2#(1)=-0.046757
suspm2#(2)=-1.2827
suspm2#(3)=-1.2864
suspm2#(4)=0.0010223
suspm2#(5)=-0.042903
suspm2#(6)=-0.021143
suspm2#(7)=-0.8287e-04
suspm2#(8)=-0.0043983
suspm2#(9)=0.42915e-04
suspm2#(10)=-0.3215
suspm2#(11)=9.65717e-05
suspm2#(12)=-4.7075

'***** body reactions,tractor weight, chassis weight and
suspension
'***** forces (not known yet) are stored in chl(14).
for x=0 to 15
  CHL#(x)=0
next x
CHL#(0)=BR#(0,n)
CHL#(1)=BR#(1,n)
CHL#(2)=BR#(2,n)
CHL#(3)=BR#(3,n)
CHL#(4)=BR#(4,n)
CHL#(5)=BR#(5,n)
CHL#(6)=BR#(6,n)
CHL#(7)=BR#(7,n)
CHL#(8)=-63666.9*cos(thetap#)

```

```
CHL#(9)=63666.9*sin(thetap#)
CHL#(10)=-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
' CHL#(12)-unknown yet (SUSPR)
```

```
a#=0
for x=0 to 11
  a#=a#+CHL#(x)*suspm2#(x)
next x
```

```
SUSPR#(n)=a#/-suspm2#(12)
```

```
CHL#(12)=SUSPR#(n)
a#=0
for x=0 to 12
  a#=a#+CHL#(x)*suspm2#(x)
next x
```

```
end sub
```

Fig. C16 Case3.bas

```
SUB CASE3
SHARED k11#() , k11i#() , k12#() , k21#() , k22#()
SHARED prescribe#() , k21k11i#() , k21k11ik12#()
SHARED k22k21k11ik12#() , k22k21k11ik12i#()
SHARED fab#() , fch#() , fch1#() , FCH2#()
SHARED BR#() , CHL#() , n , thetap# , SUSPR#() , uab#()
SHARED FLAG3 , FLAG3
SHARED const#
'*****
*****
' this routine contains the suspension force matrix for
suspension
' operating condition d
'*****
*****

'*****
' FORCES IN FCH2 ARE DETERMINED FROM CHASSIS FORCES
' STORED IN CHL#(15) AND UNSPRUNG AXLE FORCES.
'
'*****

FCH2#(0)=BR#(0,n)
FCH2#(1)=BR#(2,n)
FCH2#(2)=BR#(4,n)
FCH2#(3)=BR#(1,n)
FCH2#(4)=BR#(3,n)
FCH2#(5)=BR#(5,n)
FCH2#(6)=BR#(6,n)
FCH2#(7)=BR#(7,n)
FCH2#(8)=-2350*9.81*cos(thetap#)
FCH2#(9)=2350*9.81*sin(thetap#)
FCH2#(10)=-800*9.81*cos(thetap#)
FCH2#(11)=800*9.81*sin(thetap#)
FCH2#(12)=-800*9.81*cos(thetap#)
FCH2#(13)=800*9.81*sin(thetap#)
FCH2#(14)=-800*9.81*cos(thetap#)
FCH2#(15)=800*9.81*sin(thetap#)
FCH2#(16)=-6490*9.81*cos(thetap#)
FCH2#(17)=6490*9.81*sin(thetap#)

IF FLAG3 =1 THEN
GOTO LABEL.MISS
END IF
FLAG3=1
a#(0)=1
a#(1)=-1
a#(2)=1
a#(3)=-1
a#(4)=1
a#(5)=-1

prescribe#(0,0)=49187
prescribe#(0,1)=-2946.7
prescribe#(0,2)=2785.0
```

*Body Reactions*

*Chassis 1 forces*  
*Chassis 2 forces*

*Tractor*

prescribe#(0,3)=10570.0  
prescribe#(0,4)=-2948.0  
prescribe#(0,5)=-57919.0  
prescribe#(0,6)=-1715.2  
prescribe#(0,7)=-839.1  
prescribe#(0,8)=4599.1  
prescribe#(0,9)=-6916.7  
prescribe#(0,10)=960.4  
prescribe#(0,11)=2204.4  
prescribe#(0,12)=1155.7  
prescribe#(0,13)=1160.9  
prescribe#(0,14)=929.1  
prescribe#(0,15)=1743.3  
prescribe#(0,16)=231.6  
prescribe#(0,17)=-52.2  
prescribe#(0,18)=-5088.2  
prescribe#(0,19)=-617.0  
prescribe#(0,20)=4362.9  
prescribe#(0,21)=-738.5  
prescribe#(0,22)=2244.7  
prescribe#(0,23)=-593.7

prescribe#(1,0)=-2946.7  
prescribe#(1,1)=1654.0  
prescribe#(1,2)=-721.0  
prescribe#(1,3)=62.8  
prescribe#(1,4)=-212.9  
prescribe#(1,5)=2574.7  
prescribe#(1,6)=519.3  
prescribe#(1,7)=37.0  
prescribe#(1,8)=-865.1  
prescribe#(1,9)=454.1  
prescribe#(1,10)=-151.2  
prescribe#(1,11)=-279.9  
prescribe#(1,12)=-720.5  
prescribe#(1,13)=-9.6  
prescribe#(1,14)=-1489.3  
prescribe#(1,15)=492.8  
prescribe#(1,16)=-41.0  
prescribe#(1,17)=18.5  
prescribe#(1,18)=738.8  
prescribe#(1,19)=97.3  
prescribe#(1,20)=-778.5  
prescribe#(1,21)=460.7  
prescribe#(1,22)=-113.3  
prescribe#(1,23)=952.4

prescribe#(2,0)=2785.0  
prescribe#(2,1)=-721.0  
prescribe#(2,2)=2744.9  
prescribe#(2,3)=-2550.0  
prescribe#(2,4)=836.0  
prescribe#(2,5)=-221.0  
prescribe#(2,6)=-6.3  
prescribe#(2,7)=7.5  
prescribe#(2,8)=-329.2  
prescribe#(2,9)=-339.4  
prescribe#(2,10)=-66.5

prescribe#(2,11)=47.6  
prescribe#(2,12)=-70.9  
prescribe#(2,13)=-469.4  
prescribe#(2,14)=-38.8  
prescribe#(2,15)=-1991.9  
prescribe#(2,16)=2.3  
prescribe#(2,17)=-0.1  
prescribe#(2,18)=327.4  
prescribe#(2,19)=42.7  
prescribe#(2,20)=414.0  
prescribe#(2,21)=45.4  
prescribe#(2,22)=-459.1  
prescribe#(2,23)=24.8

prescribe#(3,0)=10569  
prescribe#(3,1)=62.9  
prescribe#(3,2)=-2550.0  
prescribe#(3,3)=53220.0  
prescribe#(3,4)=-16054.0  
prescribe#(3,5)=-61830  
prescribe#(3,6)=-1891.1  
prescribe#(3,7)=914.  
prescribe#(3,8)=-768  
prescribe#(3,9)=8538.5  
prescribe#(3,10)=-113.4  
prescribe#(3,11)=-3120.4  
prescribe#(3,12)=81.5  
prescribe#(3,13)=-1602.8  
prescribe#(3,14)=155.2  
prescribe#(3,15)=-2317.4  
prescribe#(3,16)=275.3  
prescribe#(3,17)=59.0  
prescribe#(3,18)=3176.1  
prescribe#(3,19)=73.7  
prescribe#(3,20)=1436.1  
prescribe#(3,21)=-51.5  
prescribe#(3,22)=13563.0  
prescribe#(3,23)=-98.5

prescribe#(4,0)=-2948.1  
prescribe#(4,1)=-212.9  
prescribe#(4,2)=836.0  
prescribe#(4,3)=-16051  
prescribe#(4,4)=0.25822e06  
prescribe#(4,5)=18491  
prescribe#(4,6)=950.8  
prescribe#(4,7)=-273.2  
prescribe#(4,8)=1271.0  
prescribe#(4,9)=-3129.1  
prescribe#(4,10)=-82.8  
prescribe#(4,11)=1505.7  
prescribe#(4,12)=-70.8  
prescribe#(4,13)=617.0  
prescribe#(4,14)=-31.7  
prescribe#(4,15)=493.8  
prescribe#(4,16)=-115.8  
prescribe#(4,17)=-38.0  
prescribe#(4,18)=-28689.0



prescribe#(4,19)=51.8  
prescribe#(4,20)=0.13944e06  
prescribe#(4,21)=45.1  
prescribe#(4,22)=-0.37073e06  
prescribe#(4,23)=20.2

prescribe#(5,0)=-57917.0  
prescribe#(5,1)=2575.2  
prescribe#(5,2)=-228.0  
prescribe#(5,3)=-61829  
prescribe#(5,4)=18491  
prescribe#(5,5)=0.12472e06  
prescribe#(5,6)=3672.5  
prescribe#(5,7)=-78.0  
prescribe#(5,8)=-3472.2  
prescribe#(5,9)=-1554.2  
prescribe#(5,10)=-786.9  
prescribe#(5,11)=868.8  
prescribe#(5,12)=-1194.0  
prescribe#(5,13)=422.5  
prescribe#(5,14)=-1057.5  
prescribe#(5,15)=565.1  
prescribe#(5,16)=-508.2  
prescribe#(5,17)=-6.0  
prescribe#(5,18)=2146.5  
prescribe#(5,19)=506.2  
prescribe#(5,20)=-5941.3  
prescribe#(5,21)=764.0  
prescribe#(5,22)=-15556.0  
prescribe#(5,23)=676.6

prescribe#(6,0)=-1705.3  
prescribe#(6,1)=515.9  
prescribe#(6,2)=-6.3  
prescribe#(6,3)=-1891.4  
prescribe#(6,4)=942.9  
prescribe#(6,5)=3674.3  
prescribe#(6,6)=9520.0  
prescribe#(6,7)=-8.7  
prescribe#(6,8)=-1736.2  
prescribe#(6,9)=-198.5  
prescribe#(6,10)=-234.1  
prescribe#(6,11)=171.7  
prescribe#(6,12)=-54.2  
prescribe#(6,13)=55.8  
prescribe#(6,14)=3.1  
prescribe#(6,15)=-20.2  
prescribe#(6,16)=-5626.1  
prescribe#(6,17)=-4.8  
prescribe#(6,18)=-1674.5  
prescribe#(6,19)=151.5  
prescribe#(6,20)=2309.3  
prescribe#(6,21)=36.6  
prescribe#(6,22)=-1367.1  
prescribe#(6,23)=-2.0

prescribe#(7,0)=-841.5  
prescribe#(7,1)=37.7

prescribe#(7,2)=7.5  
prescribe#(7,3)=917.9  
prescribe#(7,4)=-278.9  
prescribe#(7,5)=-71.1  
prescribe#(7,6)=1.2  
prescribe#(7,7)=2535.0  
prescribe#(7,8)=-74.1  
prescribe#(7,9)=-514.3  
prescribe#(7,10)=-14.8  
prescribe#(7,11)=49.0  
prescribe#(7,12)=-18.9  
prescribe#(7,13)=66.2  
prescribe#(7,14)=-16.0  
prescribe#(7,15)=35.9  
prescribe#(7,16)=0.6  
prescribe#(7,17)=-1004.8  
prescribe#(7,18)=67.1  
prescribe#(7,19)=9.5  
prescribe#(7,20)=10.0  
prescribe#(7,21)=12.1  
prescribe#(7,22)=255.7  
prescribe#(7,23)=10.2

prescribe#(8,0)=4598.4  
prescribe#(8,1)=-865.0  
prescribe#(8,2)=-329.2  
prescribe#(8,3)=-768.0  
prescribe#(8,4)=1271.1  
prescribe#(8,5)=-3471.0  
prescribe#(8,6)=-1702.9  
prescribe#(8,7)=-72.4  
prescribe#(8,8)=11796.0  
prescribe#(8,9)=-3193.1  
prescribe#(8,10)=-2623.5  
prescribe#(8,11)=2286.4  
prescribe#(8,12)=-1891.7  
prescribe#(8,13)=957.6  
prescribe#(8,14)=-990.3  
prescribe#(8,15)=367.5  
prescribe#(8,16)=194.7  
prescribe#(8,17)=-39.2  
prescribe#(8,18)=-11540  
prescribe#(8,19)=1669.6  
prescribe#(8,20)=4994.3  
prescribe#(8,21)=1210.0  
prescribe#(8,22)=-2218.1  
prescribe#(8,23)=633.5

prescribe#(9,0)=-6916.9  
prescribe#(9,1)=454.2  
prescribe#(9,2)=-339.4  
prescribe#(9,3)=8538.6  
prescribe#(9,4)=-3129.1  
prescribe#(9,5)=-1554.6  
prescribe#(9,6)=-167.4  
prescribe#(9,7)=-512.8  
prescribe#(9,8)=-3192.7  
prescribe#(9,9)=13055

prescribe#(9,10)=-631.7  
prescribe#(9,11)=-8454.7  
prescribe#(9,12)=-529.5  
prescribe#(9,13)=-2947.4  
prescribe#(9,14)=-319.7  
prescribe#(9,15)=-770.8  
prescribe#(9,16)=29.2  
prescribe#(9,17)=-33.1  
prescribe#(9,18)=6643.7  
prescribe#(9,19)=406.3  
prescribe#(9,20)=-3772.9  
prescribe#(9,21)=338.6  
prescribe#(9,22)=3670.0  
prescribe#(9,23)=204.5

prescribe#(10,0)=960.2  
prescribe#(10,1)=-151.2  
prescribe#(10,2)=-66.5  
prescribe#(10,3)=-113.4  
prescribe#(10,4)=-82.7  
prescribe#(10,5)=-786.7  
prescribe#(10,6)=-236.3  
prescribe#(10,7)=-14.6  
prescribe#(10,8)=-2623.5  
prescribe#(10,9)=-631.6  
prescribe#(10,10)=8260.1  
prescribe#(10,11)=439.8  
prescribe#(10,12)=-375.9  
prescribe#(10,13)=174.1  
prescribe#(10,14)=-194.3  
prescribe#(10,15)=68.8  
prescribe#(10,16)=30.1  
prescribe#(10,17)=-12.8  
prescribe#(10,18)=1655.2  
prescribe#(10,19)=-3934.8  
prescribe#(10,20)=-1006.9  
prescribe#(10,21)=240.5  
prescribe#(10,22)=327.9  
prescribe#(10,23)=124.3

prescribe#(11,0)=2204.7  
prescribe#(11,1)=-280.0  
prescribe#(11,2)=47.6  
prescribe#(11,3)=-3120.5  
prescribe#(11,4)=1505.5  
prescribe#(11,5)=868.6  
prescribe#(11,6)=142.9  
prescribe#(11,7)=47.7  
prescribe#(11,8)=2286.3  
prescribe#(11,9)=-8454.7  
prescribe#(11,10)=441.7  
prescribe#(11,11)=9424.0  
prescribe#(11,12)=362.5  
prescribe#(11,13)=-145.9  
prescribe#(11,14)=183.6  
prescribe#(11,15)=-83.1  
prescribe#(11,16)=-22.9  
prescribe#(11,17)=28.3

prescribe#(11,18)=-5413.6  
prescribe#(11,19)=-285.0  
prescribe#(11,20)=3671.9  
prescribe#(11,21)=-231.9  
prescribe#(11,22)=-2245.5  
prescribe#(11,23)=-117.4

prescribe#(12,0)=1155.4  
prescribe#(12,1)=-720.5  
prescribe#(12,2)=-70.9  
prescribe#(12,3)=81.5  
prescribe#(12,4)=-70.8  
prescribe#(12,5)=-1193.6  
prescribe#(12,6)=-53.3  
prescribe#(12,7)=-18.6  
prescribe#(12,8)=-1891.3  
prescribe#(12,9)=-529.4  
prescribe#(12,10)=-376.0  
prescribe#(12,11)=362.4  
prescribe#(12,12)=8378.1  
prescribe#(12,13)=87.5  
prescribe#(12,14)=-260.9  
prescribe#(12,15)=133.6  
prescribe#(12,16)=14.4  
prescribe#(12,17)=-4.5  
prescribe#(12,18)=1114.6  
prescribe#(12,19)=241.8  
prescribe#(12,20)=-741.3  
prescribe#(12,21)=-4009.4  
prescribe#(12,22)=251.8  
prescribe#(12,23)=166.8

prescribe#(13,0)=1161.2  
prescribe#(13,1)=-9.5  
prescribe#(13,2)=-469.3  
prescribe#(13,3)=-1602.9  
prescribe#(13,4)=616.9  
prescribe#(13,5)=421.7  
prescribe#(13,6)=53.8  
prescribe#(13,7)=65.9  
prescribe#(13,8)=957.2  
prescribe#(13,9)=-2947.2  
prescribe#(13,10)=174.3  
prescribe#(13,11)=-146.0  
prescribe#(13,12)=91.7  
prescribe#(13,13)=5968.0  
prescribe#(13,14)=105.8  
prescribe#(13,15)=-1645.0  
prescribe#(13,16)=-8.9  
prescribe#(13,17)=4.6  
prescribe#(13,18)=-2065.7  
prescribe#(13,19)=-112.1  
prescribe#(13,20)=1239.7  
prescribe#(13,21)=-60.0  
prescribe#(13,22)=-918.7  
prescribe#(13,23)=-67.6

prescribe#(14,0)=928.9

prescribe#(14,1)=-1489.3  
prescribe#(14,2)=-38.8  
prescribe#(14,3)=155.3  
prescribe#(14,4)=-31.6  
prescribe#(14,5)=-1057.3  
prescribe#(14,6)=3.2  
prescribe#(14,7)=-15.8  
prescribe#(14,8)=-990.3  
prescribe#(14,9)=-319.7  
prescribe#(14,10)=-194.3  
prescribe#(14,11)=183.6  
prescribe#(14,12)=-260.8  
prescribe#(14,13)=105.8  
prescribe#(14,14)=8545.0  
prescribe#(14,15)=48.4  
prescribe#(14,16)=6.1  
prescribe#(14,17)=-1.1  
prescribe#(14,18)=565.4  
prescribe#(14,19)=125.0  
prescribe#(14,20)=-412.5  
prescribe#(14,21)=166.8  
prescribe#(14,22)=125.5  
prescribe#(14,23)=-4118.9

prescribe#(15,0)=1743.3  
prescribe#(15,1)=492.5  
prescribe#(15,2)=-1991.9  
prescribe#(15,3)=-2317.6  
prescribe#(15,4)=493.7  
prescribe#(15,5)=564.7  
prescribe#(15,6)=-20.1  
prescribe#(15,7)=35.6  
prescribe#(15,8)=367.5  
prescribe#(15,9)=-770.7  
prescribe#(15,10)=68.8  
prescribe#(15,11)=-83.1  
prescribe#(15,12)=133.7  
prescribe#(15,13)=-1645.0  
prescribe#(15,14)=51.2  
prescribe#(15,15)=5312.0  
prescribe#(15,16)=-1.2  
prescribe#(15,17)=1.0  
prescribe#(15,18)=397.2  
prescribe#(15,19)=-44.2  
prescribe#(15,20)=-1471.5  
prescribe#(15,21)=-85.4  
prescribe#(15,22)=-352.7  
prescribe#(15,23)=-33.4

prescribe#(16,0)=231.7  
prescribe#(16,1)=-40.9  
prescribe#(16,2)=2.3  
prescribe#(16,3)=275.2  
prescribe#(16,4)=-115.5  
prescribe#(16,5)=-508.3  
prescribe#(16,6)=-5625.6  
prescribe#(16,7)=0.7  
prescribe#(16,8)=194.1

prescribe#(16,9)=27.9  
prescribe#(16,10)=29.9  
prescribe#(16,11)=-22.0  
prescribe#(16,12)=14.3  
prescribe#(16,13)=-8.8  
prescribe#(16,14)=6.1  
prescribe#(16,15)=-1.2  
prescribe#(16,16)=5910.0  
prescribe#(16,17)=0.2  
prescribe#(16,18)=172.1  
prescribe#(16,19)=-19.3  
prescribe#(16,20)=-186.4  
prescribe#(16,21)=-9.2  
prescribe#(16,22)=148.7  
prescribe#(16,23)=-3.9

prescribe#(17,0)=-52.2  
prescribe#(17,1)=18.6  
prescribe#(17,2)=-0.1  
prescribe#(17,3)=58.9  
prescribe#(17,4)=-38.0  
prescribe#(17,5)=-6.0  
prescribe#(17,6)=-3.4  
prescribe#(17,7)=-1004.6  
prescribe#(17,8)=-39.0  
prescribe#(17,9)=-32.8  
prescribe#(17,10)=-12.9  
prescribe#(17,11)=28.3  
prescribe#(17,12)=-4.5  
prescribe#(17,13)=4.6  
prescribe#(17,14)=-1.1  
prescribe#(17,15)=1.0  
prescribe#(17,16)=-0.2  
prescribe#(17,17)=1547.0  
prescribe#(17,18)=128.4  
prescribe#(17,19)=8.3  
prescribe#(17,20)=-124.2  
prescribe#(17,21)=2.9  
prescribe#(17,22)=63.4  
prescribe#(17,23)=0.7

prescribe#(18,0)=-5088.2  
prescribe#(18,1)=738.8  
prescribe#(18,2)=327.4  
prescribe#(18,3)=3175.1  
prescribe#(18,4)=-28689  
prescribe#(18,5)=2145.0  
prescribe#(18,6)=-1689.7  
prescribe#(18,7)=65.5  
prescribe#(18,8)=-11540  
prescribe#(18,9)=6644.3  
prescribe#(18,10)=1656.1  
prescribe#(18,11)=-5412.1  
prescribe#(18,12)=1114.1  
prescribe#(18,13)=-2062.2  
prescribe#(18,14)=565.4  
prescribe#(18,15)=397.2  
prescribe#(18,16)=171.0

prescribe#(18,17)=127.9  
prescribe#(18,18)=53949  
prescribe#(18,19)=-1050.8  
prescribe#(18,20)=-80065  
prescribe#(18,21)=-712.4  
prescribe#(18,22)=65800  
prescribe#(18,23)=-361.7

prescribe#(19,0)=-616.9  
prescribe#(19,1)=97.2  
prescribe#(19,2)=42.7  
prescribe#(19,3)=73.7  
prescribe#(19,4)=51.8  
prescribe#(19,5)=506.0  
prescribe#(19,6)=152.4  
prescribe#(19,7)=9.4  
prescribe#(19,8)=1669.6  
prescribe#(19,9)=406.3  
prescribe#(19,10)=-3934.2  
prescribe#(19,11)=-283.4  
prescribe#(19,12)=241.8  
prescribe#(19,13)=-112.1  
prescribe#(19,14)=125.0  
prescribe#(19,15)=-44.3  
prescribe#(19,16)=-19.4  
prescribe#(19,17)=8.2  
prescribe#(19,18)=-1050.8  
prescribe#(19,19)=2515.0  
prescribe#(19,20)=640.6  
prescribe#(19,21)=-154.7  
prescribe#(19,22)=-208.0  
prescribe#(19,23)=-80.0

prescribe#(20,0)=4362.8  
prescribe#(20,1)=-778.4  
prescribe#(20,2)=414.0  
prescribe#(20,3)=1437.2  
prescribe#(20,4)=0.13944e06  
prescribe#(20,5)=-5939.7  
prescribe#(20,6)=2332.1  
prescribe#(20,7)=11.2  
prescribe#(20,8)=4994.2  
prescribe#(20,9)=-3773.4  
prescribe#(20,10)=-1007.6  
prescribe#(20,11)=3669.8  
prescribe#(20,12)=-740.9  
prescribe#(20,13)=1239.3  
prescribe#(20,14)=-412.0  
prescribe#(20,15)=-1469.8  
prescribe#(20,16)=-184.5  
prescribe#(20,17)=-123.3  
prescribe#(20,18)=-80065  
prescribe#(20,19)=640.6  
prescribe#(20,20)=0.18397e06  
prescribe#(20,21)=473.6  
prescribe#(20,22)=-0.2488e06  
prescribe#(20,23)=263.9

prescribe#(21,0)=-738.5  
prescribe#(21,1)=460.7  
prescribe#(21,2)=45.4  
prescribe#(21,3)=-51.5  
prescribe#(21,4)=45.1  
prescribe#(21,5)=763.9  
prescribe#(21,6)=34.1  
prescribe#(21,7)=11.9  
prescribe#(21,8)=1210.0  
prescribe#(21,9)=338.6  
prescribe#(21,10)=240.5  
prescribe#(21,11)=-231.8  
prescribe#(21,12)=-4009.8  
prescribe#(21,13)=-56.4  
prescribe#(21,14)=166.8  
prescribe#(21,15)=-85.5  
prescribe#(21,16)=-9.2  
prescribe#(21,17)=2.9  
prescribe#(21,18)=-712.4  
prescribe#(21,19)=-154.7  
prescribe#(21,20)=473.6  
prescribe#(21,21)=2564.6  
prescribe#(21,22)=-160.8  
prescribe#(21,23)=-106.7

prescribe#(22,0)=2244.8  
prescribe#(22,1)=-113.4  
prescribe#(22,2)=-459.1  
prescribe#(22,3)=13560  
prescribe#(22,4)=-0.37074e06  
prescribe#(22,5)=-15556  
prescribe#(22,6)=-1403.1  
prescribe#(22,7)=249.8  
prescribe#(22,8)=-2217.9  
prescribe#(22,9)=3670.1  
prescribe#(22,10)=328.0  
prescribe#(22,11)=-2245.7  
prescribe#(22,12)=251.8  
prescribe#(22,13)=-918.9  
prescribe#(22,14)=125.6  
prescribe#(22,15)=-352.9  
prescribe#(22,16)=147.7  
prescribe#(22,17)=63.7  
prescribe#(22,18)=65800  
prescribe#(22,19)=-208.0  
prescribe#(22,20)=-0.2488e06  
prescribe#(22,21)=-160.8  
prescribe#(22,22)=0.55698e06  
prescribe#(22,23)=-80.7

prescribe#(23,0)=-593.6  
prescribe#(23,1)=952.4  
prescribe#(23,2)=24.8  
prescribe#(23,3)=-98.5  
prescribe#(23,4)=20.2  
prescribe#(23,5)=676.5  
prescribe#(23,6)=-2.0  
prescribe#(23,7)=10.1



prescribe#(23,8)=633.5  
prescribe#(23,9)=204.5  
prescribe#(23,10)=124.3  
prescribe#(23,11)=-117.4  
prescribe#(23,12)=166.9  
prescribe#(23,13)=-67.6  
prescribe#(23,14)=-4118.2  
prescribe#(23,15)=-31.4  
prescribe#(23,16)=-3.9  
prescribe#(23,17)=0.7  
prescribe#(23,18)=-361.7  
prescribe#(23,19)=-80.0  
prescribe#(23,20)=263.9  
prescribe#(23,21)=-106.7  
prescribe#(23,22)=-80.7  
prescribe#(23,23)=2633.9

prescribe#(24,0)=92.9  
prescribe#(24,1)=-12.7  
prescribe#(24,2)=-6.1  
prescribe#(24,3)=-0.04  
prescribe#(24,4)=-62.2  
prescribe#(24,5)=-89.7  
prescribe#(24,6)=-3.9  
prescribe#(24,7)=-1.4  
prescribe#(24,8)=-155.1  
prescribe#(24,9)=-50.5  
prescribe#(24,10)=-1739.6  
prescribe#(24,11)=31.9  
prescribe#(24,12)=-33.3  
prescribe#(24,13)=12.9  
prescribe#(24,14)=-17.1  
prescribe#(24,15)=5.7  
prescribe#(24,16)=1.0  
prescribe#(24,17)=-1.9  
prescribe#(24,18)=760.8  
prescribe#(24,19)=439.6  
prescribe#(24,20)=-410.2  
prescribe#(24,21)=21.3  
prescribe#(24,22)=153.3  
prescribe#(24,23)=10.9

prescribe#(25,0)=77.0  
prescribe#(25,1)=-52.0  
prescribe#(25,2)=-4.3  
prescribe#(25,3)=11.0  
prescribe#(25,4)=18.9  
prescribe#(25,5)=-85.0  
prescribe#(25,6)=-5.9  
prescribe#(25,7)=-1.1  
prescribe#(25,8)=-143.4  
prescribe#(25,9)=-28.9  
prescribe#(25,10)=-26.9  
prescribe#(25,11)=23.7  
prescribe#(25,12)=-1721.8  
prescribe#(25,13)=-9.6  
prescribe#(25,14)=-18.9

prescribe#(25,15)=12.2  
prescribe#(25,16)=1.2  
prescribe#(25,17)=-0.2  
prescribe#(25,18)=372.4  
prescribe#(25,19)=17.3  
prescribe#(25,20)=113.7  
prescribe#(25,21)=428.2  
prescribe#(25,22)=-40.3  
prescribe#(25,23)=12.1

prescribe#(26,0)=63.4  
prescribe#(26,1)=-109.0  
prescribe#(26,2)=0.6  
prescribe#(26,3)=16.0  
prescribe#(26,4)=-81.6  
prescribe#(26,5)=-77.8  
prescribe#(26,6)=0.8  
prescribe#(26,7)=-1.3  
prescribe#(26,8)=-71.3  
prescribe#(26,9)=-22.3  
prescribe#(26,10)=-14.4  
prescribe#(26,11)=14.3  
prescribe#(26,12)=-19.3  
prescribe#(26,13)=11.6  
prescribe#(26,14)=-1710.1  
prescribe#(26,15)=-11.3  
prescribe#(26,16)=0.4  
prescribe#(26,17)=-0.1  
prescribe#(26,18)=3.4  
prescribe#(26,19)=9.3  
prescribe#(26,20)=311.0  
prescribe#(26,21)=12.4  
prescribe#(26,22)=202.5  
prescribe#(26,23)=420.5

prescribe#(27,0)=-77.5  
prescribe#(27,1)=10.2  
prescribe#(27,2)=5.0  
prescribe#(27,3)=-2.0  
prescribe#(27,4)=62.7  
prescribe#(27,5)=77.5  
prescribe#(27,6)=-0.2  
prescribe#(27,7)=1.2  
prescribe#(27,8)=140.7  
prescribe#(27,9)=40.2  
prescribe#(27,10)=-303.3  
prescribe#(27,11)=-24.7  
prescribe#(27,12)=27.2  
prescribe#(27,13)=-10.0  
prescribe#(27,14)=13.9  
prescribe#(27,15)=-4.6  
prescribe#(27,16)=-0.5  
prescribe#(27,17)=1.7  
prescribe#(27,18)=-756.4  
prescribe#(27,19)=-479.3  
prescribe#(27,20)=405.2  
prescribe#(27,21)=-17.4  
prescribe#(27,22)=-152.4

prescribe#(27,23)=-8.9  
prescribe#(28,0)=-64.8  
prescribe#(28,1)=44.4  
prescribe#(28,2)=3.6  
prescribe#(28,3)=-10.2  
prescribe#(28,4)=-19.7  
prescribe#(28,5)=72.4  
prescribe#(28,6)=5.4  
prescribe#(28,7)=1.3  
prescribe#(28,8)=123.3  
prescribe#(28,9)=23.3  
prescribe#(28,10)=22.9  
prescribe#(28,11)=-19.8  
prescribe#(28,12)=-317.1  
prescribe#(28,13)=10.6  
prescribe#(28,14)=16.2  
prescribe#(28,15)=-10.8  
prescribe#(28,16)=-1.1  
prescribe#(28,17)=0.2  
prescribe#(28,18)=-360.6  
prescribe#(28,19)=-14.7  
prescribe#(28,20)=-121.5  
prescribe#(28,21)=-470.6  
prescribe#(28,22)=43.0  
prescribe#(28,23)=-10.3

prescribe#(29,0)=-53.5  
prescribe#(29,1)=93.2  
prescribe#(29,2)=-1.0  
prescribe#(29,3)=-14.3  
prescribe#(29,4)=81.2  
prescribe#(29,5)=66.6  
prescribe#(29,6)=-0.8  
prescribe#(29,7)=1.1  
prescribe#(29,8)=60.9  
prescribe#(29,9)=18.9  
prescribe#(29,10)=12.4  
prescribe#(29,11)=-12.4  
prescribe#(29,12)=16.5  
prescribe#(29,13)=-10.5  
prescribe#(29,14)=-327.0  
prescribe#(29,15)=11.8  
prescribe#(29,16)=-0.3  
prescribe#(29,17)=0.1  
prescribe#(29,18)=2.6  
prescribe#(29,19)=-7.9  
prescribe#(29,20)=-315.4  
prescribe#(29,21)=-10.6  
prescribe#(29,22)=-201.2  
prescribe#(29,23)=-464.1

'\*\*\*\*\*  
' FORCES IN FCH2 ARE DETERMINED FROM CHASSIS FORCES  
' STORED IN CHL#(15) AND UNSPRUNG AXLE FORCES.  
'  
'\*\*\*\*\*

```

'*****
' determine k11#(17,17) inverted and
' store in k11i#(17,17)
'*****
,

for x=0 to 17
  for y=0 to 17

    k11#(x,y)=prescribe#(x,y)

  next y
next x

' call invert2
for x=0 to 17
  for y=0 to 17
    k11i#(x,y)=0
  next y
next x

for x=0 to 17
  k11i#(x,x)=1
next x

for x=0 to 17

  for y=x to 17

    d#=k11#(x,y) : if d#=0 or d#=1 goto LABEL.ONE2
    for k=x to 17
      k11#(k,y)=k11#(k,y)/d#
    next k
    for k=0 to 17
      k11i#(k,y)=k11i#(k,y)/d#
    next k
LABEL.ONE2:

  next y

  if x=17 goto LABEL.TWO2

  for y=x+1 to 17

    if k11#(x,y)=0 goto LABEL.THREE2

    for k=x to 17
      k11#(k,y)=k11#(k,y)-k11#(k,x)
    next k
    for k=0 to 17
      k11i#(k,y)=k11i#(k,y)-k11i#(k,x)
    next k

LABEL.THREE2:

  next y

next x

```

```

LABEL.TWO2:

for x=0 to 17

if k11#(x,x)=1 then next x
if x<>17+1 then print "*****NOT INV*****invert2": end

for x=17 to 1 step -1

for y=x-1 to 0 step -1

d#=k11#(x,y)

for k=x to 17
k11#(k,y)=k11#(k,y)-k11#(k,x)*d#
next k
for k=0 to 17
k11i#(k,y)=k11i#(k,y)-k11i#(k,x)*d#
next k

next y

next x

'*****
' determine k21#(5,17) and multiply
' by k11i#(17,17) and
' store in k21k11i#(5,17)
'*****

for x=18 to 23
for y=0 to 17
k21#((x-18),y)=prescribe#(x,y)

next y
next x

for x=0 to 5
for y=0 to 17
k21k11i#=0.0
for z=0 to 17

k21k11i#=k21k11i#+k21#(x,z)*k11i#(z,y)

next z
k21k11i#(x,y)=k21k11i#
next y
next x

'*****
' determine k12(17,5) and multiply
' k21k11i#(5,17) by k12#(5,17) and
' store in k21k11i#k12#(5,5).
'*****
'
for x=0 to 17

```

```

for y=18 to 23
  k12#(x,(y-18))=prescribe#(x,y)
next y
next x

for x=0 to 5
  for y=0 to 5
    k21k11ik12#=0.0
    for z=0 to 17

      k21k11ik12#=k21k11ik12#+k21k11i#(x,z)*k12#(z,y)

    next z
    k21k11ik12#(x,y)=k21k11ik12#
  next y
next x

'*****
'determine k22(5,5)and subtract
'k21k11ik12#(5,5) from it and
'store in k22k21k11ik12#(5,5).
'*****

for x=18 to 23
  for y=18 to 23
    k22#((x-18),(y-18))=prescribe#(x,y)
  next y
next x

for x=0 to 5
  for y=0 to 5
    k22k21k11ik12#(x,y)=k22#(x,y)-k21k11ik12#(x,y)
  next y
next x

' CALL INVERT3

for x=0 to 5
  for y=0 to 5
    k22k21k11ik12i#(x,y)=0
  next y
next x
for x=0 to 5
  k22k21k11ik12i#(x,x)=1
next x

for x=0 to 5

  for y=x to 5

    d#=k22k21k11ik12#(x,y) : if d#=0 or d#=1 goto LABEL.ONE3
    for k=x to 5
      k22k21k11ik12#(k,y)=k22k21k11ik12#(k,y)/d#
    next k
  
```

```

    for k=0 to 5
      k22k21k11ik12i#(k,y)=k22k21k11ik12i#(k,y)/d#
    next k
LABEL.ONE3:

next y

if x=5 goto LABEL.TWO3

for y=x+1 to 5

if k22k21k11ik12#(x,y)=0 goto LABEL.THREE3

  for k=x to 5
    k22k21k11ik12#(k,y)=k22k21k11ik12#(k,y)-k22k21k11ik12#(k,x)
  next k
  for k=0 to 5
    k22k21k11ik12i#(k,y)=k22k21k11ik12i#(k,y)-k22k21k11ik12i#(k,x)
  next k

LABEL.THREE3:

  next y

next x

LABEL.TWO3:

for x=0 to 5

if k22k21k11ik12#(x,x)=1 then next x
if x<>5+1 then print "*****NOT INV*****invert3": end

  for x=5 to 1 step -1

    for y=x-1 to 0 step -1

      d#=k22k21k11ik12#(x,y)

      for k=x to 5

k22k21k11ik12#(k,y)=k22k21k11ik12#(k,y)-k22k21k11ik12#(k,x)*d#
      next k
      for k=0 to 5

k22k21k11ik12i#(k,y)=k22k21k11ik12i#(k,y)-k22k21k11ik12i#(k,x)*d#
      next k

    next y

  next x

'*****
'determine k22k21k11ik12i#(5,5) multiplied
'by airbag force unity matrix a#(5) and
'store in fab#(5).
'*****

```

```

for x=0 to 5
fab#(x)=0
  for y=0 to 5
    fab#(x)=fab#(x)+k22k21k11ik12i#(x,y)*a#(y)
  next y
' PRINT "Fab(";x;")=";fab#(x)
next x

'PRINT
'*****
'determine k22k21k11ik12i#(5,5) multiplied
'by k21k11i#(5,5) and
'store in fch#(5,5)
'*****

for x=0 to 5
  for y=0 to 17
    fch3#=0.0
    for z=0 to 5

      fch3#=fch3#+k22k21k11ik12i#(x,z)*k21k11i#(z,y)

    next z
    fch1#(x,y)=fch3#
  next y
next x

LABEL.MISS:
'*****
'determine fch1#(5,17) multiplied
'by chassis force unity matrix FCH2#(17) and
'store in fch#(17).
'*****

for x=0 to 5
fch#(x)=0
  for y=0 to 17

    fch#(x)=fch#(x)+fch1#(x,y)*FCH2#(y)

  next y
next x

IF FLAGC = 0 THEN
const1#=uab#(0,n-1)-uab#(1,n-1)+uab#(2,n-1)-uab#(3,n-1)+uab#(4
,n-1)-uab#(5,n-1)
const#=30000*(3*381+const1#)^1.57
END IF
FLAGC=1
print "constant=",const#,"*****case3"
count1#=20000
count2#=200000
step1#=(count2#-count1#)/10
eqn#=1

do until eqn# > -0.000001 and eqn# < 0.000001

f#=count1#

```



```

uab#(0,n)=fab#(0)*f#-fch#(0)
uab#(1,n)=fab#(1)*f#-fch#(1)
uab#(2,n)=fab#(2)*f#-fch#(2)
uab#(3,n)=fab#(3)*f#-fch#(3)
uab#(4,n)=fab#(4)*f#-fch#(4)
uab#(5,n)=fab#(5)*f#-fch#(5)

eqn1#=(3*381+uab#(0,n)-uab#(1,n)+uab#(2,n)-uab#(3,n)+uab#(4,n)
-uab#(5,n))

eqn#=(f#^(1/1.57))*eqn1#-(const#)^(1/1.57)

if eqn# <-0.000001 then
  count1#=count1#+step1#
end if

if eqn# >0.000001 then
  count2# =count1#
  count1# =count1#-step1#
  step1#=(count2#-count1#)/10
end if

loop

f1#=const#/((3*381+uab#(0,n)-uab#(1,n)+uab#(2,n)-uab#(3,n)+uab
#(4,n)-uab#(5,n))^1.57)
SUSPR#(n)=f#
'print
'print uab#(0,n), "case3", fch#(0), f#, fab#(0)
'print uab#(1,n)
'print uab#(2,n)
'print uab#(3,n)
'print uab#(4,n)
'print uab#(5,n)
'input azt
'stop
END SUB

```

```

SUB CASE4
S           H           A           R           E           D
BR#( ), n, thetap#, CHL#( ), SUSPR#( ), SUSPL#( ), uab#( ), uchassis#( )

DIM susp4#(13), flex4a#(5,13)
SHARED FLAGC
SHARED const#
'*****
'*****
'This routine contains the suspension force matrix and chassis
flexibility
' matrix for suspension operating condition c.
'
' Suspension flexibility matrix is formed (susp4#()).The
equation *
' is solved to give the right hand suspension force only, using
*
' the condition of zero relative displacement between the middle
axle *
' and chassis. This is determined by placing a tie bar between
the *
' middle axle and chassis and deriving a flexibility matrix
*
' which relates tie bar force to body reactions, chassis and
tractor
' self weight and suspension forces. Setting the tie bar force
to zero
' simulates the desired condition enabling suspension force to
be determined
' for the left SUSPENSION
'*****
'*****

IF FLAGC = 0 THEN
const1#=uab#(0,n-1)-uab#(1,n-1)+uab#(2,n-1)-uab#(3,n-1)+uab#(4
,n-1)-uab#(5,n-1)
const#=30000*(3*381+const1#)^1.57
END IF
FLAGC=1
'PRINT "CONST=";const#
f1#=const#/((3*381+uab#(0,n-1)-uab#(1,n-1)+uab#(2,n-1)-uab#(3,
n-1)+uab#(4,n-1)-uab#(5,n-1))^1.57)
f#=f1#

susp4#(0)=-0.046744
susp4#(1)=-0.04670
susp4#(2)=-1.2866
susp4#(3)=-1.2832
susp4#(4)=-0.0011292
susp4#(5)=-0.042928
susp4#(6)=-0.021167
susp4#(7)=-0.5722e-04
susp4#(8)=-0.0044022
susp4#(9)=-0.17166e-04
susp4#(10)=-0.3231
susp4#(11)=-1.0596e-04
susp4#(13)=-4.7075
susp4#(12)=-4.6992

```

'\*\*\*\*\* body reactions, tractor weight, chassis weight and suspension

'\*\*\*\*\* forces (not known yet) are stored in chl(14).

for x=0 to 15

  CHL#(x)=0

next x

f2#=1

do until f2# < 1 and f2# > -1

CHL#(0)=BR#(0,n)

CHL#(1)=BR#(1,n)

CHL#(2)=BR#(2,n)

CHL#(3)=BR#(3,n)

CHL#(4)=BR#(4,n)

CHL#(5)=BR#(5,n)

CHL#(6)=BR#(6,n)

CHL#(7)=BR#(7,n)

CHL#(8)=-63666.9\*cos(thetap#)

CHL#(9)=63666.9\*sin(thetap#)

CHL#(10)=-46597.5\*cos(thetap#)

CHL#(11)=46597.5\*sin(thetap#)

CHL#(12)=f#

' CHL#(13)-unknown yet (SUSPL)

a#=0

for x=0 to 12

  a#=a#+CHL#(x)\*suspm4#(x)

next x

SUSPL#(n)=a#/-suspm4#(13)

CHL#(13)=SUSPL#(n)

flex4a#(0,0)=0.02146

flex4a#(0,1)=0.00823

flex4a#(0,2)=0.45320

flex4a#(0,3)=0.91450

flex4a#(0,4)=-0.1873

flex4a#(0,5)=0.01804

flex4a#(0,6)=0.05501

flex4a#(0,7)=-0.01135

flex4a#(0,8)=0.03941

flex4a#(0,9)=-0.00941

flex4a#(0,10)=0.27982

flex4a#(0,11)=-0.02487

flex4a#(0,13)=1.7487

flex4a#(0,12)=2.5165

flex4a#(1,0)=0.084921

flex4a#(1,1)=-0.00626

flex4a#(1,2)=-0.2320

flex4a#(1,3)=-0.23140

flex4a#(1,4)=-0.04533

flex4a#(1,5)=-0.00266

flex4a#(1,6)=-0.05354

flex4a#(1,7)=-0.00316

flex4a#(1,8)=-0.04315  
flex4a#(1,9)=-0.00597  
flex4a#(1,10)=-0.01099  
flex4a#(1,11)=-0.01120  
flex4a#(1,13)=-1.8879  
flex4a#(1,12)=-2.3635

flex4a#(2,0)=0.03724  
flex4a#(2,1)=0.04131  
flex4a#(2,2)=0.62180  
flex4a#(2,3)=1.32040  
flex4a#(2,4)=-0.2182  
flex4a#(2,5)=0.03396  
flex4a#(2,6)=0.00032  
flex4a#(2,7)=-0.01333  
flex4a#(2,8)=-0.00987  
flex4a#(2,9)=-0.00675  
flex4a#(2,10)=0.30916  
flex4a#(2,11)=-0.02576  
flex4a#(2,13)=2.13540  
flex4a#(2,12)=3.19510

flex4a#(3,0)=0.02481  
flex4a#(3,1)=0.01596  
flex4a#(3,2)=0.4270  
flex4a#(3,3)=0.4961  
flex4a#(3,4)=-0.05050  
flex4a#(3,5)=0.01699  
flex4a#(3,6)=-0.008  
flex4a#(3,7)=-0.00289  
flex4a#(3,8)=-0.01161  
flex4a#(3,9)=-0.00337  
flex4a#(3,10)=0.1822  
flex4a#(3,11)=-0.01077  
flex4a#(3,13)=0.8695  
flex4a#(3,12)=0.4434

flex4a#(4,0)=0.05506  
flex4a#(4,1)=0.08421  
flex4a#(4,2)=0.8047  
flex4a#(4,3)=1.7908  
flex4a#(4,4)=-0.2501  
flex4a#(4,5)=0.05111  
flex4a#(4,6)=-0.05432  
flex4a#(4,7)=-0.0158  
flex4a#(4,8)=-0.05903  
flex4a#(4,9)=-0.004908  
flex4a#(4,10)=0.33796  
flex4a#(4,11)=-0.02616  
flex4a#(4,13)=2.5326  
flex4a#(4,12)=3.9072

flex4a#(5,0)=-0.02343  
flex4a#(5,1)=-0.02233  
flex4a#(5,2)=-0.5692  
flex4a#(5,3)=-0.3987  
flex4a#(5,4)=-0.0579  
flex4a#(5,5)=-0.01829

```
flex4a#(5,6)=0.01101
flex4a#(5,7)=-0.00289
flex4a#(5,8)=-0.014302
flex4a#(5,9)=-0.01369
flex4a#(5,10)=-0.02866
flex4a#(5,11)=-0.01002
flex4a#(5,13)=-2.3064
flex4a#(5,12)=-2.6568
```

```
for x=0 to 5
uab#(x,n)=0
  for y=0 to 13
    uab#(x,n)=uab#(x,n)+flex4a#(x,y)*CHL#(y)/1000.0
  next y
'print uab#(x,n)
next x
```

```
f#=const#/((3*381+uab#(0,n)-uab#(1,n)+uab#(2,n)-uab#(3,n)+uab#
(4,n)-uab#(5,n))1.57)
SUSPR#(n)=f#
f2#=f#-f1#
f1#=f#
loop
```

```
end sub
```

Fig. C18 Flexi.bas

```
SUB FLEXI
SHARED FLCHECK(),uchassis#(),n

'*****
***g***
' This routine calls the appropriate chassis flexibility matrix
' for the current suspension operating condition
'*****
*****

IF FLCHECK(n)=1 THEN
  CALL FLEXI1
  'PRINT "CALLING CASE1"
ELSEIF FLCHECK(n)=2 THEN
  CALL FLEXI2
  'PRINT "CALLING CASE2"
ELSEIF FLCHECK(n)=3 THEN
  'PRINT "CALLING CASE3"
  CALL FLEXI3
ELSEIF FLCHECK(n)=4 THEN
  CALL FLEXI4
  'PRINT "CALLING CASE4"
END IF

end sub
```

Fig. C19 Flex1.bas

```
SUB FLEX11
SHARED BR#( ),n,thetap#,CHL#( ),SUSPR#( ),SUSPL#( )
SHARED uchassis#( ),uab#( )
DIM flex1#(8,13),flex1a#(5,13)
'*****
*****
'This routine contains the chassis flexibility matrix for
determining
'displacements for suspension operating condition a.
'*****
*****

CHL#(0)=BR#(0,n)
CHL#(1)=BR#(1,n)
CHL#(2)=BR#(2,n)
CHL#(3)=BR#(3,n)
CHL#(4)=BR#(4,n)
CHL#(5)=BR#(5,n)
CHL#(6)=BR#(6,n)
CHL#(7)=BR#(7,n)
CHL#(8)=-63666.9*cos(thetap#)
CHL#(9)=63666.9*sin(thetap#)
CHL#(10)=-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
CHL#(12)=SUSPL#(n)
CHL#(13)=SUSPR#(n)

flex1#(0,0)=0.15622
flex1#(0,1)=0.091892
flex1#(0,2)=0.0839
flex1#(0,3)=0.0505
flex1#(0,4)=-0.1082
flex1#(0,5)=0.11628
flex1#(0,6)=-0.013469
flex1#(0,7)=0.040835
flex1#(0,8)=-0.012217
flex1#(0,9)=0.028565
flex1#(0,10)=0.002803
flex1#(0,11)=-0.004309
flex1#(0,12)=0.0926
flex1#(0,13)=0.0693

flex1#(1,0)=0.083911
flex1#(1,1)=0.050547
flex1#(1,2)=1.3528
flex1#(1,3)=0.7072
flex1#(1,4)=0.1345
flex1#(1,5)=0.044217
flex1#(1,6)=-0.079945
flex1#(1,7)=0.008794
flex1#(1,8)=-0.077668
flex1#(1,9)=0.001147
flex1#(1,10)=0.25059
flex1#(1,11)=0.01449
flex1#(1,12)=2.2495
flex1#(1,13)=1.7450
```

flex1#(2,0)=-0.10824  
flex1#(2,1)=0.10823  
flex1#(2,2)=0.1345  
flex1#(2,3)=-0.1345  
flex1#(2,4)=1.2801  
flex1#(2,5)=0  
flex1#(2,6)=0  
flex1#(2,7)=-0.083615  
flex1#(2,8)=0  
flex1#(2,9)=-0.063022  
flex1#(2,10)=0  
flex1#(2,11)=0.361328  
flex1#(2,12)=0.1051  
flex1#(2,13)=-0.1051

flex1#(3,0)=0.091889  
flex1#(3,1)=0.15622  
flex1#(3,2)=0.0505  
flex1#(3,3)=0.0839  
flex1#(3,4)=0.1082  
flex1#(3,5)=0.11628  
flex1#(3,6)=-0.013469  
flex1#(3,7)=-0.040825  
flex1#(3,8)=-0.012217  
flex1#(3,9)=-0.028555  
flex1#(3,10)=0  
flex1#(3,11)=0.004309  
flex1#(3,12)=0.0693  
flex1#(3,13)=0.0926

flex1#(4,0)=0.05054  
flex1#(4,1)=0.083924  
flex1#(4,2)=0.7072  
flex1#(4,3)=1.3528  
flex1#(4,4)=-0.1345  
flex1#(4,5)=0.044217  
flex1#(4,6)=-0.079947  
flex1#(4,7)=-0.008803  
flex1#(4,8)=-0.077671  
flex1#(4,9)=-0.001155  
flex1#(4,10)=0.25059  
flex1#(4,11)=-0.01448  
flex1#(4,12)=1.7450  
flex1#(4,13)=2.2495

flex1#(5,0)=-0.10824  
flex1#(5,1)=0.10823  
flex1#(5,2)=0.1345  
flex1#(5,3)=-0.1345  
flex1#(5,4)=1.2785  
flex1#(5,5)=0  
flex1#(5,6)=0  
flex1#(5,7)=-0.083615  
flex1#(5,8)=0  
flex1#(5,9)=-0.063022  
flex1#(5,10)=0  
flex1#(5,11)=0.3613498  
flex1#(5,12)=0.1051



flex1#(5,13)=-0.1050

flex1#(6,0)=0.11628  
flex1#(6,1)=0.11628  
flex1#(6,2)=0.0442  
flex1#(6,3)=0.0442  
flex1#(6,4)=0  
flex1#(6,5)=0.11845  
flex1#(6,6)=-0.012583  
flex1#(6,7)=0.014970  
flex1#(6,8)=-0.01119  
flex1#(6,9)=0  
flex1#(6,10)=0.0042985  
flex1#(6,11)=0  
flex1#(6,12)=0.0667  
flex1#(6,13)=0.0667

flex1#(7,0)=-0.01347  
flex1#(7,1)=-0.01347  
flex1#(7,2)=-0.0800  
flex1#(7,3)=-0.0800  
flex1#(7,4)=0  
flex1#(7,5)=-0.012583  
flex1#(7,6)=0.28262  
flex1#(7,7)=0.000408  
flex1#(7,8)=0.26394  
flex1#(7,9)=0  
flex1#(7,10)=0.05793  
flex1#(7,11)=0  
flex1#(7,12)=0.0316  
flex1#(7,13)=0.0316

flex1#(8,0)=0.040836  
flex1#(8,1)=-0.040836  
flex1#(8,2)=0.0088  
flex1#(8,3)=-0.0088  
flex1#(8,4)=-0.0836  
flex1#(8,5)=0  
flex1#(8,6)=0  
flex1#(8,7)=0.32029  
flex1#(8,8)=0  
flex1#(8,9)=0.40854  
flex1#(8,10)=0  
flex1#(8,11)=0.121884  
flex1#(8,12)=0.0069  
flex1#(8,13)=-0.0070

for x=0 to 8

  uchassis#(x,n)=0

  for y=0 to 13

    uchassis#(x,n)=uchassis#(x,n)+flex1#(x,y)\*CHL#(y)/1000.0

  next y

next x

flexla#(0,0)=0.01475  
flexla#(0,1)=-0.00232  
flexla#(0,2)=0.35340  
flexla#(0,3)=0.53990  
flexla#(0,4)=-0.10940  
flexla#(0,5)=0.01012  
flexla#(0,6)=0.05110  
flexla#(0,7)=-0.00654  
flexla#(0,8)=0.03859  
flexla#(0,9)=-0.00749  
flexla#(0,10)=0.22016  
flexla#(0,11)=-0.01764  
flexla#(0,12)=1.1810  
flexla#(0,13)=1.3474

flexla#(1,0)=0.01032  
flexla#(1,1)=-0.00408  
flexla#(1,2)=-0.21130  
flexla#(1,3)=-0.15390  
flexla#(1,4)=-0.06140  
flexla#(1,5)=-0.00102  
flexla#(1,6)=-0.05277  
flexla#(1,7)=-0.00415  
flexla#(1,8)=-0.04298  
flexla#(1,9)=-0.00636  
flexla#(1,10)=1.3456e-03  
flexla#(1,11)=-0.01269  
flexla#(1,12)=-1.7704  
flexla#(1,13)=-2.1217

flexla#(2,0)=0.02867  
flexla#(2,1)=0.02783  
flexla#(2,2)=0.49410  
flexla#(2,3)=0.84130  
flexla#(2,4)=-0.11860  
flexla#(2,5)=0.02383  
flexla#(2,6)=-0.00468  
flexla#(2,7)=-0.00717  
flexla#(2,8)=-0.01091  
flexla#(2,9)=-0.00429  
flexla#(2,10)=0.23287  
flexla#(2,11)=-0.01651  
flexla#(2,12)=1.4093  
flexla#(2,13)=1.6999

flexla#(3,0)=0.03043  
flexla#(3,1)=0.02480  
flexla#(3,2)=0.51080  
flexla#(3,3)=0.81040  
flexla#(3,4)=-0.11580  
flexla#(3,5)=0.02364  
flexla#(3,6)=-0.00472  
flexla#(3,7)=-0.00694  
flexla#(3,8)=-0.01093  
flexla#(3,9)=-0.00499  
flexla#(3,10)=0.23224  
flexla#(3,11)=-0.01684  
flexla#(3,12)=1.3457

flexla#(3,13)=1.4240

flexla#(4,0)=0.04472  
flexla#(4,1)=0.06794  
flexla#(4,2)=0.6508  
flexla#(4,3)=1.2129  
flexla#(4,4)=-0.13010  
flexla#(4,5)=0.03888  
flexla#(4,6)=-0.06035  
flexla#(4,7)=-0.00837  
flexla#(4,8)=-0.06029  
flexla#(4,9)=-0.00194  
flexla#(4,10)=0.24591  
flexla#(4,11)=-0.01501  
flexla#(4,12)=1.6568  
flexla#(4,13)=2.1038

flexla#(5,0)=-0.02188  
flexla#(5,1)=-0.01988  
flexla#(5,2)=-0.54610  
flexla#(5,3)=-0.31180  
flexla#(5,4)=-0.07600  
flexla#(5,5)=-0.01645  
flexla#(5,6)=0.01192  
flexla#(5,7)=-0.00401  
flexla#(5,8)=0.01449  
flexla#(5,9)=-0.00182  
flexla#(5,10)=-0.01482  
flexla#(5,11)=-0.01171  
flexla#(5,12)=-2.1747  
flexla#(5,13)=-2.3856

```
for x=0 to 5
  uab#(x,n)=0
  for y=0 to 13
    uab#(x,n)=uab#(x,n)+flexla#(x,y)*CHL#(y)/1000.0
  next y
next x

end sub
```

Fig. C20 Flexi2.bas

```
SUB FLEXI2
  SHARED BR#( ), n, thetap#, CHL#( ), SUSPR#( ), SUSPL#( ), uab#( )
  shared uchassis#( )
  DIM flex2#(8,12), flex2a#(5,12)
  '*****
  ' This routine contains the chassis flexibility matrix for
  determining
  ' displacements for suspension operating condition b.
  '*****
  CHL#(0)=BR#(0,n)
  CHL#(1)=BR#(1,n)
  CHL#(2)=BR#(2,n)
  CHL#(3)=BR#(3,n)
  CHL#(4)=BR#(4,n)
  CHL#(5)=BR#(5,n)
  CHL#(6)=BR#(6,n)
  CHL#(7)=BR#(7,n)
  CHL#(8)=-63666.9*cos(thetap#)
  CHL#(9)=63666.9*sin(thetap#)
  CHL#(10)=-47088*cos(thetap#)
  CHL#(11)=47088*sin(thetap#)
  CHL#(12)=SUSPR#(n)

  flex2#(0,0)=0.15671
  flex2#(0,1)=0.092201
  flex2#(0,2)=0.1011
  flex2#(0,3)=0.0551
  flex2#(0,4)=-0.1046
  flex2#(0,5)=0.11664
  flex2#(0,6)=-0.01329
  flex2#(0,7)=0.041056
  flex2#(0,8)=-0.01218
  flex2#(0,9)=0.028653
  flex2#(0,10)=0.00555
  flex2#(0,11)=-0.00398
  flex2#(0,12)=0.0954

  flex2#(1,0)=0.10114
  flex2#(1,1)=0.061504
  flex2#(1,2)=1.9650
  flex2#(1,3)=0.8703
  flex2#(1,4)=0.2617
  flex2#(1,5)=0.057164
  flex2#(1,6)=-0.073557
  flex2#(1,7)=0.01666
  flex2#(1,8)=-0.076338
  flex2#(1,9)=0.004285
  flex2#(1,10)=0.34809
  flex2#(1,11)=0.02631
  flex2#(1,12)=2.6727

  flex2#(2,0)=-0.10466
  flex2#(2,1)=0.11050
  flex2#(2,2)=0.2617
  flex2#(2,3)=-0.1006
  flex2#(2,4)=1.3065
```

flex2#(2,5)=0.002684  
flex2#(2,6)=0.001328  
flex2#(2,7)=-0.081978  
flex2#(2,8)=0.000278  
flex2#(2,9)=-0.062368  
flex2#(2,10)=0.02025  
flex2#(2,11)=0.3638  
flex2#(2,12)=0.0877

flex2#(3,0)=0.092198  
flex2#(3,1)=0.15642  
flex2#(3,2)=0.0615  
flex2#(3,3)=0.0868  
flex2#(3,4)=0.1105  
flex2#(3,5)=0.11651  
flex2#(3,6)=-0.013355  
flex2#(3,7)=-0.040683  
flex2#(3,8)=-0.012193  
flex2#(3,9)=-0.028498  
flex2#(3,10)=0.00455  
flex2#(3,11)=0.00452  
flex2#(3,12)=0.1092

flex2#(4,0)=0.055131  
flex2#(4,1)=0.086843  
flex2#(4,2)=0.8703  
flex2#(4,3)=1.3963  
flex2#(4,4)=-0.1006  
flex2#(4,5)=0.047666  
flex2#(4,6)=-0.078245  
flex2#(4,7)=-0.006707  
flex2#(4,8)=-0.077317  
flex2#(4,9)=-0.000319  
flex2#(4,10)=0.27656  
flex2#(4,11)=-0.011333  
flex2#(4,12)=2.4966

flex2#(5,0)=-0.10466  
flex2#(5,1)=0.11050  
flex2#(5,2)=0.2617  
flex2#(5,3)=-0.1006  
flex2#(5,4)=1.3049  
flex2#(5,5)=0.002684  
flex2#(5,6)=0.001328  
flex2#(5,7)=-0.081978  
flex2#(5,8)=0.000278  
flex2#(5,9)=-0.062368  
flex2#(5,10)=0.02026  
flex2#(5,11)=0.363796  
flex2#(5,12)=0.0877

flex2#(6,0)=0.11665  
flex2#(6,1)=0.11651  
flex2#(6,2)=0.0572  
flex2#(6,3)=0.0477  
flex2#(6,4)=0.0027  
flex2#(6,5)=0.11872  
flex2#(6,6)=-0.012448

```
flex2#(6,7)=0.000168
flex2#(6,8)=-0.011162
flex2#(6,9)=0.000068
flex2#(6,10)=0.00636
flex2#(6,11)=2.51086e-05
flex2#(6,12)=0.0863
```

```
flex2#(7,0)=-0.01329
flex2#(7,1)=-0.013356
flex2#(7,2)=-0.0736
flex2#(7,3)=-0.0783
flex2#(7,4)=0.0013
flex2#(7,5)=-0.012448
flex2#(7,6)=0.28269
flex2#(7,7)=0.000083
flex2#(7,8)=0.26395
flex2#(7,9)=0.000033
flex2#(7,10)=-0.05894
flex2#(7,11)=1.22324e-04
flex2#(7,12)=0.0413
```

```
flex2#(8,0)=0.041058
flex2#(8,1)=-0.040695
flex2#(8,2)=0.0167
flex2#(8,3)=-0.0067
flex2#(8,4)=-0.0820
flex2#(8,5)=0.000167
flex2#(8,6)=0.000081
flex2#(8,7)=0.62117
flex2#(8,8)=0.000015
flex2#(8,9)=0.40858
flex2#(8,10)=0.001253
flex2#(8,11)=0.122037
flex2#(8,12)=0.0050
```

```
for x=0 to 8
  uchassis#(x,n)=0
next x
```

```
for x=0 to 8
  for y=0 to 12
    uchassis#(x,n)=uchassis#(x,n)+flex2#(x,y)*CHL#(y)/1000.0
  next y
next x
```

' flexibility matrix for right suspension disps.

```
flex2a#(0,0)=0.018728
flex2a#(0,1)=0.000204
flex2a#(0,2)=0.4946
flex2a#(0,3)=0.5775
flex2a#(0,4)=-0.0801
flex2a#(0,5)=0.013101
flex2a#(0,6)=0.052578
flex2a#(0,7)=-0.004723
flex2a#(0,8)=0.0389
flex2a#(0,9)=-0.006763
```

flex2a#(0,10)=0.2426  
flex2a#(0,11)=-0.01492  
flex2a#(0,12)=1.5613

flex2a#(1,0)=0.00537  
flex2a#(1,1)=-0.007224  
flex2a#(1,2)=-0.3871  
flex2a#(1,3)=-0.2008  
flex2a#(1,4)=-0.0979  
flex2a#(1,5)=-0.004738  
flex2a#(1,6)=-0.054578  
flex2a#(1,7)=-0.00641  
flex2a#(1,8)=-0.043866  
flex2a#(1,9)=-0.007265  
flex2a#(1,10)=-0.02664  
flex2a#(1,11)=-0.01608  
flex2a#(1,12)=-2.3880

flex2a#(2,0)=0.032808  
flex2a#(2,1)=0.030458  
flex2a#(2,2)=0.6413  
flex2a#(2,3)=0.8805  
flex2a#(2,4)=-0.0881  
flex2a#(2,5)=0.026942  
flex2a#(2,6)=-0.003141  
flex2a#(2,7)=-0.005281  
flex2a#(2,8)=-0.01059  
flex2a#(2,9)=-0.003536  
flex2a#(2,10)=0.2563  
flex2a#(2,11)=-0.01367  
flex2a#(2,12)=1.9229

flex2a#(3,0)=0.030137  
flex2a#(3,1)=0.024613  
flex2a#(3,2)=0.5002  
flex2a#(3,3)=0.8076  
flex2a#(3,4)=-0.1180  
flex2a#(3,5)=0.023416  
flex2a#(3,6)=-0.00483  
flex2a#(3,7)=-0.007074  
flex2a#(3,8)=-0.01095  
flex2a#(3,9)=-0.005042  
flex2a#(3,10)=0.2306  
flex2a#(3,11)=-0.01704  
flex2a#(3,12)=1.4081

flex2a#(4,0)=0.049183  
flex2a#(4,1)=0.070781  
flex2a#(4,2)=0.8093  
flex2a#(4,3)=1.2552  
flex2a#(4,4)=-0.0971  
flex2a#(4,5)=0.042237  
flex2a#(4,6)=-0.058694  
flex2a#(4,7)=-0.006332  
flex2a#(4,8)=-0.059944  
flex2a#(4,9)=-0.001126  
flex2a#(4,10)=0.2712  
flex2a#(4,11)=-0.01195

flex2a#(4,12)=2.3441

flex2a#(5,0)=-0.030173

flex2a#(5,1)=-0.025156

flex2a#(5,2)=-0.8407

flex2a#(5,3)=-0.3903

flex2a#(5,4)=-0.1372

flex2a#(5,5)=-0.022681

flex2a#(5,6)=0.008841

flex2a#(5,7)=-0.007793

flex2a#(5,8)=0.01385

flex2a#(5,9)=-0.003325

flex2a#(5,10)=-0.06174

flex2a#(5,11)=-0.01739

flex2a#(5,12)=-2.8321

for x=0 to 5

  uab#(x,n)=0

  for y=0 to 12

    uab#(x,n)=uab#(x,n)+flex2a#(x,y)\*CHL#(y)/1000.0

  next y

next x

end sub



```

SUB FLEXI3
S           H           A           R           E           D
FCH2#( ), SUSPR#( ), k11i#( ), k12#( ), n, uchassis#( ), uab#( ), uch#( )
DIM k12uabfch2#(17), k12uab#(17)

'*****
'*****
' This routine calculates the chassis displacements for
suspension
' operating condition d
'*****
'*****

'*****
'*****
' Suspension STIFFNESS matrix is formed PRESCRIBE#().The FIRST
FIVE
' ROWS ARE MULTIPLIED BY FCH2#( ) , WHICH IS WHERE ALL THE APPLIED
LOADS ARE
' STORED , TO GIVE LEFT AND RIGHT HINGE Displacements
*
'*****
'*****

FCH2#(18)=SUSPR#(n)
FCH2#(19)=SUSPR#(n)
FCH2#(20)=SUSPR#(n)
FCH2#(21)=SUSPR#(n)
FCH2#(22)=SUSPR#(n)
FCH2#(23)=SUSPR#(n)

for x=0 to 17
  k12uab#=0.0
  for y=0 to 5
    k12uab#=k12uab#+k12#(x,y)*uab#(y,n)
  next y
  k12uab#(x)=k12uab#
next x

for x=0 to 17
  k12uabfch2#(x)=FCH2#(x)-k12uab#(x)
next x

for x=0 to 17
  uch#=0.0
  for y=0 to 17
    uch#=uch#+k11i#(x,y)*k12uabfch2#(y)
  next y
  uch#(x,n)=uch#
next x

uchassis#(0,n)=uch#(0,n)
uchassis#(1,n)=uch#(1,n)
uchassis#(2,n)=uch#(2,n)
uchassis#(3,n)=uch#(3,n)
uchassis#(4,n)=uch#(4,n)

```

```
'assuming uchassis#(5,n)=uchassis#(2,n)
uchassis#(5,n)=uch#(2,n)
uchassis#(6,n)=uch#(5,n)
uchassis#(7,n)=uch#(6,n)
uchassis#(8,n)=uch#(7,n)
```

```
uch#(18,n)=uab#(0,n)
uch#(19,n)=uab#(1,n)
uch#(20,n)=uab#(2,n)
uch#(21,n)=uab#(3,n)
uch#(22,n)=uab#(4,n)
uch#(23,n)=uab#(5,n)
```

```
end sub
```

```

SUB FLEXI4
  SHARED BR#( ),n,thetap#,CHL#( ),SUSPR#( ),SUSPL#( ),uchassis#( )
  DIM flex4#(8,13)
  '*****
  '*****
  'This routine contains the chassis flexibility matrix for
  determining
  ' displacements for suspension operating condition c
  '*****
  '*****

  CHL#(0)=BR#(0,n)
  CHL#(1)=BR#(1,n)
  CHL#(2)=BR#(2,n)
  CHL#(3)=BR#(3,n)
  CHL#(4)=BR#(4,n)
  CHL#(5)=BR#(5,n)
  CHL#(6)=BR#(6,n)
  CHL#(7)=BR#(7,n)
  CHL#(8)=-63666.9*cos(thetap#)
  CHL#(9)=63666.9*sin(thetap#)
  CHL#(10)=-47088*cos(thetap#)
  CHL#(11)=47088*sin(thetap#)
  CHL#(12)=SUSPL#(n)
  CHL#(13)=SUSPR#(n)

  flex4#(0,0)=0.15642
  flex4#(0,1)=0.092201
  flex4#(0,2)=0.0868
  flex4#(0,3)=0.0615
  flex4#(0,4)=-0.1105
  flex4#(0,5)=0.11651
  flex4#(0,6)=-0.013355
  flex4#(0,7)=0.040693
  flex4#(0,8)=-0.012194
  flex4#(0,9)=0.028507
  flex4#(0,10)=0.004545
  flex4#(0,11)=-0.0045217
  flex4#(0,12)=0.1092
  flex4#(0,13)=0.1035

  flex4#(1,0)=0.086829
  flex4#(1,1)=0.055139
  flex4#(1,2)=1.3963
  flex4#(1,3)=0.8703
  flex4#(1,4)=0.1007
  flex4#(1,5)=0.047666
  flex4#(1,6)=-0.078242
  flex4#(1,7)=0.006697
  flex4#(1,8)=-0.077314
  flex4#(1,9)=0.000309
  flex4#(1,10)=0.27656
  flex4#(1,11)=0.011344
  flex4#(1,12)=2.4966
  flex4#(1,13)=2.2539

  flex4#(2,0)=-0.11051

```

flex4#(2,1)=0.10465  
flex4#(2,2)=0.1007  
flex4#(2,3)=-0.2617  
flex4#(2,4)=1.3065  
flex4#(2,5)=-0.002694  
flex4#(2,6)=-0.001324  
flex4#(2,7)=-0.081975  
flex4#(2,8)=-0.000274  
flex4#(2,9)=-0.062363  
flex4#(2,10)=-0.02025  
flex4#(2,11)=0.363796  
flex4#(2,12)=-0.0876  
flex4#(2,13)=-0.5017

flex4#(3,0)=0.092198  
flex4#(3,1)=0.15671  
flex4#(3,2)=0.0551  
flex4#(3,3)=0.1012  
flex4#(3,4)=0.1046  
flex4#(3,5)=0.11664  
flex4#(3,6)=-0.013289  
flex4#(3,7)=-0.041046  
flex4#(3,8)=-0.012179  
flex4#(3,9)=-0.028642  
flex4#(3,10)=0.005475  
flex4#(3,11)=0.0039788  
flex4#(3,12)=0.0954  
flex4#(3,13)=0.1464

flex4#(4,0)=0.061492  
flex4#(4,1)=0.10116  
flex4#(4,2)=0.8703  
flex4#(4,3)=1.9650  
flex4#(4,4)=-0.2616  
flex4#(4,5)=0.057163  
flex4#(4,6)=-0.073561  
flex4#(4,7)=-0.016678  
flex4#(4,8)=-0.076342  
flex4#(4,9)=-0.0043  
flex4#(4,10)=0.34807  
flex4#(4,11)=-0.026295  
flex4#(4,12)=2.6727  
flex4#(4,13)=4.1598

flex4#(5,0)=-0.11051  
flex4#(5,1)=0.10465  
flex4#(5,2)=0.1007  
flex4#(5,3)=-0.2617  
flex4#(5,4)=1.3049  
flex4#(5,5)=-0.002694  
flex4#(5,6)=-0.001324  
flex4#(5,7)=-0.081975  
flex4#(5,8)=-0.000274  
flex4#(5,9)=-0.062363  
flex4#(5,10)=-0.02024  
flex4#(5,11)=0.363796  
flex4#(5,12)=-0.0876  
flex4#(5,13)=-0.5018

```
flex4#(6,0)=0.11651
flex4#(6,1)=0.11665
flex4#(6,2)=0.0477
flex4#(6,3)=0.0572
flex4#(6,4)=-0.0027
flex4#(6,5)=0.11872
flex4#(6,6)=-0.012448
flex4#(6,7)=-0.000165
flex4#(6,8)=-0.011162
flex4#(6,9)=-0.000065
flex4#(6,10)=0.0063609
flex4#(6,11)=-0.0002489
flex4#(6,12)=0.0863
flex4#(6,13)=0.1071
```

```
flex4#(7,0)=-0.013355
flex4#(7,1)=-0.013291
flex4#(7,2)=-0.0783
flex4#(7,3)=-0.0736
flex4#(7,4)=-0.0013
flex4#(7,5)=-0.012448
flex4#(7,6)=0.28268
flex4#(7,7)=-0.000082
flex4#(7,8)=0.26395
flex4#(7,9)=-0.000033
flex4#(7,10)=0.058947
flex4#(7,11)=-0.0001245
flex4#(7,12)=0.0413
flex4#(7,13)=0.0515
```

```
flex4#(8,0)=0.040696
flex4#(8,1)=-0.041058
flex4#(8,2)=0.0067
flex4#(8,3)=-0.0167
flex4#(8,4)=-0.0820
flex4#(8,5)=-0.000166
flex4#(8,6)=-0.000083
flex4#(8,7)=0.62117
flex4#(8,8)=-0.000019
flex4#(8,9)=0.40858
flex4#(8,10)=-0.001255
flex4#(8,11)=0.12204
flex4#(8,12)=-0.0050
flex4#(8,13)=-0.0315
```

```
for x=0 to 8
uchassis#(x,n)=0.0
  for y=0 to 13
```

```
uchassis#(x,n)=uchassis#(x,n)+flex4#(x,y)*CHL#(y)/1000.0
  next y
next x
```

```
end sub
```

Fig. C23 Disp.bas

```
SUB DISP
shared lhc#(),rhc#(),rb#(),uchassis#(),n
'*****
'*****
'
'
'      Hinge and ram co-ordinates are updated using chassis
displacements
' taking into consideration the origin movement. Firstly the
origin
' movement is determined from averaging the displacements of the
' two hinges in each of the co-ordinate directions.
'      Then the displacements at the hinges and ram base are
modified to
' give zero origin movement by subtracting the origin
displacements
' previously determined from their actual displacements.
'
'
'*****
'*****

' determine origin movement.

xorigin=(uchassis#(0,n)+uchassis#(3,n))/2
yorigin=(uchassis#(1,n)+uchassis#(4,n))/2
zorigin=(uchassis#(2,n)+uchassis#(5,n))/2
' print xorigin,yorigin,zorigin
' determine modified chassis displacements

lhc#(0,n+1)=lhc#(0,0)+uchassis#(0,n)-xorigin
lhc#(1,n+1)=lhc#(1,0)+uchassis#(1,n)-yorigin
lhc#(2,n+1)=lhc#(2,0)+uchassis#(2,n)-zorigin
rhc#(0,n+1)=rhc#(0,0)+uchassis#(3,n)-xorigin
rhc#(1,n+1)=rhc#(1,0)+uchassis#(4,n)-yorigin
rhc#(2,n+1)=rhc#(2,0)+uchassis#(5,n)-zorigin
rb#(0,n+1)=rb#(0,0)+uchassis#(6,n)-xorigin
rb#(1,n+1)=rb#(1,0)+uchassis#(7,n)-yorigin
rb#(2,n+1)=rb#(2,0)+uchassis#(8,n)-zorigin

end sub
```

Fig. C24 Tyre.bas

```
SUB TYRE
SHARED FLCHECK(),TYREF#(),n,tyre$

'*****
*****
' This routine calls the appropriate tyre force matrix for the
' current suspension operating condition
'*****
*****

IF FLCHECK(n)=1 THEN
  CALL TYRE1
  ' PRINT "CALLING TYRE1"
ELSEIF FLCHECK(n)=2 THEN
  CALL TYRE2
  ' PRINT "CALLING TYRE2"
ELSEIF FLCHECK(n)=3 THEN
  ' PRINT "CALLING TYRE3"
  CALL TYRE3
ELSEIF FLCHECK(n)=4 THEN
  CALL TYRE4
  ' PRINT "CALLING TYRE4"
END IF

FOR X=0 TO 5
IF TYREF#(X,n) < 0 THEN
tyre$="stop"
END IF
NEXT X

end sub
```

Fig. C25 Tyre1.bas

```
SUB TYRE1
  SHARED BR#( ), n, thetap#, CHL#( ), SUSPR#( ), SUSPL#( ), TYREF#( )
  DIM tyrel#(5,13)
  '*****
  ' This routine contains the chassis tyre force matrix for
  suspension
  ' operating condition a
  '*****
  '*****

  tyrel#(0,0)=0.0105
  tyrel#(0,1)=-0.0102
  tyrel#(0,2)=-0.0413
  tyrel#(0,3)=0.0412
  tyrel#(0,4)=-0.088
  tyrel#(0,5)=0.000138
  tyrel#(0,6)=0.0000209
  tyrel#(0,7)=-0.00593
  tyrel#(0,8)=0.00184
  tyrel#(0,9)=-0.00912
  tyrel#(0,10)=-0.0826
  tyrel#(0,11)=-0.0181
  tyrel#(0,12)=0.911
  tyrel#(0,13)=0.669

  tyrel#(1,0)=-0.0192
  tyrel#(1,1)=-0.0274
  tyrel#(1,2)=-0.854
  tyrel#(1,3)=-0.434
  tyrel#(1,4)=-0.163
  tyrel#(1,5)=-0.0214
  tyrel#(1,6)=-0.0106
  tyrel#(1,7)=-0.00978
  tyrel#(1,8)=-0.00219
  tyrel#(1,9)=-0.00709
  tyrel#(1,10)=-0.2446
  tyrel#(1,11)=-0.0238
  tyrel#(1,12)=-1.74
  tyrel#(1,13)=-1.39

  tyrel#(2,0)=-0.00137
  tyrel#(2,1)=0.00166
  tyrel#(2,2)=-0.168
  tyrel#(2,3)=0.168
  tyrel#(2,4)=-0.109
  tyrel#(2,5)=0.000139
  tyrel#(2,6)=0.0000153
  tyrel#(2,7)=-0.00583
  tyrel#(2,8)=0.0000131
  tyrel#(2,9)=-0.00266
  tyrel#(2,10)=-0.0826
  tyrel#(2,11)=-0.0167
  tyrel#(2,12)=0.810
  tyrel#(2,13)=0.770

  tyrel#(3,0)=-0.0102
  tyrel#(3,1)=0.0105
```



tyrel#(3,2)=0.0412  
tyrel#(3,3)=-0.0413  
tyrel#(3,4)=0.088  
tyrel#(3,5)=0.000138  
tyrel#(3,6)=0.0000138  
tyrel#(3,7)=0.00592  
tyrel#(3,8)=0.0000122  
tyrel#(3,9)=0.00912  
tyrel#(3,10)=-0.0826  
tyrel#(3,11)=0.0181  
tyrel#(3,12)=0.669  
tyrel#(3,13)=0.911

tyrel#(4,0)=-0.0274  
tyrel#(4,1)=-0.0192  
tyrel#(4,2)=-0.434  
tyrel#(4,3)=-0.854  
tyrel#(4,4)=0.163  
tyrel#(4,5)=-0.0214  
tyrel#(4,6)=-0.0106  
tyrel#(4,7)=0.0098  
tyrel#(4,8)=-0.00219  
tyrel#(4,9)=0.0071  
tyrel#(4,10)=-0.2446  
tyrel#(4,11)=0.0238  
tyrel#(4,12)=-1.39  
tyrel#(4,13)=-1.74

tyrel#(5,0)=0.00165  
tyrel#(5,1)=-0.00137  
tyrel#(5,2)=0.168  
tyrel#(5,3)=-0.168  
tyrel#(5,4)=0.109  
tyrel#(5,5)=0.000141  
tyrel#(5,6)=0.0000147  
tyrel#(5,7)=0.00582  
tyrel#(5,8)=0.0000128  
tyrel#(5,9)=0.00266  
tyrel#(5,10)=-0.0826  
tyrel#(5,11)=0.0167  
tyrel#(5,12)=0.770  
tyrel#(5,13)=0.810

CHL#(0)=BR#(0,n)  
CHL#(1)=BR#(1,n)  
CHL#(2)=BR#(2,n)  
CHL#(3)=BR#(3,n)  
CHL#(4)=BR#(4,n)  
CHL#(5)=BR#(5,n)  
CHL#(6)=BR#(6,n)  
CHL#(7)=BR#(7,n)  
CHL#(8)=-63666.9\*cos(thetap#)  
CHL#(9)=63666.9\*sin(thetap#)  
CHL#(10)=-47088\*cos(thetap#)  
CHL#(11)=47088\*sin(thetap#)  
CHL#(12)=SUSPL#(n)  
CHL#(13)=SUSPR#(n)

Fig. C26 Tyre2.bas

```
SUB TYRE2
SHARED BR#( ),n,thetap#,CHL#( ),SUSPR#( ),SUSPL#( ),TYREF#( )
DIM tyre2#(5,12)
' *****
*****
' This routine contains the chassis tyre force matrix for the
suspension
' operating condition b
' *****
*****

tyre2#(0,0)=0.0085
tyre2#(0,1)=-0.0115
tyre2#(0,2)=-0.112
tyre2#(0,3)=0.0224
tyre2#(0,4)=-0.103
tyre2#(0,5)=-0.00136
tyre2#(0,6)=-0.00071
tyre2#(0,7)=-0.00683
tyre2#(0,8)=-0.00013
tyre2#(0,9)=-0.00949
tyre2#(0,10)=-0.09378
tyre2#(0,11)=-0.01944
tyre2#(0,12)=0.5620

tyre2#(1,0)=-0.0134
tyre2#(1,1)=-0.0237
tyre2#(1,2)=-0.647
tyre2#(1,3)=-0.379
tyre2#(1,4)=-0.120
tyre2#(1,5)=-0.0170
tyre2#(1,6)=-0.00843
tyre2#(1,7)=-0.00712
tyre2#(1,8)=-0.00174
tyre2#(1,9)=-0.00603
tyre2#(1,10)=-0.2116
tyre2#(1,11)=-0.01979
tyre2#(1,12)=-1.0800

tyre2#(2,0)=-0.00558
tyre2#(2,1)=-0.00102
tyre2#(2,2)=-0.318
tyre2#(2,3)=0.129
tyre2#(2,4)=-0.140
tyre2#(2,5)=-0.00302
tyre2#(2,6)=-0.00154
tyre2#(2,7)=-0.00775
tyre2#(2,8)=-0.00031
tyre2#(2,9)=-0.00343
tyre2#(2,10)=-0.10623
tyre2#(2,11)=-0.01953
tyre2#(2,12)=0.544

tyre2#(3,0)=-0.00822
tyre2#(3,1)=0.0118
tyre2#(3,2)=0.112
tyre2#(3,3)=-0.0225
tyre2#(3,4)=0.103
```

```
tyre2#(3,5)=0.00163
tyre2#(3,6)=0.00075
tyre2#(3,7)=0.00683
tyre2#(3,8)=0.00016
tyre2#(3,9)=0.00949
tyre2#(3,10)=-0.07125
tyre2#(3,11)=0.01944
tyre2#(3,12)=1.02
```

```
tyre2#(4,0)=-0.0331
tyre2#(4,1)=-0.0228
tyre2#(4,2)=-0.636
tyre2#(4,3)=-0.908
tyre2#(4,4)=0.121
tyre2#(4,5)=-0.0256
tyre2#(4,6)=-0.0127
tyre2#(4,7)=0.0072
tyre2#(4,8)=-0.0026
tyre2#(4,9)=0.00607
tyre2#(4,10)=-0.27684
tyre2#(4,11)=0.01987
tyre2#(4,12)=-2.05
```

```
tyre2#(5,0)=0.00586
tyre2#(5,1)=0.0013
tyre2#(5,2)=0.318
tyre2#(5,3)=-0.129
tyre2#(5,4)=0.140
tyre2#(5,5)=0.0033
tyre2#(5,6)=0.00157
tyre2#(5,7)=0.00774
tyre2#(5,8)=0.00034
tyre2#(5,9)=0.00342
tyre2#(5,10)=-0.0588
tyre2#(5,11)=0.01953
tyre2#(5,12)=1.04
```

```
CHL#(0)=BR#(0,n)
CHL#(1)=BR#(1,n)
CHL#(2)=BR#(2,n)
CHL#(3)=BR#(3,n)
CHL#(4)=BR#(4,n)
CHL#(5)=BR#(5,n)
CHL#(6)=BR#(6,n)
CHL#(7)=BR#(7,n)
CHL#(8)=-63666.9*cos(thetap#)
CHL#(9)=63666.9*sin(thetap#)
CHL#(10)=-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
CHL#(12)=SUSPR#(n)
```

```
for x=0 to 5
  TYREF#(x,n)=0
  for y=0 to 12
```

```
    TYREF#(x,n)=TYREF#(x,n)+tyre2#(x,y)*CHL#(y)
```

```
  next y
```

```
next x
```

```
end sub
```

SUB TYRE4

SHARED BR#( ), n, thetap#, CHL#( ), SUSPR#( ), SUSPL#( ), TYREF#( )

DIM tyre4#(5,13)

\*\*\*\*\*  
\*\*\*\*\*

' This routine contains the chassis tyre force matrix for the suspension

' operating condition c

\*\*\*\*\*  
\*\*\*\*\*

tyre4#(0,0)=-0.0117  
tyre4#(0,1)=-0.00822  
tyre4#(0,2)=-0.0225  
tyre4#(0,3)=0.112  
tyre4#(0,4)=-0.103  
tyre4#(0,5)=0.00163  
tyre4#(0,6)=0.000757  
tyre4#(0,7)=-0.00683  
tyre4#(0,8)=0.000172  
tyre4#(0,9)=-0.00949  
tyre4#(0,10)=-0.07125  
tyre4#(0,11)=-0.01944  
tyre4#(0,12)=1.02  
tyre4#(0,13)=0.890

tyre4#(1,0)=-0.0228  
tyre4#(1,1)=-0.0331  
tyre4#(1,2)=-0.908  
tyre4#(1,3)=-0.636  
tyre4#(1,4)=-0.121  
tyre4#(1,5)=-0.0256  
tyre4#(1,6)=-0.0127  
tyre4#(1,7)=-0.00718  
tyre4#(1,8)=-0.00263  
tyre4#(1,9)=-0.00605  
tyre4#(1,10)=-0.2768  
tyre4#(1,11)=-0.01987  
tyre4#(1,12)=-2.05  
tyre4#(1,13)=-2.02

tyre4#(2,0)=0.0013  
tyre4#(2,1)=0.00586  
tyre4#(2,2)=-0.129  
tyre4#(2,3)=0.318  
tyre4#(2,4)=-0.140  
tyre4#(2,5)=0.0033  
tyre4#(2,6)=0.00157  
tyre4#(2,7)=-0.00775  
tyre4#(2,8)=0.000337  
tyre4#(2,9)=-0.00343  
tyre4#(2,10)=-0.0588  
tyre4#(2,11)=-0.01953  
tyre4#(2,12)=1.04  
tyre4#(2,13)=1.24

tyre4#(3,0)=-0.0115  
tyre4#(3,1)=0.0085

tyre4#(3,2)=0.0224  
tyre4#(3,3)=-0.112  
tyre4#(3,4)=0.103  
tyre4#(3,5)=-0.00136  
tyre4#(3,6)=-0.000723  
tyre4#(3,7)=0.00683  
tyre4#(3,8)=-0.000141  
tyre4#(3,9)=0.00949  
tyre4#(3,10)=-0.09378  
tyre4#(3,11)=0.01944  
tyre4#(3,12)=0.562  
tyre4#(3,13)=0.690

tyre4#(4,0)=-0.0237  
tyre4#(4,1)=-0.0134  
tyre4#(4,2)=-0.379  
tyre4#(4,3)=-0.647  
tyre4#(4,4)=0.120  
tyre4#(4,5)=-0.0170  
tyre4#(4,6)=-0.000843  
tyre4#(4,7)=0.00713  
tyre4#(4,8)=-0.00175  
tyre4#(4,9)=0.00604  
tyre4#(4,10)=-0.21160  
tyre4#(4,11)=0.01977  
tyre4#(4,12)=-1.08  
tyre4#(4,13)=-1.10

tyre4#(5,0)=-0.00102  
tyre4#(5,1)=-0.00557  
tyre4#(5,2)=0.129  
tyre4#(5,3)=-0.318  
tyre4#(5,4)=0.140  
tyre4#(5,5)=-0.00302  
tyre4#(5,6)=-0.00154  
tyre4#(5,7)=0.00775  
tyre4#(5,8)=-0.000311  
tyre4#(5,9)=0.00343  
tyre4#(5,10)=-0.10622  
tyre4#(5,11)=0.01953  
tyre4#(5,12)=0.544  
tyre4#(5,13)=0.344

CHL#(0)=BR#(0,n)  
CHL#(1)=BR#(1,n)  
CHL#(2)=BR#(2,n)  
CHL#(3)=BR#(3,n)  
CHL#(4)=BR#(4,n)  
CHL#(5)=BR#(5,n)  
CHL#(6)=BR#(6,n)  
CHL#(7)=BR#(7,n)  
CHL#(8)=-63666.9\*cos(thetap#)  
CHL#(9)=63666.9\*sin(thetap#)  
CHL#(10)=-47088\*cos(thetap#)  
CHL#(11)=47088\*sin(thetap#)  
CHL#(12)=SUSPL#(n)  
CHL#(13)=SUSPR#(n)

```
for x=0 to 5
  TYREF#(x,n)=0
  for y=0 to 13

    TYREF#(x,n)=TYREF#(x,n)+tyrel#(x,y)*CHL#(y)

  next y
next x
'for x=0 to 5
'PRINT "TYREF1=";TYREF#(x,n)
'next x
end sub
```

Fig. C29 Converge.bas

```
SUB CONVERGE
s      h      a      r      e      d
rt#( ),n,SUSPL#( ),SUSPR#( ),CONVERGED%,rl#,thetap#,TYREF#( )
,focount
SHARED tyrel#( ),concount,const#,FLCHECK( ),FILENS,thetacr
CONVERGED%=0
'*****
'*****
' This routine determines if the procedure has converged
by comparing
' the current and previous displacements of the top of
the ram. A
' displacement of < 1 mm is deemed to be convergence.
'*****
'*****

if n > 1 then

if rt#(0,n)-rt#(0,n-1) < 1 and rt#(0,n)-rt#(0,n-1) > -1
then
if rt#(1,n)-rt#(1,n-1) < 1 and rt#(1,n)-rt#(1,n-1) > -1
then
if rt#(2,n)-rt#(2,n-1) < 1 and rt#(2,n)-rt#(2,n-1) > -1
then
' convergence has occurred, iteration is terminated
'print "stop ,convergence has occurred, iteration is
terminated"

if thetacr=0 or thetacr=1 or thetacr=2 or thetacr=3 or
thetacr=4 or thetacr=5 or thetacr=6 or thetacr=7 or
thetacr=8 or thetacr=9 or thetacr=10 then
OPEN "A", #1 ,FILENS

WRITE#1,TYREF#(2,n)
WRITE#1,FLCHECK(n)

close #1
end if
concount=concount+1
CONVERGED%=1

'CALL OUTP

end if
end if
end if
end if

END SUB
```

## **APPENDIX D**

### **PROGRAM USER GUIDE**

The program user guide is aimed at giving the information necessary to enable the user to run the program, use the results, change the nine payload centre of gravity coordinates and the four payload masses and input new flexibility, stiffness and influence matrices when a new finite element model is to be considered.

#### **D1 Program Execution**

The program can be executed in two different ways. Firstly, when the user is using the Turbo Basic package, the program can be executed by the run command in the menu. The program is compiled to memory only and then run, after which the program will have to be re-compiled if further runs are required. The advantage of compiling to memory is that it is quick to swap between running and modifying a program compared to an executable program, which is then run as a stand alone program without the aid of the Turbo Basic package. Although the program can be run immediately, the executable program cannot be modified in any way. This guide will only concern itself with the former method of program execution as this is the most useful.

The disc in Fig. D1 contains the Turbo Basic program subroutines and is found attached to the inside of the rear cover. To run the program, the user must have a copy of the Turbo Basic package. To invoke the Turbo Basic package, the user must



insert this disc into an IBM compatible PC and type TB when in drive A. This command will invoke the Turbo Basic menu which the user will use to edit the subroutines and to run the program. To run the program and interpret the results, the user must follow instructions (i) to (v) below, having invoked the Turbo Basic package.

- (i) Place the cursor over the FILE menu (using the arrow keys) and press return to reveal its submenu.
- (ii) Place the cursor over the LOAD option and press return twice to give the list of files on drive A:
- (iii) Place the cursor over the filename TIPPER.BAS and press return, which loads this file into the screen editor and returns the cursor to the main menu.
- (iv) Place the cursor of the RUN menu and press return, which will compile TIPPER.BAS to memory and then run it. The compile time is approximately 1 minute and the run time for 9 payload positions and four payload masses is approximately 24 hours.
- (v) The program's rules are written to C: drive (as writing the data to the hard disk instead of the floppy disc reduces the running time). The results for each payload position and mass are stored in separate files, with 36 files required for the current program. For each ram length considered the rear left tyre force is recorded for ground slopes of  $0^\circ$  to  $10^\circ$  in steps of  $1^\circ$ . The trailer type, payload mass and position and ram length are recorded with each set of tyre force results. The results can be used in conjunction with a graphs package to obtain the stability graphs shown in Figs. 5.1(a) to 5.4(f).

## **D2 Changing program variables**

The program variables which the user can change for a given trailer design, are the 9 payload centre of gravity co-ordinates, 4 payload masses and the 36 output filenames and their destination (ie. A:, B: or C: drive). These variables are all stored in the Turbo Basic program VARIABS.BAS.

The payload centre of gravity, X, Y and Z co-ordinates are stored in the variable matrices PCGX#(8), PCGY#(8) and PCGZ#(8), respectively. The 9 X-co-ordinates are located under line 12 and can have any value in the range 0 to 8976 mm. The 9 Y-co-ordinates are located under line 28 and can have any value in the range 0 to 2028 mm. The 9 Z-co-ordinates are located under line 45 and can have any value in the -1014 to 1014 mm.

The payload masses are stored in the variable matrix PM#(3). The 4 masses are located under line 60 and can have any value in the range 0 to 25,000 Kg.

The output filenames are stored in the variable matrix PM\$(3,8) and can have up to a seven letter prefix and a three letter suffix. The output filenames for the first, second, third and fourth payload masses are located under line 72, 90, 106 and 122, respectively.

To change any of the above variables, the user must repeat steps (i) to (iii) described in Section D1 loading VARIABS.BAS, instead of TIPPER.BAS. To modify this file

the cursor must select the EDIT menu, allowing modifications to be made. When the desired modifications have been made, the user must press the ESC key to return to the main menu, where the modified file is saved. To save the file, the user must place the cursor over the FILE menu and press return, which will reveal the save command.

### **D3 Changing flexibility, stiffness and influence matrices**

To determine chassis displacements, suspension displacements, suspension forces and trailer tyre forces 13 unit load flexibility, stiffness and influence matrices are required. The matrices are generated using the lorry finite element models for suspension conditions one to four, described in Section 3.4.3.

#### **D3.1 Flexibility matrices**

The unit load flexibility matrices allow the hinges, ram/chassis contact and airbag displacements to be determined, for any loading arrangement for suspension operating conditions one, two and four. The hinge and ram/chassis contact displacement flexibility matrix variable names and their subroutine locations are shown in Table D1. The airbag displacement flexibility matrix variable names and their subroutine locations are shown in Table D2.

### **D3.2 Influence matrices**

The unit load influence matrices allow the suspension and tyre forces to be determined for any loading arrangement for suspension operating conditions one, two and four. The suspension for influence matrix variable names and their subroutine locations are shown in Table D3. The tyre force influence matrix variable names and their subroutine locations are shown in Table D4.

### **D3.3 Stiffness matrix**

The stiffness matrix allows hinge ram/chassis contact and airbag displacements and the suspension and tyre forces to be determined for any loading arrangement, for suspension operating condition d. The stiffness matrix variable name is PRESCRIBE#(29, 23) and its subroutine name is CASE3.BAS.

**Table D1**

CHASSIS DISPLACEMENT FLEXIBILITY MATRICES LOCATION				
SUSPENSION CONDITION	ONE	TWO	THREE	FOUR
SUBROUTINE NAME	FLEX1.BAS	FLEX2.BAS	CASE3.BAS	FLEX4.BAS
VARIABLE NAME	FLEXI# (8, 13)	FLEX2# (8, 13)	PRESCRIBE# (29, 33) C	FLEX4# (8, 13)

**Table D2**

AIRBAG DISPLACEMENT FLEXIBILITY MATRICES LOCATION				
SUSPENSION CONDITION	ONE	TWO	THREE	FOUR
SUBROUTINE NAME	FLEX1.BAS	FLEX2.BAS	CSE3.BAS	CASE4.BAS
VARIABLE NAME	FLEX1A# (5, 13)	FLEX2A# (5, 13)	PRESCRIBE# (29, 33)	FLEX4A# (5, 13)

**Table D3**

SUSPENSION FORGE INFLUENCE MATRICES LOCATION				
SUSPENSION CONDITION	ONE	TWO	THREE	FOUR
SUBROUTINE NAME	CASE1.BAS	CASE2.BAS	CASE3.BAS	CA54.BAS
VARIABLE NAME	SUSPM# (1, 13)	SUSPM2# (1, 13)	PRESCRIBE# (29, 33)	SUSPM4# (1, 13)

**Table D4**

TYRE FORCE INFLUENCE MATRICES LOCATION				
SUSPENSION CONDITION	ONE	TWO	THREE	FOUR
SUBROUTINE NAME	TYRE1.BAS	TYRE2.BAS	CASE3.BAS	TYRE4.BAS
VARIABLE NAME	TYRE1# (5, 13)	TYRE2# (5, 13)	PRESCRIBE# (29, 33)	TYRE4# (5, 13)

# **APPENDIX E**

## **FINITE ELEMENT ANALYSES AND FLEXIBILITY, INFLUENCE AND STIFFNESS MATRICES**

### **E1 Introduction**

To determine chassis displacements, suspension forces and tyre forces, a total of thirteen flexibility, influence and stiffness matrices are required for a given articulated tipper design. The flexibility and influence matrices are used when the suspension is operating under conditions a b or c. The stiffness matrix is used when the suspension is only operating under condition d. This appendix gives details, shown in Tables 4.3 to 4.15, of these matrices for the original chassis, and describes how the different types of matrix are obtained. Also, a typical PAFEC data file is shown in Fig. E1.

### **E2 Flexibility matrices**

The flexibility matrices, shown in Tables 4.3 to 4.8, enable the displacement of the body/chassis contact points and the airbags to be determined for any loading condition. For each matrix the top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. For Tables 4.3 to 4.5 the left column gives details of the chassis displacement to which the row coefficients correspond. For Tables 4.6 to 4.8 the left column gives details of the airbag displacement to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit force is applied in the direction of the force which corresponds to that column. The LOADS module in the PAFEC data program, shown in Fig. E1, is used to apply the unit force to the finite element mesh. The displacement at the nodes corresponding to the body/chassis contact points and the end of the airbags, resulting from the unit force form the column matrix coefficients.

## **E2 Influence matrices**

The influence matrices, shown in Tables 4.9 to 4.14, enable the suspension and tyre forces to be determined for any loading condition. For each matrix the top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. For Tables 4.9 to 4.11 the left column gives details of the suspension forces to which the row coefficients correspond. For Tables 4.12 to 4.14 the left column gives details of tyre forces to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit force is applied in the direction of the force which corresponds to that column. The LOADS module in the PAFEC data program, shown in Fig. E1, is used to apply the unit force to the finite element mesh. The forces in the tie bars resulting from the unit force form the column matrix coefficients for the suspension force influence matrices shown in Tables 4.9 to 4.11. The forces in the springs representing the trailer tyre resulting from the unit force form the column matrix coefficients for the tyre force influence matrices shown in Tables 4.12 to 4.14.



### **E3     Stiffness matrix**

The stiffness matrix, shown in Table 4.15, enables the chassis displacements, suspension forces and tyre forces, to be determined for any loading condition. The matrix top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. The left column gives details of the chassis displacements (N186-X to N314-X), chassis and tractor sprung and unsprung weight (N315-Y to N313Z), suspension forces (N195-Y to N223-Y) and tyre forces (N231-Y to N236-Y) to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit displacement is applied in the direction of the force which corresponds to that column. The PRESCRIBE.DISPLACEMENTS module in the PAFEC data program, shown in Table E1, is used to apply the unit displacement to the finite element meshes. The reaction forces resulting from the unit displacement form the column matrix coefficients for the stiffness matrix shown in Table 4.15.

STJNLD(TWOTIE.SUSPLJ)

RUNPAFEC(RUN=SUSPL)

----

TITLE CASE1 TWO TIERODS

CONTROL

OPLIB.TWOTIE

FULL.SAVE.OUTPUT

REACTIONS

CONTROL.END

NODES

AXIS=1

NODE.NUMB	X	Y	Z
-----------	---	---	---

C 1-79 : PLATE NODES

1	9535.5	-22.0	-700.0
2	9535.5	-22.0	-537.5
3	9535.5	-22.0	-375.0
4	9535.5	-22.0	-253.3
5	9535.5	-22.0	-131.5
6	9535.5	-22.0	0.0
7	9535.5	-22.0	131.5
8	9535.5	-22.0	253.3
9	9535.5	-22.0	375.0
10	9535.5	-22.0	537.5
11	9535.5	-22.0	700.0
12	9406.0	-22.0	-700.0
13	9406.0	-22.0	-375.0
14	9406.0	-22.0	-131.5
15	9406.0	-22.0	131.5
16	9406.0	-22.0	375.0
17	9406.0	-22.0	700.0
18	9276.5	-22.0	-700.0
19	9276.5	-22.0	-537.5
20	9276.5	-22.0	-375.0
21	9276.5	-22.0	-253.0
22	9276.5	-22.0	-131.5
23	9276.5	-22.0	0.0
24	9276.5	-22.0	131.5
25	9276.5	-22.0	253.0
26	9276.5	-22.0	375.0
27	9276.5	-22.0	537.5
28	9276.5	-22.0	700.0
29	9183.1	-22.0	-700.0
30	9183.1	-22.0	-375.0
31	9183.1	-22.0	-131.5
32	9183.1	-22.0	131.5
33	9183.1	-22.0	375.0
34	9183.1	-22.0	700.0
35	9089.7	-22.0	-700.0
36	9089.7	-22.0	-537.5
37	9089.7	-22.0	-375.0
38	9089.7	-22.0	-253.3
39	9089.7	-22.0	-131.5

C NODE 40 IS KING PIN

40	9089.7	-22.0	0.0
41	9089.7	-22.0	131.5
42	9089.7	-22.0	253.3
43	9089.7	-22.0	375.0
44	9089.7	-22.0	537.5

45	9089.7	-22.0	700.0
46	8977.2	-22.0	-700.0
47	8977.2	-22.0	-375.0
C NODES 48 & 49 LOCATE RAM PIN			
48	8977.2	-22.0	-131.5
49	8977.2	-22.0	131.5
50	8977.2	-22.0	375.0
51	8977.2	-22.0	700.0
52	8864.7	-22.0	-700.0
53	8864.7	-22.0	-537.5
54	8864.7	-22.0	-375.0
55	8864.7	-22.0	-253.3
56	8864.7	-22.0	-131.5
57	8864.7	-22.0	0.0
58	8864.7	-22.0	131.5
59	8864.7	-22.0	253.3
60	8864.7	-22.0	375.0
61	8864.7	-22.0	537.5
62	8864.7	-22.0	700.0
63	8741.3	-22.0	-700.0
64	8741.3	-22.0	-375.0
65	8741.3	-22.0	-131.5
66	8741.3	-22.0	131.5
67	8741.3	-22.0	375.0
68	8741.3	-22.0	700.0
69	8618.0	-22.0	-700.0
70	8618.0	-22.0	-537.5
71	8618.0	-22.0	-375.0
72	8618.0	-22.0	-253.0
73	8618.0	-22.0	-131.5
74	8618.0	-22.0	0.0
75	8618.0	-22.0	131.5
76	8618.0	-22.0	253.3
77	8618.0	-22.0	375.0
78	8618.0	-22.0	537.5
79	8618.0	-22.0	700.0
C FIFTH WHEEL BEAM NODES : 80 167			
80	9535.5	67.5	-700
81	9535.5	67.5	-537.5
82	9535.5	67.5	-375.0
83	9535.5	67.5	-253.3
84	9535.5	67.5	-131.5
85	9535.5	67.5	0.0
86	9535.5	67.5	131.5
87	9535.5	67.5	253.3
88	9535.5	67.5	375.0
89	9535.5	67.5	537.5
90	9535.5	67.5	700.0
91	9276.5	57.0	-700.0
92	9276.5	57.0	-537.5
93	9276.5	57.0	-375.0
94	9276.5	57.0	-253.3
95	9276.5	57.0	-131.5
96	9276.5	57.0	0.0
97	9276.5	57.0	131.5
98	9276.5	57.0	253.3
99	9276.5	57.0	375.0
100	9276.5	57.0	537.5

101	9276.5	57.0	700.0
102	8864.7	15.90	-131.5
103	8864.7	15.90	0.00
104	8864.7	15.90	131.5
105	8618.0	83.0	-700.0
106	8618.0	83.0	-537.5
107	8618.0	83.0	-375.0
108	8618.0	83.0	-253.3
109	8618.0	83.0	-131.5
110	8618.0	83.0	0.0
111	8618.0	83.0	131.5
112	8618.0	83.0	253.3
113	8618.0	83.0	375.0
114	8618.0	83.0	537.5
115	8618.0	83.0	700.0
116	9535.5	57.0	-700.0
117	9406.0	57.0	-700.0
118	9276.5	57.0	-700.0
119	9183.1	57.0	-700.0
120	9089.7	57.0	-700.0
121	8977.2	57.0	-700.0
122	8864.7	57.0	-700.0
123	8741.3	57.0	-700.0
124	8618.0	57.0	-700.0
125	9535.5	15.9	-375
126	9406.0	15.5	-375
127	9276.5	15.9	-375
128	9276.5	45.5	-375
129	9183.1	45.5	-375
130	9089.7	45.5	-375
131	8977.2	45.5	-375
132	8864.7	45.5	-375
133	8741.3	45.5	-375
134	8618.0	45.5	-375
135	9276.5	54.5	-131.5
136	9183.1	54.5	-131.5
137	9089.7	54.5	-131.5
138	8977.2	54.5	-131.5
139	8864.7	54.5	-131.5
140	8741.3	54.5	-131.5
141	8618.0	54.5	-131.5
142	9276.5	54.5	131.5
143	9183.1	54.5	131.5
144	9089.7	54.5	131.5
145	8977.2	54.5	131.5
146	8864.7	54.5	131.5
147	8741.3	54.5	131.5
148	8618.0	54.5	131.5
149	9535.5	15.9	375.0
150	9406.0	15.9	375.0
151	9276.5	15.9	375.0
152	9276.5	45.5	375.0
153	9183.1	45.5	375.0
154	9089.7	45.5	375.0
155	8977.2	45.5	375.0
156	8864.7	45.5	375.0
157	8741.3	45.5	375.0
158	8618.0	45.5	375.0

159	9535.5	57.0	700.0
160	9406.0	57.0	700.0
161	9276.5	57.0	700.0
162	9183.1	57.0	700.0
163	9089.7	57.0	700.0
164	8977.2	57.0	700.0
165	8864.7	57.0	700.0
166	8741.3	57.0	700.0
167	8618.0	57.0	700.0
168	8530.0	57.0	-700.0
169	7210.0	15.5	-700.0
170	6905.0	-53.0	-700.0
171	6680.0	-140.0	-700.0
172	6680.0	2.0	-700.0
173	5332.5	-170.5	-700.0
174	3985.0	-201.0	-700.0
175	3985.0	-292.0	-700.0
176	3093.0	-191.0	-700.0
177	3093.0	-297.0	-700.0
178	2670.0	-186.5	-700.0
179	2670.0	-263.0	-700.0
180	1778.0	-176.5	-700.0
181	1778.0	-268.0	-700.0
182	1355.0	-172.0	-700.0
183	1355.0	-234.0	-700.0
184	463.0	-162.0	-700.0
185	463.0	-240.0	-700.0
C NODE 186 & 205			
186	0.0	0.0	-700.0
187	8530.0	57.0	700.0
188	7210.0	15.5	700.0
189	6905.0	-53.0	700.0
190	6680.0	-140.0	700.0
191	6680.0	2.0	700.0
192	5332.5	-170.5	700.0
193	3985.0	-201.0	700.0
194	3985.0	-292.0	700.0
195	3093.0	-191.0	700.0
196	3093.0	-297.0	700.0
197	2670.0	-186.5	700.0
198	2670.0	-263.0	700.0
199	1778.0	-176.5	700.0
200	1778.0	-268.0	700.0
201	1355.0	-172.0	700.0
202	1355.0	-234.0	700.0
203	463.0	-162.5	700.0
204	463.0	-240.0	700.0
205	0.00	0.0	700.0
206	4032.75	-652.0	-700.0
207	3438.0	-652.0	-700.0
208	3093.0	-652.0	-700.0
209	2717.75	-623.0	-700.0
210	2123.0	-623.0	-700.0
211	1778.0	-623.0	-700.0
212	1402.75	-594.0	-700.0
213	808.0	-594.0	-700.0
214	463.0	-594.0	-700.0
215	4032.75	-652.0	700.0

216	3438.0	-652.0	700.0
217	3093.0	-652.0	700.0
218	2717.75	-623.0	700.0
219	2123.0	-623.0	700.0
220	1778.0	-623.0	700.0
221	1402.75	-594.0	700.0
222	808.0	-594.0	700.0
223	463.0	-594.0	700.0

C OX FREEDOM FOR SUSPENSION NODES

225	4032.75	-653.0	-700.0
226	2717.75	-624.0	-700.0
227	1402.75	-595.0	-700.0
228	4032.75	-653.0	700.0
229	2717.75	-624.0	700.0
230	1402.75	-595.05	700.0
231	3438.0	-1252.0	-925.0
232	2123.0	-1223.0	-925.0
233	808.0	-1194.0	-925.0
234	3438.0	-1252.0	925.0
235	2123.0	-1223.0	925.0
236	808.0	-1194.0	925.0

C 237            9089.7    -1272.0    0.0

238	3438.0	-652.0	-925.0
239	2123.0	-623.0	-925.0
240	808.0	-594.0	-925.0
241	3438.0	-652.0	925.0
242	2123.0	-623.0	925.0
243	808.0	-594.0	925.0
244	808.0	-530.5	-925.0
245	808.0	-530.5	-700.0
246	808.0	-530.5	700.0
247	808.0	-530.5	925.0
248	2123.0	-559.5	-925.0
249	2123.0	-559.5	-700.0
250	2123.0	-559.5	700.0
251	2123.0	-559.5	925.0
252	3438.0	-588.5	-925.0
253	3438.0	-588.5	-700
254	3438.0	-588.5	700
255	3438.0	-588.5	925.0

C TRACTOR NODES

256	7389.7	-376.7	381.5
257	7889.7	-381.3	381.5
258	8289.7	-385	381.5
259	8989.7	-391.4	381.5
260	9089.7	-392.4	381.5
261	9189.7	-393.3	381.5
262	10280.7	-403.4	381.5
263	10959.7	-409.6	381.5
264	11389.7	-413.6	381.5
265	12089.7	-420	431.5
266	12789.7	-426.4	481.5
267	13289.7	-431.1	481.5
268	7390.3	-376.7	-381.5
269	7889.7	-381.3	-381.5
270	8289.7	-385	-381.5
271	8989.7	-391.4	-381.5
272	9089.7	-392.4	-381.5

273	9189.7	-393.3	-381.5
274	10280.7	-403.4	-381.5
275	10959.7	-409.6	-381.5
276	11389.7	-413.6	-381.5
277	12089.1	-420	-431.5
278	12789.7	-426.4	-481.5
279	13289.7	-431.1	-481.5
280	8289.7	-703.5	1086
281	8289.7	-703.5	753.5
282	12089.7	-703.5	981
283	8289.7	-703.5	-753.5
284	8289.7	-703.5	-1086
285	12089.7	-703.5	-981
286	8289.7	-1240.5	1086
287	8289.7	-1240.5	753.5
288	12089.7	-1240.5	981
289	8289.7	-1240.5	-753.5
290	8289.7	-1240.5	-1086
291	12089.7	-1240.5	-981
292	9089.7	9.5	381.5
293	9089.7	9.5	0
294	9089.7	9.5	-381.5
295	9089.7	-392.4	0
296	10959.7	-340.5	0
C 297	0 0 0		
C 298	0 0 0		
299	8289.7	-703.5	381.5
300	8289.7	-703.5	-381.5
301	12089.7	-703.5	431.5
302	12089.7	-703.5	-431.5

C ROLL BARS

303	12789.7	-425.4	481.5
304	12789.7	-425.4	-481.5
305	12091.6	-703.5	431.5
306	12091.6	-703.5	-431.5
307	7889.7	-383.0	381.5
308	7889.7	-383.0	-381.5
309	8289.7	-705.0	381.5
310	8289.7	-705.0	-381.5

C MID-SUSPENSION CHASSIS NODES

311	2123	-182.6	-700
312	2123	-182.6	700
313	10959.7	-340.5	0
314	8977.2	-22.0	0

C SPRUNG CHASSIS C.OF.G

315	3985,-292,0		
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C FRONT AXLE MID POINT

316	3438	-588.5	0
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C MIDDLE AXLE MID POINT

317	2123	-559.5	0
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C REAR AXLE MID POINT

318	808	-530.5	0
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ELEMENTS

NUMB	GROUP	NUMB	ELEM. TYPE	PROPS	TOPOLOGY
1	1	44210	28	18 20 1 3	19 12 13 2
2	1	44210	28	20 22 3 5	21 13 14 4
3	1	44210	28	22 24 5 7	23 14 15 6
4	1	44210	28	24 26 7 9	25 15 16 8

5	1	44210	28	26	28	9	11	27	16	17	10
6	1	44210	28	35	37	18	20	36	29	30	19
7	1	44210	28	37	39	20	22	38	30	31	21
8	1	44210	28	39	41	22	24	40	31	32	23
9	1	44210	28	41	43	24	26	42	32	33	25
10	1	44210	28	43	45	26	28	44	33	34	27
11	1	44210	28	52	54	35	37	53	46	47	36
12	1	44210	28	54	56	37	39	55	47	48	38
13	1	44210	28	56	58	39	41	57	48	49	40
14	1	44210	28	58	60	41	43	59	49	50	42
15	1	44210	28	60	62	43	45	61	50	51	44
16	1	44210	28	69	71	52	54	70	63	64	53
17	1	44210	28	71	73	54	56	72	64	65	55
18	1	44210	28	73	75	56	58	74	65	66	57
19	1	44210	28	75	77	58	60	76	66	67	59
20	1	44210	28	77	79	60	62	78	67	68	61
21	1	34200	7	1	2	80	81				
22	1	34200	7	2	3	81	82				
23	1	34200	7	3	4	82	83				
24	1	34200	7	4	5	83	84				
25	1	34200	7	5	6	84	85				
26	1	34200	7	6	7	85	86				
27	1	34200	7	7	8	86	87				
28	1	34200	7	8	9	87	88				
29	1	34200	7	9	10	88	89				
30	1	34200	7	10	11	89	90				
31	1	34200	2	18	19	91	92				
32	1	34200	2	19	20	92	93				
33	1	34200	2	20	21	93	94				
34	1	34200	2	21	22	94	95				
35	1	34200	2	22	23	95	96				
36	1	34200	2	23	24	96	97				
37	1	34200	2	24	25	97	98				
38	1	34200	2	25	26	98	99				
39	1	34200	2	26	27	99	100				
40	1	34200	2	27	28	100	101				
41	1	34200	3	56	57	102	103				
42	1	34200	3	57	58	103	104				
43	1	34200	6	69	70	105	106				
44	1	34200	6	70	71	106	107				
45	1	34200	6	71	72	107	108				
46	1	34200	6	72	73	108	109				
47	1	34200	6	73	74	109	110				
48	1	34200	6	74	75	110	111				
49	1	34200	6	75	76	111	112				
50	1	34200	6	76	77	112	113				
51	1	34200	6	77	78	113	114				
52	1	34200	6	78	79	114	115				
53	1	34200	1	1	12	116	117				
54	1	34200	1	12	18	117	118				
55	1	34200	1	18	29	118	119				
56	1	34200	1	29	35	119	120				
57	1	34200	1	35	46	120	121				
58	1	34200	1	46	52	121	122				
59	1	34200	1	52	63	122	123				
60	1	34200	1	63	69	123	124				
61	1	34200	3	3	13	125	126				
62	1	34200	3	13	20	126	127				



63	1	34200	4	20	30	128	129
64	1	34200	4	30	37	129	130
65	1	34200	4	37	47	130	131
66	1	34200	4	47	54	131	132
67	1	34200	4	54	64	132	133
68	1	34200	4	64	71	133	134
69	1	34200	5	22	31	135	136
70	1	34200	5	31	39	136	137
71	1	34200	5	39	48	137	138
72	1	34200	5	48	56	138	139
73	1	34200	5	56	65	139	140
74	1	34200	5	65	73	140	141
75	1	34200	5	24	32	142	143
76	1	34200	5	32	41	143	144
77	1	34200	5	41	49	144	145
78	1	34200	5	49	58	145	146
79	1	34200	5	58	66	146	147
80	1	34200	5	66	75	147	148
81	1	34200	3	9	16	149	150
82	1	34200	3	16	26	150	151
83	1	34200	4	26	33	152	153
84	1	34200	4	33	43	153	154
85	1	34200	4	43	50	154	155
86	1	34200	4	50	60	155	156
87	1	34200	4	60	67	156	157
88	1	34200	4	67	77	157	158
89	1	34200	1	11	17	159	160
90	1	34200	1	17	28	160	161
91	1	34200	1	28	34	161	162
92	1	34200	1	34	45	162	163
93	1	34200	1	45	51	163	164
94	1	34200	1	51	62	164	165
95	1	34200	1	62	68	165	166
96	1	34200	1	68	79	166	167
97	1	34200	1	69	168	124	168
98	1	34200	11	168	169		
99	1	34200	12	169	170		
100	1	34200	13	170	171		
101	1	34200	14	171	173		
102	1	34200	15	173	174		
103	1	34200	16	174	176		
104	1	34200	17	176	178		
105	1	34200	18	178	311		
106	1	34200	19	180	182		
107	1	34200	20	182	184		
108	1	34200	21	184	186		
109	1	34200	1	79	187	167	187
110	1	34200	11	187	188		
111	1	34200	12	188	189		
112	1	34200	13	189	190		
113	1	34200	14	190	192		
114	1	34200	15	192	193		
115	1	34200	16	193	195		
116	1	34200	17	195	197		
117	1	34200	18	197	312		
118	1	34200	19	199	201		
119	1	34200	20	201	203		
120	1	34200	21	203	205		

121	1	34200	10	171	190	172	191
C 122	1	34200	8	174	193	175	194
123	1	34200	9	176	195	177	196
124	1	34200	8	178	197	179	198
125	1	34200	9	180	199	181	200
126	1	34200	8	182	201	183	202
127	1	34200	9	184	203	185	204
C SUSPENSION ELEMENTS LEFT				131,136,142			
128	1	34200	22	174	206		
129	2	34200	24	225	207		
130	2	34200	31	207	208		
C 131	2	30100	25	176	208		
132	2	30100	26	231	238		
133	1	34200	22	178	209		
134	2	34200	24	226	210		
135	2	34200	31	210	211		
C 136	2	30100	25	180	211		
137	2	30100	26	232	239		
138	1	34200	22	182	212		
139	2	34200	24	227	213		
140	2	34200	31	213	214		
C 141	2	30100	25	184	214		
142	2	30100	26	233	240		
C SUSPENSION ELEMENT RIGHT				146,151,156			
143	1	34200	22	193	215		
144	2	34200	24	228	216		
145	2	34200	31	216	217		
C 146	2	30100	25	195	217		
147	2	30100	26	234	241		
148	1	34200	22	197	218		
149	2	34200	24	229	219		
150	2	34200	31	219	220		
C 151	2	30100	25	199	220		
152	2	30100	26	235	242		
153	1	34200	22	201	221		
154	2	34200	24	230	222		
155	2	34200	31	222	223		
C 156	2	30100	25	203	223		
157	2	30100	26	236	243		
C AXLE TUBES							
C 159	2	34200	23	207	216	253	254
C 160	2	34200	23	210	219	249	250
C 161	2	34200	23	213	222	245	246
162	2	30100	29	231	0		
163	2	30100	29	232	0		
164	2	30100	29	233	0		
165	2	30100	29	234	0		
166	2	30100	29	235	0		
167	2	30100	29	236	0		
C ADDITIONAL AXLE TUBES							
169	2	34200	23	207	238	253	252
170	2	34200	23	210	239	249	248
171	2	34200	23	213	240	245	244
172	2	34200	23	216	241	254	255
173	2	34200	23	219	242	250	251
174	2	34200	23	222	243	246	247
C TRACTOR UNIT CHASSIS							
175	3	34200	36	256	257		

176	3	34200	36	257	258
177	3	34200	36	258	259
178	3	34200	36	259	260
179	3	34200	36	260	261
180	3	34200	36	261	262
181	3	34200	36	262	263
182	3	34200	36	263	264
183	3	34200	36	264	265
184	3	34200	36	265	266
185	3	34200	36	266	267
186	3	34200	36	267	279
187	3	34200	36	256	268
188	3	34200	36	268	269
189	3	34200	36	269	270
190	3	34200	36	270	271
191	3	34200	36	271	272
192	3	34200	36	272	273
193	3	34200	36	273	274
194	3	34200	36	274	275
195	3	34200	36	275	276
196	3	34200	36	276	277
197	3	34200	36	277	278
198	3	34200	36	278	279
199	3	34200	36	261	273
200	3	34200	36	264	276

C TRACTOR SPRINGS

C FRONT TYRES

201	3	30100	35	282	288
202	3	30100	34	288	0
203	3	30100	35	285	291
204	3	30100	34	291	0

C REAR TYRES

205	3	30100	35	280	286
206	3	30100	34	286	0
207	3	30100	35	281	287
208	3	30100	34	287	0
209	3	30100	35	283	289
210	3	30100	34	289	0
211	3	30100	35	284	290
212	3	30100	34	290	0

C FRONT SUSPENSIONS

213	3	30100	33	265	301
214	3	30100	33	277	302

C REAR SUSPENSIONS

215	3	30100	32	258	299
216	3	30100	32	270	300

C FITH WHEEL BEAMS

217	3	34200	36	292	293
218	3	34200	36	293	294

C TRACTOR AXLES

C REAR

219	3	34200	37	280	281
220	3	34200	37	281	299
221	3	34200	37	299	300
222	3	34200	37	300	283
223	3	34200	37	283	284

C FRONT

224	3	34200	37	282	301
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225 3 34200 37 301 302  
226 3 34200 37 302 285

C ROLL BARS

227 3 34200 39 303 305  
228 3 34200 29 305 306  
229 3 34200 39 306 304  
230 3 34200 38 307 309  
231 3 34200 38 309 310  
232 3 34200 38 310 308

C ADDITIONAL MAINRAIL BEAMS

233 1 34200 18 311 180  
234 1 34200 18 312 199

C MID SUSPENSION SUPPORTS

235 1 34400 40 311 210  
236 1 34400 40 312 219

C ADDITIONAL CHASSIS MASSES

237 2 30200 41 238  
238 2 30200 41 239  
239 2 30200 41 240  
240 2 30200 41 241  
241 2 30200 41 242  
242 2 30200 41 243

C TRACTOR C.OF.G BEAMS

243 3 34200 18 263 313  
244 3 34200 18 313 275

C HINGE BAR

245 1 34200 42 186 205

C RAM CENTRE 314

246 1 34200 17 48 314  
247 1 34200 17 314 49

C FRONT CROSS MEM DIVIDED TO ALLOW SPRUNG CHASSIS LOADING AT N315.

248 1 34200 8 174 315 175 315  
249 1 34200 8 193 315 194 315

C AXLES DIVIDED TO ALLOW UNSPRUNG CHASIS LOADS AT N316 N317 N318

250 2 34200 23 207 316 253 316  
251 2 34200 23 216 316 254 316  
252 2 34200 23 210 317 249 317  
253 2 34200 23 219 317 250 317  
254 2 34200 23 213 318 245 318  
255 2 34200 23 222 318 246 318  
256 1 30200 43 315

REPEATED FREEDOMS

N1 N2 DIRECTION

206 225 12345  
209 226 12345  
212 227 12345  
215 228 12345  
218 229 12345  
221 230 12345  
231 238 13  
232 239 13  
233 240 13  
234 241 13  
235 242 13  
236 243 13

C TRACTOR TYRES

282 288 13

285 291 13  
 280 286 13  
 281 287 13  
 283 289 13  
 284 290 13

C TRACTOR SUSPENSION

265 301 13  
 277 302 13  
 299 258 13  
 270 300 13

C FIFTH WHEEL

260 292 123456  
 272 294 123456

C FIFTH WHEEL/PLATE

292 43 2  
 293 40 13  
 294 37 2

C ROLL BAR NODES

305 301 123  
 306 302 123  
 307 257 123  
 308 269 123  
 309 299 123  
 310 300 123  
 303 266 123  
 304 278 123

RESTRAINTS

NODE.NUMB            DIRECTION

C CHASSIS TYRES

231	2456
232	2456
233	2456
234	2456
235	2456
236	2456

C TRACTOR TYRES

288 2456  
 291 2456  
 286 2456  
 287 2456  
 289 2456  
 290 2456

BEAMS

SECTION.NUMB	MATE.NUMB	IYY	IZZ	AXIS.NUMB
TORSIONAL.CONSTANT	AREA			
1 1	3.663E06	13.677E06	1	92.5E03
3250				
2 1	1.464E06	9.193E06	1	93.333E03
2800				
3 1	2.46E06	0.393E06	1	60.0E03
1800				
4 1	0.571E06	5.343E06	1	77.666E03
6570				
5 1	0.189E06	6.351E06	1	755.208E03
3625				
6 1	0.813E06	2.686E06	1	10.208E03
1225				
7 1	0.388E06	7.801E06	1	43.520E03

2040					
8	1	15.224E06	5.124E06	1	11.834E06
2900					
9	1	0.362E06	1.298E06	1	8.125E06
1000					
10	1	2.569E06	2.569E06	1	5.138E06
1717					
11	1	3.664E06	25.025E06	1	94229
3458					
12	1	3.665E06	69.249E06	1	98812
4008					
13	1	3.665E06	105.286E06	1	101875
4375					
14	1	3.664E06	125.099E06	1	95211
4162					
15	1	3.664E06	170.164E06	1	96512
4406					
16	1	3.664E06	186.195E06	1	96950
4488					
17	1	3.664E06	174.025E06	1	96640
4430					
18	1	3.664E06	162.496E06	1	96438
4372					
19	1	3.664E06	151.253E06	1	96128
4314					
20	1	3.664E06	140.986E06	1	95723
4258					
C 21 -HINGE BAR MAIN RAILS					
21	1	3.664E06	140.986E06	1	95723
4258					
22	1	8.919E06	17.756E06	1	24063
2875					
23	1	8.055E07	8.055E07	1	16.109E07
49700					
C 24 MOST STIFF HALF OF TRAILING ARM					
24	1	2.084E06	0.7771E06	1	1.926E06
3908					
C 31 LEAST STIFF HALF OF TRAILING ARM					
31	1	1.042E06	0.3886E06	1	0.963E06
1954					
C 36 TRACTOR FRAME					
36	2	1.585E06	27.551E06	1	30744
2598					
C 37 TRACTOR AXLES					
37	2	100E08	100E08	1	100000
10000					
C 38 REAR ROLL BAR					
38	2	133901.3	133901.3	1	267802.5
1297.2					
C 39 FRONT ROLL BAR					
39	2	397358.0	397358.0	1	794716.0
2234.6					
C MID SUSPENSION SUPPORT					
40	2	0	0	0	0
900					
42	1	1.11313E06	1.11313E06	1	2.22626E06
1995					
MATERIAL					

MATERIAL.NUMB	NU	E	RO
1	0.3	210E03	7.8E-06
2	0.3	210E03	0.0

PLATES.AND.SHELLS

PLATE.OR.SHELL.NUMBER	MATERIAL.NUMBER	THICKNESS
28	1	8

SPRINGS

NUMBER.OF.SPRING	AXIS.NUMB	KX	KY	KZ	TXX
25	1	0	197.0	0.00	0
26	1	0	1070.0	0.00	0
C 27	1	0	1060.0	0.00	2.63477E08
29	1	749.0	0.00	412.0	0
C 30	1	4360.0	0.00	4360.0	0
C	REAR TRACTOR SUSPENSION				
32	1	0	872		
C	FRONT TRACTOR SUSPENSION				
33	1	0	403		
C	TRACTOR TYRES KX & KZ				
34	1	755	0	350	
C	TRACTOR TYRES KY				
35	1	0	981		

MASSES

NUMB.OF.MASS	MASS
41	17.666
43	1220.0

LOADS

NODE.NUMB DIRECTION.OF.LOAD VALUE.OF.LOAD

C RAM FORCES

C LEFT SUSPENSION ?

176	2	1
208	2	-1
180	2	1
211	2	-1
184	2	1
214	2	-1

C 186	1	18051
C 186	2	-12495
C 186	3	1
C 205	1	-28805
C 205	2	-43486

C BODY REACTIONS

C 314	1	10754
C 314	2	-41745
C 314	3	0

C TRACTOR WEIGHT

C 313	2	-63424
C 313	3	5549

STRESS.ELEMENTS

START	FINISH
1	260

C IN.DRAW

C DRAW.NUMB	INFO.NUMB	ORIENTATION	SCALE
C 1	2	0	0.05

C OUT.DRAW

C DRAW.NUMB	PLOT.TYPE	ORIENTATION	SCALE
C 1	1	0	0.05

END.OF.DATA

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