THREE ESSAYS ON INFORMATION ECONOMICS

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This dissertation is a collection of three essays on information economics.

The first essay, "Budget Constraint and Information Transmission in Multidimensional Decision Making," illustrates how a constraint on a receiver's actions impedes information transmission from multiple senders. The constraint causes an endogenous conflict of interest between the senders and the receiver, preventing truthful revelation. Nevertheless, information can be at least partially transmitted in terms of grids in perfect Bayesian equilibrium.

The second essay, "Delegation and Retention of Authority in Organizations under Constrained Decision Making," analyzes the relationship between constraints on decision making and optimal decision making structure–centralized or decentralized decision making. Constraints on feasible decisions induce a conflict of interest between a principal and agents even if they have common preferences. The centralized decision is beneficial to the principal when the constraints weakly restrictive. However, delegation can more than compensate the principal for loss of control by exploiting the agents' information when different prior beliefs disrupt information revelation or the agents have difference preferences.

The third essay, "Tenure Reform and Quality Gap between Schools," discusses tenure reform for primary and secondary education in the United State from a game-theoretical point of view. To analyze the effect of the reform, a continuing contract (tenure) is compared with a non-continuing contract (non-tenure) based on performance evaluation. The welfare is improved after the reform, but the gap between a good and bad school becomes wider. This increased gap is caused by a unilateral transfer of qualified teachers from the good school to bad school.

Keywords: strategic information transmission, cheap talk, multi-dimensions, budget constraint, tenure reform.

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1.0 INTRODUCTION

This dissertation consists of three essays on information economics.

Chapter 2 illustrates how constraints on a receiver's actions impede information transmission from multiple senders. The uninformed receiver wants to match an action to a multidimensional state. Each sender observes only one dimension of the state space and cares only about matching the action in that dimension. The receiver has the same preference with each expert in the specific dimension, but confronts a *budget constraint* that determines feasible actions. This constraint causes a conflict of interest between the senders and the receiver, preventing truthful revelation. Nevertheless, information can be at least partially transmitted in terms of grids in perfect Bayesian equilibrium.

Chapter 3 analyzes the relationship between constraints on decision making and optimal decision making structure–centralized or decentralized decision making. A principal either makes divisional decisions or delegates them to two agents in a two-divisional organization. Constraints on feasible decisions induce conflict of interest between the principal and agents even if they have common preferences. Given the same prior belief about divisional information, truthful revelation by the agents under centralized decision making depends on the extent to which the constraints confine the principal. However, delegation can more than compensate the principal for loss of control by exploiting the agents' information when different prior beliefs disrupt information revelation or the agents have difference preferences.

Chapter 4 discusses tenure reform for primary and secondary education in United State from a game-theoretical point of view. The remarkable feature of the tenure reform is introducing objective measures of a teacher's students performance and relating their outcomes to the contract with the teacher. To analyze the effect of the reform, a continuing contract (tenure) is compared with a non-continuing contract (non-tenure) based on performance evaluation. The welfare is improved compared to the continuing contract case, but the gap between a good and bad school becomes wider under the non-continuing contract. This increased gap is caused by a unilateral transfer of a qualified teacher from the good school to bad school.

2.0 BUDGET CONSTRAINT AND INFORMATION TRANSMISSION IN MULTIDIMENSIONAL DECISION MAKING[1]

2.1 INTRODUCTION

Suppose that a decision maker (called receiver) reviews plans for multiple projects and makes decisions on them simultaneously. The receiver has no information about suitability of each plan, but can privately consult experts (senders) who specialize in each of the projects. The receiver follows the recommendations from the senders only if all of the plans are feasible, otherwise the receiver makes decisions at her discretion.[2] Then, the senders might strategically control informational quality rather than truthfully reveal the information–that is, the senders intentionally provide ambiguous information, or even exaggerate (understate) the truth, provided that the receiver cannot afford to implement their recommendations.

The purpose of this paper is to study how constraints on decision making affect information transmission from multiple senders to the receiver. We build a simple model in which the receiver consults two senders to match her action to a two-dimensional state. Te senders observe only one dimension of the state, and their observations do not overlap each other. The senders only want the receiver to match the action to the state on their own dimension. Without constraints on the receiver, the optimal actions for the senders and receiver coincide and all of them attain the best outcome. Given the constraints, however, the optimal action of the receiver might differ from the optimal actions of the senders.

To make the concept of the constraints tractable, we introduce into the model a *budget constraint* which puts a limit on the receiver's feasible actions. Technically, the budget

[1] This research is a joint work with Arya Kumar Srustidhar Chand.

[2] We use a female pronoun for the receiver and a male pronoun for senders throughout the paper.

constraint is a hyperplane that divides an action space into feasible and infeasible subsets. We can adapt the budget constraint to model the following example. Suppose a firm planning on entry into two different markets hires different consulting firms to propose and implement its entry strategies in each market. On the basis of initial proposals from the consulting firms, the entrant allocates its investment in the two markets. In this case, each consulting firm would have an incentive to exaggerate its report to attract more investment. A similar situation could arise if a policy maker investing in two public projects consults an independent expert for each project.[3] The constrained budget can simply be time rather than material resources. Two managers in two different divisions in a firm compete to attract a chief executive officer who suffer from a tight schedule for their respective divisions.

A binding budget constraint may cause a conflict of interest between the senders and the receiver. Given the belief that the constraint prevents the receiver from matching the action to the state, the senders may exaggerate the state of their own dimension to obtain a more favorable action. As a result, each sender may come to regard the other as a biased rival who tries to skew the receiver's decision against his own interest, and competes in overstating for more allocation of the budget toward his interest.[4] In other words, each sender behaves as a partisan advocate rather than a neutral and objective technocrat.

Truthful revelation depends on how strictly the feasible actions of the receiver are constrained. The senders have incomplete information about the state since the senders do not observe each other's dimension. This incomplete information creates an incentive to exaggerate the state since the senders do not know if the best action for them is feasible or not. The senders tell the truth in equilibrium only if the constraint is weak enough–the feasible action set is sufficiently large so that such an incentive is not strong.

Given the constraint that prevents truthful revelation, information is *partially* transmitted in a coarse informational structure as follows. In equilibrium, each dimension of the state space is partitioned into (up to infinite) intervals, and the senders report which interval the state is located in. The receiver takes the optimal action by combining one dimension with

[3]There is a related episode in Oslo, Norway. What would have happened if the policy makers had known that one project might be suspended or canceled due to overspending on the other project? See "They All Scream for Edvard Munch, But Oslo Can't Satisfy Demand." the Wall Street Journal, November 28, 2012.

[4]U.S. budget cut caused tensions between the Army and the National Guard. See "Budget Tubulence Hits Military Choppers." the Wall Street Journal, January 21, 2014.

the other. To sum up, information is transmitted in terms of a cell of the grid consisting of the (finite or infinite) partitions.

The quality of information transmitted in equilibrium depends on both how many cells constitute the grid and how equally the grid is partitioned. A finer grid structure (with more cells that are more equally partitioned) increases informational accuracy. Given the same form of grid (in terms of the number and the order of cells), the receiver and senders are always better off with a weaker constraint since it induces a more evenly partitioned grid.

Related literature. Given an unbounded state and action space, *informational spillover* between dimensions leads senders to reveal the state truthfully if they observe every dimension of a multidimensional state space (Battaglini, 2002). The receiver can exploit a direct conflict of interest between the senders in every dimension by punishing all senders in case of receiving different reports about the same state. However, in our model, the receiver cannot compare the reports from the senders since the senders observe only one dimension of a multidimensional state space. Given that the senders do not observe every dimension, information cannot be truthfully revealed even without any constraint on actions (Alonso, Dessein and Matouschek, 2008; Kawamura, 2011).

Given the unbounded action space and senders who observe every dimension, the shape of a state space itself determines truthful information revelation (Ambrus and Takahashi, 2008). Small punishment for minor differences in revelation may lead to robust truthful revelation even with a various form of action space (Meyer, Moreno de Barreda and Nafziger, 2013). That is, the action space can be not only a superset of the state space (as in Ambrus and Takahashi) but also a subset (like our model). However, the action space is still exogenously provided. By introducing a budget constraint on state space, we endogenously put a boundary on a feasible action set that becomes a subset of the state space.

The remainder of the paper is organized as follows. Section 2.2 introduces the basic model. Section 2.3 shows how the constraint causes an interim bias between the senders and the receiver. Section 2.4 shows when information is truthfully transmitted. Section 2.5 characterizes partially revealing equilibria. Section 2.6 discusses and extends the basic model. Section 2.7 concludes.

2.2 MODEL

There is a receiver who must take a two-dimensional action $a = (a_1, a_2) \in \mathcal{A} \subset \mathbb{R}^2$. It is optimal for the receiver to match her action to a two-dimensional state $\theta = (\theta_1, \theta_2) \in \Theta = [0, 1] \times [0, 1]$. However, the receiver only knows that the state is a random variable and is uniformly distributed on Θ.

To get some information, the receiver privately consults two informed senders, called sender 1 and sender 2, who can observe only one dimension of the state space. Without loss of generality, for $i = 1, 2$, sender i observes only θ_i.[5] Denote the family of nonempty subsets of $[0, 1]$ by M_i. The sender i reports his observation to the receiver with an *unverifiable* message $m_i \in M_i$.

The senders are only concerned with the action on the specific dimension in which they observe the state, i.e., sender i cares about only a_i. The payoff of sender i is given by $U^{S_i}(a_i, \theta_i)$ that is twice differentiable, satisfying $d^2U^{S_i}/da_i^2 < 0 < d^2U^{S_i}/da_i d\theta_i$, and $dU^{S_i}(\theta_i, \theta_i)/da_i = 0$. The receiver's payoff, $U^R(a_1, a_2, \theta_1, \theta_2)$, is maximized at $(a_1, a_2) = (\theta_1, \theta_2)$. In other words, there is no bias between the receiver and sender i in the i-th dimension. For simplicity, we assume that $U^{S_1}(\theta_1, \theta_1) = 0$, $U^{S_2}(\theta_2, \theta_2) = 0$, and $U^R = U^{S_1} + U^{S_2}$.

The receiver wants to match her action to the state, but her action is constrained. We formalize the notion of constraints on the set of feasible actions by a budget constraint that consists of a triple (p_1, p_2, w) where $p_1 \geq 0$, $p_2 \geq 0$, and $w > 0$. The budget constraint is common knowledge. The receiver has an endowment w and a_i has a cost p_i per unit. Thus, the receiver's feasible action is confined to a subset of the state space, a *budget set* $\mathcal{B}(p_1, p_2, w) = \{(a_1, a_2) | p_1 a_1 + p_2 a_2 \leq w\} \subseteq \Theta$.

The solution concept is perfect Bayesian equilibrium. Denote the communication strategy for sender i in state θ_i as a probability density function $\mu_i(m_i \mid \theta_i)$ and the strategy for the receiver as a correspondence $\alpha : m_1 \times m_2 \to \mathcal{A}$. The receiver's belief $\beta(\theta_1, \theta_2 \mid m_1, m_2)$ is a conditional probability that the receiver assigns to the state $(\theta_1, \theta_2) \in \Theta$ using Bayes' rule whenever possible after receiving the messages m_1 and m_2.

[5] A subscripted i means the i-th dimension in multidimensional variables throughout the paper. Henceforth, sender i refers to each sender when we want to distinguish them from each other but do not need to clarify whether he is sender 1 or 2.

Definition 1. *A 4-tuple* $\{\mu_1, \mu_2, \alpha, \beta\}$ *constitutes a perfect Bayesian equilibrium if*

(i) m_1 is in the support of $\mu_1(. \mid \theta_1)$ implies $m_1 \in \arg\max_{m_1} \int_{\theta_2 \in \Theta_2} U^{S_1}(\alpha_1(m_1, m_2), \theta_1) d\theta_2$;

(ii) m_2 is in the support of $\mu_2(. \mid \theta_2)$ implies $m_2 \in \arg\max_{m_2} \int_{\theta_1 \in \Theta_1} U^{S_2}(\alpha_2(m_1, m_2), \theta_2) d\theta_1$;

(iii) $\alpha(m_1, m_2) \in \arg\max_{(a_1,a_2)\in\mathcal{A}} \int_{\theta_1} \int_{\theta_2} U^R(a_1, a_2, \theta_1, \theta_2)\beta(\theta_1, \theta_2 \mid m_1, m_2) d\theta_2 d\theta_1$

subject to $p_1 a_1 + p_2 a_2 \leq w$; where,

(iv) $\beta(\theta_1, \theta_2 \mid m_1, m_2) = \dfrac{\mu(m_1 \mid \theta_1)}{\int_{\Theta_1} \mu(m_1 \mid \theta_1)\, d\theta_1} \dfrac{\mu(m_2 \mid \theta_2)}{\int_{\Theta_2} \mu(m_2 \mid \theta_2)\, d\theta_2}$.

Henceforth, we simply refer to a perfect Bayesian equilibrium as an equilibrium.

2.3 CONFLICT OF INTEREST

We show how the budget constraint may cause a conflict of interest between the receiver and the senders, and then between sender 1 and sender 2. Suppose that $p_1\theta_1 + p_2\theta_2 > w$. Given θ, for $I = S_1, S_2, R$, let $a^I(\theta)$ denote the most preferred action for I. The senders truthfully reveal the state (θ_1, θ_2) and the receiver believes their reports.

Figure 1 illustrates how the receiver responds to the senders' messages when the budget constraint prevents the receiver from matching her action to the state. The constrained receiver takes her best feasible action $a^R(\theta) = t$ on the boundary of the budget set $\mathcal{B}$, which is the closest to θ. However, the receiver's optimal action deviates from sender 1's best action $a^{S_1} = d$ and sender 2's best action $a^{S_2} = f$. Observe that $a_1^R(\theta) = t_1 < \theta_1 = a_1^{S_1}(\theta)$ and $a_2^R(\theta) = t_2 < \theta_2 = a^{S_2}(\theta)$. The receiver behaves as if she has a downward bias against the senders in each dimension. By $\chi_i(\theta) = a_i^{S_i}(\theta) - a_i^R(\theta)$, we measure the difference between the optimal action for the receiver and that for sender i in the i-th dimension. We call χ_i an *interim bias*[6] since χ_i arises from realization of the state.

The interim biases between the receiver and the senders consequently generate a further conflict of interest between sender 1 and sender 2. Figure 2 illustrates how these biases

[6] Che and Kartik (2009) show a different kind of "interim bias" that is generated by different prior beliefs between a sender and a receiver.

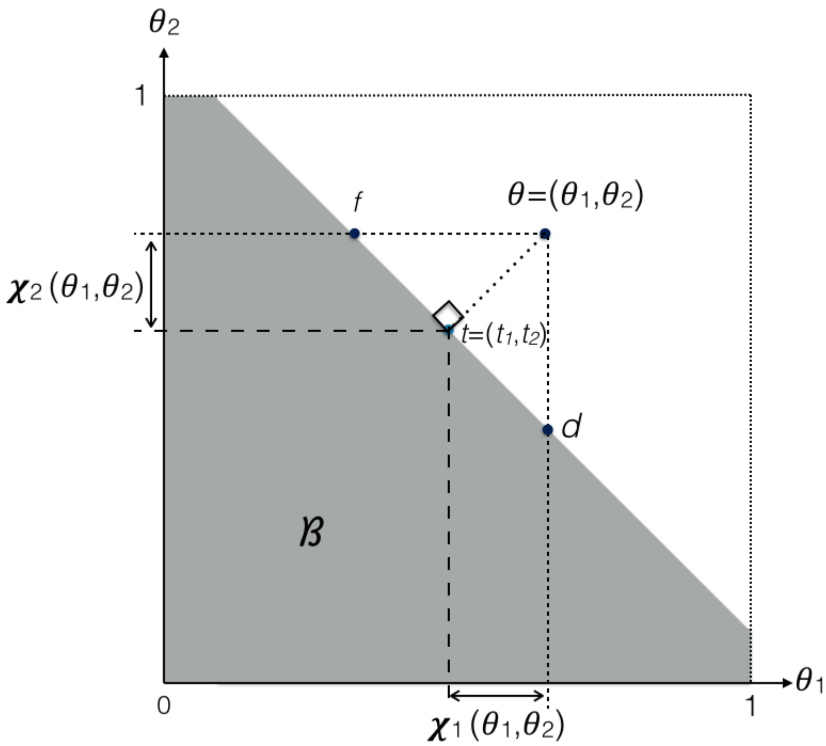

Figure 1: Truth-telling.

change when sender 1 overstates his state while sender 2 keeps revealing truthfully. Observe that for some $\theta_1' > \theta_1$, $\chi_1(\theta_1', \theta_2) < \chi_1(\theta_1, \theta_2)$ and $\chi_2(\theta_1', \theta_2) > \chi_2(\theta_1, \theta_2)$. Now, the senders compete for their own best actions by exaggerating their state.

However, interim biases do not always arise. When the receiver can match her action to the state, i.e., $p_1\theta_1 + p_2\theta_2 \leq w$, the budget constraint causes no conflict between the receiver and the senders, consequently, no conflict between sender 1 and sender 2.

2.4 TRUTHFUL INFORMATION REVELATION

In Section 2.3, we show that the senders tell the truth only if, given the state, the action is feasible to the receiver, i.e., $(\theta_1, \theta_2) \in \mathcal{B}$. Remember that when the state is beyond the

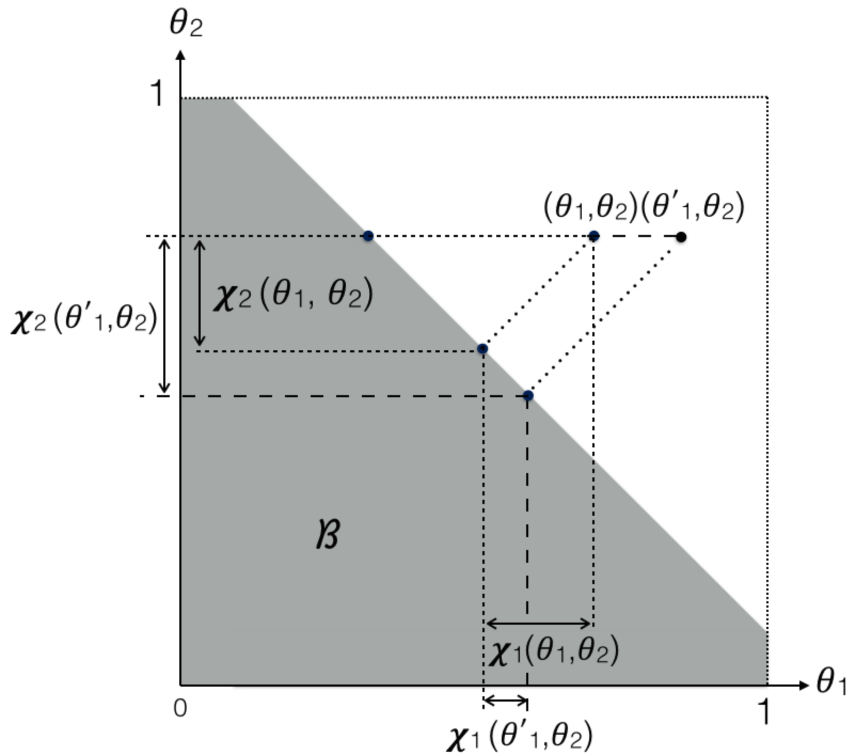

Figure 2: Overstating.

budget set, an interim bias due to the constraint, which forces the receiver into the choice like t in Figure 1, makes the senders behave as if they have an upward bias against the receiver, leading to overstating the state. Moreover, as illustrated in Figure **??**, the senders compete against each other since the interim bias in one dimension is monotonically increasing in the state of the other dimension. The remaining problem to the senders is that they do not know each other's observation.[7]

To show when the senders had better reveal the state even without the other's information[8], we begin with description of the receiver's action from each sender's point of view. Without loss of generality, we focus on sender 2. The receiver trusts both senders, and

[7]This is different from the case where a receiver is uncertain about a sender's preference. See Dziuda (2011); Li and Madarász (2008); Morgan and Stocken (2003) and Wolinsky (2003) for discussions about strategic information transmission in various points of view in the literature.

[8]Sprumont (1991) uses uniform allocation rule as a mechanism to elicit truth from the senders, here our focus is rather on the cheap talk communication.

sender 2 believes that sender 1 truthfully reports θ_1 to the receiver. For simplicity, let $p_1 = 1$, $p_2 = p > 0$, and $p \leq w$. Given the receiver's best response to the messages, sender 2 considers the action as a function of θ_1 with parameters p and m_2, $a_2(\theta_1; p, m_2)$, in the second dimension.

Given every θ_2, we show how Θ_1 is partitioned into intervals from sender 2's point of view. First, suppose that sender 2 reveals the state truthfully, i.e., $m_2 = \theta_2$. Then, Figure 3 illustrates that Θ_1 is partitioned into two intervals, $[0, \tilde{\theta}_1]$ and $(\tilde{\theta}_1, 1]$ where $\tilde{\theta}_1 = w - p\theta_2$. Notice that for $\theta_1 \in [0, \tilde{\theta}_1]$, the receiver takes sender 2's best action, $a_2(\theta_1; p, \theta_2) = a_2^{S_2}(\cdot; \theta_2) = \theta_2$, while for $\theta_1 \in (\tilde{\theta}_1, 1]$, the receiver's action is less than sender's best and decreasing in θ_1. Now, suppose that sender 2 overstates the state by $\epsilon > 0$, i.e., $m_2^{'} = \theta_2' = \theta_2 + \epsilon$. Let $\underline{\theta}_1 = w - p\theta_2'$ and $\overline{\theta}_1$ such that $\theta_2 = \arg\max U^R(\overline{\theta}_1, \theta_2')$. Then, as shown in Figure 3, Θ_1 is divided into three intervals: $[0, \underline{\theta}_1]$, $(\underline{\theta}_1, \overline{\theta}_1]$, and $(\overline{\theta}_1, 1]$. In the first interval, $[0, \underline{\theta}_1]$, the action is matched to the overstated message $a_2(\theta_1; p, \theta_2') = \theta_2'$. Then, in the second, $\theta_1 \in (\underline{\theta}_1, \overline{\theta}_1]$, the action is between the overstated and the truthful state, $\theta_2' > a_2(\theta_1; p, \theta_2') > \theta_2$. Finally, for $\theta_1 \in (\overline{\theta}_1, 1]$, the receiver takes the action that is closer to sender 2's best than one that would be taken with truth-telling, $a_2^{S_2} > a_2(\theta_1; p, \theta_2') > a_2(\theta_1; p, \theta_2)$.

The effects of overstating are twofold. Given that $U^{S_1} = -(a_1 - \theta_1)^2$, $U^{S_2} = -(a_2 - \theta_2)^2$, and $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$, Figure 4 illustrates the difference between sender 2's truth-telling payoff, $U_t^{S_2}$, indicated by a solid line, and overstating payoff, $U_\epsilon^{S_2}$, indicated by a dot line. On the one hand, overstating creates new loss in $[0, \tilde{\theta}_1]$ where there would be no loss at all without overstating. On the other hand, in $(\overline{\theta}_1, 1]$, new gain G occurs and the marginal effect is increasing in θ_1. The loss and gain cancel each other out in $(\tilde{\theta}_1, \overline{\theta}_1]$. Therefore, sender 2 needs to weigh the expected loss and gain by overstating compared to truth-telling, i.e., comparing L with G in Figure 4. Notice that as θ_2 is increasing, both $\underline{\theta}_1$ and $\overline{\theta}_1$ are decreasing, inducing that L shrinks while G expands. This implies that given the state, if overstating dominates truth-telling, i.e., $G > L$, then overstating is more beneficial than truth-telling at a higher state.

Before formally identifying whether or not the senders reveal truthfully, we provide a concept of how restrictive the budget constraint is by categorizing the constraint into two classes.

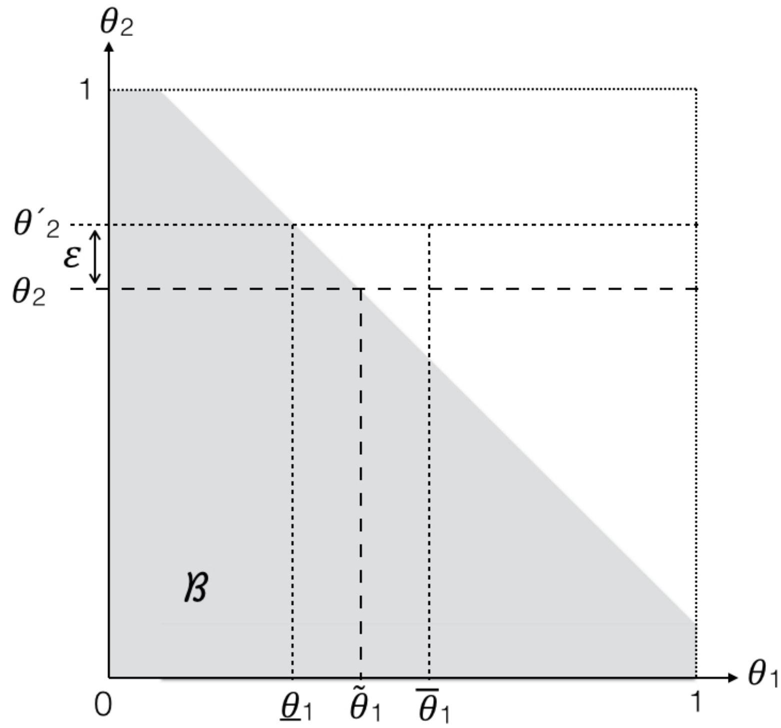

Figure 3: Partition.

Definition 2. *A budget constraint* (p_1, p_2, w) *is severe if either*

$$\sup\{a_1|(a_1, 0) \in \mathcal{B}(p_1, p_2, w)\} < \sup\{\Theta_1\} = 1, \textit{ or}$$
$$\sup\{a_2|(0, a_2) \in \mathcal{B}(p_1, p_2, w)\} < \sup\{\Theta_2\} = 1.$$

A *severe* constraint makes it impossible for the receiver to take the maximum action in either dimension. With a non-severe constraint, every action in one dimension is feasible if the receiver spends enough her endowment on that dimension. Given (p_1, p_2), w is a measure of *severity* of the constraint on the action space since $\mathcal{B}$ expands as w increases.

Without loss of generality, we focus on a *truthfully revealing equilibrium* in which the senders tell the truth, $m_1 = \theta_1$ and $m_2 = \theta_2$ for all $(\theta_1, \theta_2) \in \Theta$, and the receiver believes the senders, $\beta(\theta_1, \theta_2|m_1, m_2) = 1$.[9]

[9] Battaglini calls an equilibrium *fully* revealing if $\beta(\theta|\mu(\theta)) = 1$, and shows that, given any fully revealing equilibrium, a truthfully revealing equilibrium is outcome-equivalent to the fully revealing equilibrium.

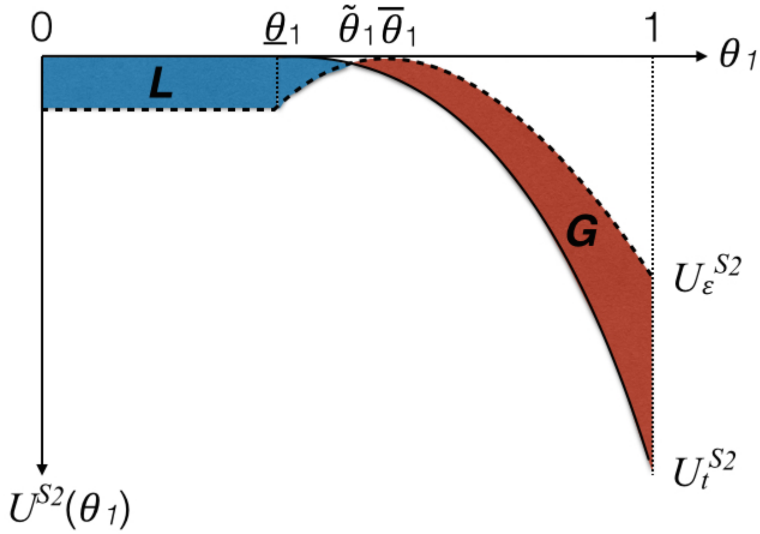

Figure 4: Expected utility.

Proposition 1. *A truthfully revealing equilibrium exists only if the constraint is non-severe.*

Proof. See A.2.1. □

Proposition 1 implies that a truthfully revealing equilibrium does not exist if the constraint is severe. Given a severe constraint, sender i would better overstate his observation rather than tell the truth without knowing θ_{-i} if $\theta_i > w/p_i$.

Proposition 2. *Given $\mathcal{B}(p_1, p_2, w) \subsetneq \Theta$, fix p_1 and p_2. Then, there exists $\underline{w} \in [\max\{p_1, p_2\}, p_1 + p_2]$ such that for all $w \geq \underline{w}$, there exists a truthfully revealing equilibrium.*

Proof. See A.2.2. □

It is not sufficient for a truthfully revealing equilibrium to exist that the constraint is not severe. Remember that in a truthfully revealing equilibrium, the senders' expected loss of overstating dominates the expected gain from truthful revelation. This incentive condition requires w to be high enough by the marginal effect due to the concavity of the utility functions. The concavity determines $\underline{w}$ that is the minimum level of endowment for truthful revelation. For example, given $p_1 = p_2 = 1$ and $U^{S_i} = -|a_i - \theta_i|^\gamma$, $\underline{w} = 4/3$ if $\gamma = 1$ and

$\underline{w} = 2$ if $\gamma = 2$. In other words, given quadratic utilities, truth-telling is possible only if there is no budget constraint on feasible actions.

2.5 PARTIAL INFORMATION REVELATION

We show that a truthfully revealing equilibrium exists under a limited circumstance in Section 2.4. However, it is still possible to transmit information in a coarse informational structure even under circumstances where a truthfully revealing equilibrium does not exist. At first, the receiver's two-dimensional decision making is regarded as two separate unidimensional decision problems. Crawford and Sobel (1982) show that for an uninformed receiver with a single biased sender in a unidimensional state space, the state space is partitioned into a finite number of intervals and messages transmit information about which interval the state lies in. In the same vein, if the i-th dimension of the state space is partitioned into intervals and sender i reports in which interval θ_i is located, the receiver can notice which cell the state belongs to by combining one dimension with the other. Now, we provide the simplest example to illustrate the characteristics of an equilibrium under the budget constraint.

Example 1 (2 × 2-grid equilibrium) . For simplicity, let $U^{S_1} = -(a_1 - \theta_1)^2$, $U^{S_2} = -(a_2 - \theta_2)^2$, and $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$. and suppose that $p_1 = p_2 = 1$ and $1/2 < w < 0.9$ (see A.1 for full characterization of a 2×2-grid equilibrium according to $0 \leq w \leq 2$). Then, there does not exist a truthfully revealing equilibrium by Proposition 1.

Nevertheless, we show that there exists a partially revealing equilibrium in this case. First, each dimension of the state space is partitioned into two intervals. In each partition, there is a point, $\bar{\theta}$, such that if $\theta < \bar{\theta}$, then the senders send a low message, $l = [0, \bar{\theta})$, and if $\theta > \bar{\theta}$, then they send a high message $h = (\bar{\theta}, 1]$. At $\theta = \bar{\theta}$, the senders are indifferent between sending l and h. Consequently, the state space is partitioned into four subspaces as shown in Figure 5.

After receiving a message pair (m_1, m_2) for $m_1, m_2 \in \{l, h\}$, the receiver updates her

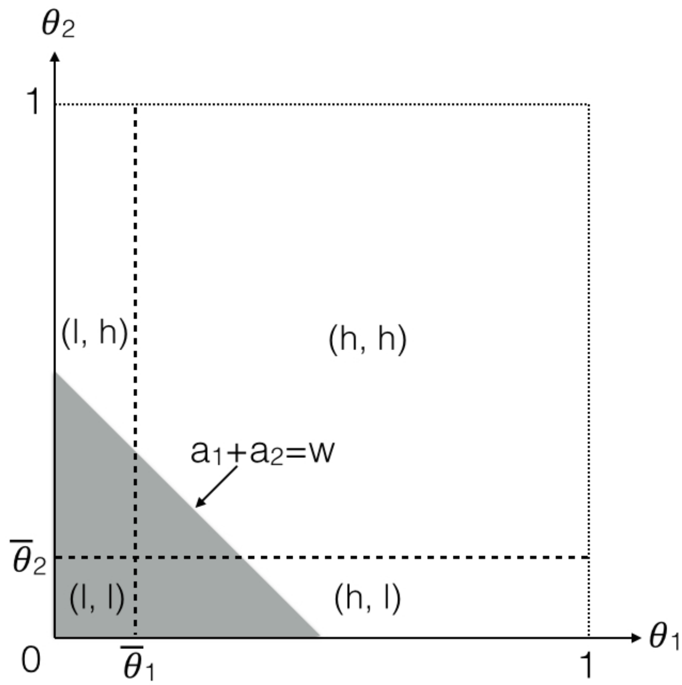

Figure 5: Equilibrium grid

belief and acts to maximize her expected payoff. Table 1 summarizes the receiver's strategy profile by each message pair when the action is restricted in three out of four cells by the constraint.

Since the senders are indifferent between sending l and h at $\theta = \bar{\theta}$, their expected payoffs must be the same at $\theta' = \bar{\theta}$ irrespective of their message:

$$\underbrace{-(\frac{\bar{\theta}}{2} - \bar{\theta})^2\bar{\theta} - (\bar{\theta} - \frac{w}{2} + \frac{1}{4})^2(1-\bar{\theta})}_{\text{expected payoff sending m=l}} = \underbrace{-(\bar{\theta} - \frac{w}{2} - \frac{1}{4})^2\bar{\theta} - (\bar{\theta} - \frac{w}{2})^2(1-\bar{\theta})}_{\text{expected payoff sending m=h}}. \tag{2.1}$$

From equation (2.1) , we identify the point $\overline{\theta}$ in the partitions:

$$\bar{\theta} = \sqrt{-1 + 4w + 4(-3 + 2w^2) - 64w + 96} \text{ for } \frac{1}{2} < w < \frac{1}{8}(3 + \sqrt{17}). \tag{2.2}$$

Notice that, given the constraint, the receiver must be able to match her action to an indifferent state, i.e., for all $w > 0$, $\bar{\theta} + \bar{\theta} \leq w$. Figure 6 illustrates what happens if the partitions fail to satisfy this condition. Observe that the state is at s. Given (l, h), the receiver

Table 1: Message-Action pairs

(m_1, m_2)	(a_1, a_2)
(l, l)	$(\frac{\bar{\theta}}{2}, \frac{\bar{\theta}}{2})$
(h, l)	$(\frac{w}{2} + \frac{1}{4}, \frac{w}{2} - \frac{1}{4})$
(l, h)	$(\frac{w}{2} - \frac{1}{4}, \frac{w}{2} + \frac{1}{4})$
(h, h)	$(\frac{w}{2}, \frac{w}{2})$

takes the action d while, given (l, l), she takes t. Since an interim bias of overstating χ_d is less than that of truth-telling χ_t, sender 2 would be better to send h even if the state belongs to l.

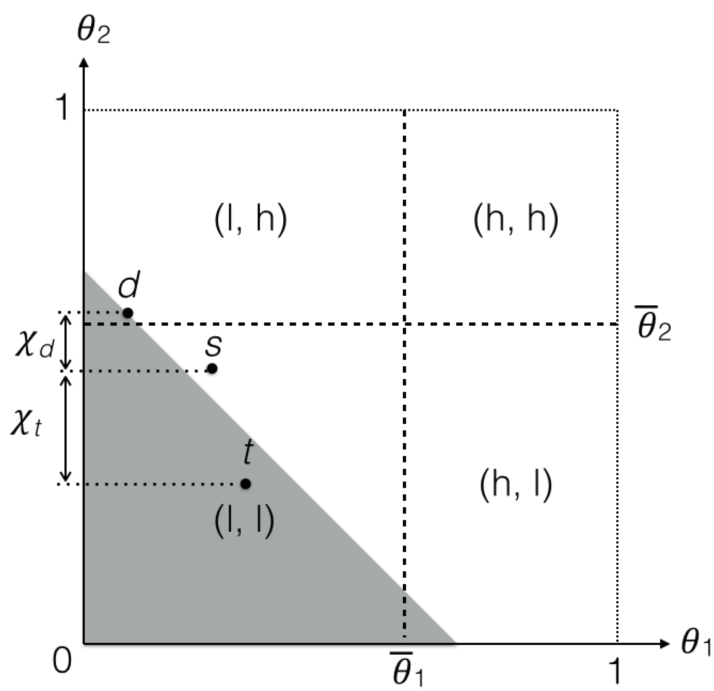

Figure 6: Non-equilbrium grid

We show how the state space is partitioned into cells of a grid in a general case. For all $n_i \in \mathbb{N}$, let $\rho_i(n_i) \equiv (\rho_i^0(n_i), \ldots, \rho_i^{n_i}(n_i))$ where $\rho_i^0(n_i) = 0 < \rho_i^1(n_i) < \cdots < \rho_i^{n_i-1}(n_i) <$

$\rho_i^{n_i}(n_i) = 1$ denote an n_i-*intervals partition* in the i-th dimension. Each interval in an n-intervals partition is measured by $\delta^j = \|\rho^j - \rho^{j-1}\|$, $j = 1, \ldots, n$. Given $\rho_1(n_1)$ and $\rho_2(n_2)$, the state space is partitioned into a rectilinear grid of $n_1 \times n_2$ cells.

Theorem 1. *Given $\mathcal{B}(p_1, p_2, w)$, for all n_1, $n_2 \in \mathbb{N}$, Θ_1 is partitioned according to $\rho_1(n_1)$ and that Θ_2 is partitioned according to $\rho_2(n_2)$. Then, there exists an $n_1 \times n_2$-grid equilibrium such that sender 1 sends a message $m_1^j \in [\rho_1^{j-1}, \rho_1^j]$ if $\theta_1 \in [\rho_1^{j-1}, \rho_1^j]$, $j = 1, \ldots, n_1$, that sender 2 sends a message $m_2^k \in [\rho_2^{k-1}, \rho_2^k]$ if $\theta_2 \in [\rho_2^{k-1}, \rho_2^k]$, $k = 1, \ldots, n_2$, and that the receiver takes an action*

$$(a_1, a_2) \in \arg\max \int_{\rho_1^{j-1}}^{\rho_1^j} \int_{\rho_2^{k-1}}^{\rho_2^k} U^R(a_1, a_2 | m_1^j, m_2^k) d\theta_2 d\theta_1 \text{ subject to } p_1 a_1 + p_2 a_2 \le w, \tag{2.3}$$

provided that partitions $\rho_1(n_1)$ and $\rho_2(n_2)$ satisfy the following condition:

$$(\textit{indifference}) \text{ for } j = 1, \ldots, n_1 \text{ and } k = 1, \ldots, n_2,$$

$$\sum_{k=1}^{n_2} [U^{S_1}(a_1(m_1^{j+1}, m_2^k), \rho_1^j) - U^{S_1}(a_1(m_1^j, m_2^k), \rho_1^j)]\delta_2^k = 0, \text{ and} \tag{2.4}$$

$$\sum_{j=1}^{n_1} [U^{S_2}(a_2(m_1^j, m_2^{k+1}), \rho_2^k) - U^{S_2}(a_2(m_1^j, m_2^k), \rho_2^k)]\delta_1^j = 0. \tag{2.5}$$

Proof. See A.2.3. □

To characterize an equilibrium, we use the technique introduced by Gordon (2010). The *indifference* conditions (2.4) and (2.5) indicate that when the state is at a cutoff point ρ^j (one of the boundaries between two adjacent intervals), the senders must be ex-ante indifferent between sending the higher message $m^{j+1} \in [\rho^j, \rho^{j+1}]$ and the lower message $m^j \in [\rho^{j-1}, \rho^j]$. Any partition ρ^n, $n \ge 2$, is represented by an element of a set $P_n = \{\rho^n = (\rho_0, \ldots, \rho_n) \in [0, 1]^{n+1} | \rho_0 = 0, \rho_1 = 1, \text{and } \rho_0 \le y_1 \le \cdots \le \rho_{n-1} \le \rho_n\}$. Let σ^n denote a correspondence that maps ρ^n to a set of vectors satisfying the indifference conditions. An equilibrium partition is the fixed point of a correspondence σ^n. In an $n_1 \times n_2$-grid equilibrium, there are $n_1 \times n_2$ distinct actions that correspond to each cell of the grid.

In equilibrium partitions, there is *monotonicity* such that the higher interval at any indifference point is wider than the lower interval, i.e. $\delta_i^j(n_i) \le \delta_i^{j+1}(n_i)$, $i = 1, 2$ and $j = 1, \ldots, n_i$ (see Claim 2 in the appendix). For reference, suppose that given the budget constraint, there exists a truthfully revealing equilibrium. Despite less efficiency, any finite intervals partition induces an equilibrium as long as the partitions are equally divided. A sequence of increasing intervals leads to a countably infinite intervals partition. This sequence can be considered as an approximation of the truthfully revealing partition. Now, suppose that the budget constraint is so severe that there is no truthfully revealing equilibrium. In Section 2.3, we showed that when the constraint limits the actions of the receiver, the senders' preferred action might be upward against the optimal action for the receiver. Truth-telling is no longer possible due to the cells in which the expectation of the state is beyond the budget constraint. In other words, the budget constraint hinders the receiver from matching the state, at least a cell that consists of the highest intervals in the partitions, $[\rho_1^{n_1-1}, 1] \times [\rho_2^{n_2-1}, 1]$. The effect of the constraint is transferred to lower intervals in turn through the indifference conditions leading to monotonicity. As a result, an equilibrium has a coarsely partitioned structure.

It is necessary for an infinite intervals partition to exist irrespective of how severely the budget constraint confines the feasible actions that there is at least one state at which a sender reveals his dimension of the state truthfully (Alonso, Dessein and Matouschek; Kawamura). In our model, this truth-telling state is zero, the lowest state. Without loss of generality, we focus on the strategic tension between sender 1 and the receiver. The most preferred actions always coincide at $\theta_1 = 0$ irrespective of θ_2, i.e., $a^{S_1}(0, \theta_2) = a^R(0, \theta_2)$. However, for $\theta_1 > 0$, the preference bias may occur depending on θ_2. At the highest state, there exists θ_2' such that the most preferred actions are always incompatible, i.e., $a^{S_1}(1, \theta_2') > a^R(1, \theta_2')$.

This feature satisfies an "outward bias" condition such that the set bounded by a sender's best actions at the boundaries of the state space includes the set bounded by the receiver's optimal actions at the boundaries of the state space, i.e., $[a^R(0), a^R(1)] \subseteq [a^S(0), a^S(1)]$ (Gordon). Given an outward bias, there exists at least one infinite intervals partition. Since there is only one state at which the senders always tell the truth, we have an countably infinite intervals partition.[10] As the number of intervals is increasing, an equilibrium partition converges to zero, $\lim_{n\to\infty} \rho^1(n) = 0$.

[10] According to Gordon, if a set of the states at which a sender tell the truth is at most countable, then the number of actions is also at most countable in equilibrium.

By adding more intervals in partitions, a countably infinite sequence increases the expected utility. A countably infinite intervals equilibrium might be regarded as an approximation of a truthfully revealing equilibrium. In general, however, the expected utility with infinite intervals partitions is not close to the expected utility when both senders tell the truth.

Corollary 1. *Suppose that given $\mathcal{B} \subsetneq \Theta$, there is no truthfully revealing equilibrium. For an $n_1 \times n_2$-grid, all n_1, $n_2 \in \mathbb{N}$, fix n_{-i}, $i = 1, 2$. Then, $\delta_i^n(n) \nrightarrow 0$ as $n \to \infty$.*

Proof. Without loss of generality, we focus on the partitions in the first dimension. Suppose that $\delta^n(n) \to 0$ as $n \to \infty$. Then, $\delta^j(n) \to 0, j = 1, \ldots, n-1$. This violates (2.4). $\square$

For a countably infinite sequence in partition to be regarded as an approximation of truth-telling, the size of each interval must converge to zero. If the size of the last interval converges to zero, then the size of the other intervals also converges to zero due to the monotonicity conditions of partitions in an equilibrium. However, the size of the last partition does not converge to zero unless the budget constraint is weak enough to encourage the senders to tell the truth.

Now, we compare two games with different budget sets. Let $\Gamma_{\mathcal{B}}$ denote a game with a budget set $\mathcal{B}$. Then, the game $\Gamma_{\mathcal{B}'}$ is *weakly nested* into the game $\Gamma_{\mathcal{B}}$ if $\mathcal{B}' \subseteq \mathcal{B}$. Let $n_1^{\Gamma_{\mathcal{B}}} \times n_2^{\Gamma_{\mathcal{B}}}$ denote the grid form of the game $\Gamma_{\mathcal{B}}$. The two games have the same *form of grids* if $n_1^{\Gamma_{\mathcal{B}}} = n_1^{\Gamma_{\mathcal{B}'}}$ and $n_2^{\Gamma_{\mathcal{B}}} = n_2^{\Gamma_{\mathcal{B}'}}$, that is, the number of intervals in each dimension is the same.

Theorem 2. *Suppose that $\Gamma_{\mathcal{B}'}$ is weakly nested into $\Gamma_{\mathcal{B}}$. Fix $n_i^{\Gamma_{\mathcal{B}}} = n_i^{\Gamma_{\mathcal{B}'}}$, $i = 1, 2$. Then, for $I = S_1, S_2, R$, $EU^I_{\Gamma_{\mathcal{B}'}} \leq EU^I_{\Gamma_{\mathcal{B}}}$.*

Proof. A.2.4 $\square$

Given the same form of grids, a grid is more equivalently partitioned as a budget set expands. If each cell in a grid has more equal size, each message in a message set is also more equally informative. This is ex-ante beneficial to the senders and receiver.

2.6 DISCUSSION AND EXTENSION

In Section 2.5, we studied the case in which the senders only partially reveal the state. However, under a special form of constraint, full revelation is always possible irrespective of the size of a budget set. This kind of constraint also can be considered as a special case of Ambrus and Takahashi. In this case, the direction of the constraint only matters as shown in Example 2.

Example 2 (unidimenstional constraint). The environment is similar to Example 1 except the constraint: $U^{S_1} = -(a_1 - \theta_1)^2$, $U^{S_2} = -(a_2 - \theta_2)^2$, $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$; $p_1 = 1$, $p_2 = 0$ and $0 < w < 1$. Suppose that the state (θ_1, θ_2) is beyond the constraint as shown in Figure 7. Sender 2 always tells the truth since the constraint only limits the action in the first dimension. Sender 1 also has no incentive to tell a lie since there is no way for him to influence the receiver. The receiver's response is the same whether sender 1 overstates, $m_1 = \overline{\theta}_1$, or understates, $m_1 = \underline{\theta}_1$.

There may also exist equilibria in which one sender tells the truth and the other sender uses partitioned messages. Contrary to an $n_1 \times n_2$-grid equilibrium, this kind of equilibrium does not alway exist, requiring that w is bigger than a threshold as shown in Example 3.

Example 3 (unidimensionally truthful revelation). The environment is the same as Example 1: $U^{S_1} = -(a_1 - \theta_1)^2$, $U^{S_2} = -(a_2 - \theta_2)^2$, $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$, and $p_1 = p_2 = 1$. Without loss of generality, we assume that sender 1 tells the truth, and sender 2 uses a equally partitioned message. Then, the maximum number of intervals is

$$n^* = \left\lfloor \frac{1}{2(2-w)} \right\rfloor$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Therefore, $w \geq 1.5$ for the receiver to get informative message in the second dimension.

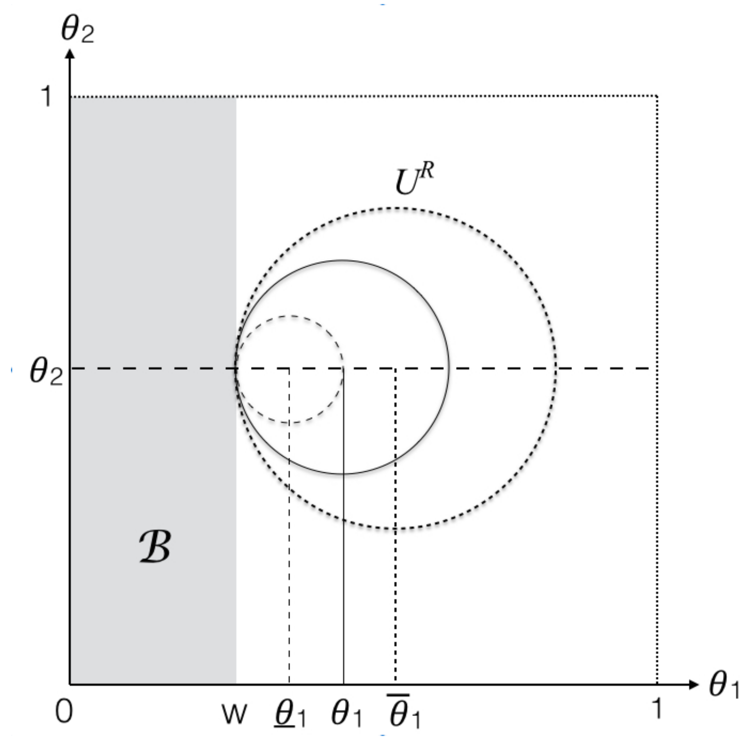

Figure 7: Unidimensional constraint.

One might think that there exists at least one interval in which the senders tell the truth. The simplest example is a "semi-revealing" equilibrium such that given a two-intervals partition, the senders reveal truthfully in one interval of the state space and pool completely in the other interval (Krishna and Morgan, 2001). However, such an equilibrium may not exist.

Example 4 (semi-revelation). The environment is the same as Example 1 except that $w < 2/3$.[11] Suppose that sender i reveals the state, i.e. $m_i = \theta_i$, if the state is less than a threshold $\overline{\theta}_i$, and sends a pooling message, $m_i = \overline{\theta}_i$, otherwise. The receiver is still a Bayesian optimizer. Suppose that $\overline{\theta}_1 + \overline{\theta}_2 < w$ and that sender 2 observes $\theta_2 = w - \overline{\theta}_1$. Figure 8 illustrates how the receiver responds to sender 2's message. Given that $m_2 = \overline{\theta}_2$, $(a_1, a_2) = (\theta_1, w - \theta_1)$ if

[11]This assumption is only for the sake of graphical simplicity and does not change the main results.

$0 \leq m_1 < \overline{\theta}_1$ and $(a_1, a_2) = (w/2, w/2)$ if $\overline{\theta}_1 \leq m_1 \leq 1$ (solid line and dot). However, if $m_2 = w - \overline{\theta}_1$, the receiver takes $a_2 = w - \overline{\theta}_1$ irrespective of m_1 (stitched line). Similarly, it is beneficial for sender i to send $m_i = \theta_i$ instead of $m_i = \overline{\theta}_i$ if $\overline{\theta}_i < \theta_i \leq w - \theta_{-i}$. Therefore, $\overline{\theta}_1$ and $\overline{\theta}_2$ must satisfy that $\overline{\theta}_1 + \overline{\theta}_2 = w$ since the constraint makes it impossible that $\overline{\theta}_1 + \overline{\theta}_2 > w$. Now, suppose that $\overline{\theta}_1 = \overline{\theta}_2 = w/2$. Given $m_2 = w/2$, $a_2 = w - m_1$ in $0 \leq m_1 < w/2$ and $a_2 = 0$ in $w/2 \leq m_1 \leq 1$ (solid line in Figure 9). However, given $m_2 = w/2 - \epsilon$, $a_2 = w/2 - \epsilon$ in $0 \leq m_1 < w/2$ and $a_2 = 0$ in $w/2 \leq m_1 \leq 1$ (stitched line). This induces that given $\theta_2 = w/2$, sender 2 would better understate the state by $\epsilon < w/4$ since the expected gain of understating dominates the expected loss.

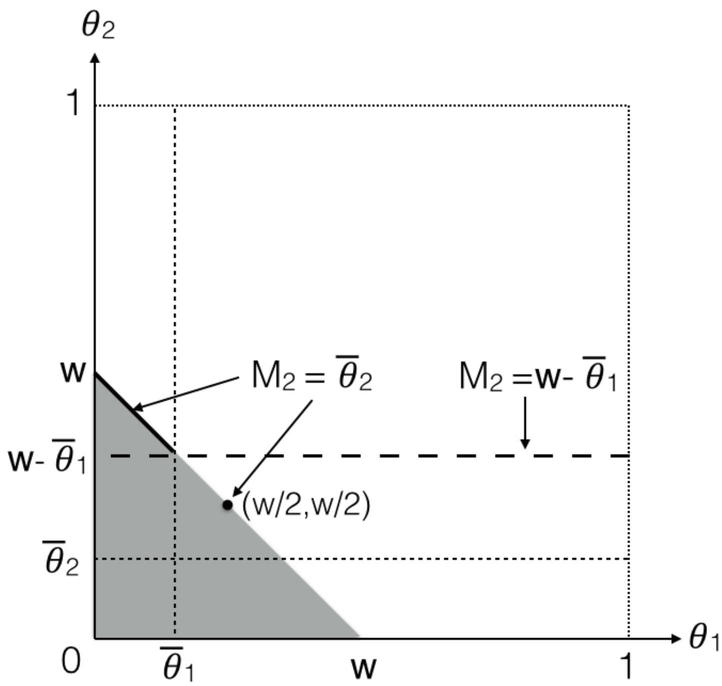

Figure 8: Separting and pooling.

In order to support the semi-revealing outcomes in an incentive-compatible way, we need the receiver to commit to message-contingent outcomes (Alonso and Matouschek, 2007, 2008; Holmström, 1984; Melumad and Shibano, 1991). In other words, given that $m_i \in [0, 1]$, if the receiver commits to taking action $a_i = m_i$ if $m_i < w/2$ and $a_i = w/2$ if $m_i \geq w/2$, then the above outcome is incentive compatible.

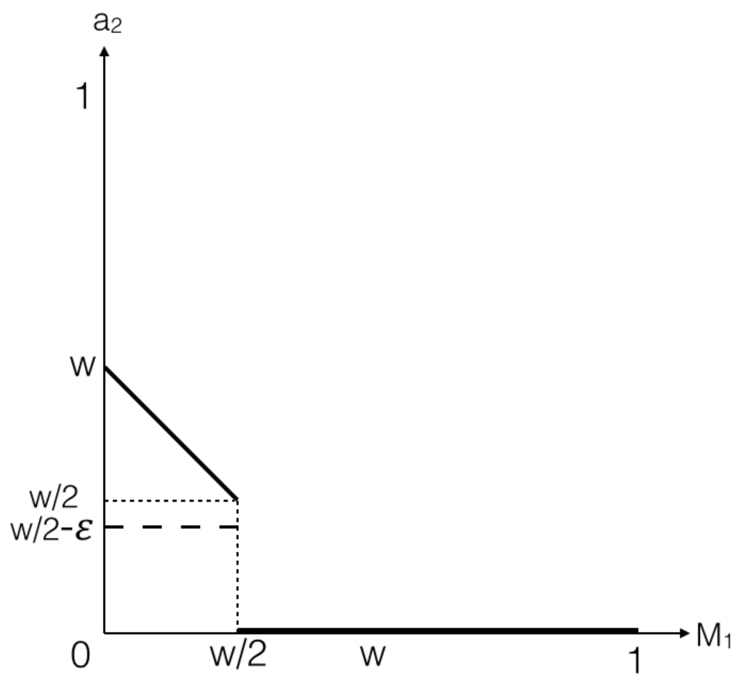

Figure 9: The expected action.

In the previous sections, we used the simplified model to focus on the effects of the constraint. Now, we extend the basic model in various directions.

More general prior distribution. Let $F(\theta_1, \theta_2)$ denote a joint cumulative distribution function. If conditional cumulative probability distributions $F_{\theta_2|\theta_1}$ and $F_{\theta_1|\theta_2}$ are concave, then we have more informative equilibrium outcomes. For a truthfully revealing equilibrium to exist, this concavity requires smaller $\underline{w}$ due to the increasing expected loss of overstating. The concavity also moves indifference points between elements in partitions toward zero in equilibrium grids. However, if the conditional distributions are convex, then we have exactly opposite results.

Heterogenous preferences. Suppose that the senders have a different preference against the receiver–for example, $U^{S_1} = -(a_1 + b_1 - \theta_1)^2$, $U^{S_2} = -(a_2 + b_2 - \theta_2)^2$, and $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$ where $-1 < b_1 < 1$ and $-1 < b_2 < 1$. The exogenous bias between sender i and the receiver, b_i, keeps sender i from revealing the state truthfully irrespective of the constraint. However, a partially revealing equilibrium can exist as long as the exogenous bias is not too large. The direction of the exogenous biases either weakens (if $b_i < 0$) or strengthens (if $b_i > 0$) the interim bias, leading the intervals in the partitions expands (shrinks).

Multidimensional space. Extending the dimension of the state space from 2 to n does not change any key characteristics of the main results if the intersection of each sender's state set is empty and the union is equal to the state space, i.e. $\cup_i \Theta_i = \Theta$ and $\bigcap_i \Theta_i = \emptyset$. The ex-ante conflict of interest between the senders happens only through the receiver's taking into account of the constraint that causes an interim bias.

Sequential transmission. Suppose that multiple senders sequentially report the state and that the latter sender can read the message from the former sender. Given the sufficiently big enough w, there exists an equilibrium in which all senders but the last reveal the state truthfully similar to Example 2. The last sender divides his dimension into an n-interval partition, and reveals where the state lies in. Definitely, the last sender benefits from informational rent of the previous senders.

2.7 CONCLUSION

We study the effect of the constraint on information transmission in an environment where the constrained receiver makes a two-dimensional decision to match with the state depending on the reports from two senders. The constraint on the receiver confines her feasible actions to match with the state, which might lead the receiver's action to deviate from the best actions of the senders. This conflict of interest may subsequently induce the senders to compete with each other for pulling the receiver toward his own interest. Therefore, the senders truthfully report the state only if the feasible action set is large enough so that the receiver is likely to match the state.

Even if the constraint makes truthful revelation impossible, we show, nevertheless, that information can be transmitted with coarse informational structure. The i-th dimension of the state space is partitioned into the intervals and sender i reports which interval the state is located in. Then, the receiver gets information in terms of a cell in the grid that consists of two intervals by combining one dimension with the other. The finer the grid, the more exactly information can be transmitted. Given the grid, the welfare of the senders and receiver increases as the constraint is weakened by allowing more feasible actions.

An interesting avenue for future research would be generalizing the form of the constraint. However, there is a trade-off between generalization and specification. The constraint shapes the feasible action set that determines both direction and magnitude of an interim bias, which implies that, given an arbitrary constraint, we might only identify whether or not a fully revealing equilibrium exists as shown by Ambrus and Takahashi. In this paper, we only use the budget constraint, one of the simplest forms of constraints, but characterized the equilibrium as fully as possible.

3.0 DELEGATION AND RETENTION OF AUTHORITY IN ORGANIZATIONS UNDER CONSTRAINED DECISION MAKING

3.1 INTRODUCTION

Suppose a multinational car maker suffers from global economic depression and severe competition. The board of directors appoints a new chief executive officer and requires her to restructure. The chief executive officer (principal) can either assign regional restructuring to the managers (agents) in each region or implement overall reorganization based on information from regional executives.[1] It is well known that there is a trade-off between "loss of control" under delegation and "loss of information" under retention of authority (Dessein, 2002). If the bias in preference between the principal and agents is sufficiently small, delegation is more beneficial to the principal while if the principal has more precise prior information, retention is more favorable. However, this result is based on the assumption that there is no constraint on the principal's decisions. That is, the principal can always make the best decision if and only if she has perfect information.

The purpose of this paper is to study how constraints on the feasible decisions of the principal affect the principal and agents under delegation and retention of authority. The starting point is that the principal may not always be able to make the best decision due to the constraints even if she is completely informed. In the previous example of the auto company, the chief executive officer can be coerced into sales of lucrative assets by impatient creditors though she knows that those assets are critical to the company's long run prospects. Her managerial decisions can be challenged or delayed by labor unions, or by

[1]We use a female pronoun for the principal and a male pronoun for the agents throughout the paper.

regional regulations or laws, or by diplomatic relations between countries.[2]

We build a simple model in which an organization consists of two divisions that are directed by two agents. The payoffs for the principal and agents depend on how well managerial decisions respond to divisional information, but the weight of each division on the entire firm's payoffs is different from each other. The agents have the only information about their own division and do not know about each other's information. The uninformed principal can either delegate divisional decisions to the agents or ask them for divisional information to use her discretion. Without the constraints, the optimal decision for the principal and agents coincides , inducing the same result between delegation and retention. However, the constraints may lead the principal to a decision that is not the best for either or both agents.

In centralized decision making, the agents reveal their information truthfully only if one of the following conditions holds. First, each agent is confident that the principal will take his own preferred decision whatever the other agent reports. Second, the principal is so severely constrained that she cannot change her decision whatever the agents report. Finally, the agents equally share the total profit of two divisions. While the first two conditions depend on a specific setting of the model, the last condition is general in a sense that ex post identical distribution resolves the ex ante strategic conflict between the agents. Since the agents do not know about each other's division, the agents reveal their information if and only if, given the same prior belief about divisional information, truth-telling is Bayesian incentive compatible.

The principal shares the common interest with the agents in the situation that the agents reveal truthfully. This leads to *full* delegation in which the principal delegates both "formal" and "real" authority (Aghion and Tirole, 1997). In other words, the agents decide whatever they like and the principal cannot overrule their decisions. However, truthful revelation (so

[2] In some countries, factory shutdowns are likely to be challaged by labor unions and become a political issue. See "In carmakers collapse, a microcosm of South Korea's woes." New York Times, Febrary 23, 2009; "Goodyears French nightmare." Time, Febrary 1, 2013; and "Goodyear sued by French workers over tire plant shutdown." Bloomberg, April 10, 2013. For a local regulation example, see "China orders polluting foreign companies to relocate." Dong-a Ilbo, July 7, 2011. Territorial disputes in East China and South China sea raise managerial risk for Chinese and Japanese companies. See "In China protests, Japanese car sales suffer." New York Times, October 9, 2012 and "China targeted by Vietnamese in fiery riots." New York Times, May 14, 2014.

full delegation) is not always possible, depending on the extent to which the constraints confine the principal and the distribution of information.

Delegation can more than compensate the principal for loss of control of the divisions by exploiting information of the agents when information revelation is impossible. The principal retains formal authority but the agents acquire real authority in *conditional* delegation. That is, the agents make proposals and the principal can only selectively approve. Nevertheless, the principal can optimally delegate authority by allowing the agents to make a choice within the specific decision set that is matched to the truthfully revealing class in centralized decision making.

Information revelation is interrupted in case of different prior beliefs between the principal and agents. For instance, the chief executive may be skeptical of optimistic revenue or profit forecasts from a regional executive. The regional executive may then compensate by inflating future projections, inciting overstatement of the other regional executive's reports. Such strategic behavior can lead to conflicts of interest even without constraints (Che and Kartik, 2009). The principal may regard the agents as ex ante biased against her no matter what information they actually have. The conflict between the principal and agents consequently causes further conflict between the agents.[3] In this case, meaningful information transmission is impossible and and only optimal delegation can mitigate the strategic tension between the principal and agents.

Related literature. Much of the literature (reviewed below) focuses on a unidimensional structure in which there is one principal and one agent. Contractual monetary transfers allow the principal for either "full commitment"–committing about both her decisions and monetary transfers– or "imperfect commitment"–only committing to transfers (Krishna and Morgan, 2008). The principal's expected payoff is in order of full commitment, optimal delegation, and imperfect commitment. Full delegation is always inferior to full commitment,

[3] In a multidivisional organization, a specific decision on a division might has a spillover effect on the others, leading to strategic conflicts. For example, in 2013, General Motors Co. decided to withdraw Chevrolet from Europe, and instead to focus on promoting the other subsidiary brands Opel and Vauxhall. This decision raised the concern of labor unions in South Korea since GM Korea exported Chevrolet branded vehicles to Europe. GM Korea responded that it might instead produce cars for export to Australia. See "GM pulls Chevy from Europe to focus on Opel, Vauxhall." Wall Street Journal, December 5, 2013; "GM hits end of the road in Australia." Wall Street Journal, December 11, 2013; and "GM likely to shift Holden production to Korea under FTA." ABC News, January 15, 2014.

but superior to imperfect commitment when the preference bias between the principal and agent is sufficiently small. The optimal contract can add to the monetary transfer an inefficient non-monetary incentive such as "money-burning" that only increases the expense of the agent without direct cost of the principal (Ambrus and Egorov, 2013). Nevertheless, to focus on decision making constraints, neither monetary transfer nor money-burning is considered in our model.

Even without monetary transfer, a principal can optimally exploit private information by committing to follow advice from the agent only if the advice satisfies a rule set by the principal (Melumad and Shibano, 1991). The optimal delegation is actually the same as the commitment, and enables the agent to make a choice within feasible decisions in a form of either "threshold"- or "interval"-delegation (Alonso and Matouschek, 2007, 2008). The optimal commitment rule depends on the principal's prior belief about information and "commitment power" that determines how costly it is when the principal overrules her commitment. The principal's preference decides the extent to which she commits to limiting her authority under retention. The less risk averse the principal is, the higher expected payoff the principal can obtain by committing to limit her feasible decisions (Kolotilin, Li and Li, 2013). However, all findings mentioned above are based on the model of a unidimensional principal and agent that is different from our multidimensional problem.

Koessler and Martimort (2012) extend the dimensionality of decisions, and show that optimal delegation can allow the principal to utilize the agent's private information even in multiple dimensions. However, their model can be considered as solving the problem of multiple principals and an agent rather than that of a principal and multiple agents since multidimensional decisions are based on unidimensional information. Moreover, optimal delegation is determined by the average and variance of the bias between the principal and agent in preference on each dimension of the decision space. There is no such a exogenous bias in our model.

The multiple dimensionality issue naturally arises when decision-relevant information is dispersed among multiple divisions within an organization. With a simple model–similar to our model–of one principal and two symmetric division managers, Alonso, Dessein and Matouschek (2008) show that delegation is preferred to retention in general. Rantakari (2008) extends their model by introducing asymmetric allocation of authority to division managers. Despite the similarity, the models in both papers differ from ours since the strategic conflicts stem from the trade-off between adaptation of decision to information within a division and coordination of decisions between the divisions. In this paper, we do not consider harmonious coordination across the divisions but focus on relative adaptation across the divisions under the constraints.

Finally, in a multidimensional environment, the shape of the space has a direct effect on information revelation if the agents have full information in every dimension. Given an unbounded state and decision space, the principal can extract truthful information from the agents by aligning her interest with the agents' interests (Battaglini, 2002). Generally, the shape of the state and decision space and inclusion relations between them determine robust truthful information revelation (Ambrus and Takahashi, 2008; Meyer, Moreno de Barreda and Nafziger, 2013). However, in our model, the effect of the shape is more limited since the agents have only unidimensional information.

The remainder of the paper is organized as follows. Section 3.2 introduces a basic model. Section 3.3 studies truthful revelation in centralizing decision making. Section 3.4 shows optimal delegation in decentralization. Section 3.5 discusses and extends the basic model. Section 3.6 concludes.

3.2 MODEL

An organization consists of two operating divisions, called division 1 and division 2, and one administration team. We call the manager who is in charge of division 1 agent 1, the manager of division 2 agent 2, and the head of the administration team principal.

Information structure. Let $\Theta \equiv [0,1] \times [0,1]$ denote the set of information. The information, random variable $\theta \in \Theta$, has a differentiable probability distribution function, $F(\cdot)$, with a density function $f(\cdot)$ supported on Θ. Each agent privately observes the information about operating his own division but does not observe the information about the other division. In other words, agent 1 observes only $\theta_1 \in \Theta_1$ whereas agent 2 observes only

$\theta_2 \in \Theta_2$.[4] Moreover, the principal knows neither θ_1 nor θ_2.

Preferences. In the divisions, decision y must be responded to θ appropriately. The profit from division i is

$$\pi_i = K_i - (y_i - \theta_i)^2 \tag{3.1}$$

where $K_i \in \mathbb{R}_+$ is the maximum. The principal does not make her own profit but shares the profits of the divisions with the agents. Let $0 < \rho < 1$ denote the weight on the principal. Then, the payoff for the principal is

$$u^P = \rho(\pi_1 + \pi_2). \tag{3.2}$$

The agents share the remaining profit after the principal takes hers; the payoff for agent 1 is

$$u^{A_1} = (1 - \rho)(\lambda\pi_1 + (1 - \lambda)\pi_2), \tag{3.3}$$

and the payoff for agent 2 is

$$u^{A_2} = (1 - \rho)((1 - \lambda)\pi_1 + \lambda\pi_2) \tag{3.4}$$

where $1/2 \leq \lambda \leq 1$ indicates the weight on their own division. Let y_i^I denote the best decision for $I = P, A_1, A_2$ in division i. Then, $y_i^P = y_i^{A_1} = y_i^{A_2}$.

Constraints. There exist constraints that put limits on feasible decisions of the organization. If the principal makes divisional decisions directly, the decisions must be chosen from a feasible decision set that is a proper subset of the information, i.e., $y \in \mathcal{Y} \subsetneq \Theta$. For the sake of simplicity, suppose that $\mathcal{Y}$ is convex. If the principal delegates a division level decision to each agent, she must make sure that the agents' decisions together are compatible with the feasible decision set.

Equilibrium concept. After observing θ_1, agent 1 sends an *unverifiable* message $m_1 \in M_1 \subset \Theta_1$ by a reporting function $\mu_1 : \Theta_1 \rightarrow m_1$. Similarly, agent 2 reports his information according to μ_2. The agents cannot observe each other's message. The principal's belief $\beta(\theta_1, \theta_2 | m_1, m_2)$ is a conditional probability that she assigns to $(\theta_1, \theta_2) \in \Theta$ using Bayes'

[4] A subscript i means the i-th dimension in multidimensional variables throughout the paper. Henceforth, we suppress a subscript for notational brevity, i.e. θ instead of θ_i, unless we need to clarify the identity of the dimension.

rule after receiving the messages m_1 and m_2. The strategy for the principal is a function $\alpha : m_1 \times m_2 \mapsto y$. Then, a perfect Bayesian equilibrium consists of 4-tuple $\{\mu_1, \mu_2, \alpha, \beta\}$ such that

$$(i)\ \mu_1(\theta_1) \in \arg\max_{m_1} E_{\theta_2}[u^{A_1}(\alpha(m_1, \mu_2), \theta)|\theta_1] \text{ for all } \theta_1 \in \Theta_1,$$

$$(ii)\ \mu_2(\theta_2) \in \arg\max_{m_2} E_{\theta_1}[u^{A_2}(\alpha(\mu_1, m_2), \theta)|\theta_2] \text{ for all } \theta_2 \in \Theta_2,$$

$$(iii)\ \alpha(m) \in \arg\max_{y} E_{\theta}[u^P(y, \theta)\beta(\theta \mid m)] \text{ subject to } (y_1, y_2) \in \mathcal{Y} \subsetneq \Theta$$

$$\text{and, given } \Theta = \{(\theta_1, \theta_2) \mid \mu_1(\theta_1) = m_1 \text{ and } \mu_2(\theta_2) = m_2\},$$

$$(iv)\ \beta(\theta \mid m) = \begin{cases} \frac{f(\theta)}{\int_\Theta f(\theta)d\theta} & \text{if } \Theta \neq \emptyset \text{ where f is a pdf.} \\ 0 & \text{otherwise.} \end{cases}$$

Through the paper, a perfect Bayesian equilibrium is simply called equilibrium.

3.3 CENTRALIZED DECISION MAKING

For the sake of clarity, first, we formalize a conflict of interest between the principal and the agents under constraints when the agents reveal their information truthfully, i.e., $(m_1, m_2) = (\theta_1, \theta_2)$. Without the constraints, truth-telling is optimal for the agents since the best decision for the principal is also the best for the agents. However, the constraints may prevent truthful revelation by confining feasible decisions.

Figure 10 illustrates when the preference bias happens and how big it is. Given θ, let $y*^P(\theta)$ denote the optimal decision for the principal and $\mathbf{p}$ the vector from θ to $y*^P$. Similarly, $y*^{A_1}(\theta)$ denotes the optimal decision for agent 1 and $\mathbf{a_1}$ is the vector from θ to $y*^{A_1}$ while $y*^{A_2}(\theta)$ denotes the optimal decision for agent 2 and $\mathbf{a_2}$ is the vector from θ to $y*^{A_2}$. Then, $\chi^{A_i,P} = \mathbf{a}_i - \mathbf{p}$ indicates a state-dependent conflict of agent i against the principal. We call this bias $\chi^{A_i,P}$ an *interim bias* since its size and direction are determined only after information is observed by the agents. The bias is measured by the Euclidean norm $\|\chi^{A_i,P}\|$.

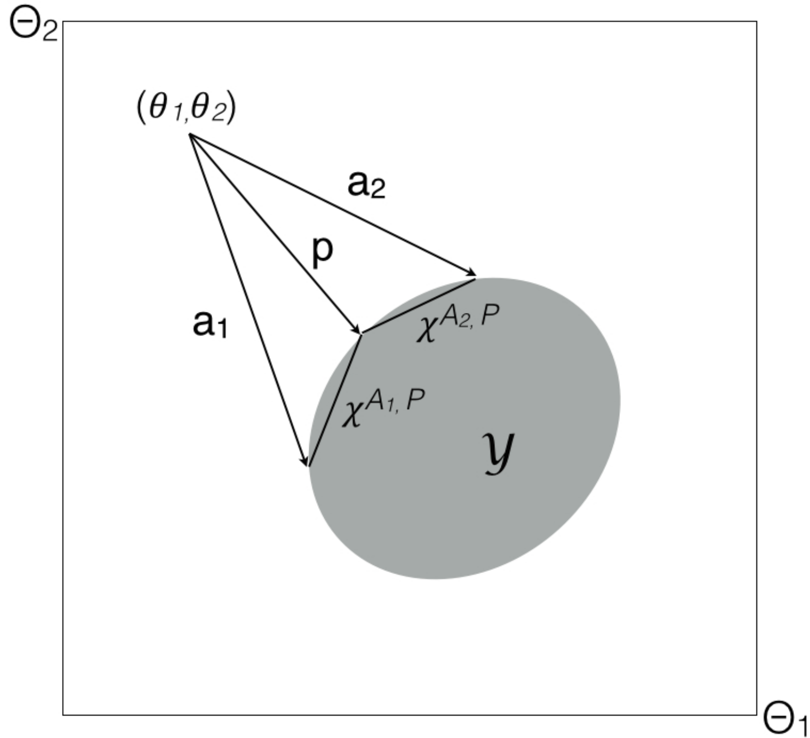

Figure 10: interim bias.

There exists a truthfully revealing equilibrium if and only if $\mathcal{Y}$ and F satisfy the following Bayesian incentive compatibility conditions: for all $(\theta_1, \theta_2), (\theta_1', \theta_2') \in \Theta$,

$$E_{\theta_2}[u^{A_1}(y(\theta_1, \theta_2), \theta)|\theta_1] \geq E_{\theta_2}[u^{A_1}(y(\theta_1', \theta_2), \theta)|\theta_1] \text{ and} \tag{3.5}$$

$$E_{\theta_1}[u^{A_1}(y(\theta_1, \theta_2), \theta)|\theta_2] \geq E_{\theta_1}[u^{A_1}(y(\theta_1, \theta_2'), \theta)|\theta_2]. \tag{3.6}$$

After the agents observe their own divisional information, they would tell the truth if and only if it is Bayesian incentive compatible. Without loss of generality, we focus on agent 1. Even if agent 1 does not observe θ_2, agent 1 learns about $f(\theta_2|\theta_1)$, a conditional probability distribution of θ_2 given θ_1. Given the principal's best response to the messages, agent 1 considers the principal's decision on division 1 as $y_1(\theta_2; m_1)$, a function of θ_2 with a parameter m_1. Then, the expected payoff of agent 1 by reporting m_1 is determined by $y(\theta; m_1)$ and

$f(\theta_2|\theta_1)$. Notice that the Bayesian incentive compatibility conditions (3.5) and (3.6) depend on both the distribution of the information set and the shape of the feasible decisions set.

Lemma 1. *For all $\theta \in \Theta$, $\mathbf{p} = \mathbf{a_1} = \mathbf{a_2}$ if and only if (i) $\theta \in \mathcal{Y}$, or (ii) $\lambda = 1/2$, or (iii) for $0 \leq \underline{y}_i \leq \overline{y}_i \leq 1$, $i = 1, 2$, $\mathcal{Y} = [\underline{y}_1, \overline{y}_1] \times [\underline{y}_2, \overline{y}_2]$.*

Proof. See B.1.1. ☐

Lemma 1 shows that the optimal decisions for the principal, agent 1, and agent 2 coincide with each other if and only if one of the following conditions holds: (i) the decisions matched to the information are feasible to the principal or (ii) the profit of each division equally contributes to the payoff for the agents or (iii) the feasible decision set for the principal has a specific shape. Given that $\lambda \neq 1/2$, $\mathcal{Y} \neq [\underline{y}_1, \overline{y}_1] \times [\underline{y}_2, \overline{y}_2]$ and $\theta \notin \mathcal{Y}$, the inconsistent optimal decisions for the principal, agent 1 and agent 2 lead to a state-dependent bias between each other. Lemma 1 implies that truthful revelation is determined by the profit share λ between the agents and the shape of the feasible decision set of the principal.

The next step is to check when each agent does not reveal his own information truthfully even without knowing the other's information.

Lemma 2. *Suppose that $\lambda \neq 1/2$ and that $\mathcal{Y} \neq [\underline{y}_1, \overline{y}_1] \times [\underline{y}_2, \overline{y}_2]$. Then, agent i tells the truth only if, for all θ_i, there exists θ'_{-i} satisfying $(\theta_i, \theta'_{-i}) \in \mathcal{Y}$.*

Proof. We prove this by contraposition. Without loss of generality, we focus on agent 1. Given θ_1 satisfying that, for all θ'_2, $(\theta_1, \theta'_2) \notin \mathcal{Y}$, $y*^{A_1} \neq y*^P$ for some θ_2 since $\mathcal{Y}$ is convex. Therefore, there exists θ'_1 such that $E_{\theta_2}[u^{A_1}(y(\theta'_1, \theta_2), \theta)|\theta_1] \geq E_{\theta_2}[u^{A_1}(y(\theta_1, \theta_2), \theta)|\theta_1]$. ☐

Lemma 2 shows a necessary condition for truthful revelation. Given θ_1, agent 1 would (over)understate his information if the principal cannot perfectly afford to respond to the divisional information, i.e. $y_1 \neq \theta_1$, whatever agent 2 reports to the principal. The remaining issue is whether agent 1 reports truthfully when the principal may or may not match the best decision for agent 1, depending on information from division 2.

Information is *fully revealed* in an equilibrium if the principal's belief is degenerate after receiving messages, i.e., $\beta(\theta|\mu(\theta)) = 1$ (Battaglini, 2002). Given a "fully revealing equilibrium," there exists a *truthfully revealing* equilibrium in which the agents tell the truth,

$m_1 = \theta_1$ and $m_2 = \theta_2$ for all $(\theta_1, \theta_2) \in \Theta$, and the principal believes the agents, $\beta(\theta|m) = 1$. Without loss of generality, we focus on a truthfully revealing equilibrium since the truthfully revealing equilibrium is outcome-equivalent to any fully revealing equilibrium.

3.4 DECENTRALIZED DECISION MAKING

3.4.1 Delegation of authority

If the principal cannot get useful information from the agents in centralized decision making, it might be better for the principal to delegate divisional decisions to the agents. In decentralized decision making, the principal makes contracts with the agents about the divisional decisions. In the contracts, the principal only decides the scope of authority within which the agents make their own decision. Formally, the optimal delegation contract for the principal is the solution of the following optimization problem:

$$\max_{\mathcal{D} \in \mathcal{P}(\Theta)} E_\theta[u^P(d^*(\theta), \theta)] \tag{3.7}$$

subject to

$$(d_1, d_2) \in \mathcal{D}_1 \times \mathcal{D}_2 \subseteq \mathcal{Y} \subsetneq \Theta, \tag{3.8}$$

$$d_1^* \in \arg\max_{d_1 \in \mathcal{D}_1} E_{\theta_2}[u^{A_1}(d_1, d_2^*, \theta_1, \theta_2)|\theta_1] \text{ and} \tag{3.9}$$

$$d_2^* \in \arg\max_{d_2 \in \mathcal{D}_2} E_{\theta_1}[u^{A_2}(d_1^*, d_2, \theta_1, \theta_2)|\theta_2] \tag{3.10}$$

where $\mathcal{P}(\Theta)$ is the power set of Θ.

Proposition 3. *There exists a solution to the delegation problem of the principal facing constraints.*

Proof. The optimization problem (3.7) is similar to the delegation problem in Holmström except that divisional decisions must be included in feasible decisions satisfying (3.8). The proof follows directly from Theorem 1 in Holmström. Since $\mathcal{Y}$ is convex, (3.8) does not have any effect on the existence of a solution. □

3.4.2 Optimal delegation

Proposition 3 shows that the delegation problem (3.7) has a solution, but does not identify what it is. To describe the characteristics of the solution, we transform the principal's delegation contract with the agents into a direct mechanism without monetary transfers as follows:

$$\max_{g(\theta)} E_\theta[u^P(g(\theta), \theta)] \tag{3.11}$$

subject to, for all $(\theta_1, \theta_2), (\theta_1', \theta_2') \in \Theta$,

$$g(\theta_1, \theta_2), g(\theta_1', \theta_2), g(\theta_1, \theta_2'), g(\theta_1', \theta_2') \in \mathcal{Y} \subsetneq \Theta, \tag{3.12}$$

$$E_{\theta_2}[u^{A_1}(g(\theta_1, \theta_2), \theta)|\theta_1] \geq E_{\theta_2}[u^{A_1}(g(\theta_1', \theta_2), \theta)|\theta_1] \text{ and} \tag{3.13}$$

$$E_{\theta_1}[u^{A_1}(g(\theta_1, \theta_2), \theta)|\theta_2] \geq E_{\theta_1}[u^{A_1}(g(\theta_1, \theta_2'), \theta)|\theta_2]. \tag{3.14}$$

Optimal delegation can be implemented by conditional retention of authority. If an outcome function $g(\theta_1, \theta_2)$ to which the principal commits in the mechanism implements the same decision (d_1, d_2) that the agent would choose from the delegation set $\mathcal{D} = \{(d_1, d_2) : (d_1, d_2) = g(\theta_1, \theta_2), \forall(\theta_1, \theta_2) \in \Theta\}$, then (3.7) and (3.11) are equivalent to each other (Holmström, 1984; Alonso and Matouschek, 2008). In other words, optimal delegation is rubber-stamping, which such that the principal approves the agents' reports if their decisions belong to the set that is determined by the principal. Figure 11 illustrates that the rubber-stamping set is made by partitioning Θ into nine subsets by a lower bound $\underline{\theta}_i$ and upper bound $\overline{\theta}_i$ in the i-th dimension. Then, the principal commits the rule: $y_i = \underline{\theta}_i$ if $m_i \leq \underline{\theta}_i$, $y_i = m_i$ if $\underline{\theta}_i < m_i < \overline{\theta}_i$, and $y_i = m_i$ if $\overline{\theta}_i \leq m_i$ as shown Figure 12.

Lemma 3. *An outcome function $g(\theta)$ satisfies the incentive compatibility conditions if, for all $\underline{\theta}_1, \overline{\theta}_1, \underline{\theta}_2$, and $\overline{\theta}_2$ such that $[\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2] \subset \mathcal{Y}$,*

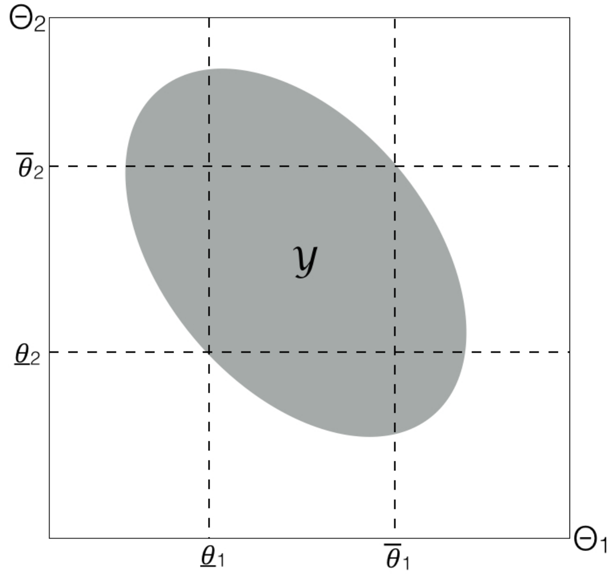

Figure 11: partition.

$$g(\theta_1,\theta_2)=\begin{cases}(\underline{\theta}_1,\underline{\theta}_2) & \textit{if } \theta_1\le\underline{\theta}_1 \textit{ and } \theta_2\le\underline{\theta}_2,\\ (\theta_1,\underline{\theta}_2) & \textit{if } \underline{\theta}_1<\theta_1<\overline{\theta}_1 \textit{ and } \theta_2\le\underline{\theta}_2,\\ (\overline{\theta}_1,\underline{\theta}_2) & \textit{if } \overline{\theta}_1\le\theta_1 \textit{ and } \theta_2\le\underline{\theta}_2,\\ (\underline{\theta}_1,\theta_2) & \textit{if } \theta_1\le\underline{\theta}_1 \textit{ and } \underline{\theta}_2<\theta_2<\overline{\theta}_2,\\ (\theta_1,\theta_2) & \textit{if } \underline{\theta}_1<\theta_1<\overline{\theta}_1 \textit{ and } \underline{\theta}_2<\theta_2<\overline{\theta}_2,\\ (\overline{\theta}_1,\theta_2) & \textit{if } \overline{\theta}_1\le\theta_1 \textit{ and } \underline{\theta}_2<\theta_2<\overline{\theta}_2,\\ (\underline{\theta}_1,\overline{\theta}_2) & \textit{if } \theta_1\le\underline{\theta}_1 \textit{ and } \overline{\theta}_2\le\theta_2,\\ (\theta_1,\overline{\theta}_2) & \textit{if } \underline{\theta}_1<\theta_1<\overline{\theta}_1 \textit{ and } \overline{\theta}_2\le\theta_2,\\ (\overline{\theta}_1,\overline{\theta}_2) & \textit{if } \overline{\theta}_1\le\theta_1 \textit{ and } \overline{\theta}_2\le\theta_2.\end{cases}$$

Proof. See B.1.2. □

Lemma 3 shows that the incentive compatible output function constitutes the truthfully revealing box that is a proper subset of the feasible decision set. Outside the box, the agents only reveal to which class their information belongs. Since any box form of the rubber-stamping set is incentive compatible, there exist multiple delegations in this partial

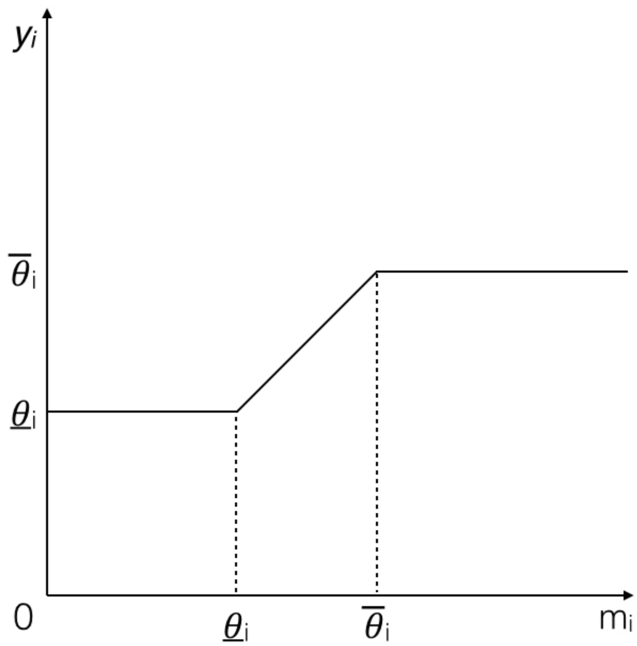

Figure 12: partial revelation.

revelation. To find the optimal rule, we need to specify (i) F, (ii) the shape of $\mathcal{Y}$, and (iii) the location of $\mathcal{Y}$ in Θ. For example, given uniform distribution of F, the bigger and the closer to the center of Θ a box $[\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$ is, the higher is the expected utility of the principal. Notice that at least two diagonal vertices of the truthfully revealing box $[\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$ must locate at the boundary of $\mathcal{Y}$, i.e. $(\underline{\theta}_1, \underline{\theta}_2), (\overline{\theta}_1, \overline{\theta}_2) \in \partial\mathcal{Y}$ or $(\overline{\theta}_1, \underline{\theta}_2), (\underline{\theta}_1, \overline{\theta}_2), \in \partial\mathcal{Y}$ since $\mathcal{Y}$ is convex.

3.4.3 Budget constraint

This subsection provides comparative statics under a specific form of constraints. We introduce a *budget constraint*, a hyperplane that divides the state space into feasible and infeasible set. In other word, under the budget constraint, $\mathcal{Y} = \{(y_1, y_2) | p_1 y_1 + p_2 y_2 \leq w\}$ where $p_1 \geq 0, p_2 \geq 0$ and $0 < w < p_1 + p_2$. For simplicity, suppose that $p_2 = 1$ and that $K_1 = K_2 = 0$ and $\rho = 1/2$ from (3.1), (3.3), and (3.4). An incentive compatible delegation rule has the following form,

$$g(\theta_1,\theta_2)=\begin{cases}(\theta_1,\theta_2) & \text{if } 0\le\theta_1<\overline{\theta}_1 \text{ and } 0\le\theta_2<\overline{\theta}_2,\\ (\overline{\theta}_1,\theta_2) & \text{if } \overline{\theta}_1\le\theta_1\le 1 \text{ and } 0\le\theta_2<\overline{\theta}_2\le 1,\\ (\theta_1,\overline{\theta}_2) & \text{if } 0\le\theta_1<\overline{\theta}_1 \text{ and } \overline{\theta}_2\le\theta_2\le 1,\\ (\overline{\theta}_1,\overline{\theta}_2) & \text{if } \overline{\theta}_1\le\theta_1\le 1 \text{ and } \overline{\theta}_2\le\theta_2\le 1,\end{cases}$$

where $\overline{\theta}_2=w-p\overline{\theta}_1$. Given this form of rule, the expected payoff for the principal is

$$E[u^P]=\frac{1}{6}((1-p^3)\overline{\theta}_1^3+3((w-1)p^2-1)\overline{\theta}_1^2-3((w-1)^2p-1)\overline{\theta}_1+w^3-3w^2+3w-2)),. \tag{3.15}$$

Now, we show the optimal delegation for the principal in term of w and p.

Proposition 4. *The optimal delegation rule is*

$$\theta_1^*=\begin{cases}\frac{w}{p} & \textit{if } 0<p<1, \textit{ and } w<p(1-\sqrt{p}),\\ 0 & \textit{if } p>1 \textit{ and } w<1-1/\sqrt{p},\\ \frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1} & \textit{otherwise.}\end{cases} \tag{3.16}$$

θ_1^* *is weakly increasing in* w *and weakly decreasing in* p.

Proof. The principal confronts the following maximization problem:

$$\begin{aligned}\max_{\overline{\theta}_1\in[0,1]} E[u^P]=&\frac{1}{6}((1-p^3)\overline{\theta}_1^3+3((w-1)p^2-1)\overline{\theta}_1^2\\ &-3((w-1)^2p-1)\overline{\theta}_1+w^3-3w^2+3w-2)).\end{aligned} \tag{3.17}$$

From (3.17),

$$\frac{dE[u^P]}{d\overline{\theta}_1}=(1-p^3)\overline{\theta}_1^2+2((w-1)p^2-1)\overline{\theta}_1-(w-1)^2p+1, \text{ and} \tag{3.18}$$

$$\frac{dE[u^P]}{d\overline{\theta}_1}=0 \text{ at } \overline{\theta}_1=\frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}. \tag{3.19}$$

Now, we check whether a corner solution exists. Suppose that $w>1$. Then,

$$E[u^P|\overline{\theta}_1=(w-1)/p,\overline{\theta}_2=1]=-\frac{1}{6}(1-w+p)^3, \tag{3.20}$$

$$E[u^P|\overline{\theta}_1=1,\overline{\theta}_2=w-p]=-\frac{(1-w+p)^3}{6p^3}, \text{ and} \tag{3.21}$$

$$E[u^P|\overline{\theta}_1=\frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1},\overline{\theta}_2=w-p\frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}]=-\frac{(1-w+p)^3}{6(1+p\sqrt{p})^3}. \tag{3.22}$$

Given $p \geq 1$,

$$\frac{E[u^P|\overline{\theta}_1 = (w-1)/p, \overline{\theta}_2 = 1]}{E[u^P|\overline{\theta}_1 = 1, \overline{\theta}_2 = w - p]} = p^3 \geq 1, \tag{3.23}$$

$$\frac{E[u^P|\overline{\theta}_1 = 1, \overline{\theta}_2 = w-p]}{E[u^P|\overline{\theta}_1 = \frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}, \overline{\theta}_2 = w - p\frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}]} = (\frac{1}{p} + \sqrt{p})^3 > 1 \tag{3.24}$$

Given $p < 1$,

$$\frac{E[u^P|\overline{\theta}_1 = (w-1)/p, \overline{\theta}_2 = 1]}{E[u^P|\overline{\theta}_1 = \frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}, \overline{\theta}_2 = w - p\frac{(w-1)\sqrt{p}+1}{p\sqrt{p}+1}]} = (1 + p\sqrt{p})^3 > 1. \tag{3.25}$$

Therefore, we have an interior solution. Now, suppose that $w \leq 1$. Similarly, we have an interior solution if (i) $0 < p < 1$ and $p - p\sqrt{p} \leq w$, (ii) $p = 1$, or (iii) $p > 1$ and $1 - 1/\sqrt{p} \leq w$. Otherwise, we have a boundary solution.

$$E[u^P|\overline{\theta}_1 = w/p, \overline{\theta}_2 = 0] = \frac{w^3 - 3pw^2 + 4p^2w - 2p^3}{6p^3}, \tag{3.26}$$

$$E[u^P|\overline{\theta}_1 = 0, \overline{\theta}_2 = w] = \frac{w^3 - 3w^2 + 4w - 2}{6} \tag{3.27}$$

For $p < 1$, $E[u^P|\overline{\theta}_1 = w/p, \overline{\theta}_2 = 0] > E[u^P|\overline{\theta}_1 = 0, \overline{\theta}_2 = w]$, and for $p > 1$, $E[u^P|\overline{\theta}_1 = w/p, \overline{\theta}_2 = 0] < E[u^P|\overline{\theta}_1 = 0, \overline{\theta}_2 = w]$. Since

$$\frac{d\theta_1^*}{dw} = \begin{cases} \frac{1}{p} > 0 & \text{for } 0 < p < 1, \text{ and } w < p(1 - \sqrt{p}), \\ 0 & \text{for } p > 1 \text{ and } w < 1 - 1/\sqrt{p}, \\ \frac{\sqrt{p}}{1+p\sqrt{p}} > 0 & \text{otherwise,} \end{cases} \tag{3.28}$$

θ_1^* is weakly increasing in w. Since,

$$\frac{d\theta_1^*}{dp} = \begin{cases} -\frac{w}{p^2} < 0 & \text{for } 0 < p < 1, \text{ and } w < p(1 - \sqrt{p}), \\ 0 & \text{for } p > 1 \text{ and } w < 1 - 1/\sqrt{p}, \\ \frac{(w-1)(1-2p\sqrt{p})-3p}{2\sqrt{p}(1+p\sqrt{p})^2} < 0 & \text{otherwise,} \end{cases} \tag{3.29}$$

θ_1^* is weakly decreasing in p. □

Corollary 2. *Fix p. The optimal payoff for the principal is increasing in w. Now, fix w. Given $w \leq 1$, the optimal payoff for the principal is constant in $p \in (1/(1-w)^2, \infty)$. Otherwise, the payoff is decreasing in p.*

Proof. From (3.15) and (3.16), the optimal payoff for the principal is

$$u^{*P} = \begin{cases} \frac{1}{6}(w^3 - 3w^2 + 3w - 2) & \text{if } w \leq 1 \text{ and } p > 1/(1-w)^2 \\ -\frac{1}{6}\frac{(1-w+p)^3}{(1+p\sqrt{p})^2} & \text{otherwise} \end{cases} \tag{3.30}$$

$$\frac{du^{*P}}{dw} = \begin{cases} (1-w)^2/2 & \text{if } w \leq 1 \text{ and } p > 1/(1-w)^2 \\ \frac{(1-w+p)^2}{2(1+p\sqrt{p})^2} & \text{otherwise} \end{cases} \tag{3.31}$$

$$\frac{du^{*P}}{dp} = \begin{cases} 0 & \text{if } w \leq 1 \text{ and } p > 1/(1-w)^2 \\ \frac{((1-w)\sqrt{p}-1)(1-w+p)^2}{2(1+p\sqrt{p})^3} & \text{otherwise} \end{cases} \tag{3.32}$$

□

3.5 DISCUSSION AND EXTENSION

3.5.1 Exogenous bias

Up to now, we have assumed that there is no preference bias between the principal and agents. Now, suppose that, without loss of generality, the profit from division 1 to agent 1 is $\pi_1^{A_1} = K_1 - (y_1 - \theta_1 - b)^2$, $b > 0$ while the profit from division 1 to the principal is the same as before, i.e. $\pi_1^P = K_1 - (y_1 - \theta_1)^2$. In other words, agent 1 has a constant positive bias against the principal in division 1. Since $y^{A_1}(\theta) \neq y^P(\theta)$ for all $\theta \in \Theta$, truthful revelation is impossible by Proposition **??**. However, the principal may still get some benefit by delegating her authority to the agents.

Delegation of authority decomposes a two-dimensional problem into two one-dimensional problems by allowing the agents to make decisions in their own division. Given divisional delegation, there is no strategic tension between the agents since the agents cannot exercise influence to each other. Now, we can regard the two-divisional delegation as two independent one-divisional delegations.

Proposition 5 (Proposition 1. of Alonso and Matouschek (2008))**.** *The principal and the agent are minimally aligned if there exists a state* $\theta^* \in (0,1)$ *such that* $E(y^P(\theta)|\theta \leq \theta^*) < y^A < E(y^P(\theta)|\theta \geq \theta^*)$*. Delegation is valuable if an only if the principal and the agent are minimally aligned.*

The minimal alignment condition by Alonso and Matouschek implies that delegation is beneficial to the principal as long as the state-independent constant bias b is small enough. Extracting exact information from the agent can more than compensate for the loss of deviation from the best decision for the principal.

3.5.2 Different prior beliefs

In Section 3.3, we assumed that the agents have the only information about their own divisions. Now, suppose that the agents do not have this information but only obtain a relative signal. Neither agent can observe the other's signal. The divisional information is drawn from a bivariate normal distribution. The principal and agents have the same prior belief about the variance of the distribution, but different beliefs about the mean: for $I = P, A_1, A_2$,

$$\begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_1^I \\ \mu_2^I \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right).$$

For simplicity, let $\mu_1^P = \mu_2^P = 0$ and $\mu_1^{A_i} = \mu_2^{A_i} > 0$. The prior beliefs are common knowledge. The signal s_i is a random variable and has a mean μ_i and a variance $\tilde{\sigma}_i^2$, i.e. $s_i \sim \mathcal{N}(\mu_i, \tilde{\sigma}_i^2)$. Given s_i, the posterior belief is

$$\theta_i | s_i \sim \mathcal{N}\left(\frac{\sigma_i^2}{\sigma_i^2 + \tilde{\sigma}_i^2} s_i + \frac{\tilde{\sigma}_i^2}{\sigma_i^2 + \tilde{\sigma}_i^2} \mu_i, \frac{\sigma_i^2 \tilde{\sigma}_i^2}{\sigma_i^2 + \tilde{\sigma}_i^2} \right).$$

This posterior belief induces the principal's expected utility as follows:

$$
\begin{aligned}
E[u^P|s_1,s_2] &= E[\rho(K_1-(y_1-\theta_1)^2+K_2-(y_2-\theta_2)^2)|s_1,s_2] \\
&= \rho\{(K_1+K_2)-E_{\theta_1}[(y_1-\theta_1)^2|s_1]-E_{\theta_2}[(y_2-\theta_2)^2|s_2]\} \\
&= \rho\{(K_1+K_2)-(y_1-E_{\theta_1}[\theta_1|s_1])^2 \\
&\quad -Var(\theta_1|s_1)-(y_2-E_{\theta_2}[\theta_2|s_2])^2-Var(\theta_2|s_2)\} \\
&= \rho\{(K_1+K_2)-(y_1-\frac{\sigma_1^2}{\sigma_1^2+\tilde{\sigma}_1^2}s_1)^2-\frac{\sigma_1^2\tilde{\sigma}_1^2}{\sigma_1^2+\tilde{\sigma}_1^2}-(y_2-\frac{\sigma_2^2}{\sigma_2^2+\tilde{\sigma}_2^2}s_2)^2-\frac{\sigma_2^2\tilde{\sigma}_2^2}{\sigma_2^2+\tilde{\sigma}_2^2}\}
\end{aligned} \tag{3.33}
$$

Therefore, the optimal decision for the principal is $\tilde{y}*^P(s)=(\sigma_1^2s_1/(\sigma_1^2+\tilde{\sigma}_1^2),\sigma_2^2s_2/(\sigma_2^2+\tilde{\sigma}_2^2))$. In this section, we focus on agent 1 without loss of generality. The expected utility for agent 1 is

$$
\begin{aligned}
E[u^{A_1}|s_1] &= E[(1-\rho)\{\lambda(K_1-(y_1-\theta_1)^2)+(1-\lambda)(K_2-(y_2-\theta_2)^2)\}|s_1] \\
&= (1-\rho)\lambda\{K_1-(y_1-\frac{\sigma_1^2}{\sigma_1^2+\tilde{\sigma}_1^2}s_1-\frac{\tilde{\sigma}_1^2}{\sigma_1^2+\tilde{\sigma}_1^2}\mu_1^{A_1})^2-\frac{\sigma_1^2\tilde{\sigma}_1^2}{\sigma_1^2+\tilde{\sigma}_1^2}\} \\
&\quad +(1-\rho)(1-\lambda)\{K_2-(y_2-\mu_2^{A_1})^2-\sigma_2^2\},
\end{aligned} \tag{3.34}
$$

and maximized by the decision $\tilde{y}*^{A_1}(s_1)=((\sigma_1^2s_1+\tilde{\sigma}_1^2\mu_1^{A_1})/(\sigma_1^2+\tilde{\sigma}_1^2),\mu_2^{A_1})$. Similarly, the optimal decision for agent 2 is $\tilde{y}*^{A_2}(s_2)=(\mu_1^{A_2},(\sigma_2^2s_2+\tilde{\sigma}_2^2\mu_2^{A_2})/(\sigma_2^2+\tilde{\sigma}_2^2))$. Notice that the interim bias between the principal and agent i is signal-independent in his own dimension and signal-dependent in the other dimension. In other words, the interim utilities have the forms

$$u^P=-(y_1-s_1')^2-(y_2-s_2')^2 \tag{3.35}$$

$$u^{A_1}=-\lambda(y_1-s_1'-b_1)^2-(1-\lambda)(y_2-x_2)^2 \tag{3.36}$$

$$u^{A_2}=-(1-\lambda)(y_1-x_1)^2-\lambda(y_2-s_2'-b_2)^2 \tag{3.37}$$

where $s_1'=\sigma_1^2s_1/(\sigma_1^2+\tilde{\sigma}_1^2)$, $s_2'=\sigma_2^2s_2/(\sigma_2^2+\tilde{\sigma}_2^2))$, $b_1=\tilde{\sigma}_1^2\mu_1^{A_1}/(\sigma_1^2+\tilde{\sigma}_1^2)$, $b_2=\tilde{\sigma}_2^2\mu_2^{A_2}/(\sigma_2^2+\tilde{\sigma}_2^2))$, $x_1=\mu_1^{A_2}$, and $x_2=\mu_2^{A_1}$. Now, the agent 1 has a constant bias in division 1 and the best decision for agent 1 in division 2 is independent of the 2-divisional signal. Therefore, information transmission is impossible and delegation is beneficial only if the bias is small enough.

3.5.3 Semi-revealing

One might think that information still can be partially transmitted even in the case in which truthful revelation is impossible. That is, there might exist a "semi-revealing" equilibrium (Krishna and Morgan, 2001). In equilibrium, the agents categorize their feasible information set into multiple classes. If information is within a certain specific class, the agents reveal truthfully. Otherwise, the agents only reveal to which class information belongs. Formally, given that $\lambda \neq 1/2$, $\underline{\theta}_1 < \overline{\theta}_1$ and $\underline{\theta}_2 < \overline{\theta}_2$ such that $[\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2] \subset \mathcal{Y}$, there exists a 4-tuple $\{\mu_1, \mu_2, \alpha, \beta\}$ such that

$$\mu_1(\theta_1) = \begin{cases} m_1 \in [0, \underline{\theta}_1] & \text{if } \theta_1 \le \underline{\theta}_1, \\ m_1 = \theta_1 & \text{if } \underline{\theta}_1 < \theta_1 < \overline{\theta}_1, \\ m_1 \in [\overline{\theta}_1, 1] & \text{if } \overline{\theta}_1 \le \theta_1; \end{cases} \quad \mu_2(\theta_2) = \begin{cases} m_2 \in [0, \underline{\theta}_2] & \text{if } \theta_2 \le \underline{\theta}_2, \\ m_2 = \theta_2 & \text{if } \underline{\theta}_2 < \theta_2 < \overline{\theta}_2, \\ m_2 \in [\overline{\theta}_2, 1] & \text{if } \overline{\theta}_2 \le \theta_2; \end{cases}$$

$$\alpha(m) \in \arg\max_y E_\theta[u^P(y, \theta)\beta(\theta \mid m)] \text{ subject to } (y_1, y_2) \in \mathcal{Y} \subsetneq \Theta;$$

$$\beta(\theta_1, \theta_2 \mid m_1, m_2) \sim U_2([\underline{m}_1, \overline{m}_1] \times [\underline{m}_2, \overline{m}_2]) \text{ where}$$

$$\underline{m}_1 = \begin{cases} 0 & \text{if } m_1 \le \underline{\theta}_1, \\ \overline{m}_1 & \text{if } \underline{\theta}_1 < m_1 < \overline{\theta}_1, \\ \overline{\theta}_1 & \text{if } \overline{\theta}_1 \le m_1; \end{cases} \qquad \overline{m}_1 = \begin{cases} \underline{\theta}_1 & \text{if } m_1 \le \underline{\theta}_1, \\ \underline{m}_1 & \text{if } \underline{\theta}_1 < m_1 < \overline{\theta}_1, \\ 1 & \text{if } \overline{\theta}_1 \le m_1; \end{cases}$$

$$\underline{m}_2 = \begin{cases} 0 & \text{if } m_2 \le \underline{\theta}_2, \\ \overline{m}_2 & \text{if } \underline{\theta}_2 < m_2 < \overline{\theta}_2, \\ \overline{\theta}_2 & \text{if } \overline{\theta}_2 \le m_2; \end{cases} \qquad \overline{m}_2 = \begin{cases} \underline{\theta}_2 & \text{if } m_2 \le \underline{\theta}_2, \\ \underline{m}_2 & \text{if } \underline{\theta}_2 < m_2 < \overline{\theta}_2, \\ 1 & \text{if } \overline{\theta}_2 \le m_2. \end{cases}$$

However, this 4-tuple cannot be an equilibrium since agent i has an incentive to deviate at $\theta_i = \underline{\theta}_i, \overline{\theta}_i$. Figure 13 illustrates why this deviation occurs by focusing on agent 1 without loss of generality. Suppose that sender 1 observes $\underline{\theta}_1$. If agent 1 tells the truth, then the principal takes action marked as the points of star shape and stitched line depend on sender 2's report, leading the interim bias $\chi(\underline{\theta}_1)$. However, agent 1 can pull the principal's decision, marked as the points of square shape and solid line, closer to his best decision by overstating his information by ϵ, i.e. $m_1 = \underline{\theta}_1 + \epsilon$.

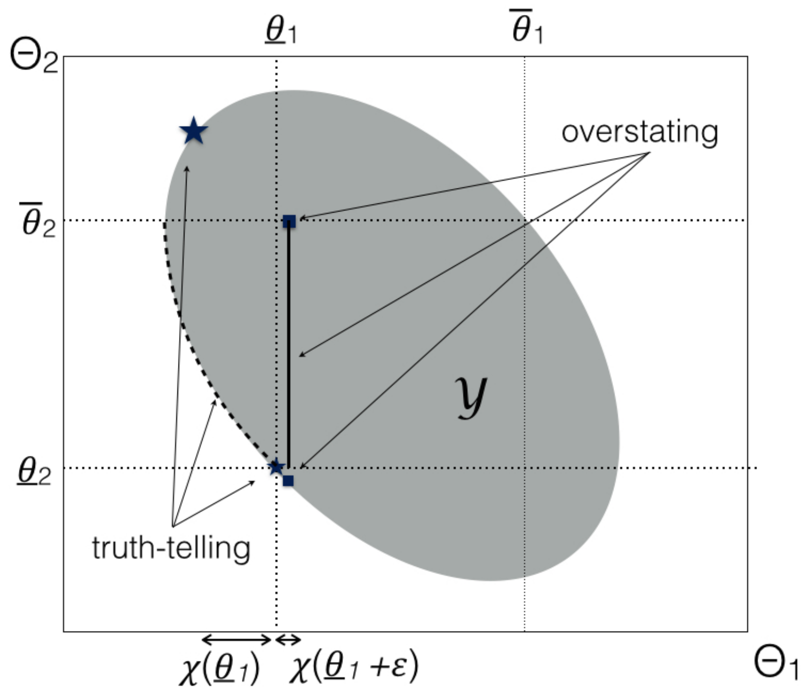

Figure 13: Semi-revelation vs conditional delegation.

3.6 CONCLUSION

We study the effect of constraints on the principal under centralized or decentralized decision making. The constraints limit the feasible decisions of the principal, which might cause strategic tension with agents who have common preferences. This strategic conflict of interest may hinder information revelation under centralized decision making. Given the same prior belief about information, truthful revelation by the agents is determined by the extent to which the constraints confine the principal and the distribution of information. In case that the agents do not reveal, the principal would be better off by conditional delegation in which the principal approves the agents' decisions only if their decisions belong to the predetermined set. Optimal delegation depends on the distribution of information, the shape of a feasible decision set, and the location of the feasible decision set on information space.

An interesting avenue for future research would be generalizing the form of the constraint.

For example, we can modify the feasible decision set into a disconnected form which responds to discrete choice models. Technically, discrete space might show different results from continuous space.

4.0 TENURE REFORM AND QUALITY GAP BETWEEN SCHOOLS

4.1 INTRODUCTION

"[T]he Challenged Statutes ['Permanent Employment Statute, Dismissal Statutes, Last-In-First Out'] violate [plaintiffs'] fundamental rights to equality of education by adversely affecting the quality of the education [plaintiffs] are afforded by the state."
- from Vergara v. California - Judgment (Superior Court of the State of California, County of Los Angeles August 27, 2014)

Teacher tenure[1] is a special form of contracts that was created to protect qualified teachers from arbitrary, unfair, and discriminatory dismissals in history. However, as seen Vergara v. California lawsuit, teacher tenure may cause a problem of keeping not only the decent but also the incompetent teachers in public schools. This problem makes education policy-makers reform tenure system in many states. This movement has been spurred by national policy initiatives such as the No Child Left Behind act (NCLB) and the Race to the Top (RTT).

The remarkable feature of the tenure reform is introducing objective measures of students performance and relating their outcomes to the contract with teachers. Teachers' performance ratings are the primary consideration for tenure grant in eleven states, and one of criteria for decisions of tenure in sixteen states. Five states require dismissal of low-performing teachers failing to improve for multiple years of remediation, and twenty seven states allow dismissal of low-performing teachers.[2]

The purpose of this article is to study the effect of switching from tenure (continuing)

[1] In the United States, tenure is also called continuing contract or permanent employment status depending on states. Some states passed laws terminating tenure, but instead made a (long and costly) due-process that effectively protects most experienced teachers (McGuinn, 2010).

[2] For detailed information, visit the website of the Education Commission of of the States, `http://www.ecs.org/html/educationIssues/teachingQuality/teacherdb_intro.asp`.

to non-continuing contract on students' welfare with a simple model. Even though it is itself interesting topic to establish a robust evaluation mechanism against noisy performance outcomes, it is not our primary concern. Rather, we pay attention to the fact that the non-continuing contract might lead to undesirable consequences such as widening the quality gab between public schools by teachers' entry, exit, or transfer following new incentives.

In our model, there are two public schools. A school consists of a principal, a teacher, and students. The principals hire teachers for two periods. The principals do not know the type of the teacher, good or bad, but they receive a binary evaluation report (indicating satisfactory or not) based on the performance of the students. The result depends on not only the teachers' quality but also the students' ability. If a good (bad) teacher is matched to a high (low) sociodemographic students, he is likely (but not always) to receive a satisfactory (unsatisfactory) evaluation. However, a good teacher is matched to a low student or vise versa, then the evaluation is uninformative–the teacher equally receives either the satisfactory or unsatisfactory. The principals keep their teachers for the second period irrespective of the evaluation under the continuing contract while they renew the contract under the non-continuing contract.

Under the non-continuing contract, the principals renew the contract only if the teachers receive a satisfactory evaluation. The welfare is improved compared to the continuing contract case, but the gap between a good and bad school becomes wider under the continuing contract. This increased gap is caused by a unilateral transfer of a qualified teacher from the good school to bad school.

Literature review. There is a literature about moral hazard problems under a multidimensional environment. Teaching is a multidimensional task. Educators are required to not only teach measurable basic skills but also promote curiosity and creative thinking. However, test-based evaluations may lead teachers to focusing on tested skills. (Holmstrom and Milgrom, 1991) show that the difficulty of measuring performance in activities other than testable skills decreases the desirability of providing incentives for better test results. Different from this literature, we focus on an one-dimensional adverse selection problem in a dynamic setting.

The successful tenure reform depends on not only the exit of bad teachers but also the replacement by good teachers. Rockoff et al. (2012) show that teachers with low evaluation are more likely to leave their school. Principals do not approve the tenure but only extend the probationary period for many teachers who would have been approved prior to the reform (Loeb, Miller and Wyckoff, 2015). However, given the extended probationary period, teachers with higher academic qualifications pretend to leave rather than stay in the same school. Even before the reform, the new hired are more likely to leave schools with lower test scores, higher poverty, and more racial minority (Scafidi, Sjoquist and Stinebrickner, 2007). The reform may give schools with low sociodemographic status more significant difficulty in hiring good teachers. This empirical evidence fits our model.

There are concerns of practical use of performance measures–for example, value-added method (VAM)–for high-stakes decisions.[3] Simple VAMs might be misleading due to a *students-teachers sorting bias* (Rothstein, 2009, 2010; Koedel and Betts, 2011).[4] Not surprisingly, there is a sorting based on a prior test score in spite of a significant variance across schools (Dieterle et al., 2015).[5] Free and reduced price-lunch (FRL) students tend to be allocated to inexperienced teachers while non-FRL students are more likely matched to the experienced (Goldhaber and Hansen, 2013). This tendency gives a disadvantage to the new hired leading to the rate of exit. Our model consider this noisy evaluation problem.

The remainder of the paper is organized as follows. Section 4.2 introduces a basic model. Section 4.3 shows a continuing contract, and Section 4.4 studies a non-continuing contract without transfers. In Section 4.5, we focus on teachers' transfer between schools. Section 4.6 concludes.

[3] American Statistical Association issued a statement on the use of value-added models in education policy (American Statistical Association, 2014) and Chetty, Friedman and Rockoff (2014) discussed the statement.

[4] Goldhaber and Chaplin (2015) check a robustness of the falsification test of Rothstein (2010), and argue that "the test proposed in Rothstein (2010) will often falsify VAMs that are unbiased and fail to falsify VAMs that are biased."

[5] Not only administrators but also teachers and parents considerably intervene in the process (Paufler and Amrein-Beardsley, 2013).

4.2 MODEL

There are two public schools. A school consists of a principal, a teacher, and students. The teachers are classified into two types, good or bad, by quality. The type set of the teachers is $T = \{g, b\}$ where g is good and b means bad. The principals do not know a teacher's type, and the probability of g is q. The students are also categorized by various factors such as a previous academic performance and demographic characteristics. The type set of the students is $\Theta = \{l, h\}$ where l denotes a low and h indicates a high sociodemographic profile. The ratio of l for each school, $\lambda = \lambda_1, \lambda_2$, distinguishes two schools. For $0 < \lambda_1 < \lambda_2 < 1$, those schools are categorized into a good (one with λ_1) and bad school (the other with λ_2).

The principals hire teachers for two periods. At the beginning of the first period, the principals hire new teachers. The teachers decide whether to stay, leave, or transfer before the second period. Under the continuing contract (tenure), the schools unconditionally continue the contract with their teachers for the remaining periods if the teachers want to stay. However, under the non-continuing contract, the principals make decisions whether to renew or terminate the contract for the second period. After terminating, the principals accept a transfer from the other school or hire a new one.

For each period, the students take exams that evaluate their academic performance by quantifiable standards. The students' academic performance y is determined by their type and their teacher's type. For $\theta \in \Theta$ and $t \in T$,

$$y(\theta, t) = \alpha_1 \mathbb{1}_{\{h\}} + \alpha_2 \mathbb{1}_{\{l\}} + \alpha_3 \mathbb{1}_{\{g\}} + \alpha_4 \mathbb{1}_{\{b\}} + \epsilon \tag{4.1}$$

where $\alpha_1 > \alpha_3 > \alpha_2 > \alpha_4$. $\mathbb{1}$ is an indicator function and ϵ is a shock with $E[\epsilon] = 0$. Equation (4.1) implies that the teachers have effect on their students' performance but their effect is dominated by the students' innate characteristics.

The teachers receive teaching evaluation for each period. The evaluation is determined by the academic achievement of the students, and is divided into two grades, satisfactory and unsatisfactory. The set of evaluation is $V = \{s, u\}$ where s means a satisfactory and u indicates an unsatisfactory grade. However, the evaluation is not perfect and the results are noisy. Suppose that $\alpha_1 + \alpha_4 = \alpha_2 + \alpha_3$ in (4.1). Then, $E[y|g, h] > E[y|g, l] = E[y|b, h] > E[y|b, l]$.

The combination of a good teacher and high profile students is likely to result in satisfactory grade while that of a bad teacher and low profile students tends to get unsatisfactory grade. The case of a good teacher and low profile students causes both grades with the same probability. It is also true for the case of a bad teacher and high profile students. In other words, the conditional probabilities of the grades are $P(s|g,h) = P(u|b,l) = \mu > 1/2$, $P(u|g,h) = P(s|b,l) = 1-\mu$, and $P(s|g,l) = P(u|g,l) = P(s|b,h) = P(u|b,h) = 1/2$.

The payoff of a principal, u^P, is determined by the students' performance that depends on their teachers' type. For simplicity, suppose that for $1/2 \leq q < 1$, $\alpha_3 = 1-q$ and $\alpha_4 = -q$ in (4.1). We assume that hiring a bad teacher is better than hiring no one. A teacher's payoff, u^A, is 1 when hired (or renewed) and 0 when not hired (dismissed) regardless of the type. However, a good teacher has a equal to or higher outside option than a bad, i.e. $0 \leq e_b \leq e_g < 1$. Table 2 summarizes the payoffs.[6]

Table 2: Payoffs

principal's	teacher's type	
decision	g	b
hiring	$(1-q, 1)$	$(-q, 1)$
no-hiring	$(-1, 0)$	$(-1, 0)$

4.3 CONTINUING CONTRACT

How does a non-continuing contract change the payoff of the principals? First of all, we consider the continuing contract case for a reference. After hiring new teachers, both schools must hold them irrespective of their type. For simplicity, suppose that there is no time discounting for the second period.

[6]The parameter q implies "threshold of reasonable doubt" (Feddersen and Pesendorfer, 1998). If the principals believe that their teachers are bad with probability higher than q, they prefer the dismissal to the renewal (Austen-Smith and Banks, 1996).

Lemma 4. *Under a continuing contract, both good and bad teachers apply for schools and principals hire them.*

Proof. Let a denote hiring a teacher and n no hiring. By assumptions, for $t \in T$,

$$u_t^A(a) > e_t \geq u_t^A(n) \text{ and } \tag{4.2}$$

$$E[u^P(a)] = 2(q(1-q) - (1-q)q) = 0 > E[u^P(n)] = -1. \tag{4.3}$$

□

From Lemma 4, the expected payoff of a principal under the continuing contract is $U_{con}^P = 0$.

4.4 NON-CONTINUING CONTRACT

Principals are Bayesian decision makers. Based on the evaluation result, the principals update their belief about the type of the teachers. For example, in the first period, the evaluation of s gives rise to a posterior belief that their teacher is a good type,

$$\begin{aligned} P(g|s) &= \frac{P(s|g)P(g)}{P(s)} \\ &= \frac{P(s|g,l)P(g)P(l) + P(s|g,h)P(g)P(h)}{P(s|g,l)P(g)P(l) + P(s|g,h)P(g)P(h) + P(s|b,l)P(b)P(l) + P(s|b,h)P(b)P(h)} \\ &= \frac{q(\lambda + 2\mu - 2\lambda\mu)}{1 + \lambda - 2\lambda\mu + q(2\mu - 1)} > q. \end{aligned} \tag{4.4}$$

Since

$$\frac{dP(g|s)}{d\mu} = \frac{2(1-q)q}{(1 + \lambda - 2\lambda\mu + q(2\mu - 1))^2} > 0 \text{ and} \tag{4.5}$$

$$\frac{dP(g|s)}{d\lambda} = \frac{(1-q)q(1-2\mu)^2}{(1 + \lambda - 2\lambda\mu + q(2\mu - 1))^2} > 0, \tag{4.6}$$

the more accurate the evaluation and the lower is the level of the bad school, the more likely the teacher is good.

The principals renew the contract if the expected payoff of renewal is bigger than or the same as that of dismissal. The set of the principal's decision in the second period is $D = \{r, d\}$ where r indicates renewal and d means dismissal. Then, the principals confront the following optimization problem: For $v \in V = \{s, u\}$, $t \in T = \{g, b\}$,

$$\max_{\delta \in D = \{r,d\}} u^P(\delta|t)P(t|v)$$

subject to

$$P(r|s)P(s|t) + P(r|u)P(u|t) \geq e_t. \tag{IR}$$

Suppose that $e_g - e_b \geq \mu - 1/2$. Then, the individual rationality conditions hold.[7]

Lemma 5. *If the teachers receive a satisfactory evaluation, the principals renew the contract, otherwise, terminate the contract.*

Proof. The solution concept is perfect Bayesian equilibrium. Suppose that the teachers receive a satisfactory evaluation. Then, the expected utility of the renewal is

$$\begin{aligned} E[u^P(r|s)] &= u^P(r|g)P(g|s) + u^P(r|b)P(b|s) \\ &= \frac{(1-q)q(2\mu-1)}{1+\lambda-2\lambda\mu+q(2\mu-1)}, \end{aligned} \tag{4.7}$$

and the expected utility of the termination is

$$E[u^P(d|s)] = u^P(d|g)P(g|s) + u^P(d|b)P(b|s) = 0. \tag{4.8}$$

Given $1/2 \leq q < 1$, $1/2 < \mu < 1$, and $0 < \lambda < 1$, $E[u^P(r|s)] > E[u^P(d|s)]$. Now, suppose that the teacher receives an unsatisfactory evaluation. Then,

$$E[u^P(d|u)] - E[u^P(r|u)] = \frac{(1-q)q(2\mu-1)}{-1+\lambda-2\lambda\mu+q(2\mu-1)} > 0. \tag{4.9}$$

$\square$

The expected payoff of a principal under a non-continuing contract is

$$U^P_{nc} = \sum_{\delta=r,d}\sum_{t=g,b} u^P(\delta|v)P(v|\lambda,t)P(t) = \frac{1}{2}(1-q)q(2\mu-1) > 0 \tag{4.10}$$

[7] See Appendix C.1 for detail.

Proposition 6. *The non-continuing contract is beneficial to the principals compared to the continuing contract. There is no difference between schools.*

Proof. It holds by 4.10. □

4.5 TRANSFER

In this section, we focus on the movement of teachers between schools. For simplicity, suppose that the teachers want to receive the satisfactory evaluation in the second period.[8] After the first period, the principals terminate the contract with unsatisfactory teachers. Now, they have to fill the vacancy by hiring a transferred or new teacher. However, satisfactory teachers want to make a contract only with the good school since the probability of receiving the satisfactory evaluation for the next period is higher.

Lemma 6. *Suppose that there is a vacancy in a good school for the second period. If a teacher in a bad school received a satisfactory evaluation, there is a transfer from the bad to good school. Suppose that there is a vacancy in a bad school for the second period. There is no transfer and the bad school hires a new teacher.*

Proof. For $\lambda_1 < \lambda_2$, $U_g^A(a|\lambda_1) > U_g^A(a|\lambda_2)$ from (C.4), and $U_b^A(a|\lambda_1) > U_b^A(a|\lambda_2)$ from (C.5). Therefore, teachers want to be hired in a good school irrespective of their type for the second period.

The expected payoff of a principal from hiring a transferred teacher who has a satisfactory evaluation is

$$\begin{aligned} U_{tr}^P(a|s) &= u^P(a|g)P(g|s) + u^P(a|b)P(b|s) \\ &= \frac{(1-q)q(2\mu-1)}{1+\lambda-2\lambda\mu+q(2\mu-1)} \end{aligned} \tag{4.11}$$

[8] Since the second period is the last period, the evaluation has no effect on teachers' employment. However, if the periods of the basic model are extended to three, the evaluation for the second period determines the expected payoff for the third period. With this assumption, our model becomes the reduced form of a three period game.

while the expected payoff from hiring a new one is.

$$U_n^P(a) = u^P(a|g)P(g) + u^P(a|b)P(b) = 0. \tag{4.12}$$

Since

$$U_{tr}^P(a|s) - U_n^P(a) > 0, \tag{4.13}$$

the principals prefer a transferred teacher. □

Figure 14 illustrates the possible outcomes on the equilibrium path for the first period. The first (second) element of the parentheses in the first row denotes the type of a teacher in the good (bad) school. The second row indicates the evaluation result of the teacher. The third row represents the type in the next period after teachers' exit, transfer, or entry.

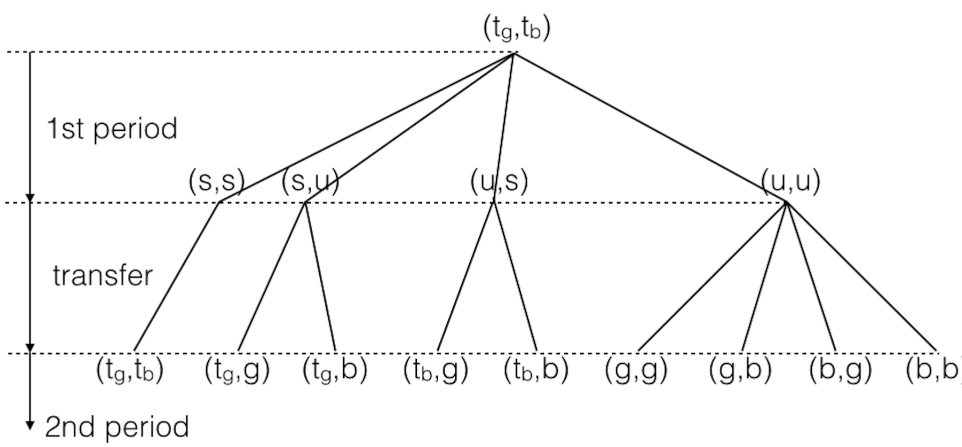

Figure 14: Extensive form on the equilibrium path for the first period

Due to the unilateral transfer from the bad to the good school, the good school hires a satisfactory teacher for the next period unless both teachers receive an unsatisfactory evaluation at the same time. However, the bad school has to hire a new teacher unless both teachers receiver a satisfactory result.

The expected payoff of the principal in the good school is

$$U_{tr}^{P_g} = \frac{1}{4}(1-q)q(2\mu - 1)(3 - (2\mu - 1)(q - \lambda_1)) > 0, \tag{4.14}$$

and the expected payoff of the principal in the bad school is

$$U_{tr}^{P_b} = \frac{1}{4}(1-q)q(2\mu-1)(1-(2\mu-1)(q-\lambda_1)) > 0. \tag{4.15}$$

Proposition 7. *Teachers' transfer is only beneficial to the good school. The gap of expected payoff is increasing in the quality of the good school.*

Proof. From (4.10), (4.14), and (4.15), $U_{tr}^{P_g} > U_{nc}^{P_g}$ and $U_{tr}^{P_b} < U_{nc}^{P_b}$.

$$\frac{d(U_{tr}^{P_g} - U_{tr}^{P_b})}{d\lambda_1} = \frac{1}{2}(1-q)q(2\mu-1)^2 > 0. \tag{4.16}$$

□

Teachers' transfer increases the payoff of the principal in the good school compared to non-transfer. However, the bad school suffer from the loss of a satisfactory teacher, leading to decreasing expected payoff.

4.6 CONCLUSION

The primary concern of this article is that introducing non-continuing contracts under the tenure reform may widen the gap between good and bad schools by good teachers' transfer from the bad to good school. The unidirectional movement may causes unequal quality of education by imbalanced distribution of the qualified teachers. High- and medium-performance teachers in low-quality schools are likely to leave than low-performance teachers (Feng, Figlio and Sass, 2010). Even if not all teachers who get a satisfactory evaluation are good due to the imperfect assessment, the good school at least gets a benefit of supplement to the vacancy with qualified teachers from the bad school. However, the bad school has no option but to hire inexperienced new ones. This is very disappointing since the additional value-added from more experience is bigger in low-quality schools (Sass et al., 2012).

APPENDIX A

CHAPTER 2

A.1 FULL CHARACTERIZATION OF A 2×2-GRID EQUILIBRIUM

For simplicity, let $U^{S_1} = -(a_1 - \theta_1)^2$, $U^{S_2} = -(a_2 - \theta_2)^2$, and $U^R = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2$ and, suppose that $p_1 = p_2 = 1$. We characterize a 2×2-grid equilibrium according to w. First, given w, the state space is partitioned into four cells. In each cell, we find the optimal solution–at the center of a cell, on the budget line or at the corner of the cell–for the receiver. Then, we confirm that these solution satisfy the indifferent conditions.

Figure 15 shows that as w decreases from 2 to 0, the equilibrium can be categorized into four cases, depending on in which cell the constraint keeps the receiver from taking the best action. For $1.5 \leq w \leq 2$, the optimization solution for the receiver in each cell must satisfy the following indifferent condition:

$$(1-\bar{\theta})(\frac{1+\bar{\theta}}{2}-\bar{\theta})^2+\bar{\theta}(\frac{1+\bar{\theta}}{2}-\bar{\theta})^2=(1-\bar{\theta})(\frac{\bar{\theta}}{2}-\bar{\theta})^2+\bar{\theta}(\frac{\bar{\theta}}{2}-\bar{\theta})^2. \tag{A.1}$$

For $0.9 \leq w < 1.5$, the different condition is

$$(1-\bar{\theta})(\frac{w}{2}-\bar{\theta})^2+\bar{\theta}(\frac{1+\bar{\theta}}{2}-\bar{\theta})^2=(1-\bar{\theta})(\frac{\bar{\theta}}{2}-\bar{\theta})^2+\bar{\theta}(\frac{\bar{\theta}}{2}-\bar{\theta})^2. \tag{A.2}$$

For $0.5 \leq w < 0.9$, the different condition is

$$(1-\bar{\theta})(\frac{w}{2}-\bar{\theta})^2+\bar{\theta}(\frac{1}{4}+\frac{w}{2}-\bar{\theta})^2=(1-\bar{\theta})(-\frac{1}{4}+\frac{w}{2}-\bar{\theta})^2+\bar{\theta}(\frac{\bar{\theta}}{2}-\bar{\theta})^2. \tag{A.3}$$

For $w < 0.5$, the different condition is

$$(1-\bar{\theta})(\frac{w}{2}-\bar{\theta})^2+\bar{\theta}(w-\bar{\theta})^2=(1-\bar{\theta})(-\bar{\theta})^2+\bar{\theta}(\frac{\bar{\theta}}{2}-\bar{\theta})^2. \tag{A.4}$$

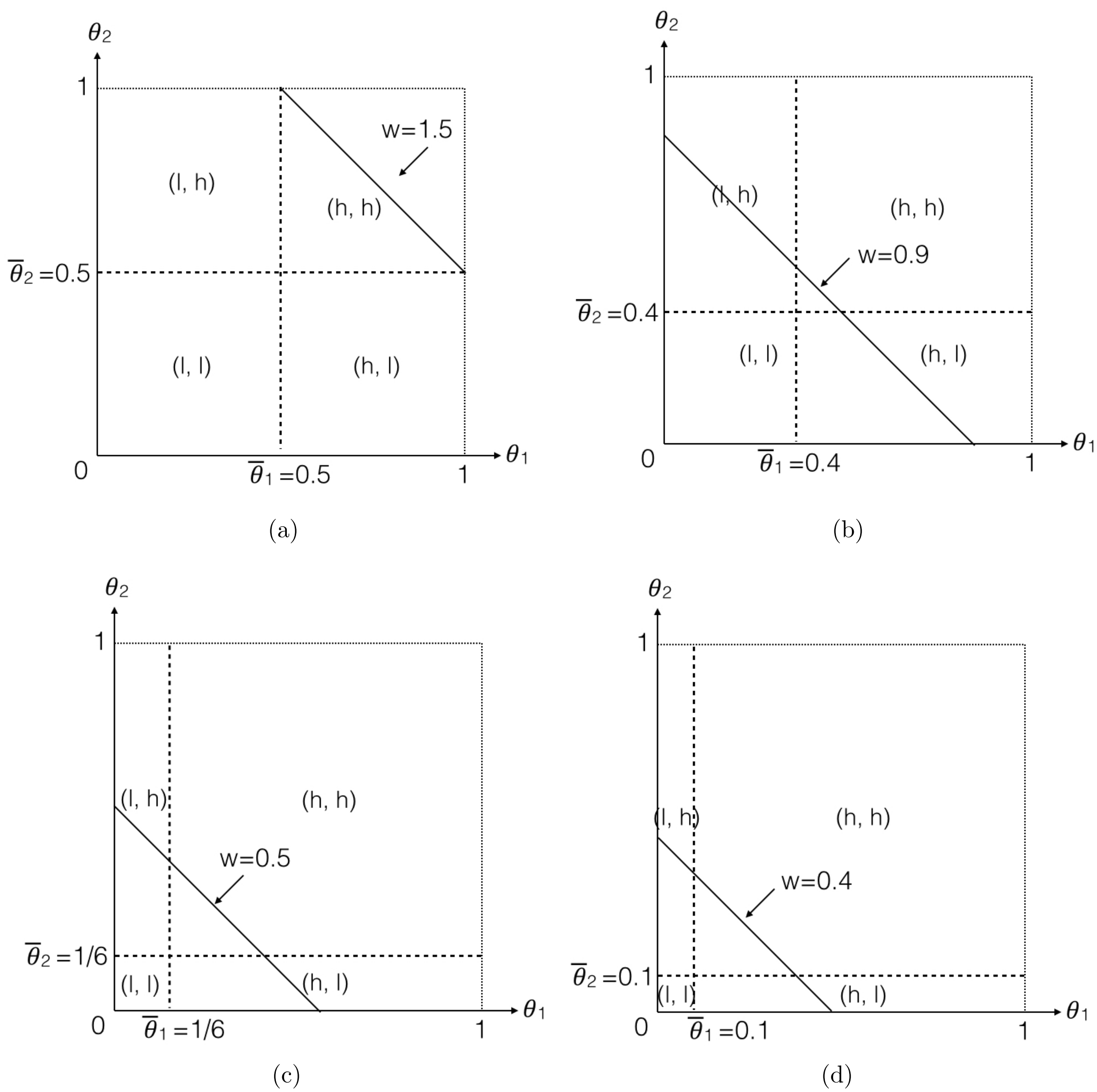

Figure 15: Given $p_1 = p_2 = 1$, a 2×2-grid according to w.

A.2 PROOFS

A.2.1 Proof of Proposition 1

First, we show that an interim bias χ_i occurs if and only if $(M_1, M_2) \notin \mathcal{B}(p_1, p_2, w)$ by Claim 1.

Claim 1. *Suppose that after observing the state $\theta \in \Theta$, the senders truthfully reveal the state, i.e., $m_1(\theta_1) = \theta_1$ and $m_2(\theta_2) = \theta_2$. Then, an interim bias between the senders and the receiver occurs if and only if $(m_1, m_2) \notin \mathcal{B}(p_1, p_2, w)$ where $p_1 > 0$ and $p_2 > 0$.*

Proof. U^R is concave since the Hessian matrix of the receiver's utility, $H(U^R)$, is negative semidefinite:

$$H(U^R) = \begin{bmatrix} U^R_{11} & U^R_{12} \\ U^R_{21} & U^R_{22} \end{bmatrix} = \begin{bmatrix} U^{S_1}_{11} & 0 \\ 0 & U^{S_2}_{22} \end{bmatrix}, \tag{A.5}$$

denoting partial derivatives by subscripts of U^I, $I = S_1, S_2, R$. Given $(m_1, m_2) = (\theta_1, \theta_2)$, if $p_1 m_1 + p_2 m_2 \leq w$, the receiver conforms to the messages, $(a_1^*, a_2^*) = (\theta_1, \theta_2)$, since $U^R_1(\theta_1, \theta_1, \theta_2, \theta_2) = U^{S_1}_1(\theta_1, \theta_1) + U^{S_2}_1(\theta_2, \theta_2) = 0$. Otherwise, the receiver takes an action such that $a^* = \max_a U^R$ where $p_1 a_1 + p_2 a_2 \leq w$. Then, there exists either an interior solution a^* satisfying

$$\frac{U^R_2}{U^R_1}\bigg|_{a=a^*} = \frac{p_2}{p_1}, \text{ and} \tag{A.6}$$

$$\sum p_i a_i^* = w \tag{A.7}$$

or a corner solution. $\square$

Without loss of generality, we focus on sender 2. For simplicity, let $p_1 = 1$ and $p_2 = p$. Then, the severe constraint means $w < max\{1, p\}$. The marginal benefit of overstating by ϵ over truth-telling θ_2 is

$$\Delta U = \int_0^1 [U^{S_2}(\theta_2 - \chi_2(\theta_1, \theta_2 + \epsilon), \theta_2) - U^{S_2}(\theta_2 - \chi_2(\theta_1, \theta_2), \theta_2)] d\theta_1. \tag{A.8}$$

Since $U_{11} < 0$, by A.6 and A.7 of Claim 1, $\Delta U > 0$ for $\theta_2 > w/p$. $\square$

A.2.2 Proof of Proposition 2

The basic setting is the same as that in the proof of Proposition 1. Suppose that $w = 1+p-\delta$ where $\delta > 0$. Then,

$$\begin{aligned}
\Delta U = &-\int_{0}^{\delta-p(1-\theta_2)} [U^{S_2}(\theta_2 - \chi_2(\theta_1,\theta_2),\theta_2) - U^{S_2}(\theta_2 - \chi_2(\theta_1,\theta_2+\epsilon),\theta_2)]d\theta_1 \\
&+ \int_{0}^{1+p(1-\theta_2-\epsilon)-\delta} U^{S_2}(\theta_2+\epsilon,\theta_2)\, d\theta_1 \\
&+ \int_{\delta-p(1-\theta_2)}^{\delta-p(1-\theta_2-\epsilon)} U^{S_2}(\theta_2 - \chi_2(\theta_1,\theta_2+\epsilon),\theta_2)\, d\theta_1. \qquad \text{(A.9)}
\end{aligned}$$

The first term is the gain of overstating while the second and the third are the losses. As $\delta \to p(1-\theta_2')$, $\Delta U < 0$. Since ΔU is a continuous function, by intermediate value theorem, $\exists w^*$ such that $\Delta U = 0$. $\square$

A.2.3 Proof of Theorem 1

First, we show that each dimension of the state is partitioned into intervals inducing discrete actions from the receiver. Suppose that, for $n_i \in \mathbb{N}$, $i = 1,2$, the i-th dimension is partitioned into n_i intervals by $\rho_i(n_i) \equiv (\rho_i^0(n_i),\ldots,\rho_i^{n_i}(n_i))$ where $\rho_i^0(n_i) = 0 < \rho_i^1(n_i) < \cdots < \rho_i^{n_i-1}(n_i) < \rho_i^{n_i}(n_i) = 1$. Given $\theta_i \in [\rho_i^{j-1},\rho_i^j]$, $j = 1,\ldots,n_i$, if sender i sends $m_i^j \in [\rho_i^{j-1},\rho_i^j]$, then the receiver's best response (2.3) induces $n_1 \times n_2$ actions.

Second, we show that senders' strategy is incentive compatible if partition $\rho_1^{n_1}$ and $\rho_2^{n_2}$ satisfy (2.4) and (2.5). Without loss of generality, given the behavior of sender 2, we focus on the strategic interaction between sender 1 and the receiver. In other words, for $\rho_1(n_1)$ and $\rho_2(n_2)$, $n_1,n_2 \in \mathbb{N}$, we fix ρ_2^k, $k = 0,\ldots,n_2$.

Claim 2. *Fix δ_2^k, $k = 1,\ldots,n_2$. Suppose that $\rho_1(n_1)$ satisfies (2.4). Then, for $j = 1,\ldots,n_1$, $\delta_1^j(n_1) \le \delta_1^{j+1}(n_1)$.*

Proof. Suppose that $\delta_1^j(n) > \delta_1^{j+1}(n)$. From (2.3), $|E_{\theta_2}[a_1^R(m_1^j|\theta_2)] - \rho_1^j| > |E_{\theta_2}[a_1^R(m^{j+1}|\theta_2] - \rho_1^j|$ contradicting (2.4). $\square$

To satisfy (2.4), we need monotonicity in ρ_1 by Claim 2.

To show the existence of equilibrium partitions, we use an extended version of the Knaster-Tarski theorem (Zhou, 1994). Any partition with $n \geq 2$ intervals is represented by a vector $\rho^n = (\rho_0, \dots, \rho_n) \in P_n \subset [0,1]^{n+1}$ such that $0 = \rho_0 \leq \rho_1 \leq \cdots \leq \rho_{n-1} \leq \rho_n = 1$. Then, P_n, $n \geq 2$, is a lattice whose least element is $(0, \dots, 0, 1)$ and greatest element is $(0, 1, \dots, 1)$. Let σ^n denote a correspondence that maps ρ^n to a set of vectors $\sigma_0(\rho^n) \times \cdots \times \sigma_n(\rho^n) = \rho^n = (\rho_0, \dots, \rho_n) \subset [0,1]^{n+1}$ such that $\sigma_0(\rho^n) = \rho_0 = 0, \sigma_n(\rho^n) = \rho_n = 1$, and $\sigma_i(\rho^n) = \rho_i = \rho^i, i = 1, \dots, n-1$, satisfying (2.4). From (2.4), $0 < \sigma_1(\rho^n) < \cdots < \sigma_{n-1}(\rho^n) < 1$ implying that $\sigma^n(\rho^n)$ is a sublattice of P_n.

Claim 3. *Suppose that, for $\rho^n \in P_n$, $\sigma^n(\rho^n)$ is a nonempty sublattice of P_n. Then, a set of fixed points of σ^n is a nonempty complete lattice.*

By Claim 2, σ_i, $i = 1, \dots, n-1$, is increasing in ρ^n. This satisfies a "ascending condition" that guarantees a nonempty set of fixed points (by Theorem 1 in Zhou). $\square$

By Claim 3, a set of fixed points of σ^n is nonempty.

Claim 4 shows that sender 1 follows his signaling rule, i.e. $m_1^j \in [\rho_1^{j-1}, \rho_1^j]$ if $\theta_1 \in [\rho_1^{j-1}, \rho_1^j]$, $j = 1, \dots, n_i$.

Claim 4. *Fix $\rho_2(n_2)$ satisfying $\delta_2^k \leq \delta_2^{k+1}$, $k = 1, \dots, n_2$. Then, $\sum_k \delta_2^k U^{S_1}(a_1^k(m_1^j, m_2^k), \theta_1)$ $> \sum_k \delta_2^k U^{S_1}(a_1^k(m_1^{-j}, m_2^k), \theta_1)$, $j = 1, \dots, n_1$.*

Proof. Given sender 2's partition ρ_2, by (2.3), the best response from the receiver is as follows: $a_1^R(\cdot, (\rho_2^0, \rho_2^1)) = a_1^1 \geq a_1^R(\cdot, (\rho_2^1, \rho_2^2)) = a_1^2 \geq \cdots \geq a_1^R(\cdot, (\rho_2^{n_2-1}, \rho_2^{n_2})) = a_1^{n_2}$. Since $U_{11}^{S_1}(a_1, \theta_1) < 0$ by assumption, $\sum_k \delta_2^k U_{11}^{S_1}(a_1^k, \theta_1) < 0$. If $\theta_1 > a_1^1$, then $\sum_k \delta_2^k U_1^{S_1}(a_1^k, \theta_1) > 0$ since $U_1^{S_1}(\theta_1, \theta_1) = 0$. Similarly, if $\theta_1 < a_1^{n_2}$, then $\sum_k \delta_2^k U_1^{S_1}(a_1^k, \theta_1) < 0$. By $\sum_k \delta_2^k U_{11}^{S_1}(a_1^k, \theta_1)$ < 0, there exists $(a_1^1, a_1^2, \dots, a_1^n)$ such that $a_1^1 \leq \theta_1 \leq a^n$ satisfying $\sum_k \delta_2^k U_1^{S_1}(a_1^k, \theta_1) = 0$. Since $U_{12}^{S_1}(a_1, \theta_1) > 0$ by assumption, $\sum_k \delta_2^k U_{12}^{S_1}(a_1^k, \theta_1) > 0$. Therefore, $\sum_k \delta_2^k U^{S_1}(a_1^k(m_1^j,$ $m_2^k), \theta_1) > \sum_k \delta_2^k U^{S_1}(a_1^k(m_1^{-j}, m_2^k), \theta_1)$. $\square$

Finally, there exists a countably infinite partition in equilibrium. We use the technique introduced by Gordon (2010). In his model, a sender has a *single-peaked* preference relation over actions, $\succsim$, if there exists a unique $a^* \in \{a^1, \dots, a^n\}$ such that $a^i < a^j \leq a^* \Rightarrow a^j \succ a^i$, and $a^j > a^i \geq a^* \Rightarrow a^i \succ a^j$. The preference is *single-crossing* if the senders who observe $\theta' > \theta$ prefer a' to a where $a' > a$, i.e. for all $\theta, \theta' \in \Theta$ such that $\theta' > \theta$, $a' \succsim_\theta a$, $\Rightarrow a' \succsim_{\theta'} a$. In our model, sender 1 has a single-peaked and single-crossing preference $\succsim_{\theta_1}$ over $(a_1^1, a_1^2, \ldots, a_1^{n_2})$ (decided by sender 2's partition), after observing θ_1 by Claim 4. Since $a_1^S(0) = a_1^R = 0$ and $[a_1^R(0), a_1^R(1)] \subseteq [a_1^S(0), a_1^S(1)]$, there exists a countably infinite partition by Theorem 4 and 5 in Gordon (2010). $\square$

A.2.4 Proof of Theorem 2

First, fix n_1, n_2 of an $n_1 \times n_2$-grid. Then, fix p_1 and p_2. Without loss of generality, we focus on sender 1. Given $w > w'$, a grid in the game Γ_w is more equally partitioned than the same form of a grid in the game $\Gamma_{w'}$. More equal size of cells in a grid increases expected utilities since utilities are single-peaked and concave. Similarly, fix p_2 and w. Given $p_1 < p_1'$, a grid in the game Γ_{p_1} is more equally partitioned than the same form of a grid in the game $\Gamma_{p_1'}$. Finally, fix p_2. Given $w > w'$ and $p_1 < p_1'$, a grid in the game $\Gamma_{p_1,w}$ is more equally partitioned than the same form of a grid in the game $\Gamma_{p_1',w'}$.$\square$

APPENDIX B

CHAPTER 3

B.1 PROOFS

B.1.1 Proof of Lemma 1

$\Leftarrow$: (i) and (ii) are obvious. (iii) Suppose that $0 < \underline{y}_1 < \overline{y}_1 < 1$ and $0 < \underline{y}_2 < \overline{y}_2 < 1$. Then, we can divide the information space $\Theta = [0,1] \times [0,1]$ divided into nine subspaces: (1) $[0, \underline{y}_1] \times [\overline{y}_2, 1]$, (2) $[\underline{y}_1, \overline{y}_1] \times [\overline{y}_2, 1]$, (3) $[\overline{y}_1, 1] \times [\overline{y}_2, 1]$, (4) $[0, \underline{y}_1] \times [\underline{y}_2, \overline{y}_2]$, (5) $[\underline{y}_1, \overline{y}_1] \times [\underline{y}_2, \overline{y}_2]$, (6) $[\overline{y}_1, 1] \times [\underline{y}_2, \overline{y}_2]$, (7) $[0, \underline{y}_1] \times [0, \underline{y}_2]$, (8) $[\underline{y}_1, \overline{y}_1] \times [0, \underline{y}_2]$, and (9) $[\overline{y}_1, 1] \times [0, \underline{y}_2]$. For all $\theta \in [0, \underline{y}_1] \times [\overline{y}_2, 1]$, $y*^P = y*^{A_1} = y*^{A_2} = (\underline{y}_1, \overline{y}_2)$. Similarly, $y*^P = y*^{A_1} = y*^{A_2}$ holds for all θ in the other subspaces. Now, suppose that $\underline{y}_1 = 0$ and $\underline{y}_2 > 0$. Then, we can divide the information space into six subspaces, and show that $y*^P = y*^{A_1} = y*^{A_2}$ holds for all θ in the other subspaces. Analogously, for all $\underline{y}_1$, $\overline{y}_1$, $\underline{y}_1$, and $\overline{y}_2$, we can divide the space into some subspaces, and show the same result in those subspaces.

$\Rightarrow$: We prove this by contraposition. If $\theta \notin \mathcal{Y}$, $\lambda \neq 1/2$, and $\mathcal{Y} \neq [\underline{y}_1, \overline{y}_1] \times [\underline{y}_2, \overline{y}_2]$, then $y*^P \neq y*^{A_1}$ or $y*^P \neq y*^{A_2}$ or $y*^{A_1} \neq y*^{A_2}$. $\square$

B.1.2 Proof of Lemma 3

Without loss of generality, we focus on agent 1. Given $\theta_1 \leq \underline{\theta}_1$, for $\theta_1' < \theta_1$ or $\theta_1 < \theta_1' \leq \underline{\theta}_1$, $E_{\theta_2}[u^{A_1}(g(\theta_1, \theta_2), \theta)|\theta_1] = E_{\theta_2}[u^{A_1}((g(\theta_1', \theta_2), \theta)|\theta_1]$, and, for $\theta_1' > \underline{\theta}_1$, $E_{\theta_2}[u^{A_1}((g(\theta_1, \theta_2), \theta)|\theta_1]$

$> E_{\theta_2}[u^{A_1}((g(\theta_1', \theta_2), \theta)|\theta_1]$. Similarly, given $\theta_1 \geq \overline{\theta}_1$, for $\theta_1' > \theta_1$ or $\overline{\theta}_1 \leq \theta_1' < \theta_1$, $E_{\theta_2}[u^{A_1}((g(\theta_1, \theta_2), \theta)|\theta_1] = E_{\theta_2}[u^{A_1}((g(\theta_1', \theta_2), \theta)|\theta_1]$, and, for $\theta_1' < \overline{\theta}_1$, $E_{\theta_2}[u^{A_1}((g(\theta_1, \theta_2), \theta)|\theta_1]$

$> E_{\theta_2}[u^{A_1}((g(\theta_1', \theta_2), \theta)|\theta_1]$. Given $\underline{\theta}_1 < \theta_1 < \overline{\theta}_1$, for all $\theta_1' \neq \theta_1$, $E_{\theta_2}[u^{A_1}(g(\theta_1, \theta_2), \theta)|\theta_1] > E_{\theta_2}[u^{A_1}((g(\theta_1', \theta_2), \theta)|\theta_1]$. $\square$

APPENDIX C

CHAPTER 4

C.1 INDIVIDUAL RATIONALITY

In Section **??**, we assumed that individual rationality holds to characterize an equilibrium. Now, this assumption is relaxed for policy implications.

To check the participation, we calculate the expected payoff by the teachers' type. We assume that the expected payoff is the same as an outside option, then the teachers stay at the school. Given the principals' strategy, the expected payoff of type t is

$$u_t^A = 1 \times P(r|t) + 0 \times P(d|t) = P(r|c)P(c|t) + P(r|u)P(u|t) = P(r|c)P(c|t). \tag{C.1}$$

Since

$$P(c|g) = P(c|g,l)P(l) + P(c|g,h)P(h) = \mu - (\mu - 1/2)\lambda \text{ and} \tag{C.2}$$

$$P(c|b) = P(c|b,l)P(l) + P(c|b,h)P(h) = 1/2 - (\mu - 1/2)\lambda, \tag{C.3}$$

$$u_g^A = \mu - (\mu - \frac{1}{2})\lambda, \text{ and} \tag{C.4}$$

$$u_b^A = \frac{1}{2} - (\mu - \frac{1}{2})\lambda. \tag{C.5}$$

Since the expected payoff graph of the bad type is parallel to that of the good type, the individual rationality condition depends on their outside options.

Lemma 7. *Suppose that the good teacher wants to stay in the school, i.e.* $u_g^A \geq e_g$*. Then, the bad type also wants to stay if his outside option is low enough compared to the good type's outside option, i.e.* $e_g - e_b \geq \mu - 1/2$*.*

Proof. From $e_g \geq e_b$ and $u_g^A \geq e_g$, $u_g^A - e_b \geq e_g - e_b \geq \mu - 1/2 = u_g^A - u_b^A$. Therefore, $u_b^A \geq e_b$. □

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