

## Research Paper

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# Detecting unbalanced bids via an improved grading-based model

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**Abstract:** Unbalanced bidding, also known as skewed bidding, is the process of increasing and/or decreasing the prices of various bid items without altering the total offered bid price. Bids can be unbalanced either mathematically (front-end loading) or materially (quantity error exploitation). Owners should be very careful when evaluating the tenders as awarding a contract to an unbalanced bid may result in severe cost overruns because the prices of those items do not reflect their true costs and markup allocations. Unbalanced bidding is still a contentious issue in the construction industry. While some researchers consider it as a legal bidding strategy in such a fierce competitive business environment, others view it as an unethical practice and claim that unbalanced bids should be disqualified. Studies regarding unbalanced bidding can be categorized into two groups: (1) the ones focusing on detecting or preventing this practice to help owners; and 2) the ones focusing on optimizing unbalanced bidding to help contractors. This study aims to develop a model, which consists of eight grading systems, to assist owners in detecting materially unbalanced bids. The proposed model is the improved version of the previous model, which was composed of five grading systems. In order to demonstrate how this grading-based model can be used by owners, an illustrative example is presented. It was found that owners can easily and successfully detect unbalanced bids via the proposed model.

**Keywords:** unbalanced bidding, materially unbalanced bids, detection model, owners

## 1 Introduction

In the construction industry, owners may select contractors through competitive bidding or direct negotiation. However, competitive bidding has long been used as a prevalent method for awarding contracts, especially in public projects. Therefore, it is commonly acknowledged that the success of construction projects and the business continuity of contractors are mainly dependent on companies' robust bidding strategies (Ahmed et al. 2016).

A bid price mainly consists of three components, namely, direct cost, indirect cost, and a bid markup, which is the sum of general overhead, profit, and contingency in percentage (Dikmen et al. 2007). Since owners tend to select contractors on the basis of the price, contractors should be very conscientious when determining their bid price. Contractors should thoroughly calculate the costs of work items and estimate the bid markup considering their bidding strategy and the contract type that will be used.

Unbalanced bidding, also known as skewed bidding, can be defined as the process of increasing the prices of some items and concurrently decreasing the prices of other items without affecting the total offered bid price, with the intention of achieving competitive advantage over rivals and thereby increasing the chance of winning the contract, minimizing the total project cost, improving cash flow, and increasing the profit of the project (Cattell et al. 2007; Afshar and Amiri 2010a; Skitmore and Cattell 2013; Hyari 2016; Hyari et al. 2016; Nikpour et al. 2017; An et al. 2018; Aziz and Aboelmagd 2019). Although this practice can be used by contractors when they are bidding lump sum or unit price contracts (e.g., Su and Lucko 2015; Hyari 2016; Hyari et al. 2016; Hyari 2017; Nikpour et al. 2017), it is predominantly used in unit price contracts as the actual quantity is not known for certain (e.g., Hoogenboom et al. 2006; Arditi and Chotibhongs 2009; Mandell and Nyström 2011; Nyström 2015; An et al. 2018).

In lump sum contracts, the contractor offers a fixed price to the owner to perform all the work items within the scope of the project. This type of contract is commonly

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used for projects in which the scope is clearly defined and the quantities of work items can be accurately estimated by the owners during the bidding process (Carty 1995; Gransberg and Riemer 2009). Payments to the contractors are made based on a percentage of work completed (Poage 2000). Lump sum contract is very risky for the contractor, whereas it offers the owner the best protection against potential cost overruns (Carty 1995; Gransberg and Riemer 2009).

On the other hand, unit price contracts are utilized where the scope of work cannot clearly be defined and the quantities of work items cannot be accurately calculated by the owners during the bidding stage (Gransberg and Riemer 2009). In general, owners provide the estimated bill of quantities of work items, and the candidates offer not only the total bid price but also the unit price for each item (Cattell et al. 2010; Cattell 2012; Nyström and Mandell 2019). Payments to the contractors are made based on actual quantities of the performed work rather than the estimated quantities (Hyari et al. 2017). Actual quantities mostly deviate from the estimated quantities, which – in turn – affects the total project cost and the contractor's profitability (Gransberg and Riemer 2009; Afshar and Amiri 2010b; Bajari et al. 2014; Hyari et al. 2017). This type of contracting system promises several advantages, such as relatively low transaction cost and transparency. On the other hand, it has some drawbacks, such as lack of incentives for innovation and the potential for receiving unbalanced bids (Nyström and Mandell 2019). This type of contract is widespread in projects involving heavy construction, such as highways, bridges, dams, pipelines, and so on, and is predominantly preferred by public agencies (Carty 1995; Poage 2000).

There are two main categories of unbalanced bids, namely, mathematically unbalanced bids and materially unbalanced bids. While mathematically unbalanced bids refer to the bids in which the markup is not proportionately allocated to each bid item's price, materially unbalanced bids refer to the bids in which not only is the markup not proportionately allocated, but the actual costs of some activities are also manipulated (Christodoulou 2008; Ardit and Chotibhongs 2009; Su and Lucko 2015).

Bids can be unbalanced mathematically via front-end loading or back-end loading, and they can be unbalanced materially via quantity error exploitation (Cattell et al. 2007; Cattell et al. 2008). Contractors may decide to use one or more of these unbalancing forms depending on their motivation.

When a contractor inflates the unit prices of the bid items scheduled to start in the early phases of the project and simultaneously deflates the unit prices of the bid

items scheduled to start in the later phases, the contractor will receive larger amounts of money for the first few progress payments. This type of price manipulation may likely reduce the financing cost of the project and thereby improve the contractor's cash flow when the value of time is considered (Wang 2004; Son et al. 2006; Cattell et al. 2008; Peterson 2009; Su and Lucko 2015; Nikpour et al. 2017). This form of unbalancing is named as "front-end loading" (Cattell et al. 2007).

Contrary to front-end loading, the practice of overpricing the items scheduled to start in the late phases of the project, along with underpricing the items scheduled to start in the early phases, is named as "back-end loading" (Cattell et al. 2007; Cattell et al. 2008; Su and Lucko 2015). This form of unbalancing is preferred by contractors who undertake long-duration projects in high-inflation-rate countries because of the fact that they can receive larger amounts of escalation in compensation for inflation (Nikpour et al. 2017; Cattell et al. 2008; Ardit and Chotibhongs 2009). Since the main idea behind mathematically unbalanced bids is improving the contractor's cash flow, they are also named as "cash flow unbalancing" (Hyari et al. 2016).

If a contractor detects mistakes in the quantities of some items estimated by the owner and/or predicts that the quantities of some items may considerably increase due to potential design changes, the contractor may tend to allocate higher prices to these items and simultaneously allocate lower prices to those items whose quantities are expected to be reduced (Cattell et al. 2007; Cattell et al. 2008; Su and Lucko 2015; Prajapati and Bhavsar 2017). In this type of unbalancing, contractors take advantage of the potential mistakes made by the owner in the quantities of some bid items and/or vagueness in design documents and, thereby, substantially increases their profit (Hyari 2016). This form of unbalancing is known as "quantity error exploitation" or "individual rate loading" (Cattell et al. 2007; Cattell et al. 2008; Su and Lucko 2015; Amusan et al. 2018).

Unbalanced bidding is still one of the most contentious issues in the construction industry. While some researchers deal with unbalanced bidding from the contractors' perspective and consider it as a legal bidding strategy in such a fierce competitive business environment, others tackle this practice from the owners' perspective and view it as an unethical practice and claim that, especially, materially unbalanced bids should be disqualified (Kenley 2003; Su and Lucko 2015).

Studies in the construction management literature and practice regarding unbalanced bidding can be categorized into two groups: (1) the ones focusing on detecting

or preventing this practice, aiming to help owners (e.g., Bell 1989; Wang 2004; Arditi and Chotibhongs 2009; Shrestha et al. 2012; Hyari 2016; Hyari et al. 2016; Nikpour et al. 2017; An et al. 2018; Polat et al. 2019); and (2) the ones focusing on optimizing unbalanced bidding, which aim to help contractors (e.g., Yizhe and Youjie 1992; Nassar 2004; Son et al. 2006; Cattell et al. 2008; Christodoulou 2008; Liu et al. 2009; Afshar and Amiri 2010a, 2010b; Mandell and Nyström 2011; Cattell et al. 2011; Bajari et al. 2014).

This study deals with the detection of unbalanced bids created by quantity error exploitation in unit price contracts. For this purpose, after reviewing existing models in the literature, an advanced model for the detection of unbalanced bids is proposed by improving the model formerly developed by Polat et al. (2019). The proposed model uses eight different grading systems for detection of unbalanced bids, whereas the previous model consisted of five grading systems. Owners may assign different weights to these grading systems according to the characteristics of their projects. After assigning weights to the grading systems, the final scores of each bidder can be calculated. All bidders can be evaluated not only according to their bid prices but also according to the calculated final scores. An illustrative example is presented to check the applicability of the proposed model in construction projects. The findings of this study indicate that the proposed model provides a marked improvement compared to the previous model. It also provides owners with a new perspective in detecting unbalanced bids during the bid evaluation phase.

## 2 Literature review: previous models for unbalanced bid detection

Unbalanced bids may cause owners to overpay, increase the risk of contractor default, demotivate the contractor for completing the project, complicate the valuation of changes during the construction phase, create unfair competition environment, and bring about delays, which in turn negatively affect the project's overall performance (Manzo and Tell 1997; Yin et al. 2010; Kenley 2003; Hyari 2017). Therefore, owners desire to detect unbalanced bids in advance in order to shield themselves from the potential adverse outcomes. In general, unbalanced bidding is not outlawed in the construction industry, but it is seen as an unethical and risky strategy. Detecting mathematically

(front-end loading or back-end loading) unbalanced bids is relatively easier than detecting materially unbalanced bids (quantity error exploitation) (Cattell et al. 2007; Su and Lucko 2015; Nikpour et al. 2017). Therefore, most of the previous research has focused on detecting mathematically unbalanced bids.

Bell (1989) developed a single percentage factor method, which precludes materially and mathematically unbalanced bids in unit price contracts. Wang (2004) proposed a procedure to handle unbalancing in lump sum contracts. This method mainly concentrates on the adjustment of rates in the estimated quantities submitted by the lowest bidder and the rates submitted by all qualified bidders. Arditi and Chotibhongs (2009) developed two different models to detect unbalanced bids created by front-end loading and quantity error exploitation. The main idea behind these models is the comparison of prices of each bid item with the owner's estimates and the average prices offered by all bidders. Shrestha et al. (2012) proposed to conduct linear correlation analysis in order to find out whether bids are mathematically unbalanced or not. Hyari (2016) developed an unbalanced bid prevention model, which uses the average unit price of all bidders to adjust unit price of every bid item submitted by each bidder. Hyari et al. (2016) provided owners with an unbalanced bid detection model, which takes into account uncertainty in estimated quantities of bid items and uses Monte Carlo simulation to measure the risk impacts of differences between actual and estimated quantities of bid items. Nikpour et al. (2017) proposed a detection tool, which develops a Bid Markup Distribution Index (BMDI) graph, to identify unbalanced bids in unit price contracts during the bid evaluation process. The developed tool also uses Monte Carlo simulation to consider the impacts of cost uncertainties and risks. Nikpour et al. (2017) developed an unbalanced bid detection tool, which is based on BMDI graph and uses Monte Carlo simulation, for the purpose of identifying unbalanced bids in unit price contracts and considering the impacts of cost uncertainties and risks during the bid evaluation process. An et al. (2018) developed an unascertained model, which uses the owner's estimated price as evaluation criterion and identifies unbalanced bids. Polat et al. (2019) proposed an unbalanced bid detection model, which contains five different grading systems and aims to assist owners in detecting unbalanced bids. The proposed model enables owners to calculate the individual grades of each bidder or calculate the final score of each bidder by assigning different weights to these grading systems according to the project characteristics or their own needs.

### 3 The improved grading-based model for unbalanced bid detection

The objectives of this study include the following: (1) defining new grading systems to extend the existing approach in detection of unbalanced bids; and (2) validating the new grading systems through an illustrative example. In order to achieve these objectives, the model formerly developed by Polat et al. (2019), which consisted of five grading systems, was modified by adding three new grading systems.

In the model of Polat et al. (2019), the first grading system compares the ratio of each activity’s total price in the bid price offered by each bidder with the one estimated by the owner, the second grading system compares the unit price of each activity offered by each bidder with the ones estimated by the owner, the third grading system compares the unit price of each activity offered by each bidder with the average of unit prices offered by all bidders, the fourth grading system compares the bid price offered by the bidder with the estimated construction cost (ECC), and the fifth grading system compares the sum of total prices offered by bidders for those activities whose quantities may likely increase during the construction phase with the ones estimated by the owner.

In this study, major and minor bid items have been defined. If a bid item’s total price is  $\geq 5\%$  of the owner’s ECC, this bid item is called as a “major bid item”. On the other hand, if a bid item’s total price is  $< 5\%$  of the ECC, this bid item is called as a “minor bid item”.

Polat et al. (2019) described five grading systems for detection of unbalanced bids in detail. For a thorough description of the five grading systems, readers are directed to Polat et al. (2019). The modified approach proposed in this study adopts the same grading systems used by Polat et al. (2019), but the following grading systems are added for the detection of unbalanced bids.

#### 3.1 Sixth grading system

The main idea behind this grading system is to compare the unit price ( $bup_{imjkl}$ ) of each major activity  $i_{mjk}$  ( $k = 1, 2, \dots, K$ ) offered by each bidder  $l$  with the ones estimated by the owner ( $oup_{imjk}$ ). First, the major activities ( $i_{mj}$ ) whose total price is  $\geq 5\%$  of the ECC are determined. Then, the sixth comparison ratio ( $r_{6kl}$ ) for the  $k^{th}$  major activity and the  $l^{th}$  bidder is calculated using Equation 1.

$$r_{6kl} = \frac{bup_{imjkl}}{oup_{imjk}} \tag{1}$$

where  $r_{6kl}$  is the sixth comparison ratio for the  $k^{th}$  major activity and the  $l^{th}$  bidder,  $bup_{imjkl}$  is the unit price of the  $k^{th}$  major activity offered by the  $l^{th}$  bidder, and  $oup_{imjk}$  is the unit price of the  $k^{th}$  major activity estimated by the owner.

Each bidder  $l$  receives a grade for each major activity ( $g_{6imjkl}$ ) according to the value of the sixth comparison ratio ( $r_{6kl}$ ) based on the intervals given in Table 1, which was formerly used by Polat et al. (2019) for the grading systems 1, 2, 3, and 5 and is now used for the new grading systems 6, 7, and 8. Then, the total score of the  $l^{th}$  bidder received from the sixth grading system ( $BTS_{6l}$ ) is found by using Equations 2–6.

$$BP_l = \sum_{i=1}^n (bup_{li} \times q_i) \tag{2}$$

where  $BP_l$  is the total bid price offered by the  $l^{th}$  bidder,  $bup_{li}$  is the unit price of the  $i^{th}$  activity offered by the  $l^{th}$

Tab. 1: Grade values for grading systems 6, 7, and 8

Comparison ratio	Grade	Comparison ratio	Grade
$r \leq 0.9$	42	$1.005 < r \leq 1.010$	20
$0.900 < r \leq 0.905$	41	$1.010 < r \leq 1.015$	19
$0.905 < r \leq 0.910$	40	$1.015 < r \leq 1.020$	18
$0.910 < r \leq 0.915$	39	$1.020 < r \leq 1.025$	17
$0.915 < r \leq 0.920$	38	$1.025 < r \leq 1.030$	16
$0.920 < r \leq 0.925$	37	$1.030 < r \leq 1.035$	15
$0.925 < r \leq 0.930$	36	$1.035 < r \leq 1.040$	14
$0.930 < r \leq 0.935$	35	$1.040 < r \leq 1.045$	13
$0.935 < r \leq 0.940$	34	$1.045 < r \leq 1.050$	12
$0.940 < r \leq 0.945$	33	$1.050 < r \leq 1.055$	11
$0.945 < r \leq 0.950$	32	$1.055 < r \leq 1.060$	10
$0.950 < r \leq 0.955$	31	$1.060 < r \leq 1.065$	9
$0.955 < r \leq 0.960$	30	$1.065 < r \leq 1.070$	8
$0.960 < r \leq 0.965$	29	$1.070 < r \leq 1.075$	7
$0.965 < r \leq 0.970$	28	$1.075 < r \leq 1.080$	6
$0.970 < r \leq 0.975$	27	$1.080 < r \leq 1.085$	5
$0.975 < r \leq 0.980$	26	$1.085 < r \leq 1.090$	4
$0.980 < r \leq 0.985$	25	$1.090 < r \leq 1.095$	3
$0.985 < r \leq 0.990$	24	$1.095 < r \leq 1.100$	2
$0.990 < r \leq 0.995$	23	$1.100 < r$	1
$0.995 < r \leq 1.000$	22		
$1.000 < r \leq 1.005$	21		

Source: Polat et al. (2019).

bidder, and  $q_i$  is the quantity of the  $i^{\text{th}}$  activity estimated by the owner.

$$br_{imjkl} = \frac{bup_{imjkl} \times q_{imjk}}{BP_l} \quad (3)$$

where  $br_{imjkl}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the  $l^{\text{th}}$  bidder in his total bid price  $BP_l$ ,  $bup_{imjkl}$  is the unit price of the  $k^{\text{th}}$  major activity offered by the  $l^{\text{th}}$  bidder,  $q_{imjk}$  is the quantity of the  $k^{\text{th}}$  major activity, and  $BP_l$  is the total bid price offered by the  $l^{\text{th}}$  bidder.

$$ECC = \sum_{i=1}^n (oup_i \times q_i) \quad (4)$$

where  $ECC$  is the estimated construction cost,  $oup_i$  is the unit price of the  $i^{\text{th}}$  activity, and  $q_i$  is the quantity of the  $i^{\text{th}}$  activity estimated by the owner.

$$or_{imjk} = \frac{oup_{imjk} \times q_{imjk}}{ECC} \quad (5)$$

where  $or_{imjk}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the owner in the  $ECC$ ,  $oup_{imjk}$  is the unit price of the  $k^{\text{th}}$  major activity estimated by the owner,  $q_{imjk}$  is the quantity of the  $k^{\text{th}}$  major activity, and  $ECC$  is the estimated construction cost.

$$BTS_{6l} = \frac{\sum_{k=1}^K (br_{imjkl} \times g_{6imjkl})}{\sum_{k=1}^K (or_{imjk}) \times g_{max}} \quad (6)$$

where  $BTS_{6l}$  is the total score of the  $l^{\text{th}}$  bidder received from the sixth grading system,  $br_{imjkl}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the  $l^{\text{th}}$  bidder in his total bid price  $BP_l$ ,  $g_{6imjkl}$  is the grade received by each bidder  $l$  for the  $k^{\text{th}}$  major activity according to the value of the sixth comparison ratio ( $r_{6kl}$ ) based on the intervals given in Table 1,  $or_{imjk}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the owner in the estimated construction cost  $ECC$ , and  $g_{max}$  is the maximum value of the sixth grading system, which is equal to 42.

### 3.2 Seventh grading system

The main idea behind this grading system is to compare the unit price of the  $k^{\text{th}}$  major activity offered by the  $l^{\text{th}}$  bidder ( $bup_{imjkl}$ ) with the average of the unit prices offered by all bidders for the  $k^{\text{th}}$  major activity ( $aup_{imjk}$ ). The average unit price of the  $k^{\text{th}}$  major activity ( $aup_{imjk}$ ) and the seventh comparison ratio ( $r_{7kl}$ ) for the  $k^{\text{th}}$  major

activity and the  $l^{\text{th}}$  bidder is calculated using Equations 7 and 8, respectively.

$$aup_{imjk} = \frac{\sum_{l=1}^L bup_{imjkl}}{L} \quad (7)$$

where  $aup_{imjk}$  is the average of the unit prices offered by all bidders for the  $k^{\text{th}}$  major activity,  $bup_{imjkl}$  is the unit price of the  $k^{\text{th}}$  major activity offered by the  $l^{\text{th}}$  bidder, and  $L$  is the total number of bidders.

$$r_{7kl} = \frac{bup_{imjkl}}{aup_{imjk}} \quad (8)$$

where  $r_{7kl}$  is the seventh comparison ratio for the  $k^{\text{th}}$  major activity and the  $l^{\text{th}}$  bidder,  $bup_{imjkl}$  is the unit price of the  $k^{\text{th}}$  major activity offered by the  $l^{\text{th}}$  bidder, and  $aup_{imjk}$  is the average of the unit prices offered by all bidders for the  $k^{\text{th}}$  major activity.

Each bidder  $l$  receives a grade for each major activity ( $g_{7imjkl}$ ) according to the value of the seventh comparison ratio ( $r_{7kl}$ ) based on the intervals given in Table 1 and then the total score of the  $l^{\text{th}}$  bidder received from the seventh grading system ( $BTS_{7l}$ ) is found by using Equation 9.

$$BTS_{7l} = \frac{\sum_{k=1}^K (br_{imjkl} \times g_{7imjkl})}{\sum_{k=1}^K (or_{imjk}) \times g_{max}} \quad (9)$$

where  $BTS_{7l}$  is the total score of the  $l^{\text{th}}$  bidder received from the seventh grading system,  $br_{imjkl}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the  $l^{\text{th}}$  bidder in his total bid price  $BP_l$ ,  $g_{7imjkl}$  is the grade received by each bidder  $l$  for the  $k^{\text{th}}$  major activity according to the value of the seventh comparison ratio ( $r_{7kl}$ ) based on the intervals given in Table 1,  $or_{imjk}$  is the ratio of the  $k^{\text{th}}$  major activity estimated by the owner in the  $ECC$ , and  $g_{max}$  is the maximum value of the seventh grading system, which equals 42.

### 3.3 Eighth grading system

The main idea behind this grading system is to compare the ratio of the sum of all major activities' total prices offered by the  $l^{\text{th}}$  bidder ( $sbr_{imjkl}$ ) to that of minor activities ( $sbr_{imnp}$ ) on the one hand with the ones ( $sor_{imjk}$ ,  $sor_{imnp}$ ) estimated by the owner on the other. First the minor activities ( $i_{mnp}$ ) ( $p = 1, 2, 3, \dots, P$ ) whose total price is <5% of the  $ECC$  are determined. Then, the eighth comparison ratio ( $r_{8l}$ ) for the  $l^{\text{th}}$  bidder is calculated using Equations 10–12.

$$\begin{aligned}
 sbr_{i_{mjkl}} &= \sum_{k=1}^K (bup_{imjkl} \times q_{i_{mjkl}}); sbr_{i_{mnp}} \\
 &= \sum_{p=1}^P (bup_{imnp} \times q_{i_{mnp}}) \quad (10)
 \end{aligned}$$

where  $sbr_{i_{mjkl}}$  is the sum of the major activities' prices offered by the  $l^{th}$  bidder,  $bup_{imjkl}$  is the unit price of the  $k^{th}$  major activity offered by the  $l^{th}$  bidder,  $q_{i_{mjkl}}$  is the quantity of the  $k^{th}$  major activity,  $sbr_{i_{mnp}}$  is the sum of the minor activities' prices offered by the  $l^{th}$  bidder,  $bup_{imnp}$  is the unit price of the  $p^{th}$  minor activity offered by the  $l^{th}$  bidder, and  $q_{i_{mnp}}$  is the quantity of the  $p^{th}$  minor activity.

$$\begin{aligned}
 sor_{i_{mjkl}} &= \sum_{k=1}^K (oup_{imjkl} \times q_{i_{mjkl}}); sor_{i_{mnp}} \\
 &= \sum_{p=1}^P (oup_{imnp} \times q_{i_{mnp}}) \quad (11)
 \end{aligned}$$

where  $sor_{i_{mjkl}}$  is the sum of the major activities' prices estimated by the owner,  $oup_{imjkl}$  is the unit price of the  $k^{th}$  major activity estimated by the owner,  $q_{i_{mjkl}}$  is the quantity of the  $k^{th}$  major activity,  $sor_{i_{mnp}}$  is the sum of the minor activities' prices estimated by the owner,  $oup_{imnp}$  is the unit price of the  $p^{th}$  minor activity estimated by the owner, and  $q_{i_{mnp}}$  is the quantity of the  $p^{th}$  minor activity.

Then, the eighth comparison ratio ( $r_{8l}$ ) is calculated using Equation 12.

$$r_{8l} = \frac{(sbr_{i_{mjkl}} \div sbr_{i_{mnp}})}{(sor_{i_{mjkl}} \div sor_{i_{mnp}})} \quad (12)$$

where  $r_{8l}$  is the eighth comparison ratio for the  $l^{th}$  bidder,  $sbr_{i_{mjkl}}$  is the sum of the major activities' prices offered by the  $l^{th}$  bidder,  $sbr_{i_{mnp}}$  is the sum of the minor activities' prices offered by the  $l^{th}$  bidder,  $sor_{i_{mjkl}}$  is the sum of the major activities' prices estimated by the owner, and  $sor_{i_{mnp}}$  is the sum of the minor activities' prices estimated by the owner.

Each bidder  $l$  receives a grade for each major activity ( $g_{8l}$ ) according to the value of the eighth comparison ratio ( $r_{8l}$ ) based on the intervals given in Table 1, and then the total score of the  $l^{th}$  bidder received from the eighth grading system ( $BTS_{8l}$ ) is found by using Equation 13.

$$BTS_{8l} = \frac{g_{8l}}{g_{max}} \times 100 \quad (13)$$

where  $BTS_{8l}$  is the total score of the  $l^{th}$  bidder received from the eighth grading system,  $g_{8l}$  is the grade received by each bidder  $l$  according to the value of the eighth comparison ratio ( $r_{8l}$ ) based on the intervals given in Table 1, and  $g_{max}$  is the maximum value of the eighth grading system, which equals 42.

For all grading systems, a comparison rate is calculated. Bidders receive grades depending on these ratios. The grading table (Table 1) allows the owner to evaluate the bidders objectively. In grading systems 6, 7, and 8, a comparison rate that is  $>1.050$  corresponds to the lowest grade ( $g_{min}=1$ ), whereas a comparison rate  $<0.950$  corresponds to the highest grade ( $g_{max}=42$ ). Finally, each bidder's final score can be calculated by assigning weights to each grading system according to a preferred criterion (Equation 14). The evaluation of the bidders will be based on these final scores.

$$FS_l = \sum_{j=1}^8 (w_j \times BTS_{jl}) \quad (14)$$

where  $w_j$  is the weight for the  $j^{th}$  grading system and  $BTS_{jl}$  is the total score of the  $l^{th}$  bidder according to the  $j^{th}$  grading system.

## 4 Illustrative example

An illustrative example used by Polat et al. (2019) is presented to validate the applicability of the proposed model in construction projects. The presented example comprises 72 activities; five out of these 72 activities are identified as "major activities", while others are identified as "minor activities". The unit price of each activity estimated by the owner are taken from "The Construction and Installation Unit Prices Book", published by the Turkish Ministry of Environment and Urban Planning. The units, quantities, and unit prices of these 72 activities, as estimated by the owner (O) and offered by eight bidders (B), are presented in Table 2.

The ECC provided by the owner is 13,766,619.41 TL, and the bid prices offered by the eight bidders are 14,043,276.86 TL ( $BP_1$ ), 13,826,569.14 TL ( $BP_2$ ), 13,389,997.59 TL ( $BP_3$ ), 13,624,850.19 TL ( $BP_4$ ), 13,947,114.50 TL ( $BP_5$ ), 13,622,893.85 TL ( $BP_6$ ), 13,641,083.17 TL ( $BP_7$ ), and 13,538,572.61 TL ( $BP_8$ ), respectively.

The weights assigned to the grading systems by Polat et al. (2019) were 20% for the first grading system, 15% for the second one, 10% for the third one, 15% for the fourth one, and 40% for the fifth one. In this study, different weights are assigned due to the addition of three

Tab. 2: Input data for illustrative example

Activity ID	Unit	Quantity	Unit prices (in TL [Turkish Liras])								
			O	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>
A <sub>1</sub>	m <sup>3</sup>	700	14.38	14.75	13.07	15.50	15.56	14.92	13.64	14.88	14.66
A <sub>2</sub>	m <sup>3</sup>	365	38.83	41.91	41.25	42.38	35.58	41.11	38.14	40.41	36.32
A <sub>3</sub>	m <sup>3</sup>	850	2.84	3.08	2.59	2.96	2.63	2.98	2.84	2.86	2.68
A <sub>4</sub>	m <sup>3</sup>	736	4.83	5.29	4.57	5.27	5.15	4.36	4.46	4.62	4.72
A <sub>5</sub>	M	198	67.70	65.94	63.94	74.14	65.43	69.76	68.75	69.39	62.33
A <sub>6</sub>	m <sup>3</sup>	59	31.88	29.82	28.96	29.66	29.41	33.12	29.21	28.89	29.34
A <sub>7</sub>	m <sup>3</sup>	150	14.19	13.60	13.21	14.34	13.67	13.78	15.17	13.64	15.11
A <sub>8</sub>	m <sup>3</sup>	90	29.19	26.97	28.68	29.95	31.42	30.40	31.12	29.52	30.38
A <sub>9</sub>	m <sup>3</sup>	2000	178.63	170.44	183.53	173.77	166.04	189.94	187.99	175.84	169.66
A <sub>10</sub>	M	1200	335.43	330.84	362.48	336.29	333.25	322.75	318.94	329.43	353.08
A <sub>11</sub>	M	650	68.40	65.53	74.79	69.38	63.26	64.79	69.13	72.10	73.25
A <sub>12</sub>	m <sup>3</sup>	350	52.20	51.26	47.00	54.89	56.72	47.53	54.14	52.51	56.04
A <sub>13</sub>	m <sup>3</sup>	100	86.29	84.53	92.11	85.72	91.16	88.14	85.52	88.90	82.79
A <sub>14</sub>	m <sup>3</sup>	360	121.63	112.82	133.19	131.99	130.73	122.52	128.27	111.87	124.88
A <sub>15</sub>	M	36	29.19	29.64	28.97	30.16	27.98	28.77	31.89	29.44	27.92
A <sub>16</sub>	M	40	33.40	35.59	34.05	32.36	36.04	35.12	36.57	32.56	36.23
A <sub>17</sub>	m <sup>2</sup>	1000	22.18	20.98	23.41	20.95	23.78	22.35	23.19	23.00	20.95
A <sub>18</sub>	m <sup>2</sup>	750	23.24	23.34	21.41	23.86	21.85	21.32	24.63	21.88	22.87
A <sub>19</sub>	m <sup>2</sup>	635	31.39	32.96	31.66	29.76	30.56	32.36	28.90	29.47	33.33
A <sub>20</sub>	m <sup>2</sup>	400	35.64	36.75	37.55	36.58	39.03	35.38	37.56	35.40	36.21
A <sub>21</sub>	m <sup>2</sup>	348	38.05	39.78	38.77	39.28	35.44	35.79	39.19	39.63	35.08
A <sub>22</sub>	m <sup>2</sup>	250	50.16	50.34	49.73	52.14	45.85	52.27	49.12	50.30	49.46
A <sub>23</sub>	m <sup>2</sup>	100	26.56	26.26	27.36	26.88	25.07	24.98	27.50	24.59	27.82
A <sub>24</sub>	m <sup>2</sup>	150	35.63	36.26	34.12	37.72	34.84	35.66	35.98	32.82	35.03
A <sub>25</sub>	m <sup>2</sup>	75	23.61	24.87	21.25	21.81	24.29	23.15	23.41	24.72	24.89
A <sub>26</sub>	m <sup>2</sup>	98	28.59	25.89	26.65	28.53	28.52	31.41	28.69	28.21	30.46
A <sub>27</sub>	m <sup>2</sup>	50	27.29	27.41	25.95	29.84	26.82	28.77	25.32	25.65	24.73
A <sub>28</sub>	m <sup>2</sup>	43	29.98	30.20	29.84	27.50	30.75	28.48	32.57	28.26	30.09
A <sub>29</sub>	m <sup>2</sup>	66	44.61	45.92	47.46	44.67	40.65	48.92	42.76	43.53	41.53
A <sub>30</sub>	m <sup>2</sup>	40	58.94	54.01	56.03	53.11	59.19	59.65	54.99	59.98	60.40
A <sub>31</sub>	m <sup>2</sup>	40	39.54	43.20	42.80	39.07	37.86	38.69	42.66	41.16	39.66
A <sub>32</sub>	m <sup>2</sup>	100	40.24	42.55	39.59	41.02	42.52	43.04	41.36	41.56	36.69
A <sub>33</sub>	m <sup>2</sup>	450	1.94	1.75	2.08	2.01	1.94	2.00	1.99	2.01	1.75
A <sub>34</sub>	m <sup>2</sup>	350	2.35	2.17	2.48	2.39	2.27	2.55	2.20	2.58	2.19
A <sub>35</sub>	m <sup>2</sup>	40	16.91	15.70	15.29	15.72	16.19	15.85	15.66	16.05	17.10
A <sub>36</sub>	m <sup>2</sup>	60	20.71	18.91	20.89	20.97	20.51	21.01	22.31	20.95	20.96

Continued

Tab. 2: Continued

Activity ID	Unit	Quantity	Unit prices (in TL [Turkish Liras])								
			O	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub>	B <sub>7</sub>	B <sub>8</sub>
A <sub>37</sub>	m <sup>2</sup>	50	14.68	15.50	14.91	13.82	13.55	13.29	14.37	14.57	13.84
A <sub>38</sub>	m <sup>2</sup>	1000	27.71	26.86	27.56	26.32	29.55	26.55	26.78	27.64	28.43
A <sub>39</sub>	m <sup>2</sup>	450	43.24	39.04	46.78	45.19	46.00	43.59	45.01	45.12	47.13
A <sub>40</sub>	m <sup>2</sup>	900	32.39	34.31	29.90	34.25	33.58	32.77	29.28	31.71	29.34
A <sub>41</sub>	m <sup>2</sup>	650	33.90	33.90	30.71	36.87	36.48	30.83	35.46	31.94	31.03
A <sub>42</sub>	m <sup>2</sup>	100	6.29	6.84	5.71	6.33	5.92	5.68	5.97	6.86	6.02
A <sub>43</sub>	m <sup>2</sup>	1000	1.29	1.29	1.31	1.23	1.40	1.36	1.22	1.19	1.18
A <sub>44</sub>	m <sup>2</sup>	150	7.33	7.01	6.67	6.64	7.45	7.91	7.12	7.29	7.64
A <sub>45</sub>	m <sup>2</sup>	2000	11.78	11.58	12.45	11.39	11.86	12.38	11.66	12.90	12.94
A <sub>46</sub>	m <sup>2</sup>	1600	30.04	29.74	30.87	31.84	27.61	30.66	31.46	29.77	30.38
A <sub>47</sub>	m <sup>2</sup>	2000	29.56	30.55	30.33	30.01	32.43	28.17	30.23	29.49	27.07
A <sub>48</sub>	m <sup>3</sup>	600	4.59	4.65	4.72	4.72	4.68	4.50	4.27	4.87	4.15
A <sub>49</sub>	m <sup>3</sup>	450	5.84	5.36	6.42	5.29	5.88	6.11	5.41	6.06	5.64
A <sub>50</sub>	m <sup>2</sup>	750	4.83	5.29	4.61	4.97	5.18	4.77	4.79	4.86	5.21
A <sub>51</sub>	m <sup>2</sup>	1600	115.81	107.85	117.90	125.97	116.49	107.78	122.21	120.78	120.28
A <sub>52</sub>	m <sup>2</sup>	650	136.51	142.73	139.82	133.89	126.59	128.11	149.12	141.47	126.39
A <sub>53</sub>	m <sup>2</sup>	650	88.36	89.02	81.44	91.52	92.22	93.89	89.25	79.90	81.71
A <sub>54</sub>	m <sup>2</sup>	250	123.24	133.28	134.02	112.88	133.04	134.99	119.83	131.82	114.27
A <sub>55</sub>	m <sup>2</sup>	690	50.34	48.72	47.31	53.42	49.00	46.54	53.07	51.53	45.89
A <sub>56</sub>	m <sup>2</sup>	600	170.88	157.50	161.31	160.47	179.91	166.13	182.67	164.95	182.67
A <sub>57</sub>	m <sup>2</sup>	350	319.38	338.86	338.76	344.12	339.48	325.54	350.95	306.97	302.08
A <sub>58</sub>	m <sup>2</sup>	400	250.09	261.30	253.70	264.69	249.89	244.55	265.10	264.46	226.00
A <sub>59</sub>	ton	1300	2096.56	2127.33	2152.31	1941.54	2089.57	2077.30	2026.49	1988.62	2045.57
A <sub>60</sub>	ton	1650	2017.94	2140.37	2143.31	1877.71	1975.24	1998.95	1887.98	2160.53	1974.92
A <sub>61</sub>	ton	350	1972.66	1871.37	1796.76	2120.92	2169.36	1789.12	2155.85	1837.37	1987.05
A <sub>62</sub>	ton	1000	1939.23	1985.11	1832.36	1762.37	1999.50	2115.62	2044.00	1860.08	1918.36
A <sub>63</sub>	ton	1150	1914.79	1914.13	1780.31	2038.14	1781.33	1987.31	1875.30	1810.11	1885.75
A <sub>64</sub>	ton	200	3386.01	3642.50	3635.81	3425.98	3236.22	3658.23	3346.48	3591.23	3238.26
A <sub>65</sub>	Kg	4000	8.64	9.39	8.07	8.58	7.87	8.65	9.13	8.38	7.97
A <sub>66</sub>	m <sup>2</sup>	2000	9.59	9.16	10.29	10.06	9.74	10.16	9.55	8.82	10.51
A <sub>67</sub>	m <sup>2</sup>	600	13.00	12.96	13.32	14.30	14.00	13.68	11.84	12.36	14.07
A <sub>68</sub>	m <sup>2</sup>	150	5.23	5.62	5.34	5.53	5.32	5.06	4.91	5.17	5.45
A <sub>69</sub>	m <sup>2</sup>	2000	15.65	14.26	16.26	14.72	16.25	15.96	14.87	15.75	16.47
A <sub>70</sub>	m <sup>2</sup>	2000	18.56	19.20	16.81	18.13	19.78	18.95	17.37	17.73	18.78
A <sub>71</sub>	m <sup>2</sup>	700	28.60	27.78	27.36	26.28	31.23	30.24	27.97	27.13	28.49
A <sub>72</sub>	m <sup>2</sup>	2000	20.88	21.98	22.75	22.95	19.38	19.91	21.36	19.67	22.51

Source: Polat et al. (2019).



Tab. 3: Final scores of bidders

Bidder ID	Grading system number								Final Score	Ranking
	1 (15%)	2 (10%)	3 (5%)	4 (10%)	5 (30%)	6 (5%)	7 (5%)	8 (20%)		
B <sub>1</sub>	51.35	41.16	39.12	31.82	50.00	40.19	36.59	45.24	44.84	8
B <sub>2</sub>	49.30	47.62	45.27	50.00	66.67	51.24	47.13	69.05	58.15	1
B <sub>3</sub>	49.52	62.05	59.75	77.27	2.38	66.41	62.42	76.19	46.74	6
B <sub>4</sub>	50.73	55.00	53.22	63.64	54.76	54.74	51.55	50.00	53.88	4
B <sub>5</sub>	50.23	43.29	42.19	40.91	42.86	43.47	41.00	52.38	45.62	7
B <sub>6</sub>	49.74	54.88	53.38	63.64	52.38	57.57	54.73	64.29	56.17	3
B <sub>7</sub>	49.59	53.83	52.26	59.09	38.10	55.75	52.74	61.90	50.58	5
B <sub>8</sub>	50.35	58.99	57.40	68.18	61.90	59.61	56.58	52.38	58.00	2

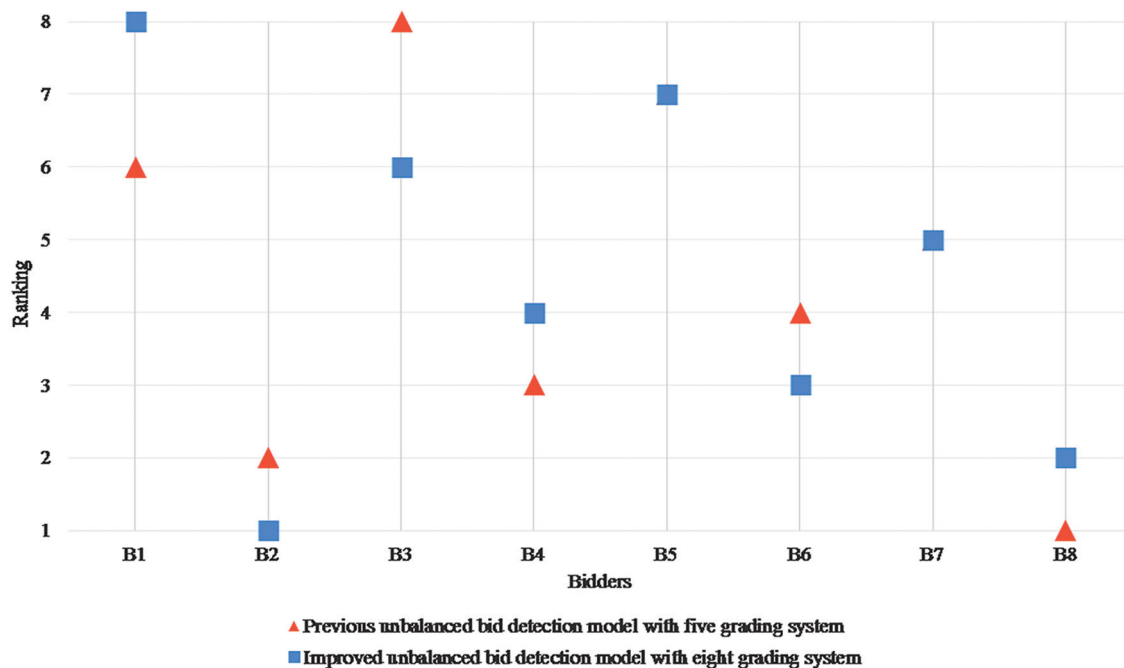


Fig. 1: Comparison of the ranking of bidders according to the previous model developed by Polat et al. (2019) and the current model.

new grading systems to the proposed model. An expert, who works as a contracting engineer in a public agency and is responsible for evaluating whether the proposed bids are balanced or not, was consulted, and weights were assigned to the developed grading systems based on his opinion and experience. In this study, weights were assigned as 15% for first grading system, 10% for the second one, 5% for the third one, 10% for the fourth one, 30% for the fifth one, 5% for the sixth one, 5% for the seventh one, and 20% for the eighth one. The owner can assign different weights to the grading systems according to the project characteristics. The final scores calculated for the eight bidders are presented in Table 3.

Based on the results presented in Table 3, Bidder 2 ( $B_2$ ) has the highest final score, whereas Bidder 1 ( $B_1$ ) has the lowest final score. Although  $B_3$  offered the lowest bid price, it is ranked sixth according to the final scores. Moreover,  $B_1$  has the lowest final score while offering the highest bid price. Finally,  $B_1$  submits the highest bid price and the most unbalanced bid, whereas  $B_2$  submits the most balanced bid despite not offering the lowest bid price. In other words,  $B_2$  is the most appropriate bidder for the owner.

The findings of this study reveal that the improved model provides a different ranking of bidders than the one obtained by the model in Polat et al. (2019). The ranking

of the bidders according to the final scores is presented in Figure 1.

As seen in Figure 1, the improved unbalanced bid detection model provides a different ranking of bidders than the one obtained by Polat et al.'s (2019) model. For instance, while Bidder 8 ( $B_8$ ) ranked first and Bidder 2 ( $B_2$ ) ranked second according to the previous model, Bidder 2 ( $B_2$ ) ranked first and Bidder 8 ( $B_8$ ) ranked second according to the proposed improved model. The main reason behind this variation is the addition of new grading systems and the changes in the weights assigned to the grading systems.

## 5 Conclusions

Unbalanced bidding is one of the most important problems for owners in the construction industry. Not only does it create an environment of unfair competition but also increases the project cost, considering the time value of money. If owners detect unbalancing when evaluating bids, they have the right to disqualify the unbalanced bids. Since the bidder who offers the lowest bid price is predominantly awarded the project and the unit price of each bid item is usually not taken into consideration, it is not an easy task for owners to detect the existence of unbalancing. Moreover, it is more difficult for owners to detect materially unbalanced bids (i.e., quantity error exploitation) when compared with the mathematically unbalanced ones (front-end loading and back-end loading). A fairer competition environment can be created as long as the owners are armed with a tool that can help them in detecting unbalanced bids easily.

The main aim of this study was to propose an improved version of the unbalanced bid detection model developed by Polat et al. (2019). The proposed model uses eight different grading systems. Owners may assign different weights to these grading systems according to the characteristics of their projects. After assigning weights to each grading system, the final scores of bidders can be calculated. All bidders can be evaluated not only according to the offered bid prices but also according to the calculated final scores. While the submitted bid with the highest final score indicates that it is the most balanced one, the submitted bid with the lowest final score indicates that it is the most unbalanced one. This study highly recommends the owners to consider not only the submitted bid price but also the balance score of the bidders when evaluating the bids, in order to shield themselves from the potential problems arising from unbalanced bids.

An illustrative example is also presented to validate the applicability of the proposed model. It has been observed that the improved model can not only detect unbalanced bids but also provide a different ranking of bidders than the one obtained by the model of Polat et al. (2019). However, this study is limited by the fact that it only focuses on materially unbalanced bids in unit price contracts. In future studies, the model can be further developed so that it can also deal with the detection of mathematically unbalanced bids.

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