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Critical stress of oval foil winding with epoxy coated insulation determined using measured equivalent modulus of elasticity

Nina Meško[a], Branimir Ćućić[a] and Damir Žarko[b]

[a]Končar Distribution and Special Transformers Inc, Zagreb, Croatia; [b]Faculty of Electrical Engineering and Computing, University of Zagreb, Zagreb, Croatia

ABSTRACT

In the distribution transformers design oval windings are used due to economic advantages. On the other hand, such windings are more susceptible to radial forces in a short circuit. A diamond dotted paper with an epoxy coating is used in order to increase the stiffness of the winding. Despite that, winding failure may occur during the short circuit, e.g. buckling of inner winding. Because of a very thin foil conductor (typically 0.5–2 mm), the most critical is inner low voltage foil winding which can collapse due to radial forces at stresses far below the elastic limit of conductor material. This paper shows an analytical approach to the calculation of critical stress in inner oval foil winding with epoxy coated insulation. Critical stress was calculated using the equation for free buckling of round winding. Equivalent Young's modulus of elasticity was obtained experimentally from the testing of the sample model loaded with bending force on a tensile test machine. A total of 12 test samples were formed from aluminium foil conductor and diamond dotted paper and cured at the temperature of 105°C. The results were successfully verified on distribution transformers subjected to short circuit withstand tests.

1. Introduction

External short circuits in the power system can cause high currents and accordingly high electromagnetic forces in the transformer. Mechanical stress caused by electromagnetic forces can damage the transformer. Despite the requirements which need to be achieved in order for the transformer to successfully pass the short circuit test [1] and the transformer manufacturer's efforts to secure the transformers from malfunction due to the short circuit, external short circuits in the power systems are still one of the most common causes of the transformer malfunction [2,3]. Bergonzi et al. [4] claim that every fourth power transformer does not pass the short circuit test.

Transformer failure not only generates the cost of acquiring the new unit or the cost of repair, but also the cost of the undelivered energy in the power network. This is the reason why there is a constant effort in improving the transformer's ability to withstand short circuits.

2. Related work

There are many papers and books written about forces, stresses and criteria for designing transformers with traditional round windings. Waters [5] and Kulkarni [2] give an overall classical approach to this subject. In [3] Bertagnolli gives the overview of the short

circuit causes, manufacturing experiences and criteria for improving the chances of passing the short circuit testing. In [6] factors of safety for the radial and axial forces, effects of temperature, moisture and aging and ways of improving short circuit withstand capability are described. Bakshi and Kulkarni [7] give buckling strength analysis of inner windings of power transformers under radial short-circuit forces, while Geissler [8] presents the finite element analysis of radial buckling strength with the focus on continuously transposed conductors (CTCs).

On the other hand, for the transformers with the non-round windings there is a lack of literature especially those related to stress calculation and design criteria for composite winding structures. The CIGRE Working Group 12.19 [9] gives critical buckling stress for inner oval winding based on a "long column collapse" assuming the round part of oval winding to be unsupported. The paper [10] focusses on inner oval winding with the foil conductor giving an analytical approach for mechanical strength calculation. In [11] it can be found that the short-circuit design of rectangular windings is generally worked out empirically and that the real strength of the composite structure of a winding may depend almost entirely on the bond.

This paper extends [10] by applying the principle given in [9]. The focus will be on determining the critical stress of inner foil winding with epoxy coated

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CONTACT Nina Meško ✉ nina.mesko@koncar-dst.hr ✉ Končar Distribution and Special Transformers Inc, Zagreb, Croatia

insulation using the equivalent Young's modulus of elasticity obtained experimentally from the deflection of 12 sample models loaded with bending force on a tensile test machine. These windings are usual in distribution transformers.

3. Forces and stresses in windings

3.1. Forces in windings

When a wire carrying an electric current I is placed in a magnetic field with flux density B, it experiences the force F. For the segment of wire of length l this force is defined by the vector product

$$F = I \cdot l \times B \tag{1}$$

where F, l and B are the vectors (denoted by a bold letter). The direction of the vector l is along the wire, aligned with the direction of the current flow I.

The force in the transformer winding F is usually calculated in the radial (F_R) and axial (F_A) direction. The main geometry and typical transformer leakage flux pattern are shown in Figure 1. Vectors l, B and F are equal

$$l = \pm l \cdot a_t \tag{2}$$

$$Bs = B_R \cdot a_R + B_A \cdot a_A \tag{3}$$

$$F = NI \cdot l \times B = \pm NI \cdot l \cdot (B_A \cdot a_R - B_R \cdot a_A)$$
$$= F_R \cdot a_R + F_A \cdot a_A \tag{4}$$

where:

a_R, a_A, a_t – unit vectors in radial, axial and tangential direction, respectively

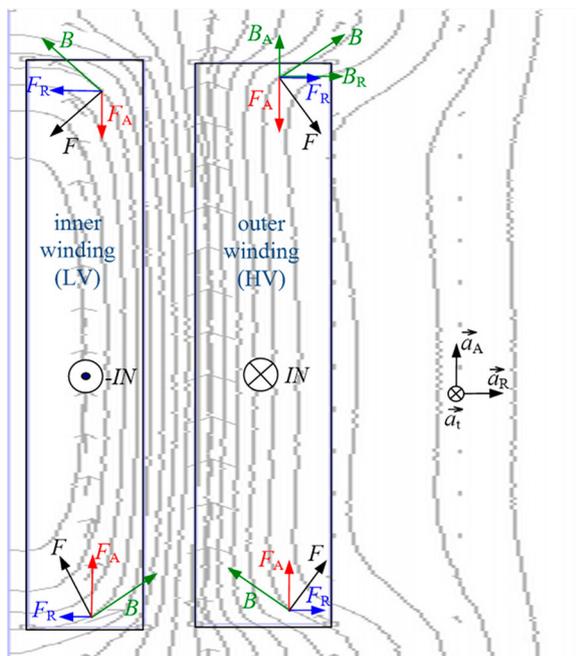F – force on the winding, N

F_R – radial component of the force, N

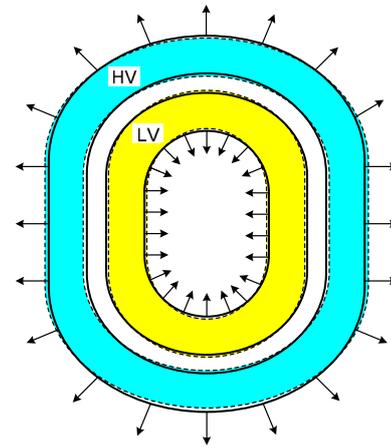**Figure 1.** Leakage flux and forces in windings.

Figure 2. Oval windings subjected to radial short-circuit forces.

F_A – axial component of the force, N

B_R – radial component of the flux density, T

B_A – axial component of the flux density, T

The axial component of the flux density B_A induces the radial component of the force F_R while the radial component of the flux density B_R induces the axial component of the force F_A. In this paper, the focus is on the radial force. Radial force F_R tends to move the inner winding inwards but also tends to move the outer winding outwards. Therefore, radial forces cause the compressive stress on the inner windings and tensile stress on the outer windings.

Figure 2 illustrates the behaviour of oval windings subjected to radial short circuit forces. These radial forces cause the inner winding of the transformer to be pushed inwardly against the core, and the outer winding to be expanded outwardly. The curved part of the winding is less liable to deformation since its shape is closer to circular. On the other hand, the forces acting in the same direction are concentrated along the straight part which is why the outer winding (HV) tends to become circular [12]. Therefore, this is the area where the largest change in winding geometry is expected.

3.2. Force calculation

The critical stress is normally defined for a circular ring so the equivalent radial force (from which the compressive stress is extracted) is calculated only for the round parts of the winding.

Total radial force in the round parts of the winding is a sum of all radial forces affecting N_L individual layers or all N turns in the winding

$$F_R = \sum_{i=1}^{N_L} F_{Ri} = \sum_{j=1}^{N} F_{Rj} = NB_A I\ell \tag{5}$$

where F_R is the radial force in N, N is the number of turns, B_A is the mean axial component of the flux density in the winding in T, I is the current flowing in a turn

in kA and l is the mean circumference of the round parts of the winding in mm ($l = D\pi$). The diameter D is the mean value of the inner and outer diameter of round parts of the inner LV winding in mm.

Mean flux density in the case of two-winding configuration is approximately one half of the flux density B_0 in the main cooling duct between the lower voltage (LV) and higher voltage (HV) windings

$$B_A = \frac{1}{2}B_0 = \frac{1}{2}\mu_0\frac{NI}{H_{12}}K_R \cdot 10^6 \qquad (6)$$

where H_{12} is the mean electrical height of both windings in mm, K_R is the Rogowski factor, and μ_0 is the permeability of vacuum ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).

Total radial force for both round parts of the winding is

$$F_R = \frac{1}{2}\frac{\mu_0(NI)^2 l K_R}{H_{12}} \cdot 10^6 = \frac{1}{2}\frac{\mu_0(NI)^2 D\pi K_R}{H_{12}} \cdot 10^6 \qquad (7)$$

The radial force is usually calculated for the highest current load so in this case for current I the peak value of the test current I_p is used.

Short circuit failure in distribution transformers sometimes occurs only in the outermost part of the inner winding (winding is divided into parts by cooling duct(s)). The inner foil winding in the distribution transformers is usually designed with 0, 1 or 2 cooling ducts. Figure 3 shows an example of the buckling of 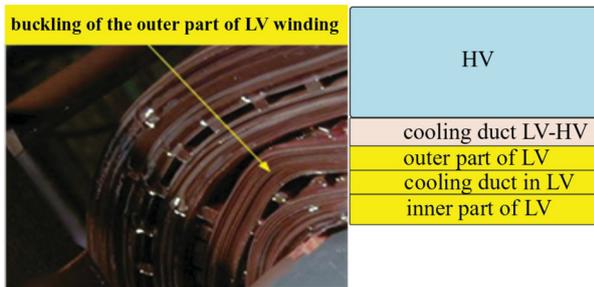the outer part of the LV foil winding (with 1 cooling duct), while the inner part is unaffected. Mean compressive radial stress (section 0) in the outermost part of the inner winding is higher than the mean compressive stress in the whole winding (by 60% in the case of one cooling duct in the foil winding [10]) due to the higher flux density. Furthermore, critical stress slightly decreases in the outer part of the LV winding. Therefore, compressive and critical stresses are to be calculated in the outermost part of the inner winding.

When calculating the total radial force in the outer part of the winding, instead of N and B_0, $k_N \cdot N$ and $k_B \cdot B_0$ should be used in (5) and (6). According to Figure 4(b) which corresponds to the case of 1 cooling duct shown in Figure 3, when the radial force is calculated in the outer half of the LV winding only half of the turns are included and the mean flux density there is 3/4 of the flux density in the main cooling duct (B_0), therefore $k_N = 1/2$ and $k_B = 3/4$. In the case of two cooling ducts in the inner LV foil winding, we have $k_N = 1/3$ and $k_B = 5/6$ for the outer third of the inner winding (Figure 4(c)).

Radial force in the outer part of the inner winding is calculated as follows:

$$\begin{aligned} F_R &= k_N k_B \frac{\mu_0(NI)^2 l K_R}{H_{12}} \cdot 10^6 \\ &= k_N k_B \frac{\mu_0(NI)^2 K_R}{H_{12}}(k_{D1}D_1 \\ &\quad + k_{D2}D_2)\pi \cdot 10^6 \end{aligned} \qquad (8)$$

where D_1 and D_2 are the inner and outer diameter of the inner LV winding in mm, while k_N, k_B, k_{D1}, and k_{D2} are the constants according to Table 1. Diameters D_1 and D_2 are calculated from the winding circumference.

Figure 3. Short circuit failure in the outer part of the inner LV foil winding.

Table 1. Constants for the inner winding.

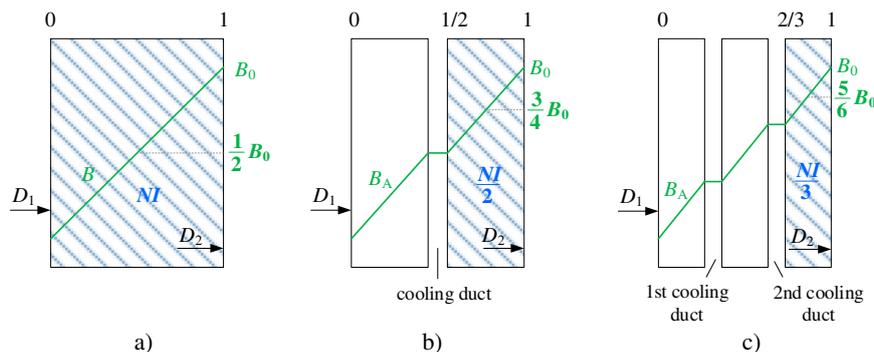| Constant	Whole inner winding (without cooling ducts)	Outer half of the inner winding (1 cooling duct)	Outer third of the inner winding (2 cooling ducts)
k_N	1	1/2	1/3
k_B	1/2	3/4	5/6
k_{D1}	1/2	1/4	1/6
k_{D2}	1/2	3/4	5/6

Figure 4. Parts of the inner LV winding in which the radial force is calculated for the cases of (a) 0, (b) 1 and (c) 2 cooling ducts.

3.3. Compressive stress and buckling

3.3.1. Compressive stress calculation

Radial force F_R causes compressive stress in cross-section of the inner winding. Figure 5 shows the inner winding subjected to an external radial pressure p_R which causes the compressive stress F_c/S in the winding cross-section.

Equivalent compressive force F_c in the cross-section is obtained by integrating all elementary forces acting in the same direction

$$F_c = \int_0^{\pi/2} dF_R \sin\alpha = \int_0^{\pi/2} p_R H_W dl \sin\alpha$$

$$= \int_0^{\pi/2} p_R H_W r d\alpha \sin\alpha = p_R H_W r = \frac{F_R}{2r\pi H_W} H_W r$$

$$F_c = \frac{F_R}{2\pi} \tag{9}$$

where p_R is the radial pressure on the perimeter of the winding induced by the radial force F_R, while H_W is the height of the winding.

Compressive stress in the cross-section S of the observed part of the inner winding is

$$\sigma_c = \frac{F_c}{S} = \frac{F_R}{2\pi S}$$

$$= k_N k_B \frac{\mu_0(NI)^2 K_R}{2SH_{12}}(k_{D1}D_1 + k_{D2}D_2) \cdot 10^6 \tag{10}$$

The cross-section S can be written as

$$S = k_N N (S_F + S_{DPP}) = k_N N (t_F + t_{DPP})h_F \tag{11}$$

where t_F and h_F are the thickness and height of foil conductor, respectively, while t_{DPP} is the thickness of layer insulation.

Compressive stress in the observed part of inner winding can be rewritten as

$$\sigma_c = k_B \frac{\mu_0 NI^2 K_R}{2(t_F + t_{DPP})h_F H_{12}}(k_{D1}D_1 + k_{D2}D_2) \cdot 10^6 \tag{12}$$

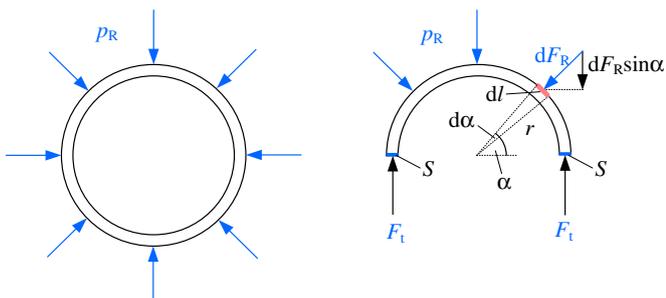Compressive stress for the whole winding is ($k_B = k_{D1} = k_{D2} = 1/2$)

$$\sigma_c = \frac{\mu_0 NI^2 K_R}{8(t_F + t_{DPP})h_F H_{12}}(D_1 + D_2) \cdot 10^6 \tag{13}$$

In the previous equations, stress is expressed in N/mm², current in kA, dimensions of the conductor and winding in mm and the cross-section in mm².

According to the standard IEC 60076-5 [1], compressive stress must not exceed

$$\sigma_c \leq 0.35 R_{p02} \quad \text{for the conductor without epoxy} \tag{14}$$

$$\sigma_c \leq 0.60 R_{p02} \quad \text{for the conductor with epoxy} \tag{15}$$

where R_{p02} is the yield stress of the conductor material.

3.3.2. Buckling

When a structure is subjected to compressive stress, buckling may occur. According to [13], a circular ring or tube can collapse due to external pressure at stresses far below the elastic limit of the material.

3.3.2.1. Free buckling of a thin ring.
For the thin ring, which is not supported from the inside (Figure 6), the critical hoop stress is defined as

$$\sigma_{cr} = E\left(\frac{t}{D}\right)^2 \tag{16}$$

where t is the thickness of the ring in mm, D is the diameter of the ring in mm and E is Young's modulus of the material in GPa.

This form of buckling without any supports from the inside is called free buckling. Critical stress in the ring is proportional to the square of the ring thickness and inversely proportional to the square diameter of the ring. The thicker the ring is and the smaller the diameter is, the more difficult is for the ring to buckle.

3.3.2.2. Forced buckling.
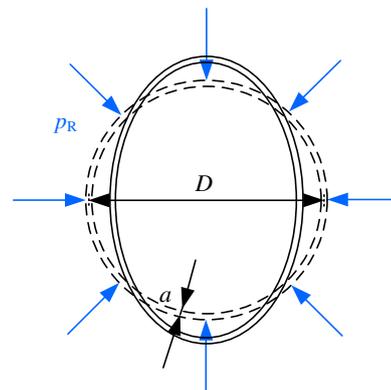Even if a ring has support on the inside (Figure 7), it can buckle. This sort of buckling is called forced buckling.

Figure 5. Compressive stress.

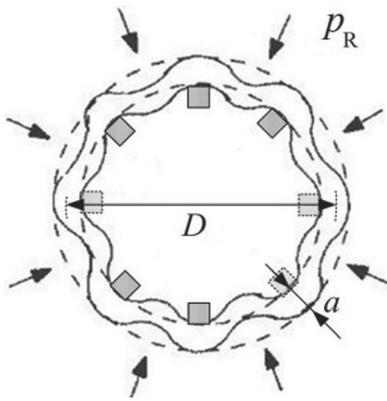**Figure 6.** Free buckling of a circular ring.

Figure 7. Forced buckling.

Critical stress in a ring which leads to forced buckling is

$$\sigma_{\text{cr\_forced}} = E\left(\frac{t}{D}\right)^2 \frac{n^2 - 4}{12} \qquad (17)$$

where n is the number of supports. Since there are usually around 20 supports, critical stress in a case of forced buckling is $20^2/12 \approx 33$ times higher than in the case of a free buckling. It's worth mentioning that when the number of supports is $n = 4$ (in every quarter of the circle), critical stress is equal as for the free buckling.

3.3.2.3. The most common form of buckling deformation of the inner oval foil winding. The most common form of deformation of the inner oval foil winding subjected to an inward force is shown in Figure 8. This could be explained with a fact that the foil winding or outer part of foil winding is not supported around the entire circumference. Due to vibration and opposite direction of forces (LV winding is pushed towards the core and HV winding in the opposite direction), supportive sticks in the cooling duct between LV and
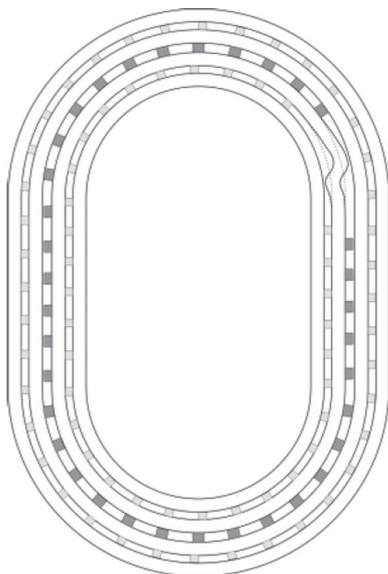

Figure 8. The most common form of buckling.

HV winding could lose their position which triggers the buckling of the winding. Based on factory experience gained by testing a large number of distribution transformers with foil winding, this phenomenon usually happens where the straight part of the oval winding becomes circular.

4. The proposed method for determination of the critical stress

For oval and unsupported winding critical stress can be calculated by using the expression (18) for the free buckling [10]. In this paper, the focus is on foil windings with epoxy coated layer insulation (diamond dotted press paper, DPP) which implies that the equivalent modulus of elasticity E_{equiv} must be taken for the composite structure of the conductor material (aluminium) and epoxy coated layer insulation (as discussed later in section 0).

To avoid buckling of the winding, the following criterion must be fulfilled [10]:

$$\sigma_c \leq \sigma_{\text{cr}} = E_{\text{equiv}} \left(\frac{t}{D}\right)^2 \qquad (18)$$

where t is the thickness of the foil winding or outer part of the foil winding and D is the mean diameter of the winding or the outer part of the winding (only round part of the oval is considered).

4.1. Equivalent modulus of elasticity in the linear region

Young's modulus E describes tensile elasticity, or the tendency of an object to deform along an axis when opposing forces are applied along that axis; it is defined as the ratio of tensile stress to tensile strain. Young's modulus of aluminium is a constant of the material, but for the composite structure (aluminium + epoxy coated paper) the equivalent modulus of elasticity depends on the winding dimensions and needs to be obtained experimentally.

4.1.1. Models of the foil windings

For experimental determination of the equivalent Young's modulus, 12 samples each of dimensions 250 × 20 mm (length × width) were used. Thickness of a sample ranging from 6 to 35 mm was selected in a way to represent the real thickness of the winding or outer part of foil winding. This was achieved by varying the conductor thickness from 0.4 to 1.35 mm. Since the aluminium is today usually used as a conductor material in the windings of distribution transformers, the aluminium foil was used. Two diamond dotted press papers (DPP) with a total thickness of 0.1 mm were inserted between each two foil conductors (Figure 9). Samples were cured at a temperature of 105°C.
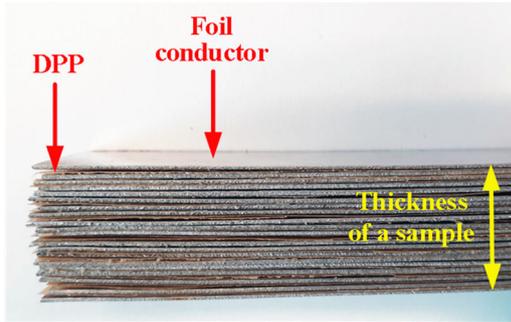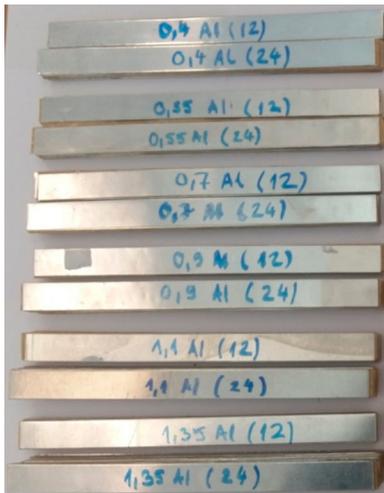
Figure 9. Structure of a test sample.

Figure 10. Tested samples.

Figure 11. Bending of a test sample on a tensile test machine.

width of the sample in mm, t is the thickness of the sample in mm, and Δy is the change in deflection in the linear region (in mm) if the applied force is increased by ΔF (in N).

The typical graph obtained from the tensile machine is shown in Figure 12.

5. Results and discussion

5.1. Experimental results (equivalent modulus of elasticity in the linear region)

Measured equivalent modulus of elasticity E of foil winding with epoxy coated insulation in the linear region (expressed in relation to base modulus of elasticity E_b of foil conductor, $E_b = 65$ GPa) is shown in Figure 13.

5.2. Tangential modulus of elasticity and critical stress calculation

According to [14], the appropriate modulus to use in Equation (18) is the tangential modulus of elasticity E_t (beyond the linear region). Compared to the equivalent modulus of elasticity in the linear region, the tangential modulus is lower because of the lower slope of a curve in Figure 12. As shown in Figure 12, the factor of reduction is 3,5. Decreased equivalent tangential modulus of elasticity E_t is obtained by dividing the modulus E from linear range by the reduction factor k ($k > 1$).

$$E_t = \frac{E}{k} \qquad (20)$$

Critical stress is obtained from the following equation:

$$\sigma_{cr} = E_t \left(\frac{t}{D} \right)^2 = \frac{E}{k} \left(\frac{t}{D} \right)^2 \qquad (21)$$

5.3. Verification of experimentally determined critical stress margin

For the verification process, only the transformers with oval foil windings stressed far below the elastic limit of the material and yield point R_{p02} were considered. In another words, if the transformer failed the short circuit withstand test, it was because of the buckling of inner foil winding (higher compressive stress than critical).

All samples are shown in Figure 10. Every sample is marked in the following way (e.g. 0,7 Al (24)):

- the first decimal number denotes foil thickness,
- Al denotes conductor material,
- number in brackets denotes the number of individual conductors in a sample.

For example, the sample marked as 0.7 Al (24) means that 24 aluminium foil conductors of thickness 0.7 mm were used in the model.

4.1.2. Determining the equivalent modulus of elasticity in linear region

Equivalent Young's modulus E is determined by applying the theory valid for the simply supported beam loaded with the bending force F concentrated in the middle of the beam.

E is obtained by measuring the deflection y of the sample model loaded in bending on tensile test machine shown in Figure 11

$$E = \frac{250 \, l^3 \Delta F}{w \, t^3 \Delta y} \qquad (19)$$

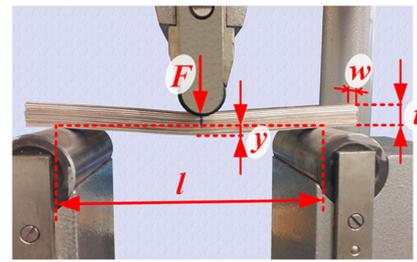where E is the equivalent modulus of elasticity in GPa, l is the distance between the supports in mm, w is the
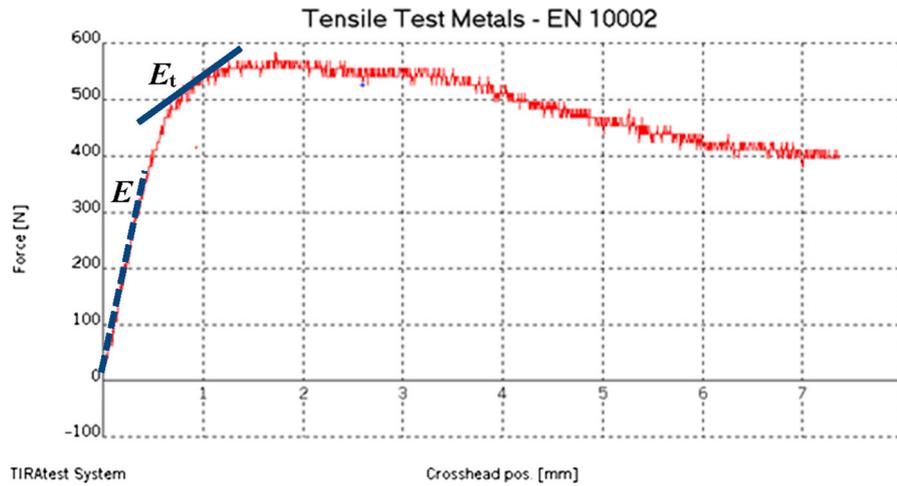
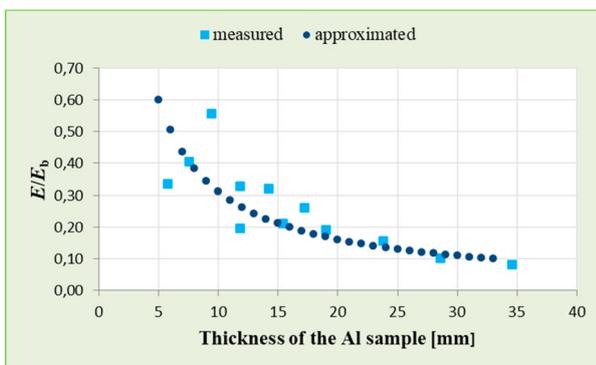**Figure 12.** The graph obtained from the tensile machine.

Figure 13. Equivalent modulus of elasticity of epoxy foil winding in linear range expressed in relation to modulus of elasticity of aluminium foil conductor.
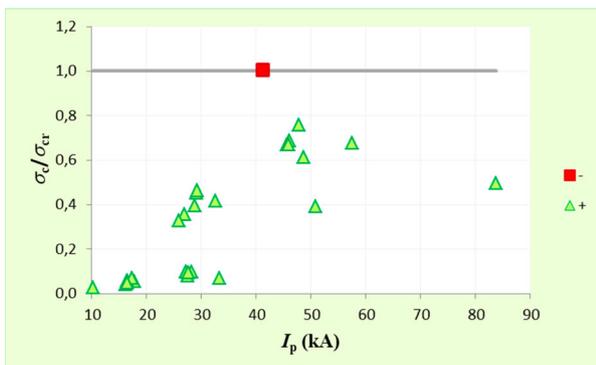

Figure 14. Sample transformers grouped according to the calculated compressive stress with respect to experimentally determined critical stress with markers showing short circuit test passability.

Figure 14 shows the ratio between compressive stress σ_c, calculated according to (12), and critical stress σ_{cr}, calculated according to (21), in inner foil winding for a group of 27 tested distribution transformers in the power range 250–1600 kVA. The ratio σ_c/σ_{cr} is shown as a function of test current peak value I_p. Red squares denote transformers that failed the test, and green triangles the transformers that successfully passed the short

circuit withstand test. As the ratio σ_c/σ_{cr} approaches 1 and especially when it exceeds 1, the situation becomes critical and buckling may occur although it does not necessarily have to. It depends mostly whether the position of supports is lost or not. Nevertheless, such design should be avoided or additional supports must be provided in the clamping system in order to prevent the outer winding to be expanded outwardly.

6. Conclusion

The analytical approach to the calculation of critical stress in inner oval foil winding with epoxy coated insulation in distribution transformers is presented. The critical stress depends on the equivalent tangential Young's modulus of elasticity of the composite structure (aluminium + epoxy coated paper) which was determined from the modulus of elasticity in the linear region by measuring the deflection of the samples on a tensile test machine.

It was demonstrated on a group of 27 distribution transformers subjected to short circuit withstand test that calculated compressive stress in the outer part of the inner LV foil winding correlates well with the critical stress obtained for the composite structure. In other words, the buckling of the inner oval foil winding consistently does not occur in the transformer units whose compressive stress is lower than critical stress determined using the methodology described in the paper.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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