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UDC 519.8:339.1:336.6:519.61
Original scientific paper
Received: 13.03.2009.

Optimal production-shipment decisions for the finite production rate model with scrap

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SUMMARY

This paper is concerned with the decision-making on the optimal production batch size and optimal number of shipments for a finite production rate model with random scrap rate. The classic finite production rate (FPR) model assumes a continuous inventory issuing policy for satisfying product demand and perfect quality for all items produced. However, in a real life vendor-buyer integrated production-inventory system, a multiple shipment policy is practically used in lieu of the continuous issuing policy, and it is inevitable to generate defective items during a production run. All nonconforming items produced are assumed to be scrap, and the finished (perfect quality) products can only be delivered to customers if the whole lot is quality assured at the end of the production run. The fixed-quantity multiple instalments of the finished batch are delivered to customers at a fixed interval of time. Mathematical modelling is employed and the renewal reward theorem is used to cope with the variable production cycle length. The long-run average cost for the proposed model is derived, and its convexity is proved by the use of the Hessian matrix equations. A closed-form optimal production-shipment policy for such an imperfect FPR model is obtained and a special case is discussed. Finally, a numerical example is provided to demonstrate the model's practical usage.

Key words: *manufacturing, multiple shipments, FPR model, scrap, optimal batch size, inventory.*

1. INTRODUCTION

The purpose of this study is to determine jointly the optimal production lot size and optimal number of shipments for a finite production rate (FPR) model with scrap. The FPR model is commonly used in the manufacturing sector to assist firms in minimizing overall production-inventory costs [1,2]. It employed a mathematical model to describe the important trade-off between the fixed set-up costs and inventory holding costs when items are produced in batch at a finite replenishment rate. The FPR model derives an economic production quantity that minimizes the long-run average production-inventory cost per unit time.

The classical FPR model (also known as the economic production quantity (EPQ) model) assumes a "continuous" inventory issuing policy for satisfying product demand. However, in a real life vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly used at customer's request. Goyal [3] first studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier, and an example was provided to illustrate his proposed method. Studies have since been carried out to address various aspects of vendor-buyer supply chain optimization issues [4-12]. Jamal and Sarker [4] determined an economic manufacturing quantity and recommended a raw material ordering policy to deliver a fixed amount of finished products at a regular interval within the production cycle time. They estimated the batch size from the lower bound concept of the JIT delivery amount and developed an algorithm for computing the optimal or near optimal batch sizes for both manufacturing and raw material ordering policies. Lu [6] investigated a one-vendor multi-buyer integrated inventory model with the objective of minimizing vendor's total annual cost subject to the maximum costs that the buyers may be prepared to incur. Lu's model required to know buyer's annual demand and previous order frequency. As a result, an optimal solution for the one-vendor one-buyer case was obtained and a heuristic approach for the one-vendor multi-buyer case was also provided. Viswanathan [8] examined the integrated vendor-buyer inventory models with two different strategies that had been proposed in the literature for the problem: one where each replenishing quantity delivered to the buyer is identical and the other strategy where at each delivery all the inventory available with the vendor is supplied to the buyer. He showed that there is no one strategy that obtains the best solution for all possible problem parameters. His study presented the results of a detailed numerical investigation that analyzed the relative performance of the two strategies for various problem parameters. Hill [9] examined a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers. The objective is to determine a purchasing and production schedule which minimizes the overall costs of purchasing, manufacturing, and stockholding. Diponegoro and Sarker [11] determined ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. They also developed a lower bound on the optimal solution for problem with lost sale. Ouyang et al. [12] studied the single-vendor single-buyer integrated production inventory models with stochastic demand in controllable lead time. They assumed that shortage during lead time is permitted and lead time can be reduced at an added cost, and developed iterative procedures for finding the optimal policies accordingly.

In addition to the continuous issuing policy, the "perfection production" is another unrealistic assumption of the classic FPR model. However, in a real life manufacturing environment, owing to various different factors, it is inevitable to generate nonconforming items during a production run. Studies have been carried out to enhance the conventional FPR model by addressing the issue of imperfect quality production [13-20]. The defective items produced, sometimes can be reworked and repaired; therefore, total production-inventory costs can be significantly reduced. For instance, production processes in printed circuit board assembly (PCBA), or in plastic injection molding, or in other industries such as chemical, textiles, metal components, etc., sometimes employ rework as an acceptable process in terms of level of product quality [21-27]. Yu and Bricker [21] presented an informative application of Markov Chain Analysis to a multistage manufacturing problem. Liu and Yang [22] considered a lot-sizing problem in a single-stage imperfect production system where the job processing is failure-prone. The processing may generate two types of defects: reworkable and scrap. The production process will switch between new jobs and rework jobs and both new-job processing time and rework time are random. They discussed the optimal lot-sizing control, under a class of operating policies, to maximize the average profit over an infinite time horizon. They demonstrated the existence of an optimal lot size and developed an algorithm for determining an optimal lot size. Jamal et al. [24] studies the optimal production batch size with rework process at a single-stage production system. Both cases of rework being completed within the same production cycle and rework being done after N cycles are examined. Mathematical models for each case were developed; the optimal batch sizes and total system costs were derived respectively. Chiu [26] derived the optimal lot size and backorder level for an EMQ model with backlogging, random defective rate, scrap, and imperfect rework process.

Little attention has been paid to study the joint decision-makings on the optimal production batch size and optimal number of shipments for an FPR model with random scrap rate, therefore this paper is intended to bridge the gap.

2. MATHEMATICAL MODELLING AND ANALYSIS

This study considers a production system where the process may randomly produce an x portion of defective items at a production rate d. All items produced are screened and inspection cost per item is included in the unit production cost C. All nonconforming items are assumed to be scrap and are discarded at the end of production. In order to prevent shortages from occurring, the constant production rate P must be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P\text{-}d\text{-}\lambda)>0$ or $(1\text{-}x\text{-}\lambda/P)>0$. The production rate of scrap items d can be expressed as $d=Px$. Unlike the classic FPR model assuming a continuous issuing policy for satisfying demand, this research considers a multi-shipment policy. It is also assumed that the finished items can only be delivered to customers if the whole lot is quality assured at the end of production. Fixed quantity n instalments of the finished batch are delivered to customers at a fixed interval of time during the production downtime t_2 (refer to Figure 1).

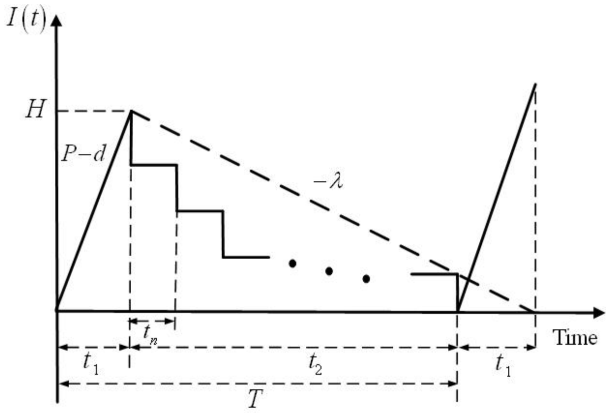

Fig. 1 On-hand inventory of perfect quality items in FPR model with scrap and a multiple shipment policy

The following cost parameters are also included in this paper: the disposal cost per scrap item C_S, setup cost K, unit holding cost h, unit production cost C, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation is listed in Appendix 1. From Figure 1, the following parameters can be obtained directly [23, 25]:

$$T = \frac{Q}{\lambda}(1-x) \tag{1}$$

$$t_1 = \frac{Q}{P} = \frac{H}{P-d} \tag{2}$$

$$t_2 = nt_n = T - t_1 = Q\left(\frac{(1-x)}{\lambda} - \frac{1}{P}\right) \tag{3}$$

$$T = t_1 + t_2 \tag{4}$$

$$H = (P-d)t_1 = (P-d)\frac{Q}{P} = (1-x)Q \tag{5}$$

The on-hand inventory of scrap items during production uptime t_1 are illustrated in Figure 2. One notes that maximum level of on-hand scrap items is dt_1, and:

$$dt_1 = Pxt_1 = xQ. \tag{6}$$

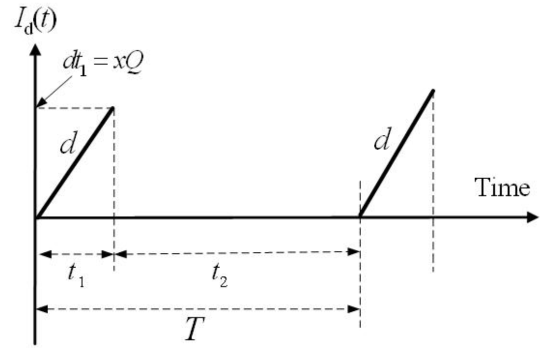

Fig. 2 On-hand inventory of scrap items in FPR model with scrap and a multiple shipment policy

Cost for each delivery is:

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{7}$$

Total delivery costs for n shipments in a cycle are:

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H = nK_1 + C_T Q(1-x) \tag{8}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_2 where n fixed-quantity instalments of the finished batch are delivered to customers at a fixed interval of time, are as follows (see Appendix 2 for derivations):

$$h\left(\frac{n-1}{2n}\right)Ht_2 \tag{9}$$

The variable holding costs for finished products kept by the customer during the delivery time t_2, are as follows (see Appendix 3 for the detailed computations):

$$\frac{h_2}{2}\left[\frac{H}{n}t_2 + T(H - \lambda t_2)\right] \tag{10}$$

Total production-inventory-delivery cost per cycle $TC(Q,n)$ consists of the setup cost, variable production cost, disposal cost, fixed and variable delivery cost, holding cost during production uptime t_1, and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time t_2. Therefore, $TC(Q,n)$ is:

$$\begin{aligned} TC(Q,n) &= CQ + K + C_S[xQ] + C_T[Q(1-x)] + nK_1 + \\ &+ h\left[\frac{H+dt_1}{2}(t_1) + \left(\frac{n-1}{2n}\right)Ht_2\right] + \frac{h_2}{2}\left[\frac{H}{n}t_2 + T(H-\lambda t_2)\right] \end{aligned} \tag{11}$$

The proportion x of scrap items is assumed to be a random variable with a known probability density function. In order to take the randomness of defective rate into account, the expected values of x can be used in the cost analysis. Substituting all parameters from Eqs. (1) to (10) in $TC(Q,n)$, the expected cost $E[TCU(Q,n)]$ can be obtained (see Appendix 4 for computations):

$$E\left[TCU(Q,n)\right]=\frac{E\left[TC(Q,n)\right]}{E[T]}=\frac{C\lambda}{1-E[x]}+\frac{(K+nK_1)\lambda}{Q(1-E[x])}+\frac{C_S E[x]\lambda}{(1-E[x])}+C_T\lambda+\frac{hQ\lambda}{2P(1-E[x])}+$$
$$+\left(\frac{n-1}{n}\right)\left[\frac{hQ(1-E[x])}{2}-\frac{hQ\lambda}{2P}\right]+\frac{h_2Q}{2}\left[\left(\frac{1}{n}\right)(1-E[x])+\left(\frac{n-1}{n}\right)\frac{\lambda}{P}\right] \tag{12}$$

3. PROOF OF CONVEXITY

For the proof of convexity of $E[TCU(Q,n)]$, one can use the Hessian matrix equations [28, 29] and verify whether the following condition (Eq. (13)) holds or not:

$$[Q\quad n]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}>0 \tag{13}$$

From Eq. (12), one obtains the following terms:

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q}=-\frac{K\lambda}{Q^2(1-E[x])}-\frac{nK_1\lambda}{Q^2(1-E[x])}+\frac{h\lambda}{2P(1-E[x])}+\left(1-\frac{1}{n}\right)\frac{h_2\lambda}{2P}+$$
$$+\left(1-\frac{1}{n}\right)\left[\frac{h(1-E[x])}{2}-\frac{h\lambda}{2P}\right]+\left(\frac{1}{n}\right)\frac{h_2}{2}(1-E[x]) \tag{14}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2}=\frac{2(K+nK_1)\lambda}{Q^3(1-E[x])} \tag{15}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n}=\frac{K_1\lambda}{Q(1-E[x])}-\frac{1}{n^2}(h_2-h)\left[\frac{Q(1-E[x])}{2}-\frac{Q\lambda}{2P}\right] \tag{16}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}=\frac{1}{n^3}(h_2-h)\left[Q(1-E[x])-\frac{Q\lambda}{P}\right] \tag{17}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}=-\frac{K_1\lambda}{Q^2(1-E[x])}+\left(\frac{1}{n^2}\right)\left[\frac{h(1-E[x])}{2}-\frac{h\lambda}{2P}\right]-\left(\frac{1}{n^2}\right)\frac{h_2}{2}(1-E[x])+\left(\frac{1}{n^2}\right)\frac{h_2\lambda}{2P} \tag{18}$$

Substituting Eqs. (15), (17) and (18) in the Hessian matrix equations, one has:

$$[Q\quad n]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2K\lambda}{Q(1-E[x])}>0 \tag{19}$$

Equation (19) is resulting positive, because K, λ, $(1\text{-}E[x])$, and Q are all positive. Hence, it follows that the expected integrated costs $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. Therefore, the convexity of $E[TCU(Q,n)]$ is proved, and there exists a minimum of $E[TCU(Q,n)]$.

4. DERIVATION OF THE OPTIMAL SOLUTIONS

To derive jointly the optimal production lot size Q^* and optimal number of shipments n^*, one can differentiate $E[TCU(Q,n)]$ with respect to Q and with respect to n, and solve the linear system of Eqs. (14) and (16) by setting these partial derivatives equal to zero.

With further derivations, one obtains:

$$\frac{(K+nK_1)\lambda}{Q^2(1-E[x])}=\frac{h\lambda}{2P(1-E[x])}+\left(\frac{n-1}{2n}\right)\left[h(1-E[x])+(h_2-h)\left(\frac{\lambda}{P}\right)\right]+\frac{h_2}{2n}(1-E[x]) \quad (20)$$

or:

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{\frac{h\lambda}{P}+\left[\left(\frac{n-1}{n}\right)h+\left(\frac{1}{n}\right)h_2\right](1-E[x])^2+\left(\frac{n-1}{n}\right)(h_2-h)\left(\frac{\lambda}{P}\right)(1-E[x])\right\}}} \quad (21)$$

and:

$$n^*=\sqrt{\frac{K(h_2-h)(1-E[x])\left[(1-E[x])-(\lambda/P)\right]}{K_1\left[h\lambda E[x]/P+h(1-E[x])^2+h_2(\lambda/P)(1-E[x])\right]}} \quad (22)$$

4.1 Special Case

Suppose that all items produced are of perfect quality, i.e. $x=0$, then the proposed model becomes the same as the classic FPR model with multiple shipments. Total cost per cycle is:

$$TC_1(Q,n)=CQ+K+C_TQ+nK_1+h\left[\frac{H}{2}(t_1)+\left(\frac{n-1}{2n}\right)Ht_2\right]+\frac{h_2}{2}\left[\frac{H}{n}t_2+T(H-\lambda t_2)\right] \quad (23)$$

The expected production-inventory-delivery cost per unit time for this special model is:

$$E[TCU_1(Q,n)]=\frac{E[TC_1(Q,n)]}{E[T]}=C\lambda+\frac{(K+nK_1)\lambda}{Q}+C_T\lambda+\frac{hQ\lambda}{2P}+ +\left(\frac{n-1}{n}\right)\left[\frac{hQ}{2}-\frac{hQ\lambda}{2P}\right]+\frac{h_2Q}{2}\left[\left(\frac{1}{n}\right)+\left(\frac{n-1}{n}\right)\frac{\lambda}{P}\right] \quad (24)$$

The convexity of $E[TCU_1(Q,n)]$ can also be proved and the optimal solutions to this special model can be obtained as follows:

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{h+\left(\frac{1}{n}\right)\left(1-\frac{\lambda}{P}\right)(h_2-h)+(h_2)\left(\frac{\lambda}{P}\right)\right\}}} \quad (25)$$

$$n^*=\sqrt{\frac{K(h_2-h)\left[1-(\lambda/P)\right]}{K_1\left[h+h_2(\lambda/P)\right]}} \quad (26)$$

5. NUMERICAL EXAMPLE

Assume that a manufactured item can be produced at a rate of *60,000* units per year and it has experienced a flat demand rate of *3,400* units per year. There is a random scrap rate during the production uptime which follows a uniform distribution over the interval [*0*, *0.3*]. Additional parameters used in this example are:

K = *$20,000* per production run,
C = *$100* per item,
C_S = *$20*, repaired cost for each item reworked,
K_1 = *$4,350* per shipment, a fixed cost,
h_2 = *$80* per item kept at the customer's end per unit time,
h = *$20* per item per year,
C_T = *$0.1* per item delivered.

From Eqs. (22), (21) and (12), the optimal number of delivery $n^*=3$, the optimal production lot size $Q^*=2,652$, and the long-run average cost $E[TCU(Q^*,n^*)]=\$512,047$ can be obtained. It may be noted that n^* should practically be an integer number, so in this example $n^*=3$ is rounded off from its original value *3.1733* computed by Eq. (22). Also, because $E[TCU(Q^*,n^*)]$ is not necessarily symmetrical on both sides of n^*, in the case of n^* falling closer to the midpoint of two integers, we suggest that both integer numbers should be plugged into Eq. (12), and select whichever integer value gives the minimum cost as n^*.

The effect of variation of the number of shipments n on the long-run integrated cost function $E[TCU(Q,n)]$ is depicted in Figure 3. It is noted that in this numerical example, the optimal integer number of shipments $n^*=3$.

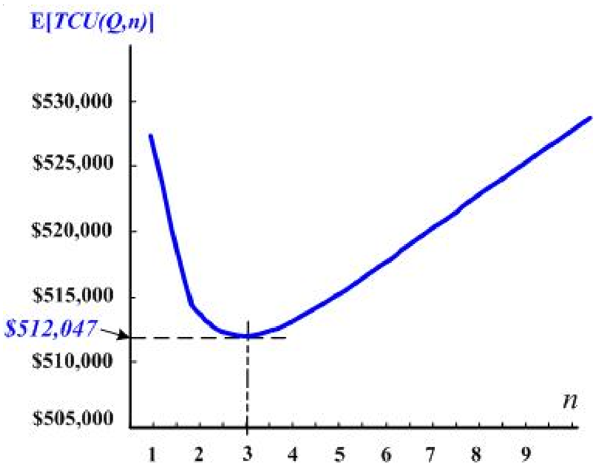

Fig. 3 Variation of the number of shipments effects on the long-run integrated cost function E[TCU(Q,n)]

Variation of the random scrap rate x effects on the long-run integrated cost function $E[TCU(Q^*,n^*)]$ is illustrated in Figure 4. It is noted that as the random scrap rate x increases, the value of the long-run cost function $E[TCU(Q^*,n^*)]$ increases significantly. Figure 5 shows the convexity of the long-run integrated cost function $E[TCU(Q,n^*=3)]$.

Furthermore, the optimal solutions for the special case can be obtained by using Eqs. (26), (25) and (24) respectively: the optimal number of delivery $n^*=3$ (is rounded off from *3.257*), the optimal lot size $Q^*=2,276$, and the long-run integrated costs $E[TCU_1(Q^*,n^*)]=\$439,101$.

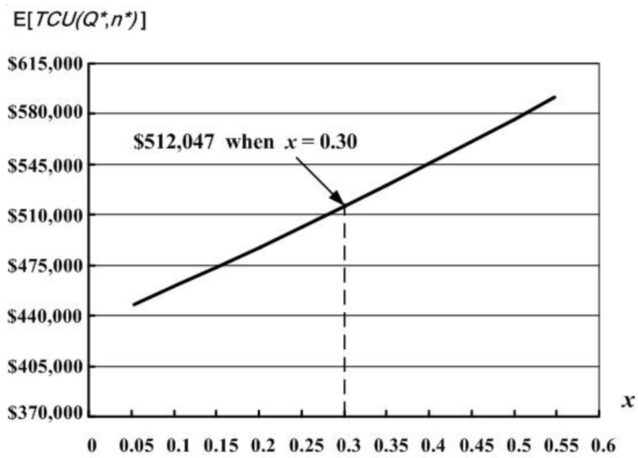

Fig. 4 Variation of random scrap rate effects on the long-run integrated cost function E[TCU(Q,n*)]*

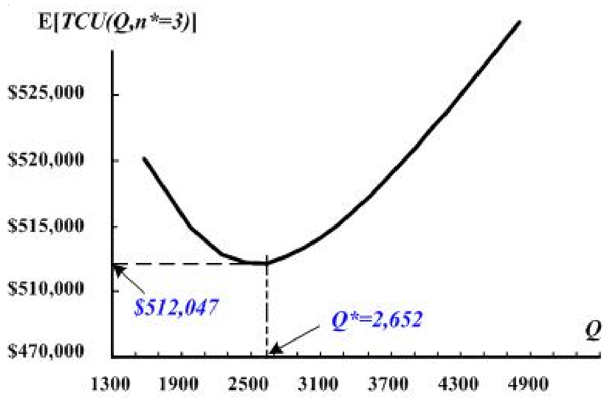

Fig. 5 Convexity of the long-run integrated cost function E[TCU(Q,n=3)]*

6. CONCLUDING REMARKS

The classic FPR model implicitly assumes a continuous inventory issuing policy and perfect quality production for all items produced. However, in a real life vendor-buyer integrated production-inventory system, multi-shipment policy is practically used in lieu of the continuous issuing policy, and generation of defective items during a production run is inevitable. The present study combines a multiple shipment policy and quality assurance into an FPR model with scrap. Mathematical modeling and analysis are used. The renewal reward theorem is employed to cope with the variable cycle length. The long-run integrated production-inventory-delivery cost per unit time for the proposed model is derived, and its convexity is proved by the use of the Hessian matrix equations. A closed-form solution of the optimal production-shipment policy for such an imperfect FPR model is obtained.

It may be noted that without an in-depth investigation and robust analysis of such a realistic system, the optimal production-shipment policy that minimizes the long-run average integrated costs cannot be obtained, and insight regarding the effects of system parameters (as depicted in Figures 4 to 6) cannot be gained. One interesting topic for future research will be to investigate the effect on optimal production-shipment policy when shortages are allowed and backordered in such a realistic system.

ACKNOWLEDGEMENTS

The authors greatly appreciate the National Science Council of Taiwan for supporting this research under grant number: NSC-97-2410-H-324-013-MY2.

APPENDIX 1

Notation:

T – production cycle length,

Q – production lot size, a decision variable, to be determined for each cycle,

n – number of fixed quantity installments of the finished batch to be delivered to customers, a decision variable, to be determined for each cycle,

t_1 – the production uptime for the proposed FPR model,

t_2 – time required for delivering all quality assured finished products,

H – maximum level of on-hand inventory in units when regular production process ends,

t_n – a fixed interval of time between each instalment of finished products delivered during production downtime t_2,

$I(t)$ – on-hand inventory of perfect quality items at time t,

$I_d(t)$ –on-hand inventory of scrap items at time t,

$TC(Q,n)$ – total production-inventory-delivery costs per cycle for the proposed model,

$TC_1(Q,n)$ – total production-inventory-delivery per cycle when no defective items produced (i.e. the special case - classic FPR model with multi-delivery policy),

$E[TCU(Q,n)]$ – the long-run average costs per unit time for the proposed model,

$E[TCU_1(Q,n)]$ – the long-run average costs per unit time for model in the special case.

APPENDIX 2

Computations of the manufacturer's holding cost of finished products during t_2 (i.e. Eq. (9)) are as follows:

(1) When $n=1$, total holding cost in delivery time = 0;

(2) When $n=2$, total holding costs in delivery time become (see Figure 6):

$$h\left(\frac{H}{2}\times\frac{t_2}{2}\right)=h\left(\frac{1}{2^2}\right)Ht_2 \tag{27}$$

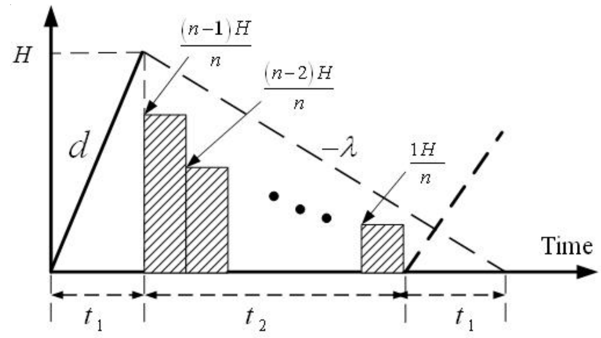

Fig. 6 On-hand inventory of the finished items kept by manufacturer during time t_2 in FPR model with scrap and a multiple shipment policy

(3) When $n=3$, total holding costs in delivery time are:

$$h\left(\frac{2H}{3}\times\frac{t_2}{3}+\frac{1H}{3}\times\frac{t_2}{3}\right)=h\left(\frac{2+1}{3^2}\right)Ht_2 \tag{28}$$

(4) When $n=4$, total holding costs in delivery time become:

$$h\left(\frac{3H}{4}\times\frac{t_2}{4}+\frac{2H}{4}\times\frac{t_2}{4}+\frac{1H}{4}\times\frac{t_2}{4}\right)=h\left(\frac{3+2+1}{4^2}\right)Ht_2 \tag{29}$$

Therefore, the following general term for total holding costs during delivery time t_2 can be obtained by mathematical induction:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1}i\right)Ht_2=h\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_2=h\left(\frac{n-1}{2n}\right)Ht_2 \tag{30}$$

APPENDIX 3

Computations of the customer's holding cost during time t_2 (i.e. Eq. (10)) are as follows.

Because n instalments (fixed quantity D) of the finished lot are delivered to customer at a fixed interval of time t_n, one has the following:

$$D=\frac{H}{n} \tag{31}$$

$$t_n=\frac{t_2}{n} \tag{32}$$

At the customer's end, the demand between shipments is (λt_n), if we let I denote number of items that will be left over after satisfying the demand during each fixed interval of time t_n (refer to Figure 7), then:

$$I=D-\lambda t_n \tag{33}$$

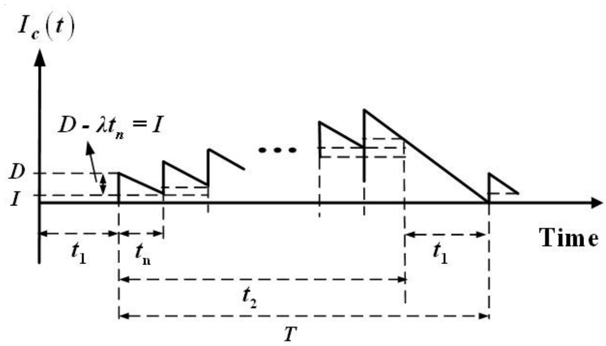

Fig. 7 On-hand inventory of the finished items kept by customer during time t_2 in FPR model with scrap and a multiple shipment policy

From Figure 7, one can calculate the average inventory as follows:

$$Average\ inventory = \left[\left(\frac{D+I}{2}\right)t_n\right] + \left[\frac{(D+I)+\left[(D+I)-\lambda t_n\right]}{2}t_n\right] + \left[\frac{(D+2I)+\left[(D+2I)-\lambda t_n\right]}{2}t_n\right] + \dots$$

$$\dots + \left[\frac{\left[D+(n-1)I\right]+\left[\left[D+(n-1)I\right]-\lambda t_n\right]}{2}t_n\right] + \left(\frac{nI}{2}\right)(t_1) \tag{34}$$

or:

$$Average\ inventory = \left(D - \frac{\lambda}{2}t_n\right)t_n + \left(D + I - \frac{\lambda}{2}t_n\right)t_n + \left(D + 2I - \frac{\lambda}{2}t_n\right)t_n + \dots$$

$$\dots + \left(D + (n-1)I - \frac{\lambda}{2}t_n\right)t_n + \left(\frac{nI}{2}\right)(t_1) =$$

$$= n\left(D - \frac{\lambda}{2}t_n\right)t_n + \frac{n(n-1)}{2}It_n + \frac{nI}{2}(t_1) \tag{35}$$

Substituting Eqs. (31) through (33) into Eq. (35), general term for average inventory at the customer's end can be obtained:

$$Average\ inventory = n\left(\frac{H}{n} - \frac{\lambda}{2}t_n\right)t_n + \frac{n(n-1)}{2}\left(\frac{H}{n} - \lambda t_n\right)t_n + \frac{n}{2}\left(\frac{H}{n} - \lambda t_n\right)(t_1) =$$

$$= Ht_n - \frac{n\lambda}{2}t_n^2 + Ht_n\frac{(n-1)}{2} - \frac{n(n-1)}{2}\lambda t_n^2 + \frac{H}{2}(t_1) - \frac{n}{2}(\lambda t_n)(t_1) =$$

$$= 1/2\left[(Ht_2/n) + T(H - \lambda t_2)\right] \tag{36}$$

APPENDIX 4

Computation of Eq. (12):
Recall Eq. (11) as follows:

$$TC(Q,n) = CQ + K + C_S[xQ] + C_T[Q(1-x)] + nK_1 +$$

$$+ h\left[\frac{H + dt_1}{2}(t_1) + \left(\frac{n-1}{2n}\right)Ht_2\right] + \frac{h_2}{2}\left[\frac{H}{n}t_2 + T(H - \lambda t_2)\right] \tag{11}$$

then:

$$TC(Q,n) = CQ + K + nK_1 + C_S[xQ] + C_T[Q(1-x)] + \frac{hQ^2}{2P} + \left(\frac{n-1}{n}\right)\left[\frac{hQ^2(1-x)^2}{2\lambda} - \frac{hQ^2(1-x)}{2P}\right] + \frac{h_2Q^2}{2n}\left[\frac{(1-x)^2}{\lambda} + \frac{(n-1)(1-x)}{P}\right] \tag{37}$$

Since:

$$E[TCU(Q)] = \frac{E[TC(Q)]}{E[T]} \tag{38}$$

with further derivations, one obtains Eq. (12) as follows:

$$E[TCU(Q,n)] = \frac{E[TC(Q,n)]}{E[T]} = \frac{C\lambda}{1-E[x]} + \frac{(K+nK_1)\lambda}{Q(1-E[x])} + \frac{C_S E[x]\lambda}{(1-E[x])} + C_T\lambda + \frac{hQ\lambda}{2P(1-E[x])} + \left(\frac{n-1}{n}\right)\left[\frac{hQ(1-E[x])}{2} - \frac{hQ\lambda}{2P}\right] + \frac{h_2Q}{2}\left[\left(\frac{1}{n}\right)(1-E[x]) + \left(\frac{n-1}{n}\right)\frac{\lambda}{P}\right] \tag{12}$$

REFERENCES

[1] S. Nahmias, *Production & Operations Analysis*, 5th edition, McGraw-Hill, New York, 2005.

[2] R.J. Tersine, *Principles of Inventory and Materials Management*, PTR Prentice-Hall, New Jersey, pp. 121-129, 1994.

[3] S.K. Goyal, Integrated inventory model for a single supplier-single customer problem, *Int. Journal of Production Research*, Vol. 15, pp. 107-111, 1977.

[4] A.M.M. Jamal and B.R. Sarker, Optimal batch size for a production system operating under a just-in-time delivery system, *Int. Journal of Production Economics*, Vol. 32, No. 2, pp. 255-260, 1993.

[5] B.R. Sarker and G.R. Parija, An optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy, *Journal of the Operational Research Society*, Vol. 45, pp. 891–900, 1994.

[6] L. Lu, A one-vendor multi-buyer integrated inventory model, *European Journal of Operational Research*, Vol. 81, No. 2, pp. 312-323, 1995.

[7] P.C. Yang and H.M. Wee, A single-vendor and multiple-buyers production-inventory policy for a deteriorating item, *European Journal of Operational Research*, Vol. 143, No. 3, pp. 570-581, 2002.

[8] S. Viswanathan, Optimal strategy for the integrated vendor-buyer inventory model, *European Journal of Operational Research*, Vol. 105, pp. 38-42, 1998.

[9] R.M. Hill, Optimizing a production system with a fixed delivery schedule, *Journal of the Operational Research Society*, Vol. 47, pp. 954-960, 1996.

[10] M. Khouja, Optimizing inventory decisions in a multi-stage multi-customer supply chain, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 39, No. 3, pp. 193-208, 2003.

[11] A. Diponegoro and B.R. Sarker, Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries, *Computers and Operations Research*, Vol. 33, No. 8, pp. 2387-2404, 2006.

[12] L.-Y. Ouyang, K.-S. Wu and C.-H. Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time, *Int. Journal of Production Economics*, Vol. 92, No. 3, pp. 255-266, 2004.

[13] R.E. Barlow and F. Proschan, *Mathematical Theory of Reliability*, John Wiley, New York, 1965.

[14] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions*, Vol. 18, pp. 48-55, 1986.

[15] T. Bielecki and P.R. Kumar, Optimality of zero-inventory policies for unreliable production facility, *Operations Research*, Vol. 36, pp. 532-541, 1988.

[16] S.W. Chiu, Ch.-K. Ting and Y.-Sh.P. Chiu, Optimal order policy for EOQ model under shortage level constraint, *Int. Journal for Engineering Modelling*, Vol. 18, No. 1-2, pp. 41-46, 2005.

[17] M. Berg, M.J.M. Posner and H. Zhao, Production-inventory systems with unreliable machines, *Operations Research*, Vol. 42, pp. 111-118, 1994.

[18] Y.-Sh.P. Chiu, C.-Y. Tseng, W.-C. Liu and Ch.-K. Ting, Economic manufacturing quantity model with imperfect rework and random breakdown under abort/resume policy, *P I Mech E Part B: Journal of Engineering Manufacture*, Vol. 223, No. 2, pp. 183-194, 2009.

[19] T. Boone, R. Ganeshan, Y. Guo and J.K. Ord, The impact of imperfect processes on production run times, *Decision Sciences*, Vol. 31, pp. 773-785, 2000.

[20] S.W. Chiu, Y.-Sh.P. Chiu and C.-C. Shih, Determining expedited time and cost of the end product with defective component parts using critical path method (CPM) and time-costing method, *Journal of Scientific & Industrial Research*, Vol. 65, No. 9, pp. 695-701, 2006.

[21] K.-Y.C. Yu and D.L. Bricker, Analysis of a Markov chain model of a multistage manufacturing system with inspection, rejection, and rework, *IIE Transactions*, Vol. 25, No. 1, pp. 109-112, 1993.

[22] J.J. Liu and P. Yang, Optimal lot-sizing in an imperfect production system with homogeneous reworkable jobs, *European Journal of Operational Research*, Vol. 91, No. 3, pp. 517-527, 1996.

[23] P.A. Hayek and M.K. Salameh, Production lot sizing with the reworking of imperfect quality items produced, *Production Planning and Control*, Vol. 12, pp. 584-590, 2001.

[24] A.M.M. Jamal, B.R. Sarker and S. Mondal, Optimal manufacturing batch size with rework process at a single-stage production system, *Computers & Industrial Engineering*, Vol. 47, pp. 77-89, 2004.

[25] Y.P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging, *Engineering Optimization*, Vol. 35, No. 4, pp. 427-437, 2003.

[26] S.W. Chiu and Y.-Sh.P. Chiu, Mathematical modeling for production system with backlogging and failure in repair, *Journal of Scientific & Industrial Research*, Vol. 65, pp. 499-506, 2006.

[27] S.W. Chiu, Optimal replenishment policy for imperfect quality EMQ model with rework and backlogging, *Applied Stochastic Models Business Industry*, Vol. 23, pp. 165-178, 2007.

[28] R.L. Rardin, *Optimization in Operations Research*, Prentice-Hall, New Jersey, 1998.

[29] F.S. Hillier and G.J. Lieberman, *Introduction to Operations Research*, McGraw Hill, New York, 2001.

DONOŠENJE ODLUKE O OPTIMALNOJ PROIZVODNJI I ISPORUCI ZA MODEL OGRANIČENE PROIZVODNJE S OTPACIMA

SAŽETAK

Ovaj rad se bavi donošenjem odluka o optimalnoj količini serijske proizvodnje, te o optimalnom broju isporuka za model ograničene proizvodnje sa slučajnim omjerom otpadaka. Klasični model ograničene proizvodnje (FPR) pretpostavlja politiku kontinuiranog izdavanja inventara u svrhu zadovoljenja potražnje za proizvodom kao i savršenom kvalitetom svega proizvedenog. Međutim, u stvarnom prodavač-kupac integriranom proizvodno-skladištenom sustavu, koristi se politika višekratne isporuke umjesto politike kontinuirane isporuke pa je neizbježna pojava oštećenih proizvoda za vrijeme proizvodnje. Svi neodgovarajući predmeti smatraju se otpacima, a dovršeni (savršene kvalitete) prozvodi mogu se isporučiti kupcima ako je sveukupna proizvedena količina dokazano kvalitetna na kraju proizvodnje. Fiksna količina višestrukih obroka konačne serije isporučuje se kupcima u fiksnom vremenskom intervalu. Da bi se savladala promjenjiva duljina ciklusa, korišten je matematički model i "renewal reward" teorem. Dobiven je dugoročni prosječni trošak predloženog modela, a njegova konveksnost je dokazana pomoću Hesseovih matrica. Postignut je zatvoreni oblik politike optimalne proizvodnje i isporuke za jedan takav nesavršeni FPR model, a prodiskutiran je jedan poseban slučaj. Konačno, pokazan je jedan numerički primjer kako bi se ukazalo na praktično korištenje modela.

Ključne riječi: *proizvodnja, mnogostruke isporuke, FPR model, otpadak, optimalna količina serije, inventar.*