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A multi-product FPR model with rework and an improved delivery policy

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SUMMARY

A multi-item finite production rate (FPR) model with rework and an improved delivery policy is examined in this paper. Unlike the classic FPR model whose purpose is to derive the most economic lot size for a single-product production system with perfect quality and a continuous issuing policy, this paper considers a production of multiple products on a single machine, rework of all nonconforming items produced, and a cost-reduction, multi-delivery policy. We extend the work of Chiu et al. [1] by incorporating an improved n+1 shipment policy into their model. According to such a policy, one extra delivery of finished items is made during vendor's production uptime to satisfy product demands during the period of vendor's uptime and rework time. When the rest of the production lot is quality assured and the rework has been finished as well, n fixed-quantity installments of finished items are delivered to customers. The objectives are to determine an optimal, common-production cycle time that minimizes the long-run average system cost per time unit, study the effects of rework and the improved delivery policy on the optimal production. Mathematical modelling and analysis is used to derive a closed-form, optimal, common-cycle time. Finally, practical usages of the obtained results are demonstrated by a numerical example.

Key words: *finite production rate model, multi-item system, common cycle time, rework, optimization, multi-delivery.*

1. INTRODUCTION

The classic finite production rate (FPR) model aims at deriving the most economic production lot size for a single-product production system with no nonconforming items produced and a continuous issuing policy of its finished items [2-3]. In real-life vendor-buyer integrated systems, however, for the purpose of maximizing machine utilization vendors often make plan to produce multiple products in turn on a single machine. Gordon and Surkis [4] have presented a simple and practical approach to determine control policies for a multi-item inventory environment where the items are ordered from a single supplier and the demand for items are subject to severe fluctuations. The time between orders can either be fixed or based on the accumulation of a fixed-order quantity for all products. Their model balances carrying and stock-out costs. An operational system structure has been developed and a simulation procedure used to determine an appropriate value of the inventory factor in the model. Rosenblatt and Finger [5] have studied the problem of multi-item production in a single facility. The proposed facility was an electrochemical machining system and the products examined were impact sockets of various sizes for power wrenches. A grouping procedure of the various items was adopted. A modified version of an existing algorithm was applied to ensure production cycle times which are multiples of the shortest production cycle time. Tamura [6] has proposed an approximation procedure to solve the production planning problem for a multistage production system which produces many different components and assembles them into finished products under capacity limitations. A generalized production planning model was built using mixed-integer programming. The solution procedure was approximated by a linear programming method. Different algorithms were developed in detail for a two-stage production problem. Numerical example was provided to examine the validity and efficiency of the proposed algorithms. Zipkin [7] explored the performance of a multi-item production-inventory system. He compared two alternative policies, representing different modes of collecting and utilizing information, then derived a closed-form measure of performance for one of them, the familiar first-come-first-served (FCFS) policy, and proposed a comparable approximation for the other, the longest-queue (LQ) policy. These results were illustrated, tested through simulations, and used to address several basic managerial issues. Moon and Silver [8] studied a multi-item newsvendor problem subject to a budget constraint on the total value of replenishment quantities. Dynamic programming procedures were presented for two situations: (1) where the end item demand distributions are assumed known as the normally distributed demand; and (2) a distribution free approach where only the first two moments of distributions are assumed known. In addition, simple and efficient heuristic algorithms were developed. Computational experiments based on a set of test problems showed that the performance of the heuristics was excellent. Studies related to various aspects of multi-item production planning and optimization issues can also be found in Refs. [9-12].

Also, in real-production environments due to various unpredictable factors, generation of defective items in any given production run is inevitable. Mak [13] used mathematical modelling to study an inventory system where the number of units of acceptable quality in a replenishment lot is uncertain and the demand is partially captive. He assumed that the fraction of the demand during the stock-out period which can be backordered is a random variable whose probability distribution is known. The optimal replenishment policy was synthesized for such a system. A numerical example was given to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Gopalan and Kannan [14] considered the manufacturing, inspections and rework activities as a two-stage, transfer-line production system. They analyzed some transient state characteristics of such a system subject to an initial buffer of infinite capacity, inspection at both inter- and end-stages, and rework. A stochastic model was developed to investigate their system. Explicit analytical expressions for some of the system characteristics were obtained using the state-space method and regeneration point technique. Inderfurth et al. [15] studied a deterministic problem of planning the production of new and recovering of defective items of the same product manufactured on the same facility. The processing of a batch includes two stages: the regular production and the rework process. While waiting for rework, defective items deteriorate and there is a given deterioration time limit. Deterioration results in an increase in time and cost for performing rework processes. The objective of their study was to find batch sizes and positions of items to be reworked such that overall production-inventory costs are minimized. A polynomial, dynamic programming algorithm was presented to solve this problem. Many studies have since been conducted to address different aspects of imperfect production systems as well as quality assurance issues in production [16-20].

Assumption of the continuous inventory issuing policy is another unrealistic assumption in the classic FPR model. In real production-shipment systems, it is common for vendors to adopt multiple or periodic delivery policy for transporting finished goods to customers. Schwarz et al. [21] studied the fill-rate of a one-warehouse, N-identical retailer distribution system. An approximation model was adopted from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As a result, properties of fill-rate policy were suggested to provide management when looking into system optimization. Hahm and Yano [22] determined the frequency of production and delivery of a single component with the objective of minimizing the long-run average cost per unit time. They considered the following costs: production setup costs, inventory-holding costs for both the supplier and customer, and transportation costs. They proved that the ratio between the production interval and delivery interval must be an integer in an optimal solution. They used these results to describe situations in which it is optimal to have synchronized production and delivery, and discussed the ramifications of these conditions on strategies for setup cost and time reductions. Sarker and Khan [23] considered a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. A general cost model was formulated considering both raw materials and finished products. Using this model, a simple procedure was developed to determine an optimal ordering policy for procurement of raw materials as well as manufacturing batch size, to minimize the total cost of meeting customers' demands in time. Sucky [24] focused on supply chain management from the perspective of inventory management. The coordination of order and production policies between customers and suppliers in supply chains was of his particular interest. The study provided several bargaining models, depending on alternative production policies of the supplier. With the bargaining models, offered cooperative policy and offered side payment can be derived. Chiu et al. [1] derived an optimal common-production cycle time for a multi-item finite production rate (FPR) model with rework and multi-shipment policy. They focused on a multi-item, production-delivery integrated system under a common-cycle time policy, a rework process of all nonconforming items, and deliveries of n fixed-quantity installments of the finished lots upon completion of reworks. As a result, a closed-form optimal cycle time that minimizes the long-run average system cost is obtained. Additional studies that addressed various aspects of periodic or multiple deliveries issues in vendor-buyer integrated systems can also be found in Refs. [25-28].

For the purpose of reducing inventory-holding cost, this paper extends the specific multi-item FPR model of Chiu et al. [1] by incorporating an improved *n+1* shipment policy into their model. According to such a policy, one extra delivery of finished items is made during vendor's production uptime to satisfy product demands for the periods of vendor's uptime and rework time. At the end of rework and when the rest of the production lot is quality assured, n fixed-quantity installments of finished items are delivered to customers. The objectives are to determine the optimal-common production cycle time that minimizes the long-run average system cost per time unit, and study the effects of rework and the improved delivery policy on the optimal cycle time and system costs.

2. MODEL DESCRIPTION AND FORMULATIONS

A multi-item finite production rate (FPR) model with rework and an improved delivery policy is studied in this paper. Description of the model is as follows: suppose a producer plans L products to be made in turn on a single machine and during the production process for each product i (where $i = 1, 2,\ldots, L$), there is a x_i portion of nonconforming items being randomly produced at a rate d_{1i}. All items are screened and the inspection cost is included in production cost unit C_i. It is assumed that all nonconforming items are repairable at a rate of P_{2i} at the end of regular production, in each cycle with reworking cost unit C_{Ri}. Shortages are not allowed in the proposed model, so the constant production rate for product i, P_{1i} must satisfiy $(P_{1i} - d_{1i} - \lambda_i) > 0$, where λ_i is the annual demand rate for product i, and d_{1i} can be expressed as $d_{1i} = x_i P_{1i}$.

Aiming at reducing inventory holding cost, unlike prior work of Chiu et al. [1] and assuming a multi-shipment policy wherein the delivery of finished items starts right after the end of rework, this research adopts an improved *n+1* multi-shipment policy. According to the proposed *n+1* delivery policy, an initial shipment of finished goods is delivered to meet customers' product demands during producer's uptime and reworking time. Upon the completion of the rework, n fixed-quantity installments of the finished products are transported to customers, at a fixed time interval t_n (see Figure 1).

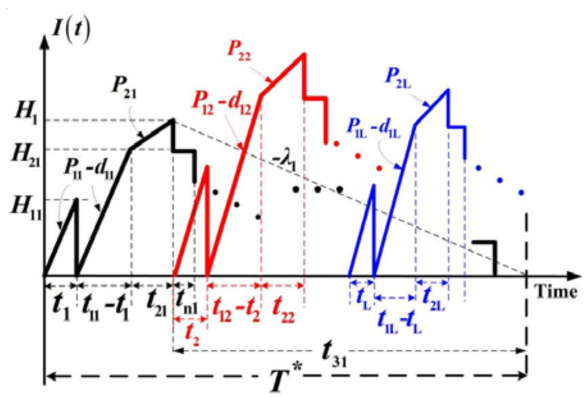

Fig. 1 On-hand inventory of perfect quality items for product i in the proposed model

Other cost parameters used in this study include: the production setup cost K_i, unit holding cost h_i, holding cost h_{1i} per reworked item, the fixed delivery cost K_{1i} for product i per delivery, and unit shipping cost C_{Ti} for each product i. In addition, the following notation is used in modeling and analysis:

T = common-production cycle length - the decision variable,

t_i = time required for producing items to meet demand of product i during producer's uptime t_{1i} and reworking time t_{2i},

t_{1i} = the production uptime for product i,

t_{2i} = the reworking time for product i,

t_{3i} = the delivery time for product i,

H_{1i} = level of on-hand inventory in units for meeting demand of product i during producer's uptime t_{1i} and reworking time t_{2i},

H_{2i} = maximum level of on-hand inventory for product i when the regular production ends,

H_i = maximum level of on-hand inventory in units for product i when the rework process ends,

n = number of fixed-quantity installments of the finished batch to be delivered to customers in each cycle, it is assumed to be a constant for all products,

Q_i = production lot size per cycle for product i,

t_{ni} = fixed interval of time between each installment of finished product i being delivered during t_{3i},

$I(t)$ = on-hand inventory of perfect quality items at time t,

$I_D(t)_i$= on-hand inventory of defective items for product i at time t,

$TC(Q_i)$ = total production-inventory-delivery costs per cycle for product i,

$E[TCU(Q_i)]$ = total expected production-inventory-delivery costs per unit time for L products in the proposed system,

$E[TCU(T)]$ = total expected production-inventory-delivery costs per unit time for L products in the proposed system using common-production cycle time T as decision variable.

From Figure 1, for any $i=1, 2,\ldots, L$, the following formulas can be obtained directly:

$$T = t_{1i} + t_{2i} + t_{3i} = \frac{Q_i}{\lambda_i} \tag{1}$$

$$H_{1i} = \lambda_i (t_{1i} + t_{2i}) \tag{2}$$

$$H_{2i} = (P_{1i} - d_{1i})(t_{1i} - t_i) \tag{3}$$

$$H_i = H_{2i} + (P_{2i}) t_{2i} \tag{4}$$

$$t_i = \frac{H_{1i}}{P_{1i} - d_{1i}} = \frac{\lambda_i (t_{1i} + t_{2i})}{P_{1i} - d_{1i}} \tag{5}$$

$$t_{1i} = \frac{Q_i}{P_{1i}} = \frac{H_{1i} + H_{2i}}{P_{1i} - d_{1i}} \tag{6}$$

$$t_{2i} = \frac{H_i - H_{2i}}{P_{2i}} \tag{7}$$

$$t_{3i} = T - (t_{1i} + t_{2i}) = n t_{ni} \tag{8}$$

$$\lambda = \sum_{i=1}^{L} \lambda_i \tag{9}$$

From Figure 2, for any $i=1,2,\ldots,L$, the following formulas can be directly obtained:

$$d_{1i} \cdot t_{1i} = x_i \cdot Q_i \tag{10}$$

$$t_{2i} = \frac{x_i Q_i}{P_{2i}} \tag{11}$$

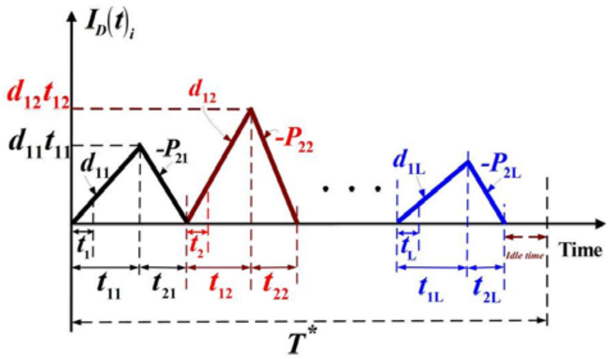

Fig. 2 On-hand inventory of defective items for product i in the proposed model

Total delivery costs for product i (i.e., $n+1$ shipments) in a production cycle are:

$$(n+1) K_{1i} + C_{Ti} Q_i \tag{12}$$

The variable holding costs for finished items of product i during delivery time t_{3i} are as follows [25]:

$$h_i \left(\frac{n-1}{2n} \right) H_i \cdot t_{3i} \tag{13}$$

Total production-inventory-delivery cost per cycle for L products consists of the setup cost, variable production cost, rework cost, fixed and variable delivery cost, holding cost during production uptime t_{1i} and rework time t_{2i}, and holding cost for finished goods kept during the delivery time t_{3i}. Therefore, total $TC(Qi)$ for L products is:

$$\sum_{i=1}^{L} TC(Q_i) = \sum_{i=1}^{L} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{Ri}(x_i Q_i) + (n+1) K_{1i} + \\ + C_{Ti} Q_i + h_{1i} \left[\frac{d_{1i} t_{1i}}{2} \cdot (t_{2i}) \right] \end{array} \right\} + \sum_{i=1}^{L} \left\{ h_i \left[\begin{array}{l} \frac{H_{1i}}{2}(t_i) + \frac{H_{2i}}{2}(t_{1i} - t_i) + \frac{d_{1i} t_{1i}}{2}(t_{1i}) + \\ + \frac{H_{2i} + H_i}{2}(t_{2i}) + \left(\frac{n-1}{2n} \right) H_i t_{3i} \end{array} \right] \right\} \tag{14}$$

In this study, the defective rate x is assumed to be a random variable with a known probability density function. In order to take the randomness of x into account, the expected value of x is used. Substituting all parameters from Eqs. (1) to (13) in Eq. (14), and with further derivations expected $E[TCU(Qi)]$ can be obtained as follows:

$$E[TCU(T)] = \sum_{i=1}^{L} \left\{ \begin{array}{l} C_i \lambda_i + \frac{K_i}{T} + C_{Ri} \lambda_i E[x_i] + C_{Ti} \lambda_i + \\ + \frac{(n+1) K_{1i}}{T} + \frac{h_{1i} T \lambda_i^2 E(x_i)^2}{2 P_{2i}} \end{array} \right\} + \sum_{i=1}^{L} \left\{ \frac{h_i T \lambda_i^2}{2} \left\{ \begin{array}{l} \lambda_i \left[\frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}} \right]^2 \left[\frac{2\lambda_i}{P_{1i}(1 - E[x_i])} - 1 \right] - \\ - \frac{E[x_i]}{P_{2i}} \left[\frac{1}{P_{1i}} - [1 - E[x_i]] \right] + \frac{1}{P_{1i}} + \\ + \left[\frac{1}{\lambda_i} - \frac{2}{P_{1i}} - \frac{E[x_i]}{P_{2i}} \right] - \\ - \left(\frac{1}{n} \right) \left[\begin{array}{l} \left[\frac{1}{\lambda_i} - \frac{2}{P_{1i}} - \frac{E[x_i]}{P_{2i}} \right] \\ + \lambda_i \left[\frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}} \right]^2 \end{array} \right] \end{array} \right\} \right\} \tag{15}$$

Let:

$$\begin{array}{l} E_{1i} = \left[\frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}} \right]; \\ E_{2i} = \left[\frac{1}{\lambda_i} - \frac{2}{P_{1i}} - \frac{E[x_i]}{P_{2i}} \right]; \\ E_{3i} = \frac{2\lambda_i}{P_{1i}[1 - E[x_i]]} \end{array} \tag{16}$$

Then Eq. (15) becomes:

$$E\left[TCU\left(T\right)\right]= \\ =\sum_{i=1}^{L}\left\{\begin{array}{l} C_i\lambda_i+\frac{K_i}{T}+C_{Ri}\lambda_i E\left[x_i\right]+C_{Ti}\lambda_i+\frac{(n+1)K_{1i}}{T}+ \\ +\frac{h_{1i}T\lambda_i^2 E\left[x_i\right]^2}{2P_{2i}} \end{array}\right\}+ \\ +\sum_{i=1}^{L}\left\{+\frac{h_iT\lambda_i^2}{2}\left\{\begin{array}{l} \lambda_i E_{1i}^2\cdot\left[E_{3i}-1\right]-\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_i E_{1i}^2\right]- \\ -\frac{E\left[x_i\right]}{P_{2i}}\cdot\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}+E_{2i} \end{array}\right\}\right\} \tag{17}$$

3. OPTIMAL PRODUCTION CYCLE TIME

In order to obtain an optimal common-cycle time T^*, one must first prove the existence of minimum of the expected cost function $E[TCU(T)]$. By differentiating $E[TCU(T)]$ with respect to T, first and second derivative are obtained as:

$$\frac{dE\left[TCU(T)\right]}{dT}= \\ =\sum_{i=1}^{L}\left\{\begin{array}{l} -\frac{K_i}{T^2}-\frac{(n+1)K_{1i}}{T^2}+\frac{h_{1i}\lambda_i^2 E\left[x_i\right]^2}{2P_{2i}}+ \\ +\frac{h_i\lambda_i^2}{2}\left\{\begin{array}{l} \lambda_i E_{1i}^2\cdot\left[E_{3i}-1\right]+\frac{1}{P_{1i}}+E_{2i}- \\ -\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_i E_{1i}^2\right]- \\ -\frac{E\left[x_i\right]}{P_{2i}}\cdot\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right] \end{array}\right\} \end{array}\right\} \tag{18}$$

$$\frac{d^2E\left[TCU(T)\right]}{dT^2}=\sum_{i=1}^{L}\left\{\frac{2\left[K_i+(n+1)K_{1i}\right]}{T^3}\right\} \tag{19}$$

It can be seen that Eq. (19) turns out positive because K_i, n, K_{1i} and T are all positive. The second derivative of $E[TCU(T)]$ with respect to T is greater than zero. Thus, $E[TCU(T)]$ is a convex function for all T different from zero.

The optimal common-production cycle time T^* can be obtained by setting the first derivative of $E[TCU(T)]$ equal to zero:

$$\frac{dE\left[TCU(T)\right]}{dT}= \\ =\sum_{i=1}^{L}\left\{\begin{array}{l} -\frac{K_i}{T^2}-\frac{(n+1)K_{1i}}{T^2}+\frac{h_{1i}\lambda_i^2 E\left[x_i\right]^2}{2P_{2i}}+ \\ +\frac{h_i\lambda_i^2}{2}\left\{\begin{array}{l} \lambda_i E_{1i}^2\cdot\left[E_{3i}-1\right]+\frac{1}{P_{1i}}+E_{2i}- \\ -\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_i E_{1i}^2\right]- \\ -\frac{E\left[x_i\right]}{P_{2i}}\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right] \end{array}\right\} \end{array}\right\}=0 \tag{20}$$

or:

$$\frac{1}{T^2}\sum_{i=1}^{L}\left\{K_i+(n+1)K_{1i}\right\}= \\ =\sum_{i=1}^{L}\left\{\begin{array}{l} +\frac{h_i\lambda_i^2}{2}\left\{\begin{array}{l} \lambda_i E_{1i}^2\left[E_{3i}-1\right]-\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_i E_{1i}^2\right]- \\ -\frac{E\left[x_i\right]}{P_{2i}}\cdot\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}+E_{2i} \end{array}\right\}+ \\ +\frac{h_{1i}\lambda_i^2 E\left[x_i\right]^2}{2P_{2i}} \end{array}\right\} \tag{21}$$

With further derivations, one obtains:

$$T^*=\sqrt{\frac{2\sum_{i=1}^{L}\left\{K_i+(n+1)K_{1i}\right\}}{\sum_{i=1}^{L}\left\{\begin{array}{l} h_i\lambda_i^2\left\{\begin{array}{l} \lambda_i E_{1i}^2\left[E_{3i}-1\right]-\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_i E_{1i}^2\right]- \\ -\frac{E\left[x_i\right]}{P_{2i}}\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}+E_{2i} \end{array}\right\}+ \\ +\frac{h_{1i}\lambda_i^2 E\left[x_i\right]^2}{P_{2i}} \end{array}\right\}}} \tag{22}$$

4. NUMERICAL EXAMPLE

In order to show a direct comparison with the results obtained by Chiu et al.'s model [1], the same example has been taken as the one examined in their study. Reconsider that a producer plans a routine production schedule to produce five products in turn on a single machine under a common-production cycle policy. The production rates P_{1i} for each product are *58000*, *59000*, *60000*, *61000* and *62000*, respectively. Demand rates λ_i for five different products are *3000*, *3200*, *3400*, *3600* and *3800* per year, respectively.

During the production, there are random defective rates x_i for each product which follow the uniform distribution over the intervals of [*0*, *0.05*], [*0*, *010*], [*0*, *0.15*], [*0*, *020*] and [*0*, *0.25*], respectively. All defective items are considered to be repairable at the rates P_{2i} of *1800*, *2000*, *2200*, *2400* and *2600* items per year, respectively, with additional reworking costs of *$50*, *$55*, *$60*, *$65* and *$70* per item.

For the purpose of reducing inventory-holding costs, this paper incorporates an *n+1* delivery policy, according to which, one extra delivery of finished items is made during producer's uptime to satisfy product demands for the period of producer's uptime and reworking time (see Fig 1). Once the rest of the production lot is quality assured at the end of the rework process, *n* fixed-quantity installments of finished items are delivered to customers at fixed time intervals during t_3. Additional values of system parameters are given as follows:

C_i = unit production costs are *$80*, *$90*, *$100*, *$110* and *$120*,

h_i = unit holding costs are *$10*, *$15*, *$20*, *$25* and *$30*,

h_{1i} = unit holding costs per reworked are *$30*, *$35*, *$40*, *$45* and *$50*,

K_i = the production setup costs are *$3800*, *$3900*, *$4000*, *$4100* and *$4200*,

C_{Ti} = unit transportation costs are *$0.1*, *$0.2*, *$0.3*, *$0.4* and *$0.5*,

K_{1i} = fixed costs per delivery are *$1800*, *$1900*, *$2000*, *$2100* and *$2200*,

n = number of shipments per cycle, in this study it is assumed to be *3* (i.e. *n+1=4*),

To compute an optimal common production cycle time T^*, one can apply Eq. (22) and find $T^*=0.7238$ (years), and the total expected production-inventory-delivery costs per time unit for L products for the proposed system is $E[TCU(T^*=0.7238)]=\$1,975,584$. Effects of the variation of the common-production cycle time T on the system cost $E[TCU(T)]$ are illustrated in Figure 3.

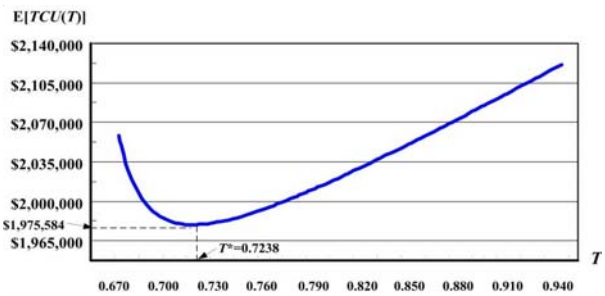

Fig. 3 Variation of common-cycle time T effects on the system cost E[TCU(T)]

The purpose of this study aims at reducing producer's holding cost in each product made during the production uptime, reworking time, and the delivery time (see Figure 4 for details).

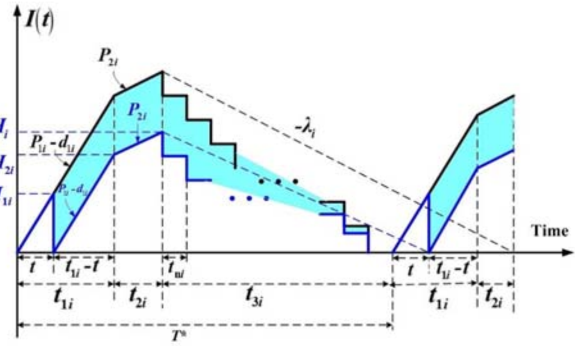

Fig. 4 Expected reduction in producer's inventory-holding costs (light blue-shaded area) for each product i in the proposed model, in comparison with [1]

Percentage of reduction in producer's inventory-holding costs for 5 different products is illustrated in the numerical example in Figure 5. The same common cycle time $T=0.6026$ and the same number of deliveries $n=4$ (i.e. $(n+1)=4$ in our model) have been used as in the previous study [1]. In summary, this study realizes a significant total savings of *$33,342* (i.e. *$2,008,926* [1] – *$1,975,584*) or *11.54*% of total other related costs (i.e. $E[TCU(T)]–(\lambda C)$: the system cost excludes variable production cost).

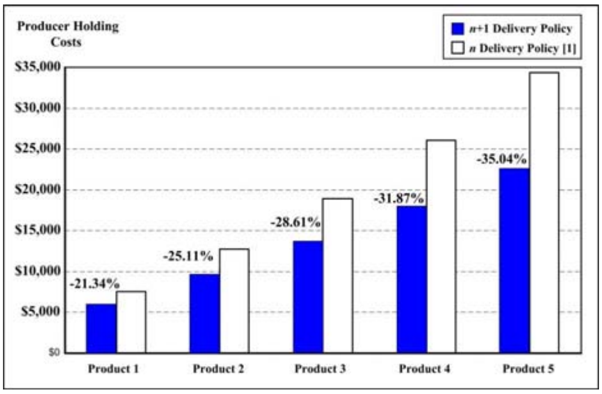

Fig. 5 Percentage of reduction in producer's inventory-holding costs for 5 different products in the numerical example

5. CONCLUDING REMARKS

This study integrates a cost-reduction delivery policy into a multi-item finite production rate (FPR) model with rework [1] for the purpose of lowering producer's inventory-holding costs. Mathematical modelling along with the differential calculus is employed to deal with the problem. First, the expected integrated production-inventory-delivery cost per unit time is derived and it is proved to be convex. Second, the closed-form optimal common-cycle time that minimizes the long-run expected system cost is obtained. Finally, a numerical example is provided to show practical usage of our research results and to demonstrate significant savings in producer's inventory-holding costs in comparison with the previous study [1].

ACKNOWLEDGEMENTS

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MODEL OGRANIČENE PROIZVODNJE RAZNOVRSNIH PROIZVODA S PROCESOM OBRADE ROBE S GREŠKOM TE POBOLJŠANIM SUSTAVOM ISPORUKE

SAŽETAK

U ovome se radu razmatra model ograničene proizvodnje raznovrsnih proizvoda s procesom obrade robe s greškom te poboljšanim sustavom isporuke. Za razliku od konvencionalnog modela ograničene proizvodnje koji obuhvaća neprekidnu, serijsku proizvodnju jednog proizvoda iznimno visoke kvalitete, u ovome se radu razmatra mogućnost proivodnje više različitih proizvoda pomoću istog stroja, obrada svih proizvoda s greškom te ekonomičniji sustav višestruke isporuke robe. Proširujemo rad Chiua i suradnika integrirajući u njihov model poboljšan sustav isporuke, n+1, prema kojemu se jedna dodatna pošiljka gotovih proizvoda isporučuje tijekom proizvođačeva rada kako bi se zadovoljila potražnja za proizvodima za vrijeme kada proizvođač obrađuje proizvode s greškom. Jednom kada ostatak proizvoda posjeduje garantiranu kvalitetu te je proces obrade nevaljale robe dovršen, n obroka fiksne količine gotovih proizvoda se isporučuje kupcima. Cilj ovoga rada je definirati optimalan zajednički proizvodni ciklus koji bi smanjio dugoročni prosječni trošak sustava po jedinici vremena te analizirati utjecaj obrade oštećenih proizvoda i poboljšanog sustava isporuke robe na optimalan proces proizvodnje. U tu svrhu korišteno je matematičko modeliranje i analiza čime je dobiven zatvoren oblik optimalnog, zajedničkog vremenskog ciklusa. Naposlijetku, numeričkim primjerom je prikazana praktična primjena dobivenih rezultata.

Ključne riječi*: model ograničene proizvodnje robe, sustav proizvodnje raznovrsnih proizvoda, zajednički vremenski ciklus, proces obrade proizvoda s greškom, optimizacija, višestruka isporuka.*