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Solving finite production rate model with scrap and multiple shipments using algebraic approach

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SUMMARY

This paper solves a finite production rate (FPR) model with scrap and multiple shipments using an algebraic method. Classic FPR model assumes a continuous inventory issuing policy to satisfy demand and perfect quality production for all items produced. However, in real life vendor-buyer integrated production-inventory system, multiple shipment policy is practically used in lieu of a continuous issuing policy and generation of defective items during production run is inevitable. In this study, it is assumed that all defective items are scrap and the perfect quality items can only be delivered to customers if the whole lot is quality assured at the end of the production run. A conventional approach for solving the FPR model is the use of differential calculus on the long-run average cost function with the need to prove optimality first. This paper demonstrates that optimal lot size and its overall costs for the aforementioned FPR model can be derived without derivatives. As a result, it enables students or practitioners who have little knowledge of calculus to understand and to handle with ease the real-life FPR model.

Key words: *FPR model, production, optimal lot size, scrap, multiple shipments.*

1. INTRODUCTION

The aim of this paper is to determine optimal lot size for finite production rate (FPR) model with random scrap rate and multiple shipments of the perfect quality items. In the manufacturing sector, when products are produced in-house instead of being acquired from outside suppliers, the FPR model is often utilized to cope with the finite production-inventory replenishment rate in order to minimize the expected overall cost per unit time, Refs. [1-2]. The classic FPR model (also known as economic manufacturing quantity (EMQ) model) assumes that all items produced are of perfect quality. However, in real-life production systems, due to process deterioration or other factors, generation of imperfect quality items is inevitable. Many studies have been carried out to enhance the conventional EMQ model by addressing the issue of produced imperfect quality items, Refs. [3-11].

Another unrealistic assumption of classic FPR model is the “continuous” inventory issuing policy for satisfying product demand. In real life vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly used. Goyal [12] first studied the integrated inventory model for a single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. He gave example to illustrate his proposed method. Studies have since been carried out to address various aspects of vendor-buyer supply chain optimization issues, Refs. [12-18]. Lu [13] studied a one-vendor multi-buyer integrated inventory model with the objective of minimizing vendor's total annual cost subject to the maximum costs that the buyers may be prepared to incur. Lu's model required to know buyer's annual demand and previous order frequency. As a result, an optimal solution for the one-vendor one-buyer case was obtained and a heuristic approach for the one-vendor multi-buyer case was also provided. Diponegoro and Sarker [16] determined an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. They also developed a lower bound on the optimal solution for problem with lost sale. Chiu et al. [18] examined a finite production rate model with random scrap rate and multiple shipments. They used mathematical modeling and conventional derivatives on the cost function to prove optimality, and they derived the optimal production lot size.

A few recent studies [19-21] presented algebraic approaches for solving classic economic order quantity (EOQ) and EMQ model without reference to the use of derivatives, neither applying the first-order nor the second-order differentiations [19-20]. This paper uses such an algebraic approach [21] to solve FPR model with random scrap and multiple shipments (as it was examined by Chiu et al. [18]) and it demonstrates that optimal lot size, and expected overall costs can be derived by using the algebraic method.

2. THE PROPOSED FPR MODEL

Recall the FPR model with random scrap and multiple deliveries using the conventional approach [18], in a production system where the process may randomly produce x portion of defective items at a production rate d. All items produced are screened and inspection cost per item is included in the unit production cost C. All nonconforming items are assumed to be scrap. In order to avoid shortage from occurring, the constant production rate P has to be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P-d-\lambda)>0$. The production rate of scrap items d can be expressed as $d=Px$.

Unlike the classic FPR model assuming a continuous issuing policy for satisfying demand, a multi-delivery policy is considered. It is assumed that the finished items can only be delivered to customers when the whole lot is quality assured at the end of production. Fixed quantity n installments of the finished batch are delivered at a fixed interval of time during the production downtime [18]. Cost parameters considered in this study include: the disposal cost per scrap item C_S, setup cost K, unit holding cost h, unit production cost C, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers.

Additional notation includes production lot size Q and cycle length T, production uptime t_1, time required for delivering all quality assured finished products t_2, the maximum level of on-hand inventory H in units when regular production process ends, the total costs per cycle $TC(Q)$, and the long-run average costs per unit time $E[TCU(Q)]$. Figure 1 depicts the level of on-hand inventory of perfect quality items in the proposed FRP.

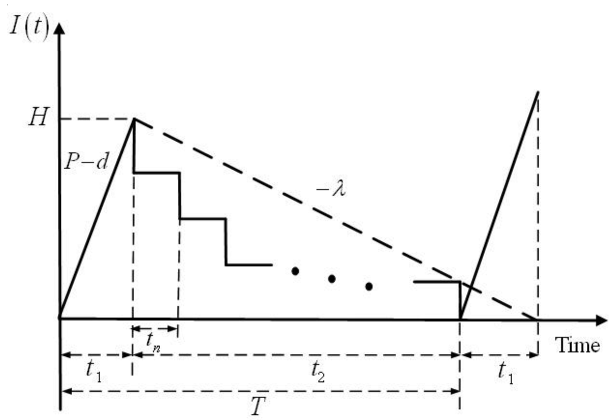

Fig. 1 On-hand inventory of perfect quality items in FPR model with scrap and a multiple delivery policy

By using mathematical modeling and analysis [18], the total costs per cycle $TC(Q)$ and the long-run average production-inventory-delivery costs $E[TCU(Q)]$ can be derived as follows:

$$TC(Q) = CQ + K + C_S[xQ] + nK_1 + C_T[Q(1-x)] + h\left[\frac{H+dt_1}{2}(t_1)\right] + h\left(\frac{n-1}{2n}\right)Ht_2 \quad (1)$$

$$E[TCU(Q)] = \frac{C\lambda}{1-E[x]} + \frac{(K+nK_1)\lambda}{Q(1-E[x])} + \frac{C_S E[x]\lambda}{(1-E[x])} + C_T\lambda + \frac{hQ\lambda}{2P(1-E[x])} + \left(\frac{n-1}{n}\right)\left[\frac{hQ(1-E[x])}{2} - \frac{hQ\lambda}{2P}\right] \quad (2)$$

3. SOLVING PROPOSED FRP MODEL USING ALGEBRAIC APPROACH

Instead of using conventional differential calculus on the long-run average cost function $E[TCU(Q)]$ for optimality and solution to proposed FPR model, this study employs algebraic approach. Let β_0, β_1, and β_2 denote the following:

$$\beta_0 = \frac{C\lambda}{1-E[x]} + \frac{C_S E[x]\lambda}{(1-E[x])} + C_T\lambda \tag{3}$$

$$\beta_1 = \frac{(K+nK_1)\lambda}{(1-E[x])} \tag{4}$$

$$\beta_2 = \frac{h\lambda}{2P(1-E[x])} + \left(\frac{n-1}{n}\right)\left[\frac{h(1-E[x])}{2} - \frac{h\lambda}{2P}\right] \tag{5}$$

Substituting Eqs. (3) to (5) into Eq. (2), one has:

$$E[TCU(Q)] = \beta_0 + \beta_1 Q^{-1} + \beta_2 Q \tag{6}$$

By further rearrangement of Eq. (6), one obtains:

$$\begin{aligned} E[TCU(Q)] &= \beta_0 + Q\left[\beta_1 Q^{-2} + \beta_2\right] = \\ &= \beta_0 + Q\left[\left(\sqrt{\beta_1}Q^{-1}\right)^2 - 2\cdot\left(\sqrt{\beta_1}Q^{-1}\right)\cdot\sqrt{\beta_2} + \left(\sqrt{\beta_2}\right)^2\right] + \\ &\quad + 2\cdot\sqrt{\beta_1}\cdot\sqrt{\beta_2} = \\ &= \beta_0 + Q\left[\sqrt{\beta_1}Q^{-1} - \sqrt{\beta_2}\right]^2 + 2\cdot\sqrt{\beta_1}\cdot\sqrt{\beta_2} \end{aligned} \tag{7}$$

From Eq. (7), if the following square term is zero, then $E[TCU(Q)]$ has the minimum value:

$$\sqrt{\beta_1}Q^{-1} - \sqrt{\beta_2} = 0 \tag{8}$$

or:

$$\sqrt{\frac{\beta_1}{\beta_2}} = Q \tag{9}$$

Substituting Eqs. (4) to (5) into Eq. (9), one obtains the optimal production lot size as follows:

$$Q^* = \sqrt{\frac{2(K+nK_1)\lambda}{\frac{h\lambda}{P} + \left(\frac{n-1}{n}\right)(1-E[x])\left[h(1-E[x]) - \frac{h\lambda}{P}\right]}} \tag{10}$$

It is noted that Eq. (10) is identical to Eq. (15) in [18]. It follows that optimal $E[TCU(Q^*)]$ is (from Eq. (7)):

$$E[TCU(Q^*)] = \beta_0 + 2\cdot\sqrt{\beta_1}\cdot\sqrt{\beta_2} \tag{11}$$

The long-run average costs per unit time $E[TCU(Q^*)]$ revealed in Eq. (11), is a simpler form in comparison with that of Eq. (2).

4. VERIFICATION BY NUMERICAL EXAMPLE

The aforementioned results derived by this study (i.e. Eqs. (10) and (11)) can be verified using the same numerical example given by Chiu et al. [18]. Reconsider that a manufactured item has experienced a flat demand rate of *3,400* units per year and it can be produced at a rate of *60,000* units per year. There is a random scrap rate during the production uptime, which follows uniform distribution over the interval [*0, 0.3*]. Other parameters considered by this example are given as follows:

- K = *$20,000* per production run,
- C = *$100* per item,
- C_S = *$20*, repaired cost for each item reworked,
- K_1 = *$4,400* per shipment, a fixed cost,
- h = *$20* per item per year,
- n = 4 installments of the finished batch are delivered per cycle,
- C_T = *$0.1* per item delivered.

Using Eqs. (10) and (11), one obtains that $Q^*=4,768$ and E[TCU(Q^*)]=*$475,432*. Both of them are identical as shown in Ref. [18].

5. CONCLUSION

This study shows that optimal production lot size Q^* and the long-run average production - inventory - delivery costs $E[TCU(Q^*)]$ for the proposed FPR model can be derived using the algebraic approach (i.e. without derivatives). As a result, it also presents a simpler form for the long-run average costs $E[TCU(Q^*)]$ (as revealed in Eq. (11)). This simplified method enables, practitioners or students with little or no knowledge of calculus to understand or handle with ease the real-life FPR model.

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7. REFERENCES

[1] E.A. Silver, D.F. Pyke and R. Peterson, *Inventory Management and Production Planning and Scheduling*, John Wiley & Sons, New York, pp. 151-172, 1998.

[2] S. Nahmias, *Production and Operations Analysis*, 5th edition, McGraw-Hill Irwin, Inc., New York, pp. 195-223, 2005.

[3] R.E. Barlow and F. Proschan, *Mathematical Theory of Reliability*, John Wiley, New York, 1965.

[4] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions*, Vol. 18, pp. 48-55, 1986.

[5] T. Boone, R. Ganeshan, Y. Guo and J.K. Ord, The impact of imperfect processes on production run times, *Decision Sciences*, Vol. 31, pp. 773-785, 2000.

[6] S.W. Chiu, Ch.-K. Ting and Y.-Sh.P. Chiu, Optimal order policy for EOQ model under shortage level constraint, *Int. J. for Engineering Modelling*, Vol. 18, No. 1-2, pp. 41-46, 2005.

[7] Y.-Sh.P. Chiu, Y.-L. Lien and Ch.-A.K. Lin, Incorporating machine reliability issue and backlogging into the EMQ model - Part I: Random breakdown occurring in backorder filling time, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 1-13, 2009.

[8] S.W. Chiu, Y.-Sh.P. Chiu, H.-J. Chuang and Ch.-H. Lee, Incorporating machine reliability issue and backlogging into the EMQ model - Part II: Random breakdown occurring in inventory piling time, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 15-24, 2009.

[9] S.W. Chiu, H.-D. Lin, Ch.-B. Cheng and Ch.-L. Chung, Optimal production- shipment decisions for the finite production rate model with scrap, *Int. J. for Engineering Modelling*, Vol. 22, No. 1-4, pp. 25-34, 2009.

[10] Y.-Sh.P. Chiu, K.-K. Chen, F.-T. Cheng and M.-F. Wu, Optimization of the finite production rate model with scrap, rework and stochastic machine breakdown, *Computers and Mathematics with Applications*, Vol. 59, Issue 2, pp. 919-932, 2010.

[11] Y.-Sh.P. Chiu and C.-K. Ting, A note on 'Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns', *European Journal of Operational Research*, Vol. 201, No. 2, pp. 641-643, 2010.

[12] S.K. Goyal, Integrated inventory Model for a single supplier-single customer problem, *Int. J. of Production Research*, Vol. 15, pp. 107-111, 1977.

[13] L. Lu, A one-vendor multi-buyer integrated inventory model, *European Journal of Operational Research*, Vol. 81, pp. 312-323, 1995.

[14] B.R. Sarker and G.R. Parija, An optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy, *Journal of the Operational Research Society*, Vol. 45, pp. 891–900, 1994.

[15] Y.-Sh.P. Chiu, S.W. Chiu, C.-Y. Li and C.-K. Ting, Incorporating multi-delivery policy and quality assurance into economic production lot size problem, *Journal of Scientific & Industrial Research*, Vol. 68, pp. 505-512, 2009.

[16] A. Diponegoro and B.R. Sarker, Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries, *Computers and Operations Research*, Vol. 33, No. 8, pp. 2387-2404, 2006.

[17] M. Khouja, Optimizing inventory decisions in a multi-stage multi-customer supply chain, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 39, No. 3, pp. 193-208, 2003.

[18] Y.-Sh.P. Chiu, S.W. Chiu, Ch.-A.K. Lin and C.-Y. Chang, Finite production rate model with random scrap rate and multiple shipments, Proc. of the 10th International Conference on Automation Technology, National Cheng Kung University, Taiwan, p. 91, 2009.

[19] R.W. Grubbström and A. Erdem, The EOQ with backlogging derived without derivatives, *Int. J. of Production Economics*, Vol. 59, pp. 529-530, 1999.

[20] L.E. Cárdenas-Barrón, The economic production quantity (EPQ) with shortage derived algebraically, *Int. J. of Production Economics*, Vol. 70, pp. 289-292, 2001.

[21] Y.-Sh.P. Chiu, K.-K. Chen, C.-K. Ting and V. Chiu, Economic lot sizing with imperfect rework derived without derivatives, *Int. J. for Engineering Modelling*, Vol. 20, No. 1-4, pp. 61-65, 2007.

RJEŠAVANJE MODELA OGRANIĆENE PROIZVODNJE S OTPACIMA I VIŠESTRUKIM ISPORUKAMA KORISTEĆI ALGEBARSKI PRISTUP

SAŽETAK

Ovaj rad se bavi rješavanjem modela ograničene proizvodnje s otpacima i višestrukim isporukama koristeći algebarsku metodu. Klasičan model ograničene proizvodnje (FPR) pretpostavlja politiku kontinuiranoga izdavanja inventara u svrhu zadovoljavanja potražnje za proizvodom kao i savršenom kvalitetom svega proizvedenog. Međutim, u stvarnom prodavač-kupac integriranom proizvodno-skladišnom sustavu koristi se politika višekratne isporuke umjesto politike kontinuirane isporuke pa je neizbježan pojam oštećenih proizvoda tijekom proizvodnje. U ovom radu se pretpostavlja da su svi neispravni predmeti otpaci, a da se samo predmeti savršene kvalitete mogu isporučiti kupcima ako je sveukupna proizvedena količina dokazano kvalitetna na kraju proizvodnje. Konvencionalan pristup rješavanju FPR modela je korištenje diferencijalnog računa za izračunati dugoročan prosječan trošak, ponajprije zbog potrebe dokazivanja optimalnosti. Ovaj rad dokazuje da se ukupna količina i ukupan trošak za ranije navedeni FPR model može izračunati bez izvedenica. U konačnici to omogućava studentima i stručnjacima bolje razumijevanje i lakoću korištenja FPR modela u stvarnom životu.

Ključne riječi: *FPR model, proizvodnja, optimalna sveukupna količina, otpadak, višestruke isporuke.*