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An easy approach to derive EOQ and EPQ models with shortage and defective items

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SUMMARY

Huang [Journal of Statistics and Management Systems, Vol. 6, No. 2, pp. 171-180, 2003.] studied the EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) models with backlogging and defective items using the algebraic approach. He assumed that a 100% inspection policy and the known proportion of defective items was removed after the screening process prior to storage or use. In this paper, we will offer another simple approach to find both the optimal lot size and backorder level under the minimized total relevant cost per unit time.

Key words: *arithmetic-geometric mean inequality approach, algebraic approach, EOQ, EPQ, shortage, defective item.*

1. INTRODUCTION

The economic order quantity (EOQ) model with/without shortages and economic production quantity (EPQ) model with/without shortages are widely used by practitioners as a decision-making tool for the control of inventory. However, the assumptions of the EOQ / EPQ models are rarely met. This has led many researchers to study the EOQ / EPQ extensively under realistic situations.

A common unrealistic assumption of the EOQ / EPQ is that all units produced or purchased are of good quality. We know that it is difficult to produce or purchase items of a 100% good quality. Recently, Huang [1] studied the EOQ and EPQ models with backlogging and defective items. He assumed that 100% inspection policy and the known proportion of defective items was removed after the screening process prior to storage or use. In addition, Huang [1] used the algebraic method to determine the optimal solution by minimizing the annual relevant cost. In previous several papers, the EOQ and EPQ formulae for the shortage case have been derived using differential calculus and solving two simultaneous equations with the need to prove optimality conditions with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. Grubbström and Erdem [2] and Cárdenas-Barrón [3] showed that the formulae for the EOQ and EPQ with backlogging were derived without differential calculus. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. But Ronald et al. [4] thought that their algebraic procedure was too sophisticated to be absorbed by ordinary readers. Hence, Ronald et al. [4] derived a procedure to transform a two-variable problem into two steps, and then, in each step they solve a one-variable problem using only the algebraic method without calculus. Recently, Chang et al. [5] rewrote the objective function of Ronald et al. [4] so that the usual skill of completing the square can solve the problem without using their sophisticated method. All previously mentioned articles use algebraic optimization.

Although Huang [1] used the easily algebraic approach to find the optimal solution, his method had the same problem as Grubbström and Erdem [2] and Cárdenas-Barrón [3]. Recently, Tu et al. [6] used another simple arithmetic-geometric mean inequality approach to investigate the retailer's EOQ under trade credit period depending on the order quantity. Therefore, in this paper, we provide the same as Tu et al. [6] easy-to-understand and simple-to-apply arithmetic-geometric mean inequality approach without using derivatives to obtain the optimal solution. This approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

2. DERIVATION OF THE LOT SIZE AND BACKORDER LEVEL

The following notation and assumptions are the same as Huang's [1] which will be used further on in this paper.

Notation

- Q – order quantity (EOQ model) / production quantity (EPQ model) (including defective items),
- V – maximum on-hand inventory level (including defective items),
- L – maximum shortage (backorder) level (including defective items),
- D – demand rate for nondefective items, units per time,
- P – production rate for nondefective items, units per time ($P>D$),
- A – ordering cost / setup cost,
- I – the fixed inspection cost incurred with each lot,
- i – unit inspection cost,
- h – unit stock holding cost per unit per time,
- b – unit shortage cost per unit short per time,
- k – the known percentage of defective items in Q,
- $TAC(V,L)$ – total relevant cost per unit time.

Assumptions

(1) Demand rate is known and constant.

(2) Production rate is known and constant.

(3) Time period is infinite.

(4) Each lot purchased / produced contains a known proportion of defectives removed prior to storage or use.

(5) The screening time for items is so fast that we can neglect it. The inspection cost consists of a fixed per lot inspection cost and a fixed unit inspection cost.

We use here an easy and a simple arithmetic–geometric-mean-inequality (AGM) approach the same as Tu et al. [6] to obtain the optimal lot size and backorder level that minimizes the annual total relevant cost. The arithmetic–geometric mean inequality is as follows. For any of two real positive numbers, say x and y, the arithmetic mean $(x+y)/2$ is always greater than or equal to the geometric mean $\sqrt{xy}$. Namely,

$$\frac{x+y}{2} \geq \sqrt{xy}$$

The equation holds only if $x=y$.

Model I : EOQ model (shown in Figure 1)

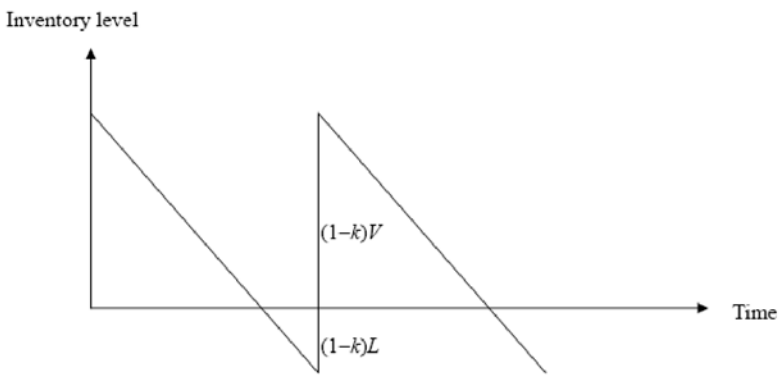

Fig. 1 The inventory level of Model I

From Eq. (2) in Huang [1] we know that the total relevant cost per unit time, $TAC(V,L)$, can be expressed as:

$$TAC(V,L) = \frac{D}{(1-k)(V+L)}\left[A + I + (V+L)i + \frac{(1-k)^2 V^2 h}{2D} + \frac{(1-k)^2 L^2 b}{2D}\right] \quad (1)$$

For convenience, we let $Q=V+L$ and substitute into Eq. (1).

Then, we can obtain:

$$TAC(Q,L)=\frac{D}{(1-k)Q}\left[A+I+Qi+\frac{(1-k)^2(Q-L)^2h}{2D}+\frac{(1-k)^2L^2b}{2D}\right] \tag{2}$$

Then we rewrite Eq. (2) as:

$$TAC(Q,L)=\frac{(1-k)(h+b)}{2Q}\left[L-\frac{h}{h+b}Q\right]^2+\frac{(1-k)hbQ}{2(h+b)}+\frac{D(A+I)}{(1-k)Q}+\frac{Di}{1-k} \tag{3}$$

It implies that when Q is given, we can set L as:

$$L=\left(\frac{h}{h+b}\right)Q$$

to get the minimum value of $TAC(Q,L)$ as follows:

$$TAC[Q,L(Q)]=\frac{(1-k)hbQ}{2(h+b)}+\frac{D(A+I)}{(1-k)Q}+\frac{Di}{1-k} \tag{4}$$

By using the AGM approach we can easily obtain that in Eq. (4):

$$\begin{aligned}TAC[Q,L(Q)]&=\frac{(1-k)hbQ}{2(h+b)}+\frac{D(A+I)}{(1-k)Q}+\frac{Di}{1-k}\geq\\ &\geq 2\sqrt{\frac{(1-k)hbQ}{2(h+b)}\times\frac{D(A+I)}{(1-k)Q}}+\frac{Di}{1-k}=\sqrt{\frac{2hbD(A+I)}{(h+b)}}+\frac{Di}{1-k}\end{aligned} \tag{5}$$

When the equality:

$$\frac{(1-k)hbQ}{2(h+b)}=\frac{D(A+I)}{(1-k)Q} \tag{6}$$

holds, $TAC[Q, L(Q)]$ has a minimum. Then, we can find the optimal ordering quantity:

$$EOQ(Q^*)=\frac{1}{1-k}\sqrt{\frac{2(A+I)D}{h}}\sqrt{\frac{h+b}{b}}=\frac{1}{1-k}\sqrt{\frac{2(A+I)Db}{h(h+b)}}+\frac{1}{1-k}\sqrt{\frac{2(A+I)Dh}{b(h+b)}} \tag{7}$$

and putting Q^* into $L=\left(\frac{h}{h+b}\right)Q$ one can obtain the optimal allowable backorder level:

$$L^*=\frac{1}{1-k}\sqrt{\frac{2(A+I)Dh}{b(h+b)}} \tag{8}$$

Therefore, the minimum value of total relevant cost per unit time, $TAC(Q^*, L^*)$ is:

$$TAC(Q^*,L^*)=\sqrt{\frac{2hbD(A+I)}{(h+b)}}+\frac{Di}{1-k} \tag{9}$$

Equations (7) to (9), in this paper, are the same as Eqs. (6), (5) and (7) in Huang [1], respectively.

Model II : EPQ model (shown in Figure 2)

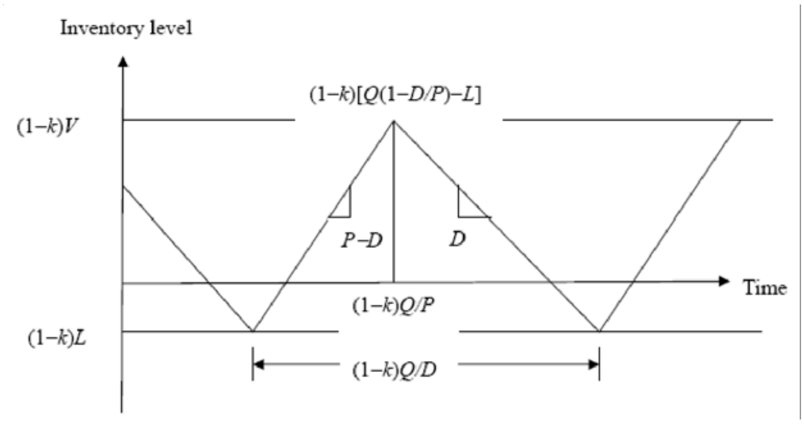

Figure 2. The inventory level of Model II

For convenience, let $\rho = (1 - \frac{D}{P})$, $Q = \frac{V + L}{\rho}$ and Eq. (9) in Huang [1] we know that the total relevant cost per unit time, *TAC*(*Q*,*L*), can be expressed as:

$$TAC(Q,L) = \frac{D}{(1-k)Q\rho}\left[A + I + \frac{(1-k)^2(Q\rho - L)^2 h}{2D\rho} + \frac{(1-k)^2 L^2 b}{2D\rho}\right] + \frac{Di}{1-k} \tag{10}$$

Then we rewrite Eq. (10) as:

$$TAC(Q,L) = \frac{(1-k)(h+b)}{2Q\rho^2}\left[L - \frac{h\rho}{h+b}Q\right]^2 + \frac{(1-k)hbQ}{2(h+b)} + \frac{D(A+I)}{(1-k)Q\rho} + \frac{Di}{1-k} \tag{11}$$

It implies that when *Q* is given, we can set *L* as:

$$L = \left(\frac{h\rho}{h+b}\right)Q$$

to get the minimum value of *TAC*(*Q*,*L*) as follows:

$$TAC[Q,L(Q)] = \frac{(1-k)hbQ}{2(h+b)} + \frac{D(A+I)}{(1-k)Q\rho} + \frac{Di}{1-k} \tag{12}$$

By using the AGM approach we can easily obtain that in Eq. (12):

$$\begin{aligned} TAC[Q,L(Q)] &= \frac{(1-k)hbQ}{2(h+b)} + \frac{D(A+I)}{(1-k)Q\rho} + \frac{Di}{1-k} \geq \\ &\geq 2\sqrt{\frac{(1-k)hbQ}{2(h+b)} \times \frac{D(A+I)}{(1-k)Q\rho}} + \frac{Di}{1-k} = \sqrt{\frac{2hbD(A+I)}{(h+b)\rho}} + \frac{Di}{1-k} \end{aligned} \tag{13}$$

When the equality:

$$\frac{(1-k)hbQ}{2(h+b)} = \frac{D(A+I)}{(1-k)Q\rho} \tag{14}$$

holds, *TAC*[*Q*,*L*(*Q*)] has a minimum. Then, we can find the optimal production quantity:

$$EPQ(Q^*) = \frac{1}{1-k}\sqrt{\frac{2(A+I)D}{h\rho}}\sqrt{\frac{h+b}{b}} \tag{15}$$

and putting Q* into $L = \left(\frac{h\rho}{h+b}\right)Q$ one can obtain the optimal allowable backorder level:

$$L^* = \frac{1}{1-k}\sqrt{\frac{2(A+I)Dh\rho}{b(h+b)}} \tag{16}$$

Therefore, the minimum value of total relevant cost per unit time, *TAC*(*Q**,*L**) is:

$$TAC(Q^*,L^*) = \sqrt{\frac{2hbD(A+I)}{(h+b)\rho}} + \frac{Di}{1-k} \tag{17}$$

Eqs. (15) to (17), in this paper, are the same as Eqs. (16), (15) and (17) in Huang [1], respectively.

3. CONCLUSIONS

This paper improves Huang's [1] algebraic procedure and offers another simple AGM approach to find the optimal ordering quantity (EOQ model) and the optimal production quantity (EPQ model) with shortages and defective items. Using this improved approach presented in this paper, we can find the optimal ordering quantity and the optimal production quantity without using differential calculus. This should also mean that this simple AGM approach is a more accessible approach to ease the learning of basic inventory theories to younger students who lack the knowledge of calculus.

4. REFERENCES

[1] Y.F. Huang, The deterministic inventory models with shortages and defective items derived without derivatives, *J. Stat. Managt. Sys.*, Vol. 6, pp. 171-180, 2003.

[2] R.W. Grubbström and A. Erdem, The EOQ with backlogging derived without derivatives, *Int. J. Prod. Eco.*, Vol. 59, pp. 529-530, 1999.

[3] L.E. Cárdenas-Barrón, The economic production quantity (EPQ) with shortage derived algebraically, *Int. J. Prod. Eco.*, Vol. 70, pp. 289-292, 2001.

[4] R.J. Ronald, G.K. Yang and P. Chu, The EOQ and EPQ models with shortages derived without derivatives, *Int. J. Prod. Eco.*, Vol. 92, pp. 181-184, 2004.

[5] S.K.J. Chang, J.P.C. Chuang and H.J. Chen, Short comments on technical note: The EOQ and EPQ models with shortages derived without derivatives, *Int. J. Prod. Eco.*, Vol. 97, pp. 241-243, 2005.

[6] Y.C. Tu, K.H. Hsu and Y.F. Huang, Retailer's economic order quantity model under trade credit period depending on the order quantity without calculus, *J. Appl. Sci.*, Vol. 8, pp. 2785-2787, 2008.

JEDNOSTAVAN PRISTUP ODREĐIVANJU EOQ I EPQ MODELA S NEDOSTACIMA I NEISPRAVNIM PREDMETIMA

SAŽETAK

Huang (Journal od Statistics and Management Systems, Vol. 6, No. 2, pp. 171-180, 2003.) je proučavao EOQ (veličina ekonomičnog reda) i EPQ (veličina ekonomične proizvodnje) modele s izostalim istraživanjima i neispravnim predmetima koristeći algebarski pristup. Pretpostavio je da je stopostotnim nadziranjem i detaljnim ispitivanjem bila odstranjena poznata količina neispravnih predmeta prije skladištenja ili upotrebe. U ovom radu nudimo jedan drugačiji jednostavni pristup da bi se odredila optimalna količina robe kao i razina zaostalih narudžbi u uvjetima umanjenja ukupnog relevantnog troška u jedinici vremena.

Ključne riječi: *aritmetičko-geometrijski pristup srednje nejednakosti, algebarski pristup, EOQ, EPQ, nedostatak, neispravni predmeti.*