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A note on ‘impacts of random scrap rate on production system in supply chain environment with a specific shipping policy’

Yuan-Shyi Peter Chiu⁽¹⁾, Hong-Dar Lin⁽¹⁾, Feng-Tsung Cheng⁽²⁾ and Singa Wang Chiu⁽³⁾

⁽¹⁾Department of Industrial Engineering, Chaoyang University of Technology, Wufong, Taichung 413, TAIWAN

⁽²⁾Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung 407, TAIWAN

⁽³⁾Department of Business Administration, Chaoyang University of Technology,
Gifong East Road, Taichung 413, TAIWAN, e-mail: swang@cyut.edu.tw

SUMMARY

This paper employs an alternative approach to reexamine the impacts of random scrap rate on production system in supply chain environment with a specific shipping policy. A straightforward approach in terms of a two-phase algebraic derivation is proposed in this study to replace the conventional method with the need of applying first-order and second-order differentiations to the system cost function for proof of convexity before derivation of the optimal production-shipment policy. The research result of this study is confirmed that is identical to what was obtained by Cheng et al. [1] where they used the conventional method to solve the same problem. The proposed approach is helpful for practitioners, who may not have sufficient knowledge of differential calculus to understand such an integrated production-shipment system in supply chain environment.

Key words: optimization, production-shipment system, random scrap rate, lot size, multi-distribution, algebraic approach, production planning.

1. INTRODUCTION

The most economical production lot size was first proposed by Taft [2], to assist manufacturing firms in minimizing total production costs (this is also known as the economic production quantity (EPQ) model). The EPQ model implicitly assumes that all items produced are of perfect quality. However, in real world production settings, due to different factors generation of nonconforming items seems inevitable. For this reason, many studies have been carried out during the past decades, to address the imperfect production and its related issues [3-13]. Another unrealistic assumption of classic EPQ model is the continuous inventory issuing policy, for in vendor-buyer integrated production-shipment system, the periodic deliveries instead of continuous policy is often

used. Research has since been focused on addressing issues of various aspects of multi-deliveries in supply chain optimization [14-22].

Cheng et al. [1] investigated the impacts of random scrap rate on production system in supply chain environment with a specific shipping policy. They used the differential calculus along with Hessian matrix equations to derive the optimal production batch size and number of deliveries for such a specific vendor-buyer system with random scrap rate. In a recent paper, Grubbström and Erdem [23], presented an algebraic approach to solve the economic order quantity (EOQ) model with backlogging, without reference to the use of derivatives. Other studies that have applied the same or similar method include for example Refs. [24-27]. This paper applies the same alternative approach to reexamine the model of Cheng et al. [1]. As a result,

the optimal replenishment lot size, number of deliveries, and the long-run average cost can all be derived without using differential calculus or the Hessian matrix equations.

2. PROBLEM STATEMENT AND ASSUMPTION

In this study, an alternative approach is adopted to reexamine Cheng et al.'s model [1] as stated earlier. To ease the readability, this study adopts the exact notation as used in Cheng et al. [1]. Description of the model is as follows. Consider a real life production system where process may randomly produce a portion x of scrap items at a rate d . Under regular operating schedule, the constant production rate P is larger than the sum of demand rate λ and production rate of defective items d (i.e. $(P-d-\lambda) > 0$). Cost variables include unit production cost C , setup cost K per production run, buyer's unit holding cost h_2 , disposal cost per scrap item C_S , vendor's unit holding cost h , delivery cost C_T per item, and a fixed delivery cost K_1 per shipment. Additional notation includes:

- n - number of fixed quantity installments of finished batch to be delivered to customer during time t_2 , one of the decision variables to be determined for each cycle,
- Q - replenishment lot size, another decision variable to be determined for each cycle,
- T - cycle length,
- H - the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1 ,
- H_1 - maximum level of on-hand inventory in units when regular production ends,
- t - the production time needed for producing enough perfect items for satisfying product demand during the production uptime t_1 ,
- t_1 - the production uptime for the proposed model,
- t_2 - time required for delivering the remaining perfect quality finished products,

- t_n - a fixed interval of time between each installment of products delivered during time t_2 ,
- I - demand during production time t , i.e. $I=\lambda t$,
- D - demand during production uptime t_1 , i.e. $D=\lambda t_1$,
- $I(t)$ - on-hand inventory of perfect quality items at time t ,
- $TC(Q,n)$ - total production-inventory-delivery costs per cycle for the proposed model,
- $E[TCU(Q,n)]$ - the long-run average costs per unit time for the proposed model.

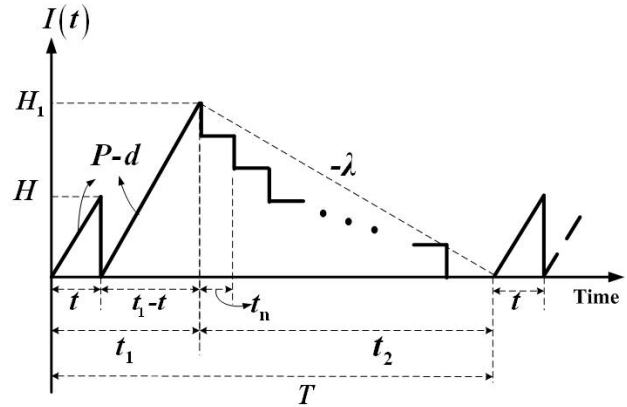


Fig. 1 The vendor's on-hand inventory of perfect quality items for the proposed model with random scrap and $(n+1)$ delivery policy [1]

Refer to Cheng et al. [1], Figure 1 depicts the vendor's on-hand inventory of perfect quality items. For the purpose of easing readability, this paper adopted the same basic formulations as that in Ref. [1]. Total production-inventory-delivery cost per cycle $TC(Q,n)$ of the proposed model consists of the setup cost, variable manufacturing cost, variable disposal cost, $(n+1)$ fixed and variable shipping cost, holding cost for perfect quality items during t_1 , holding cost for scrap items during t_1 , vendor's holding cost for finished goods during the delivery time t_2 , and buyer's holding cost. Using the same formulation procedures, one has $TC(Q,n)$ as follows (see Eq. (1) in Ref. [1]):

$$\begin{aligned}
 TC(Q,n) = & K + CQ + C_S[xQ] + (n+1)K_1 + C_TQ(1-x) + \\
 & + h \left[\frac{Ht}{2} + \frac{H_1(t_1-t)}{2} + \frac{dt_1}{2}(t_1) \right] + h \left[\left(\frac{n-1}{2n} \right) H_1 t_2 \right] + \\
 & + h_2 \left[\frac{H(t_1)}{2} + n \left(\frac{D+2I}{2} \right) t_n \right] \tag{1}
 \end{aligned}$$

Taking into the randomness of defective rate x , one can use the expected values of x in cost analysis and obtains $E[TCU(Q,n)]$ as follows (see Eq. (2) in Ref. [1]):

$$\begin{aligned}
 E[TCU(Q,n)] &= \frac{E[TC(Q,n)]}{E[T]} = \frac{C\lambda}{1-E(x)} + \frac{[(n+1)K_I + K]\lambda}{Q[1-E(x)]} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda + \\
 &+ \frac{hQ}{2} \left\{ \frac{2\lambda^3}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) - \frac{\lambda^2}{P^2[1-E(x)]} + [1-E(x)] - \frac{\lambda[1-2E(x)]}{P[1-E(x)]} \right\} + \\
 &\left\{ -\left(\frac{I}{n}\right) \left[[1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \right\} + \\
 &+ \frac{h_2Q}{2n} \left[[1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] + \\
 &+ h_2Q\lambda^2 \left[\frac{I}{P^2} \left[E\left(\frac{I}{1-x}\right) + \frac{I}{[1-E(x)]} \right] - \frac{\lambda}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) \right] \tag{2}
 \end{aligned}$$

With further arrangement, one has:

$$\begin{aligned}
 E[TCU(Q,n)] &= \left[\frac{C\lambda}{1-E(x)} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda \right] + \frac{nK_I\lambda}{Q[1-E(x)]} + \frac{(K_I + K)\lambda}{Q[1-E(x)]} + \\
 &+ Q \left\{ \frac{h}{2} \left[\frac{2\lambda^3}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) - \frac{\lambda^2}{P^2[1-E(x)]} + [1-E(x)] - \frac{\lambda[1-2E(x)]}{P[1-E(x)]} \right] \right\} + \\
 &\left\{ +h_2\lambda^2 \left[\frac{I}{P^2} \left[E\left(\frac{I}{1-x}\right) + \frac{I}{[1-E(x)]} \right] - \frac{\lambda}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) \right] \right\} + \\
 &+ \frac{Q}{2n} (h_2 - h) \left[[1-E(x)] - \frac{2\lambda}{P[1-E(x)]} + \frac{\lambda^2}{P^2[1-E(x)]} \right] \tag{3}
 \end{aligned}$$

3. THE PROPOSED APPROACH

In this section, a two-phase algebraic approach is employed to derive optimal replenishment lot size and the optimal number of deliveries. One notes Eq. (3) has two decision variables Q and n , and they are in terms of coefficients associated with nQ^{-1} , Q^{-1} , Q , and Qn^{-1} . First let $\pi_1, \pi_2, \pi_3, \pi_4$ and π_5 denote the following:

$$\pi_1 = \frac{C\lambda}{1-E(x)} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda \tag{4}$$

$$\pi_2 = \frac{K_I\lambda}{[1-E(x)]} \tag{5}$$

$$\pi_3 = \frac{(K_I + K)\lambda}{[1-E(x)]} \tag{6}$$

$$\pi_4 = \left\{ \frac{h}{2} \left[\frac{2\lambda^3}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) - \frac{\lambda^2}{P^2[1-E(x)]} + [1-E(x)] - \frac{\lambda[1-2E(x)]}{P[1-E(x)]} \right] \right\} + \\
 \left\{ +h_2\lambda^2 \left[\frac{I}{P^2} \left[E\left(\frac{I}{1-x}\right) + \frac{I}{[1-E(x)]} \right] - \frac{\lambda}{P^3[1-E(x)]} E\left(\frac{I}{1-x}\right) \right] \right\} \tag{7}$$

$$\pi_5 = \frac{(h_2 - h)}{2} \left[[1 - E(x)] - \frac{2\lambda}{P[1 - E(x)]} + \frac{\lambda^2}{P^2[1 - E(x)]} \right] \tag{8}$$

Then Eq. (3) becomes:

$$E[TCU(Q, n)] = \pi_1 + nQ^{-1}\pi_2 + Q^{-1}\pi_3 + Q\pi_4 + Qn^{-1}\pi_5 \tag{9}$$

With further rearrangement of Eq. (9) one has:

$$E[TCU(Q, n)] = \pi_1 + Q^{-1}[\pi_3 + \pi_4Q^2] + Qn^{-1}[\pi_2(Q^{-1}n)^2 + \pi_5] \tag{10}$$

$$E[TCU(Q, n)] = \pi_1 + Q^{-1}[(\sqrt{\pi_3})^2 + (Q\sqrt{\pi_4})^2 - 2(\sqrt{\pi_3})(Q\sqrt{\pi_4}) + 2(\sqrt{\pi_3})(Q\sqrt{\pi_4})] + Qn^{-1}[\sqrt{\pi_2}(Q^{-1}n)^2 + (\sqrt{\pi_5})^2 - 2[\sqrt{\pi_2}(Q^{-1}n)](\sqrt{\pi_5}) + 2[\sqrt{\pi_2}(Q^{-1}n)](\sqrt{\pi_5})] \tag{11}$$

Therefore, one obtains:

$$E[TCU(Q, n)] = \pi_1 + Q^{-1}[(\sqrt{\pi_3}) - (Q\sqrt{\pi_4})]^2 + Qn^{-1}[(\sqrt{\pi_5}) - \sqrt{\pi_2}(Q^{-1}n)]^2 + 2\sqrt{\pi_3 \cdot \pi_4} + 2\sqrt{\pi_2 \cdot \pi_5} \tag{12}$$

One notes that $E[TCU(Q, n)]$ is minimized if the second and the third square terms in Eq. (12) equal zeros. That is:

$$(\sqrt{\pi_3}) - (Q\sqrt{\pi_4}) = 0 \tag{13}$$

and:

$$(\sqrt{\pi_5}) - \sqrt{\pi_2}(Q^{-1}n) = 0 \tag{14}$$

or:

$$Q^* = \sqrt{\frac{\pi_3}{\pi_4}} \tag{15}$$

and:

$$n^* = \sqrt{\frac{\pi_5}{\pi_2}} \cdot Q^* \tag{16}$$

or:

$$n^* = \sqrt{\frac{\pi_5}{\pi_2}} \cdot \sqrt{\frac{\pi_3}{\pi_4}} \tag{17}$$

3.1 Results and discussion

Substituting Eqs. (5) to (8) in Eq. (17) and with further derivations, one has:

$$n^* = \sqrt{\frac{\frac{(K_1 + K)\lambda(h_2 - h)}{2} \left[[1 - E(x)] - \frac{2\lambda}{P[1 - E(x)]} + \frac{\lambda^2}{P^2[1 - E(x)]} \right]}{\left\{ K_1\lambda \left[\frac{h}{2} \left\{ \frac{2\lambda^3}{P^3[1 - E(x)]} E\left(\frac{1}{1-x}\right) - \frac{\lambda^2}{P^2[1 - E(x)]} + [1 - E(x)] - \frac{\lambda[1 - 2E(x)]}{P[1 - E(x)]} \right\} + h_2\lambda^2 \left[\frac{1}{P^2} \left[E\left(\frac{1}{1-x}\right) + \frac{1}{[1 - E(x)]} \right] - \frac{\lambda}{P^3[1 - E(x)]} E\left(\frac{1}{1-x}\right) \right] \right\}}} \right.} \tag{18}$$

One notes that Eqs. (15) and (18) are the theoretical minimum points for Q and n based on derivations of the long-run cost function $E[TCU(Q,n)]$. However, n in Eq. (18) is most likely to be a real number; but in real-life the number of deliveries n can only take on integer value. Therefore, the following second phase derivation is required.

Let n^+ denote the smallest integer greater than or equal to n (derived from Eq. (18)) and n^- denote the largest integer less than or equal to n . Because n^* is known as either n^+ or n^- , so one can now retreat $E[TCU(Q,n)]$ as a cost function with single decision variable Q . Similarly, considering n as constant, Eq. (9) can be rearranged as:

$$E[TCU(Q,n)] = \pi_1 + Q^{-1}(n\pi_2 + \pi_3) + Q(\pi_4 + n^{-1}\pi_5) \tag{19}$$

Then:

$$E[TCU(Q,n)] = \pi_1 + Q \left[\left[Q^{-1} \sqrt{n\pi_2 + \pi_3} \right]^2 + \left(\sqrt{\pi_4 + n^{-1}\pi_5} \right)^2 \right] \tag{20}$$

or:

$$E[TCU(Q,n)] = \pi_1 + Q \left\{ \left[Q^{-1} \sqrt{n\pi_2 + \pi_3} \right] - \left(\sqrt{\pi_4 + n^{-1}\pi_5} \right) \right\}^2 + 2\sqrt{n\pi_2 + \pi_3} \sqrt{\pi_4 + n^{-1}\pi_5} \tag{21}$$

Similarly, $E[TCU(Q,n)]$ is minimized if the second square term in Eq. (21) equals zero. That is:

$$Q = \frac{\sqrt{n\pi_2 + \pi_3}}{\sqrt{\pi_4 + n^{-1}\pi_5}} \tag{22}$$

Substituting Eqs. (5) to (8) in Eq. (22) and with further derivations, one has:

$$Q^* = \sqrt{\frac{2\lambda[(n+1)K_1 + K]}{h \left\{ \frac{2\lambda^3}{P^3} \cdot E\left(\frac{1}{1-x}\right) - \frac{\lambda^2}{P^2} + [1-E(x)]^2 - \frac{\lambda[1-2E(x)]}{P} \right\} + \frac{(h_2-h)}{n} \left\{ [1-E(x)]^2 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right\} + h_2\lambda^2 \left\{ \frac{2}{P^2} \cdot E\left(\frac{1}{1-x}\right) \left[[1-E(x)] - \frac{\lambda}{P} \right] + \frac{1}{P^2} \right\}}} \tag{23}$$

One notes that the difference between Eqs. (18) and (23) is that Eq. (18) is the minimal Q based on the real number of n (which is not practical or unusable); while Eq. (23) is our optimal lot size which is based on a realistic number of deliveries n (which is an integer). It is also noted that Eqs. (23) and (18) are identical to what were derived by Cheng et al. in Ref. [1]. It follows that the long-run average cost $E[TCU(Q^*,n^*)]$ is:

$$E[TCU(Q^*,n^*)] = \pi_1 + 2\sqrt{n\pi_2 + \pi_3} \sqrt{\pi_4 + n^{-1}\pi_5} \tag{24}$$

4. RECONFIRMING RESULTLS USING NUMERICAL EXAMPLE

The aforementioned results are verified by using the same numerical example as in Ref. [1]. Consider the following system parameters:

$P = 60000$ items per year,

$\lambda = 3400$ items per year,

$x =$ a random scrap rate is assumed to be uniformly distributed over interval $[0, 0.3]$,

$C = \$100$ per item,

$K = \$20000$ per production run,

$C_S = \$20$ disposal cost for each scrap item,

$h = \$20$ per item per year,

$C_T = \$0.1$ per item delivered,

$K_1 = \$4400$ per shipment, a fixed cost.

$h_2 = \$80$ per item kept at the buyer's end, per unit time.

From Eq. (18) one obtains $n^*=3.84$. To determine the optimal integer value of n , two adjacent integer numbers and its corresponding lot sizes (i.e. Eq. (23)) are plugged in Eqs. (9) respectively.

One has $E[TCU(Q=3073, n=3)]=\$509702$ and $E[TCU(Q=3455, n=4)]=\$509012$. Therefore, the optimal

production lot size $Q^*=3455$, the optimal number of deliveries $n^*=4$, and the long-run average cost $E[TCU(Q^*,n^*)]=\$509012$. It is noted that they are identical to what were obtained by Cheng et al. in Ref. [1].

5. CONCLUDING REMARKS

Cheng et al. [1] used the conventional method (i.e. used the differential calculus along the Hessian matrix equations) to derive the optimal production-shipment policy for a production system in supply chain environment with a specific shipping policy and random scrap rate. This paper reexamines their study by using an alternative algebraic approach. Such a straightforward derivation allows practitioners, who may not have sufficient knowledge of differential calculus, to understand such an integrated production-shipment system with ease.

6. REFERENCES

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**OSVRT NA "UTJECAJE SLUČAJNOG OMJERA OTPADAKA NA SUSTAV
PROIZVODNJE U OKOLIŠU DISTRIBUTIVNE MREŽE
SA SPECIFIČNOM POLITIKOM ISPORUKE"**

SAŽETAK

Ovaj članak primjenjuje alternativni pristup u ponovnom istraživanju utjecaja slučajnog omjera otpadaka na sustav proizvodnje u okolišu distributivne mreže sa specifičnom politikom isporuke. Predložen je jednostavan pristup u obliku dvofazne algebarske derivacije kojim bi se zamijenila konvencionalna metoda u kojoj je trebalo izvršiti prvu i drugu derivaciju funkcije sistemskih troškova radi dokazivanja konveksnosti prije deriviranja optimalne politike proizvodnja-isporuka. Rezultati istraživanja u ovoj studiji su potvrđeni sličnošću s rezultatima koje je dobio Cheng i drugi [1] pri čemu su oni rabili konvencionalnu metodu za rješenje iste zadaće. Predloženi pristup je koristan praktičarima koji nemaju dovoljno znanja o diferencijalnom računu da bi razumijeli jedan takav integrirani sustav proizvodnja-isporuka u okolišu distributivne mreže.

Ključne riječi: optimizacija, sustav proizvodnja-isporuka, slučajni omjer otpadaka, količina robe, višestruka distribucija, algebarski pristup, planiranje proizvodnje.