UDC 519.86:336.645:519.61
Original scientific paper
Received: 27.10.2011.

Effect of variable shipping frequency on production-distribution policy in a vendor-buyer integrated system

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SUMMARY

This paper investigates the effect of variable shipping frequency on production-distribution policy in a vendor-buyer integrated system. In a recent article Chiu et al. [1] derived the optimal replenishment lot size for an economic production quantity problem with multi-delivery and quality assurance, based on an assumption that the number of shipment is a given constant. However, in a vendor-buyer integrated system in supply chain environment, joint determination of replenishment lot size and number of shipments may help such a system to gain significant competitive advantage in terms of becoming a low-cost producer as well as having tight linkage to customer. For this reason, the present study extends the work of Chiu et al. [1] by considering shipping frequency as one of the decision variables and incorporating customer's stock holding cost into system cost analysis. Hessian matrix equations are employed to certify the convexity of cost function that contains two decision variables, and the effect of variable shipping frequency on production-distribution policy is investigated. A numerical example is provided to demonstrate practical usage of the research result.

Key words: *variable shipping policy, production-distribution system, supply chains, vendor-buyer system, lot size, rework.*

1. INTRODUCTION

The finite production rate (FPR) model [2] employs a mathematical technique to describe an important trade-off between fixed production setup cost and inventory holding cost, and to derive the optimal batch size that minimizes the long-run average cost per unit time. The FPR model is also known as the economic production quantity (EPQ) model or the economic manufacturing quantity (EMQ) model [3-4]. It is often used in the manufacturing sector when products are produced in-house (with non-instantaneous inventory replenishment rate) instead of being acquired from outside suppliers (with unloading treated as instantaneous replenishment rate). The classical FPR model assumes that all items produced are of perfect quality. However, in real world production systems, due to process deterioration or other factors, generation of imperfect quality items during a production run is inevitable. Studies [5-13] have been carried out to enhance the classic FPR model by addressing the issue of imperfect quality items produced.

The nonconforming items, sometimes, can be reworked and repaired; hence total production-inventory costs can be reduced [14-24]. Gopalan and Kannan [14] considered manufacturing, inspections, and rework activities as a two-stage transfer-line production system. They analyzed some of the transient state characteristics of such a two-stage production system subject to an initial buffer of infinite capacity, inspection at both the inter- and end-stages and rework. A stochastic model was developed to investigate the system. Explicit analytical expressions for some of the system characteristics have been obtained using the state-space method and regeneration point technique. Grosfeld-Nir and Gerchak [17] examined multistage production system where defective units can be reworked repeatedly at every stage. The yield of each stage is uncertain, so several production runs may need to be attempted until the quantity of finished products is sufficient. The trade-off at each stage is between using small lots, possibly necessitating repeated rework set-ups and large lots, which may result in costly overproduction. They showed that a multistage system where only one of the stages requires a set-up (a "single-bottleneck system") can be reduced to a single-stage system. They also proved that it is best to make the "bottle-neck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems, where all stages require set-ups, optimally.

Another unrealistic assumption of the classic FPR model is "continuous" inventory issuing policy for satisfying product demand. In real life vendor-buyer integrated production-inventory system, at customer's request, multiple or periodic deliveries of finished products are commonly used [25-37]. Goyal [25] first studied the integrated inventory model for a single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. He gave example to illustrate his proposed method. Hill [26] studied a model in which a manufacturing firm purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers. The objective is to determine a purchasing and production schedule which minimizes total cost of purchasing, manufacturing and stockholding. Diponegoro and Sarker [30] determined an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. A lower bound on the optimal solution was also developed for problem with lost sale. It was shown that the solution and the lower bound were consistently tight. Chiu et al. [1] derived the optimal replenishment lot size for an economic production quantity problem with multi-delivery and quality assurance, based on an assumption that the number of shipment is a given constant.

However, in a vendor-buyer integrated system in supply chain environment, joint determination of replenishment lot size and number of shipments may help such a system to gain significant competitive advantage in terms of becoming a low-cost producer as well as having tight linkage to customer. For this reason, the present study extends the work of Chiu et al. [1] by considering shipping frequency as one of the decision variables and incorporating customer's stock holding cost into system cost analysis, and investigates effect of variable shipping frequency on production-distribution policy in supply chain environment.

2. PROBLEM DESCRIPTION AND MATHEMATICAL MODELING

Consider a production-distribution system where the manufacturing process may randomly produce a portion x of defective items at a production rate d. All items produced are screened, and inspection cost per item is included in the unit production cost C. Among these defective items, a θ portion is assumed to be scrap and the other portion can be reworked and repaired at a rate P_1, in each cycle when regular production ends. In order to avoid shortage from occurring, the constant production rate P has to be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P-d-\lambda) > 0$ or $(1-x-\lambda/P) > 0$; where $d = Px$. It is assumed that the finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Fixed quantity n installments of the finished batch are delivered to the customer at a fixed interval of time during production downtime t_3 (see Figure 1).

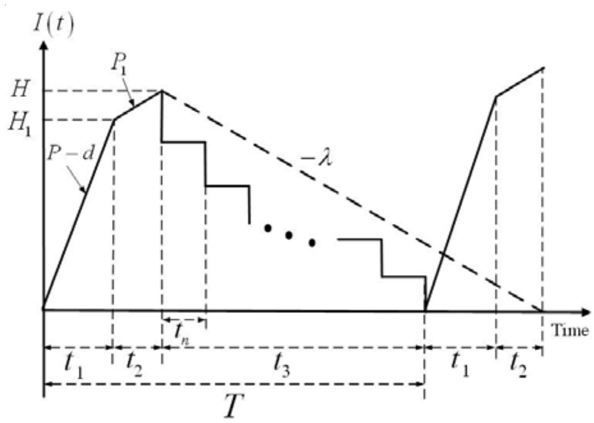

Fig. 1 On-hand inventory of perfect quality items in an integrated finite production rate model with multi-shipment policy, scrap and rework

The cost parameters considered in proposed model include: fixed delivery cost K_1 per shipment, unit delivery cost C_T per item shipped, setup cost K, unit holding cost h, unit production cost C, disposal cost per scrap item C_S, unit rework cost C_R, holding cost h_1 for each reworked item, holding cost h_2 for each item kept by customer. Additional notation used in this paper is listed in Appendix A. From Figure 1, the following parameters and derivations can be obtained directly:

$$t_1 = \frac{Q}{P} \tag{1}$$

$$H_1 = (P-d)t_1 = (P-d)\frac{Q}{P} = (1-x)Q \tag{2}$$

$$H = H_1 + P_1 t_2 = Q(1-\theta x) \tag{3}$$

$$T = t_1 + t_2 + t_3 \tag{4}$$

The maximum level of defective items dt_1, the time needed for rework t_2 and the time needed for delivery t_3 are:

$$dt_1 = Pxt_1 = xQ. \tag{5}$$

$$t_2 = \frac{xQ(1-\theta)}{P_1} \tag{6}$$

$$t_3 = nt_n = T - (t_1 + t_2) = Q\left(\frac{(1-\theta x)}{\lambda} - \frac{1}{P} - \frac{x(1-\theta)}{P_1}\right) \tag{7}$$

A θ portion among nonconforming items is assumed to be scrap and can be obtain as:

$$\theta dt_1 = \theta Pxt_1 = \theta xQ \tag{8}$$

The other repairable portion (1-θ) is reworked right after the production uptime t_1 ends.

The delivery cost per shipment is:

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{9}$$

Total distribution costs for n shipments in a cycle are:

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H = nK_1 + C_T Q(1-\theta x) \tag{10}$$

Total production-inventory-delivery cost per cycle $TC(Q,n)$ consists of variable production cost, setup cost, variable rework cost, disposal cost, fixed and variable delivery cost, holding cost during uptime t_1 and rework time t_2, holding cost for items reworked, and holding cost for finished goods kept by both manufacturer and customer during t_3, when n fixed-quantity installments of the finished batch are delivered to customer at a fixed interval of time. Therefore, $TC(Q,n)$ is (see Appendix B for the computation of customer's holding cost and see Appendix in Ref. [1] for the computation of manufacturer's holding cost; i.e. the last two terms of Eq. (11)):

$$\begin{aligned} TC(Q,n) &= CQ + K + C_R\left[x(1-\theta)Q\right] + C_S\left[x\theta Q\right] + nK_1 + C_T\left[Q(1-\theta x)\right] + h_1 \cdot \frac{P_1 \cdot t_2}{2} \cdot (t_2) + \\ &+ h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2)\right] + h\left(\frac{n-1}{2n}\right)Ht_3 + \frac{h_2}{2}\left[\frac{H}{n}t_3 + T(H - \lambda t_3)\right] \end{aligned} \tag{11}$$

It is noted [1] that Eq. (11) now contains two decision variables and incorporates customer's stock holding cost, and the objective here is to jointly derive the optimal production lot size Q^* and the optimal number of delivery n^*.

The proportion x of defective items is assumed to be a random variable with a known probability density function. In order to take the randomness of defective rate into account, the expected values of x can be used in the inventory cost analysis. Substituting all related parameters from Eqs. (1) to (10) in $TC(Q,n)$, the expected production-inventory-delivery cost per unit time $E[TCU(Q,n)]$ can be obtained:

$$\begin{aligned} E\left[TCU(Q,n)\right] &= \frac{E\left[TC(Q,n)\right]}{E[T]} = \frac{C\lambda}{1-\theta E[x]} + \frac{(K+nK_1)\lambda}{Q(1-\theta E[x])} + \frac{C_R E[x](1-\theta)\lambda}{(1-\theta E[x])} + \frac{C_S E[x]\theta\lambda}{(1-\theta E[x])} + \\ &+ \frac{hQ\lambda}{2P(1-\theta E[x])} + \frac{Q\lambda E[x]}{2P_1(1-\theta E[x])}\left\{h\left[(2-E[x]-\theta E[x])(1-\theta)\right] + h_1 E[x](1-\theta)^2\right\} + \\ &+ C_T\lambda + \left(1-\frac{1}{n}\right)\frac{Q(h_2-h)}{2}\left\{\frac{\lambda}{P} + \frac{E[x]\lambda(1-\theta)}{P_1}\right\} + \frac{Q(1-\theta E[x])}{2}\left(h + \frac{h_2-h}{n}\right) \end{aligned} \tag{12}$$

3. JOINTLY DETERMINING PRODUCTION-DISTRIBUTION POLICY

For the proof of convexity of $E[TCU(Q,n)]$, one can use Hessian matrix equations [38] and obtain the following (see Appendix C for detailed computation):

$$\begin{bmatrix} Q & n \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q \partial n} \\ \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q \partial n} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2K\lambda}{Q(1-\theta E[x])} > 0 \tag{13}$$

Equation (13) is resulting positive, because K, λ, Q, and $(1-\theta E[x])$ are all positive. Hence, the expected integrated costs $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. It follows that for the optimal production lot size Q^* and the optimal number of delivery n^*, one can differentiate $E[TCU(Q,n)]$ with respect to Q and with respect to n, and solve the linear system of Eqs. (14) and (15), by setting these partial derivatives equal to zero:

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q}=-\frac{\lambda(K+nK_1)}{Q^2\left(1-\theta E[x]\right)}+\frac{h\lambda}{2P\left(1-\theta E[x]\right)}+\left(1-\frac{1}{n}\right)\frac{(h_2-h)\lambda}{2}\left[\frac{1}{P}+\frac{E[x](1-\theta)}{P_1}\right]+$$
$$+\frac{\lambda E[x](1-\theta)}{2P_1\left(1-\theta E[x]\right)}\left\{h\left[2-E[x]-\theta E[x]\right]+h_1E[x](1-\theta)\right\}+\frac{\left(1-\theta E[x]\right)}{2}\left[h+\frac{(h_2-h)}{n}\right] \quad (14)$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n}=\frac{K_1\lambda}{Q\left(1-\theta E[x]\right)}-\frac{(h_2-h)}{n^2}\left[\frac{Q\left(1-\theta E[x]\right)}{2}-\frac{Q\lambda}{2P}-\frac{QE[x](1-\theta)\lambda}{2P_1}\right] \quad (15)$$

By setting Eq. (14) equal to zero, one has:

$$\frac{(K+nK_1)\lambda}{Q^2\left(1-\theta E[x]\right)}=\frac{\lambda}{2\left(1-\theta E[x]\right)}\left\{h\left[\frac{1}{P}+\frac{E[x]}{P_1}\left[2-E[x]-\theta E[x]\right](1-\theta)\right]+\frac{h_1\left(E[x]\right)^2(1-\theta)^2}{P_1}\right\}+$$
$$+\left(\frac{n-1}{2n}\right)\left[h\left(1-\theta E[x]\right)+(h_2-h)\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right]+\frac{h_2}{2n}\left(1-\theta E[x]\right) \quad (16)$$

After rearrangement, one obtains the following optimal production lot size Q^*:

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{\begin{array}{l}\frac{h\lambda}{P}+\frac{\lambda E[x](1-\theta)}{P_1}\left[h\left(2-E[x]-\theta E[x]\right)+h_1E[x](1-\theta)\right]+\left[h+\left(\frac{h_2-h}{n}\right)\right]\left(1-\theta E[x]\right)^2\\+\left(\frac{n-1}{n}\right)(h_2-h)\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\left(1-\theta E[x]\right)\end{array}\right\}}} \quad (17)$$

By setting Eq. (15) equal to zero, one has:

$$n^2=\frac{Q^2(h_2-h)\left[\left(1-\theta E[x]\right)^2-\left(1-\theta E[x]\right)\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right]}{2K_1\lambda} \quad (18)$$

Substituting Eq. (17) in Eq. (18), one obtains:

$$n^2=\frac{K(h_2-h)\left[\left(1-\theta E[x]\right)^2-\left(1-\theta E[x]\right)\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right]}{K_1\left\{\begin{array}{l}\frac{h\lambda}{P}+\frac{h\lambda}{P_1}\left[2E[x]-\left(E[x]\right)^2-\theta\left(E[x]\right)^2\right](1-\theta)+h\left(1-\theta E[x]\right)^2+\\+(h_2-h)\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\left(1-\theta E[x]\right)+\frac{h_1\left(E[x]\right)^2\lambda(1-\theta)^2}{P_1}\end{array}\right\}} \quad (19)$$

After rearrangement one obtains the optimal number of delivery n^* as:

$$n^*=\sqrt{\frac{K(h_2-h)\left[\left(1-\theta E[x]\right)-\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right]}{K_1\left\{\frac{h\lambda E[x]}{\left(1-\theta E[x]\right)}\left[\frac{\theta}{P}+\frac{\left(1-E[x]\right)(1-\theta)}{P_1}\right]+h\left(1-\theta E[x]\right)+\frac{h_1\left(E[x]\right)^2\lambda(1-\theta)^2}{P_1\left(1-\theta E[x]\right)}+h_2\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right\}}} \quad (20)$$

3.1 The special case with $x=0$

Analysis of the similar model without considering random defective rate: If all items produced are of perfect quality, then the proposed model becomes the same as classic finite production rate model with multi-delivery policy. When $x=0$, the total cost per cycle is:

$$TC_1(Q,n) = CQ + K + C_T Q + nK_1 + h\frac{H}{2}(t_1) + h\left(\frac{n-1}{2n}\right)Ht_2 + \frac{h_2}{2}\left[\frac{H}{n}t_2 + T(H-\lambda t_2)\right] \tag{21}$$

The expected production-inventory-delivery cost per unit time for this special model can be derived as follows:

$$E\left[TCU_1(Q,n)\right] = C\lambda + \frac{(K+nK_1)\lambda}{Q} + C_T\lambda + \frac{hQ\lambda}{2P} + \left(\frac{n-1}{n}\right)\left(\frac{hQ}{2} - \frac{hQ\lambda}{2P}\right) + \left(\frac{1}{n}\right)\frac{h_2 Q}{2} + \left(1-\frac{1}{n}\right)\frac{h_2 Q\lambda}{2P} \tag{22}$$

Convexity of $E[TCU_1(Q,n)]$ can also be proved and the optimal solutions to this special model can be obtained as follows:

$$Q^* = \sqrt{\frac{2(K+nK_1)\lambda}{\left\{\frac{h\lambda}{P} + \left(\frac{n-1}{n}\right)\left[h + (h_2-h)\left(\frac{\lambda}{P}\right)\right] + \left(\frac{1}{n}\right)h_2\right\}}} \tag{23}$$

$$n^* = \sqrt{\frac{K(h_2-h)\left[1-(\lambda/P)\right]}{K_1\left[h + h_2(\lambda/P)\right]}} \tag{24}$$

4. NUMERICAL EXAMPLE WITH FURTHER DISCUSSION

Suppose that a product has a flat demand rate of *3,400* units per year. This item can be produced at a rate of *60,000* units per year. During the manufacturing process, a random nonconforming rate x is assumed, which follows a uniform distribution over the interval [*0*, *0.3*]. Among the defective items, a portion $\theta=0.1$ is considered to be scrap and the other portion can be reworked and repaired, at *a* rate $P_1=2{,}100$ units per year. Additional values of parameters include:

- h = *\$20* per item per year,
- h_1 = *\$40* per item reworked per unit time (year),
- h_2 = *\$80* per item kept at the customer's end per unit time,
- K = *\$20,000* per production run,
- C = *\$100* per item,
- C_T = *\$0.1* per item delivered,
- K_1 = *\$4,350* per shipment, a fixed cost,
- C_R = *\$60* repaired cost for each item reworked,
- C_S = *\$20* disposal cost for each scrap item.

From Eqs. (20), (17) and (12), one obtains: the optimal number of delivery $n^*=2$, the optimal production lot size $Q^*=1{,}707$, and the long-run average cost $E[TCU(Q^*,n^*)]$= *\$490,585*. The effects of variation of production lot size on the expected costs $E[TCU(Q,n^*=2)]$ and on the components of $E[TCU(Q,n^*=2)]$ are illustrated in Figure 2.

The effect of variation of shipping frequency n on the $E[TCU(Q^*,n^*)]$ and on various components of $E[TCU(Q^*,n^*)]$ are analyzed and depicted in Figure 3. It may be seen that as shipping frequency n increases, total delivery cost goes up significantly.

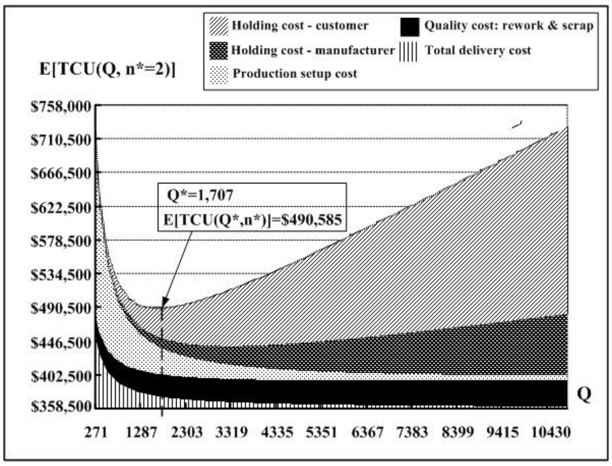

Fig. 2 Variation of lot size effects on the cost function E[TCU(Q,n=2)] and on the various components of E[TCU(Q,n*=2)]*

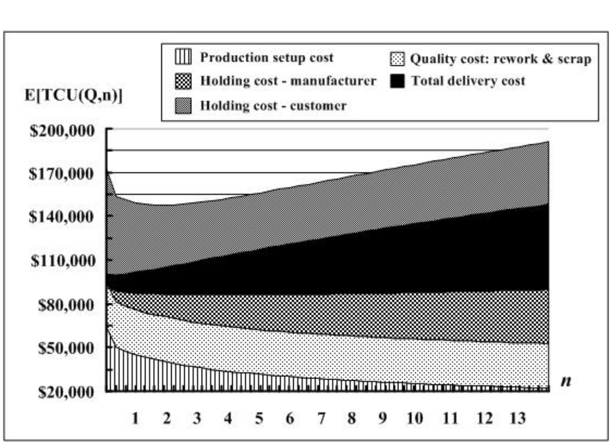

Fig. 3 Variation of shipping frequency n on the E[TCU(Q,n*)] and on the various components of E[TCU(Q*,n*)]*

Similarly, for the special case (i.e. situation when all items produced are of perfect quality), the optimal number of delivery $n^*=3$ (is rounded off from $3{,}257$), the optimal lot size $Q^*=2{,}276$, and the long-run average cost $E[TCU1(Q^*,n^*)]=\$439{,}101$ can also be obtained by using Eqs. (24), (23) and (22).

5. CONCLUSIONS

The present paper extends the work of Chiu et al. [1] by considering shipping frequency as one of the decision variables and incorporating customer's stock holding cost into system cost analysis. Hessian matrix equations are employed to certify the convexity of cost function that contains two decision variables, and the effect of variable shipping frequency on production-distribution policy is investigated. As a result, the closed-form solutions in terms of the optimal replenishment lot size and the optimal number of shipments are derived. It may be noted that without an in-depth investigation and robust analysis of such a production-distribution system, the optimal inventory replenishment and delivery policy cannot be obtained. Neither can the insight regarding the effects of variable shipping frequency and other system parameters be clearly gained (Figures 2 and 3). For future research, one can study effect of backlogging on the operating policy of the system.

6. ACKNOWLEDGEMENTS

The authors greatly appreciate the support of the National Science Council (NSC) of Taiwan under grant number: NSC 99-2221-E-324-017.

APPENDIX A

Notation:

H_1 – maximum level of on-hand inventory in units when regular production process ends,

H – the maximum level of on-hand inventory in units when rework process finishes,

t_1 – the production uptime for the proposed finite production rate model,

t_2 – time required for reworking of defective items,

t_3 – time required for delivering all quality assured finished products,

T – cycle length,

Q – production lot size, to be determined for each cycle,

D – number of finished items (fixed quantity) to be distributed to customer per delivery,

n – number of fixed quantity installments of the finished batch to be delivered to customer, to be determined for each cycle,

t_n – a fixed interval of time (between each installment of finished products to be delivered to customer during production downtime t_3),

$I(t)$ – on-hand inventory of perfect quality items at time t,

$I_d(t)$ – on-hand inventory of defective items at time t,

$I_c(t)$ – on-hand inventory at the customer's end at time t,

$TC(Q,n)$ – total production-inventory-delivery costs per cycle for the proposed model,

$TC_1(Q,n)$ – total production-inventory-delivery per cycle when no defective items produced (i.e. special case: the classic finite production rate model with multi-delivery policy),

$E[TCU(Q,n)]$ – the long-run average costs per unit time for the proposed model,

$E[TCU_1(Q,n)]$ – the long-run average costs per unit time for model in the special case.

APPENDIX B

Computations of the customer's holding cost during t_3 are as follows.

Because n installments (fixed quantity D) of the finished lot are delivered to customer at a fixed interval of time t_n, one has the following:

$$D = \frac{H}{n} \tag{B-1}$$

$$t_n = \frac{t_3}{n} \tag{B-2}$$

At the customer's end, the demand between shipments is (λt_n), and let I denote number of items that will be left over after satisfying the demand during each fixed interval of time t_n (see Figure 4), then:

$$I = D - \lambda t_n \tag{B-3}$$

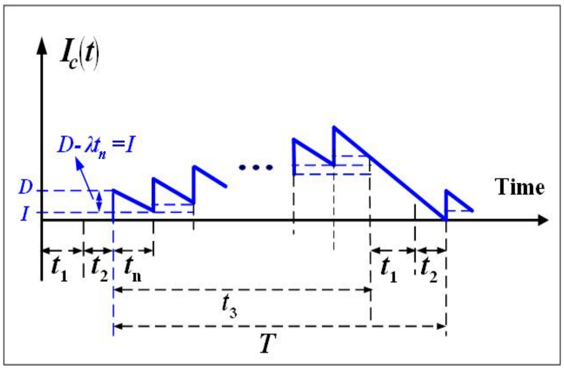

Fig. 4 On-hand inventory at the customer's end when n installments of the finished batch are delivered

From Figure 4 one can calculate the average inventory as follows:

$$Average\ inventory = \left[\left(\frac{D+I}{2}\right)t_n\right] + \left[\frac{(D+I)+\left[(D+I)-\lambda t_n\right]}{2}t_n\right] + \left[\frac{(D+2I)+\left[(D+2I)-\lambda t_n\right]}{2}t_n\right] +$$

$$+ \ \ldots \ + \left[\frac{\left[D+(n-1)I\right]+\left[\left[D+(n-1)I\right]-\lambda t_n\right]}{2}t_n\right] + \left(\frac{nI}{2}\right)(t_1+t_2) \tag{B-4}$$

Substituting Eq. (B-3) in Eq. (B-4), the average inventory becomes:

$$Average\ inventory = \left(D-\frac{\lambda}{2}t_n\right)t_n + \left(D+I-\frac{\lambda}{2}t_n\right)t_n + \left(D+2I-\frac{\lambda}{2}t_n\right)t_n + \ldots + \left(D+(n-1)I-\frac{\lambda}{2}t_n\right)t_n +$$

$$+\left(\frac{nI}{2}\right)(t_1+t_2) =$$

$$= n\left(D-\frac{\lambda}{2}t_n\right)t_n + \frac{n(n-1)}{2}It_n + \frac{nI}{2}(t_1+t_2) \tag{B-5}$$

Substituting Eq. (B-1) through (B-3) in Eq. (B-5), the following general term for average inventory at the customer's end can be obtained:

$$Average\ inventory = n\left(\frac{H}{n}-\frac{\lambda}{2}t_n\right)t_n + \frac{n(n-1)}{2}\left(\frac{H}{n}-\lambda t_n\right)t_n + \frac{n}{2}\left(\frac{H}{n}-\lambda t_n\right)(t_1+t_2) =$$

$$= Ht_n - \frac{n\lambda}{2}t_n^{\,2} + Ht_n\frac{(n-1)}{2} - \frac{n(n-1)}{2}\lambda t_n^{\,2} + \frac{H}{2}(t_1+t_2) - \frac{n}{2}(\lambda t_n)(t_1+t_2) =$$

$$= \frac{Ht_3}{n} - \frac{\lambda t_3^{\,2}}{2n} + \frac{(n-1)Ht_3}{2n} - \frac{(n-1)\lambda t_3^{\,2}}{2n} + \frac{H}{2}(t_1+t_2) - \frac{\lambda t_3}{2}(t_1+t_2) =$$

$$= \frac{1}{2}\left[\frac{Ht_3}{n} + T(H-\lambda t_3)\right] \tag{B-6}$$

Hence, the holding cost for items kept at the customer's end is (i.e. the last term in Eq. (11)):

$$\frac{h_2}{2}\left[\frac{Ht_3}{n} + T(H-\lambda t_3)\right] \tag{B-7}$$

APPENDIX C

Computational procedures for Eq. (13) are as follows:

From Eq. (12), the following partial derivative can be obtained:

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q} = -\frac{\lambda(K+nK_1)}{Q^2\left(1-\theta E[x]\right)} + \frac{h\lambda}{2P\left(1-\theta E[x]\right)} + \left(1-\frac{1}{n}\right)\frac{(h_2-h)\lambda}{2}\left[\frac{1}{P}+\frac{E[x](1-\theta)}{P_1}\right] +$$

$$+\frac{\lambda E[x](1-\theta)}{2P_1\left(1-\theta E[x]\right)}\left\{h\left[2-E[x]-\theta E[x]\right]+h_1E[x](1-\theta)\right\} + \frac{\left(1-\theta E[x]\right)}{2}\left[h+\frac{(h_2-h)}{n}\right] \tag{C-1}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} = \frac{2(K+nK_1)\lambda}{Q^3\left(1-\theta E[x]\right)} \tag{C-2}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n} = \frac{K_1\lambda}{Q\left(1-\theta E[x]\right)} - \frac{(h_2-h)}{n^2}\left[\frac{Q\left(1-\theta E[x]\right)}{2} - \frac{Q\lambda}{2P} - \frac{QE[x](1-\theta)\lambda}{2P_1}\right] \tag{C-3}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2} = \frac{1}{n^3}(h_2-h)\left[Q\left(1-\theta E[x]\right) - \frac{Q\lambda}{P} - \frac{QE[x]\lambda(1-\theta)}{P_1}\right] \tag{C-4}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n}=-\frac{K_1\lambda}{Q^2\left(1-\theta E\left[x\right]\right)}-\frac{1}{n^2}\left(h_2-h\right)\left[\frac{\left(1-\theta E\left[x\right]\right)}{2}-\frac{\lambda}{2P}-\frac{E\left[x\right]\left(1-\theta\right)\lambda}{2P_1}\right] \tag{C-5}$$

Substituting Eqs. (C-1) through (C-5) in Hessian matrix, Eq. (13), one obtains:

$$\begin{bmatrix}Q & n\end{bmatrix}\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=$$

$$=\left\{\begin{matrix}\frac{2\left(K+nK_1\right)\lambda}{Q\left(1-\theta E\left[x\right]\right)}+\frac{\left(h_2-h\right)}{n}\left[Q\left(1-\theta E\left[x\right]\right)-\frac{Q\lambda}{P}-\frac{QE\left[x\right]\lambda\left(1-\theta\right)}{P_1}\right]\\ -\frac{2nK_1\lambda}{Q\left(1-\theta E\left[x\right]\right)}-\frac{\left(h_2-h\right)}{n}\left[Q\left(1-\theta E\left[x\right]\right)-\frac{Q\lambda}{P}-\frac{QE\left[x\right]\lambda\left(1-\theta\right)}{P_1}\right]\end{matrix}\right\}=\frac{2K\lambda}{Q\left(1-\theta E\left[x\right]\right)}>0 \tag{13}$$

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EFEKT PROMJENJIVE OTPREMNE UČESTALOSTI NA POLITIKU PROIZVODNJE I DISTRIBUCIJE U INTEGRIRANOM SUSTAVU PRODAVAČ-KUPAC

SAŽETAK

Ovaj članak istražuje efekt promjenjive otpremne učestalosti na politiku proizvodnje i distribucije u integriranom sustavu prodavač-kupac. U nedavno objelodanjenom članku Chiu et al. [1] su izveli optimalno nadopunjavanje količine robe za problem ekonomične proizvodnje količine s višestrukom dostavom i osiguranjem kvalitete utemeljeno na pretpostavci da je broj otprema zadana konstanta. Međutim, u integriranom sustavu prodavač-kupac unutar okruženja distributivne mreže, zajednička odlučnost o nadopunjavanju količine robe i broj otprema mogu pomoći takvom sustavu da dobije značajne kompetitativne prednosti u smislu da postane jeftiniji proizvođač kao i da uspostavi uske veze s kupcem. Iz tog razloga, ovaj članak proširuje istraživanja koja su proveli Chiu et al. [1] uzimajući u obzir otpremnu učestalost kao jednu od odlučujućih varijabli kao i uključujući troškove držanja zaliha korisnika usluge u sistemsku analizu troškova. Za potvrdu konveksnosti funkcije troška koja sadrži dvije odlučujuće varijable korištene su Hessove matrice, a ujedno je istražen i efekt promjenjive otpremne učestalosti na politiku proizvodnje i distribucije. Jednim numeričkim primjerom pokazana je praktična primjena rezultata istraživanja.

***Ključne riječi**:* *promjenjiva otpremna politika, sustav proizvodnja-distribucija, distributivne mreže, sustav prodavač-kupac, količina robe, ponovni rad.*