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A single-producer multi-retailer integrated inventory model with a rework process

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SUMMARY

This study considers a single-producer multi-retailer integrated inventory model with the reworking of random defective items produced. The objective is to find the optimal production lot size and optimal number of shipments that minimizes total expected costs for such a specific supply chains system. It is assumed that a product is manufactured by a producer. All items are screened for quality purpose and random nonconforming items will be picked up and reworked at the end of regular production in each cycle. After the entire lot is quality assured, multiple shipments will be delivered synchronously to m different retailers in each production cycle. Each retailer has its own annual product demand, unit stock holding cost, and fixed and variable delivery costs. Mathematical modeling and analysis is used to deal with the proposed model and to derive the expected system cost. Hessian matrix equations are employed to prove the convexity of the cost function. As a result, a closed-form optimal replenishment-delivery policy for such a specific single-producer multi-retailer integrated inventory model is obtained. A numerical example is provided to show the practical usage of the proposed model.

Key words: *supply chains, optimization, production, lot size, multiple retailers, multiple shipments, rework.*

1. INTRODUCTION

In real life supply chains environments, it is common to have a manufacturer who supplies a product to several retailers. In such internal types of supply chains, management would like to figure out the best production-shipment policy in order to minimize the total expected system costs. Schwarz [1] first studied a one-warehouse *N*-retailer deterministic inventory system with the objective of deriving the stocking policy that minimizes the long-run average system cost per unit time. The optimal solutions along with a few necessary properties are derived for such a one-retailer and *N* identical retailer problems. Heuristic solutions for the general problem were also suggested. Production lot size was not considered in his model. Goyal [2] considered an integrated inventory model for a single supplier-single customer problem. A method was proposed for solving those inventory problems, wherein a product made by a single supplier is procured by a single customer. A numerical example was provided to verify his solution process. Banerjee [3] investigated a joint economic lot-size model for purchaser and vendor, with the focus on minimizing the joint total relevant costs. He concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either party. Kim and Hwang [4] developed a formulation of a quantity discount pricing schedule for a supplier. They assumed a single incremental discount system and proposed an algorithm for deriving an optimal discount schedule. They investigated cases in which both the discount rate and the break point are unknown and either one is prescribed, using a numerical example to illustrate their algorithm. Parija and Sarker [5] determined an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to multiple customers, with a fixed-quantity at a fixed time-interval to each of the customers. In their model, an optimal multi-ordering policy for procurement of raw materials for a single manufacturing system is developed to minimize the total cost incurred due to raw materials and finished goods inventories. A closed-form solution to the problem was obtained for the minimal total cost and the algorithm was demonstrated for multiple customer systems. Cetinkaya and Lee [6] presented an analytical model for coordinating inventory and transportation decisions in vendor-managed inventory (VMI) systems. They considered a vendor realizing a sequence of random demands from a group of retailers located in a given geographical region. They assumed that the vendor has the autonomy of holding small orders until an agreeable dispatch time with the expectation that an economical consolidated dispatch quantity accumulates. As a result, the actual inventory requirements at the vendor are partly dictated by the parameters of the shipment-release policy in use. The optimum replenishment quantity and dispatch frequency were simultaneously derived, and a renewal theoretic model for the case of Poisson demands was developed together with some analytical results for their model. Other studies in the related fields have also been extensively carried out to address various aspects of vendor-buyer supply chain issues [7-14].

Another special focus of the present study is the manufacturer's product quality. The classic economic production quantity (EPQ) model assumes a perfect production [15-17]. However, in a real life manufacturing environment, different unpredictable factors likely result in the production of a random number of defective items. In this study all nonconforming items are reworked and repaired in order to assure the entire finished lot has the expected quality. Many studies have been conducted during past decades to address different aspects of imperfect production systems with quality assurance issues [18-26].

The purpose of this study is to simultaneously determine the optimal production lot size and optimal number of shipments that minimizes the total expected system costs for such a single-producer multi-retailer integrated inventory system with a rework process. As little attention has been paid to this area, this paper is intended to bridge the gap.

2. DESCRIPTION AND MODELLING

This study examines a single-producer multi-retailer integrated inventory system with a rework process. We assume that a product can be made at an annual production rate P by the producer, and the production process may randomly generate an x portion of nonconforming items at a production rate d. All items produced are screened and the inspection expense is included in the unit production cost C. All defective items are assumed to be re-workable at a rate of P_1, and a rework process starts right after the end of regular production, in each cycle. Under the normal operation, to prevent shortages from occurring, the constant production rate P must satisfies $(P\text{-}d\text{-}\lambda)>0$ or $(1\text{-}x\text{-}\lambda/P)>0$, where λ is the sum of annual demands of retailers and d can be expressed as $d=Px$. Unlike the classic EPQ model that assumes a continuous inventory issuing policy for satisfying demand, this study considers a multi-shipment policy; finished items can only be delivered to the retailers when the entire lot is quality assured at the end of the rework process. Each retailer has its own annual demand rate λ_i. Fixed quantity n installments of the finished batch are delivered to multiple retailers synchronously at a fixed interval of time during the downtime t_3 (refer to Figures 1 and 2). Cost parameters used in this study are as follows: the production setup cost K, unit holding cost h, unit production cost C, unit cost C_R and unit holding cost h_1 for each reworked item, unit disposal cost C_S, the fixed delivery cost K_{1i} per shipment delivered to retailer i, unit holding cost h_{2i} for item kept by retailer i, and unit shipping cost C_i for item shipped to retailer i. Additional notations are listed below:

- H_1 – level of on-hand inventory in units when regular production process ends,
- H – maximum level of on-hand inventory in units when the rework process ends,
- t_1 – the production uptime for the proposed system,
- t_2 – time required for reworking the nonconforming items produced in each cycle,
- t_3 – time required for delivering all quality assured finished products to retailers,
- Q – production lot size per cycle, a decision variable (to be determined),
- n – number of fixed quantity installments of the finished batch to be delivered to retailers for each cycle, a decision variable (to be determined),
- m – number of retailers,
- tn – a fixed interval of time between each installment of finished products delivered during production downtime t_2,
- T – production cycle length,
- $I(t)$ – on-hand inventory of perfect quality items at time t,
- $I_c(t)$ – on-hand inventory at the retailers at time t,
- $TC(Q,n)$ – total production-inventory-delivery costs per cycle for the proposed system,
- $E[TCU(Q,n)]$ – total expected production-

inventory-delivery costs per unit time for the proposed system.

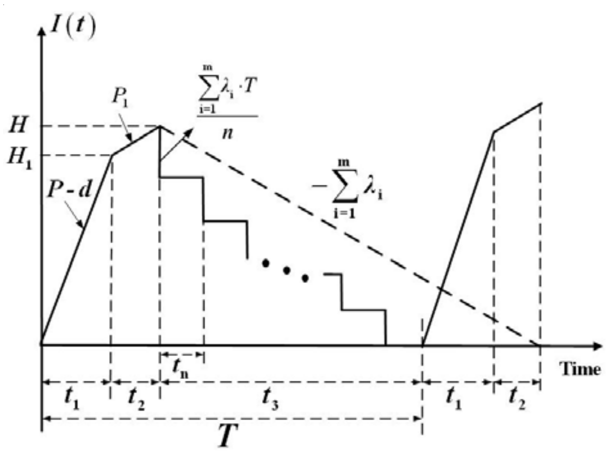

Fig. 1 On-hand inventory of perfect quality items in producer side

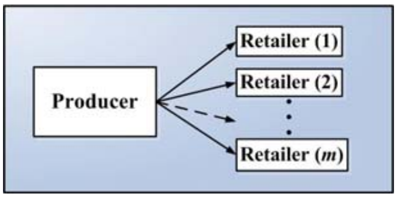

Fig. 2 A single-producer multiple-retailer integrated system

The following equations can be directly obtained from Figure 1:

$$t_1 = \frac{Q}{P} = \frac{H_1}{P-d} \tag{1}$$

$$t_2 = \frac{xQ}{P_1} \tag{2}$$

$$t_3 = nt_n = T - (t_1 + t_2) = Q\left(\frac{1}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right) \tag{3}$$

$$T = t_1 + t_2 + t_3 = \frac{Q}{\lambda} \tag{4}$$

$$H_1 = (P-d)t_1 = (P-d)\frac{Q}{P} = (1-x)Q \tag{5}$$

$$H = H_1 + P_1 t_2 = Q \tag{6}$$

$$\lambda = \sum_{i=1}^{m} \lambda_i \tag{7}$$

The on-hand inventory of scrap items during production uptime t_1 is:

$$dt_1 = Pxt_1 = xQ \tag{8}$$

Cost for each delivery to m retailers is:

$$\sum_{i=1}^{m} K_{1i} + \frac{1}{n}\left(\sum_{i=1}^{m} C_i \lambda_i T\right) \tag{9}$$

Total delivery costs of n shipments to m retailers in a production cycle are:

$$n\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_i \lambda_i T \tag{10}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_2 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows [27]:

$$h\left(\frac{n-1}{2n}\right)Ht_3 \tag{11}$$

Total stock holding costs for products kept by the retailers during the cycle are (see Figure 3 and Appendix for details):

$$\frac{1}{2}\sum_{i=1}^{m} h_{2i}\lambda_i \left[\frac{Tt_3}{n} + (t_1 + t_2)T\right] \tag{12}$$

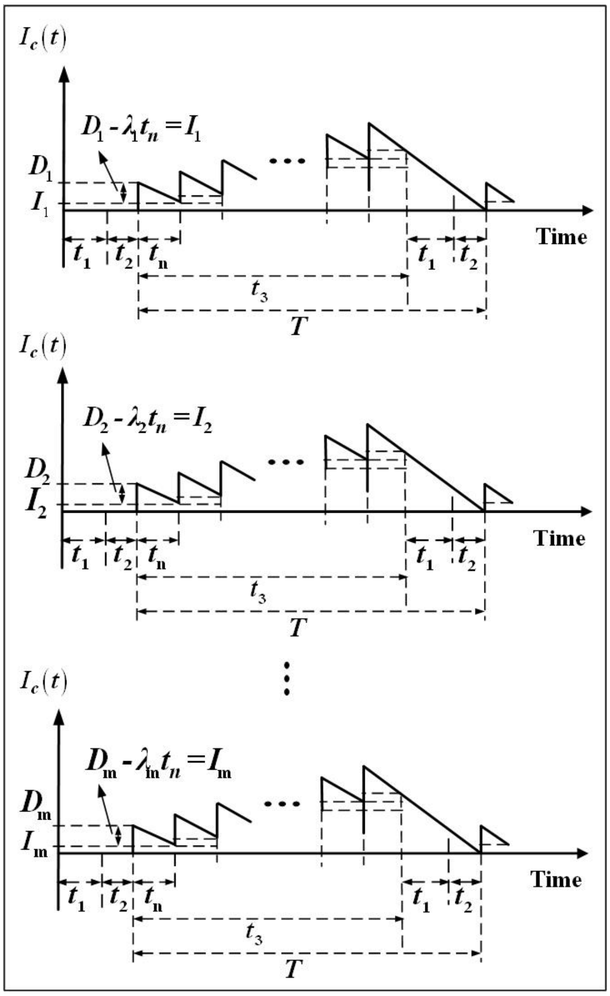

Fig. 3 On-hand inventory in m retailer sides

Total production-inventory-delivery cost per cycle $TC(Q,n)$ consists of the setup cost, variable production cost, the cost for the reworking, disposal cost, the fixed and variable delivery cost, holding cost during production uptime t_1 and reworking time t_2, and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time t_3. $TC(Q,n)$ is:

$$TC(Q,n) = CQ + K + C_R[xQ] + n\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_i\lambda_i T + h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2) + \left(\frac{n-1}{2n}\right)Ht_3\right] + $$
$$+ h_1\frac{P_1 \cdot t_2}{2}(t_2) + \frac{1}{2}\sum_{i=1}^{m} h_{2i}\lambda_i\left[\frac{Tt_3}{n} + (t_1 + t_2)T\right] \tag{13}$$

In order to take the randomness of defective rate into account, the expected values of x are used in cost analysis of this study. Substituting all parameters from Eqs. (1) to (12) in $TC(Q,n)$, and with further derivations, the expected cost $E[TCU(Q,n)]$ can be obtained as follows:

$$E\left[TCU(Q,n)\right] = C\sum_{i=1}^{m}\lambda_i + \frac{1}{Q}\left(K + n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i + C_R E[x]\sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}C_i\lambda_i + $$
$$+ \frac{h}{2}\left(Q\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{P} + \frac{1}{P_1}\left(2E[x] - (E[x])^2\right)\right] + \left(\frac{n-1}{2n}\right)\left(hQ\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i} - \frac{1}{P} - \frac{E[x]}{P_1}\right] + $$
$$+ \frac{h_1}{2}\left(\frac{1}{P_1}\right)(E[x])^2 \cdot Q\sum_{i=1}^{m}\lambda_i + \left(\frac{n-1}{2n}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right)\left[\frac{1}{P} + \frac{E[x]}{P_1}\right] + \left(\frac{1}{2n}\right)\frac{1}{\sum_{i=1}^{m}\lambda_i}\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right) \tag{14}$$

3. DERIVING THE OPTIMAL POLICY

3.1 Proof of convexity

The Hessian matrix equations [28] are employed here to prove the convexity of $E[TCU(Q,n)]$; namely, to verify whether the following condition (Eq. (14)) holds or not:

$$[Q \quad n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q,n)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} > 0 \tag{15}$$

The following is obtained from Eq. (14):

$$\frac{\partial E[TCU(Q,n)]}{\partial Q} = -\frac{\left(K + n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{Q^2} + \frac{h}{2}\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P} + \frac{1}{P_1}\left[\left(2E[x] - (E[x])^2\right)\right]\right] + $$
$$+ \left(\frac{n-1}{n}\right)\left(\frac{h}{2}\right)\left[1 - \sum_{i=1}^{m}\lambda_i\left(\frac{1}{P} + \frac{E[x]}{P_1}\right)\right] + \frac{h_1}{2}\left(\frac{1}{P_1}\right)(E[x])^2 \cdot \sum_{i=1}^{m}\lambda_i + $$
$$+ \left(\frac{1}{2n}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i\right)\left[(n-1)\left[\frac{1}{P} + \frac{E[x]}{P_1}\right] + \left(\sum_{i=1}^{m}\lambda_i\right)^{-1}\right] \tag{16}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2}=\frac{2\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{Q^3} \tag{17}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n}=\frac{\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q}-\left(\frac{Q}{2n^2}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E\left[x\right]}{P_1}\right] \tag{18}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2}=\left(\frac{Q}{n^3}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E\left[x\right]}{P_1}\right] \tag{19}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n}=-\frac{\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q^2}-\left(\frac{1}{2n^2}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E\left[x\right]}{P_1}\right] \tag{20}$$

If Eqs. (17), (19) and (20) are substituted into Eq. (15), then:

$$\left[Q\quad n\right]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2K\sum_{i=1}^{m}\lambda_i}{Q}>0 \tag{21}$$

Equation (21) is resulting positive, because K, λ, and Q are all positive. Hence, $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. Therefore, the convexity of $E[TCU(Q,n)]$ is proved, and there exists a minimum of $E[TCU(Q,n)]$.

3.2 Deriving the optimal policy

To simultaneously determine the production-delivery policy for the proposed single-producer multi-retailer integrated inventory model with a rework process, one can solve the linear system of Eqs. (16) and (18) by setting these partial derivatives equal to zero. With further derivations one obtains:

$$Q^*=\sqrt{\frac{2\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{\left\{\begin{array}{l}h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[\left(2E\left[x\right]-\left(E\left[x\right]\right)^2\right)\right]\right]+h_1\left(E\left[x\right]\right)^2\sum_{i=1}^{m}\lambda_i\frac{1}{P_1}+\\ +\left(\frac{n-1}{n}\right)\left[h+\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P}+\frac{E\left[x\right]}{P_1}\right)\right]+\sum_{i=1}^{m}h_{2i}\lambda_i\left(n\sum_{i=1}^{m}\lambda_i\right)^{-1}\end{array}\right\}}} \tag{22}$$

and:

$$n^{*}=\sqrt{\frac{K\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left[\left(\sum_{i=1}^{m}\lambda_i\right)^{-1}-\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right]}{\sum_{i=1}^{m}K_{1i}\left\{\begin{array}{l}h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[2E[x]-\left(E[x]\right)^2\right]\right]+h+\\+\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)+h_1\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i\left(\frac{1}{P_1}\right)\end{array}\right\}}} \tag{23}$$

It should be noted that in a real-world situation the number of deliveries takes on integer values only; however, Eq. (23) results in a real number. In order to determine the integer value of n^* that minimizes the expected system cost, two adjacent integers to n must be examined respectively [29]. Let n^+ denote the smallest integer greater than or equal to n (derived from Eq.(23)) and let n^- denote the largest integer less than or equal to n. Substitute n^+ and n^- respectively in Eq. (22), then apply the resulting (Q, n^+) and (Q, n^-) in Eq. (14) respectively. Then, the one that gives the minimum long-run average cost is selected as the optimal replenishment-distribution policy (Q^*, n^*). An example is provided in the next section to show the practical use of the obtained results.

4. NUMERICAL EXAMPLE

Consider that a product can be made by a producer at a production rate (P) of *60,000* units per year and the annual demands λ_i of this product from *5* different retailers are *650*, *350*, *450*, *800*, and *750* units respectively (total demand is *3000* units per year). There is a random defective rate during the production uptime which follows a uniform distribution over the interval [*0, 0.3*]. All nonconforming items are repairable during the rework process at a rate (P_1) of *3600* units per year. Values of additional parameters are:

- K = *$35000* per production run,
- C = *$100* per item,
- h = unit holding cost per item at the producer side, *$25* per item per year,
- h_1 = unit holding cost per item reworked, *$60* per item per year,
- C_R = *$60*, cost for each items reworked,
- K_{1i} = the fixed delivery cost per shipment for retailer i, they are *$400*, *$100*, *$300*, *$450*, and *$250* respectively,
- h_{2i} = unit holding cost for item kept by retailer i, they are *$70*, *$80*, *$75*, *$60*, and *$65* per item respectively,
- C_i = unit transportation cost for item delivered to retailer i, they are *$0.5*, *$0.4*, *$0.3*, *$0.2*, and *$0.1* respectively.

First determine the optimal integer number of delivery for the proposed model by computing Eq. (23), one has $n=4.51$. Then, examine the aforementioned two adjacent integers to n and apply Eq. (22) to obtain (Q, n^+) = (*2310,5*) and (Q, n^-) = (*2228,4*). Finally, substitute these (Q, n^+) and (Q, n^-) in Eq. (14) respectively. Choosing the one that gives the minimum system cost, one obtains the optimal number of delivery $n^* = 5$, the optimal replenishment $Q^*=2310$, and total expected cost $E[TCU(Q^*,n^*)]$ = *$438,211*.

Variation of Q and n effects on the optimal $E[TCU(Q^*,n^*)]$ are illustrated in Figure 4. Variation of random defective rate effects on the optimal (Q^*,n^*) policy and on the system cost $E[TCU(Q^*,n^*)]$ are depicted in Figure 5. It is noted that as the random defective rate x increases, the optimal production lot size Q^* decreases, while the expected system cost $E[TCU(Q^*,n^*)]$ increases significantly. It should also be noted that the optimal number of delivery n^* decreases (and takes on an integer number only) as Q^* decreases.

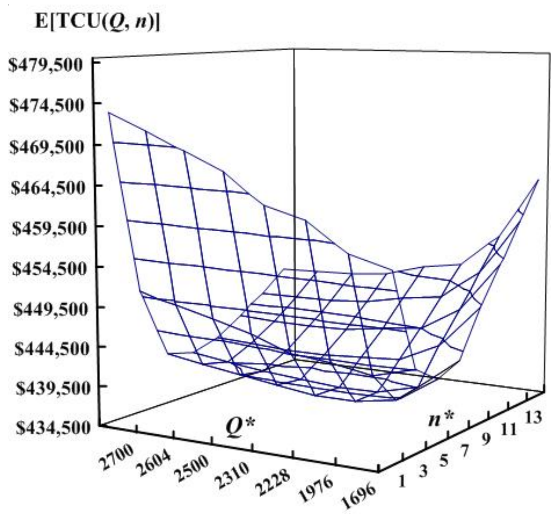

Fig. 4. Variation of Q and n effects on the optimal E[TCU(Q,n*)]*

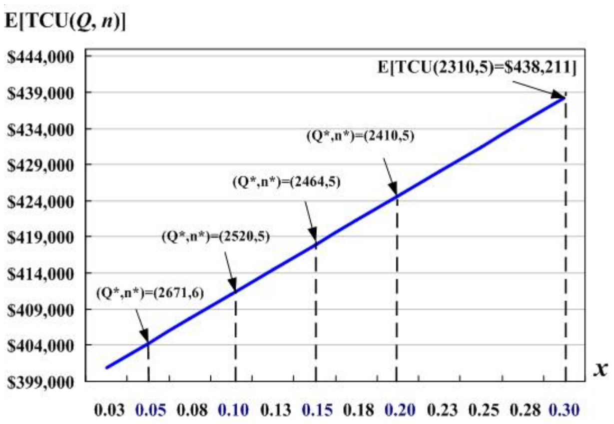

Fig. 5. Variation of random defective rate effects on the optimal (Q,n*) policy and on the system cost E[TCU(Q*,n*)]*

5. CONCLUDING REMARKS

This study considers a single-producer multi-retailer integrated inventory model with a rework process. In real life supply chains environments, it is common to have a vendor who supplies a product to multiple retailers. Generation of nonconforming items seems to be inevitable during the production process. Management of such an intra supply-chain system would certainly like to figure out the best replenishment-distribution policy in order to minimize the long-run average system cost. A solution procedure that uses mathematical modeling and analysis to deal with the aforementioned supply chain system is proposed. A closed-form solution of the optimal replenishment-distribution policy is obtained. Effects of various system parameters on the optimal solution are investigated (Figures 4 and 5) in order to provide the management with some insights on this specific single-producer multi-retailer integrated inventory model.

6. ACKNOWLEDGEMENTS

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7. APPENDIX

From Figure 1, because n installments (fixed quantity D) of the finished lot are delivered to customer at a fixed interval of time t_n, one has:

$$D_i = \frac{H}{n} \tag{A-1}$$

$$t_n = \frac{t_3}{n} \tag{A-2}$$

From Figure 2, computations of the retailers' holding cost (Eq. (12)) are as follows:

$$h_{21}\left[\frac{n(D_1-I_1)t_n}{2}+\frac{n(n+1)}{2}I_1\cdot t_n+\frac{nI_1(t_1+t_2)}{2}\right]+$$
$$+h_{22}\left[\frac{n(D_2-I_2)t_n}{2}+\frac{n(n+1)}{2}I_2\cdot t_n+\frac{nI_2(t_1+t_2)}{2}\right]+$$
$$+\ \ ....\ \ +$$
$$+h_{2m}\left[\frac{n(D_m-I_m)t_n}{2}+\frac{n(n+1)}{2}I_m\cdot t_n+\frac{nI_m(t_1+t_2)}{2}\right]=$$
$$=\sum_{i=1}^{m}h_{2i}\left[\frac{n(D_i-I_i)t_n}{2}+\frac{n(n+1)}{2}I_i\cdot t_n+\frac{nI_i(t_1)+t_2}{2}\right]=$$
$$=\frac{\sum_{i=1}^{m}h_{2i}}{2}\left[n(D_i-I_i)t_n+n(n+1)I_it_n+nI_i(t_1+t_2)\right] \tag{A-3}$$

Because n installments (fixed quantity D) of the finished lot are delivered to customer at a fixed interval of time t_n, one has the following:

$$t_1+t_2=\frac{nI_i}{\lambda_i} \tag{A-4}$$

$$D_i=\lambda_i t_n+I_i \tag{A-5}$$

where I_i denotes the number of left over items for each retailer that will be left over after demand has been satisfied during each fixed interval of time t_n (see Figure 3). The retailers' holding cost, Eq. (A-3), becomes:

$$\frac{1}{2}\sum_{i=1}^{m}h_{2i}\left[(D_i-I_i)t_3+(n+1)I_it_3+\lambda_i(t_1+t_2)^2\right]=$$
$$=\frac{1}{2}\sum_{i=1}^{m}h_{2i}\left[(\lambda_i t_n+I_i)t_3+\lambda_i(t_1+t_2)t_3+\lambda_i(t_1+t_2)^2\right]=$$
$$=\frac{1}{2}\sum_{i=1}^{m}h_{2i}\lambda_i\left[\frac{Tt_3}{n}+(t_1+t_2)T\right] \tag{A-6}$$

8. REFERENCES

[1] L.B. Schwarz, A simple continuous review deterministic one-warehouse N-retailer inventory problem, *Management Science*, Vol. 19, pp. 555-566, 1973.

[2] S.K. Goyal, Integrated inventory model for a single supplier-single customer problem, *Int. Journal of Production Research*, Vol. 15, pp. 107-111, 1977.

[3] A. Banerjee, A joint economic-lot-size model for purchaser and vendor, *Decision Sciences*, Vol. 17, No. 3, pp. 292-311, 1986.

[4] K.H. Kim and H. Hwang, An incremental discount pricing schedule with multiple customers and single price break, *European Journal of Operational Research*, Vol. 35, No. 1, pp. 71-79, 1988.

[5] G.R. Parija and B.R. Sarker, Operations planning in a supply chain system with fixed-interval deliveries of finished goods to multiple customers, *IIE Transactions*, Vol. 31, No. 11, pp. 1075-1082, 1999.

[6] S. Cetinkaya and C.Y. Lee, Stock replenishment and shipment scheduling for vendor managed inventory systems, *Management Science*, Vol. 46, No. 2, pp. 217-232, 2000.

[7] A. Banerjee, J.S. Burton, Coordinated vs. independent inventory replenishment policies for a vendor and multiple buyers, *International Journal of Production Economics*, No. 35, pp. 215-222, 1994.

[8] M.-J. Yao, C.-C. Chiou, On a replenishment coordination model in an integrated supply chain with one vendor and multiple buyers, *European Journal of Operational Research*, Vol. 159, pp. 406-419, 2004.

[9] M.A. Hoque, Synchronization in the single-manufacturer multi-buyer integrated inventory supply chain, *European Journal of Operational Research*, Vol. 188, No. 3, pp. 811-825, 2008.

[10] B.R. Sarker and A. Diponegoro, Optimal production plans and shipment schedules in a supply- chain system with multiple suppliers and multiple buyers, *European Journal of Operational Research*, Vol. 194, No. 3, pp. 753-773, 2009.

[11] Y.-Sh.P. Chiu, S.-C. Liu, C.-L. Chiu and H.-H. Chang, Mathematical modelling for determining the replenishment policy for EMQ model with rework and multiple shipments, *Mathematical and Computer Modelling*, Vol. 54, pp. 2165-2174, 2011.

[12] S.W. Chiu, Y.-Sh.P. Chiu and J.-C. Yang, Combining an alternative multi-delivery policy into economic production lot size problem with partial rework, *Expert Systems with Applications*, Vol. 39, pp. 2578-2583, 2012.

[13] A.A. Taleizadeh, S.T.A. Niaki and A. Makui, Multiproduct multiple-buyer single-vendor supply chain problem with stochastic demand, variable lead-time, and multi-chance constraint, *Expert Systems with Applications*, Vol. 39, No. 5, pp. 5338-5348, 2012.

[14] K.-K. Chen, M.-F. Wu, S.W. Chiu, and C.-H. Lee, Alternative approach for solving replenishment lot size problem with discontinuous issuing policy and rework, *Expert Systems with Applications*, Vol. 39, No. 2, pp. 2232-2235, 2012.

[15] E.W. Taft, The most economical production lot, *Iron Age*, Vol. 101, pp. 1410-1412, 1918.

[16] S. Nahmias, *Production and Operations Analysis*, McGraw-Hill Co. Inc., New York, 2009.

[17] F.S. Hillier and G.J. Lieberman, *Introduction to Operations Research*, McGraw Hill, New York, 2001.

[18] T. Bielecki and P.R. Kumar, Optimality of zero-inventory policies for unreliable production facility, *Operations Research*, Vol. 36, pp. 532-541, 1988.

[19] T. Boone, R. Ganeshan, Y. Guo and J.K. Ord, The impact of imperfect processes on production run times, *Decision Sciences*, Vol. 31, No. 4, pp. 773-785, 2000.

[20] R.H. Teunter and S.D.P. Flapper, Lot-sizing for a single-stage single-product production system with rework of perishable production defectives, *OR Spectrum*, Vol. 25, No. 1, pp. 85-96, 2003.

[21] S.W. Chiu, C.-K. Ting, and Y.-Sh.P. Chiu, Optimal order policy for EOQ model under shortage level constraint, *Int. J. for Engineering Modelling*, Vol. 18, No. 1-2, pp. 41-46, 2005.

[22] H.-D. Lin, Y.-Sh.P. Chiu and C.-K. Ting, A note on optimal replenishment policy for imperfect quality EMQ model with rework and backlogging, *Computers and Mathematics with Applications*, Vol. 56, No. 11, pp. 2819-2824, 2008.

[23] S.W. Chiu, K.-K. Chen and J.-C. Yang, Optimal replenishment policy for manufacturing systems with failure in rework, backlogging, and random breakdown, *Mathematical and Computer Modelling of Dynamical Systems*, Vol. 15, No. 3, pp. 255-274, 2009.

[24] S.S. Sana, A production–inventory model in an imperfect production process, *European Journal of Operational Research*, Vol. 200, No. 2, pp. 451-464, 2010.

[25] T.J. Lee, S.W. Chiu and H.-H. Chang, On improving replenishment lot size of an integrated manufacturing system with discontinuous issuing policy and imperfect rework, *American Journal of Industrial and Business Management*, Vol. 1, No. 1, pp. 20-29, 2011.

[26] Y.-Sh.P. Chiu, K.-K. Chen and C.-K. Ting, Replenishment run time problem with machine breakdown and failure in rework, *Expert Systems with Applications*, Vol. 39, pp. 1291-1297, 2012.

[27] S.W. Chiu, H.-D. Lin, C.-B. Cheng and C.-L. Chung, Optimal production-shipment decisions for the finite production rate model with scrap, *Int. J. for Engineering Modelling*, Vol. 22, pp. 25-34, 2009.

[28] R.L. Rardin, *Optimization in Operations Research*, Prentice-Hall, New Jersey, 1998.

[29] Y.-Sh.P. Chiu, F.-T. Cheng and H.-H. Chang, Remarks on optimization process of manufacturing system with stochastic breakdown and rework, *Applied Mathematics Letters*, Vol. 10, pp. 1152-1155, 2010.

INTEGRIRANI MODEL SKLADIŠTENJA KOJI UKLJUČUJE JEDNOG PROIZVOĐAČA, VIŠE TRGOVACA NA MALO I POSTUPAK DORADE PROIZVODA

SAŽETAK

Ovaj članak razmatra integrirani model skladištenja koji obuhvaća jednog proizvođača, više trgovaca na malo i doradu slučajno izabranih neispravnih proizvoda. Cilj je pronaći optimalnu sveukupnu količinu proizvodnje i optimalni broj pošiljki koji minimaliziraju ukupne očekivane troškove za takve specifične sustave opskrbnih lanaca. Pretpostavlja se da je proizvođač izradio proizvod. Svi proizvodi se provjeravaju u smislu kontrole kvalitete, a slučajno izabrani neispravni proizvodi će se izdvojiti i doraditi na kraju redovne proizvodnja u svakom ciklusu. Nakon što je dokazana kvaliteta cjelokupno proizvedene količine, višestruke pošiljke će biti istovremeno isporučene na m različitih trgovaca na malo u svakom proizvodnom ciklusu. Svaki trgovac na malo ima svoju godišnju potrebu za proizvodom, jedinični trošak držanja zaliha te fiksne i varijabilne troškove isporuke. Kako bi se dobio očekivani trošak sustava korišteno je matematičko modeliranje i analiza. Za potvrdu konveksnosti funkcije troška korištene su Hesseove matrice. Kao rezultat dobiven je zatvoreni oblik politike optimalnog nadopunjavanja zaliha i isporuke za takav specifični integrirani model skladištenja koji obuhvaća jednog proizvođača i više trgovaca na malo. Prikazan je i numerički promjer kako bi se demonstrirala praktična primjena predloženog modela.

***Ključne riječi**:* *opskrbni lanci, optimizacija, proizvodnja, sveukupna količina proizvodnje, više trgovaca na malo, višestruke isporuke, dorada proizvoda.*