ESSAYS ON DYNAMIC PRICING AND BUNDLING IN SUBSCRIPTION MARKETS

by

Nabita Penmetsa

B.E in Mechanical Engineering, Osmania University, 2005
M.B.A, Indian Institute of Technology, Madras, 2007

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This dissertation was presented

by

Nabita Penmetsa

It was defended on

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and approved by

Esther Gal-Or, Glenn Stinson Chair in Competitiveness, Professor of Business Administration and Economics

Jennifer Shang, Professor of Business Administration,

Tansev Geylani, Associate Professor of Business Administration

Mustafa Akan, Assistant Professor of Operations Management, Tepper School of Business

Dissertation Chair: Jerrold H. May, Professor of Business Administration, Katz Graduate School of Business

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Nabita Penmetsa, PhD

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This dissertation consists of two essays that investigate dynamic pricing and bundling strategies in subscription markets. In the first essay, we analyze the dynamic price discrimination strategies of a monopolist offering new services on a subscription basis. In subscription markets, the pricing policy can be based on customers' past purchase behavior (behavioral price discrimination) and time of purchase (intertemporal price discrimination). In the presence of uncertainty regarding the value of new features and heterogeneity in consumer valuations of the existing features, we investigate the profits and rate of adoption of new technology that can be achieved with each pricing strategy. When the prior heterogeneity in consumer valuation of the existing features is relatively large, the monopolist can improve his profits by committing to ignore consumer past behavior and varying prices based only on time. We also study the role of commitment power of the monopolist to announce future prices and correlation in valuations of the new and existing features.

In the second essay, we investigate the multi-product pricing strategies of a sequentially innovating monopolist introducing new services. The new service can either represent a new functionality not directly related to the existing service or an enhancement to the existing services. When the existing service is offered in multiple versions, the monopolist can sell the new service separately or bundle the new service with some or all versions of the primary service. We analyze two pricing strategies that represent the two extremes of a spectrum of bundling strategies that a monopolist offering such services can practice: Discriminative Bundling (DB) and Independent Pricing (IP). Using the discriminative bundling (DB) strategy, a service provider offering multiple versions of the primary service bundles the new service only with higher versions of the primary service while selling it separately to remaining customers. Using the independent pricing strategy (IP), the service provider offers the new service separately to all consumers including those buying lower and higher end versions. We find that the comparison of the two strategies in terms of profits depends on the nature of the new service and the general distribution of consumer valuations for the new and the primary services.

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1 GENERAL INTRODUCTION

Cloud computing is transforming the way software is delivered and priced. Software companies are moving away from the traditional model of selling boxed software for a fixed price to delivering software as a service for a recurring subscription fee. The existing pricing literature treats software as an experience good. However, the move to a subscription model represents a unique opportunity to analyze the pricing of an experience good in a repeat purchase setting. In the first essay, we analyze the dynamic price discrimination strategies of a monopolist offering new services on a subscription basis. Access to customer subscription history allows the monopolist to design pricing policies that can be based on customers' past purchase behavior and the time of purchase. Uncertainty regarding the value of new features and heterogeneity in consumer valuation of the existing features creates intertemporal incentives that influence profits and the rate of adoption of new technology. We find that the comparison of pricing regimes critically depends on whether or not the monopolist finds it optimal to encourage all consumers to adopt the new technology early. They differ only when the prior heterogeneity in consumer valuation of the existing features is relatively large, in which case the monopolist finds it optimal to serve only part of the population of consumers having relatively high valuation. The monopolist can improve his profits by committing to ignore consumer past behavior and varying prices based only on time. Moreover, if a stronger commitment to never utilize any price discrimination is feasible, the profits of the monopolist are even higher. We also investigate the effect of positive correlation between the consumer valuations of the existing and new features of the technology. We find that as correlation increases, the gap in profits of the various regimes narrows while the ranking remains the same. In particular, with perfect correlation all time inconsistency issues that arise due to lack of commitment disappear completely with all regimes, and the "First Best" outcome is attainable.

In the second essay, we investigate the pricing strategies of a monopolist using more than one form of price discrimination mechanism. Specifically, we analyze the pricing strategies of a monopolist offering multiple versions of a primary service and investing in new services. When introducing new services, the monopolist can sell the new service separately or bundle the new service with some or all versions of the primary service. We focus on two pricing strategies that represent the two extremes of the various types of bundling that a monopolist offering such services can practice: Discriminative Bundling (DB) and Independent Pricing (IP). Using the discriminative bundling (DB) strategy, a service provider offering multiple versions of the primary service bundles the new service only with higher versions of the primary service and sells the new service separately to remaining customers. Using the independent pricing strategy (IP), the service provider offers the new service separately to all consumers including those buying lower and higher end versions. We find that the comparison of the two strategies in terms of profits depends on the nature of the new service and the general distribution of consumer valuations for the new and the primary services. When the new service represents an enhancement to the primary service, consumer valuations for the new service are correlated with their valuations for the primary service. In such cases, a discriminative bundling strategy always leads to lower profits when compared to independent pricing. When the new service represents a service extension that is unrelated to the primary service, consumer valuations for the new service are independent of their valuations for the primary service. In addition to the correlation between consumer valuations for the new and the primary service, the nature of the relationship between the new and the primary service can also influence the comparison of pricing strategies. When there is a difference in the value derived from using the new service with and without the premium version, discriminative bundling can lead to higher profits when compared to the independent pricing strategy when customer heterogeneity for the primary service is sufficiently large and the degree of complementarity or substitutability is sufficiently high.

2 ESSAY 1: DYNAMIC PRICING OF SERVICE UPGRADES IN SUBSCRIPTION MARKETS

2.1 INTRODUCTION

The introduction of subscription services in the software industry is disrupting the traditional model of selling software for a one-time fixed payment. Microsoft and Adobe, for instance, have started offering their new version of Office and Creative Suite software, respectively, on a subscription basis. A subscription model differs from the traditional model in two important ways. First, in a subscription environment consumers are required to renew their subscription after every period in order to continue using the software. Second, subscriptions allow a service provider to more successfully collect and maintain customer data. This information can be used, for instance, to identify whether a user is a first time customer or a returning customer who is in the market for renewal. The service provider can use this information to design pricing strategies that are based on the time of purchase (vary the subscription price over time) and the customer's purchase history (offer different prices to new and returning customers), thus giving rise to three possible price discrimination regimes: (i) Behavioral price discrimination (B) where the seller can offer different prices to new and experienced customers, (ii) Intertemporal price discrimination (I) where the seller can vary prices based only on time, and (iii) Behavioral and Intertemporal price discrimination combined (BI), where the seller can vary prices based upon both behavior and time.

There exists wide variation in the way technology companies are pricing their subscriptions of new software. Adobe, for instance, offers introductory discounts on the subscription of Creative Cloud to academic users and existing users of CS6 software. Similarly, Intuit offers introductory discounts to new customers on subscriptions of Quickbooks Online and Quickbooks Accountant. The introductory discounts are valid only for one subscription period and returning customers who want to renew their subscriptions are required to pay the then market price of the software, which tends to be higher than the introductory price. In contrast, some firms stay away from such introductory offers that favor new customers over returning customers. For example, Microsoft does not offer any special discounts to new customers on the subscription of Office 365. However, most of these firms mention in their offer that the current price of the subscription is valid only for a limited period of time and is subject to change, indicating that the firms wish to preserve their flexibility to vary prices in the future. Although the existing literature presents some insights on the relative profitability of such pricing strategies (see, for instance, Fudenberg et al. (2006) and Jing (2011)) there still exists incomplete understanding of the intertemporal incentives of firms and consumers in an environment where consumers gain information about a new technology from their experience of using it. In particular, the literature has not considered how the growth prospects of the market or the extent of uncertainty facing the consumer affect the comparison among the various pricing regimes.

Our objective is to close this gap in the literature by focusing on the pricing of a new service in a subscription environment with forward-looking consumers who are uncertain of their valuation of the new service. New software such as Adobe Creative Cloud can be considered a bundle of new and existing features. While consumers are familiar with the existing features of the software, they face significant uncertainty with respect to the value of the new features. Consumers who buy the service gain experience with the new features and use the information gathered in making future purchasing decisions. Contingent upon the pricing strategy of the firm, consumers can strategically decide also on the timing of their first subscription to the new service. In particular, they can choose between subscribing to the service immediately and postponing their subscription to a later date, if they expect prices to decline over time. Postponing the subscription prevents them, however, from experimenting with the new features and assessing their added value. Whether this added information influences their future behavior depends upon their valuation of the existing features. Consumers with very high valuation for the existing features may renew their subscription even if their valuation of the new features is relatively low. For instance, professional web designers who tend to have relatively high valuation for Adobe's creative applications may renew their subscription to Creative Cloud even if they find that the quality of the new applications is not impressive.

We explore how the different pricing regimes affect the profitability of a monopolistic firm, the extent of market coverage, and the pattern of prices over time when the monopolist does not have the power to commit to future prices. Given that in subscription markets the firm can observe past behavior of its customers and can also change prices as time elapses, in order to implement a pricing regime that excludes either past behavior or time or both variables, it is necessary for the firm to utilize a commitment device that prohibits it from using some or all of the observable variables at its disposal. In the absence of such commitment devices, the monopolist will utilize both behavior and time in designing its pricing policy. We also derive the equilibrium when the monopolist has full commitment power to credibly announce future prices to new and returning customers. We use this full commitment case as a benchmark to illustrate the characteristics of the "First Best" outcome that ensures the highest profits for the monopolist. Under the "First Best", the monopolist rewards the loyalty of returning customers by charging them lower prices, and also commits to never cutting prices to new customers in future periods. We compare the various pricing regimes without commitment against this "First Best" benchmark and demonstrate that this comparison critically depends upon the extent of coverage of the market and the overall uncertainty facing the consumers regarding their valuation of the new features.

We start by analyzing the pricing regimes when the market is less than fully covered, implying that there is always a segment of consumers who have not experienced as of yet the new service. This regime arises in our model if the heterogeneity in consumer valuations of the existing features of the service is sufficiently large. We demonstrate that in such growing markets, if the monopolist has the flexibility to choose among the three price discrimination regimes, his profits are highest if only time and not past behavior is utilized in pricing. This regime of intertemporal price discrimination is characterized by a declining schedule of prices over time as the monopolist seeks to expand his market by making his service more affordable to low valuation consumers. In contrast to this declining schedule of prices, when only behavior is used as a basis for discrimination, the monopolist charges returning customers higher prices than new customers. Given their choice to subscribe to the new service a second time, returning customers implicitly reveal that their valuation of the service is relatively high, thus providing an incentive for the monopolist to charge them a higher price. Even though the regime of behavioral price discrimination is less profitable for the monopolist than intertemporal price discrimination, it does encourage a bigger number of consumers to adopt the new technology early because of the low introductory prices that the monopolist charges in this case. We also find that utilizing both behavior and time as bases for discrimination has the most adverse effect on the profits of the monopolist. Anticipation of an increase in prices to experienced customers makes learning less attractive for consumers and anticipation of a decrease in prices to new customers in the future encourages more consumers to postpone their purchase to a later period. We further demonstrate that if the monopolist has stronger commitment powers that permit him to commit to a constant price irrespective of the period of purchase or the purchase history of the consumer, his profits are even higher than with any price discrimination regime, including that of intertemporal price discrimination.

We continue by analyzing the pricing regimes when the market is fully covered, namely when the entire customer base adopts the new service early and the potential for further growth disappears. In our model this regime arises when the extent of heterogeneity in the consumer valuation of the existing features of the service is relatively small. Because all consumers adopt the new technology early, the monopolist does not have an incentive to cut prices to new customers in later periods in order to expand his market. Moreover, customers in later periods are all returning customers and the distinction between new and returning customers disappears. As a result, all pricing regimes yield identical profits and patterns of prices over time. Specifically, prices rise over time under either behavioral or intertemporal price discrimination, as repeat purchases by consumers signal to the monopolist that returning customers have high valuation for the product. Recall, that this trend of prices contradicts the trend under the "First Best" where loyalty is rewarded in the form of lower prices. As a result, in the absence of commitment power the monopolist still cannot achieve the "First Best" outcome even when the market is fully covered. It is noteworthy that the environment and results we obtain with full market coverage resemble that derived in Jing (2011).

Finally, we analyze the effect of correlation between the valuations of the new and existing features on the comparison among the different pricing regimes. In the presence of correlation, the consumer's familiarity with her valuation of the basic features helps her resolve some of the uncertainty regarding her valuation of the new features, and the importance of consumer learning declines. We find that as the degree of correlation increases, while the ranking of the various pricing schemes remains as in the case of no correlation, the difference in profitability between the best and worst performing pricing regimes diminishes. The difference disappears completely in the extreme case, when the basic and new valuations are perfectly correlated, and therefore, the consumer faces no uncertainty. We find that with perfect correlation all pricing regimes are equivalent and yield the "First Best" outcome, even when the monopolist does not have commitment power to set future prices. It is interesting that this result is similar to that derived in the Durable Good Monopolist literature (Coase (1972) and Bulow (1983)), where it has been demonstrated that by leasing instead of selling the product the monopolist can overcome all time inconsistency issues that arise due to his lack of commitment power. Like in the case of perfect correlation in our model, the Durable Good Monopoly literature focuses on an environment where consumers face no uncertainty regarding their preferences.

2.2 LITERATURE REVIEW

Our work is primarily related to two streams of literature: dynamic pricing of experience goods and behavior based price discrimination. Nelson (1970) was the first to introduce the concept of an experience good. When buying such a good consumers are initially uncertain of their valuation of the product, but gain information about this valuation after purchasing and using the product. An early study that derives the implications of such consumer learning on the path of pricing chosen by the firm is Shapiro (1983). Subsequent studies in this area have focused on deriving the optimal pricing path with strategic consumers and heterogeneity in consumer population in terms of experience with the good (Cremer 1984) and rate of learning (Bergemann and Välimäki (2006)). While these studies focus on a monopoly setting, Villas-Boas (2004, 2006) considers learning in a competitive environment. He demonstrates that the pricing path and the profitability of each firm depend on the extent of negative or positive skewness of the distribution function of consumer valuations.

In subscription markets, the identity of buyers is not anonymous and service providers can distinguish between new and repeat customers. As a result, providers can price discriminate between new and existing customers. Behavior based price discrimination has been addressed in the literature both in monopolistic and competitive settings. Fudenberg et al. (2006) provides a comprehensive review of the literature in this area. Studies that investigate the role of behavior based price discrimination in competitive markets focus primarily on the ability of a firm to poach the customers of its competitors by offering them better deals than those offered to the existing customers of the firm. In some of these studies the driving force in enticing customers to switch relates to the existence of switching costs (see for instance, Chen (1997), Shaffer and Zhang (2000), and Taylor (2003)). In other studies enticing customers relates to horizontal product differentiation among competitors (see, for instance, Chen and Pearcy (2010), Fudenberg and Tirole (2000), Pazgal and Soberman (2008), Shin and Sudhir (2010) and Villas-Boas (1999)).

At the intersection of the above two streams, Caminal (2012), Cremer (1984) and Jing (2011) analyze behavior based price discrimination in a monopoly setting when consumers are uncertain of their valuation of the new product and experience with the product reduces this uncertainty. Caminal (2012) and Cremer (1984) analyze the optimality of coupons and loyalty programs that reward returning customers over new customers. They assume that the monopolist has the power to commit to future prices. In contrast, similar to us, Jing (2011) evaluates time contingent pricing and behavior based price discrimination when the monopolist does not have the power to commit to future prices.

As in the experience good pricing literature, Jing (2011) assumes that consumers are completely uninformed of their valuations prior to gaining experience with the new product and are, therefore, identical prior to purchasing the good for the first time. The price chosen by the monopolist leads to either none or all of the consumers buying the service in an early period. Because it is optimal for the monopolist to induce the entire population of consumers to start experimenting with the product early, in later periods all consumers are returning customers. In a given period, therefore, the monopolist faces either all new or all returning customers and the distinction between new or returning customers in a given period disappears. In contrast to this setting, we assume that consumers are heterogeneous even before they learn their idiosyncratic valuations of the new features of the service. We find that when this initial heterogeneity is relatively high, it is optimal for the monopolist to sell only to a segment of the population with sufficiently high valuation of the basic service. Because part of the market remains uncovered, in later periods the monopolist is tempted to cut his price to new customers in order to expand his market, thus giving rise to price discrimination based on time. Such incentives do not arise in Jing (2011) where the entire population of consumers is attracted right away. It is noteworthy that our environment becomes identical to that modeled in this previous paper when the extent of prior heterogeneity among consumers is relatively small and the monopolist finds it optimal to cover the entire market right away. As a result, we obtain the same results as Jing in this case. Specifically, any form of price discrimination (behavioral or intertemporal) yields identical profits for the monopolist.

Another stream of literature that is related to our environment is that of dynamic product design when firms do not have commitment power (see, for instance, Dhebar (1994), Fudenberg and Tirole (1998), Kornish (2001), Krishnan and Ramachandran (2011), and Sankaranarayanan (2007)). These papers all assume a durable good monopolist that can offer technological improvements or upgrades to the product in the future. The monopolist cannot commit, however, to the extent of improvement or the future price of the product. The lack of commitment to future prices that we assume in our paper is extended in this stream of literature to include also lack of commitment power regarding the quality of the product in the future.

2.3 MODEL AND PRELIMINARIES

A monopolist is offering a new service that consists of some new and existing features, on a subscription basis. Consumers have heterogeneous valuations for the existing features of the service. We designate this heterogeneity by a parameter θ which is uniformly distributed in the population over the interval $[0, \theta_H]$. While each consumer knows her own valuation of the existing version, the monopolist is only familiar with the distribution of this valuation in the population. We designate the value of new features by a parameter q. The valuation of the new features is unknown to both the monopolist and the consumers. However, consumers can learn their valuation of the new features after using the new version for one subscription cycle. The realization of this value to the consumer varies across different consumers. For example, Adobe Creative Cloud includes a new web development tool called Muse. Customer reviews indicate that some users who tried the new tool value the new feature highly due to ease of use but some users were unhappy about the speed and performance aspects of the tool[1]. We assume that the distribution of this value is uniform over the interval $[0, q_H]$. The reservation price of a consumer of type θ for the new service is $\theta + q$. We assume that q and θ are independently distributed, and that the distributions of these variables in the population are common knowledge. We also assume that the marginal cost of providing the new version is negligible. This is a valid assumption for many software products for which most of the costs are fixed in nature and relate to the cost of developing the product. We consider a two-period model where in each period the monopolist has to make pricing decisions. The two-period assumption is justified given that the lifecycle of software tends to be rather limited, as it is often replaced by totally new products or because the hardware on which it is installed becomes obsolete.

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[1] Customer reviews of the Adobe Creative Cloud membership on the Amazon website vary tremendously. Of 43 reviews published on February 12, 2014, 19 gave it 5 stars and 11 gave it only 1 star. See all reviews at the link http://www.amazon.com/Adobe-Creative-Membership-Pre-Paid-Product/product-reviews/B007W76ZLW A similar spread in evaluation exists also on the Adobe website http://www.adobe.com/products/creativecloud/reviews.html

We consider an environment where the monopolist cannot credibly commit to future prices. As a result, under behavioral price discrimination he cannot credibly commit to the price returning customers will face in the future. Similarly, under intertemporal price discrimination or the combined intertemporal and behavioral price discrimination he cannot commit to future prices any type of customer (old or new) will face. In the absence of commitment power, the monopolist chooses future prices when the future arrives. It is noteworthy that the regimes of exclusively behavioral or exclusively intertemporal price discrimination (regimes B or I) correspond to environments where the monopolist does have some commitment power. With behavioral price discrimination the monopolist can utilize a mechanism to credibly communicate to new customers that he will never charge lower prices to other new customers in the future. An example of such a mechanism is to offer new customers a refund if prices are ever lowered to other new customers in the future. Similarly, with intertemporal price discrimination the monopolist possesses a credible way of convincing customers that their level of experience with the new service does not affect the prices they will face in the future. This being the case in spite of the fact that the monopolist can identify consumer's past behavior based upon their subscription history. Using contemporaneous Most Favored Customer clauses (Cooper (1986)) that promise to reimburse customers if any other customer is offered a lower price in a given period may offer a vehicle to implement this type of partial price discrimination regime. The bi-dimensional price discrimination regime BI does not require any commitment power on the part of the monopolist. In this case the monopolist does not preclude himself from incorporating in his pricing any available information about either the past subscription history of the customer or the period of time when the service is purchased. Regardless of the regime considered we assume that the monopolist cannot commit at the present to prices he will charge in the future. In the absence of such commitment power, we will derive the equilibrium by backward induction.

In the first period the monopolist sells the new version to consumers who have no experience with the new features. In the second period the monopolist faces two groups of consumers: (i) New consumers who still have no experience with the new service and (ii) Repeat consumers who bought the new service in period 1 and have, therefore, experience using the new features. Depending on the form of price discrimination regime used by the monopolist, the price of the service in the second period may be different for new and returning consumers. In the most general case i.e., the BI regime, the monopolist can choose three different prices: an introductory price P_{1N} in the first period for all consumers, a price for new consumers in the second period P_{2N}, and a price for returning consumers in the second period P_{2R}. However, in the B and I regimes which are characterized by only one form of price discrimination, the choice of the monopolist is reduced to only two prices. For example, in the B regime, the price to new customers remains the same in the second period $P_{1N} = P_{2N=} P_N$, therefore, the monopolist chooses a price P_N in the first period and a price P_R for returning customers in the second period. Similarly, in the I regime, since new and returning customer are offered the same price in the second period $P_{2N} = P_{2R} = P_2$, and the choice of the monopolist is reduced to only two prices P_1 and P_2. We describe our modelling approach by focusing on the case of price discrimination based on both time and behavior (BI Regime). We use a similar approach to obtain the equilibrium with the uni-dimensional price discrimination regimes by imposing the additional constraints, $P_{2N} = P_{1N} = P_N$ in the B regime and $P_{2N} = P_{2R} = P_2$ in the I regime.

Focusing on the BI regime we start by considering the consumer's choice. When deciding on whether to buy the new version in the first period consumers are forward looking and try to assess their expected utility over both periods. Each consumer knows her valuation parameter θ right from the start. However, because the valuation of the new features is unknown in the first period, all consumers use the prior distribution to calculate its mean as $\frac{q_H}{2}$. In the second period, consumers who bought the new version in the first period learn the realization of q and decide on whether to continue buying the new version or not. In particular, if the realization of q is relatively low the consumer may decide not to buy the service in the second period. For high realizations of q, however, consumers will continue to buy the service. Consumers who did not buy the new version in the first period are still uncertain of the value q in the second period as well, and continue to use the mean of the prior distribution of q in assessing the average value of the added features. Since the utility derived from the existing version in each period is θ, a consumer of type θ assesses her net surplus in the first period when buying the subscription as $\frac{q_H}{2} + \theta - P_{1N}$. The evaluation of the second period net surplus depends on whether or not the consumer purchased the new version in the first period. For those who bought the new version in the first period, the net surplus when buying it a second time is given by $q + \theta - P_{2R}$. Note that an informed consumer will continue to buy the service in the second period only if $q > (P_{2R} - \theta)$. Because consumers are forward looking, in calculating their aggregate expected utility over both periods they take into account the possibility that they may not continue to buy the service in the second period even if they choose to purchase it in the first period. For those who did not buy the new product in the first period, the net surplus in period 2 is given by $Max\left\{\theta + \frac{q_H}{2} - P_{2N}\,, 0\right\}$. Ignoring discounting of future payoffs in the aggregation, a consumer of type θ receives the following expected utilities from buying and not buying the new service in period 1.

$$EU(\theta|Buy\ in\ period\ 1) = \frac{q_H}{2} + \theta - P_{1N} + \int_{P_{2R}-\theta}^{q_H} \frac{q+\theta-P_{2R}}{q_H} dq + \int_0^{P_{2R}-\theta} \frac{0}{q_H} dq. \quad (1)$$

$$EU(\theta|Not\ buy\ in\ period\ 1) = 0 + Max\{\frac{q_H}{2} + \theta - P_{2N}, 0\}. \quad (2)$$

Note that a consumer who did not buy the new version in the first period may face a new price P_{2N} if she considers buying it in the second period. However, only if $P_{2N} < P_{1N}$, a consumer will find it optimal to postpone the purchase to the second period. From (1) and (2) it follows that if $P_{2N} \geq P_{1N}$ any consumer who chooses to buy the new version will do so in the first stage. As noted earlier, the two integral expressions in (1) reflect the idea that when anticipating the expected utility derived in the second period, strategic consumers know that they will be able to incorporate information obtained as a result of their purchasing decision in the first period. In the B regime, the monopolist does not vary prices over time and charges new customers the same price that he charged in the first period ($P_{2N} = P_{1N}$). Therefore, for a consumer who did not buy the new service in the first period, the net surplus in period 2 is equal to zero.

Because we consider a growing market that is far from reaching saturation, we assume that in both periods there is a group of consumers with no experience of using the new version. Hence, the market is less than fully covered in each period, and in the second period the population of consumers can be divided into two segments: experienced and inexperienced consumers. To identify these two segments, we have to solve for the consumer of type θ^* who is indifferent between buying and not buying in period 1, namely a consumer who satisfies the equation $EU(\theta^*|Buy\ in\ period\ 1) = EU(\theta^*|Not\ buy\ in\ period\ 1)$. Because the added benefit of buying is increasing in θ all consumers of type $\theta < \theta^*$ will not buy the new service in the first period and all consumers of type $\theta > \theta^*$ will buy it in the first period.

Recall that we consider an environment where the monopolist cannot credibly commit to the future price he will charge any type of customer. Instead, he optimally chooses the price new and returning customers pay when the future arrives. Consistent with this assumption, we solve the game by backward induction starting by characterizing the equilibrium of the second stage. Such an approach guarantees that the strategies chosen in the second stage are credible (such strategies have been referred to in the literature as close-loop strategies.) Adobe and Intuit, for instance, offer trial agreements to new customers without committing to future prices once the agreements expire. Our modeling approach is consistent with this reality.

In the second stage, the monopolist can observe the size of the segment of consumers who bought the new version in the first stage, $\theta_H - \theta^*$, and potentially may continue to do so in the second stage. Note that the monopolist takes the size of this segment as given at the beginning of the second stage. In particular, his choice of P_{2R} cannot change the size of the segment of experienced buyers who consider buying the service again. However, the price that the monopolist charges returning customers determines who in the region $[\theta^*, \theta_H]$ will continue to buy the service, and under what circumstances. In particular, the second period price determines whether the repeat purchase decision of a returning customer is conditional on her experience with the new features. We use the term conditional buying to characterize circumstances under which a consumer utilizes her past experience with the new service in deciding whether to continue buying the service for a second time. Whether conditional buying occurs depends upon the price P_{2R} that the monopolist charges returning customers. For instance, if the monopolist lowers P_{2R} significantly so that $P_{2R} < \theta^*$, there is no conditional buying by any type of customer because irrespective of their past experience all consumers will continue to buy the service again given that their expected net payoff (for any $\theta > \theta^*$) is positive for all q realizations. We consider the following two cases that can arise at the equilibrium with less than full coverage, contingent upon the value of P_{2R}:[2]

a) $\theta^* < P_{2R} < \theta_H$ – Conditional buying by lower tail. Only consumers having very low valuation for the existing version condition their continued purchase upon the q realization. Consumers having high valuation continue to buy irrespective of the realized value of q.
b) $\theta_H < P_{2R} < \theta^* + q_H$ – Conditional buying by all. All consumers condition their repeat purchase decision on q irrespective of their valuation for the existing version.

If the monopolist chooses to attract new customers in the second stage $P_{2N} < \theta^* + \frac{q_H}{2}$ and a new segment of consumers defined by the interval $[\theta_n, \theta^*]$ purchases the new version in the second stage, where $\theta_n = P_{2N} - \frac{q_H}{2}$. In the following derivations we investigate which of the above two conditional buying regimes can arise when the market is less than fully covered in both stages of the game.

Assuming conditional buying by every customer type, we can write the payoff function of the monopolist in the second stage as follows:

$$\pi_2^{BI} = \int_{P_{2N}-\frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H \theta_H} dq d\theta. \tag{3}$$

Note that each experienced customer sometimes buys the service a second time when $q > P_{2R} - \theta$ and sometimes does not when $q < P_{2R} - \theta$. Optimizing the objective (3) with respect to P_{2N} and P_{2R} yields the second stage equilibrium prices. For a given θ^* and conditional buying by each customer type, the second period equilibrium prices for the service and the second stage equilibrium profit are given as:

[2] There are two additional cases that may arise at the equilibrium depending upon the choice of P_{2R} as follows:
(i) $P_{2R} < \theta^*$ – No conditional buying by any type. All returning customers continue to purchase irrespective of the q realization.
(ii) $\theta^* + q_H < P_{2R} < \theta_H + q_H$ – Conditional buying by upper tail For such a high P_{2R}, consumers having low valuation never purchase the service again and those having high valuation condition their purchase decision on the value of q.

It can be shown, however, that these two cases are inconsistent with less than full coverage of the market. The proof of this result can be provided by the authors upon request.

$$P_{2N}^* = \frac{q_H+q_L}{4} + \frac{\theta^*}{2}, \quad P_{2R}^* = \frac{q_H}{2} + \frac{\theta_H+\theta^*}{4}. \tag{4}$$

$$\pi_2^{*BI} = \frac{1}{\theta_H}\left[(P_{2N}^*)^2 + \frac{\theta_H-\theta^*}{q_H}(P_{2R}^*)^2\right]. \tag{5}$$

Given the optimal solution derived in the second stage, in the first stage the monopolist chooses the price to all customers P_{1N} to maximize his aggregate profits over the two periods (again, we ignore discounting in the aggregation). He knows, in particular, that the choice of price in this period will affect the number of consumers who choose to experiment with the new version, i.e., the segment $\theta_H - \theta^*$. The consumer of type θ^* is indifferent between buying and not buying the new service in period 1, namely $EU(\theta^*|Buy\ in\ period\ 1) = EU(\theta^*|Not\ buy\ in\ period\ 1)$ for this consumer, which from (1) and (2) implies that:

$$\frac{q_H}{2} + \theta^* - P_{1N} + \frac{[q_H - P_{2R}^* + \theta^*]^2}{2q_H} = \frac{q_H}{2} + \theta^* - P_{2N}^*. \tag{6}$$

Note that consumers have rational expectations and can anticipate the future prices P_{2N}^* and P_{2R}^* as derived in (4) by solving the optimization problem of the monopolist in the second stage. The last expression on the left hand side of (6) incorporates this information to measure the added expected utility of the consumer in the second period as a result of buying the new service in the first period.

Substituting into (6) the optimal second period prices derived from (4), we can solve for θ^* in terms of P_{1N}, the first period price charged by the monopolist. The solution for θ^* in terms of P_{1N} will determine the size of the segment of consumers who choose to experiment with the new version in the first period $(\theta_H - \theta^*)$. This solution captures the traditional tradeoff between price and quantity that a demand function represents. When the monopolist raises the price P_{1N} that he charges in the first stage fewer consumers purchase the service and experiment with the new features, namely $(\theta_H - \theta^*)$ declines. The functional relationship that ties θ^* to P_{1N} is obtained from the utility optimization of the consumers. The monopolist knows that consumers are sophisticated and can anticipate the expected benefit they will derive from buying the service not only in the current period, but in the subsequent period as well. Hence, the added benefit obtained by the consumer over both periods as expressed in (6) is used by the monopolist in deriving the first period demand for the new version.

When choosing the first period price the monopolist maximizes his profits over both periods as follows:

$$\max_{P_{1N}} \pi^{BI} = \frac{(\theta_H - \theta^*)}{\theta_H} P_{1N} + \pi_2^{*BI}. \tag{7}$$

where π_2^{*BI} is given in (5). Optimizing (7) with respect to P_{1N}, while incorporating the tradeoff between price and quantity given in (6), yields the results reported in Proposition 1 for the BI regime. The results for the B and I regimes can be similarly obtained.

<u>Proposition 1</u>

When the market is less than fully covered in both stages of the game:

(i) There does not exist an equilibrium with conditional buying by all returning consumers under the BI and I regimes.

(ii) Under the B regime such an equilibrium exists when $1.482 < \frac{q_H}{\theta_H} < 1.603$.

According to part (i) of the proposition, in the BI and I regimes, when the market is less than fully covered, the second period price that the monopolist chooses does not support conditional buying by all returning customers. Under these two regimes, the monopolist has an incentive to lower prices to new customers in the second period in order to expand his market. These lower prices put downward pressure on the prices the monopolist can charge returning customers as well. As a result, the price to returning customers can never exceed θ_H and conditional buying by all cannot arise. In contrast, in the B regime, for larger values of the ratio $\frac{q_H}{\theta_H}$, there exists an equilibrium that leads to all consumers engaging in conditional buying.

2.3.1 "First Best" Outcome

We now consider an environment where the monopolist does have the power to commit to future prices in order to derive a benchmark for the "First Best" pricing of the monopolist. Lack of commitment power implies that the monopolist does not have the ability to credibly commit to prices he will charge in the future. In particular, he cannot credibly commit to the price returning customers will face in the second stage when selling the service to new consumers who are still unfamiliar with the new version. In contrast, when the monopolist does have commitment power, he cannot renege on promises regarding future prices without incurring significant costs. The costs may be related to loss of reputation or damages incurred when breaking a written contract with customers. If such commitment is feasible it leads to the highest profits that can be obtained in our environment. We will use this "First Best" outcome as a benchmark in evaluating the various price discrimination regimes without commitment. To derive the "First Best" we allow the monopolist full flexibility in choosing prices; namely prices can vary by period and by the purchase history of the customer. The monopolist chooses these prices in the first period and is fully committed to them. Because the monopolist can credibly communicate the future prices P_{2N} and

P_{2R}, consumers no longer have to infer them from the second stage profit maximization of the monopolist. Rather, they are guaranteed that the monopolist will not renege on his promised prices.

Assuming less than full coverage of the market and conditional buying by all returning customers, the payoff function for the two periods can be expressed as follows:

$$\max_{P_{1N},P_{2N},P_{2R}} \frac{(\theta_H-\theta^*)}{\theta_H} P_{1N} + \int_{P_{2N}-\frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H\theta_H} dq d\theta.$$

We report the result of the maximization and characterize the "First Best" outcome in Proposition 2.

Proposition 2

(i) When the monopolist has full commitment power to set future prices and the market is less than fully covered, the equilibrium is characterized by conditional buying by only the lower tail of returning customers. The monopolist sets the same price to new customers in both periods, implying that no new customers are attracted in the second period.

(ii) Under the "First Best" the prices set by the monopolist are:

$P_{1N} = P_{2N} = \theta^* + \frac{q_H}{2} + \frac{(q_H+\theta^*-P_{2R})^2}{2q_H}$,

$P_{2R} = \frac{2\theta_H-\theta^*}{3}$, where $\theta^* = \frac{4\theta_H-(9-3\sqrt{5})q_H}{8}$.

The expected profits of the monopolist over both periods can be expressed as:

$E\pi^* = \frac{P_{2R}^2(2P_{2R}-(\theta_H+\theta^*))}{2q_H\theta_H} + \frac{(\theta_H-\theta^*)[2q_H^2+4q_H\theta^*+\theta^{*2}]}{2q_H\theta_H}$.

(iii) The equilibrium with less than full coverage arises when $\frac{q_H}{\theta_H} < 1.745$.

(iv) $P_{1N} > P_{2R}$.

When the monopolist can commit to future prices at the beginning of the first period, he never chooses to attract new customers in the second stage. As a result, price discrimination is based only upon the past behavior of the consumer. The result that the monopolist does not find it optimal to attract new customers in the future is similar to that derived in the Durable Good Monopolist literature (Coase (1972), Stokey (1979)). In this literature, the profits of the monopolist are highest if he can credibly commit not to lower prices to new customers in the future. Hence, regardless of whether the monopolist sells a durable good (in the Durable Good Monopolist literature) or an experience based non-durable good (in the current setting) such a commitment reduces the incentives of new customers to postpone their purchase. Similar to the nonexistence result of conditional buying by all that is reported in part (i) of Proposition 1, here as well, when the market is less than fully covered conditional buying by all returning customers cannot arise under the "First Best" outcome. Instead, the price set by the monopolist in the second stage is sufficiently low so that there is always a segment of returning customers who buy the service a second time for all q realizations.

Part (iii) of the proposition states that the monopolist finds it optimal to abandon part of the market only when there is sufficient prior heterogeneity in the consumer valuation of the existing features, namely only when the ratio $\frac{q_H}{\theta_H}$ is sufficiently small. The last part of the Proposition indicates that with commitment power the monopolist chooses to reward loyalty by cutting prices to returning customers. The cut in price is so significant, in fact, that returning customers of very high valuation for the existing version choose to purchase the service again for all values of q. In contrast, in Section 4 we show that when the monopolist cannot commit to future prices and can base pricing on the purchase history of the customer (regimes B and BI), he charges returning customers a higher price than new customers.

2.4 PRICE DISCRIMINATION REGIMES WITHOUT COMMITMENT POWER TO FUTURE PRICES

Given that conditional buying by all can never arise under the BI and I regimes when the market is less than fully covered in both periods, next we explore the possible existence of the second type of conditional buying; specifically, conditional buying by the lower tail of returning customers. We start by considering the BI regime that arises when the monopolist has no capacity to commit to ignoring some observables when setting prices. Subsequently, we consider the regimes when some partial commitment is feasible. Specifically, in the B regime the commitment is that the monopolist will ignore the period of purchase when offering a deal to a new customer. Hence, "early adopters" or "late adopters" are treated equally. In the I regime the commitment is that the monopolist will not treat returning customers differently from new customers.

2.4.1 Behavioral and Intertemporal Price Discrimination (BI Regime):

Under the BI regime, the monopolist price discriminates based on the customer's past purchase history and time of purchase. Assuming that the market is less than fully covered, the second stage payoff function of the monopolist when he uses both behavioral and intertemporal price discrimination can be expressed as follows:

$$\pi_2^{BI} = \int_{P_{2N}-\frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{P_{2R}} \frac{1}{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H} dq\, d\theta + \int_{P_{2R}}^{\theta_H} \frac{P_{2R}}{\theta_H} d\theta.$$

The revenues of the monopolist in the second stage accrue from three different segments of consumers: a segment of new consumers, $\theta \epsilon \left[P_{2N} - \frac{q_H}{2}, \theta^*\right]$, a segment of returning consumers whose decision to purchase again depends on the q realization, $\theta \epsilon [\theta^*, P_{2R}]$, and a segment of returning consumers who always buy the service a second time, $\theta \epsilon [P_{2R}, \theta_H]$. Optimizing the second stage profit with respect to P_{2N}

and P_{2R} and substituting the optimal prices back into π_2^{BI}, we can express the second stage profit as a function of θ^* as follows:

$$\pi_2^{BI*} = \frac{{P_{2N}^*}^2}{\theta_H} + \frac{{P_{2R}^*}^2 (P_{2N}^* - \theta^*)}{\theta_H q_H}$$

Using the relationship between first and second period prices that is implied from the utility equation (6), and the optimal second period profit as derived above, the first stage optimization problem of the monopolist can be expressed as:

$$\max_{\theta^*} \pi^{BI} = \frac{(\theta_H - \theta^*)}{\theta_H}\left[P_{2N}^* + \frac{(q_H - P_{2R}^* + \theta^*)^2}{2q_H}\right] + \pi_2^{BI*}.$$

The result of the optimization is summarized in the next proposition.

Proposition 3

Under behavioral and intertemporal price discrimination (BI) with less than full coverage of the market:

(i) The prices set by the monopolist can be expressed as:

$$P_{2N}^* = \frac{\theta^*}{2} + \frac{q_H}{4},$$

$$P_{2R}^* = \frac{2\theta^* + \sqrt{{\theta^*}^2 + 6q_H(\theta_H - \theta^*)}}{3},$$

$P_{1N}^* = \frac{\theta^*}{2} + \frac{q_H}{4} + \frac{(q_H + \theta^* - P_{2R}^*)^2}{2q_H}$, where θ^* solves the equation:

$$\frac{1}{2}\frac{(\theta_H - \theta^*)}{\theta_H} + \frac{(q_H - P_{2R}^* + \theta^*)(q_H + P_{2R}^* - \theta^*)}{q_H(3P_{2R}^* - 2\theta^*)}\frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_{2R}^* + \theta^*)(q_H + P_{2R}^* + \theta^*)}{2\, q_H \theta_H} = 0.$$

(ii) The BI equilibrium with less than full coverage exists if: $\frac{q_H}{\theta_H} < 0.8322$.

(iii) The expected profits of the monopolist over both periods can be expressed as:

$$E\pi^* = \frac{P_{1N}^*(\theta_H - \theta^*)}{\theta_H} + \frac{{P_{2N}^*}^2}{\theta_H} + \frac{{P_{2R}^*}^2(P_{2R}^* - \theta^*)}{\theta_H q_H}.$$

(iv) $P_{1N}^* > P_{2N}^*$ and $P_{2R}^* > P_{1N}^*$.

From part (iv) of the proposition, the monopolist lowers his price to new customers in the second period in order to expand his market. In comparison to the prices they pay in the first period, experienced customers face higher prices in the second period. This implies that the pricing strategy of the monopolist does not reward loyalty. Specifically, the monopolist charges a low price to new consumers in the first period in order to encourage them to experiment with the new service. Once these consumers choose to continue the service a second time, the monopolist charges them a higher price. The monopolist takes advantage of the fact that returning customers tend to have higher valuations for the new service as can be implied by their decision to buy the service a second time. Because the monopolist does not cover the entire market in the first period he also has an incentive to lower prices further to new consumers in the second period in order to encourage consumers with relatively low valuation for the basic features to change their minds and experiment with the new version. Note that in the absence of commitment power, the monopolist violates two characteristics of the optimal pricing under the "First Best" outcome. First, instead of rewarding the loyalty of returning customers the monopolist actually penalizes loyalty. Second, instead of keeping prices constant over time to new customers in order to discourage them from postponing their purchase, the monopolist cuts prices to new customers in the second stage.

According to part (ii) of the Proposition, the equilibrium leads to less than full coverage in both periods only when the value of $\frac{q_H}{\theta_H}$ is sufficiently small (less than 0.8322.) Hence, like in the "First Best" case, only if the prior heterogeneity in the consumer valuation of the existing features is sufficiently large in comparison to the heterogeneity in their valuation of the new features, the monopolist finds it optimal to serve only a portion of the population of consumers.

2.4.2 Behavioral Price Discrimination (B Regime)

In this section, we derive the equilibrium for behavioral price discrimination where the monopolist price discriminates based only on the past purchase behavior of a consumer by charging different prices to new and returning customers. We designate these prices by P_N and P_R, for new and repeat customers, respectively. Consumers are forward-looking and incorporate the future expected benefit derived in the second period in their decision on whether to buy the new version in the first period. The overall expected utilities over both periods when buying and not buying the new service, respectively, are as follows:

$$EU(\theta|Buy\ in\ period\ 1) = \frac{q_H}{2} + \theta - P_N + \int_{P_R-\theta}^{q_H} \frac{q+\theta-P_R}{q_H} dq + \int_0^{P_R-\theta} \frac{0}{q_H} dq. \tag{8}$$

$$EU(\theta|Not\ buy\ in\ period\ 1) = 0 + Max\left\{\theta + \frac{q_H}{2} - P_N\ ,\ 0\right\} \tag{9}$$

Note that a consumer who did not buy the new version in the first period remains a new consumer in the second period as well thus still facing the price P_N if she considers buying in the second period. This explains the second term in the expected utility of such a consumer in (9). However, if this consumer chooses not to buy the new version in the first period she will continue to do so in the second period as well because she faces the same price for the service in either period and has no additional information about the upgraded service. As a result, no new customers will be attracted in the second period under this regime.

With conditional buying by only the lower tail of returning customers, the second period expected profit of the monopolist can be written as follows:

$$\pi_2^B = \int_{\theta^*}^{P_R} \frac{1}{\theta_H} \int_{P_R-\theta}^{q_H} \frac{P_R}{q_H} dq\, d\theta + \int_{P_R}^{\theta_H} \frac{P_R}{\theta_H} d\theta. \tag{10}$$

Optimizing the objective (10) with respect to P_R yields the second stage equilibrium price. For a given θ^*, the second period equilibrium price for the service and the second stage equilibrium profit are summarized as follows:

$$P_R^* = \frac{2\theta^* + \sqrt{\theta^{*2} + 6q_H(\theta_H - \theta^*)}}{3},$$

$$\pi_2^{*B} = \frac{P_R^{*2}(P_R^* - \theta^*)}{\theta_H q_H}.$$

In the first stage the monopolist chooses P_N to maximize his expected profits over both periods.

$$\max_{P_N} \pi^B = \frac{(\theta_H - \theta^*)}{\theta_H} P_N + \pi_2^{*B}, \tag{11}$$

Substituting into the equation $EU(\theta | Buy\ in\ period\ 1) = EU(\theta | Not\ buy\ in\ period\ 1)$ the expression derived for P_R^* from the second stage equilibrium, we can obtain the relationship between P_N and θ^*, the threshold consumer who is indifferent between buying and not buying the new version as follows:

$$P_N = \theta^* + \frac{q_H}{2} + \frac{(q_H + \theta^* - P_R^*)^2}{2q_H}. \tag{12}$$

The monopolist incorporates this relationship in maximizing π^B. The result of the maximization is reported in Proposition 4.

Proposition 4

Under behavioral price discrimination (B) with less than full coverage of the market and conditional buying by the lower tail:

(i) The prices set by the monopolist can be expressed as:

$$P_R^* = \frac{2\theta^* + \sqrt{\theta^{*2} + 6q_H(\theta_H - \theta^*)}}{3},$$

$P_N^* = \theta^* + \frac{q_H}{2} + \frac{(q_H + \theta^* - P_R^*)^2}{2q_H}$, where θ^* solves the equation:

$$\theta_H - 2\theta^* - \frac{q_H}{2} + \frac{(q_H - P_R^* + \theta^*)(q_H + P_R^* - \theta^*)}{q_H(3P_R^* - 2\theta^*)} \frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_R^* + \theta^*)(q_H + P_R^* + \theta^*)}{2\, q_H \theta_H} = 0.$$

(ii) The equilibrium with less than full coverage and conditional buying by the lower tail exists if: $\frac{q_H}{\theta_H} < 1.47588$.

(iii) The expected profits of the monopolist can be expressed as:

$$E\pi^* = \frac{P_N^*(\theta_H - \theta^*)}{\theta_H} + \frac{{P_R^*}^2(P_R^* - \theta^*)}{\theta_H q_H}.$$

(iv) $P_R^* > P_N^*$.

As in the bi-dimensional regime BI in the uni-dimensional regime B, as well, the price of the new service for returning customers is higher than it is for new customers. As in the earlier regime, the monopolist reduces the price to new consumers in order to encourage them to experiment with the new version. Once these consumers choose to renew their subscription a second time, the monopolist charges them a higher price. The monopolist takes advantage of the fact that returning customers tend to have higher valuations for the service, as can be implied by their decision to buy the service a second time. This is in contrast to the "First Best" case, where the monopolist charges returning customers a lower price than new customers. In essence, under behavioral price discrimination, the monopolist rewards loyalty when he can commit to future prices, but punishes loyalty when he does not have such commitment power.

In contrast to the BI regime, however, the monopolist does not attract any new customers in the second stage because he does not lower the price to new customers in this stage, given that only past behavior and not time is utilized as a price discrimination device. Note that according to Proposition 1, when $1.6 > \frac{q_H}{\theta_H} > 1.48$, there is conditional buying by all returning customers under the B regime, an outcome that can never arise under the BI regime when the market is less than fully covered.

2.4.3 Intertemporal Price Discrimination (I Regime)

Under intertemporal price discrimination, the monopolist price discriminates based on time only by charging different prices in the first and second periods. He does not discriminate in his pricing, however, between new and returning customers in spite of the fact that he has access to the subscription histories of different consumers. Let P_1 represent the price of the new version in the first period and P_2 its price in the second period. Similar to the analysis in the previous cases, we have to derive the added expected utility from buying the new version in the first period in order to determine the segment of consumers who buy in the first period.

$$EU(\theta|Buy\ in\ period\ 1) = \frac{q_H}{2} + \theta - P_1 + \int_{P_2-\theta}^{q_H} \frac{q+\theta-P_2}{q_H} dq + \int_0^{P_2-\theta} \frac{0}{q_H} dq. \quad (13)$$

$$EU(\theta|Not\ buy\ in\ period\ 1) = 0 + Max\left\{\theta + \frac{q_H}{2} - P_2\,,\ 0\right\} \quad (14)$$

Unlike in the B regime but similar to the BI regime, with intertemporal price discrimination a consumer who did not buy the new version in the first period may face a new price P_2 in the second period. As a result, consumers may choose to buy the new version in the second period even though they did not do so in the first. However, only if $P_2 < P_1$ there might be some consumers who choose to postpone their purchase to the second period. From (13) and (14) if $P_2 \geq P_1$, $EU(\theta|Buy\ in\ period\ 1) > EU(\theta|Not\ buy\ in\ period\ 1)$ for all θ values, implying that all the consumers who choose to buy the new version, do it in the first period. If the monopolist chooses to attract new customers in the second stage $P_2 < \theta^* + \frac{q_H}{2}$ and a new segment of consumers defined by the interval $[\theta_n, \theta^*]$ purchases the new version in the second stage, where $\theta_n = P_2 - \frac{q_H}{2}$.

With conditional buying by only the lower tail of returning customers, we can express the second period expected profit of the monopolist as follows:

$$\pi_2^I = \int_{P_2 - \frac{q_H + q_L}{2}}^{\theta^*} \frac{P_2}{\theta_H} d\theta + \int_{\theta^*}^{P_2} \frac{1}{\theta_H} \int_{P_2-\theta}^{q_H} \frac{P_2}{q_H} dq\, d\theta + \int_{P_2}^{\theta_H} \frac{P_2}{\theta_H} d\theta. \quad (15)$$

The monopolist chooses P_2 to maximize (15). Differentiating π_2^I with respect to P_2 and solving the first order condition for P_2, yields the second period equilibrium price and profits as a function of θ^* as summarized below:

$$P_2^* = \frac{2(\theta^* - q_H) + \sqrt{\theta^{*2} - 8\theta^* q_H + 7{q_H}^2 + 6 q_H \theta_H}}{3},$$

$$\pi_2^{*I} = \frac{{P_2^*}^2 (q_H + P_2^* - \theta^*)}{\theta_H q_H}.$$

The expansion of the market will occur at the equilibrium only if the solution for P_2^* satisfies the inequality $P_2^* < \theta^* + \frac{q_H}{2}$, which holds for all values of $\frac{q_H}{\theta_H}$ that support this equilibrium.

In the first stage the monopolist chooses P_1 to maximize his profits over both periods. Assuming that an expansion of the market occurs implies the following optimization:

$$\max_{P_1} \pi^I = \frac{(\theta_H - \theta^*)}{\theta_H} P_1 + \pi_2^{*I}, \quad (16)$$

The value of θ^* can be expressed in terms of the price P_1 by solving the equation $EU(\theta|Buy\ in\ period\ 1) = EU(\theta|Not\ buy\ in\ period\ 1)$ (from (13) and (14)) after substituting P_2^* from the second stage equilibrium. Proposition 5 is implied by this maximization:

Proposition 5

Under intertemporal price discrimination (I) with less than full coverage of the market:

(i) The prices set by the monopolist can be expressed as:

$$P_2^* = \frac{2(\theta^* - q_H) + \sqrt{\theta^{*2} - 8\theta^* q_H + 7q_H{}^2 + 6q_H\theta_H}}{3},$$

$P_1^* = P_2^* + \frac{(q_H + \theta^* - P_2^*)^2}{2q_H}$, where θ^* solves the equation:

$$\left(\frac{q_H + P_2^*}{3P_2^* - 2\theta^* + 2q_H}\right)\frac{(\theta_H - \theta^*)}{\theta_H} + \frac{(q_H - P_2^* + \theta^*)(q_H + P_2^* - \theta^*)}{q_H(3P_2^* - 2\theta^* + 2q_H)}\frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_2^* + \theta^*)(q_H + P_2^* + \theta^*)}{2\,q_H\theta_H} = 0.$$

(ii) The equilibrium with less than full coverage exists if: $\frac{q_H}{\theta_H} < 1.28367$.

(iii) The expected profits of the monopolist can be expressed as:

$$E\pi^* = \frac{P_1^*(\theta_H - \theta^*)}{\theta_H} + \frac{P_2^{*2}(q_H + P_2^* - \theta^*)}{\theta_H q_H}.$$

(iv) $P_1^* > P_2^*$.

Similar to the BI regime, with intertemporal discrimination as well, the monopolist finds it optimal to lower the price to new customers in the second stage in order to expand his market coverage. Because low valuation consumers choose not to purchase the service in the first period the monopolist is tempted to lower the price further in the second period in order to encourage some of these low valuation consumers to join the market. However, in contrast to the BI regime the monopolist does not discriminate based upon past behavior of the consumers, and as a result, both new and returning customers benefit from the same lower price charged in the second stage. The pattern of prices to returning customers is similar, therefore, to that derived under the "First Best" outcome, where the monopolist rewards returning customers by offering them a lower price in the second period. However, under intertemporal price discrimination the same lower price is offered also to new customers, thus leading to some customers postponing their purchase to the second period.

2.4.4 Comparison of the Three Price Discrimination Regimes

In Proposition 6 we compare the three price discrimination regimes in terms of profitability and market coverage.

Proposition 6

(i) $\pi^I > \pi^B > \pi^{BI}$.

(ii) $(\theta_H - \theta^*)^B > (\theta_H - \theta^*)^I > (\theta_H - \theta^*)^{BI}$.

According to Proposition 6 when the monopolist cannot commit to future prices his profits are highest if he bases discrimination only on time, and lowest under the two dimensional price discrimination regime that incorporates both behavior and time in pricing. Behavioral price discrimination that bases discrimination only on the past subscription history of the consumer is ranked as intermediate between the other two regimes. Intertemporal price discrimination duplicates the pattern of prices in the "First Best" outcome by rewarding returning customers with a lower price in the second period. However, in contrast to the “First Best”, the same lower price is offered also to new customers in the second period. This introduces two counteracting effects on the incentives of consumers to purchase the new service early. On the one hand, low valuation consumers for the basic version can anticipate that the monopolist will lower prices in the future in order to expand his market coverage, thus providing them an incentive to postpone their purchase in order to benefit from the lower prices. On the other hand, however, high valuation consumers for the basic service do not have to worry about being held hostage by the monopolist when they choose to renew their subscription. They have a stronger incentive, therefore, to experiment with the service in the first period, thus providing them with the opportunity to assess their valuation of the new features early and allowing them also to benefit from an extra period of consumption. According to part (i) of the Proposition, the latter favorable effect is so significant that the regime of intertemporal price discrimination yields the highest profits to the monopolist. According to part (ii) of the Proposition, it is regime B, however, that encourages the biggest number of customers to buy early as there is no incentive for a new customer to postpone purchase to the second period. The BI regime yields the smallest number of customers buying the service early because of the combined effect of encouraging low valuation consumers (for the basic version) to postpone the purchase in order to benefit from lower prices to new customers in the future and high valuation consumers (for the basic version) to be less inclined to purchase early because of their concern of facing higher prices as returning customers in the future.

From the results reported in Proposition 6 it follows that the monopolist benefits most from the regime where only time is incorporated in pricing. Recall, however, that this regime may not be easy to implement. The monopolist has to find a credible way of convincing consumers that he will never use their purchase history to charge higher prices to experienced customers. Using anonymizing technologies that make it hard to identify repeat customers may offer the monopolist a vehicle to implement this regime. Alternatively, contemporaneous Most Favored Customer clauses that offer to reimburse customers if any other customer is charged a lower price in a given period can also facilitate implementing this regime. In the absence of such commitment devices, the most likely regime to arise is regime BI, where both behavioral and intertemporal price discrimination arises. Because the monopolist sells subscription services he can observe customers’ past history and distinguish between new and returning customers. In addition, because he has limited ability to commit to future prices, he will have an incentive to vary the price to new consumers over time in order to expand his market and attract additional relatively low valuation consumers. Hence, the bi-dimensional price discrimination regime (regime BI) is the most sensible outcome that is likely to arise in this case.

2.4.5 No Price Discrimination

In the addition to the B and I regimes where the monopolist possesses a mechanism to partially commit to different segments of the consumers, we now consider another possible regime where the monopolist can commit never to utilize price discrimination; either based on time or on purchase history. Specifically, he can commit to selling the service for the same price in both periods and credibly communicate to consumers that the price he sets for the product early on will remain the same in the future irrespective of whether they are new or returning customers. Note that this type of commitment is actually stronger than the one utilized under the B or I regimes, given that the same price has to be credibly promised to two different types of consumers. However, it falls short of the full commitment case assumed under the "First Best" outcome, in which case the monopolist has full flexibility to vary prices across groups and credibly communicate to each group future prices early in the first period. We characterize in Proposition 7 the equilibrium that arises when a policy of no price discrimination is feasible.

Proposition 7

When a commitment to practice a policy of no price discrimination is feasible, the monopolist sets the same price for both periods and no new customers are attracted in the second period. When the market is less than fully covered in both periods, there is conditional buying by the lower tail only. The price set by the monopolist and his expected profits are given as:

$P_1 = P_2 = \frac{q_H}{4} + \frac{\theta_H}{2}$,

$E\pi^* = \frac{\theta_H}{2}\left(1 + \frac{q_H}{2\theta_H}\right)^2$.

The equilibrium with less than full coverage exists when $\frac{q_H}{\theta_H} < 1.488$.

In Table 1 we conduct numerical calculations to illustrate the characterization of the three price discrimination regimes as well as the no price discrimination regime. Note that in order to obtain prices, the entries in the Table should be multiplied by θ_H. To obtain profits, the profit columns should be multiplied by ${\theta_H}^2$.

	BI Regime					B Regime				I Regime				No Price Discrimination			With Commitment			
q_H/θ_H	θ^*	P_{2N}	P_{2R}	P_{1N}	Profit	θ^*	P_R	P_N	profit	θ^*	P_2	P_1	profit	θ^*	$P_2=P_1$	Profit	θ^*	P_R	P_N	Profit
0.100	0.578	0.314	0.640	0.321	0.492	0.457	0.549	0.508	0.550	0.492	0.501	0.542	0.550	0.466	0.300	0.551	0.471	0.510	0.540	0.551
0.200	0.556	0.328	0.676	0.343	0.536	0.422	0.593	0.524	0.602	0.472	0.506	0.574	0.603	0.433	0.350	0.605	0.443	0.519	0.581	0.605
0.300	0.534	0.342	0.709	0.368	0.583	0.390	0.633	0.546	0.657	0.446	0.513	0.603	0.657	0.399	0.400	0.661	0.414	0.529	0.621	0.662
0.400	0.512	0.356	0.740	0.393	0.631	0.359	0.670	0.569	0.713	0.417	0.523	0.631	0.715	0.366	0.450	0.720	0.385	0.538	0.662	0.722
0.500	0.490	0.370	0.770	0.418	0.683	0.330	0.705	0.595	0.772	0.386	0.535	0.658	0.775	0.332	0.500	0.781	0.357	0.548	0.702	0.784
0.600	0.468	0.384	0.799	0.444	0.736	0.300	0.739	0.622	0.834	0.355	0.547	0.686	0.837	0.299	0.550	0.845	0.328	0.557	0.743	0.849
0.700	0.446	0.398	0.827	0.470	0.792	0.271	0.771	0.650	0.898	0.323	0.560	0.713	0.903	0.265	0.600	0.911	0.299	0.567	0.783	0.917
0.800	0.423	0.412	0.854	0.497	0.849	0.242	0.802	0.678	0.965	0.290	0.573	0.740	0.971	0.231	0.650	0.980	0.271	0.576	0.824	0.987

Table 1: Comparison of Price Discrimination Regimes

According to the numerical results reported in Table 1, a commitment to "no price discrimination" policy yields higher profits to the monopolist than either one of the three regimes of price discrimination we have considered. In particular, “no price discrimination” dominates the most profitable intertemporal price discrimination regime I. The main reason being, that a constant price over both periods eliminates the incentive of consumers to postpone their purchase. Note, however, that profits with a constant price are still lower than profits that accrue under the “First Best” outcome, when the monopolist has full commitment power and chooses to reward the loyalty of returning customers by charging them a lower price.

It is noteworthy that the ranking of the different pricing regimes in terms of their profitability would likely remain the same if we considered a model with more than two periods of consumption, as long as the market remained less than fully covered in each of the periods. With less than full market coverage, the monopolist would still have incentives to cut prices to new customers in later periods in order to expand his market coverage. However, with multiple periods this incentive would likely weaken over time as a bigger portion of consumers had already joined the market. In addition, in the absence of commitment power, the monopolist would still have incentive to hike the price to returning customers even in a multi-period model. This incentive would likely weaken, once again, over time as the expanded market coverage implies that a bigger number of consumers with relatively low valuations of the basic service become part of the group of returning customers.

2.5 PRICING WITH FULL MARKET COVERAGE

So far, we focused attention on analyzing the different pricing regimes when there is less than full coverage of the market in both periods. Less than full coverage implies that in each period there are some consumers who have not tried the new service as of yet. Hence, our focus has been on relatively new and growing markets that are far from reaching their saturation stage. In the present section we investigate how the different pricing regimes compare when the market is fully covered, namely when the market has reached a more mature stage where all consumers have tried the new service. Using the notation we use in the paper, when $\theta^* = 0$. It is noteworthy that with full coverage of the market, the monopolist no longer has incentives to lower his price to new consumers in the second period (under the BI and I regimes) in order to attract consumers with relatively low valuation for the basic service who choose to refrain from consumption in the first period. In the absence of such incentives on the part of the monopolist, consumers no longer have any reason to postpone their purchase to the second stage in anticipation of lower prices in the future. As well, full coverage of the market implies also that our environment becomes very similar to the environment modeled in Jing (2011). Specifically, Jing assumes that there is uni-dimensional heterogeneity in the consumer population in terms of their valuations, and that apriori, all consumers are identical. The consumers’ uncertainty is fully resolved after experimenting with the product for one period. The monopolist designs his pricing to induce all consumers to engage in experimentation. In our model the heterogeneity of consumers is bi-dimensional, and even before consumers experiment with the new service, their valuation of the basic service is different. The pricing of the monopolist determines which segment of consumers chooses to experiment early. However, when the market is fully covered, the latter strategic choice of the monopolist disappears, and like in Jing, the monopolist designs his pricing to induce all the consumers to experiment with the product early.

When there is full market coverage in the first period, the market is no longer segmented into those who buy and do not buy in the first period, and the monopolist faces only returning customers in the second period. The pricing problem of the monopolist reduces to finding a price to inexperienced customers in the first period and a price to experienced customers in the second period. The BI regime is equivalent, therefore, to the B regime, as there are no new customers in the second period. Moreover, because there are no incentives to cut the price to new consumers in the second period, the I regime is also equivalent to the other two price discrimination regimes.

A consumer of type $\theta^* = 0$, evaluates her expected surplus as the sum of her expected utility in the first period and her expected utility of possibly buying the service again in the second period, contingent on her experience with the new features in the first period. The monopolist chooses the first period price to extract the entire expected surplus of a consumer of type $\theta^* = 0$. Specifically,

$$\frac{q_H}{2} + \theta^* - P_N + \int_{P_R-\theta^*}^{q_H} \frac{q+\theta^*-P_R}{q_H} dq + \int_0^{P_R-\theta^*} \frac{0}{q_H} dq = 0. \tag{17}$$

Following a similar approach to that used in deriving the equilibrium with less than full coverage, the two-stage optimization yields the equilibrium characterized in Proposition 8.

<u>Proposition 8</u>

When $\frac{q_H}{\theta_H} > 1.47588$, $\theta^* = 0$ and the market is fully covered under all pricing regimes. In this case, all three regimes yield identical prices and profits. Specifically,

(i) When $1.47588 < \frac{q_H}{\theta_H} < 1.5$ there is conditional buying by only the lower tail of returning customers, and:

$$P_1 = P_N = \frac{q_H}{2} + \frac{\left[q_H - \sqrt{\frac{2}{3} q_H \theta_H}\right]^2}{2 q_H},$$

$$P_2 = P_R = \sqrt{\frac{2}{3} q_H \theta_H},$$

$$E\pi^* = q_H + \frac{\theta_H}{3} - \frac{1}{3}\sqrt{\frac{2}{3} q_H \theta_H},$$

$$P_2 > P_1.$$

(ii) When $1.5 < \frac{q_H}{\theta_H}$, there is conditional buying by all returning customers, and:

$P_1 = P_N = \frac{20q_H{}^2 - 4q_H\theta_H + \theta_H{}^2}{32q_H}$,

$P_2 = P_R = \frac{q_H}{2} + \frac{\theta_H}{4}$,

$E\pi^* = \frac{28q_H{}^2 + 4q_H\theta_H + 3\theta_H{}^2}{32q_H}$,

$P_2 > P_1$ if $\frac{q_H}{\theta_H} < 2.9$.

(iii) When the market is fully covered and the monopolist has full commitment power to set future prices, the "First Best" is characterized by conditional buying by only the lower tail of returning customers, and:

$P_1 = \frac{18q_H{}^2 - 12q_H\theta_H + 4\theta_H{}^2}{18q_H}$,

$P_2 = \frac{2}{3}\theta_H$,

$P_1 > P_2$.

According to Proposition 8 when the extent of prior heterogeneity in the consumer valuation is relatively small (i.e., $\frac{q_H}{\theta_H} > 1.47588$) the monopolist finds it optimal to serve the entire market right away. In this case the equivalence result we report in the Proposition is similar to that derived in Jing when the price to returning customers (referred to as "full information price" in Jing) exceeds the consumer's average valuation of the product. In this case, Jing finds that behavioral and intertemporal price discrimination schemes lead to identical pricing and profits. Even though in his comparison, Jing identifies two distinct cases contingent upon whether the price to returning customers is higher or lower than the average valuation, for the uniform distribution we consider, the former is always the case, and the two pricing mechanisms are equivalent in Jing as well. Note that Jing makes a different assumption concerning consumer expectations than we make in our model. In our model, with behavioral price discrimination, consumers expect the monopolist to offer a different price to new consumers only when there are some unserved customers still to attract (i.e., θ^*>0). However, when the market is fully covered θ^*=0, the consumer does not expect any reduction in prices to new customers in the future (Stokey (1979) makes a similar assumption in her model.) As a result, the equivalence between the B and I regimes would continue to hold in our model even if the distribution functions of the valuations were not uniform. With behavioral price discrimination in Jing, however, even if at the equilibrium no consumer postpones her purchase to the second stage, a consumer still expects that the monopolist has a special price for a deviating consumer who decides to postpone her purchasing of the product to the second stage. Hence, it is this type of "out of equilibrium" pricing that introduces a difference between the B and I regimes for some distribution functions. At any rate, for the uniform distribution, the different assumptions concerning consumer expectations lead to the same equivalence result reported in Proposition 8.

When the market is fully covered the average valuation of a consumer of type θ^*=0 is $\left(0+\frac{q_H}{2}\right)$. It is easy to show from the expressions derived for P_2 in parts (i) and (ii) of the Proposition that $P_2 > \frac{q_H}{2}$, and the price to returning customers exceeds, indeed, the average valuation of the threshold consumer. Note that like in the case with less than full market coverage, the "First Best" outcome with full coverage is still characterized by rewarding the loyalty of consumers (in part (iii) of the Proposition $P_1 > P_2$.) In contrast, in the absence of commitment power, the pattern of prices reported in parts (i) and (ii) of the Proposition is opposite to that derived under the "First Best", and returning customers face a higher price than new customers.

2.6 EFFECT OF CORRELATION BETWEEN THE VALUATIONS OF THE BASIC AND NEW FEATURES

In the analysis so far, we assumed that there is no correlation between the consumer's valuations of the new and existing features. In the absence of any correlation, the only way the consumer can learn about her valuation of the new features is by experimenting with the new service. The consumer's familiarity with her preference for the basic features does not provide her with any valuable information regarding her valuation of the new features. However, if θ and q are not independently distributed, the value of θ does provide valuable information about the value of q, and the importance of learning via experimentation declines. In the extreme case of perfect correlation between θ and q, the consumer can predict her valuation of the new features before even using the new service, and there is no additional learning derived by the consumer when using the new service. In order to assess the effect of such absence of learning, we now consider this extreme case of perfect correlation by assuming that the marginal distributions of θ and q are as we assumed so far (i.e., uniform over$[0,\theta_H]$ and$[0,q_H]$, respectively.) However, a given value of θ uniquely determines the value of q according to the equation $q=\theta\frac{q_H}{\theta_H}$. Hence, the value of q is set at the same decile as that of θ and $\Pr(\theta \le \theta_0) = \Pr\left(q \le \ \theta_0\frac{q_H}{\theta_H}\right) = \frac{\theta_0}{\theta_H}$. In Proposition 9, we characterize the equilibrium with perfect correlation.

<u>Proposition 9</u>

When there is perfect correlation between the consumer's valuations of the basic and new features and the market is less than fully covered, all three pricing regimes are equivalent and identical to the optimal pricing under the "First Best" outcome. Specifically,

$P_{1N} = P_{2N} = P_{2R} = \frac{\theta_H+q_H}{2}$.

$E\pi^* = \frac{\theta_H + q_H}{2}$.

When there is perfect correlation between θ and q, the monopolist charges the same price to new and returning customers and does not vary prices over time. This pricing is also optimal under the "First Best" outcome, when the monopolist has full commitment power. Hence, in the absence of any learning on the part of consumers, it is completely inconsequential whether the monopolist has the capacity to commit to future prices. The "First Best" outcome can be achieved even without any such capacity. This result is reminiscent of a similar result derived in the Durable Good Monopoly literature. In this literature it has been established that by reverting to leasing instead of outright selling of the durable good the monopolist can restore the "First Best" outcome, even in the absence of commitment power to setting future prices. The subscription environment we consider in the present model is similar to the leasing proposed in this previous literature. However, in the durable good monopolist literature it is assumed that the consumer faces no uncertainty and is fully familiar with her valuation of the product. The existence of full correlation in our setting eliminates the uncertainty facing the consumer because she can fully infer the value of q from the observation of θ. It is not surprising, therefore, that like the leasing result obtained in the Durable Good Monopolist literature, in our setting of subscription with perfect correlation, as well, the "First Best" outcome can be achieved even without commitment power. This result indicates that the uncertainty facing the consumer regarding her valuation of the new features and the process of learning via experimentation is the driving force behind the divergence of the results under the various pricing regimes we consider.

Having considered the extreme cases of independence and perfect correlation between the valuations, we now develop a modelling approach that allows us to consider an intermediate case of partial correlation between the valuations. This will allow us to assess the effect of increased correlation on the different pricing regimes we consider.

To consider intermediate values of correlation between θ and q, we introduce a parameter h that measures the probability that θ and q are independently distributed. The complementary probability of $(1-h)$ measures the probability that there is perfect correlation between θ and q, as specified in Proposition 9. Both the monopolist and the consumer are aware of these probabilities and incorporate them in their evaluations. For example, a consumer of type θ expects the value of new service to be $\frac{q_H}{2}$ with probability h and $\theta \frac{q_H}{\theta_H}$ with probability $(1-h)$. Therefore, in the case of the BI regime, the expected utility that a consumer of type θ receives from buying and not buying the upgraded service in period 1 can be expressed as follows:

$EU(\theta|Buy\ in\ period\ 1) = h\left[\theta + \frac{q_H}{2} - P_{1N} + \int_{P_{2R}-\theta}^{q_H} \frac{\theta+q-P_{2R}}{\theta_H}\right] + (1-h)\left[Max\left\{\theta\left(1+\frac{q_H}{\theta_H}\right) - P_{1N}, 2\theta\left(1+\frac{q_H}{\theta_H}\right) - P_{1N} - P_{2R}\right\}\right].$

$EU(\theta|Not\ Buy\ in\ period\ 1) = max\left\{0, h\left(\theta + \frac{q_H}{2}\right) + (1-h)\theta\left(1+\frac{q_H}{\theta_H}\right) - P_{2N}\right\}.$

Note that in case of perfect correlation (with probability $1-h$), a consumer who bought the new service in the first period may or may not buy it again. A consumer of type θ whose valuation of q is equal $\theta\frac{q_H}{\theta_H}$ will not buy the new service a second time for the price P_{2R} if $\theta\left(1+\frac{q_H}{\theta_H}\right) < P_{2R}$ and will buy it a second time in the opposite case.

Assuming less than full market coverage in both periods, the equilibrium with no correlation leads to learning by only the lower tail of returning customers. Assuming that this remains true with partial correlation as well, the second stage payoff function of the monopolist in the BI regime can be expressed as follows:

$$\pi_2^{BI} = \begin{cases} \int_{\hat{\theta}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + h\left[\int_{\theta^*}^{P_{2R}} \frac{1}{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H} dq\, d\theta + \int_{P_{2R}}^{\theta_H} \frac{P_{2R}}{\theta_H} d\theta\right] + (1-h)\int_{\theta_{second}}^{\theta_H} \frac{P_{2R}}{\theta_H} d\theta & \text{when } P_{2R} > \theta^*\left(1+\frac{q_H}{\theta_H}\right) \\ \int_{\hat{\theta}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + h\left[\int_{\theta^*}^{P_{2R}} \frac{1}{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H} dq\, d\theta + \int_{P_{2R}}^{\theta_H} \frac{P_{2R}}{\theta_H} d\theta\right] + (1-h)\frac{(\theta_H-\theta^*)}{\theta_H} & \text{when } P_{2R} < \theta^*\left(1+\frac{q_H}{\theta_H}\right) \end{cases} \quad (18)$$

The first part of the payoff function represents the profit from new customers in the second period. The cutoff value $\hat{\theta}$ represents the threshold customer in the second stage who is indifferent between buying and not buying in the second period. The utility of the threshold consumer can be written as $h\left(\hat{\theta} + \frac{q_H}{2}\right) + (1-h)\hat{\theta}\left(1+\frac{q_H}{\theta_H}\right) - P_{2N} = 0$, the solution to which yields the threshold $\hat{\theta}$. The second integral represents the profit from returning customers if θ and q are independent. The third integral represents the profit from returning customers when there is perfect correlation between θ and q. The expression for this integral depends on the comparison between the values of P_{2R} and $\theta^*\left(1+\frac{q_H}{\theta_H}\right)$. If $P_{2R} > \theta^*\left(1+\frac{q_H}{\theta_H}\right)$, a consumer of type θ^* will not buy the service a second time if $q = \theta^*\frac{q_H}{\theta_H}$. Moreover, only when $\theta > \frac{P_{2R}}{\left(1+\frac{q_H}{\theta_H}\right)} \equiv \theta_{second}$ an experienced consumer will buy the service a second time, where $\theta_{second} > \theta^*$. If $P_{2R} < \theta^*\left(1+\frac{q_H}{\theta_H}\right)$, all the consumers in the region $[\theta^*, \theta_H]$ will buy the service a second time in case of perfect correlation. The equation that characterized the indifferent consumer θ^* can be written as:

$$h\left[\theta^* + \frac{q_H}{2} - P_{1N} + \int_{P_{2R}-\theta^*}^{q_H} \frac{\theta^*+q-P_{2R}}{\theta_H}\right] + (1-h)Max\left\{\theta^*\left(1+\frac{q_H}{\theta_H}\right) - P_{1N}, 2\theta^*\left(1+\frac{q_H}{\theta_H}\right) - P_{1N} - P_{2R}\right\} = \left[h\left(\theta^* + \frac{q_H}{2}\right) + (1-h)\left(\theta^*\left(1+\frac{q_H}{\theta_H}\right)\right) - P_{2N}\right]. \tag{19}$$

Optimizing the second stage profit (18) with respect to P_{2R} and P_{2N} and utilizing (19) to express the first period price as a function of θ^* leads to an optimization problem in the decision variable θ^*. Using the same approach, we derive the equilibrium for the B and I regimes. The analytical approach to solving for θ^* is cumbersome. Therefore, we perform a numerical analysis to compare the three regimes for varying levels of the parameter h that measures the degree of independence between the distributions of θ and q. The results of the numerical analysis are summarized in Table 2.

Parameters		BI Regime				B Regime				I Regime				No Price Discrimination			
q_H/θ_H	h	θ^*	$\theta^*\left(1+\frac{q_H}{\theta_H}\right)$	P_{2R}	Profit	θ^*	$\theta^*\left(1+\frac{q_H}{\theta_H}\right)$	P_R	Profit	θ^*	$\theta^*\left(1+\frac{q_H}{\theta_H}\right)$	P_2	Profit	θ^*	$\theta^*\left(1+\frac{q_H}{\theta_H}\right)$	P_S	Profit
0.2	0	0.5000	0.6000	0.6000	0.6000	0.5000	0.6000	0.6000	0.6000	0.5000	0.6000	0.6000	0.6000	0.5000	0.6000	0.6000	0.6000
0.2	0.2	0.5856	0.7027	0.7027	0.5381	0.4835	0.5801	0.6122	0.5994	0.4961	0.5954	0.5658	0.5995	0.4867	0.5841	0.5893	0.5998
0.2	0.4	0.5713	0.6855	0.6855	0.5370	0.4668	0.5602	0.6129	0.5992	0.4921	0.5905	0.5457	0.5994	0.4732	0.5679	0.5789	0.6002
0.2	0.5	0.5641	0.6770	0.6770	0.5368	0.4589	0.5507	0.6110	0.5993	0.4895	0.5874	0.5377	0.5996	0.4665	0.5598	0.5738	0.6007
0.2	0.6	0.5570	0.6684	0.6684	0.5367	0.4512	0.5415	0.6083	0.5996	0.4865	0.5838	0.5306	0.6000	0.4597	0.5516	0.5689	0.6013
0.2	0.7	0.5499	0.6599	0.6599	0.5369	0.4438	0.5325	0.6050	0.6001	0.4832	0.5799	0.5239	0.6004	0.4530	0.5435	0.5640	0.6020
0.2	0.8	0.5428	0.6514	0.6514	0.5373	0.4365	0.5238	0.6012	0.6007	0.4796	0.5755	0.5176	0.6010	0.4462	0.5355	0.5592	0.6029
0.2	0.9	0.5357	0.6428	0.6428	0.5379	0.4293	0.5151	0.5970	0.6015	0.4757	0.5708	0.5115	0.6018	0.4395	0.5274	0.5546	0.6039
0.2	1	0.5285	0.6342	0.6342	0.5360	0.4221	0.5065	0.5926	0.6024	0.4715	0.5658	0.5055	0.6026	0.4328	0.5194	0.5500	0.6050
0.4	0	0.5000	0.7000	0.7000	0.7000	0.5000	0.7000	0.7000	0.7000	0.5000	0.7000	0.7000	0.7000	0.5000	0.7000	0.7000	0.7000
0.4	0.2	0.5731	0.8024	0.8024	0.6253	0.4739	0.6635	0.7077	0.6985	0.4895	0.6852	0.6434	0.6988	0.4755	0.6657	0.6774	0.6992
0.4	0.4	0.5469	0.7656	0.7656	0.6237	0.4466	0.6252	0.7057	0.6987	0.4768	0.6675	0.6062	0.6994	0.4494	0.6292	0.6560	0.7008
0.4	0.5	0.5339	0.7474	0.7474	0.6241	0.4326	0.6056	0.7022	0.6996	0.4693	0.6570	0.5973	0.7001	0.4359	0.6103	0.6457	0.7025
0.4	0.6	0.5209	0.7292	0.7292	0.6254	0.4185	0.5858	0.6975	0.7011	0.4610	0.6454	0.5764	0.7022	0.4222	0.5911	0.6358	0.7049
0.4	0.7	0.5079	0.7111	0.7111	0.6275	0.4041	0.5658	0.6918	0.7032	0.4517	0.6324	0.5627	0.7043	0.4083	0.5717	0.6263	0.7078
0.4	0.8	0.4949	0.6929	0.6929	0.6305	0.3895	0.5453	0.6852	0.7058	0.4413	0.6179	0.5495	0.7071	0.3943	0.5520	0.6172	0.7113
0.4	0.9	0.5325	0.7455	0.7476	0.6294	0.3747	0.5245	0.6779	0.7092	0.4298	0.6017	0.5364	0.7105	0.3801	0.5321	0.6084	0.7153
0.4	1	0.4685	0.6559	0.6559	0.6392	0.3595	0.5033	0.6700	0.7132	0.4168	0.5835	0.5233	0.7147	0.3657	0.5120	0.6000	0.7200
0.6	0	0.5000	0.8000	0.8000	0.8000	0.5000	0.8000	0.8000	0.8000	0.5000	0.8000	0.8000	0.8000	0.5000	0.8000	0.8000	0.8000
0.6	0.2	0.5621	0.8993	0.8993	0.7115	0.4659	0.7454	0.7994	0.7972	0.4846	0.7753	0.7210	0.7976	0.4657	0.7452	0.7644	0.7981
0.6	0.4	0.5257	0.8412	0.8412	0.7100	0.4291	0.6866	0.7914	0.7985	0.4646	0.7433	0.6687	0.7996	0.4280	0.6848	0.7314	0.8015
0.6	0.5	0.5079	0.8126	0.8126	0.7120	0.4098	0.6556	0.7852	0.8009	0.4524	0.7238	0.6575	0.8005	0.4080	0.6529	0.7160	0.8053
0.6	0.6	0.4901	0.7841	0.7841	0.7157	0.3897	0.6235	0.7779	0.8046	0.4384	0.7015	0.6262	0.8061	0.3874	0.6198	0.7013	0.8104
0.6	0.7	0.4722	0.7556	0.7556	0.7214	0.3688	0.5901	0.7695	0.8096	0.4224	0.6758	0.6065	0.8113	0.3661	0.5858	0.6874	0.8169
0.6	0.8	0.4542	0.7267	0.7267	0.7291	0.3471	0.5553	0.7601	0.8161	0.4037	0.6459	0.5871	0.8181	0.3443	0.5508	0.6742	0.8249
0.6	0.9	0.4956	0.7929	0.8085	0.7286	0.3242	0.5188	0.7498	0.8241	0.3816	0.6106	0.5674	0.8266	0.3217	0.5148	0.6617	0.8342
0.6	1	0.4172	0.6675	0.6675	0.7511	0.3002	0.4804	0.7386	0.8338	0.3547	0.5675	0.5470	0.8374	0.2985	0.4776	0.6500	0.8450
0.8	0	0.5000	0.9000	0.9000	0.9000	0.5000	0.9000	0.9000	0.9000	0.5000	0.9000	0.9000	0.9000	0.5000	0.9000	0.9000	0.9000
0.8	0.2	0.5521	0.9937	0.9937	0.7968	0.4591	0.8263	0.8885	0.8954	0.4807	0.8653	0.7978	0.8961	0.4571	0.8228	0.8504	0.8964
0.8	0.4	0.5072	0.9129	0.9129	0.7959	0.4138	0.7449	0.8726	0.8983	0.4546	0.8183	0.7311	0.8996	0.4087	0.7357	0.8055	0.9021
0.8	0.5	0.4853	0.8735	0.8735	0.8002	0.3894	0.7010	0.8632	0.9029	0.4382	0.7888	0.7172	0.9008	0.3825	0.6885	0.7849	0.9086
0.8	0.6	0.4635	0.8342	0.8342	0.8076	0.3637	0.6546	0.8529	0.9098	0.4191	0.7543	0.6771	0.9113	0.3550	0.6391	0.7655	0.9176
0.8	0.7	0.4415	0.7948	0.7948	0.8184	0.3364	0.6055	0.8417	0.9192	0.3964	0.7134	0.6519	0.9207	0.3263	0.5873	0.7474	0.9291
0.8	0.8	0.4193	0.7547	0.7547	0.8328	0.3073	0.5531	0.8296	0.9313	0.3689	0.6641	0.6269	0.9332	0.2962	0.5332	0.7305	0.9433
0.8	0.9	0.4579	0.8242	0.8662	0.8338	0.2760	0.4968	0.8165	0.9463	0.3348	0.6026	0.6012	0.9495	0.2646	0.4763	0.7147	0.9602
0.8	1	0.3726	0.6707	0.6707	0.8739	0.2421	0.4358	0.8023	0.9646	0.2900	0.5221	0.5734	0.9707	0.2314	0.4165	0.7000	0.9800

Table 2: Comparison of Price Discrimination Regimes in the Presence of Correlation.

In the Table we consider only values of the ratio $\frac{q_H}{\theta_H}$ that support the existence of the equilibrium with less than full market coverage under all regimes when h=1, the case we considered so far (according to Propositions 3-5 $\frac{q_H}{\theta_H} < 0.8322$). For this range of values, it can be verified that under the BI regime for small values of h the monopolist sets the price to returning customers equal to $\theta^*\left(1+\frac{q_H}{\theta_H}\right)$, so that with full correlation each returning customer ends up purchasing the new service a second time. For bigger degrees of independence, when h is close to 1, the price to returning customers is strictly higher than $\theta^*\left(1+\frac{q_H}{\theta_H}\right)$, implying that in the case of full correlation only a subset of returning customers with relatively high valuations for the basic service buy the new service a second time. We also find that in the B regime the price to returning customers is set strictly above $\theta^*\left(1+\frac{q_H}{\theta_H}\right)$ and in the I regime, it is set strictly below $\theta^*\left(1+\frac{q_H}{\theta_H}\right)$, implying that with full correlation and the B regime only a subset of returning customers and in the I regime all returning customers buy the service a second time.

In addition to the three price discrimination regimes we also include in the Table the regime of "no price discrimination" that we discussed in Section 4.5. From the Table we can observe that the ranking of the regimes remains the same as established for the case that h=1, even in the presence of correlation between θ and q. We can observe that perfect correlation (i.e., h=0) leads to equivalence among the four pricing regimes, consistent with the result reported in Proposition 9. As the extent of correlation decreases (h increases) the difference in profits among the regimes increases. This indicates that as the extent of correlation decreases, the importance of learning through experience is more significant, thus leading to greater divergence in profitability among the regimes.

2.7 CONCLUSION

Coase (1972) suggested that a monopolist selling a durable good can overcome time inconsistency issues related to lack of commitment power by leasing the durable good. Although the current trend of selling software subscriptions corresponds to such a leasing strategy, we demonstrate in this paper that it does not necessarily resolve the time inconsistency problem when there is uncertainty regarding the value of the new technology. This uncertainty creates intertemporal incentives for the consumers and the monopolist that are absent in the earlier literature. We investigate the effect of such uncertainty on price discrimination strategies when selling a new technology in a subscription environment. When the monopolist lacks the ability to commit to future prices, we find that the choice of optimal pricing strategy depends on whether or not the entire population of consumers chooses to adopt the new technology early. If there is a segment of the population who postpone the purchase of the technology, we find that the best price discrimination strategy is intertemporal price discrimination, where only time, and not past behavior of the customer, serves as a basis for discrimination. Moreover, if the monopolist can credibly commit never to adopt any form of price discrimination, his profits are even higher. Even though it may be difficult to enforce commitment devices that promise not to differentiate between new and returning customers or not to vary prices in the future, especially in the B2C markets, firms can establish a reputation for following such policies by not offering introductory discounts that tend to favor new over returning customers and by not varying prices over time. When the entire population of consumers chooses to adopt the technology early, we find that intertemporal and behavioral price discrimination strategies yield identical profits. Regardless of whether or not all consumers are "early adopters" though, the monopolist cannot achieve the "First Best" outcome that requires full commitment power to credibly communicate future prices. However, when the consumer's familiarity with the existing features of the technology provides her with a perfect signal of her valuation of the new features of the technology (because there is possibly perfect correlation between the valuations) the consumer no longer faces any uncertainly and Coase's prediction is restored, namely the "First Best" outcome is attainable even without any commitment device.

We make several simplifying assumptions to illustrate the effect of learning on the pricing strategies. Relaxing some of these assumptions is unlikely to change our qualitative results. For instance, the assumption that valuations are uniformly distributed in the population keeps the model analytically tractable, while allowing us to measure the extent of heterogeneity in the consumer population in terms of a single parameter; namely the spread of the distribution. Relaxing this assumption by considering a general distribution is unlikely to change our qualitative results. For simplification, we also consider a two period model to illustrate the comparison among the pricing regimes. Extending the model to multiple periods is unlikely to provide any additional insights, as we illustrate our results in both growing and saturated markets. In addition, we do not allow the monopolist to alleviate some of the uncertainty facing the consumer regarding the value of the new technology. In practice, most service providers offer free trials for a limited time to encourage consumers to experiment in a risk free environment. However, the trials, being limited in nature, are unlikely to resolve completely the uncertainty facing the consumer.

3 ESSAY 2: VERSIONING AND EX-POST BUNDLING OF NEW SERVICES IN SUBSCRIPTION MARKETS

3.1 INTRODUCTION

Driven by cost effectiveness and ease of implementation, cloud computing is driving the consumerization of Information technology. Companies offering software as a service (SaaS) such as Salesforce, Jive, Workday, and Intacct offer cloud based services, ranging from CRM and marketing to accounting and HR solutions. There is large variability in the pricing schemes utilized by these firms, and innovation in pricing may be the driver of growth and profitability in the highly competitive market. Most of the cloud based software services are offered on a subscription basis, ranging from monthly to annual contracts. Versioning is a very popular strategy in this space, with services providers offering several editions of their flagship services, to address the distinct requirements of businesses of different sizes. For example, Salesforce offers 5 versions of its flagship CRM software called Sales Cloud. Following the success of their flagship services, companies invest in new services and enhanced features that help to deepen their existing customer relationships. For example, Salesforce extended its CRM software to include software that supports customer service and marketing functions. When introducing new services, the important decision facing a multi-product or multi-service firm is to choose the right selling strategy. The service provider can either sell the new service on its own, or offer the new service as a part of the existing offerings.

The new service may be offered along with the primary service in different ways. It may be included with all versions of the primary service, or selectively, with only the higher versions of the primary service. A common practice among firms offering cloud-based services is to offer the new service at no additional cost with higher versions of the primary service and to sell the new service separately for customers subscribing to lower-end versions. For example, Salesforce offers new services, such as Salesforce Identity and Private App Exchange, at no additional cost to customers subscribing to the premium versions of Sales Cloud. The premium version of the primary service can therefore be considered as a bundle comprised of the primary service at a higher quality level and future service enhancements in the form of new services. This can be treated as a special case of the traditional mixed bundling strategy. In the traditional mixed bundling strategy, the service provider offers for sale both a bundle and individual products, so that consumers who are interested in both products may buy the bundle, and consumers who are interested in one product but not in the other may buy that product separately. However, in the context of the examples cited above where the primary service is offered in multiple versions, bundling may be more complicated. While the new service is offered separately in addition to being offered as a part of the bundle, the primary service is not offered separately at the same quality level as the one included in the bundle. We can apply insights from traditional bundling literature to analyze the pricing of new service introductions. However, the pricing problem differs from the existing bundling literature in the following way: new services are introduced at a later point in time after the quality levels and the prices for the various versions of the primary service are announced. The service provider lacks the flexibility to vary the price of primary service when new services are introduced in a later period due to the presence of price agreements that restrict price changes over time. The service provider should therefore, account for future service enhancements when choosing the price schedule for the primary service.

In this paper, our goal is to investigate the pricing strategies of a service provider offering vertically differentiated variants of a service and introducing new services that either enhance or extend the existing service offerings. We analyze two strategies that are commonly practiced by firms selling information goods and software services: Discriminative Bundling (DB) and Independent Pricing (IP). Using the discriminative bundling strategy (DB), a service provider, offering multiple versions of the primary service, offers new services at no additional cost to customers buying premium versions of the primary service, and sells the new service separately to remaining customers. Using the independent pricing strategy (IP), the service provider offers the new service separately to all consumers, including those buying lower and higher end versions. There may be other variations of such bundling strategies in practice, depending on the number of versions of the primary service, and the compatibility of the new service with the primary service. For example, one less common form of bundling is bundling the new service with all versions of the primary service, in addition to selling new service separately for non-subscribers. We focus our attention on the most general cases of discriminative bundling and independent pricing, as the insights from comparing the two extreme cases can be applied to other less common cases as well. The price of the premium version reflects future enhancements.

We analyze the pricing strategies in a two-period monopoly setting. We assume that a service provider offers only two versions of the primary service: a basic version and a premium version. In the first period, the service provider announces the quality levels of the basic and of the premium versions, and the corresponding schedule of prices. We assume that the quality levels of the two versions are exogenously determined, and that the monopolist chooses the optimal price for the basic and the premium versions based on the quality levels. The monopolist introduces a new service in the second period, and has a choice between selling the new service independently to all customers and offering the new service at no additional cost to customers subscribing to the premium version. A consumer can derive a positive value from using the new service either on its own or in combination with the primary service. At the beginning of the second period, the monopolist announces the pricing strategy and the corresponding standalone price for the new service when sold separately. Consumers make their purchase decisions based on the price schedule for the primary service, and the standalone price for the new service. We assume that the price of the basic and the premium versions in the second period remain the same as in the first period to reflect the commitment issues in practice. Firms have very little flexibility to change the subscription prices of the primary services from one period to another due to the complexity inherent in subscription contracts and the potential negative impact of strategic consumer behavior on profits (Coase (1972)). It is not uncommon to see firms maintaining the same subscription prices for more than five years. The monopolist can anticipate future service enhancements and account for them in choosing the current period prices. Therefore, the monopolist chooses the optimal price schedule for the primary service by taking into account the future enhancements and the pricing strategy for the new service. In the second period, when introducing the new service, the monopolist only chooses the optimal standalone price for the new service, based on the first period prices of the primary service.

As shown in the existing literature (Bhargava and Choudhary (2001)), versioning of the primary service is optimal only when consumer valuations for the service are non-linear in nature. Non-linear valuations can explain two important features of information goods and services: (1) higher consumer types gain a greater increase in utility than do lower types for the same increase in the quality of service, and (2) consumers do not necessarily gain greater value from increasing the quality of service, due to the disutility from including too many features (Thompson et.al. (2005)). Therefore, consumer valuations for the primary service may be described by a non-linear function. The primary service can be versioned based on the number of features included in each version, or by the amount of usage allowed in each version. For example, Sales Cloud is versioned based on the type and number of features offered in each version, while Force.com is versioned based on the number of custom objects that are included in each version[3]. While the primary service is suitable for versioning, the new service may not be suitable for versioning. For example, a new service, such as the Private App exchange is not suitable for versioning as it represents a unique functionality that can aid a consumer in achieving a clearly defined goal. In addition, depending on the nature of new service, consumers'valuations for the primary and the new service may be correlated and the degree of correlation may depend on the nature of the new service. For new services that represent a direct extension to the primary service, valuations may be positively correlated and the degree of correlation may be relatively high. On the other hand, for new services that represent a radically new functionality not directly related to the primary service, the degree of correlation may be relatively low and may even be negative. We, therefore, model consumer valuations for the new

[3] The website provides details of various products of Salesforce and the available versions. http://www.salesforce.com/crm/editions-pricing.jsp.

service using a linear function, and analyze the effect of the nature of the relationship between the new and primary service on the optimality of pricing policies.

We contribute to the literature on bundling and versioning by combining the unique aspects of each pricing strategy. We compare the discriminative bundling (DB) and the independent pricing (IP) strategies in terms of their profitability for two types of valuation function for the new service. We first analyze the pricing strategies, assuming that the new service is a direct extension to the primary service and consumer valuations for the new service are perfectly correlated with their preference for the primary service. All consumers agree on the quality level of the new service, and the heterogeneity arises only due to the heterogeneity in consumer's preference for the primary service. In such a setting, we find that discriminative bundling unambiguously leads to lower profits when compared to the independent pricing strategy. Mixed bundling allows a monopolist offering two services to more efficiently extract the surplus from fringe customers who have very high valuation for one service and not the other. However, when consumer valuations for the two services are positively correlated, this cannot be realized as consumers with very high valuation for one service also have a high valuation for the other service. When the monopolist has the flexibility to price the bundle so as to maximize the second period profit, there is no difference in profits that can be achieved from the DB and IP strategies. This result is similar to the findings in the existing literature. However, when the monopolist does not have the flexibility to vary the price of the premium version over time, he charges a lower price for the bundle when compared to the no commitment price to balance the loss in revenue from selling the premium version to fewer consumers in the first period. Due to the presence of perfect correlation between consumer valuations for the new and the primary service, the standalone price for the new service directly depends on the price of the bundle and it is also lower when compared to the no commitment case. The decrease in the price of the new service, combined with a less than proportionate increase in the price of the premium version, lead to lower profits in the DB strategy.

We next analyze the case where the new service is a radically new functionality not directly related to the primary service. Although, we ideally want to study the case where the valuation for the new service is completely independent of the primary service, it is too complex to solve analytically and does not lead to closed form solutions. We therefore, assume that the preference for the new service within a segment of consumers is independent of their preference for the primary service but introduce an idiosyncratic component in the valuation for the new service that depends on whether the new service is used separately, or along with the premium version. When the value that can be obtained from the new service by using it along with the premium version is higher (lower) than the value that be obtained by using it without the premium version, it represents complementarity (substitutability) between the new and primary services. This approach allows us to understand how the nature of the relationship between the new and primary services influences the pricing strategies. In this setting, we find that the comparison of profits in the DB and IP strategies is not unambiguous as it was in the previous case. Specifically, the DB strategy may lead to higher profits than the IP strategy when the heterogeneity among consumer preferences for the primary service is sufficiently large, and when there is sufficiently large difference in the value that can be obtained from the new service when consumed as a part of the bundle and when consumed on its own. Even in this case, commitment to the price of the primary service restricts the monopolist from charging the optimal bundle price and leads to a lower price for the premium version in both periods. However, when valuation for the new service is not directly related to the valuation for the primary service, the monopolist can at least, extract the complete surplus from consumers buying the new service separately.

3.2 LITERATURE REVIEW

Versioning and bundling have been studied extensively in the context of information goods and services. Versioning is the practice of offering vertically differentiated variants of a product at different prices. Earlier studies on vertical differentiation (see Bhargava and Choudhary (2001) for a review) show that it is an optimal strategy, in the presence of non-zero marginal costs that increase either proportionally, or more rapidly, as quality increases. In the context of information goods, where marginal costs are negligible, earlier studies show that versioning is not optimal when consumer valuations are linear (Bhargava and Choudhary (2001), Varian (1997)). Subsequent studies analyzed the effect of non-linear consumer valuations (Bhargava and Choudhary (2004)), positive network effects (Bhargava and Choudhary (2004) and Jing (2007)), competition (Jones and Mendelson (2011) and X. Wei and Nault (2006)) and antipiracy goals (Chellappa and Shivendu (2005)), and show that versioning of information goods is optimal under certain conditions. In a recent paper (X. D. Wei & Nault (2014)) study the optimality of versioning when consumers with horizontal taste preferences are linked by vertical group tastes.

Bundling, as a price discrimination strategy, has received tremendous interest from researchers and practitioners alike (see Venkatesh and Mahajan (2009) for a detailed review of the literature in bundling). In the context of information goods, where marginal costs are negligible, Bakos and Brynjolfsson (1999) show that bundling a large number of unrelated information goods is optimal. On the other hand, Geng, Stinchcombe, and Whinston (2005) show that pure bundling is sub-optimal when consumer valuations decrease with an increase in the number of products in the bundle. Wu, Hitt, Chen, and Anandalingam (2008) develop a non-linear mixed integer programming approach to price a customized bundle of information goods. However, none of the above studies consider the combination of bundling and versioning that is quite popular in cloud based software services. We bridge the gap in the literature by developing a two-period model, with price commitment, that reflects the decision process of service providers in practice. Similar to the existing literature, we find that bundling is less profitable than independent pricing when consumer valuations for the new and the primary service are positively correlated. Since the monopolist starts charging the bundle price from the first period even before the introduction of new service, a higher bundle price will lead to a lower number of consumers buying the premium version in the first period. Therefore, the monopolist chooses an optimal price for the bundle that maximizes overall profits in contrast to a bundle price that maximizes only the current period profits. However, when the value derived from the new service is different when consumed as a part of the bundle and on its own, bundling can lead to higher profits when compared to independent pricing when there is sufficiently large heterogeneity in consumer preferences for the primary service and in the presence of strong complementarity or substitutability between the new and the primary service. This is in contrast to findings in existing literature (Venkatesh and Kamakura (2003)), where mixed bundling leads to lower profits when compared to pure components when the bundled products are strong substitutes. When products are strong substitutes, the bundle is less valuable and consumers buy their preferred product separately. The divergence in the results is due to the fact that the type of mixed bundling we consider varies from the traditional definition of mixed bundling. Unlike previous studies, in our setting a consumer who has a high preference for the premium version does not have the option of buying the premium version outside the bundle. Therefore, even in the presence of strong substitutability, bundling the new service with the premium version does not discourage consumers from buying the bundle. .

3.3 MODEL SETUP

A monopolist offers a primary service at more than one quality level. The quality level may represent the number of features or the amount of good that can be used by the customer. Without loss of generality, we assume that the monopolist offers two versions of the service: a basic version (L) of lower quality $v_L > 0$ and a premium version (H) of higher quality v_H $(v_H > v_L)$. All consumers agree that the quality of the premium version is higher than the quality of the basic version. However, consumers are heterogeneous in their preferences for the services of the firm. We denote the heterogeneity by a parameter θ that is distributed uniformly over the interval $[0, \theta_H]$. Let P_L denote the price of the basic version (L) and P_H the price of the premium version (H). We assume that the utility that a consumer of type θ derives from using the service of quality v can be represented by the following function:

$$U(\theta, v) = \theta v - \frac{v^2}{2}.$$

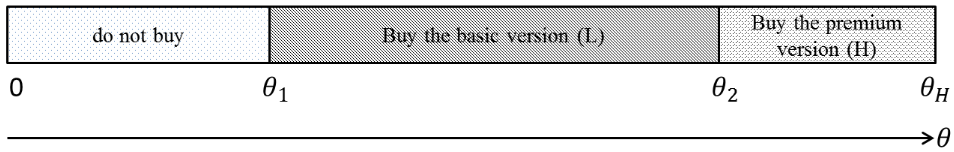
Higher customer types or customers with a high preference for the services of the firm get greater utility from using the same quality as $U_{\theta}(\theta, v) > 0 \; for \; all \; v > 0$. In addition it satisfies the Spence-Mirrlees single-crossing property. A consumer of type θ derives a utility of $U(\theta, v)$ in each period of consuming the service. In contrast to the utility form described in Sundararajan (2005) and Hui et.al. (2007), we assume that the marginal utility of a consumer decreases after attaining a maximum. Each customer of type θ has a unique quality level v^{*} that maximizes her utility. This indicates that a customer does not necessarily gain higher utility from an increase in the quality level of the service, and may even suffer a decrease in utility from consuming a product with $v > v^{*}$. The literature on information services pricing indicates that consumer utility from information goods and software services is best described by a non-linear utility function. The assumption of a non-linear utility function helps describe the important feature of the service that high type consumers derive a larger increase in value than do lower types from the same increase in the quality of the service. In addition, it is not optimal for a monopolist to offer more than one version or quality level of a service unless the marginal utility of a customer is non-constant (Bhargava and Choudhary (2004)).

The monopolist announces the quality levels and price schedule for the various versions of the primary service at the beginning of the first period. Based on the quality levels and prices, consumers choose the version that maximizes their surplus. This segments the consumer population into three groups: those who do not subscribe to any version, those who subscribe to the basic version, and those who subscribe to the premium version.

3.3.1 Market Segmentation with Pure Versioning

Given the two versions L and H with quality levels v_L and v_H, and their corresponding prices P_L and P_H, the surplus derived by a consumer of type θ is given by $U(\theta, v) - P$. The segmentation of the market is as shown in Fig 1. There are two cutoff points that divide the population of consumer into three segments. Let θ_1 denote the type of threshold customer who is indifferent between not subscribing to either version and subscribing to the lower quality version L. Let θ_2 represent the type of threshold customer who is indifferent between buying the basic (L) and the premium version (H).

Figure 1: Segmentation of consumers under pure versioning.



We can derive the threshold customer type θ_1 and θ_2, who is indifferent between buying the basic version and not buying at all and who is indifferent between buying the basic version and the premium version respectively, by solving the following equations:

$$U(\theta_1, v_L) - P_L = 0, \tag{1}$$

$$U(\theta_2, v_L) - P_L = U(\theta_2, v_H) - P_H \,. \tag{2}$$

Equations (1) and (2) also ensure that the incentive compatibility constraint and individual rationality conditions are automatically satisfied.

Determination of the cutoff points yields the size of the customer segments for the basic version and for the premium version. We can derive the payoff function of the monopolist by establishing the profit that can be gained from each of these segments as follows:

$$\max_{P_L,P_H} \pi_1 = \frac{1}{\theta_H}\int_{\theta_1}^{\theta_2} P_L \, d\theta + \frac{1}{\theta_H}\int_{\theta_2}^{\theta_H} P_H \, d\theta \tag{3}$$

The result of the maximization is summarized in Proposition 1.

Proposition 1

A monopolist offering a service at two qualities levels v_L and v_H can maximize his profits by charging the following prices at the equilibrium:

$$P_L = \frac{1}{4} v_L (2\theta_H - v_L).$$

$$P_H = \frac{1}{4} v_H (2\theta_H - v_H).$$

The equilibrium exists only when $\theta_H > \frac{v_H + v_L}{2}$.

Pure versioning is optimal only when the heterogeneity among consumers measured by the parameter θ_H, is greater than the average quality of the primary service. In the absence of sufficient level of heterogeneity among consumers, it is not profitable for the monopolist to offer a lower quality version. Pure versioning does not take into account any future product or service enhancements. However, in practice, firms continuously invest in developing new services and features that can either complement the existing service or expand the services of the firm. In the next section, we model the decision problem of a monopolist offering a new service in the second period, and analyze the pricing strategies.

3.4 PRICING NEW SERVICE INTRODUCTIONS

The monopolist introduces a new service (N) in the second period. The new service can be used on its own, or it can be used in conjunction with the primary service. The new service may represent a limited functionality that is not suitable for versioning and can be offered only at a single quality level. Let q denote the quality level of the new service. The utility that a consumer derives from using the new service is linear in nature, so that all consumers, irrespective of whether they are low or high types, derive the same increase in utility from a marginal increase in the quality of the service. We, therefore, assume that the consumer utility from using the new service is of the form $U(\theta, q) = \theta q$. For example, Salesforce introduced new features, such as Salesforce Identity and Private App Exchange, which represent a functionality that is independent of the primary service. Those features are offered at a single quality level, and both non-subscribers and subscribers of the primary service can derive a positive value from using the services.

The monopolist may sell the new service in different ways. He may sell the new service on its own independent of the primary service or bundle the new feature with the primary service for no additional price. Depending on the approach, bundling the new service with the primary service leads to an increase in surplus achieved by the consumer in the second period. Although consumer valuation for the primary service does not vary from one period to another, the additional value from the new service can shift the segmentation of consumers inherited from the first period. For example, bundling the new service with all versions of the primary service may encourage some non-subscribers with sufficiently high preferences to begin subscribing to the basic version in the second period. Similarly bundling the new service with only the premium version may encourage subscribers of the basic version to upgrade to the premium version in the second period. We describe each of these strategies in detail, in the next section, and derive the equilibrium for each case.

3.4.1 Independent Pricing Equilibrium

In independent pricing, the monopolist sells the new service separately to all consumers. The pricing of the new service is completely independent of the price schedule for the primary service. Both subscribers and non-subscribers of the primary service buy the new service separately if their valuation for the service is sufficiently high. If we let P_N denote the price of the new feature offered as a standalone service, a consumer of type θ will buy the new service if $\theta q > P_N$.

The monopolist can choose the price for new service by solving the following maximization problem:

$$\max_{P_N} \pi_2 = \int_{\frac{P_N}{q}}^{\theta_H} \frac{P_N}{\theta_H} d\theta + \pi_1. \tag{4}$$

Solving the optimization problem in one variable yields the optimal price for the new service $P_N = \frac{\theta_H q}{2}$. The profit from the new service is $\frac{\theta_H q}{4}$.

The second period payoff to the monopolist is the sum of the profits from the primary service and from the new service. As consumer valuations for the primary service do not change from one period to another, the payoff from the primary service remains the same as in the case of pure versioning. The monopolist continues to charge the same optimal prices for the basic and the premium versions as described in Proposition 1. The new service leads to an expansion in the segment of consumers who buy the services offered by the monopolist, as $\frac{P_N}{q} < \theta_1$.

3.4.2 Discriminative Bundling Equilibrium

Using discriminative bundling, which is a form of mixed bundling, a firm offers both a bundle and pure components for sale to a heterogeneous group of consumers. However, using discriminative bundling, the monopolist offers the bundle to a select group of customers, and sells the services separately to the remaining customers. In the context of versioning, the firm may offer the bundle only to subscribers of premium versions, while selling the component services separately to remaining customers. For example, Salesforce bundles new features such as Salesforce Identity and Private App Exchange, with premium versions of Sales Cloud. Subscribers to the lower end versions are required to pay for the new features separately.

While the existing literature provides extensive analyses of versioning and bundling, considering the two strategies separately, our work extends the literature by analyzing pricing strategies that combine the two popular price discrimination mechanisms. Although versioning can be considered as the offering bundles of various sizes depending on the number of features included in each, the separation in time between announcing the price schedule for the primary service and introducing the new service lead to interesting dynamics behavior of the firm and the consumers. As the new service is introduced after the quality levels and the price schedule are announced, the monopolist can observe the segmentation of customers in the first period based on the announced prices, and use this information in making the second period decisions. We consider a two-period model, in which the monopolist can account for future expected service enhancements, and choose the price schedule for the primary service that optimizes current and future profits. By observing the segmentation of customers based on the prices announced in the first period, the monopolist can choose the optimal price for the new service in the second period. This modelling approach allows us to take into account the commitment issues that firms face with respect to the pricing of subscription services.

We begin with the second period, and derive the optimal price for the new service as a function of the optimal first period prices for the basic and premium versions. The monopolist inherits the segmentation of consumers from the first period based on the prices for the basic and premium versions, and uses that information to define the segment of consumers who buy the new service. Using the discriminative bundling strategy, the monopolist bundles the new service with the premium version of the primary service. Let P_N be the standalone price of the new service for subscribers of the basic version and non-subscribers to the existing service. Non-subscribers and subscribers to the basic version will buy the new feature if $\theta q > P_N$. Because the price schedule for the primary service remains unchanged in the second period, some consumers who are currently subscribing to the basic version may upgrade to the premium version if the increase in utility from using the new service is greater than the difference in surplus from consuming the basic and premium versions. However, the relative prices of the premium version, the basic version, and the new service determine whether any existing consumers buy the new service separately. When the price of the premium version is greater than the sum of the basic version and the new service, there will be a segment of existing customers who buy the new service separately. On the other hand, if the price of the premium version is less than the sum of the prices for the basic version and the new service, none of the existing customers of the basic version will buy the new service separately. Depending on the relative prices for the primary service and for the new service, three different cases might arise at the equilibrium, as follows:

i) $P_H > P_L + P_N$.

ii) $P_H \leq P_L + P_N$ and $\frac{P_N}{q} < \theta_1$.

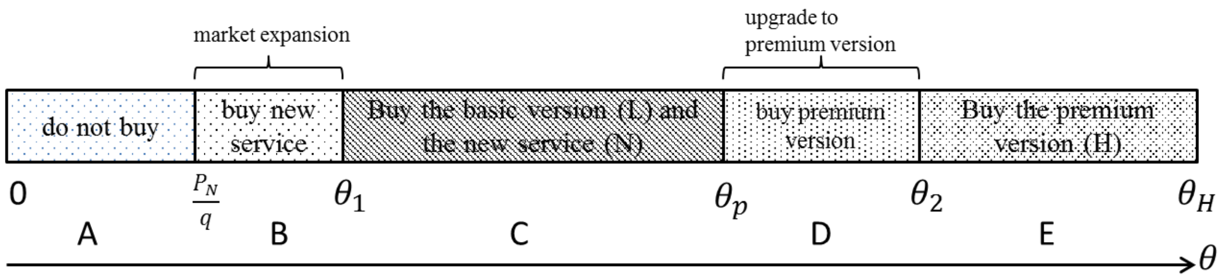
iii) $P_H \leq P_L + P_N$ and $\frac{P_N}{q} > \theta_1$.

We analyze each of these cases in detail, and derive conditions on the parameter values that give rise to each type of equilibrium.

Case (i): $P_H > P_L + P_N$.

When the premium version is more expensive than buying the basic version and the new service separately, some basic version subscribers continue with the same version in the second period also and buy the new service separately. Basic version subscribers with sufficiently high θ will upgrade to the premium version, as the additional utility from the new service θq makes the premium version more attractive. Depending on the prices for the new and primary service, there may be five different segments of consumers in the second period as shown in Fig 2.

Figure 2: Segmentation of consumers in the second period when $P_H > P_L + P_N$.



Segment A is the segment of consumers who do not buy any service. Segment B is the segment of consumers who buy only the new service. Segment B is non-empty only if the price for the new service is sufficiently low, specifically $\frac{P_N}{q} < \theta_1$. Segment C is the segment of consumers who continue to buy the basic version; they buy the new service separately if $P_N < \theta q$. Segment D is the segment of consumers who upgrade to the premium version in the second period. Segment E is the segment of consumers who continue to buy the premium version, as they now get a better service at the same price. Based on these segments, we can derive the payoff function of the monopolist. The second stage optimization problem of the monopolist may be expressed as follows:

$$\max_{P_N} \pi_2 = \int_{\frac{P_N}{q}}^{\theta_p} \frac{P_N}{\theta_H} d\theta + \int_{\theta_1}^{\theta_p} \frac{P_L}{\theta_H} d\theta + \int_{\theta_p}^{\theta_2} \frac{P_H}{q_H} d\theta + \int_{\theta_2}^{\theta_H} \frac{P_H}{q_H} d\theta. \tag{5}$$

The thresholds θ_1 and θ_2 in the second period payoff function are inherited from the first stage, and solve the utility equations in (1) and (2). The θ_p in the payoff function represents the preference of the threshold customer who is indifferent between shifting to the premium version and continuing with the basic version in the second period and buying the new separately. The threshold θ_p can be derived by solving the following utility equation:

$$U(\theta_p, v_L) - P_L + \theta_p q - P_N = U(\theta_p, v_H) + \theta_p q - P_H. \tag{6}$$

The solution of the equation yields the expression for θ_p as a function of the first period prices and the quality of the new service as follows:

$$\hat{\theta} = \frac{(v_H{}^2 - v_L{}^2)}{2(q + v_H - v_L)} + \frac{(P_H - P_L)}{(q + v_H - v_L)}. \tag{7}$$

The first integral in the payoff function given in (5) represents the revenue from selling the new service separately at price P_N to non-subscribers and basic version subscribers of the primary service. The second integral represents the payoff from subscribers of the basic version. The third integral represents the additional payoff from subscribers who shift from the basic version to the premium version after the introduction of the new service. The last integral represents the revenue from customers who continue to subscribe to the premium version.

Solving the second stage maximization problem yields the equilibrium standalone price of the new service as a function of the first period price for the basic and premium versions P_L and P_H as follows:

$$P_N = \frac{q\left[2(P_H - P_L) + v_H{}^2 - v_L{}^2\right]}{4(q + v_H - v_L)}. \tag{8}$$

Note that the standalone price for the new service depends on the difference in the price of the premium and the basic versions. The firm can charge a higher price for the new service when the difference is large. The standalone price for the new service is also an increasing function of the ratio of the quality level of the new service and the difference in the quality levels of the basic and premium versions. As the quality level of the new service increases, the monopolist can charge a higher standalone price for the new service. Substituting the optimal standalone price P_N in the second period payoff function (5) yields the optimal second stage profit as a function of the first period prices for the basic and the premium versions.

Lemma 1

The equilibrium with $P_H > P_L + P_N$ exists when the values of θ_H and $\frac{q}{v_H - v_L}$ fall into one of the regions described below:

a) $\frac{v_H + v_L}{2} < \theta_H < \frac{(2v_H + v_L)}{2}$ and $0 < \frac{q}{v_H - v_L} < 4\left(\frac{\theta_H}{v_H + v_L} - \frac{1}{2}\right)$ or

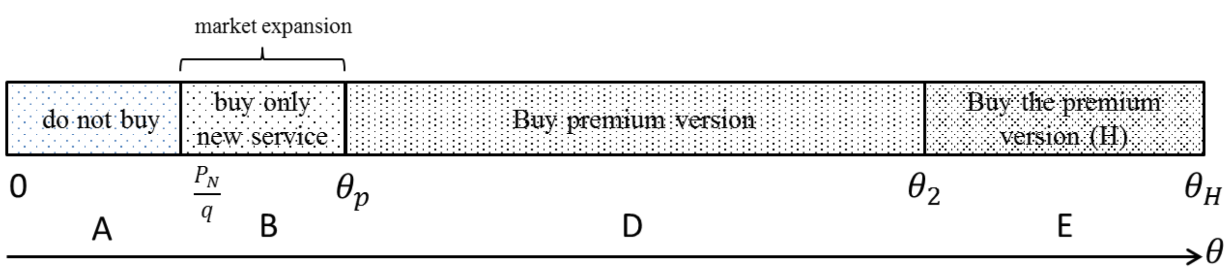
b) $\theta_H > \frac{(2v_H + v_L)}{2}$ and $0 < \frac{q}{v_H - v_L} < \frac{2v_H}{2\theta_H - v_H}$.

For $P_H > P_L + P_N$ to arise at the equilibrium, the heterogeneity among consumers needs to be higher than the average quality of the primary service, the same condition that supports simple versioning. In addition, it requires that the quality level of the new service is sufficiently low. Whereas greater heterogeneity supports finer segmentation in the second period, low quality of the new service allows the monopolist to charge a higher price for the premium version, when compared to the price for the new service and for the basic version. Substituting the optimal standalone price P_N from (8) in the second period payoff function (5) yields the optimal second stage profit.

Case (ii): $P_H \leq P_L + P_N$ and $\frac{P_N}{q} < \theta_1$.

When $P_H \leq P_L + P_N$, buying the premium version is less expensive than buying the basic version and the new service separately. Therefore, consumers with sufficiently high valuations for both the primary service and the new service will upgrade to the premium version in the second period. When the price of the new service is sufficiently low, specifically, $\frac{P_N}{q} < \theta_1$, all the basic version subscribers derive a positive surplus from buying the new service, and, therefore, find it optimal to upgrade to the premium version at a lower price. Therefore, there are no buyers for the basic version in the second period, leading to only three segments of consumers in the second period, as shown in Fig 3.

Figure 3: Segmentation of consumers in the second period when $P_H \leq P_L + P_N$ and $\frac{P_N}{q} < \theta_1$.



Segment A is the segment of consumers who do not buy any service. Segment B is the segment of consumers who buy only the new service. D is the segment of consumers who buy the premium version in the second period, and segment E is the segment of consumers who continue to buy the premium service. Therefore, consumers have a choice between buying only the new service or buying the premium version. Based on these segments, the second stage optimization problem of the monopolist can be expressed as follows:

$$\max_{P_N} \pi_2 = \int_{\frac{P_N}{q}}^{\theta_p} \frac{P_N}{\theta_H} d\theta + \int_{\theta_p}^{\theta_H} \frac{P_H}{q_H} d\theta. \tag{9}$$

The limit θ_p represents the type of the threshold customer who is indifferent between buying only the new service and upgrading to the premium version. The threshold customer type can be derived by solving the following utility equation that compares the utility gained from buying the new service and from upgrading to the premium version:

$$\theta_p q - P_N = U(\theta_p, v_H) + \theta_p q - P_H. \tag{10}$$

The solution to equation (10) yields an expression for θ_p as a function of the first period price for the premium version, the price for the new service, and the quality level of premium version as follows:

$$\theta_p = \frac{P_H - P_N}{v_H} + \frac{v_H}{2}. \tag{11}$$

Solving the maximization problem in (9) yields the equilibrium standalone price of the new service as a function of the first period price for the premium versions P_H as follows:

$$P_N = \frac{q(4P_H + v_H{}^2)}{4(q + v_H)}. \tag{12}$$

Equation (12) shows that the optimal price for the new service depends only on the price for the premium version and the quality levels of the new service and the premium version. As the quality level of the new service increases in comparison to the quality level of the premium version, the monopolist can charge a higher price for the new service. This encourages more consumers to upgrade to the premium version. Substituting the optimal price for new service from (12) in the payoff function in (9) yields the optimal second stage profit as a function of the first period prices.

Lemma 2

The equilibrium with $P_H \leq P_L + P_N$ and $\frac{P_N}{q} < \theta_1$ exists when the values of θ_H and $\frac{q}{v_H - v_L}$ fall into one of the regions described below:

a) $\frac{v_H}{4} < \theta_H \leq \frac{v_H}{2}$ and $q > \frac{2v_H(v_H - 2\theta_H)}{v_H - 4\theta_H}$

b) $\frac{v_H}{2} < \theta_H \leq \frac{v_H + v_L}{2}$ and $q > 0$.

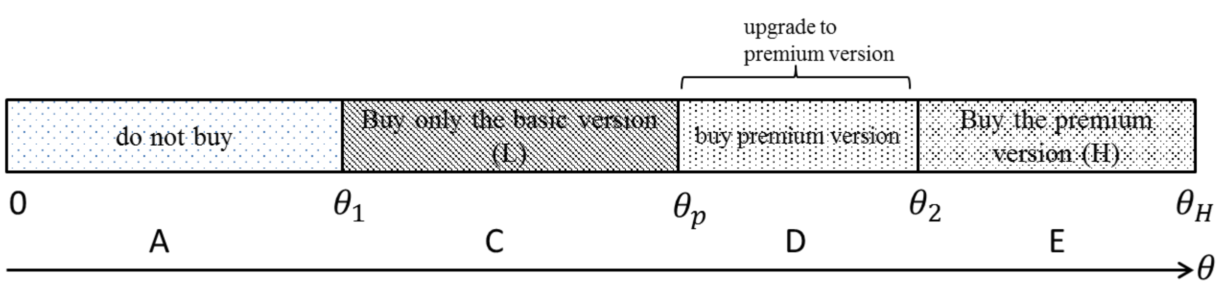
c) $\theta_H > \frac{v_H + v_L}{2}$ and $\frac{q}{v_H - v_L} > \frac{4v_H\left(\theta_H - \frac{v_H + v_L}{2}\right)}{v_H{}^2 - v_L{}^2 + 4v_L\theta_H}$.

Unlike the previous case, the equilibrium with $P_H \leq P_L + P_N$ and $\frac{P_N}{q} < \theta_1$ exists even when the heterogeneity among consumers is smaller than the average quality of the primary services $\theta_H < \frac{v_H + v_L}{2}$, thereby expanding the region of parameter values that support versioning. The equilibrium exists even when the heterogeneity among customers is low, because there is no versioning in the second period as all the basic version subscribers upgrade to the premium version. In addition to lower heterogeneity, we also need the quality of new service to be sufficiently high, so that consumers have a higher incentive to upgrade to the premium version.

Case (iii): $P_H \leq P_L + P_N$ and $\frac{P_N}{q} > \theta_1$.

Similar to the previous case, the premium version is less expensive than buying the basic and the new service separately. However, when the price of new service is sufficiently high, such that $\frac{P_N}{q} > \theta_1$, some of the low type basic version subscribers are not interested in buying the new service. The higher type consumers, who are interested in the new service, may, however, upgrade to the premium version at a lower price, and get both the primary service and the new service. Therefore, no consumers buy the new service separately. This leads to three customers segments in the second period, as shown in Fig 4.

Figure 4: Segmentation of consumers when $P_H \leq P_L + P_N$ and $\frac{P_N}{q} > \theta_1$.



There is no expansion of the market in the second period, as those who did not buy in the first period also do not buy in the second period. Low type consumers who bought the basic version in the first period continue with the basic version. Basic version subscribers with sufficiently high $\theta > \frac{P_N}{q}$ upgrade to the premium version. Based on these segments, the second stage payoff function of the monopolist can be expressed as follows:

$$\pi_2 = \int_{\theta_1}^{\theta_p} \frac{P_L}{\theta_H} d\theta + \int_{\theta_p}^{\theta_H} \frac{P_H}{q_H} d\theta. \tag{13}$$

The limit θ_1 is inherited from the first stage, θ_p represents the type of threshold customer who is indifferent between buying the basic version and upgrading to the premium version. θ_p can be derived by solving the following utility equation:

$$U(\theta_p, v_L) - P_L = U(\theta_p, v_H) + \theta_p q - P_H. \tag{14}$$

The solution to (14) yields an expression for θ_p as a function of the first period prices and the quality of new service, as follows:

$$\theta_p = \frac{P_H - P_L}{v_H - v_L + q} + \frac{{v_H}^2 - {v_L}^2}{2(v_H - v_L + q)}. \tag{15}$$

The monopolist does not have a decision variable in the second period, as there no customers for the new service in this case. The second period profit is a function only of the first period prices for the primary services.

Lemma 3

The equilibrium with $P_H \leq P_L + P_N$ and $\frac{P_N}{q} > \theta_1$ exists when the values of θ_H and $\frac{q}{v_H - v_L}$ are in the range described below:

$$0 < \frac{q}{v_H - v_L} < \frac{2v_H}{v_H + v_L} \text{ and } \frac{(v_H + v_L)[q + 2(v_H - v_L)]}{4(q + v_H - v_L)} < \theta_H < \frac{(q + 2(v_H - v_L))(v_H(v_H - v_L) - qv_L)}{2q(q + v_H - v_L)}.$$

Similar to case (ii), the equilibrium exists even when the heterogeneity among consumers is lower than the average quality level of the primary services. Note that segmentation fails when there is not sufficient heterogeneity among consumers. However, the quality level for the new service should be sufficiently small to ensure that the price for premium version is not too high, and there is still a segment of consumers buying the basic version.

We compare the three cases to find any overlapping regions that support more than one type of equilibrium and compare the profits with the cases. We summarize the findings in Proposition 2.

Proposition 2

The regions that support each of the three cases that may arise at the equilibrium with discriminative bundling are non-exclusive. In the region where more than one type of equilibrium may exist:

a) Case (iii) is always dominated by case (i)

b) Case (i) dominates case (ii) for all values of q when $v_L > (3-\sqrt{5})\theta_H$.

c) Case (ii) dominates case (i) for all values of q when $v_L > \left(\frac{\sqrt{17}-3}{2}\right)\theta_H$ and $v_H < \frac{8\theta_H{}^2 - v_L(v_L+\theta_H) - \theta_H\sqrt{v_L{}^2+32\theta_H{}^2}}{v_L+2\theta_H}$.

d) For intermediate values of v_L and v_H, case (i) dominates case (ii) when $\frac{q}{v_H-v_L} < \frac{v_H\left[\sqrt{v_L{}^2+32\theta_H{}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8\theta_H{}^2}$.

When the new service is bundled with only the premium version, it is always profitable for the monopolist to sell the new service separately in the second period. Case (iii) is always less profitable than case (i) because the price for the premium version is the same in both cases, but the monopolist is losing the revenue he can gain from selling the new service separately under case (iii). This result also indicates that pure bundling of the new service with the primary service is always less profitable than selling the new service separately in addition to bundling it with the premium version. Therefore it is profitable for the monopolist to sell the new service at a sufficiently low price such that there is a segment of consumers who either buy only the new service or buy both the basic version and the new service. However, when the quality of the basic version is sufficiently small, it is optimal for the monopolist to sell the premium version at a price sufficiently low that all basic version consumers upgrade to the premium version.

First Stage Analysis:

Given the optimal second stage profit as a function of the first period prices for the basic and premium versions, the monopolist can now choose the optimal prices for the basic and premium versions that maximize current and future profits. The first stage optimization problem of the monopolist can be expressed as follows:

$$\max_{P_L,P_H} \int_{\theta_1}^{\theta_2} \frac{P_L}{\theta_H} d\theta + \int_{\theta_2}^{\theta_H} \frac{P_H}{q_H} d\theta + \pi_2. \tag{16}$$

The result of the optimization is summarized in Proposition 1.

<u>Proposition 3</u>

Using discriminative mixed bundling, the monopolist charges the following prices for the basic and premium versions of the primary service and the new service:

(i) When $v_L > (3-\sqrt{5})\theta_H$ or $\frac{q}{v_H - v_L} < \frac{v_H\left[\sqrt{{v_L}^2+32{\theta_H}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8{\theta_H}^2}$.

$P_L = \frac{1}{4}v_L(2\theta_H - v_L)$.

$P_H = \frac{2v_H(v_H-v_L)(2\theta_H-v_H)+2q\theta_H(v_H-v_L)+qv_H(2\theta_H-v_H)}{4(q+2(v_H-v_L))}$.

$P_N = \frac{q(v_H-v_L)\theta_H}{q+2(v_H-v_L)}$.

(ii) When $(v_L > \left(\frac{\sqrt{17}-3}{2}\right)\theta_H \text{ and } v_H < \frac{8{\theta_H}^2-v_L(v_L+\theta_H)-\theta_H\sqrt{{v_L}^2+32{\theta_H}^2}}{v_L+2\theta_H})$ or $\frac{q}{v_H-v_L} > \frac{v_H\left[\sqrt{{v_L}^2+32{\theta_H}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8{\theta_H}^2}$.

$P_L = \frac{v_L[4\theta_H(q+v_H)-v_L(q+2v_H)]}{4(q+2v_H)}$.

$P_H = \frac{4\theta_H v_H(q+v_H)-{v_H}^2(q+2v_H)}{4(q+2v_H)}$.

$P_N = \frac{qv_H\theta_H}{(q+2v_H)}$.

The primary difference between traditional bundling and bundling in the presence of versioning is that the premium version is more valuable than the basic version and the new service together. In contrast to the existing literature, a consumer cannot reproduce the bundle by buying the bundled products separately This allows for the premium version to be priced higher than the sum of the prices for the basic version and the new service. On the other hand, restricting the price of premium version to be lower than the sum of the prices for the basic version and the new service eliminates the segment of consumers who buy the basic version and the new service separately. However, it is always optimal for the monopolist to sell the new service separately to non-subscribers and subscribers to the basic version, or to both. We compare the prices and profits under the DB regime against the benchmark case of pricing the new service independently in the next section.

3.4.3 Comparison of Equilibria

In this subsection, we compare the equilibria of the two pricing strategies with respect to prices and profits.

Proposition 4

Using the superscripts DB and IP to represent the discriminative bundling and independent pricing strategies, the comparison of prices across the three regimes are as follows:

$P_L^{DB} \geq P_L^{IP}, P_H^{DB} > P_H^{IP}, P_N^{DB} < P_N^{IP}$.

The price charged for the basic version under the DB regime is either equal to or greater than the price charged under the IP regime. Specifically, the P_L under the DB and IP regimes is the same when the quality level of the new service is low and equilibrium under the DB regime leads to $P_H > P_L + P_N$. When the quality level of the new service is high, the equilibrium under the DB regime leads to case (ii), in which no customers buy the basic version in the second period. Therefore, the monopolist increases the price for the basic version, when compared to the IP regime, to encourage more consumers to upgrade to the premium version. The price for the premium version under the DB regime is higher than the price charged under the IP regime as the premium version receives an enhancement in the second period from the new service. On the other hand, the price of the new service is lower under the DB regime, as the potential market segment for the new service is limited to non-subscribers and subscribers to the basic version.

Proposition 5

The comparison of profits from using discriminative bundling and independent pricing yields the following result:

$\pi^{DB} < \pi^{IP}$.

Independent pricing is always more profitable than discriminative bundling when a consumer's valuation for the new service is perfectly correlated with her valuation for the primary service. Due to price commitment, the monopolist chooses the bundle price that optimizes overall profits in the first and second periods. Using the DB strategy, the monopolist charges a higher price for the premium version when compared to the IP strategy as the premium version receives an enhancement in the second period due to the inclusion of new service. The monopolist begins charging a higher price for the premium version from the first period. As a result, fewer consumers buy the premium version in the first period when compared to the IP environment. In the second period, as bundling the new service makes the premium version more appealing, some basic version subscribers upgrade to the premium version, thereby increasing the segment of consumers buying the premium version. The reduction in the size of the segment of consumers buying the premium version in the first period is exactly equal to the increase in the second period, due to the linear form of valuation function for the new service. By bundling the new service at no additional cost, the monopolist is losing the revenue he can gain from selling the new service separately to the premium version subscribers. Due to the price commitment, the monopolist cannot price the premium version at the optimal bundle price that can be obtained from optimizing just the current period profits. In order to balance the loss in market share for the premium version in the first period, the monopolist chooses a lower bundle price. As the consumer valuations for the new service are perfectly correlated with valuations for the primary service, the standalone price for the new service in the second period is also lower when compared to the price under the IP strategy. The decrease in the price of new service when compared to the IP strategy is given by $\frac{1}{r+2}$, where $r = \frac{q}{v_H - v_L}$. However, the increase in the price of the premium version over the price under the IP strategy is equal to $\frac{1}{2(r+2)}$. The extent of decrease in the price of new service and increase in the price of the premium version under the DB strategy depends on the ratio of the quality of new service and the difference in the quality levels of the basic and the premium versions. The lower price for the new service, combined with an incommensurate increase in the price of the premium version, leads to lower profits using the DB strategy, when compared to pricing the new service independently.

3.5 EFFECT OF INDEPENDENT VALUATIONS AND THE NATURE OF RELATIONSHIP BETWEEN THE NEW AND THE PRIMARY SERVICE

In the previous section, we assumed that all consumers agree on the quality level of a new service, differing only in their preference for the services offered by the monopolist. The form of valuation function that we consider for the new service yields a prefect correlation between consumer valuations for the new and the primary service. This approach is suitable to describe new services that represent enhancements to the primary service and therefore consumer preferences for the new service are correlated with their preference for the primary service. For example, offering an analytics service or a social media feature may represent an enhancement to the primary CRM software. However, for new services that represent a radically new functionality, consumers' valuation for the new service may be independent of their valuation for primary service. For example, Salesforce introduced a new service called Force.com that allows companies to build apps for customers and employees. The new service is not directly related to the primary CRM software and consumers' preference for the new service may be independent of their preference for the primary service. In addition to the degree of correlation between the new and the primary service, the new service can either complement or substitute the primary service depending on the nature of service. For example, consumers using the new service as a part of the premium version may derive a value that is different from the value derived by consumers buying the new service separately. The difference may be due to the value that can be gained from using the new service along with other features that are only offered in the premium version. For example, Salesforce bundles the new service Private App Exchange with higher versions of Sales Cloud that also include features that allow users to build custom apps. The new service may provide additional value to users of the premium version, as it offers a platform upon which they can organize and manage access to apps they create using the premium version. The difference in value gained by premium version and basic version buyers may even represent the disutility that a premium version subscriber can suffer from adding too many features in the premium version. In this section, we develop a modelling approach to analyze the effect of the nature of the relationship between the new and primary services on comparison pricing policies.

Assuming that a buyer derives a different value from the new service when purchased separately and when offered as a part of the premium version, let q_s represent the value of the new service when purchased separately and q_p the value when offered as a part of the premium version. When $q_p > q_s$, the new service is more valuable when offered with the premium version and less valuable when sold separately to basic version subscribers and to non-subscribers. This is the case with new services that complement existing services. Similarly when $q_p < q_s$, the new service is less valuable when offered with the premium version and more valuable to basic version subscribers and to non-subscribers. This is the case with new services that substitute the features in the premium version. Using the same notation for prices as in the previous section, non-subscribers and subscribers to the basic version buy the new service if $q_s > P_N$.

3.5.1 Independent Pricing Equilibrium

As the valuation for new service is binary, the monopolist can select a price for the new service that either encourages all consumers to buy the new service, or only a segment of consumers, depending on the relative ordering of q_s and q_p. When $q_s < q_p$, the monopolist can cover the entire market for the new service by charging $P_N = q_s$, or can sell only to the premium version subscribers by charging $P_N = q_p$. When $q_s > q_p$, the monopolist can sell the new service to all consumers by charging $P_N = q_p$, or can sell to a sub-segment by charging $P_N = q_s$. Considering all the different cases, the equilibrium with independent pricing is summarized in Proposition 6.

<u>Proposition 6</u>

When a consumer's valuation for the new service is independent of her valuation for the primary service and the consumer may derive a higher or lower value from the new service when used along with the premium version, the equilibrium with independent pricing can be characterized as follows:

(i) $q_s < q_p$:

When $q_s > q_p \frac{\left[\frac{q_p}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$P_L = \frac{1}{4} v_L(2\theta_H - v_L)$.

$P_H = \frac{1}{4} v_H(2\theta_H - v_H)$.

$P_N = q_s$.

When $q_s < q_p \frac{\left[\frac{q_p}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$P_L = \frac{1}{4} v_L(2\theta_H - v_L)$.

$P_H = \frac{1}{4}\left[-q_p - {v_H}^2 + 2v_H\theta_H\right]$.

$P_N = q_p$.

(ii) $q_s > q_p$

When $q_p > q_s \frac{\left[\frac{q_s}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$P_L = \frac{1}{4} v_L(2\theta_H - v_L)$.

$P_H = \frac{1}{4} v_H(2\theta_H - v_H)$.

$P_N = q_p$.

When $q_p < q_s \frac{\left[\frac{q_s}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$

$P_L = \frac{1}{4} v_L(2\theta_H - v_L)$.

$P_H = \frac{1}{4}[q_s - {v_H}^2 + 2v_H\theta_H]$.

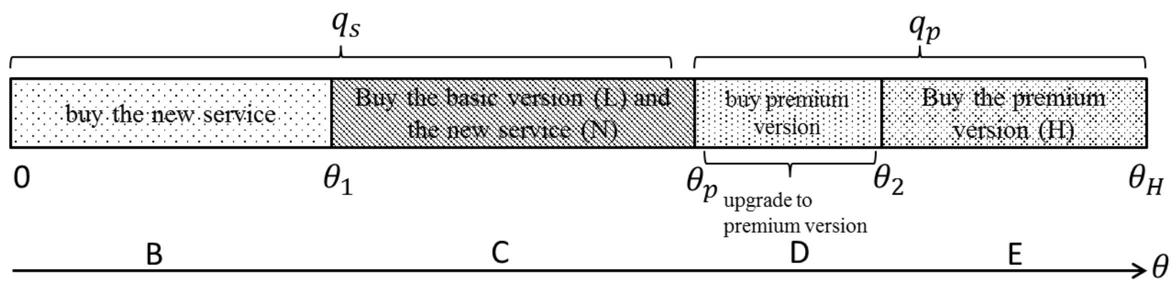
$P_N = q_s$.

Depending on the nature of relationship between the new and the primary service, the equilibrium with independent pricing yields four different cases. When the monopolist prices the new service sufficiently low to cover the entire market, the price for the basic and the premium versions are independent of the valuation for the new service, and they are the same as the prices with pure versioning. When the monopolist sells the new service only to a sub-segment of consumers, depending on whether $q_s > q_p$ or $q_s > q_p$, the prices for the basic and the premium versions are adjusted to account for the size of the segment buying the new service. For example, in case (ii), the price of the premium version is lowered so that there is a larger segment of premium version consumers for the new service in the second period at price $P_N = q_p$.

3.5.2 Discriminative Bundling Equilibrium

Unlike the IP regime, the relative ordering of the quality levels does not influence the price charged for the new service, as premium version consumers do not need to pay for the new service separately. As all consumers in the interval $[0, \theta_2]$ have a valuation equal to q_s for the new service, the monopolist can either cover the entire market by charging a price $P_N = q_s$ or not sell the new service separately by charging a higher price. The segmentation of consumers in the second period, after the introduction of new service, can be described by Fig 5.

Figure 5: Segmentation of consumers in the second period under horizontal differentiation.



There are four segments in the second period. Segment B consumers buy only the new service. Segment C is the group of consumers who buy both the basic version and the new service. Segment D is the segment of buyers who upgrade to the premium version, and Segment E consists of buyers who continue buying the premium version. A basic version subscriber upgrades to the premium version if $U(\theta_p, v_L) + q_s - P_L - P_N > U(\theta_p, v_H) + q_p - P_H$. The thresholds θ_1 and θ_2 are inherited from the first period and are a function only of the prices for the primary service. Using an approach similar to that described in Section 3, we derive the equilibrium with discriminative bundling, considering the more general case of $P_H > P_L + P_N$ that yields full segmentation.

<u>**Proposition 7**</u>

When a consumer's valuation for the new service is independent of her valuation for the primary service and the consumer may derive a higher or lower value from the new service when used along with the premium version, following is the characterization of equilibrium for the DB regime, assuming $P_H > P_L + P_N$:

$P_L = \frac{1}{4} v_L (2\theta_H - v_L)$.

$P_H = \frac{q_s+q_p}{4} + \frac{v_H}{4}(2\theta_H - v_H)$.

$P_N = q_s$.

The equilibrium exists when one of the following conditions holds:

a) $q_s < q_p$ and $\theta_H > \frac{q_s+q_p}{2(v_H-v_L)} + \frac{v_H+v_L}{2}$.

b) $q_s > q_p$ and $\theta_H > \frac{3q_s-q_p}{2(v_H-v_L)} + \frac{v_H+v_L}{2}$.

The price for the basic version remains the same as the price without bundling. The price for the premium version is higher than the price without bundling, and it depends on the distribution of valuations for the new service. The monopolist can capture the entire consumer surplus from the new service unlike the previous case in which valuations are perfectly correlated. The equilibrium characterization of the DB strategy is independent of the ordering of q_s and q_p. The equilibrium exists only when the heterogeneity among consumers is sufficiently high. The lower bound on θ is required to support segmentation, and to satisfy the assumption about prices, $P_H > P_L + P_N$. The heterogeneity should be greater than the average quality level of the primary services due to the presence of idiosyncratic valuations for the new service. Specifically, when the valuation for the new service increases when compared to the difference in the quality levels of the basic and premium versions, the lower bound on θ_H is more restrictive. When the new service is less valuable when offered with the premium version, there should be sufficiently large heterogeneity, with respect to θ, to justify a higher price for the premium version.

3.5.3 Comparison of Equilibria

Proposition 8

The comparison of profits under the independent pricing and discriminative bundling regimes yields the following result when the valuation of new service is independent of the valuation for the primary service within a segment of consumers:

$\pi^{DB} > \pi^{IP}$, when there is sufficiently large differential in the value realized from the new service when consumed as a part of the bundle and on its own, and there is sufficiently large heterogeneity among consumers with respect to the primary service.

When consumers' valuation for the new service is not perfectly correlated with that of the primary service, the comparison of profits under the DB, and the IP strategies is not completely unambiguous.

Discriminative bundling may lead to higher profits when compared to the IP strategy, when there is sufficiently large heterogeneity in consumers' valuations for the primary service and the difference in the valuation of the new service between subscribers to the premium version and other customers, is sufficiently large. When the valuation for the new service is independent of the valuation for the primary service within a segment of consumers, the monopolist does not need to decrease the standalone price for new service, in order to attract the remaining customers. In fact, the monopolist can capture the complete surplus from consumers buying the new service separately in this formulation. The monopolist charges a higher price for the premium version in the first period when compared to the price charged under the IP regime and therefore sells the premium version to a smaller segment of consumers. However, he gains additional consumers for the premium version in the second period by offering the new service at no additional cost. This motivates some consumers to upgrade to the premium version. Unlike the case with perfect correlation, the number of consumers that the monopolist loses in the first period is not the same as the number of consumers that upgrade to the premium version in the second period. The difference depends on the quality levels q_s and q_p. Therefore, with independence in the valuations of the new and the primary service, the comparison of profits in the IP and the DB regimes is not unambiguous as it was in the case of perfect correlation. The comparison depends on the degree of complementarity or substitutability as given by the difference between q_s and q_p, and the extent of heterogeneity among consumers. A greater heterogeneity in consumer valuations for the primary services leads to a larger segment of consumers buying the premium version, and a higher price differential between the basic and premium versions. Therefore, with sufficient large θ_H, the revenue gain that can be achieved using the DB strategy from charging a higher price for the premium version and encouraging a larger number of consumers to purchase the premium version, is greater than the revenue that is lost by offering the new service at no additional cost to premium version subscribers. When the new service is much more valuable to a basic version subscriber or a non-subscriber than it is to a premium version subscriber, it is more profitable for the monopolist under the IP strategy to sell only to the non-premium version subscribers at a higher price than selling to all consumers at a lower price. Using the DB strategy, however, the monopolist can continue to capture the surplus from non-premium version subscribers, while also charging a higher price for the premium version. Therefore when q_s is sufficiently greater than q_s indicating a high degree of substitutability between the new and the primary service, the DB strategy is more profitable than the IP strategy. This result contradicts the findings in existing literature, where mixed bundling is not profitable when products are strong substitutes. The contradiction in results can be explained by the difference in the type of mixed bundling that we consider. In the existing literature, consumers who like one product more than the other have an option of buying their preferred product separately. In the current setting, the premium version is not sold separately outside the bundle. Therefore consumers who have a high valuation for the premium version will end up buying the premium version at a higher price even if they have a low valuation for the new service.

3.6 CONCLUSION

In this paper, we provide insights on an important decision facing a multi-product firm, that of pricing new products or service extensions. The decision is complicated for a firm offering information goods or software services, which are often offered at more than one quality level. In such a setting, the service provider is often faced with a difficult choice of whether to sell the new service on its own or to bundle the new service with all or only certain versions of the existing services. We analyze two strategies commonly used in practice: Bundling the new service with only the higher versions of the existing service (Discriminative Bundling) and selling the new service separately on its own (Independent Pricing). In the absence of any marginal costs of offering the new service in addition to the existing service, we find that the optimal strategy depends on the nature of new service. When a new service represents an enhancement to the primary service and consumers' valuation for the new and the primary service are highly correlated, bundling the new service with the premium version is less profitable than selling it separately to all consumers. When the new service represents a new feature that is not related to the primary service, and the value that a consumer derives from using the new service along with the premium version may be higher or lower than the value obtained by using the new service without the premium version, bundling the new service is more profitable than independent pricing under certain conditions. Specifically, when the heterogeneity in consumer valuations is sufficiently large and the difference in the value realized from using the new service with and without the premium version of the primary service is sufficiently large, bundling leads to higher profits, by capturing the consumer surplus more efficiently. Our results provide valuable and useful insights to firms offering information goods and software services, by laying out recommendations for the optimal pricing strategy to use, based on the nature of the new services and general customer distributions.

Our work is a first step in the direction of analyzing ex-post bundling of new services with the existing versions of the primary service. We make several simplifying assumptions that may influence our results. The presence of factors such as competition, network effects, transaction costs, and consumer learning, in practice, may lead to different conclusions, and may explain practices in the industry that may seem sub-optimal in the context of our model. For example, we assume that there are no marginal costs associated with selling the new service on its own or in the bundle. However, the presence of transaction costs and asymmetry in the costs associated with delivering the primary service for basic and premium version subscribers may influence our results. We assume that consumers are completely aware of their valuations for the new service. Uncertainty with respect to consumer valuations may potentially make the pricing problem dynamic in nature. We also assume certain functional forms to describe consumer valuations for the existing and new services; a different form of the valuation function for either of the services may influence our results. In addition, with the entry of new firms, cloud based software and information services industries are becoming increasingly competitive. Competition among service providers may be an important factor in the pricing decisions of new services. We assume that the firm holds a monopoly position in both the primary and new services. However, competition in either or both services may drive the firm towards a different pricing strategy. Innovations in service delivery and pricing models open fertile areas for future research, and some of the issues that we raise, may offer interesting directions for future research in multi-product pricing.

APPENDIX

Proofs for Essay 1

Proof of Proposition 1:

(i) The second stage payoff function of the monopolist when he uses both intertemporal and behavioral price discrimination is given by (3). The first order conditions for P_{2R} and P_{2N} are as follows:

$$\frac{\partial \pi_2^{BI}}{\partial P_{2R}} = q_H - 2P_{2R} + \frac{\theta_H + \theta^*}{2} = 0,$$

$$\frac{\partial \pi_2^{BI}}{\partial P_{2N}} = \theta^* - 2P_{2N} + \frac{q_H}{2} = 0.$$

The solution to the above equations yields the second stage equilibrium prices P_{2R}^* and P_{2N}^* as follows:

$$P_{2R}^* = \frac{q_H}{2} + \frac{\theta_H + \theta^*}{4}, \quad \text{(A1)}$$

$$P_{2N}^* = \frac{\theta^*}{2} + \frac{q_H}{4}. \quad \text{(A2)}$$

The second stage profits can be expressed as:

$$\pi_2^* = \frac{{P_{2N}^*}^2}{\theta_H} + \frac{(\theta_H - \theta^*){P_{2R}^*}^2}{q_H \theta_H}.$$

We can express the first stage equilibrium price as a function of the second stage equilibrium prices by substituting the second period prices derived above, in the utility equation that defines the indifferent consumer of type θ^* (from (1) and (2)) as follows:

$$P_{1N} = P_{2N}^* + \frac{(q_H - P_{2R}^* + \theta^*)^2}{2q_H}.$$

Substituting P_{1N} in the first stage payoff function (7) yields an optimization problem in θ^* as:

$$\max_{\theta^*} \pi^{BI} = \frac{(\theta_H - \theta^*)}{\theta_H}\left[P_{2N}^* + \frac{(q_H - P_{2R}^* + \theta^*)^2}{2q_H}\right] + \frac{{P_{2N}^*}^2}{\theta_H} + \frac{{P_{2R}^*}^2(P_{2R}^* - \theta^*)}{\theta_H q_H}.$$

Solving the first order condition for θ^* yields two roots. Choosing the root that satisfies the second order condition yields the solution for θ^* as follows:

$$\theta^* = \frac{1}{33}\left(13\theta_H - 28q_H + 2\sqrt{97{q_H}^2 + 82\theta_H q_H + {\theta_H}^2}\right). \quad \text{(A3)}$$

In order to derive this solution we assume that all customer types engage in conditional buying. To satisfy this assumption, the second stage equilibrium price for returning customers P_{2R}^* (from (A1)) should satisfy the condition described in case (c) of the main text, specifically $\theta_H < P_{2R}^* < \theta^* + q_H$. In addition, we need the conditions $0 < \theta^* < \theta_H$ and $0 < P_{2N}^* - \frac{q_H}{2} < \theta^*$ to ensure that the market is not fully covered in the first and second stages. In summary, we need the following conditions to support this equilibrium.

1) $\theta_H < P_{2R}^* < \theta^* + q_H$.

2) $0 < \theta^* < \theta_H$.

3) $0 < P_{2N}^* - \frac{q_H}{2} < \theta^*$.

In order to satisfy the first two conditions, $\frac{q_H}{\theta_H}$ values should fall in the range given by $\frac{13-\sqrt{57}}{4} < \frac{q_H}{\theta_H} < \frac{5}{2}$. However, in this range of $\frac{q_H}{\theta_H}$ values $P_{2N}^* - \frac{q_H}{2} < 0$, contradicting the assumption of less than full market coverage in both periods. Therefore, conditional buying by all returning customers does not occur at the equilibrium when there is less than full market coverage in both periods.

(ii) Using a similar approach, we can derive the equilibrium with only behavioral (B) and only intertemporal (I) price discrimination regimes by substituting $P_{2N} = P_{1N}$ and $P_{2N} = P_{2R} = P_2$, respectively. Similar to the BI regime, we can show that under purely intertemporal price discrimination (I) regime, when there is less than full market coverage in both periods, conditional learning by all returning customers does not occur at the equilibrium. However, for purely behavioral price discrimination (B) regime, conditional buying by all returning customers is consistent with the assumption of less than full market coverage in both periods when $1.482 < \frac{q_H}{\theta_H} < 1.62$.

Proof of Proposition 2:

(i), (ii) When the monopolist has full commitment power to set future prices, he announces the price to new and returning customers at the beginning of the first period. Assuming that the monopolist uses both behavioral and intertemporal price discrimination when he can commit to prices in both periods at the beginning of the game and that there is learning by all returning customers, the payoff function for the two periods is given by:

$$\max_{P_{1N},P_{2N},P_{2R}} \frac{(\theta_H-\theta^*)}{\theta_H} P_{1N} + \int_{P_{2N}-\frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H \theta_H} dq d\theta.$$

We can solve the maximization problem in three variables by deriving the first order conditions as follows:

$$\frac{\partial \pi^{BI}}{\partial P_{1N}} = (\theta_H - \theta^*) - \frac{\partial \theta^*}{\partial P_{1N}} \left[P_{1N} - P_{2N} + \frac{P_{2R}}{q_H}(q_H - P_{2R} + \theta^*) \right] = 0. \tag{A4}$$

$$\frac{\partial \pi^{BI}}{\partial P_{2N}} = \theta^* - 2P_{2N} + \frac{q_H}{2} - \frac{\partial \theta^*}{\partial P_{2N}} \left[P_{1N} - P_{2N} + \frac{P_{2R}}{q_H}(q_H - P_{2R} + \theta^*) \right] = 0. \tag{A5}$$

$$\frac{\partial \pi^{BI}}{\partial P_{2R}} = (\theta_H - \theta^*) \left(q_H - 2P_{2R} + \frac{\theta_H+\theta^*}{2} \right) - \frac{\partial \theta^*}{\partial P_{2R}} \left[P_{1N} - P_{2N} + \frac{P_{2R}}{q_H}(q_H - P_{2R} + \theta^*) \right] = 0. \tag{A6}$$

The expressions for $\frac{\partial \theta^*}{\partial P_{1N}}$, $\frac{\partial \theta^*}{\partial P_{2N}}$ and $\frac{\partial \theta^*}{\partial P_{2R}}$ can be obtained from the utility equation defining the indifferent consumer that is derived from (6). Solving (A4), (A5) and (A6) simultaneously yields the equilibrium prices in terms of θ^* as follows:

$P_{2N}^{*} = \frac{\theta_H}{2} + \frac{q_H}{4}$. (A7)

$P_{2R}^{*} = \frac{\theta_H - \theta^*}{2}$.

$P_{1N}^{*} = P_{2N}^{*} + \frac{(q_H - P_{2R}^{*} + \theta^*)^2}{2q_H}$.

However, to support learning by all returning customers, $\theta_H < P_{2R}^{*} < \theta^* + q_H$. The expression for P_{2R}^{*} in (A7) is inconsistent with the inequality $P_{2R}^{*} > \theta_H$ for any $\theta^* > 0$. Hence with less than full coverage it is impossible for this equilibrium to arise.

Considering a regime with conditional learning by the lower tail only yields the payoff functions:

$\max_{P_{1N}, P_{2N}, P_{2R}} \frac{(\theta_H - \theta^*)}{\theta_H} P_{1N} + \int_{P_{2N} - \frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{P_{2R}} \int_{P_{2R} - \theta}^{q_H} \frac{P_{2R}}{q_H \theta_H} dq d\theta + \int_{P_{2R}}^{\theta_H} \frac{P_{2R}}{q_H \theta_H} d\theta,$

where the relationship between P_{1N} and P_{2N} ensures that a consumer of type θ^* is indifferent between buying in period 1 or postponing consumption to period 2. Hence,

$P_{1N} = P_{2N} + \frac{(q_H - P_{2R} + \theta^*)^2}{2q_H}$.

Substituting the expression for P_{1N} back into the objective, we obtain a maximization problem in P_{2N}, P_{2R} and θ^*. Optimizing with respect to the three variables yields:

$P_{2N}^{*} = \frac{\theta_H}{2} + \frac{q_H}{4}$, (A8)

$P_{2R}^{*} = \frac{8\theta_H + q_H(\sqrt{89} - 9)}{16}$,

$\theta^* = \frac{24\theta_H - q_H(7 + \sqrt{89})}{48}$. (A9)

However to sustain an equilibrium with expansion of the market in the second period (i.e., $P_{2N}^{*} < P_{1N}^{*}$) it is necessary that $P_{2N}^{*} - \frac{q_H}{2} < \theta^*$, which is inconsistent with (A8) and (A9). Hence, $P_{1N}^{*} = P_{2N}^{*}$ and no new consumers are attracted to the market in the second period. The choice of the monopolist reduces, therefore, to a choice of two prices: price to new customers P_N and price to returning customers P_R. This choice is equivalent to the one facing the monopolist in the uni-dimensional behavioral regime.

When there is conditional buying only by the lower tail of customers, the payoff function for the two periods is given by:

$\pi^C = \frac{(\theta_H - \theta^*)}{\theta_H} P_N + \int_{\theta^*}^{P_R} \int_{P_R - \theta}^{q_H} \frac{P_R}{q_H \theta_H} dq\, d\theta + \int_{P_R}^{\theta_H} \frac{P_R}{\theta_H} d\theta.$

$\max_{P_N, P_R} \frac{(\theta_H - \theta^*)}{\theta_H} P_N + P_R(P_R - \theta^*)\left(q_H - P_R + \frac{P_R + \theta^*}{2}\right) + (\theta_H - P_R)P_R.$ (A10)

Based on the consumer utility equation, we can express the relationship between the first and second period price as follows:

$$P_N = \theta^* + \frac{q_H}{2} + \frac{(q_H - P_R + \theta^*)^2}{2q_H}. \tag{A11}$$

Substituting the expression for P_N from (A11) in the profit function (A10), we can write the first stage maximization problem in two variables P_R and θ^* as follows:

$$\max_{P_R,\theta^*} \frac{(\theta_H - \theta^*)}{\theta_H}\left(\theta^* + \frac{q_H}{2} + \frac{(q_H - P_R + \theta^*)^2}{2q_H}\right) + P_R(P_R - \theta^*)\left(q_H - P_R + \frac{P_R + \theta^*}{2}\right) + (\theta_H - P_R)P_R.$$

The first order conditions in P_R and θ^* can be derived as follows:

$$\frac{\partial \pi^C}{\partial P_R} = \theta_H - 2P_R + \frac{P_R(q_H - P_R + \theta^*)}{q_H} + \frac{(P_R - \theta^*)\left(q_H - \frac{P_R}{2} + \frac{\theta^*}{2}\right)}{q_H} - \frac{(\theta_H - \theta^*)(q_H - P_R + \theta^*)}{q_H}. \tag{A12}$$

$$\frac{\partial \pi^C}{\partial \theta^*} = -\left(\theta^* + \frac{q_H}{2} + \frac{(q_H - P_R + \theta^*)^2}{2q_H}\right) + (\theta_H - \theta^*)\left(1 + \frac{q_H - P_R + \theta^*}{q_H}\right) - \frac{P_R(q_H - P_R + \theta^*)}{q_H}. \tag{A13}$$

Solving the first order conditions (A12) and (A13) yields the solution for second period price P_R^* and the threshold customer θ^* as reported in Proposition 2. We choose the solution for P_R^* and θ^* to satisfy the second order conditions.

The solution for P_R^* and θ^* should satisfy the following conditions that are required to ensure conditional learning only by the lower tail of returning customers and less than full market coverage.

1) $P_R^* < \theta_H$.
2) $\theta^* < P_R^* < \theta^* + q_H$.
3) $0 < \theta^* < \theta_H$.

Condition (1) and (2) are always satisfied by the solution for P_R^* and θ^* for $\theta^* > 0$. The condition $\theta^* > 0$ is valid only when $\frac{q_H}{\theta_H} < \frac{4}{3-\sqrt{5}}$.

(iii) Substituting the solution for prices back into (A10) yields the expected profits.

(iv) It is easy to demonstrate that $P_N^* > P_R^*$.

Proof of Proposition 3:

(i), (ii) and (iii) The second stage payoff function when there is active learning only by the lower tail of the distribution of consumers from is as follows:

$$\pi_2^{BI} = \int_{P_{2N}-\frac{q_H}{2}}^{\theta^*} \frac{P_{2N}}{\theta_H} d\theta + \int_{\theta^*}^{P_{2R}} \frac{1}{\theta_H} \int_{P_{2R}-\theta}^{q_H} \frac{P_{2R}}{q_H} dq\, d\theta + \int_{P_{2R}}^{\theta_H} \frac{P_{2R}}{\theta_H} d\theta.$$

The first order conditions can be derived as follows:

$$\frac{\partial \pi_2^{BI}}{\partial P_{2N}} = \theta^* + \frac{q_H}{2} - 2P_{2N} = 0. \tag{A14}$$

$$\frac{\partial \pi_2^{BI}}{\partial P_{2R}} = \theta_H - 2P_{2R} + \frac{P_{2R}(q_H - P_{2R} + \theta^*)}{q_H} + \frac{(P_{2R} - \theta^*)\left(q_H - \frac{P_{2R}}{2} + \frac{\theta^*}{2}\right)}{q_H}. \tag{A15}$$

Solving the first order conditions (A14) and (A15) yields the solution for P_{2N}^* and P_{2R}^* as reported in Proposition 3. Using the first order condition (A14) and (A15), it is possible to express the second stage optimized profits as:

$$\pi_2^* = \frac{{P_{2N}^*}^2}{\theta_H} + \frac{{P_{2R}^*}^2(P_{2R}^* - \theta^*)}{\theta_H q_H}.$$

The relationship between second stage optimal prices and the first period price P_{1N} can be obtained from the utility equation for the indifferent consumer described in (6):

$$P_{1N} = P_{2N}^* + \frac{(q_H - P_{2R}^* + \theta^*)^2}{2q_H} \qquad \text{(A16)}$$

Using this expression for P_{1N}, the first stage optimization problem can be written as follows:

$$\max_{\theta^*} \pi^{BI} = \frac{(\theta_H - \theta^*)}{\theta_H}\left[P_{2N}^* + \frac{(q_H - P_{2R}^* + \theta^*)^2}{2q_H}\right] + \frac{{P_{2N}^*}^2}{\theta_H} + \frac{{P_{2R}^*}^2(P_{2R}^* - \theta^*)}{\theta_H q_H}. \qquad \text{(A17)}$$

Substituting the solution for P_{2N}^* and P_{2R}^* from the second stage optimization in (A17) leads to a maximization problem in just one variable θ^*. The first order condition in terms of θ^* can be expressed as follows:

$$\frac{1}{2}\frac{(\theta_H - \theta^*)}{\theta_H} + \frac{(q_H - P_{2R} + \theta^*)(q_H + P_{2R} - \theta^*)}{q_H(3P_{2R} - 2\theta^*)}\frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_{2R} + \theta^*)(q_H + P_{2R} + \theta^*)}{2\, q_H \theta_H} = 0 \qquad \text{(A18)}$$

In order to support conditional learning only by the lower tail of experienced customers and less than full market coverage in the BI regime, we need additional constraints on the second period price to new customers, P_{2N}^*, in addition to the constraints on P_{2R}^* and θ^* that we discussed in the proof of Proposition 2. First, we require $P_{2N}^* > \frac{q_H}{2}$ to ensure that there is less than full market coverage in the second period. Second, $P_{2N}^* < \theta^* + \frac{q_H}{2}$ is required to ensure that new customers will find it attractive to buy in the second period. It is obvious from the solution for P_{2N}^*, that the latter condition is always true.

To ensure the conditions necessary to support this equilibrium, the following inequalities on θ^* have to be satisfied:

1) $P_{2N}^* > \frac{q_H}{2} \Rightarrow \theta^* > \frac{q_H}{2}$.
2) $P_{2R}^* < \theta_H \Rightarrow \theta^* < 3\theta_H - 2q_H$.
3) $\theta^* < P_{2R}^* < \theta^* + q_H \Rightarrow \theta^* > \frac{\theta_H}{2} - \frac{3}{4}q_H$.
4) $0 < \theta^* < \theta_H$.

Combining all the four conditions, the lower and upper bounds on θ^* can be expressed as: $Max\left\{\frac{q_H}{2}, \frac{\theta_H}{2} - \frac{3}{4}q_H\right\} < \theta^* < Min\{\theta_H, 3\theta_H - 2q_H\}$. This can be simplified to indicate the range of parameter values and the corresponding bounds on θ^* as below:

a) $For\ \frac{q_H}{\theta_H} < \frac{2}{5},\ \frac{\theta_H}{2} - \frac{3}{4}q_H < \theta^* < \theta_H.$

b) $For\ \frac{2}{5} < \frac{q_H}{\theta_H} < 1, \frac{q_H}{2} < \theta^* < \theta_H.$

c) $For\ 1 < \frac{q_H}{\theta_H} < \frac{6}{5}, \frac{q_H}{2} < \theta^* < 3\theta_H - 2q_H.$

We can now evaluate the first order condition for θ^* at the lower bound (LB) and upper bound (UB) and derive the range of $\frac{q_H}{\theta_H}$ values that support this equilibrium. If $\frac{\partial \pi^{BI}}{\partial \theta^*}\Big|_{LB} > 0$, the optimized solution for θ^* satisfies the constraint that it is bigger than LB. If $\frac{\partial \pi^{BI}}{\partial \theta^*}\Big|_{UB} < 0$, the optimized solution for θ^* satisfies the constraint that it is smaller than UB.

a) When $\frac{q_H}{\theta_H} < \frac{2}{5}$,

$\frac{\partial \pi^{BI}}{\partial \theta^*} < 0$ at the upperbound $\theta^* = \theta_H$ for all values of $\frac{q_H}{\theta_H} > 0$.

$\frac{\partial \pi^{BI}}{\partial \theta^*} > 0$ at the lower bound $\theta^* = \frac{\theta_H}{2} - \frac{3}{4}q_H$, for all values of $0 < \frac{q_H}{\theta_H} < \frac{2}{5}$.

b) When $\frac{2}{5} < \frac{q_H}{\theta_H} < 1$,

$\frac{\partial \pi^{BI}}{\partial \theta^*} > 0$ at the lower bound $\theta^* = \frac{q_H}{2}$, for $\frac{2}{5} < \frac{q_H}{\theta_H} < 0.8322$ and the upper bound is the same as the previous case.

c) When $1 < \frac{q_H}{\theta_H} < \frac{6}{5}$,

The lower bound does not satisfy the first order condition and therefore this range of $\frac{q_H}{\theta_H}$ values does not support the equilibrium.

Combining all the three scenarios, the equilibrium with behavioral and intertemporal price discrimination exists only if $0 < \frac{q_H}{\theta_H} < 0.8322$.

(iv) From (A16) it follows immediately that $P_{1N} > P_{2N}$. The comparison of P_{2R} and P_{1N} is more involved. However after some algebraic manipulations it can be established that $P_{2R} > P_{1N}$ in the range of feasible values of $\frac{q_H}{\theta_H}$.

Proof of Proposition 4:

(i), (ii) and (iii) Under the behavioral price discrimination regime, there is no change in the price to new customers in the second period leading to $P_{1N} = P_{2N}$. Therefore the second stage payoff function is just the profit that can be obtained from returning customers.

$$\pi_2^B = \int_{\theta^*}^{P_R} \frac{1}{\theta_H} \int_{P_R-\theta}^{q_H} \frac{P_R}{q_H} dq\, d\theta + \int_{P_R}^{\theta_H} \frac{P_R}{\theta_H} d\theta.$$

The first order condition with respect to the second period price P_R is the same as first order condition (A15) in the BI regime. Solving the first order condition leads to the solution for P_R^* as reported in the Proposition. The second period optimized profits can be expressed as:

$$\pi_2^* = \frac{{P_R^*}^2 (P_R^* - \theta^*)}{\theta_H q_H}.$$

The relationship between second stage optimal price P_R^* and the first period price P_N can be obtained from the utility equation for the indifferent consumer described in (12):

$$P_N = \theta^* + \frac{q_H}{2} + \frac{(q_H - P_R^* + \theta^*)^2}{2q_H}.$$

Using this expression for P_N, the first stage optimization problem can be written as follows:

$$\max_{\theta^*} \pi^B = \frac{(\theta_H - \theta^*)}{\theta_H}\left[\theta^* + \frac{q_H}{2} + \frac{(q_H - P_R^* + \theta^*)^2}{2q_H}\right] + \frac{{P_R^*}^2 (P_R^* - \theta^*)}{\theta_H q_H}.$$

Substituting the solution for P_R^* from the second stage optimization leads to a maximization problem in one variable θ^*. The first order condition in terms of θ^* can be expressed as follows:

$$\theta_H - 2\theta^* - \frac{q_H}{2} + \frac{(q_H - P_R^* + \theta^*)(q_H + P_R^* - \theta^*)}{q_H(3P_R^* - 2\theta^*)} \frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_R^* + \theta^*)(q_H + P_R^* + \theta^*)}{2\, q_H \theta_H} = 0 \qquad \text{(A19)}$$

Following a similar approach to the one used in the proof of Proposition 3 for the BI regime, we can write the conditions on P_R^* and θ^* that are required to support conditional learning by only the lower tail of returning customers and less than full market coverage in both periods. Notice that there are fewer conditions to support the B regime as there is no market expansion in the second period. Substituting the solution for P_R^*, the equilibrium conditions can be expressed as follows:

1) $P_R^* < \theta_H \Rightarrow \theta^* < 3\theta_H - 2q_H$.
2) $\theta^* < P_R^* < \theta^* + q_H \Rightarrow \theta^* > \frac{\theta_H}{2} - \frac{3}{4} q_H$.

3) $0 < \theta^* < \theta_H$.

Combining all the four conditions, the lower and upper bounds on θ^* can be expressed as: $\frac{\theta_H}{2} - \frac{3}{4} q_H < \theta^* < Min\{\theta_H, 3\theta_H - 2q_H\}$. This can be simplified to indicate the range of parameter values and the corresponding bounds on θ^* as below:

a) $For\ \frac{q_H}{\theta_H} < \frac{2}{3},\ \frac{\theta_H}{2} - \frac{3}{4} q_H < \theta^* < \theta_H$.

b) $For\ \frac{2}{3} < \frac{q_H}{\theta_H} < 1, 0 < \theta^* < \theta_H$.

c) $For\ 1 < \frac{q_H}{\theta_H} < \frac{3}{2}, 0 < \theta^* < 3\theta_H - 2q_H$.

We can now evaluate the first order condition at each of the bounds and derive the range of $\frac{q_H}{\theta_H}$ values that support this equilibrium.

a) When $\frac{q_H}{\theta_H} < \frac{2}{3}$,

$\frac{\partial \pi^{BI}}{\partial \theta^*} < 0$ at the upperbound $\theta^* = \theta_H$ for all values of $\frac{q_H}{\theta_H} > 0$.

$\frac{\partial \pi^{BI}}{\partial \theta^*} > 0$ at the lower bound $\theta^* = \frac{\theta_H}{2} - \frac{3}{4} q_H$, for all values of $0 < \frac{q_H}{\theta_H} < \frac{2}{3}$.

b) When $\frac{2}{3} < \frac{q_H}{\theta_H} < 1$,

$\frac{\partial \pi^{BI}}{\partial \theta^*} > 0$ at the lower bound $\theta^* = 0$, for $\frac{2}{3} < \frac{q_H}{\theta_H} < 1$ and the upper bound is the same as in the previous case.

c) When $1 < \frac{q_H}{\theta_H} < \frac{3}{2}$,

The lower bound is the same as in the previous case. $\frac{\partial \pi^{BI}}{\partial \theta^*} < 0$ at the upper bound $\theta^* = 3\theta_H - 2q_H$ for $1 < \frac{q_H}{\theta_H} < 1.47588$.

Combining all the three scenarios, the equilibrium with behavioral price discrimination exists only if $0 < \frac{q_H}{\theta_H} < 1.47588$.

(iv) The comparison of the prices yields that $P_R > P_N$ for all $\frac{q_H}{\theta_H}$ values that support the equilibrium.

Proof of Proposition 5:

(i), (ii) and (iii) Under the Intertemporal price discrimination regime, both new and returning customers are offered the same price in the second period leading to $P_{2N} = P_{2R} = P_2$. Therefore the second stage payoff function is the sum of profit from new and returning customers as expressed below:

$$\pi_2^B = \int_{P_2-\frac{q_H}{2}}^{\theta^*} \frac{P_2}{\theta_H} d\theta + \int_{\theta^*}^{P_2} \frac{1}{\theta_H} \int_{P_2-\theta}^{q_H} \frac{P_2}{q_H} dq\, d\theta + \int_{P_2}^{\theta_H} \frac{P_2}{\theta_H} d\theta.$$

The first order condition with respect to the second period price P_2 can be derived as follows:

$$\frac{\partial \pi_2^B}{\partial P_2} = \theta^* - 2P_2 + \frac{q_H}{2} + \frac{P_2(q_H - P_2 + \theta^*)}{q_H} + \frac{(P_2-\theta^*)\left(q_H - \frac{P_2}{2} + \frac{\theta^*}{2}\right)}{q_H} + \theta_H - 2P_2 = 0.$$

Using this first order condition, the optimized second stage profits can be written as:

$$\pi_2^* = \frac{{P_2^*}^2(q_H + P_2^* - \theta^*)}{\theta_H q_H}.$$

The relationship between second stage optimal price P_2^* and the first period price P_1 can be obtained from the utility equation for the indifferent consumer as follows (from (13) and (14)):

$$P_1 = P_2^* + \frac{(q_H - P_2^* + \theta^*)^2}{2q_H}$$

Using this expression for P_1, the first stage optimization problem can be written as follows:

$$\max_{\theta^*} \pi^I = \frac{(\theta_H - \theta^*)}{\theta_H}\left[P_2^* + \frac{(q_H - P_2^* + \theta^*)^2}{2q_H}\right] + \frac{{P_2^*}^2(q_H + P_2^* - \theta^*)}{\theta_H q_H}. \quad \text{(A20)}$$

Substituting the solution for P_2^* from the second stage optimization in (A20) leads to a maximization problem in one variable θ^*. The first order condition in terms of θ^* can be expressed as follows:

$$\left(\frac{q_H + P_2^*}{3P_2^* - 2\theta^* + 2q_H}\right)\frac{(\theta_H - \theta^*)}{\theta_H} + \frac{(q_H - P_2^* + \theta^*)(q_H + P_2^* - \theta^*)}{q_H(3P_2^* - 2\theta^* + 2q_H)}\frac{(\theta_H - \theta^*)}{\theta_H} - \frac{(q_H - P_2^* + \theta^*)(q_H + P_2^* + \theta^*)}{2\, q_H \theta_H} = 0 \quad \text{(A21)}$$

Similar to the BI and B regimes, we can write the conditions on P_2^* and θ^* that are required in order to support conditional learning only by the lower tail of returning customers and less than full market coverage in both periods. In addition to the constraints on P_2^* that ensure conditional learning only by the lower tail of returning customers and an interior solution for θ^*, we also require $P_2^* > \frac{q_H}{2}$ and $P_2^* < \theta^* + \frac{q_H}{2}$ to ensure that the price to new customers is not too high and that the market expansion does not lead to complete coverage. Substituting the solution for P_2^*, the equilibrium conditions can be expressed as follows:

1) $P_2^* > \frac{q_H}{2} \Rightarrow \theta^* > q_H - \frac{1}{2}\sqrt{q_H(8\theta_H - 3q_H)}$.
2) $\theta^* < P_2^* < \theta^* + \frac{q_H}{2} \Rightarrow \frac{8}{20}\theta_H - \frac{7}{20}q_H < \theta^* < \frac{\theta_H}{2} + \frac{q_H}{4}$.

3) $P_2^* < \theta_H => \theta^* < 3\theta_H - q_H$.

4) $0 < \theta^* < \theta_H$.

Combining all the four conditions, the lower and upper bounds on θ^* can be expressed as: $\frac{\theta_H}{2} - \frac{3}{4}q_H < \theta^* < Min\{\theta_H, 3\theta_H - 2q_H\}$. This can be simplified to indicate the range of parameter values and the corresponding bounds on θ^* as below:

a) $For\ \frac{q_H}{\theta_H} < \frac{8}{7},\ \frac{1}{20}(8\theta_H - 7q_H) < \theta^* < \frac{\theta_H}{2} + \frac{q_H}{4}$.

b) $For\ \frac{8}{7} < \frac{q_H}{\theta_H} < 2, q_H - \frac{1}{2}\sqrt{q_H(8\theta_H - 3q_H)} < \theta^* < \frac{\theta_H}{2} + \frac{q_H}{4}$.

We can now evaluate the first order condition at each of the bounds and derive the range of $\frac{q_H}{\theta_H}$ values that support this equilibrium.

a) When $\frac{q_H}{\theta_H} < \frac{8}{7}$,

$\frac{\partial \pi^I}{\partial \theta^*} < 0$ at the upperbound $\theta^* = \frac{\theta_H}{2} + \frac{q_H}{4}$ for all values of $\frac{q_H}{\theta_H} > 0$.

$\frac{\partial \pi^I}{\partial \theta^*} > 0$ at the lower bound $\theta^* = \frac{1}{20}(8\theta_H - 7q_H)$, for all values of $\frac{q_H}{\theta_H} > 0$.

b) When $\frac{8}{7} < \frac{q_H}{\theta_H} < 2$,

$\frac{\partial \pi^I}{\partial \theta^*} > 0$ at the lower bound $\theta^* = q_H - \frac{1}{2}\sqrt{q_H(8\theta_H - 3q_H)}$, for $\frac{8}{7} < \frac{q_H}{\theta_H} < 1.28367$ and the upper bound is the same as in the previous case.

Therefore, the only condition that restricts the range of $\frac{q_H}{\theta_H}$ values that support intertemporal price discrimination regime is $P_2^* > \frac{q_H}{2}$. The equilibrium with intertemporal price discrimination exists only if $0 < \frac{q_H}{\theta_H} < 1.28367$.

(iv) The comparison of prices is obvious from the requirement that a consumer of type θ^* is indifferent between buying in periods1 and 2.

Proof of Proposition 6:

(i) and (ii): We utilize the first order conditions to compare the three regimes in terms of profits and the extent of market coverage. We first compare θ^* in the BI and B regimes.

In order to prove $\theta^{*BI} > \theta^{*B}$, it is sufficient to show that $\left(\frac{\partial \pi^B}{\partial \theta^*}\right)_{\theta^*=\theta^{*BI}} < 0$ in the range of parameter values that support the BI and B regimes.

$$\left(\frac{\partial \pi^B}{\partial \theta^*}\right)_{\theta^*=\theta^{*BI}} = \theta_H - 2\theta^{*BI} - \frac{q_H}{2} + \frac{(q_H - P_{2R}^* + \theta^{*BI})(q_H + P_{2R}^* - \theta^{*BI})(\theta_H - \theta^{*BI})}{q_H(3P_{2R}^* - 2\theta^{*BI})} - \frac{(q_H - P_{2R}^* + \theta^{*BI})(q_H + P_{2R}^* + \theta^{*BI})}{2\, q_H}.$$

But from (A18), we know that $\frac{(q_H - P_{2R}^* + \theta^{*BI})(q_H + P_{2R}^* - \theta^{*BI})(\theta_H - \theta^{*BI})}{q_H(3P_{2R}^* - 2\theta^{*BI})} - \frac{(q_H - P_{2R}^* + \theta^{*BI})(q_H + P_{2R}^* + \theta^{*BI})}{2\, q_H \theta_H} = -\frac{1}{2}(\theta_H - \theta^{*BI})$. Therefore $\left(\frac{\partial \pi^B}{\partial \theta^*}\right)_{\theta^*=\theta^{*BI}}$, can be simplified as follows:

$$\left(\frac{\partial \pi^B}{\partial \theta^*}\right)_{\theta^*=\theta^{*BI}} = \theta_H - 2\theta^{*BI} - \frac{q_H}{2} - \frac{1}{2}(\theta_H - \theta^{*BI}) = \frac{\theta_H}{2} - \frac{q_H}{2} - \frac{3}{2}\theta^{*BI}.$$

$\left(\frac{\partial \pi^B}{\partial \theta^*}\right)_{\theta^*=\theta^{*BI}} < 0$ when $\theta^{*BI} > \frac{\theta_H - q_H}{3}$. We can show that $\frac{\theta_H - q_H}{3}$ is smaller than the lower bounds for θ^{*BI} that support the BI regime leading to the conclusion that $\theta^{*BI} > \frac{\theta_H - q_H}{3}$ for all parameter values that support the equilibrium. As a result, $\theta^{*B} < \theta^{*BI}$. We can use a similar approach to demonstrate that $\theta^{*I} < \theta^{*BI}$. The comparison of θ^{*I} and θ^{*B} is more complicated, and we therefore, perform the comparison using numerical results reported in Table 1.

<u>Comparison of profits:</u>

We can evaluate the profit in the B regime at $\theta = \theta^{*BI}$. Since $\theta^{*BI} > \theta^{*B}$, ${\pi^B}_{\theta=\theta^{*BI}} < \pi^{*B}$. If we can show that $\pi^{*BI} < {\pi^B}_{\theta=\theta^{BI}}$ unambiguously for the range of parameter values that support the equilibrium, it follows that $\pi^{*BI} < \pi^{*B}$.

$$\pi^{*BI} - {\pi^B}_{\theta=\theta^{BI}} = \frac{(\theta_H - \theta^{*BI})}{\theta_H}\left(\frac{\theta^{*BI}}{2} + \frac{q_H}{4} - \theta^{*BI} - \frac{q_H}{2}\right) + \frac{1}{\theta_H}\left(\frac{\theta^{*BI}}{2} + \frac{q_H}{4}\right)^2. \tag{A22}$$

Expression (A21) can be simplified as follows:

$$\pi^{*BI} - {\pi^B}_{\theta=\theta^{BI}} = \left(\frac{\theta^{*BI}}{2} + \frac{q_H}{4}\right)\left(\frac{3}{2}\theta^{*BI} - \theta_H + \frac{q_H}{4}\right).$$

$$\pi^{*BI} - {\pi^B}_{\theta=\theta^{BI}} < 0 \; when \; \theta^{*BI} < \frac{2}{3}\theta_H - \frac{q_H}{6}.$$

We can show that $\left.\frac{\partial \pi^{BI}}{\partial \theta^*}\right|_{\theta^{*BI}=\frac{2}{3}\theta_H - \frac{q_H}{6}} < 0$ in the range of $\frac{q_H}{\theta_H}$ values that support the BI regime, thereby implying that $\theta^{*BI} < \frac{2}{3}\theta_H - \frac{q_H}{6}$ and $\pi^{*BI} < {\pi^B}_{\theta=\theta^{BI}}$.

The comparison of π^{*B} and π^{*I} is more complicated, and we therefore, perform the comparison using numerical results reported in Table 1.

Proof of Proposition 7:

When the monopolist can commit to setting the same price in both periods, the common price P_S can be expressed in terms of θ^* using the customer utility equation as follows:

$\theta^* + \frac{q_H}{2} - P_S + \frac{(q_H - P_S + \theta^*)^2}{2q_H} = 0.$

This is a quadratic equation with two roots. We choose the root that satisfies the condition that $P_S^* < \theta^* + q_H$, implying that type θ^* sometimes buys a second time.

$P_S^* = 2q_H - \sqrt{2}q_H + \theta^*.$ (A23)

Assuming conditional learning by all, the payoff function can be expressed as follows:

$\max_{\theta^*} \pi^S = (\theta_H - \theta^*)P_S^* + \int_{\theta^*}^{P_S^*} \int_{P_S^* - \theta}^{q_H} \frac{P_S^*}{q_H \theta_H} dq\, d\theta.$ (A24)

Substituting the solution for P_S^*, (A24) represents a maximization problem in one variable θ^*. Solving the first order conditions yields a solution for θ^* that contradicts the assumption of conditional learning by all returning customers.

Therefore, we now solve the equilibrium assuming conditional learning by the lower tail of returning customers only. The expression for P_S^* in terms of θ^* (from (A23)) remains the same. However, the payoff function can now be expressed as:

$\max_{\theta^*} \pi^S = (\theta_H - \theta^*)P_S^* + \int_{\theta^*}^{P_S^*} \int_{P_S^* - \theta}^{q_H} \frac{P_S^*}{q_H \theta_H} dq\, d\theta + \int_{P_S^*}^{\theta_H} \frac{P_S^*}{\theta_H} d\theta.$

Substituting the expression for P_S^* from (A23) and solving the first order conditions for θ^* yields the equilibrium reported in Proposition 8. In order to support the equilibrium with conditional learning only by the lower tail of returning customers, we need $\theta^* < P_S^* < \theta^* + q_H$ and $P_S^* < \theta_H$. We can show that the solution $P_S^* = \frac{\theta_H}{2} + \frac{q_H}{4}$ satisfies these conditions for $0 < \frac{q_H}{\theta_H} < 1.488$.

Proof of Proposition 8:

(i), (ii) When there is full coverage of the market in the first period, $\theta^* = 0$. All three pricing regimes are similar, as there are no new customers in the second stage. The monopolist's pricing problem reduces to choosing a first period price and second period that is common to all customers. We derive the equilibrium, assuming conditional buying by all returning customers.

The second period payoff function when there is conditional learning by all customers can be expressed as follows:

$\max_{P_R} \pi_2 = \int_0^{\theta_H} \int_{P_R - \theta}^{q_H} \frac{P_R}{q_H \theta_H} dq d\theta.$

Solving the first order condition as given below yields the solution for P_R as reported in Proposition 7.

$$\frac{\partial \pi_2}{\partial P_{2R}} = q_H - 2P_R + \frac{\theta_H}{2}. \tag{A25}$$

In order to support conditional buying by all returning customers, we need $\theta_H < P_R < q_H$. The solution for P_R satisfies this condition for $\frac{q_H}{\theta_H} > \frac{3}{2}$. Therefore under full market coverage, conditional buying by all returning customers occurs at the equilibrium when $\frac{q_H}{\theta_H} > \frac{3}{2}$.

The first period price can be derived directly from the consumer utility equation as follows:

$$P_N^* = \frac{q_H}{2} + \frac{\left(\frac{q_H}{2} - \frac{\theta_H}{4}\right)^2}{2q_H}. \tag{A26}$$

From (A25) and (A26), the expected profit under the equilibrium can be expressed as follows:

$$\pi = \left(\frac{q_H}{2} + \frac{\left(\frac{q_H}{2} - \frac{\theta_H}{4}\right)^2}{2q_H} + \frac{1}{q_H}\left(\frac{q_H}{2} + \frac{\theta_H}{4}\right)^2\right) = \frac{28q_H^2 + 4q_H\theta_H + 3\theta_H^2}{32q_H}.$$

The comparison of prices yields that $P_R^* > P_N^*$ for $\frac{q_H}{\theta_H} < 2.9$.

For $\frac{q_H}{\theta_H} < \frac{3}{2}$, only the lower tail of customers engages in conditional buying and the upper tail of customers buys the service again irrespective of their q realization. In this case, the second stage payoff function of the monopolist can be expressed as follows:

$$\max_{P_R} \pi_2 = \int_0^{P_R} \int_{P_R}^{q_H} \frac{P_R}{q_H \theta_H} dq\, d\theta + \int_{P_R}^{\theta_H} \frac{P_R}{\theta_H} d\theta.$$

Solving the first order condition yields the solution for P_R as reported in Proposition 7.

In order to support conditional buying by the lower tail of customers only, we require $0 < P_R < q_H$ and $P_R < \theta_H$. The solution for P_R satisfies the two conditions when $1.47588 < \frac{q_H}{\theta_H} < \frac{3}{2}$.

The first period price derived from the customer utility equation is given by the following expression:

$$P_N^* = \frac{q_H}{2} + \frac{\left(q_H - \sqrt{\frac{2}{3} q_H \theta_H}\right)^2}{2q_H}.$$

Substituting the two prices, yields the equilibrium profit as reported in the Proposition.

(iii) When the monopolist has commitment power to announce future prices and assuming conditional buying by all, the equilibrium can be derived by solving the payoff function as follows:

$$\max_{P_R} \pi_2 = \left(\frac{q_H}{2} + \frac{(q_H - P_R)^2}{2q_H}\right) + \int_0^{\theta_H} \int_{P_{R-\theta}}^{q_H} \frac{P_R}{q_H \theta_H} dq d\theta.$$

Solving the optimization problem yields the solution for P_R^*. Substituting this solution in the customer utility equation yields the first period price P_N^*.

However, we can show the solution for P_R^* does not satisfy the conditions required for conditional buying by all customers, namely $\theta_H < P_R^* < q_H$. Therefore we derive the equilibrium with conditional learning by the lower tail of customers only. We can write the optimization problem as follows:

$$\max_{P_R} \pi_2 = \theta_H \left(\frac{q_H}{2} + \frac{(q_H - P_R)^2}{2q_H}\right) + \int_0^{P_R} \int_{P_R - \theta}^{q_H} \frac{P_R}{q_H \theta_H} dq\, d\theta + \int_{P_R - \theta}^{\theta_H} \frac{P_R}{\theta_H} d\theta.$$

Solving the first order condition yields the solution for P_R^* as reported in Proposition 7. Substituting the solution for P_R^* in the utility equation yields the optimal first period price. A comparison of prices yields that $P_N^* > P_R^*$ for $\frac{q_H}{\theta_H} > 1.13$. Without commitment, full coverage arises when $\frac{q_H}{\theta_H} > 1.43$. Hence, with commitment $P_N^* > P_R^*$ whenever full coverage arises (with or without commitment).

Proof of Proposition 9:

When there is perfect correlation between consumer's valuation of the basic features θ and the new features q, there is no uncertainty associated with the valuation of new features. The value of new features to a consumer of type θ is given by $\theta \frac{q_H}{\theta_H}$. We can, therefore, write the utility equation for a consumer of type θ^*as follows:

$$2\theta^* \left(1 + \frac{q_H}{\theta_H}\right) - P_{1N} - P_{2R} = \theta^* \left(1 + \frac{q_H}{\theta_H}\right) - P_{2N}. \tag{A27}$$

(i) <u>Equilibrium with Commitment Power:</u>

Assuming that the monopolist uses both behavioral and intertemporal price discrimination (BI) regimes, the payoff function for the two periods can be expressed as follows:

$$\pi_2 = \frac{(\theta_H - \theta^*)}{\theta_H} [P_{1N} + P_{2R}] + \frac{(\theta^* - \tilde{\theta})}{\theta_H} P_{2N}.$$

From the utility equation (A27), we can express the relationship between the prices and θ^* as follows:

$$\theta^* = \frac{P_{1N} + P_{2R} - P_{2N}}{\left(1 + \frac{q_H}{\theta_H}\right)}.$$

Similarly $\tilde{\theta}$ can be expressed as $\tilde{\theta} = \frac{P_{2N}}{\left(1 + \frac{q_H}{\theta_H}\right)}$.

Substituting the expressions for θ^* and $\tilde{\theta}$, the optimization problem in P_{1N}, P_{2R} and P_{2N} can be written as:

$$\max_{P_{1N}, P_{2R} P_{2N},} \left[\theta_H - \frac{P_{1N}+P_{2R}-P_{2N}}{\left(1+\frac{q_H}{\theta_H}\right)}\right] \frac{P_{1N}+P_{2R}}{\theta_H} + \left(\frac{P_{1N}+P_{2R}-2P_{2N}}{\left(1+\frac{q_H}{\theta_H}\right)}\right) \frac{P_{2N}}{\theta_H}.$$

Optimizing with respect to the prices, yields the following results:

$P_{1N} + P_{2R} = \theta_H + q_H$,

$P_{2N} = \frac{P_{1N}+P_{2R}}{2}$, which leads to the result $\theta^* = \tilde{\theta}$, implying there are no new customers in the second stage.

Substituting the optimal prices in the payoff functions yields the expected profit with commitment power:

$E\pi^* = \frac{\theta_H+q_H}{2}$.

(ii) Equilibrium without Commitment Power:

In the BI regime, the second stage payoff function for the monopolist can be expressed as follows:

$$\pi_2 = \begin{cases} \frac{P_{2N}}{\theta_H}\left(\theta^* - \frac{P_{2N}}{1+\frac{q_H}{\theta_H}}\right) + \frac{P_{2R}}{\theta_H}(\theta_H - \theta^*) \; if \; P_{2R} \leq \theta^*\left(1+\frac{q_H}{\theta_H}\right) \\ \frac{P_{2N}}{\theta_H}\left(\theta^* - \frac{P_{2N}}{1+\frac{q_H}{\theta_H}}\right) + \frac{P_{2R}}{\theta_H}\left(\theta_H - \frac{P_{2R}}{1+\frac{q_H}{\theta_H}}\right) \; if P_{2R} \geq \theta^*\left(1+\frac{q_H}{\theta_H}\right) \end{cases}$$

Optimizing with respect to P_{2R} and P_{2N} yields the solution for second period prices as follows:

$$P_{2R}^* = \begin{cases} \theta^*\left(1+\frac{q_H}{\theta_H}\right) \; if P_{2R} \leq \theta^*\left(1+\frac{q_H}{\theta_H}\right) \\ \frac{\theta_H+q_H}{2} \; if \; P_{2R} \geq \theta^*\left(1+\frac{q_H}{\theta_H}\right) \end{cases}. \tag{A28}$$

$$P_{2N}^* = \frac{\theta^*}{2}\left(1+\frac{q_H}{\theta_H}\right). \tag{A29}$$

From the utility equation that defines the indifferent consumer (A27), we can express the relationship between the first and second period prices as follows:

$P_{1N} = P_{2N}^* - P_{2R}^* + \theta^*\left(1+\frac{q_H}{\theta_H}\right)$.

Substituting the expression for P_{2R}^* and P_{2N}^* from (A28) and (A29), the first period price can be derived as follows:

$P_{1N} = \frac{\theta^*}{2}\left(1+\frac{q_H}{\theta_H}\right)$.

Since $P_{1N} = P^*_{2N}$, there are no new customers in the second stage and the utility equation defining the indifferent customer can be expressed as :

$$2\theta^* \left(1 + \frac{q_H}{\theta_H}\right) - P_{1N} - P^*_{2R} = 0,$$

which leads to the solution for first period price $P_{1N} = \theta^* \left(1 + \frac{q_H}{\theta_H}\right)$.

Substituting P_{1N} and P^*_{2R}, the first stage payoff function can be expressed as follows:

$$\max_{\theta^*} \frac{(\theta_H - \theta^*)}{\theta_H} 2\theta^* \left(1 + \frac{q_H}{\theta_H}\right).$$

Optimizing with respect to θ^* yields the solution $\theta^* = \frac{\theta_H}{2}$. Substituting this solution yields the expected profit in the BI regime as follows:

$$E\pi^* = \frac{\theta_H + q_H}{2}.$$

We can show that the optimal prices and the expected profit remain the same in the region $\theta^* < \frac{\theta_H}{2}$.

Using a similar approach, we can show that the intertemporal (I) and behavioral (B) price discrimination regimes are equivalent and lead to the same profit as the first best case.

Proofs for Essay 2

Proof of Proposition 1:

Solving equations (1) and (2) yields the expression for the thresholds θ_1 and θ_2 as follows:

$$\theta_1 = \frac{P_L}{v_L} + \frac{v_L}{2}. \tag{A1}$$

$$\theta_2 = \frac{P_H - P_L}{v_H - v_L} + \frac{v_H + v_L}{2}. \tag{A2}$$

Substituting the expression for θ_1 and θ_2 in the payoff function (3), the first order conditions with respect to P_L and P_H can expressed as follows:

$$\frac{P_L\left(-\frac{1}{v_H - v_L} - \frac{1}{v_L}\right)}{\theta_H} + \frac{P_H}{(v_H - v_L)\theta_H} + \frac{\frac{P_H - P_L}{v_H - v_L} + \frac{v_H + v_L}{2} - \left(\frac{P_L}{v_L} + \frac{v_L}{2}\right)}{\theta_H} = 0. \tag{A3}$$

$$-\frac{P_H - P_L}{(v_H - v_L)\theta_H} + \frac{\theta_H - \left(\frac{P_H - P_L}{v_H - v_L} + \frac{v_H + v_L}{2}\right)}{\theta_H}. \tag{A4}$$

Solving equations (A3) and (A4) yields the optimal prices under pure versioning as reported in Proposition 1.

The equilibrium with pure versioning exists only when $P_L < P_H$ and $\theta_1 < \theta_2 < \theta_H$. Substituting the optimal prices and solving the inequality $P_L < P_H$ yields a lower bound on the range of parameter values that support the equilibrium.

Proof of Lemma 1:

Substituting θ_1 and θ_2 from (A1) and (A2) and θ_p from (7), in the second stage payoff function given in (5) yields an optimization in one variable P_N. The first order condition with respect to P_N can be expressed as follows:

$$\frac{-P_N\left(\frac{1}{q} + \frac{1}{v_H - v_L}\right)}{\theta_H} + \frac{P_H - P_L}{(v_H - v_L)\theta_H} + \frac{\frac{(v_H^2 - v_L^2)}{2(q + v_H - v_L)} + \frac{(P_H - P_L)}{(q + v_H - v_L)} - \frac{P_N}{q}}{\theta_H}. \tag{A4}$$

Solving (A4) yields the optimal price P_N as a function of P_L and P_H in (8).

Substituting P_N from (8), the second stage optimal profits can be expressed as a function of P_L and P_H as follows:

$$\pi_2{}^* = \frac{q\left[4(P_H - P_L) + v_H{}^2 - v_L{}^2\right]}{4(q + v_H - v_L)(v_H - v_L)}\left(P_H - P_L + \frac{v_H{}^2 - v_L{}^2}{4}\right) + \theta_H P_H - (P_H - P_L)\left(\frac{v_H - v_L}{2} + \frac{P_H - P_L}{v_H - v_L}\right) - P_L\left(\frac{P_L}{v_L} + \frac{v_L}{2}\right). \tag{A5}$$

Substituting (A5) back in the first stage payoff function (16) yields a maximization problem in P_L and P_H. The first order conditions with respect to P_L and P_H can be expressed as follows:

$$-2P_L\left(\frac{1}{v_H-v_L}+\frac{1}{v_L}\right)+\frac{2P_H}{(v_H-v_L)}-\frac{v_L}{2}+\frac{v_H-v_L}{2}+\frac{-4P_Lq+4(P_Hv_L-P_Lv_H)-qv_L^2+v_Hv_L(v_H-v_L)}{2(q+v_H-v_L)v_L}=0. \quad \text{(A6)}$$

$$\theta_H-\frac{2(P_H-P_L)}{v_H-v_L}-\frac{v_H+v_L}{2}+\frac{-4(P_H-P_L)-\left(v_H^2-v_L^2-2\theta_H(q+v_H-v_L)\right)}{2(q+v_H-v_L)}=0. \quad \text{(A7)}$$

Solving the first order conditions yields the optimal first period prices for the basic and premium versions as follows:

$$P_L=\frac{1}{4}v_L(2\theta_H-v_L). \quad \text{(A8)}$$

$$P_H=\frac{2v_H(v_H-v_L)(2\theta_H-v_H)+2q\theta_H(v_H-v_L)+qv_H(2\theta_H-v_H)}{4(q+2(v_H-v_L))}. \quad \text{(A9)}$$

To derive the equilibrium we assumed that $P_H > P_L + P_N$. In order to obtain the segmentation described in Fig 2 at the equilibrium, we require $\frac{P_N}{q} < \theta_p$ and $\theta_1 < \theta_p$. In addition, we also need the condition $0 < \theta_1 < \theta_2 < \theta_H$, to support versioning in the first period.

Substituting the optimal prices form (8), (A8) and (A9),

(i) $P_H > P_L + P_N$ holds when $\theta_H > \frac{v_H+v_L}{2}$ and $0 < \frac{q}{v_H-v_L} < \frac{-2(v_H+v_L)+4\theta_H}{v_H+v_L}$.

(ii) $\theta_1 < \theta_p$ is true when $v_H > 2\theta_H$ or ($v_H > 2\theta_H$ and $0 < \frac{q}{v_H-v_L} < \frac{2v_H}{2\theta_H-v_H}$).

$\frac{P_N}{q} < \theta_p$ is always true in the range of parameter values that support the above two conditions.

Combining (i) and (ii) yields the regions of parameter values that support this equilibrium as described in Lemma 1.

Lemma 2 and Lemma 3 can be derived using a similar approach.

Proof of Proposition 2:

The region of parameter values that support case (i) and case (iii) overlap when:

$\theta_H > \frac{v_H+v_L}{2}$ and $0 < \frac{q}{v_H-v_L} < \frac{\sqrt{8v_H^2+16v_Hv_L+9v_L^2}-3v_L}{2(v_H+2v_L)}$.

Comparing the overall profits that the monopolist can achieve in case (i) and case (iii), we can express the difference in profits as follows:

$$\pi_{case(i)}-\pi_{case(iii)}=\frac{q(v_H-v_L)(v_H+v_L)^2}{16(q+v_H-v_L)\theta_H}>0.$$

We can therefore, eliminate case (iii) and compare case (i) and (ii) in the region of parameter values that support both cases.

The region of parameter values that support case (i) and case (ii) overlap when:

a) $\frac{v_H+v_L}{2} < \theta_H < \frac{2v_H+v_L}{2}$ and $\frac{2v_H(2\theta_H-v_H-v_L)}{v_H^2-v_L^2+4v_L\theta_H} < \frac{q}{v_H-v_L} < \frac{-2(v_H+v_L)+4\theta_H}{v_H+v_L}$ or,

b) $\frac{2v_H+v_L}{2} < \theta_H < \frac{1}{4}(2v_H+3v_L)+\frac{1}{4}\sqrt{4v_H^2+8v_Hv_L+5v_L^2}$ and $\frac{2v_H(2\theta_H-v_H-v_L)}{v_H^2-v_L^2+4v_L\theta_H} < \frac{q}{v_H-v_L} < \frac{2v_H}{2\theta_H-v_H}$.

Comparing the overall profits that the monopolist can achieve in case (i) and case (ii), we can express the difference in profits as follows:

$$\pi_{case(i)} - \pi_{case(ii)} = \frac{v_H(v_H-v_L)[4v_H(v_H-v_L)+2q(2v_H-v_L)+q^2]-8\theta_H{}^2q^2}{16(q+2v_H)(q+2v_H-2v_L)\theta_H}.$$

$$\pi_{case(i)} - \pi_{case(ii)} > 0 \text{ when } 0 < \frac{q}{v_H-v_L} < \frac{v_H\left[\sqrt{v_L{}^2+32\theta_H{}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8\theta_H{}^2}.$$

Comparing the upper bound on $\frac{q}{v_H-v_L}$ with the region where both case (i) and case (ii) exist:

a) $\frac{v_H\left[\sqrt{v_L{}^2+32\theta_H{}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8\theta_H{}^2} > \frac{-2(v_H+v_L)+4\theta_H}{v_H+v_L}$ when $v_L > (3-\sqrt{5})\theta_H$ in which case $\pi_{case(i)} > \pi_{case(ii)}$ for all values of q.

b) $\frac{v_H\left[\sqrt{v_L{}^2+32\theta_H{}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8\theta_H{}^2} < \frac{2v_H(2\theta_H-v_H-v_L)}{v_H^2-v_L^2+4v_L\theta_H}$ when $0 < v_L < \left(\frac{\sqrt{17}-3}{2}\right)\theta_H$ and $v_H < \frac{8\theta\mathrm{h}^2-v_L(v_L+\theta_H)-\theta_H\sqrt{v_L{}^2+32\theta_H{}^2}}{v_L+2\theta_H}$, in which case $\pi_{case(i)} < \pi_{case(ii)}$ for all values of q.

c) For intermediate values of v_L and v_H, $\pi_{case(i)} > \pi_{case(ii)}$ when $0 < \frac{q}{v_H-v_L} < \frac{v_H\left[\sqrt{v_L{}^2+32\theta_H{}^2}-(2v_H-v_L)\right]}{v_H(v_H-v_L)-8\theta_H{}^2}$.

Proof of Proposition 3:

The proof for Proposition 3 directly follows from the proof of Lemma 1 and Lemma 2.

Proof of Proposition 4:

Comparing the price of new service under the DB and IP regimes:

Case (i):

$P_N^{DB} = \frac{q\theta_H}{\frac{q}{v_H - v_L}+2} < \frac{q\theta_H}{2} = P_N^{IP}$.

$P_L^{DB} = P_L^{IP} = \frac{1}{4} v_L(2\theta_H - v_L)$.

$P_H^{DB} - P_H^{IP} = \frac{q(v_H - v_L)\theta_H}{2(q+2(v_H - v_L))} > 0$.

Case (ii):

$P_N^{DB} = \frac{q\theta_H}{\frac{q}{v_H}+2} < \frac{q\theta_H}{2} = P_N^{IP}$.

$P_L^{DB} - P_L^{IP} = \frac{q v_L \theta_H}{2q+4v_H} > 0$.

$P_H^{DB} - P_H^{IP} = \frac{q v_H \theta_H}{2q+4v_H} > 0$.

Proof of Proposition 5:

Comparing the overall profits with discriminative bundling and independent pricing regimes,

Case (i): $\pi^{DB} - \pi^{IP} = -\frac{q^2\theta_H}{4q+8(v_H - v_L)} < 0$.

Case (ii): $\pi^{DB} - \pi^{IP} = -\frac{q v_H v_L (v_H - v_L) + 2{v_H}^2 v_L (v_H - v_L) + 4q^2 \theta \mathrm{h}^2}{16(q+2v_H)\theta_H} < 0$.

Proof of Proposition 6:

(a) When $q_s < q_p$, the monopolist can either charge $P_N = q_s$ and cover the entire market or charge $P_N = q_p$ and sell the new service only to premium version subscribers.

(i) $P_N = q_s$.

The monopolist sells the new service to all consumers and the payoff function can be expressed as follows:

$$\pi = 2\left(\int_{\frac{P_L}{v_L}+\frac{v_L}{2}}^{\frac{(P_H - P_L)}{v_H - v_L}+\frac{(v_H + v_L)}{2}} \frac{P_L}{\theta_H} d\theta + \int_{\frac{(P_H - P_L)}{v_H - v_L}+\frac{(v_H + v_L)}{2}}^{\theta_H} \frac{P_H}{\theta_H} d\theta\right) + \int_0^{\theta_H} \frac{q_s}{\theta_H} d\theta.$$

The first order conditions with respect to P_L and P_H are the same as (A3) and (A4), which yields the same price for the basic and premium version as pure versioning.

(ii) $P_N = q_p$

The monopolist sells the new service only to the premium version customers and payoff function can be expressed as follows:

$$\pi = 2\left(\int_{\frac{P_L}{v_L}+\frac{v_L}{2}}^{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}} \frac{P_L}{\theta_H} d\theta + \int_{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}}^{\theta_H} \frac{P_H}{\theta_H} d\theta\right) + \int_{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}}^{\theta_H} \frac{q_p}{\theta_H} d\theta.$$

The first order conditions with respect to P_L and P_H can be expressed as follows:

$$2\left(\frac{P_L\left(-\frac{1}{v_H-v_L}-\frac{1}{v_L}\right)}{\theta_H} + \frac{P_H}{(v_H-v_L)\theta_H} + \frac{\frac{P_H-P_L}{v_H-v_L}+\frac{v_H+v_L}{2}-\left(\frac{P_L}{v_L}+\frac{v_L}{2}\right)}{\theta_H}\right) + \frac{q_p}{(v_H-v_L)\theta_H} = 0. \quad \text{(A10)}$$

$$2\left(-\frac{P_H-P_L}{(v_H-v_L)\theta_H} + \frac{\theta_H-\left(\frac{P_H-P_L}{v_H-v_L}+\frac{v_H+v_L}{2}\right)}{\theta_H}\right) - \frac{q_p}{(v_H-v_L)\theta_H} = 0. \quad \text{(A11)}$$

Solving the above two equations yields the optimal prices as reported in Proposition 6.

Comparing the overall profits in the above two cases, it is fairly straightforward to show that $P_N = q_s$ leads to higher profits than $P_N = q_p$ when $q_s > q_p \frac{\left[\frac{q_p}{v_H-v_L}-2(v_H+v_L)+4\theta_H\right]}{8\theta_H}$.

(b) When $q_s > q_p$, the monopolist can either charge $P_N = q_p$ and cover the entire market or charge $P_N = q_s$ and sell the new service only to non-premium version customers.

(i) $P_N = q_p$.

The monopolist sells the new service to all consumers and the payoff function can be expressed as follows:

$$\pi = 2\left(\int_{\frac{P_L}{v_L}+\frac{v_L}{2}}^{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}} \frac{P_L}{\theta_H} d\theta + \int_{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}}^{\theta_H} \frac{P_H}{\theta_H} d\theta\right) + \int_0^{\theta_H} \frac{q_p}{\theta_H} d\theta.$$

The first order conditions with respect to P_L and P_H are the same as (A3) and (A4), which yields the same price for the basic and premium version as pure versioning.

(ii) $P_N = q_s$.

The monopolist sells the new service only to the basic version customers and non-subscribers. The payoff function can be expressed as follows:

$$\pi = 2\left(\int_{\frac{P_L}{v_L}+\frac{v_L}{2}}^{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}} \frac{P_L}{\theta_H} d\theta + \int_{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}}^{\theta_H} \frac{P_H}{\theta_H} d\theta\right) + \int_0^{\frac{(P_H-P_L)}{v_H-v_L}+\frac{(v_H+v_L)}{2}} \frac{q_s}{\theta_H} d\theta.$$

The first order conditions with respect to P_L and P_H can be expressed as follows:

$$2\left(\frac{P_L\left(-\frac{1}{v_H-v_L}-\frac{1}{v_L}\right)}{\theta_H} + \frac{P_H}{(v_H-v_L)\theta_H} + \frac{\frac{P_H-P_L}{v_H-v_L}+\frac{v_H+v_L}{2}-\left(\frac{P_L}{v_L}+\frac{v_L}{2}\right)}{\theta_H}\right) - \frac{q_s}{(v_H-v_L)\theta_H} = 0. \quad \text{(A12)}$$

$$2\left(-\frac{P_H-P_L}{(v_H-v_L)\theta_H}+\frac{\theta_H-\left(\frac{P_H-P_L}{v_H-v_L}+\frac{v_H+v_L}{2}\right)}{\theta_H}\right)+\frac{q_s}{(v_H-v_L)\theta_H}=0. \tag{A13}$$

Solving the above two equations yields the optimal prices as reported in Proposition 6.

Similar to (a), it is fairly straightforward to show that, $P_N = q_p$ leads to higher profits than $P_N = q_s$ when $q_p > q_s \frac{\left[\frac{q_s}{v_H-v_L}-2(v_H+v_L)+4\theta_H\right]}{8\theta_H}$.

Proof of Proposition 7:

Assuming, $P_H > P_L + P_N$, the second stage optimization problem is as given in (5).

While θ_1 and θ_2 are inherited from the first period, the threshold θ_p can be derived by solving the following utility equation:

$$\theta_p v_L - \frac{{v_L}^2}{2} + q_s - P_L - P_N = \theta_p v_H - \frac{{v_H}^2}{2} + q_p - P_H. \tag{A14}$$

The solution to (A14) yields $\theta_p = \frac{(v_H+v_L)}{2} + \frac{(q_s-q_p)}{v_H-v_L} + \frac{(P_H-P_L-P_N)}{v_H-v_L}$.

Since all consumers in the interval $[0, \theta_p]$ have the same valuation q_s for the new service, the optimal price for the new service when sold separately, $P_N = q_s$.

The second period payoff function can therefore, be expresses as a function of P_L and P_H as follows:

$$\pi_2 = -\frac{({P_L}^2 v_H + {P_H}^2 v_L)}{v_L(v_H-v_L)\theta_H} + \frac{2(q_s+q_p)(P_H-P_L)-2q_s q_p+({v_H}^2-{v_L}^2)(q_s-P_H)-(v_H-v_L)(P_H\theta_H-P_L)+4P_H P_L}{2(v_H-v_L)\theta_H}. \tag{A15}$$

Substituting π_2 from (A15) in the first period payoff function given in (16) yields the overall profit with horizontal differentiation.

The first order conditions with respect to P_L and P_H can be derived as follows:

$$-2P_L\left(\frac{1}{v_H-v_L}+\frac{1}{v_L}\right)+\frac{2P_H}{(v_H-v_L)}+v_H-\frac{q_s+q_p}{v_H-v_L}+\frac{2(P_H v_L-P_L v_H)}{(v_H-v_L)v_L}=0. \tag{A16}$$

$$2\theta_H - 4\left(\frac{P_H-P_L}{v_H-v_L}\right)-(v_H+v_L)+\frac{q_s+q_p}{v_H-v_L}=0. \tag{A17}$$

Solving equations (A16) and (A17) yields the optimal prices as reported in Proposition 7.

We assumed that $P_H > P_L + q_s$ in deriving the equilibrium and $0 < \theta_1 < \theta_p < \theta_2 < \theta_H$ is required to support versioning in the first period and second periods.

Substituting the optimal prices and solving the inequalities yields the region of parameter values that support this equilibrium as described in Proposition 7.

Proof of Proposition 8:

Comparing the profits with DB and IP, we can express the difference in profits as follows:

a) $q_s < q_p$

When $q_s > q_p \frac{\left[\frac{q_p}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$\pi^{DB} - \pi^{IP} = \frac{3\left(q_s{}^2 + q_p{}^2\right) - 10 q_s q_p - 4\left(q_p - q_s\right)\left(v_H{}^2 - v_L{}^2 - 2\theta_H\right) - v_H{}^2\left(v_H{}^2 - v_L{}^2\right) + v_H v_L{}^2\left(v_H - v_L\right) + 4 v_H \theta_H\left(v_H - \theta_H\right)}{16\left(v_H - v_L\right)\theta_H}$.

The expression on the right-hand side is positive when the following condition holds:

$q_p > \frac{1}{3}\left[5q_s + 2(v_H{}^2 - v_L{}^2) - 4\theta_H(v_H - v_L) + \sqrt{16 q_s{}^2 + 8 q_s(v_H - v_L)(v_H + v_L - 2\theta_H) + \mathrm{A}}\right]$. (A18)

Where $\mathrm{A} \equiv (v_H - v_L)(7 v_H{}^3 + 7 v_H{}^2 v_L - 7 v_H v_L{}^2 - 4\mathrm{vl}^3 - 4\theta_H(7 v_H{}^2 - 4 v_L{}^2 - 7 v_H \theta_H + 4 v_L \theta_H)$

However, in the region of parameter values that support the DB regime, (A18) is true only when:

$\theta_H > \frac{6 v_H{}^2 - 7 v_L{}^2 + v_H\sqrt{\frac{3 v_L(v_H - v_L)}{2}}}{2(6 v_H - 7 v_L)}$.

When $q_s < q_p \frac{\left[\frac{q_p}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$\pi^{DB} - \pi^{IP} =$
$\frac{(3 q_s{}^2 + q_p{}^2) - 10 q_s q_p + 4\left(q_p - q_s\right)\left(v_H{}^2 - v_L{}^2 - 2\theta_H(v_H - v_L)\right) - v_H{}^2\left(v_H{}^2 - v_L{}^2\right) + v_H v_L{}^2\left(v_H - v_L\right) + 4 v_H \theta_H\left(v_H - \theta_H\right)}{16\left(v_H - v_L\right)\theta_H}$.

The expression on the right-hand side is positive when the following condition holds:

$q_p >$
$5q_s + \sqrt{}(22 q_s{}^2 - 4 q_s(v_H - v_L)(v_H + v_L - 2\theta_H) + v_H{}^2(v_H{}^2 - v_L{}^2) - v_H v_L{}^2(v_H - v_L) + 4 v_H \theta_H\big(v_H(v_H +$
$v_L) + \theta_H(v_H - v_L)\big)$. (A19)

However, in the region of parameter values that support the DB regime, (A19) is true only when:

$\theta_H > \frac{5 v_H{}^2 - 3 v_L{}^2}{2(v_H + v_L)} - \sqrt{\frac{(v_H - v_L)(8 v_H{}^3 - v_H{}^2 v_L - 16 v_H v_L{}^2 - 8 v_L{}^3)}{2\left(v_H + v_L\right)^2}}$.

b) $q_s > q_p$

When $q_p > q_s \frac{\left[\frac{q_s}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$$\pi^{DB} - \pi^{IP} = \frac{3(q_s{}^2 + q_p{}^2) - 10q_s q_p - 4(q_p - q_s)(v_H{}^2 - v_L{}^2 + 2\theta_H) - v_H{}^2(v_H{}^2 - v_L{}^2) + v_H v_L{}^2(v_H - v_L) + 4v_H\theta_H(v_H - \theta_H)}{16(v_H - v_L)\theta_H}.$$

The expression on the right-hand side is positive when the following condition holds:

$$q_s > \frac{1}{3}\left[5q_p - 2(v_H{}^2 - v_L{}^2) - 4\theta_H(v_H - v_L) + \sqrt{16q_p{}^2 - 8q_p(v_H - v_L)(v_H + v_L + 2\theta_H) + \mathrm{B}}\right]. \quad \text{(A20)}$$

$where\ B \equiv (v_H - v_L)(7v_H{}^3 + 7v_H{}^2 v_L - 7v_H v_L{}^2 - 4v_L{}^3 - 4\theta_H(v_H{}^2 - 4v_L{}^2 + 7v_H\theta_H - 4v_L\theta_H)$.

However, in the region of parameter values that support the DB regime, (A20) is true only when:

$$\theta_H > -\frac{2v_H{}^2 + v_L{}^2}{2(2v_H - 5v_L)} - \sqrt{\frac{(v_H - v_L)(8v_H{}^3 - v_H{}^2 v_L - 20v_H v_L{}^2 - 8v_L{}^3)}{2(2v_H - 5v_L)^2}}.$$

When $q_p < q_s \frac{\left[\frac{q_s}{v_H - v_L} - 2(v_H + v_L) + 4\theta_H\right]}{8\theta_H}$.

$$\pi^{DB} - \pi^{IP} = \frac{(3q_p{}^2 + q_s{}^2) - 10q_s q_p + 4q_p\left(v_H{}^2 - v_L{}^2 - 2\theta_H(v_H - v_L)\right) - v_H{}^2(v_H{}^2 - v_L{}^2) + v_H v_L{}^2(v_H - v_L) + 4v_H\theta_H(v_H - \theta_H)}{16(v_H - v_L)\theta_H}.$$

The expression on the right-hand side is positive when the following condition holds:

$q_s > 5q_p + \surd(22q_p{}^2 + 4q_p(v_H - v_L)(v_H + v_L - 2\theta_H) + v_H{}^2(v_H{}^2 - v_L{}^2) - v_H v_L{}^2(v_H - v_L) - 4v_H\theta_H(v_H - v_L)(v_H - \theta_H)$. (A21)

However, (A21) is not true in the region of parameter values that support the DB regime.

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