Sys Rev Pharm 2020; 11(2): 230 – 236
A multifaceted review journal in the field of pharmacy
E-ISSN 0976-2779 P-ISSN 0975-8453

Original Research

New *X*-bar Control Chart Using Skewness Correction Method for Skewed Distributions with Application in Healthcare

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Article History: *Submitted: 10.12.2019* *Revised: 10.01.2020* *Accepted: 01.02.2020*

ABSTRACT
Control chart has been long-established among the highly reputable tools in statistical process control (SPC) with extensive industrial application. Shewhart chart is one of the most popular charts, but its reliability is arguable when dealing with skewed data, due to inflated false alarm rate (Type I error). In alleviating the problem, this study has developed a new *X-bar* control chart for monitoring of process mean using skewness correction (SC) method for skewed distributions, thus named as SC- $\bar{X}_S$ control chart. The SC method is incorporated into the standard Shewhart's *X-bar* chart, leading to the proposed univariate SC- $\bar{X}_S$ to monitor the process mean of skewed data. It offers asymmetric control limits using the usual three sigma and the same known function of the skewness estimated from subgroups without assuming any distribution. The chart's constants, and skewness correction factor are computed via numerical integration. To evaluate the strength and weakness of the charts, several conditions are created from different types of distributions and subgroup sizes. The SC- $\bar{X}_S$ performance evaluation based on the false alarm rates (FAR) and probability of out-of-control (OOC) detection are accomplished using Monte Carlo simulation in SAS version 9.4. To illustrate its applicability, a real data on healthcare is employed. Its FAR performance is compared to the established charts: weighted variance *X-bar R*(WV- $\bar{X}_R$); weighted variance *X-bar S*(WV- $\bar{X}_S$); and standard *X-bar S* (ST- $\bar{X}_S$). In aspect of the probability of OOC detection, the SC- $\bar{X}_S$ is contended by the exact *S* chart. Extensive simulation study shows that the proposed SC- $\bar{X}_S$ chart performs well in terms of FAR in almost all the degrees of skewness and sample sizes, *n*. In terms of the probability of OOC detection, it provides the closest values to those of the exact chart. It offers substantial enhancement over the established charts, and thus signifies as a preferred alternative especially in cases of skewed data.
Keywords: Control Charts, False Alarm Rate, Mean Shift, Skewness Correction, Skewed Distribution.
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DOI: 10.5530/srp.2020.2.35

BACKGROUND/OBJECTIVES AND GOALS

The main purposes of control charts are to assess whether or not a process is stable (in-control) – if not stable or out-of-control (OOC), then bring it into in-control state, and to continue monitoring to ensure process stability is maintained. The popular Shewhart $\bar{X}$, cumulative sum (CUSUM), exponentially weighted moving average (EWMA) and other charts assume the quality characteristic is normally distributed. When the assumption is violated, the performance of these charts is arguable, owing to the sensitivity of $\bar{X}$ in handling non-normal data. In real world scenarios, however, data normality is hardly attained; for instance, measurements taken from chemical processes and lifetimes of electrical or electronic components are often skewed.

Ideally, an OOC situation must be detected immediately upon its occurrence, and each OOC signal triggered must be reliable. In other words, on top of fast detection, there should also be only very few false alarms. Shewhart chart is among the most popular charts, but its reliability is arguable when dealing with skewed data, due to inflated false alarm rate (FAR), also known as Type I error. Obliviously, such will detriment process monitoring when the normality assumption is severely violated. In alleviating this problem, heuristic $\bar{X}$ charts have been developed for process monitoring of the mean from skewed data. Bai and Choi (1995) introduced the weighted variance (WV– $\bar{X}$) chart, while Chang and Bai (2001) designed various charts based on weighted standard deviation (WSD), namely WSD- $\bar{X}$, WSD-CUSUM and WSD-EWMA charts. Another method using skewness correction (SC) factor was incorporated into $\bar{X}$ by Chan and Cui (2003), creating SC- $\bar{X}$ chart. Although these charts could mediate the problem of highly inflated FAR of the original Shewhart $\bar{X}$ chart in situations of non-symmetrical data, the rates were still disputable. Later, Khoo et al. (2008) proposed synthetic WV- $\bar{X}$ chart which performed excellently in controlling FAR while achieving high probability of OOC detection. Another alternative for $\bar{X}$ chart which is gaining popularity is the synthetic $\bar{X}$ chart. Originally suggested by Wu and Spedding (2000), it combines Shewhart $\bar{X}$ chart and conforming run length (CRL) chart, making it effective in process mean shift detection.

This paper is an extension to the previous work on WV- $\bar{X}$ (Khoo et al., 2008) by integrating skewness correction factor to the control limits of Shewhart chart based on standard deviation, namely $\bar{X}_S$ charts. The advantages of using skewness correction factor are to enable faster process shift detection in both the mean and standard deviation, while simultaneously reducing FAR. In practice, however, the distributions of *X* and *S* are often unknown; thus, making it difficult for the users to adopt the chart constants: d_2', d_3', C_4^*, B_3^* and B_4^*, which are based on normality distribution. Theoretically, SC charts should be able to overcome the problem of inflated FAR associated with Shewhart charts when dealing with skewed process data.

The research objectives areto (i) construct a new univariate control chart for monitoring process mean using SC method, denoted as SC- $\bar{X}_S$, (ii) compare false alarm rate and probability of OOCdetection of SC- $\bar{X}_S$ with established charts: weighted variance *X-bar R* chart (WV- $\bar{X}_R$); weighted variance *X-bar S* chart (WV- $\bar{X}_S$); and standard *X-bar S* chart (ST- $\bar{X}_S$), and (iii) apply the proposed charts on the length of stay due to vaginal delivery complications.

METHODS

Simulations are carried out to assess the performance of the new charts with regards to FAR (Type I error) and the probability of shift detection in the process mean. To evaluate the strengths and weaknesses of the charts, several conditions are created from different types of distributions and subgroup sizes. Gamma, lognormal and Burr distributions are used, since they embody a vast range of skewness as their parameters are varied, while subgroups of size 5, 7, 10 and 15 are considered for SC- $\bar{X}_S$ chart. The data sets are generated using SAS 9.4 version and the proposed charts are then compared with the established charts for skewed data. The computation of the FAR and shift detection rates when the process is in-control and out-of-control (OOC), are performed using a Monte Carlo simulation in SAS version 9.4.

The false alarm rate (FAR) is formulated by the proportion of points falling beyond the control limits when the process is actually stable (in-control). Meanwhile, the probability of OOC detection is defined as the proportion of points falling beyond the control limits when the process mean has shifted. In this section, the computation of the probability of OOC detection is considered under the assumption that the shift of mean happens instantaneously upon completing the sample inspection or at the start of inspecting a new sample (subgroup). All the charts are designed based on a desired FAR of 0.0027, which is equivalent to the standard in-control average run length (ARL) of 370.

In the case of known parameters, the limits of all the charts are computed using their mean and standard deviation of stable (in-control) process parameters. The FAR and probability of OOC detection are calculated based on 1000 simulated subgroups each of size, *n* for both "in-control" and "OOC" situations. For the case involving unknown parameters, the FAR and probability of OOC detection are obtained as follows. For a specific distribution, firstly, 30 "in-control" subgroups, each of size, *n,* are produced in order to estimate the chart's control limits. Then 1000 "in-control" subgroups each having the same size, *n*, are generated from the same distribution and the FAR are computed for those charts. This procedure is repeated 10,000 times and the average FAR for the charts considered are computed. Similarly, these steps are followed for computing the average probability of OOC detection using the "OOC" subgroups.

RESULTS

The results and findings are reported in several sections: the constructions of the SC-$\bar{X}$ chart, its performance evaluation in comparison with the other existing charts, and an application to real hospital data.

Skewness Correction for Mean (SC-$\bar{X}$)

Skewness correction (SC) method is employed to remedy the Shewhart chart in accordance to the level of skewness of a random variable *X*. It offers asymmetric control limits using the usual three sigma and the same known function of the skewness estimated from subgroups without assuming any distribution. The SC method refers to the Cornish–Fisher expansion (Chan & Cui, 2003). The upper control and lower control limits of the SC-$\bar{X}$ chart are respectively:

$$UCL_{SC\text{-}\bar{X}} = \mu_X + \left(3 + \frac{4\alpha_3 / \left(3\sqrt{n}\right)}{1 + 0.2\alpha_3^2 / n} \right) \frac{\sigma_X}{\sqrt{n}}$$

(1)

and

$$LCL_{SC\text{-}\bar{X}} = \mu_X + \left(-3 + \frac{4\alpha_3 / \left(3\sqrt{n}\right)}{1 + 0.2\alpha_3^2 / n} \right) \frac{\sigma_X}{\sqrt{n}}$$

(2)

Here, α_3 is the skewness coefficient of the random variable *X*.

In cases of unknown values of the process parameters, the control limits of the SC-$\bar{X}$ chart are computed as:

$$UCL_{SC-\bar{X}} = \bar{\bar{X}} + \left(3 + \frac{4\hat{\alpha}_3 / \left(3\sqrt{n}\right)}{1 + 0.2\hat{\alpha}_3^2 / n} \right) \frac{\bar{S}}{C_4^* \sqrt{n}}$$

(3)

and

$$LCL_{SC-\bar{X}} = \bar{\bar{X}} + \left(-3 + \frac{4\hat{\alpha}_3 / \left(3\sqrt{n}\right)}{1 + 0.2\hat{\alpha}_3^2 / n} \right) \frac{\bar{S}}{C_4^* \sqrt{n}}$$

(4)

Where $\bar{\bar{X}}$ and $\bar{S}$ are the sample grand mean and the average sample standard deviation, respectively. Here, $c_4^* = \frac{E(S)}{\sigma_X}$. Also note that $\hat{\alpha}_3$ is the estimator of the skewness coefficient, α_3 which can be estimated using the following formula: $\hat{\alpha}_3 = \frac{1}{nm-3} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{X_{ij} - \bar{\bar{X}}}{S_X} \right)^3$,

where $S_X = \sqrt{\frac{1}{nm-1} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(X_{ij} - \bar{\bar{X}} \right)^2}$, where *m* is the number of samples (subgroups) and *n* is the number of observations in each sample or better known as sample size.

Performance Evaluation

In evaluating the performance of the charts, the false alarm rate (FAR) and the probability of OOC detection are considered. The latter measures the ability of a chart in responding to a process shift of a certain magnitude, δ. To cover a wide range of skewness, Weibull, lognormal and gamma distributions are chosen in this study.

The subsequent tables provide the FAR (Type I error) and the probability of OOC detection for all the new charts in comparison with other existing charts. It is to be noted that for each setting, the single smallest value of the error which indicates the best chart, is printed in bold text (excluding multiple smallest values shared among more than one chart). To facilitate reading, the first column of chart is reserved for the proposed chart being evaluated.

Tables 3a and 3b present the FAR (Type I error) of the SC-$\bar{X}_S$ chart and existing charts for skewed distribution when sample sizes, n = 5, 7 and n = 10, 15, respectively. For comparison purposes, the normal distribution is included alongside the Weibull and gamma distributions. Looking at the bolded values, it is evident that the new SC-$\bar{X}_S$ chart produces the smallest FAR (Type I error) in almost all the scenarios investigated. In fact, at small sample sizes of 5 and 7 (Table 3a), SC-$\bar{X}_S$ chart seems to give the best Type I errors for all settings.

Table 3a: False alarm rates (Type I errors) comparison (n = 5, 7)

Distribution	α_3	Sample size, *n*									
		5					7				
		SC − $\bar{X}_S$	SC − $\bar{X}_R$	WV − $\bar{X}_R$	WV − $\bar{X}_S$	ST − $\bar{X}_S$	SC − $\bar{X}_S$	SC − $\bar{X}_R$	WV − $\bar{X}_R$	WV − $\bar{X}_S$	ST − $\bar{X}_S$
normal	0.0	0.0037	0.0039	0.0037	0.0038	0.0038	**0.0034**	0.0035	0.0035	0.0035	0.0035
Weibull (β)											
3.6286	0.0	**0.0030**	0.0032	0.0031	0.0032	0.0047	**0.0029**	0.0033	0.0033	0.0032	0.0039
2.2266	0.5	**0.0031**	0.0032	0.0033	0.0032	0.0057	**0.0030**	0.0034	0.0034	0.0031	0.0044
1.5688	1.0	**0.0033**	0.0035	0.0042	0.0041	0.0089	**0.0031**	0.0036	0.0039	0.0035	0.0067
1.2123	1.5	**0.0037**	0.0040	0.0055	0.0052	0.0137	**0.0035**	0.0040	0.0045	0.0042	0.0104
0.9987	2.0	**0.0046**	0.0048	0.0067	0.0066	0.0192	**0.0041**	0.0047	0.0055	0.0051	0.0148
0.8598	2.5	**0.0056**	0.0059	0.0080	0.0078	0.0251	**0.0048**	0.0056	0.0064	0.0059	0.0197
0.7637	3.0	**0.0067**	0.0072	0.0089	0.0088	0.0308	**0.0057**	0.0066	0.0076	0.0068	0.0247
gamma (α)											
38000	0.0	**0.0037**	0.0039	0.0037	0.0038	0.0038	**0.0035**	0.0037	0.0036	0.0036	0.0036
15.4	0.5	0.0040	0.0040	0.0040	0.0040	0.0048	0.0037	0.0038	0.0037	0.0037	0.0043
3.913	1.0	0.0040	0.0040	0.0044	0.0045	0.0071	**0.0037**	0.0039	0.0042	0.0040	0.0062
1.788	1.5	**0.0043**	0.0045	0.0057	0.0055	0.0112	**0.0039**	0.0042	0.0048	0.0046	0.0094
0.983	2.0	**0.0046**	0.0050	0.0069	0.0066	0.0164	**0.0040**	0.0047	0.0055	0.0050	0.0133
0.648	2.5	**0.0051**	0.0055	0.0075	0.0072	0.0223	**0.0044**	0.0054	0.0060	0.0054	0.0233
0.442	3.0	**0.0063**	0.0070	0.0076	0.0076	0.0293	**0.0050**	0.0060	0.0071	0.0054	0.0233

Table 3b: False alarm rates (Type I errors) comparison (n = 10, 15)

Distribution	α_3	Sample size, *n*									
		10					15				
		SC − $\bar{X}_S$	SC − $\bar{X}_R$	WV − $\bar{X}_R$	WV − $\bar{X}_S$	ST − $\bar{X}_S$	SC − $\bar{X}_S$	SC − $\bar{X}_R$	WV − $\bar{X}_R$	WV − $\bar{X}_S$	ST − $\bar{X}_S$
normal	0.0	**0.0033**	0.0034	0.0034	0.0036	0.0034	**0.0033**	0.0039	0.0036	0.0034	0.0034
Weibull (β)											
3.6286	0.0	**0.0030**	0.0033	0.0033	0.0035	0.0031	0.0031	0.0042	0.0037	0.0036	0.0031
2.2266	0.5	**0.0030**	0.0034	0.0035	0.0034	0.0035	**0.0031**	0.0044	0.0038	0.0038	0.0034
1.5688	1.0	**0.0031**	0.0037	0.0035	0.0038	0.0050	**0.0032**	0.0040	0.0039	0.0039	0.0045
1.2123	1.5	**0.0034**	0.0044	0.0038	0.0039	0.0075	**0.0033**	0.0042	0.0041	0.0041	0.0062
0.9987	2.0	**0.0037**	0.0046	0.0044	0.0043	0.0107	**0.0036**	0.0050	0.0043	0.0038	0.0085
0.8598	2.5	**0.0043**	0.0052	0.0051	0.0051	0.0145	**0.0040**	0.0057	0.0046	0.0041	0.0112
0.7637	3.0	**0.0050**	0.0060	0.0061	0.0058	0.0183	**0.0046**	0.0062	0.0052	0.0047	0.0142
gamma (α)											
38000	0.0	0.0034	0.0034	0.0034	0.0039	0.0035	**0.0033**	0.0039	0.0036	0.0039	0.0034
15.4	0.5	0.0036	**0.0035**	**0.0035**	0.0040	0.0040	**0.0034**	0.0040	0.0035	0.0043	0.0038
3.913	1.0	**0.0036**	0.0038	0.0038	0.0042	0.0053	**0.0035**	0.0037	0.0037	0.0043	0.0048
1.788	1.5	**0.0036**	0.0043	0.0039	0.0044	0.0076	**0.0035**	0.0040	0.0039	0.0042	0.0062
0.983	2.0	**0.0038**	0.0047	0.0045	0.0043	0.0109	**0.0035**	0.0049	0.0042	0.0036	0.0085
0.648	2.5	**0.0041**	0.0052	0.0049	0.0047	0.0143	0.0037	0.0057	0.0043	0.0037	0.0112
0.442	3.0	**0.0042**	0.0056	0.0057	0.0046	0.0186	0.0038	0.0060	0.0045	**0.0033**	0.0143

The following Table 4 shows the OOC detection probabilities for the proposed SC- $\bar{X}_S$ chart and existing charts considering exponential distribution when shape parameter of Weibull distribution, $\beta = 1$ and magnitude of shift, δ from 0 to 3.

Table 4: OOC detection probabilities of various mean control charts: Weibull, β= 1; n = 5; α_3 = 2; δ = 0.0, 3.0 (0.25).

δ	$SC-\bar{X}_S$	Exact method	$WV-\bar{X}_R$	$WV-\bar{X}_S$	$ST-\bar{X}_S$
0.00	0.0021	**0.0027**	0.0052	0.0052	0.0092
0.25	0.0044	**0.0034**	0.0124	0.0124	0.0216
0.50	0.0109	**0.0084**	0.0283	0.0283	0.0484
0.75	0.0246	**0.0190**	0.0620	0.0620	0.1017
1.00	0.0547	**0.0431**	0.1292	0.1292	0.2014
1.25	0.1143	**0.0916**	0.2474	0.2474	0.3636
1.50	0.2233	**0.1835**	0.4333	0.4333	0.5885
1.75	0.3959	**0.3358**	0.6689	0.6689	0.8215
2.00	0.6279	**0.5532**	0.8856	0.8856	0.9696
2.25	0.8550	**0.7917**	0.9897	0.9897	0.9999
2.50	0.9811	**0.9563**	1.0000	1.0000	1.0000
2.75	1.0000	**0.9994**	1.0000	1.0000	1.0000
3.00	1.0000	**1.0000**	1.0000	1.0000	1.0000

In terms of the probability of OOC detection, the most accurate is that of the exact method. Hence, the values obtained for each setting shall be compared against those of the Exact method (in bold). Among the four charts, the proposed SC- $\bar{X}_S$ chart yields value closest to the exact method, indicating its better performance than the other three charts.

Application on Data on Birth Problems and Delivery Complications

The applicability of the charts in real life situation is put to test using real health-related data to monitor and improve public health services. Hospital length of stay data are often highly skewed, hence are suitable for the proposed charts. In this study, data on the length of stay are based on patients warded due to birth problems and delivery complications from a local hospital. There are two categories of problems/complications faces by patient during or before the birth which have potential for unusually long lengths of stay: birth problems and delivery complications. Birth problems are typically detected before giving birth such as *Caesarean (LSCS), Forceps, Vacuum, Twin and Lower Body Weight*. Meanwhile, delivery complications faced by patients during the births are *Post-Partum Hemorrhoids (PPH), Manual Removal of Placenta (MRP) and Perineal Tear 3rd/4th Degree*. It is to be noted that normal vaginal birth or known as spontaneous vertex delivery (SVD) data are excluded in this study.

The data available in this study were recorded from March 2016 to July 2016 comprise 748 patients. The length of stay for an individual patient is computed in days and measured by the difference between date of admission and dates of discharge. A patient who stays a night is regarded as having one day stay. A 'day only' admission has a length of stay of zero. The total number of patients admitted due to birth problems and delivery complications for each of the 17 weeks study is presented in Table 5.

Table 5: Summary of Birth Problems and Delivery Complications Data

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Number of Patients	53	47	48	41	36	59	50	33	45	44	59	45	37	44	34	46	27

The week is identified by the date of discharge in order to compute the length of stay. It is observed that the total number of patients discharged in a week varies between 27 to 59.

Meanwhile, Figure 1 below shows the histogram for the length of stay (days) of patients with birth problems and delivery complications.

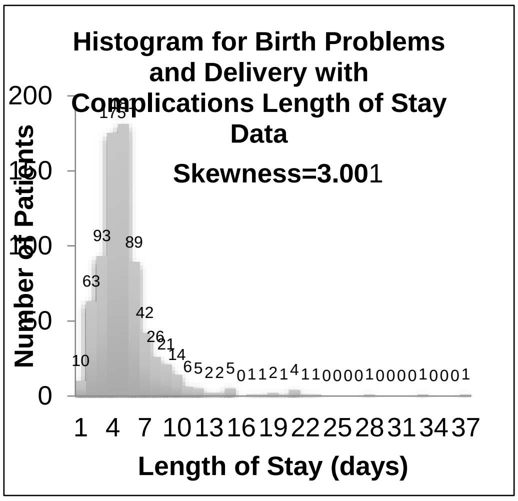

Figure 1: Summary of Length of Stay (days).

The pattern of histogram shown in Figure 1 indicates highly right-skewed data where the distribution's peak is off the center and its tail stretches away on the right side. In fact, the calculated skewness coefficient is $\alpha_3 = 3.001$. Hence, this data set is very suitable to illustrate the application of the proposed SC- $\bar{X}_S$ chart. To apply the proposed charts on the data, the following steps are taken. Figure 2 represents the steps for establishing SC-$\bar{X}$ and SC-S (Atta et al., 2017) control charts into applications of the length of stay for birth problems and delivery complications.

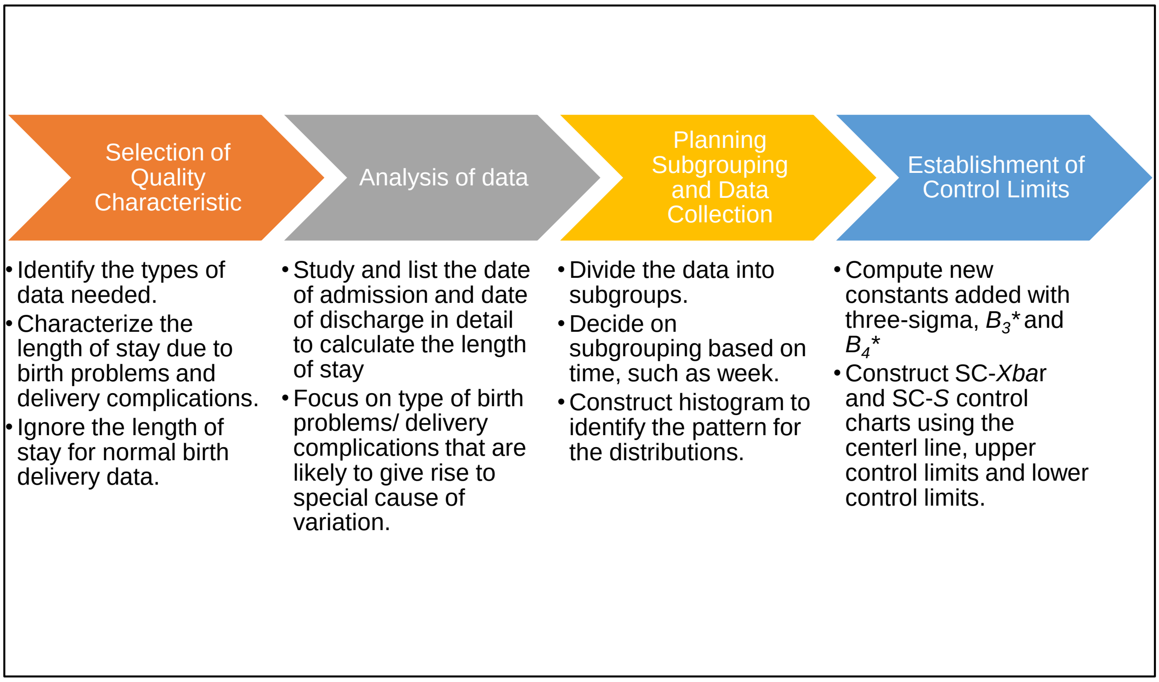

Figure 2: Steps to Establish the Control Limits of SC-$\bar{X}$

In this case, the length of stay is computed in the preliminary analysis before constructing the control charts, as shown in Figure 2. It is to be noted that the new constants of skewness correction factor, B_3^* and B_4^* of the SC-$\overline{X}_S$ and SC-S charts respectively, are based on computation in Sections 3.4.1 and 3.4.2 earlier.

Similar to the research of Dubois (1991), the interest in using the SC-$\bar{X}$ and SC-S chart in this study is to monitor and investigate an unusually long length of stay due to birth problems and delivery complications. Although the standard practice is to construct control chart using a minimum 20 subgroups, only 17 subgroups (17 weeks) are available in our study. Next, by using simple random sampling, a sample size, n = 5 is taken for each week. The factors of skewness correction, c_4 and d_4 are computed in order to calculate the control limits. The constructed SC-$\bar{X}$ control chart is shown in Figure 3.

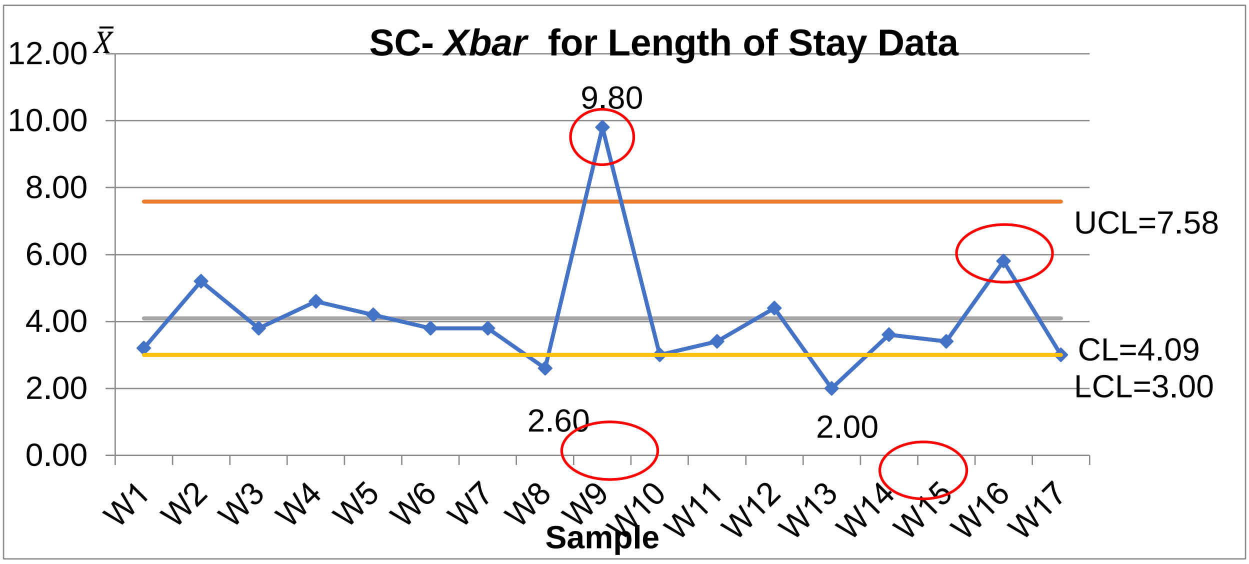

Figure 3: SC-$\bar{X}$ Chart (UCL = 7.58; LCL = 3.00; CL = 4.09, n = 5; c_4 =1.49)

There exist points which fall outside the control limits: Weeks 8, 9 and 13, indicating patients who stay longer than usual. Referring to the point beyond the UCL, in Week 9, a patient was discharged after 36 days due to cesarean, fibro sarcoma at buttock and external hemorrhoid.

CONCLUSION

In summary, all the results strongly indicate that the proposed control chart in this research has better performance compared to all the four established control charts for skewed populations. Simulation studies prove that it provides false alarm rates (Type I errors) and probability of out-of-control detection that are lower than existing charts for skewed distributions for all levels of skewness and sample sizes. Application to real hospital data set proves its applicability in monitoring process mean of a skewed data. Hospital management as well as researchers can adopt our proposed control chart in order to monitor and improve quality characteristics if the data do not follow a normal distribution. Other potential application areas are chemical processes, reaction times in experiments, number of claims by insurance customers and lifetimes of electronic components.

ACKNOWLEDGEMENT

The authors would like to thank the Ministry of Education, Malaysia and Universiti Utara Malaysia (UUM) for the financial support under the Fundamental Research Grant Scheme (FRGS/2/2013: S/O 12904). Gratitude goes to the local hospital for providing the real data set.

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