TRANSSHIPMENT IN DECENTRALIZED SUPPLY CHAINS

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Transshipment in Decentralized Supply Chains

by

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Abstract

Transshipment is the practice of sharing common resources among supply chain members in order to mitigate the risks of uncertain demands. The main theme of this thesis is the transshipment problem in decentralized supply chains. The members of decentralized supply chains are self-interested agents who do not necessarily consider the efficiency of the whole chain, and need contracts that specify the details of their cooperation. We provide a systematic overview of coordinating contracts in supply chains before focusing on three specific questions concerning the decentralized transshipment problem.

The first problem addressed by this thesis is to find *coordinating transshipment contracts* for supply chains with two agents. We propose a transshipment contract that always coordinates the general two-agent supply chains. This mechanism relies on an implicit pricing mechanism, i.e. agents initially agree on a formula for setting the transshipment prices, and once quantity decisions have been made and prior to the realization of demands, they fix the transshipment prices.

The second problem is to find coordinating contracts with a *pricing mechanism* in supply chains with more than two agents. We propose a mechanism for deriving the transshipment prices based on the coordinating allocation rule introduced by Anupindi et al. (2001). With the transshipment prices being set, the agents are free to match their residuals based on their individual preferences. It has been shown that with the transshipment prices derived from the proposed mechanism, the optimum transshipment patterns are always pair-wise stable, i.e. there are no pairs of agents that can be jointly better off by unilaterally deviating from the optimum transshipment patterns.

The third problem pertains to the effects of *cooperation costs* on transshipment games. Despite its practical relevance, the issue of cooperation costs has not been addressed in the supply chain contracting literature thus far. We study the cooperative transshipment game with symmetric newsvendors having normally distributed independent demands. We provide characterization of optimal individual quantities, the maximum expected profits, and individual allocations for these games. These results, though interesting by themselves, are only a point of departure for studying the games with cooperation costs. We provide conditions for stability (non-emptiness of the core) of these games under two governance network structures, i.e. clique and hub.

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N	Set of agents
i, j	Generic members of N
n	Cardinality of N
Q	A subset of N
S	Set of transshipment sellers
B	Set of transshipment buyers
r_i	Market selling price of a unit of agent i's products
c_i	Production cost of a unit of agent i's products
ν_i	Salvage value of a unit of agent i's products
t_{ij}	Unit transportation cost from agent i to agent j
h_i	Lost sale penalty for a unit of agent i's demands
X_i	Production/order quantity of agent i
s_{ij}	Transshipment price that agent i charges agent j for a unit of its products
p_{ij}	Marginal transshipment profit obtained by a unit transshipment from i to j
D_i	A random variable representing agent i's market demand
H_i	Agent i's surplus products after realization of market demand
E_i	Agent i's unsatisfied demand after realization of market demand
W_{ij}	Transshipment quantity from agent i to agent j
ϕ	Probability Density Function (PDF) of standard normal distribution
Φ	Cumulative Density Function (CDF) of standard normal distribution
π_i	Realized (actual) profit of agent i
J_i	Expected profit of agent i
J_Q	Total expected profit of agents in Q
^{NC}	The superscript indicating non-cooperative mode
^{DC}	The superscript indicating decentralized cooperative mode

c	The superscript indicating centralized non-cooperative mode
u_i	Utility function of agent i
d_i	Utility of disagreement scenario for agent i
γ_i	Bargaining power of agent i
α_i, β_i	Agent i's allocation of total profit
Δ	Increment in individual allocation of total profit
v	Characteristic function
$v(Q)$	Value of characteristic function v for a coalition Q
K	Cooperation costs per link
E	Expected value operator

Chapter 1

Introduction

A supply chain is the set of entities involved in the design of new products and services, procuring raw materials, transforming them into semi-finished and finished products, and delivering them to the end customer (Swaminathan and Tayur, 2003). In a broad sense a supply chain consists of two or more legally separated organizations, being linked by material, information and financial flows. These organizations may be firms producing parts, components and end products, logistic service providers and even the ultimate customer, and in a narrow sense the term supply chain is also applied to a large company with several sites often located in different countries (Stadtler and Kilger, 2008). The main underlying tenet of Supply Chain Management (SCM) is that organizations can improve their performance in terms of higher profit levels and customer satisfaction, and lower lead-times and uncertainties through integration and collaboration with other organizations who are parts of the same supply system. Therefore, as discussed by Lee (2004), top-performing supply chains possess three qualities: (1) great supply chains are agile and they react speedily to sudden changes in demand or supply, (2) they adapt over time as market structures and strategies evolve, and (3) they align the interests of all the firms in the supply chain so that companies optimize the chain's performance when they maximize their interests.

The ultimate goal in managing supply chains is to better serve the market. In a recent study, Fawcett et al. (2008) found that the top four perceived benefits of SCM are improvements in responding to customer requests, on-time delivery, customer satisfaction, and order fulfillment lead-time. The same study also highlights that after the inadequacy of required information systems, the most important barrier to achieving the SCM benefits is the lack of clear supply chain guidelines. Therefore, the challenge in managing supply chains is not just the aspiration to improve the efficiency of the whole supply chain, but the mechanisms to actually coordinate the many complex processes spanning across it. Without appropriate mechanisms, uncoordinated supply chains may suffer drastic inefficiencies. Narayanan and Raman (2004) elaborate the example of *Cisco Systems, Inc.* and show how the lack of coordination mechanisms resulted in 2.5 billion dollars of inventory write-offs.

Transshipment is the practice of sharing common resources among different agents in supply chains in order to mitigate the risks associated with uncertain demands. In manufacturing, transshipment is typical in industries wherein the volatile market demands should be met by utilizing pre-specified production capacities/quantities. In retailing, transshipment of inventories can also boost the service level while reducing inventory costs. The time lag between decisions on production/order quantities and the realization of random demands—which could be due to long procurement lead-times or technological constraints—makes the initial decisions an inflexible parameter at the time of demand realization. The option to transship provides the agents with the opportunity to improve efficiency both at the individual and network levels.

Transshipment can be implemented in a variety of circumstances when uncertainties about external factors cannot be adequately handled in advance. For many production supply chains, procurement of raw materials and parts with long lead-times in anticipation of random market demand is a major concern, both for supply chain agents and ultimate customers in some cases such as H1N1 vaccines (Hirschler and Kelland, 2009). As the volatility of market demand increases, the risk of mismatch between the stacked resources and actual demand escalates. An example of transshipment practice is discernible in the oil industry where volatility of demands and limitation of regional refinery capacities make transshipment a reasonable practice (Dempster et al., 2000). Other examples of transshipment in the retailing industry come from automobile dealer networks (Zhao et al., 2005), computer retailing (Shao et al., 2008), construction machinery (Rao et al., 2000), and apparel (Mogre et al., 2009). Although in most cases transshipment is done by physically moving products and inventories from one agent to another, this feature is not necessary. In *virtual* transshipment, the customers of one agent may be served directly from another agent. This type of transshipment is common in the electricity markets (Yang and Qin, 2007).

Traditionally, operations management deals with centralized systems where it is assumed that a single agent chooses all the necessary actions and makes all the relevant decisions for the whole system. Therefore, optimization is the primary concern for decision makers. However, decisions in real supply chains are usually decentralized. This is either because the supply chain is comprised of agents with different preferences (e.g. different ownerships), or a large number of decisions add to system complexity to the point that centralized decision making and control are infeasible—so the decisions must be distributed among *autonomous* agents. The issue here is that, when agents individually optimize their decisions, supply chain efficiency is not necessarily maximized. Hence, coordination becomes a major problem. In decentralized systems, the major goal is to design appropriate coordinating mechanisms so that individual decisions are coordinated. These mechanisms are either contractual mechanisms (among separately-owned interacting agents) or performance evaluation measures (among interacting agents with the same ownership structure). In both cases, a coordinating mechanism transforms the agents' objectives so that they would be aligned with the integrated supply chain objectives. The fundamental working hypothesis is that each agent, being rational, maximizes its individual objective. Therefore, a coordinating mechanism needs to ensure that individual decisions result in supply chain's maximum efficiency. The main tool for studying the decision making processes of rational agents is *game theory*. The analysis of the transshipment problem is significantly complicated in decentralized supply chains where transshipments are done among self-interested rational agents. The purpose of this thesis is to study the contractual mechanisms for coordinating the transshipments in decentralized supply chains.

When supply chain agents intend to cooperate with each other, they need contracts that specify the details of their cooperation. Although contracts have been studied in law, economics, and marketing disciplines, their study in operations management and SCM takes a rather different approach: "What distinguishes SCM contract analysis may be its focus on operational details, requiring more explicit modeling of materials flows and complicating factors such as uncertainty in the supply or demand of products, forecasting and the possibility of revising those forecasts, constrained production capacity, and penalties for overtime and expediting" (Tsay et al., 1999, p. 302). In SCM, the issue of contracts and their effects on agents' decisions becomes central once one approaches a supply chain as the *nexus-of-contracts* (Whang, 1995). This emphasizes that a supply chain is a collection of self-interested agents bound together through a set of contracts. This thesis mainly investigates transshipment contracts and their effects on the supply chain efficiency.

When optimization of the system's total efficiency is (at least partially) in conflict with agents' incentives, reconciliation of these conflicts is the goal of *coordinating contracts*. A coordinating contract has three characteristics:

(a) the set of supply chain optimum decisions should be a *pure Nash equilibrium*;

(b) it should divide the supply chain profits arbitrarily among the agents; and

(c) it should be worth adopting (Cachon, 2003).

Supply chain coordination through contracts has been a burgeoning area of research in recent years. In spite of rapid development of research, there are only a few structured analyses of assumptions, methods, and real-life-applicability of results in this field. In Chapter 2, a systematic framework of contracting in supply chain context is presented. The aim of that chapter is to provide a systematic overview of coordinating contracts in supply chains through highlighting the main concepts, assumptions, methods, and presenting the state-of-the-art research in this field.

The first question addressed by this thesis is to find coordinating transshipment contracts for a supply chain with only two agents. In Chapter 3, we study a supply chain with two independent agents producing a similar product and cooperating through transshipment. Previous research shows that only under a certain range of problem parameters, a set of *linear transshipment prices* (i.e. transshipment prices that are fixed before the decisions on production/order quantities have been made) could be found which induce the agents to decide their production quantities so that the total expected profit of the two agents equals the maximum expected profit of the centralized supply chain. However, even though such transshipment prices do exist, they result in exclusive divisions of total expected profits and thus they cannot accommodate the arbitrary division of total expected profits due to different bargaining powers of the agents (the second coordination requirement in Cachon's definition (Cachon, 2003)). Using the Generalized Nash Bargaining Solution, we model the negotiation between the agents over the division of total expected profit resulting from their cooperation, and derive a coordinating contract for this setting. This contract has an implicit pricing mechanism and should be carried out in two rounds. In the first round, the agents set the transshipment prices as an implicit function of their production quantities, and in the second round, after the agents individually decide their quantities, they fix the negotiated transshipment prices by selecting them among all the possible transshipment prices.

The second question is to investigate the coordinating contracts with pricing mechanisms in supply chains with more than two agents. This question is studied in Chapter 4. The contracts which are based on allocation rules require agents to be able to take advantage of side payments (which may be infeasible in some situations). From the implementation point of view, these contracts also need a governing agent to collect and redistribute the realized profits among the members of the coalition. In order to avoid these difficulties, the agents can turn to the contracts with pricing mechanisms. Then, whenever a transshipment between an agent with surplus and another one with outstanding demand happens, the latter pays the former a sum proportional to the amount transshipped. The advantage is that the additional institution required for *redistribution* of extra profits becomes unnecessary—agents who are involved in a transshipment transaction can handle the redistributions without incentive-aligning side payments. As this thesis' main contribution to this question, we show that transshipments among several agents resembles a matching game in a two-sided market where the supply and demand values are real numbers. We have derived a pricing mechanism with which optimal transshipment patterns are always *pair-wise stable* solutions to the corresponding matching process, i.e. given the transshipment prices, no pairs of agents can simultaneously improve their profits by mutually deviating from the optimal transshipment patterns.

The third question pertains to the effects of cooperation costs on transshipment games. Chapter 5 addresses the cooperative transshipment game with symmetric newsvendors having independent and normally distributed demands. The cooperative transshipment game without cooperation costs has been well studied in the literature, however, general analytical results for it seem out of reach at the moment. We provide characterizations of optimal individual quantities, the maximum expected profits, and individual allocations for these games. In particular, we prove that though individual allocations grow with the coalition size they diminish at the same time according to *two laws of diminishing individual allocations*. These results though interesting by themselves are only a point of departure for studying the games with cooperation costs. In reality, when agents seek to cooperate with each other, they have to incur negotiation and governance costs, e.g. monitoring and infrastructure. The cooperation costs depend on the cooperation network structure. We consider two: (1) Clique network structure, where all the agents in the coalition are directly linked to each other; and (2) Hub network structure, where the agents are linked to a designated coordinator agent. We provide the necessary and sufficient conditions for the cost per link necessary to render the core of the game non-empty for both network structures. These maximum admissible costs are always decreasing for cliques, however, increasing or exhibiting a unimodal pattern for hubs. To the best of our knowledge, these results are the first to incorporate cooperation costs in the analysis of transshipment games in the operational research and operations management literature.

1.1 Transshipment Games

At this point, it is worthwhile to distinguish among the variations of transshipment games which are analyzed in different sections of the thesis. The notation used in this thesis is listed on pages xi and xii.

1.1.1 Non-cooperative Transshipment Game

A non-cooperative transshipment game is a stochastic game. In a two-agent non-cooperative transshipment game, it will be shown that agent i's expected profit equals

$$J_i^{DC}(\mathbf{s},\mathbf{X}) = \mathsf{E}\big[r_i \min\{X_i, D_i\} + \nu_i H_i - c_i X_i + (s_{ij} - t_{ij} - \nu_i)\min[(D_j - X_j)^+, (X_i - D_i)^+] + (r_i - s_{ji})\min[(D_i - X_i)^+, (X_j - D_j)^+]\big] \quad (1.1)$$

Chapter 3 analyzes this game for a supply chain with two agents.[1]

1.1.2 Non-cooperative/cooperative Transshipment Game

A non-cooperative/cooperative transshipment game is a two-stage game. The first stage game is a stochastic non-cooperative game, and the second stage game, which is played after the realization of demands, is a deterministic cooperative game. This game was first formulated by Anupindi et al. (2001). The profit function for each individual agent is

$$J_i^{DC}(\mathbf{X}) = \mathsf{E}\left[r_i \min\{X_i, D_i\} + \nu_i H_i - c_i X_i + \alpha_i(\mathbf{X},\mathbf{D})\right] \quad (1.2)$$

where $\alpha_i(\mathbf{X},\mathbf{D})$ represents agent i's allocation of the second stage deterministic cooperative game, i.e., *ex post* cooperative transshipment game. For given $\mathbf{X}$ and $\mathbf{D}$, the *ex post* cooperative transshipment game assigns to any sub-coalition $Q \subseteq N$ the value

[1] The original game considered in Chapter 3 also incorporates the lost-sale penalties which, for the ease of comparison, are excluded from this formulation.

R_Q equals to

$$
\begin{aligned}
R_Q(\mathbf{X}, \mathbf{D}) \quad = \quad & \max_{\mathbf{W}} \sum_{i \in Q} \sum_{j \in Q} p_{ij} W_{ij} \\
s.t. \quad & \\
& \sum_{j \in Q} W_{ij} \leq H_i, \forall i \in Q \\
& \sum_{i \in Q} W_{ij} \leq E_j, \forall j \in Q \\
& W_{ij} \geq 0, \forall i, j \in Q.
\end{aligned} \qquad (1.3)
$$

The *ex post* cooperative transshipment game is also known as the **Owen Game** (Owen, 1975). Chapter 4 analyzes the non-cooperative/cooperative transshipment game.

1.1.3 Cooperative Transshipment Game

A cooperative transshipment game (or the *ex ante* cooperative transshipment game when the reference is not immediately clear from the context) is a cooperative game with a stochastic characteristic function (Slikker et al., 2005). The *ex ante* cooperative transshipment game assigns to any coalition $Q \subseteq N$ the value $\hat{J}_Q$ which is given by

$$
\hat{J}_Q = \max_{\mathbf{X}} J_Q(\mathbf{X}) = \max_{\mathbf{X}} \mathsf{E}\left[\sum_{i \in Q} (r_i \min(X_i, D_i) + \nu_i H_i - c_i X_i) + R_Q(\mathbf{X}, \mathbf{D})\right] \qquad (1.4)
$$

where for given $\mathbf{X}$ and $\mathbf{D}$,

$$
\begin{aligned}
R_Q(\mathbf{X},\mathbf{D}) \quad &= \quad \max_{\mathbf{W}} \sum_{i\in Q}\sum_{j\in Q} p_{ij}W_{ij} \qquad (1.5)\\
&s.t.\\
&\sum_{j\in Q} W_{ij} \le H_i, \forall i \in Q\\
&\sum_{i\in Q} W_{ij} \le E_j, \forall j \in Q\\
&W_{ij} \ge 0, \forall i,j \in Q
\end{aligned}
$$

Chapter 5 studies this game in supply chains with symmetric newsvendor agents facing independent and normally distributed demands.

1.2 Centralized Transshipment Problem

In Chapters 3, 4, and 5, all transshipment games are compared with the centralized transshipment problem. The centralized transshipment problem is the following stochastic optimization problem:

$$
\max_{\mathbf{X}} J(\mathbf{X}) = \max_{\mathbf{X}} \mathsf{E}\left[\sum_{i\in N}(r_i \min(X_i, D_i) + \nu_i H_i - c_i X_i) + R_N(\mathbf{X},\mathbf{D})\right] \qquad (1.6)
$$

where for given $\mathbf{X}$ and $\mathbf{D}$,

$$
\begin{aligned}
R_N(\mathbf{X},\mathbf{D}) \quad &= \quad \max_{\mathbf{W}} \sum_{i\in N}\sum_{j\in N} p_{ij}W_{ij} \qquad (1.7)\\
&s.t.\\
&\sum_{j\in N} W_{ij} \le H_i, \forall i \in N\\
&\sum_{i\in N} W_{ij} \le E_j, \forall j \in N\\
&W_{ij} \ge 0, \forall i,j \in N.
\end{aligned}
$$

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Chapter 2

Coordinating Contracts in Supply Chain Management: A Review of Methods and Literature

Summary: *Supply chain coordination through contracts has been a burgeoning area of research in recent years. In spite of rapid development of research, there are only a few structured analyses of assumptions, methods, and applicability of insights in this field. The aim of this chapter is to provide a systematic overview of coordinating contracts in supply chains through highlighting the main concepts, assumptions, methods, and present the state-of-the-art research in this field.*

2.1 Introduction

The Supply Chain Management (SCM) paradigm asserts that when making decentralized decisions, the efficiency of the whole system should be taken into consideration. When decision making is decentralized, i.e. decisions are made by independent agents comprising the chain, optimizing the system's total efficiency might be in conflict with the agents' incentives. Therefore, coordinating the agents' decisions becomes a major issue. By viewing a supply chain as a nexus-of-contracts (Wang and Parlar, 1994), i.e. a group of rational agents interacting with each other according to pre-specified rules, more efficient SCM is achieved by designing appropriate contracts coordinating the agents' decisions. This is the main objective of research on coordinating contracts in supply chains. Although contracts have been studied in law, economics, and marketing disciplines, their study in SCM takes a rather different approach:

> What distinguishes SCM contract analysis may be its focus on operational details, requiring more explicit modeling of materials flows and complicating factors such as uncertainty in the supply or demand of products, forecasting and the possibility of revising those forecasts, constrained production capacity, and penalties for overtime and expediting (Tsay et al., 1999).

A contract specifies the mechanism for governing the interaction contingencies among agents. It manifests the exchange of promises regarding the actions which are to be done in time. Necessarily, contracts must be *enforceable*, i.e. the agents' refrainment from fulfilling their promises should be ruled out (or made highly improbable). For a contract to be enforceable, its terms (the mutual promises), should be *verifiable* by an enforcing body. However, the verifiability of a contract's terms is dependent on the enforcing body. If a contract's terms are verifiable by a *court of law*, that contract would be a *legal* contract.

Supply chain contracts are not always required to be legal. Several papers in the literature consider contracts among independent agents that are divisions of the same company and a higher level manager can verify the rendition of lateral promises (e.g. Chen (1999), Lee and Whang (1999), and Zhang (2006)). Nevertheless, the process of contract design should explicitly point out the verifying ability of the enforcing agent. Two approaches to verification are detectable in the literature: direct, and indirect. In direct verification, the conditions regarding the fulfillment of contract terms must be *observed*. However, in indirect verification, the aforementioned conditions may be *inferred*. In reality, the verification process is a mixture of the two approaches. An example of direct verification is the delivery of the products ordered from a supplier by a retailer. The retailer can observe, i.e. count, the number of products received. Indirect verifications are achieved when a certain action is considered to be necessary (or self-enforcing) for a rational agent. For example, a manufacturer can verify that if the market selling price is greater than the total production cost and salvage value, the retailer would satisfy market demand as much as it can.

The study of supply chain contracts is an interdisciplinary research area. For the most part, it is a synthesis of *inventory theory* (e.g. Zipkin (2000)), *game theory* (e.g. Owen (1995)), and *contract economics theory* (e.g. Brousseau and Glachant (2002)). In spite of rapid development of research on supply chain contracting and coordination, there are only a few structured analyses of the assumptions, methods, and the implications of insights in this field. Relevant examples include Li and Wang (2007), Chan et al. (2004), Li and Wang (2007), and Gomez-Padilla et al. (2005). The aim of this chapter is to provide a general overview of coordinating contracts in supply chains through highlighting the main concepts, assumptions, methods, and presenting the state-of-the-art research in this field. The chapter intends to provide a non-technical framework encompassing the most important components of these theories.

The rest of this chapter is organized as follows. In Section 2.2, the concept of coordination in SCM contracting is elaborated. Section 2.3 provides a classification scheme for coordinating contract in supply chains. Some of the well-known contractual mechanisms in SCM are introduced in Section 2.4. Section 2.5 contains a review of recent literature based on the proposed classification scheme. Section 2.6 discusses several issues with regard to coordinating contracts in SCM and finally, Section 2.7 introduces some directions for future research in this area.

2.2 Coordination and Supply Chain Contracts

As a rule of thumb, the efficiency of a centralized decision making system is superior to that of a decentralized system, all other things being equal. A well-known justification of the latter is the double marginalization conundrum (Spengler, 1950). The incompatible incentives of agents in a decentralized system make the decisions that are optimal for the agents sub-optimal for the whole chain. In the decentralized supply chain literature, coordination refers to the equivalence of agents' individually-optimal decisions[2] with the optimal decisions of the (centralized) supply chain[3]. The incompatibility of incentives in decentralized supply chains stems from the fundamental characteristic of agents, i.e., *rationality.* The rationality of individuals implies that each agent seeks to maximize its own *utility*, and moreover, each agent is able to calculate its optimal decisions, which lead to the maximization of its utility, given the

[2]We have elaborated on various supply chain decisions in Section 2.3.1.4.

[3]Note that in the centralized supply chain literature, coordination refers to the derivation of supply chain's optimal decisions (see Thomas and Griffin (1996) for a literature review of coordination in centralized supply chains). Therefore, in centralized supply chains, coordination is in fact a global optimization problem, while in decentralized supply chains the latter is a mechanism design problem.

information it has[4]. As the result, the agents do not undertake the supply chain optimal decisions unless they know that those decisions are also optimal for themselves. In order to coordinate a supply chain, a contract must transform the agents' utility functions in a way that the supply chain optimal decisions would also be optimal for the agents. However, this is only one necessary condition for a contract to be *coordinating*. Another necessary condition is that a contract must not be forced upon agents; they must willfully *accept* the contract. The literature contains at least two approaches to formulating the acceptability condition of a contract. The first approach implies that a contract is acceptable if it leads to the utility of each agent being above a certain acceptable level for that agent. These levels can be interpreted differently, e.g. reservation profits, opportunity costs, outside options, or *status quo* utilities. The second approach demands that not only should an acceptable contract guarantee minimum amounts of utilities to the agents, but it also must divide the extra utilities in a *fair* manner among them[5]. Cachon (2003) states three conditions that a coordinating contract should meet:

(1) with a coordinating contract, the set of supply chain optimum decisions should be a *pure Nash equilibrium*;

(2) it should divide the supply chain profits (utilities in general) arbitrarily among the agents; and

(3) it should be worth adopting.

The first condition is concerned with the transformation of agents' utility functions. Although this definition does not directly specify the acceptability condition, the

[4]For further discussions on the concept of rationality and proposed critiques see Osborne and Rubinstein (1994).

[5]One approach to fairness is to consider it as the correspondence between the bargaining powers (yet another hard-to-define concept) and the agents' utilities. See Nagarajan and Sošić (2008) for elaboration on this issue.

second condition implies that if a contract can divide the supply chain profits among agents in any manner, at least one of those division schemes should be acceptable to all agents[6]. Unfortunately, the criteria for assessing the third condition are rather vague, but it could be taken as the combination of other qualitative acceptability conditions yet to be formalized.

Alternatively, Gan et al. (2004) define coordinating contract as

> a contract which the agents of a supply chain agree upon and the optimizing decisions of the agents under the contract satisfy each agent's reservation payoff *[minimum acceptable utilities]* constraint and lead to Pareto-optimal decisions and Pareto-optimal sharing rule.

This definition formulates the acceptability condition according to the first approach stated earlier (satisfaction of minimum acceptable utilities). One drawback of this approach is that it does not indicate how one contract should be agreed by the agents in cases where there exists multiple contracts with Pareto-optimal sharing rules which satisfy the agent's minimum acceptable utilities. Gan et al. (2004) also define *flexible* coordinating contract as a coordinating contract such that by adjustment of some parameters, it could lead to any Pareto-optimal sharing rule.

Despite the different interpretations of acceptability condition of a coordinating contract in Cachon (2003) and Gan et al. (2004), the fundamental notions in both definitions are similar. That is, with a coordinating contract, agents' optimum decisions must be the same as the supply chain's optimum decisions, and the contract should divide the resultant payoffs among them so that all agents are satisfied and as the result they would accept the contract. We provide two variations of the concept of

[6]For the cases with two agents, there is always an acceptable division schemes among all the possible divisions. However, for the cases with more than two agents, this might not hold. In particular, this definition does not address the possibility of coalition formation among the agents. This issue is further discussed in Section 3.2.2.

coordination:

- Weak Coordination: If a contract could achieve the equivalence of agents' optimal decisions (pure Nash equilibrium) and the supply chain's optimal decisions, and at the same time it satisfies the minimum acceptable utilities for all agents, then the contract is *weakly coordinating.*

- Strong Coordination: If a contract could achieve the equivalence of agents' optimal individual decisions (pure Nash equilibrium) and the supply chain's optimal solution, and at the same time it could divide the total supply chain payoff in any manner among the agents, then the contract is *strongly coordinating.*

The relationship between the two definitions is that if a weakly coordinating contract is also flexible, then it is strongly coordinating as well.

2.3 Methodology of Coordinating Contracts

The purpose of this section is to provide a taxonomy of supply chain contracting problems and an overview of methods used in analyzing the coordinating ability of contracts.

2.3.1 Classification of Problems

Numerous parameters impact how contracts affect collaborative performance of supply chain agents. However, in order to retain tractability, only a few of those parameters can be abstracted and investigated simultaneously in a model. The result is a plethora of models with various combinations of parameters. Here, we present a list of the most important classes of parameters which have been considered in the literature.

Supply Chain Topology

A supply chain consists of several business entities (agents) with certain kinds of flows among them (such as material, information, and money) that can be represented by a network. Despite the complex structure of an average-sized real world supply chain, the contracting literature focuses on small chunks of such networks comprising of few nodes (representing supply chain agents) and the flows between them. In many cases, supply chain contracts are considered to be centered around a focal node and the immediate predecessors and/or successors which form a hierarchy of *tiers*. We refer to this aspect as *supply chain topology*. The common topologies in supply chain contracting literature are as follows.

- **Two-tier topology with two nodes:** The majority of studies in the supply chain contracting literature consider this topology. The nodes might represent a supplier and a manufacturer, or a producer and a retailer, *etc.* This topology resembles a *bilateral monopoly.*[7] The well-known coordinating contracts for supply chains mainly address this topology (see Section 2.4).

- **One-tier topology with several nodes:** The contracts with this topology deal with horizontal collaboration among several independent agents that are in the same supply chain tier (all retailers, or manufacturers for instance). The collaboration is through pooling resources in order to balance the outstanding demands and surplus resources. In *sub-contracting* literature, the flow of resources among any two agents are only in one way. However, in the *transshipment* literature, the flows are bilateral. Although the agents collaborate with

[7] A bilateral monopoly consists of two vertically-dependent agents: an upstream supplier (a ŞmonopolistŤ) that sells all its output to a downstream buyer (a ŞmonopsonistŤ) that acquires all its supply of an essential input from the monopolist. Their relationship is symmetric. Both have market power, and neither can survive without the other; therefore, the agents necessarily deal with each other, negotiate and conclude contracts, and settle prices and quantities (Ingene and Parry, 2004, p. 32).

one another, still, they may compete over some aspects of their business, e.g. order quantities (Rudi et al., 2001) or their market selling prices (Zhao and Atkins, 2009). An important aspect of the supply chain models with this topology is whether the collaboration among the agents happes prior to the realization of the demand afterwards.

- **Two-tier topology with several nodes:** The contracts with this topology address the interactions among a focal node and several other nodes all being located in an adjacent tier. Therefore this topology is comprised of either one upstream node that supplies several downstream nodes, or one downstream node that is being supplied from several upstream nodes. The nodes in the same tier may compete with one another over the limited capacity of the other tier's resources (as in Cachon and Lariviere (1999)), or on market prices (as in Deneckere et al. (1997)), *etc.* In more elaborate models the nodes in the same tier are assumed to pool resources, e.g. Ulku et al. (2007).

- **More general topologies** Assuming more than two tiers in an independently owned serial supply chain system will drastically increase the complexity of analysis of coordinating contracts. To the best of our knowledge there are only a few papers which consider these topologies. As an example, Zijm and Timmer (2008) study the coordination problem in a three-tier supply chain with three nodes. However, they assume separate contracts governing the interactions between the node in adjacent tiers.

Supply Chain Environment

The supply chain environment is the collection of external factors affecting the supply chains' decisions. Some of the most relevant dimensions of supply chain environment are as follows:

- **Certainty/Uncertainty of environment:** Usually, the uncertainty of supply chain environment refers to the market demands. Two broad categories are deterministic and stochastic market demands. Sarmah et al. (2006) review the contracts with quantity-discount policies in deterministic demand environment. In deterministic systems, the coordination might pertain to the timing of orders (Klastorin et al., 2002). The coordinating contracts with uncertain market demand environment mostly consider continuous probability functions. An example of coordination with discrete demand distributions is Zhao et al. (2006) which consider a one-tier supply chain with two nodes and Poisson demand arrival rates. Recently, Xu and Zhai (2010) study the general properties of coordination in a two-tier, two-node topology with fuzzy demands. The other source of uncertainty about the supply chain environment is associated with the supply chain's input. The supply chain contracting literature has considered uncertain delivery times (e.g. (Zimmer, 2002)) and uncertain delivered quantities (e.g. (He and Zhang, 2008)). The latter is also referred to as *random yield.*

- **Sensitivity of environment to supply chain decisions:** In many supply chain models, market demands are assumed to be sensitive to some decision variables internal to the chain. Among others, the decision on market selling price and marketing efforts are the most addressed. For example, in addition to choosing the order size, a retailer facing price-sensitive market demand should also decide its selling price. This, in turn, affects the coordinating ability of the contract between the retailer and its supplier. Yano and Gilbert (2005) and Chan et al. (2004) review the literature on supply chain contracts with price sensitive market demands. When the market demand is affected by the marketing effort of a downstream agent—which is unverifiable by the chain—a coordinating contract should induce the supply chain's optimal level of marketing effort. He et al. (2009) explore coordinating contracts for a two-tier, two-node topology with both price and marketing effort sensitive market demand. Another factor that could affect the market demand is the stock level. Sajadieh et al. (2010) address the issue of coordination in the supply chain where the amount of stock displayed to customers has a positive effect on demand.

- **Dependencies among agents in the same tier:** The individual decisions of agents who operate in the same supply chain tier may affect each other. These dependencies add another dimension to the complexity of models. Competition, and correlated market demands are among factors that amount to dependencies among agents in the same tier. Multiple nodes in a particular tier may compete over their market shares (when they are operating in the same market), or supplier's quotas (when the supplier's capacity is restricted), or fill rates. Cachon and Lariviere (1999) investigate the supply chain coordination in the supply chain where the downstream agents compete over the limited supplier's capacity. Hartman and Dror (2005) analyze the cooperation among many newsvendors with dependent demands.

Length of Contract

The length of a contract is the duration of time that the contracting agents are assumed to uphold the contract. Therefore, the contract terms are not *re-negotiated* during the length of a contract. This has a crucial effect on modeling the underlying supply chain problem. The effective length of a supply chain contract can be compared with the number of inventory replenishment periods. Accordingly, there is a close affinity between the length of a supply chain contract and the modeling approach. The two main classes are:

- **Single period models:** A large number of supply chain contracts has been devised for the single period supply chain model, i.e. the newsvendor model with its numerous variations (Khouja, 1999). This family of supply chain models is specially appropriate for the supply chains with perishable products, short selling seasons, and long procurement lead-times. Nevertheless, the analytical simplicity of single period supply chain models has given rise to the popularity of contracts with one period length. Cachon and Lariviere (2005) outline several coordinating contracts for the standard newsvendor model. Hu et al. (2007) consider a single period model with limited and uncertain supplier's capacity. Cachon (2003) provides an excellent literature review on coordinating contracts for this family of models. Cachon (2004) addresses coordination in a single-period model with two replenishment opportunities for the downstream agent.

- **Multi-period models:** The multi-period models could simply be the combination of two consecutive newsvendor models (Barnes-Schuster et al., 2002), or they might consist of several stocking periods. The multi-period models are mainly based on the multi-echelon model of Clark and Scarf (1960). Among the early papers that address the multi-period supply chain contracts is Cachon and Zipkin (1999) which offers a coordinating contract based on the end-of-period inventory information at different agents.

Supply Chain Decisions

Among the numerous decision variables that are critical in managing supply chains, the supply chain contracting literature commonly concentrates on those that are related to capacity, order size, market selling price, marketing efforts, contract type, lead-times, quality, review period, and stocking policy. For a more detailed analysis of supply chain decision variables see Tsay et al. (1999). Considering the multiplicity of decision makers in decentralized supply chains, an important aspect of supply chain decisions is the distribution of decision making responsibilities among supply chain agents. Although traditionally some decision variables are attributed to certain supply chain entities, e.g. responsibility of deciding the order size to the downstream agent (buyer), many cases with less conventional approaches have also been investigated in the literature. For example, in an insightful paper Lariviere and Porteus (2001) assume that the upstream agent chooses the order size while the downstream agent picks the buying price. Hence, the distribution of decision rights among supply chain agents falls, at least partially, within the purview of the modeler.

Another aspect of this issue is related to the right of non-compliance among supply chain agents. Generally, whenever one contracting agent requests something from another agent, the latter may have the right to not comply with the former's request. In supply chain contracting literature, the allotment of compliance rights is, in fact, the choice of the modeler. Cachon and Lariviere (2001) refer to this issue as *compliance regimen*. Accordingly, there are two classes of compliance regimes: *voluntary* and *forced*. Cachon and Lariviere (2001) use these terms with respect to the responsibility of a supplier to completely fill the manufacturer's order. In this context, if the model gives the supplier the right to decide the fraction of manufacturer's order to deliver, then the system would be under voluntary compliance regimen. In other words, under voluntary compliance regimen, an agent has the right to decide whether to fulfill or not to fulfill the requests it receives. Under the forced compliance regime, on the other hand, an agent is obligated to fulfill the requests it receives.[8] Therefore, whether explicitly or implicitly, the compliance regimens of all the mutual promises in a supply chain contract should be indicated. If a contract can coordinate a specific supply chain setting under a voluntary-compliance regime, it could coordinate under

[8]It is because non-compliance would be penalized. The penalties (or other forms of threats) are implicitly assumed to be large enough so that, in theory, non-compliance never occurs.

the forced-compliance regime as well. The opposite might not be the case.

Characterization of Supply Chain Agents

Earlier in this chapter, *rationality* has been addressed as an underlying characteristic of the agents. Two other aspects of supply chain agents' characteristics pertain to their *utility functions* and attitudes toward risk. Utility functions reflect preferences of agents which, in turn, determine their decision making criteria. In the supply chain contracting literature, it is conventional to assume that the utilities of agents are solely a function of monetary payoffs. That is, agents only care about the amount of profit they make. Nevertheless, there has been a recent trend in considering utility functions which reflect agents' social preferences as well. For instance, supply chain agents may also care about *fairness* in a mutual business relationship (Cui et al., 2007). Other examples include *inequity aversion* (Cui et al., 2007) and *status seeking* among agents (Loch and Wu, 2008).

In decision making in uncertain environments, the analysis of agents' decision making process requires knowledge about their attitudes toward risk. Two types of such attitudes have been considered in the literature: *risk-neutrality*, and *risk-aversion*.[9] For a risk-neutral agent, a certain payoff of M is equally preferred as an uncertain payoff with the same *expected* value M, while a risk-averse agent prefers the certain payoff M. Hence, the objective of a risk-neutral agent is to maximize its expected profit (or equivalently to minimize its expected cost). While there is only one measure for risk-neutrality, risk-aversiveness can be reflected in many (theoretically infinite) ways. Among the objectives studied for risk-averse agents are the minimization of variance of profits (Chen and Parlar, 2007)), and the minimization of mean-variance difference (Gan et al., 2004; Choi et al., 2008). Van Mieghem (2003) reviewed the

[9] To the best of author's knowledge, *risk-taking* attitudes have never been considered in supply chain contracting literature.

literature on capacity investments considering the issue of risk-aversion. The general characteristics of supply chain contracts with risk-averse agents are studied in Gan et al. (2004).

Information Structure in Supply Chains

Information structure pertains to the agents' knowledge in comparison to the collective knowledge of agents in the supply chain. When all the information about supply chain is simultaneously known by every agent, the information structure is said to be *complete* or *symmetric*. On the other hand, if some agents have some information that the other agents do not, the information structure is *incomplete* or *asymmetric*. The pieces of information that are known only by an agent is that agent's *private* information.

In general, coordination under incomplete information is more complex than coordination under complete information. One approach to deal with incomplete information structure is to assume certain *types* of agents each with known characteristics (c.f. Harsanyi and Selten (1972)). Although the agents do not know what types of agents they are facing, the probability that an unknown agent is of a particular type is assumed to be common knowledge. A coordinating contract in these supply chains is comprised of a menu of contracts designed in a way that will make the agents with private information choose the only contract that result in the supply chain optimum decisions. Therefore, a coordinating contract in an incomplete information setting will result in the truthful revelation of private information. Several papers study supply chain contracts under asymmetric information. Corbett and Tang (1999) assume a two-tier, two-node supply chain with deterministic and price-sensitive demand function where the upstream agent does not know the exact cost structure of the downstream agent. They investigate the effect of contracts with different pricing mechanisms on the overall efficiency of the chain. Corbett et al. (2004) study a supply chain with two agents where the supplier does not know the retailer's internal cost. Cachon and Lariviere (2001) analyze a supply chain contracting problem where the information regarding the probability distribution of market demand is the private information of the downstream agent. Burnetas et al. (2007) introduce a coordinating quantity-discount policy in a two-tier two-node topology where the upstream agent does not have the information regarding the demand distribution of the downstream agent. The risk sharing contract of Gan et al. (2005) can coordinate when the upstream agent does not know how risk averse the downstream agent is. Burnetas et al. (2007) introduce an all-unit discount policy that results in coordination of a two-tier two-node topology supply chain in one period. Sucky (2006) analyzes a two-tier two-node supply chain in a deterministic environment under a forced compliance regimen. Assuming that the upstream agent is uncertain about the downstream agent's cost structure, he shows that coordination can be achieved through bargaining and with the help of side payments.

2.3.2 Analytical Methods of Coordinating Contracts

The ability of a contract to coordinate a supply chain is completely context-dependent. Contracts can be distinguished at two layers: the *contract template*, and the *contract setup*. At the outer layer, the contract template provides a holistic view of interactions among the agents involved in a contract and points out the variables that the contract is based upon. The second layer, i.e. the contract setup, specifies the particular setup of contract variables for a given contract template. Consider the famous wholesale-price contract as an example. The contract template declares that the buyer should pay the seller a fixed price for a unit of ordered product. The contract setup, on the other hand, specifies the exact unit price in the contract. The goal of this section is to answer two important questions:

(1) How is contract template obtained? and

(2) How is the coordinating ability of a contract analyzed?

In most cases, the contract templates are inspired by the structure of contracts which are being used in practice. The alternative approach requires more creativity; that is, the modeler *invents* a contract template by specifying the hypothetical interactions among the agents. However, justifying the practicality of such a contract template is rather challenging. Some of the most well-known contract templates are introduced in the next section.

Game theory is the fundamental tool for investigating the coordinating ability of a contract, with specified template and setup, in a given supply chain setting. For a brief review of related game theory concepts in supply chain contracts see Cachon and Netessine (2006) and Chinchuluun et al. (2008). Accordingly, one should analyze whether the contract can be set up so that it could induce all the agents to select the supply chain's optimal decision, and whether the resultant division scheme of supply chain profits are acceptable to them. The latter is addressed in two different cases: contracts between two agents, and contracts among more than two agents.

Contracts Between Two Agents

When there are only two agents involved in a contract, an assessment of the coordinating ability of a contract should concentrate on two issues: first, the negotiation process over a contract, and second, the effect of the negotiated contract on agents' decisions. The most common procedure used in the literature is the Stackelberg game. This approach simplifies the analysis of negotiation process between the agents by assuming that one agent (the leader) gives a take-it-or-leave-it offer, including the contract template and setup, to the other agent (the follower) who has the right to either accept or reject the offer. A Stackelberg supply chain game is played as follows. Anticipating the follower's minimum acceptable (expected) profit, the leader offers a contract setup that (1) induces the follower to choose the supply chain optimum decisions and (2) results in the follower's minimum acceptable (expected) profit level. This approach is suitable for situations where the leader has significantly more power and the interactions between the agents are restricted. In general, the idea of the follower either completely accepting the contract or wholly rejecting it without any further negotiations may seem too restrictive.

Another approach to analyze the negotiation process over a contract is to consider an explicit bargaining process. The bargaining process shall specify the exact contract setup which leads to an acceptable split of the maximum supply chain profits. Two approaches which have been used in the literature are Strategic Negotiation (Rubinstein, 1982) and Axiomatic Negotiation (Nash, 1950). With Strategic Negotiation (Sequential Bargaining), after a contract has been offered by an agent, the other agent could offer a new contract (counter-offer) if it is not acceptable to the latter. Considering the value of time (or agents' patience), this bargaining process has been proven (Rubinstein, 1982) to converge to a mutually acceptable contract setup. For a review of the implementation of strategic negotiation in supply chain contracts see Wu (2004). With Axiomatic Negotiation approach, the bargaining solution is developed by considering axioms that correspond to the desirable properties of negotiation outcomes. The bargaining solution can be thought as the suggestion of an unbiased arbitrator. Hence, a contract is proven to be coordinating if the underlying negotiation problem has a feasible solution. A recent example of implementation of this approach is Hezarkhani and Kubiak (2010b) which uses the generalized Nash bargaining solution (Muthoo, 1996) in a transshipping supply chain (see Chapter 3). Nagarajan and Sošić

(2008) review the literature of bargaining and negotiation in supply chains.

Contracts Among Several Agents

The analysis of coordinating contracts becomes more complex as the number of participants in the contract increases. The principle approach to study the contracts among several agents is *cooperative game theory*. The cooperative game theory approach to contracts provides mechanism for the distribution of total payoff that is generated by the coalition of all supply chain agents, i.e., *grand coalition*. The acceptability of a contract to the agents implies that not only should it provide each agent with its minimum acceptable payoff, but also it must eliminate the incentives for the agents to form sub-coalitions and gain more profits in that way. In other words, in the n-agent case, the coordinating contract should meet some *stability* criteria with regard to the distribution of grand coalition's payoff among the agents.

One of the most natural stability concepts is the concept of *core* (Peleg, 1995). If a contract could distribute the grand coalition's payoff among the agents so that no subset of agents could be better off by forming a sub-coalition, then that distribution mechanism would be in the core of the corresponding cooperative game. However, it might be the case that no such distribution mechanism can be found. Nevertheless, there are alternative stability concepts that can be used in conjunction with other solution concepts in cooperative game theory, e.g. Shapley value, nucleus, bargaining set, *etc.* (Owen, 1995). Slikker et al. (2005) study the stochastic cooperative games with newsvendors who can also pool resources through transshipments and show that the core of this class of supply chain problems is non-empty. Ozen et al. (2009) provide a general framework for cooperation under uncertainty. Brandenburger and Stuart (2007) study *bi-form games*. The bi-form games are to model the supply chains wherein a set of agents face individual and correlated decision making problems followed by a cooperative stage. In a one-tier several agent topology, Anupindi et al. (2001) introduce an allocation rule in the core of the second stage transshipment game. An alternative allocation rule has been proposed in Sošić (2006) which redistributes the extra profit generated through the transshipments according to the Shapley value. Although the resultant allocation is not necessarily in the core, it could result in the *farsighted stability* of the grand coalition, i.e. the agents do not form sub-coalitions since they take into the consideration other agents' reactions as well. Chen and Zhang (2009) approach the transshipment problem as a two stage cooperative game, and show that the problem of finding an allocation in the core of n-agent transshipment game is NP-hard. Hezarkhani and Kubiak (2010a) adopted the concept of pair-wise stability (Baiou and Balinski, 2002), a non-cooperative solution concept derived from the matching problem in two-sided markets, into the transshipment problem with many agents (Chapter 4 is an edited version of this paper).

2.4 Well-known Contract Templates for Supply Chains

The typical solution to incompatible incentives in a supply chain is for the agents to agree to a set of transfer payments that modifies their incentives, and hence modifies their behavior (Cachon, 1999). Additionally, the flow of goods and materials might also be subject to modification (as in a *buyback* contract). This section addresses some of the well-known contract templates in supply chains. We start with one of the most basic supply chain contracts, i.e. wholesale-price contract, in a basic supply chain (single-period model with risk-neutral agents, independent demands, and symmetric information structure) and address the coordinating components which can be added to it in order to achieve coordination in various supply chains.

2.4.1 Wholesale-price Contracts

In the simplest supply chain, the wholesale-price contract requires the buyer to pay a fixed and quantity-independent price to the seller for each unit purchased. Although the wholesale-price contract fails to coordinate supply chains in a simple two-tier topology with two nodes, it is the most common contract in practice—perhaps because of its simplicity.

In the standard newsvendor supply chain, two types of wholesale-price contracts are possible. First, the downstream agent has to place orders *before* the realization of uncertain market demand and the upstream agent provides products accordingly. Second, the downstream agent can place its order *after* observing the actual market demand while the upstream agent should prepare itself in advance for meeting it. Although in both cases the integrated system is a standard newsvendor model, they are different with respect to allocation of risk between the two agents. Cachon (2004) calls the first type *push* and second type *pull* wholesale-price contracts. Lariviere and Porteus (2001) analyze the properties of push wholesale-price contracts where the upstream agent can satisfy all the downstream agent's orders and it acts as the Stackelberg leader offering the wholesale price to the downstream agent who determines the order quantity. Note that with this contract, the seller gets a risk-less sum of money before realization of market demand and the buyer faces all the risk associated with the uncertainty market demand. Cachon and Netessine (2004) analyze the pull contract where the upstream agent has to decide its capacity level before receiving the downstream agent's orders. As the authors conclude, both types of wholesale price contracts fail to coordinate the supply chain. In fact, the only wholesale-price in the push setting which induces the downstream agent to place the optimal centralized order size, leaves the upstream agent with no profit, thus, the wholesale price contract cannot satisfy the acceptability condition of coordination, i.e., it cannot result in weak coordination.

2.4.2 Contracts with Discount Policies

Discount policies, i.e. quantity-dependent unit prices, are well-known coordinating components in supply chain contracts. There are several forms of discount policies; see Dolan (1987) for a review. Discount policies are the main coordinating components in supply chains with *deterministic* demand. Jeuland and Shugan (1983) address the problem of coordination in the two-tier two-node topology and propose a coordinating quantity-discount contract. As they show, there are several coordinating quantity discount contracts which lead to different split schemes for extra profits generated through cooperation. Klastorin et al. (2002) consider a two-tier supply chain with one upstream agent and several downstream agents and show a discount policy that can coordinate the ordering times of downstream agents so that the supply chain can save holding costs at the upstream level. Cachon (2003) incorporates the quantity discount component in a standard newsvendor supply chain and demonstrates its coordinating ability in a two-tier topology with two nodes. In his model, the mutually acceptable division of supply chain profits is determined by a Nash bargaining mechanism between the two agents.

2.4.3 Contracts with Return Policies

With the return policies the seller promises to compensate the buyer for unsold quantities. One might ask why contracts with return policies are needed while quantity discount contract are just as well coordinating. First,

> [b]uy-back payments play a very important role in channel coordination when the multi-retailer supply chain is considered. When retailers

serve markets of different sizes, the manufacturer can attain the profits of a coordinated channel only if he can charge different wholesale prices to each outlet. However, in the US such a practice is restricted by the Robinson Patman Act which protects the retailers against price discrimination by the manufacturers. It is shown that the buy-back payments for used products provide a second degree of freedom for the manufacturer to differentiate the average wholesale price charged to each retail outlet, and thereby attain the coordinated channel profits in a decentralized supply chain. (Debo et al., 2004)

Second, with the return policies the upstream agent is also bearing the risk associated with the market demand so the downstream agent prefers it to a quantity discount contract with the same expected profit.

The variations of return policies depend upon the amount of leftover inventory which can be returned and the amount of compensation—the ratio of unit compensation fee to the original purchase price. Pasternack (1985) shows that in a single-period supply chain with risk-neutral agents, the return policies that allow for full leftover return and partial compensation can coordinate the supply chain. Other variations of return policies are (1) unlimited return and full compensation, (2) limited return and full compensation, and (3) limited return and partial compensation. In the newsvendor supply chain, Pasternack (1985) also proves that the return policies that allow for full return and full compensation cannot be coordinating. In the same setting, Cachon (2003) shows that partial return and full compensation policy cannot be coordinating, while partial return and partial compensation can. Su (2009) study the impact of full returns policies and partial returns policies on supply chain performance. He demonstrates that consumer returns policies may distort incentives under common supply contracts and proposes strategies to coordinate the supply chain.

2.4.4 Revenue Sharing Contracts

In revenue sharing contracts, the downstream agent commits to return a pre-negotiated portion of its realized profits to the upstream agent. The successful implementation of these contracts is reported in the video rental industry (Cachon and Lariviere, 2005). The revenue sharing contract can also coordinate the price-sensitive newsvendor supply chain (Cachon and Netessine, 2004). Qin and Yang (2008) consider a two-tier, two-node topology and analyze the revenue sharing contract as a Stackelberg game and conclude that, in order to achieve coordination, the agent that keeps more than half the revenue should serve as the leader of the Stackelberg game. Yao et al. (2008b) study a two-tier, three-node topology where the downstream agents compete over setting the market selling prices. They combine the Stackelberg game among the upstream and downstream agents and the Bayesian Nash game between the two downstream agents and investigate the effect of different revenue-sharing contracts on supply chain performance.

A particular case of revenue sharing—widely known as *consignment* contracts (Wang et al., 2004)—is the instance where the *ownership* of goods do not change with their delivery to the downstream agent, i.e. the upstream agent remains the owner. Then, the upstream agent pays the downstream agent a commission for each sold item. Wang et al. (2004) investigate the performance of consignment contracts, i.e. supplier and retailer's respective shares of total profit, when the demand is sensitive to the market selling price.

2.4.5 Rebate Contracts

In rebate contracts, the upstream agent rewards the downstream agent for every unit sold. Therefore, in some sense, a rebate policy resembles a return policy: while in buyback contracts the downstream agent is compensated for unsold units, in rebate contracts the latter is rewarded for the units sold. Accordingly, different rebate policies can be implemented: (1) policies that reward for all units sold, and (2) policies that reward for sold units only above a threshold. In single-period supply chains, Taylor (2002) shows that the second class of rebate policies can achieve coordination. Chen et al. (2007) consider the rebate contract in a two-tier, two-node topology with price-sensitive demands and find that the mail-in rebates (which is payed upon request) may benefit the upstream agent while instant-rebates (which includes every interaction) may not.

2.4.6 Contracts with Side Payments

Although the notion of side payment has a clear definition in game theory[10], its use in supply chain contracting literature is somewhat inconsistent[11]. We define side payments as the lump-sum monetary transfers among the contracting agents which are independent of amount of trade and used as compensation and incentive alignment mechanisms. In order to clarify the issue consider two contracts introduced earlier: the wholesale-price, and the revenue sharing contracts. In the wholesale-price contract, the amount of money transferred from the buyer to the seller is a linear function of units purchased. On the other hand, in the revenue sharing contract the downstream agent pays the upstream agent a lump-sum of money after the realization of its profits. According to this definition, the latter is a side payment while the former is not.

Examples of side-payment contracts among two agents include two-part tariff (where limited side-payments are allowed, e.g. Zaccour (2008)) and option contracts (e.g. Barnes-Schuster et al. (2002)). In general, the contracts that rely on allocations of

[10]In game theory terminology, side payment is defined as the exchange of a perfectly dividable common good that is capable of transferring utility (Aumann, 2000).

[11]Two alternative definitions are proposed in Rubin and Carter (1990) and Taylor (2002).

realized profits take advantage of side payments. Hence, almost all the contracts with more than two contracting agents, which utilize profit-allocation mechanisms, are contracts with side payments. Although the inclusion of side payments in supply chain contracts could facilitate coordination, they may be infeasible in some situations, e.g. in some cases they might be prohibited by law (Leng and Zhu, 2009).

2.5 Literature Review and Discussion

This section classifies the recent literature on coordinating supply chain contracts. The classification scheme has been explained in earlier sections. The papers wherein the analysis does not result in coordination have not been considered. The literature review is presented through extensive tables (Table 2.1 and 2.2). In order to summarize the information in the tables, we use the following notation. In the Topology column, the xT/yN represents the number of tiers and nodes of the topology. For instance, 2T/2N represents two tiers with two nodes topology. In the Contract Length column, x-p shows the number of periods in the model (n-p stands for multiple-periods). In the Agent Characteristics column, Risk-N and Risk-A represent risk-neutral and risk-averse agents respectively.

The large number of variables that can be included in analyzing the contractual situation limits the comprehensiveness of this classification scheme. Moreover, several other important aspects of supply chain contracts cannot be quantitatively analyzed. Some of those aspects are: the *applicability*, i.e. the possibility of implementation of a contract in a given real world context, the *verifiability*, i.e. availability of mechanisms for verifying the lateral promises stated in the contract, and the *ease of implementation*, i.e. the effort which is required to apply a contract in real world supply chains. In fact, there is no known measure to compare coordinating contracts for specific supply

Reference	Supply Chain Problem						Coordinating Contract
	Topology	Contract Length	Decision Variables	Agents Charact.	Environment	Information Structure	
Cachon and Netessine (2004)	2T/2N	1-p	Order size, Selling price	Risk-N	Stochastic & price-sensitive demand	Complete	Revenue-sharing contract
Taylor (2002)	2T/2N	1-p	Order size	Risk-N	Stochastic demand	Complete	Rebate contract
Tsay (1999)	2T/2N	1-p	Order sizes	Risk-N	Stochastic demand	Complete	Quantity-flexibility contract
Bernstein and Federgruen (2005)	2T/2N	1-p	Order size, Selling price	Risk-N	Stochastic & price-sensitive demand	Complete	Price-discount sharing contract
Tomlin (2003)	2T/2N	1-p	Order size, Capacities	Risk-N	Stochastic & price-sensitive demand	Complete	Quantity-Premium price only contract
Gan et al. (2005)	2T/2N	1-p	Order size	Risk-A & Risk-N	Stochastic demand	Complete	Risk sharing contract
Ying and Choi (2010)	2T/2N	1-p	Order size	Risk-A	Stochastic demand	Complete	Profit sharing contract
Hanany et al. (2010)	1T/2N	1-p	Order sizes	Risk-N	Stochastic demand	Complete	Transshipment fund mechanism
Chakravarty and Zhang (2007)	1T/2N	1-p	Order sizes	Risk-N	Stochastic demand	Complete	Return policies contract
Leng and Zhu (2009)	1T/2N	1-p	Order sizes	Risk-N	Stochastic demand	Complete	Side payments
Anupindi et al. (2001)	1T/nN	1-p	Order sizes	Risk-N	Stochastic demand	Complete	Side payments
Donohue (2000)	2T/2N	1-p (2 production stages)	Order sizes	Risk-N	Stochastic demand	Complete	Wholesale-price with *ex ante* return option
Cachon (2004)	2T/2N	1-p (2 production stages)	Order sizes	Risk-N	Stochastic demand	Complete	Advance-purchase discount contract
Barnes-Schuster et al. (2002)	2T/2N	2-p	Order sizes	Risk-N	Stochastic demand	Complete	Option contract
Yao et al. (2008a)	2T/2N	1-p	Order size	Risk-N	Stochastic and price-sensitive demand	Complete	Return policy contract
Ha and Tong (2008)	2T/2N	1-p	Capacity, Service level, Effort	Risk-N	Stochastic and effort-sensitive demand	Complete	Revenue sharing contract

Table 2.1: Supply Chain Contracting Literature

Reference	**Supply Chain Problem**						**Coordinating Contract**
	Topology	Contract Length	Decision Variables	Agents' Characteristics	Environment	Information Structure	
Cachon (2003)	2T/nN(1 upstream & n downstream)	1-p	Order sizes, Selling prices	Risk-N	Stochastic & price-sensitive demand	Complete	Buyback Contract & Resale price maintenance Contract
Cachon and Zipkin (1999)	2T/2N	n-p	Order sizes	Risk-N	Stochastic demand	Complete	Linear transfer payment contract
Zhang (2006)	2T/3N (2 upstream & 1 downstream)	n-p	Order sizes	Risk-N	Stochastic demand	Complete	Linear transfer payment contract
Zijm and Timmer (2008)	3T/3N	n-p	Order sizes	Risk-N	Stochastic demand	Complete	Side payments
Ding and Chen (2008)	3T/3N	1-p	Order size	Risk-N	Stochastic demand	Complete	Return policy contract
Caldentey and Wein (2003)	2T/2N	n-p	Order sizes	Risk-N	Stochastic demand, Make-To-Order Queue	Complete	Linear transfer payment contract
Burnetas et al. (2007)	2T/2N	1-p	Order size	Risk-N	Stochastic demand	Incomplete	All-unit discount policy
Sucky (2006)	2T/2N	N/A	Lot sizes	N/A	Deterministic demand	Incomplete	Side payments
Burnetas et al. (2007)	2T/2N	1-p	Order size	Risk-N	Deterministic demand	Incomplete	Side payments
Shin and Benton (2007)	2T/2N	1-p	Order size	N/A	Deterministic demand	Complete	Quantity discount contract
Zou et al. (2008)	2T/3N (2 upstream & 1 downstream)	2-p	Order size	Risk-N	Stochastic demand	Complete	Buyback contract
He and Zhang (2008)	2T/2N	1-p	Order size	Risk-N	Stochastic demand, Random yield	Complete	Risk-sharing contract
Krishnan and Winter (2010)	2T/3N(1 upstream & 2 downstream)	1-p	Order size, Selling price	Risk-N	Stochastic & price-sensitive demand	Complete	Buyback contract
Ryu and Yucesan (2010)	2T/2N	1-p	Order size	Risk-N	Fuzzy demand	Complete	Buyback, Quantity-discount, Revenue-sharing contracts

Table 2.2: Supply Chain Contracting Literature (Cont'd)

chains.

One of the weak points of coordinating supply chain contracts is their sensitivity to context. In this respect, the over-simplification of a problem may result in serious flaws. In fact, the supply chain contracts which coordinate in a particular theoretic supply chain (under certain simplifications), may lead to very different results when implemented in real world situations. Cachon and Kok (2010) show that well-known coordinating contracts such as quantity-discount and two-part tariffs could worsen the performance of supply chain when applied in a two-tier topology with multiple competing suppliers. Accordingly, one should be very cautious when implementing these insights into practice.

A common assumption in the supply chain contracting literature is that the process of contracting does not have any significant costs. However, there are several costs associated with the contracting process, e.g. costs related to writing down the contracts and their monitoring and enforcements costs. In addition, the literature does not consider the costs that the contracting agents incur in order to collaborate with each other. Many studies have shown that cooperation among supply chain agents requires costly infrastructure for information sharing, process and resource coordination, and performance measurements (c.f. McLaren et al. (2002)). Therefore, without considering such realistic costs, the practical benefits of coordinating contracts would be unclear and inconclusive. The research must find the conditions under which additional profits which result from implementing a coordinating contract are actually significant. Chapter 5 in this thesis incorporates the concept of cooperation costs into the analysis of transshipments in supply chains.

Despite the growing number of analytical studies on supply chain contracts, there are only a few empirical studies aiming at validation of the theoretical predictions in this area. In a laboratory study, Katok and Wu (2009) show that the effect of coordinating

Chapter	Supply Chain Problem						Coordinating Contract
	Topology	Contract Length	Decision Variables	Agents Charact.	Environment	Information Structure	
3	1T/2N	1-p	Production/Inventory quantities	Risk-N	Stochastic demand	Complete	Implicit pricing
4	1T/nN	1-p	Production/Inventory quantities	Risk-N	Stochastic demand	Complete	Pricing mechanism
5	1T/nN	1-p	Production/Inventory quantities	Risk-N	Stochastic demand & positive cooperation costs	Complete	Side payments

Table 2.3: Contributions of the thesis

contracts on supply chain efficiency is smaller than what is predicted analytically. On the other hand, the small number of empirical research papers in this area almost certainly indicates that the actual decision making process in supply chains is hugely influenced by bounded rationality, anchoring, experience, and insufficiently adjusted heuristics (e.g. Schweitzer and Cachon (2000), Bolton and Katok (2008), and Ben-zion et al. (2008)). Additionally, the empirical studies of supply chain contracts do not reach beyond the laboratory tests—perhaps due to the sensitivity of necessary information.

The main focus of this thesis is coordination in transshipment problems. Table 2.3 depicts the contributions of different chapters of this thesis to the existing literature on supply chain contracting, according to the proposed classification scheme. Chapter 3 addresses a single tier supply chain problem with two agents. Under the assumption of risk neutrality, we propose a contract which, drawing upon an implicit pricing mechanism, coordinates the production/Inventory quantities. Chapter 4 studies the transshipment problem with n agents. The decision variable to coordinate is again production/inventory quantities. Finally, Chapter 5 address the coordination in n-agent transshipment problem with positive cooperation costs.

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Chapter 3

Coordinating Transshipment Problem With Two Agents

Summary: *This chapter studies a supply chain with two independent agents producing/ordering an homogeneous product and cooperating through transshipment. Previous studies of this chain show that only under certain conditions, linear transshipment prices could be found that induce the companies to choose the first best production quantities. Moreover, even if such transshipment prices do exist, they result in a unique division of total expected profit and thus they cannot accommodate arbitrary divisions of the profit. Using the Generalized Nash Bargaining Solution, we derive coordinating transshipment prices that* always *give rise to a coordinating contract for the chain. This contract relies on an implicit pricing mechanism.*

3.1 Introduction

Generally, cooperation between agents in a supply chain falls into two major categories: vertical and horizontal. The vertical cooperation is defined as concerted practices between agents operating at different levels of supply chain, e.g. manufacturer-wholesaler, supplier-retailer (Cruijssen et al., 2007). Most of the previous research on supply chain contracts addresses vertical cooperation. In *wholesale price* contracts, the seller offers a wholesale price to the buyer. If the buyer accepts the contract, it will pay the seller for each purchased unit (Lariviere and Porteus, 2001). *Quantity discount* contracts are generally similar to the wholesale price contracts except that the seller offers a price which is dependent on the buyer's order quantity (see Cachon (2003)). In *buyback* contracts the seller offers a contract with a fixed unit price along with a buyback unit price. With this contract, the buyer pays the seller for each unit purchased, and after the resolution of uncertainties, the seller compensates for the buyer's unsold units (Pasternack, 1985). In *revenue sharing* contracts, the buyer receives a unit wholesale price (which is less than its marginal cost) before the realization of demand, and then it gets a portion of retailer's profit after the realization of demand. Except for the wholesale price contract, the rest of these contracts can be designed as coordinating contracts.

On the other hand, the horizontal cooperation is defined as the collaboration between agents operating at the same level(s) in the supply chain, e.g. retailers, distributors, or transportation agencies (Cruijssen et al., 2007). An instance of horizontal cooperation is *transshipment*. Whenever agents have to stock up their resources in anticipation of uncertain demands, they might end up in two situations. First, in case of high demands they encounter unsatisfied demand which causes either lost sales or backorder costs. Second, in case of low demands, they confront the costs of surplus resources, e.g. holding costs or reduced sale prices. By transshipment an agent has the chance to use another agent's surplus resources whenever it faces unsatisfied demand. An example of this practice is discernible in the oil industry where volatility of demands and limitation of regional refinery capacities make transshipment a reasonable practice (Dempster et al., 2000). The popularity of this practice is growing thanks to advances in information and communication technologies. To the best of our knowledge, previous research does not provide any coordinating contract for the transshipment problem. This chapter proposes such a contract for a supply chain with two agents.

The main question addressed in this chapter is the existence of transshipment prices which

(a) rational agents can agree upon prior to the realization of demands; and

(b) give rise to the coordination of production decisions.

We use the Generalized Nash Bargaining Solution (Roth, 1979; Nagarajan and Sošić, 2008) to develop a model for the negotiation over the division of total expected profit resulting from the agents' cooperation. We prove that there exists a contract for determining the transshipment prices which coordinates the production decisions, and also divides the total expected profit between the agents based on their bargaining powers. Our approach implies that this contract must have two rounds (see Figure 3.1). In the first round, the agents accept a *condition*, i.e. a pricing formula which is an implicit function of their quantity decisions, for determining the transshipment prices which is an implicit function of later decisions on their production quantities. In the second round, after the agents individually made their production decisions, they fix the negotiated transshipment prices by selecting them among all the possible coordinating transshipment prices. The pricing mechanism in this contract is, in fact, an *implicit pricing mechanism*. We show that the proposed contract is a coordinating

Figure 3.1: Sequence of Actions in the Proposed Two-agent Contract

contract.

The rest of this chapter is organized as follows. Section 3.2 provides a brief literature review and the chapter's motivation; Section 3.3 presents the basic framework and notation; Section 3.4 formulates the mathematical model of the transshipment problem; Section 3.5 illustrates the details of the proposed contract; Section 3.6 compares our mechanism with the mechanisms previously proposed for this problem; and finally, Section 3.7 contains concluding remarks.

3.2 Literature Review

Horizontal cooperation has been explored previously in different forms, e.g. subcontracting and outsourcing (Van Mieghem, 1999), lateral capacity or resource exchange (Chakravarty and Zhang, 2007; Krajewska et al., 2007), and transshipment. There are two main streams of research in the transshipment problem. In the *ex post* transshipment, it is assumed that the transshipment is done after the demand realization (Krishnan and Rao, 1965; Tagaras, 1989; Herer and Rashit, 1999; Rudi et al., 2001; Hu et al., 2007). The other stream assumes that agents transship based on their updated demand forecasts and before the observation of actual demands, i.e., *ex ante* transshipment (Das, 1975; Gross, 1963; Chod and Rudi, 2006). We focus on the former in this chapter.

Traditionally, most of the research on the transshipment problem assume a *central-*

ized supply chain with a single decision maker (Krishnan and Rao, 1965; Tagaras, 1989; Herer and Rashit, 1999). In the *decentralized* supply chain, agents are owned or managed independently, and there are potential conflicts of interests. Thus, the main instrument for analyzing the decentralized supply chains becomes *game theory*. Perhaps one of the first papers which utilize the game theory concepts in operations management context is Parlar (1988). He developed a model for the single-period transshipment problem and derived the ordering quantities using the Nash Equilibrium. However, this research does not consider any transshipment prices other than the market selling prices.

Using game theory in a decentralized supply chain, Van Mieghem (1999) examines the subcontracting problem where an agent can use the subcontractor's capacity when its demand exceeds its own capacity. He analyzes the initial investment decisions under three different contract types: price-only contracts, incomplete contracts, and state-dependent price-only contracts. In his analysis of the state-dependent price-only contracts (states are defined with respect to the actual demands) he suggests a mechanism for deriving the transshipment prices that can result in the initial investment levels which maximize the centralized profit. However, with his state-dependent price-only contracts, the determination of the transshipment prices requires knowledge about the actual demands.

Rudi et al. (2001) study a single-period transshipment problem with two independent retailers. They derive the transshipment prices that cause the independent retailers to choose the supply chain optimal production/order quantities. However, Hu et al. (2007) prove that such transshipment prices may exist only under certain conditions, thus not always. Therefore, Hu et al. (2007) conclude that

> firms that would like to coordinate multiple locations may have to resort to other mechanisms than solely relying on linear transshipment

prices (p. 1294).

This conclusion motivates the development of the implicit pricing mechanism in this chapter. Moreover, even if such transshipment prices exist, they lead to a singular division of total expected profit that might be unacceptable to at least one of the agents. Hence, these transshipment prices do not give rise to a coordinating contract according to Cachon's definition. Instead of assuming exogenous transshipment prices, we model the negotiation over the total expected profit resulting from cooperation between agents. We propose a coordinating contract with an implicit pricing mechanism that *always* leads to the first best quantities being the Nash equilibrium, and accommodates the division of total expected profit according to the agents' bargaining powers. Finally, we show that the agents may have several choices when fixing the transshipment prices.

An alternative approach to coordinate the transshipment problem employs cooperative game theory. This approach advocates that once the agents have decided their quantities and the market demand has been observed, they form coalitions, transship the surplus, if any, and divide the extra profits resulting from the transshipment. Anupindi et al. (2001) provide an allocation rule based on the dual prices of residuals, i.e. the dual allocation rule, in the core of corresponding cooperative game. Still, as Huang and Sošić (2010b) show, the dual allocation rule is unable to coordinate the general supply chain with two agents.

Although most of the previous research on supply chain contracts use the Stackelberg game for analyzing the dynamics between the parties (see Cachon (2003)), this chapter uses the concept of Generalized Nash Bargaining Solution. The rationale is that prior to the realization of demands, neither agent knows if it has unsatisfied demands or surplus products. Therefore, the Stackelberg game is not suitable in the supply chain where neither agent has some distinctive characteristics for being the *leader*. Clearly, if the agents wait until they receive some updated information about their demands, they might be able to later distinguish the leader as the seller (or alternatively as the buyer) as in Chakravarty and Zhang (2007).

3.3 Notation and Framework

Consider a system with two risk-neutral newsvendor agents ($i, j = 1, 2$) producing an homogeneous product ($i \neq j$ throughout the chapter). The agents decide their production/order quantities, $\mathbf{X} = (X_1, X_2)$, prior to the realization of random demands, $\mathbf{D} = (D_1, D_2)$. The $\mathbf{D}$ has a bivariate continuous and twice differentiable density function with its support on positive reals. The unit production costs, selling prices, and salvage values are denoted by $\mathbf{c} = (c_1, c_2)$, $\mathbf{r} = (r_1, r_2)$, and $\boldsymbol{\nu} = (\nu_1, \nu_2)$ respectively. We assume $0 \leq \boldsymbol{\nu} < \mathbf{c} < \mathbf{r}$. The agents are penalized at the rate $\mathbf{h} = (h_1, h_2)$ for each unit of unsatisfied demand.

We study a single-period model with two stages. At the beginning of stage one, agents agree on the way to set the transshipment prices, $\mathbf{s} = (s_{12}, s_{21})$, where s_{12} is the unit price that 2 should pay in order to receive a unit of 1's surplus product. The agents decide their quantities individually and independently afterwards. At the beginning of stage two, demands are realized and agents carry out the transshipments. When i transships to j, the former incurs a unit transportation cost, $t_{ij} \geq 0$. Let $\mathbf{t} = (t_{12}, t_{21})$. To assure that the transshipment occurs only if one agent has unsatisfied demands and the other has surplus, it is commonly assumed (see Rudi et al. (2001) for example) that $c_i < c_j + t_{ji}$, $\nu_i < \nu_j + t_{ji}$, and $r_i + h_i < r_j + h_j + t_{ji}$ for $i, j = 1, 2$. The transshipment is feasible if neither agent is worse off by doing it. From the transshipment-receiver agent's viewpoint, a transshipment price is feasible if it is less than or equal to the market selling price plus the lost sale penalty. From the transshipment-sender agent's viewpoint, a transshipment price is feasible if it is greater than or equal to the transportation cost plus the salvage value. Therefore, transshipment prices are feasible if

$$t_{ij} + \nu_i \le s_{ij} \le r_j + h_j \tag{3.1}$$

for $i, j = 1, 2$. We assume that $t_{ij} + \nu_i < r_j + h_j$ for $i, j = 1, 2$.

3.4 The Model

We formulate the individual expected profits, $\mathbf{J} = (J_1, J_2)$, in the non-cooperative mode (without transshipment) and the decentralized cooperative mode (with transshipment). We use the superscripts NC and DC to distinguish between non-cooperative and decentralized cooperative modes respectively.

Non-cooperative mode The individual expected profits in the non-cooperative mode are

$$J_i^{NC}(X_i) = \mathsf{E}\left[r_i \min(D_i, X_i) + \nu_i (X_i - D_i)^+ - h_i (D_i - X_i)^+ - c_i X_i\right] \tag{3.2}$$

for $i = 1, 2$ where $x^+ = \max(x, 0)$. The optimum quantities in this mode, $\mathbf{X}^{NC} = (X_1^{NC}, X_2^{NC})$, are simply the critical fractiles of the corresponding newsvendor problems. Therefore, for all X_i, $J_i^{NC}(X_i) \le J_i^{NC}(X_i^{NC})$ with $i = 1, 2$. The total expected profit in this mode is denoted by

$$J_T^{NC}(\mathbf{X}) = J_1^{NC}(X_1) + J_2^{NC}(X_2).$$

Cooperative mode In the cooperative mode, after the realization of demands, if one agent has some unsatisfied demand and the other has some surplus products, they carry out the transshipment. Similar to Hu et al. (2007), we use the following additional notation:

$W_{ij}(\mathbf{X}) = \min[(D_j - X_j)^+, (X_i - D_i)^+]$: transshipment quantity from i to j after the realization of demands. It is the smaller of the two values of the unsatisfied demand of agent j, $(D_j - X_j)^+$, and the surplus products of agent i, $(X_i - D_i)^+$.

$D_i^s(\mathbf{X}) = \min(D_i, X_i) + W_{ji}(\mathbf{X})$: the demand that agent i can satisfy after transshipment.

$I_i(\mathbf{X}) = (X_i - D_i)^+ - W_{ij}(\mathbf{X})$: surplus products of agent i after transshipment.

$D_i^u(\mathbf{X}) = (D_i - X_i)^+ - W_{ji}(\mathbf{X})$: unsatisfied demand at i after transshipment. The individual profit functions are

$$J_i^{DC}(\mathbf{s}, \mathbf{X}) = \mathsf{E}\left[r_i D_i^s(\mathbf{X}) - s_{ji} W_{ji}(\mathbf{X}) + (s_{ij} - t_{ij}) W_{ij}(\mathbf{X}) + \nu_i I_i(\mathbf{X}) - h_i D_i^u(\mathbf{X}) - c_i X_i\right] \tag{3.3}$$

for $i, j = 1, 2$. By some rearrangement and simplification (see Appendix) we have

$$J_1^{DC}(\mathbf{s}, \mathbf{X}) = (s_{12} - t_{12} - \nu_1)\Gamma_{12}(\mathbf{X}) + (r_1 + h_1 - s_{21})\Gamma_{21}(\mathbf{X}) + J_1^{NC}(X_1) \tag{3.4}$$

$$J_2^{DC}(\mathbf{s}, \mathbf{X}) = (r_2 + h_2 - s_{12})\Gamma_{12}(\mathbf{X}) + (s_{21} - t_{21} - \nu_2)\Gamma_{21}(\mathbf{X}) + J_2^{NC}(X_2), \tag{3.5}$$

where $\Gamma_{12}(\mathbf{X}) = \mathsf{E}[W_{12}(\mathbf{X}, \mathbf{D})]$ and $\Gamma_{21}(\mathbf{X}) = \mathsf{E}[W_{21}(\mathbf{X}, \mathbf{D})]$. The total expected profit in the cooperative mode is independent of the transshipment prices and equals

$$J_T^{DC}(\mathbf{X}) = (r_2 + h_2 - t_{12} - \nu_1)\Gamma_{12}(\mathbf{X}) + (r_1 + h_1 - t_{21} - \nu_2)\Gamma_{21}(\mathbf{X}) + J_1^{NC}(X_1) + J_2^{NC}(X_2). \tag{3.6}$$

The optimum quantities, i.e. the first best quantities, are $\mathbf{X}^C = (X_1^C, X_2^C)$. The concavity of $J_T^{DC}(\mathbf{X})$ with respect to $\mathbf{X}$ is shown in Pasternack and Drezner (1991) where a method for calculating $\mathbf{X}^C$ is also given.

3.5 The Contract

Axiomatic bargaining was first proposed by John Nash (1951). Consider two players (here 1 and 2) who either reach an agreement or fail to do so and then the disagreement occurs. A bargaining problem is a pair $(\mathbf{F}, \mathbf{d})$ where $\mathbf{F}$ is a closed convex subset of $\mathbb{R}^2$ consisting of the set of all utility pairs, $\mathbf{u} = (u_1, u_2)$, that are the utilities of the bargaining scenarios, and $\mathbf{d} = (d_1, d_2)$ are the utilities in the disagreement scenario. If for both players the utilities of agreement scenarios are greater than that of the disagreement scenario, then players have an incentive to reach an agreement and cooperate with each other. Nash proves that by considering certain axioms about players' preferences and utility functions as well as the bargaining outcomes, the bargaining solution can be uniquely determined. These axioms are (Osborne and Rubinstein, 1990):

(a) individual rationality,

(b) Pareto-efficiency,

(c) invariance to equivalent utility representations,

(d) independence of irrelevant alternatives, and

(e) symmetry.

The Nash Bargaining Solution (NBS), denoted by $f(\mathbf{F}, \mathbf{d})$, can then be derived through solving the following system

$$f(\mathbf{F}, \mathbf{d}) = \underset{\mathbf{u}=(u_1,u_2)\in\mathbf{F},\, \mathbf{u}\geq\mathbf{d}}{\arg\max} \; (u_1 - d_1)(u_2 - d_2). \tag{3.7}$$

By relaxing the symmetry axiom, the remaining Nash Bargaining axioms determine a bargaining solution derived by solving the following system.

$$f_g(\mathbf{F}, \mathbf{d}) = \underset{\mathbf{u}=(u_1,u_2)\in\mathbf{F},\, \mathbf{u}\geq\mathbf{d}}{\arg\max} (u_1 - d_1)^{\gamma}(u_2 - d_2)^{1-\gamma} \tag{3.8}$$

where $0 < \gamma < 1$ is the player 1's bargaining power and $1 - \gamma$ is player 2's bargaining power (Roth, 1979). Note that in this model the γ is assumed to be known *a priori*. The solution to (3.8) is called the Generalized Nash Bargaining Solution (GNBS) (Nagarajan and Sošić, 2008).

3.5.1 The Implicit Pricing Mechanism

In (3.8), set $u_i \equiv J_i^{DC}(\mathbf{s}, \mathbf{X})$ and $d_i \equiv J_i^{NC}(X_i^{NC})$ for $i = 1, 2$. Observe that for all $\mathbf{s}$ we have $J_i^{DC}(\mathbf{s}, \mathbf{X}^{NC}) \geq J_i^{NC}(X_i^{NC})$. Thus, there are always $\mathbf{s}$ and $\mathbf{X}$ such that $J_i^{DC}(\mathbf{s}, \mathbf{X}) \geq J_i^{NC}(X_i^{NC})$. For those $\mathbf{s}$ and $\mathbf{X}$ the GNBS can be formulated as

$$f_g = \arg\max_{\mathbf{s}} \left[J_1^{DC}(\mathbf{s}, \mathbf{X}) - J_1^{NC}(X_1^{NC})\right]^{\gamma}\left[J_2^{DC}(\mathbf{s}, \mathbf{X}) - J_2^{NC}(X_2^{NC})\right]^{1-\gamma}. \tag{3.9}$$

Lemma 3.1 (The GNBS condition). *For any* $\mathbf{X}$, *the transshipment prices which solve (3.9),* $\mathbf{s}^* = (s_{12}^*, s_{21}^*)$, *satisfy the following condition*

$$\begin{aligned}\Gamma_{12}(\mathbf{X})s_{12}^* - \Gamma_{21}(\mathbf{X})s_{21}^* = &\left[\gamma(r_2 + h_2) + (1-\gamma)(t_{12} + \nu_1)\right]\Gamma_{12}(\mathbf{X}) - \left[(1-\gamma)(r_1 + h_1) + \gamma(t_{21} + \nu_2)\right]\Gamma_{21}(\mathbf{X}) \\ &+ \gamma\left[J_2^{NC}(X_2) - J_2^{NC}(X_2^{NC})\right] - (1-\gamma)\left[J_1^{NC}(X_1) - J_1^{NC}(X_1^{NC})\right].\end{aligned} \tag{3.10}$$

Proof. First, we note that (3.9) is concave on $\mathbf{s}$ (see Appendix). If both $\Gamma_{12}(\mathbf{X})$ and $\Gamma_{21}(\mathbf{X})$ are nonzero, the GNBS condition can be obtained by setting either of the first order conditions, which are provided in the proof of concavity of (3.9) in the Appendix, to zero and solving it. If either of the $\Gamma_{12}(\mathbf{X})$ and $\Gamma_{21}(\mathbf{X})$ is zero, then either of the first order conditions is always zero and the GNBS condition can be obtained by setting the other equation to zero and solving it. □

Note that the transshipment prices which meet the GNBS condition in (3.10) are implicit functions of $\mathbf{X}$. Therefore, $\mathbf{s}^*(\mathbf{X})$ is an *implicit pricing mechanism.* This implies a two-round contract detailed in Figure 3.1. In round one, the agents accept $\mathbf{s}^*(\mathbf{X})$ and then individually decide their quantities; in round two, they fix the transshipment prices by selecting a point using the implicit pricing mechanism. By Lemma 3.1, for any $\mathbf{X}$, if both $\Gamma_{12}(\mathbf{X}) \neq 0$ and $\Gamma_{21}(\mathbf{X}) \neq 0$, the agents will have several alternatives for fixing $\mathbf{s}^*(\mathbf{X})$ since $s_{12}^*(\mathbf{X})$ and $s_{21}^*(\mathbf{X})$ lie on the line defined by (3.10). However, if either $\Gamma_{12}(\mathbf{X}) = 0$ or $\Gamma_{21}(\mathbf{X}) = 0$ (but not both), then one of the transshipment prices disappears from the equation (3.10) and consequently there will be only one choice for $\mathbf{s}^*(\mathbf{X})$. The case with $\Gamma_{12}(\mathbf{X}) = \Gamma_{21}(\mathbf{X}) = 0$ is trivial because then neither agent expects any transshipments. The resultant transshipment prices will be referred to as the *negotiated transshipment prices.*

3.5.2 Deciding the Quantities

When individually deciding their quantities, the agents undergo a game. The *individual* optimum quantities are thus determined by the Nash equilibrium, $\mathbf{X}^{DC} = (X_1^{DC}, X_2^{DC})$, which is the intersection point of the agents' *reaction functions*[12] (Fudenberg and Tirole, 2002).

Rudi et al. (2001) argue that there is a unique set of *linear transshipment prices* (transshipment prices that are fixed before the decisions on production/order quantities has been made) that results in the Nash equilibrium being equal to the first best quantities. Hu et al. (2007) refute this claim by proving that these special linear transshipment prices do not necessarily exist (we shall return to their counter-example in Section 3.6). Moreover, even if such linear transshipment prices do exist, they can

[12] A reaction function specifies the decision of an agent as a function of other agents' decisions.

only divide the supply chain profit between the agents in one way for there is a one-to-one correspondence between linear transshipment prices and the division of total expected profit (Hu et al., 2007).

We now show that with the implicit pricing mechanism, for any combination of bargaining powers, the game to select the quantities *always* results in the first best quantities.

Lemma 3.2. *With* $\mathbf{s}^*(\mathbf{X})$, *the expected individual profits are*

$$J_1^{DC}(\mathbf{s}^*(\mathbf{X}),\mathbf{X}) = \gamma J_T^{DC}(\mathbf{X}) + \left[(1-\gamma)J_1^{NC}(X_1^{NC}) - \gamma J_2^{NC}(X_2^{NC})\right]. \tag{3.11}$$

$$J_2^{DC}(\mathbf{s}^*(\mathbf{X}),\mathbf{X}) = (1-\gamma)J_T^{DC}(\mathbf{X}) - \left[(1-\gamma)J_1^{NC}(X_1^{NC}) - \gamma J_2^{NC}(X_2^{NC})\right]. \tag{3.12}$$

Proof. At the point $\mathbf{s}^*(\mathbf{X})$, (3.4) can be rewritten as

$$J_1^{DC}(\mathbf{s}^*(\mathbf{X}),\mathbf{X}) = \Gamma_{12}(\mathbf{X})s_{12}^* - \Gamma_{21}(\mathbf{X})s_{21}^* - (t_{12}+\nu_1)\Gamma_{12}(\mathbf{X}) + (r_1+h_1)\Gamma_{21}(\mathbf{X}) + J_1^{NC}(X_1) \tag{3.13}$$

Substituting (3.10) in (3.13) one obtains (3.11). By applying the same procedure to (3.5) one can get (3.12). □

Lemma 3.2 states that with $\mathbf{s}^*(\mathbf{X})$, the expected individual profit for each agent equals its maximum expected profit in the non-cooperative mode, $J_i^{NC}(X_i^{NC})$, plus a fraction (γ for agent 1 and $1-\gamma$ for agent 2) of expected extra profit resulting from the cooperation, i.e., $J_T^{DC}(\mathbf{X}) - J_T^{NC}(\mathbf{X}^{NC})$. We have the following theorem.

Theorem 3.1. *With* $\mathbf{s}^*(\mathbf{X})$, $\mathbf{X}^{DC} = \mathbf{X}^{C}$.

Proof. The agents' reaction functions are $X_i^{DC} = \arg\max_{X_i} J_i^{DC}(\mathbf{X})$ for $i,j = 1,2$. The solution to the system of first order conditions,

$$\left\{\partial J_1^{DC}(\mathbf{X},\mathbf{s})/\partial X_1 = 0, \partial J_2^{DC}(\mathbf{X},\mathbf{s})/\partial X_2 = 0\right\},$$

is the Nash equilibrium. By substituting (3.11) and (3.12) and simplification, the latter is equivalent to

$$\left\{\partial J_T^{DC}(\mathbf{X})/\partial X_1 = 0, \partial J_T^{DC}(\mathbf{X})/\partial X_2 = 0\right\}.$$

The solution to the last system is $\mathbf{X}^C$. □

Thus, if the implicit pricing mechanism is implemented, the Nash equilibrium quantities equals the first best quantities.

3.5.3 Fixing the Negotiated Transshipment Prices

After the quantities have been decided, the agents should fix the negotiated transshipment prices—according to the implicit pricing mechanism in (3.10)—so that they also meet the feasibility conditions given in (3.1). Let $\mathbf{\Omega}(\mathbf{X})$ be the set of all such transshipment prices for a given $\mathbf{X}$.

Lemma 3.3. *For a given* $\mathbf{X}$,

$$\mathbf{\Omega}(\mathbf{X}) = \begin{cases} \max(L_1, t_{21} + \nu_2) \leq s_{21}^* \leq \min(L_2, r_1 + h_1) & \text{if } \Gamma_{12}(\mathbf{X}) \neq 0 \text{ and } \Gamma_{21}(\mathbf{X}) \neq 0 \\ t_{21} + \nu_2 \leq s_{21}^* \leq r_1 + h_1 & \text{if } \Gamma_{12}(\mathbf{X}) = 0 \text{ and } \Gamma_{21}(\mathbf{X}) \neq 0 \\ t_{12} + \nu_1 \leq s_{12}^* \leq r_2 + h_2 & \text{if } \Gamma_{12}(\mathbf{X}) \neq 0 \text{ and } \Gamma_{21}(\mathbf{X}) = 0 \end{cases} \tag{3.14}$$

where

$$\begin{aligned} L_1 =& -\gamma\left(r_2 + h_2 - t_{12} - \nu_1\right)\frac{\Gamma_{12}(\mathbf{X})}{\Gamma_{21}(\mathbf{X})} + (1-\gamma)(r_1 + h_1) + \gamma(t_{21} + \nu_2) \\ &- \frac{\gamma[J_2^{NC}(X_2) - J_2^{NC}(X_2^{NC})] - (1-\gamma)[J_1^{NC}(X_1) - J_1^{NC}(X_1^{NC})]}{\Gamma_{21}(\mathbf{X})} \\ L_2 =& (1-\gamma)\left(r_2 + h_2 - t_{12} - \nu_1\right)\frac{\Gamma_{12}(\mathbf{X})}{\Gamma_{21}(\mathbf{X})} + (1-\gamma)(r_1 + h_1) + \gamma(t_{21} + \nu_2) \\ &- \frac{\gamma[J_2^{NC}(X_2) - J_2^{NC}(X_2^{NC})] - (1-\gamma)[J_1^{NC}(X_1) - J_1^{NC}(X_1^{NC})]}{\Gamma_{21}(\mathbf{X})}. \end{aligned} \tag{3.15}$$

Proof. $\Omega(\mathbf{X})$ is defined by the GNBS condition and the feasibility constraints for the transshipment prices. For the first case, substituting the s_{12}^* from the GNBS condition into the feasibility condition for s_{12}, the $\Omega(\mathbf{X})$ becomes the intersection of $L_1 \le s_{21}^* \le L_2$ and $t_{21} + \nu_2 \le s_{21}^* \le r_1 + h_1$. The latter is equivalent to

$$\max(L_1, t_{21} + \nu_2) \le s_{21}^* \le \min(L_2, r_1 + h_1).$$

The second and third cases follow subsequently. □

Note that when either $\Gamma_{12}(\mathbf{X}) = 0$ or $\Gamma_{21}(\mathbf{X}) = 0$ the feasibility condition in (3.1) solely determines the boundaries. However, when both $\Gamma_{12}(\mathbf{X})$ and $\Gamma_{21}(\mathbf{X})$ are positive, the GNBS condition enforces further restriction on the boundaries. The following theorem ensures that for the first best quantities, i.e. $\mathbf{X}^C$, feasible negotiated transshipment prices can *always* be found.

Theorem 3.2. *$\Omega(\mathbf{X}^C)$ is non-empty.*

Proof. Assume that $\Gamma_{12}(\mathbf{X}) \neq 0$ and $\Gamma_{21}(\mathbf{X}) \neq 0$. In order to prove that $\Omega(\mathbf{X}^C)$ is non-empty, it is sufficient to show that

$$\max(L_1, t_{21} + \nu_2) < \min(L_2, r_1 + h_1)$$

for $\mathbf{X} = \mathbf{X}^C$. From the assumptions of our model, we know that $t_{12} + \nu_1 < r_2 + h_2$ and $t_{21} + \nu_2 < r_1 + h_1$. This directly results in $L_1 < L_2$. To show that $L_1 < r_1 + h_1$ is equivalent to show that

$$J_T^{DC}(\mathbf{X}^C) - J_T^{NC}(\mathbf{X}^C) + \frac{1}{\gamma}\left(J_1^{NC}(X_1^{NC}) - J_1^{NC}(X_1^C)\right) > 0.$$

We know that

$$J_T^{DC}(\mathbf{X}^C) > J_T^{NC}(\mathbf{X}^C).$$

Since

$$J_1^{NC}(X_1^{NC}) - J_1^{NC}(X_1^C) \geq 0,$$

we have $L_1 < r_1 + h_1$. To show that $t_{21} + \nu_2 < L_2$ is equivalent to show that

$$J_T^{DC}(\mathbf{X}^C) - J_T^{NC}(\mathbf{X}^C) + \frac{1}{1-\gamma}\left(J_2^{NC}(X_2^{NC}) - J_2^{NC}(X_2^C)\right) > 0.$$

With

$$J_T^{DC}(\mathbf{X}^C) > J_T^{NC}(\mathbf{X}^C)$$

and

$$J_2^{NC}(X_2^{NC}) - J_2^{NC}(X_2^C) \geq 0,$$

we can see that $t_{21} + \nu_2 < L_2$. Now consider that $\Gamma_{12}(\mathbf{X}) = 0$. In this case, it needs to be shown that

$$t_{21} + \nu_2 \leq s_{21}^*(\mathbf{X}^C) \leq r_1 + h_1.$$

Based on the previous part, the proof is straightforward. The case where $\Gamma_{21}(\mathbf{X}) = 0$ is similar. □

Whenever there are multiple possibilities for selecting the transshipment prices, the choice among them does not affect the individual expected profits and can be done arbitrarily and possibly by using a secondary criterion, for instance the variances of the agents' individual profits.

Characteristics	Agent 1	Agent 2
Demand Distribution	Truncated Normal Dist.(100,50)	Truncated Normal Dist.(200,100)
Selling Price	$r_1 = 20$	$r_2 = 25$
Lost Sale Penalty	$h_1 = 5$	$h_2 = 8$
Transportation Cost	$t_{12} = 6$	$t_{21} = 6$
Salvage Value	$\nu_1 = 8$	$\nu_2 = 9$
Unit Cost of Production	$c_1 = 10$	$c_2 = 12$

Table 3.1: Description of Example 1

Optimum quantities	$X_1^{NC} = 160.02$	$X_2^{NC} = 316.43$
Maximum Individual Expected Profit	$J_1^{NC}(X_1^{NC}) = 864.34$	$J_2^{NC}(X_2^{NC}) = 2191.0$
Maximum Total Expected Profit (Centralized)	$J_T^{NC}(\mathbf{X}^{NC}) = 3055.34$	

Table 3.2: Example 1: The Outcome in the Non-Cooperative Mode

Special Case: Symmetric Agents

For two completely symmetric agents, i.e. when all the parameters as well as the bargaining powers are equal, we have

$$[J_2^{NC}(X_2^{DC}) - J_2^{NC}(X_2^{NC})] = [J_1^{NC}(X_1^{DC}) - J_1^{NC}(X_1^{NC})]$$

in (3.10). Therefore, when the agents fix the transshipment prices, they can always pick them equal

$$\mathbf{s}^* = (\frac{r+h+\nu+t}{2}, \frac{r+h+\nu+t}{2})$$

which is *independent* of the realization of demands.

3.5.4 An Example

Consider two agents described in Table 3.1. We assume that they have independent truncated Normal demand distributions. Table 3.2 yields their expected profits in the non-cooperative mode. In the cooperative mode, the negotiated transshipment prices

Optimum Quantities (Centralized)	$X_1^C = 181.14 \quad X_2^C = 269.01$
Maximum Total Expected Profit (Centralized)	$J_T^{DC}(\mathbf{X}^C) = 3181.1$

Table 3.3: Example 1: Centralized Solution

meet the following GNBS condition:

$$\Gamma_{12}(\mathbf{X})s_{12}^* - \Gamma_{21}(\mathbf{X})s_{21}^* = 23.5\Gamma_{12}(\mathbf{X}) - 20\Gamma_{21}(\mathbf{X}) + \frac{1}{2}[J_2^{NC}(X_2) - J_1^{NC}(X_1) - 1326.66]. \quad (3.16)$$

Table 3.3 shows the optimum quantities and the total expected profit in the centralized supply chain. By Theorem 3.1, the optimum individual quantities with the negotiated transshipment prices in (3.16) are those in Table 3.3. Next, the agents could choose a specific set of transshipment prices by picking any point on the line $s_{12}^* - 0.074s_{21}^* = 19.435$ with $15 \le s_{21}^* \le 25$ (e.g. $\mathbf{s}^* = (20.915, 20)$).

3.6 Linear versus Implicit Pricing Mechanism

We are now ready to illustrate the difference between the linear pricing mechanism presented by Rudi et al. (2001) and Hu et al. (2007), the dual allocation mechanism of Anupindi et al. (2001) and Huang and Sošić (2010b), and our implicit pricing mechanism. We use the example proposed in Hu et al. (2007) as an the instance where no linear transshipment prices could be found that induce the agents to choose the first best quantities. We also show how our implicit pricing mechanism leads to the coordination of this system.

Consider two agents with the characteristics given in Table 3.4. In the non-cooperative mode, the agents' expected profit would be $J_1^{DC}(X_1 = (1,2,3)) = (6,9,8.8)$ and $J_2^{DC}(X_2 = (1,2,3)) = (6,5,4)$. Therefore, $\mathbf{X}^{NC} = (2,1)$. Now assume that the two agents can transship. The best policy in the centralized supply chain is $\mathbf{X}^C = (1,3)$ which gives rise to the total profit of 16.48.

Characteristics	Agent 1	Agent 2
Demand Distribution	(1,2,3) with probabilities (0.3,0.32,0.38)	Deterministic (1)
Selling Price	$r_1 = 11$	$r_2 = 11$
Lost Sale Penalty	$h_1 = 0$	$h_2 = 0$
Transportation Cost	$t_{12} = 4$	$t_{21} = 1$
Salvage Value	$\nu_1 = 1$	$\nu_2 = 4$
Unit Cost of Production	$c_1 = 5$	$c_2 = 5$

Table 3.4: The Hu et al. (2007) Counter-example

(a)

X_1	X_2 = 1	X_2 = 2	X_2 = 3
1	6	$13.7 - .07s_{21}$	$17.88 - 1.08s_{21}$
2	9	$13.18 - 0.38s_{21}$	$13.18 - 0.38s_{21}$
3	8.8	8.8	8.8

(b)

X_1	X_2 = 1	X_2 = 2	X_2 = 3
1	6	$1.5 + .07s_{21}$	$-1.4 + 1.08s_{21}$
2	6	$3.1 + 0.38s_{21}$	$0.58 + 0.38s_{21}$
3	6	5	4

Table 3.5: Individual Expected Profits with Linear Transshipment Prices

Now consider the corresponding decentralized system with transshipments. Table 3.5 shows the expected profits for the two agents as a function of the transshipment price, **s**, and the quantities, **X**. The linear transshipment price is by definition the same for each entry of the Tables 3.5(a) and 3.5(b) (see Hu et al. (2007)).

Hu et al. (2007) prove that there is no linear transshipment price, s_{21}, that induces the agents to set their quantities as the first best. In fact, when $s_{21} \in [5, 145/19)$, the Nash equilibrium is $\mathbf{X}^{DC} = (2, 1)$ with joint profits of 15, and when $s_{21} \in [145/19, 11)$, $\mathbf{X}^{DC} = (2, 2)$ with joint profits of 16.28.

Table 3.6 shows the individual expected profits calculated according to the dual allocation mechanism of Anupindi et al. (2001). Thus, this mechanism results in the Nash equilibrium $\mathbf{X}^{DC} = (2, 2)$ with joint profits of 16.28. Therefore, the condition for the existence of coordinating dual allocation for two agents given in Huang and Sošić (2010b) is not satisfied. Thus, the agents are unable to attain $J_T^{DC}(\mathbf{X}^C) = 16.48$ either with the linear transshipment prices or the dual allocation mechanism.

Now assume that s_{21} is set as by our implicit pricing mechanism. The implicit pricing

X_1	X_2 = 1	2	3
1	6	6	10.56
2	9	9	11.28
3	8.8	8.8	8.8

(a)

X_1	X_2 = 1	2	3
1	6	10.2	5.92
2	6	7.28	4
3	6	5	4

(b)

Table 3.6: Individual Expected Profits with Dual allocation Mechanism

X_1	X_2 = 1	2	3
1	6	9.1	9.74
2	9	9.64	9.14
3	8.8	8.8	8.8

(a)

X_1	X_2 = 1	2	3
1	6	6.1	6.74
2	6	6.64	4.62
3	6	5	4

(b)

Table 3.7: The Agents' Expected Profits with $s^*_{21}(\mathbf{Q})$

mechanism is obtained form the GNBS condition in (3.10). In this example $\Gamma_{12}(\mathbf{X}) = 0$ thus $\Gamma_{21}(\mathbf{X})$ is equivalent to the coefficients of s_{21} in Table 3.5(b). Assuming $\gamma = 0.5$, the GNBS condition becomes

$$s^*_{21}(\mathbf{X}) = 8 + \frac{J_1^{NC}(X_1) - J_2^{NC}(X_2) - 3}{2\Gamma_{21}(\mathbf{X})}.$$

Substituting the respective values of $s^*_{21}(\mathbf{X})$ in Table 3.5, one obtains the expected individual profits in Table 3.7(a) and 3.7(b). Then, the Nash equilibrium is $\mathbf{X}^E = (1,3)$ that is exactly the same as the first best solution. The total expected profit in this case is also 16.48. Therefore, it can be seen that this implicit pricing mechanism leads to the coordination of the system.

3.7 Comments

The contract proposed in this chapter is limited to the two-agent supply chain. A possible extension to the supply chain with $n > 2$ agents needs to deal with two new key features: (1) the sensitivity of optimal transshipment patterns to actual demands, and (2) the possibility of coalitions formed by subsets on n agents. The coordination of transshipment problem with these two new features remains a challenging open problem. We leave these questions for the future research.

Recently, Huang and Sošić (2010b) developed several heuristics for setting the transshipment prices in a general n-agent supply chain. Those heuristics are developed so that the extra profits from transshipments mimic the allocations in the core of the *ex post* cooperative transshipment game. A centralized depot handles the transshipments in their contract. In the next chapter, we address this problem in detail and introduce a mechanism for coordinating the transshipment problem in a general n-agent supply chain.

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Chapter 4

Coordinating the Multi-agent Transshipment Problem

Summary: *The decentralized transshipment problem is a two-stage decision making problem where the agents first choose their individual production levels in anticipation of random demands and after demand realizations they pool residuals via transshipment. The coordination will be achieved if at optimality all the decision variables, i.e. production/order quantities and transshipment patterns, in the decentralized supply chain are the same as those of centralized supply chain. This chapter studies the coordination via* transshipment prices. *We propose a procedure for deriving the transshipment prices based on the coordinating allocation rule introduced by Anupindi et al. (2001). With the transshipment prices being set, the agents are free to match their residuals based on their individual preferences. We draw upon the concept of pair-wise stability to capture the dynamics of corresponding matching process. As the main result of this chapter, we show that with the derived transshipment prices, the optimum*

transshipment patterns are always pair-wise stable, i.e. there are no pairs of agents that can be jointly better off by unilaterally deviating from the optimum transshipment patterns.

4.1 Introduction

The multi-agent transshipment problem is coordinated if (a) every agent sets its production/order quantity equal to the centrally optimum amount for that agent, and (b) the transshipment pattern, i.e. the union of individual transshipments among the agents, in the decentralized problem is the same as the optimum transshipment patterns.

Under some conditions on the demand distribution functions, Anupindi et al. (2001) propose a coordinating contract that operates upon an *allocation rule* that specifies each agent's share of the extra profit generated through the transshipments. They argue that if an allocation rule in the core of the *ex post* transshipment game could be found, the optimum transshipment patterns would be also optimal for all the agents involved. Granot and Sošić (2003) show that this contract may not support the voluntary engagement of all the surplus products and unsatisfied demands in the transshipment stage. In other words, some agents might be better off by announcing only a portion of their surplus products or unsatisfied demands at the time of transshipments. However, in a repeated setting, the agents are willing to share all of their residuals in an equilibrium whenever the discount factor is large enough (Huang and Sošić, 2010a). An alternative allocation rule has been proposed in (Sošić, 2006). The rule redistributes the extra profit generated through the transshipments according to the Shapley value. Although the resultant allocation is not necessarily in the core, it could result in the *farsighted stability* of the grand coalition.

The contracts based on the allocation mechanisms require that the agents be able to take advantage of side payments (which may not be possible in all situations). From the implementation point of view, these contracts also need a governing party to collect the realized profits and redistribute them among the members of the coalition. In order to avoid these difficulties, the agents can turn to the contracts with pricing mechanisms. Then, whenever a transshipment between an agent with surplus and another agent with unsatisfied demand happens, the latter pays the former a sum proportional to the amount transshipped. The advantage of the pricing mechanism is that the additional institution for redistribution of extra profits is unnecessary—agents who are involved in a transshipment transaction can handle the "redistributions" without incentive-aligning side payments. Moreover, in this way, the amount of extra profits that is generated through transshipments between any two agents is divided completely between them.

Despite the appealing properties of pricing mechanisms, finding coordinating contracts based on them is challenging. Hu et al. (2007) show that linear transshipment prices, i.e. the transshipment prices which are fixed before the decisions on production quantities are made, may not be coordinating even with only two agents participating. In the general case with more than two agents, Huang and Sošić (2010b) show that the transshipment prices which are fixed before the decisions on production quantities cannot coordinate the system. They also propose some heuristic approaches for finding the transshipment prices which result in better performance in the decentralized system. In Chapter 3, a contract based on an implicit pricing mechanism that could coordinate the transshipment problem with two agents has been proposed. With an implicit pricing mechanism, agents initially agree on a formula for setting the transshipment prices as a function of their decisions on production quantities, and once those decisions have been made and prior to the realization of demands, they fix the transshipment prices. As they prove, this postponement in fixing the transshipment prices give rise to the coordination of the system. In this chapter, we take the coordinating allocation rule introduced in Anupindi et al. (2001) and introduce an equivalent pricing mechanism based on this rule. With the transshipment prices being set, the agents are free to match their surplus products and unsatisfied demands based on their individual preferences. This resembles a matching game in a two-sided market where the supply and demand values are real numbers (see Baiou and Balinski (2002)). We show that with the derived pricing mechanism the optimal transshipment patterns are always *pair-wise stable* solutions to the corresponding matching process, i.e. given the transshipment prices, no pairs of agents can simultaneously improve their profits by mutually deviating from the optimal transshipment patterns.

The rest of this chapter is organized as follows. Section 4.2 provides a detailed description of the problem. In Section 4.3 the optimal solution in the centralized system is addressed. Section 4.4 addresses the decentralized system with the allocation rule mechanism. Section 4.5 presents the transshipment prices derived from the coordinating allocation rule of Anupindi et al. (2001). Section 4.6 discusses the matching process that results in the formation of transshipment patterns and introduces the concept of pair-wise stability. It also demonstrates the pair-wise stability of the optimum transshipment patterns with the transshipment prices developed in the preceding sections. An example has been given in Section 4.7. Finally, Section 4.8 gives some concluding remarks.

4.2 Problem Statement

There are n newsvendor agents producing a homogeneous product in anticipation of random demands. We index the agents with $i \in N = \{1, ..., n\}$. The parameters r_i,

c_i, and ν_i respectively represent the unit selling prices, production costs, and salvage values for the agents. We study a single period production-transshipment model. We assume that there is no competition over setting the selling prices during the course of our analysis. We represent the vector of random demands by $\mathbf{D} = \{D_i | i \in N\}$. The joint PDF of demand is continuous and twice differentiable. Before the realization of market demands, the agents decide on their production quantities denoted by the vector $\mathbf{X} = \{X_i | i \in N\}$. After the realization of market demands, each agent encounters either surplus products, $H_i = \max\{X_i - D_i, 0\}$, or unsatisfied demand, $E_i = \max\{D_i - X_i, 0\}$. Accordingly, the agents with surplus products transship to the agents with unsatisfied demands. The amount of products transshipped among agents is denoted by $\mathbf{W} = \{W_{ij} | i, j \in N\}$ where W_{ij} is the amount that i transships to j ($i \neq j$ throughout this chapter). When products are transshipped from i to j, a unit transportation cost, t_{ij}, is incurred by agent i.

4.3 Centralized Mode

If the aforementioned system is managed by a single decision maker, the optimal decisions will be obtained by analyzing the two stage stochastic decision making problem. Following the backward induction process, the system's total profit for given values of $\mathbf{X}$, $\mathbf{D}$, and $\mathbf{W}$ is

$$\pi^C(\mathbf{X}, \mathbf{D}, \mathbf{W}) = \sum_{i \in N} \left(r_i \min\{X_i, D_i\} + \nu_i H_i - c_i X_i \right) + \sum_{i \in N} \sum_{j \in N} p_{ij} W_{ij}. \qquad (4.1)$$

where $p_{ij} = r_j - \nu_i - t_{ij}$ is the marginal profit due to a unit of transshipments from i to j. The optimal transshipment pattern is obtained by solving the following linear program

$$\begin{aligned}
\pi^C(\mathbf{X},\mathbf{D}) &= \max_{\mathbf{W}} \pi^C(\mathbf{X},\mathbf{D},\mathbf{W}) \\
s.t. & \\
& \sum_{j\in N} W_{ij} \le H_i, \forall i \in N \\
& \sum_{i\in N} W_{ij} \le E_j, \forall j \in N \\
& W_{ij} \ge 0, \forall i,j \in N. \qquad (4.2)
\end{aligned}$$

The optimal solution of (4.2) is referred to as the optimal transshipment pattern and denoted by $\mathbf{W}^*$.

The optimal production quantities then can be found by first calculating the expected value of $\pi^C(\mathbf{X},\mathbf{D})$ over $\mathbf{D}$, i.e. $J^C(\mathbf{X}) = \mathsf{E}[\pi^C(\mathbf{X},\mathbf{D})]$, for each given $\mathbf{X}$, and second finding the value of $\mathbf{X}$ which maximizes the $J^C(\mathbf{X})$. Note that the latter is a concave function with respect to $\mathbf{X}$ (see Huang and Sošić (2010b)). The vector of system optimal production quantities is denoted by $\mathbf{X}^*$.

4.4 Decentralized Mode

In the decentralized mode, the agents are considered to be self-interested and individually managed. The outcomes of collaborations among the agents in this mode are specified by the collaboration mechanism, i.e. the *contract*. Following the non-cooperative/cooperative framework in Anupindi et al. (2001), we consider contracts with the allocation mechanisms where the agents individually and non-cooperatively decide on their production quantities and after the realization of demand, cooperatively decide the transshipment patterns. The contract specifies a rule for redistributing the maximum attainable profits due to transshipments among the agents in the second stage.

The acceptability of allocations to the agents can be analyzed through the concept of *core*. Assume that after the realization of demand, the agents can form coalitions and carry out the transshipments among them in the best possible way, and then redistribute the resulting profits in any way. Let $Q \subseteq N$ be a sub-coalition of agents. For given values of $\mathbf{X}$ and $\mathbf{D}$, the maximum attainable profit through transshipments for the coalition Q is

$$\begin{aligned} R_Q(\mathbf{X},\mathbf{D}) &= \max_{\mathbf{W}} \sum_{i\in Q}\sum_{j\in Q} p_{ij}W_{ij} \\ s.t. & \\ & \sum_{j\in Q} W_{ij} \le H_i, \forall i \in Q \\ & \sum_{i\in Q} W_{ij} \le E_j, \forall j \in Q \\ & W_{ij} \ge 0, \forall i,j \in Q. \end{aligned} \tag{4.3}$$

We call this the *ex post* cooperative transshipment game. For the grand coalition ($Q = N$), the optimal transshipment pattern obtained from the latter is equivalent to those in (4.2). An allocation rule $\alpha_i(\mathbf{X},\mathbf{D}), \forall i \in N$ is in the core of *cooperative transshipment game*—a game with characteristic function given in (4.3)—if

$$\sum_{i\in Q} \alpha_i(\mathbf{X},\mathbf{D}) \ge R_Q(\mathbf{X},\mathbf{D}), \forall Q \subset N, \tag{4.4}$$

$$\sum_{i\in N} \alpha_i(\mathbf{X},\mathbf{D}) = R_N(\mathbf{X},\mathbf{D}). \tag{4.5}$$

The concept of core is perhaps the most appealing stability concept in the cooperative game theory: given an allocation rule in the core, the formation of grand coalition is guaranteed. Because the transshipments in a coalition are carried out to maximize the coalition's total profit, whenever the grand coalition is formed, the transshipment patterns would be the same as those of $\mathbf{W}^*$. Hence, an allocation rule in the core will result in the coordination of transshipment patterns. Following the results of Owen (1975), the following allocation rule is always in the core of the cooperative transshipment game:

$$\alpha_i^d(\mathbf{X}, \mathbf{D}) = \lambda_i^* H_i + \mu_i^* E_i, \forall i \in N \tag{4.6}$$

where for $i \in N$, λ_i^* and μ_i^* are the optimal dual solution of (4.3) with $Q = N$. This allocation rules is referred to as the *dual allocation rule* Anupindi et al. (2001).

In order to find the individually optimum decisions on production quantities, first note the individual profit functions for given values of $\mathbf{X}$ and $\mathbf{D}$, that is

$$\pi_i^{DC}(\mathbf{X}, \mathbf{D}) = r_i \min\{X_i, D_i\} + \nu_i H_i - c_i X_i + \alpha_i(\mathbf{X}, \mathbf{D}), \tag{4.7}$$

where $\alpha_i(\mathbf{X}, \mathbf{D})$ represents the agent i's allocation of the second stage cooperative game (not necessarily in the core). Let $J_i^{DC}(\mathbf{X}) = \mathsf{E}[\pi_i^{DC}(\mathbf{X}, \mathbf{D})]$ as the expected profit to agent i in the decentralized mode given $\mathbf{X}$. The optimal policy for the agents is the Nash Equilibrium (NE) on $\mathbf{X}$ in the corresponding non-cooperative game.[13] Anupindi et al. (2001) construct an allocation rule which results in the coordination of decisions on production quantities. Theorem 4.1 follows from their Corollary 5.1. It introduces an allocation rule in the core of the second stage cooperative game which also coordinates the production quantities in the decentralized mode.

Theorem 4.1. *Consider the following allocation rule:*

$$\alpha_i^c(\mathbf{X}, \mathbf{D}) = \alpha_i^f(\mathbf{X}, \mathbf{D}) - \alpha_i^f(\mathbf{X}^*, \mathbf{D}) + \alpha_i^d(\mathbf{X}^*, \mathbf{D}) \tag{4.8}$$

[13]With the Nash Equilibrium production quantities, $\mathbf{X}^{NE}$, we have $J_i^{DC}(\mathbf{X}^{NE}) \geq J_i^{DC}(\mathbf{X}_{-i}^{NE} \cap X_i), \forall X_i, \forall i \in N$ where $\mathbf{X}_{-i}^{NE}$ is the vector of production quantities with its i'th element removed.

where

$$\alpha_i^f(\mathbf{X},\mathbf{D}) = \gamma_i \pi^C(\mathbf{X},\mathbf{D}) - (r_i \min\{X_i, D_i\} + \nu_i H_i - c_i X_i), \tag{4.9}$$

and for all i, $\gamma_i \geq 0$ *and* $\sum_{i\in N} \gamma_i = 1$. *Then, this allocation rule is in the core of* ex post *cooperative transshipment game. Also, if* $J_i^{DC}(\mathbf{X})$ *is simultaneously continuous in* $\mathbf{X}$, *the demand densities belong to the class of Polya Frequency Functions of order 2, and* $\pi_i^{DC}(\mathbf{X},\mathbf{D})$ *is unimodal in* X_i *for every* $\mathbf{X}_{-i}$, *then with this allocation rule the Nash equilibrium on production quantities will be unique and the same as the optimal production quantities.*

Therefore, the allocation rule $\alpha_i^c(\mathbf{X},\mathbf{D})$ is coordinating the two stage transshipment problem.

4.5 Transshipment Prices Based on Coordinating allocation Rule

One of the major practical drawbacks of contracts which solely rely on the allocation rules is the need for a governing party to collect and redistribute the profits due to transshipments. A more convenient and practically appealing mechanism is a pricing mechanism. With a pricing mechanism (i) the total profit generated by transshipments between two agents is distributed only between those two, and (ii) the sum of money paid by the transshipment-receiver to the transshipment-sender is a linear function of the amount transshipped. In this section we propose a procedure to derive a pricing mechanism for the transshipment game based on the coordinating allocation rule in Theorem 4.1. The derived pricing mechanism can facilitate the implementation of the contract.

After the realization of demand, the set of newsvendor agents, $\mathbf{N}$, can be divided into the set of transshipment sellers $\mathbf{S} = \{i | H_i > 0\}$, and the set of transshipment buyers $\mathbf{B} = \{j | E_j > 0\}$. The following lemma was shown in Sánchez-Soriano (2006).

Lemma 4.1. *(Proposition 5 in Sánchez-Soriano (2006)) If $\mathbf{z} = \{z_1, ..., z_n\}$ is an allocation in the core of* ex post *cooperative transshipment game, and $\mathbf{W}^*$ is an optimal solution of the transportation problem in (4.2), then the following system will have a solution with $U_{ij} \geq 0$ and $V_{ij} \geq 0$ for all $i \in \mathbf{S}$ and $j \in \mathbf{B}$*

$$\begin{aligned} z_i &= \textstyle\sum_{j \in \mathbf{B}} U_{ij}, && \forall i \in \mathbf{S} \\ z_j &= \textstyle\sum_{i \in \mathbf{S}} V_{ij}, && \forall j \in \mathbf{B} \\ U_{ij} + V_{ij} &= p_{ij} W^*_{ij}, && \forall i \in \mathbf{S}, \forall j \in \mathbf{B} \end{aligned} \tag{4.10}$$

The U_{ij} and V_{ij} are in fact pair-wise allocations of profit that is generated by transshipments between i and j (see Sánchez-Soriano (2006)). The idea is to divide the profit generated by each buyer-seller pair solely between them so that the total profit gained by every agent equals its allocation in the core. We have the following corollary.

Corollary 4.1. *Let $\mathbf{W}^*$ be an optimal primal solution of (4.2). The following system has a solution with $U_{ij} \geq 0$ and $V_{ij} \geq 0$ for all $i \in \mathbf{S}$ and $j \in \mathbf{B}$:*

$$\begin{aligned} \alpha^c_i(\mathbf{X}^*, \mathbf{D}) &= \textstyle\sum_{j \in \mathbf{B}} U_{ij}, && \forall i \in \mathbf{S} \\ \alpha^c_j(\mathbf{X}^*, \mathbf{D}) &= \textstyle\sum_{i \in \mathbf{S}} V_{ij}, && \forall j \in \mathbf{B} \\ U_{ij} + V_{ij} &= p_{ij} W^*_{ij}, && \forall i \in \mathbf{S}, \forall j \in \mathbf{B} \end{aligned} \tag{4.11}$$

The latter is straightforward by noting that according to Theorem 4.1, $\alpha^c_i(\mathbf{X}^*, \mathbf{D})$ is an allocation in the core of *ex post* cooperative transshipment game.

The pair-wise allocations can be used to develop a pricing mechanism. Let s_{ij} be the transshipment price which is paid by j to i for a unit transshipment. With a unit transshipment from i to j, the marginal profit to agent i would be the transshipment price minus the i's salvage value minus the transportation cost from i to j, i.e. $u_{ij} = s_{ij} - \nu_i - t_{ij}$. Thus, u_{ij} is the marginal profit to agent i when transshipping a unit to j. On the other hand, the agent j resells the product acquired through the transshipment to its customers. Thus $v_{ij} = r_j - s_{ij}$ is the marginal profit to the agent j when receiving a unit from i. The transformation

$$U_{ij} = u_{ij} W_{ij}^* = (s_{ij} - \nu_i - t_{ij}) W_{ij}^*$$

and

$$V_{ij} = v_{ij} W_{ij}^* = (r_j - s_{ij}) W_{ij}^*$$

with $\nu_i + t_{ij} \leq s_{ij} \leq r_j$ connects the pair-wise allocations and the transshipment prices. We have the following lemma.

Lemma 4.2. *Let* $U_{ij} = (s_{ij} - \nu_i - t_{ij}) W_{ij}^*$ *and* $V_{ij} = (r_j - s_{ij}) W_{ij}^*$. *A solution to the system (4.11) is as follows: for* $i \in \mathbf{S}$ *and* $j \in \mathbf{B}$ *such that* $W_{ij}^* > 0$, $s_{ij}^* = \lambda_i^* + \nu_i + t_{ij} = r_j - \mu_j^*$.

Proof. First, note that for all $i \in \mathbf{S}$, $\alpha_i^c(\mathbf{X}^*, \mathbf{D}) = \lambda_i^* H_i$ and for all $j \in \mathbf{B}$, $\alpha_j^c(\mathbf{X}^*, \mathbf{D}) = \mu_j^* E_j$. Second, from the complementary slackness we have

$$\lambda_i^* \left(H_i - \sum_j W_{ij}^* \right) = 0,$$

and

$$\mu_j^* \left(E_j - \sum_i W_{ij}^* \right) = 0.$$

Hence,

$$\lambda_i^* H_i = \lambda_i^* \sum_{j \in \mathbf{B}} W_{ij}^*$$

and

$$\mu_j^* E_j = \mu_j^* \sum_{i \in \mathbf{S}} W_{ij}^*.$$

Also, by definitions of U_{ij} and V_{ij}, for $i \in \mathbf{S}$ and $j \in \mathbf{B}$, we have

$$U_{ij} + V_{ij} = p_{ij} W_{ij}^*.$$

Therefore, (4.11) is equivalent to

$$\begin{aligned} \lambda_i^* \sum_{j \in \mathbf{B}} W_{ij}^* &= \sum_{j \in \mathbf{B}} (s_{ij} - \nu_i - t_{ij}) W_{ij}^*, \forall i \in \mathbf{S} \\ \mu_j^* \sum_{i \in \mathbf{S}} W_{ij}^* &= \sum_{i \in \mathbf{S}} (r_j - s_{ij}) W_{ij}^*, \forall j \in \mathbf{B}. \end{aligned} \quad (4.12)$$

This in turn implies

$$\begin{aligned} \sum_{j \in \mathbf{B}} s_{ij} W_{ij}^* &= \sum_{j \in \mathbf{B}} (\lambda_i^* + \nu_i + t_{ij}) W_{ij}^*, \forall i \in \mathbf{S} \\ \sum_{i \in \mathbf{S}} s_{ij} W_{ij}^* &= \sum_{i \in \mathbf{S}} \left(r_j - \mu_j^*\right) W_{ij}^*, \forall j \in \mathbf{B}. \end{aligned} \quad (4.13)$$

Therefore, for $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that $W_{ij}^* > 0$, the solution of the latter system of equations is $s_{ij}^* = \lambda_i^* + \nu_i + t_{ij}$ or, equivalently $s_{ij}^* = r_j - \mu_j^*$. The complementary slackness conditions of the dual guarantee that for $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that $W_{ij}^* > 0$, we have $\lambda_i^* + \mu_j^* = r_j - \nu_i - t_{ij}$ and therefore both ways for defining s_{ij}^* will result in the same values. □

Note that for $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that $W_{ij}^* = 0$ any value of s_{ij} is part of the optimal solution of (4.13). Next, we analyze the transshipment patterns that will result from these transshipment prices.

4.6 Formation of Transshipment Patterns

Cooperative game theory requires that the individual players in the coalition grant their decision making rights to the coalition. An alternative approach to analyze the n-player transshipment game is to consider that the sellers and buyers are free to search the market and match their surplus products and unmet demands based on their individual preferences that stem from the given transshipment prices. Then the question is "what would be the outcome of this matching process in terms of transshipment patterns?"

This problem is an instance of network formation in the two-sided markets where buyers and sellers match their trade quantities. In this supply chain, any transshipment requires the mutual decision of a buyer and a seller with respect to the amount transshipped. The fact that mutual consent is needed to form a single transshipment is generally a hurdle for trying to use any off-the-shelf non-cooperative game theoretic approach Jackson (2005). There are several approaches to model these game situations. In the supply chain where each seller has a unit of product and each buyer needs a unit of product, Jackson (2005) summarizes the approaches taken in the literature. In spite of the multiplicity of approaches, the concept of *pair-wise stability* is perhaps the most tractable.

In the context of transshipment problem where the buyers and sellers can transship any amounts between themselves, Baiou and Balinski (2002) develop the concept of pair-wise stability. In short, this approach proposes that the outcome of matching surplus products and unsatisfied demands between buyers and sellers should necessarily be pair-wise stable with regards to the individual preferences:

> a solution is *stable* if no pair of opposite agents can increase the number of units they exchange, perhaps by giving up trades with less preferred

agents (Baiou and Balinski, 2002).[14]

Although their definition of stability is based on the ordinal preferences of agents, we propose an alternative cardinal approach to reflect the preference orderings via the transshipment prices.

Let us assume that the agents are provided with a set of transshipment prices, $\mathbf{s} = \{s_{ij} | i \in \mathbf{S}, j \in \mathbf{B}\}$. We define the preferences of each agent over the agents on the opposite side of the transshipment market as follows.

- For $i \in \mathbf{S}$, transshipping to j' is preferred over j ($j' >_i j$) if $u_{ij'} > u_{ij} \geq 0$. If $u_{ij'} = u_{ij} \geq 0$, then i is indifferent between transshipping to j or j'. The set $j^{\geq_i} = \{j' \in \mathbf{B} | j' \geq_i j\}$ contains all the buyers that are at least as preferable as j to i.

- For $j \in \mathbf{B}$, receiving transshipments from i' is preferred over i ($i' >_j i$) if $v_{i'j} > v_{ij} \geq 0$. If $v_{i'j} = v_{ij} \geq 0$, then j is indifferent between receiving transshipments from i or i'. The set $i^{\geq_j} = \{i' \in \mathbf{S} | i' \geq_j i\}$ contains the sellers that are at least as preferable as i to j.

Here we present the definition of pair-wise stability[15]:

Definition 1. *A transshipment pattern* $\mathbf{W} = \{W_{ij} | i \in \mathbf{S}, j \in \mathbf{B}\}$ *is pair-wise stable if for every* i *and* j *with* $u_{ij} \geq 0$ *and* $v_{ij} \geq 0$*:*

$$W_{ij} < \min\{H_i, E_j\} \text{ implies } \sum_{j \in j^{\geq_i}} W_{ij} = H_i \text{ or } \sum_{i \in i^{\geq_j}} W_{ij} = E_j \tag{4.14}$$

[14]Although Baiou and Balinski (2002) use the term "stability", we use the term "pair-wise stability" in order to avoid confusion between different types of stability, e.g. core stability, farsighted stability, *etc.*

[15]The original definition of Baiou and Balinski (2002) is based on *ordinal* preferences of the agents. We have slightly modified their definition to address the *cardinal* preferences.

This definition states that with a stable transshipment pattern, if the amount of transshipments between i and j is less than the maximum amount that they can transship between themselves, i.e. $\min\{H_i, E_j\}$, then it must be the case that either i has transshipped its surplus products to the agents which it considers to be at least as preferable as j, or j has received transshipments from the agents which it considers to be at least as preferable as i. If for some i and j the latter does not hold, they can together unilaterally improve their individual marginal profits. Specially, the value of W_{ij} may be increased by $\delta > 0$, and $W_{ij'}$ for some $j' <_i j$ and $W_{i'j}$ for some $i' <_j i$ may both be decreased (if necessary) by δ Baiou and Balinski (2002).

Remark 1. *For pair $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that either $u_{ij} < 0$ or $v_{ij} < 0$, $W_{ij} = 0$ is the only pair-wise stable transshipment pattern. One side can always improve by refraining from participating in the transshipment.*

At this point, one may ask whether there are transshipment prices with which the optimal solution, $\mathbf{W}^*$, is a pair-wise stable transshipment pattern for the decentralized system. The answer to this question is affirmative.

Theorem 4.2. *For $i \in \mathbf{S}$ and $j \in \mathbf{B}$, if $W^*_{ij} > 0$, define $s^*_{ij} = \lambda^*_i + \nu_i + t_{ij} = r_j - \mu^*_j$ and if $W^*_{ij} = 0$, define $s^*_{ij} = 0$. Then, the optimal solution, $\mathbf{W}^*$, is a pair-wise stable transshipment pattern for the corresponding decentralized transshipment system.*

Proof. It is straightforward to check that with these transshipment prices, for $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that $W^*_{ij} > 0$, $u^*_{ij} = \lambda^*_i$, and $v^*_{ij} = \mu^*_j$.

Also, for $i \in \mathbf{S}$ and $j \in \mathbf{B}$ such that $W^*_{ij} = 0$, $u^*_{ij} = -\nu_i - t_{ij}$, and $v^*_{ij} = r_j$.

Next, we analyze the preference orderings that result from s^*_{ij}. For any given seller $i \in \mathbf{S}$, for all $j \in \mathbf{B}$ such that $W^*_{ij} > 0$ we have $u^*_{ij} = \lambda^*_i \geq 0$, and for all $j \in \mathbf{B}$ such that $W^*_{ij} = 0$ we have $u^*_{ij} < 0$. Therefore, i has no preference for the buyer j such that

$W_{ij}^* = 0$ and is indifferent to all the other buyers, i.e.

$$j^{2_i} = \{j' | W_{ij'}^* > 0\}.$$

With respect to buyers, for any given buyer $j \in \mathbf{B}$, for all $i \in \mathbf{S}$ such that $W_{ij}^* > 0$ we have $v_{ij}^* = \mu_j^* \geq 0$. For all i such that $W_{ij}^* = 0$ we have $v_{ij}^* = r_j > \mu_j^*$. Therefore, if $W_{ij}^* > 0$, then $i^{2_j} = \mathbf{S}$ and if $W_{ij}^* = 0$, then

$$i^{2_j} = \{i | W_{ij}^* = 0\}.$$

In order to prove the pair-wise stability of $\mathbf{W}^*$ with proposed transshipment prices, first consider the buyer-seller pairs such that $W_{ij}^* = 0$. In this case, since $u_{ij}^* < 0$, $W_{ij}^* = 0$ is stable (see Remark 4.1).

For the buyer-seller pairs with $W_{ij}^* > 0$, we proceed by contradiction. Suppose $W_{ij}^* > 0$ is not pair-wise stable. Then

$$W_{ij}^* < \min\{H_i, E_j\}$$

and both $\sum_{j \in j^{2_i}} W_{ij}^* < H_i$ and $\sum_{i \in i^{2_j}} W_{ij}^* < E_j$. Thus, i and j can simultaneously improve by transshipping an additional amount of

$$\bar{W}_{ij} = \min\left\{H_i - \sum_{j | W_{ij}^* > 0} W_{ij}^*, E_j - \sum_{i \in \mathbf{S}} W_{ij}^*\right\}$$

between themselves. Note that i and j can tranship $\bar{W}_{ij}$ without altering their transshipment amounts with other agents. This additional transshipment increases the system's total profit by $p_{ij}\bar{W}_{ij}$. The latter contradicts the optimality of W_{ij}^*. Therefore, $\mathbf{W}^*$ must be pair-wise stable. □

	r_i	ν_i	t_{ij}				H_i	E_i
			$j=1$	$j=2$	$j=3$	$j=4$		
$i=1$	6	0	0	1	2	0.75	10	0
$i=2$	5	0	1	0	1.5	1.5	0	7
$i=3$	7	0	0.9	1.1	0	1.2	0	9
$i=4$	8	0	2	0.7	1.8	0	5	0

Table 4.1: An Example of Transshipment Among Four Agents

4.7 An Example

We illustrate our approach to derive the transshipment prices in the second stage through an example. Consider the supply chain with four agents. Since we focus on the second stage, we assume that the decisions on production quantities have been already made and the demands have also been realized. Accordingly, there are two sellers and two buyers in the system. The parameters are given in Table 4.1. The optimal transshipment pattern in the centralized mode is

$$\mathbf{W}^* = \{W_{12}^* = 1, W_{13}^* = 9, W_{42}^* = 5\}.$$

The dual optimal solution is $\lambda_1^* = 4, \lambda_4^* = 4.3$ and $\mu_2^* = 0, \mu_3^* = 1$. The dual allocation for this problem is $A = \{40, 0, 9, 21.5\}$. Following the Theorem 4.2, we set $s_{12} = 5, s_{13} = 6, s_{42} = 5$, and $s_{43} = 0$. With these transshipment prices, the marginal profits due to transshipments are shown in Figure 4.1. The preference ordering for the agents are thus as follows: agent 1 is indifferent between transshipping to 2 or 3, agent 4's only preferred partner is agent 2 (since $u_{43} < 0$), agent 2 is indifferent between receiving transshipments from 1 or 4, and finally, agent 3 prefers 4 over 1. To check the stability of optimal solution with the above mentioned transshipment prices see figure below. The numbers on each link show the unit profit to the corresponding agent. One can

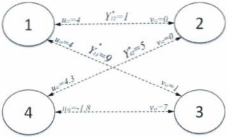


Figure 4.1: Optimal and Pair-Wise Stable Transshipment Patterns

check that with

$$\mathbf{W}^* = \{W_{12}^* = 1, W_{13}^* = 9, W_{42}^* = 5\},$$

no pairs of sellers and buyers can improve their profits by unilateral deviation from the optimal transshipment pattern.

4.8 Comments

One of the main assumptions in this model is that the agents do not incur any cost when deciding to cooperate with each other. However, in reality, there are several types of costs that have to be incurred in order to establish and main relationships between independent agents. In the next chapter, we explicitly include the cooperation costs into the analysis of decentralized transshipment problem.

The following chapter is an edited version of:

B. Hezarkhani and W. Kubiak. Symmetric newsvendor transshipment games with cooperation costs. *To be submitted*

Chapter 5

Symmetric Newsvendor Transshipment Games with Cooperation Costs

Summary: *In a transshipment game, supply chain agents cooperate to transship surplus products after demand realization. The problem has been well studied in the literature, however, general analytical results for it seem out of reach at the moment. In this chapter, we study the cooperative transshipment game with symmetric newsvendors having normally distributed independent demands. We provide characterization of optimal individual quantities, the maximum expected profits, and individual allocations for these games. In particular, we prove that though individual allocations grow with the coalition size they diminish at the same time according to* two laws of diminishing individual allocations. *These results though interesting by themselves are only a point of departure for studying the games with cooperation costs. The cooperation costs depend on*

the cooperation network structure. The chapter considers two, the clique and the hub, and provides the necessary and sufficient conditions for the cost per link necessary to render the core of the game non-empty for either. These maximum admissible costs are always decreasing for cliques, however, increasing or exhibiting a unimodal pattern for hubs.

5.1 Introduction

A transshipment game is concerned with a group of newsvendors who sell a similar product in separate markets and who are willing to reduce their uncertain demand risks by participating in agreements that allow them to share unsold products among themselves. In *responsive transshipment*, which is the focus of this chapter, newsvendors have the option to transship surplus products, if any, *after* the realization of market demands to other newsvendors. The individual newsvendors thus need to decide their optimal production/order quantities, and then to decide how to transship surplus products after the realization of market demands. In a decentralized supply chain, these decisions are functions of a cooperation *mechanism* that newsvendors agree upon. The efficiency of such a mechanism is determined by comparing the quantity decisions that the mechanism leads to with the quantity decisions that are optimal for the centralized system. A mechanism that makes the decentralized system quantity decisions the same as those of the centralized system is called a *coordinating* mechanism. A mechanism is essentially a contract in a supply chain viewed as nexus-of-contracts. As it is discussed in Chapter 2, the growing literature on supply chain contracts seeks to design coordinating contracts (see also Hezarkhani and Kubiak (2010c), Li and Wang (2007), or Gomez-Padilla et al. (2005)).

A common assumption made in previous studies of the transshipment game is that cooperation among newsvendors is costless. However, in reality, when newsvendors cooperate with each other, they incur costs associated with negotiations and governance, e.g. common infrastructure and monitoring. The aim of this chapter is to include cooperation costs into the analysis of cooperative transshipment game.

"[C]ollective decision making processes are often relatively costly" (Williamson, 1975, p. 45). The crucial importance of cooperation costs in economic analysis has been known for a long time. The pioneering paper of Coase (Coase, 1937) on transaction costs and the works of Williamson (e.g. Williamson (1975))—that have given rise to the transaction cost theory—attest to this claim. The costs that are incurred whenever economic agents cooperate with each other will determine the nature of their mutual operations. Adrian and Press (1968) introduce eight cost groups that are inherent in collective decision making: (1) information costs, (2) responsibility costs, (3) inter-game costs, (4) costs of division of payoffs, (5) dissonance costs (6) inertia costs, (7) time costs, and (8) persuasion costs. To the best of our knowledge, the costs of cooperation among agents have been assumed away from all the supply chain contracting models, including transshipment models, in the literature thus far. Nevertheless, a number of studies point to the importance of this issue. In an empirical study, Grover and Malhotra (2003) examine the drivers and effects of transaction costs on supply chains and emphasize underutilization of the transaction cost theory in supply chain literature. Voß and Schneidereit (2002) provide a classification scheme for supply chain contracts and consider their interdependencies with transaction cost economics. In another empirical study, Artz and Brush (2000) examine the factors affecting cooperation costs. They show that asset specificity and environmental uncertainty directly increase cooperation costs, and also that by altering the behavioral orientation of the coalition, the relational norms lower exchange costs.

The transshipment game without cooperation cost has been well studied in the literature (Paterson et al. (2011) provide a review of the literature). Rather than using non-cooperative game theory and drawing upon pricing mechanisms as the primary coordinating mechanism—which is traditionally applied in two-agent supply chains, e.g. Rudi et al. (2001), Hu et al. (2007), Huang and Sošić (2010b), Hezarkhani and Kubiak (2010b) (Chapter 3), and Hanany et al. (2010)—we employ cooperative game theory and its *allocation rule* mechanisms in this chapter. The main advantage in so doing is that cooperative game theory simplifies the analysis of cooperation among the agents by taking a holistic approach. Chapter 4 shows an example of implementations of price mechanisms in multi-agent transshipment game (see also Hezarkhani and Kubiak (2010a)). An allocation rule specifies each agents' share of total profit generated by agents' coalition. Then, if all agents are satisfied with their allocations, the coalition is *stable*. Thus, it is beneficial to all agents to maximize the coalition's total profit. Although there are various interpretations of the stability concept in game theory (see Jackson (2005) for a review of literature), we use the concept of *core* as the measure of stability in transshipment coalitions (Owen, 1995). Nagarajan and Sošić (2008) provide a survey of applications of various game theoretic concepts in OM.

The literature on the transshipment game contains two different game setups. Anupindi et al. (2001) study a two-stage non-cooperative/cooperative setup where they give an allocation rule to distribute the profits realized by transshipments after the demand realization among newsvendors. However, with this rule the newsvendors have incentives to both deviate from the centrally optimal transshipment patterns (Sošić, 2006), and break apart from the coalition after the realization of demands (Suakkaphong and Dror, 2010). Another approach to the transshipment problem allows the characteristic function to be expected payoffs. For a general overview of stochastic cooperative games see Suijs et al. (1999). Slikker et al. (2005) prove the core non-emptiness for the transshipment games with the characteristic function being expected payoffs, and Chen and Zhang (2009) generalize this result to games with concave ordering cost. The translation of expected allocations in the core into realized allocations does not necessarily guarantee stability, however, the distribution of realized allocations can be done in a way that they remain in sync with the expected allocations. For example, Charnes and Granot (1977) introduce a mechanism that minimizes the total objections of agents to the difference between their expected and realized allocations. In order to model the impact of cooperation costs in transshipment game, we draw upon the inter-organizational governance literature which argues that the network of external contracts is the most important facet of an organization's environment (c.f. Smith-Doerr and Powell (2005)), which determines the costs that an organization incurs to cooperate with its environment. The economic actions are embedded in networks of relationships among agents. These networks affect the economic performance through inter-firm resource pooling, cooperation, and coordinated adaptation (Uzzi, 1996). Gulati (1998) suggests considering the implications of network structure. Zaheer and Venkatraman (1995) argue that the cost of coordinating exchange is a function of both the network structure and the process. As the network structure is a determinant of the cooperation costs in coalitions, we consider it as a variable in our model. Rosenkopf and Schilling (2007) study the network structures in different coalitions across various industries. The network structures differ with respect to the level of connectedness of their members and the number of connections among them. Van den Nouweland (2005) studies the strategic formation of cooperative networks with positive costs for establishing links among agents. We base our analysis in this chapter on the assumption that cooperation costs in transshipment games is determined by the structure of a network connecting participating newsvendors. Then it follows that the total cooperation cost among a coalition of agents is a function of total number of links in the network of the coalition. Accordingly, we consider two different typical structures for networks in transshipment games: (1) Clique network structure where a link needs to be established between any pair of agents in the coalition, and (2) Hub network structure where the connections among agents are established through a central coordinator agent, i.e., each agent is linked to the central coordinating agent.

We demonstrate that transshipment games with symmetric newsvendor agents facing independent and normally distributed demands fall into three categories: over-mean, under-mean, and mean games. The category depends on the critical fractile of a single newsvendor. We show that individual quantity in over-mean games of any size is over-mean, optimal individual quantity in under-mean games of any size is under-mean, and individual optimal quantity in mean games of any size is mean. As the game size grows these individual optimal quantities get closer to the demand distribution mean for the over- and under-mean games. However, for either category we show a threshold value t^* of the transportation cost t such that the individual optimal quantity actually converges to the distribution mean if the transportation cost does not exceed the threshold, and to a value determined by a t-dependent critical fractile otherwise. Irrespective of the category, the individual allocations grow as more newsvendors join in the grand coalition, that is as the size of the game grows. However, we prove two laws of diminishing individual allocations that accompany this growth. We claim that the absolute individual gain resulting from the grand coalition being joined in by one more newsvendor strictly decreases. This law is key for the analysis of games with clique networks, and it does not depend on transportation cost, t. The other claim is that the absolute gains make up a convex sequence (Hazewinkel, 2002) up to a certain threshold grand coalition size n^* and a concave sequence from that threshold on. The threshold depends on the transportation cost t so that higher transportation costs result in a smaller threshold. This law is key for the games with hub networks. The threshold may not exist in which case the sequence remains convex for any grand coalition size. We show that this is the case for small transportation cost, that is t less than t^*.

Unlike the transshipment game without cooperation costs, transshipment games with cooperation costs may have empty cores. This depends both on the network structure and the cooperation cost per link, K, in the network. We develop a sufficient and necessary condition for non-emptiness of the core of games with cooperation costs, and give a sufficient and necessary condition for the cost per link to guarantee a non-empty core in these games. These conditions can be translated into the maximum admissible cost per link that guarantees a non-empty core. This cost depends on the network structure. It decreases for the clique so that for any given cost per link K one can determine the largest game with non-empty core, all larger games would not be stable as their cores would be empty. The cost is either increasing or unimodal for the hub. In the latter case it actually increases up to the critical grand coalition of size n^{**} and then decreases from that size on. Consequently, with the hub network, newsvendors may look for a critical mass in terms of their number first in order to be able to guarantee non-empty core for their game for a given cost per link. This may, however, only happen prior to n^{**}, which always happens if $n^{**} = \infty$. Moreover, we show that $n^{**} \geq n^*$. Thus, if a finite n^* does not exist, then neither does a finite n^{**}. Finally, we show that for costless transportation n^{**} does not exist, that is n^{**} happens at infinity. Thus, the maximum admissible cost increases asymptotically to a certain finite value which it never attains. We show a similar result for the mean games. In both these cases, the grant coalition size must be large enough to be able to afford a given cooperation cost per link below the limit. However, if the cost per link is at the limit or above it any game's core is empty. We illustrate these results with some computational experiments.

The rest of this chapter is organized as follows. Section 5.2 briefly introduces the general transshipment game, and Section 5.2.1 tailors it to symmetric newsvendors. Section 5.3 demonstrates the general properties of optimal quantities in symmetric newsvendor transshipment games with independent and normally distributed demands. Section 5.4 studies the general properties of maximum expected profits in symmetric newsvendor transshipment games with independent and normally distributed demands. It determines the characteristic functions of these games as well as individual allocations in the cores of the games. It then proceeds to show that the individual allocation, though growing with the size of coalitions, are subject to two laws that diminish the growth. These two laws are key to the transshipment games with cooperation costs studied in Section 5.5. The section determines the characteristic functions of symmetric newsvendor transshipment games with cooperation costs for the clique and the hub and gives a necessary and sufficient condition for non-empty core in these games. This condition is then studied in Section 5.5.1 with the aim to determine the maximum admissible cost per link that renders a non-empty core for positive transportation costs. Section 5.5.2 studies the same problem under the assumption of costless transportation, and Section 5.5.3 does it for mean newsvendors. Finally, Section 5.6 provides some directions for further research.

5.2 The Transshipment Game

Consider a set N of n newsvendors agents. The agents need to decide their production/order quantities (simply quantities hereafter), X_i, in anticipation of a continuous and twice differentiable random demand D_i with mean μ_i and standard deviation σ_i, $i \in N$. For each newsvendor, the market selling price, purchasing cost, and salvage value are r_i, c_i, and ν_i respectively ($\nu_i < c_i < r_i$). The newsvendors have the option to form a transshipment coalition to transship their otherwise surplus products to other members of the coalition after the realization of demands. In order to transship one unit of product from newsvendor i to newsvendor j, both members of the same coalition, the transportation cost t_{ij} is incurred by either i or j. The W_{ij} is the quantity transshipped from newsvendor i to j. In order to avoid trivial scenarios, we assume that for all $i, j \in N$, $c_i < c_j + t_{ji}$, $\nu_i < \nu_j + t_{ji}$, $r_i < r_j + t_{ji}$, and $t_{ij} < r_j - \nu_i$. We denote by $\mathbf{X}$, $\mathbf{D}$, and $\mathbf{W}$ vectors of production quantities, random demands, and quantities transshipped, respectively, for newsvendors in N.

The transshipment game without cooperation costs (Slikker et al., 2005) is a cooperative game $(\dot{J}, N)$, with the characteristic function $\dot{J} : 2^N \to \mathbb{R}$, which assigns to any sub-coalition $Q \subseteq N$ the value $\dot{J}_Q$ of that sub-coalition equal to

$$\dot{J}_Q = \max_{\mathbf{X}} J_Q(\mathbf{X}) = \max_{\mathbf{X}} \mathsf{E}\left[\sum_{i \in Q} (r_i \min(X_i, D_i) + \nu_i H_i - c_i X_i) + R_Q(\mathbf{X}, \mathbf{D})\right] \quad (5.1)$$

where for given $\mathbf{X}$ and $\mathbf{D}$,

$$\begin{aligned} R_Q(\mathbf{X}, \mathbf{D}) \quad = \quad & \max_{\mathbf{W}} \sum_{i \in Q} \sum_{j \in Q} p_{ij} W_{ij} \qquad (5.2) \\ s.t. \quad & \\ & \sum_{j \in Q} W_{ij} \le H_i, \forall i \in Q \\ & \sum_{i \in Q} W_{ij} \le E_j, \forall j \in Q \\ & W_{ij} \ge 0, \forall i, j \in Q, \end{aligned}$$

and $H_i = \max(X_i - D_i, 0)$ is newsvendor i surplus, $E_i = \max(D_i - X_i, 0)$ is newsvendor i unsatisfied demand, finally $p_{ij} = r_j - \nu_i - t_{ij}$ is the marginal transshipment profit resulting from transshipping one unit from newsvendor i to newsvendor j.

Let β_i, $i \in N$, be the individual allocation that newsvendor i receives in a grand coalition, that is the coalition containing all newsvendors in N. The allocations β_i, $i \in N$, are said to be in the core of the transshipment game if and only if $\sum_{i \in Q} \beta_i \geq \dot{J}_Q$ for all $Q \subset N$, and $\sum_{i \in N} \beta_i = \dot{J}_N$. That is, a coalitional game has a non-empty core if allocations can be found such that for any subset of agents, the sum of their allocations is at least as much as the value of the sub-coalition made of that subset of agents. The following key theorem by Slikker et al. (2005) ensures a non-empty core for any transshipment game.

Theorem 5.1. *(Slikker et al., 2005) The transshipment game with the characteristic function defined in (5.1) has a non-empty core.*

This theorem implies that it is always to the benefit of individual newsvendors, more precisely never to their disadvantage, to form infinitely large coalitions as long as there is no cooperation costs involved in forming the coalitions.

5.2.1 Transshipment Games with Symmetric Newsvendors

The transshipment game with symmetric newsvendors, being a special case of the transshipment games, has always non-empty core by Theorem 5.1. By the newsvendor symmetry any individual allocations β_i, $i \in N$, in the core of the cooperative game played by n newsvendors must equal $1/n$-th share of the grand coalition maximum expected profit $\dot{J}_N = \dot{J}_n$. Therefore, we need to study this profit to determine the core of the game. This is done in Sections 5.3 and 5.4. However, we need to derive a formula for $J_N(\mathbf{X}) = J_n(\mathbf{X})$ for symmetric newsvendors from (5.1) first. This is done in this section.

The symmetry of newsvendors ensures that any unit transshipment between any two newsvendors results in the same profit $p = r - \nu - t > 0$ for the coalition, which allows us to suppress the newsvendor indices in p_{ij}. Therefore, the $R_n(\mathbf{X}, \mathbf{D})$ in (5.2) is maximized by transshipments that result in either no surplus or no unsatisfied demand in the coalition. Thus maximum extra profit obtained through transshipments equals $R_n(\mathbf{X}, \mathbf{D}) = p\min\left(\sum_{i\in N} H_i, \sum_{i\in N} E_i\right)$. Therefore, the expected profit of the grand coalition, $J_n(\mathbf{X})$ can be simplified as following.

$$J_n(\mathbf{X}) = \mathsf{E}\left[\sum_{i=1}^{n}\left(r\min(X_i, D_i) + \nu H_i - cX_i\right) + p\min\left(\sum_{i=1}^{n} H_i, \sum_{i=1}^{n} E_i\right)\right] \tag{5.3}$$

The following lemma allows for further simplification of $J_n(\mathbf{X})$.

Lemma 5.1.

$$\min\left(\sum_{i=1}^{n} H_i, \sum_{i=1}^{n} E_i\right) = \min\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} D_i\right) - \sum_{i=1}^{n}\min\left(X_i, D_i\right) \tag{5.4}$$

Proof. First note that $\min(A, B) + C = \min(A + C, B + C)$. Then,

$$\begin{aligned}
&\min\left(\sum_{i=1}^{n} H_i, \sum_{i=1}^{n} E_i\right) + \sum_{i=1}^{n}\min\left(X_i, D_i\right) = \\
&\min\left(\sum_{i=1}^{n} H_i + \sum_{i=1}^{n}\min\left(X_i, D_i\right), \sum_{i=1}^{n} E_i + \sum_{i=1}^{n}\min\left(X_i, D_i\right)\right) = \\
&\min\left(\sum_{i=1}^{n}\left[\max\left(X_i - D_i, 0\right) + \min\left(X_i, D_i\right)\right], \sum_{i=1}^{n}\left[\max\left(D_i - X_i, 0\right) + \min\left(X_i, D_i\right)\right]\right) = \\
&\min\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} D_i\right).
\end{aligned} \tag{5.5}$$

The last step can be verified as follows. Suppose $X_i \geq D_i$. Then $\max(X_i - D_i, 0) = X_i - D_i$ and $\min(X_i, D_i) = D_i$ and they sum up to X_i. Similar argument holds for the case where $X_i < D_i$. □

The expected profit of the grand coalition, $J_n(\mathbf{X})$, can be simplified to

$$J_n(\mathbf{X}) = \sum_{i=1}^{n}(-cX_i + (r-p)\mathsf{E}[\min(X_i, D_i)] + \nu\mathsf{E}[H_i]) + p\mathsf{E}\left[\min\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} D_i\right)\right]. \quad (5.6)$$

Due to the anonymity and symmetry of newsvendors, the production quantities X_i making up the vector $\mathbf{X}$ can be replaced by the single variable quantity X, similarly the random demands D_i making up the vector $\mathbf{D}$ can be replaced by the single random variable D, and thus the expected profit can be further simplified to

$$J_n(X) = -ncX + n(r-p)\mathsf{E}[\min(X, D)] + n\nu\mathsf{E}[\max(X-D, 0)] + p\mathsf{E}[\min(nX, nD)]. \quad (5.7)$$

Furthermore, we have (see Appendix for detailed derivations)

$$\mathsf{E}[\min(X, D)] = X - \int_0^X F_D(\xi)d\xi = \mu - I_D(X), \quad (5.8)$$

$$\mathsf{E}[\max(X-D, 0)] = \int_0^X F_D(\xi)d\xi = I_D(X) + X - \mu, \quad (5.9)$$

$$\mathsf{E}[\min(nX, Z)] = nX - \int_0^{nX} F_Z(\xi)d\xi = n\mu - I_Z(nX) \quad (5.10)$$

where F_D (f_D) and F_Z (f_Z) are CDFs (PDFs) of the random demand variables D and $Z = nD$ respectively, and

$$I_D(X) = \int_X^{\infty} (\xi - X) f_D(\xi) d\xi$$

and

$$I_Z(X) = \int_{nX}^{\infty} (\xi - nX) f_Z(\xi) d\xi$$

are the well known loss functions (see (Porteus, 2002)). The equation (5.7) can then be simplified to

$$J_n(X) = n(r-c)X - nt\int_0^X F_D(\xi)d\xi - p\int_0^{nX} F_Z(\xi)d\xi, \tag{5.11}$$

which is used in deriving the key condition for optimal production quantities in Section 5.3, or equivalently to,

$$J_n(X) = n(\nu - c)X + n(r-\nu)\mu - ntI_D(X) - pI_Z(nX), \tag{5.12}$$

which is used in deriving a formula for the maximum expected profit $\hat{J}_n$ in Section 5.4.

5.3 Optimal Quantities with Independent and Normally Distributed Demands

From now on we assume that newsvendor demands are independent and normally distributed. The main motivation behind this assumption comes from the fact that normal distribution is a strictly *stable* distribution (Fristedt and Gray, 1997), that is the total demand $Z = \sum_{i=1}^{n} D_i = nD$ is *normally* distributed with $\mu_Z = n\mu$ and $\sigma_Z^2 = n\sigma^2$, with a closed formula for density. Moreover, Alfaro and Corbett (2003) show that normal distribution is a good approximation of general distribution functions in transshipment problem. Dong and Rudi (2004) also restrict their analysis to normal distributions when analyzing the effect of transshipment among two agents and the upstream supplier. Our main goal in this section is to characterize the optimal production levels for transshipment games with n symmetric newsvendors, or just games of size n for simplicity.

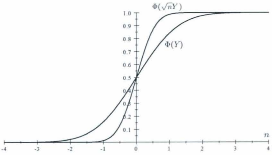


Figure 5.1: Functions $\Phi(Y)$ and $\Phi(\sqrt{n}Y)$

It can be observed from (5.11) that, since the second derivative of $J_n(X)$ with respect to X is always negative, the optimal quantity can be found from the first order condition

$$dJ_n(X)/dX = n(r-c) - ntF_D(X) - npF_Z(nX) = 0. \tag{5.13}$$

Let $\dot{X}_n$ be a solution to (5.13). Also, let ϕ and Φ be the PDF and CDF of the standard normal distribution respectively. Using the transformation

$$\dot{Y}_n = (\dot{X}_n - \mu)/\sigma$$

for $n \geq 1$, and (5.13), the equation

$$r - c = t\Phi(\dot{Y}_n) + p\Phi(\sqrt{n}\dot{Y}_n), \tag{5.14}$$

characterizes the optimal quantity for a transshipment game of size n (see Appendix for the detailed derivations). Figure 5.1 depicts the relative behavior of functions $\Phi(Y)$ and $\Phi(\sqrt{n}Y)$.

A game of size one is equivalent to a single newsvendor for which the optimal quantity is obviously $\dot{Y}_1 = \Phi^{-1}\left(\frac{r-c}{r-\nu}\right)$. If the fraction $\frac{r-c}{r-\nu}$ is less than 0.5, i.e. $r - c < c - \nu$, then the optimal quantity for a single newsvendor is less than the demand mean μ, hence we refer to this type of newsvendor as an *under-mean newsvendor*. If $r - c > c - \nu$, then the optimal quantity for a single newsvendor is larger than demand mean μ, hence we call this type of newsvendor an *over-mean newsvendor*. The case with $r - c = c - \nu$ implies $\dot{Y}_1 = 0$. Then, the optimal quantity for a single newsvendor equals the demand mean μ, hence we call this type of newsvendor a *mean newsvendor*. We extend these three categories of newsvendors to the transshipment games by saying that the transshipment game of size n is under-mean, over-mean, and mean if $\dot{Y}_n < 0$, $\dot{Y}_n > 0$, and $\dot{Y}_n = 0$ respectively. Observe that by (5.14), we have $\dot{Y}_n = \frac{1}{\sqrt{n}}\dot{Y}_1$ for $t = 0$. Then, the grand coalition of n newsvendors boils down to a single newsvendor with demand of $Z = nD$. Therefore, from this point on we exclude $t = 0$ from our analysis in this section. The following lemma shows that the game category for any n is determined by the category of a single newsvendor game, and remains unchanged for all size games.

Lemma 5.2. *For $n \geq 1$,*

- *If $\dot{Y}_1 > 0$, then $\dot{Y}_n > 0$.*
- *If $\dot{Y}_1 < 0$, then $\dot{Y}_n < 0$.*
- *If $\dot{Y}_1 = 0$, then $\dot{Y}_n = 0$.*

Proof. The proof is by contradiction. Consider the first proposition. Suppose that $\dot{Y}_1 > 0$ and $\dot{Y}_{n'} \leq 0$ for some $n' \geq 2$. Then, either $0 < \Phi(\sqrt{n'}\dot{Y}_{n'}) < \Phi(\dot{Y}_{n'}) < \frac{1}{2}$ or $\Phi(\sqrt{n'}\dot{Y}_{n'}) = \Phi(\dot{Y}_{n'}) = \frac{1}{2}$. In the former case, let $\Phi(\sqrt{n'}\dot{Y}_{n'}) = \rho\Phi(\dot{Y}_{n'})$ where $0 < \rho < 1$. The equation (5.14) then simplifies to $\Phi(\dot{Y}_{n'}) = \frac{r-c}{t+\rho(r-\nu-t)}$, and thus, $\frac{r-c}{t+\rho(r-\nu-t)} < \frac{1}{2}$. On the hand, since $r-\nu-t>0$, then $\frac{r-c}{r-\nu}<\frac{r-c}{t+\rho(r-\nu-t)}$. However, for $\dot{Y}_1>0$ we have $\frac{r-c}{r-\nu}>\frac{1}{2}$, and thus $\frac{1}{2}<\frac{r-c}{t+\rho(r-\nu-t)}$ which leads to a contradiction. In the latter case, equation (5.14) simplifies to $\frac{r-c}{r-\nu}=\frac{1}{2}$ which also leads to a contradiction since $\dot{Y}_1>0$. Therefore, if $\dot{Y}_1>0$ then $\dot{Y}_n>0$ for all $n\geq 2$.

Now, consider the second proposition. Suppose that $\dot{Y}_1<0$ and $\dot{Y}_{n'}\geq 0$ for some $n'\geq 2$. Then, either $\frac{1}{2}<\Phi(\dot{Y}_{n'})<\Phi(\sqrt{n'}\dot{Y}_{n'})<2\Phi(\dot{Y}_{n'})$ or $\Phi(\sqrt{n'}\dot{Y}_{n'})=\Phi(\dot{Y}_{n'})=\frac{1}{2}$. In the former case, let $\Phi(\sqrt{n'}\dot{Y}_{n'})=\kappa\Phi(\dot{Y}_{n'})$ where $1<\kappa<2$. The equation (5.14) then simplifies to $\Phi(\dot{Y}_{n'})=\frac{r-c}{t+\kappa(r-\nu-t)}$, and thus, $\frac{r-c}{t+\kappa(r-\nu-t)}>\frac{1}{2}$. On the hand, since $r-\nu-t>0$, then $\frac{r-c}{t+\kappa(r-\nu-t)}<\frac{r-c}{r-\nu}$. However, since $\dot{Y}_1<0$, then $\frac{r-c}{r-\nu}<\frac{1}{2}$, which leads to a contradiction. In the latter case, the equation (5.14) simplifies to $\frac{r-c}{r-\nu}=\frac{1}{2}$ which also leads to a contradiction since $\dot{Y}_1<0$. Therefore, if $\dot{Y}_1<0$ then $\dot{Y}_n<0$ for all $n\geq 2$.

Finally, consider the last proposition. Suppose that $\dot{Y}_1=0$ and $\dot{Y}_{n'}\neq 0$ for some $n'\geq 2$. Then, either $\Phi(\sqrt{n'}\dot{Y}_{n'})<\Phi(\dot{Y}_{n'})<\frac{1}{2}$ or $\frac{1}{2}<\Phi(\dot{Y}_{n'})<\Phi(\sqrt{n'}\dot{Y}_{n'})<2\Phi(\dot{Y}_{n'})$. Since $r-\nu-t>0$, then we have $\frac{r-c}{r-\nu}<\frac{r-c}{t+\rho(r-\nu-t)}<\frac{1}{2}$, in the former case, and $\frac{1}{2}<\frac{r-c}{t+\kappa(r-\nu-t)}<\frac{r-c}{r-\nu}$ in the latter case. On the other hand, since $\dot{Y}_1=0$, then $\frac{r-c}{r-\nu}=\frac{1}{2}$ which leads to a contradiction in both cases. Therefore, if $\dot{Y}_1=0$ then $\dot{Y}_n=0$ for all $n\geq 2$. □

We now show that the over-mean games reduce their optimal quantities as their size grows. These optimal quantities get closer to the demand mean μ. Similarly, the under-mean games increase their optimal quantities as their size grows again getting closer to the demand mean μ. Finally, the mean games keep their optimal production levels equal μ for all game sizes which follows from Lemma 5.2. We have the following theorem.

Theorem 5.2. *We have the following:*

- *For over-mean games,* $\dot{Y}_1>\dot{Y}_2>\ldots>\dot{Y}_n>\ldots>0$.
- *For under-mean games,* $\dot{Y}_1<\dot{Y}_2<\ldots<\dot{Y}_n<\ldots<0$.

Proof. The proof is by contradiction. The system of equations obtained from the equation (5.14) for any pair n and $n-1$, $n \geq 2$ implies that

$$t\Phi(\dot{Y}_{n-1}) + p\Phi(\sqrt{n-1}\dot{Y}_{n-1}) = t\Phi(\dot{Y}_n) + p\Phi(\sqrt{n}\dot{Y}_n).$$

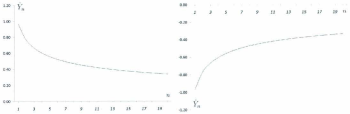
First, consider over-mean games. Suppose that $\dot{Y}_{n'-1} \leq \dot{Y}_{n'}$ for some $n' \geq 2$. Since Φ is strictly increasing, we have $\Phi(\dot{Y}_{n'-1}) \leq \Phi(\dot{Y}_{n'})$. By Lemma 1, $\dot{Y}_n > 0$ for all $n \geq 1$, thus we also get $\sqrt{n'-1}\dot{Y}_{n'-1} < \sqrt{n}\dot{Y}_{n'}$, which implies $\Phi(\sqrt{n'-1}\dot{Y}_{n'-1}) < \Phi(\sqrt{n'}\dot{Y}_{n'})$. Hence, $t\Phi(\dot{Y}_{n'-1}) + p\Phi(\sqrt{n'-1}\dot{Y}_{n'-1}) < t\Phi(\dot{Y}_{n'}) + p\Phi(\sqrt{n'}\dot{Y}_{n'})$ which leads to a contradiction. Therefore, $\dot{Y}_{n-1} > \dot{Y}_n$ for all $n \geq 2$.

Second, consider under-mean games. Suppose that $\dot{Y}_{n'-1} \geq \dot{Y}_{n'}$ for some $n' \geq 2$. We have $\Phi(\dot{Y}_{n'-1}) \geq \Phi(\dot{Y}_{n'})$. By Lemma 1, $\dot{Y}_n < 0$ for all $n \geq 1$, thus we also get $\sqrt{n'-1}\dot{Y}_{n'-1} > \sqrt{n'}\dot{Y}_{n'}$ which implies $\Phi(\sqrt{n'-1}\dot{Y}_{n'-1}) > \Phi(\sqrt{n'}\dot{Y}_{n'})$. Hence, $t\Phi(\dot{Y}_{n'-1}) + p\Phi(\sqrt{n'-1}\dot{Y}_{n'-1}) > t\Phi(\dot{Y}_{n'}) + p\Phi(\sqrt{n'}\dot{Y}_{n'})$ which leads to a contradiction. Therefore, $\dot{Y}_{n-1} < \dot{Y}_n$ for all $n \geq 2$. □

Figure 5.2 shows the values of $\dot{Y}_n$ for two instances of transshipment games. Obviously, the optimal quantities are decreasing for the over-mean game (Figure 5.2 (a)) and increasing for the under-mean game (Figure 5.2 (b)).

Although the risk pooling mechanism naturally embedded in a coalition—revealed in Lemma 5.2 and Theorem 5.2—makes the mean μ a natural target for the optimal production quantity in a coalition, this optimal quantity does *not* necessarily converge to the mean μ as the coalition size grows. This is shown in Theorem 5.4 presented later in this section. Before proving this theorem one needs to investigate the sequence $\sqrt{n}\dot{Y}_n$ first.

Theorem 5.3. *For games of size n and $n-l$, $n \geq 2$ and $1 \leq l < n$, other things being equal, we have* $\frac{\dot{Y}_{n-l}}{\dot{Y}_n} < \sqrt{\frac{n}{n-l}}$.



(a) $r = 40, c = 15, v = 10, t = 10$ (b) $r = 40, c = 35, v = 10, t = 10$

Figure 5.2: Values of $\dot{Y}_n$ for Two Instances of Transshipment Games

Proof. The system of equations obtained by considering the equation (5.14) for any pair n and $n-l$, $n \geq 2$ and $1 \leq l < n$, leads to

$$\frac{\Phi(\dot{Y}_n) - \Phi(\dot{Y}_{n-l})}{\Phi(\sqrt{n-l}\dot{Y}_{n-l}) - \Phi(\sqrt{n}\dot{Y}_n)} = \frac{p}{t} > 0$$

By Theorem 5.2, if $\dot{Y}_1 > 0$, then we have $\Phi(\dot{Y}_n) - \Phi(\dot{Y}_{n-l}) < 0$ for $1 \leq l < n$. Therefore, the denominator must be negative as well, thus $\Phi(\sqrt{n-l}\dot{Y}_{n-l}) - \Phi(\sqrt{n}\dot{Y}_n) < 0$. Since Φ is strictly increasing, we get $\frac{\dot{Y}_{n-l}}{\dot{Y}_n} < \sqrt{\frac{n}{n-l}}$. If $\dot{Y}_1 < 0$, then again by Theorem 2 we have $\Phi(\dot{Y}_n) - \Phi(\dot{Y}_{n-l}) > 0$ for $1 \leq l < n$. Hence, the denominator must be positive as well, thus $\Phi(\sqrt{n-l}\dot{Y}_{n-l}) - \Phi(\sqrt{n}\dot{Y}_n) > 0$ which result in $\frac{\dot{Y}_{n-l}}{\dot{Y}_n} < \sqrt{\frac{n}{n-l}}$. □

This leads to the following corollary.

Corollary 5.1. *We have the following:*

- *For over-mean games,* $0 < \dot{Y}_1 < \sqrt{2}\dot{Y}_2 < ... < \sqrt{n}\dot{Y}_n < ....$
- *For under-mean games,* $0 > \dot{Y}_1 > \sqrt{2}\dot{Y}_2 > ... > \sqrt{n}\dot{Y}_n > ....$

Theorem 5.2 and Corollary 5.1 show a "complementary" behavior of the sequences $\dot{Y}_n$ and $\sqrt{n}\dot{Y}_n$; whenever one of them is *descending* the other must be *ascending*. This must be so in order to satisfy the equation (5.14). We now focus on the question: where do these two sequences tend to as the game size grows? We begin with the following technical lemma.

Lemma 5.3. *If $lim_{n\to\infty}|\dot{Y}_n| = a > 0$, then $lim_{n\to\infty}|\sqrt{n}\dot{Y}_n| = \infty$.*

Proof. If a sequence a_n diverges to ∞ and a sequence b_n is bounded below by K, then $a_n b_n$ diverges to ∞, provided $K > 0$ (Kosmala, 1998). Since $lim_{n\to\infty}\sqrt{n} = \infty$, if $lim_{n\to\infty}|\dot{Y}_n| = a > 0$ then it must be the case that $lim_{n\to\infty}|\sqrt{n}\dot{Y}_n| = \infty$. □

We are now ready to prove the main result.

Theorem 5.4. *Let $\lim_{n\to\infty}\dot{Y}_n = a$ and $\lim_{n\to\infty}\sqrt{n}\dot{Y}_n = b$. Then for over-mean games, we have $t < r - \nu < 2(r - c)$, $2(c - \nu) < r - \nu$ and*

$$\begin{cases} a = 0, b = \Phi^{-1}\left(\frac{r-c-t/2}{r-\nu-t}\right) < \infty & \text{if } t < 2(c-\nu) \\ a = \Phi^{-1}\left(\frac{-c+\nu+t}{t}\right) \geq 0, b = \infty & \text{if } t \geq 2(c-\nu) \end{cases}$$

and for under-mean games, we have $t < r - \nu < 2(c - \nu)$, $2(r - c) < r - \nu$ and

$$\begin{cases} a = 0, b = \Phi^{-1}\left(\frac{r-c-t/2}{r-\nu-t}\right) > -\infty & \text{if } t < 2(r-c) \\ a = \Phi^{-1}\left(\frac{r-c}{t}\right) \leq 0, b = -\infty & \text{if } t \geq 2(r-c) \end{cases}$$

Proof. Proof by contrapositive. A contrapositive proof of $A \to B$ is $\neg B \to \neg A$. That is $A \to B \Leftrightarrow \neg B \to \neg A$. For the purpose of our proof assume that $B_1 \vee B_2$ (only one can be true). Then $\neg B_1 \to \neg A \Leftrightarrow B_2 \to \neg A$.

First, consider over-mean games. Then, we have $t < r - \nu < 2(r - c)$ and $2(c - \nu) < r - \nu$. By Lemma 5.3, there are only two possible scenarios for a and b as n tends to infinity: $\{a = 0$ and $b < \infty\}$, or $\{a \geq 0$ and $b = \infty\}$.

To prove that if $t < 2(c - \nu)$, then $a = 0$ and $b < \infty$, it must be shown that if $a \geq 0$ and $b = \infty$, then $t \geq 2(c - \nu)$. If $a \geq 0$ and $b = \infty$ then the equation (5.14) becomes

$r-c=t\Phi(a)+(r-\nu-t)$, or $\Phi(a)=\frac{-c+\nu+t}{t}$. By Lemma 5.2, $\dot{Y}_1 > \dot{Y}_2 > ... > \dot{Y}_n ... \geq a \geq 0$, and moreover $\Phi(\dot{Y}_1)=\frac{r-c}{r-\nu}$ and $\Phi(0)=1/2$, thus a must satisfy $\frac{1}{2} \leq \Phi(a) < \frac{r-c}{r-\nu}$. The right hand side always holds since $t < r-\nu$. In order for the left hand side to hold, we must have $t \geq 2(c-\nu)$. This proves if $t < 2(c-\nu)$ then $a=0$ and $b<\infty$. In this case the equation (5.14) becomes $r-c=t/2+(r-\nu-t)\Phi(b)$, or $b=\Phi^{-1}\left(\frac{r-c-t/2}{r-\nu-t}\right)$.

To prove that if $t \geq 2(c-\nu)$, then $a \geq 0$ and $b=\infty$, we must show that if $a=0$ and $b<\infty$, then $t<2(c-\nu)$. If $a=0$ and $b<\infty$, then the equation (5.14) becomes $r-c=t/2+(r-\nu-t)\ \Phi(b)$, or $\Phi(b)=\frac{r-c-t/2}{r-\nu-t}$. By Corollary 5.1, $\dot{Y}_1 < \sqrt{2}\dot{Y}_2 < ... < \sqrt{n}\dot{Y}_n ... \leq b < \infty$, and $\Phi(\infty)=1$, thus b must satisfy $\frac{r-c}{r-\nu} < \Phi(b) < 1$. The left hand side holds since for over-mean games $t < r-\nu$. In order for the right hand side to hold, we must have $t < 2(c-\nu)$. This proves that if $t \geq 2(c-\nu)$ then $a \geq 0$ and $b=\infty$. In this case the equation (5.14) becomes $r-c=t\Phi(a)+(r-\nu-t)$, or $a=\Phi^{-1}\left(\frac{-c+\nu+t}{t}\right)$.

Now consider coalitions of under-mean games. Then, we have $t < r-\nu < 2(c-\nu)$ and $2(r-c) < r-\nu$. By Lemma 5.3, there are only two possible scenarios for a and b as n tends to infinity: $\{a=0 \text{ and } b>-\infty\}$, or $\{a \leq 0 \text{ and } b=-\infty\}$.

To prove that if $t<2(r-c)$, then $a=0$ and $b>-\infty$, it must be shown that if $a \leq 0$ and $b=-\infty$, then $t \geq 2(r-c)$. If $a \leq 0$ and $b=-\infty$, then the equation (5.14) becomes $r-c=t\Phi(a)$, or $\Phi(a)=\frac{r-c}{t}$. By Lemma 5.2, $\dot{Y}_1 < \dot{Y}_2 < ... < \dot{Y}_n ... \leq a \leq 0$, $\Phi(\dot{Y}_1)=\frac{r-c}{r-\nu}$, and moreover $\Phi(0)=1/2$, thus a must satisfy $\frac{r-c}{r-\nu} < \Phi(a) \leq \frac{1}{2}$. The left hand side holds since $t < r-\nu$. In order for the right hand side to hold, we must have $t \geq 2(r-c)$. This proves that if $t<2(r-c)$ then $a=0$ and $b>-\infty$. In this case the equation (5.14) becomes $r-c=t/2+(r-\nu-t)\Phi(b)$, or $b=\Phi^{-1}\left(\frac{r-c-t/2}{r-\nu-t}\right)$.

To prove that if $t \geq 2(r-c)$, then $a \leq 0$ and $b=-\infty$, it must be shown that if $a=0$ and $b>-\infty$, then $t<2(r-c)$. If $a=0$ and $b>-\infty$ then the equation (5.14) becomes $r-c=t/2+(r-\nu-t)\ \Phi(b)$, or $\Phi(b)=\frac{r-c-t/2}{r-\nu-t}$. By Corollary 5.1, $\dot{Y}_1 > \sqrt{2}\dot{Y}_2 > ... > \sqrt{n}\dot{Y}_n ... \geq b > -\infty$, and $\Phi(-\infty)=0$, thus b must satisfy $0<\Phi(b)<\frac{r-c}{r-\nu}$. Since $t<r-\nu$,

(a) Over-mean games (b) Under-mean games

Figure 5.3: $lim_{n\to\infty}\acute{Y}_n$ as a function of t

then for the left hand side to hold we must have $t < 2(r-c)$. The right hand side holds for the under-mean games. This proves that if $t \geq 2(r-c)$ then $a \leq 0$ and $b = -\infty$. In this case the equation (5.14) becomes $r - c = t\Phi(a)$, or $a = \Phi^{-1}\left(\frac{r-c}{t}\right)$. □

Figure 5.3 shows the limit $a = \lim_{n\to\infty}\acute{Y}_n$ as a function of t for over-mean and under-mean games. It follows from Theorem 5.4 that sufficiently low transportation cost, that is the cost not *exceeding* $2(c-\nu)$ for the over-mean games and not *exceeding* $2(r-c)$ for the under-mean games, allows the optimum quantity to converge to the demand mean μ as the game size grows. Therefore, sufficiently large games become practically *mean* games for these sufficiently *low* costs.

On the other hand, for the over-mean games, the more the transportation cost exceeds $2(c-\nu)$, moving up towards $r-\nu$, the closer the optimal quantities become to $\acute{Y}_1 = \Phi^{-1}\left(\frac{r-c}{r-\nu}\right)$ for sufficiently large games. Then, the optimal quantities of newsvendors in sufficiently large games become practically *indistinguishable* from the optimal quantities for a *single* over-mean newsvendor game. Therefore, other newsvendors in a sufficiently large game make ever-disappearing difference in setting up optimal quantity for any individual newsvendor who sets it close to $\acute{Y}_1$.

Similarly, for the under-mean newsvendors, the more the transportation cost exceeds $2(r-c)$, moving up towards $r-\nu$, the closer the optimal production quantities become to $\dot{Y}_1 = \Phi^{-1}\left(\frac{r-c}{r-\nu}\right)$ for sufficiently large games. This time, the optimal production quantities of individual newsvendors in sufficiently large games become practically *indistinguishable* from the optimal quantities for a *single* under-mean newsvendor. Therefore, again, other newsvendors in a sufficiently large game make ever-disappearing difference in setting up optimal quantity for any individual newsvendor who sets it close to $\dot{Y}_1$.

5.4 Characteristic Functions and Individual allocations

We now derive a formula for the maximum expected profit $\dot{J}_n$, and the individual allocation β_n in the game of size n. Let $I(X) = \int_X^\infty (\xi - X)\phi(\xi)d\xi$ be the unit normal loss function. Using the transformation $Y = (X-\mu)/\sigma$, we have

$$I_D(X) = E\left[\max(D-X,0)\right] = \sigma E\left[\max\left(\frac{D-\mu}{\sigma} - Y, 0\right)\right] = \sigma I(Y),$$

$$I_Z(nX) = E\left[\max(Z-nX,0)\right] = \sqrt{n}\sigma E\left[\max\left(\frac{Z-n\mu}{\sqrt{n}\sigma} - \sqrt{n}Y, 0\right)\right] = \sqrt{n}\sigma I(\sqrt{n}Y).$$

Then, (5.12) can be rewritten as

$$J_n(Y) = n(r-c)\mu - n(c-\nu)\sigma Y - nt\sigma I(Y) - p\sqrt{n}\sigma I(\sqrt{n}Y). \quad (5.15)$$

For standard normal distribution, we have

$$I(Y) = \phi(Y) - Y(1 - \Phi(Y)) \quad (5.16)$$

(Porteus, 2002; Hartman and Dror, 2005). This relation is easily verifiable by noting that $\phi'(Y) = -Y\phi(Y)$. By applying (5.16) to (5.15) we get

$$J_n(Y) = n(r-c)(\mu+\sigma Y) - nt\sigma\left(\phi(Y) + Y\Phi(Y)\right) - np\sigma\left(\frac{\sqrt{n}}{n}\phi(\sqrt{n}Y) + Y\Phi(\sqrt{n}Y)\right). \tag{5.17}$$

Finally, by setting Y to $\dot{Y}_n$ in (5.17), and then applying the optimality conditions in (5.14), a closed form expression for the maximum expected profits for normal distributions is follows

$$\dot{J}_n = n(r-c)\mu - \sigma\left(nt\phi(\dot{Y}_n) + \sqrt{n}p\phi(\sqrt{n}\dot{Y}_n)\right). \tag{5.18}$$

Although in general finding an allocation in the core of a transshipment game is NP-hard (Chen and Zhang, 2009), for symmetric newsvendors there is only one core allocation possible, the one with all individual allocations equal to $1/n$-th of the $\dot{J}_n$. That is

$$\beta_n = \dot{J}_n/n = (r-c)\mu - \sigma\left(t\phi(\dot{Y}_n) + \frac{p}{\sqrt{n}}\phi(\sqrt{n}\dot{Y}_n)\right). \tag{5.19}$$

The following result follows from Theorem 5.1.

Lemma 5.4. *For all* $1 \le l < n$, $t\phi(\dot{Y}_n) + \frac{1}{\sqrt{n}}p\phi(\sqrt{n}\dot{Y}_n) \le t\phi(\dot{Y}_l) + \frac{1}{\sqrt{l}}p\phi(\sqrt{l}\dot{Y}_l)$.

Proof. To any coalition of size l we allocate $l\beta_n = l\frac{\dot{J}_n}{n}$. Therefore, in order for the allocation β_n to be in the core of this transshipment game, we must have $l\beta_n \ge \dot{J}_l$, for any $1 \le l < n$. Since the allocation β_n is unique, and by Theorem 1 the core is non-empty, the lemma follows. □

A technical note is in order at this point. Equation (5.18), for large values of σ/μ, does not guarantee that the $\dot{J}_n$ is positive. This is due to the fact that under normal distribution with relatively large standard deviations, negative market demands are likely to occur which is not quite meaningful in our setting. In order to avoid such circumstances, it suffices to assume that

$$\sigma \leq (r-c)\mu / \left[(r-\nu)\phi\left(\Phi^{-1}\left(\frac{r-c}{r-\nu}\right)\right)\right].$$

From Lemma 5.4 it is straightforward to check that this assumption leads to $\dot{J}_n \geq 0$ for all n.

5.4.1 The Laws of Diminishing Individual allocations

In this section, we show that the individual allocation β_n increases as n grows, that is

$$\beta_1 < \beta_2 < \ldots < \beta_n < \ldots. \tag{5.20}$$

However, there are two laws of diminishing individual allocations that accompany this growth. The first is concerned with the absolute gains

$$\Delta_n = \beta_n - \beta_{n-1}$$

which diminish as the size of grand coalition n grows, that is

$$\Delta_2 > \Delta_3 > \ldots > \Delta_n > \ldots. \tag{5.21}$$

The second imposes a lower bound of $\frac{n+1}{n-1}$ on the ratio of the absolute gains Δ_n over Δ_{n+1}, that is

$$\frac{\Delta_n}{\Delta_{n+1}} \geq \frac{n+1}{n-1}. \tag{5.22}$$

While the first law ensures that these ratios are always higher than 1, the second sharpens this lower bound showing that the absolute gain Δ_n is at least $\frac{2}{n-1}100\%$

higher than Δ_{n+1}. More precisely, the second law states that if there exists the grand coalition size n^* such that

$$\frac{\Delta^*_n}{\Delta_{n^*+1}} \geq \frac{n^*+1}{n^*-1},$$

then the sharper lower bound of (5.22) holds for all $n \geq n^*$. This critical grand coalition size n^* depends on the transportation cost t and the ratio p/t. The critical size is studied later in the section. It suffices to say for now that our computational experiments, see Table 5.1, show that the critical n^* decreases as t grows. However, we show that the critical n^* does not exist for $t < 2(c-\nu)$ in the case of over-mean games, and for $t < 2(r-c)$ in the case of under-mean games (we return to the examples in Table 5.1 after we define n^{**} in Section 5.5.1). These observations indicate that the high transportation costs precipitate the critical grand coalition n^*, and consequently the second law of diminishing individual allocations. We prove later that the critical n^* does not exists for either $t = 0$ or mean games. We leave the case of $t = 0$ for Section 5.5 and assume that $t > 0$ in this section. Both laws of diminishing individual allocations are key for determining the cooperation costs newsvendors can afford to pay to form a grand coalition in Section 5.5. The first is key for the clique cooperation network, the second for the hub. We now prove the two laws. Let $\beta(x)$ be the extension of β_n to the set of positive real numbers. We begin with the following result.

Theorem 5.5. *$\beta(x)$ is a strictly increasing and strictly concave function of x.*

Proof. The first derivative of $\beta(x)$ is

$$\frac{d\beta(x)}{dx} = -\sigma\left[t\frac{d\phi(\dot{Y}_x)}{dx} + p\left(-\frac{1}{2x\sqrt{x}}\phi(\sqrt{x}\dot{Y}_x) + \frac{1}{\sqrt{x}}\frac{d\phi(\sqrt{x}\dot{Y}_x)}{dx}\right)\right] \tag{5.23}$$

Instance		n^*	n^{**}	Instance		n^*	n^{**}	Instance		n^*	n^{**}
$[r,c,\nu,\mu,\sigma]=[40,15,10,100,50]$	$t=8$	$NA^{\dagger}$	-	$[r,c,\nu,\mu,\sigma]=[40,20,15,100,50]$	$t=8$	$NA^{\dagger}$	-	$[r,c,\nu,\mu,\sigma]=[40,15,10,100,20]$	$t=8$	$NA^{\dagger}$	-
	$t=10$	$NA^{\dagger}$	-		$t=10$	$NA^{\dagger}$	-		$t=10$	$NA^{\dagger}$	-
	$t=11$	49	$NA^{\ddagger}$		$t=11$	50	$NA^{\ddagger}$		$t=11$	49	$NA^{\ddagger}$
	$t=12$	17	63		$t=12$	17	65		$t=12$	17	63
	$t=15$	5	11		$t=16$	4	8		$t=15$	5	11
	$t=22$	2	3		$t=24$	2	3		$t=22$	2	3
$[r,c,\nu,\mu,\sigma]=[40,20,10,100,50]$	$t=18$	$NA^{\dagger}$	-	$[r,c,\nu,\mu,\sigma]=[40,27,25,100,50]$	$t=3$	$NA^{\dagger}$	-	$[r,c,\nu,\mu,\sigma]=[40,20,15,60,20]$	$t=8$	$NA^{\dagger}$	-
	$t=20$	$NA^{\dagger}$	-		$t=4$	$NA^{\dagger}$	-		$t=10$	$NA^{\dagger}$	-
	$t=21$	192	$NA^{\ddagger}$		$t=5$	12	39		$t=11$	50	$NA^{\ddagger}$
	$t=23$	31	73		$t=6$	5	10		$t=12$	17	65
	$t=26$	11	32		$t=7$	3	6		$t=16$	4	8
	$t=29$	7	15		$t=8$	2	3		$t=24$	2	3

† Based on the experiments with largest $n = 1000$

‡ Based on the experiments with largest $n = 100$

Table 5.1: Values of n^* and n^{**} for some Instances of Transshipment Game

We have

$$\frac{d\phi(\sqrt{x}\dot{Y}_x)}{dx} = -\left(\frac{1}{2\sqrt{x}}\dot{Y}_x + \sqrt{x}\frac{d\dot{Y}_x}{dx}\right)\sqrt{x}\dot{Y}_x\phi(\sqrt{x}\dot{Y}_x) = -\phi(\sqrt{x}\dot{Y}_x)\left(\frac{1}{2}\dot{Y}_x^2 + x\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right).$$

Therefore,

$$\begin{aligned}\frac{d\beta(x)}{dx} &= -\sigma\left[-t\frac{d\dot{Y}_x}{dx}\dot{Y}_x\phi(\dot{Y}_x) + p\left(-\frac{1}{2x\sqrt{x}}\phi(\sqrt{x}\dot{Y}_x) - \frac{1}{\sqrt{x}}\phi(\sqrt{x}\dot{Y}_x)\left(\frac{1}{2}\dot{Y}_x^2 + x\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)\right)\right] \\ &= -\sigma\left[-t\frac{d\dot{Y}_x}{dx}\dot{Y}_x\phi(\dot{Y}_x) - p\phi(\sqrt{x}\dot{Y}_x)\left(\frac{1}{2x\sqrt{x}} + \frac{1}{2\sqrt{x}}\dot{Y}_x^2 + \sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)\right] \\ &= -\sigma\left[-\frac{d\dot{Y}_x}{dx}\dot{Y}_x\left(t\phi(\dot{Y}_x) - p\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)\right) + \frac{p}{2x\sqrt{x}}\phi(\sqrt{x}\dot{Y}_x) + \frac{p}{2\sqrt{x}}\dot{Y}_x^2\phi(\sqrt{x}\dot{Y}_x)\right] \quad (5.24)\end{aligned}$$

From equation (5.14), we define $G(\dot{Y}_x, x) = t\Phi(\dot{Y}_x) + p\Phi(\sqrt{x}\dot{Y}_x) - (r-c)$ as the implicit function which obtains $\dot{Y}_x$. Figure 5.4 shows the graph of this function for an instance of transshipment game. As it is observable in Figure 5.4 and according to (5.25), $\dot{Y}_x$ is continuous on x and always has finite slope which allow us to use the *Implicit*

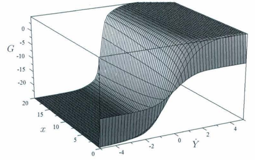


Figure 5.4: $G(\dot{Y}_x, x)$ for an Instance with $r = 40$, $c = 15$, $\nu = 10$, and $t = 10$

Function Theorem. We have

$$\frac{d\dot{Y}_x}{dx} = -\frac{\partial G(\dot{Y}_x, x)/\partial x}{\partial G(\dot{Y}_x, x)/\partial \dot{Y}_x} = -\frac{p\dot{Y}_x\phi(\sqrt{x}\dot{Y}_x)}{2\sqrt{x}\Lambda_x} \tag{5.25}$$

where $\Lambda_x = t\phi(\dot{Y}_x) + \sqrt{x}p\phi(\sqrt{x}\dot{Y}_x)$. Note that $\frac{d\dot{Y}_x}{dx}\dot{Y}_x \leq 0$. Hence, the first derivative of $\beta(x)$ is simplified to

$$\frac{d\beta(x)}{dx} = -\sigma\left[\frac{p\dot{Y}_x^2\phi(\sqrt{x}\dot{Y}_x)}{2\sqrt{x}} - \frac{p\phi(\sqrt{x}\dot{Y}_x)}{2x\sqrt{x}} - \frac{p\dot{Y}_x^2\phi(\sqrt{x}\dot{Y}_x)}{2\sqrt{x}}\right] = \frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x\sqrt{x}} \tag{5.26}$$

The latter equation is obviously positive which proves that $\beta(x)$ is strictly increasing.

The second derivative of $\beta(x)$ is

$$\begin{aligned}
\frac{d^2\beta(x)}{dx^2} &= \frac{\sigma p}{2}\frac{d}{dx}\left(\frac{\phi(\sqrt{x}\dot{Y}_x)}{x\sqrt{x}}\right) \\
&= \frac{\sigma p}{2}\frac{x\sqrt{x}\frac{d\phi(\sqrt{x}\dot{Y}_x)}{dx} - \frac{3}{2}\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)}{x^3} \\
&= \frac{\sigma p}{2x^3}\left(x\sqrt{x}\left[-\phi(\sqrt{x}\dot{Y}_x)\left(\frac{1}{2}\dot{Y}_x^2 + x\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)\right] - \frac{3}{2}\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)\right) \\
&= -\frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x^2\sqrt{x}}\left(x\left[\frac{1}{2}\dot{Y}_x^2 + x\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right] + \frac{3}{2}\right)
\end{aligned}$$

Note that by replacing the $\frac{d\dot{Y}_x}{dx}$ with its explicit formula we get

$$\frac{1}{2}\dot{Y}_x^2 + x\frac{d\dot{Y}_x}{dx}\dot{Y}_x = \frac{1}{2}\dot{Y}_x^2 - x\frac{p\dot{Y}_x\phi(\sqrt{x}\dot{Y}_x)}{2\sqrt{x}\Lambda_x}\dot{Y}_x = \frac{1}{2}\dot{Y}_x^2\left(1 - \frac{\sqrt{x}p\phi(\sqrt{x}\dot{Y}_x)}{\Lambda_x}\right) = \frac{t\dot{Y}_x^2\phi(\dot{Y}_x)}{2\Lambda_x}$$

and consequently,

$$\frac{d^2\beta(x)}{dx^2} = -\frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x^2\sqrt{x}}\left(\frac{xt\dot{Y}_x^2\phi(\dot{Y}_x)}{2\Lambda_x} + \frac{3}{2}\right) \tag{5.27}$$

Thus, $\frac{d^2\beta(x)}{dx^2} < 0$ which shows that $\beta(x)$ is strictly concave. □

We have the following first law of diminishing individual allocations.

Theorem 5.6 (First Law of Diminishing Individual allocations). $\frac{\Delta_n}{\Delta_{n+1}} > 1$ *for* $n \geq 2$.

Proof. By Theorem 5.5, $\beta(x)$ is strictly concave. Thus, $2\beta_n > \beta_{n+1} + \beta_{n-1}$ or equivalently $\beta_n - \beta_{n-1} > \beta_{n+1} - \beta_n$ which proves the theorem. □

In order to prove the second law of diminishing individual allocations we need to consider the sequence $\dot{J}_n = n\beta_n$. By introducing $\dot{J}(x) = x\beta(x)$ as the extension over positive real numbers, we have the following result.

Lemma 5.5. *Let* $S(x) = (x\dot{Y}_x^2 - 1)\frac{\phi(\dot{Y}_x)}{\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)}$. $\dot{J}(x)$ *is concave if* $S(x) \geq p/t$, *and strictly convex if* $S(x) < p/t$.

Proof. First note that $\frac{d^2\dot{J}(x)}{dx^2} = 2\frac{d\beta(x)}{dx} + x\frac{d^2\beta(x)}{dx^2}$. By substituting (5.26) and (5.27) we have

$$\begin{aligned}\frac{d^2\dot{J}(x)}{dx^2} &= 2\left(\frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x\sqrt{x}}\right) + x\left(-\frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x^2\sqrt{x}}\left(\frac{xt\dot{Y}_x^2\phi(\dot{Y}_x)}{2\Lambda_x} + \frac{3}{2}\right)\right)\\ &= \frac{\sigma p\phi(\sqrt{x}\dot{Y}_x)}{2x\sqrt{x}}\left[\frac{1}{2} - \frac{xt\dot{Y}_x^2\phi(\dot{Y}_x)}{2\Lambda_x}\right]\end{aligned}$$

Therefore, $\dot{J}(x)$ is concave if and only if $\frac{d^2\dot{J}(x)}{dx^2} \leq 0$, that is if and only if $xt\dot{Y}_x^2\phi(\dot{Y}_x) \geq \Lambda_x$ which is equivalent to

$$S(x) = (x\dot{Y}_x^2 - 1)\frac{\phi(\dot{Y}_x)}{\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)} \geq \frac{p}{t}.$$

Finally, $\dot{J}(x)$ is strictly convex if and only if $\frac{d^2\dot{J}(x)}{dx^2} > 0$, that is if and only if $S(x) < p/t$. □

The function $S(x)$ is not monotone in general. Its behavior depends on the parameters p and t. Figure 5.5 depicts $S(x)$ for some values of these parameters. Although in all instances $S(x)$ starts as an increasing function, it does not necessarily remain increasing as x grows. In Figure 5.5 (a) and (e), the function becomes decreasing after a certain value of x. If $S(x)$ is an increasing function for some p and t, and it reaches the critical value of p/t, then it remains above this value. The key observation shown in Lemma 5.6 is that, if $S(x)$ is not monotone, then it *never* reaches the critical value p/t. This trait of behavior is also observable in Figure 5.5. In Figure 5.5 (a), $S(x)$ reaches its maximum of ≈ 0.16 at $x \approx 10$ while the critical value is $p/t = 5$ for this instance. Also, in Figure 5.5 (e), the function $S(x)$ reaches its maximum of ≈ 4.4

at $x \approx 5$ while the critical value is $p/t = 25$ for this instance. Lemma 5.6 formalizes this behavior.

Lemma 5.6. *At any x such that $\frac{dS(x)}{dx} = 0$, $S(x) < p/t$.*

Proof. First note that at any x such that $x\dot{Y}_x^2 \le 1$, $S(x)$ is non-positive and thus $S(x) < p/t$. Therefore, we assume without loss of generality that $x\dot{Y}_x^2 > 1$ for the rest of the proof. We have $\phi(\dot{Y}_x)/\phi(\sqrt{x}\dot{Y}_x) = e^{\frac{1}{2}(x-1)\dot{Y}_x^2}$. Hence

$$\begin{aligned}
\frac{dS(x)}{dx} &= \frac{d}{dx}\left(\frac{(x\dot{Y}_x^2-1)e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{\sqrt{x}}\right)\\
&= \frac{1}{x}\left(\sqrt{x}\left[(\dot{Y}_x^2+2x\frac{d\dot{Y}_x}{dx}\dot{Y}_x)e^{\frac{1}{2}(x-1)\dot{Y}_x^2}+\left(\frac{1}{2}\dot{Y}_x^2+\frac{1}{2}(x-1)2\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)(x\dot{Y}_x^2-1)e^{\frac{1}{2}(x-1)\dot{Y}_x^2}\right]-\frac{1}{2\sqrt{x}}(x\dot{Y}_x^2-1)e^{\frac{1}{2}(x-1)\dot{Y}_x^2}\right)\\
&= \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{x}\left[(\sqrt{x}\dot{Y}_x^2+2x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x)+\left(\frac{1}{2}\dot{Y}_x^2+(x-1)\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)(x\sqrt{x}\dot{Y}_x^2-\sqrt{x})-\frac{\sqrt{x}}{2}\dot{Y}_x^2+\frac{1}{2\sqrt{x}}\right]\\
&= \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{x}\left[\sqrt{x}\dot{Y}_x^2+2x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\left(\frac{1}{2}\dot{Y}_x^2+(x-1)\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)x\sqrt{x}\dot{Y}_x^2-\left(\frac{1}{2}\dot{Y}_x^2+(x-1)\frac{d\dot{Y}_x}{dx}\dot{Y}_x\right)\sqrt{x}-\frac{\sqrt{x}}{2}\dot{Y}_x^2+\frac{1}{2\sqrt{x}}\right]\\
&= \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{x}\left[\frac{\sqrt{x}}{2}\dot{Y}_x^2+2x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\frac{1}{2}x\sqrt{x}\dot{Y}_x^4+(x-1)x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x^3-\frac{1}{2}\sqrt{x}\dot{Y}_x^2-(x-1)\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\frac{1}{2\sqrt{x}}\right]\\
&= \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{x}\left[2x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\frac{1}{2}n\sqrt{x}\dot{Y}_x^4+(x-1)x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x^3-x\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x+\frac{1}{2\sqrt{x}}\right]
\end{aligned}$$

or

$$\frac{dS(x)}{dx} = \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{x}\left[\sqrt{x}\frac{d\dot{Y}_x}{dx}\dot{Y}_x\left(x+1+(x-1)x\dot{Y}_x^2\right)+\frac{1}{2}x\sqrt{x}\dot{Y}_x^4+\frac{1}{2\sqrt{x}}\right]$$

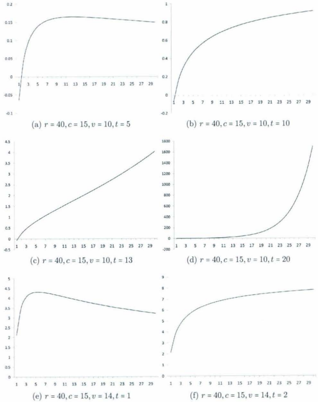
Using (5.25) the latter simplifies to

$$\frac{dS(x)}{dx} = \frac{e^{\frac{1}{2}(x-1)\dot{Y}_x^2}}{2x\sqrt{x}\Lambda_x}\left[\sqrt{x}p\phi(\sqrt{x}\dot{Y}_x)\left(\dot{Y}_x^2-1\right)\left(x\dot{Y}_x^2-1\right)+t\phi(\dot{Y}_x)\left(x^2\dot{Y}_x^4+1\right)\right].$$

If $\left(\dot{Y}_x^2-1\right)\left(x\dot{Y}_x^2-1\right) \ge 0$, then $\frac{dS(x)}{dx} > 0$. Therefore, we assume without loss of generality that $\dot{Y}_x^2 < 1$ for the rest of the proof. Next, we have $\frac{dS(x)}{dx} = 0$ if and only if

$$-\frac{\phi(\dot{Y}_x)}{\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)}\frac{x^2\dot{Y}_x^4+1}{\left(\dot{Y}_x^2-1\right)\left(x\dot{Y}_x^2-1\right)} = \frac{p}{t}.$$

Thus to complete the proof its suffices to show that the following inequality holds for



(a) $r = 40, c = 15, v = 10, t = 5$

(b) $r = 40, c = 15, v = 10, t = 10$

(c) $r = 40, c = 15, v = 10, t = 13$

(d) $r = 40, c = 15, v = 10, t = 20$

(e) $r = 40, c = 15, v = 14, t = 1$

(f) $r = 40, c = 15, v = 14, t = 2$

Figure 5.5: The Function $S(x)$ with Respect to Different Parameters

$x \geq 1$

$$\frac{\phi(\dot{Y}_x)}{\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)} \frac{x^2\dot{Y}_x^4 + 1}{\left(1 - \dot{Y}_x^2\right)\left(x\dot{Y}_x^2 - 1\right)} > \left(x\dot{Y}_x^2 - 1\right) \frac{\phi(\dot{Y}_x)}{\sqrt{x}\phi(\sqrt{x}\dot{Y}_x)}.$$

This simplifies to

$$x^2\dot{Y}_x^4 + 1 > \left(1 - \dot{Y}_x^2\right)\left(x\dot{Y}_x^2 - 1\right)^2$$

or

$$-x^2\dot{Y}_x^4 + 2x\dot{Y}_x^2 - (2x+1) < 0$$

which holds for $x \geq 0$ since $\dot{Y}_x^2 < 1$. This proves that for any x such that $\frac{dS(x)}{dx} = 0$, we have $S(x) < p/t$. □

Lemma 5.7. *If $S(x^*) = p/t$ for some $x^* > 1$, then $S(x) \geq p/t$ for all $x \geq x^*$ and $S(x) < p/t$ for $1 \leq x < x^*$.*

Proof. From the proof of Lemma 5.6, we have

$$\begin{aligned}\left(\frac{dS(x)}{dx}\right)_{x=1} &= \frac{1}{2\Lambda_1}\left[p\phi(\dot{Y}_1)\left(-\dot{Y}_1^2 - \dot{Y}_1^2 + \dot{Y}_1^4 + 1\right) + t\phi(\dot{Y}_1)\left(\dot{Y}_1^4 + 1\right)\right] \\ &= \frac{1}{2\Lambda_1}\left[p\left(\dot{Y}_1^2 - 1\right)^2 + t\left(\dot{Y}_1^4 + 1\right)\right] > 0\end{aligned}$$

If $\frac{dS(x)}{dx} \geq 0$ for $x \geq 1$, then the lemma obviously holds. Otherwise, let $\left(\frac{dS(x)}{dx}\right)_{x=x'} < 0$ for some $x' > 1$. Then by the Intermediate Value Theorem for Derivatives we have $\left(\frac{dS(x)}{dx}\right)_{x=c} = 0$ for some $1 < c < x'$. Thus by Lemma 5.6 $S(c) < \frac{p}{t}$ for any such c which implies $S(1) < \frac{p}{t}$. Therefore, $x^* > 1$ if x^* exists. Now for any two points $1 < a < b$ such that $S(a) = S(b) = \frac{p}{t}$ we have, by the Rolle's Theorem, a point $a < d < b$ such that $\left(\frac{dS(x)}{dx}\right)_{x=d} = 0$. For any such point we have $S(d) < \frac{p}{t}$ by Lemma 5.6. Consequently, $\left(\frac{dS(x)}{dx}\right)_{x=a} = 0$ which leads to a contradiction by Lemma 5.6. □

Theorem 5.7. *If $S(x^*) = p/t$ for some $x^* \geq 1$, then $\dot{J}(x)$ is concave for $x \geq x^*$ and convex for $1 \leq x < x^*$.*

Proof. The proof directly follows from Lemma 5.6 and Lemma 5.7. □

We have the following second law of diminishing individual allocations.

Theorem 5.8 (Second Law of Diminishing Individual allocations). *If $S(x^*) = p/t$ for some $x^* > 1$, then $\frac{\Delta_n}{\Delta_{n+1}} \geq \frac{n+1}{n-1}$ for $n \geq n^*$, and $1 < \frac{\Delta_n}{\Delta_{n+1}} < \frac{n+1}{n-1}$ for $2 \leq n < n^*$, where n^* equals either $\lfloor x^* \rfloor$ or $\lceil x^* \rceil$ or $\lceil x^* \rceil + 1$.*

Proof. By Theorem 5.7, we have $2\dot{J}_n \geq \dot{J}_{n+1} + \dot{J}_{n-1}$ for $n \geq \lceil x^* \rceil + 1$. Thus, $2n\beta_n \geq (n+1)\beta_{n+1} + (n-1)\beta_{n-1}$ and consequently $(n-1)(\beta_n - \beta_{n-1}) \geq (n+1)(\beta_{n+1} - \beta_n)$. Therefore $\frac{\Delta_n}{\Delta_{n+1}} \geq \frac{n+1}{n-1}$ for $n \geq \lceil x^* \rceil + 1$. Also, by Theorem 5.7, we have $2\dot{J}_n < \dot{J}_{n+1} + \dot{J}_{n-1}$ for $2 \leq n \leq \lfloor x^* \rfloor - 1$. Thus, $2n\beta_n < (n+1)\beta_{n+1} + (n-1)\beta_{n-1}$ and consequently $(n-1)(\beta_n - \beta_{n-1}) < (n+1)(\beta_{n+1} - \beta_n)$. Therefore, by Theorem 5.6, we have $1 < \frac{\Delta_n}{\Delta_{n+1}} < \frac{n+1}{n-1}$ for $2 \leq n \leq \lfloor x^* \rfloor - 1$. It remains to consider $\lfloor x^* \rfloor$ and $\lceil x^* \rceil$. Assume, x^* is not an integer, thus $\lfloor x^* \rfloor \neq \lceil x^* \rceil$. Let $L(x)$ be the straight line connecting points $(\lfloor x^* \rfloor, \dot{J}(\lfloor x^* \rfloor))$ and $(\lceil x^* \rceil + 1, \dot{J}(\lceil x^* \rceil + 1))$, and let $M(x)$ be the straight line connecting points $(\lfloor x^* \rfloor - 1, \dot{J}(\lfloor x^* \rfloor - 1))$ and $(\lceil x^* \rceil, \dot{J}(\lceil x^* \rceil))$. we have the following four cases to consider.

If $L(\lceil x^* \rceil) \leq \dot{J}(\lceil x^* \rceil)$ and $M(\lfloor x^* \rfloor) > \dot{J}(\lfloor x^* \rfloor)$, then $\frac{\Delta_{\lceil x^* \rceil}}{\Delta_{\lceil x^* \rceil + 1}} \geq \frac{\lceil x^* \rceil + 1}{\lceil x^* \rceil}$, and $\frac{\Delta_{\lfloor x^* \rfloor}}{\Delta_{\lfloor x^* \rfloor + 1}} < \frac{\lfloor x^* \rfloor + 1}{\lfloor x^* \rfloor - 1}$. Thus, $n^* = \lceil x^* \rceil$.

If $L(\lceil x^* \rceil) \leq \dot{J}(\lceil x^* \rceil)$ and $M(\lfloor x^* \rfloor) \leq \dot{J}(\lfloor x^* \rfloor)$, then $\frac{\Delta_{\lceil x^* \rceil}}{\Delta_{\lceil x^* \rceil + 1}} \geq \frac{\lceil x^* \rceil + 1}{\lceil x^* \rceil}$, and $\frac{\Delta_{\lfloor x^* \rfloor}}{\Delta_{\lfloor x^* \rfloor + 1}} \geq \frac{\lfloor x^* \rfloor + 1}{\lfloor x^* \rfloor - 1}$. Thus, $n^* = \lfloor x^* \rfloor$.

If $L(\lceil x^* \rceil) > \dot{J}(\lceil x^* \rceil)$ and $M(\lfloor x^* \rfloor) > \dot{J}(\lfloor x^* \rfloor)$, then $\frac{\Delta_{\lceil x^* \rceil}}{\Delta_{\lceil x^* \rceil + 1}} < \frac{\lceil x^* \rceil + 1}{\lceil x^* \rceil}$, and $\frac{\Delta_{\lfloor x^* \rfloor}}{\Delta_{\lfloor x^* \rfloor + 1}} < \frac{\lfloor x^* \rfloor + 1}{\lfloor x^* \rfloor - 1}$. Thus, $n^* = \lceil x^* \rceil + 1$.

If $L(\lceil x^* \rceil) > \dot{J}(\lceil x^* \rceil)$ and $M(\lfloor x^* \rfloor) \leq \dot{J}(\lfloor x^* \rfloor)$, then there is $x' > \lceil x^* \rceil$ such that $L(x) > \dot{J}(x)$ for $\lceil x^* \rceil - 1 < x < x'$, and $x'' \leq \lfloor x^* \rfloor$ such that $M(x) < \dot{J}(x)$ for $x'' < x \leq \lfloor x^* \rfloor$. We now show that this leads to a contradiction. First, consider the straight line $P(x)$ which is the part of $L(x)$ between $(\lfloor x^* \rfloor, \dot{J}(\lfloor x^* \rfloor))$ and $(\lceil x^* \rceil, \dot{J}(\lceil x^* \rceil))$, and the straight $Q(x)$ line connecting $(\lfloor x^* \rfloor - 1, \dot{J}(\lfloor x^* \rfloor - 1))$ and $(\lfloor x^* \rfloor, \dot{J}(\lfloor x^* \rfloor))$. The $\dot{J}(x)$ remains below $P(x)$ for $\lfloor x^* \rfloor < x < x'$ by definition of x', and $\dot{J}(x)$ remains below $Q(x)$ for $\lfloor x^* \rfloor - 1 < x < \lfloor x^* \rfloor$ because $\dot{J}(x)$ is convex there. Now consider $M(x)$, it stays above $Q(x)$ for $\lfloor x^* \rfloor - 1 < x < \lfloor x^* \rfloor$ since $\dot{J}(x)$ is a strictly increasing function and thus $\dot{J}(\lfloor x^* \rfloor) < \dot{J}(\lceil x^* \rceil)$. Therefore, we have $x'' > \lfloor x^* \rfloor$ which leads to a contradiction. Finally, consider $\lfloor x^* \rfloor = \lceil x^* \rceil$, then x^* is an integer. we have two cases to consider. If $L(x^*) \leq \dot{J}(x^*)$, then $\frac{\Delta_{x^*}}{\Delta_{x^*+1}} \geq \frac{x^*+1}{x^*-1}$. Thus, $n^* = x^*$. Otherwise, $L(x^*) > \dot{J}(x^*)$ and then $\frac{\Delta_{x^*}}{\Delta_{x^*+1}} < \frac{x^*+1}{x^*-1}$. Thus, $n^* = x^* + 1$. □

We have the following result with respect to the existence of n^*.

Theorem 5.9. *For over-mean games with $t < 2(c - \nu)$, under-mean games with $t < 2(r - c)$, and mean games no $n^* < \infty$ exists.*

Proof. From Theorem 5.7, it is clear that the existence of n^* depends on the existence of x^*. Consider over-mean games with $t < 2(c - \nu)$ and assume that there exist $x^* < \infty$. According to Lemma 5.7, for all $x \geq x^*$ it must be the case that $S(x) \geq p/t > 0$. However, by Theorem 5.4 we have $\lim_{x \to \infty} S(x) = 0$ for $t < 2(c - \nu)$ which leads to a contradiction. Hence, there exist no $x^* < \infty$ and thus no $n^* < \infty$. A similar argument proves the theorem for the under-mean games with $t < 2(r - c)$. Moreover, in mean games we have $\dot{Y}_x = 0$ and therefore $S(x) < 0 < p/t$. By Theorem 5.8, then there would be no $n^* < \infty$ for mean games. □

5.5 Games with Cooperation Costs

In the transshipment game of size n with cooperation cost any coalition of l, $1 \leq l \leq n$, symmetric newsvendors incurs cost $\mathbf{K}_l$ needed for it to form. The characteristic function, $\ddot{J} : 2^N \to \mathbb{R}$, of the transshipment game with cooperation costs is defined by setting $\ddot{J}_l = \dot{J}_l - \mathbf{K}_l$ for any coalition of size $1 \le l \le n$. Since the newsvendors are anonymous and symmetric, there is only one allocation possible in the core, if one exists, namely the one with all individual allocations equal to $\frac{1}{n}$-th of the $\ddot{J}_n$. Thus, the individual allocations must be $\alpha_n = \frac{\ddot{J}_n}{n} = \beta_n - \frac{1}{n}\mathbf{K}_n$. Hence, any coalition of size l gets $l\alpha_n = l\frac{\ddot{J}_n}{n}$ allocated. Therefore, in order for the allocation α_n to be in the core of a transshipment game with grand coalition of size n and cooperation costs, we must have $l\alpha_n \ge \ddot{J}_l$, for any $1 \le l < n$, and $n\alpha_n = \ddot{J}_n$. The latter condition is satisfied by definition of α_n, the former reduces to

$$\alpha_n \ge \alpha_l, \ \forall l < n. \tag{5.28}$$

Therefore, the core of the transshipment game with cooperation costs is non-empty if and only if the condition (5.28) is satisfied. Let $\Psi' = \{(\ddot{J}, n) | n \in \mathbb{N}\}$ as the set of all such transshipment games. We intend to analyze the impact of coalition size n on the stability of games in Ψ' under the assumption that the total cooperation cost for a coalition is proportional to the total number of links the coalition creates in its cooperation network. We consider two alternative cooperation networks: (1) Clique network, and (2) Hub network (Figure 5.6). By abstracting various types of costs, we presume that the cooperation costs are lump sum monetary amounts which represent the investments that any given pair of newsvendors make in order to establish a bilateral link in the network. Let K be the per-link cooperation cost. In the *Clique* network, each pair of newsvendors is connected by a separate link. The total number of links in a clique network with n newsvendors is thus $n(n-1)/2$ and the total cooperation cost is $\mathbf{K}_n^{clique} = \frac{n(n-1)}{2}K$. The condition (5.28) then becomes $\beta_n - \beta_l \ge \frac{n-l}{2}K$ for all $l < n$. Therefore, the core of the transshipment game with the clique network is non-empty if and only if the cost per link K satisfies the following

Figure 5.6: Different Network Structures for Coalitions

inequality

$$K \le 2 \min_{l<n} \frac{\beta_n - \beta_l}{n-l}. \tag{5.29}$$

We define the maximum admissible cost per link for the clique network of size n as

$$K_n^{clique} = 2 \min_{l<n} \frac{\beta_n - \beta_l}{n-l}. \tag{5.30}$$

We prove that the maximum admissible costs K_n^{clique} is always attained at $l = n-1$ in Section 5.5.1. The *Hub* network portrays the situation wherein the transshipments are coordinated through a designated newsvendor all other newsvendors are connected only to that designated newsvendor. The total number of links in the hub network is then $n-1$ and the cooperation cost is $\mathbf{K}_n^{hub} = (n-1)K$. The condition (5.28) then becomes $\beta_n - \beta_l \ge \frac{n-l}{nl} K$ for all $l < n$. Therefore, the core of the transshipment game with the hub network is non-empty if and only if the cost per link K satisfies the following inequality

$$K \le n \min_{l<n} \frac{l(\beta_n - \beta_l)}{n-l}. \tag{5.31}$$

We define the maximum admissible cost per link for the hub network of size n as

$$K_n^{hub} = n \min_{l<n} \frac{l(\beta_n - \beta_l)}{n-l}. \tag{5.32}$$

We show, in Section 5.5.1 that the maximum admissible cost K_n^{hub} is either always attained at $l = 1$ or there is a size n^{**} such that the minimum is attained at $l = 1$ for all games with fewer than n^{**} newsvendors and at $l = n - 1$ for all games with at least n^{**} newsvendors. This bipolar effect for the hub network is a consequence of the second law of diminishing individual allocations given in Theorem 5.8.

5.5.1 Positive Transportation Costs

We begin with a theorem that is a consequence of the laws of diminishing individual allocations. The theorem is key in determining the maximum admissible costs for both clique and hub networks. Let n^{**} be the smallest n such that $\frac{\beta_n - \beta_{n-1}}{\beta_n - \beta_1} < \frac{1}{(n-1)^2}$ if such n exists and infinity otherwise.

Theorem 5.10. *We have*

$$\frac{\beta_n - \beta_l}{\beta_n - \beta_1} \geq \frac{n-l}{l(n-1)}$$

for $n < n^{**}$ *and* $l < n$, *and*

$$\frac{\beta_n - \beta_l}{\beta_n - \beta_{n-1}} \geq \frac{(n-l)(n-1)}{l}$$

for $n \geq n^{**}$ *and* $l < n$. *Moreover* $n^{**} > n^*$.

Proof. We first show that $\frac{\beta_n - \beta_l}{\beta_n - \beta_1} \geq \frac{n-l}{l(n-1)}$ for all $n < n^{**}$ and for all $l < n$. The proof is by induction. Clearly, the inequality holds for $n = 2$. Assume that it holds for $2 \leq n$ and for all $l < n$, and additionally $n + 1 < n^{**}$, We prove that then it holds for $n + 1$. We have $\frac{\beta_{n+1} - \beta_l}{\beta_{n+1} - \beta_1} = \frac{\beta_{n+1} - \beta_n}{\beta_{n+1} - \beta_1} + \frac{\beta_n - \beta_l}{\beta_{n+1} - \beta_1}$. Since by the inductive assumption $\frac{\beta_n - \beta_l}{\beta_n - \beta_1} \geq \frac{n-l}{l(n-1)}$

for all $l < n$, then $\beta_n - \beta_l \geq (\beta_n - \beta_1)\frac{n-l}{l(n-1)}$ for all $l < n$. Thus,

$$\begin{aligned}
\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_1} &\geq \frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1} + \left(\frac{\beta_n-\beta_1}{\beta_{n+1}-\beta_1}\right)\frac{n-l}{l(n-1)} \\
&= \frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1} + \left(1 - \frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1}\right)\frac{n-l}{l(n-1)} \\
&= \frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1}\left(\frac{n(l-1)}{l(n-1)}\right) + \frac{n-l}{l(n-1)}
\end{aligned}$$

for all $l < n$. By assumption $n+1 < n^{**}$, thus $\frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1} \geq \frac{1}{n^2}$, which implies

$$\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_1} \geq \frac{(l-1)}{nl(n-1)} + \frac{n-l}{l(n-1)} = \frac{n+1-l}{nl}$$

for all $l < n+1$. Thus the inequality holds for $n+1$ and by induction for all $n < n^{**}$. This ends the proof if $n^{**} = \infty$. Therefore, let us now assume $n^{**} < \infty$. We now show that if $\frac{\beta_n-\beta_l}{\beta_n-\beta_1} \geq \frac{n-l}{l(n-1)}$ for all $l < n$, and $n^{**} = n+1$, then $\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_n} \geq \frac{(n+1-l)n}{l}$ for all $l < n+1$. To see this, note that $\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_n} = \frac{\beta_{n+1}-\beta_1}{\beta_{n+1}-\beta_n} - \frac{\beta_l-\beta_1}{\beta_{n+1}-\beta_n}$. Since $\frac{\beta_n-\beta_l}{\beta_n-\beta_1} \geq \frac{n-l}{l(n-1)}$ for all $l < n$, then $\frac{\beta_l-\beta_1}{\beta_n-\beta_1} \leq 1 - \frac{n-l}{l(n-1)}$ for all $l < n$. Thus

$$\begin{aligned}
\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_n} &\geq \frac{\beta_{n+1}-\beta_1}{\beta_{n+1}-\beta_n} - \frac{\beta_n-\beta_1}{\beta_{n+1}-\beta_n}\left(1 - \frac{n-l}{l(n-1)}\right) \\
&= \frac{\beta_{n+1}-\beta_1}{\beta_{n+1}-\beta_n} - \frac{\beta_n-\beta_1}{\beta_{n+1}-\beta_n} + \left(\frac{\beta_n-\beta_1}{\beta_{n+1}-\beta_n}\right)\frac{n-l}{l(n-1)} \\
&= 1 + \left(\frac{\beta_n-\beta_1}{\beta_{n+1}-\beta_n}\right)\frac{n-l}{l(n-1)} = 1 + \left(\frac{\beta_{n+1}-\beta_1}{\beta_{n+1}-\beta_n} - 1\right)\frac{n-l}{l(n-1)}
\end{aligned}$$

for all $l < n$. However, $\frac{\beta_{n+1}-\beta_n}{\beta_{n+1}-\beta_1} < \frac{1}{n^2}$, which implies

$$\frac{\beta_{n+1}-\beta_l}{\beta_{n+1}-\beta_n} \geq 1 + \left(n^2 - 1\right)\frac{n-l}{l(n-1)} = \frac{(n+1-l)n}{l}$$

for all $l < n$. Moreover, the last inequality for $l = n-1$ implies $\frac{\Delta_n}{\Delta_{n+1}} \geq \frac{n+1}{n-1}$, that is $\frac{\Delta_{n^{**}-1}}{\Delta_{n^{**}}} \geq \frac{n^{**}}{n^{**}-2}$. According to Theorem 8, it must be the case that $n^{**} - 1 \geq n^*$. Thus,

$n^{**} > n^*$.

Finally, we show that $\frac{\beta_n - \beta_l}{\beta_n - \beta_{n-1}} \geq \frac{(n-l)(n-1)}{l}$ for $n \geq n^{**}$ and all $l < n$ by induction. We have just shown that this inequality holds for $n = n^{**}$. Now, we assume that it holds for all $n \geq n^{**}$ and prove that it also holds for $n+1$, i.e., $\frac{\beta_{n+1} - \beta_l}{\beta_{n+1} - \beta_n} \geq \frac{(n+1-l)n}{l}$ for all $l < n+1$.

To see this, observe that $\frac{\beta_{n+1} - \beta_l}{\beta_{n+1} - \beta_n} = 1 + \frac{\beta_n - \beta_l}{\beta_{n+1} - \beta_n} = 1 + \left(\frac{\beta_n - \beta_l}{\beta_n - \beta_{n-1}}\right)\left(\frac{\beta_n - \beta_{n-1}}{\beta_{n+1} - \beta_n}\right)$. Since by the inductive assumption $\frac{\beta_n - \beta_l}{\beta_n - \beta_{n-1}} \geq \frac{(n-l)(n-1)}{l}$, then

$$
\begin{aligned}
\frac{\beta_{n+1} - \beta_l}{\beta_{n+1} - \beta_n} &\geq 1 + \frac{(n-l)(n-1)}{l}\left(\frac{\beta_n - \beta_{n-1}}{\beta_{n+1} - \beta_n}\right) \\
&= 1 + \frac{(n-l)(n-1)}{l}\frac{\Delta_n}{\Delta_{n+1}}
\end{aligned}
$$

Since $n+1 > n^{**} > n^*$, then by Theorem 5.8, we have $\frac{\Delta_n}{\Delta_{n+1}} \geq \frac{n+1}{n-1}$. Therefore,

$$
\frac{\beta_{n+1} - \beta_l}{\beta_{n+1} - \beta_n} \geq 1 + \frac{(n-l)(n-1)}{l}\frac{(n+1)}{(n-1)} = \frac{(n-l+1)n}{l}
$$

for all $l < n+1$. Thus, the inequality holds for $n+1$, and by induction for all $n \geq n^{**}$. This ends the proof of the theorem. □

Table 5.1 also demonstrates the values of n^{**} for some instances of transshipment games.

Clique Network

We are now ready to determine the maximum admissible cost per link for the clique network.

Theorem 5.11. $K_n^{clique} = 2(\beta_n - \beta_{n-1})$.

Proof. By (5.30) we need to show that $\beta_n - \beta_{n-1} \leq \frac{\beta_n - \beta_l}{n-l}$ for all $l < n$, which is equivalent to $\frac{\beta_n-\beta_l}{\beta_n-\beta_{n-1}} \geq n-l$ for all $l < n$. We have

$$\frac{\beta_n-\beta_l}{\beta_n-\beta_{n-1}} = \frac{\beta_n-\beta_{n-1}}{\beta_n-\beta_{n-1}} + \frac{\beta_{n-1}-\beta_{n-2}}{\beta_n-\beta_{n-1}} + \ldots + \frac{\beta_{l+1}-\beta_l}{\beta_n-\beta_{n-1}} = \frac{\Delta_n}{\Delta_n} + \frac{\Delta_{n-1}}{\Delta_n} + \ldots + \frac{\Delta_{l+1}}{\Delta_n}.$$

By Theorem 5.6, $\Delta_n \leq \Delta_{n-1} \leq \ldots \leq \Delta_{l+1}$, thus the right hand side sums up to at least $n-l$, which proves the theorem. □

Furthermore, the maximum admissible cost for the clique networks is decreasing and tends to 0 as the number of newsvendors grows.

Theorem 5.12. *K_n^{clique} is decreasing on n, and $\lim_{n\to\infty} K_n^{clique} = 0$.*

Proof. By Theorem 5.11, it needs to be shown $\beta_n - \beta_{n-1} \leq \beta_{n-1} - \beta_{n-2}$, for $n > 2$, which is equivalent to $\Delta_n \leq \Delta_{n-1}$ for $n > 2$. The latter holds by Theorem 5.6. Moreover, by Theorems 5.11 $\lim_{n\to\infty} K_n^{clique} = 2\lim_{n\to\infty}(\beta_n - \beta_{n-1}) = 2(\lim_{n\to\infty}\beta_n - \lim_{n\to\infty}\beta_{n-1}) = 0$. □

The following corollaries follow immediately from Theorem 5.12.

Corollary 5.2. *For the clique, given the cooperation cost per link K, there is a maximum transshipment game size $S(K)$ such that all transshipment games larger than $S(K)$ have empty cores, and all transshipment games of size not exceeding $S(K)$ have non-empty cores.*

Corollary 5.3. *For the clique, all transshipment games have non-empty cores only if the cooperation cost per link $K = 0$.*

Hub Network

The maximum admissible cost for the hub networks is determined as follows.

Theorem 5.13. *We have*

$$K_n^{hub} = \frac{n(\beta_n - \beta_1)}{n-1}$$

for $n < n^{**}$, and

$$K_n^{hub} = n(n-1)(\beta_n - \beta_{n-1})$$

for $n \geq n^{**}$.

Proof. It needs to be shown that $\frac{n(\beta_n-\beta_1)}{n-1} \leq \frac{nl(\beta_n-\beta_l)}{n-l}$ for $n < n^{**}$ and $l < n$, which is equivalent to $\frac{\beta_n-\beta_l}{\beta_n-\beta_1} \geq \frac{n-l}{l}\frac{1}{n-1}$ for all $l < n$. The latter holds by Theorem 5.10. For $n \geq n^{**}$, we need to show $n(n-1)(\beta_n - \beta_{n-1}) \leq \frac{nl(\beta_n-\beta_l)}{n-l}$ for $l < n$, which is equivalent to $\frac{\beta_n-\beta_l}{\beta_n-\beta_{n-1}} \geq \frac{n-l}{l}(n-1)$ for all $l < n$. The latter holds by Theorem 5.10. □

Furthermore, we have the following theorem.

Theorem 5.14. *K_n^{hub} is increasing on n for $n < n^{**}$, and decreasing on n for $n \geq n^{**}$.*

Proof. We have

$$\frac{\beta_n - \beta_1}{\beta_{n-1} - \beta_1} = \frac{1}{1 - \frac{\beta_n-\beta_{n-1}}{\beta_n-\beta_1}}.$$

By setting $l = n-1$ in Theorem 5.10, we obtain $\frac{\beta_n-\beta_{n-1}}{\beta_n-\beta_1} \geq \frac{1}{(n-1)^2}$ for $n < n^{**}$. Thus,

$$\frac{\beta_n - \beta_1}{\beta_{n-1} - \beta_1} \geq \frac{1}{1 - \frac{1}{(n-1)^2}} = \frac{(n-1)^2}{n(n-2)},$$

for $n < n^{**}$. Hence, the last inequality implies immediately that $\frac{n(\beta_n-\beta_1)}{n-1} \geq \frac{(n-1)(\beta_{n-1}-\beta_1)}{n-2}$ for $n < n^{**}$. Therefore, by Theorem 5.13, $K_n^{hub} \geq K_{n-1}^{hub}$ for $n < n^{**}$.

In order to have $K_n^{hub} \leq K_{n-1}^{hub}$ for $n \geq n^{**}$, we need $n(n-1)(\beta_n - \beta_{n-1}) \leq (n-1)(n-2)(\beta_{n-1}-\beta_{n-2})$ for $n \geq n^{**}$ by Theorem 5.13. This inequality is equivalent to $\frac{\beta_n-\beta_{n-1}}{\beta_{n-1}-\beta_{n-2}} \leq \frac{n-2}{n}$ for $n \geq n^{**}$. Theorem 5.10 for $n \geq n^{**}$ and $l = n-2$ gives $\frac{\beta_n-\beta_{n-2}}{\beta_n-\beta_{n-1}} \geq \frac{2(n-1)}{n-2}$. Finally,

$$\frac{\beta_n - \beta_{n-2}}{\beta_n - \beta_{n-1}} = 1 + \frac{\beta_{n-1} - \beta_{n-2}}{\beta_n - \beta_{n-1}} \geq \frac{2(n-1)}{n-2}$$

	n	1	2	3	4	5	6	7	8	9	10	11	12	13
	J_n	2125	4429	6756	9096	11443	13795	16152	18511	20873	23237	25603	27971	30340
$t = 5$	K_n^{clique}	NA	178	75	44	29	21	16	13	11	9	8	7	6
	K_n^{hub}	NA	178	190	198	204	209	213	216	218	221	223	224	226
	J_n	2125	4389	6667	8950	11238	13527	15818	18110	20403	22697	24992	27287	29582
$t = 10$	K_n^{clique}	NA	139	55	31	20	14	10	8	7	5	4	4	3
	K_n^{hub}	NA	139	146	150	153	155	157	158	160	161	161	162	163
	J_n	2125	4351	6583	8817	11050	13284	15517	17750	19982	22215	24446	26678	28909
$t = 15$	K_n^{clique}	NA	101	37	19	12	8	6	4	3	2	2	2	1
	K_n^{hub}	NA	101	104	105	106	106	107	107	107	107	104	100	96
	J_n	2125	4316	6507	8696	10885	13072	15259	17444	19629	21814	23998	26181	28365
$t = 20$	K_n^{clique}	NA	65	22	10	6	3	2	1	1	1	0	0	0
	K_n^{hub}	NA	65	65	63	58	52	46	40	35	30	26	22	19
	J_n	2125	4282	6437	8591	10744	12896	15047	17198	19349	21500	23650	25800	27950
$t = 25$	K_n^{clique}	NA	32	10	4	2	1	1	0	0	0	0	0	0
	K_n^{hub}	NA	32	29	24	20	16	12	9	7	5	4	3	2

Table 5.2: An Example of Over-mean Games ($r = 40$, $c = 15$, and $\nu = 10$)

which implies $\frac{\beta_n - \beta_{n-1}}{\beta_{n-1} - \beta_{n-2}} \le \frac{n-2}{n}$ for $n \ge n^{**}$ as required. □

We have the following asymptotic results about the maximum admissible costs with positive transportation costs under hub network structure.

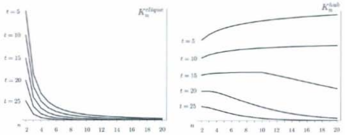
Theorem 5.15. *We have* $\lim_{n\to\infty} K_n^{hub} \le \sigma\left[(r-\nu)\phi(\dot{Y}_1) - t\phi(a)\right]$.

Proof. We have $\lim_{n\to\infty} \min_{l<n} \frac{nl(\beta_n - \beta_l)}{n-l} \le \lim_{n\to\infty} \frac{n(\beta_n - \beta_1)}{n-1}$. Also,

$$\lim_{n\to\infty} \frac{n(\beta_n - \beta_1)}{n-1} = \lim_{n\to\infty} \frac{\sigma n}{n-1} \lim_{n\to\infty}\left[(r-\nu)\phi(\dot{Y}_1) - t\phi(\dot{Y}_n) - p\frac{\phi(\sqrt{n}\dot{Y}_n)}{\sqrt{n}}\right]$$
$$= \sigma \lim_{n\to\infty}\left[(r-\nu)\phi(\dot{Y}_1) - t\phi(\dot{Y}_n) - p\frac{\phi(\sqrt{n}\dot{Y}_n)}{\sqrt{n}}\right]$$

The last converges to $\sigma\left[(r-\nu)\phi(\dot{Y}_1) - t\phi(a)\right]$. □

Theorem 5.9 gives sufficient conditions for the inequality in Theorem 5.15 to become an equality. Table 5.2 and Figure 5.7 show K_n^{clique} and K_n^{hub} as functions of the number of newsvendors n for various transportation costs t. The K_n^{hub} may or may not have a single maximum. If it does the number of newsvendors n^{**} at the maximum depends on t, e.g. $n^{**} = 10$ for $t = 15$ and $n^{**} = 3$ for $t = 20$. By Theorem 5.9, $n^{**} = \infty$ for $t = 5$. Our experiments up to $n = 100$ do not yield the n^{**} for $t = 10$, therefore, it is possible that for this value the maximum happens at larger values of n or not at all.



(a) Clique network

(b) Hub network

Figure 5.7: Example of Maximum Admissible Cost per Link as a Function of n ($r = 40$, $c = 15$, and $\nu = 10$)

5.5.2 Free Transportations

In this section we assume that the transshipments are free, that is, we let $t = 0$. Although the assumption of free transshipments is rather restrictive, it portraits *virtual* transshipments, where the transportation of goods can be done with no significant costs, or where the producers re-direct customers to each other instead of transshipping the actual product (see Wang and Parlar (1994) for an example of the latter). By setting $t = 0$ in (5.14) we obtain

$$\dot{Y}_n = \frac{1}{\sqrt{n}}\Phi^{-1}\left(\frac{r-c}{r-\nu}\right) = \frac{1}{\sqrt{n}}\dot{Y}_1, \tag{5.33}$$

and by setting $t = 0$ in (5.18) and using (5.33) we get

$$\beta_n = (r-c)\mu - \sigma(r-\nu)\phi(\dot{Y}_1)\frac{\sqrt{n}}{n}. \tag{5.34}$$

Clique Network

The maximum admissible cost for cliques with costless transportations is as follows.

Theorem 5.16. $K_n^{clique} = 2\sigma(r-\nu)\phi(\dot{Y}_1)\left(\frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n}}\right)$.

Proof. By (5.34) and Theorem 5.11. □

Hub Network

By (5.34) the condition (5.32) for the hub networks becomes

$$K \le \min_{l<n} \sigma(r-\nu)\phi(\dot{Y}_1)\left(\frac{\sqrt{nl}}{\sqrt{l}+\sqrt{n}}\right). \tag{5.35}$$

Therefore, the maximum admissible cost for hubs with costless transportations is as follows.

Theorem 5.17. $K_n^{hub} = \sigma(r-\nu)\phi(\dot{Y}_1)\left(\frac{\sqrt{n}}{1+\sqrt{n}}\right)$.

Proof. It needs to be shown that $\frac{\sqrt{nl}}{\sqrt{l}+\sqrt{n}}$ is increasing in l and thus attains minimum at $l=1$. Thus, it suffices to show that $\frac{\sqrt{nl}}{\sqrt{l}+\sqrt{n}} > \frac{\sqrt{n(l-1)}}{\sqrt{l-1}+\sqrt{n}}$. The latter inequality holds since $\sqrt{l} > \sqrt{l-1}$. Therefore, $\frac{\sqrt{nl}}{\sqrt{l}+\sqrt{n}}$ is increasing in l and attains its minimum at $l=1$. □

Contrary to the clique networks, this cost increases as the game size grows which follows from the following theorem.

Theorem 5.18. *K_n^{hub} is an increasing function of n. Moreover, $\lim_{n\to\infty} K_n^{hub} = \sigma(r-\nu)\phi(\dot{Y}_1)$.*

Proof. We need to show that $K_n^{hub} < K_{n+1}^{hub}$. This inequality holds since $\frac{\sqrt{n}}{1+\sqrt{n}} < \frac{\sqrt{n+1}}{1+\sqrt{n+1}}$. Obviously, $\lim_{n\to\infty} \frac{\sqrt{n}}{1+\sqrt{n}} = 1$, and thus $\lim_{n\to\infty} K_n^{hub} = \sigma(r-\nu)\phi(\dot{Y}_1)$. □

Corollary 5.4. *If the transshipment game with n newsvendors has a non-empty core for cost per link K, then so do all larger games with the same cost per link.*

Theorems 5.17 and 5.18 also imply that the number of symmetric newsvendors may be *insufficient*, for a given cost per link K, to have a stable coalition. In other words, if n' symmetric newsvendors consider cooperating in a game with cooperation cost per link K, then $K_{n'}^{hub} < K$ proves that their grand coalition is too expensive to form for it is simply too small. Therefore, searching for more symmetric newsvendors willing to join in and expand the game could make the transshipment game worth playing. To find the size of this minimal expansion one needs to solve the equation $K = \sigma(r-\nu)\phi(\dot{Y}_1)\left(\frac{\sqrt{n}}{1+\sqrt{n}}\right)$, to determine n using the bisection method and then round up the solution to the closest integer. Finally, subtracting n' would give the required expansion size. Clearly, only if the cost per link K does not exceed $\sigma(r-\nu)\phi(\dot{Y}_1)\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)$ all size games are worth playing for all of them have non-empty cores.

5.5.3 Mean Newsvendors

We now consider an important case of mean symmetric newsvendors. For a mean newsvendor marginal profit equals the marginal loss of unsold items, that is $r-c=c-\nu$. In this case, by Lemma 1, $\dot{Y}_n = 0$ for $n \geq 1$. Therefore, the maximum expected profit in (5.18) becomes

$$\beta_n = (r-c)\mu - \frac{\sigma}{\sqrt{2\pi}}\left(t + \frac{\sqrt{n}}{n}p\right). \tag{5.36}$$

Clique Network

We have the following maximum admissible cost for the clique network of mean newsvendors.

Theorem 5.19. $K_n^{clique} = \frac{2\sigma p}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n}}\right)$.

Proof. By (5.36) and Theorem 5.11. □

The key difference between the maximum admissible cost in this theorem and the one in Theorem 5.16 is that the former depends on t. Therefore, the higher transportation costs the fewer newsvendors can play a transshipment game with non-empty core for a given cooperation cost per link K.

Hub Network

We have the following maximum admissible cost for the hub network of mean newsvendors.

Theorem 5.20. $K_n^{hub} = \frac{\sigma p}{\sqrt{2\pi}}\left(\frac{\sqrt{n}}{1+\sqrt{n}}\right)$.

Proof. By (5.36), $\frac{nl(\beta_n-\beta_l)}{n-l} = \frac{\sigma p}{\sqrt{2\pi}}\left(\frac{\sqrt{nl}}{\sqrt{n}+\sqrt{l}}\right)$. As shown in the proof of Theorem 5.17, $\frac{\sqrt{nl}}{\sqrt{n}+\sqrt{l}}$ is increasing on l and attains its minimum at $l = 1$. Therefore, the maximum admissible cooperation cost per link of the hub network structure for which the transshipment game with n symmetric newsvendors has non-empty core is $K_n^{hub} = \frac{\sigma(r-\nu-t)}{\sqrt{2\pi}}\left(\frac{\sqrt{n}}{1+\sqrt{n}}\right)$. □

For any fixed transportation cost t, the counterparts of Theorem 5.18 and Corollary 5.4 hold for mean symmetric vendors. Again, the key difference is that maximum admissible cost in this case depends on t. Therefore, the higher transportation costs the fewer symmetric newsvendors *suffices* to play a transshipment game with non-empty core for a given cooperation cost per link.

5.6 Comments

The stability of the games with asymmetric agents and arbitrary network structures can only be determined numerically through the examination of all possible sub coalitions and their comparison with the individual allocations under grand coalition. This, even if possible in theory, can only be done for limited game sizes in practice due to the problem of computational intractability. Therefore, there is a great need for the insight obtained analytically which this chapter is motivated by.

This chapter is the first to incorporate cooperation costs in the analysis of decentralized transshipment games in the operational research and operations management literature. We believe that including the cooperation costs into the game theory based supply chain models provides, and will continue to provide, new and interesting insights into their possible application in real-life supply chain coordination and management.

Chapter 6

Conclusions and Open Problems

The opportunities for research on supply chain contracting and coordination are numerous—as partly shown in Chapter 2. In fact, the research on supply chain contracts is still in its infancy and there is plenty of room for building upon the current research and expanding it. The analysis of the literature in Chapter 2 reveals that most of the coordinating contracts require the following preliminary conditions: (1) rationality of the players, (2) absence of contracting costs, (3) complete knowledge structure, (4) risk neutrality, and (5) profit orientedness. However, most of these assumptions, if not all, do not provide an adequate realistic picture of the supply chains in which they ought to be applied. Agents might not know how to optimize their decisions or they may not have the sufficient computational power to actually calculate them. The information sharing among the agents is very limited. Agents' behavior is opportunistic and there are various types of agents with regard to their utilities. Therefore, unless the gap between the theory and the practice does not close, the insights achieved from the research will be questionable. Among the possibilities for future research in this area are: (1) incorporating the under-analyzed aspects of supply chain contracting, e.g. verifiability and compliance; (2) refining the definition of acceptability in coordinating contracts; (3) considering more general utility functions of supply chain members in order to capture realistic decision making criteria; (4) investigating more complex supply chain topologies; and (5) strengthening the usefulness of theoretical insights through empirical and case-based studies.

With respect to the decentralized transshipment problem, in Chapter 3, we proposed a contract with an implicit pricing mechanism (demonstrated in Lemma 3.1) that can coordinate the transshipments in a two-agent supply chain. This contract has several desirable properties. First, the implicit pricing mechanism gives rise to the choice of the best production quantities (see Theorem 3.1). This is particularly important because the linear pricing mechanisms in Rudi et al. (2001), Hu et al. (2007), and Huang and Sošić (2010b) do not necessarily lead to the Nash equilibrium being the best production quantities. Second, the implicit pricing mechanism allows for an arbitrary division of total expected extra profit according to the bargaining powers. Third, when the agents fix the negotiated transshipment prices they usually have multiple alternatives to choose from (as Theorem 3.2 implies). Thus, a secondary criterion can also be used to fine-tune the choice of transshipment prices. We suggest the minimization of the variances of the agents' individual profits. A direction for generalization is to include the agents' competition when they choose their market selling prices. Recently, Zhao and Atkins (2009) analyze the transshipment prices in a two-agent supply chain where price-sensitive demand functions reflect the competition over the selling prices.

We have addressed the decentralized transshipment problem with n agents in Chapter 4. The contracts based on allocation rules address the coordination for this problem but the practical difficulties of allocation rules motivated our approach. The contracts with transshipment prices provide more flexibility by letting the individual agents choose their transshipment partners. The allocation rule proposed in Anupindi et al.

(2001) has the desirable property of both being in the core of the second stage cooperative game and coordinating the individual decisions on production quantities. For those reasons, we have constructed our transshipment prices (as shown in Lemma 4.2) upon those allocations. We showed that with the transshipment prices derived from this allocation rule, the optimum transshipment patterns are always pair-wise stable (see Theorem 4.2). Moreover, by carrying out the optimum transshipment patterns, each agent receives a profit which equals the Anupindi et al. (2001) allocation for that agent (see Corollary 4.1). The contribution of Chapter 4 is to implement a solution concept from the network games in two-sided markets for the first time in analyzing the decentralized transshipment problem.

Chapter 5 of the thesis incorporates the costs of cooperation into the analysis of the stability of decentralized transshipment games in coordinated supply chains. In order to obtain provable results, we have considered supply chains with symmetric newsvendors and independent and normally distributed demands. Assuming cooperation cost to be directly proportional to the number of links in the coalition network, we examine two general network structures: *Clique*, where all agents are connected to each other, and *Hub*, where all agents are solely connected to a designated agent. We provide the conditions for the stability of such games. Drawing upon the two laws of diminishing individual allocations (Theorem 5.6 and Theorem 5.8), we demonstrate that under the clique structure, the stability of symmetric transshipment games becomes more susceptible to the cooperation costs as the number of participating newsvendors increases (see Theorem 5.12 and Corollary 5.2). However, this effect is bi-polar under the hub structure, that is, while increasing the size of game, up to a certain size, enables newsvendors to handle larger cooperation cost per links, and this increase in size after some threshold will negatively impact the stability of the grand coalitions (see Theorem 5.14). Though the characteristic function in the transshipment games studied in Chapter 5 are expected values of possible allocations, which is also the case for the games studied in Slikker et al. (2005) and Chen and Zhang (2009), we realize that an adequate link between these games and the deterministic games with the characteristic function determined by the realization of demands still needs to be established. An immediate important direction for further research is to study connected networks that fall between the clique and the hub. Yet another is the extension of the model to include correlations between newsvendors' demands. Also, it remains open whether or not the existence of a finite n^* implies the existence of a finite n^{**}. Finally, the transshipment games with cooperation costs played by asymmetric newsvendors remain a great challenge for analytical treatment for now. They remain so even under the assumption that demands are normal and independent though with different means and standard deviations. However, some questions motivated by this chapter may be a lesser challenge and yet provide interesting insights. One such a question is when would the game be over-mean, under-mean, or mean? Or when does the maximum admissible cost per link for hubs remain unimodal? These questions are left for future research.

Bibliography

C.R. Adrian and C. Press. Decision costs in coalition formation. *The American Political Science Review*, 62(2):556–563, 1968.

J.A. Alfaro and C.J. Corbett. The value of SKU rationalization (The pooling effect under suboptimal inventory policies). *Production Operations Management Journal*, 12(1):12–29, 2003.

R. Anupindi, Y. Bassok, and E. Zemel. A general framework for the study of decentralized distribution systems. *Manufacturing & Service Operations Management*, 3(4):349–368, 2001.

K.W. Artz and T.H. Brush. Asset specificity, uncertainty and relational norms: an examination of coordination costs in collaborative strategic alliances. *Journal of Economic Behavior & Organization*, 41(4):337–362, 2000.

R.J. Aumann. *Collected papers: Volume 2*, chapter A survey of cooperative games without side payments. MIT Press, 2000.

M. Baiou and M. Balinski. Erratum: The stable allocation (or ordinal transportation) problem. *Mathematics of Operations Research*, 27(4):662–680, 2002.

D. Barnes-Schuster, Y. Bassok, and R. Anupindi. Coordination and flexibility in supply contracts with options. *Manufacturing & Service Operations Management*, 4(3):171–207, 2002.

U. Benzion, Y. Cohen, R. Peled, and T. Shavit. Decision-making and the newsvendor problem: an experimental study. *Journal of the Operational Research Society*, 59(9): 1281–1287, 2008.

F. Bernstein and A. Federgruen. Decentralized supply chains with competing retailers under demand uncertainty. *Management Science*, 51(1):18–29, 2005.

G.E. Bolton and E. Katok. Learning by doing in the newsvendor problem: A laboratory investigation of the role of experience and feedback. *Manufacturing & Service Operations Management*, 10(3):519–538, 2008.

A. Brandenburger and H. Stuart. Biform games. *Management Science*, 53(4):537–549, 2007.

E. Brousseau and J. M. Glachant. *The economics of contracts: theories and applications*. Cambridge University Press, 2002.

A. Burnetas, S.M. Gilbert, and C.E. Smith. Quantity discounts in single-period supply contracts with asymmetric demand information. *IIE Transactions*, 39(5):465–479, 2007.

G.P. Cachon. *Quantitative models for supply chain management*, chapter Competitive supply chain inventory management. Kluwer, 1999.

G.P. Cachon. *Handbooks in Operations Research and Management Science: Supply chain management*, chapter Supply Chain Coordination with Contracts. North-Holland, 2003.

G.P. Cachon. The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts. *Management Science*, 50(2):222–238, 2004.

G.P. Cachon and A.G. Kok. Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination. *Management Science*, 56(3):571–589, 2010.

G.P. Cachon and M.A. Lariviere. Capacity choice and allocation: strategic behavior and supply chain performance. *Management Science*, 45(8):1091–1108, 1999.

G.P. Cachon and M.A. Lariviere. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Science*, 47(5):629–646, 2001.

G.P. Cachon and M.A. Lariviere. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, 51(1):30–44, 2005.

G.P. Cachon and S. Netessine. *Supply Chain Analysis in the eBusiness Era*, chapter Game theory in supply chain analysis. Kluwer, 2004.

G.P. Cachon and S. Netessine. *Tutorials in Operations Research*, chapter Game theory in supply chain analysis. INFORMS, 2006.

G.P. Cachon and P.H. Zipkin. Competitive and cooperative inventory policies in a two-stage supply chain. *Management science*, 45(7):936–953, 1999.

R. Caldentey and L. M. Wein. Analysis of a decentralized production-inventory system. *Manufacturing & Service Operations Management*, 5(1):1–17, 2003.

A.K. Chakravarty and J. Zhang. Lateral capacity exchange and its impact on capacity investment decisions. *Naval Research Logistics*, 54(6):632–644, 2007.

L.M.A. Chan, Z.J.M. Shen, D. Simchi-Levi, and J.L. Swann. *Handbook of quantitative supply chain analysis: Modeling in the E-Business Era*, chapter Coordination of pricing and inventory decisions: A survey and classification. Kluwer, 2004.

A. Charnes and D. Granot. Coalitional and chance-constrained solutions to n-person games, II: Two-stage solutions. *Operations Research*, 25(6):1013–1019, 1977.

F. Chen. Decentralized supply chains subject to information delays. *Management Science*, 45(8):1076–1090, 1999.

F. Chen and M. Parlar. Value of a put option to the risk-averse newsvendor. *IIE Transactions*, 39(5):481–500, 2007.

X. Chen and J. Zhang. A stochastic programming duality approach to inventory centralization games. *Operations Research*, 57(4):840–851, 2009.

X. Chen, C.L. Li, B.D. Rhee, and D. Simchi-Levi. The impact of manufacturer rebates on supply chain profits. *Naval Research Logistics*, 54(6):667–680, 2007.

A. Chinchuluun, A. Karakitsiou, and A. Mavrommati. *Pareto optimality, game theory and equilibria*, chapter Game Theory Models and Their Applications in Inventory Management and Supply Chain. Springer, 2008.

J. Chod and N. Rudi. Investment, trading, and pricing under forecast updating. *Management Science*, 52(12):1913–1929, 2006.

T.M. Choi, D. Li, H. Yan, and C.H. Chiu. Channel coordination in supply chains with agents having mean-variance objectives. *Omega*, 36(4):565–576, 2008.

A.J. Clark and H. Scarf. Optimal policies for a multi-echelon inventory problem. *Management Science*, 50(12):475–490, 1960.

R.H. Coase. The nature of the firm. *Economica*, 4(16):386–405, 1937.

C.J. Corbett and C.S. Tang. *Quantitative models for supply chain management*, chapter Designing supply contracts: contract type and information symmetry. Kluwer, 1999.

C.J. Corbett, D. Zhou, and C.S. Tang. Designing supply contracts: Contract type and information asymmetry. *Management Science*, 50(4):550–559, 2004.

F. Cruijssen, M. Cools, and W. Dullaert. Horizontal cooperation in logistics: Opportunities and impediments. *Transportation Research Part E*, 43(2):129–142, 2007.

T.H. Cui, J.S. Raju, and Z.J. Zhang. Fairness and channel coordination. *Management Science*, 53(8):1303–1314, 2007.

C. Das. Supply and redistribution rules for two-location inventory systems: one-period analysis. *Management Science*, 21(7):765–775, 1975.

L. G. Debo, R. C. Savaskan, and L. N. Van Wassenhove. *Reverse Logistics, Quantitative Models for Closed Loop Supply Chains*, chapter Coordination in closed-loop supply chains. Springer-Verlag, Berlin, 2004.

M.A.H. Dempster, H. N. Pedron, E.A. Medova, J.E. Scott, and A. Sembos. Planning logistics operations in the oil industry. *Journal of the Operational Reasearch Society*, 51 (11):1271–1288, 2000.

R. Deneckere, H. Marvel, and J. Peck. Demand uncertainty and price maintenance: amrkdowns as destructive competition. *American Economic Review*, 87(4):619–41, 1997.

D. Ding and J. Chen. Coordinating a three level supply chain with flexible return policies. *Omega*, 36(5):865–876, 2008.

R.J. Dolan. Quantity discounts: managerial issues and research opportunities. *Marketing Science*, 6(1):1–22, 1987.

L. Dong and N. Rudi. Who benefits from transshipment? exogenous vs. endogenous wholesale prices. *Management Science*, 50(5):645–657, 2004.

K.L. Donohue. Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Science*, 46(11):1397–1411, 2000.

S.E. Fawcett, G.M. Magnan, and M.W. McCarter. Benefits, barriers, and bridges to effective supply chain management. *Supply Chain Management: An International Journal*, 13(1): 35–48, 2008.

B. Fristedt and L.F. Gray. *A modern approach to probability theory.* Springer, 1997.

D. Fudenberg and J. Tirole. *Game Theory.* MIT Press, 2002.

X. Gan, S. P. Sethi, and H. Yan. Coordination of supply chains with risk-averse agents. *Production and Operations Management*, 13(2):135–149, 2004.

X. Gan, S. P. Sethi, and H. Yan. Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer. *Production & Operations Management*, 14(1):80–89, 2005.

A. Gomez-Padilla, J. Duvallet, and D. Daniel Llerena. *Research methodologies in supply chain management*, chapter Contract typology as a research method in supply chain management. Physica-Verlag HD, 2005.

D. Granot and G. Sošić. A three-stage model for a decentralized distribution system of retailers. *Operations Research*, 51(5):771–784, 2003.

D. Gross. Centralized inventory control in multilocation supply systems. *Multistage Inventory Models and Techniques*, pages 47–84, 1963.

V. Grover and M.K. Malhotra. Transaction cost framework in operations and supply chain management research: theory and measurement. *Journal of Operations management*, 21 (4):457–473, 2003.

R. Gulati. Alliances and networks. *Strategic management journal*, 19(4):293–317, 1998.

A.Y. Ha and S. Tong. Revenue sharing contracts in a supply chain with uncontractible actions. *Naval Research Logistics*, 55(5):419–431, 2008.

E. Hanany, M. Tzur, and A. Levran. The transshipment fund mechanism: Coordinating the decentralized multilocation transshipment problem. *Naval Research Logistics*, 57(4): 342–353, 2010.

J.C. Harsanyi and R. Selten. A generalized Nash solution for two-person bargaining games with incomplete information. *Management Science*, 18(5):80–106, 1972.

B. C. Hartman and M. Dror. Allocation of gains from inventory centralization in newsvendor environments. *IIE Transactions*, 37(2):93–107, 2005.

M. Hazewinkel. *Encyclopaedia of Mathematics*. Kluwer Academic Publishers, 2002.

Y. He and J. Zhang. Random yield risk sharing in a two-level supply chain. *International Journal of Production Economics*, 112(2):769–781, 2008.

Y. He, X. Zhao, L. Zhao, and J. He. Coordinating a supply chain with effort and price dependent stochastic demand. *Applied Mathematical Modelling*, 33(6):2777–2790, 2009.

Y.T. Herer and A. Rashit. Lateral stock transshipments in a two-location inventory system with fixed and joint replenishment costs. *Naval Research Logistics*, 46(5):525–547, 1999.

B. Hezarkhani and W. Kubiak. Symmetric newsvendor transshipment games with cooperation costs. *To be submitted.*

B. Hezarkhani and W. Kubiak. Transshipment prices and pair-wise stability in coordinating the decentralized transshipment problem. In *BQGT '10: Proceedings of the Behavioral and Quantitative Game Theory*, pages 1–6, 2010a.

B. Hezarkhani and W. Kubiak. A coordinating contract for transshipment in a two-company supply chain. *European Journal of Operational Research*, 207(1):232–237, 2010b.

B. Hezarkhani and W. Kubiak. Coordinating contracts in SCM: A review of methods and literature. *Decision Making in Manufacturing and Services*, 4(1-2):5–28, 2010c.

B. Hirschler and K. Kelland. EU states may share scant H1N1 vaccine supplies. *Reuters*, Available at http://www.reuters.com /article/GCA-SwineFlu/ idUSTRE58E4HT20090915?sp=true, 2009.

X. Hu, I. Duenyas, and R. Kapuscinski. Existence of coordinating transshipment prices in a two-location inventory model. *Management Science*, 53(8):1289–1302, 2007.

X.H. Huang and G. Sošić. Repeated newsvendor game with transshipments under dual allocations. *European Journal of Operational Research*, 204(2):274–284, 2010a.

X.H. Huang and G. Sošić. Transshipment of Inventories: Dual Allocations vs. Transshipment Prices. *to appear in Manufacturing & Service Operations Management*, 12(2):299–318, 2010b.

C. A. Ingene and M. E. Parry. *Mathematical models of distribution channels*. Springer US, 2004.

M.O. Jackson. *Group formation in economics: Networks, clubs, and coalitions*, chapter A survey of models of network formation: Stability and efficiency. Cambridge, MA: Cambridge University Press, 2005.

A. P. Jeuland and S. M. Shugan. Managing channel profits. *Marketing Science*, 2(3): 239–272, 1983.

E. Katok and D.Y. Wu. Contracting in supply chains: A laboratory investigation. *Management Science*, 55(12):1953–1968, 2009.

M. Khouja. The single-period (news-vendor) problem: literature review and suggestions for future research. *Omega*, 27(5):537–553, 1999.

T.D. Klastorin, K. Moinzadeh, and J. Son. Coordinating orders in supply chains through price discounts. *IIE Transactions*, 34(8):679–689, 2002.

W.A. Kosmala. *Advanced calculus: A friendly approach*. Prentice Hall, 1998.

M.A. Krajewska, H. Kopfer, G. Laporte, S. Ropke, and G. Zaccour. Horizontal cooperation among freight carriers: request allocation and profit sharing. *Journal of Operational Research Society*, 59(11):1483–1491, 2007.

H. Krishnan and R.A. Winter. Inventory dynamics and supply chain coordination. *Management Science*, 56(1):141–147, 2010.

K.S. Krishnan and V.R.K. Rao. Inventory control in n warehouses. *The Journal of Industrial Engineering*, 16:212–215, 1965.

M.A. Lariviere and E.L. Porteus. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing and Service Operations Management*, 3(4):293–305, 2001.

H.L. Lee. The triple-A supply chain. *Harvard Business Review*, 82(10):102–113, 2004.

H.L. Lee and S. Whang. Decentralized multi-echelon supply chains: Incentives and information. *Management Science*, 45(5):633–640, 1999.

M. Leng and A. Zhu. Side-payment contracts in two-person non zero-sum supply chain games: Review, discussion and applications. *European Journal of Operational Research*, 196(2):600–618, 2009.

X. Li and Q. Wang. Coordination mechanisms of supply chain systems. *European Journal of Operational Research*, 179(1):1–16, 2007.

C.H. Loch and Y. Wu. Social Preferences and Supply Chain Performance: An Experimental Study. *Management Science*, 54(11):1835–1849, 2008.

T. McLaren, M. Head, and Y. Yuan. Supply chain collaboration alternatives: understanding the expected costs and benefits. *Internet Research: Electronic Networking Applications and Policy*, 12(4):348–364, 2002.

R. Mogre, A. Perego, and A. Tumino. RFID-enabled lateral trans-shipments in the fashion & apparel supply chain. Technical report, www.rfidsolutioncenter.it, 2009.

A. Muthoo. A bargaining model based on the commitment tactic. *Journal of Economic Theory*, 69(1):134–152, 1996.

M. Nagarajan and G. Sošić. Game-theoretic analysis of cooperation among supply chain agents: review and extensions. *European Journal of Operational Research*, 187(3):719–745, 2008.

V.G. Narayanan and A. Raman. Aligning incentives in supply chains. *Harvard Business Review*, 82(11):94 – 102, 2004.

J.F. Nash. The bargaining problem. *Econometrica: Journal of the Econometric Society*, 18 (2):155–162, 1950.

J.F. Nash. Noncooperative games. *The Annals of Mathematics*, 54(2):286–295, 1951.

M.J. Osborne and A. Rubinstein. *Bargaining and Markets*. Academic Press, Inc., 1990.

M.J. Osborne and A. Rubinstein. *A course in game theory*. MIT press, 1994.

G. Owen. On the core of linear production games. *Mathematical programming*, 9(1):358–370, 1975.

G. Owen. *Game Theory*. Academic Press, 1995.

U. Ozen, M. Slikker, and H. Norde. A general framework for cooperation under uncertainty. *Operations Research Letters*, 37(3):148–154, 2009.

M. Parlar. Game theoretic analysis of the subsitutable product inventory problem with random demands. *Naval Research Logistics*, 35:397–409, 1988.

B.A. Pasternack. Optimal pricing and return policies for perishable commodities. *Marketing Science*, 4(2):166–176, 1985.

B.A. Pasternack and Z. Drezner. Optimal inventory policies for substitutable commodities with stochastic demand. *Naval Research Logistics*, 38:221–240, 1991.

C. Paterson, G. Kiesmuller, R. Teunter, and K. Glazebrook. Inventory models with lateral transshipments: A review. *European Journal of Operational Research*, 210(2):125–136, 2011.

B. Peleg. An axiomatization of the core of cooperative games without side payments. *Game and Economic Theory: Selected Contributions in Honor of Robert J. Aumann*, 1995.

E.L. Porteus. *Foundations of stochastic inventory theory*. Stanford University Press, 2002.

Z. Qin and J. Yang. Analysis of a revenue-sharing contract in supply chain management. *International Journal of Logistics Research and Applications*, 11(1):17–29, 2008.

U. Rao, A. Scheller-Wolf, and S. Tayur. Development of a rapid-response supply chain at Caterpillar. *Operations Research*, 48(2):189–204, 2000.

L. Rosenkopf and M.A. Schilling. Comparing alliance network structure across industries: observations and explanations. *Strategic Entrepreneurship Journal*, 1(3-4):191–209, 2007.

A.E. Roth. *Axiomatic models of bargaining*. Springer-Verlag Berlin, 1979.

P.A. Rubin and J.R. Carter. Joint optimality in buyer-supplier negotiations:. *Journal of purchasing and materials management*, 26(2):20–26, 1990.

A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society*, 50(1):97–109, 1982.

N. Rudi, S. Kapur, and D.F. Pyke. A two-location inventory model with transshipment and local decision making. *Management Science*, 47(12):1668–1680, 2001.

K. Ryu and E. Yucesan. A fuzzy newsvendor approach to supply chain coordination. *European Journal of Operational Research*, 200(2):421–438, 2010.

M.S. Sajadieh, A. Thorstenson, and M.R.A. Jokar. An integrated vendor-buyer model with stock-dependent demand. *Transportation Research Part E: Logistics and Transportation Review*, 46(6):963–974, 2010.

J. Sánchez-Soriano. Pairwise solutions and the core of transportation situations. *European Journal of Operational Research*, 175(1):101–110, 2006.

S.P. Sarmah, D. Acharya, and S.K. Goyal. Buyer vendor coordination models in supply chain management. *European Journal of Operational Research*, 175(1):1–15, 2006.

M.E. Schweitzer and G.P. Cachon. Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Science*, 46(3):404–420, 2000.

J. Shao, H. Krishnan, and S.T. McCormick. Incentives for Transshipment in Decentralized Supply Chains with Competing Retailers. Working Paper, Sauder School of Business, University of British Columbia, 2008.

H. Shin and W.C. Benton. A quantity discount approach to supply chain coordination. *European Journal of Operational Research*, 180(2):601–616, 2007.

M. Slikker, J. Fransoo, and M. Wouters. Cooperation between multiple news-vendors with transshipments. *European Journal of Operational Research*, 167(2):370–380, 2005.

L. Smith-Doerr and W.W. Powell. *The handbook of economic sociology*, volume 2, chapter Networks and economic life, pages 379–402. Princeton University Press, 2005.

G. Sošić. Transshipment of inventories among retailers: Myopic vs. farsighted stability. *Management Science*, 52(10):1493–1508, 2006.

J. Spengler. Vertical integration amnd antitrust policy. *Journal of Political Economy*, 58 (4):347–352, 1950.

H. Stadtler and C. Kilger. *Supply chain management and advanced planning: concepts, models, software, and case studies.* Springer Verlag, 2008.

X. Su. Consumer returns policies and supply chain performance. *Manufacturing & Service Operations Management*, 11(4):595–612, 2009.

N. Suakkaphong and M. Dror. Managing decentralized inventory and transshipment. *TOP*, pages 1–27, 2010.

E. Sucky. A bargaining model with asymmetric information for a single supplier-single buyer problem. *European Journal of Operational Research*, 171(2):516–535, 2006.

J. Suijs, P. Borm, A. De Waegenaere, and S. Tijs. Cooperative games with stochastic payoffs. *European Journal of Operational Research*, 113(1):193–205, 1999.

J.M. Swaminathan and S.R. Tayur. Models for supply chains in e-business. *Management Science*, 49(10):1387–1406, 2003.

G. Tagaras. Effects of pooling on the optimization and service levels of two-location inventory systems. *IIE Transactions*, 21(3):250–257, 1989.

T.A. Taylor. Supply chain coordination under channel rebates with sales effort effects. *Management Science*, 48(8):992–1007, 2002.

D.J. Thomas and P.M. Griffin. Coordinated supply chain management. *European Journal of Operational Research*, 94(1):1–15, 1996.

B. Tomlin. Capacity Investments in Supply Chains: Sharing the Gain Rather Than Sharing the Pain. *Manufacturing and Service Operations Management*, 5(4):317–333, 2003.

A.A. Tsay. The quantity flexibility contract and supplier-customer incentives. *Management Science*, 45(10):1339–1358, 1999.

A.A. Tsay, S. Nahmias, and N. Agrawal. *Quantitative models for supply chain management*, chapter Modeling supply chain contracts: A review. Kluwer, 1999.

S. Ulku, B. Toktay, and E. Yucesan. Risk ownership in contract manufacturing. *Manufacturing & Service Operations Management*, 9(3):225–241, 2007.

B. Uzzi. The sources and consequences of embeddedness for the economic performance of organizations: The network effect. *American Sociological Review*, 61(4):674–698, 1996.

A. Van den Nouweland. *Group formation in economics: networks, clubs and coalitions*, chapter Models of network formation in cooperative games. Cambridge University Press, 2005.

J.A. Van Mieghem. Coordinating investment, production, and subcontracting. *Management Science*, 45(7):954–971, July 1999.

J.A. Van Mieghem. Capacity management, investment, and hedging: review and recent developments. *Manufacturing & Service Operations Management*, 5(4):269–302, 2003.

S. Voß and G. Schneidereit. *Cost management in supply chains*, chapter Interdependencies between supply contracts and transaction costs, pages 253–272. Springer, 2002.

Q. Wang and M. Parlar. A three-person game theory model arising in stochastic inventory control theory. *European Journal of Operational Research*, 76(1):83–97, 1994.

Y. Wang, L. Jiang, and Z.J. Shen. Channel performance under consignment contract with revenue sharing. *Management Science*, 50(1):34–47, 2004.

S. Whang. Coordination in operations: a taxonomy. *Journal of Operations Management*, 12(3-4):413–422, 1995.

O.E. Williamson. *Markets and hierarchies, analysis and antitrust implications.* Free Press New York, 1975.

S.D. Wu. *Handbook of quantitative supply chain analysis: Modeling in the E-Business Era*, chapter Supply chain intermediation: A bargaining theoretic framework, pages 67–115. Kluwer, 2004.

R. Xu and X. Zhai. Analysis of supply chain coordination under fuzzy demand in a two-stage supply chain. *Applied Mathematical Modelling*, 34(1):129–139, 2010.

J. Yang and Z. Qin. Capacitated production control with virtual lateral transshipments. *Operations Research*, 55(6):1104–1119, 2007.

C. Yano and S. Gilbert. Coordinated pricing and production/procurement decisions: A review. *Managing Business Interfaces*, 16(2):65–103, 2005.

Z. Yao, S.C.H. Leung, and K.K. Lai. Analysis of the impact of price-sensitivity factors on the returns policy in coordinating supply chain. *European Journal of Operational Research*, 187(1):275–282, 2008a.

Z. Yao, S.C.H. Leung, and K.K. Lai. Manufacturer's revenue-sharing contract and retail competition. *European Journal of Operational Research*, 186(2):637–651, 2008b.

W. Ying and T. Choi. Mean-variance analysis of supply chains under wholesale pricing and profit sharing schemes. *European Journal of Operational Research*, 204(2):255–262, 2010.

G. Zaccour. On the coordination of dynamic marketing channels and two-part tariffs. *Automatica*, 44(5):1233–1239, 2008.

A. Zaheer and N. Venkatraman. Relational governance as an interorganizational strategy: An empirical test of the role of trust in economic exchange. *Strategic Management Journal*, 16(5):373–392, 1995.

F. Zhang. Competition, Cooperation, and Information Sharing in a Two-Echelon Assembly System. *Manufacturing & Service Operations Management*, 8(3):273–291, 2006.

H. Zhao, V. Deshpande, and J.K. Ryan. Inventory sharing and rationing in decentralized dealer networks. *Management Science*, 51(4):531–547, 2005.

H. Zhao, V. Deshpande, and J.K. Ryan. Emergency transshipment in decentralized dealer networks: When to send and accept transshipment requests. *Naval Research Logistics*, 53(6):547–567, 2006.

X. Zhao and D. Atkins. Transshipment between competing retailers. *IIE Transactions*, 41 (8):665–676, 2009.

H. Zijm and J. Timmer. Coordination mechanisms for inventory control in three-echelon serial and distribution systems. *Annals of Operations Research*, 158(1):161–182, 2008.

K. Zimmer. Supply chain coordination with uncertain just-in-time delivery. *International journal of production economics*, 77(1):1–15, 2002.

P.H. Zipkin. *Foundations of inventory management*. McGraw-Hill Boston, 2000.

X. Zou, S. Pokharel, and R. Piplani. A two-period supply contract model for a decentralized assembly system. *European Journal of Operational Research*, 187(1):257–274, 2008.

Appendix

Derivation of (3.4) and (3.5)

From (3.3) we have

$$\begin{aligned} J_i^{DC}(\mathbf{s},\mathbf{X}) = &\mathsf{E}\Big[r_i \min\left((D_i, X_i) + W_{ji}(\mathbf{X})\right) - s_{ji}W_{ji}(\mathbf{X}) + (s_{ij} - t_{ij})W_{ij}(\mathbf{X}) + \\ &\nu_i\left((X_i - D_i)^+ - W_{ij}(\mathbf{X})\right) - h_i\left((D_i - X_i)^+ - W_{ji}(\mathbf{X})\right) - c_i X_i\Big] \end{aligned}$$

By rearranging we get

$$\begin{aligned} J_i^{DC}(\mathbf{s},\mathbf{X}) = &\mathsf{E}\Big[(s_{ij} - t_{ij} - \nu_i)W_{ij}(\mathbf{X}) + (r_i + h_i - s_{ji})W_{ji}(\mathbf{X}) + \\ &r_i \min(D_i, X_i) + \nu_i(X_i - D_i)^+ - h_i(D_i - X_i)^+ - c_i X_i\Big] \end{aligned}$$

which is equivalent to

$$\begin{aligned} J_i^{DC}(\mathbf{s},\mathbf{X}) = &\mathsf{E}\left[(s_{ij} - t_{ij} - \nu_i)W_{ij}(\mathbf{X}) + (r_i + h_i - s_{ji})W_{ji}(\mathbf{X})\right] + \\ &\mathsf{E}\left[r_i \min(D_i, X_i) + \nu_i(X_i - D_i)^+ - h_i(D_i - X_i)^+ - c_i X_i\right]. \end{aligned}$$

For $i, j = 1, 2$, the latter results in (3.4) and (3.5).

Concavity of (3.9)

Let

$$J_i^E(\mathbf{s},\mathbf{X}) = J_i^{DC}(\mathbf{s},\mathbf{X}) - J_i^{NC}(X_i^{NC}) \tag{6.1}$$

for $i = 1, 2$. We have

$$\frac{\partial f_g}{\partial s_{12}} = \frac{\partial\left([J_1^E(\mathbf{s},\mathbf{X})]^\gamma [J_2^E(\mathbf{s},\mathbf{X})]^{1-\gamma}\right)}{\partial s_{12}} = \Gamma_{12}(\mathbf{X})[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-1}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma}\left(\gamma[J_2^E(\mathbf{s},\mathbf{X}) + J_1^E(\mathbf{s},\mathbf{X})] - J_1^E(\mathbf{s},\mathbf{X})\right)$$

$$\frac{\partial f_g}{\partial s_{21}} = \frac{\partial\left([J_1^E(\mathbf{s},\mathbf{X})]^\gamma [J_2^E(\mathbf{s},\mathbf{X})]^{1-\gamma}\right)}{\partial s_{21}} = \Gamma_{21}(\mathbf{X})[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-1}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma}\left(-\gamma[J_2^E(\mathbf{s},\mathbf{X}) + J_1^E(\mathbf{s},\mathbf{X})] + J_1^E(\mathbf{s},\mathbf{X})\right)$$

Second order conditions are

$$\frac{\partial f_g}{\partial s_{12}^2} = \frac{\partial^2 \left([J_1^E(\mathbf{s},\mathbf{X})]^\gamma [J_2^E(\mathbf{s},\mathbf{X})]^{1-\gamma}\right)}{\partial s_{12}^2} = (\gamma^2-\gamma)\Gamma_{12}^2(\mathbf{X})[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-2}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma-1}\left(J_2^E(\mathbf{s},\mathbf{X})+J_1^E(\mathbf{s},\mathbf{X})\right)^2$$

$$\frac{\partial f_g}{\partial s_{21}^2} = \frac{\partial^2 \left([J_1^E(\mathbf{s},\mathbf{X})]^\gamma [J_2^E(\mathbf{s},\mathbf{X})]^{1-\gamma}\right)}{\partial s_{21}^2} = (\gamma^2-\gamma)\Gamma_{21}^2(\mathbf{X})[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-2}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma-1}\left(J_2^E(\mathbf{s},\mathbf{X})+J_1^E(\mathbf{s},\mathbf{X})\right)^2$$

$$\frac{\partial f_g}{\partial s_{12}\partial s_{21}} = \frac{\partial^2 \left([J_1^E(\mathbf{s},\mathbf{X})]^\gamma [J_2^E(\mathbf{s},\mathbf{X})]^{1-\gamma}\right)}{\partial s_{12}\partial s_{21}} = -(\gamma^2-\gamma)\Gamma_{12}(\mathbf{X})\Gamma_{21}(\mathbf{X})[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-2}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma-1}\left(J_2^E(\mathbf{s},\mathbf{X})+J_1^E(\mathbf{s},\mathbf{X})\right)^2$$

The terms $\Gamma_{12}(\mathbf{X})$, $\Gamma_{21}(\mathbf{X})$, $J_1^E(\mathbf{s},\mathbf{X})$, and $J_2^E(\mathbf{s},\mathbf{X})$ are non-negative. Also we have $\gamma^2-\gamma < 0$. The Hessian matrix is

$$\begin{aligned} H(f_g) &= \begin{bmatrix} \frac{\partial f_g}{\partial s_{12}^2} & \frac{\partial f_g}{\partial s_{12}\partial s_{21}} \\ \frac{\partial f_g}{\partial s_{12}\partial s_{21}} & \frac{\partial f_g}{\partial s_{21}^2} \end{bmatrix} \\ &= \begin{bmatrix} \Gamma_{12}^2(\mathbf{X})R(\mathbf{X}) & -\Gamma_{12}(\mathbf{X})\Gamma_{21}(\mathbf{X})R(\mathbf{X}) \\ \Gamma_{12}(\mathbf{X})\Gamma_{21}(\mathbf{X})R(\mathbf{X}) & \Gamma_{21}^2(\mathbf{X})R(\mathbf{X}) \end{bmatrix} \end{aligned}$$

where $R(\mathbf{X}) = (\gamma^2-\gamma)[J_1^E(\mathbf{s},\mathbf{X})]^{\gamma-2}[J_2^E(\mathbf{s},\mathbf{X})]^{-\gamma-1}\left(J_2^E(\mathbf{s},\mathbf{X})+J_1^E(\mathbf{s},\mathbf{X})\right)^2 < 0$. Then, it is clear that $\mathbf{x}H(f_g)\mathbf{x}^T$ is non-positive. Therefore, the Hessian matrix is negative semi-definite and f_g is concave on $\mathbf{s}$.

Derivation of 5.8, 5.9, and 5.10

$$\begin{aligned} \mathsf{E}[\min(X,D)] &= X\int_X^\infty f(D)dD + \int_{-\infty}^X Df(D)dD \\ &= X(1-F(X)) + XF(X) - \int_{-\infty}^X F(D)dD \\ &= X - \int_{-\infty}^X F(D)dD \end{aligned}$$

$$
\begin{aligned}
\mathsf{E}[\max(X-D,0)] &= \int_{-\infty}^{X}(X-D)f(D)dD \\
&= X\int_{-\infty}^{X}f(D)dD - \int_{-\infty}^{X}Df(D)dD \\
&= XF(X) - XF(X) + \int_{-\infty}^{X}F(D)dD \\
&= \int_{-\infty}^{X}F(D)dD
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{E}[\min(nX,Z)] &= nX\int_{nX}^{\infty}f_Z(Z)dZ + \int_{-\infty}^{nX}Zf_Z(Z)dZ \\
&= nX(1-F_Z(nX)) + nXF_Z(nX) - \int_{-\infty}^{nX}F_Z(Z)dZ \\
&= nX - \int_{-\infty}^{nX}F_Z(Z)dZ
\end{aligned}
$$

$$
\begin{aligned}
I_D(X) &= \int_{X}^{\infty}(\xi - X)f_D(\xi)d\xi \\
&= \int_{X}^{\infty}\xi f_D(\xi)d\xi - X\int_{X}^{\infty}f_D(\xi)d\xi \\
&= \mu - \int_{-\infty}^{X}\xi f_D(\xi)d\xi - X(1-F_D(X)) \\
&= \mu - \int_{-\infty}^{X}\bar{F}_D(\xi)d\xi
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{E}[\min(X,D)] &= \int_{-\infty}^{X}\bar{F}_D(\xi)d\xi = \mu - E[\max(D-X,0)] = \mu - I_D(X) \\
\mathsf{E}[\max(X-D,0)] &= \int_{-\infty}^{X}(X-\xi)f_D(\xi)d\xi = \int_{-\infty}^{X}F_D(\xi)\xi = I_D(X) + X - \mu \\
\mathsf{E}[\min(nX,Z)] &= \int_{-\infty}^{nX}\bar{F}_Z(\xi)d\xi = n\mu - I_Z(nX)
\end{aligned}
$$

Derivation of 5.14

Assume y is a normal random variables with mean μ_y and standard deviation σ_y. Then, the random variable $x = \alpha y + \beta$ is also a normal random variable with mean $\mu_x = \alpha\mu_y + \beta$ and standard deviation $\sigma_x = \alpha\sigma_y$. Hence, if y is a standard normal random variable, the, x

would be a normal random variables with mean α and standard deviation β.

Now, $F_x(X) = P\{x \le X\} = P\{\sigma y + \mu \le X\} = P\{y \le \frac{X-\mu}{\sigma}\} = \Phi(Y)$, where $Y = \frac{X-\mu}{\sigma}$.

Similarly, if $z = nx$ is the random variable which is the summation of n normal random variables with mean μ and standard deviation σ, then $\mu_z = n\mu$ and $\sigma_z = \sqrt{n}\sigma$. Now, $F_z(nX) = P\{z \le nX\} = P\{\sqrt{n}\sigma y + n\mu \le nX\} = P\{y \le \frac{nX-n\mu}{\sqrt{n}\sigma}\} = \Phi(\sqrt{n}Y)$, where $Y = \frac{X-\mu}{\sigma}$.