RJVs and Price Collusion under Endogenous Product Di®erentiation

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Abstract

We characterise the interplay between ¯rms' decisions in product development, be it joint or independent, and their ensuing repeated price behaviour, either collusive or Bertrand-Nash. Firms face a choice between participating in a joint venture inventing a single product, and in independent ventures developing their respective products which can be either horizontally or vertically di®erentiated. We prove that joint product development and the resulting lack of horizontal product di®erentiation may destabilise collusion, whilst ¯rms' R&D decisions have no bearings on collusive stability in the vertical di®erentiation setting. We also discover the non-monotone dependence of ¯rms' venture decisions at the development stage upon their intertemporal preferences, as well as upon consumers' willingness to pay.

Keywords : R&D, product innovation, collusive stability, time discount factor, optimal punishment.

JEL classi¯cation : D43, L13, O31.

1 Introduction

Whilst public authorities explicitly prohibit collusive market behaviour, there is scarce evidence that they discourage cooperation in R&D activities. As to the latter, there indeed exist several examples of policy measures meant to stimulate the formation of research joint ventures (RJVs henceforth).[1] If cooperation in innovation activities may induce collusion in the product market, then the above mentioned tendency to encourage cooperative R&D but to discourage market collusion will render itself inconsistent.

There exists a wide literature concerning the e®ects of product di®erentiation on the stability of implicit collusion either in output levels or in prices (Deneckere, 1983 ; Chang, 1991, 1992 ; Rothschild, 1992 ; Ross, 1992 ; Friedman and Thisse, 1993 ; Häckner, 1994, 1995, 1996 ; Lambertini, 1997a ; Albk and Lambertini, 1998 ; inter alia). There also have been studies dealing with R&D in di®erentiated markets. A few of them, including Motta (1992) and Rosenkranz (1995), consider cooperation in the development phase. On the other hand, the e®ectiveness of RJVs in eliminating e®ort duplication has been well noted in a large number of contributions (Katz, 1986 ; d'Aspremont and Jacquemin, 1988, 1990 ; Kamien et al., 1992 ; Suzumura, 1992 ; inter alia).

So far, however, few serious attempts have been made to consolidate these two streams of research. Among these few pioneering studies are Martin (1995), and Cabral (1996). The former analyses the strategic e®ects of an RJV aimed at achieving a process innovation for an existing product, when the product is marketed by ¯rms engaging in Cournot behaviour. Martin shows that the presence of cooperation in process innovation enhances cartel stability, which can overbalance the welfare advantage of eliminating e®ort duplication through the RJV. His ¯nding has potential implications in the case of product innovation as well. Cabral, on the other hand, proves the existence of those cases where competitive pricing is needed to sustain more e±cient R&D agreements.

Our e®ort in this paper broadly follows Martin's, except that we take into account the possible e®ects of product di®erentiation resulting from the presence or the absence of cooperation in product development, as opposed to Martin's analysis of process development. In particular, unlike most of the existing literature on repeated games under product di®erentiation, we explicitly model the e®ort-saving e®ects of RJVs, which a®ect ¯rms' incentives as well as social welfare. Namely, in an RJV, ¯rms share the costs of product development by jointly developing a single product. An RJV thereby eliminates e®ort duplication, whilst it o®ers no product di®erentiation : all participant ¯rms will

[1] See the National Cooperative Research Act in the US ; EC Commission (1990) ; and, for Japan, Goto and Wakasugi (1988).

have to market one identical product. Independent ventures do the opposite: each ¯rm bears the full costs of innovating its respective product, in return for the possibility of product di®erentiation.

In brief, we investigate the bearings of product innovation, either through an RJV or through independent ventures, on ¯rms' ability to build an implicit cartel in the market phase and maintain it over time. We prove that ¯rms' R&D decisions, insofar as they a®ect the degree of vertical di®erentiation only, have no bearings on collusion stability. Horizontally di®erentiated independent ventures can enhance the sustainability of price collusion in marketing if ¯rms collude by choosing that subgame perfect equilibrium which maximises the sum of ¯rms' discounted streams of pro¯ts.

Note also that ¯rms' decision between joint and independent ventures at the development stage can be non-monotone in their intertemporal preferences as well as in consumers' willingness to pay, due to the fact that the collusive stability in the marketing stage can be a®ected by product di®erentiation.

The paper is organised as follows. The general structure of the game is laid out in section 2. The horizontal di®erentiation setting is closely analysed in sections 3. Then, the vertical di®erentiation model is discussed in section 4. Section 5 discusses brie°y the di®erence between our ¯ndings and existing results in the literature. Finally, Section 6 provides concluding remarks.

2 The model

We consider a duopoly with two a priori identical ¯rms playing the following three-stage game. The entire game is embedded in the discrete time structure $t = 0; 1; 2; \cdots$. The ¯rst two stages take place at $t = 0$, both are for product innovation in its broad sense.

The ¯rst stage is for initial venture decisions, where ¯rms choose between independent and joint ventures. An RJV is formed if and only if both ¯rms agree to stay in it; otherwise if at least one of them disagrees with an RJV, then each of the two ¯rms forms an independent venture.

The second stage describes product development. Products are located in the relevant space, which we assume to be unidimensional. Depending upon whether such a space is horizontal or vertical, we discuss two separate versions of the model in sections 3 and 4, respectively. In either version:

² The two ¯rms jointly develop a single product if they decided on a joint venture in the previous stage. The joint venture serves as a uni¯ed decision maker only in this second stage. Namely, the RJV chooses a product so as to maximise the sum of the two ¯rms' discounted streams of pro¯ts. The two ¯rms also bear symmetrically the cost of product development.

² Each of the two ¯rms independently chooses a product and develops it if the two ¯rms decided on independent ventures in the ¯rst stage. In this case, each ¯rm bears the full development cost of its own product. Note in particular, the noncooperativeness of the ¯rms' product decisions does not necessarily preclude the possibility that their decisions can still be implicitly collusive, rewarding or penalising particular product pro¯les through their ensuing market behaviour.

Then ¯nally, the third stage is a Bertrand supergame t = 1; 2; ¢¢¢. Throughout the game, the discount factor ± 2 [0; 1) is common to both ¯rms. In establishing the critical threshold of the discount factor stabilising price collusion, we follow the optimal punishment strategy as de¯ned by Abreu (1986).

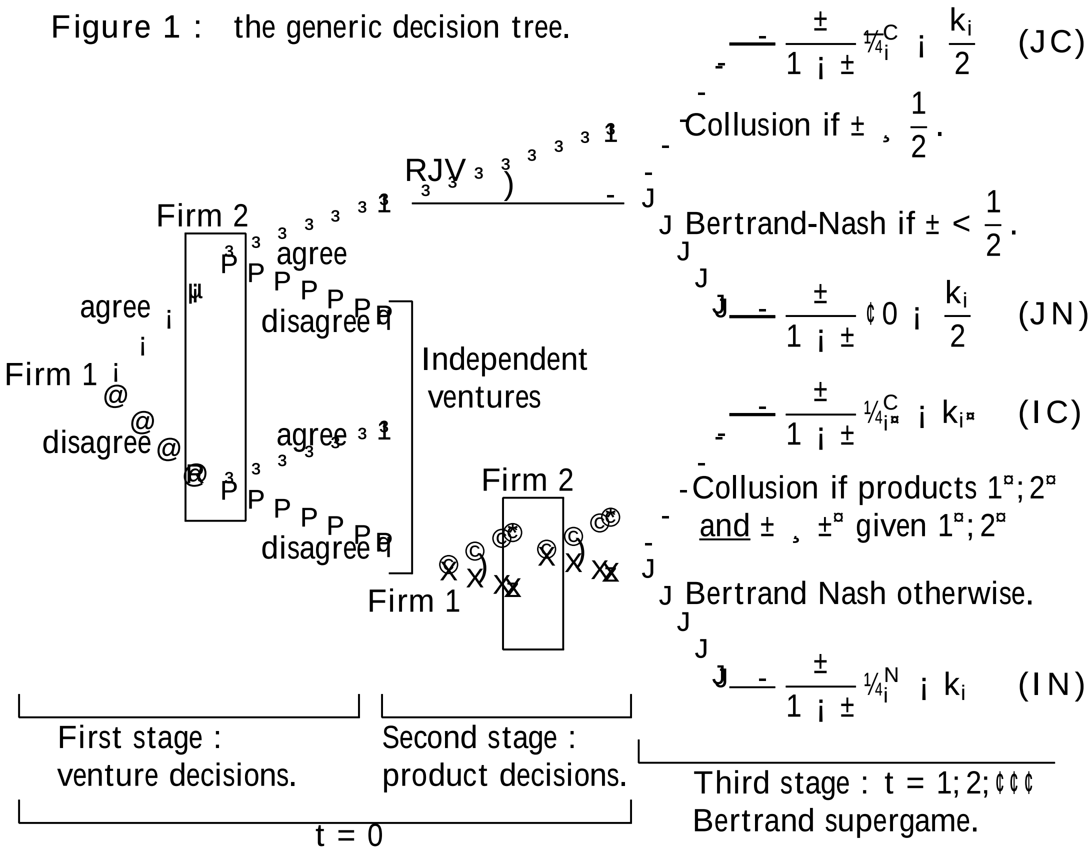
Observe that, when ¯rms choose a joint venture, they supply the market with an undi®erentiated product, thereby to sustain collusion in the resulting perfect Bertrand market, the discount factor ± needs to be $\frac{1}{2}$ or above. In this case, the predictions o®ered by the conventional folk theorem and by Abreu's optimal punishment coincide (Lambson, 1987). If ± < $\frac{1}{2}$, there is no prospect of colluding at any prices other than one-shot Bertrand-Nash equilibrium prices.

On the other hand, when ¯rms choose independent ventures, their collusive or noncollusive pricing behaviour in the Bertrand supergame can be made contingent upon the product portfolio they have selected. By colluding in marketing if and only if a particular product portfolio 1¤; 2¤ has been selected (i.e., ¯rm 1 has chosen product 1¤ and ¯rm 2 has chosen 2¤), ¯rms may be able to sustain the particular product pro¯le as part of a subgame perfect equilibrium. Obviously, depending upon which product portfolio to sustain, there can be countlessly many subgame perfect equilibria of this structure. Among them, we focus on pro¯t e±cient ones, i.e., those equilibria yielding the highest possible discounted pro¯ts (greater details shall be shown in sections 3 and 4). Thereby, even though their product decisions as well as pricing actions are entirely noncooperative, the ¯rms can e®ectively collude both in product portfolio and in the ensuing Bertrand supergame, as an outcome of a purely noncooperative subgame perfect equilibrium.

Hence, a general picture of the decision problem facing the two ¯rms is provided by ¯gure 1, where the discounted stream of net pro¯ts for each ¯rm is listed as (JC), (JN), (IC), and (IN), with J; I; C and N standing for joint venture, independent ventures, collusion and one-shot Bertrand-Nash behaviour, respectively. In the picture, the sub-trees for the supergame in the third stage are suppressed and replaced with binary equilibrium outcomes: either collusion or Bertrand-Nash.[2] k_i is the development cost of product i.

Note that both collusive profits and Bertrand-Nash profits vary depending upon the product portfolio. In the case of undifferentiated products, Bertrand-Nash profits are nil, and in calculating collusive profits π_i^C we assume that the two firms will set an identical price to split demand evenly, thereby equalising profits, when colluding in prices. In independent ventures, firms collude in prices and earn collusive profits $\pi_{i^*}^C$ if and only if collusive portfolio $1^*; 2^*$ has been chosen; otherwise they repeat one-shot Bertrand-Nash equilibrium, earning π_i^N which depends upon the portfolio. Let δ^* denote the critical discount factor sustaining collusion, which may depend upon the given pair of products.

Figure 1 : the generic decision tree.



In the following two sections we solve this tree backward to identify pure strategy subgame perfect equilibrium (simply "equilibrium" hereinafter unless otherwise specified).

[2] Note that our purpose in this paper is to analyse firms' R&D and marketing behaviour without mixing them with entry/exit decisions. To this end, we assume no possibility of exiting even when operative profits are literally below zero. This can be justified, for example, in the presence of substantial exit costs.

3 The horizontal differentiation setting

We adopt the spatial location model due to d'Aspremont, Gabszewicz and Thisse (1979). Consumers are uniformly distributed over the unit line segment $[0;1]$. In every period, each consumer buys one unit of the product that maximises his net utility:

$$U = s - d_i^2 - p_i;$$

where s is gross surplus, p_i is the price charged by firm i and d_i is the distance between the consumer and firm i. We assume that, if a consumer is indifferent between the two firms' products, then he randomises his purchase with probability one half from each firm. This implies that, if the two firms locate at the same site and choose the same price, then they split the demand evenly. We also assume $s \geq 5/4$,[3] and normalise the marginal production cost to nil.

The development cost of a product is a positive constant k independent of the location. Therefore, an RJV pays k jointly, each firm bearing $k/2$, whereas an independent venture also pays k in the second stage, at $t = 0$.

The game is solved by backward induction in the following subsections 3.1 through 3.3.

3.1 Subgame ensuing independent ventures

When firms undertake independent ventures, each of them bears the full development cost k. Although the two firms' location choices are mutually independent and noncooperative, as aforementioned, they may still be able to enforce a particular pair of locations using their ensuing marketing behaviour as a rewarding/punishing device. Namely, in the marketing supergame, firms collude in prices only if they have chosen a prescribed pair of locations. Especially, when there are more than one pair of locations enforceable by this means, hereinafter we analyse the most profit-efficient symmetric one among such location pairs.

In this horizontal product space setting we assume that, if in the first stage the two firms choose independent ventures, then in the latter two stages they play that subgame perfect equilibrium which prescribes the following.

1. The most profitable symmetric profile of locations and prices, as long as δ is sufficiently high in order to sustain such a profile through Abreu's optimal punishment.

[3] This ensures that full market coverage obtains at the noncooperative one-shot equilibrium in prices, if firms locate in 0 and 1, respectively.

2. If δ does not suffice to sustain the above 1., then the most profitable among those symmetric location pairs starting from which the collusion at the joint profit maximal (i.e., monopoly) price level is sustainable by Abreu's optimal punishment given δ.

3. If the set of all those symmetric location pairs in 2. is empty, i.e., if δ is so low that collusion at the monopoly price is unsustainable starting from any location pair at all, then the most profitable location pair anticipating one-shot Bertrand-Nash equilibrium pricing.

Let $\bar{\delta}[s]$ define the critical threshold between cases 1 and 2, and $\underline{\delta}[s]$ the threshold between cases 2 and 3, respectively. Then:

Lemma 1:

1. When $\delta \geq \bar{\delta}[s]$, firms locate at $\frac{1}{4}; \frac{3}{4}$ and price at $s - \frac{1}{16}$.
2. When $\underline{\delta}[s] \leq \delta \leq \bar{\delta}[s]$, firms locate at such location a and $1 - a$ that $\delta = \delta^*$, where a decreases in s.
3. When $\delta < \underline{\delta}[s]$, firms locate at endpoints 0 and 1 and play the related one-shot equilibrium price.
4. $0 \leq \underline{\delta}[s] < \bar{\delta}[s] < \frac{1}{2}$ for any $s \geq \frac{5}{4}$, where the equality holds only when $s = \frac{5}{4}$.

Proof: See appendix 7.1. It is algebraically straightforward to verify that, whenever $\delta \geq \underline{\delta}[s]$, each firm has no strict incentive to deviate in location and endure the one-shot Nash equilibrium outcome later, as opposed to complying with the prescribed location and colluding later. ■

The reason why $a \in (1/4; 1/2]$ does not occur is because the critical threshold for cartel stability is increasing in a, while cartel profits are the same for any $a = 1/4 \pm \varepsilon$, with $\varepsilon \in [0; 1/4]$. Therefore, it is convenient for firms to relocate farther apart and increase differentiation, rather than the opposite (see also Häckner, 1995; 1996).

3.2 Subgame ensuing a joint venture

Turn now to the case of a joint venture. Observe that location is no longer a strategic instrument for each firm, since the two firms commit to develop an identical product, locating at the same point in the product space. The game thereby reduces into a straightforward Bertrand supergame, where the critical threshold of the discount factor is 1/2. As a consequence, the choice between Bertrand-Nash and collusive pricing depends exclusively on firms' time preferences.

If $\delta \in [1/2; 1)$, the two firms' joint collusive profits are maximised when they together locate at $1/2$, entailing the profit $\pi_i^C = (s - 1/4)/2$ per firm, per period. Otherwise, if $\delta \in [0; 1/2)$, Bertrand-Nash profits are nil irrespective of the firms' location in the product space as long as their products are undifferentiated. Notice that this involves a loss, equal to half the development cost, for each firm. The option to stay out is assumed away, namely, the initial investment is thought of as irreversible and firms can avoid loosing it if and only if (JN) is not the equilibrium outcome.

3.3 Initial venture decisions

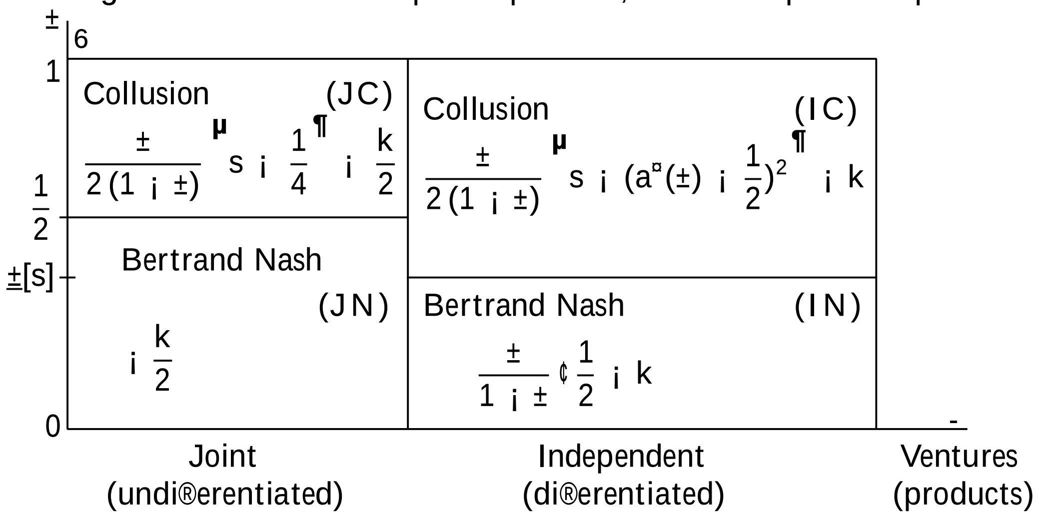
From the above Lemma 1, we have observed that, after independent ventures firms can collude whenever $\delta \geq \delta^*$, whereas after a joint venture they can collude when and only when $\delta \geq 1/2$. Therefore, item 4 of Lemma 1 immediately proves the following.

Proposition 1: The range of time discount factors over which price collusion ensuing a joint venture is sustainable is a proper subset of that where collusion ensuing independent ventures is sustainable.

In plain words, a joint venture, when it hinders horizontal product differentiation, serves to destabilise price collusion in the marketing supergame.

The resulting discounted profits per firm appear as in Figure 2.

Figure 2 : Discounted profits per firm, horizontal product space.



where

$$a^*(\delta) = \begin{cases} \frac{1}{4} & \text{if} \quad \delta \in \left[\bar{\delta}[s]; 1\right); \\ \arg_a \{\delta = \delta^* \mid s\} & \text{if} \quad \delta \in \left[\underline{\delta}[s]; \bar{\delta}[s]\right). \end{cases}$$

The dependence of firms' innovative venture decisions on the time discount factor δ, the gross surplus s and the development cost k is identified by the following proposition, using a three-regime taxonomy based upon the level of time preferences.

Proposition II :

1. $\delta \in \left[0; \bar{\delta}[s]\right)$. In this regime, firms repeat the one-shot Bertrand-Nash equilibrium under both independent and joint venture cases. Therefore, the joint venture is chosen over independent ventures if and only if $(JN) \geq (IN)$, i.e.

$$k > \frac{\delta}{1-\delta}. \tag{1}$$

2. $\delta \in \left[\underline{\delta}[s]; \bar{\delta}[s]\right)$. In this regime, firms collude in prices only under independent ventures, not if they undertake a joint venture. Hence, the joint venture is preferred if and only if $(JN) > (IC)$, i.e.

$$k > \frac{\delta}{1-\delta}\left(s - \frac{1}{4} + a^*(\delta)\,(1 - a^*(\delta))\right). \tag{2}$$

3. $\delta \in \left[\bar{\delta}[s]; 1\right)$. In this regime, firms always collude. As a result, the joint venture is undertaken if and only if $(JC) > (IC)$, i.e.

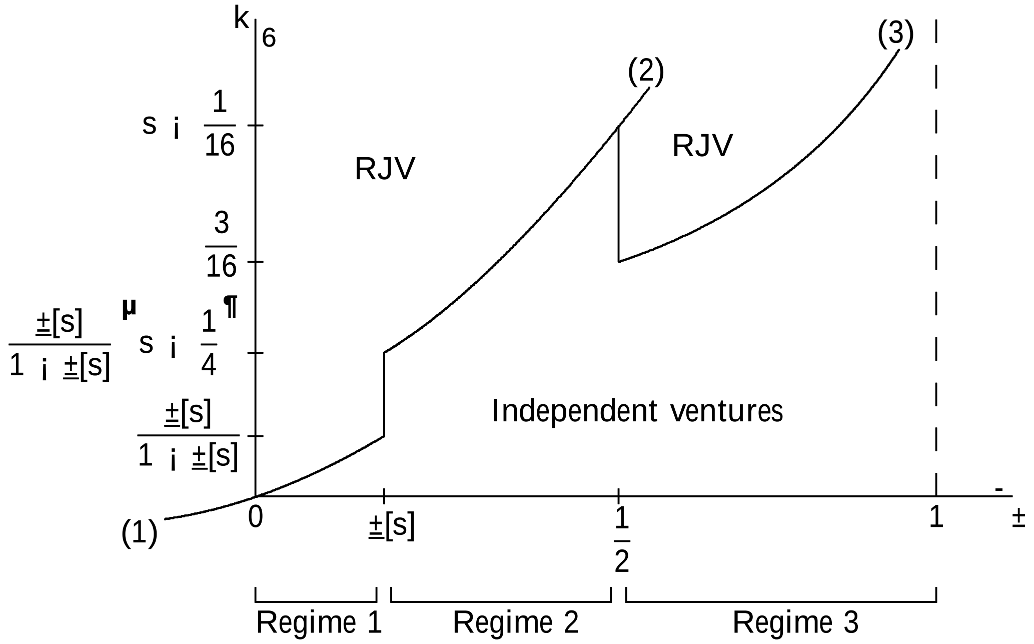
$$k > \frac{\delta}{1-\delta} a^*(\delta)\,(1 - a^*(\delta))$$

which, noting that $a^*(\delta) = \frac{1}{4}$ in this range of δ, can be rewritten into

$$k > \frac{3\delta}{16(1-\delta)}. \tag{3}$$

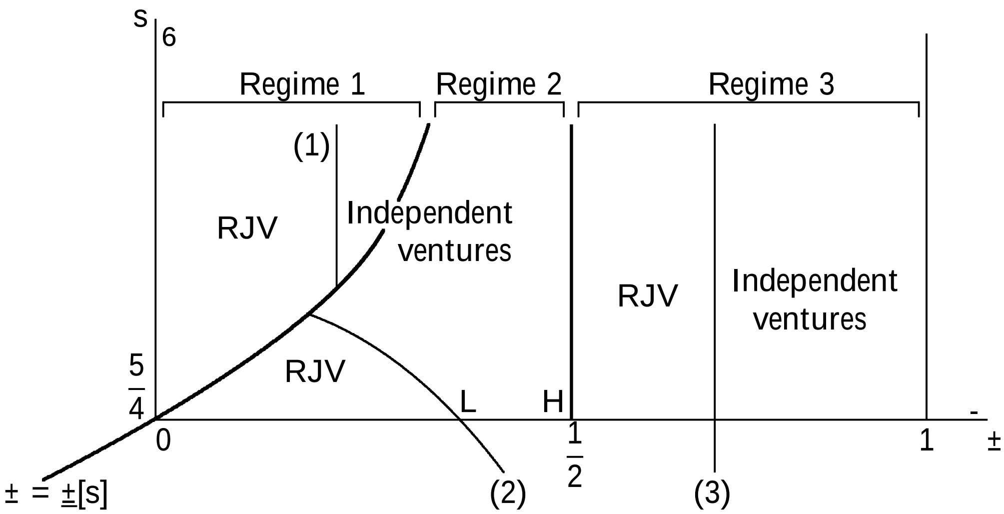
Figure 3 plots the venture cost k against the discount factor δ, given s. Overall, independent ventures tend to become increasingly attractive as δ grows. However, in regime 2, the condition for independent venture is loosened (inequality (2)) as compared to the adjacent areas (inequality conditions (1) and (3)). The driving force is the fact that when δ lies in this regime, firms can sustain collusion if and only if they have chosen independent ventures. Observe that the firms' indifference threshold in k between joint and independent ventures is monotone in their time preferences over the interval $\delta \in [0; 1/2)$, as well as over the interval $\delta \in [1/2; 1)$:

Figure 3 : Comparative statics on ¯rms' venture decisions in the parametric plane f±; kg.



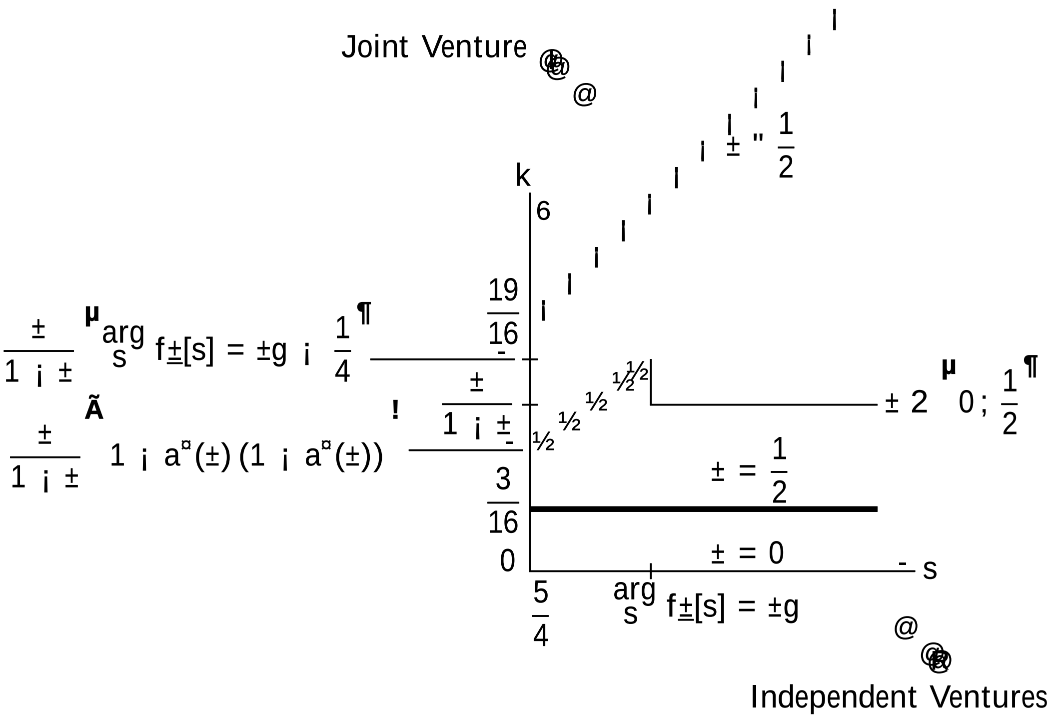
Consider now the relation between s and ±, given k. Figure 4 plots the gross surplus s against the discount factor ±, given the venture cost k. Once again, independent ventures become more pro¯table in regime 2 relative to the adjacent areas. When $k > \frac{3}{16}$, the boundary (3) lies to the right of H. Also, when $k < \frac{19}{16}$, the boundary (2) intersects with the boundary $s = \frac{5}{4}$ at L to the left of H. Thus, as long as $\frac{3}{16} < k < \frac{19}{16}$, ¯rms' venture decisions are non-monotone in ± for any $s ¸ \frac{5}{4}$.

Figure 4 : Comparative statics on ¯rms' venture decisions in the parametric plane f±; sg.



Finally, Figure 5 plots the venture cost k against the gross surplus s. Here, ¯rms' indifference boundary between joint and independent ventures shifts up as ± increases from 0 to 1/2. The horizontal portion to the right of the kinks correspond to regime 1, and the up-sloping portion to the left of the kinks correspond to regimes 2 (the kinked locus is meant to represent a generic ± 2 (0; 1=2)). The boundary jumps down when ± reaches 1/2; thereafter, parallely shifts up again as ± increases further from 1/2 to 1 (regime 3). This discontinuity re°ects the fact that as ± exits regime 2 and enters regime s, the extra bene¯t of collusive stability o®ered by product di®erentiation becomes no longer relevant.

Figure 5 : Firms' indi®erence boundary between joint and independent ventures, drawn on a cost (k) - bene¯t (s) plane given ±.



Note that these observations imply the following.

Corollary i : For any given k; s such that

$$\frac{3}{16} < k < s ¡ \frac{1}{16};$$

¯rms' decisions between joint and independent ventures become non-monotone in the discount factor ±.

Corollary ii : For any given k; ± such that

$$\frac{±}{1 ¡ ±} < k < \frac{±}{1 ¡ ±} \left(\arg_s f±[s] = ±g ¡ \frac{1}{4} \right);$$

¯rms' venture decisions are non-monotone in the gross surplus s.

4 The vertical differentiation setting

We adopt a model of vertical differentiation in the vein of Gabszewicz and Thisse (1979), Shaked and Sutton (1982), inter alia. A unit mass of consumers are uniformly distributed over the interval $[0, \Theta]$, representing their marginal willingness to pay for quality. Each consumer buys at most one unit of the product that maximises his net utility:

$$U = \theta q_i - p_i, \qquad \theta \in [0, \Theta],$$

where q_i and p_i identify the quality and the price of product i. The market is supplied by two single-product firms producing qualities $q_1, q_2 \in (0, \bar{q}]$, where $\bar{q}$ is the highest quality level which is technologically feasible. Without loss of generality we assume $q_1 \geq q_2$ throughout this section. Also, if a consumer is indifferent between the two firms' products, then he randomises his purchase with probability one half from each firm. This implies that, if the two firms' qualities and prices are identical, then they split the market evenly. We assume[4]

$$\frac{\delta}{8(1-\delta)} \Theta \bar{q} > k_1 - k_2 \tag{4}$$

and also that the marginal production cost is nil.

The development cost of a product is a non-decreasing function of its quality in the following way. The cost of product innovation is a constant k_1 if it is the highest quality being produced. Otherwise, the development cost is $k_2 \in (0, k_1)$. This describes the economic situation where there is a unilateral externality that the technology adopted by a high-quality firm can be partially imitated by a lower-quality firm, but not vice versa.[5] This naturally implies that, if a joint venture is undertaken, each firm bears $k_1/2$.

The game can be solved backward, similarly to section 3.

4.1 Subgame ensuing independent ventures

We assume equilibrium selection criteria mostly analogous to those in our horizontal differentiation setting (see section 3.1) except that, by the nature of vertical differentiation,

[4] This assumption, that the total surplus $\Theta \bar{q}$ is sufficiently high, is somewhat parallel to the assumption $s \geq \frac{5}{4}$ in the horizontal differentiation model (section 3), even though full market coverage is no longer guaranteed in the vertical differentiation model. See appendix 7.2 for computational details.

[5] This is observationally equivalent to a perhaps more intuitive assumption that a first entrant must innovate a product from scratch, paying k_1, whilst a subsequent entrant can innovate a product on the ground of its predecessor's technological heritage, saving the development cost down to k_2 where $0 < k_2 < k_1$. See Lemma 2-ii in appendix 7.3.

¯rms' product portfolio is not \symmetric" unless they produce an identical quality.

Namely, if in the ¯rst stage the two ¯rms choose independent ventures, then in the latter two stages they play that subgame perfect equilibrium which prescribes the following.

1. The pro¯le of qualities and prices which maximises the two ¯rms' aggregate pro¯ts, as long as ± is su±ciently high in order to sustain such a pro¯le through Abreu's optimal punishment.
2. If ± does not su±ce to sustain the above 1., then the most pro¯table among those quality pairs starting from which the collusion at the joint pro¯t maximal (i.e., monopoly) price level is sustainable by Abreu's optimal punishment given ±.
3. If the set of all those symmetric location pairs in 2. is empty, i.e., if ± is so low that collusion at the monopoly price is unsustainable starting from any quality pair at all, then the most pro¯table quality pair anticipating one-shot Bertrand-Nash equilibrium pricing.

Our ¯ndings in this vertical di®erentiation setting is qualitatively quite distinct from those in the horizontal product space in several ways. In particular, item 2. in the above taxonomy turns out to be vacuous.

Lemma 2 :

² If ± ¸ $\frac{1}{2}$, both ¯rms develop $\bar{q}$ in the second stage, and collude in prices in the ensuing marketing stage.

² If ± < $\frac{1}{2}$, then $q_1 = \bar{q}$, $q_2 = \frac{4}{7}\bar{q}$ followed by Bertrand-Nash competition in the marketing stage, is the unique (up to the two ¯rms' permutation) pure strategy equilibrium.

Proof : See appendix 7.2. ■

4.2 Subgame ensuing a joint venture

In the case of a joint venture, the quality is no longer a strategic variable for each ¯rm. The two ¯rms commit to develop an identical product, which reduces the subgame into a simple Bertrand supergame without product di®erentiation, where the critical threshold of the discount factor is 1/2 in order to sustain price collusion. The choice between the one-shot Bertrand-Nash equilibrium and collusive pricing thereby depends solely upon firms' time preferences.

If $\delta \in [0, 1/2)$, the one-shot Bertrand-Nash equilibrium profits are nil irrespective of the firms' location in the product space as long as their products are undifferentiated. Otherwise, if $\delta \in [1/2, 1)$, the two firms' joint collusive profits are maximised as follows.

Lemma 3 : If firms engage in a joint venture anticipating price collusion in the market supergame, then both firms develop $\bar{q}$ in the second stage. Collusion is sustainable iff $\delta \geq 1/2$.

Proof : Step 1 of appendix 7.2, except that each firm's initial R&D expense is no longer k_1 but now $\frac{k_1}{2}$ instead, proves Lemma 3. ■

4.3 Initial venture decisions

Lemmata 2 and 3 imply:

Proposition III : The critical threshold of the discount factor in sustaining price collusion is always $\delta^* = 1/2$ irrespective of firms' initial venture decisions.

The relevant per period profits when firms adopt independent ventures and compete à la Bertrand-Nash are $\pi_1^N = 7\Theta\bar{q}/48$ and $\pi_2^N = \Theta\bar{q}/48$. Obviously, if firms undertake a joint venture and then play Bertrand-Nash, their stage profits in the marketing supergame are nil. Otherwise, if firms collude in prices, their individual per period profit is $\pi_i^C = \Theta\bar{q}/8$, irrespective of their venture decisions. Hence the discounted profits are summarised in Figure 6.

Figure 6 : Discounted profits per firm, vertical product space.

δ	Joint (undifferentiated)	Independent (undifferentiated)	Independent (differentiated)
$[1/2, 1)$	Collusion (JC) $\frac{\delta}{1-\delta} \cdot \frac{\Theta\bar{q}}{8} - \frac{k_1}{2}$	Collusion (IC) $\frac{\delta}{1-\delta} \cdot \frac{\Theta\bar{q}}{8} - k_1$	
$[0, 1/2)$	Bertrand-Nash (JN) $-\frac{k_1}{2}$		Bertrand-Nash (IN) $\frac{\delta}{1-\delta} \cdot \frac{7\Theta\bar{q}}{48} - k_1$; $\frac{\delta}{1-\delta} \cdot \frac{\Theta\bar{q}}{48} - k_2$

Ventures (products)

Clearly, if $\delta \in [1/2, 1)$, firms are going to collude anyway, so that the venture decisions have no relevance as to the quality that is going to be marketed. Otherwise, when $\delta \in [0, 1/2)$, we assume that firms choose independent ventures if and only if they fail to agree on a joint venture at $\bar{q}$, in which case the firm who disagrees switches to a quality strictly lower than $\bar{q}$.

Proposition IV :

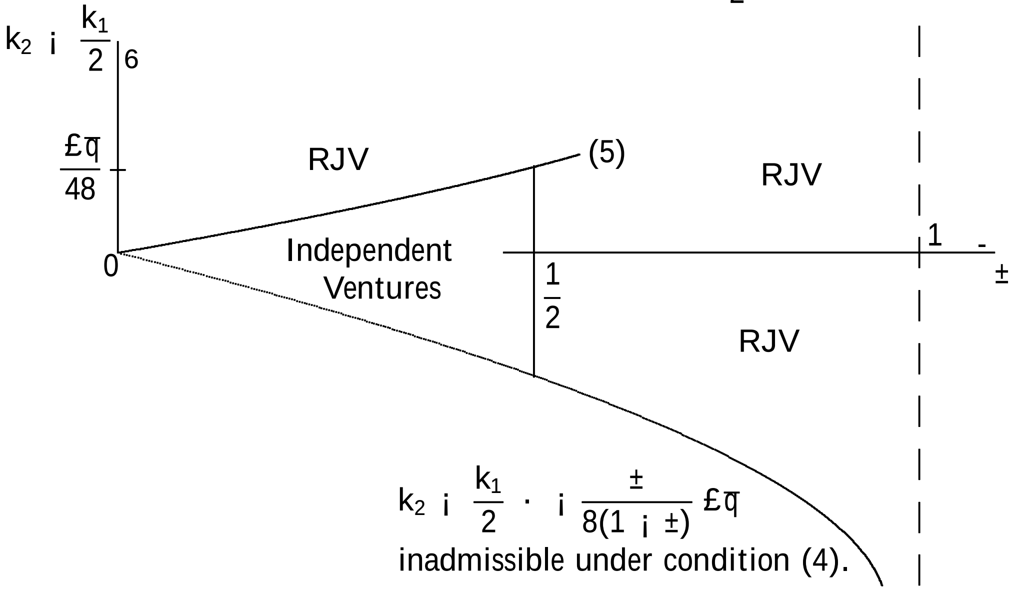
1. $\delta \in [0, 1/2)$. In this regime, firms are unable to collude in prices. Hence, the relevant comparison involves Bertrand-Nash competition with either a joint venture or independent ventures. Therefore, independent ventures take place if and only if $(IN) \geq (JN)$ for the lower quality firm (due to the above assumption), i.e.

$$\frac{\delta}{1-\delta} \cdot \frac{\theta \bar{q}}{48} - k_2 \geq -\frac{k_1}{2}. \tag{5}$$

2. $\delta \in [1/2, 1)$. In this regime, firms collude in prices regardless of their venture decisions. Therefore, as long as the venture cost is strictly positive, a joint venture always dominates independent ventures.

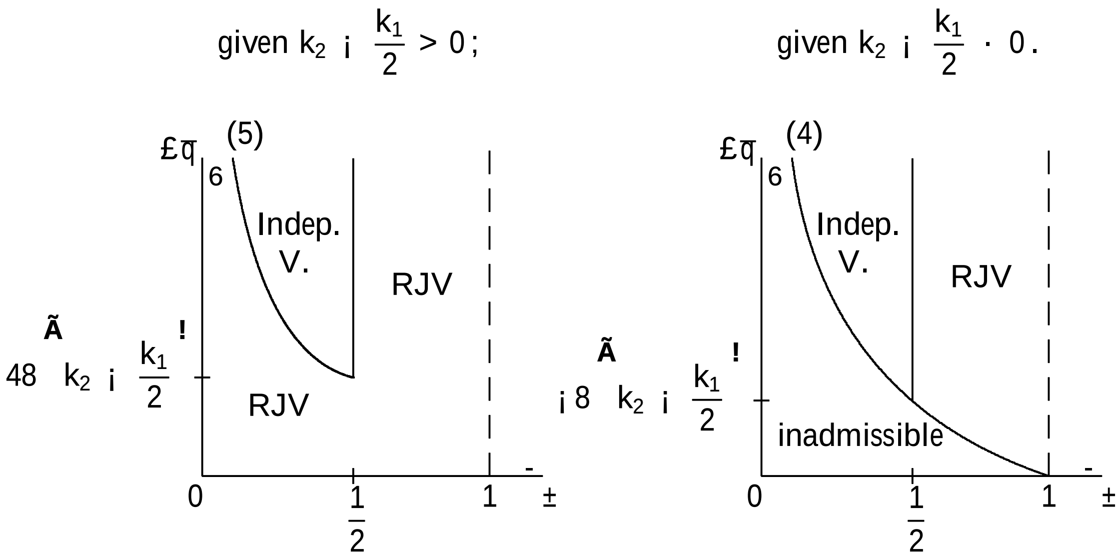
Figure 7 is a vertical-differentiation analogue of Figure 3, plotting the product development cost differential $k_2 - \frac{k_1}{2}$ against the discount factor δ, given the gross surplus $\theta \bar{q}$. Within the range $0 \leq \delta < 1/2$, independent ventures tend to become more attractive as δ grows from 0 towards 1/2, according to the inequality condition (5). This reflects the fact that, as firms become increasingly forward looking, the reduction in initial venture costs made possible by an RJV decreases its importance. Once $\delta \geq 1/2$, on the other hand, a joint venture is unambiguously more profitable than independent ventures.

Figure 7 : Comparative statics on ¯rms' venture decisions in the parametric plane $f±; k_2 ¡ \frac{k_1}{2} g$.



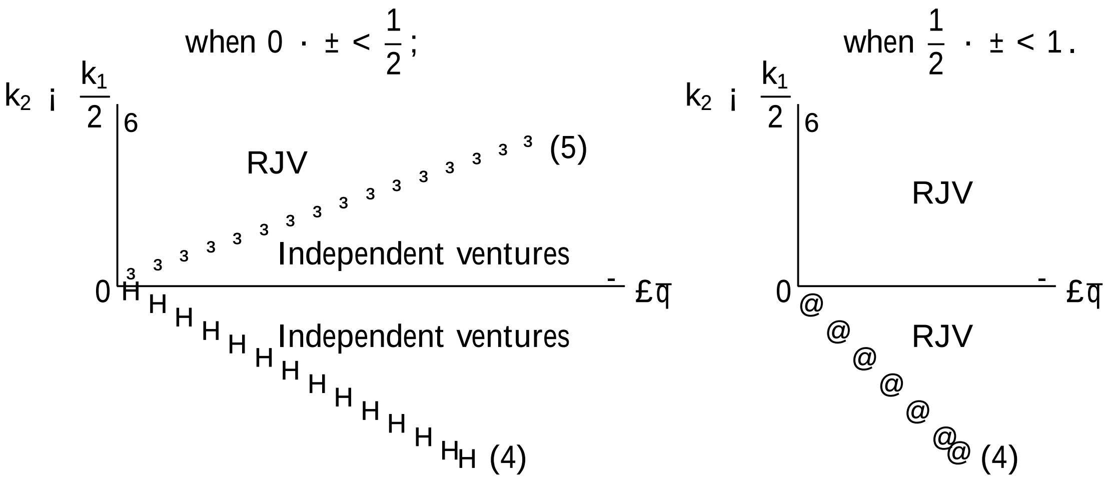
Turn now to the relation between £q and ±. Figure 8 plots the gross surplus £q against the discount factor ±, given the innovation cost di®erential $k_2 ¡ \frac{k_1}{2}$. Clearly from the two diagrams in Figure 8, ¯rms' venture decisions are monotone in the total surplus £q, and is dependent upon £q when and only when $k_2 ¡ \frac{k_1}{2} > 0$ and $± < \frac{1}{2}$.

Figure 8 : Comparative statics on ¯rms' venture decisions in the parametric plane f±; £qg



Finally, Figure 9 plots the venture cost di®erential k_2 ¡ $\frac{k_1}{2}$ against the gross surplus £q̄. Here, the ¯rms' indi®erence boundary between joint and independent ventures rotates counterclockwise as ± increases from 0 towards 1/2, according to the inequality condition (5). Once ± reaches 1/2, condition (5) becomes irrelevant, thereby the indi®erence boundary disappears. This discontinuity re°ects the fact that, once ± exceeds the threshold 1/2, an RJV unambiguously dominates independent ventures exactly by the innovation cost saving k_1=2 (see Figure 6).

Figure 9 : Firms' indi®erence boundary between joint and independent ventures, drawn on a cost (k_2 ¡ $\frac{k_1}{2}$) - bene¯t (£q̄) plane given ±



These observations imply the following.

Corollary iii : When 0 < k_2 ¡ $\frac{k_1}{2}$ < $\frac{£\bar{q}}{48}$, ¯rms' venture decisions are non-monotone in the discount factor ±.

5 Discussion relating to literature

5.1 Horizontal product space

Connoisseurs may have noticed that the subgame ensuing independent ventures in the horizontal product space reminds Friedman and Thisse (1993). Our observation in Lemma 1, which also a®ects Propositions I, II and Corollaries i, ii, is nevertheless dissimilar to Friedman and Thisse. The reason is as follows. The key di®erence between their analysis and ours is the timing when collusive behaviour commences.

Friedman and Thisse stands on the assumption (or, in other words, equilibrium selection criterion) that, in the marketing supergame, firms collude in prices so as to maximise their joint profits given any location pair they have chosen. Based upon this premise, back in the second stage, each firm locates according to individual incentives. Thereby any location decision is not subject to punishment through pricing behaviour. In this sense, collusive behaviour does not commence until the third (marketing) stage.

In our paper, on the contrary, we focus on such equilibria that, if a firm deviates from the prescribed location in the second stage, then both firms compete in marketing by playing the one-shot Bertrand-Nash equilibrium every marketing period. This serves as a punishment against the location deviation. In this sense, collusive behaviour commences in the second (location) stage onwards. The reason why we consider this class of equilibria is because this can entail a more profit-efficient subgame perfect equilibrium outcome.[6]

If we applied a similar analysis to Friedman and Thisse, then our results would be altered accordingly. Firstly, Lemma 1 would be replaced with the following.

Lemma 1* :

- When $\delta \geq \frac{1}{2}$, the two firms' equilibrium locations coincide at $\frac{1}{2}$, and in the marketing stage, $\pi_i^C = \frac{1}{2}\left(s - \frac{1}{4}\right)$ so as to maximise joint profits between the two firms.
- Otherwise, when $\delta < \frac{1}{2}$, they locate at the endpoints of the unit segment and play the one-shot Bertrand-Nash equilibrium at each $t \in [1, \infty)$.

It is algebraically straightforward to verify that this result, including the critical discount factor $\delta^* = \frac{1}{2}$, stands unaffected by the difference in penal codes to sustain price collusion — one-shot Nash reversion in Friedman and Thisse, and Abreu's optimal punishment in our analysis.[7] Also see d'Aspremont, Gabszewicz and Thisse (1979) as for the second half of Lemma 1*.

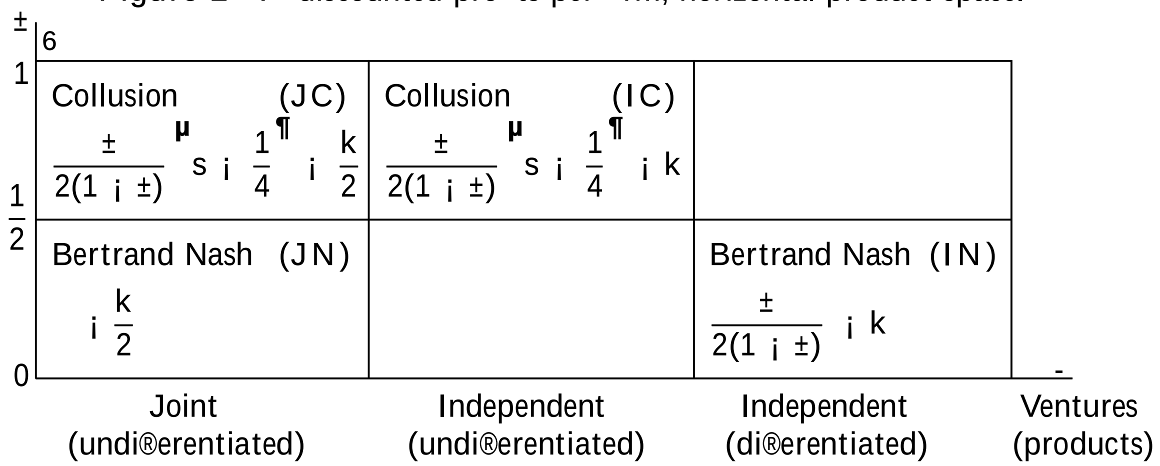
Consequently, Propositions I, II and Figures 2, 3 would be replaced with the following.

[6] One might argue that our punishment scheme against location deviations is not renegotiation proof. Note in general, however, that any punishment using pricing behaviour as an enforcement device, is renegotiation disproof, whether it is against location deviation or price deviation. Hence we find no reason why location decisions cannot be collusive.

[7] These two penal schemes offer the same critical discount factor in a Bertrand supergame when products are perfect substitutes (i.e., located at the same point). See Lambertini and Sasaki (1998).

Proposition I*: The range of time discount factors over which the price collusion in the binary equilibrium is sustainable is $\delta \in [1/2; 1]$ irrespective of firms' venture decisions in the first stage.

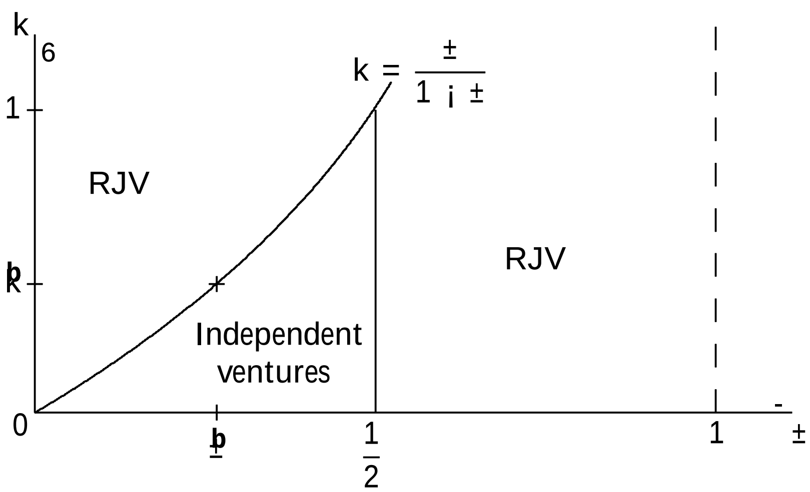
Figure 2*: discounted profits per firm, horizontal product space.



Proposition II*:

- $\delta \in [0; 1/2)$. In this regime, firms are unable to collude. Hence, a joint venture is preferred if and only if (JN) > (IN); i.e., if $k > \frac{\delta}{1-\delta}$.
- $\delta \in [1/2; 1)$. In this regime, firms always collude. As a result, since a joint venture has a cost-saving effect vis à vis independent ventures, while both choices ensure the same stream of operative (collusive) profits, a joint venture is always preferred.

Figure 3*: Comparative statics on firms' venture decisions in the parametric plane $\{\delta; k\}$.



Note in particular that the dependence of ¯rms' innovative venture decisions on the gross surplus s would disappear if we assumed against collusion in location as Friedman and Thisse does. Hence Corollary ii would disappear, and Corollary i would be altered as follows.

Corollary i* : For any given k 2 (0; 1), ¯rms decisions between joint and independent ventures become non-monotone in the discount factor ±.

5.2 Vertical product space

Analogously to the horizontal case, we can either allow or prohibit collusion in location when the product space is vertical. However, the vertical di®erentiation game does not entail observationally distinct outcomes between these two forms of collusion (see Appendix 7.3).

The intuitive reason why these two forms of collusion yield observationally distinct outcomes in the horizontal di®erentiation game is because it is pro¯table to cover the whole market, thereby it is joint pro¯t e±cient for independent ventures to locate far apart from each other so as to cover separate parts of the horizontal segment. In the vertical di®erentiation game, it is no longer pro¯table to cover the low end of the consumers' distribution, so that independent ventures cannot enhance their joint collusive pro¯ts analogously by di®erentiating away from each other.

6 Concluding remarks

We have analysed the unfolding of R&D and market behaviour of ¯rms in a possibly di®erentiated duopoly either horizontally or vertically, alternatively. We have mapped the e®ects of intertemporal preferences, the technology of product development and consumers' willingness to pay on ¯rms' venture decisions as well as on price behaviour over the entire parameter space.

In particular, we have learnt that the interlink between ¯rms' R&D decisions and their prospective ability to collude in marketing hinges crutially upon the form of collusion | more concretely, whether they are to collude in locations and prices, or in prices only. Insofar as ¯rms are to collude whenever possible, given any product portfolio they have chosen, the set of those discount factors under which collusion is strategically sustainable

(± ¸ $\frac{1}{2}$ in our model; see section 5) stands entirely una®ected by the ¯rms' initial choice between joint and independent ventures. This result holds in both horizontal and vertical di®erentiation settings.

On the other hand, if the ¯rms are to collude more e±ciently, then their decisions in product innovation may in°uence their collusive prospects, through the e®ect that horizontal di®erentiation can enhance collusive stability. Namely, by choosing that subgame perfect equlibrium which prescribes price collusion only in the particular subgame commencing from that location pair which maximises the discounted sum of the ¯rms' total pro¯ts, the ¯rms can e®ectively enforce such a pro¯t-e±cient location pair as part of collusive equilibrium path. This enforcement of horizontal di®erentiation enhances not only the ¯rms' collusive pro¯ts, but also the stability of collusion by lowering the critical discount factor. This e®ect is present only with horizontal di®erentiation; in the vertical di®erentiation game, there is no hope in the direction of lowering the critical discount factor by this means.

In brief, the qualitative di®erence between horizontal and vertical product spaces, in relation to the presence or absence of the interactive relations between ¯rms' decisions in product innovation and their ability to sustain price collusion in the ensuing marketing supergame, can be attributed not entirely to the intrinsic di®erence in construction of these two product spaces, but also largely to the way ¯rms collude in the Bertrand supergame. It is only when ¯rms collude e±ciently that they can better stabilise price collusion by developing horizontally, but not vertically, di®erentiated products by investing in independent ventures; hence, the ultimate choice between joint and independent ventures critically depends upon the trade-o® between the cost-saving e®ect of an RJV and the pro-collusive e®ect of independent ventures. In all other cases | i.e., when ¯rms do not punish location deviations, or when the product space is vertical, or both | the choice between joint and independent product innovation does not involve any prospect to stabilise price collusion in the ensuing marketing stage.

Finally, contrary to some of the earlier beliefs, we have established that the relationship between product di®erentiation and the discount factor can indeed be non-monotone. This seemingly counterintuitive result stems from the balance between cost consideration in product development and ¯rms' ability to sustain future collusion, be there any interactive forces between these two e®ects or not.

7 Appendix

7.1 Proof of Lemma 1

Firms 1 and 2 locate at a and $1-b$. Without loss of generality we assume $a \leq 1-b$.

It is straightforward to verify that, insofar as $s \geq 5/4$, it is always profitable for firms, whether pricing collusively or competitively, to cover the entire market, i.e., all consumers in $[0;1]$ should prefer buying to not buying.

The generic location x of the consumer who is indifferent between the two products is defined by

$$s-(x-a)^2-p_1=s-(1-b-x)^2-p_2:$$

Whenever there is a unique x satisfying this condition, which occurs only if $a<1-b$, the following demand system obtains :

$$y_1=\frac{1-b+a}{2}+\frac{p_2-p_1}{2(1-b-a)}; \quad y_2=1-y_1:$$

Otherwise, if there is no such $x \in [0;1]$, one of the firms will take over the whole market. Hence the complete demand system is

$$y_1 = \max\left\{0;\min\left\{\frac{1-b+a}{2}+\frac{p_2-p_1}{2(1-b-a)};1\right\}\right\}; \tag{6}$$

$$y_1 = \max\left\{0;\min\left\{\frac{1-a+b}{2}+\frac{p_1-p_2}{2(1-a-b)};1\right\}\right\}: \tag{7}$$

The two firms are to choose that subgame perfect equilibrium which yields the highest joint discounted profits. The most profitable outcome consists of a location pair $a^C;1-b^C$ and the price pair $p_1^C;p_2^C$. The subgame perfect equilibrium sustaining this outcome, when δ is sufficiently high, is as follows.

- The two firms collude at $p_1^C;p_2^C$ in the marketing supergame if and only if $a=a^C;b=b^C$ has been selected in the second stage. Otherwise, if either $a \neq a^C$ or $b \neq b^C$ has been detected, then they simply repeat the one-shot Bertrand-Nash equilibrium resulting from the location pair $a;1-b$.

- Once the equilibrium location decisions $a=a^C;b=b^C$ have been ovserved, then the firms play $p_1^C;p_2^C$ until any deviation is detected. Once a deviation is detected, then Abreu's optimal punishment comes in effect.

The next step is to examine the stability of such collusion. From the symmetric structure of the game, it is apparent that the most profitable subgame perfect equilibrium must be a symmetric profile. When Abreu's optimal punishment is considered, finding the optimal punishment price p^p as well as the critical threshold of the discount factor δ^* involves solving the following system of simultaneous equations:

$$\pi_i^d(p^C) - \pi_i^C = \delta^*(\pi_i^C - \pi_i(p^p)); \tag{8}$$

$$\pi_i^d(p^p) - \pi_i(p^p) = \delta^*(\pi_i^C - \pi_i(p^p)); \tag{9}$$

where π^C is the collusive profit per firm, per period, p^C is the collusive price, and $\pi_i^d(p_{-i})$ is the profit resulting from the one-shot best response against p_{-i}. As in Chang (1991, 1992), Ross (1992) and Häckner (1995, 1996), we define the collusive profile in terms of a generic pair of symmetric locations a^C and $1 - a^C$ and solve the system (8)-(9) by plugging $a = b = a^C$ into (6)-(7), to obtain p^p and δ^*.

First consider item 1 of Lemma 1. The most profitable outcome (whether it is an equilibrium outcome or not) in marketing is

$$a = b = \frac{1}{4}; \quad p_1 = p_2 = s - \frac{1}{16} \tag{10}$$

as Bonanno (1987) proves. In section 3.1 we define the sustainability condition for this price collusion as $\delta \geq \bar{\delta}[s]$. It can be verified from (8)-(9) that $\bar{\delta}[s]$ increases in the total surplus s. In particular, $\bar{\delta}[s] \uparrow \frac{1}{2}$ as $s \uparrow \infty$. On the other hand, when $a^C \in [0, \frac{1}{4}]$ and $\frac{5}{4} \leq s \leq \frac{25}{16}$, the quantity sold by the deviator from the collusive price is not bound by the upperlimit (= unity), hence the solution to (8)-(9) is

$$p^p = -\frac{(4s + 8a^C - 5)^2}{64(1 - 2a^C)}; \qquad \delta^* = \frac{(4s + 8a^C - 5)^2}{(4s - 8a^C + 3)^2} \tag{11}$$

with the deviation output

$$y_i^d(p^p) = \frac{4((a^C)^2 + a^C - s) - 3}{16(2a^C - 1)}. \tag{12}$$

Letting $a^C = \frac{1}{4}$ in (11) we obtain $\bar{\delta}[s] = \left(\frac{4s - 3}{4s + 1}\right)^2$. Especially, $\bar{\delta}\left(\frac{5}{4}\right) = \frac{1}{9}$.

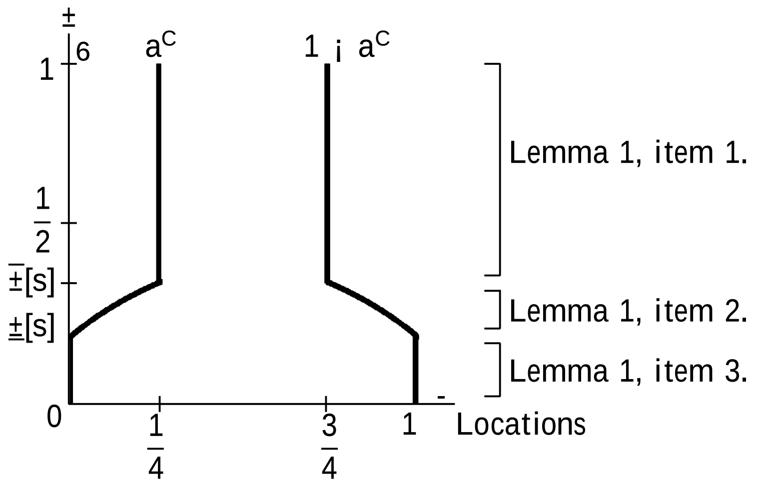
Now proceed to item 2 of in Lemma 1. From (6)-(7) and (8)-(9), δ^* strictly increases in $a^C \in [0, \frac{1}{4}]$ given s. Hence $\underline{\delta}[s]$ is the critical discount factor when $a^C = 0$. This directly implies that, when $\underline{\delta}[s] \leq \delta < \bar{\delta}[s]$, firms locate a^C and $1 - a^C$ such that

$$a^C = \arg_a [\delta^* = \delta \mid s]; \tag{13}$$

which increases in δ. In particular, firms choose $a^C = 0$ when $\delta = \underline{\delta}[s]$. Plugging $a^C = 0$ into (11), it can be verified that $\underline{\delta}[s] = \left(\frac{4s - 5}{4s + 3}\right)^2$. The deviation output $y_i^d(p^p) < 1$ for all $s < \frac{13}{4}$, with $\underline{\pm}\left(\frac{5}{4}\right) = 0$. For all s ¸ $\frac{13}{4}$; the deviation output is the entire market, $y_i^d(p^p) = 1$.

When $\pm < \underline{\pm}[s]$, price collusion at maximum di®erentiation is unsustainable by means of Abreu's optimal punishment. Hence item 3 of Lemma 1 comes in e®ect. When ¯rms are unable to collude, they play the well known two-stage subgame perfect equilibrium yielding maximum di®erentiation (d'Aspremont, Gabszewicz and Thisse, 1979).

Figure 10 : Endogenous horizontal di®erentiation (independent ventures).



Hence, ¯rms choose to collude whenever $\pm$ ¸ $\underline{\pm}[s]$. In summary, the foregoing discussion establishes that price collusion ensuing independent ventures becomes easier to sustain when (i) product di®erentiation is large, and (ii) gross individual surplus s is low.

7.2 Proof of Lemma 2

In each period of the market supergame, the demand functions obtain as follows. When $q_1 > q_2$ (which can occur only when the two ¯rms develop their products independently), we identify the marginal willingness to pay of the consumer who is indi®erent between buying the high-quality good and buying the low-quality good, denoted by h, and that between buying the low-quality good and not buying at all, denoted by l, as:

$$h = \frac{p_1 ¡ p_2}{q_1 ¡ q_2}; \qquad l = \frac{p_2}{q_2}: \tag{14}$$

Hence, the demand functions are unravelled as follows:

$$y_1 = \frac{£ ¡ \min f h; £ g}{£}; \qquad y_2 = \frac{\min f h; £ g ¡ \min f l; £ g}{£}: \tag{15}$$

On the other hand, if the two firms produce an identical quality $q_1 = q_2$, if $p_1 \neq p_2$ then whichever firm charging the higher price sells nil; otherwise if $p_1 = p_2$ they split the market evenly, attracting

$$y_1 = y_2 = \frac{1}{2\theta}\left(\theta - \min\left\{\frac{p_1}{q_1}, \theta\right\}\right) \tag{16}$$

customers each.

Under the assumption that unit variable cost of production is nil, the per period profit of firm i is $\pi_i = p_i y_i$.

The first half of Lemma 2 can be proven through the following three steps.

Step 1 : When $q_1 = q_2$, each firm pays the innovation cost k_1. The joint profit between the two firms per marketing period is

$$\pi_1 + \pi_2 = \frac{\min\{p_1, p_2\}}{\theta}\left(\theta - \frac{\min\{p_1, p_2\}}{q_1}\right) \tag{17}$$

as long as

$$\frac{\min\{p_1, p_2\}}{q_1} \leq \theta$$

(see (16)). The first-order derivative

$$\frac{\partial(\pi_1 + \pi_2)}{\partial \min\{p_1, p_2\}} = 0$$

is satisfied at $\min\{p_1, p_2\} = \frac{\theta q_1}{2}$, with which the joint profit (17) increases in q_1, attaining its maximum when $q_1 = \overline{q}$. It is straightforward to verify that the resulting joint profit

$$\pi_1 + \pi_2 = \frac{\min\{p_1, p_2\}}{\theta}\left(\theta - \frac{\min\{p_1, p_2\}}{\overline{q}}\right)$$

is no less than the supremum of the sum of (19) over the range (18). Hence $q_1 = q_2 = \overline{q}$, $\min\{p_1, p_2\} = \frac{\theta\overline{q}}{2}$ is joint profit maximal.[8] We assume that colluding firms split the demand evenly by setting $p_1 = p_2 = \frac{\theta\overline{q}}{2}$.

Step 2 : We now need to prove that there is no equilibrium with $q_1 > q_2$. By (14), when $q_1 > q_2$ each firm attracts a strictly positive demand iff

$$\frac{p_2}{q_2} < \frac{p_1 - p_2}{q_1 - q_2} < \theta: \tag{18}$$

[8] An analogous result has been derived by Rosenkranz (1995, p. 13) in a model of vertical differentiation under the assumption of full market coverage.

Insofar as these conditions are satisfied, by demand functions (15), per period profit functions are

$$\pi_1 = \frac{p_1}{\Theta}\left(\Theta - \frac{p_1 - p_2}{q_1 - q_2}\right); \qquad \pi_2 = \frac{p_2}{\Theta}\left(\frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}\right). \tag{19}$$

Within the range (18), the system of first-order conditions

$$\frac{\partial}{\partial p_1}(\pi_1 + \pi_2) = \frac{\partial}{\partial p_2}(\pi_1 + \pi_2) = 0$$

has no interior solution. The only solution is the limiting solution on the boundary of the range (18):

$$p_1 \to \frac{\Theta q_1}{2}; \qquad p_2 \to \frac{\Theta q_2}{2}$$

which implies that the lower quality attracts zero demand.

Step 3 : Finally, we need to ascertain that a firm does not have a strict incentive to deviate from $\bar{q}$ to a lower quality without expecting any positive demand. When both firms produce $\bar{q}$, each of them earns the discounted net profit

$$-k_1 + \frac{\delta}{8(1 - \delta)}\Theta\bar{q}. \tag{20}$$

If a firm deviates to a lower quality $\bar{q} - \varepsilon$, the firm's net discounted profit becomes simply $-k_2$, which is strictly lower than (20) under the assumption (4). Hence, the deviation is unprofitable.

Note also that, whenever $q_1 = q_2$ (whether they are at $\bar{q}$ or not) collusion is sustainable if and only if $\delta \geq \frac{1}{2}$. This, in conjunction with above step 2, implies that item 2 in the trichotomy preceding Lemma 2 (see section 4.1) is vacuous. This completes the proof of the first half of Lemma 2.

On the other hand, the second half of the lemma can be proven using in part the following Lemma 2-i.

Lemma 2-i : If firms undertake independent ventures and anticipate Bertrand-Nash competition in marketing, then any pure-strategy equilibrium must have $q_2 = \frac{4}{7}q_1$ in the second stage.

Proof : The profit functions at the first stage are (cf. Choi and Shin, 1992):

$$\pi_1 = \frac{4\Theta q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2}; \qquad \pi_2 = \frac{\Theta q_1 q_2(q_1 - q_2)}{(4q_1 - q_2)^2}.$$

It can be immediately verified that, as $\partial \pi_2 / \partial q_2 = 0$ if $q_2 = 4q_1/7$; the solution to the leader's problem defined as

$$\max_{q_1} \pi_1 \left(q_1 \middle| q_2 = \frac{4}{7} q_1 \right)$$

is $q_1 = \bar{q}$, i.e., it coincides with the Nash best reply identified by Choi and Shin. ■

The remainder of the proof of the second half of Lemma 2 is to ascertain that, given $q_2 = \frac{4}{7}\bar{q}$, firm 1 does not have a strict incentive to deviate from $q_1 = \bar{q}$ to $q_1 = \frac{16}{49}\bar{q}$, in the latter case firms indeed switch labels since we always refer to the higher quality firm as "firm 1".

$$\text{If} \quad q_1 = \bar{q};\ q_2 = \frac{4}{7}\bar{q}; \quad \text{then} \ \pi_1 = \frac{7}{48} \times \bar{q}:$$
$$\text{If} \quad q_1 = \frac{4}{7}\bar{q};\ q_2 = \frac{16}{49}\bar{q}; \quad \text{then} \ \pi_2 = \frac{1}{84} \times \bar{q}:$$

Hence the condition for no deviation is

$$\frac{\delta}{1-\delta} \cdot \frac{7}{48} \times \bar{q} - k_1 \quad \geq \quad \frac{\delta}{1-\delta} \cdot \frac{1}{84} \times \bar{q} - k_2;$$

which simplifies into

$$\frac{15\delta}{112(1-\delta)} \times \bar{q} \quad \geq \quad k_1 - k_2:$$

Obviously, this is always satisfied under assumption (4). This completes the proof of the second half of Lemma 2.

7.3 Supplementary note on unilateral spillover externality

Consider the following alternative game as a thought experiment.

Definition : Game Γ_B is a three-stage game which is identical to the vertical differentiation game in section 4 except that, in the second stage, independent ventures are to locate their products sequentially, firm 1 first and then firm 2 second, and that the costs of product innovation for these two firms are k_1 and k_2 respectively.

Lemma 2-ii : In Game Γ_B, if firms choose independent ventures and anticipate Bertrand-Nash competition in the marketing stage, then in equilibrium $q_1 = \bar{q}$ and $q_2 = 4\bar{q}/7$.

Proof is identical to the proof of Lemma 2 except that the condition (4) is no longer relevant. ■

Comparing Lemma 2-ii with Lemma 2 in section 4, the following can be veri¯ed.

Corollary iv: Whenever condition (4) is satis¯ed, the game $¡_B$ and the vertical di®erentiation game described in section 4 are observationally equivalent.

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