On technology competition*

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Abstract

One of the key features of our economies consists of the coexistence of different technologies supplying similar products and services. Also, we often observe that an 'old' technology is improved when a new one, doing basically the same things, appears. Behind this process of improvement of the old technology almost always lies an intentional research activity carried out by the owner of the old technology, whose aim is to delay as much as possible the loss of its market share. We focus our attention on this competition process and we supply a formal model capable of describing the delay of the overtaking of the new technology over the old one.

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1. Introduction

Technological change is undoubtedly one of the driving forces of our economies, and two striking features of our production systems consist of technological variety on the one hand, and technological persistence on the other. The main aim of the present paper is to analyze the process of competition between two technologies, one which we will call 'old' and the other which will be defined as 'new', while reviewing some of the determinants of both variety and mechanisms which favour persistence. The interest in such a topic arises from the observation that there exist technologies which experience a life much longer than expected, as they are - at least technologically - overwhelmingly outperformed by new and emerging ones.

Economic theory has already hinted, sometimes long ago, at some mechanisms which contribute to prolong the life of 'old' technologies, and in particular learning-by-doing (Smith, 1776; Arrow, 1962) and learning-by-using (Rosenberg, 1982), to which we ought to add costs analysis as at first developed by Frankel (1955) and, more thoroughly, by Salter (1969).

We have to emphasise the fact that economic theory has found it difficult for long to deal with phenomena concerning technological change, its origins and its diffusion, not to mention the problems related to increasing returns. At first technological change was seen as 'manna from heaven', while adoption and diffusion of best-practice technology were considered to be instantaneous. Things have changed dramatically since the late 1980s, and different schools of thought have contributed to the analysis.

What we are mainly interested in here is, in particular, that sort of mechanism, which implies intentional action pointing at improving the performance of an 'old' technology, as stimulated by the emergence of a new one, devoted to supply the same sort of operations or services.

The paper is divided as follows: section two hints at technological variety as a characteristic feature of our socio-economic environment; section three explicitly recalls examples and models pointing at 'old' and 'new' technologies competing; section four contains the model proposed and the simulations which describe the process of competition between two technologies via improved performance; the fifth and final section contains the conclusions.

2. Technological variety, technological competition and economics

Ever since the first Industrial Revolution occurred in the late 18th century, economic and technological forces have shaped our socio-economic systems, and it is very difficult to disentangle the two forces. Economic incentives have strong effects on the direction undertaken by technological creation and development, but the way in which technology can change is not free from constraints. On the other hand, technology opens economic opportunities otherwise unthinkable. The whole matter is made more complex by the existence of rules, standards and limits of various kind, which in turn, contribute to 'bias' the evolution of technology itself.

If, on the one hand technology can only be developed along specific paths – the notions of technological systems, technological paradigms and trajectories are analytical tools pointing at these limits – on the other hand we observe that often the same results can be obtained by means of completely different technologies. Let us consider a few examples.

We can produce electricity in many different ways, by burning different kinds of fuel, exploiting nuclear reactions, falling water or by means of photovoltaic cells; moreover, often within each technology different paths are open, so, for instance, two different types of photovoltaic cells are available, 'photox cells' based on cuprous oxide and 'photronic cells' based on the use of selenium, the result being always the production of electricity. Electricity itself, at first, was produced and used separately as 'direct current' was opposed to 'alternating current'; in this case devices were realized after a while in order to make use of either type of current.

Sound recording also is characterised by a rich technological history. Different kinds of tape and materials for disks have been used for recording, while different techniques have been used through time for the recording of phonograph records (mechanical, electrical, digital). Before the occurrence of the digital revolution we took for granted for a long while that, as long as a tape was involved in playing music, the medium was essentially a plastic tape coated with magnetizable iron oxide particles. However, as Morton has pointed out, there has been a (short) period, immediately after World War II, during which people spoke of *wire recorder* rather than of *tape recorders.* The wire recorder owed its name to "the ability of this machine to capture sound in the form of magnetic impulses on a very fine steel wire" (Morton, 1998, p. 213). Other aspects related to sound recording and reproduction are important: monophonic, stereophonic and quadraphonic systems have been developed and adopted with different success; in fact while monophonic disk recording and reproducing systems were quickly supplanted by strereophonic ones, quadraphonic systems never reached proper market success.

Technological competition can also be looked at from a different perspective, making reference to broader principles, such as the case of digital and analog ways of getting a particular result. In fact, one of the most important forms of competition of these years is actually the one between analog and digital systems.

Digital and analog systems can often provide similar services, but with a different efficiency and quality of the result. A typical path, however, is that of digital technologies supplanting – sometimes gradually, sometimes suddenly – the existing analog ones.

Some attention ought to be devoted to 'bridging technologies' which are systematically developed through time between techniques or technologies held to be, at first, incompatible. It must be pointed out that it is generally an economic incentive that pushes towards this forms of technology creativity. In fact, often massive investments run the risk of being made suddenly obsolescent, so that intentional efforts are devoted to make complementary, rather than alternative, two (or more) techniques/technologies. We have already hinted at the example of direct current and alternate current, but there exist other examples.

One we all are familiar with is modems, which belong in the family of analog-to-digital converter devices. A *modem* – which owes its name to the contraction of the words *mo*dulation-*dem*odulation – in fact, is a device capable of converting the digital signals produced by computers into the analog signals that telephone circuits are designed to carry (EST, 1996, article modem). The reason for developing such a device lies in that, despite the availability of all-digital transmission networks, there exists a massive worldwide analog telephone network which constitute the most widespread and readily available facility for data transmission, besides voice. Could we forget the economic problem, it would be much more efficient to make use of all-digital transmission networks, which would guarantee higher speed and clearer transmission, also for human speech. However, the analog network was already there, and it could be used to promote digital technology (EST, 1996, article modem).

Analog and digital co-existence has been developed not only in terms of translation of signals, so that two devices based on a different technology can communicate, but in terms of constructing palindromic machines: during the 1960s in fact hybrid, or analog-digital, computers were produced.

The 1990s might have even opened a new way of computing, as we can already speak of DNA or molecular computers, characterized by high speed, capability of information storage and an extremely efficient use of energy:

> "For example, one gram of DNA, which when dry would occupy a volume of approximately one cubic centimeter, can store as much information as approximately one trillion CDs. [...] Molecular computers also have the potential for extraordinary energy efficiency. In principle one joule is sufficient for approximately 2×10^{19} ligation operations. [...] Existing supercomputers are far less efficient, executing at most 10^9 operations per joule" (Adleman, 1998, p. 41).

Many more examples could easily be supplied. The ones indicated above make it clear that technological variety, change and competition constitute three structural feautures of our economies, while economic forces affect the whole process.

3. The coexistence of old and new technologies

There exist economic and technological reasons which justify the coexistence of old and new technologies. Two basic types of forces can be identified: (i) forces which actually affect factors' productivity and (ii) forces which depend on sunk costs.

The two examples often referred to with respect to the first type of force are learning-by-doing and learning-by-using.

Learning-by-doing occurs at different levels of production, and concerns both individuals and work processes or organizations as a whole. Typical examples have become the ones referred to by Arrow (1962), i.e. the Horndal iron works in Sweden – where productivity rose by 2% per annum for fifteen year, despite the fact that no investment had occurred –, and the production of airframes – to produce the n^{th}airframe of a given type the amount of labour required is proportional to $n^{-1/3}$. In this case factors' productivity increases with cumulative output, and the existing techniques/technologies can survive for long, even when a new one emerges. We can expect learning-by-doing to occur at any level of technological complexity.

Learning-by-using has been referred to by Rosenberg (1982) as that process of learning, and form of disembodied technological change, which concerns the use of machinery, that is the process whereby we learn how to use identical machines in more efficient ways. Typical examples are the ones related to the use of machine tools, whose productivity – given identical machines – increases with their use.

The two categories of learning just mentioned can contribute to define the innovative content of 'simple' technology reproduction. In fact, what we learn through production can affect the way in which the technology which is being used can change. Technology reproduction can be looked at from at least two perspectives: *symbolic* and *material*. By symbolic reproduction we mean the codified form of transmission of technological knowledge such as the one which we find in technical textbooks, in manuals of use of machines or also in internal reports; however when we materially reproduce our means of production as well as products, we are likely to discover new ways of doing things. The material reproduction of technology contains a high degree of tacit knowledge, part of which can be codified at some future time. Both processes lead to improvements of the existing technologies and products, and thus contribute to prolong their life.

If technology reproduction often leads to improvements of both products and processes of production, there also exist lock-in phenomena which my lead to long-run inefficiency due to early adoption of a particular technology. About ten years ago Brian Arthur presented a model in which two technologies compete for a market; both technologies display 'naturally' increasing returns to adoption "in that the more they are adopted the more experience is gained with them, and the more they are improved" (Arthur, 1989, p. 116). Thus, Arthur argues, given two technologies which become available at the same time, simple adoption could lead to a process which leads the worst technology to corner the market – a random walk with absorbing barriers. Examples usually indicated concern video recorders (betamax *vs* VHS), keyboards (the so-called qwerty keyboard) or the distance between the rails in Britain.

The comment above leads us to the point of pervasiveness of a technology. The more a technology is pervasive and diffused, the more difficult will be to switch to a new one. It is not only a problem of the direct costs associated with substitution, but of all of the connected necessary changes, institutional, organizational, and so on. Any new technology, in order to be adopted, must offer a series of incentives in terms of both economic and non-economic conditions.

But let us turn our attention to the explicit analysis of the effects of sunk costs on keeping alive an old technology. We make use here of Frankel's (1955) article which clarifies why from a purely technological viewpoint an old technology can happily survive together with a new, more efficient, one. Let us summarize his main results.

Frankel's analysis focuses on process innovation, and he begins with use of two cost categories: (i) past or sunk outlays, which "denote expenditures already made which are allocable to some future production period and which must be recovered from future revenues", and (ii) future outlays, which "denote all costs that will be incurred over some future production period". For an old method already in use, past outlays always are some positive amount, while for a new method not yet adopted, past outlays are zero, and total costs are simply given by future outlays (Frankel, 1955, p. 298).

Given these initial definitions, and by defining P = past outlays of the old method, F = future outlays of the old method, $F+P$ = total costs of the old method, T = total costs of the new method, R_1 = total revenue from the old method, R_2 = total revenue from the new method, the two following criteria emerge:

if $[R_1 - (F + P)] < (R_2 - T) < (R_1 - F)$ the old method should be continued in use until the fixed plant wears out;

if $(R_2 - T) > (R_1 - F)$ the old method should be immediately replaced (Frankel, 1955).

Later in the paper he explains a model in which the further aspect of technology: interrelatedness is taken into account. As an explanatory device he uses the case of a plant in which there exist a number of components (machines) which are interrelated, that is new ones need be accommodated into an existing framework. Should this be the case, the only possibility would be to replace the worn-out machine with a duplicate of itself, which in turn, would mean that innovation could not take place (Frankel, 1955, pp. 301-304). Despite this, Frankel shows that there still exists the possibility of a technologically outdated plant to survive as long as total unit costs of the new method do not fall below a certain percentage of those of the old method – the percentage depending on the degree of interrelatedness, the discount rate, the durability of capital, the ratio of total variable to total fixed costs of the old method, the ratio of the fixed costs of the new to the fixed costs of the old method and the elasticity of demand. Frankel (1955, p. 303) shows a set of examples in which the old technology would remain in place until total unit costs of the new method do not fall below 18.6% of those of the old method .

Salter's model represents a milestone in clarifying *theoretically* the adjustment process which takes place within an industry in which there occurs technological change. In its simplest version, the model considers the coexistence of different techniques characterized by different cost structures. First of all, this makes it clear that it is inappropriate to discuss productivity movements of an industry in terms of a representative firm, as the dispersion around the average is too important to be excluded from the analysis. Then, by defining the price of the commodity produced as the sum of operating costs (labour and materials rewarded according to their prices) and capital costs (which include normal profits), one obtains the price which defines the oldest plants that can remain in operation. The marginal firm will be that one whose operating costs equal the price – at this technological stage. As soon as a new best-practice technique will become available, this marginal firm will have to leave the market as the price would go down, thus rendering its operating costs higher than the new price (Salter, 1969, pp. 58-59).

In this model there is room for the coexistence between different techniques characterized by a broad range of cost structure. The reason why they can coexist lies in that old and new techniques compete upon unequal terms: in fact, while new plants will be constructed when gross revenue is expected to cover the supply price of all factors of production, thus *including* capital, for plants already in existence the capital at work has no longer a supply price. Put it another way, bygones are bygones. There remains that these "unequal terms of competition give existing capital equipment a lease of life even when outmoded" (Salter, 1969, p. 62).

A more complete model, in which firms differ under many respects, i.e. unit costs, propensity to accumulate and even products characteristics is presented by Metcalfe (1998).

To sum up we can note that:

> "Nothing in our understanding of the economics of investments decisions suggests that new technology is necessarily *economically* superior to existing technology or that today is always the best time to invest in new technology. Delay may well be rational." (Metcalfe, 1997, p. 128, emphasis added).

The rationality of the delay – he continues – which affects the adoption and diffusion processes, may lie in the systemic connections, expected gains from delays and even capacity-expansion decisions.

Recent theorizing has concentrated on quite a few sides of technology, i.e. technology creation, technological change and persistence, and technology's relationship with growth (Barro and Sala-i-Martin, 1995; Stoneman, 1995; Metcalfe, 1998; Nelson, 1998). However, as far as we know, there does not exist a formal model which takes explicitly into account the competition which can often be observed between two (or more) technologies in providing the same sort of product or service.

4. The model

The phenomenon we want to model is that of the intentional reaction which develops on the side of an 'old' technology whenever a new one appears. This phenomenon cannot be considered as marginal, as the history of technology supplies many cases of a reaction of this kind, and this happens today as it happened long ago. We can think of the competition between artisans and early machinery, the mechanization process of entire work processes on the one hand and the persistence of the putting-out system on the other hand, the developments in analog technologies at the appearance of digital ones. The general process is constituted by the fact that the development of the old technology is engendered by its eventual supplanter.

Before illustrating our model, let us briefly recall three clear examples, the first referring to the sailship, the second referring to the so-called ADSL technology in data transfer, and the third to superconductors. The first example is recounted by Gilfillan (1935) who showed how the 'old' sailship was quickly improved as steamships emerged during the 19th century. Improvements concerned nearly all of the components and materials of the sailship which was thus transformed from a basically wooden structure to a basically metallic one, whose carrying capability was massively improved. As Gilfillan pointed out:

> "Large size was 5- to 800 tons in 1800, and for three centuries previously; by mid-century it was 2,000, carrying hundreds, even 1,000 emigrants; and today 3- to 5,000 tons register. Iron was used first for bolts, then in diagonal straps for strengthening the hull, then in knees (1810), breast-hooks and pillars, then for frames (composite build 1850), and for the whole hull, 1838, followed much later by steel, and in time by iron (1840) and steel (1863) in masts, spars and standing and even running rigging." (Gilfillan, 1935, pp. 156-157)

The process which saw sailing ships disappearing was thus one characterised by a process of technological progress, and carrying capacity was multiplied by more than six times in less than a century. In this the disappearance of the sailship was delayed by quite a long period. Eventually, however, the process reached its final point, as the new technology was overwhelmingly superior to the old one, as the steam-engine applied to ships could also be massively improved. The basic point is that:

> "It is paradoxical, but on examination logical, that this noble flowering of the sailing ship, this apotheosis during her decline and just before extermination, was partly vouchsafed by her supplanter, the steamer." (Gilfillan, 1935, p. 156)

Coming to the 1990s, we consider the case of the medium of data transmission. Two basic sets of technologies are available: (i) the traditional ones, based on ordinary copper-wire telephone lines and TV cables, and (ii) the new ones based on fibre optics. The main benefits of fibre optics consist of high bandwidth, low cost and small diameter of the cables – i.e. fiber optics outperform both copper and TV cables in each relevant characteristic.. Given this technological situation one would expect a quick overtaking process of fiber optics over wires.

For instance, in 1995 the Italian (then monopolistic) telephone company launched a campaign worth nearly 8bn dollars in order to fix fibre optics in 10m Italian homes, so that huge quantities of data could travel along the new fibres. However, in its early stages the plan was scrapped: in 1996 broadband modems appeared – in the United States – capable of overcoming the bandwidth limits of the copper wire. The development of such a device was actually stimulated by the need of transferring more and more data (internet, digital TV and whatever sort of data can be transmitted) from providers to homes by means of the existing plain old telephone system (and cable TV networks), which represents the most widespread network in the world, as it started long ago, and virtually all families (of the rich countries) do possess a telephone line. Actually, two broadband contenders have appeared, one for TV cables and another one for telephone lines – the latter technology is called asymmetric digital subscriber line or ADSL (Halfhill, 1996; Brownstein, 1997). These innovations were stimulated by the laggard component of the transmission system, at a time in which fiber optics were competitive under many respects. What is important is that an intentional research effort was started and, despite the fact that fiber optics are still more efficient than copper, the leap due to the breaking of the limited bandwidth of copper wires has rendered their life much longer.

The third example refers to the use of superconductors as better materials for building computers' circuits. This new promising technology has not (yet) reached the market in spite of the enormous potential of superconductivity. Among the many proposed applications there are logical elements for digital circuits. The so-called *Josephson junctions*, essentially two superconductors separated by a tiny non-superconductor barrier (Barone and Paternò, 1982), are recognized as devices capable of operating at a speed about two orders of magnitude higher than conventional silicium-based semiconducting circuits and about one order of magnitude higher than the new (and not yet fully marketed) gallium-arsenide compounds (Hayakawa, 1990). Perhaps, and even more important, the superconducting devices are capable to operate at very low levels of power dissipation per gate, perhaps three orders of magnitude lower than semiconducting devices (Hayakawa, 1990), so easing the miniaturizing problems and, in principle, making the speed limit even higher. Finally, the discovery of a new generation of superconductors, the so-called High Tc Superconductors, has further moved this physical speed limits upward. The question, repeated at each conference on applied superconductivity, is always: "why the superconducting revolution has not come yet?"

One possible element is that improvements in semiconductors have been so fast that it is difficult to estimate if and when their betterment will slow down. Not only in everyday life we witness to constantly improved processors sold at constant or decreasing prices, but also the data on top-performing computers (the ones for which the refrigeration necessary for superconductors is presumably not an issue) clearly demonstrate that the clock speed has increased exponentially through time. Just as an example in 1970 the top super-computers used a clock frequency just above $1MHz$, while nowadays computers have clock frequency at least around $1000MHz$, the evolution nicely following an exponential law (Malone, 1995). Thus the achievements of Josephson computers have been frustrated by the impression to "shoot at a moving target". On the other hand, also the performances of *prototype* Josephson devices have been impressive; just to quote two examples:

(i) Since the 1980s a superconducting computer based on Josephson technology was capable of impressive – at the time – performances, i.e. 4 bit X 256-word, 8 bit address RAM and 1,000 MHz clock speed (Takada, 1991). The work was carried out essentially by the American IBM project (1969-83) (Anacker and Zappe, 1972) and the Japanese MITI project (1981-91) (Kroger, 1986). The physical limitation of these projects was that of the so-called *latching* (or *voltage-state*) logic, which is about few thousands of MHz.

(ii) A new generation of devices based on Single Flux Quantum dynamics vortex has been proposed by Soviet scientists as early as 1985 (Likharev et al., 1991). The work has been continued in the USA – after the collapse of the Soviet Union – leading to prototype logical circuits as fast as *500,000MHz* thus supporting the claim that a peta-flop computer is now possible with Josephson technology (Likharev, 1999).

However, despite these potentialities, semiconductors still constitute the centre of attention of computer makers. One explanation is that until the semiconductor devices will have not reached their physical upper limit there will be little room for research on radically new devices. In other words, until the semiconductor technology will be able to make steady improvements, it will be more profitable for companies to improve the old technology rather than funding projects to convert a prototype into a marketable product. This, in turn, is due to two basic factors: (a) the size of the already existing companies focusing their R&D on traditional semiconductors and miniaturization; (b) the marginal improvement per unit of money invested in the established technology could be greater than the respective increment obtained in an intrinsically more difficult technology. The latter statement might be stronger when both technologies are far away from the ultimate physical limits.

What we want to emphasize here is that the delay in the introduction of a new technology cannot be just the result of inertia or some other non-rational choice, but rather the result of day-by-day logical and economically sound decisions.

This is the kind of problems we have in mind, and hereafter we propose a model that – we hope – sheds some light on the dynamics which develop within the competition process between two technologies.

Thus, in our model, two technologies exist which provide the same service; we will call A the 'old', and B the 'new' technology; technology and firm will be used as synonymous, thus we will refer to A (and B) as either technology or firm. Both technologies are characterized by an upper limit in terms of performance; both technologies can be improved by means of an R&D activity, however, being A the older technology it is characterized by both a lower upper limit, and a closer proximity to the upper limit itself with respect to B (when B will appear).

The initial situation is given by the existence of A only, which allows the production of a certain quantity of output sold at a certain price. We do not investigate the growth of output (put it another way, demand always matches supply, whatever quantity is supplied), as we are interested in the market share of the two technologies; as far as price is concerned – but we will not deal explicitly with its dynamics – it is performance-related and higher than costs, that is profits are positive by assumption.

Firm A earns positive profits $G_A(t)$, which depend on its market share S_A and on its technological performance, which is synthesised by our term P_A. In the same way, we will define S_B, $G_B(t)$ and P_B as the share, the profits and the technological performance of B, respectively. Total profits and share are standardized to 1.

If we define S_A (S_B) as the fraction of the market captured by the technology A (B), we assume that this fraction depends on the performances of the two technologies in the following simple way:

$$S_A(t) = \frac{P_A(t)}{P_A(t) + P_B(t)} \tag{4.1}$$

and consequently A's profits are simply proportional to its market share:

$$G_A(t) = \pi \frac{P_A(t)}{P_A(t) + P_B(t)} \tag{4.2}$$

where π represents the profits of the market as a whole which, as already hinted above, we standardize to 1, so that our equation (4.2) becomes $G_A(t) = \frac{P_A(t)}{P_A(t)+P_B(t)}$. At first B's performance, share and profits equal 0, and $G_A = 1$, i.e. the existing (not yet definable as old) technology takes all. We assume that no part of profits is reinvested at all in any R&D activity until there emerges the competition of B.

Analogously, we can write for B:

$$G_B(t) = \pi \frac{P_B(t)}{P_A(t) + P_B(t)} \tag{4.3}$$

and given that profits are standardized to 1, we can write:

$$G_B(t) = S_B(t) = \frac{P_B(t)}{P_A(t) + P_B(t)} \tag{4.4}$$

that is, B's profits depend on its own performance as well as on A's performance. It is important to notice that the market share depends only on the performances *at the same time*, and therefore in this model it is not included any effect due to the tendency of the market to keep or 'prefer' an old technology.

Technological performance can be improved only by investing in intentional R&D activity, whose funding comes from profits; the percentage of profits invested in R&D, in turn, depends on the 'dangerousness', i.e. on the share, of the rival technology.

The dynamics of the performance of technology A can thus be written as:

$$dP_A(t) = E_A G_A \frac{P_B(t)}{P_A(t)+P_B(t)} e^{-\frac{P_A^M}{P_A^M - P_A(t)}} dt \tag{4.5}$$

where E_A is a sort of 'efficiency converter' of the money spent in R&D, that is the higher E_A the more effective the expenditure in R&D, G_A are the profits earned by technology A, $P_B/(P_A + P_B)$ is the weight of technology B performance which affects the amount of R&D performed by A (that is the higher P_B the more A will invest on R&D), while $\exp\left[-P_A^M/(P_A^M - P_A)\right]$ is the weight which limits A's performance to its maximum value, P_A^M (that means that once technology A has reached its best performance its rate of performance improvement approaches 0).

Of course, we will write a similar expression for technology B:

$$dP_B(t) = E_B G_B \frac{P_A(t)}{P_A(t)+P_B(t)} e^{-\frac{P_B^M}{P_B^M - P_B(t)}} dt \tag{4.6}$$

Thus, by taking into account that $G_A = P_A/(P_A + P_B)$ and that $G_B = P_B/(P_A + P_B)$ [i.e., equation (4.3)] the rate of change of the performance of the two technologies can be rewritten as:

$$\frac{dP_A(t)}{dt} = E_A \frac{P_A(t)P_B(t)}{[P_A(t)+P_B(t)]^2} e^{-\frac{P_A^M}{P_A^M - P_A(t)}} \tag{4.7}$$

$$\frac{dP_B(t)}{dt} = E_B \frac{P_A(t)P_B(t)}{[P_A(t)+P_B(t)]^2} e^{-\frac{P_B^M}{P_B^M - P_B(t)}} \tag{4.8}$$

We can further decrease the number of parameters by noting that renormalizing time ($t \longrightarrow E_B t$) and introducing the factor $\gamma = E_A/E_B$, equations (4.7-4.8) become:

$$\frac{dP_A(t)}{dt} = \gamma \frac{P_A(t)P_B(t)}{[P_A(t)+P_B(t)]^2} e^{-\frac{P_A^M}{P_A^M - P_A(t)}} \tag{4.9}$$

$$\frac{dP_B(t)}{dt} = \frac{P_A(t)P_B(t)}{[P_A(t)+P_B(t)]^2} e^{-\frac{P_B^M}{P_B^M - P_B(t)}} \tag{4.10}$$

The meaning of the factor γ is to discriminate between the case in which technology A is more efficient in using money to improve its performance ($\gamma > 1$) to the case when technology A is less efficient ($\gamma < 1$). In the following we will assume γ to be 1, that is both technologies have the same capability of transforming money into improved performance via R&D activity.

Two analytical considerations can be derived from the structure of equations (4.9-4.10): (i) the system has no inertia, i.e. it does not show any tendency to behave according to its previous 'motion'; (ii) when the performance of both technologies is far away from the asymptotic values ($P_A << P_A^M$, $P_B << P_B^M$) the increments that one gets in the performance itself are proportional with respect to the initial values.

While the first consideration is a direct consequence of the fact that the differential equations are of the first order and is ultimately due to the choice (4.1), the second is not so straightforward. To prove it we note that if the arguments of the exponential terms are both approximately 1, the right-hand-sides are simply proportional, and one can write:

$$\frac{dP_A(t)}{dt} = \gamma \frac{dP_B(t))}{dt} \tag{4.11}$$

that can be readily integrated and gives:

$$P_A(t) - P_A(0) = \gamma \left[P_B(t) - P_B(0) \right]. \tag{4.12}$$

The other side of the technological competition consists of the change of the market share of technology A relative to technology B which decreases dramatically, until it reaches the asymptotic value which we obtain as:

$$\frac{G_A}{G_B} = \frac{P_A^M}{P_B^M}. \tag{4.13}$$

We are now able to run the simulations which depict the performance and the share of both technologies through time. In the simulations we assume that once B emerges, both A and B invest money on R&D activities at the same rate. However, as we will see in the pictures and in table one, despite the fact that A, starting from a dominant position invests at the same rate as B, A's performance cannot be improved as fast as it would be necessary to keep up with B's. Put it another way, A and B invest the same percentage of their profits, but A's pie is a decreasing one, while B's is increasing. Also, let us remind that γ is assumed to be 1.

The behavior shown in Fig. 1 is general, but the actual values of t_1 and t_2 depend upon the initial conditions. For this simulation we have used initial values for both technologies far away from the asymptotic value, and in particular a very small value for the initial performances (and therefore for the initial earnings) of technology B.

It is quite interesting the performance behavior described in figure 1. Technology A (the solid line) starts reacting when B (dashed line) comes into existence; had we not technology B, A's performance would be represented by a horizontal line parallel to the time axis (the dotted line is a continuation of such performance) and the expected time of technological overtaking should be t_1. A's performance however increases, so that the overtaking of B is delayed in time, and the overtaking occurs at a relatively higher level of performance and at a later time, t_2. And this is the phenomenon which we wanted to model: the dynamics engendered by the emergence of the new technology which stimulates the performance of the old one.

In figure two the solid line depicts the market share of technology A, while the dashed line describes technology B's share; we have to know that according to equation (4.5) the two lines also reflect the fraction of profits devoted to R&D by the other technology, i.e. B's line denotes the fraction of profits devoted to R&D by A, and conversely.

Table one supplies some figures for some critical values of time; as one can see, at first A gets all of the profits being its performance $P_A = 10$ and $P_B \approxeq 0$ (0.0001); after 463 periods (t_1) B would have overtaken A, had A not undertaken its R&D activity; at this point in time the performance of A is still much higher than B's ($P_A = 18.5$ as compared with $P_B = 10$). The two technologies have the same performance and share at $t = 792$, with $P_A = P_B = 39$ and both have a 50% share; it is worthwhile noting that at this stage they are investing 25% of their profits on R&D. Finally, as $t \to \infty$, both technologies will have reached their upper limit in terms of performance ($P_A = 100$, $P_B = 1000$), while the share of A reduces to 9% and B's has grown to its maximum of 91%; at this stage the R&D activity absorbs a mere 8% of the profits.

A final comment is needed to point out that even if we considered both $\gamma > 1$ (i.e. $E_A > E_B$ which implies that A is more efficient than B in transforming R&D into improvements) *and* a higher investment of A with respect to B on R&D, the final position would be the same. What would happen would simply be an overtaking of B over A at a later stage.

5. Conclusions

We started this paper by pointing at a recurring phenomenon observed in economic reality, i.e. the persistence of 'old' technologies despite the emergence of new ones capable of offering the same services in a more efficient way.

The kind of mechanism we have modeled consists of the new lease of life which comes from the intentional effort activated by the 'owners' of the old technology in order to give their technology one more chance – and examples range from the sailship during the 19th century to nowadays broadband modems.

As we have seen, eventually the new technology overtakes the old one; however, this overtaking occurs at a later time than would have occurred if the old technology had not been improved, and then overtaking occurs at a higher level of the performance for both technologies – a higher performance that would have never been reached, had the new technology never appeared. We need to stress that this delay in overtaking in performance (t_2 rather than t_1 in terms of figure 1) is a qualitative result which does not depend upon the parameters' values.

The model is also capable of depicting the evolution of the market share of the two technologies which is reflected in their different performance through time, with the old technology experiencing a dramatic fall as the new one emerges and is improved. The old technology's share, though, does not fall to zero.

It is worthwhile stressing that there does not occur lock-in as there does not exist inertia in this system; put it another way, the final equilibrium does not depend on history. Also, the final equilibrium, i.e. the share of the two technologies depends on the asymptotic values of the best performance that each technology can reach (P_A^M and P_B^M).

By way of conclusion, we would like to stress the fact that despite its simplistic assumptions and despite being based on simulations, the model can give an account of the real processes of competition which develop between new and old technologies.

Figure captions

- Fig. 1 Performances of technology A (solid line) and B (dashed line) as a function of time. The dotted line represents the supposed constant behavior of technology A. In the figure are also indicated the overtaking times of the two technologies supposing that technology A does not react (t_1) and the actual behavior (t_2). Parameters of the simulations are: $\gamma = 1$, $P_A^M = 100$,

$P_B^M = 1000$. Initial conditions: $P_A(0) = 10$, $P_B(0) = 10^{-4}$.

- Fig. 2 Market shares in the same conditions as Fig. 1.
- Table 1: critical values of t.

Table 1 Critical values of t

	t	P_A	P_B	$\frac{P_A}{P_A+P_B}$	$\frac{P_B}{P_A+P_B}$	Π_A	Π_B	$R\&D_{A,B} = \frac{P_A P_B}{(P_A+P_B)^2}$
t_0	0	10	10^{-4}	$\simeq 1$	$\simeq 0$	1	0	0
t_1	463	18.5	10	64.9%	35.1%	64.9%	35.1%	22.8%
t_2	792	39	39	50.0%	50.0%	50.0%	50.0%	25.0%
t_3	1500	60.6	98.0	38.2%	61.8%	38.2%	61.8%	23.6%
$t \to \infty$	∞	100	1000	9.0%	91.0%	9.0%	91.0%	8.0%