Transportation Technology in a Duopoly Model of International Trade*

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September 24, 2001

Abstract

In this paper I will evaluate the role of R&D investment in transport and communication in a duopoly with trade. I will in fact consider the strategic behaviour of two firms located in two different countries. They can activate R&D investments in order to improve the technology of the transportation process. Transport and communication (TC) costs are of the iceberg type, i.e only a fraction of the good shipped abroad reaches the foreign market. I will then study a game in which firms may priorly commit themselves to a certain level of R&D investment and then they play in the market. As for the market game, I will consider both a Cournot duopoly with homogeneous products and a Bertrand duopoly with differentiated goods. In both models, my analysis suggests that firms are willing to invest in transport and communication technology when such a strategy turns out to be efficient, i.e when it does not imply an excessive cost. More precisely, a variety of equilibria will arise as a result of different levels of TC R&D efficiency. If the cost is low, the game has an equilibrium in dominant strategies where both firms invest in TC and maximize the aggregate profit. As the cost increases, the game becomes a prisoner's dilemma; both firms still invest in TC but they do not reach the Pareto-efficient solution. For even higher levels of the cost required, the game shows an equilibrium in dominant strategies where no firm finances TC R&D and the aggregate profit is maximized.

JEL Classification: D43, F12, L13, O31.
Keywords: R&D, transport and communication, trade

*I thank for helpful discussions and suggestions Ai Ting Goh, Luca Lambertini and Gianpaolo Rossini. The usual disclaimer applies.

1 Introduction

Models of imperfect competition serve more and more to revisit and to reconsider standard economic theories. International trade is a field of study where this kind of approach has become widely employed. In this paper, I will analyse a model of trade between countries by using the standard tools of oligopoly theory. I will in fact consider a duopoly game both *à la Cournot* and *à la Bertrand* with firms facing the possibility of investing in R&D.

Firms spend substantial amounts on research and development (R&D). Investments in R&D are generally classified into two types, process innovation and product innovation. The activity of product innovation consists in the development of technologies for producing new products or for increasing the quality of the existing ones. As a consequence, in most of the cases it decreases the degree of substitutability between rival products in oligopolies. As far as process innovation is concerned, it aims at decreasing the costs of producing existing products. Literature has considered the different degree of efficiency of process innovating R&D between the Cournot and the Bertrand setting. An established result states that there is an excess of process-innovating R&D under Cournot competition, while the opposite holds under Bertrand competition (Brander & Spencer,1983; Dixon, 1985).

In this paper I will not follow the conventional line of research but I will focus on a particular type of process innovation. I will in fact investigate the role of an investment in Transport and Communication (TC) R&D. This type of R&D can be generally thought to affect transport and communication costs, which may be interpreted in terms of the distance between the consumer and the producer. My choice is based on different reasons: firstly, in many circumstances the costs associated to the process-innovating activity may be very high and firms may be discouraged from pursuing such an activity. Firms can thus think of investing in transport and communication technology in order to enlarge their market by serving more consumers. Secondly, in the era of information revolution the exploitation of Internet may allow firms to enlarge the linkages to their consumers. As Shapiro and Varian (1999) appreciate, "*Information technology is about information and the associated technology*". By investing in computers and more advanced logistics a firm can then manipulate informations and reach a kind of one-to-one relation with consumers. In this way she could ship the product to those consumers really interested in it and/or to design the product in such a way to match consumers' requirements. Finally, several works (Krugman and Venables, 1990; Martin and Rogers, 1995) have emphasized the impact of different types of infrastructures on trade patterns and industrial competition. These infrastructures can be interpreted in a broad sense as encompassing any facility, service or good that can facilitate the juncture between producers and consumers. Poor infrastructures impose high shipping costs and then a large portion of the goods can be "lost on the way". By investing in communication and transport specific R&D, a firm may then increase this fraction and enlarge her market share.

Transport and communication costs are assumed to be of the 'iceberg' form invented by Samuelson (1954) and widely used in trade theory (Helpman and Krugman, 1985; Krugman, 1990). In the second part of this paper I will also link aspects of product differentiation with the analysis of transportation technologies. Modern theories of product differentiation have been very much influenced by Hotelling (1929), who described product and price competition through a spatial framework. He also tackled the issue of transport cost by considering the costs that consumers have to sustain to reach the closest firm. A very interesting approach was also used by Launhardt (1885), whose contribution has been recently acknowledged. In fact, Launhardt proposed a simple spatial duopoly model which considered both horizontal and vertical product differentiation. Furthermore, he paid attention to the influence of differences in transportation costs. He thus recognized the possibility of different form of heterogeneity among firms, associated either to location or to transportation technology. Recently Thisse and Dos Santos Ferreira (1996) expanded Launhardt model by allowing firms to choose their transportation cost technologies.

So far, however, the topic of strategic investment to reduce the burden of TC costs has been rather neglected. This is quite surprising given the role played by new technologies, which can make cross border trade less expensive. Lambertini, Mantovani and Rossini (2001) analyse R&D activity in transport and communication technology in a domestic framework. Lambertini and Rossini (2001) investigate the role of TC R&D in a Cournot duopoly with trade. This paper represents an attempt to expand their results by considering also a Bertrand duopoly with trade. I will in fact examine different scenarios in which firms behave symmetrically or asymmetrically depending on their decision to commit to an investment in transportation technology. Firms play then a two stage game. In the first stage they face a binary choice and decide whether to invest in TC or not. In the second stage they compete in the market, either in a Cournot setting with homogeneous goods and in a Bertrand setting with differentiated goods. The solution of the game comes from backward induction. In both models I will find that firms are willing to invest in transport and communication technology for reasonable levels of the R&D required, even if this could give rise to a prisoner's dilemma where firms do not maximize the aggregate profit.

This paper is organized as follows. The two-stage game will be analysed in the following two sections, first in a Cournot setting (section 2) and then in a Bertrand setting (section 3). Conclusions and final remarks are in section 4.

2 The Cournot case

I will start by considering a market where firms sell a homogeneous product and choose quantities as strategic variables. Despite of its simplicity, this model will provide a very useful introductive analysis on the TC investment issue. Moreover, its main results will be confirmed also in the next section, where I will study a model with Bertrand competition and differentiated goods. In what follows, I will first define the setting and then I will look for the subgame perfect Nash equilibria of the two-stage game.

2.1 The setting

I consider two countries, Home and Foreign, indicated respectively by H and F. Assume that in each country there is only one firm; firm i is located in country H while firm j is located in country F. Both firms produce a homogeneous good which can be sold both in the domestic and in the foreign market. Market competition takes place as a Cournot game where each firm chooses the profit-maximizing quantity for each country separately. A crucial element is then what Helpman (1982) refers to as a 'segmented market' perception: each firm considers each country as a separate market and makes distinct quantity decision for each. Furthermore, there are transport costs incurred in exporting goods from one country to the other but not within the country. In other words, I assume that transportation costs only affect international trade.

Different methods have been suggested to model the transportation costs. One way is to include them in the cost function (Brander and Krugman 1983); domestic marginal cost is a constant c, while the marginal cost of export is $c\,/\,t$, $t \in [0, 1]$. This approach is appropriate to model a pure process-innovating

R&D investment, but in my model I treat a particular kind of innovation, which influences the quantity *effectively* shipped abroad. As a consequence, when a quantity q_i (q_j) is produced, only a fraction $t \in]0,1]$ of the product reaches the consumers in the foreign market. I might think of $(1-t)$ as the "waste" of product during the freight process. In this way I will be able to analyse the type of R&D that I have in mind. I assume linear market demand for the two homogeneous goods with a unitary reservation price:

$$p^H = 1 - q_i^H - t_j^H q_j^H \tag{1}$$

$$p^F = 1 - q_j^F - t_i^F q_i^F \tag{2}$$

where q_i^H (q_i^F) is the quantity produced by firm i and sold in market H (F) and similarly for q_j^H (q_j^F), while t_j^H $(t_i^F) \in]0,1]$ indicates the fraction which arrives at destination when firm j *(i)* ships the product to country H *(F)*. For the sake of simplicity, I will assume a perfect symmetry of firms in the ability of delivering the goods and therefore $t_j^H = t_i^F = t$. The initial t is then exogenously given to firms and it can be conceived in terms of the state of public facilities and infrastructures. Furthermore, firms may priorly decide whether to invest in transportation technology or not. Through a private investment in transport specific R&D, a firm can increase the fraction of the product which arrives in the foreign country, thus enlarging her market. I assume that a firm which invests in transportation technology can deliver the entire product to the customers in the other country, thus no portion is lost in the way $(t = 1)$. Capital expenditure is represented by $k > 0$ for both firms. In the case where the firm does not invest in transportation technology, she will deliver only a portion $t \in]0,1[$ of the product. Marginal cost is constant and it is equal to c.

In this first part I will then analyse a model of international duopoly *à la Cournot* where two firms located in two different countries produce a homogeneous good and play a two-stage game. In the first stage they have to decide whether to invest in transport technology or not, while in the second stage they compete non-cooperatively in the market by setting the quantities. The solution of the game comes from backward induction. Following the framework above, I would need to analyse 4 cases, according to the investment choices by firms.

2.1.1 Case A: none invests in R&D

In this case the demand functions for country H and F are obtained by simplifying 1 and 2, given the assumption $t_j^H = t_i^F = t$. I then have:

$$p^H = 1 - q_i^H - tq_j^H, \tag{3}$$

$$p^F = 1 - q_j^F - tq_i^F. \tag{4}$$

Profit functions are given by:

$$\pi_i = \left(1 - q_i^H - tq_j^H\right) q_i^H + \left(1 - q_j^F - tq_i^F\right) tq_i^F - c\left(q_i^H + q_i^F\right), \tag{5}$$

$$\pi_j = \left(1 - q_j^F - tq_i^F\right) q_j^F + \left(1 - q_i^H - tq_j^H\right) tq_j^H - c\left(q_j^F + q_j^H\right). \tag{6}$$

Each firm maximizes her profit with respect to the quantity sold on the domestic market and to the quantity sold abroad. From market stage first order conditions (FOCs)[1] I get the following equilibrium quantities:

$$(q_i^H)^* = (q_j^F)^* = \frac{c + t - 2ct}{3t} \tag{7}$$

$$(q_j^H)^* = (q_i^F)^* = \frac{c\,(t-2) + t}{3t^2}. \tag{8}$$

Non-negativity constraints on quantities imply that $(q_i^H)^* = (q_j^F)^* \geq 0$ if $t \geq \frac{c}{2c-1}$ and $(q_i^F)^* = (q_j^H)^* \geq 0$ if $t \geq \frac{2c}{1+c}$. An additional constraint is given by the non-negativity of the marginal revenues for the good shipped abroad

[1] First Order Conditions are::

$$\frac{\partial_A \pi_i}{\partial q_i} = 0, \; \frac{\partial_A \pi_i}{\partial q_j} = 0, \; \frac{\partial_A \pi_j}{\partial q_j} = 0, \; \frac{\partial_A \pi_j}{\partial q_i} = 0.$$

Second order conditions are always satisfied, as it can be easily checked in this and subsequent cases.

and this requires that $t \geq c$. The comparison of the threshold levels of t gives $\frac{2c}{1+c} > \frac{c}{2c-1} > c$ and therefore the binding constraint is $t \geq \frac{2c}{1+c}$.

By substituting the equilibrium quantities into the profit functions I obtain the equilibrium total profits:

$$_c\pi_i^A = {}_c\pi_j^A = \frac{2t^2 - 2ct\,(1+t) + c^2\left(5 - 8t + 5t^2\right)}{9t^2} \tag{9}$$

where low case c indicates profits found in the Cournot setting and superscript A indicates the case under consideration. A similar notation will be used also in the following cases.

2.1.2 Case B: firm j invests in R&D while firm i does not

In this case there is only one firm that undertakes R&D investments. More precisely, firm j decides to allocate resources to the transport and communication technology in order to deliver abroad the 'entire' product. On the contrary, firm i does not invest at all. Demand functions become then:

$$p^H = 1 - q_i^H - q_j^H, \tag{10}$$

$$p^F = 1 - q_j^F - tq_i^F. \tag{11}$$

Profit functions are given by:

$$\pi_i = \left(1 - q_i^H - q_j^H\right) q_i^H + \left(1 - q_j^F - tq_i^F\right) tq_i^F - c\left(q_i^H + q_i^F\right) \tag{12}$$

$$\pi_j = \left(1 - q_j^F - tq_i^F\right) q_j^F + \left(1 - q_i^H - q_j^H\right) q_j^H - c\left(q_j^F + q_j^H\right) - k \tag{13}$$

By applying the same method as before, I use market stage FOCs to find the optimal quantities:

$$\left(q_i^H\right)^* = \frac{1-c}{3},\ \left(q_j^H\right)^* = \frac{1-c}{3},\ \left(q_i^F\right)^* = \frac{c\,(t-2)+t}{3t^2},\ \left(q_j^F\right)^* = \frac{c+t-2ct}{3t} \tag{14}$$

As for non-negativity constraints, $(q_i^H)^* = (q_j^H)^* \geq 0$ if $c \leq 1$; moreover, $(q_i^F)^* \geq 0$ if $t \geq \frac{2c}{1+c}$, $(q_j^F)^* \geq 0$ if $t \geq \frac{c}{2c-1}$, as in case A, and $t \geq c$ for firm i which does not invest in TC. Hence the constraint is $t \geq \frac{2c}{1+c}$, with $c \leq 1$.

By substituting into the original profit function I get the equilibrium total profits:

$$ {}_c\pi_i^B = \frac{2\left[t^2 - 2ct + c^2\left(2 - 2t + t^2\right)\right]}{9t^2} \tag{15} $$

$$ {}_c\pi_j^B = \frac{2c\left(1-3t\right)t + 2t^2 + c^2\left(1 - 4t + 5t^2\right)}{9t^2} - k. \tag{16} $$

2.1.3 Case C : firm i invests in R&D while firm j does not

One can easily recognize that case C is the reverse of case B. Without entering into the calculations, it is evident that:

$$ {}_c\pi_i^C = \pi_j^B \, , \tag{17} $$

$$ {}_c\pi_j^* = \pi_i^B . \tag{18} $$

2.1.4 Case D.c : both firms invest in R&D

This is the symmetric case where both firms undertake transport and communication technology investments. The demand functions are:

$$ p^H = 1 - q_i^H - q_j^H , \tag{19} $$

$$ p^F = 1 - q_j^F - q_i^F . \tag{20} $$

Profits are then given by:

$$\pi_i = \left(1 - q_i^H - q_j^H\right) q_i^H + \left(1 - q_j^F - q_i^F\right) q_i^F - c\left(q_i^H + q_i^F\right) - k \tag{21}$$

$$\pi_j = \left(1 - q_j^F - q_i^F\right) q_j^F + \left(1 - q_i^H - q_j^H\right) q_j^H - c\left(q_j^F + q_j^H\right) - k. \tag{22}$$

From profit-maximization I get the equilibrium quantities:

$$\left(q_i^H\right)^* = \left(q_j^H\right)^* = \left(q_i^F\right)^* = \left(q_j^F\right)^* = \frac{1-c}{3} \tag{23}$$

which are always non-negative for $c \leq 1$. The corresponding equilibrium profits are then:

$$_c\pi_i^D = {_c\pi_j^D} = \frac{2 - 4c + 2c^2}{9} - k \tag{24}$$

2.2 The solution of the game

To find the solution of the two-stage game I consider the reduced form of the game represented in normal form in matrix 1. The feasibility condition requires $t \geq \frac{2c}{1+c}$, with $c \leq 1$, as I get from the condition of non-negativity both of quantities and of marginal revenues for the product shipped in the foreign market. Let me now analyse the game represented below:

		firm j	
		0	k
firm i	0	$_c\pi_i^A = {_c\pi_j^A}$	$_c\pi_i^B \quad {_c\pi_j^B}$
	k	$_c\pi_i^C \quad {_c\pi_j^C}$	$_c\pi_i^D = {_c\pi_j^D}$

Matrix 1

The above game presents different solutions according to the productivity of TC R&D. By dividing the admissable set of k into three regions, I can derive the following result:

Proposition 1 *In an international Cournot duopoly game where transportation costs only affect international trade, firms decide to invest in transport and communication technology according to the efficiency of the R&D required. In particular, three different symmetric solutions can be found: (i) at high levels of efficiency there is an equilibrium in dominant strategies in which both firms invest in TC R&D and maximize their aggregate payoff; (ii) for lower levels of efficiency the game is a prisoner's dilemma with both firms investing in TC R&D without being able to maximize their total payoff; (iii) for even lower levels of efficiency the game shows an equilibrium in dominant strategies where no firm undertakes TC R&D and the aggregate payoff is maximized.*

Proof. I prove the above preposition by comparing the equilibrium profits appearing in Matrix 1. Let me start from the principal diagonal. I have that ${}_c\pi_{i,j}^D \geq {}_c\pi_{i,j}^A$ if

$$k \leq k_1 \leq \frac{c\,(t-1)\,(5c-2t-3ct)}{9t^2}.$$

I then compare ${}_c\pi_i^A$ with ${}_c\pi_i^C$ (or ${}_c\pi_j^A$ with ${}_c\pi_j^B$). It appears that ${}_c\pi_i^A \leq {}_c\pi_i^C$ $({}_c\pi_j^A \leq {}_c\pi_j^B)$ if

$$k \leq k_2 \leq \frac{4c\,(1-t)\,(t-c)}{9t^2}.$$

Moreover, ${}_c\pi_i^B \leq {}_c\pi_i^D$ (and ${}_c\pi_j^C \leq {}_c\pi_j^D$) if

$$k \leq k_3 \leq \frac{4c\,(c-t)\,(t-1)}{9t^2} = k_2.$$

In the acceptable region of parameters it is always true that $k_1 < k_2 = k_3$.

Three cases have to be considered: (i) when $k \leq k_1$, for firm i ${}_c\pi_i^C \geq {}_c\pi_i^A$ and ${}_c\pi_i^D \geq {}_c\pi_i^B$, while for firm j ${}_c\pi_j^B \geq {}_c\pi_j^A$ and ${}_c\pi_j^D \geq {}_c\pi_j^C$. As a consequence, both firms invest in R&D and they maximize the aggregate profit, being ${}_c\pi_{i,j}^D \geq {}_c\pi_{i,j}^A$. (ii) For $k_1 < k \leq k_2$ the only thing which changes with respect to the previous case is that ${}_c\pi_{i,j}^D \leq {}_c\pi_{i,j}^A$. Both firms still invest in R&D but they do not reach the Pareto optimum, which would require the choice of no investment in TC R&D for both firms. (iii) For lower levels of efficiency, i.e. for $k > k_2$, it holds the opposite of the case (i), because ${}_c\pi_i^C \leq {}_c\pi_i^A$ and ${}_c\pi_i^D \leq {}_c\pi_i^B$ for firm i, while for firm j ${}_c\pi_j^B \leq {}_c\pi_j^A$ and ${}_c\pi_j^D \leq {}_c\pi_j^C$. None invests in R&D, thus maximizing the aggregate profit, given that ${}_c\pi_{i,j}^A \geq {}_c\pi_{i,j}^D$.

It can be easily checked that the sequence of payoffs presented is invariant as the value of the parameter c changes within its admissable range. ■

I then found two symmetric subgame perfect Nash equilibria, where both firms either invest in TC (for $k \leq k_2$) or do not invest (for $k > k_2$). Firms competing in quantities and producing homogeneous goods have an incentive to undertake TC R&D if the advantage they get is fairly high, i.e. when a low R&D expenditure is sufficient to eliminate the additional costs related to the shipping of the product abroad. The positive effect deriving from the increase of the sales in the foreign market overcomes the negative effect due to the reduction in prices in both markets and the R&D cost. Apart from that, however, a simultaneous investment in TC R&D tends to increase the competition within both markets, because firms can deliver a higher quantity of the product to the same consumers, thus lowering the price. This consideration could be useful in explaining why the region for which such an equilibrium is Pareto dominant is limited to the case of a very efficient R&D expenditure, where a very low level of k ($k \leq k_1$) is sufficient to increase the percentage t up to 1, thus eliminating the waste due to the transport process. In all the other circumstances, firms would yield a higher aggregate profit by not investing in TC. In the interval $k_1 < k \leq k_2$, in fact, firms still invest in TC but they would yield a higher profit by not investing at all. The last symmetric NE consists in both firms not undertaking any kind of R&D activity and this occurs for particularly high levels of k ($k > k_2$).

3 The Bertrand case

In this second part I will introduce two basic changes with respect to the previous case. The game is still played in two stages and in the first one firms decide whether to invest in TC or not, but in the second stage they now compete non-cooperatively in the segmented markets by setting prices. Furthermore, firms produce and sell a differentiated product and not an homogeneous one as before. I will then try to link aspects of product differentiation with the analysis of transportation technologies.

As for product differentiation, after the work of Lancaster (1979), it became clear that products can be identified by two different interpretations of their position in the space of characteristics. Two products are said to be *horizontally differentiated* when they own different characteristics so that, if supplied at the same price, they both obtain a positive market share. Different consumers cannot unambiguously rank the products they prefer because they may have different tastes. On the other hand, products are said to be *vertically differentiated* when they own different amounts of the same characteristics so that, if offered at the same price, only one product is sold. In my model I will consider products which are differentiated in an horizontal sense, i.e. they own different characteristics attracting consumers' tastes. By using the *spatial metaphora*, distance from the 'ideal' product can be recasted as physical distance from the firm selling the product, as specified by Hotelling (1929). Such an interpretation turns out to be useful in the kind of model I consider, where firms located away from consumers try to reduce the physical distance by improving the transport technology.

As I will show, the main results of the Cournot model with homogeneous goods will be confirmed here, with both firms investing in TC only for efficient levels of the R&D expenditure required, and not investing otherwise. However, it will be interesting to investigate the role played by the product differentiation in determining the intervals where the different symmetric equilibria hold.

I will follow the same structure of the first part, first defining the setting and then finding subgame perfect Nash equilibria of the game.

3.1 The setting

I still consider firm i located in country H and firm j located in country F. They produce a differentiated good which is sold both in the domestic and in the foreign market. Market competition takes place non-cooperatively as a Bertrand game where each firm chooses the profit-maximizing price for each country separately. As before, there are transportation costs which only affect international trade. When a quantity q_i (q_j) is delivered abroad, only a fraction t of it arrives at destination. As to R&D competition, I still consider a binary strategy set for both firms, which can eliminate the freight losses by investing a fixed amount k in TC R&D.

As before, marginal cost is constant and it is equal to c and reservation prices are unitary. Non-negativity of the marginal revenues for the product shipped abroad still requires that $t \geq c$. Demand functions are still assumed to be linear. However, due to the product differentiation introduced in this second model, I have to consider four inverse demand functions, two for each market where firms operate:

$$p_i^H = 1 - q_i^H - t_j^H \gamma q_j^H, \tag{25}$$

$$p_j^H = 1 - t_j^H q_j^F - \gamma q_i^H, \tag{26}$$

$$p_i^F = 1 - t_i^F q_i^F - \gamma q_j^F, \tag{27}$$

$$p_j^F = 1 - q_j^F - t_i^F \gamma q_i^F. \tag{28}$$

As for the notations, p_i^H (p_i^F) is the price set by firm i in market H (F) and similarly for p_j^H (p_j^F) , while t_j^H $(t_i^F) \in]0,1]$ indicates the fraction which arrives at destination when firm j *(i)* ships the product to country H *(F)*. The analysis is simplified by assuming $t_j^H = t_i^F = t$. Moreover, $t = 1$ in case of investment on TC R&D, as introduced before. Finally, the parameter $\gamma \in [0, 1]$ represents product substitutability as perceived by consumers. The degree of product differentiation decreases with the parameter γ[2], whose value is assumed to be exogenously given.

In this second part I will then describe an international duopoly *à la Bertrand* where two firms located in different countries play a one-shot two-stage game in TC innovation and marketing. In the first stage they have to decide whether to invest in transport technology or not, while in the second stage they compete in the market by setting prices. The solution of the game comes from backward induction.

As in the previous model, I need to distinguish 4 cases, according to the investment decisions taken by firms.

3.1.1 Case A: none invests in R&D

In this second model I consider firms which maximize their profit with respect to the prices set in the market. This implies that I need to express the profit functions in terms of prices. The first step requires then the conversion

[2] As $\gamma \to 1$, products become perfect substitutes, while the opposite holds as $\gamma \to 0$.

of the inverse demand functions into their direct form. By taking into account the assumption $t_j^H = t_i^F = t$, from 25 and 26 on the one side, and from 27 and 28 on the other side, I get:

$$q_i^H = \frac{1}{1+\gamma} - \frac{p_i^H}{1-\gamma^2} + \frac{\gamma\, p_j^H}{1-\gamma^2}, \tag{29}$$

$$q_j^H = \frac{1}{(1+\gamma)\, t} - \frac{p_j^H}{(1-\gamma^2)\, t} + \frac{\gamma\, p_i^H}{(1-\gamma^2)\, t}, \tag{30}$$

$$q_j^F = \frac{1}{1+\gamma} - \frac{p_j^F}{1-\gamma^2} + \frac{\gamma\, p_i^F}{1-\gamma^2}, \tag{31}$$

$$q_i^F = \frac{1}{(1+\gamma)\, t} - \frac{p_i^F}{(1-\gamma^2)\, t} + \frac{\gamma\, p_j^F}{(1-\gamma^2)\, t}. \tag{32}$$

Profit functions are given by:

$$\pi_i = p_i^H q_i^H + p_i^F t\, q_i^F - c\,\left(q_i^H + q_i^F\right) \tag{33}$$

$$\pi_j = p_j^F q_j^F + p_j^H t\, q_j^H - c\,(q_j^F + q_j^H) \tag{34}$$

Each firm maximizes her profit with respect to the prices set both in the domestic and in the foreign market. By substituting in 33 and 34 the demand functions 29, 30, 31 and 32, I can express the profits as functions of the prices. From market stage first order conditions (FOCs)[3] I obtain the following equilibrium prices:

[3] First Order Conditions are::

$$\frac{\partial_A \pi_i}{\partial p_i^H} = 0,\ \frac{\partial_A \pi_i}{\partial p_i^F} = 0,\ \frac{\partial_A \pi_j}{\partial p_j^F} = 0,\ \frac{\partial_A \pi_j}{\partial p_j^H} = 0.$$

Second order conditions are always satisfied, as it can be easily checked in this and subsequent cases.

$$(p_i^H)^* = (p_j^F)^* = \frac{c\gamma + 2t + 2ct - \gamma t - \gamma^2 t}{4t - \gamma^2 t} \qquad (35)$$

$$(p_j^H)^* = (p_i^F)^* = \frac{2c + 2t + c\gamma t - \gamma t - \gamma^2 t}{4t - \gamma^2 t} \qquad (36)$$

It can be easily demonstrated that the above price levels are always positive in the admissable region of parameters considered.

By substituting the equilibrium prices into the profit functions, I obtain the equilibrium total profits:

$$_b\pi_i^A = {_b\pi_j^A} = \frac{\nu_1}{(\gamma^2 - 4)^2 (\gamma^2 - 1)t^2}, \qquad (37)$$

where $\nu_1 = (3c^2\gamma^2 - 4c^2 - c^2\gamma^4 + 8ct - 8c\gamma t + 8c^2\gamma t - 6c\gamma^2 t +$
$+4c\gamma^3 t - 4c^2\gamma^3 t + 2c\gamma^4 t - 8t^2 + +8ct^2 - 4c^2t^2 + 8\gamma t^2 +$
$-8c\gamma t^2 + 6\gamma^2 t^2 - 6c\gamma^2 t^2 + +3c^2\gamma^2 t^2 - 4\gamma^3 t^2 + 4c\gamma^3 t^2 +$
$-2\gamma^4 t^2 + 2c\gamma^4 t^2 - c^2\gamma^4 t^2)$.

Low case b stands for the profits obtained in the Bertrand setting, while superscript A refers to the case A under consideration. A similar notation will be also used in the following cases.

3.1.2 Case B: firm j invests in R&D while firm i does not

This is the asymmetric case where firm j invests in TC, while firm i does not invest at all. Inverse demand functions 25, 26 are therefore simplified by putting $t_j^H = t = 1$. As in the previous case, I derive the direct demand functions:

$$q_i^H = \frac{1}{1+\gamma} - \frac{p_i^H}{1-\gamma^2} + \frac{\gamma p_j^H}{1-\gamma^2}, \qquad (38)$$

$$q_j^H = \frac{1}{(1+\gamma)} - \frac{p_j^H}{(1-\gamma^2)} + \frac{\gamma p_i^H}{(1-\gamma^2)}, \qquad (39)$$

$$q_j^F = \frac{1}{1+\gamma} - \frac{p_j^F}{1-\gamma^2} + \frac{\gamma\, p_i^F}{1-\gamma^2}, \tag{40}$$

$$q_i^F = \frac{1}{(1+\gamma)\; t} - \frac{p_i^F}{(1-\gamma^2)\; t} + \frac{\gamma\, p_j^F}{(1-\gamma^2)\; t}. \tag{41}$$

Profit functions are:

$$\pi_i = p_i^H q_i^H + p_i^F t\, q_i^F - c\left(q_i^H + q_i^F\right) \tag{42}$$

$$\pi_j = p_j^F q_j^F + p_j^H q_j^H - c(q_j^F + q_j^H) - k \tag{43}$$

By applying the same method as before, I use the FOCs of the market stage to find the optimal prices:

$$\left(p_i^H\right)^* = \left(p_j^H\right)^* = \frac{\gamma - c - 1}{\gamma - 2}, \tag{44}$$

$$\left(p_i^F\right)^* = \frac{2c\, + 2t + c\,\gamma\, t - \gamma\, t - \gamma^2\, t}{4t - \gamma^2 t} \tag{45}$$

$$(p_j^F)^* = \frac{c\,\gamma + 2t + 2c\, t - \gamma\, t - \gamma^2\, t}{4t - \gamma^2\, t} \tag{46}$$

As it can be seen, the only thing which changes with respect to the previous case is the level of q_j^H, which increases because of the investment in TC undertaken by firm j, thus leading to a decrease of $\left(p_j^H\right)^*$. Non-negativity constraints on prices are always positive in the admissable region of parameters, as it can be easily checked. I can then substitute into the original profit functions to obtain the equilibrium profits:

$$_b\pi_i^B = \frac{(c-1)\,(\gamma - c - 1)}{4 - 3\gamma^2 + \gamma^3} + \frac{\nu_2}{(\gamma^2\; - 4)^2\,(\gamma^2\; - 1)t^2} + \frac{\nu_3}{(4 - 5\gamma^2 + \gamma^4)t^2} \tag{47}$$

and

$$ {}_b\pi_j^B = \frac{(c-1)\,(\gamma - c - 1)}{4 - 3\gamma^2 + \gamma^3} + \frac{\nu_4}{(\gamma^2\, - 4)^2\,(\gamma^2\, - 1)t^2} - \frac{\nu_5}{(4 - 5\gamma^2 + \gamma^4)t} - k, \quad (48) $$

where
$\nu_2 = (2t - 2c\, + c\,\gamma\,t + c\gamma^2 - \gamma\,t - \gamma^2\,t)(\gamma\,t - 2c\, - 2t - c\,\gamma\,t + \gamma^2\,t),$
$\nu_3 = c\,(2c\, - c\,\gamma^2\, - 2t + \gamma\,t - c\,\gamma\,t + \gamma^2\,t - 2t^2 + 2ct^2 + \gamma t^2 +$
$\quad -c\gamma t^2 + \gamma^2\,t^2 - c\,\gamma^2\,t^2),$
$\nu_4 = (\gamma\,t - 2t - 2ct\, - c\,\gamma\, + \gamma^2\,t)(c\gamma\, - 2c\,t + 2t - \gamma\,t - \gamma^2\,t + c\,\gamma^2\,t),$
$\nu_5 = c\,(c\,\gamma + 4t - 4c\,t - 2\gamma\,t + c\gamma t - 2\gamma^2 t + 2c\gamma^2 t).$

3.1.3 Case C: firm i invests in R&D while firm j does not

As it can be easily noticed, case C is the reverse of case B. Without developing all the calculations, it is evident that:

$$ {}_b\pi_i^C = {}_b\pi_j^B \quad (49) $$

$$ {}_b\pi_j^C = {}_b\pi_i^B. \quad (50) $$

3.1.4 Case D: both firms invest in R&D

This is the symmetric case where both firms invest in TC. By rewriting the inverse demand functions and considering that $t_j^H = t_i^F = 1$, I get:

$$ q_i^H = \frac{1}{1+\gamma} - \frac{p_i^H}{1-\gamma^2} + \frac{\gamma\,p_j^H}{1-\gamma^2}, \quad (51) $$

$$ q_j^H = \frac{1}{1+\gamma} - \frac{p_j^H}{(1-\gamma^2)} + \frac{\gamma\,p_i^H}{(1-\gamma^2)}, \quad (52) $$

$$q_j^F = \frac{1}{1+\gamma} - \frac{p_j^F}{1-\gamma^2} + \frac{\gamma\, p_i^F}{1-\gamma^2}, \tag{53}$$

$$q_i^F = \frac{1}{1+\gamma} - \frac{p_i^F}{(1-\gamma^2)} + \frac{\gamma\, p_j^F}{(1-\gamma^2)}. \tag{54}$$

Profits functions are now given by:

$$\pi_i = p_i^H q_i^H + p_i^F q_i^F - c\left(q_i^H + q_i^F\right) - k \tag{55}$$

$$\pi_j = p_j^F q_j^F + p_j^H q_j^H - c\,(q_j^F + q_j^H) - k. \tag{56}$$

From profit-maximization I get the following equilibrium prices:

$$\left(p_i^H\right)^* = \left(p_j^H\right)^* = \left(p_i^F\right)^* = \left(p_j^F\right)^* = \frac{\gamma - c - 1}{\gamma - 2} \tag{57}$$

which are always non-negative. The corresponding equilibrium profits are then:

$$_b\pi_i^D = {}_b\pi_j^D = \frac{2(1-2c+c^2)(1-\gamma)}{4-3\gamma^2+\gamma^3} - k \tag{58}$$

3.2 The solution of the game

To find the solution of the two stage game I consider again the reduced form of the game represented in normal form in Matrix 2:

		firm j	
		0	k
firm i	0	$_b\pi_i^A = {}_b\pi_j^A$	$_b\pi_i^B \quad {}_b\pi_j^B$
	k	$_b\pi_i^C \quad {}_b\pi_j^C$	$_b\pi_i^D = {}_b\pi_j^D$

Matrix 2

From the reduced form of the game I can derive the following result:

Proposition 2 *In an international Bertrand duopoly game with transportation costs that only affect international trade, firms decide to invest in transport and communication technology according to the efficiency of the R&D required. In particular, the same conclusions as in Proposition 1 hold, even if the threshold levels of the R&D cost k are different.*

Proof. I prove the above preposition by comparing the equilibrium profits appearing in Matrix 2. In the principal diagonal ${}_{b}\pi_{i,j}^{D} \geq {}_{b}\pi_{i,j}^{A}$ if

$$k \leq k_4 = \frac{c(1-t)\,\nu_6}{(\gamma^2-4)^2\,(\gamma^2-1)t^2},$$

where $\nu_6 = 4c - 3c\gamma^2 + c\gamma^4 - 8t + 4ct + 8\gamma t - 8c\gamma t + 6\gamma^2 t - 3c\gamma^2 t +$
$- 4\gamma^3 t + 4c\gamma^3 t - 2\gamma^4 t + c\gamma^4 t.$

Then, by comparing ${}_{b}\pi_{i}^{A}$ with ${}_{b}\pi_{i}^{C}$ (or ${}_{b}\pi_{j}^{A}$ with ${}_{b}\pi_{j}^{B}$), it appears that ${}_{b}\pi_{i}^{A} \leq {}_{b}\pi_{i}^{C}$ (${}_{b}\pi_{j}^{A} \leq {}_{b}\pi_{j}^{B}$) if

$$k \leq k_5 = \frac{c(t-1)(2-\gamma^2)\,\nu_7}{(\gamma^2-4)^2\,(\gamma^2-1)t^2},$$

where $\nu_7 = c\gamma^2 - 2c + 4t - 2ct - 2\gamma t + 2c\gamma t - 2\gamma^2 t + c\gamma^2 t.$

Moreover, ${}_{b}\pi_{i}^{B} \leq {}_{b}\pi_{i}^{D}$ (and ${}_{b}\pi_{j}^{C} \leq {}_{b}\pi_{j}^{D}$) if

$$k \leq k_6 = k_5.$$

In the acceptable region of parameters it is always true that $k_4 < k_5 = k_6$, as simple algebra shows.

Three cases have to be considered: (i) when $k \leq k_4$, for firm i ${}_{b}\pi_{i}^{C} \geq {}_{b}\pi_{i}^{A}$ and ${}_{b}\pi_{i}^{D} \geq {}_{b}\pi_{i}^{B}$, while for firm j ${}_{b}\pi_{j}^{B} \geq {}_{b}\pi_{j}^{A}$ and ${}_{b}\pi_{j}^{A} \geq {}_{b}\pi_{j}^{C}$. Both firms invest in R&D, thus giving rise to a unique solution in dominant strategies which is also Pareto-efficient, being ${}_{b}\pi_{i,j}^{D} \geq {}_{b}\pi_{i,j}^{A}$. (ii) For $k_4 < k \leq k_5$ the only thing which changes with respect to the previous case is that ${}_{b}\pi_{i,j}^{D} \leq {}_{b}\pi_{i,j}^{A}$. Both firms still invest in R&D but they do not reach the Pareto optimum. (iii) For lower levels of efficiency, i.e. for $k > k_5$, we have in practise the opposite of the case (i); neither firm i nor firm j invests in R&D and the game has an equilibrium in dominant strategies where the aggregate payoff of the firms is maximized by not investing in TCRD, given that ${}_{b}\pi_{i,j}^{A} \geq {}_{b}\pi_{i,j}^{D}$.

It can be easily checked that the sequence of payoffs presented is invariant as the value of the parameter c varies within its admissable range. ■

I then demonstrated the validity of the results found before also in the case of Bertrand competition with differentiated products. There are still two symmetric subgame perfect Nash equilibria: both firms invest in TC R&D when it requires a low cost ($k \leq k_5$), while they do not invest for $k > k_5$. As before, there is an interval ($k_4 < k \leq k_5$) where the game shows a prisoner's dilemma; firms invest in TC without maximizing their total payoff.

It would be interesting to analyse the impact of the product differentiation parameter γ on the intervals of k appearing above. By differentiating k_4 and k_5 with respect to k, one gets:

$$\frac{\partial k_4}{\partial \gamma} < 0, \tag{59}$$

$$\frac{\partial k_5}{\partial \gamma} < 0. \tag{60}$$

As a consequence, as γ increases, i.e. when products become less differentiated, the interval in which both firms invest in TC shrinks and it is then sufficient a lower value of k than before to make them abandon the TC R&D project. An investment in TC R&D reduces the distance separating the firm from the consumers located abroad. Physical distance can be interpreted as distance from the 'ideal' product, as I mentioned before. By considering horizontal product differentiation, therefore, an interesting feature deriving from TC R&D activity is that products becomes less differentiated. In this second model, in fact, distance between firms and consumers embodies two components, one due to horizontal product differentiation (measured by γ) and the other due to transport costs (measured by t). The former is exogenously given, while the latter can be influenced by firms through an investment in TC R&D. This provides an explanation to the fact that firms do not invest when products are close substitutes. An investment in TC R&D eliminates the additional burden of the shipping phase of the product. Natural barriers which separate different countries vanish, allowing more products to arrive at destination from abroad. Prices are therefore pushed down. Furthermore, the product which enters in higher quantity than before is a close substitute to the existing one, and this make competition fiercer. Remind that I consider here a Bertrand model with competition in prices. The gain deriving from an increase of sales in the foreign market does not compensate anymore the negative effects due to the cut in prices in both markets and to the R&D cost. In the limit case of a market with perfect substitutes goods ($\gamma = 1$), an investment in TC R&D could give rise to the *Bertrand paradox*, with both firms charging a price equal to the marginal cost, thus getting a zero profit. On the contrary, the more the products are differentiated, i.e. the more γ approaches 0, the more firms are willing to finance TC R&D projects. Given the same R&D expenditure k, in fact, firms increase the quantity sold in the foreign market but prices still remain sufficiently high due to the strong product differentiation.

4 Conclusions

In this paper I analysed a model of international duopoly where two firms play a two stage game. In the first stage they decide whether to invest in transport and communication technology or not, while in the second stage they compete in the market. As for the market game, I considered both a Cournot duopoly with homogeneous goods and a Bertrand duopoly with differentiated goods. A very important feature of my model is the specification of the kind of R&D which can be financed. I considered the possibility for firms to invest in transport and communication R&D, which aims at improving any facility related to the export of the goods abroad. I assumed in fact that transportation costs only affect international trade and then a firm which sells in the foreign market bears additional costs, not only because of distance but also because of differences of various kind existing among countries. For example, a firm which exports her good could find in the foreign market different networks of connection, different administrative procedures and so on. By investing in TC R&D, however, a firm can reduce such costs and reach a higher fraction of consumers.

As for the model, the analysis revealed the presence of different subgame perfection Nash equilibria, depending on the relative efficiency of the R&D effort. Such an efficiency has been measured by the value of the parameter k, the cost of doing R&D. Depending on its level, I found two symmetric equilibria: both firms invest in TC when such a strategy is very effective, i.e for low levels of k, while they do not invest at all for high levels of k. For intermediate values of k the game gives rise to a prisoner's dilemma, where both firms invest in TC but they do not maximize their aggregate profit. These results are valid both in the Cournot case and in the Bertrand one, thus confirming the intuition of the model.

The case where both firms invest in TC R&D deserves a further investigation. A possible explanation should be that firms can exploit network externalities. As I introduced at the very beginning, in this paper I considered a kind of R&D activity which affects the transport and communication technology of a firm. If both firms invest in TC R&D, they can benefit from the fact that more and more people are using the same network of connection (see the example of Internet). In other words, there is a sort of positive feedback from user to user which turns out to be beneficial also for firms.

In the Bertrand case, furthermore, I tried to link aspects of product differentiation with the issue of transportation costs. Physical distance between firms and consumers was expressed by two parameters, γ and t, indicating respectively the degree of horizontal product differentiation and the impact of transport costs. The more the products are differentiated, the more are firms willing to finance TC R&D projects, which allow to increase the sales in the foreign market without arriving to an excessive drop in prices. On the contrary, when products are close substitutes, an investment in TC R&D leads to a price war which turns out to be damaging for firms.

I would remind that the results obtained are strongly influenced by the assumptions introduced in my analysis. In fact, two simplifications were very useful but their impact on the results has to be taken into account. Firstly, I assumed that investing a fixed amount k in TC is sufficient to eliminate the waste of product during the freight phase, thus leading to $t = 1$. A first extension should then consider the parameter t as a positive function of the R&D expenditure. Secondly, I assumed that the parameter γ measuring the product differentiation was exogenously determined. It would be interesting to model a game with firms facing the possibility of investing simultaneously in product innovation R&D and in transportation technology. The topic of product vs. process innovation has surprisingly found scanty attention in the literature, since the two kinds of innovation has been usually treated either separately or in aggregate. However, some recent contributions have been recently devoted to this issue (Rosenkranz 1996; Bonanno and Haworth 1998; Filippini and Martini, 2000).

Even if both extensions would add more insight into my analysis, I think that the results appearing in this paper are interesting, especially because they constitute an attempt to model a topic which has not been widely explored.

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