Prospect Theory and the Law of Small Numbers in the Evaluation of Asset Prices

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June 1, 2005

Abstract

We develop a model of one representative agent and one asset. The agent evaluates the earnings according to Prospect Theory and he does not know exactly the stochastic process generating earnings. While the earnings are generated by a random walk process, the agent considers a Markovian process, according to which firm's earnings move between two regimes, represented by a mean-reverting process and a trend process, as in Barberis, Shleifer and Vishny (1998). We study how an agent who is loss averse evaluates the price of a stock when she takes into account the wrong stochastic process. This twofold departure from rationality determines permanent effects on stock prices, even in long run. First, the model shows that agent who evaluates the asset according to Prospect Theory consistently underestimates the asset, due to loss aversion bias. This is shown under two different assumption regarding the functional form of utility. A kinked linear utility function (as in Bernatzi and Thaler, 1985) and the original and more general specification of Kahneman and Tversky (1979) are used. The model allows to explain observed phenomenon in the cross-section earnings return distribution. We solve this model and according to Barberis et all (1998), we evaluate the framework by using artificial data sets of earnings and prices simulated from the model. For plausible range of parameter values, it generates the empirical predictions of overreaction and underreaction observed in the data are explained.

JEL:G12, G14

Key words: Investor sentiment, Loss Aversion, Overreaction, Underreaction

1 Introduction

Behavioral Finance attempts at explaining some observed financial phenomenon by studying models in which agents are not fully rational. The departures from traditional paradigm used by Behavioral Finance literature are mainly two: on one side beliefs are not assumed to be formed according to standard rational Bayesian paradigm and on the other, preferences are not consistent with subjective expected utility. The present paper tries at establishing a connection between these two strands of literature: the one based on beliefs and the other one based on changes in preferences. Here we try to investigate the effect of the interaction between different psychological biases exhibited by individuals on asset prices. The paper is motivated by experimental evidence both of cognitive psychology and experimental economics.

We develop a model of one representative agent and one asset. The agent evaluates the earnings according to Prospect Theory and she does not know exactly the stochastic process generating earnings. While the earnings are generated by a random walk process, the agent considers a Markovian process, according to which firm's earnings move between two regimes, represented by a mean-reverting process and a trend process, as in Barberis, Shleifer and Vishny (1998). We study how an agent who is loss averse evaluates the price of a stock when she takes into account the wrong stochastic process. In this paper we address these issues by developing an asset pricing model endowed with prospect theory.

The design of the model is compatible with main evidence from psychology literature. On one side, the idea that people care about changes in financial wealth and they exhibit loss aversion over wealth changes constitutes a central feature of Prospect Theory, used here to describe the preferences of the agent in this single-agent economy. On the other side, the model tries at capturing cognitive limitation exhibited by agents in the task of processing information. In particular, they follow representativeness and anchoring heuristics in the formation of beliefs regarding the earnings process.

We show that this twofold departure from rationality determines permanent effects on stock prices, even in long run. The stock price deviates from the one under full rationality for the presence of two biases: one captures the effect on the stock price due to the systematic error in the beliefs regarding the stochastic process underlying the earnings; the second effect on the stock price is due to loss aversion. Under the first characterization we use a kinked linear utility function, according to Bernatzi and Thaler (1985) and we show that an agent who evaluates the asset according to Prospect Theory consistently underevaluates the asset, because of the presence of loss aversion bias. Under the second functional form, we take into account the diminishing sensitivity attitude, modelled under the assumption of concavity over gains and convexity over losses as in the original specification of Kahneman and Tversky (1979). We show that the stock price can be both undervalued and overvalued: since even gains are weighted proportionally more, the deviation from the fundamental price can go in both directions, depending on the sequence of realised earnings.

Secondly, the model allows to explain observed phenomenon in the cross-section earnings return distribution. We solve this model and according to Barberis et all (1998), we evaluate the framework by using artificial data sets of earnings and prices simulated from the model. This allows to show that, for plausible range of parameter values, it generates the empirical predictions of overreaction and underreaction observed in the data, as pointed out by a number of empirical studies on U.S. market.

2 Related literature

The present work is closely related with the literature on Behavioral Finance. Broadly speaking, behavioral finance attempts at explaining some observed financial phenomenon by studying models in which agents are not fully rational. Behavioral Finance turns to extensive experimental evidence compiled by cognitive psychologists on the systematic biases exhibited by persons on preferences and how they form beliefs. In particular, the behavioral finance literature departs from traditional paradigm mainly in two ways: on one side beliefs are not assumed to be formed according to probability theory, as for example Bayes' law and on the other, preferences are not consistent with subjective expected utility. The paper tries at establishing a connection between these two strands of literature: the one based on beliefs and the other one based on changes in preferences. We examine this issue in the context of a finance application.

Regarding departures from Bayesian beliefs, a number of works have tried to incorporate psychological biases exhibited by persons when assessing the likelihood of an event into analytical frameworks. Barberis, Shleifer and Vishny (1998), which constitutes one of the main beliefs-based model, is the paper more directly related to the present work. Barberis et all study how an agent evaluates the price of a stock when she makes systematic errors when using public information to form expectations of future cash flows. In particular the authors address the question considering an agent that takes into account the wrong stochastic process. They build a model that incorporates two updating biases, known as conservatism, which indicates the tendency to underweight new information relative to priors and representativeness, in the version of the law of small numbers, which captures the tendency to make too much inference from small samples, since people expect even small samples to exhibit the properties of the parent population. Barberis et all show that these departures from rationality determine permanent effects on stock prices, even in long run and allow to explain observed phenomenon in the cross-section earnings return distribution. In particular, the introduction of these biases explains the phenomenon of underreaction and overreaction observed empirically on cross-section of average returns. Another beliefs-based model, which attempts at explaining the same empirical evidence, is the work of Daniel, Hirshleifer and Subrahmanyam (1998, 2001) which focus attention on the presence of biases in the interpretation of private information, instead of public information.

Regarding departures from "standard preferences", Prospect Theory developed by Kahneman and Tversky (1979) has been widely used within the paradigm of alternative behavior under uncertainty. Bernatzi and Thaler (1995) constitues the first attempt to use loss aversion to explain the well known equity premium puzzle[1], which simply says that investors demand a high premium in order to hold stocks, despite stocks offer high average returns and a low covariance with consumption. Bernatzi and Thaler (1995) examine how an agent characterized by preferences modelled a la Prospect theory allocates her financial wealth optimally between T-bills and the stock market, estimating the portfolio evaluation period to be one year. The so-called "myopic loss aversion", i.e. the combination of loss aversion and frequent evaluations of portofolio returns, is the key element used to explain the equity premium puzzle: investors require a high premium as a compensation for risk if they are loss averse over annual changes in financial wealth, since they fear large drops in financial wealth evaluated each year. Barberis, Huang and Santos (2001) develop a dynamic equilibrium model of stock returns, where the Bernatzi and Thaler (1995) framework constitutes the building

[1] This phenomenon has been called equity premium puzzle since the work of Mehra and Prescott (1985), who first noticed and studied it.

block. The investors get utility both from consumption and changes in the value of holdings of the risky asset over a period of one year. In particular, they modify the utility function introducing a state variable keeping tracks of past gains and losses and anchoring to this state variable the parameter which measures sensitivity to losses. This is successful in explaining both the volatility puzzle and the equity premium puzzle.

3 Some psychological evidence

The model presented here is motivated by empirical evidence of cognitive psychology, that documents a number of cognitive biases displayed by people when they are asked to form beliefs and make decisions given their beliefs. The cognitive limitations taken into account in the formal model are representativeness heuristic and conservatism regarding biases on beliefs and loss aversion related to biases on preferences.

According to the *representativeness heuristic*, identified by Kahneman and Tversky (1974), people evaluate the probability that a data set A was generated by a model B or that an object A belongs to a class B according to the degree by which A reflects the characteristics of B or is similar to B. Despite being useful in order to reduce the evaluation process to simpler tasks, the representativeness heuristics may cause biases which affect substantially estimates and beliefs assessments. A bias generated by representativeness is constitued by the *base rate neglect*, that arises when people are asked to assess the conditional probability of an event. In fact, people applying the representativeness heuristics, underestimate the probability of the conditioned event (base rate). To illustrate this bias, Kahneman and Tversky (1974) asked subjects in the sample to assess which event is more likely on the basis of a given description. The two events are: a person named Linda is "a bank teller" (statement A) or "a bank teller and is active in the feminist movement"(statement B). The description on Linda's personality assigned to subjects in the experiment is as follows:

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Subjects typically assign greater probability to B, showing a clear violation of Bayes' law. This can be explained in the light of representativeness heuristic: people consider Linda's description representative of a feminist and therefore they put too much weight on the resemblance of Linda with statement B, despite the fact that statement B is the union event, and hence less likely than the single event belonging to the union, according to standard probability theory. In fact, subjects tend to overweight Linda's description, because of the close matching up with the idealized view of a "feminist" and as a consequence, they underestimate the statistical base rate evidence of the fraction of population belonging to this profession.

Another bias is represented by the *sample size neglect*, according to which people do not take into account (fully) the size of the sample from which observations are drawn when estimating the probability of an event and believe that even small samples can reflect the properties of the parent population. This means that people believe that a small sample can be representative as a large one. In literature, this belief is known as the "law of small numbers", as in Rabin (2002). The implication is that people tend to infer too quickly on the basis of the sample information on the data generating process.

When forming their beliefs, people exhibit another deviation relative to standard Bayesian rationality, known as *conservatism,* identified by Edwards (1968), according to which people tend to underweight the statistical base rate relative to sample evidence. When sample evidence does not allow to recognize easily the data generating process, people react too little to sample evidence and they tend to rely too much on their priors. In particular, according to experimental psychological evidence, people update their posteriors in the right direction, but in an insufficient way relative to the predictions of standard Bayesian theory.

In the present work we are interested in the representativeness heuristic and conservatism because they offer a behavioral alternative explanation to observed empirical phenomenon in the cross-section earnings return distribution. As pointed out in Barberis et all (1998), representativeness heuristic and the law of small numbers can be considered suggestive of the observed empirical financial phenomenon of overreaction. Financial overreaction stems from the fact that individuals do react too much to long and consistent strings of information on firms' earnings. Following the representativeness heuristic, individuals consider a consistent long sequence of positive (negative) information representative of a positive (negative) trend on firm's growth rate. This is due to the fact that people recognize "order in chaos", i.e. they believe to see patterns in a truly random sequence and consequently, they infer incorrectly that the earnings are expected to grow in the future at the same positive (negative) rate. This pushes prices up (down) too much in case of a positive (negative) news sequence; as a consequence, firms become over(under)valued and investors earn lower (higher) rates of return on their investment than the expected ones. This generates the phenomenon of financial overreaction. On the other hand, conservatism appears to be suggestive of the observed empirical phenomenon of underreaction. When investors receive a good (negative) piece of information on firm's earnings, they tend to disregard the information to be noisy, for example because it contains many temporary components. Therefore, investors rely more on their initial firm's evaluation and update their beliefs only by too little. Consequently, in case of positive (negative) news, returns rate on investment will be higher (lower) than the expected one, generating underreaction[2].

Empirical evidence from cognitive psychology shows that individuals exhibit biases relative not only to standard Bayesian rationality, but even with respect to expected utility theory. *Prospect theory*, developed by the seminal contribution of Kahneman and Tversky (1979), incorporates into individual's preferences some of the main observations made by psychology on human behaviour, among which particular relevance is given to the presence of *reference points*, *loss aversion*, *status quo bias* and *diminishing sensitivity*. Loss aversion indicates the tendency of individuals to evaluate the disutility deriving from a loss more than the utility assigned to a same sized gain. The notion of loss aversion[3] does not coincide with the one of risk aversion: while the latter is modelled through the concavity of the utility function, loss aversion is taken into account in the utility function by introducing a kink, i.e a non differentiable point. The kink is fixed in correspondence to the reference point, where the slope of the utility function changes sharply,

[2]Mullainathan (2001) develops an economic model trying to fit together conservatism and representativeness heuristic, despite the fact that these two biases appear to move into opposite directions.

[3]Rabin and Thaler (2001) suggest to incorporate loss aversion to overcome the theoretical problem presented by expected utility theory and pointed out by Calibration Theorem (Rabin 2000). Loss aversion allows to explain the so-called "first order risk aversion", which indicates the presence of an intense risk aversion in lotteries of modest entity. A concave utility function implies the so-called "second order risk aversion", measured through the well-known Arrow-Pratt index, according to which a significative risk aversion is recorded in lotteries of large entity and a substantial neutrality towards risk in lotteries of small dimensions.

capturing the higher weight assigned by preferences to losses with respect to gains. Prospect theory inserts explicitly the presence of the so-called reference points into the functional form of utility. Psychological studies show that individuals evaluate their satisfaction level not in terms of an absolute evaluation of reached result, but in relative terms with respect to desired or expected results. The utility experienced by an agent from her consumption activity depends on the reference point she takes into account, for example the level of past consumption or the future one, estimated on the basis of her expectations. Besides loss aversion, psychological evidence documents the phenomenon of "diminishing sensitivity", according to which the marginal utility derived from a gain (net of the reference point) diminuishes as the gain increases. This implies that the utility function of monetary wealth results to be concave over the region of gains and convex over the region of losses. Therefore, an agent exhibits an attitude of risk aversion towards gains and a preference for risk loving with respect to losses. The utility function is S-shaped, whose slope diminishes the further the wealth level gets from the reference point. Prospect theory conjugates analytical tractability with empirical evidence on psychological studies and explanatory power of evidence from experimental economics. For these reasons, the utility function according to prospect theory is used in a number of economic applications, in order to evaluate the economic implications of the hypothesis of loss aversion in various frameworks of choice.

4 A formal model

We examine a one-security pricing model with one representative agent. We focus the attention only on the demand side of the market.

The formal setting presented here is analogous to the one of Barberis et all (1998). The earnings of the security at time t are given by $N_t = N_{t-1} + y_t$, where y_t is a stochastic shock to earnings. We assume that all earnings are paid out as dividends[4]. We assume that y_t follows a random walk. The investor does not know exactly the stochastic process generating y_t. In particular, we assume that she believes that y_t can either take the value $+y$ or $-y$ and is generated by one of the two models, described by the following transition matrices.

Model 1	$y_{t+1} = y$	$y_{t+1} = -y$	Model 2	$y_{t+1} = y$	$y_{t+1} = -y$
$y_t = y$	π_L	$1 - \pi_L$	$y_t = y$	π_H	$1 - \pi_H$
$y_t = -y$	$1 - \pi_L$	π_L	$y_t = -y$	$1 - \pi_H$	π_H

Both models are a one-period Markov process, where the earnings shock at period $t+1$, y_{t+1}, depends only on the shock occured in the previous period, at t. The two processes differ in the transition probabilities: Model 1 is apt to describe a mean-reverting process, under the assumption $0 < \pi_L < 0,5$ and Model 2 describes a trend process under the assumption $0,5 < \pi_H < 1$. Under these assumptions on the parameters, under Model 1, it is more likely that a positive shock is followed by a negative one, while under Model 2 the shocks tend to persist and therefore it is more likely that a shock of the same sign occurs. Note that Model 1 is suggestive of conservatism, while Model 2 of representativeness heuristic.

The investor assumes to know the Markovian process, describing the transition from Model 1 to Model 2, according to the realisations of the stochastic variable y_t. The Markovian process, described in the following matrix, is assumed to be one period: the state of the world about which stochastic model is believed to describe the evolution of y_t is a function only of the state of world in the previous period.

	$s_{t+1} = 1$	$s_{t+1} = 2$
$s_t = 1$	$1 - \lambda_1$	λ_1
$s_t = 2$	λ_2	$1 - \lambda_2$

[4] Earnings and dividends are used indifferently from now on.

We denote the state of the world with s_t, which equals $i = 1, 2$ whenthe investor believes Model i describes the pattern followed by y_t. λ_i describes the probability of transition from state i to state j, $j \neq i$, i.e. the probability of switching between the two regimes. We assume λ_2 to be higher than λ_1, which means that the investor thinks Model 1 is more likely to be the right model describing the earnings generating process than Model 2. In the next sections no results rely on this assumption.

In this model, there is no learning regarding the stochastic process generating earnings: the probabilistic structure of the model remains unchanged since the investor believes to know exactly π_H and π_L. All agent's learning is about her beliefs regarding which state of the world is the true model generating earnings. The representative investor updates her beliefs about the state of the world in a Bayesian way: she observes the earnings shock y_t occurred in each period and she calculates the probability of y_t being generated by Model 1, using the new observation to update her beliefs about Model 1 at date $t-1$.

We label $q_t = \Pr(s_t = 1 \mid y_t, y_{t-1}, q_{t-1})$ and q_{t+1} is calculated using Bayes' rule. As shown in Barberis et all (1998),

$$q_{t+1} = \frac{\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right) \Pr\left(y_{t-1} \mid s_{t+1} = 1, y_t\right)}{\begin{array}{c}\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right) \Pr\left(y_{t-1} \mid s_{t+1} = 1, y_t\right) \\ + \left(\lambda_1 q_t + \left(1-\lambda_2\right)\left(1-q_t\right)\right) \Pr\left(y_{t-1} \mid s_{t+1} = 2, y_t\right)\end{array}} \tag{1}$$

If y_{t+1} exhibits same sign as the shock occured in period t,

$$q_{t+1} = \frac{\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right) \pi_L}{\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right) \pi_L + \left(\lambda_1 q_t + \left(1-\lambda_2\right)\left(1-q_t\right)\right) \pi_H} \tag{2}$$

If y_{t+1} exhibits opposite sign as the shock occured in period t,

$$q_{t+1} = \frac{\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right)\left(1-\pi_L\right)}{\begin{array}{c}\left(\left(1-\lambda_1\right) q_t + \lambda_2 \left(1-q_t\right)\right)\left(1-\pi_L\right) \\ + \left(\lambda_1 q_t + \left(1-\lambda_2\right)\left(1-q_t\right)\right)\left(1-\pi_H\right)\end{array}} \tag{3}$$

q_{t+1} decreases after a shock of the same sign, since the investor gives more weight to Model 2, while it increases after a shock of opposite sign, which in turns means that the investor assigns a higher weight to Model 1.

4.1 Preferences

We depart from Barberis et all setting (1998) by endowing the investor with a utility function a la Prospect theory.

We assume the agent evaluates dividends according to the following utility function:

$$v(x) = \begin{array}{c} x \quad if\ x \geq x^{\star} \\ x + l(x - x^{\star}) \quad if\ x < x^{\star} \end{array} \tag{4}$$

The agent gets utility from two sources: the first term in this preference specification represents utility over earnings x paid out by the firm, which is a standard feature of asset pricing models. This is all the utility the agent retrieves from her investment in case the realised earnings x are higher than agent's reference point x^*. In the opposite case, i.e. when realised earnings are lower than the reference point, the investor experiences a loss, which is proportional to the differential between realized earnings and reference point by a factor l, interpreted as a measure of sensitivity to loss aversion. The utility function is kinked at the reference point $x^{\star}$: this means that the investor evaluates differently gains and losses. We assume $l > 1$, i.e. the agent retrieves a higher level of disutility from a loss with respect to a same sized gain. In other words, the utility function displays loss aversion, which is increasing in the parameter l. Finally, note that we consider here a simplified version of prospect theory utility function, which is reduced to a kinked linear utility function, analogous to Bernatzi and Thaler (1995). A different framework will be considered in the next section, where the utility function is allowed to be concave over gains and convex over losses, capturing the attitude of diminishing sensitivity displayed by subjects in experimental settings.

Introducing the presence of a reference point x^* into the preference specification raises a delicate issue regarding which level of x^* should be fixed. The reference point x^* can be interpreted as an aspiration level for the investor, i.e. the desired level of earnings needed to consider satisfyicing the investment return. Otherwise x^*can be interpreted as the expected earnings level paid out by the firm in each period. In the model presented here, we set the reference level x^* to be equal to y, the value of earnings in case of a positive realization[5]. This means that the investor aims at realising positive dividends in each period. This is the highest reference level we could choose, which is associated with the largest loss the agent can experience. Hence the results stated in the following sections are robust to changes in the reference point, since the only other reasonable x^* the agent could choose would be lower than y; for example the investor could fix her aspiration level equal to the expected value of earnings[6]. Note that expected earnings would be zero, if the investor takes into account the true stochastic process generating earnings, a random walk model without drift.

Note that taking into account the stochastic process generating earnings in the period $t+i$ according to investor's beliefs and setting the reference point equal to $y^* = y$, the utility function can take only two possible values:

$$v\left(y_{t+i}\right) = \begin{array}{c} y \quad if \ y_{t+i} = y \\ -y\left(1+2l\right) \quad if \ y_{t+i} = -y \end{array} \tag{5}$$

Note that the preference specification here does not introduce any form of irrationality even if preferences are non standard; in fact, the agent experiences utility from sources different from earnings consumption, as the psychological loss from failing to reach the aspiration level y^*.

4.2 Solution of the model

Within the framework of one representative agent and one security, the asset price is given by agent's evaluation of the asset. The price of the security is given by the expected discounted utility of the dividends over the infinite horizon:

$$P_t = E_t\left\{\sum_{j=1}^{\infty}\frac{v\left(N_{t+j}\right)}{\left(1+\delta\right)^j}\right\} \tag{6}$$

[5]Remember here that dividends can either take the value y or $-y$.

[6]Note that the expected earnings value paid by a firm in each period must belong to the interval $\{-y; y\}$.

Note that the expectations are taken over agent's beliefs on the stochastic process generating earnings. Because the agent doesn't realize her mistake, her evaluation of the security will differ from the correct value of the asset, in absence of loss aversion and errors in beliefs formation. The following proposition shows the effect of biases exhibited by the agent when evaluating the security.

Proposition 1 *The asset price satisfies*

$$P_t^{PT} = \frac{v(N_t)}{\delta} + y_t(p_1 - p_2 q_t) - 2l y_t\left(p_1^{'} - p_2^{'} q_t\right) \quad \text{if} \quad y_t > 0 \tag{7}$$

$$P_t^{PT} = \frac{v(N_t)}{\delta} + y_t(p_1 - p_2 q_t) + 2l y_t\left(p_1^{''} - p_2^{''} q_t\right) \quad \text{if} \quad y_t < 0 \tag{8}$$

if we assume the investor believes that earnings act according to the Markovian process described above and evaluates the earnings according to Prospect Utility Function as in 5.

Proof. See Appendix. ■

The formula for P_t has the following explanation. In case the agent knows that the earnings shock follows a random walk, the security price would simply reduce to $\frac{v(N_t)}{\delta}$, the expected discounted utility associated with the random walk process over the infinite horizon. The asset price deviates from its fundamental value because of the presence of two biases, regarding beliefs formation and loss aversion. Because the investor does not know the correct stochastic process followed by earnings, the second term $y_t(p_1 - p_2 q_t)$ captures the systematic bias of the uninformed investor. Instead of a random walk model, the investor uses a regime switching model to forecast earnings The third component, either $-2l y_t\left(p_1^{'} - p_2^{'} q_t\right)$ if $y_t > 0$ or $2l y_t\left(p_1^{''} - p_2^{''} q_t\right)$ if $y_t < 0$, captures the bias due to loss aversion, as it is shown in Proposition 6.

According to a huge body of empirical evidence, overreaction and underreaction characterize the cross-section distribution of earnings returns. The model presented here offers an explanation of these empirical phenomena, based on the bias exhibited by the agent when forming beliefs. The presence of the attitude of loss aversion does not undermine the channel through which these phenomena are generated, even if it affects the evaluation of the security price.

Overreaction occurs when the average return on firm's stock after a consistent series of good news is lower than the average return after a series of negative announcements. In the model here, we need to define overreaction in terms of a string of positive realizations of earnings shocks y_t.

Definition 2 *Overreaction means that the expected return following a series of positive shocks is smaller than the expected return following a series of negative shocks.*

$$E_t(P_{t+1} - P_t \mid y_t = y_{t-1} = ... = y_{t-j} = y) - E_t(P_{t+1} - P_t \mid y_t = y_{t-1} = ... = y_{t-j} = -y) < 0 \quad (9)$$

When a positive series of good news is announced (or a long string of positive earnings is realized), the investor tends to believe that that firm belongs to a subset of "good companies", whose earnings are assumed to grow at a positive rate even in the future. By using the representativeness heuristics, she regards the trend process, labelled as Model 2, more likely to describe the true earnings generating process. This pushes the security price up, above the true evaluation. Since earnings shocks follow a random walk, negative realizations will occur, contradicting investor's optimism and as a consequence, the investor gets a lower return from the security after a series of positive news. This is a clear violation of the semi-strong form of market efficiency, since the investor may realise positive profits using stale information and trading the security after the earnings announcement. Early studies as Poterba and Summers (1988) and Cutler et all (1991) discover evidence of negative autocorrelation in stock returns over horizons of three to five years and predictability of returns on the basis of book to market ratio. De Bondt and Thaler (1985) find that stocks with a long series of poor returns over a period of three years offer consistently higher returns than stocks with very high returns. The result is robust to changes in the measure of stock evaluation, as the ratio of market value to cash flow or market value to book value of assets.

Underreaction occurs when the average return on a firm's stock following a positive earnings announcement is higher than the average return after a bad news. In the model presented here, we need to define underreaction in terms of a single positive realization of the earnings shock.

Definition 3 *Underreaction means that the expected return following one positive shock is higher than the expected return following one negative shock*

$$E_t\left(P_{t+1} - P_t \mid y_t = y\right) - E_t\left(P_{t+1} - P_t \mid y_t = -y\right) > 0 \tag{10}$$

After observing a single good news, the investor tends to disregard the information contained in the news as noisy, by using conservatism. As a consequence, the stock underreacts to the announcement of a single good news; in the subsequent periods the mistake is partially corrected and the price of the security goes up gradually. The investor realises therefore a positive return. Bernard (1992) finds that U.S. stocks with higher standardized unexpected earnings (SUE)[7] earn higher returns in the period after the announcement; this means that agents underreact to earnings news and stale information on SUE has predictive power for future stock returns, a violation of the semi-strong market efficiency hypothesis. In the same direction points the evidence elaborated by Jeedgadesh and Titman (1993), which shows a positive autocorrelation between a cross section of U.S. stock returns over a period of six month due to a slow incorporation of information into prices and hence due to underreaction.

Proposition 4 *The price function determined in Proposition 1 exhibits both underreaction and overreaction under the same conditions stated in Barberis et all.*

Proof. See Appendix ■

The following proposition stated and proved in Barberis et all (1998) gives the conditions for underreaction and overreaction on the parameters underlying the model, such that conditions 9 and 10 are satisfied.

Proposition 5 *Proposition 2 in Barberis et all (1998)*

Suppose the underlying parameters π_L, π_H, λ_1 and λ_2 satisfy

$\underline{k}p_2 < p_1 < \overline{k}p_2$,

$p_2 \geq 0$,

where

[7] Standardized unexpected earnings (SUE) is defined as the difference between a firm's earnings in a given quarter and its earnings during the quarter a year before, scaled by the standard deviation of firm's earnings.

$$\underline{k} = \underline{q} + \tfrac{1}{2}\overline{\Delta}\left(\underline{q}\right),$$
$$\overline{k} = \underline{q}^{e} + \tfrac{1}{2}\left(c_1 + c_2 q_*\right),$$
$$c_1 = \frac{\overline{\Delta}\left(\underline{q}\right)\overline{q} - \underline{\Delta}\left(\overline{q}\right)\underline{q}}{\overline{q} - \underline{q}}$$
$$c_2 = \frac{\underline{\Delta}\left(\overline{q}\right)\overline{q} - \overline{\Delta}\left(\underline{q}\right)}{\overline{q} - \underline{q}}$$
$$q_* = \begin{matrix} \overline{q}^{e} & if & c_2 < 0 \\ \underline{q}^{e} & if & c_2 \geq 0 \end{matrix}$$

where $\underline{q}^{e}$ *and* $\overline{q}^{e}$ *are bounds on the unconditional mean of the random variable* q_t. *Then the conditions for both underreaction and overreaction are satisfied.*

Proof. See Barberis et all (1998). ■

Here we show that the introduction of loss aversion affects the evaluation of the stock with permanent effects. The effect on the price of the security is stated in the following proposition.

Proposition 6 *Under the assumption that the investor evaluates earnings according to Prospect Utility Function as in 5 and under the condition that guarantees both underreaction and overreaction, the security price is always underevalued with respect to the evaluation assigned by a risk neutral investor.*

Proof. See Appendix. ■

From the comparison we obtain the result that the stock price is always undervalued with respect to a risk neutral investor. The intuition comes straightly from the fact that for each probability structure, i.e. for any realization of investor's beliefs, the loss averse agent is assigning a negative weight to losses, a weight higher than the one assigned to a same-sized gain. Because of the negative weight given to losses, the loss-averse investor always has lower evaluation of the security according to Kahneman and Tversky prospect theory. This comes from the fact that the investor tends to assign more weight to the trend model and regards positive results to be more likely than negative results; because of the negative weight assigned to losses, a lower evaluation of the stock under prospect theory is induced. In case of underreaction, exactly the opposite phenomenon occurs: negative results are regarded more likely than the positive ones.

5 A different set-up

In this section we address the question on the role of loss aversion in the evaluation of asset price in a different framework. Here we consider a model with one representative agent and one security, as in the previous section and we define agent's preferences by incorporating the attitude of diminishing sensitivity. On one side, we move closer to the original Kahneman and Tversky's specification (1979), and we fit better with psychological evidence describing attitude towards risk. On the other, a kinked linear utility function allows to obtain sharper predictions on agent's evaluation of security, because of a simpler structure of the model.

5.1 The utility function

We consider the following specification of the prospect theory utility function. Besides loss aversion, we observe the phenomenon of "diminishing sensitivity", according to which the marginal utility deriving from a gain (net of the reference point) diminuishes as the gain increases[8]. This implies that the utility function of monetary wealth results to be concave on the region of gains and convex on the region of losses. Therefore, an agent exhibits an attitude of risk aversion towards gains and a preference for risk loving with respect to losses. The utility function is S-shaped, whose slope diminishes the further the wealth level gets from the reference point.

Analytically, we consider the following expression:

$$v(x) = \begin{array}{c} x^{\alpha} \quad if\ x \geq 0 \\ (-\lambda)\, x^{\alpha} \quad if\ x < 0 \end{array} \tag{11}$$

With no loss of generality, we assume α is an even number. This is a simplification with respect to the original formulation of Kahneman and Tversky (1979): while we modify directly the agent's evaluation of utility retrieved from earnings taking into account the attitude of loss aversion and diminishing sensitivity, we do not introduce any nonlinear probability transformation to capture the tendency of individuals to overweight small probabilities.

[8] The phenomenon of diminishing sensitivity can be explained by the following "real life" example given by Kahneman and Tversky (ANNO). "It is easier to distinguish the difference between a change of 3° and a change 6° of degrees in room temperature than it is to discriminate between a change of 13° and a change of 16°".

According to the functional form in 11, earnings are evaluated in terms of gains and losses with respect to a reference point and each individual assigns to losses a higher weight than to same sized gains, exhibiting therefore loss aversion. We assume $\lambda > 1$. The utility function has a kink (a non differentiable point) in correspondence to the reference point, where the marginal value of a loss is larger than the marginal value of a same sized gain. The phenomenon of diminishing sensitivity is guaranteed by the power function, which requires the utility function to be S-shaped so that the individual is risk loving on the region of losses and risk averse on the region of gains.

5.2 Solution of the model

Within the framework of one representative agent and one security, the price of the stock is calculated as the expected discounted value of utility of earnings. The following Proposition offers a general expression of the price of the security. Note that the general formulation of the utility function in 11 does not allow any more to simplify the problem as in the previous section, since we lose the addivity in the payoffs, due to the non-linearity in earnings of the utility function.

Proposition 7 *If the investor believes that the process generating earnings shocks satisfies the Markovian process described above and she evaluates the earnings according to Prospect Theory utility function as in 11, the price of the security satisfies the following equation:*

$$P_t^{PT} = \sum_{j=1}^{\infty}\sum_{k=0}^{j} \frac{(-\lambda)^I \left[ky_t + (j-k)(-y_t)\right]^{\alpha}}{(1+\delta)^j} \cdot \binom{j}{k} \left(\prod_{i=1}^{k} \overline{\gamma}' Q^i q^t\right) \left(\prod_{i=k+1}^{j+1} \underline{\gamma}' Q^i q^t\right) \tag{12}$$

where I is the indicator function which takes the value 0 *if* $[ky + (j+1-k)(-y)] \geq 0$ *and* 1 *if* $[ky + (j+1-k)(-y)] < 0$

Proof. See the appendix. ■

We notice immediately that P_t can either take negative or positive values, while P_t would be zero if the investor uses the random walk process (under the assumption $N_t = 0$). This is due to the specific version of the prospect theory utility function used here.

We need to show that there is no conclusive comparison with Shleifer model, in case the agent is characterised by a prospect theory utility function as in 11. In case the agent is risk neutral, the price of the security takes the following expression:

$$P_t^{RN} = \sum_{j=1}^{\infty}\sum_{k=0}^{j}\frac{[ky_t + (j-k)(-y_t)]}{(1+\delta)^j}\cdot\binom{j}{k}\left(\prod_{i=1}^{k}\overline{\gamma}'Q^i q^t\right)\left(\prod_{i=k+1}^{j+1}\underline{\gamma}'Q^i q^t\right) \quad (13)$$

After some calculations, it is possible to show that the difference between the two prices can be expressed as follows:

$$P_t^{PT} - P_t^{RN} = \sum_{j=1}^{\infty}\frac{1}{(1+\delta)^j}\cdot$$

$$\left\{\begin{array}{l}\sum_{k=0}^{\frac{j}{2}}(-\lambda)(2k-j)(-y)\left[((2k-j)(-y))^{\alpha-1}-1\right]\cdot\binom{j}{k}\left(\prod_{i=1}^{k}\overline{\gamma}'Q^i q^t\right)\left(\prod_{i=k+1}^{j+1}\underline{\gamma}'Q^i q^t\right)\\ \qquad +\sum_{k=\frac{j}{2}+1}^{j}(2k-j)\,y\left[((2k-j)\,y)^{\alpha-1}-1\right]\cdot\binom{j}{k}\left(\prod_{i=1}^{k}\overline{\gamma}'Q^i q^t\right)\left(\prod_{i=k+1}^{j+1}\underline{\gamma}'Q^i q^t\right)\end{array}\right\} \quad (14)$$

From the inspection of 14 it appears clearly that it is not possible to predict whether the price in case the agent displays loss aversion will be undervalued or overvalued than the one under risk neutrality.

5.3 Some simulation results

In this section we evaluate numerically the model presented above, since we do not obtain any conclusive results in terms of comparison between security price under prospect theory and risk neutrality. We run a simulation analogous to the one of Barberis et all (1998). We simulate artificial data sets of earnings and prices from our model. We choose parameters values to satisfy conditions to generate overreaction and underreaction, as stated in Proposition 5. We fix $\pi_H = \frac{1}{3}$ so that Model 1 is a mean-reverting model and $\pi_L = \frac{3}{4}$ a trend model. Moreover, we assume $\lambda_1 = 0.1$ and $\lambda_2 = 0.3$ so that the agent considers Model 1 to be more likely than Model 2. Finally,

λ is assumed to be $2,25$ according to Kahneman and Tversky's empirical estimates; this indicates that losses are weighted more than double than gains of same size.

We simulate a stream of earnings, using a binomial model that gives earnings equal either to y or $-y$ with a given probability. In alternative, we generate earnings stream with a random walk process. We set the initial level of earnings N_1 to be equal to zero and we generate 2,000 earnings sequences. Each sequence corresponds to an hypothetical firm and is constitued by six earnings realization. According to Bernatzi and Thaler (1995), who estimated the average period for the evaluation of returns on financial portfolio investment to be one year, we can interpret a period in the model to coincide with one year, after which an earnings realization occurs. Given the values chosen for the parameters, we use the simulated earnings data to calculate prices and returns, according to the model presented in section 5.2. We use the same data to generate prices and returns according to Barberis et all (1998) model.

We address the question on the role of loss aversion when this attitude interplays with the presence of psychological biases affecting beliefs formation. For each n period in the sample, from one to four, we form two portfolios. One portfolio consists of all firms with positive earnings changes in every period n considered in the sample, while the other is formed by all firms with negative earnings changes in every period n. For each firm in each portfolio, we calculate the difference between the two prices calculated according to P_t^{PT} as in 12 and P_t^{RN} as in 13. The difference $P_t^{PT} - P_t^{RN}$ results to be consistently positive for each firm belonging to the first portfolio and negative for each one in the second portfolio. We check the robustness of the results under different assumption on q, the prior probability that Model 1 describes the true earnings generating process.

We calculate the difference between returns on the two portfolios in the year after formation and in every of the subsequent n years in the sample. Label the difference as $r_+^n - r_-^n$. We observe the pattern expected for overreaction and underreaction. According to 10, the underreaction condition, the average return following a positive earnings shock is greater than the average return following a negative shock; the simulated data generates this phenomenon in returns on portfolio. The difference becomes negative as the number of shocks increases, satisfying the condition for overreaction stated in 9.

Earnings sort		
$r_+^1 - r_-^1$	0.0522	
$r_+^2 - r_-^2$	0.0245	(Table 1)
$r_+^3 - r_-^3$	-0.0175	
$r_+^4 - r_-^4$	-0.0478	

With respect to Barberis et all (1998), not only the magnitudes of the numbers in the table are quite reasonable, but their absolute values are closer to those found in the empirical literature. Note that we report only point estimates, without addressing their statistic significance; we would need otherwise to impose more structure, such as on the cross-sectional covariance properties of earnings changes.

6 Concluding comments

We have presented a model of one security The model is motivated by psychological evidence, in particular two are the main ideas which inspire the framework: loss aversion as presented by Kahneman and Tversky (1979) and the tendency of people to put too much weight on the strength of the evidence presented and too little weight to its statistical weight. We show that in case the utility function is modelled according to a kinked linear utility function as in Bernatzi and Thaler (1992), the price is systematically undervalued with respect to a risk neutral agent, who commits the same systematic error when forming beliefs regarding the stochastic process generating earnings. The deviation becomes unclear in case we model the preferences for embedding the attitude of diminishing sensitivity. Simulation results show that the price of the stock is overvalued after a sufficiently long sequence of positive results and undervalued after a sufficiently long sequence of negative stock earnings realization. Moreover, the model is consistent with empirical evidence concerning underreaction and overreaction.

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7 Appendix

Proof. of Proposition 1. The asset price is given by the expected discounted value of the utility of the earnings over the infinite horizon. Because of the stochastic process generating earnings and because of the linearity of the utility function, we can rewrite the expression of the price in the following way: $P_t = E_t\left[\frac{v(N_t)+v(y_{t+1})}{(1+\delta)} + \frac{v(N_t)+v(y_{t+1})+v(y_{t+2})}{(1+\delta)^2} + ...\right] = \frac{v(N_t)}{\delta} + \frac{1}{\delta}\sum_{j=1}^{\infty} E_t\left[\frac{v(y_{t+j})}{(1+\delta)^{j-1}}\right]$

We need to calculate an expression for $E_t[v(y_{t+j})]$. We keep the same probabilistic structure as in Barberis et all (1998):

$$E_t[v(y_{t+j}) \mid \phi_t] = \begin{array}{l} y_t \Pr(y_{t+j} = y_t \mid \phi_t) + (-y_t)(1+2l)\Pr(y_{t+j} = -y_t \mid \phi_t) \;\; if \;\; y_t = y \\ -y_t \Pr(y_{t+j} = -y_t \mid \phi_t) + (y_t)(1+2l)\Pr(y_{t+j} = y_t \mid \phi_t) \;\; if \;\; y_t = -y \end{array}$$

where:

$\Pr(y_{t+j} = y_t \mid \phi_t) = \Pr(y_{t+j} = y_t, s_{t+j} = 1 \mid \phi_t) + \Pr(y_{t+j} = y_t, s_{t+j} = 2 \mid \phi_t) = q_1^{t+j} + q_3^{t+j}$

$\Pr(y_{t+j} = -y_t \mid \phi_t) = \Pr(y_{t+j} = -y_t, s_{t+j} = 1 \mid \phi_t) + \Pr(y_{t+j} = -y_t, s_{t+j} = 2 \mid \phi_t) = q_2^{t+j} + q_4^{t+j}$

where we denote as $q^{t+j} = \left(q_1^{t+j}, q_2^{t+j}, q_3^{t+j}, q_4^{t+j}\right)'$ and ϕ_t denotes the information owned by the investor at date t that consists of all past observations of earnings

We note that
$q^{t+j} = Qq^{t+j-1} = Q^j q^t$
where

$$Q' = \begin{array}{c|cccc} & (1) & (2) & (3) & (4) \\ (1) & (1-\lambda_1)\pi_L & (1-\lambda_1)(1-\pi_L) & \lambda_1\pi_H & \lambda_1(1-\pi_H) \\ (2) & (1-\lambda_1)(1-\pi_L) & (1-\lambda_1)\pi_L & \lambda_1(1-\pi_H) & \lambda_1\pi_H \\ (3) & \lambda_2\pi_L & \lambda_2(1-\pi_L) & (1-\lambda_2)\pi_H & (1-\lambda_2)(1-\pi_H) \\ (4) & \lambda_2(1-\pi_L) & \lambda_2\pi_L & (1-\lambda_2)(1-\pi_H) & (1-\lambda_2)\pi_H \end{array}$$

$$q^t = \begin{array}{c} q_t \\ 0 \\ 1-q_t \\ 0 \end{array} \quad \text{and } q_t = \Pr(s_t = 1 \mid y_t, y_{t-1}, q_{t-1})$$

The probability structure of the model is recursive, since the probability of earnings shock at date $t+j$ is given by the probability of earnings shock at date t times the transition matrix to j-th power.

In particular, we can rewrite
$\Pr(y_{t+j} = y_t \mid \phi_t) = q_1^{t+j} + q_3^{t+j} = \overline{\gamma}' q^{t+j}$
$\Pr(y_{t+j} = -y_t \mid \phi_t) = q_2^{t+j} + q_4^{t+j} = \underline{\gamma}' q^{t+j}$
where $\overline{\gamma}' = (1,0,1,0)$ and $\underline{\gamma}' = (0,1,0,1)$

We have that the expected utility of earnings at date $t+j$ is given by the following expression:
$E_t[v(y_{t+j}) \mid \phi_t] = y_t\overline{\gamma}'Q^jq^t - y_t(1+2l)\underline{\gamma}'Q^jq^t$ if $y_t > 0$
$E_t[v(y_{t+j}) \mid \phi_t] = y_t(1+2l)\overline{\gamma}'Q^jq^t - y_t\underline{\gamma}'Q^jq^t$ if $y_t < 0$
Therefore:
$$P_t^{PT} = \frac{v(N_t)}{\delta} + y_t(p_1 - p_2q_t) - 2ly_t\left(p_1^{'} - p_2^{'}q_t\right) \text{ if } y_t > 0$$
$$P_t^{PT} = \frac{v(N_t)}{\delta} + y_t(p_1 - p_2q_t) + 2ly_t\left(p_1^{''} - p_2^{''}q_t\right) \text{ if } y_t < 0$$
where p_1 and p_2 are given by the following two expressions:
$$p_1 = \frac{1}{\delta}\left(\gamma_0'(1+\delta)[I(1+\delta) - Q]^{-1}Q\gamma_1\right)$$
$$p_2 = -\frac{1}{\delta}\left(\gamma_0'(1+\delta)[I(1+\delta) - Q]^{-1}Q\gamma_2\right)$$
where:
$\gamma_0' = (1,-1,1,-1)$,
$\gamma_1' = (0,0,1,0)$,
$\gamma_2' = (1,0,-1,0)$.
where p_1' and p_2' are given by the following two expressions:
$$p_1' = \frac{1}{\delta}\left(\underline{\gamma}'(1+\delta)[I(1+\delta) - Q]^{-1}Q\gamma_1\right)$$

$p_2' = -\frac{1}{\delta}\left(\underline{\gamma}'(1+\delta)[I(1+\delta)-Q]^{-1}Q\gamma_2\right)$

where p_1'' and p_2'' are given by the following two expressions:

$p_1'' = \frac{1}{\delta}\left(\overline{\gamma}'(1+\delta)[I(1+\delta)-Q]^{-1}Q\gamma_1\right)$

$p_2'' = -\frac{1}{\delta}\left(\overline{\gamma}'(1+\delta)[I(1+\delta)-Q]^{-1}Q\gamma_2\right)$ ■

Proof. of Proposition 4.We need to calculate the function

$f(q) = E_t\left(P_{t+1}^{PT} - P_t^{PT} \mid y_t = +y, q_t = q\right) - E_t\left(P_{t+1}^{PT} - P_t^{PT} \mid y_t = -y, q_t = q\right)$

As noticed above, P_t^{PT} corresponds to P_t^{RN} plus a bias due to loss aversion. Therefore, the first part of the analysis corresponds quasi entirely to the one produced in Barberis et all (1998)[9], where:

$$P_{t+1}^{RN} - P_t^{RN} = \frac{v(y_{t+1})}{\delta} + (y_{t+1}-y_t)(p_1 - p_2 q_t) - y_t p_2 (q_{t-1}-q_t) - (y_{t+1}-y_t) p_2 (q_{t+1}-q_t)$$

In expectations:

$$E_t\left(P_{t+1}^{RN} - P_t^{RN} \mid y_t = +y, q_t = q\right) = \tfrac{1}{2}\left\{\frac{y}{\delta} + yp_2\underline{\Delta}(q)\right\} +$$

$$+\tfrac{1}{2}\left\{-\frac{(1+2l)\,y}{\delta} - 2y(p_1 - p_2 q) - yp_2\overline{\Delta}(q) + 2yp_2\overline{\Delta}(q)\right\} =$$

$$= \tfrac{1}{2}yp_2\left(\overline{\Delta}(q) + \underline{\Delta}(q)\right) + y(p_2 q - p_1) - \frac{ly}{\delta}$$

$$E_t\left(P_{t+1}^{RN} - P_t^{RN} \mid y_t = -y, q_t = q\right) = \tfrac{1}{2}\left\{\frac{y}{\delta} + 2y(p_1 - p_2 q) + yp_2\underline{\Delta}(q) + 2yp_2\overline{\Delta}(q)\right\} +$$

$$+\tfrac{1}{2}\left\{-\frac{(1+2l)\,y}{\delta} - yp_2\underline{\Delta}(q) + \right\} =$$

$$= y(p_1 - p_2 q) - \tfrac{1}{2}yp_2\left(\overline{\Delta}(q) + \underline{\Delta}(q)\right) - \frac{ly}{\delta}$$

Then;

$f_1(q) = \left(P_{t+1}^{RN} - P_t^{RN} \mid y_t = +y, q_t = q\right) - \left(P_{t+1}^{RN} - P_t^{RN} \mid y_t = -y, q_t = q\right) = 2y(p_2 q - p_1) + yp_2\left(\overline{\Delta}(q) + \underline{\Delta}(q)\right)$

We need to evaluate the difference in the bias due to loss aversion. Label it as $f_2(q)$. As before we need to distinguish between two cases:

$y_t > 0$: we need to distinguish between:

$y_{t+1} = y_t = +y > 0$ same sign

$-2l\left[y_{t+1}(p_1' - p_2' q_{t+1}) - y_t(p_1' - p_2' q_t)\right] =$

$= -2l\left[(y_{t+1}-y_t)(p_1' - p_2' q_t) - y_t p_2'(q_{t+1}-q_t) - (y_{t+1}-y_t)p_2'(q_{t+1}-q_t)\right]$

In expectations: this happens with probability $\frac{1}{2}$ it becomes: $-lyp_2'\underline{\Delta}(q)$

$y_{t+1} = -y; y_t = +y$ opposite sign

[9] except for the evaluation of

$2l\left[y_{t+1}\left(p_1'' - p_2''q_{t+1}\right) + y_t\left(p_1' - p_2'q_t\right)\right] =$ [10]

$= 2l\left[y_{t+1}\left(p_1'' - p_2''q_{t+1}\right) - y_t\left(p_1'' - p_2''q_t\right) + y_t\left(p_1'' - p_2''q_{t+1}\right) + y_t\left(p_1' - p_2'q_t\right)\right]$

$$= 2l\left[\begin{array}{c}(y_{t+1} - y_t)\left(p_1'' - p_2''q_t\right) - y_t p_2''\left(q_{t+1} - q_t\right) - \left(y_{t+1} - y_t\right)p_2''\left(q_{t+1} - q_t\right) \\ +y_t\left(p_1'' - p_2''q_t\right) + y_t\left(p_1' - p_2'q_t\right)\end{array}\right]$$

In expected terms, with probability $\frac{1}{2}$:

$l\left[-2y\left(p_1'' - p_2''q\right) - yp_2''\overline{\Delta}(q) + 2yp_2''\overline{\Delta}(q) + y\left(p_1'' - p_2''q\right) + y\left(p_1' - p_2'q\right)\right] =$

$= l\left[yp_2''\overline{\Delta}(q) - y\left(p_1 - p_2q\right)\right]$

The overall term coming from $l\left[-yp_2'\underline{\Delta}(q) + yp_2''\overline{\Delta}(q) - y\left(p_1 - p_2q\right)\right]$

$y_t < 0$: we need to distinguish between:

$y_{t+1} = y_t = -y < 0$ same sign

$2l\left[y_{t+1}\left(p_1'' - p_2''q_{t+1}\right) - y_t\left(p_1'' - p_2''q_t\right)\right] =$

$= 2l\left[(y_{t+1} - y_t)\left(p_1'' - p_2''q_t\right) - y_t p_2''\left(q_{t+1} - q_t\right) - \left(y_{t+1} - y_t\right)p_2''\left(q_{t+1} - q_t\right)\right]$

In expectations: this happens with probability $\frac{1}{2}$ it becomes: $-lyp_2''\underline{\Delta}(q)$

$y_{t+1} = +y; y_t = -y$ opposite sign

$-2l\left[y_{t+1}\left(p_1' - p_2'q_{t+1}\right) + y_t\left(p_1'' - p_2''q_t\right)\right] =$

$= -2l\left[y_{t+1}\left(p_1' - p_2'q_{t+1}\right) - y_t\left(p_1' - p_2'q_t\right) + y_t\left(p_1'' - p_2''q_{t+1}\right) + y_t\left(p_1' - p_2'q_t\right)\right]$

$$= -2l\left[\begin{array}{c}(y_{t+1} - y_t)\left(p_1' - p_2'q_t\right) - y_t p_2'\left(q_{t+1} - q_t\right) - \left(y_{t+1} - y_t\right)p_2'\left(q_{t+1} - q_t\right) \\ +y_t\left(p_1'' - p_2''q_t\right) + y_t\left(p_1' - p_2'q_t\right)\end{array}\right]$$

In expected terms, with probability $\frac{1}{2}$:

$-l\left[2y\left(p_1' - p_2'q\right) + yp_2'\overline{\Delta}(q) - 2yp_2'\overline{\Delta}(q) - y\left(p_1'' - p_2''q\right) - y\left(p_1' - p_2'q\right)\right] =$

$= -l\left[-yp_2'\overline{\Delta}(q) - y\left(p_1 - p_2q\right)\right]$

The overall term coming from $-l\left[yp_2''\underline{\Delta}(q) - yp_2'\overline{\Delta}(q) - y\left(p_1 - p_2q\right)\right]$

Summing up the two components we get:

$f_2(q) = l\left[2y\left((p_2q - p_1) + yp_2\left[\underline{\Delta}(q) + \overline{\Delta}(q)\right]\right)\right]$

Therefore, the overall

$f^{PT}(q) = E_t\left(P_{t+1}^{PT} - P_t^{PT} \mid y_t = +y, q_t = q\right) - E_t\left(P_{t+1}^{PT} - P_t^{PT} \mid y_t = -y, q_t = q\right) = f_1(q) + f_2(q)$

$= (1 + l)\left\{2y\left((p_2q - p_1) + yp_2\left[\underline{\Delta}(q) + \overline{\Delta}(q)\right]\right)\right\} = (1 + l)f_1(q)$

As shown, the function $f^{PT}(q)$ is equal to the $f_1(q)$ times a positive costant function of the disappointment parameter. It follows that the analysis of the conditions under which $f^{PT}(q)$ exhibits underreaction and overreaction corresponds to $f_1(q)$ exhibits underreaction and overreaction, as stated in Barberis et all (1998). ■

Proof. of Proposition 6. The proof comes obviously by comparing the security price under standard assumption of risk neutrality (according to

[10] We use the fact that $p_i = p_i'' - p_i'$ where $i = 1, 2$

Barberis et all (1998)) and under prospect theory.

Let us denote with P^{RN} the security price evaluated by a risk neutral investor with the same beliefs of the investor considered here. In particular, we note that according to Barberis et all:

$$P_t^{RN} = \frac{N_t}{\delta} + y_t \left(p_1 - p_2 q_t\right) \tag{15}$$

The security price deviates systematically from the correct value due to the bias caused by the attitude of loss aversion exhibited by the agent. By a direct comparison, it follows straightly:

$$P_t^{PT} - P_t^{RN} = \frac{v\left(N_t\right) - N_t}{\delta} - 2ly_t \left(p_1^{'} - p_2^{'} q_t\right) \text{ if } y_t > 0 \tag{16}$$

$$P_t^{PT} - P_t^{RN} = \frac{v\left(N_t\right) - N_t}{\delta} + 2ly_t \left(p_1^{''} - p_2^{''} q_t\right) \text{ if } y_t < 0 \tag{17}$$

Both 16 and 17 depend on the condition on underreaction and overreaction. Note that $v\left(N_t\right) - N_t \leq 0$, because of the particular functional form chosen for prospect utility function. Assume for simplicity $N_t = 0$. Note that the term premultiplying $\left(p_1^{i} - p_2^{i} q_t\right)$ is negative both when a negative or a positive shock occurs. It follows immediately that for each y_t , i.e. independently of whether a positive or negative shock occurs, P_t^{PT} is undervalued with respect to the case of a risk neutral investor. ■

Proof. of Proposition 7. Within this setting with one representative investor, the security price is given by the expected discounted value of utility of future earnings, in other words: $P_t = E_t \left\{ \sum_{j=1}^{\infty} \frac{v\left(N_{t+j}\right)}{(1+\delta)^j} \right\}$. Given the assumption on the utility function describing investor's preference over earnings, we cannot rely on the additivity over the stochastic shocks y_t as in the previous section. We need to determine an expression of $N_{t+j.}$ Because of the stochastic process driving earnings, we can write:

$N_{t+j} = N_t + y_{t+1} + y_{t+2} + ... + y_{t+j}$

We assume $N_t = 0$ for simplicity. Since $N_{t+j.}$ is the sum of j-th shocks, which can only take two values, $+y$ and $-y$,

$N_{t+j} = ky_t + (j-k)\left(-y_t\right)$

where k indicates the number of same-sign shocks occurred and $j-k$ the number of shocks with opposite sign with respect to y_t.

Recalling:
$\Pr\left(y_{t+j} = y_t \mid \phi_t\right) = q_1^{t+j} + q_3^{t+j} = \overline{\gamma}' Q^j q^t$
$\Pr\left(y_{t+j} = -y_t \mid \phi_t\right) = q_2^{t+j} + q_4^{t+j} = \underline{\gamma}' Q^j q^t$
then $\Pr ob\left(N_{t+j} = ky_t + (j-k)(-y_t)\right) = \binom{j}{k} \prod_{i=1}^{k} \left(\overline{\gamma}' Q^i q^t\right) \prod_{i=k+1}^{j} \left(\underline{\gamma}' Q^j q^t\right)$
since there are $\binom{j}{k}$ times to combine k same sign shocks.
Hence, it follows straightly the security price function:

$$P_t = \sum_{j=1}^{\infty} \sum_{k=0}^{j} \frac{(-\lambda)^I \left[ky_t + (j-k)(-y_t)\right]^\alpha}{(1+\delta)^j} \cdot \binom{j}{k} \left(\prod_{i=1}^{k} \overline{\gamma}' Q^i q^t\right) \left(\prod_{i=k+1}^{j+1} \underline{\gamma}' Q^i q^t\right)$$

where $\prod(\cdot)$ equals 1 when $i < k$ or $j > k$. ■