Price Competition over Boundedly Rational Agents

Barbara Luppi*
University of Bologna and LSE, STICERD

Abstract

We develop a model to study market interaction between rational firms on one side of the market and boundedly rational consumers on the other. A special feature of bounded rationality is modelled here: from psychological evidence, people tend to group events or numbers into categories; therefore we consider consumers who partition the price space into connected sets and regard each price belonging to the same set as equal.

According to Rubinstein (1993), we endogenize the choice of the price partition by consumers, who determine the optimal price partition given the constraint imposed on their ability to process information on prices. We develop a model with two firms and two states of nature. We show that we depart from classical Bertrand result when consumers are characterized by a bound on the finiteness of price partition inferior to the cardinality of the space of world states. In other words, in presence of consumers who can partition the price space into two sets and with two states of the world, firms find optimal to set price above marginal cost, making positive profits. The intuition of the result can be explained as follows: when a consumer chooses the price partition, she faces a trade off between the detection of the state of nature and the detection of a deviating behavior of the firm in a given state of nature.

Keywords: bounded rationality, Bertrand competition

JEL Classification: D43, D81

*Corresponding address: University of Bologna, Department of Economics, Piazza Scaravilli, 2. email: bluppi@economia.unibo.it

1 Introduction

In economic models decision makers are assumed to be fully rational, in the sense that economic agents are not limited in their ability to process information or perform calculations. In most of the models of economic theory behavioral differences among consumers are due to differences in preferences or in the information they possess. Wide psychological evidence indicates instead that human beings depart systematically from full rationality: cognitive limitations affect individual ability to recognize prices and therefore to process the information contained in price offers. We aim at investigating the role of bounded rationality into industrial organization framework and studying how bounded rationality affects market performances of firms and determines departures from well-established results.

This paper studies market interaction between rational firms on one side of the market and boundedly rational consumers on the other. While firms are assumed to be standard profit-maximizer competing a la Bertrand, consumers are assumed to be boundedly rational in the way they process information on prices. On one hand, the presence of bounded rationality makes consumers vulnerable to exploitation by rational firms; on the other hand, the competition among firms reduces the incentive to take advantage of consumers in order to beat the competitors and gain market share. This paper aims at investigating the interplay of these two opposite forces.

We develop a stilized model, where we characterize explicitly the procedural element of bounded rationality in the decisional procedure. The paper is motivated by psychological evidence on consumer information processing: studies of Rosch and Mervis (1975) enlights the existence of a common heuristic: in environments with abundance of information, people exhibit the tendency of grouping events, objects or numbers into categories on the basis of perceived similarities. We attempt at incorporating this heuristic by modelling explicitly the choice of the price partition made by consumers. In particular, we develop a partitional model on the price space and consumers are assumed to be bounded in their ability to process information over prices. As in Rubinstein (1993), this translates into the assumption that exists a bound on the finiteness of price space partition, i.e. on the number of sets into which the price space can be divided by consumers. The agent chooses her optimal information structure, i.e. the optimal price partition, given a constraint on her ability to process information. We depart from Rubinstein setting by modelling the price competition among perfectly rational firms. While Rubinstein examines the optimal pricing schedule of a monopolist who discriminates among consumers, here we study the optimal pricing choice of firms competing a la Bertrand. The main novelty of the paper consists in addressing directly the question on the role of a procedural element of boundedly rationality on the competitive mechanism and the effects on the equilibrium configuration.

In the first part of the paper, we develop a very simple model, where we investigate this issue under the assumption that the price partition of each consumer is exogenously given. Despite this simplifying assumption, firms' behavior departs systematically from standard Bertrand result: the equilibrium price never collapses to marginal cost, so that the competitive incentive to undercut the rival does not outweight completely the incentive to exploit the bounded rationality feature displayed by consumers. The result is robust both to the introduction of heterogeneity on firm's cost structure and on consumers' side, modelled in terms of differences in the price partition.

In the second part, we endogenize the choice of the price partition by consumers, who determine the optimal price partition given the constraint imposed on their ability to process information on prices. We develop a model with two firms and two states of nature. We show that under certain values of the parameters, we depart from classical Bertrand result when consumers are characterized by a bound on the finiteness of price partition inferior to the cardinality of the space of world states. In other words, in presence of consumers who can partition the price space into two sets and with two states of the world, firms find optimal to set price above marginal cost, making positive profits. The intuition of the result can be explained as follows: when a consumer chooses the price partition, she faces a trade off between the detection of the state of nature and the detection of a deviation of the firm in a given state of nature.

The plan of the paper is as follows. In section 2 we discuss the related literature. In section 3 we present the model under the assumption of a price partition assigned exogenously to each consumers. The choice of price partition is endogenised in section 4. Section 5 concludes.

2 Related literature

The present paper is closely related with the literature on bounded rationality. The paper takes as a departure point Rubinstein (1993), where he models the endogenous choice of a price partition by an agent characterized by a bound on his ability to process information on prices and examines the price chosen by a monopolistic firm which cannot discriminate price among consumers on the market of an indivisible good. The model is particularly interesting because of two characteristics: it constitues one of the few and earliest attempt to incorporate elements of bounded rationality into the field of industrial organization. Secondly, it provides a modelling tool for bounded rationality, according to which the agents are bounded on the ability to process information on the *endogenous* price set by the monopolist in equilibrium. While differences in information are modelled by differences in partitions of the relevant state space, here differences in the ability to process information are modelled as differences in the constraints on the family of partitions available to the consumers.

Dow (1991) characterizes the choice of a price partition in a two stage game, where the consumer visits two stores following a fixed order and without the possibility of stopping his search. The consumer cannot exactly remember the price quoted at the first store, but only remembers to which partition the quoted price belongs. The main difference with respect to Rubinstein (1993) relies on the fact that the decision problem of the agent is not embedded into an equilibrium analysis, but the price is exogenously drawn from a random probability distribution, exogenously given and identical for both firms.

Other papers attempt at integrating elements of bounded rationality into industrial organization framework. Spiegler (2003), (2004) study competition among rational firms over boundedly rational consumers, where the element of bounded rationality captures a cognitive limitation in they way agents process information in stochastic environments. A particular kind of limited rationality is taken into account: the consumers choose according to the $S(K)$ procedure modelled by Osborne and Rubinstein (1998), i.e. they sample every alternative K times and choose the best alternative in the sample. This procedure can be interpreted in the light of the "representativeness" heuristic, (Kahneman and Tversky, 1979), which captures people's attitude to use limited experience to form assessments, neglecting base rates and sample size. Spiegler (2003) considers a market for a treatment, where a continuum of patients recover with a given exogenous probability whether or not they acquire a quack's treatment. Spiegler shows that a market for quacks arises when agents make incorrect inference according to the $S(K)$ procedure and quacks end up charging positive prices and inflicting a welfare loss on consumers. Spiegler (2004) studies a market consisting of n profit-maximising firms and a continuum of agents, where each firm offers lotteries over divisions of the firm-agent interaction of a unit surplus. Analitically each firm chooses a cumulative distribution function over the unit interval. A consumer chooses the best lottery in his sample formed with an $S(1)$ procedure. The main result is that a higher number of competitors increases only the noisiness of the distribution without changing the expected value of the lottery to the consumers. This implies that standard intuition on competition fails to apply here.

Other papers have studied the interaction between rational and boundedly rational agents. Piccione and Rubinstein (2004) model differences among agents in their ability to recognise temporal patterns of prices: they use the concept of De Brunijin sequences in a dynamic model of markets to demonstrate the existence of equilibria in which prices fluctuate over time independently of the fundamentals and only competent agents can recognise the price patterns. Fershtam and Kalai (1993) study competitive behavior of firms which operate in multiple markets and whose ability to implement complex competition strategies in these market is bounded; this is the only model that incorporates elements of bounded rationality on firm's side.

Another literature strand to which the paper refers is the field of behavioral economics: Laibson and Gabaix (2004) study the effect of imperfect consumer knowledge on the pricing schedule offered by a firm on the market of a single good. They define consumer's knowledge to be imperfect as the situation where consumers are not able to anticipate "shrouded" product attributes or consumers have noisy evaluations of product attributes. This "ad hoc" assumption constitues a critical point since the deviation from standard competition results rely entirely on an exogenous element describing consumer's limitation to process information. Della Vigna and Malmendier (2004) study contract design in the face of consumers who have time-inconsistent preferences and are only partially aware of that and develop an application to health club industry.

Relevant is the literature on equilibrium in markets with search. Within the search literature, Salop (1977) tries at explaining price dispersion in a market where consumer know the prices available but do not know what stores charges what price. Each consumer can either choose to buy a unity good from a store randomly chosen or to incur a fixed search cost in order to get informed about which stores charge the lowest price. This literature is related because the search cost can be reinterpreted as the mental cost of processing information, instead of the physical cost of sampling. Under the assumption of a correlation between consumers' search costs and other consumers' characteristics, Salop shows that a monopolist finds optimal to charge different prices.

3 A toy model: the case of exogenous price partition

This section is devoted to the analysis of a simple model where we show that introducing an element of bounded rationality changes completely the result of standard models. Here we consider the effect of an element of bounded rationality displayed by consumers on the equilibrium configuration of Bertrand model of competition among rational firms. The literature on consumer information processing suggests that observed price information is encoded, evaluated, integrated into memory and finally recalled, and all these processes exhibit potential inaccuracies. Studies of Rosch and Mervis (1975) enlights the existence of a common heuristic: in environments with abundance of information, people exhibit the tendency of grouping events, objects or numbers into categories on the basis of perceived similarities. Following Simon (1982), we model explicitly a procedural aspects of decision making, consisting in the choice of the price partition and we introduce a specific element of bounded rationality, in terms of a limitation of the set of possible price partitions available to consumers. Here we start with a simplified framework where the price partition is exogenously assigned to each consumer and that will be extended in the next section.

We consider the market for a single good, consisting of two sellers and N consumers, each of whom is interested in consuming only one unity of the good. Two identical firms compete a la Bertrand, offering the same homogenous good and have the same marginal cost, c.

Here, we introduce the key element of the model: while firms are assumed to be perfectly rational, the consumers are boundedly rational, in the sense they are limited in their ability to process information given in a price offer made in the market. As in Rubinstein (1993), the bounded rationality of consumers is modelled in terms of a bound on the family of partitions of prices recognized by the agent: the agent can only recognize whether the price belongs to a certain interval, but does not remember the exact price of the good. We constrain the information structure to be a partition of the price set consisting of connected intervals. Agents are assumed to be homogenous, in the sense that they are characterized by the same price partition and the same number of sets in the partition (i.e. the number of cutting points +1). In this section we consider the simplest price partition, with only one cutting point, labelled with x. In other words, agents are characterized by the following decision rule: buy if and only if $p \leq x$, i.e. buy only if the price is lower or equal than an (exogenously given) price threshold x[1]. We are assuming that consumers have an exogenously fixed price partition, and in particular that they partition the price space only into two regions. This partition can be interpreted as the division of the price space into two regions corresponding to the high and low price regions, such that the consumer finds convenient to buy the good only if the price is regarded to be low, i.e. if the price is lower than the threshold x. Otherwise, the consumer is not willing to buy the commodity if she believes the price to be high, since it is above the threshold.

We characterise the equilibrium price chosen by firms who compete a la Bertrand.

Proposition 1 *The equilibrium is set at* $p_1 = p_2 = x \geq c$

Proof. See Appendix ■

In this very simple model, at equilibrium both firms set a price higher than the marginal cost and make positive profits at equilibrium. Here, standard intuition on competition fails to apply and the equilibrium price configuration deviates systematically from standard Bertrand result where firms have a symmetric cost structure. This departure standard result is motivated by the presence of a cognitive limitation on consumer's side: because agents cannot recognize prices below the exogenous cutting point x in the partition, firms do not have any incentive to undercut prices below x. The bounded perception of prices by the consumers introduces a countervailing incentive with respect to standard Bertrand model: on one side, firms display an incentive to undercut in order to gain market share; on the other side firms have an incentive to increase prices below the cutting point up to x, since this increase of the price does not have any effect on sales share, but will only have a positive effect on profit margin per unit sold. The interplay of

[1]Note that the only interesting case is when $x \geq c$.

these two opposed incentives drives the equilibrium result. Obviously, when $x = c$, we simply collapse to the standard Bertrand case.

Analogous result is obtained when we introduce an asymmetry in the cost structure on firm's side. Suppose without loss of generality that firm 1 has a constant marginal cost $c_1 < c_2$, under the assumption $c_1 < c_2 \leq x^2$. The following Proposition establishes the market equilibrium configuration.

Proposition 2 *The equilibrium is set at* $p_1 = p_2 = x \geq c_2 > c_1$.

Proof. See Appendix. ■

Both firms make positive profits at equilibrium, even though firm 1 is making more profits than firm 2. Again, and even more clearly, we move away from Bertrand classical result: at equilibrium both firms are active on the market, even the one with the largest marginal cost and both charge price strictly above marginal costs and equal to the cutting point x. Firms face the same trade-off between the incentive to undercut rival's price and to increase price up to the threshold. The equilibrium configuration in Proposition 2 is the result of the fact that the latter incentive outweighs the former one. The interplay of these two forces drives the result according to which the inefficient firm has positive market share at equilibrium. Results are robust to the introduction of heterogeneity on consumers' side[3].

4 Endogenizing the Choice of Price Partition

In this section we endogenise the choice of price partition made by consumers and we study the issue of whether firms can still take advantage of this element of bounded rationality despite competition forces.The crucial feature of the model relies on the fact that the agents are bounded on the ability to process information on endogenous equilibrium prices

The main idea of the paper is to model explicitly the tradeoff between the detection of a state of nature and the deviation of firm in a given state of nature. The consumer chooses the optimal price partition on equilibrium prices: when choosing the cutoff the consumer can try to detect the state of

[2] This is again the only non trivial case. In fact, under a different assumption on relevant economic parameters, $c_1 < x \leq c_2$, the firm with lowest marginal cost would have an incentive to undercut the rival, excluding her from the market and having positive profits charging a price equal to $x \leq p = c_2 - \varepsilon$

[3] See proof in Appendix 2 .

nature realized or to fix the cutoff in order to detect anticompetitive deviation of the firm.

The idea I have in mind can be explained through a very simple example. Every day I go to work and I do my shopping on the way to work or back home. I can reach work just following two different routes. A supermarket is located on each route. Therefore the consumer can visit just a single store on each route. I can go to the supermarket when there a lot persons decide to do their shopping or, at the contrary, during times of the day when fewer persons do shopping.

I can interpret the state of world "the supermarket is crowded" or " the supermarket is not crowded", and this what I label high state or low state of world.

We depart from Rubinstein (JPE, 1993) by considering two firms competing for the market of the good a la Bertrand. Firms choose their prices both simultaneously and non cooperatively. We consider a market for a single good, consisting of two sellers and N consumers, each of whom is interested in consuming only one unity of the good.

Two states of nature, labelled H and L, may occur. Consumers and firms have the same initial belief that the probabilities of state H and L are π_H and π_L. Once nature selects, only the firms get informed about which state of nature is realized. Firms have a symmetric cost structure: the costant marginal cost of producing in H state is c_H and in L state is c_L, set equal to zero with no loss of generality. Consumers' evaluation of one unity of the commodity is equal to v_H in H state and v_L in L state. It is assumed that $v_H > v_L$. Here, it is assumed that all consumers are able to determine only one cutting point: they can divide the price space in two connected sets and are able to attach the order "buy" or "not buy" to each of the two sets. Therefore the consumers can decide "always buy","never buy","buy if and only if $p \leq p^*$","buy iff $p \geq p^*$","buy iff $p < p^*$" and "buy iff $p > p^*$"

The timing of the game is as follows:

Stage 1: Each firm announces its pricing policy. Firms announce their pricing policy simultaneously.

Stage 2: Each consumer selects a partition (given the constraint on consumer's type) for each firm.

Stage 3: Nature selects.

Stage 4: Each consumer gets information about the cell in his partition which includes announced price and decide whether or not to purchase the good and decides to buy from the firm characterized by the lower index.

Each firm commits itself to the pricing policy announced in stage 1 of the game and the firm's offer is determined determined by the pricing policy to which the seller has committed itself, after uncertainty is solved in stage 3.

The following Proposition characterises the equilibrium configuration in pure strategy where each firm selects a price for each state of the world.

The timing of the game is as follows. In the first stage of the game, firms compete in prices and deliver binding pricing offers to a representative consumer.

The consumer processes the information contained in the pricing offers submitted by firms and chooses the price partition in order to minimize her expected cost to buy the good, given the cutoff rule.Nature chooses the state of the world and each firm is obliged to deliver the good at the announced price. Consumer buys the good according to the cutoff rule chosen in stage 2.

Assumptions on parameters: $v_H \geq c_H > v_L \geq c_L$.

Lemma 1 *Given a price configuration such that* $\lambda^* = \widetilde{\lambda}$ *and the consumer finds optimal to buy in both states of world, each firm has no incentive to undercut the rival firm in state* L *in such a way that the consumer will still find optimal to choose the same cutoff, i.e.* $\lambda^* = \widetilde{\lambda}$

Proof. *Trivial. The deviating firm lowers the profit margin per unit without increasing market share.* ■

Lemma 2 *Given a price configuration such that* $\lambda^* = c_H$ *and the consumer finds optimal to buy in both states of world, each firm has no incentive to undercut the rival firm in state* H *in such a way that* $p_{jL} < p\prime_{iH} < c_H$

Proof. Obvious. The deviating firm would sustain a loss for sure in state H, without increasing market share in state L. ■

The following Proposition characterizes the equilibrium of the model.

Proposition 3 *For parameters values* $v_H - c_H \geq v_L - c_L$ *and* $c_H < \pi_L v_L + \pi_H v_H$*, the unique Nash equilibrium of the game is a symmetric configuration where firms charge prices equal to* $p_{iH} = c_H$ *and* $p_{iL} = v_L$ *and the representative consumer chooses a cutoff equal to* c_H*. For parameters values* $v_H - c_H \geq v_L - c_L$ *and* $c_H \geq \pi_L v_L + \pi_H v_H$*, the unique Nash equilibrium of the game is a symmetric configuration where firms charge prices equal to* $p_{iH} = c_H$ *and* $p_{iL} = v_L + \dfrac{\pi_H}{\pi_L}(v_H - c_H)$ *and the representative consumer* *chooses a cutoff equal to c_H.For parameters values $v_H - c_H < v_L - c_L$, the unique Nash equilibrium of the game is a symmetric configuration where firms charge prices equal to $p_{iH} = c_H$ and $p_{iL} = c_L$ and the representative consumer chooses a cutoff equal to c_H.*

Proof. *See Appendix.* ■

When the profit margin per unit in high state is lower than the one in the low state, the Nash Equilibrium coincides with the one established in standard Bertrand model: each firm set a price equal to the marginal cost in each state of the world, i.e. $p_{iH} = c_H$ and $p_{iL} = v_L$ for each firm $i = 1, 2$. However, when the unit profit margin is higher in state H than in state L, the Nash equilibrium departs systematically from Bertrand result. In the case the cost in state H is low enogh, each firm will realize profits in the low state, by setting the price in the low state equal to the monopolistic one, i.e. $p_{iL} = v_L$. In the opposite case, each firm is able to realise half of the cartel profits: each firm can take advantage of consumers by setting a price equal to c_H in state H, but a price higher than v_L in state L, in such a way that the firm can get half profits she would get as she prices at consumer's evaluation in each state of nature[4].

These results are coherent with empirical evidence regarding profit margins on retail and wholesale prices for large supermarkets chains, as pointed out in Chevalier et all (2005): they find that prices fall on average during seasonal demand peaks for a product, largely due to changes in retail margins. Retail margins for a specific good are showed to fall during peak demand periods for that good, even if these periods do not coincide with aggregate demand peaks for the retailer.

[4]Note that the Nash equilibrium is unique for each parameter interval.

5 Conclusions

We have developed a model in order to study market interaction between rational firms on one side of the market and boundedly rational consumers on the other. We consider a special feature of bounded rationality: from psychological evidence, people tend to group events or numbers into categories, so here we consider consumers who partition the price space into connected sets and regard each price belonging to the same set as equal. In presence of this feature of bounded rationality, we expect that on one hand, the presence of bounded rationality makes consumers vulnerable to exploitation by rational firms; on the other hand, the competition among firms reduces the incentive to take advantage of consumers in order to beat the competitors and gain market share. This paper shows that we depart systematically from standard Bertrand result, i.e. even in presence of a symmetric cost structure profits do not collapse to zero and prices charged by firms are well above the marginal cost

References

[1] Chen, Y, G. Iyer and A. Pazgal "Limited Memory and Market Competition",(2004) mimeo

[2] Chevalier, J. A., P. Rossi and A. K. Kashyap (2005), "Why don't Prices Rise During Periods of Peak Demand?" Evidence from Scanner Data", Working paper 12, Yale School of Management.

[3] Della Vigna S. and U. Malmendier (2004), "Contract Design and Self-Control: Theory and Evidence", Quarterly Journal of Economics, forthcoming

[4] Dow, J. (1991), Review of Economic Studies

[5] Gabaix X. and D. Laibson (2003), "Some Industrial Organization with Boundedly Rational Consumers", mimeo

[6] Fershtman C. and E. Kalai (1993), "Complexity Considerations and Market behavior" RAND Journal of Economics 24, 224-235

[7] Osborne and Rubinstein (1998), "Games with Procedurally Rational Players", American Economic Review 88, 834-849

[8] Piccione, M. and A. Rubinstein (2003), "Modeling the Economic Interaction of Agents with Diverse Abilities to Recognize Equilibrium Patterns", Journal of European Economic Association 1, 212-223

[9] Rubinstein A. (1993), "On Price Recognition and Computational Complexity in a Monopolistic Model", Journal of Political Economy 101, 473-484

[10] Salop (1977)

[11] Spiegler (2003), "The Market for Quacks", mimeo

[12] Spiegler (2004) "Competition over Non-Bayesian Agents", mimeo

6 Appendix

6.1 Appendix 1: Detailed proofs

Proof. of Proposition 1 Consider the following two cases: $p_1 > p_2 > x > c$ or $p_1 = p_2 > x > c$.

No firm has a positive demand and its profit is zero. On the other hand, if firm 1 charges $p_1 = x - \varepsilon$, it obtains the entire market, with profits equal to $N(x - \varepsilon - c)$ and has a positive profit margin of $x - \varepsilon - c$. Therefore, firm 1 does not charge the optimal price.

Consider $p_1 > x > p_2 > c$. Firm 1 has zero market share and zero profit. By reducing its price slightly to $x - \varepsilon$, firm 1 gains half of the market and earns positive profits $\frac{1}{2}N(x - \varepsilon - c)$. Firm 1 does not need to reduce its price below p_2, given that consumers cannot recognize the difference between x and any price inferior to x belonging to the partition associated with the order to buy.

Consider $x > p_1 = p_2 = c$. Firm 1 can increase its profit by charging the price $c + \varepsilon$, since it earns positive profits, without losing market share.

The only equilibrium configuration is represented by $p_1 = p_2 = x > c$, associated with positive profits of both firms. ■

Proof. of Proposition 2 As above. Note only that firm 1 will never find optimal to cut price below c_2, since it would not increase market share, given the price partition of the consumers, but the decrease of the price has only the effect to lower its profit margin, without increasing market share. ■

Proof. of Proposition 3 [5] The objective function of the consumer is

$$\max_{\{\lambda,\gamma_s\}} \sum_{s=H,L} \pi_s (v_s - p_{is}) \gamma_s$$

where π_s is the exogenous probability that state of nature s occurs, v_s is consumer's evaluation of the good in state of nature s and γ_s[6] is an indicator variable defined as:

$$\gamma_s = \begin{cases} 0 \text{ if } NOT\ BUY \text{ in state } s \\ 1 \qquad\quad \text{if } BUY \text{ in state } s \end{cases}$$

[5] In the proof we make use of two simplifying observations when we calculate equilibrium cutoff: $p_{iL} \leq p_{iH}$ for every i and the equilibrium price vector is symmetric, i.e. firms charge the same price in the same state of world.

[6] Note that γ_s and λ depends on optimal price vector at equilibrium, i.e.
$\lambda = f(p_{1L}, p_{1H}, p_{2L}, p_{2H})$
$\gamma_s = f(p_{1L}, p_{1H}, p_{2L}, p_{2H}; \lambda)$

With no loss of generality, assume $\pi_H \geq \pi_L$ ■

Case 1

$$p_{is} \leq v_s \ \forall s, i$$

The optimal cutoff is

$$\lambda = \begin{cases} \max\limits_{s=H,L} \left[\min\limits_i p_{is}\right] & if \ \pi_H \left(v_H - \underline{p}_H\right) \geq \frac{\pi_L}{2}\Delta_L \\ \min\limits_i p_{iL} & otherwise \end{cases}$$

$$\gamma_s = \begin{cases} 1, \forall s \ if \ \pi_H \left(v_H - \underline{p}_H\right) \geq \frac{\pi_L}{2}\Delta_L \\ 1 \ for \ s = L, 0 \ for \ s = H \quad otherwise \end{cases}$$

where $\Delta_L = \overline{p}_L - \underline{p}_L$ *and* $\overline{p}_s = \max\limits_i p_{is}, \underline{p}_s = \min\limits_i p_{is}$,*for* $s = L, H$. *The consumer chooses the cutoff* λ *that maximises consumer's expected utility. Given the cutoff rule the consumer can either choose to buy the good in each state of the world or only in state L; note that the consumer cannot decide to buy only in state* H *due to cutoff rule*[7]. *Under the first cutoff value, the consumer rules out the highest price in the price set and she receives:*

$$EU = \frac{\pi_L}{2}\left(v_L - \underline{p}_L\right) + \frac{\pi_L}{2}\left(v_L - \overline{p}_L\right) + \pi_H \left(v_H - \underline{p}_H\right)$$

while under the second cutoff configuration the consumer rules out to buy in high state and the best she can do is to buy at cheapest price in low state, with an expected utility of:

$$EU = \pi_L \left(v_L - \underline{p}_L\right)$$

The consumer will choose the cutoff that maximises her expected utility.

Proposition 4 ***Proof.***

[7] Note that the consumer cannot exclude with certainty to buy from firm charging price p_{is} associated with minimal net utility $\min\limits_{i,s}(v_s - p_{is})$ Given the cutoff rule, she can only exclude to buy the good in state of nature H if $p_{iH} > p_{iL} \ \forall i$, given the assumption $c_H > c_L$.

Case 2

$$p_{iH} > v_H, p_{jH} \leq v_H \ for \ i \neq j, p_{iL} \leq v_L \ \forall i$$

The optimal cutoff is

$$\lambda = \begin{cases} \max\limits_{s=H,L} \left[\min\limits_i p_{is} \right] \ if \ \pi_H \left(v_H - \underline{p}_H \right) \geq \frac{\pi_L}{2} \Delta_L \\ \min\limits_i p_{iL} \qquad otherwise \end{cases}$$

$$\gamma_s = \begin{cases} 1, \forall s \ if \ \pi_H \left(v_H - \underline{p}_H \right) \geq \frac{\pi_L}{2} \Delta_L \\ 1 \ for \ s = L, 0 \ for \ s = H \qquad otherwise \end{cases}$$

As before.

Case 3

$$p_{iH} \leq v_H \ \forall i, p_{iL} > v_L, p_{jL} \leq v_L \ for \ i \neq j$$

The optimal cutoff is

$$\lambda = \begin{cases} \max\limits_{s=H,L} \left[\min\limits_i p_{is} \right], \ if \ \pi_H \left(v_H - \underline{p}_H \right) \geq \frac{\pi_L}{2} \Delta_L \\ \min\limits_i \left(p_{iL} \right) \ otherwise \end{cases}$$

with

$$\gamma_s = \begin{cases} 1, \forall s \ if \ \pi_H \left(v_H - \underline{p}_H \right) \geq \frac{\pi_L}{2} \Delta_L \\ 1 \ for \ s = L, 0 \ for \ s = H \ \ otherwise \end{cases}$$

and expected utility equal to

$$EU = \frac{\pi_L}{2} \left(v_L - \underline{p}_L \right) + \frac{\pi_L}{2} \left(v_L - \overline{p}_L \right) + \pi_H \left(v_H - \underline{p}_H \right)$$

in the first case and

$$EU = \pi_L \left(v_L - \underline{p}_L \right)$$

in the second case.

Case 4

$$p_{iH} > v_H \ \forall i, p_{iL} \leq v_L \ \forall i$$

The optimal cutoff is

$$\lambda = \min_i \left(p_{iL} \right)$$

$$\gamma_s = \begin{cases} 1, s = L \\ 0, s = H \end{cases}$$

The consumer finds profitable to buy only in low state since she can rule out the state of world with negative net utility in state H and attaining the highest expected utility, given the price vector, equal to:

$$EU = \pi_L \left(v_L - \underline{p}_L\right)$$

Case 5

$$p_{iH} > v_H, p_{jH} \leq v_H \ for \ i \neq j, p_{iL} > v_L, p_{jL} \leq v_L \ for \ i \neq j$$

The optimal cutoff is

$$\lambda = \begin{cases} \max_s \left[\min_i p_{is}\right], \ if \ \pi_H \left(v_H - \underline{p}_H\right) \geq \frac{\pi_L}{2} \Delta_L \\ \min_i \left(p_{iL}\right) \ otherwise \end{cases}$$

with

$$\gamma_s = \begin{cases} 1, \forall s \ if \ \pi_H \left(v_H - \underline{p}_H\right) \geq \frac{\pi_L}{2} \Delta_L \\ 1 \ for \ s = L, 0 \ for \ s = H \ \ otherwise \end{cases}$$

and expected utility equal to

$$EU = \frac{\pi_L}{2} \left(v_L - \underline{p}_L\right) + \frac{\pi_L}{2} \left(v_L - \overline{p}_L\right) + \pi_H \left(v_H - \underline{p}_H\right)$$

in the first case and

$$EU = \pi_L \left(v_L - \underline{p}_L\right)$$

in the second case. The condition follows straight from the comparison of the two expected utility; the consumer chooses the cutoff that maximises her expected utility.

Case 6

$$p_{iL} > v_L \ \forall i, p_{iH} \leq v_H \ \forall i$$

The optimal cutoff rule is:

$$\lambda = \begin{cases} \max_s \left[\min_i p_{is}\right], \ if \ \frac{\pi_L}{2} \left[\left(v_L - \underline{p}_L\right) + \left(v_L - \overline{p}_L\right)\right] + \pi_H \left(v_H - \underline{p}_H\right) \geq 0 \\ \max_{i,s} \left(p_{is}\right) + \varepsilon \qquad otherwise \end{cases}$$

$$\gamma_s = \begin{cases} 1\ \forall s\ if\ \frac{\pi_L}{2}\left[\left(v_L - \underline{p}_L\right) + \left(v_L - \overline{p}_L\right)\right] + \pi_H\left(v_H - \underline{p}_H\right) \geq 0 \\ 0\ \forall s \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

in the first case and $\gamma_s =$*in the second case.*

Case 7

$$p_{iL} > v_L, p_{jL} \leq v_L\ for\ i \neq j, p_{iH} > v_H\ \forall i$$

The optimal cutoff is:

$$\lambda = \min_i p_{iL}$$

$$\gamma_L = 1, \gamma_H = 0$$

with expected utility equal to

$$EU = \pi_L\left(v_L - \underline{p}_L\right)$$

Case 8

$$p_{iL} > v_L\ \forall i, p_{iH} > v_H, p_{jH} \leq v_H\ for\ i \neq j$$

The optimal cutoff is

$$\lambda = \begin{cases} \max_{s=H,L}\left[\min_i p_{is}\right], if\ \frac{\pi_L}{2}\sum_i\left(v_L - p_{iL}\right) + \pi_H\left(v_H - \underline{p}_H\right) \geq 0 \\ \max_{i,s}\left(p_{is}\right) + \epsilon,\ otherwise \end{cases}$$

with

$$\gamma_s = \begin{cases} 1, \forall s\ if\ \frac{\pi_L}{2}\sum_i\left(v_L - p_{iL}\right) + \pi_H\left(v_H - \underline{p}_H\right) \geq 0 \\ 0\ \forall s \qquad\qquad\qquad\qquad otherwise \end{cases}$$

expected utility equal to

$$EU = \frac{\pi_L}{2}\left(v_L - \underline{p}_L\right) + \frac{\pi_L}{2}\left(v_L - \overline{p}_L\right) + \pi_H\left(v_H - \underline{p}_H\right)$$

in the first case and

$$EU = \pi_L\left(v_L - \underline{p}_L\right)$$

in the second case. The condition follows immediately from the comparison of the two expected utility.

Case 9

$$p_{iL} > v_L \ \forall i, p_{iH} > v_H \ \forall i$$

The optimal cutoff is:

$$\lambda = \max_{i,s} (p_{is}) + \varepsilon$$

$$\gamma_s = 0 \ \forall s$$

with expected utility equal to $EU = 0$. *The consumer is better off by choosing the outside option not to buy the good, since she gets a zero expected utility instead of a negative one.*

The second part of the proof characterizes the optimal behaviour of the firm. In stage 1 firms compete in prices and set the price of the good in both states of nature in order to maximise total expected profits. We analyse the optimal behavior of the firm under each case.
Under Case 1, the monopolistic price configuration according to which the monopolistic price, equal to consumer's evaluation of the good, v_s *for* $s = L, H$ *is charged in both states of the world, cannot be an equilibrium. Under the monopolistic price configuration, firm' s expected profits is equal to* $E(\Pi_i) = \frac{1}{2} \sum_{s=L,H} \pi_s (v_s - c_s)$. *Each firm has a unilateral incentive to deviate in state* H: *each firm gains all the market in state* H *and increases profits above (half) the monopolist level by slightly undercutting the rival and setting a price* $p\prime_{iH} = v_H - \varepsilon$ *with* ε *small enough in state* H. *Firms will find optimal to undercut in state* H, *triggering out a price war in the high state of world. This means that* v_H *cannot be an equilibrium price. According to Lemma 1, given a price configuration, each firm has no incentive to deviate only in state L and slightly undercut the rival in state L, since it would not induce a change in the optimal cutoff chosen by consumer; therefore he would simply lose profit margin per unit without gaining market share.*
A candidate Nash equilibrium is the price configuration

$$\{p_{is}^*\}_{s=L,H \ i=1,2} = [p_{iL}^* = v_L; p_{iH}^* = c_H]$$

corresponding to firms' profits equal to

$$E\pi_i^* = \frac{\pi_L}{2} (v_L - c_L) \ \ i = 1, 2$$

and consumer's cutoff

$$\lambda^* = c_H$$

Note that according to Lemma 1, firms do not have an incentive to slightly undercut price in state L since they would only lose profit margin per unit, without gaining market share and they cannot increase price in state H, since they do not gain any market share.
We need to rule out three possible deviations for this price configuration to be an equilibrium. First, each firm can undercut p_{iL} so low that consumer finds profitable to deviate from cutoff and buy only in state L. Suppose firm i deviates and charges $\underline{p}\prime_{iL}$. Then, consumer chooses cutoff to maximise expected utility: she will set the cutoff at $\lambda\prime = \underline{p}\prime_{iL}$ if $EU\prime = \pi_L \left(v_L - \underline{p}\prime_{iL}\right) > EU^* = \frac{\pi_L}{2}\left(v_L - \underline{p}\prime_{iL}\right) + \pi_H \left(v_H - c_H\right)$. The consumer chooses λ to maximise her expected utility. To rule out this deviation, it is sufficient to assume

$$v_H - c_H \geq v_L - c_L$$

The second deviation available to each firm goes in the opposite direction: each firm can deviate unilaterally in state L and increase price above v_L without inducing a change in λ^*. Each firm has an incentive to deviate and charge a price up to $p\prime_L = v_L + \frac{\pi_H}{\pi_L}\left(v_H - c_H\right)$. This configuration cannot be for sure an equilibrium if $p\prime_L \geq c_H$, since the consumer can detect the deviation and rule out to buy from the deviating firm in state L. This happens when

$$c_H \geq \pi_L v_L + \pi_H v_H$$

By Lemma 2 we know that the firm has no incentive to undercut p_H in state H below c_H, unless he deviates in both states simultaneously setting the same price in state L and H. Suppose the firm i deviates and sets $p\prime_{iH} = p\prime_{iL} = v_L - \varepsilon$, with expected profits equal to $E\pi\prime_i = v_L - \varepsilon - (\pi_L c_L + \pi_H c_H)$. To rule out this deviation, $E\pi\prime_i < E\pi_i^*$, which requires in the limiting condition that

$$c_H > v_L + \frac{\pi_L}{2\pi_H}\left(v_L - c_L\right)$$

which is always satisfied for $(v_L - c_L) \to 0$ or c_H very high.
In case

$$v_H - c_H \geq v_L - c_L$$

$$c_H \leq \pi_L v_L + \pi_H v_H$$

the equilibrium price configuration is

$$\{p_{is}^*\}_{s=L,H;\ i=1,2} = \left[p_{iL}^* = v_L + \frac{\pi_H}{\pi_L}(v_H - c_H)\,; p_{iH}^* = c_H\right]$$

with expected profits

$$E\pi_i^* = \frac{\pi_H}{2}(v_H - c_H) + \frac{\pi_L}{2}(v_L - c_L)\,,\ i = 1,2$$

and consumer's cutoff

$$\lambda^* = c_H$$

If

$$c_H \leq v_L + \frac{\pi_L}{2\pi_H}(v_L - c_L)$$

the equilibrium price configuration is

$$\{p_{is}^*\}_{s=L,H;\ i=1,2} = [p_{iL}^* = c_L; p_{iH}^* = c_H]$$

with expected profits

$$E\pi_i^* = 0\ i = 1,2$$

and consumer's cutoff

$$\lambda^* = c_H$$

Under Case 2, firm i can profitably increase profits by setting p_{iH} equal to rival's level. So we are back to case 1.

Under Case 3, suppose the price vector is such that consumer chooses as a cutoff $\lambda = \min_i (p_{iL})$. The firm charging the highest p_{iL} does not sell in any state of world and can increase profits by undercutting the rival in state L or by decreasing p_{iH} so that the cutoff chosen by consumers is $\lambda = \max_s \left[\min_i (p_{iL})\right]$, in which case the consumer buys in both states of nature. As in case 1. On the other side, firm charging lowest price in L state can increase the price up to rival's price without losing market share, but this would collapse either to case 6

Under Case 4 each firm can profitably deviate by setting a price in state H equal to v_H and serve all the market in high state of nature (back to case 2

Under Case 5 the firm charging the highest price in state H can profitably increase profits by undercutting the rival in state H (back to case 3)

Under Case 6 suppose the price vector is such that the consumer never finds optimal to buy. Each firm has a unilateral incentive to deviate to reduce price either in L or in H state of nature. Suppose the firm deviates in state L, we collapse to cases 3 or 1.

Under Case 7: as in case 5.

Under Case 8 suppose the price vector is such that the consumer never finds optimal to buy. Each firm has a unilateral incentive to deviate to reduce price either in L or in H state of nature. Suppose the price vector is such that consumer finds optimal to buy in both states of nature. The firm with highest price in state H finds profitable to deviate and undercut the rival in state H, gaining all the market (and we collapse back to case 6).

Under Case 9, each firm is charging a price above consumer's evaluation and has a zero market share. Each firm can profitably deviate by setting a price just equal to the monopolistic price and gain all the market. We collapse in any of the previous cases.

■

6.2 Appendix 2: The case of heterogenous consumers

We examine the effect of introducing heterogeneity on consumer's side within the simple setting presented in the above section. Consumers are assumed to be characterized by a different (exogenous) partition of the price space.

Assume type i consumer acts according to the following decision rule: buy if and only if $p \leq x_i$, i.e. buy only if the price is lower or equal than an (exogenously given) price threshold x_i. We assume there are only two types of consumers, and one consumer for each type and without loss of generality that $x_2 < x_1$.

We consider firms characterized by a symmetric cost structure: let us assume the relevant economic parameters satisfy the following relation: $c \leq x_2 < x_1$. The next Proposition characterises the equilibrium configuration.

Proposition 5 *There is a unique equilibrium, given by:* $p_1 = p_2 = x_2$ *if* $x_1 - x_2 < (x_2 - c)$, *or* $p_1 = p_2 = x_1$ *if* $x_1 - x_2 > 2(x_2 - c)$; *there are two equilibrium at* $p_1 = x_1$ *and* $p_2 = x_2$ *or at* $p_1 = x_2$ *and* $p_2 = x_1$ *if* $(x_2 - c) < x_1 - x_2 < 2(x_2 - c)$

Proof. Consider the following case: $p_1 = p_{:2} = x_1$. Both firms sell one unit of the good to type 1 consumer with equal probability, but nothing to type 2 consumer, achieving a profit level equal to $\pi_i = \frac{1}{2}(x_1 - c)$ for $i = 1, 2$. Suppose firm 1 deviates by charging a price $p_1 = x_2$: by lowering the price from x_1 to x_2, firm 1 gains in terms of sale volume, since it sells one unit for sure to type 2 consumer, while it lowers the margin on the unit sold to type 1 consumer, which buys with equal probability from either firm 1 or 2. Note that by lowering the price charged on the market, firm 1 does not steal the market of type 1 consumer to firm 2, since type 1 cannot distinguish among prices inferior or equal to x_1. Therefore, firm 1 has an unilateral incentive deviate if $\pi'_1 = \frac{1}{2}(x_2 - c) + (x_2 - c) > \frac{1}{2}(x_1 - c)$, i.e. when $x_2 - c > \frac{1}{2}(x_1 - x_2)$. This condition requires that the profit margin on one unit sold to type 2 consumer exceeds the loss (reduction of the margin) in expected terms of selling to type 1 consumer. Note that firms have no incentive to cut price below x_2, since this would lead only to a reduction in the margin per unit without increasing firm's market share. Moreover, the condition stated hold for both firms, given the symmetric cost structure.

Consider the opposite case: $p_1 = p_2 = x_2$: both firms sell one unit to both type 1 and 2 consumers with equal probability, receiveng an expected profit equal to $\pi_i = x_2 - c$, for $i = 1, 2$. The only possible deviation is to raise the price to x_1. Suppose firm 1 deviates and charges x_1: firm 1 will sell only to type 1 consumer, since type 2 consumer can detect that firm 2 charges a lower price. This happens with probability $\frac{1}{2}$, because type 1 consumers cannot recognize prices below to the cutoff point. Then firm 1's profit after deviation becomes $\pi_1' = \frac{1}{2}(x_1 - c)$. Therefore, this deviation is optimal under the condition $\frac{1}{2}(x_1 - c) \geq x_2 - c$. This means that by increasing the profit margin per unit sold, the deviating firm gets a gain higher than the loss due the reduction in sales volume because he does not sell to type 2 consumers, who can detect the firm charging the lowest price.

For intermediate values of the cutting point differential $(x_1 - x_2)$, $(x_2 - c) < x_1 - x_2 < 2(x_2 - c)$, two equilibria arise where firms coordinate to play the opposite strategies in terms of prices. In this case, suppose firm 1 charges x_1 and firm 2 x_2. Firm 1 sells only to type 1 consumers with $\frac{1}{2}$ probability, while firm 2 sells one unit of the good for sure to type 2 consumers and to type 1 with probability $\frac{1}{2}$. Profits yielded by this price configuration are $\pi_1 = \frac{1}{2}(x_1 - c)$ and $\pi_2 = \frac{3}{2}(x_2 - c)$. There are two possible deviations: firm 1 can reduce the price charged to x_2, getting profits equal to $\pi_1' = (x_2 - c)$ or firm 2 can increase the price to x_1, getting profits equal to $\pi_2' = \frac{1}{2}(x_1 - c)$. The condition stated above is determined by imposing standard condition to determine the non-optimality of unilateral deviation. ■

Contrary to standard Bertrand equilibrium configuration, note that when $c \leq x_2 < x_1$, it is not obvious that the equilibrium will occur at the lowest cutting point $p_1 = p_2 = x_2 > c$, because of the taughness of price competition among firms. The above proposition shows that the equilibrium configuration prevailing on the market depends on the marginal cost c and the cutting point differential $\Delta x = x_1 - x_2$.[8]. This result is driven by the the interplay of two opposite incentives faced by each firm: on one hand, each firm wants to undercut the rival in order to increase its market share, on the other hand each firm has the incentive to charge the highest price within the price partition, since each consumer cannot recognize prices below the cutting point of the price partition.

The result is even more striking when we introduce heterogeneity in firms' cost structure. Within the simple setting considered here, heterogeneity on firms' side can be introduced, by assuming firms differ in their cost structure.

[8] Under the assumption there are N_1 type 1 consumers and N_2 type 2 consumers, $p_1 = p_2 = x_2$ is the equilibrium under the condition $N_2(x_2 - c) > \frac{1}{2}N_1(x_1 - x_2)$, ie. total net profit on selling to type 2 consumer is greater than the loss (in expected terms) of reducing the margin to type 1 consumers.

With no loss of generality, we assume $c_2 < c_1$. Two interesting cases may arises, depending on the value of the cost parameters. Propositions 4 and 5 determine the equilibrium price configurations for different parameter values.

Proposition 6 *When $c_2 \leq x_2 < c_1 \leq x_1$, there is a unique equilibrium at $p_1 = p_2 = x_1$ if $x_2 - c_2 < \frac{1}{2}(x_1 - x_2)$ or at $p_1 = x_1$ and $p_2 = x_2$ otherwise.*

Proof. Since $x_2 < c_1 \leq x_1$, firm 1 finds always optimal to fix $p_1 = x_1$, getting $\pi_1 = \frac{1}{2}(x_1 - c_1)$[9]. Firm 2 can either decide to fix $p_2 = x_1$, selling only to type 1 consumers with a profit level of $\pi_2 = \frac{1}{2}(x_1 - c_1)$, since both firms have equal probability to sell to type 1 consumers or set $p_2 = x_2$, getting $\pi_2 = (x_2 - c_2) + \frac{1}{2}(x_2 - c_2)$. Two equilibrium configuration may arise at $p_1 = p_2 = x_1$ if $x_2 - c_2 < \frac{1}{2}(x_1 - x_2)$ and at $p_1 = x_1$ and $p_2 = x_2$ otherwise ■

Firm 2 has a clear cost advantage with respect to firm 1, since it is the only firm who finds profitable to sell to both consumer types, since $c_2 \leq x_2 < x_1$. Despite the presence of this cost advantage, firm 2 cannot exclude firm 1 from the market and moreover, it is not always clear that firm 2 will find convenient to charge the lower price, at least to gain all the market of type 2 consumers. The predictions are even less sharp in case of a lower cost advantage in favour of firm 2. Multiple equilibria arise according to different parameters assumptions.

Proposition 7 *When $c_2 \leq c_1 \leq x_2 < x_1$, multiple equilibria arise.*

Proof. When $c_2 \leq c_1 \leq x_2 < x_1$, firms 1 and 2 compete in prices and they can find convenient to set either p_i equal to x_1 or x_2, since both firms have marginal cost lower than x_i for $i = 1, 2$. The payoffs associated with this game are represented by the following matrix:

$$\begin{array}{ccc} & x_1 & x_2 \\ x_1 & \frac{1}{2}(x_1 - c_1); \frac{1}{2}(x_1 - c_1) & \frac{1}{2}(x_1 - c_1); (x_2 - c_2) + \frac{1}{2}(x_2 - c_2) \\ x_2 & x_2 - c_1 + \frac{1}{2}(x_2 - c_1); \frac{1}{2}(x_1 - c_2) & (x_2 - c_2); (x_2 - c_2) \end{array}$$

Different equilibria may arise depending on the value of the parameters describing consumers' partition of price space and firms'cost structure. The following matrix gives the value of the parameter for which the 4 possible outcomes may arise as Nash equilibria of the game that we have considered above

[9] Since firm 1 has only $\frac{1}{2}$ probability of getting the market of type 1 consumers under $p_2 \leq x_1$

	x_1	x_2
x_1	$x_1 > \overline{x}^2$	$\underline{x}^1 < x_1 < \overline{x}^2$
x_2	$\underline{x}^2 < x_1 < \overline{x}^1$	$x_1 < \underline{x}^1$

The payoff matrix associated with game with heterogenous consumers (with fixed exogenous partition) and heterogeneous firms is the following:

	x_1	x_2
x_1	$\frac{1}{2}(x_1 - c_1); \frac{1}{2}(x_1 - c_1)$	$\frac{1}{2}(x_1 - c_1); (x_2 - c_2) + \frac{1}{2}(x_2 - c_2)$
x_2	$x_2 - c_1 + \frac{1}{2}(x_2 - c_1); \frac{1}{2}(x_1 - c_2)$	$(x_2 - c_2); (x_2 - c_2)$

Different equilibria may arise depending on the value of the parameters describing consumers' partition of price space and firms'cost structure.

	x_1	x_2
x_1	$x_1 > \overline{x}^2$	$\underline{x}^1 < x_1 < \overline{x}^2$
x_2	$\underline{x}^2 < x_1 < \overline{x}^1$	$x_1 < \underline{x}^1$

$p_1 = p_2 = x_1$. For firm 1, $\frac{1}{2}(x_1 - c_1) > x_2 - c_1 + \frac{1}{2}(x_2 - c_1) \Rightarrow x_1 > 3x_2 - 2c_1 \equiv \overline{x}^1$. For firm 2, $\frac{1}{2}(x_1 - c_2) > x_2 - c_2 + \frac{1}{2}(x_2 - c_2) \Rightarrow x_1 > 3x_2 - 2c_2 \equiv \overline{x}^2$.

$p_1 = p_2 = x_2$. For firm1, $(x_2 - c_1) > \frac{1}{2}(x_1 - c_1) \Rightarrow x_1 < 2x_2 - c_1 \equiv \underline{x}^1$ For firm 2, $(x_2 - c_2) > \frac{1}{2}(x_1 - c_2) \Rightarrow x_1 < 2x_2 - c_2 \equiv \underline{x}^2$.

$p_1 = x_1; p_2 = x_2$. Given $p_2 = x_2$, for firm 1 $(x_2 - c_1) < \frac{1}{2}(x_1 - c_1) \Rightarrow x_1 > \underline{x}^1$. Given $p_1 = x_1$, for firm 2, $\frac{1}{2}(x_1 - c_2) < x_2 - c_2 + \frac{1}{2}(x_2 - c_2) \Rightarrow x_1 < \overline{x}^2$

$p_1 = x_2; p_2 = x_1$. Given $p_2 = x_1$, for firm 1 $\frac{1}{2}(x_1 - c_1) < x_2 - c_1 + \frac{1}{2}(x_2 - c_1) \Rightarrow x_1 < \overline{x}^1$. Given $p_1 = x_2$, for firm 2, $\frac{1}{2}(x_1 - c_2) > x_2 - c_2 \Rightarrow x_1 < \underline{x}^2$

The ordering of the cutoff points is the following: $\overline{x}^2 > \overline{x}^1$. Moreover $\overline{x}^2 > \underline{x}^2$ and $\overline{x}^1 > \underline{x}^1$.. Therefore, $\overline{x}^2 > \overline{x}^1 > \underline{x}^1$. Need to distinguish two cases: when $\underline{x}^2 > \overline{x}^1$, in which case there arise multiple equilibria (both $p_1 = x_1, p_2 = x_2$ or $p_1 = x_2, p_2 = x_1$ may arise) or when $\underline{x}^2 < \overline{x}^1$, in which case $p_1 = x_2, p_2 = x_1$ is ruled out as an equilibrium. ∎