Private and Public Consumption and Counter-Ciclical Fiscal Policy

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Abstract

In this paper we set up a NK-DSGE model in which the consumer derives utility from a consumption basket made of public and private consumption, there is monopolistic competiton in the intermediate good sector, Calvo-Yun price setting, endogenous labour supply and Taylor rule for monetary policy. We analyse the effects of a countercilical fiscal policy rule and how varying the degree of counterciclicality affects the determinacy of the rational expectations equilbrium and the ability to respond to technological and government spending shocks. A broad conclusion is that desiderability of pro/counter ciclical stance of fiscal policy cannot accurately be drawn without taking into account the quality of the public expenditure and the way it interacts with private consumption.

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1 Introduction

The debate on fiscal policy rules and their interactions with monetary policy is one of the most recent and vibrant field of research in modern macroeconomics. On the theoretical side, the debate was triggered by the seminal work of Leeper (1991) and continued by a huge and growing literature that had its milestones in Woodford (1996,1998,2001), Benigno and Woodford (2002) and many others. In the real world the most visible and important application of this growing literature is the experience of the European Monetary Union (EMU) and the Stability and Growth Pact (SGP), which was designed to maintain fiscal discipline in the EMU and whose implementation and reform are among the most pressing political and economic issues in the institutional building of the European Union.

Generally speaking, fiscal discipline is a pre-condition for low interest rate, to achieve permanent reduction of distorsionary taxation and, as the literature on Fiscal Theory of the Price Level has recently emphasized[1] , for price stability. In a monetary union with decentralized fiscal framework[2] , it's necessary to prevent free-riding problems and to allow the conduct of a common monetary policy .Since its introduction, and with an intensity directly proportional to the difficulties to comply with its prescriptions, the SGP has been charged with many allegations. While many of them seems highly questionable (such as the proposals of the golden rule or exclusions of some kind of public expenditure from the official computations), and some ambiguous (such as the medium term target of "close to balance"), some others definitely need a closer look and more economic-based considerations.

More specifically, one of the main aims of the SGP was to provide countries with the necessary and sufficient incentive to implement countercyclical fiscal policies, possibly with a much-needed degree of automatism and enforceability, given time-inconsistency problems (Chari and Kehoe,2003) and political economy-related considerations (Alesina and Drazen,1991, Alesina and Perotti,1995, Alesina, Perotti, Tavares 1998). Economic theory has in fact achieved a widespread consensus on the beneficial effects of countercyclical fiscal policy, insofar as it provides stabilization over the business

[1] Sims(1994), Cochrane (1999), Woodford (1995)

[2] An important, and often neglected, issue is that the need for fiscal policy rules arises directly from the highly decentralized setting of EMU as far as fiscal issues are concerned and the ultimate intergovernmental nature of the EU as a whole, aspects that give rise to potential moral hazard problems. If the European integration goes further towards a truly federal system, all this discussion on SGP would obviously be different and even no longer useful.

cycle, enables the economy to effectively fight recessions, and it is consistent with optimal tax-smoothing (Barro, 1979). According to many observers, one of the main problems regards the insufficient effort the Pact produces in order to avoid asymmetry and procyclical bias in the conduct of national fiscal policies: it does not ensure fiscal consolidation during good times and induces pro-cyclical adjustments during an economic downturn or, at least, it does not seem to do the job in a symmetric way. Accusation of tendence to procylicality is made by Coricelli and Ercolani (2002) and confirmed by Orbàn and Szapàrt (2004), and also by Buti, Eijffinger,Franco (2003), Balassone, Monacelli (2000),at least for the lack of automatism during "good times".

So the literature and the policy-making has achieved a widespread consensus on the importance of counter-ciclicality in fiscal policy practice; this paper is concerned with the insertion of this feature into the standard New Keynesian General Equilibrium framework and the analysis of how it changes the stability of rational expectations equilibrium, its sensibility to variations in policy rule and structural parameters, and the ability to respond to stochastic shocks.

The theoretical methodology we used is the adoption of a counterciclical dynamics for government expenditure into a fairly standard New Keynesian general equilibrium model, with monopolistic competition, Calvo-Yun pricing, endogenous labour supply, Taylor rule for monetary policy and the insertion of useful government expenditure. The latter is one the three main approaches that are being used in order to break the Ricardian equivalence (which would deliver virtually no effect of fiscal policy on real variables), so to replicate with general equilibrium models the main empirical regularities concerning government variables; the other two approaches are finite horizon (or overlapping generations) models, and rule of thumb-agents (or credit constrained) models. This paper chooses to concentrate on the useful government expenditure approach, in line with contributions such as Garelli (2001), Bouakez, Rebei (2003), Gali, Monacelli (2005), ; our methodology implies the use of a consumption basket made of a CES aggregate of public and private consumption. This allows us to analyse the properties of the model and our results for a different combination of preferences parameters leading alternatively to complementarity or substitutability between public and private consumption. Our main conclusion is that we cannot properly assess the desiderability of prociclical or counterciclical fiscal policy without making reference to the way public expenditure interacts with private consumption. That is, without considering a more appropriate disaggregation of government expenditure along different categories which have different impact on economic variables.

The paper is organized as follows. Section 2 outlines the theoretical set up, section 3 deals with the resulting system of equations in which the economy collapses and the issue of complementarity / substitutability between public and private consumption; section 4 is concerned with stability analysis, section 5 with impulse response. Section 6 concludes and lays the foundation for future research work.

2 The model

2.1 Households and the demand-side

The economy is composed of a continuum of infinitely-lived individuals, whose measure is normalized to unity. Each of them consumes a consumption basket $\tilde{C}_t$and supplies labour N_t to a continuum of monopolistically-competitive intermediate firms. Wealth is allocated into one-period bonds (B_t).

The instantaneous utility function amounts to be:

$$U_t = \frac{\tilde{C}_t^{1-\gamma}}{1-\gamma} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n} \tag{1}$$

The consumption basket is a mix of public and private consumption:

$$\tilde{C}_t = \left[\theta C_t^{\frac{v-1}{v}} + (1-\theta) G_t^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}} \tag{2}$$

The representative household choose $\left[C_{t+i}, N_{t+i}, \frac{B_{t+i}}{P_{t+i}} \right]_{i=0}^{\infty}$ to solve:

$$\max E_t \sum_{i=0}^{\infty} \beta^i U_{t+i} \tag{3}$$

subject to the budget constraint:

$$C_t = \frac{W_t}{P_t} N_t + \Pi_t - T_t - \frac{\left(\frac{1}{i_t}\right) B_t - B_{t-1}}{P_t} \tag{4}$$

with:

Π_t= real-profits from the firms

T_t= lump-sum taxes
i_t= nominal interest rate
$\gamma, \gamma_m, \gamma_n > 0$

Fisher-parity holds:

$$R_{t+1} = i_t E_t \left(\frac{P_t}{P_{t+1}} \right) \tag{5}$$

with R_{t+1}being the real interest rate.

Bellman equation is:

$$V \left(\frac{B_{t-1}}{P_t} \right) = \max \left[U_t + \beta E_t V \left(\frac{B_t}{P_{t+1}} \right) \right] \tag{6}$$

First order conditions with respect to consumption:

$$\frac{C_{t+1}}{C_t} = (\beta E_t R_{t+1})^{\nu} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{1-\gamma\nu} \tag{7}$$

First order condition with respect to labour supply:

$$\frac{W_t}{P_t} \tilde{C}_t^{-\gamma} = a_n N_t^{\gamma_n} \tag{8}$$

Loglinearizing equation(7):

$$c_t = -\nu r_{t+1} + (1 - \gamma\nu)\tilde{c}_t - (1 - \gamma\nu) E_t \tilde{c}_{t+1} + E_t c_{t+1} \tag{9}$$

Loglinearizing (2):

$$\tilde{c}_t = \theta \left(\frac{C}{\tilde{C}} \right)^{\frac{\nu-1}{\nu}} c_t + (1 - \theta) \left(\frac{G}{\tilde{C}} \right)^{\frac{\nu-1}{\nu}} g_t \tag{10}$$

shifting one period-forward:

$$E_t \tilde{c}_{t+1} = \theta \left(\frac{C}{\tilde{C}} \right)^{\frac{\nu-1}{\nu}} E_t c_{t+1} + (1 - \theta) \left(\frac{G}{\tilde{C}} \right)^{\frac{\nu-1}{\nu}} E_t g_{t+1} \tag{11}$$

Plugging (10) and (11) into (9) I get:

$$c_t = -\frac{\nu}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} r_{t+1} - \frac{\left[(1-\gamma\nu)(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}(E_t g_{t+1} - g_t) + E_t c_{t+1} \tag{12}$$

(12) is the Euler equation for this case.
Defining:

$$\frac{\nu}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = \Phi$$

$$\frac{\left[(1-\gamma\nu)(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = \Omega$$

$$c_t = -\Phi r_{t+1} - \Omega(E_t g_{t+1} - g_t) + E_t c_{t+1} \tag{13}$$

Aggregate resource constraint is:

$$Y = C + G$$

Log-linearization leads to:

$$y_t = \frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{G}}{\bar{Y}} g_t \tag{14}$$

and

$$E_t c_{t+1} = \frac{Y}{C} E_t y_{t+1} - \frac{G}{C} E_t g_{t+1} \tag{15}$$

Putting (13) and (15) into (14):

$$y_t = -\frac{\bar{C}}{\bar{Y}} \Phi r_{t+1} + E_t y_{t+1} + \left[\frac{\bar{G} + \bar{C}\ \Omega}{\bar{Y}}\right] [g_t - E_t g_{t+1}] \tag{16}$$

(16) is the IS curve of this economy.

2.2 Firms and the supply-side

There are two kind of firms: final goods producers and intermediate goods producers.

2.2.1 Final goods producers

Final goods producers are perfectly competitive firms producing an homogeneous good $Y_t$using intermediate goods, since there are a continuum of intermediate goods producers of measure unity, each producing a differentiated input for final goods production. Let $Y_t(f)$ being the input produced by intermediate goods firm f and z the types available; the production function that transforms intermediate goods into final output is:

$$Y_t = \left[\int_0^1 Y_t^f(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{17}$$

with $\varepsilon > 1$ being the elasticity of substitution between intermediate goods. We can see that this CES production function exhibits diminishing marginal product, a property that will drive the firms to diversify and produce all the intermediate goods available.

The final good producer will minimize its cost; therefore it will choose $Y_t^f(z)$ to:

$$\min \int_0^1 P_t(z) Y_t^f(z) dz \tag{18}$$

subject to the production function (17).

The Langrangian results into:

$$L = \int_0^1 P_t(z) Y_t^f(z) dz - \lambda \left[\int_0^1 Y_t^f(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{19}$$

The first order condition with respect to $Y_t^f(z)$ is:

$$P_t(z) = \lambda \left(\frac{Y_t^f(z)}{Y_t^f} \right)^{-\frac{1}{\varepsilon}} \tag{20}$$

To proceed now we need to solve for the Lagrange multiplier; to do so, following Clarida, Galì and Gertler (1999), we re-write the FOC of this maximization problem in a different way, looking at the Langrangian:

$$P_t(z) = \lambda \frac{\delta Y_t^f}{\delta Y_t^f(z)} \tag{21}$$

Multiplying both sides for the term $\frac{Y_t^f(z)}{Y_t^f}$ we obtain:

$$\frac{1}{Y_t^f} P_t(z) Y_t^f(z) = \lambda \frac{\delta Y_t^f}{\delta Y_t^f(z)} \frac{Y_t^f(z)}{Y_t^f} \tag{22}$$

Integrating through:

$$\frac{1}{Y_t^f} \int_0^1 P_t(z) Y_t^f(z) dz = \lambda \int_0^1 \frac{\delta Y_t^f}{\delta Y_t^f(z)} \frac{Y_t^f(z)}{Y_t^f} dz \tag{23}$$

Considering that:
$\int_0^1 P_t(z) Y_t^f(z) dz =$ total cost of production $= E_t$
$\int_0^1 \frac{\delta Y_t^f}{\delta Y_t^f(z)} \frac{Y_t^f(z)}{Y_t^f} dz = 1$
then (23) becomes:

$$\frac{E_t}{Y_t^f} = \lambda \tag{24}$$

Since the final goods firms operate in perfect competition, total cost of production must be equal to the total value of goods sold:

$$E_t = P_t Y_t^f \tag{25}$$

Se we can combine to get:

$$P_t = \lambda \tag{26}$$

Plugging (26) into (20)

$$P_t(z) = P_t \left(\frac{Y_t^f(z)}{Y_t^f} \right)^{-\frac{1}{\varepsilon}} \tag{27}$$

$$Y_t^f(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t^f \tag{28}$$

Integrating over all final good firms we can obtain the total demand of intermediate good z:

$$Y_t(z) = \int_0^1 Y_t^f(z) df \tag{29}$$

Plugging (28) into (29):

$$Y_t(z) = \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t^f \, df \tag{30}$$

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} \int_0^1 Y_t^f \, df \tag{31}$$

As $\int_0^1 Y_t^f df = Y_t$ (the sum of single firms output gives the total output of the sector)

we are finally able to obtain the following total demand curve for intermediate good z

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \tag{32}$$

If we plug it into the production function of firm f we obtain, after a few algebrical manipulations, an expressions for the aggregate price index:

$$P_t = \left[\int_0^1 P_t(z)^{\frac{1}{1-\varepsilon}} dz \right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

2.2.2 Intermediate goods producers

Intermediate goods firms are monopolistically competitive and have the following standard constant-return-to-scale production function, with $A_t$being the technological parameter following a stationary process.

$$Y_t(f) = A_t N_t \tag{34}$$

and set prices on a Calvo-Yun based staggered framework, with $(1 - \zeta)$ being the probability that each period the firm adjusts its price, and ζ being obviously the probability that it keeps prices constant. The adjustment probability is independent across time and across firms, and the average number of periods a price remain fixed is given by $\frac{1}{1-\zeta}$. Technology follows a standard AR(1) process.

Intermediate firms maximize expected discounted profits subject to the production function (34) and the demand curve they face (32); in the flexible-prices equilibrium the maximization problem leads to the usual condition:

$$MC_t = \frac{1}{1+\mu} \tag{35}$$

with $\frac{1}{1+\mu} = \frac{\varepsilon}{1-\varepsilon}$being the mark-up, following Dixit and Stiglitz (1977).

With sticky prices, the relevant maximization problem for the firm becomes:

$$\max \sum_{i=0}^{i=\infty} (\zeta\beta)^i E_t \Lambda_{t,i} \left[\frac{P_t(f) - MC_{t+i}^n(f)}{P_{t+i}} Y_{t,t+i}(f) \right] \tag{36}$$

with:
$MC_t^n(f) = P_t MC_t(f) =$ nominal marginal cost
$\Lambda_{t,i} = \left(\frac{C_{t+i}}{Ct}\right)^{-\gamma}$ =stochastic discount factor
subject to (32) and (34).

Plugging the constraints into the objective function and recalling the condition coming from cost minimization[3]:

$$\frac{\frac{W}{P}}{A} = MC \tag{37}$$

we get the optimal price:

$$P_t^* = (1+\mu) \sum_{i=0}^{i=\infty} \vartheta_{t,i} MC_{t+1}^n \tag{38}$$

with:
$\vartheta_{t,i} = f(\Lambda_{t,i}, Y_{t+i})$
that is, the optimal price equals the steady-state mark-up times a weighted average of expected future nominal marginal costs; the weights depend on how much the firm discounts future cash flows in each period $t+i$ (taking into account that prices remain fixed along the way) and on the revenue expected in each period.

Considering also that the aggregate price index is a combination of price charged by those firms who get to change their prices and those who do not:

$$P_t = \left[\zeta P_t^{1-\varepsilon} + (1-\zeta) P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{39}$$

[3] This condition comes from the simple cost minimization problem in a no-capital situation with constant return to scale technology: min $\frac{W/P}{N}$subject to $AN - Y = 0$ and remembering that the Langrange multiplier, being the derivative of total cost with respect to Y, is the real marginal cost MC.

2.3 Potential output

Let us start by looking for an equilibrium on the labour market. This has to imply that firms'wage decisions based on cost minimization problem must be equal to the wage decision by households based on their optimizing behaviour. From the firms'point of view we have the usual condition coming from cost minimization:

$$\frac{\frac{W_t}{P_t}}{A_t} = MC_t \tag{40}$$

which, combined with the steady-state expression for marginal cost (35) becomes:

$$\frac{W_t}{P_t} = \frac{A_t}{1+\mu} \tag{41}$$

Combining with households'optimal labour supply (8):

$$\frac{A_t}{1+\mu} = a_n \frac{N_t^{\gamma_n}}{\tilde{C}_t^{-\gamma}} \tag{42}$$

using the production function (34), after few simple algebrical manipulations:

$$Y_t = A_t^{\frac{1+\gamma_n}{\gamma_n}} a_n^{-\frac{1}{\gamma_n}} (1+\mu)^{-\frac{1}{\gamma_n}} \tilde{C}^{-\frac{\gamma}{\gamma_n}} \tag{43}$$

which is, in level, the supply function of our economy.
Solving for the steady-state mark-up:

$$(1+\mu) = Y_t^{-\gamma_n} A_t^{\gamma_n+1} \tilde{C}_t^{-\gamma} a_n^{-1} \tag{44}$$

Loglinearizing it:

$$\mu_t = (\gamma_n + 1)a_t - \gamma\, \tilde{c}_t - \gamma_n y_t \tag{45}$$

Using the loglinearized basket of consumption (10)and the loglinearized resource constraint (14):

$$\mu_t = (\gamma_n+1)a_t - \gamma \left[\theta \left(\frac{\bar{C}}{\tilde{\bar{C}}}\right)^{\frac{\nu-1}{\nu}} \left[\frac{\bar{Y}}{\bar{C}} y_t - \frac{\bar{G}}{\bar{C}} g_t\right] + (1-\theta)\left(\frac{\bar{G}}{\tilde{\bar{C}}}\right)^{\frac{\nu-1}{\nu}} g_t\right] - \gamma_n y_t \tag{46}$$

To find an expression for potential output let us set $\mu = 0$ (since at Y^*log-deviations from steady state mark up are equal to zero) and solve for y_t^* :

$$y_t^* = \frac{\gamma_n + 1}{\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}} + \gamma_n} a_t + \frac{\left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{G}}{\bar{C}} - \gamma(1-\theta)\left(\frac{G}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left(\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}} + \gamma_n\right)} g_t \quad (47)$$

Equation (47)is the expression for potential output in log-deviations from the steady-state. The coefficient on g_t, indicating the positive effect of fiscal policy on potential output, may cause some concern regarding its plausibility; nevertheless, as we will show in the next sections, with the parametrization we used the above coefficient will endogenously turn out to have second-order effect, as its value is very close to zero.

2.4 New Keynesian Phillips Curve

In order to get the New Keynesian Phillips curve, let us solve (47) for $a_t$and plug it back into (46):

$$\mu = \left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}} + \gamma_n\right](y_t^* - y_t)$$

Considering the loglinearization of the mark-up (35) :

$$\mu = -mc \quad (48)$$

$$mc = \left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}} + \gamma_n\right](y_t - y_t^*) \quad (49)$$

If we go back to the firm sector and plug the expression for optimal price P_t^* (equation 38) into the aggregate price index (equation 39), after some long but standard algebra and dropping all the terms involving a product of two or more variables in log-deviation from the steady-state[4], we get to the standard formulation of the New Keynesian Phillips Curve:

[4]That is because we are not interested in second-order terms.

$$\pi_t = \left[\frac{(1-\zeta)(1-\beta\zeta)}{\theta}\right] mc_t + \beta E_t \pi_{t+1} \tag{50}$$

Plugging 49:

$$\pi_t = \left[\frac{(1-\zeta)(1-\beta\zeta)}{\theta}\right] \left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} \frac{\bar{Y}}{\bar{C}} + \gamma_n\right] (y_t - y_t^*) + \beta E_t \pi_{t+1} \tag{51}$$

Renaming the parameters according to:

$\left[\frac{(1-\zeta)(1-\beta\zeta)}{\zeta}\right] = \lambda$ component depending on probability of price adjustment

$\left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} \frac{\bar{Y}}{\bar{C}} + \gamma_n\right] = \delta$ component depending on preferences parameters

$\lambda\delta = k$

we have the standard New Keynesian Phillips curve, microfounded just as we find in Woodford (2003):

$$\pi_t = k(y_t - y_t^*) + \beta E_t \pi_{t+1} \tag{52}$$

2.5 Monetary policy rule

As for monetary policy, we skip the usual derivations stemming from most studies on optimal monetary policy, and we assume (in line with the whole literature) a Taylor-like monetary policy rule for the evolution of the nominal interest rate:

$$i_t = \phi_\pi \pi_t + \phi_x (y_t - y_t^*) \tag{53}$$

where obviously interest rate responds to movements in current inflation and current output gap.

We thus are ready to express the equations of our economy in a full-system.

2.6 Fiscal policy rule

I need a fiscal rule to close it. A wide literature (and well-known policy debate, concerning especially the European Union and the Stability and Growth Pact) emphasizes the importance for governments to stick to counter-cyclicality of fiscal policy regimes, in order to stabilize output over the cycle, smooth taxation and let automatic stabilizers work properly and effectively.Lately, this issue has gained particular importance in international organization (such as ECB, OECD, IMF) policy reccomandations. Following this approach,we assume that government applies a counter-cyclical log-linear fiscal policy rule of the kind:

$$\frac{G_t}{\bar{G}} = \left(\frac{Y_t}{Y^*}\right)^{-\alpha} \tag{54}$$

where α is the parameter determining the intensity of the countercicilicality of the fiscal policy rule.

Loglinearization of (54) leads to:

$$g_t = -\alpha(y_t - y_t^*) \tag{55}$$

3 The system of equations

The economy is described by the system of equations (47),(52),(53),(16),(55)

In matrix form:

$$E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \left[\begin{Bmatrix} 1 + \frac{c_1(\beta\phi_x+\kappa)}{\beta(1-\alpha b_2+c_2\alpha)} & \frac{c_1(\beta\phi_\pi-1)(1-\alpha b_2)}{\beta(1-\alpha b_2+c_2\alpha)} \\ \frac{-\kappa}{\beta(1-\alpha b_2)} & \beta^{-1} \end{Bmatrix}\right] \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \frac{-c_1 b_1(\beta\phi_x+\kappa)-\beta\alpha c_2 b_1}{\beta(1-\alpha b_2+c_2\alpha)} \\ \frac{\kappa b_1}{\beta(1-\alpha b_2)} \end{bmatrix} \begin{bmatrix} a_t \end{bmatrix}$$

with the parameters:

$$\frac{\gamma_n+1}{\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{C}+\gamma_n} = b_1$$

$$\frac{\left[\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{G}}{C}-\gamma(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left(\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{C}+\gamma_n\right)} = b_2$$

$$\frac{\bar{C}}{\bar{Y}}\Phi = \frac{\bar{C}}{\bar{Y}}\frac{\nu}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = c_1$$

$$\left[\frac{\bar{G}+\bar{C}\ \Omega}{\bar{Y}}\right] = \frac{\bar{G}}{\bar{Y}} + \frac{\bar{C}}{\bar{Y}}\frac{\left[(1-\gamma\nu)(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = c_2$$

3.1 Complementarity / substitutability

Take again (1):

$$U_t = \frac{\left[\left[\theta C_t^{\frac{\upsilon-1}{\upsilon}} + (1-\theta) G_t^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}}\right]^{1-\gamma}}{1-\gamma} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n}$$

Considering the basket (2):

$$\tilde{C}_t = \left[\theta C_t^{\frac{\upsilon-1}{\upsilon}} + (1-\theta) G_t^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}}$$

Let us calculate the marginal utility of private consumption:

$$\frac{\partial U}{\partial C_t} = \theta C_t^{-\frac{1}{\nu}} \tilde{C}_t^{-\gamma+\frac{1}{\nu}} \tag{56}$$

Loglinearizing:

$$\theta C_t^{-\frac{1}{\nu}} \tilde{C}_t^{-\gamma+\frac{1}{\nu}} (1 + -\frac{1}{\nu} c_t + (\frac{1}{\nu} - \gamma) \tilde{c}_t) \tag{57}$$

Plugging the loglinearized basket (10), after few algebrical manipulations:

$$\theta \left(\frac{C}{\tilde{C}}\right)^{-\frac{1}{\nu}} \tilde{C}^{-\gamma} - \theta \left(\frac{C}{\tilde{C}}\right)^{-\frac{1}{\nu}} \tilde{C}^{-\gamma} \left[\frac{1}{\upsilon} - \theta \left(\frac{C}{\tilde{C}}\right)^{-\frac{1}{\nu}+1} (\frac{1}{\nu} - \gamma)\right] c_t + \tag{58}$$
$$+\theta \left(\frac{C}{\tilde{C}}\right)^{-\frac{1}{\nu}} \tilde{C}^{-\gamma} \left(\frac{1}{\nu} - \gamma\right) (1-\theta) \left(\frac{G}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} g_t$$

Let us study the sign of the coefficient on g_t :

$$\text{sign } \theta \left(\frac{C}{\tilde{C}}\right)^{-\frac{1}{\nu}} \tilde{C}^{-\gamma} (\frac{1}{\nu} - \gamma)(1-\theta) \left(\frac{G}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} = \text{sign} \left(\frac{1}{\nu} - \gamma\right)$$

If $\frac{1}{\nu} - \gamma > 0$
$\frac{1}{\nu} > \gamma$
$\nu < \frac{1}{\gamma}$ or, in the terminology below, $v < \sigma$
then private and public consumption are complements, because increasing g_t increases the marginal utility of private consumption.

So now I have a benchmark for the calibration; whenever the value for ν is lower than $\frac{1}{\gamma}$ I have complementarity, and the wider the gap the greater the complementarity.

4 Simulations

Here is the list of fixed parameters for calibration:

inverse of labour supply elast.	γ_n	1.5
gov.consumption to GDP	$\frac{\bar{G}}{\bar{Y}}$	0.25
priv.consumption to GDP	$\frac{\bar{C}}{\bar{Y}}$	0.75
GDP to gov.cons.	$\frac{\bar{Y}}{\bar{G}}$	4
GDP to priv.cons.	$\frac{\bar{Y}}{\bar{C}}$	1.34
public to private cons.	$\frac{\bar{G}}{\bar{C}}$	0.33
output coeff. Taylor rule	ϕ_x	0.5
output coeff. Phillips curve	κ	0.043
weight in the basket	θ	0.75

The degree of complementarity / substitutability between public and private consumption depends on the sign and magnitude of $\frac{1}{\nu} - \gamma$; we conduct several experiments by varying those two parameters, thereby checking the range of fiscal policy parameters consistent with stability of rational expectations equilibrium. With the variation of ν, also the ratio of private/public consumption and the basket of consumption $(\frac{C}{\bar{C}}, \frac{G}{\bar{C}})$ vary accordingly.[5] Note that, in all the below experiments, the coefficient b_2 (indicating the positive effect of government expenditure on potential output level) is close to 0.01.

In the following experiments, we vary the coefficient of monetary policy on inflation (ϕ_π) (simulating the standard case, the aggressive case, the very aggressive case, and the loose case), for different values of the elasticity of substitution between C and G (ν) and the risk adversion parameter (γ):

4.1 Complementarity

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
BENCH	0.5	1.5	1.5	$-1 < \alpha < 3$
*case*1	0.5	1.5	2.5	$-1 < \alpha < 3$
*case*2	0.5	1.5	0.7	*never*

The inclusion of fiscal policy does not seem to modify the Taylor principle: the coefficient on inflation, still needs to be greater than one for the equilibrium to be determinate. On the other hand, even when monetary policy is aggressive, the range of parameters consistent with stability calls for counterciclicality (or moderate prociclicality).

[5] Since they are function of θ and ν, so whenever we vary the elasticity of substitution we also have to change those ratios.

Increasing the degree of complementarity by lowering the risk adversion parameter does not seem to change anything in terms of determinacy.

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*3	0.25	1.5	1.5	$-1 < \alpha < 3$
*case*4	0.25	1.5	2.5	$-1 < \alpha < 3$
*case*5	0.25	1.5	0.7	*never*

Increasing the degree of complementarity by lowering the elasticity of substitution:

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*6	0.5	0.9	1.5	$-1 < \alpha < 3$
*case*7	0.5	0.9	2.5	$-1 < \alpha < 3$
*case*8	0.5	0.9	0.7	*never*

Moving both, so further widening the degree of complementarity:

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*9	0.25	0.9	1.5	$-1 < \alpha < 3$
*case*10	0.25	0.9	2.5	$-1 < \alpha < 3$
*case*11	0.25	0.9	0.7	*never*

So far we have seen what happens to determinacy in case public and private consumption are complements in the utility function; determinacy is ensured for all the values of fiscal policy parameter leading to counterciclicality, and for a limited prociclical response. This result holds if we wide the degree of complementarity, lowering the risk adversion parameter and/or the elasticity of substitution. Let us see what happens now if we induce substitutability.

4.2 Substitutability

Let us start by raising γ above 1, (so that $\nu > \frac{1}{\gamma}$), giving it a large value that is often assigned in some of the literature:

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*12	13	1.5	1.5	$-2 < \alpha < 3$
*case*13	13	1.5	2.5	$-2 < \alpha < 3$
*case*14	13	1.5	0.7	*never*

If I induce substitutability by keeping γconstant and raising ν:

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*18	0.5	3	1.5	$-1.5 < \alpha < 3$
*case*19	0.5	4	1.5	$-1.8 < \alpha < 3$
*case*20	0.5	5	1.5	$-2 < \alpha < 3$
*case*21	0.5	9	1.5	*always*

Increasing the degree of substitutability by raising the elasticity of substitution makes prociclical fiscal policy also determinate; equivalently, increasing complementarity (=decreasing substitutability) makes only counterciclical f.p. determinate. The same result is obtained, although in a much slower pace, by raising γ, as the previous table shows.

Let'see if this results holds also with different monetary policy stance.

TITLE	γ	v	ϕ_π	*STABILITY PARAMETERS*
*case*22	0.5	3	2.5	$-1.5 < \alpha < 3$
*case*23	0.5	4	2.5	$-1.8 < \alpha < 3$
*case*24	0.5	5	2.5	$-2 < \alpha < 3$
*case*25	0.5	9	2.5	$-2.5 < \alpha < 3$

And with passive monetary policy, it is never determinate, no matter how substitute $G$and C are.

The main conclusion we can draw is that increasing the substitutability between public and private consumption (no matter the source of that increase) allows for stability of prociclical fiscal policy, in addition to counterciclical, which is the only stability rule in case of complementarity. The intuition behind this result is that if government expenditure is prociclical, namely it is directly proportional to the dynamics of actual output, the inflationary pressure is dumped by the optimizing behaviour of consumers, who endogenously decrease private consumption in response to an increase in public consumption (because of the complementarity in the utility function).

5 Response to shocks

5.1 Technological shocks

Let us now analyze the way α affects the optimal response to technological shocks.

Re-write the system so that:

$$\begin{aligned} E_t y_{t+1} &= \left(1 + \frac{\beta c_1 \phi_x + \kappa c_1}{\beta(1-\alpha b_2 + c_2\alpha)}\right) y_t + \left(\frac{c_1(\beta\phi_\pi - 1)(1-\alpha b_2)}{\beta(1-\alpha b_2 + c_2\alpha)}\right)\pi_t + \left(\frac{-c_1 b_1(\beta\phi_x + \kappa) - \beta\alpha c_2 b}{\beta(1-\alpha b_2 + c_2\alpha)}\right. \\ E_t\pi_{t+1} &= -\frac{\kappa}{\beta(1-\alpha b_2)} y_t + \beta^{-1}\pi_t + \frac{\kappa b_1}{\beta(1-\alpha b_2)} a_t \\ a_t &= \rho a_{t-1} + \varepsilon_t^a \end{aligned}$$

with $\varepsilon_t^a \sim (0, \sigma^2)$

In matrix form:

$$AE_t y_{t+1} = By_t + Cx_t$$

is:

$$\begin{Bmatrix} 1 & 0 & -a_3 \\ 0 & 1 & -b_3 \\ 0 & 0 & 1 \end{Bmatrix} E_t \begin{Bmatrix} y_{t+1} \\ \pi_{t+1} \\ a_t \end{Bmatrix} = \begin{Bmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & \rho \end{Bmatrix} \begin{Bmatrix} y_t \\ \pi_t \\ a_{t-1} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \varepsilon_t^a$$

5.1.1 Complementarity

Let us start from the benchmark case, in which the risk adversion coefficient (γ)is set at 0.5 and the elasticity of substitution between private and public consumption (v)is set at 1.5. So, following the derived rule of thumb $v \lesseqgtr \frac{1}{\gamma}$, we have complementarity. Here are the results, for different values of the fiscal policy pararamer α :

FIGURE 1 HERE

The counterciclical case (upper panel) shows us that the more counterciclical fiscal policy is, the greater the impact on output; persistance of the shocks, however, is always the same. On the other hand, inflation response increases with counterciclicality. Prociclicality (lower panel) is good for inflation, which decreases (the more prociclical fiscal policy is the greater the reduction), but not good for output, whose response is very small and decreases with the degree of prociclicality. Obviously, the inflationary bias is reduced the more aggressive monetary policy becomes, but at expense of the output response (which is reduced).

Let us see what happens when we vary the degree of substitutability/ complementarity between public and private consumption.

Let us widen the degree of complementarity by lowering v to 0.9:

FIGURE 2 HERE

We can see that increasing the degree of complementarity between private and public consumption increases the response of output and inflation to the technological shock; this effect is greater the more counterciclical fiscal policy is. We do not show it here, but the same results are obtained if the increase in complementarity comes from an decrease in the risk adversion coefficient γ instead of v.

5.1.2 Substitutability

Let us induce substitutabilty by raising the elasticity of substitution ν to 9 :

FIGURE 3 HERE

Response of output and inflation is lower (with respect to the complementarity case), but still the role of counterciclical fiscal policy is preserved: the greater α, the greater the response of output and inflation.

We can then conclude that counterciclical fiscal policy enhances the response of output and inflation to technological shocks; in particular, the more counterciclical it is, the bigger the effect. If private and public consumption are complements, these effects are further increased; if they are substitutes, the signs of the effects are the same, but of much lower magnitude. Prociclical fiscal policy has a much lower positive effect on output, and a negative effect on inflation.

5.2 Government expenditure shocks

This section analyses what happens in case of shocks to the fiscal policy rule; in order to do so, we add a i.i.d. shock ε_t^g to equation (55), so to capture all the deviations from rationality that policy-makers might be induced to, due for example to political or lobbying pressures. We also have to re-write the system in a 7X7 form (see Appendix B).

Here are the results in the standard case:

FIGURE 4 HERE

As we can see, both in case of counterciclicality and prociclicality we have approximately the same effect: the immediate impact is a substantial raise in the output gap, due to the fact that actual output reacts to government expenditure shock much more intensively than potential; then the resulting inflationary pressure brings down output gap , and the whole movement is absorbed within a couple of periods. The only significant difference is the dynamics of public expenditure: in case of counterciclicality, its jump is obviously smaller than the prociclical case, and it is reduced as the degree of counterciclicality increase (the opposite happens in the prociclical case).

Here are the results for a stronger degree of complementarity (obtained by lowering γ to 0.25 and v to 0.6):

FIGURE 5 HERE

As we can see, under counterciclical regime increasing complementarity raises the response of actual output and reduces the dynamic of government expenditure (since the complementary increase in private consumption increases output which in turn, under a counterciclical fiscal policy rule, reduces public expenditure). Under prociclical regime, on the other hand, actual output's response is enhanced (for the same reason), but government expenditure response is more and more severe.

What happens in case of substitutability?

Intuitively, output will react less, since the innovation in public expenditure is now matched by a decrease in private consumption, which in turn dumpens the output's reaction. This will also be reflected in a smoother jump of public expenditure as we increase the absolute value of α, both under prociclical and counterciclical regime. These the results (for $v = 9$):

FIGURE 6 HERE

6 Conclusions

This paper is concerned with a more detailed analysis on when counterciclical fiscal policy rule (lately recommended by most observers) is actually desirable. We presented a New Keynesian dynamic stochastic general equilibrium model with government expenditure entering the utility function of the representative household through the consumption basket and a fiscal policy rule depending on output gap. Our results can be summarized as follows.

As far as stability properties are concerned, the modified set up delivers stability of rational expectation equilibrium mainly for parameters showing counterciclicality of fiscal policy rule. However, increasing the degree of substitutability between private and public consumption, delivers stability also for values of fiscal policy rule consistent with a prociclical stance, because optimizing behaviour of agents (who decrease private consumption in response to a government expenditure shock because of complementarity) takes care of dumpening the resulting inflationary bias of fiscal policy.

As far as response to stochastic shocks, we showed that counterciclical fiscal policy amplifies the effects on output (but also on inflation), and this effect is bigger the wider is the degree of complementarity between public and private consumption; prociclical fiscal policy, on the other hand, is better for inflation but reduces considerably the positive effect of technological shock on output. On the other hand, under substitutability between private and public consumption, responses are qualitatively the same, but quantitatively much less significant.

We also analysed the effects of government expenditure shocks, concluding that a positive fiscal policy shocks impacts more on actual output under complementarity, as perfectly intuitive; in this case, counterciclical fiscal policy helps to dumpen the subsequent response of public expenditure (which follows output), whereas a prociclical stance increases the amplitude of the initial fluctuations. Also in this case, substitutability lowers the response of economic variables, including public expenditure. It seems therefore that effects on government deficit of an unexpected increase in public expenditure (captured by fiscal policy shock) are themselves depending on the relationship between private and public consumption: in case of complementarity, effects on deficits can be very severe if policy makers are tempted by prociclical stance; on the other hand, under substitutability, they trade a lower effect on deficit with a lower response of output.

The main result of this paper is that it really seems to make little sense to discuss prociclicality or countericlicality of fiscal policy without distinguish between the different categories of the government expenditure, and their properties.For example, government expenditure on a public good (which is complementary to private consumption) is consistent with counter-ciclical fiscal policy and amplifies the response of output to technological and fiscal shocks; on the other hand, financing a merit good (which seems to be featured by substitutability) delivers stability under a prociclical stance, and helps smoothing the response of economic variables to stochastic shocks.

Following these results, future extensions include a more realistic analysis of fiscal policy, both on the revenue side (inserting distorsionary taxation on labour, capital and consumption) and on the expenditure side (disaggregating government consumption in a more detailed fashion), in line with contributions such as Fiorito, Kollintzas (2004), Forni, Monteforte, Sessa (2006) and Lopez-Salido, Rabanal (2006).

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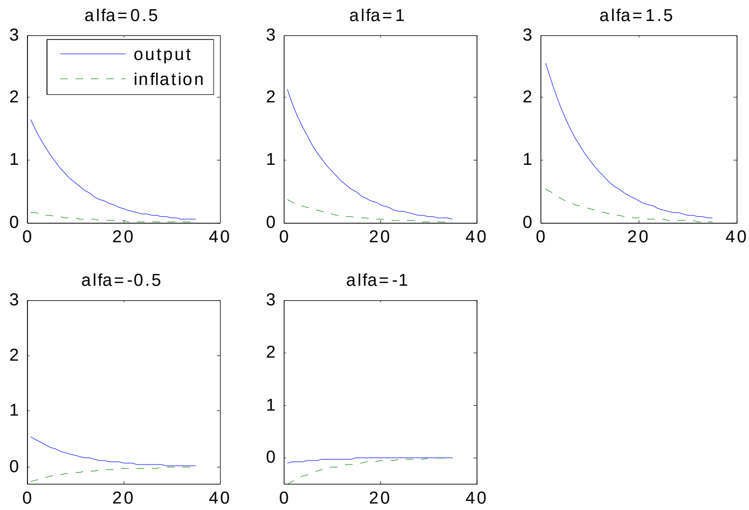

Figure 1:

8 Tables of figures

FIGURE 1: response to technological shock in the standard case
FIGURE 2: response to technological shock increasing complementarity
FIGURE 3: response to technological shock increasing substitutability
FIGURE 4: response to government expenditure shock in the standard case
FIGURE 5: response to gov.exp. shock increasing complementarity
FIGURE 6: response to gov.exp shock increasing substitutability

9 Appendix A

Then the system is:

$$y_t^* = b_1 a_t + b_2 g_t \tag{59}$$

$$\pi_t = k(y_t - y_t^*) + \beta E_t \pi_{t+1} \tag{60}$$

$$i_t = \phi_\pi \pi_t + \phi_x (y_t - y_t^*) \tag{61}$$

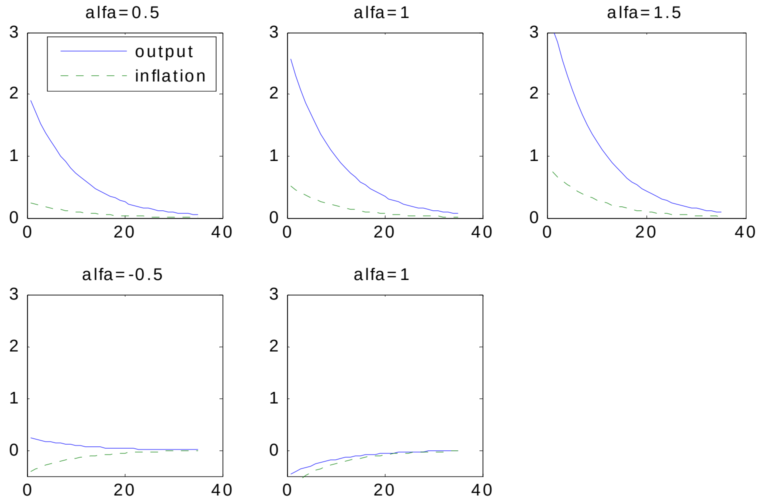

Figure 2:

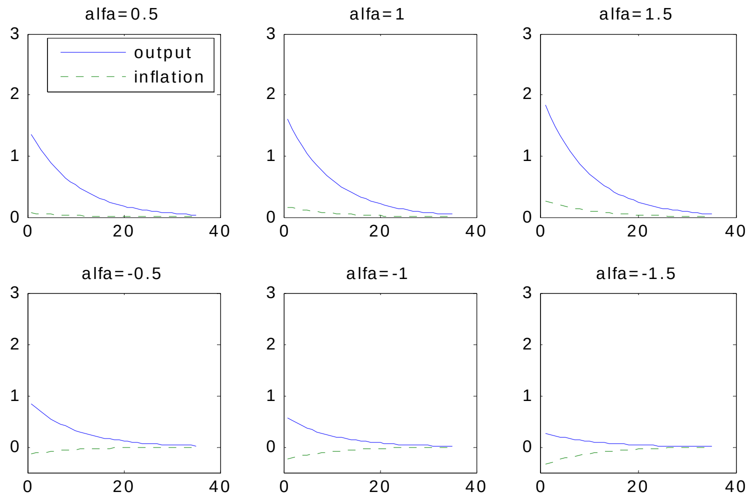

Figure 3:

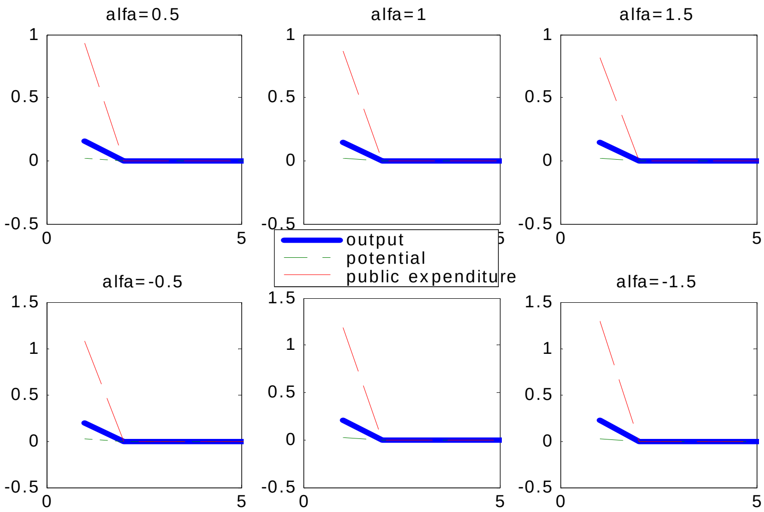

Figure 4:

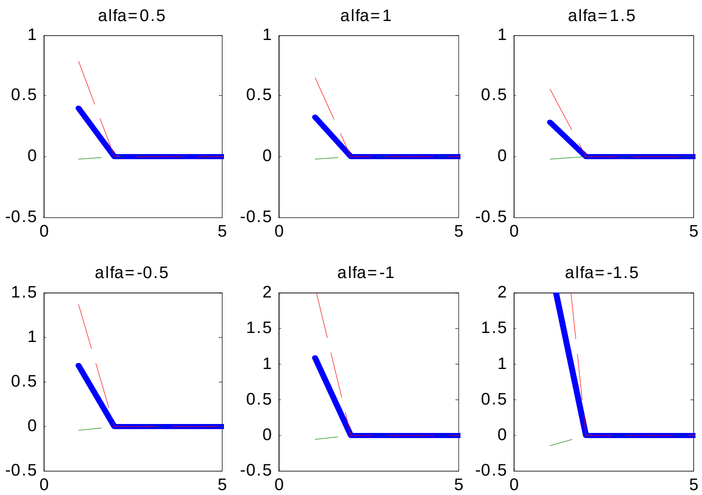

Figure 5:

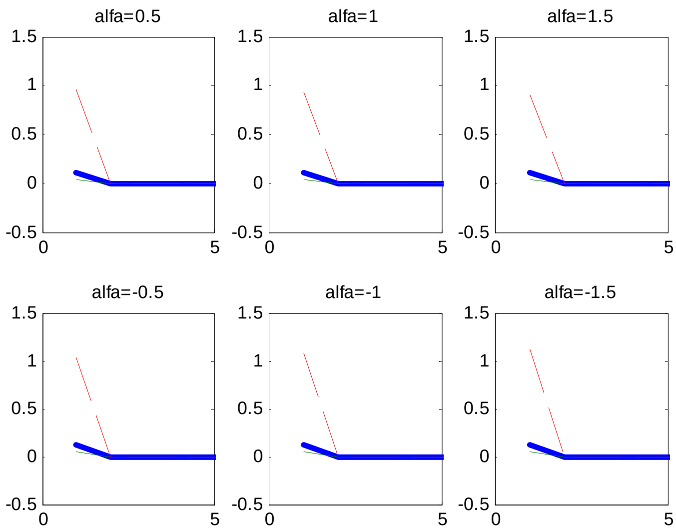

Figure 6:

$$y_t = -c_1(i_t - E_t\pi_{t+1}) + E_t y_{t+1} + c_2\left[g_t - E_t g_{t+1}\right] \tag{62}$$

$$g_t = -\alpha(y_t - y_t^*) \tag{63}$$

10 First equation

Update (63):

$$E_t g_{t+1} = -\alpha E_t y_{t+1} + \alpha E_t y^*_{t+1}$$

We know from (59) that:

$$E_t y^*_{t+1} = b_2 E_t g_{t+1}$$

since $E_t a_{t+1} = 0$
Plugging in:

$$\begin{aligned} E_t g_{t+1} &= -\alpha E_t y_{t+1} + \alpha b_2 E_t g_{t+1} \\ (1 - \alpha b_2) E_t g_{t+1} &= -\alpha E_t y_{t+1} \end{aligned}$$

$$E_t g_{t+1} = \frac{-\alpha}{(1-\alpha b_2)} E_t y_{t+1} \tag{64}$$

Let us put (63) into (59):

$$\begin{aligned} y_t^* &= b_1 a_t + b_2[-\alpha(y_t - y_t^*)] \\ y_t^* &= b_1 a_t - \alpha b_2 y_t + \alpha b_2 y_t^* \\ (1-\alpha b_2) y_t^* &= b_1 a_t - \alpha b_2 y_t \end{aligned}$$

$$y_t^* = \frac{b_1}{(1-\alpha b_2)} a_t - \frac{\alpha b_2}{1-\alpha b_2} y_t \tag{65}$$

Let us now plug (61),(63), and (64) into (62):

$y_t = -c_1 i_t + c_1 E_t \pi_{t+1} + E_t y_{t+1} + c_2 g_t - c_2 E_t g_{t+1}$

$y_t = -c_1 [\phi_\pi \pi_t + \phi_x (y_t - y_t^*)] + c_1 E_t \pi_{t+1} + E_t y_{t+1} + c_2[-\alpha(y_t - y_t^*)] - c_2[\frac{-\alpha}{(1-\alpha b_2)} E_t y_{t+1}]$

$y_t = -c_1 \phi_\pi \pi_t - c_1 \phi_x (y_t - y_t^*) + c_1 E_t \pi_{t+1} + E_t y_{t+1} - \alpha c_2 (y_t - y_t^*) + \frac{c_2 \alpha}{(1-\alpha b_2)} E_t y_{t+1}$

$y_t = -c_1 \phi_\pi \pi_t - c_1 \phi_x y_t + c_1 \phi_x y_t^* + c_1 E_t \pi_{t+1} + E_t y_{t+1} - \alpha c_2 y_t + \alpha c_2 y_t^* + \frac{c_2 \alpha}{(1-\alpha b_2)} E_t y_{t+1}$

$(1 + \frac{c_2 \alpha}{(1-\alpha b_2)}) E_t y_{t+1} = (1 + c_1 \phi_x + \alpha c_2) y_t - (c_1 \phi_x + \alpha c_2) y_t^* + c_1 \phi_\pi \pi_t - c_1 E_t \pi_{t+1}$

Plug (65) in: (looking only at $(c_1 \phi_x + \alpha c_2) y_t^*$) :

$$\begin{aligned} (c_1 \phi_x + \alpha c_2) y_t^* &= (c_1 \phi_x + \alpha c_2) \left[\frac{b_1}{(1-\alpha b_2)} a_t - \frac{\alpha b_2}{1-\alpha b_2} y_t \right] \\ &= \frac{c_1 \phi_x b_1}{1-\alpha b_2} a_t - \frac{c_1 \phi_x \alpha b_2}{1-\alpha b_2} y_t + \frac{\alpha c_2 b_1}{1-\alpha b_2} a_t - \frac{\alpha^2 c_2 b_2}{1-\alpha b_2} y_t \\ &= \frac{c_1 \phi_x b_1 + \alpha c_2 b_1}{1-\alpha b_2} a_t - \frac{c_1 \phi_x \alpha b_2 + \alpha^2 c_2 b_2}{1-\alpha b_2} y_t \end{aligned}$$

So the whole thing becomes:

$\left(\frac{1-\alpha b_2 + c_2 \alpha}{1-\alpha b_2} \right) E_t y_{t+1} = \left(\frac{(1-\alpha b_2)(1 + c_1 \phi_x + \alpha c_2) + c_1 \phi_x \alpha b_2 + \alpha^2 c_2 b_2}{1-\alpha b_2} \right) y_t + c_1 \phi_\pi \pi_t - \frac{c_1 \phi_x b_1 + \alpha c_2 b_1}{1-\alpha b_2} a_t - c_1 E_t \pi_{t+1}$

Focus on $c_1 E_t \pi_{t+1}$:

considering (60):

$$
\begin{aligned}
c_1 E_t \pi_{t+1} &= \frac{c_1\pi_t - c_1 k(y_t - y_t^*)}{\beta} \\
&= \frac{c_1\pi_t}{\beta} + \frac{-\kappa c_1 y_t + \kappa c_1 y_t^*}{\beta} \\
&= \frac{c_1\pi_t}{\beta} + \frac{-\kappa c_1 y_t}{\beta} + \frac{\kappa c_1}{\beta}\left[\frac{b_1}{(1-\alpha b_2)} a_t - \frac{\alpha b_2}{1-\alpha b_2} y_t\right] \\
&= \frac{c_1\pi_t}{\beta} - \frac{\kappa c_1 y_t}{\beta} + \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t - \frac{\kappa c_1 \alpha b_2}{\beta(1-\alpha b_2)} y_t \\
&= \left(\frac{-\kappa c_1}{\beta} - \frac{\kappa c_1 \alpha b_2}{\beta(1-\alpha b_2)}\right) y_t + \frac{c_1\pi_t}{\beta} + \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t \\
&= \left(\frac{-\kappa c_1(1-\alpha b_2) - \kappa c_1 \alpha b_2}{\beta(1-\alpha b_2)}\right) y_t + \frac{c_1\pi_t}{\beta} + \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t \\
&= \left(\frac{-\kappa c_1 + \kappa c_1 \alpha b_2 - \kappa c_1 \alpha b_2}{\beta(1-\alpha b_2)}\right) y_t + \frac{c_1\pi_t}{\beta} + \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t \\
&= \left(\frac{-\kappa c_1}{\beta(1-\alpha b_2)}\right) y_t + \frac{c_1\pi_t}{\beta} + \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t
\end{aligned}
$$

Plug into the whole thing:

$$
\begin{aligned}
\left(\frac{1-\alpha b_2 + c_2\alpha}{1-\alpha b_2}\right) E_t y_{t+1} &= \left(\frac{(1-\alpha b_2)(1+c_1\phi_x+\alpha c_2) + c_1\phi_x \alpha b_2 + \alpha^2 c_2 b_2}{1-\alpha b_2}\right) y_t + c_1\phi_\pi \pi_t \\
&\quad - \frac{c_1\phi_x b_1 + \alpha c_2 b_1}{1-\alpha b_2} a_t + \left(\frac{\kappa c_1}{\beta(1-\alpha b_2)}\right) y_t - \frac{c_1\pi_t}{\beta} - \frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)} a_t
\end{aligned}
$$

Focus on the terms in y_t :

$$
\left(\frac{\beta(1-\alpha b_2)(1+c_1\phi_x+\alpha c_2) + \beta c_1\phi_x\alpha b_2 + \beta\alpha^2 c_2 b_2 + \kappa c_1}{\beta(1-\alpha b_2)}\right) y_t
$$

$$
\left(\frac{(\beta-\beta\alpha b_2)(1+c_1\phi_x+\alpha c_2) + \beta c_1\phi_x\alpha b_2 + \beta\alpha^2 c_2 b_2 + \kappa c_1}{\beta(1-\alpha b_2)}\right) y_t
$$

$$
\left(\frac{\beta + \beta c_1\phi_x + \beta\alpha c_2 - \beta\alpha b_2 - \beta\alpha b_2 c_1\phi_x - \beta\alpha^2 b_2 c_2 + \beta c_1\phi_x\alpha b_2 + \beta\alpha^2 c_2 b_2 + \kappa c_1}{\beta(1-\alpha b_2)}\right) y_t
$$

$$
\left(\frac{\beta + \beta c_1\phi_x + \beta\alpha c_2 - \beta\alpha b_2 + \kappa c_1}{\beta(1-\alpha b_2)}\right) y_t
$$

Focusing on the terms in a_t :

$$-\frac{c_1\phi_x b_1+\alpha c_2 b_1}{1-\alpha b_2}a_t-\frac{\kappa c_1 b_1}{\beta(1-\alpha b_2)}a_t=\frac{-\beta c_1\phi_x b_1-\beta\alpha c_2 b_1-\kappa c_1 b_1}{\beta(1-\alpha b_2)}a_t$$

Focusing on the terms in π_t :

$$c_1\phi_\pi\pi_t-\frac{c_1\pi_t}{\beta}=(c_1\phi_\pi-c_1\beta^{-1})\pi_t$$

Putting everything in:

$$E_t y_{t+1}=\left(1+\frac{\beta c_1\phi_x+\kappa c_1}{\beta(1-\alpha b_2+c_2\alpha)}\right)y_t+\left(\frac{c_1(\beta\phi_\pi-1)(1-\alpha b_2)}{\beta(1-\alpha b_2+c_2\alpha)}\right)\pi_t+\left(\frac{-c_1 b_1(\beta\phi_x+\kappa)-\beta\alpha c_2 b_1}{\beta(1-\alpha b_2+c_2\alpha)}\right)a_t$$

11 Second equation.

Take 60:

$$\begin{aligned}E_t\pi_{t+1} &= \frac{\pi_t-\kappa(y_t-y_t^*)}{\beta}\\ E_t\pi_{t+1} &= \beta^{-1}\pi_t-\kappa\beta^{-1}y_t+\kappa\beta^{-1}y_t^*\end{aligned}$$

Use the expression for potential output:

$$\begin{aligned}E_t\pi_{t+1} &= \beta^{-1}\pi_t-\kappa\beta^{-1}y_t+\kappa\beta^{-1}\left(\frac{b_1}{(1-\alpha b_2)}a_t-\frac{\alpha b_2}{1-\alpha b_2}y_t\right)\\ E_t\pi_{t+1} &= \beta^{-1}\pi_t-\kappa\beta^{-1}y_t+\frac{\kappa\beta^{-1}b_1}{(1-\alpha b_2)}a_t-\frac{\kappa\beta^{-1}\alpha b_2}{1-\alpha b_2}y_t\\ E_t\pi_{t+1} &= \beta^{-1}\pi_t-\frac{\kappa\beta^{-1}(1-\alpha b_2)+\kappa\beta^{-1}\alpha b_2}{1-\alpha b_2}y_t+\frac{\kappa\beta^{-1}b_1}{(1-\alpha b_2)}a_t\\ E_t\pi_{t+1} &= \beta^{-1}\pi_t-\frac{\kappa\beta^{-1}-\kappa\beta^{-1}\alpha b_2+\kappa\beta^{-1}\alpha b_2}{1-\alpha b_2}y_t+\frac{\kappa\beta^{-1}b_1}{(1-\alpha b_2)}a_t\end{aligned}$$

$$E_t\pi_{t+1}=\beta^{-1}\pi_t-\frac{\kappa}{\beta(1-\alpha b_2)}y_t+\frac{\kappa b_1}{\beta(1-\alpha b_2)}a_t$$

So the system is:

$$
\begin{aligned}
E_t y_{t+1} &= \left(1 + \frac{c_1(\beta\phi_x + \kappa)}{\beta(1-\alpha b_2 + c_2\alpha)}\right) y_t + \left(\frac{c_1(\beta\phi_\pi - 1)(1-\alpha b_2)}{\beta(1-\alpha b_2 + c_2\alpha)}\right)\pi_t + \left(\frac{-c_1 b_1(\beta\phi_x + \kappa) - \beta\alpha c_2 b}{\beta(1-\alpha b_2 + c_2\alpha)}\right. \\
E_t \pi_{t+1} &= -\frac{\kappa}{\beta(1-\alpha b_2)} y_t + \beta^{-1}\pi_t + \frac{\kappa b_1}{\beta(1-\alpha b_2)} a_t
\end{aligned}
$$

12 Appendix B - the system for analysis on output gap

The system is in the form:

$$A E_t Y_{t+1} = B Y_t + C X_t$$

with the vector $Y_t$being:

$$Y_t = [y_t, \pi_t, y_t^*, x_t, g_t, i_{t-1}, a_{t-1}]$$

The system in a convenient form is:

$$
\begin{aligned}
E_t y_{t+1} + \frac{\bar{C}}{\bar{Y}}\Phi E_t \pi_{t+1} - \left[\frac{\bar{G} + \bar{C}\ \Omega}{\bar{Y}}\right] E_t g_{t+1} - \frac{\bar{C}}{\bar{Y}}\Phi i_t &= y_t - \left[\frac{\bar{G} + \bar{C}\ \Omega}{\bar{Y}}\right] g_t \\
\beta E_t \pi_{t+1} &= \pi_t - k x_t \\
i_t &= \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_x x_t \\
\frac{\gamma_n + 1}{\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} \frac{\bar{Y}}{\bar{C}} + \gamma_n} a_t &= y_t^* - \frac{\left[\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} \frac{\bar{G}}{\bar{C}} - \gamma(1-\theta)\left(\frac{G}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left(\gamma\theta\left(\frac{C}{\tilde{C}}\right)^{\frac{\nu-1}{\nu}} \frac{\bar{Y}}{\bar{C}} + \gamma_n\right)} g_t \\
0 &= g_t + \alpha x_t - \varepsilon_t \\
a_t &= \rho a_{t-1} + \varepsilon_t \\
0 &= x_t - y_t + y_t^*
\end{aligned}
$$

In matrix form:

$$
\left\{\begin{matrix}
1 & \frac{\bar{C}}{\bar{Y}}\Phi & 0 & 0 & -\left[\frac{\bar{G}+\bar{C}\ \Omega}{\bar{Y}}\right] & -\frac{\bar{C}}{\bar{Y}}\Phi & 0 \\
0 & \beta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\gamma_n+1}{\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}}+\gamma_n} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{matrix}\right\} E_t \left\{\begin{matrix} y_{t+1} \\ \pi_{t+1} \\ y^*_{t+1} \\ x_{t+1} \\ g_{t+1} \\ i_t \\ a_t \end{matrix}\right\} =
$$

$$
\left\{\begin{matrix}
1 & 0 & 0 & 0 & -\left[\frac{\bar{G}+\bar{C}\ \Omega}{\bar{Y}}\right] & 0 & 0 \\
0 & 1 & 0 & -\kappa & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -\frac{\left[\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{G}}{\bar{C}}-\gamma(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left(\gamma\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\frac{\bar{Y}}{\bar{C}}+\gamma_n\right)} & 0 & 0 \\
-1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha & 1 & 0 & 0 \\
0 & \phi_\pi & 0 & \phi_x & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho
\end{matrix}\right\} \left\{\begin{matrix} y_t \\ \pi_t \\ y^*_t \\ x_t \\ g_t \\ i_{t-1} \\ a_{t-1} \end{matrix}\right\} + \left\{\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{matrix}\right\} \epsilon_t
$$

$$
\frac{\nu}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = \Phi
$$

$$
\frac{\left[(1-\gamma\nu)(1-\theta)\left(\frac{G}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]}{\left[1-(1-\gamma\nu)\theta\left(\frac{C}{\bar{C}}\right)^{\frac{\nu-1}{\nu}}\right]} = \Omega
$$