The fuzzy value of patent litigation under imprecise information

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Abstract. The vague notion of "probabilistic patents" is formalized through a model which combines real option theory and a fuzzy methodology. The imprecise estimates the patent holder possesses about her future profits, the validity and scope of the patent, the litigation costs, the court's decision etc. under a regime of imperfect enforcement of property rights are specified modelling uncertainty with fuzzy sets. Such methodology is embedded within a real option approach, where the value of a patent includes the option value of litigation. We study how the value of a patent is affected by the timing and incidence of litigation. The main results are compared with the empirical findings of previous results.

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1 Introduction

A central theme in the recent debate on patent reform is the need for policy-makers to explicitly recognize the policy dimension of legal uncertainty. Virtually, all property rights contain some elements of uncertainty and both litigation and settlement decisions occur in an environment characterized by imprecise information ([4],[26],[22],[12]). Fuzzy boundaries of the patent property right have been identified as a main cause of the recent explosion of patent litigation ([5],[6],[7]). The acknowledgement of the effects of uncertainty is outlined as an important element that law should accurately take into account. Recently, the European Commission has started studying the feasibility of alternative schemes against patent litigation risks, including insurance, in view of the dramatic explosion of patent litigation ([9]). As far as patent policy is concerned, it is emphasized that the effects of uncertainty should be incorporated in regulation and enforcement rules. This issue requires an accurate quantitative determination of the patent value. In this paper we propose a new comprehensive model, focusing on the different sources of uncertainty. Our main result is a valuation formula for patents which can be used for practical applications.

A patent is usually defined as a right to make exclusive use of an innovation at a predetermined cost for a predetermined period of time, i.e. the life of the patent. The patent holder may commercialize some products or licence her technology or use it for further developments. As such it is viewed as a real option, which is considered a most suitable way to model patents nowadays ([11]). The interpretation of patents as real options presupposes an enforceable property right. Yet, an increased number of patents have registered a high frequency of disputes and litigation involving patent holders and alleged infringers, so that the risk that a patent will be declared invalid is substantial. There is a wide variation across patents in their exposure to risk: as [19] and [2] have shown through detailed empirical evidence, for high-value patents and specific types of patentees the litigation risk can be quite high, in some cases almost offsetting what would otherwise be the R&D incentive provided by patent ownership. Bessen and Meurer [7] in a most comprehensive empirical research have found that technology that rely heavily on software are vexed by huge patent litigation costs, so that "the patent system has turned from a source of net subsidy to R&D to a net tax" ([6], page 1) Moreover, "roughly half of all litigated patents are found to be invalid, including some of great commercial significance" ([20], page 76). Thus, because of uncertainty in the enforcement of property rights, it has been stated that " a patent does not confer upon its owner the right to exclude but rather a right *to try* to exclude by asserting the patent in court" ([20], page 75). Accordingly, the clarification of the norms about intellectual property right has been indicated as the main challenge for lawyers and politicians in the next decades ([18]).

Because of imperfect enforcement of property rights, most patents represent highly uncertain or probabilistic property rights. Lemley and Shapiro ([20]) use the term *probabilistic patents.* Modelling patents as probabilistic rights requires to rethink how to reform the patent granting process and the patent litigation procedures.

In this paper we translate the vague notion of probabilistic patents into a mathematical model, where the valuation of patents can be performed by a combination of real options and a fuzzy methodology. In order to capture the notion of vagueness about the validity and scope of patents under a regime of imperfect enforcement of property rights, we introduce a notion of uncertainty, alternative to probability theory, through the theory of fuzzy sets. In this way, we are able to capture the vague and imprecise estimates the patent holder possesses about her future profits, the validity of the patent, the litigation costs, the court's decision. Moreover, we embed such methodology within a real option approach, where the value of a patent includes the option value of litigation.

There are various papers applying the theory of real options to the valuation of patents although very few of them introduce the patent enforcement process explicitly. [23] first estimated the distribution of the returns earned from holding patents as options which are renewed at alternative ages and require renewal fees. [8] builds on [23] and derives empirical predictions on the relationship between patents and market uncertainty. [25] implements a simulation model to value patents as complex options, taking into account uncertainty in the cost-to-completion of the project and the possibility of abandoning the project. [27] models sequential real options, analyzing the patenting decision and its effects in research, development and commercialization. [28], [17], [14] also investigate the patenting decision under technological and market uncertainty with competing firms. None of the above-mentioned papers introduce the risk of litigation. To the best of our knowledge, the only analyses of the option value of litigation are [21] and [3]. However, [21] is mainly focused on the timing and incidence of patent litigation and is concerned with the empirical estimates of patent litigation. [3] develops at length both the theory and the numerical implementation of some jump-diffusion models, where the risk of litigation is exogenously given and negatively affects the value of the patent in the form of discontinuities or jumps in the value process. He also addresses some issues of endogenous patent risk through a model where the patent holder possesses full knowledge about the probability distribution of the litigation risk.

Our paper is the first that combines a real option to litigate with a fuzzy valuation. The need for a fuzzy valuation comes from the common observation that patent claims are sometimes vaguely defined, the outcomes of a trial are difficult to forecast, legal costs are not easily predictable, it may be years before litigation is concluded, there may be divergence in parties' expectations about the court decision, future cash flows from commercialization are imprecise. Although the existing literature has identified the main determinants of litigation, it has not investigated how the value of a patent is affected by the timing and incidence of litigation under an appropriate framework of uncertainty. In this paper a novel approach to patents evaluation is presented and is based on applying fuzzy sets and fuzzy arithmetic to capture imprecise information in a way which is suitable for applicability too. Section 2 presents the model of a patent under imperfect enforcement of property rights, where the relevant parameters are fuzzy. The model is solved analytically for infinitely lived patents and in Section 3 the main results are compared with the empirical findings of previous studies. Finally, Section 4 concludes.

2 The fuzzy valuation method

In what follows let us introduce the basic method that is used in our application. For our purpose, let us introduce the essential definitions here and refer to the classical literature on fuzzy-stochastic theory, starting from [29], [15], [16], [24]. A fuzzy number is a fuzzy set (depicted with tilde) of the real line R, which is commonly defined by a normal, upper-semicontinuous, fuzzy convex membership function $\mu : R \rightarrow [0,1]$ of compact support. Fuzzy numbers can be interpreted as possibility distributions. While in classical set theory an element either belongs to a set or does not belong to a set, in fuzzy set theory we allow for a gradation of belonging, that can describe vague, imprecise or inaccurate knowledge about some estimates or data.

The γ-cut of a fuzzy number is given by: $\widetilde{\mu}_\gamma = \{x \epsilon R \mid \widetilde{\mu}(x) \geqq \gamma\}$, $\gamma \epsilon (0,1]$, and $\widetilde{\mu}_0 = cl\,\{x \epsilon R \mid \widetilde{\mu}(x) \geqq 0\}$, where cl denotes the closure of an interval. Let us write the closed intervals as $\widetilde{\mu}_\gamma = \left[\widetilde{\mu}_\gamma^-, \widetilde{\mu}_\gamma^+\right]$ for $\gamma \epsilon [0,1]$,so that $\widetilde{\mu}_\gamma^-$ denotes the left-hand side and $\widetilde{\mu}_\gamma^+$ the right-hand side of the γ-cut. Given two fuzzy numbers, $\widetilde{\mu}$ and $\widetilde{\eta}$, the partial order $\succsim$ on fuzzy numbers can be defined such that $\widetilde{\mu} \succsim \widetilde{\eta}$ means that $\widetilde{\mu}_\gamma^- \geq \widetilde{\eta}_\gamma^-$ and $\widetilde{\mu}_\gamma^+ \geq \widetilde{\eta}_\gamma^+$ for all $\gamma \epsilon [0,1]$. The arithmetic operations on two fuzzy numbers can be defined in the standard way, in terms of the γ-cuts for $\gamma \epsilon [0,1]$. In particular, for fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$ the addition and subtraction $\widetilde{\mu} \pm \widetilde{\eta}$ and the scalar multiplication $a\mu$, where $a \geq 0$, are fuzzy numbers as follows:

$$(\widetilde{\mu} + \widetilde{\eta})_\gamma = \left[\widetilde{\mu}_\gamma^- + \widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+ + \widetilde{\eta}_\gamma^+\right],$$
$$(\widetilde{\mu} - \widetilde{\eta})_\gamma = \left[\widetilde{\mu}_\gamma^- - \widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+ - \widetilde{\eta}_\gamma^-\right],$$
$$(a\widetilde{\mu})_\gamma = \left[a\widetilde{\mu}_\gamma^-, a\widetilde{\mu}_\gamma^+\right].$$

Moreover, multiplication between two fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$ is given by:

$$(\widetilde{\mu}\widetilde{\eta})_\gamma = \left[(\widetilde{\mu}\widetilde{\eta})_\gamma^-, (\widetilde{\mu}\widetilde{\eta})_\gamma^+\right],$$

where $(\widetilde{\mu}\widetilde{\eta})_\gamma^- = \min\left[\widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^+\right]$
and $(\widetilde{\mu}\widetilde{\eta})_\gamma^+ = \max\left[\widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^+\right]$.

Division between two fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$ is given by:

$$\left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma = \left[\left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma^-, \left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma^+\right],$$

where $\left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma^- = \min\left[\frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^+}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^+}\right]$ and $\left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma^+ = \max\left[\frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^+}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^+}\right]$.

Let (Ω, Π, A) be a probability space. A fuzzy-number-valued map $\widetilde{X}$ is called a fuzzy random variable if $\left\{(\omega, x)\ \epsilon\ \Omega \times R \mid \widetilde{X}(\omega)(x) \geq \gamma\right\}$ is measurable for all $\gamma \epsilon [0,1]$. It is called integrably bounded if both $\omega \rightarrow$

$\widetilde{X_\gamma}^-(\omega)$ and $\omega \rightarrow \widetilde{X_\gamma}^+(\omega)$ are integrable for all $\gamma\epsilon[0,1]$. The expectation $E(\widetilde{X})$ of the integrably bounded fuzzy random variable $\widetilde{X}$ is also defined by a fuzzy number

$E(\widetilde{X})(x) = \sup_{\gamma\epsilon(0,1)} \min\left\{\gamma, \mathbf{1}_{E(\widetilde{X})_\gamma}(x)\right\}, x\epsilon R,$
where $E(\widetilde{X})_\gamma = \left[\int_\Omega \widetilde{X_\gamma}^-(\omega)d\Pi(\omega), \int_\Omega \widetilde{X_\gamma}^+(\omega)d\Pi(\omega)\right]$, $\gamma\epsilon[0,1]$.

Let us now introduce the valuation method, which is based on fuzzy variables (see, f.e. [10]) along the lines in [1].
Suppose an innovator owns a patent allowing her to generate additional cash flows from commercializing some product. For simplicity, let us assume that the protection period is infinite, which facilitates the derivation of a closed-form solution. Commercialization is related with some expected income which fluctuates randomly. This income can be either in the form of royalties or in the form of increased revenues from the ability to exclude others from the market. Let P denote the net cash flow resulting from the patent, which is described by the following stochastic dynamics:

$$dP_t = P_t(\mu dt + \sigma dW_t)$$

where $\mu < r$ is the appreciation rate, r is the risk-free interest rate and σ is the volatility ($\mu\epsilon R$, $\sigma > 0$) and W_t is a standard Wiener process ([14]). In particular, let $\left\{\widetilde{P}_t\right\}_{t\geq 0}$ be a fuzzy stochastic process, which is specified as follows:
$\widetilde{P}_t(\omega)(x) = \max\left\{1 - \mid \frac{x-P_t(\omega)}{cP_t(\omega)} \mid, 0\right\}$, $0 < c < 1$,

that is, the fuzzy random variable $\widetilde{P}_t$ is of the triangular type, with centre $P_t(\omega)$, and left-width and right-width $cP_t(\omega)$. A triangular fuzzy number with centre $P_t(\omega)$ may be seen as a fuzzy quantity "approximately equal to $P_t(\omega)$". Observe that the fuzziness in the process increases if c is bigger. The choice of a triangle-type shape is not restrictive at all and is mainly adopted to facilitate computation. Moreover, it has a nice interpretation, in that represents the net cash flows under pessimistic (i.e. left side) and optimistic (i.e. right side) estimates. The γ-cuts of $\widetilde{P}_t(\omega)(x)$ are $\widetilde{P}^{\pm}_{t,\gamma}(\omega) = \left[\widetilde{P}^{-}_{t,\gamma}(\omega), \widetilde{P}^{+}_{t,\gamma}(\omega)\right] = [(1-(1-\gamma)c)P_t(\omega), (1+(1-\gamma)c)P_t(\omega)]$.

However, as argued in [20], the value of a patent depends not only on the uncertainty about the commercial significance of the innovation being patented, but also on the uncertainty about the validity and scope of the legal right being granted. The latter introduces the notion of "probabilistic patents". A patent does not confer an absolute right to exclude others from infringement; on the other hand, the actual scope (for which patent law often allows to use vague claim language; see [6]) and validity of a patent right and even whether the patent right will withstand litigation at all are uncertain and contingent issues. Therefore, following [21], a patent can be described as a portfolio consisting of two assets: an asset paying a stochastic cash flow P and an option to litigate. Note however, that [21] considers the case of a patent infringement (and ultimately the uncertainty over the validity of the patent), while our analysis applies to challenge suits too. Indeed, we compute both the option value of litigation held by the challenger, his threshold value and the value of the patent for the patent-holder. Since the option to litigate/sue can be exercised anytime prior to patent expiration, the decision to litigate can be modeled within a real option analysis.

2.1 The patent value

Let us formalize the notion of a probabilistic patent. Based on the alleged infringement, a challenger may decide to litigate at any time $\tau\epsilon(t,\infty)$ and if successful receives a fraction θ of future net cash flows, which is determined by court and not known in advance by the two parties (if unsuccessful, nothing is received). Litigation may end up being successful or not: let p denote the probability that the challenger will be successful, as from the beliefs of the patent-holder. In what follows, we assume that both θ and p are fuzzy numbers. It is in practice impossible to have an ex-ante absolutely correct estimate of the parameters that describe the structure of litigation. Thus, imprecision in such future estimates depends on how abstract or vague patent claims and boundaries are and is formalized through fuzzy sets. Specifically, $\widetilde{\theta}(x) = \max\left\{1- \mid \frac{x-\theta}{b\theta} \mid, 0\right\}$, that is, it has a symmetric triangle-type shape, with centre θ and width $b\theta$, and $\widetilde{p}(x) = \max\left\{1- \mid \frac{x-p}{dp} \mid, 0\right\}$, that is, it has a symmetric triangle-type shape, with centre p and width dp, where $0 < b, d$ and $b+d-bd < 1$. Finally, let $\widetilde{L}_i$, $i = 1, 2$ denote the litigation costs incurred by the patent-holder ($i = 1$) and the challenger ($i = 2$). Both are fuzzy numbers, so that $\widetilde{L}_i = \max\left\{1- \mid \frac{x-L_i}{f_i L_i} \mid, 0\right\}$ that is, it has a symmetric triangle-type shape, with centre L_i and width $f_i L_i$,where $0 < f_i < 1$.

We can specify the γ-cuts, that is,

$\widetilde{\theta}^{\pm}_{\gamma} = \left[\widetilde{\theta}^{-}_{\gamma}, \widetilde{\theta}^{+}_{\gamma}\right] = [(1-(1-\gamma)b)\theta, (1+(1-\gamma)b)\theta]$;

$\widetilde{p}^{\pm}_{\gamma} = \left[\widetilde{p}^{-}_{\gamma}, \widetilde{p}^{+}_{\gamma}\right] = [(1-(1-\gamma)d)p, (1+(1-\gamma)d)p]$;

$\widetilde{L}^{\pm}_{i,\gamma} = \left[\widetilde{L}^{-}_{i,\gamma}, \widetilde{L}^{+}_{i,\gamma}\right] = [(1-(1-\gamma)f_i)L_i, (1+(1-\gamma)f_i)L_i]$.

All agents are assumed to follow a policy of value-maximization. The optimal litigation time $\tau*$ is chosen by the challenger in response to the resolution of uncertainty related to P_t over time. The optimal stopping time $\tau*$ is defined as the first time P_t exceeds a critical level which is sufficiently high to justify the cost of litigation. More specifically, we will find $\tau* = \inf\left[t : P*_\gamma^\pm < P_t\right]$, where $P*_\gamma^\pm$ is obtained in Proposition 1.

The value of the patent under the risk of litigation for the patent-holder can be specified as follows:

$\widetilde{V}_\gamma(P_t, t) =$
$= \sup_{\tau\epsilon(t,\infty)} E\left\{\int_t^\infty e^{-r(s-t)} P_s ds - \int_\tau^\infty e^{-r(s-t)}(p\theta P_s)ds - e^{-r(\tau-t)} L_1\right\}.$

which is equivalent to the expected present value of cash flows from commercialization minus the option to litigate gained by the challenger, minus the present value of additional litigation costs.

We can prove Proposition 1, where the value of the patent is specified in terms of the both ends of the γ-cuts:

Proposition 1 *The payoff from commercializing under imperfect patent protection due to the risk of litigation is given by:*

$\widetilde{V}_\gamma^\pm(P_t, t) =$
$\left[\frac{\widetilde{P}_{t,\gamma}^-}{r-\mu} - \widetilde{\widetilde{L}}_{1,\gamma}^+ - \frac{(p\theta\Pi_t)_\gamma^+}{r-\mu}, \frac{\widetilde{P}_{t,\gamma}^+}{r-\mu} - \widetilde{\widetilde{L}}_{1,\gamma}^- - \frac{(p\theta P_t)_\gamma^-}{r-\mu}\right]$, *if* $P_t > P*_\gamma^\pm$,
$\left[\frac{\widetilde{P}_{t,\gamma}^-}{r-\mu} - (\widetilde{\widetilde{L}}_{1,\gamma}^+ + \frac{\varepsilon}{\varepsilon-1}\widetilde{\widetilde{L}}_{2,\gamma}^+)((\frac{P_t}{P*_\gamma^-})^\varepsilon), \frac{\widetilde{P}_{t,\gamma}^+}{r-\mu} - (\widetilde{\widetilde{L}}_{1,\gamma}^- + \frac{\varepsilon}{\varepsilon-1}\widetilde{\widetilde{L}}_{2,\gamma}^-)((\frac{P_t}{P*_\gamma^+})^\varepsilon)\right]$,
if $P_t \le P*_\gamma^\pm$,
where $P*_\gamma^\pm = \frac{\epsilon(r-\mu)}{\epsilon-1}\left[\frac{(1-(1-\gamma)f_2)L_2}{\theta p(1+(1-\gamma)(b+d+bd(1-\gamma))}, \frac{(1+(1-\gamma)f_2)L_2}{\theta p(1-(1-\gamma)(b+d-bd(1-\gamma))}\right]$
and $\epsilon = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 1.$

Proof. It follows from standard arguments in the theory of real options (see [11]), in view of the fact that the option value of litigation held by the challenger and denoted by $\widetilde{O}_\gamma^\pm(P_t, t)$ satisfies the following partial differential equation:

$$\frac{1}{2}\sigma^2 P_t^2 \partial_{P_t}^2 \widetilde{O}_\gamma^\pm + \mu P_t \partial_{P_t} \widetilde{O}_\gamma^\pm - r\widetilde{O}_\gamma^\pm - (p\theta\Pi_t)_\gamma^\pm = 0$$

with the final condition $-\widetilde{\widetilde{L}}_{2,\gamma}^\pm$ if $P_t > P*_\gamma^\pm$. Then $P*_\gamma^\pm$ is obtained as the solution of this Cauchy problem, if we take into account the expression for γ-cuts of fuzzy numbers. Finally, $\widetilde{V}_\gamma^\pm(P_t, t)$ can be easily obtained observing that $\widetilde{V}_\gamma^\pm(P_t, t) = \frac{\widetilde{P}_{t,\gamma}^\pm}{r-\mu} - \widetilde{O}_\gamma^\pm(P_t, t) - E\left[e^{-r(\tau*-t)}(\widetilde{\widetilde{L}}_{1,\gamma}^+ + \widetilde{\widetilde{L}}_{2,\gamma}^+)\right]$
■

It is apparent that the patent holder's value is smaller under the risk of litigation than it would be under perfect patent protection (equivalent

to $p = \theta = L_i = 0$). $P*_\gamma^\pm$represents the critical value between the stopping region where litigation occurs (for $P_t > P*_\gamma^\pm$) and the continuation region (for $P_t < P*_\gamma^\pm$). Observe that $P*_\gamma^\pm$is included in the set
$\frac{\epsilon(r-\mu)}{\epsilon-1}\left[\frac{(1-f_2)L_2}{\theta p(1+b+d+bd)}, \frac{(1+f_2)L_2}{\theta p(1-b-d+bd)}\right]$
which is positive. Note that because of the fuzzy modelling litigation occurs if the cash flow resulting from the patent is larger than $P*_\gamma^+$,while if it is less that $P*_\gamma^-$then waiting becomes the optimal strategy. In the intermediate range we cannot conclude for any of the two occurrences, so that this area will be called the "indecision area".

3 The role of imprecise information: results and discussion

In this section a numerical implementation is performed to evaluate the role of imprecise information on litigation decisions. To this purpose, let us concentrate on the characteristics of the critical value and perform a sensitivity analysis. The membership function for $\widetilde{P}*$ is plotted in Figures 1-6. Observe that the shape of the critical value is asymmetric to the right, implying that litigation is postponed on average in the fuzzy model in comparison with the non-fuzzy model, where the critical value is the crisp value $P*_1^\pm = \frac{\epsilon(r-\mu)}{\epsilon-1}\frac{L_2}{\theta p}$. Since on average the fuzzy threshold value is larger than without fuzziness, the decision about litigation in a fuzzy context differs from the decision in a non-fuzzy model.

It is useful to study the effects of the model parameters on the critical value. Figures 1-6 allow us to view the results graphically. It is shown how the fuzzy shape of the critical value changes as L_2 changes (Figure 1), as p changes (Figure 3), as θ changes (Figure 5). The dashed curves represent the shape of the critical value for the highest value of the parameters, the thin solid curve for the intermediate value and the thick solid curve gives the lowest value. For $\gamma = 1$ we obtain the crisp value, which is increasing in L_2 (Figure 1) and decreasing in p (Figure 3) and θ(Figure 5). Therefore, the challenger hastens litigation if his cost of litigation decreases, the probability of successful litigation increases, the fraction of future net cash-flows increases. Notice that the shape of the critical value is thinner (i.e. there is less dispersion from the crisp value) for the parameter values which have a positive impact on the challenger's value, it is thicker (i.e. there is more dispersion from the crisp value) for the parameter values which have a negative impact on the challenger's value. It implies that the "indecision area" about litigation becomes bigger for the parameter values reducing the challenger's value.

Figures 2,4,6 display the impact of "fuzziness". In Figure 2 an increase in fuzziness is measured by an increase in f_2, in Figure 4 by an increase in d and in Figure 6 by an increase in b. The dashed curves represent the shape of the critical value for the highest value of the parameters, the thin solid curve for the intermediate value and the thick solid curve gives the lowest value. Note that as fuzziness increases, the fuzzy shape of the critical value enlarges, the membership function becomes asymmetric and shifts to the right, implying that for conjectures on the left side (optimistic regarding the litigation costs, pessimistic regarding the probability of success and the fraction of future cash flows) the decision is not affected much by an increase in fuzziness, while for conjectures on the right side (pessimistic regarding the litigation costs, optimistic regarding the probability of success and the fraction of future cash flows), the decision is affected more. For conjectures on the left (right) side an increase in fuzziness decreases (increases) the perceived value of the patent. As a consequence, for left-side challengers imprecise information makes waiting less valuable, and litigation occurs earlier; for right-side challengers waiting becomes more valuable, and litigation occurs later. On average, in view of the asymmetric shape of the critical value, litigation tends to be postponed. These results are aligned with the literature on real options and ambiguity aversion and contrast with the effects of volatility of standard real options, where an increase in volatility always leads to an increase in the value of the investment opportunity.

In summary, the following testable implications can be found: (i) higher p, θ lead to more litigation (in keeping with [21]); (ii) the greater the fuzziness over the patent "strength" - probability of patent validity (p) and patent scope (θ)- the more delayed becomes litigation; (iii) the greater the fuzziness over the profit flow obtained with the patent the less likely becomes litigation (in keeping with [19]). It appears in empirical research ([7]) that industries characterized by complex technologies (e.g. electronics, computers; see [13]) or by less precise claims (software) have higher litigation rates than industries where patents have very well-defined boundaries (e.g. chemical compounds, pharmaceuticals). Since it is expected that p and θ are larger if patent boundaries are fuzzier, our results seem to be consistent with this empirical evidence too.

[insert Figures 1-6 here]

4 Conclusion

The notion of probabilistic patents requires a deep understanding of patent risk and an appropriate way of modelling the different sources of uncertainty, regarding both the commercial significance of the invention being patented and the validity and scope of the legal right being granted. Recent debates on the appropriate methods for calculating infringement damages have engaged the patent community: actually, patent litigation is one piece of the patent reform puzzle. In this paper we provide a valuation formula for patents within a framework that combines a real option to litigate with a fuzzy methodology and give a specific characterization of legal uncertainty. Our model is consistent with the view that the strength of patent protection and the presence of more codified and defined boundaries are crucial determinants of the patent value, the probability of licensing and patent validity.

A comprehensive model and the resulting valuation formula can have significant implications for patent policies. To the extent that the effects of uncertainty and enforcement are neglected in patent policy, incentives for innovation may become inconsistent, issues of patent invalidity arbitrary and the economic cost of patent litigation will be exacerbated. The contribution of uncertainty to total patent value and its effect on the litigation decisions need to be taken into account to set a good patent system and make it an effective tool for providing positive incentives.

In this paper we have pointed out a few relevant differences with respect to traditional real option modelling of patents. These are due to imprecise information about the driving parameters, that is, the probability of successful litigation, the gains from successful infringement suits or patent challenges, the litigation costs. A realistic estimate of these parameter values, which our model makes possible, is necessary to ensure that patent policy will be effective.

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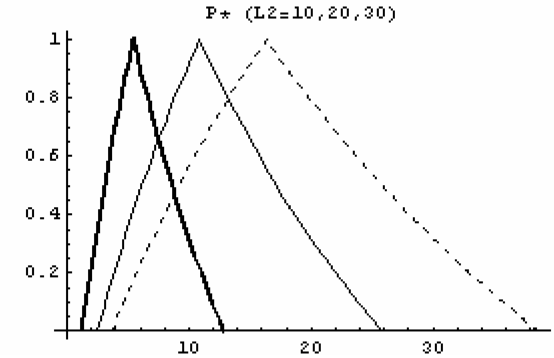


Figure 1

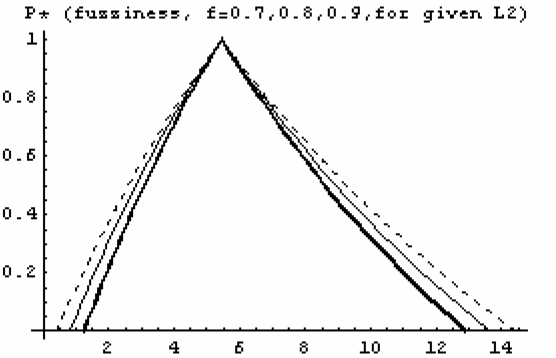
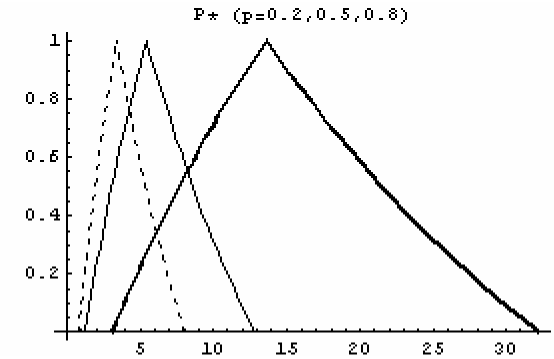


Figure 2



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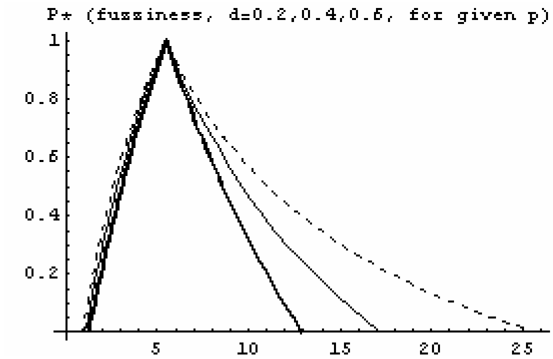

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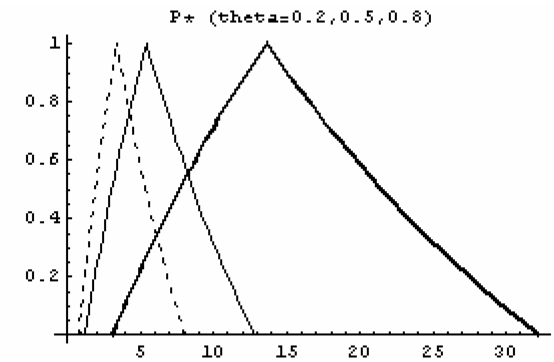

Figure 5

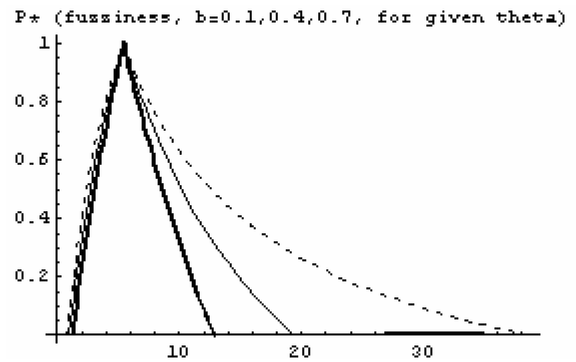

Figure 6