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Abstract

This paper aims at participating in the long-lasting debate about the analytical foundations of the Cournot equilibrium. In a homogeneous oligopoly, under standard regularity conditions, we prove that Cournot-Nash emerges both under (i) price competition and Cournot conjectures; and (ii) supply function competition with *ex post* market clearing. We demonstrate both results within a model of exogenous product differentiation.
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1 Introduction

One of the key aspects of the debate around Cournot and Bertrand models lies in the widespread opinion according to which the former model needs an auctioneer.[1] The auctioneer, indeed, might represent the answer to the central question arising under quantity competition in oligopoly about what mechanism is the market price established. Hence, in modelling homogeneous oligopolies, one seems facing a dilemma which is well summarised by Friedman (1977, p. 39): "one is faced with a choice between Cournot's version in which firms use the 'wrong' variable and the model behaves reasonably, and Bertrand's version in which firms use the 'correct' variable and the model behaves absurdly".

In an influential paper, Kreps and Scheinkman (1983) have proposed a way out from the impasse. They show that a capacity-constrained price-setting game yields the Cournot equilibrium. However, for a subset of admissible capacity levels, the use of mixed strategies at the price subgame is required. Moreno and Ubeda (2006) circumvent this difficulty by using the notion of reservation price to define a firm's supply curve as best reply to the rivals' aggregate supply, in such a way that a pure-strategy equilibrium always exists and coincides with Cournot.

In this paper we show that the Cournot equilibrium may result from either (i) Bertrand competition coupled with Cournot-Nash conjectures, or (ii) supply function competition with ex post market clearing. As for (i), we use the same approach as in Novshek (1980) and rely on the invertibility of the demand function, which, coupled with the Cournot-Nash conjecture (whereby the effects on price of a change in individual and aggregate output coincide), implies the attainment of the Cournot outcome at equilibrium. As for (ii), we assume firms compete in supply functions as in Klemperer and Meyer (1989) and the related growing literature,[2] except that we do not impose market clearing before firms solve for the Nash equilibrium, but after. We show that the resulting equilibrium coincides with Cournot. We prove both results with homogeneous as well as (exogenously) differentiated

[1] See Lambertini and Mosca (2014) for an updated account.

[2] See, for instance, Bolle (1992), Gilbert and Newbery (1992), Delgado and Moreno (2004), Ciarreta and Gutierrez-Hita (2006), Vives (2011), Bolle *et al.* (2013) and Holmberg *et al.* (2013). In particular, Bolle (1992) and Gilbert and Newbery (1992) are among the first to apply supply function competition to wholesale electricity markets.

products[3] relying on single-stage games generating pure-strategy equilibria only. Our approach looks then simpler than that requiring an upstream stage modelling the choice of capacity, followed by either price competition (Kreps and Scheinkman, 1983) or supply competition based on reservation prices (Moreno and Ubeda, 2006).

The paper is organised as follows. In section 2, we set up a general model of homogeneous oligopoly and prove the Kreps and Scheinkman result in a simple single-stage price game. In section 3, we investigate the supply function equilibrium. Under the standard *ex ante* market clearing condition, we rank such equilibrium between Bertrand and Cournot in a simpler way than Klemperer and Meyer (1989). Moreover, we establish there our central result: competition in supplty functions under *ex post* market clearing yields the Cournot-Nash equilibrium. In section 4, we use Singh and Vives's (1984) model to extend our main results to differentiated oligopoly. Section 5 concludes.

2 Price competition with Cournot conjectures

We consider an oligopolistic market where a population of single-product firms $\mathcal{N} = \{1, 2, 3, ..., n-1, n\}$ produces a homogeneous good whose inverse demand function is $p(Q)$, where p is price and Q is total output. We assume that $p(Q)$ is invertible for all $Q \geq 0$, with

i] $p(0) > 0$;

ii] $\partial p(Q)/\partial Q \equiv p'(Q) < 0$ for all $Q \geq 0$;

iii] $\partial^2 p(Q)/\partial Q^2 \equiv p''(Q) \leq 0$ for all $Q \geq 0$.

Production entails a cost function $C_i(q_i) > 0$ for all $q_i > 0$, with the following properties for all i:

[3] To the best of our knowledge, ours is the first attempt at modelling supply function competition in a differentiated oligopoly, Indeed, since Grossman (1981) and Klemperer and Meyer (1989), supply function competition has always been investigated in homogeneous oligopolies. It is worth stressing from now that, while under homogeneous goods the market clearing condition is unique, under product differentiation there exist as many market clearing conditions as the number of varieties being marketed, irrespective of the number of firms. For more on this, see section 4 below.

iv] $C_i(0) = 0$;

v] $\partial C_i(q_i)/\partial q_i \equiv C_i'(q_i) > 0$ for all $q_i \geq 0$;

vi] $\partial^2 C_i(q_i)/\partial q_i^2 \equiv C_i''(q_i) \geq 0$ for all $q_i \geq 0$.

The individual profit function of firm i is then $\pi_i = p(Q)\, q_i - C_i(q_i)$.

Competition in output levels yields the well known Novshek (1980) first order condition (FOC):

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + q_i p'(Q) - C_i'(q_i) = 0 \tag{1}$$

which, using the Cournot-Nash conjecture whereby $p'(Q) = \partial p(Q)/\partial q_i$, can be rewritten as follows:

$$p(Q)\left[1 + \frac{q_i p'(Q)}{p(Q)}\right] = C_i'(q_i)\,. \tag{2}$$

On the other hand, if firms compete in prices, the expression of individual profits becomes $\pi_i = pq_i(p) - C_i(q_i(p))$. Accordingly, the effect of a variation in price on firm i's profits is described by

$$\frac{\partial \pi_i}{\partial p} = q_i(p) + [p - C_i'(q_i(p))]\, q_i'(p)\,. \tag{3}$$

Now, observe that solving (2) w.r.t. q_i, one obtains:

$$q_i^* = \frac{C_i'(q_i) - p(Q)}{p'(Q)} \tag{4}$$

which implicitly identifies the unique optimal quantity in the Cournot model.

Let's go back to (3). For a moment, suppose firms sell differentiated varieties, in such a way that firm i's demand function is $q_i(p_i, \mathbf{p}_{-i})$, where $\mathbf{p}_{-i}$ is the vector of the $n-1$ prices of i's rivals. Assume that

$$\begin{gathered} \frac{\partial q_i(\cdot)}{\partial p_i} < 0\,;\ \frac{\partial q_i(\cdot)}{\partial p_j} > 0 \\ \left|\frac{\partial q_i(\cdot)}{\partial p_i}\right| > \frac{\partial q_i(\cdot)}{\partial p_j} \end{gathered} \tag{5}$$

for all $j \neq i$, i.e., the direct effect prevails on the cross effect. The profit function of firm i is

$$\pi_i = p_i q_i(p_i, \mathbf{p}_{-i}) - C_i(q_i(p_i, \mathbf{p}_{-i})) \tag{6}$$

and the relevant FOC is

$$\frac{\partial \pi_i}{\partial p_i} = q_i(p_i, \mathbf{p}_{-i}) + [p_i - C_i'(q_i(p_i, \mathbf{p}_{-i}))]\, q_i'(p_i, \mathbf{p}_{-i}) \tag{7}$$

where $q_i'(p_i, \mathbf{p}_{-i}) = \partial q_i(p_i, \mathbf{p}_{-i}) / \partial p_i$. To come back to the homogeneous good case, one has to take the limit of (6-7), and consider that, as varieties become identical, the price is unique. Hence, (7) becomes:

$$\frac{\partial \pi_i}{\partial p} = q_i(p) + [p - C_i'(q_i(p))]\, q_i'(p) \tag{8}$$

with $q_i'(p) < 0$. Seen with the eyes of a Cournot player, $q_i'(p)$ is the inverse of $p'(Q) = \partial p / \partial q_i$, which measures the effect on price of an output variation along the demand curve. Therefore, since $[p - C_i'(q_i(p))]\, q_i'(p) < 0$ for all $p > C_i'(q_i(p))$, we obtain:

$$\frac{\partial \pi_i}{\partial p} = 0 \Leftrightarrow q_i = \frac{C_i'(q_i) - p(Q)}{p'(Q)} \equiv q_i^* \tag{9}$$

while

$$\frac{\partial \pi_i}{\partial p} = q_i > 0 \tag{10}$$

if $p = C_i'(q_i)$. We may therefore claim:

Lemma 1 *Under assumptions [i-vi], the invertibility of the demand function and Cournot-Nash conjectures yield $\partial \pi_i / \partial p = 0$ in correspondence of the Cournot-Nash output level.*

The claim in Lemma 1 can be interpreted as follows. Jointly, (9-10) show two related facts:

- If a firm calculates the impact of a change in price on its profits and then adopts the Cournot-Nash conjecture, then the first derivative of π_i w.r.t. market price is nil in correspondence of the optimal output implicitly identified by (1) in the Cournot game. That is, imposing

$\partial\pi_i/\partial p = 0$ and solving for q_i in the Bertrand game yields the Cournot outcome as in Kreps and Scheinkman (1983), as long as the interplay between price and individual output is evaluated along the demand function. In turn, referring to the capacity-building game examined by Kreps and Scheinkman, this amounts to saying that if firms look for the output level at which they should stop accumulating capacity, they may just examine the effect of a variation of price on individual profits, impose Cournot conjectures and nullify the relevant derivative.

- When price equals marginal cost, the first derivative of the profit function w.r.t. price is not vanishing. This involves marginal cost pricing being a corner solution, Pareto-inefficient for firms, as they would like to escape from it by shrinking output levels and raising market price.

3 Supply function competition

Let us now consider competition in supply functions. Following Klemperer and Meyer (1989), we define the supply function of firm i as $S_i(p)$, with $S_i'(p) > 0$ and $S_i''(p) \geq 0$ for all $i = 1, 2, 3, ..., n$, so that the profit function becomes $\pi_i = pS_i(p) - C_i(S_i(p))$. Then, assuming that there is a unique market clearing price and imposing the market clearing condition according to which total demand $D(p)$ must equal industry supply $\mathbf{S}(p) = \sum_{i=1}^{n} S_i(p)$, firm i's maximization problem can be written as

$$\max_p \pi_i = p\left[D(p) - \mathbf{S}_{-i}(p)\right] - C_i\left(D(p) - \mathbf{S}_{-i}(p)\right) \tag{11}$$

where $\mathbf{S}_{-i}(p) \equiv \sum_{j \neq i} S_j(p)$ and the resulting FOC

$$\frac{\partial \pi_i}{\partial p} = D(p) - \mathbf{S}_{-i}(p) + \left[p - C_i'\left(D(p) - \mathbf{S}_{-i}(p)\right)\right]\left[D'(p) - \mathbf{S}_{-i}'(p)\right] = 0 \tag{12}$$

delivers:

$$\mathbf{S}_{-i}'(p) = \frac{q_i}{p - C_i'(q_i)} + D'(p) \tag{13}$$

in which $q_i \equiv D(p) - \mathbf{S}_{-i}$ at the market-clearing price (cf. Klemperer and Meyer, 1989, p. 1248). In order to carry out a comparison of the different equilibria, we can consider the situation where all firms are symmetric, whereby $q_i = q$, $\mathbf{S}_{-i}'(p) = (n-1)S'(p)$, and $C_i'(q_i) = C'(q)$.

Compare first Cournot and supply function competition and plug q^* into the r.h.s. of (12) to obtain

$$\frac{C'(q)-p(Q)}{p'(Q)}+\left[p-C_i'\left(D(p)-\mathbf{S}_{-i}(p)\right)\right]\left[D'(p)-\mathbf{S}_{-i}'(p)\right] \tag{14}$$
$$=\frac{C'(q)-p}{p'(Q)}+\left[p-C'(q)\right]S'(p)$$

since $D'(p) = n \cdot S'(p)$ and $\mathbf{S}'_{-i}(p) = (n-1) \cdot S'(p)$ at the market-clearing price. For competition in supply function to yield higher output and lower profits than Cournot competition, (14) must be *positive* (recall that under supply function equilibria the FOC is taken on price). Indeed, so it is, since $[C'(q) - p]/p'(Q) = q^* > 0$, $p > C'(q)$ and $S'(p) > 0$.

The comparison between Bertrand and supply function equilibria is straightforward, since at the Bertrand-Nash equilibrium marginal cost pricing obtains and consequently (3) reduces to $\partial\pi_i/\partial p = q > 0$ for any q including that implicitly identified by (13). Hence, the Bertrand equilibrium delivers higher output and lower profits than supply function competition.

The foregoing analysis amounts to a different, arguably simpler, proof of a result already attained by Klemperer and Meyer (1989, pp. 1258-60):

Proposition 2 *Under assumptions [i-vi], the individual and industry output and price emerging at the supply function equilibrium are intermediate between those generated by price and quantity competition.*

Now, still assuming that a unique market clearing price exists, we examine what happens under supply function competition without imposing the market clearing condition *ex ante*. To do so, we introduce the concept of notional price $\widehat{p} = f\left(S_i(p), \mathbf{S}_{-i}(p)\right)$, which is the price that all firms expect to prevail as a function of the vector of their supplies, in such a way that in the unique equilibrium their expectation must be confirmed and the notional price and the market clearing price coincide. As a consequence, firm i's maximisation problem becomes

$$\max_p \pi_i = \widehat{p}(\cdot)\, S_i(p) - C_i\left(S_i(p)\right) \tag{15}$$

and the relevant FOC is

$$\frac{\partial \pi_i}{\partial p} = \widehat{p}(\cdot)\, S_i'(p) + S_i(p)\, \widehat{p}'(\cdot)\, \mathbf{S}'(p) - C_i'\left(S_i(p)\right) S_i'(p) = 0 \tag{16}$$

In order to impose market clearing *ex post*, we require $\widehat{p}(\cdot) = p$. This, together with symmetry across firms, allows us to rewrite (16) as follows:

$$\frac{\partial \pi}{\partial p} = pS' + Sp'\mathbf{S}' - C'S' = 0 \tag{17}$$

The last step amounts to noticing that, at equilibrium, $p'\mathbf{S}' = 1$ and therefore the solution to the above equation is

$$p = \frac{C'S' - S}{S'} \tag{18}$$

which coincides with the Cournot equilibrium price $p^* = [C'(q)\, q' - q] / q'$ as $S = q$ at the market clearing price.

The above discussion proves our central result:

Proposition 3 *Under assumptions [i-vi], imposing ex post market clearing under supply function competition yields the Cournot equilibrium.*

There is an interesting implication of the above result as for the long-standing debate initiated by Bertrand's critique to the Cournot assumption of firms setting quantities, and the seeming lack of an auctioneer. The core issue is not the need of a third agent (other than firms or consumers) in charge of setting the price, but rather whether firms impose market clearing *before* or *after* taking FOCs w.r.t. the relevant market variable.

Lemma 1 and Proposition 3 jointly imply:

Corollary 4 *Under Cournot-Nash conjectures, both Bertrand competition and supply function competition (with ex post market claering) collapse into the Cournot equilibrium.*

4 Product differentiation in oligopoly

In this section, we illustrate the extension of the above results when product differentiation enters the picture. To this end, we use the differentiated oligopoly version of the duopoly model introduced by Singh and Vives (1984). The utility function of the representative consumer is

$$U = a\sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\sigma \sum_{j\neq i} q_i q_j\right) \tag{19}$$

where $a > 0$ and parameter $\sigma \in (0, 1]$ measures the degree of product substitutability, i.e., σ is an inverse measure of product differentiation. When $\sigma = 1$, the product is homogeneous.[4] The direct demand functions resulting from the constrained maximisation problem are:

$$q_i = \max\left\{0, \frac{a}{1+\sigma(n-1)} - \frac{p_i[1+\sigma(n-2)] - \sigma\sum_{j\neq i} p_j}{(1-\sigma)[1+\sigma(n-1)]}\right\} \forall\, i = 1, 2, ..., n. \tag{20}$$

System (20) can be inverted to yield the demand system needed to model Cournot competition:

$$p_i = a - q_i - \sigma \sum_{j\neq i} q_j \,\forall\, i = 1, 2, ..., n. \tag{21}$$

Note that (20) satisfies the properties in (5).

On the supply side, all single-product firms operate with the same technology summarised by the convex cost function $C_i = cq_i^2/2$, $i = 1, 2, ..., n$, with $c > 0$.

We set out with Cournot competition. The problem of firm i is

$$\max_{q_i} \pi_i = \left(a - q_i - \sigma\sum_{j\neq i} q_j\right) q_i - \frac{cq_i^2}{2} \tag{22}$$

The optimal individual output in the symmetric Cournot-Nash (CN) equilibrium is

$$q^{CN} = \frac{a}{2 + c + \sigma(n-1)} \tag{23}$$

the corresponding price is

$$p^{CN} = \frac{a(1+c)}{2 + c + \sigma(n-1)} \tag{24}$$

and profits are

$$\pi^{CN} = \frac{a^2(2+c)}{2[2 + c + \sigma(n-1)]^2} \tag{25}$$

[4]If $\sigma = 0$, the two varieties do not interact and firms are separate monopolists. We also disregard the range $\sigma \in [-1, 0)$, where products are complements.

Under price competition, the relevant demand system is (20). Bertrand-Nash (BN) equilibrium magnitudes are:

$$p^{BN} = \frac{a\left[(1+c)(1+\sigma(n-2)) - \sigma^2(n-1)\right]}{c\left[1+\sigma(n-2)\right] + \left[2+\sigma(n-3)\right]\left[1+\sigma(n-1)\right]} \tag{26}$$

$$q^{BN} = \frac{a\left[1+\sigma(n-2)\right]}{c\left[1+\sigma(n-2)\right] + \left[2+\sigma(n-3)\right]\left[1+\sigma(n-1)\right]} \tag{27}$$

$$\pi^{BN} = \frac{a^2\left[1+\sigma(n-2)\right]\left[(2+c)(1+\sigma(n-2)) - 2\sigma^2(n-1)\right]}{2\left[c\left[1+\sigma(n-2)\right] + \left[2+\sigma(n-3)\right]\left[1+\sigma(n-1)\right]\right]^2} \tag{28}$$

We now consider competition in supply functions. We confine our attention to the case of linear supply functions, adopting the procedure suggested by Ciarreta and Gutierrez-Hita (2006). The supply function of firm i writes $S_i = \beta_i p_i$, and the *ex ante* market clearing condition is $S_i = q_i$ for all $i = 1, 2, ..., n$, where q_i is defined as in (20). The presence of n varieties requires imposing n market-clearing conditions, one for each variety. Consider the individual demand function defined in (20). Whenever $q_i > 0$, market clearing requires $\beta_i p_i = q_i$:

$$\beta_i p_i = \frac{a}{1+\sigma(n-1)} - \frac{p_i\left[1+\sigma(n-2)\right] - \sigma\sum_{j\neq i} p_j}{(1-\sigma)\left[1+\sigma(n-1)\right]} \tag{29}$$

Solving the system of n equations defined by (29) delivers the market-clearing price for each variety $i = 1, 2, ..., n$:

$$p_i = \frac{a\left[1+(1-\sigma)\beta_j\right]}{1+\beta_i + \left[1+(n-2)\sigma\right]\beta_j + (1-\sigma)\left[1+(n-1)\sigma\right]\beta_i\beta_j} \tag{30}$$

where, to simplify the exposition, we have set $\sum_{j\neq i}\beta_j = (n-1)\beta_j$. The profit function of firm i is defined as

$$\pi_i = p_i S_i - \frac{cS_i^2}{2} = \beta_i p_i^2 - \frac{c\beta_i^2 p_i^2}{2} \tag{31}$$

where p_i is (30). Maximising π_i w.r.t. β_i and solving the resulting FOC under the symmetry condition $\beta_j = \beta_i$ delivers:

$$\beta_{ea}^{SF} = \frac{(n-2)\sigma - c + \sqrt{(2+c)\left[2+c+2(n-2)\sigma\right] + \left[n(n-8)+8\right]\sigma^2}}{2\left[c(1+(n-2)\sigma) + (1-\sigma)(1+(n-1)\sigma)\right]} \tag{32}$$

where superscript SF mnemonics for *supply function* and subscript ea stands for *ex ante*. Note that $\beta_{ea}^{SF} \in \mathbb{R}^{+}$ for all $n \geq 2$. The equilibrium supply function is:

$$S_{ea}^{SF} = \frac{2a}{2 + c + n\sigma + \sqrt{(2+c)\left[2 + c + 2(n-2)\sigma\right] + \left[n(n-8)+8\right]\sigma^2}} \tag{33}$$

Using the above expression one can easily obtain the corresponding equilibrium profits π_{ea}^{SF}.

We now turn to *ex post* market clearing. Not to impose the ex ante market clearing condition (29) entails substituting $q_i = \beta_i p_i$ into the individual profit function, which therefore becomes:

$$\pi_i = \left(a - \beta_i p_i - \sigma \sum_{j \neq i} \beta_j p_j\right) \beta_i p_i - \frac{c\beta_i^2 p_i^2}{2} \tag{34}$$

Taking the FOC on β_i and imposing symmetry across β's (but not yet across prices), one obtains the equilibrium level of the slope of the supply function:

$$\beta^*\left(p_i, \mathbf{p}_{-i}\right) = \frac{a}{(2+c)\, p_i + \sigma \sum_{j \neq i} p_j} \tag{35}$$

where $\mathbf{p}_{-i}$ is the vector of the prices set by the $n-1$ rivals of firm i. Imposing now symmetry across prices entails that $\beta^* = a/\left[2 + c + \sigma(n-1)\right]$, and solving the *ex post* market clearing condition

$$p = a - \beta^* p \left[1 + \sigma(n-1)\right] \tag{36}$$

we obtain

$$p_{ep}^{SF} = \frac{a(1+c)}{2 + c + \sigma(n-1)} \tag{37}$$

where subscript ep mnemonics for *ex post* market clearing. Now we can simplify all of the relevant equilibrium expressions, which can be written as follows:

$$\beta_{ep}^{SF} = \frac{1}{1+c}\, ; \; q_{ep}^{SF} = \frac{a}{2 + c + \sigma(n-1)}$$

$$\pi_{ep}^{SF} = \frac{a^2(2+c)}{2\left[2 + c + \sigma(n-1)\right]^2} \tag{38}$$

and it is apparent that $p_{ep}^{SF} = p^{CN}$, $q_{ep}^{SF} = q^{CN}$ and $\pi_{ep}^{SF} = \pi^{CN}$. Hence, we have proved that supply function competition with ex post market clearing yields Cournot equilibrium also in a differentiated oligopoly.

The next step consists in showing that Lemma 1 holds true also under product differentiation. To this aim, we have to model the equivalent of Cournot conjectures in the differentiated Bertrand model based upon (20). We proceed as follows. If a price-setting firm, say i, anticipates that, in equilibrium, all prices must coincide, then (20) rewrites as

$$q_i|_{p_j=p_i} = \frac{a-p_i}{1+\sigma(n-1)} \tag{39}$$

If the anticipation, generating (39), is plugged into the inverse demand function (21), the latter rewrites as

$$p_i = a - q_i|_{p_j=p_i} - \sigma\sum_{j\neq i} q_j|_{p_k=p_j} = a - \frac{a-p_i}{1+\sigma(n-1)} - \sigma\sum_{j\neq i}\frac{a-p_j}{1+\sigma(n-1)} \tag{40}$$

Using (39-40), the profit function of firm i becomes

$$\pi_i = q_i|_{p_j=p_i}\left(a - q_i|_{p_j=p_i} - \sigma\sum_{j\neq i} q_j|_{p_k=p_j}\right) - \frac{c\left(q_i|_{p_j=p_i}\right)^2}{2} = \tag{41}$$

$$\frac{a-p_i}{1+\sigma(n-1)}\left(a - \frac{a-p_i}{1+\sigma(n-1)} - \sigma\sum_{j\neq i}\frac{a-p_j}{1+\sigma(n-1)}\right) - \frac{c(a-p_i)^2}{2[1+\sigma(n-1)]^2}$$

The relevant FOC w.r.t. p_i is

$$\frac{\partial \pi_i}{\partial p_i} = \frac{a(1+c) - (2+c)p_i - \sigma\sum_{j\neq i} p_j}{[1+\sigma(n-1)]^2} = 0 \tag{42}$$

Imposing symmetry across prices and solving the above equation one obtains the equilibrium price

$$p^* = \frac{a(1+c)}{2+c+\sigma(n-1)} = p^{CN} \tag{43}$$

The remaining equilibrium magnitudes confirm the coincidence between this equilibrium outcome and the Cournot-Nash one.

5 Concluding remarks

What we have shown is that the Cournot equilibrium can be reached along three alternative routes. The first and obvious is the game in the output space, *by the book*. The remaining two emerge from settings which are seemingly more competitive, wherein firms must be explicitly concerned with the equilibrium levels of price(s), something they are not required to do in the Cournot model without an auctioneer. The first new route we have explored consists in imposing Cournot conjectures in the price-setting game, which also reproduces Kreps and Scheinkman's (1983) result in a single-stage model. The second route amounts to impose *ex post* market-clearing in the game where firms compete in (linear) supply functions. All of this holds true irrespective of the degree of product differentiation.

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