ISSN 2282-6483

Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

To Know or Not To Know: Strategic Inattention and Endogenous Market Structure

Roberto Cellini
Luca Lambertini
Gianmarco I.P. Ottaviano

Quaderni - Working Paper DSE N°987

To Know or Not To Know: Strategic Inattention and Endogenous Market Structure*

Roberto Cellini[(a)] Luca Lambertini[(b)]

Gianmarco I.P. Ottaviano[(b,c)]

February 2015

Abstract

We model an industry in which a discrete number of firms choose the output of their differentiated products deciding whether or not to consider the impact of their decisions on aggregate output. We show that two threshold numbers of firms exist such that: below the lower one there is a unique equilibrium in which all firms consider their aggregate impact as in standard oligopoly; above the higher threshold there is a unique equilibrium in which all firms disregard that impact as in standard monopolistic competition; between the two thresholds there are two equilibria, one in which all firms consider their aggregate impact and the other in which they do not. We then show that our model of 'strategic inattention' is isomorphic to a model of 'strategic delegation' with managerial compensation based on relative profit performance.

Keywords: information; strategic interaction; monopolistic competition; oligopoly; delegation.

JEL Codes: D43, L13.

---

*Contacts: [(a)]University of Catania, cellini@unict.it; [(b)]University of Bologna, luca.lambertini@unibo.it, gianmarco.ottaviano@unibo.it; [(c)]London School of Economics, g.i.ottaviano@lse.ac.uk. We would like to thank Herbert Dawid, Stephen Martin, Juan Prieto Rodriguez, Jacques-Francois Thisse as well as participants at various seminars for useful comments and suggestions on a previous draft. The usual disclaimer applies.

1 Introduction

Oligopoly models *à la* Cournot assume that firms choose their output levels considering the impact of their individual choices on overall industry output. In monopolistic competition models firms are assumed to take, instead, industry output as given. With a continuum of firms this assumption is inconsequential as there is no individual impact on the industry. Differently, with a discrete number of firms each firm does have an individual impact on the industry, but monopolistic competition models make the behavioral assumption that the firm neglects this piece of information.

A common way to justify this neglect is to argue that, for some purposes, monopolistic competition provides a convenient approximation to the exact Cournot equilibrium when there is a 'large number' of firms so that the individual impact of any of them on the industry can be considered negligible in practice (Dixit and Stiglitz, 1977). This justification has been criticized as being both mathematically inconsistent (see, e.g., Keen and Standish, 2006) and not necessarily innocuous in terms of approximating the correct market outcome (see, e.g., d'Aspremont *et al.*, 1996). Be that as it may, the 'large number' fix is widely used and probably all that could be said on its implications has already been said. What has not been discussed yet is, instead, another natural but very different justification of why a discrete number of firms may disregard the impact of their individual choices on aggregate output: there might well be circumstances in which a firm's profit maximizing choice is indeed to strategically neglect that piece of information so that an industry equilibrium emerges in which a discrete number of firms choose to behave as monopolistic competitors rather than as oligopolists. The aim of this paper is to fill the gap by characterizing those circumstances in a partial equilibrium model of an industry in which oligopolistic or monopolistically competitive market structures arise endogenously in equilibrium from firms' decisions on 'strategic inattention'.

Specifically, we consider an industry in which a discrete number of firms supply horizontally differentiated products. Demand is linear in quantity consumed and total cost is quadratic in the quantity produced. Firms are single-product profit-maximizers and play a non-cooperative two-stage game. In the first stage, they simultaneously decide whether or not to use information that industry output equals the sum of their individual outputs. In the second stage, they choose their output levels based on information that in the first stage they decided to use. If they decided to consider their individual impact on industry output, in the second stage market structure corresponds to a familiar Cournot oligopoly. If they chose instead to neglect that impact, in the second stage market structure corresponds to familiar monopolistic competition.

In equilibrium, either or both alternative market structures may emerge, depending on the values of demand and cost parameters, and crucially on the number of firms. For a given number of firms, high (low) product differentiation and weak (strong) negative reaction of marginal cost to scale give rise to a unique subgame perfect equilibrium in pure strategies characterized by oligopoly (monopolistic competition); an oligopolistic and a monopolistically competitive subgame perfect equilibria in pure strategies coexist, instead, for intermediate product differentiation and moderate reaction of marginal cost to scale. For given demand and cost parameters, oligopoly (monopolistic competition) arises in the subgame perfect equilibrium in pure strategies when the number of firms is low (large) while an oligopolistic and a monopolistically competitive subgame perfect equilibria in pure strategies coexist for an intermediate number of firms. The outcome in which some firms behave as oligopolists and others as monopolistic competitors never arises in a subgame perfect equilibrium in pure strategies. We show that it may, however, emerge in a subgame perfect equilibrium in mixed strategies.

It is important to stress the difference between these results and the traditional view of monopolistic competition as an approximation to the exact oligopolistic equilibrium when market structure is given and there is a 'large number' of firms. Our novel insight is that for a wide range of parameter values monopolistic competition emerges as 'the' equilibrium market structure when firms can decide to strategically neglect the aggregate impact of their choices. Indeed, the set of parameter values for which monopolistic competition is the only equilibrium market structure is much wider than the set for which oligopoly is the only one. We also show that, rather than being a mere intellectual curiosity, a firm's strategic neglect of its individual impact on the industry is implied by a simple realistic managerial contract based on Relative Performance Evaluation (RPE), in which firms benchmark their managers' performance in terms of own profit against rivals' average profit (see, e.g., Aggarwal and Samwick, 1999).

Our analysis contributes to four strands of literature. First, a key feature of our model is that the market regime (oligopoly or monopolistic competition) is determined endogenously by the strategic choices of firms. In this respect, our analysis is related to the literature on 'endogenous market structure'. In this literature what is endogenous is the number of competing firms with a focus on their entry and exit decisions in an oligopolistic setting (see, e.g., Etro, 2008, 2011, 2012; Dunne *et al.*, 2013). This focus on entry and exit is also the hallmark of old and new models of monopolistic competition (see, e.g., Spence, 1976; Dixit and Stiglitz, 1977; Ottaviano *et al.*, 2002; Behrens and Murata, 2007). With respect to all these works, we take the reverse angle. Keeping the number of firms fixed, we study how oligopoly or monopolistic competition endogenously arise in equilibrium from the decisions of firms to use or ignore the aggregate impact of their individual output choices.

Second, in our model information on a firm's individual impact on the aggregate is freely accessible. Still, it can be individually convenient for a firm to ignore that piece of information, which therefore has negative value for the firm. In this respect, our analysis contributes to the literature on the value of information in games. Kamien *et al.* (1990), Bassan *et al.* (1997) and Bassan *et al.* (2003) are all examples showing that information is not relevant *per se*, but rather for the way it affects players' best replies to rivals. Similarly, Kadane *et al.* (1996) show that a Bayesian agent may find it rational to pay not to see some pieces of information. Safra and Sugarik (1993) make a similar point for cases in which agents do not choose according to the expected utility principle. In Barros (1997), by ignoring information on the actions taken by their (sale) agents, oligopolistic principals forgo the possibility of appropriating the agents' benefits from their relation specific investments, which ends up increasing the principals' expected profits. Differently from these works, we investigate the strategic benefits firms may enjoy from ignoring pieces of available information concerning their interdependence. Lastly, limited ability to process information has also been used to explain why agents may not make full use of available information. For example, in 'rational inattention' models available information is not used because agents' ability to translate information into action is assumed to be constrained by a finite 'capacity' to process information (see, e.g., Sims, 2010, for a survey). In our model information is freely processable, so processing capacity is not an issue and inattention arises, instead, from strategic behavior.

Third, in our model whether all firms or only a subset of them decide to consider or to ignore their individual impact on aggregate output is an endogenous outcome. In this respect, our analysis also speaks to the studies on the interactions among 'asymmetric' firms that may differ along several dimensions such as size, objectives and organization. Chirco *et al.* (2013) provide a review of theoretical, empirical, and experimental works supporting the coexistence of heterogeneous motives for firms in an oligopolistic market, with specific attention to the delegation of market operations to managers. More directly connected to our paper, Kokovin *et al.* (2014) present a model in which oligopolistic and monopolistically competitive firms interact simultaneously in markets with differentiated products. In their model, however, the asymmetry across firms is given and linked to exogenous differences in firm size, with large oligopolists interacting with a fringe of small monopolistic competitors (see, also, Shimomura and Thisse, 2012). In the same vein, Anderson *et al.* (2013) develop and use the concept of 'aggregative game' to analyze the free entry of firms in markets where oligopolistic and monopolistically competitive producers coexist. Differently from these works, we do not assume that exogenous differences in firm size determine whether or not firms ignore their individual impacts on aggregate outcomes.

Fourth and last, we contribute to the literature on strategic delegation games. In our model the first-stage choice of neglecting information on the aggregate impact of individual output supports more aggressive production decisions in the second stage. The idea that decisions made in the first stage of a game can be used to commit to more aggressive behavior in the second stage has a long tradition in oligopoly models. In particular, from a mathematical point of view, the problem our firms solve closely resembles the strategic choice of delegation contracts to managers analyzed by Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Fumas (1992), Aggarwal and Samwick (1999) and Miller and Pazgal (2001) among others. We show that the conditions that dictate the emergence of oligopoly or monopolistic competition as equilibrium market structures in our game are the same that support strategic delegation to managers through a simple realistic RPE contract (see, e.g., Gibbons and Murphy, 1990; Murphy, 1999).

The rest of the paper is organized as follows. Section 2 presents the model, characterizes its subgame perfect equilibria in pure strategies for an arbitrary number of firms, and discusses comparative statics with respect to demand and cost parameters as well as to the number of firms. Second 3 zooms in for a close-up of the two-firm case to provide an intuitive discussion of our findings and a characterization of subgame perfect equilibria in mixed strategies. Section 4 investigates the formal connections between our results and those in the literature on strategic delegation through managerial contracts. Section 5 provides some concluding remarks.

2 A Model of Strategic Inattention

Consider an industry in which n single-product firms (indexed $h = 1, ..., n$) sell n horizontally differentiated products facing linear inverse demand

$$p_h = a - \beta q_h - \sigma Q, \tag{1}$$

where p_h and q_h are the price and the output level of firm h, while $Q = \sum_{h=1}^{n} q_h$ is industry output.[1] Total cost is assumed to be a quadratic function of output:

$$c_h = cq_h + bq_h^2.$$

While the demand parameters are assumed to be positive, the cost parameters are assumed to be non-negative with $c < a$.[2] To make future expressions less cumbersome, it is useful to define the following positive bundling parameters

$$\alpha \equiv \frac{a-c}{\beta+b}, \; \eta \equiv \frac{\sigma}{\beta+b}, \; \gamma \equiv \beta + b$$

so that the profit of firm i can be written as

$$\pi_h = \gamma \left(\alpha - q_h - \eta Q \right) q_h. \tag{2}$$

where all parameters are again positive. Among them, as we will see, the key parameter will turn out to be η. This measures the impact of aggregate output Q on the firm's profit margin π_h/q_h relative to the impact of own output q_h. Equivalently, it measures the (absolute value of the) change in q_h needed to keep the profit margin unchanged for a given change in Q. Intuitively, η measures the dependence of the firm's profit on the industry aggregate. In the limit case $\eta = 0$ the firm's profit is independent from aggregate output. As η grows, the firm's profit increasingly depend on aggregate output. Hence, we call η the 'aggregate dependence' parameter. In the case of constant marginal cost ($b = 0$), η is a pure demand parameter inversely

[1] In the case of a discrete number of firms, (1) corresponds to the demand function first introduced by Bowley (1924) and then revisited, *inter alia*, by Spence (1976), Dixit (1979) and Singh and Vives (1984). Ottaviano *et al.* (2002) and Melitz and Ottaviano (2008) have adapted it to the case of a continuum of firms.

[2] We assume linear demands because this specification yields straightforward comparative statics. In light of Aggarwal and Samwick (1999), the results in this section can be generalized to nonlinear demand functions as long as $\partial p_h(q_h, Q)/\partial q_h < 0$ and $\partial p_h(q_h, Q)/\partial Q < 0$.

measuring the extent of product differentiation: stronger product differentiation implies lower aggregate dependence. With increasing marginal cost ($b > 0$), aggregate dependence is also affected by the gradient of marginal cost.

Market structure is endogenized by assuming that firms play a two-stage game, with both stages characterised by complete, symmetric and imperfect information. In the second stage, firms $h = 1, ..., n$ simultaneously maximize profit π_h with respect to their output level q_h. In the first stage, they simultaneously decide whether in the second-stage profit maximization they will take the condition $Q = \sum_{i=h}^{n} q_h$ into account or they will rather consider Q as a given parameter. We look for subgame perfect equilibria in pure strategies focusing on symmetric outcomes.

2.1 Second Stage: How Much Output

Solving backwards, we start with the second stage and consider a generic composition of the industry in which k firms have decided to neglect the aggregate impact of their individual output choices and $n - k$ firms have decided to consider it. For parsimony, we call the former 'myopic' and the latter 'smart'. Industry output can then be expressed as

$$Q = \sum_{i=1}^{k} q_i^m + \sum_{j=k+1}^{n} q_j^s \tag{3}$$

where q_i^m is the output of myopic firm i with $i = 1, ..., k$ and q_j^s is the output of smart firm j with $j = k + 1, ..., n$.

Given (2), the FOC for profit maximization by a smart firm is

$$\frac{\partial \pi_i^s}{\partial q_i^s} = \gamma \left[\alpha - (2 + \eta)\, q_i^s - \eta Q\right] = 0, \tag{4}$$

with SOC satisfied for all parameter values.

After imposing symmetry $q_i^m = q^m$ for all $i = 1, ..., k$ and $q_j^s = q^s$ for all $j = k + 1, ..., n$, (3) becomes $Q = kq^m + (n - k)q^s$ and thus (4) can be rewritten as

$$\alpha - \left[2 + \eta\,(n - k + 1)\right] q^s - \eta k q^m = 0. \tag{5}$$

Analogously, the FOC by a myopic firm is

$$\frac{\partial \pi_i}{\partial q_i} = \gamma \left(\alpha - 2q_i^m - \eta Q\right) = 0, \tag{6}$$

with SOC satisfied for all parameter values. Using (3) with symmetry, (6) can be rewritten as

$$\alpha - (2 + \eta k)\, q^m - \eta\, (n - k)\, q^s = 0. \tag{7}$$

Conditions (5) and (7) together imply that the equilibrium output levels of smart and myopic firms are

$$q^s(k, n) = \frac{2\alpha}{4 + 2\eta\,(n+1) + \eta^2 k} \tag{8}$$

and

$$q^m(k, n) = \frac{(2 + \eta)\,\alpha}{4 + 2\eta\,(n+1) + \eta^2 k} \tag{9}$$

respectively, with equilibrium industry output

$$Q = \frac{(2n + \eta k)\,\alpha}{4 + 2\eta\,(n+1) + \eta^2 k}. \tag{10}$$

These expressions show that industry output increases with the number of firms (n) and the fraction of them that are myopic (k) whereas individual output falls with both n and k. They also shows that myopic firms are bigger ($q^m(k, n) > q^s(k, n)$), the more so the larger the aggregate dependence parameter η.

Substituting (8) and (9) in (2) gives equilibrium profits

$$\pi^s\,(k, n) = \frac{4\alpha^2\gamma\,(1 + \eta)}{\left[4 + 2\eta\,(n+1) + \eta^2 k\right]^2} \tag{11}$$

and

$$\pi^m\,(k, n) = \frac{\alpha^2\gamma\,(2 + \eta)^2}{\left[4 + 2\eta\,(n+1) + \eta^2 k\right]^2}, \tag{12}$$

which reveal that myopic firms not only supply more output but also earn higher profit. Hence, neglecting their individual impact on industry output, for given n and k myopic firms behave more aggressively in terms of output and this pays in terms of profit.

2.2 First Stage: To Know or Not To Know

In the first stage, firms simultaneously decide whether in the second stage they will be myopic or smart. Given n firms, there exists a subgame perfect equilibrium partition $\{k, n-k\}$ in which k myopic firms coexist with $n-k$ smart firms if and only if no myopic firm has a unilateral incentive to become smart because

$$\pi^s (k-1, n) - \pi^m (k, n) < 0 \tag{13}$$

and no smart firm has an incentive to become myopic because

$$\pi^m (k+1, n) - \pi^s (k, n) < 0. \tag{14}$$

The following result holds:

Lemma 1 *Consider an industry in which n firms compete by choosing the output levels of their differentiated products. In making this choice, firms can decide whether or not to take the impact of their individual choices on aggregate output into account. Then, no subgame perfect equilibrium in pure strategies exists in which some firms take the aggregate impact of their individual choices into account while others do not* $(0 < k < n)$.

Proof. Consider any given partition $\{k, n-k\}$ with $k \in (0, n)$. Given (11) and (12), condition (13) is satisfied if and only if

$$k - 1 > \frac{2(2+\eta)\sqrt{\eta+1} - 2\eta(n-1)}{\eta^2},$$

while condition (14) is satisfied if and only if

$$0 < k < \frac{2(2+\eta)\sqrt{\eta+1} - 2\eta(n-1)}{\eta^2}.$$

Hence, (11) and (12) cannot be satisfied at the same time and for all parameter values either a smart or a myopic firm has a unilateral incentive to deviate from $\{k, n-k\}$. ■

Then, we can prove:

Proposition 2 *Consider an industry in which n firms compete by choosing the output levels of their differentiated products. In making this choice, firms can decide whether or not to take the impact of their individual choices on aggregate output into account. Define $n_L \equiv 1 + \frac{2}{\eta}\sqrt{\eta+1}$ and $n_H \equiv 1 + \left(1+\frac{2}{\eta}\right)\sqrt{\eta+1}$. Then, for $1 < n < n_L$ there exists a unique subgame perfect equilibrium in pure strategies in which all firms take the aggregate impact of their individual choices into account ($k = 0$). For $n > n_H$ there exists a unique subgame perfect equilibrium in pure strategies in which all firms do not take the aggregate impact of their individual choices into account ($k = n$). For $n_L \leq n \leq n_H$ there are two subgame perfect equilibria in pure strategies, one in which all firms take the aggregate impact of their individual choices into account and the other in which all firms do not.*

Proof. Consider first the partition $\{k, n-k\} = \{0, n\}$ in which all firms take the aggregate impact of their individual choices into account. Unilateral deviation from this 'fully smart' outcome pays if and only if the profit of a single myopic firm is larger than any firm's profit when all firms are smart. Formally, this happens for

$$\pi^m(1,n) - \pi^s(0,n) > 0$$

with (11) and (12) implying

$$\pi^m(1,n) - \pi^s(0,n) = \alpha^2\gamma\eta^2 \frac{-8\eta + n^2\eta^2 - 4\eta^2 - \eta^3 - 2n\eta^2 - 4}{(\eta + n\eta + 2)^2 (2\eta + 2n\eta + \eta^2 + 4)^2}.$$

This expression is positive if and only if its numerator is positive, which is the case for

$$n - 1 > \left(1 + \frac{2}{\eta}\right)\sqrt{\eta+1}.$$

Consider now the partition $\{k, n-k\} = \{n, 0\}$ in which all firms do not take the aggregate impact of their individual choices into account. Unilateral deviation from this 'fully myopic' outcome pays if and only if the profit of a single smart firm is larger than the profit of any firm's profit when all firms are myopic. The difference between these profits corresponds to

$$\pi^s(n-1,n) - \pi^m(n,n) = \alpha^2\gamma\eta^2 \frac{4\eta - n^2\eta^2 - \eta^2 + 2n\eta^2 + 4}{(n\eta + 2)^2 (2\eta + 2n\eta - \eta^2 + n\eta^2 + 4)^2}.$$

This expression is positive if and only if its numerator is positive, which is the case for

$$0 < n - 1 < \frac{2}{\eta}\sqrt{\eta + 1}.$$

Hence, for $0 < n-1 < \frac{2}{\eta}\sqrt{\eta+1}$ unilateral deviation from $\{k, n-k\} = \{n, 0\}$ pays whereas unilateral deviation from $\{k, n-k\} = \{0, n\}$ does not, and for $n - 1 > \left(1 + \frac{2}{\eta}\right)\sqrt{\eta+1}$ unilateral deviation from $\{k, n-k\} = \{0, n\}$ pays whereas unilateral deviation from $\{k, n-k\} = \{n, 0\}$ does not. Together with Lemma 1, this implies the results stated in the proposition, if one defines $n_L \equiv 1 + \frac{2}{\eta}\sqrt{\eta+1}$ and $n_H \equiv 1 + \left(1 + \frac{2}{\eta}\right)\sqrt{\eta+1}$. ■

Based on Proposition 2, only two equilibrium market structures exist. The first is 'monopolistic competition', in which all firms are myopic with firm output and profit respectively equal to

$$q^m(n, n) = \frac{\alpha}{2 + \eta n} \text{ and } \pi^m(n, n) = \frac{\alpha^2\gamma}{(2 + \eta n)^2}. \tag{15}$$

The second is 'oligopoly', in which all firms are smart with output and profit respectively equal to

$$q^s(0, n) = \frac{\alpha}{2 + \eta(n+1)} \text{ and } \pi^s(0, n) = \frac{\alpha^2\gamma(1+\eta)}{[2 + \eta(n+1)]^2}. \tag{16}$$

Comparing (15) and (16) shows that firms are bigger in terms of output but earn smaller profit under monopolistic competition than under oligopoly.

Figure 1 provides a graphical representation of the parametrical conditions supporting the three alternative outcomes described in Proposition 2 with n on the vertical axis and η on the horizontal one. The two downward sloping curves depict n_L and n_H as functions of η with the former lying below the latter. These curves partition the parameter space in three areas. Above n_H the unique subgame perfect equilibrium features only myopic firms. Below n_L, the unique subgame perfect equilibrium features only smart firms. Between the n_L and n_H there are two subgame perfect equilibria in which all firms are either myopic or smart. Hence, large n and large η support monopolistic competition as the unique equilibrium market structure whereas small n and small η support oligopoly as the unique equilibrium market structure. For intermediate value of n and η both monopolistic competition and oligopoly are equilibrium market structures.

The figure shows that, for any given degree of aggregate dependence η, there exists a threshold number of firms above which monopolistic competition is an equilibrium market structure. This number is the closest integer to n_L from above and need not be very large except for very small values of η. For instance, it equals 22 for $\eta = 0.1$, 9 for $\eta = 0.3$, 6 for $\eta = 0.5$, and 5 for $\eta = 0.9$. Moreover, for any given η, there also exists a threshold number of firms above which monopolistic competition emerges as the unique equilibrium market structure. This number is the closest integer to n_H from above and need not be very large except for very small or very large values of η. For instance, it equals 24 for $\eta = 0.1$, 10 for $\eta = 0.3$, 8 for $\eta = 0.5$, 5 for $\eta = 0.9$ and does not rise back to 24 until around $\eta = 500$. Note that this is very different from saying that in the limit monopolistic competition 'approximates' oligopoly as n goes to infinity. Instead, for $n > n_H$ monopolistic competition is 'the' equilibrium outcome rather than its approximation. Note also that the minimum of the n_H curve corresponds to $\eta = 1 + \sqrt{5}$ implying $n_H = 4.3302$ so that $n \geq 5$ is a necessary condition for monopolistic competition to be the unique equilibrium market structure.

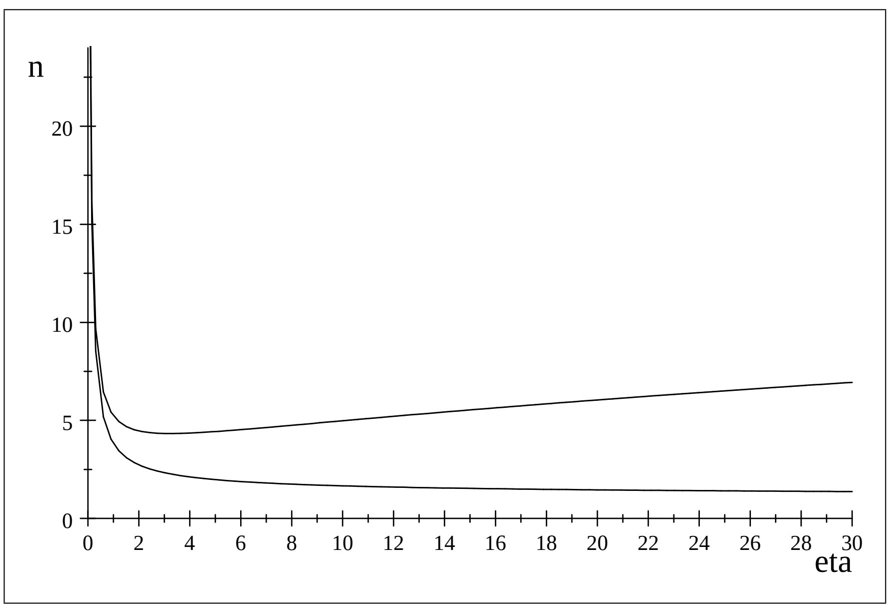

Figure 1. Oligopoly vs. Monopolistic Competition

3 The Two Firms Case

To further discuss the intuition behind Proposition 2, it is useful to focus on the simple case of two firms. This also comes handy in showing how a 'mixed' market structure, in which some firms are myopic while others are smart, can emerge as a subgame perfect equilibrium in mixed strategies.

3.1 The Strategic Implications of Inattention

Proposition 2 shows that monopolistic competition emerges as *an* equilibrium market structure when aggregate dependence is strong (large η) and there are many firms (large n). In both cases the profit a firm can make is influenced a lot by how much its competitors produce. An intuitive explanation of the proposition can be given comparing the 'best replies' of smart and myopic firms as defined by (4) and (6). However, as η and n work in the same direction, intuition is better served by focusing on the simple case of two firms only, indexed i and j.

Consider first the oligopolistic market regime, in which both firms take their individual impact on total output into account. In the case of two firms, the FOC for firm i's profit maximization becomes

$$\frac{\partial \pi_i^s}{\partial q_i^s} = \gamma \left[\alpha - 2\left(1+\eta\right) q_i^s - \eta q_j^s \right] = 0, \tag{17}$$

which defines firm i's 'best reply' to a given q_j^s as the linear function

$$q_i^s = \frac{\alpha}{2(1+\eta)} - \frac{\eta}{2(1+\eta)} q_j^s. \tag{18}$$

The symmetric equilibrium output level therefore equals

$$q^{ss} \equiv q^s\left(0,2\right) = \frac{\alpha}{2+3\eta},$$

with associated equilibrium profit

$$\pi^{ss} \equiv \pi^s\left(0,2\right) = \frac{\alpha^2 \gamma \left(1+\eta\right)}{\left(2+3\eta\right)^2}, \tag{19}$$

where superscript ss denotes that both firms are 'smart'.

Next, consider the monopolistically competitive market regime, in which both firms take Q as given. In this case, the FOC for profit maximization becomes

$$\frac{\partial \pi_i^m}{\partial q_i^m} = \gamma \left(\alpha - 2q_i^m - \eta Q^m\right) = \gamma \left[\alpha - (2+\eta)\, q_i^m - \eta q_j^m\right] = 0, \tag{20}$$

where the second equality is obtained after imposing $Q^m = q_i^m + q_j^m$. This defines firm i's 'best reply' to a given q_j^m when it neglects its individual impact on industry output as the linear function

$$q_i^m = \frac{\alpha}{2+\eta} - \frac{\eta}{2+\eta} q_j^m. \tag{21}$$

The symmetric equilibrium output level in this case equals

$$q^{mm} \equiv q^m (2,2) = \frac{\alpha}{2\,(1+\eta)},$$

with associated equilibrium profit

$$\pi^{mm} \equiv \pi^m (2,2) = \frac{\alpha^2 \gamma}{4\,(1+\eta)^2}. \tag{22}$$

where superscript mm denotes that both firms are 'myopic'.

Comparing the 'best replies' (18) and (21) reveals that the latter has larger intercept and larger slope than the former. Hence, to any given output level of its competitor, a firm replies by producing more when it is myopic than when it is smart. This is why, as already discussed, a monopolistic competitor is more 'aggressive' than an oligopolist that internalizes the indirect impact of larger individual output on own profit through its effects on industry output. As a result, in the symmetric equilibrium output is larger and profit is smaller under monopolistic competition: $q^{mm} > q^{ss}$ and $\pi^{mm} < \pi^{ss}$.

Last, consider a mixed market regime, in which firm i is smart while firm j is myopic. The FOCs of the two firms now are:

$$\begin{aligned} \frac{\partial \pi_i^s}{\partial q_i^s} &= \gamma \left[\alpha - 2\,(1+\eta)\, q_i^s - \eta q_j^m\right] = 0; \\ \frac{\partial \pi_j^m}{\partial q_j^m} &= \gamma \left(\alpha - 2q_j^m - \eta Q\right) = 0. \end{aligned} \tag{23}$$

These, after imposing $Q = q_i^s + q_j^m$, imply equilibrium output levels

$$q^{sm} \equiv q^s(1,2) = \frac{2\alpha}{4+6\eta+\eta^2} \text{ and } q^{ms} \equiv q^m(1,2) = \frac{\alpha(2+\eta)}{4+6\eta+\eta^2}, \tag{24}$$

with associated equilibrium profits

$$\pi^{sm} \equiv \pi^s(1,2) = \frac{4\alpha^2\gamma(1+\eta)}{(4+6\eta+\eta^2)^2} \text{ and } \pi^{ms} \equiv \pi^m(1,2) = \frac{\alpha^2\gamma(2+\eta)^2}{(4+6\eta+\eta^2)^2} \tag{25}$$

for smart firm i and myopic firm j respectively. In (24) and (25) superscript sm (ms) on a firm's outcomes indicate that the firm is smart (myopic) while its competitor is myopic (smart). It is readily verified that these expressions imply $q^{sm} < q^{ms}$ and $\pi^{sm} < \pi^{ms}$: in equilibrium the myopic firm supplies larger output and gains higher profit than its smart rival.

At first sight it may seem counterintuitive that the myopic firm performs better that the smart one, and that information has thus a negative value. Why this happens can be, nonetheless, understood by recalling that the best replies (18) and (21) require a myopic firm to act more aggressively than a smart firm.

Figure 2 provides a graphical representation of what this implies. In the figure q_i and q_j are measured along the horizontal and vertical axes respectively. The downward sloping lines represent the firms' best replies: the two thin lines correspond to the case in which firms are smart; the thick ones to case when they are myopic. The oligopoly outcome ($q_i^s = q_j^s = q^{ss}$) and the monopolistic competition outcome ($q_i^m = q_j^m = q^{mm}$) can be found at the crossing of the two thin lines and the two thick lines respectively. Both crossings are on the 45-degree line passing through the origin (not drawn to avoid cluttering the figure) where firms produce the same level of output. The remaining two crossings involve a thin and a thick lines. They are symmetric around the 45-degree line passing through the origin. The one above this line entails $q_i^s = q^{sm}$ and $q_j^m = q^{ms}$ with $q^{ms} > q^{sm}$; the other crossing is its mirror image.

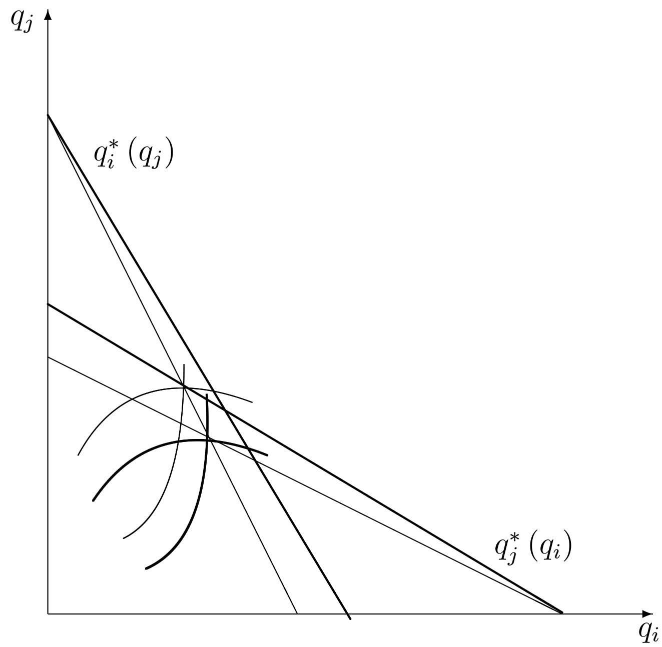

Figure 2 Best replies: Smart vs myopic

3.2 Mixed Strategies and 'Mixed Market Structure'

When two equilibria exist in which all firms are either myopic or smart, the second stage has the features of a 'coordination game' and thus admits a mixed strategy equilibrium. Again, this is most readily characterized in the case of two firms. For this case the second stage can be summarized as in **Matrix 1**, where firms i and j are confronted with a choice between being smart (s) or being myopic (m) and their payoffs correspond to (19), (22) and (25).

Matrix 1: The first stage

		j: s	j: m
i	s	π^{ss}, π^{ss}	π^{sm}, π^{ms}
	m	π^{ms}, π^{sm}	π^{mm}, π^{mm}

Applying Proposition 2 with $n = 2$, the outcome in which both firms are smart is always a subgame perfect equilibrium in pure strategies while the outcome in which both firms are myopic is a subgame perfect equilibrium in pure strategies if and only if

$$\eta > 2\left(1+\sqrt{2}\right). \tag{26}$$

This defined the relevant interval of aggregate dependence for the existence of a mixed strategy subgame perfect equilibrium.

Specifically, define the probabilities that firm h attaches to pure strategies s and m as $\mathfrak{p}_{hs}$ and $\mathfrak{p}_{hm} = 1 - \mathfrak{p}_{hs}$ respectively, with $\mathfrak{p}_{hs} \in [0,1]$ and $h = i, j$. Given the symmetry existing *a priori* between firms, we can impose $\mathfrak{p}_{is} = \mathfrak{p}_{js} = \mathfrak{p}_s$ and solve for $\mathfrak{p}_s$ the indifference condition between being smart and myopic

$$\mathfrak{p}_s \pi^{ss} + (1-\mathfrak{p}_s)\,\pi^{sm} = \mathfrak{p}_s \pi^{ms} + (1-\mathfrak{p}_s)\,\pi^{mm} \tag{27}$$

where the left (right) hand side is a firm's expected profit when it is smart (myopic). Solving (27) for $\mathfrak{p}_s$ gives the equilibrium probability

$$\mathfrak{p}_s^* = \frac{\pi^{sm} - \pi^{mm}}{\pi^{sm} - \pi^{mm} + \pi^{ms} - \pi^{ss}} = \frac{(2+3\eta)^2\,(\eta^2 - 4\eta - 4)}{\eta^2\,(16 + 44\eta + 33\eta^2 + 4\eta^3)} \tag{28}$$

Accordingly, given that we have $\mathfrak{p}_s^* \in (0,1)$ as long as (26) holds, there exists a positive probability that a firm chooses to be smart while the other chooses to be myopic. As long as (26) holds, the probability $(1-\mathfrak{p}_s)^2$ of the monopolistic competition outcome (mm) is larger than the probability $2\mathfrak{p}_s\,(1-\mathfrak{p}_s)$ of a mixed outcome (sm and ms), and this is larger than the probability $(\mathfrak{p}_s)^2$ of the oligopolistic outcome (ss). Moreover, as η grows, $(1-\mathfrak{p}_s)^2$ also grows whereas $2\mathfrak{p}_s\,(1-\mathfrak{p}_s)$ and $(\mathfrak{p}_s)^2$ fall, with the former falling faster than the latter. Hence, the probability differentials between these three outcomes increase when aggregate dependence increases.

4 Strategic Delegation as Strategic Inattention

Our **Figure 2** looks pretty much like Figure 2 in Aggarwal and Samwick (1999), which compares the best replies under standard differentiated Cournot duopoly with the best replies when firms owners delegate output choices to managers through compensation based on a linear combination of own and rival's profits. This suggests the possible existence of a fundamental isomorphism between our model of strategic inattention and their models of strategic delegation. The aim of this section is to nail down such isomorphism.

Consider the following delegation model. As in Section (2) there are n firms – indexed $h = 1, ..., n$ – that produce differentiated products and choose output so as to maximize profit. Firm owners play a two-stage game with both stages characterised by complete, symmetric and imperfect information. In the first stage, firm owners simultaneously decide whether or not to delegate profit maximization to managers. In the second stage, firms simultaneously maximize profit with respect to output.

Delegation can be implemented through a simple and realistic Relative Performance Evaluation (RPE) contract in which the manager of a firm is rewarded (penalized) for profit above (below) industry average.[3] This assumes that the manager's action choice at the second stage is not contractible, whereas profits are contractible.[4] Specifically, firm h can offer its manager compensation

$$w_h = k_h + \pi_h - \overline{\pi}$$

where k_h is a constant unrelated to performance optimally chosen by the firm's owners and

$$\overline{\pi} \equiv \frac{\sum_{z=1}^{n} \pi_z}{n}$$

is average industry profit. We assume that the managerial labor market is competitive and managers have a reservation wage w'.

Solving backwards, if firm owners decide not to delegate, the second stage delivers the standard Cournot outcome described in Section 2 with equilibrium output and profit given by (16). If firm owners decide instead to delegate, in the second stage the managers of firms $h = 1, ..., n$ simultaneously

[3] This simple contract would not be the optimal contract if firm owners were allowed to choose the weights attached to own and average industry profits in the manager's compensation. In this case, as shown by Aggarwal and Samwick (1999) and Miller and Pazgal (2001) in a two-firm setup, the optimal weights would depend on demand parameters. We do not allow firms owners to choose weights but only to decide between no delegation and delegation via our simple profit-based RPE contract.

[4] This could be justified by inroducing a common additive shock affecting firms' profits as in Aggarwal and Samwick (1999). The shock would make it impossible to perfectly infer the manager's noncontractible action choice from profit. We prefer to leave this justification implicit in order to streamline the presentation.

choose output q_h so as to maximize w_h. In particular, given (2) the manager of firm h maximizes

$$w_h = k_h + \gamma\left(\alpha - q_h - \eta Q\right) q_h - \frac{\sum_{z=1}^{n} \gamma\left(\alpha - q_z - \eta Q\right) q_z}{n},$$

with FOC

$$\gamma\left[\alpha - \left(2+\eta\right) q_h - Q\eta\right] - \frac{\gamma\left(\alpha - 2q_h - 2\eta Q\right)}{n} = 0 \tag{29}$$

In the first stage, owners of firms $h = 1, ..., n$ simultaneously choose k_h so as to maximize $\pi_h - w_h$, compare profit under the profit maximizing delegation contract k_h with profit under Cournot, and decide whether to delegate or not. As the market for managers is competitive, k_hs are chosen so that the managers are held to their reservation wage w'. After imposing symmetry $Q = nq$, (29) can then be solved for output and profit under delegation to yield

$$q^d = \frac{\alpha}{2+\eta n} \text{ and } \pi^d = \frac{\alpha^2 \gamma}{\left(n\eta + 2\right)^2}. \tag{30}$$

Comparing (30) with (15) reveals that strategic delegation under the simplest profit-based RPE contract leads to the same outcome as strategic neglect of the aggregate impact of firms' individual output choices. Hence, we can state:

Proposition 3 *Consider an industry in which n firms compete by choosing the output levels of their differentiated products. Firms can decide whether or not to delegate output decision to managers through a contract in which managers' compensation is based on their profit performance relative to industry average. Define $n_L \equiv 1 + \frac{2}{\eta}\sqrt{\eta+1}$ and $n_H \equiv 1 + \left(1 + \frac{2}{\eta}\right)\sqrt{\eta+1}$. Then, for $1 < n < n_L$ there exists a unique subgame perfect equilibrium in pure strategies in which all firms do not delegate. For $n > n_H$ there exists a unique subgame perfect equilibrium in pure strategies in which all firms delegate. For $n_L \leq n \leq n_H$ there are two subgame perfect equilibria in pure strategies, one in which all firms delegate and the other in which all firms do not.*

Hence, Propositions 2 and 3 together imply:

Corollary 4 *Consider an industry in which n firms compete by choosing the output levels of their differentiated products. A game in which firms decide whether or not to take into account the effect of their individual choices on aggregate output is isomorphic to a game in which they decide whether or not to delegate output choices to managers with compensation based on own profit performance relative to industry average.*

An interesting implication concerns the literature on the 'divisionalization' of multi-product firms (see, e.g., Baye *et al.*, 1996; Ziss, 1998). The creation by a multi-product firm of 'divisions' that compete independently in the market each peddling its own product can be seen as a commitment to disregard aggregate dependence. Divisionalization entails pros and cons, but the strategic incentive for the firm to divisionalize rests on its commitment to more aggressive behavior, which increases the firm's market share at the expenses of its rivals. A model in which divisionalization can arise as the optimal choice for a big (oligopolistic) firm that competes with a fringe of small monopolistically competitive rivals is proposed by Kokovin *et al.* (2014). In their model, depending on demand parameters, the big firm may find it convenient to be broken down into horizontal profit-maximizing divisions that behave like monopolistically competitive units. Corollary 4 then implies that divisionalization can be implemented by firm owners through managerial contracts based on relative profit performance.

5 Conclusion

We have modeled the endogenous emergence of market structure in an industry where a discrete number of firms compete by choosing the output levels of their differentiated products. In making this choice, they can strategically decide whether or not to consider the impact of their individual decisions on aggregate output.

We have shown that there exist two threshold numbers of firms such that: when the number of firms in the industry falls below the lower threshold, there is a unique subgame perfect equilibrium in pure strategies in which all firms take the aggregate impact of their individual choices into account as in standard oligopoly; when the number of firms falls above the higher threshold, there is a unique subgame perfect equilibrium in pure strategies in which all firms disregard their aggregate impact as in standard monopolistic competition; when the number of firms falls between the two thresholds, there are two subgame perfect equilibria in pure strategies, one in which all firms consider their aggregate impact and the other in which they do not.

In terms of comparative statics, we have found that the lower threshold decreases with the relative importance of aggregate output for individual firm profit as dictated by product differentiation and the gradient of marginal cost. In particular, if product differentiation is weak and marginal cost does not increase steeply with production, oligopoly emerges as the unique equilibrium market structure only when the number of firms is very small. The relation between the higher threshold and the relative importance of aggregate output is, instead, U-shaped.

We have also shown that our model of 'strategic inattention' is isomorphic to a model of 'strategic delegation' of output choices by firm owners to managers in which managerial compensation is based on relative profit performance. Accordingly, even in the presence of only few firms, 'strategic delegation' can lead to the emergence of monopolistic competition as the equilibrium market structure by *de facto* implementing 'strategic inattention'. In this respect, one should observe less delegation based on relative profit performance in industries characterized by the presence of few firms, strong product differentiation and steep marginal cost.

Three final comments are in order. First, only with mixed strategies we have been able to generate equilibrium outcomes in which some firms consider while others neglect their aggregate impacts. These 'mixed' outcomes may be quite relevant in practice and would be easy to generate with pure strategies if one allowed for firm heterogeneity and 'rational inattention' due to costly information acquisition and processing. Whether this would also be possible with 'strategic inattention' in the absence of any cost of acquiring and processing information is an interesting direction of future research. Second, we have considered 'strategic inattention' and 'strategic delegation' with relative performance evaluation in the case of single-product firms. It may be interesting to extend the analysis to the case of multi-product firms that can choose whether to neglect the individual impact of a product output on firm or industry total output. Third, our analysis has been based on a static model. A dynamic approach could be used to investigate on the intertemporal dimension of strategic inattention.

References

[1] Aggarwal, R. and A. Samwick (1999), "Executive compensation, strategic competition, and relative performance evaluation: Theory and evidence", *Journal of Finance*, **54**, 1999-2043.

[2] Anderson S., N. Erkal and D. Piccinin (2013), "Aggregative oligopoly games with entry", CEPR Discussion paper 9511.

[3] Barros F. (1997) "Asymmetric information as commitment in oligopoly", *European Economic Review*, **41**, 207-25.

[4] Bassan, B. and M. Scarsini (1995), "On the value of information in multi-agent decision theory", *Journal of Mathematical Economics*, **24**, 557-76.

[5] Bassan, B., M. Scarsini and S. Zamir (1997), "I don't want to know! Can it be rational?", Discussion paper no. 158, Center for Rationality and Interactive Decision Theory, The Hebrew University of Jerusalem.

[6] Bassan, B., O. Gossner, M. Scarsini and S. Zamir (2003), "Positive value of information in games", *International Journal of Game Theory*, **32**, 17-31.

[7] Baye M.R., K. Crocker and J. Ju (1996), "Divisionalization, franchising and disinvesture incentives in oligopoly", *American Economic Review*, **86**, 223-36.

[8] Behrens, K. and Y. Murata (2007), "General equilibrium models of monopolistic competition: a new approach", *Journal of Economic Theory*, **136**, 776-787.

[9] Bowley, A.L. (1924), *The mathematical groundwork of economics*, Oxford, Oxford University Press.

[10] Chirco, A., C. Colombo and M. Scrimitore (2013), "Quantity competition, endogenous motives and behavioral heterogeneity", *Theory and Decisions*, **74**, 55-74.

[11] d'Aspremont, C., R. Dos Santos Ferreira and L.-A. Gerard-Varet (1996), "On the Dixit-Stiglitz Model of Monopolistic Competition", *American Economic Review*, **86**, 623-629.

[12] d'Aspremont, C., A. Jacquemin, J.J. Gabszewicz and J. Weymark (1983), "On the stability of collusive price leadership", *Canadian Journal of Economics*, **16**, 17-25.

[13] Dixit, A.K. (1979), "A model of duopoly suggesting the theory of entry barriers", *Bell Journal of Economics*, **10**, 20-32.

[14] Donsimoni, M.-P., N.S. Economides and H.M. Polemarchakis (1986), "Stable cartels", *International Economic Review*, **27**, 317-27.

[15] Dunne, T., S.D. Klimek, M.J. Roberts and D. Yi Xu (2013), "Entry, exit, and the determinants of market structure", *RAND Journal of Economics*, **44**, 462-87.

[16] Etro, F. (2008), "Stackelberg Competition with Endogenous Entry", *Economic Journal*, **118**, 1670-97.

[17] Etro, F. (2011), "Endogenous Market Structures and Contract Theory: Delegation, Principal-Agent Contracts, Screening, Franchising and Tying", *European Economic Review*, **55**, 463-79.

[18] Etro, F. (2012), "Endogenous Market Structure and Welfare", Ca' Foscari Univeristy of Venice Working paper 2012/12.

[19] Fershtman, C. and K. Judd (1987), "Equilibrium incentives in oligopoly", *American Economic Review*, **77**, 927-40.

[20] Fumas, V. (1992), "Relative performance evaluation of management: The effects on industrial competition and risk sharing", *International Journal of Industrial Organization*, **10**, 473-489.

[21] Gibbons, R., and K. Murphy (1990), "Relative performance evaluation for chief executive officers", *Industrial & Labor Relations Review*, **43**, 30S-51S.

[22] Kadane, J.B., M.J. Shervish and T. Seidenfel (1996), "Reasoning to a forgone conclusion", *Journal of the American Statistical Association*, **91**, 1228-35.

[23] Keen, S. and R. Standish (2006), "Profit maximization, industry structure, and competition: A critique of neoclassical theory", *Physica A*, **370**, 81-85.

[24] Kamien, M.I., Y. Tauman and S. Zamir (1990), "On the value of information in a strategic conflict", *Games and Economic Behavior*, **2**, 129-53.

[25] Kokovin, S., M. Parenti, J.-F. Thisse and E. Zhelobodko (2014), "Markets with dominant firms and competitive fringes", CORE, mimeo.

[26] Lambertini, L. and M. Trombetta (2002), "Delegation and firms' ability to collude", *Journal of Economic Behavior and Organization*, **47**, 359-73.

[27] Melitz, M. and G.I.P. Ottaviano (2008), "Market size, trade, and productivity", *Review of Economic Studies*, **75**, 295-316.

[28] Miller, N.H. and A.I. Pazgal (2001), "The equivalence of price and quantity competition with delegation", *RAND Journal of Economics*, **32**, 284-301.

[29] Murphy, K. (1999), "Executive compensation", *Handbook of Labor Economics*, **3**, Amsterdam, Elsevier, 2485-2563.

[30] Ottaviano, G.I.P., T. Tabuchi and J.-F. Thisse (2002), "Agglomeration and trade revisited", *International Economic Review*, **43**, 409-36.

[31] Parenti, M. (2013), "Large and small firms in a global market: David vs. Goliath", CORE Discussion paper 2013/107

[32] Safra, Z. and Sulganik E. (1993), "The value of information with general preference relations", Working paper 34-93 Tel Aviv University, Faculty of Management.

[33] Shimomura, K.-I. and J.-F. Thisse (2012), "Competition among the big and the small", *RAND Journal of Economics*, **43**, 329-47.

[34] Singh, N., and X. Vives (1984), "Price and quantity competition in a differentiated duopoly", *RAND Journal of Economics*, **15**, 546-54.

[35] Sims, C.A. (2010), "Rational inattention and monetary economics", *Handbook of Monetary Economics*, **3**, 155-181.

[36] Sklivas, S. (1987), "The strategic choice of management incentives", *RAND Journal of Economics*, **18**, 452-58.

[37] Spence, A.M. (1976), “Product differentiation and welfare”, *American Economic Review*, **66**, 407-14.

[38] Vickers, J. (1985), “Delegation and the theory of the firm”, *Economic Journal* (Conference Supplement), **95**, 138-47.

Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

Strada Maggiore 45
40125 Bologna - Italy
Tel. +39 051 2092604
Fax +39 051 2092664
http://www.dse.unibo.it