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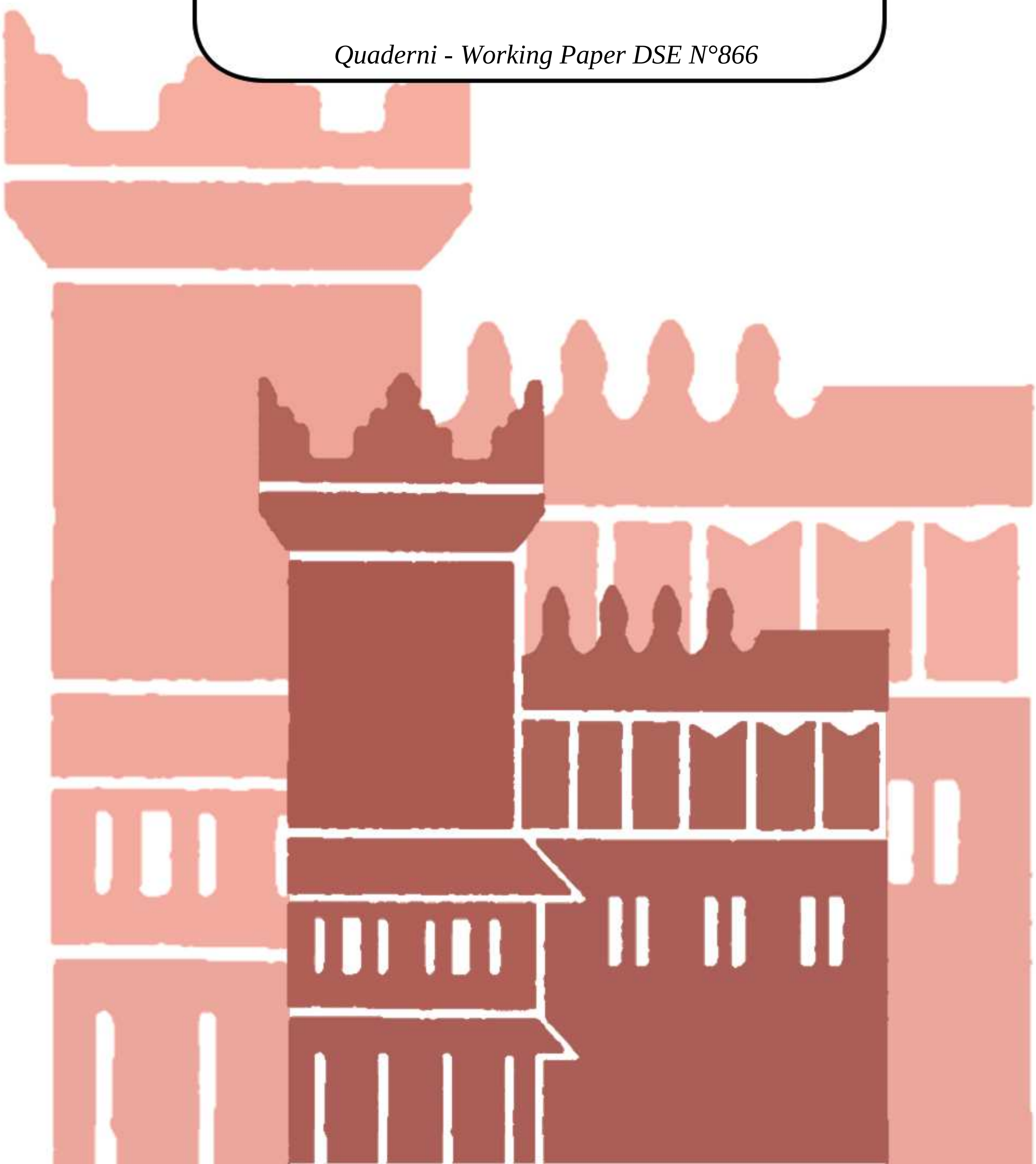

Bidimensional Screening with Intrinsically Motivated Workers*

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Abstract

We study the screening problem of a firm that needs to hire a worker to produce output and that observes neither the productive ability nor the intrinsic motivation of the job applicant. We completely characterize the set of optimal contracts according to whether motivation or ability is the main determinant of the worker's performance. We show that it is always in the firm's interest to hire all types of worker and to offer different contracts to different types of employees. Interestingly, when motivation is very high, incentives force the firm to pay higher informational rents, to increase effort distorsions for motivated workers, and to offer a strictly positive wage to workers enjoying a positive utility from effort provision, who thus become paid volunteers. These results suggest that, from the principal's viewpoint, very high motivation might not be a desirable worker's characteristic.

Jel classification: D82, D86, J31, M55.

Key-words: bidimensional screening, self-selection, intrinsic motivation, skills.

1 Introduction

A recent literature addresses the issue of the selection of applicants in a labor market where potential workers can be intrinsically motivated for the job, as in the market for civil servants, health professionals and teachers (Handy and Katz 1998, Delfgaauw and Dur 2007, 2010 Francois 2000, Heyes 2005). A shared view from this literature is that high wages are necessary to attract applicants with high skills, but this comes at the cost of employing workers who are less motivated for the task to be performed.

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Conversely, low monetary wages select highly motivated workers, who might not necessarily be talented or skilled. This suggests that firms are not able to design optimal compensation practices in order to screen potential applicant according to both productive ability and motivation, despite the fact that the workers' overall performance depends on the interplay of both characteristics. We depart from this view claiming that, in a world where workers' attributes are not observable, a firm can succeed in solving this problem offering simple contracts based on one screening instrument only and a non-linear wage.

Take the market for nurses, where hospitals typically offer contracts characterized by a different number of working hours: in the U.S., part-time contracts require about 24 hours each week, full-time nurses work an average of about 43 hours a week; moreover nurses can choose paid voluntary overtime up to a total amount that cannot exceed 60 hours a week.[1] As for nurses' monetary compensation, the total salary they receive is generally represented by an hourly wage that depends on the number of hours worked per day: it encompasses part-time penalties and/or overtime premia. We show that such simple contracts, defined only by the number of hours worked per week and by the total salary, are likely to enable the hospital to screen applicants with respect to two different dimensions of private information, namely ability and intrinsic motivation. In particular, our model predicts that high-ability motivated applicants choose the contract with the largest voluntary overtime and low-ability non-motivated nurses are targeted to part-time contracts.

As is well known, also workers' career concerns can be used as a screening device. Typically, workers self-select into different career paths: some of them accept tasks involving strong performance evaluation in exchange for more likely and faster promotions; some others prefer a slower progress up the job ladder together with lower pay and almost no performance evaluation. In the academia, for instance, junior professors can either choose tenure-track positions, which require them to demonstrate, within a short time span, a strong record of published research, grant funding, teaching and administrative service or positions off the tenure track (such as lecturer or adjunct professor), which require them to teach full- or part-time but with few or no research responsibilities. Here an optimal contract consists of the career path and the overall compensation. Intuitively, tenure-track positions are targeted to attract the best researchers.

We investigate the problem of the selection of workers whose overall performance results from the interplay of skills and motivation and we thus contribute to the existing literature by explicitly accounting for the bidimensional nature of workers' private information. Indeed, the literature either deals with the

[1] Bae (2012) presents a quantitative survey data collected from registered nurses who worked in hospitals as staff nurses in North Carolina and West Virginia in 2010-2011. Concerning overtime, the author shows that 33.2% of nurses working overtime are choosing to perform voluntary paid overtime; among them, 42% are working overtime more than 12 hours a week. Interestingly, the survey also considers the reasons reported by nurses as to why they worked overtime. Nearly half (46.3%) of nurses choosing voluntary overtime declared that they "like to work overtime".

selection of workers who are privately informed about their motivation only (see Heyes 2005 and Delfgaauw and Dur 2007) or considers asymmetric information on both motivation and skills but does not explicitly solve the bidimensional screening problem, because the principal is only asked to hire a limited set of types (see Delfgaauw and Dur 2008). A detailed description of the related literature is provided in a separate section which follows.

We consider a principal-agent relationship where agents' skills (or productive ability) and intrinsic motivation are independently and discretely distributed, and take two possible values. Productive ability lowers the worker's cost of providing effort whereas motivation is interpreted as the worker's enjoyment of her personal contribution to the firm's outcome or as a non-monetary benefit accruing to the worker when performing a given task. Since worker's characteristics can not be observed by the employer, they can not be contracted upon. Instead, we assume that the firm can observe and verify the effort levels provided by different types of workers. Thus, the employer offers a menu of contracts consisting in different combinations of wage rate and effort provision. Our goal is then to describe the set of optimal screening contracts and, in particular, to analyze which types of workers are hired and which are the optimal compensation practices that the firm adopts.[2]

The complete characterization of optimal contracts allows us to deliver some novel and interesting insights. Despite having only one instrument (the observable effort level) at his disposal, the firm always offers contracts that entail full separation and full participation of types (whenever they exist) which dominate menus with pooling or exclusion. Thus, when full separation and full participation of types is feasible, screening is not too costly for the firm, neither in terms of information rents that the principal leaves to the most motivated and/or most able type, nor in terms of distortions of effort levels that less motivated and/or less able types are required to provide. From this viewpoint, our results stand in contrast with the literature on multidimensional screening which predicts that bunching and/or exclusion are inevitable.

As for the optimal wage schemes, under full information, high-ability non-motivated workers are always paid the highest wage, while motivated low-ability employees are always paid the lowest salary. Under asymmetric information, however, there is a reversal in the ranking of rewards: high-skilled motivated workers receive the highest transfer, while low-skilled non motivated worker obtain the lowest reward. This is because motivated workers are able to mimic non-motivated ones and thus require an information rent for truthful revelation, which increases their compensation.

Our results are driven by the relative importance of the difference in motivation *vis à vis* the difference in ability, which influences the principal's "preference ordering" over the possible types. High-skilled motivated workers are unambiguously the best types, since they provide the highest possible level of

[2] With a slight loss of generality, our analysis could be entirely rephrased in terms of a governmental agency (the principal) willing to hire a manager (the agent) who might be endowed with public service motivation.

effort and contribute the most to output production, low-skilled non-motivated employees are the worst types, while there is no natural ranking of intermediate types. Accordingly, there are two possible states of the world to be studied. The first one is characterized by heterogeneity in motivation prevailing over heterogeneity in ability (meaning that motivation has a larger impact than ability on the worker's effort provision and on the firm's output), in which case the low-ability motivated worker is asked to provide a higher effort than the high-ability non-motivated type. The second is characterized by heterogeneity in ability being more significant than heterogeneity in motivation (meaning that ability contributes relatively more to effort provision) so that high-skilled non-motivated workers are induced to exert higher effort than low-skilled motivated ones.

When motivation has a larger impact than ability on effort provision, we obtain an intriguing result: for intermediate types, wages might not increase monotonically in effort provision, in which case low-skilled motivated workers become "paid volunteers" at the optimal contract, because they always earn a strictly positive salary (given the information rents that they receive) even if they enjoy a net utility from effort provision.

Conversely, when ability has a larger impact than motivation on effort provision, but vocation is still high, a tension realizes: on the one hand, at any optimal contract, the high-skilled non-motivated worker is required to provide a higher effort than the low-skilled motivated one; on the other hand, as motivation increases, the motivated worker faces a diminishing disutility of effort so that it becomes more and more convenient to increase her effort and more and more difficult to meet the previous monotonicity condition. This tension generates a counter-intuitive pattern of binding incentive compatibility constraints and leads to the most interesting classes of solutions where: (*i*) the rents paid by the principal to the different types of workers are lower than in the case in which motivation prevails, (*ii*) there is no distortion in the effort provided by low-skilled motivated workers, together with the standard result of no distortion at the top. In this situation, we find that, although wages increase monotonically in effort levels, indirect utilities (i.e. information rents) are monotonically decreasing in effort for intermediate types. Hence, low-skilled motivated workers exert a lower effort, gain a lower salary but are more satisfied than skilled, non motivated types.

To conclude, we obtain an unexpected result: when motivation is very high, all worker types receive higher informational rents and the low-skilled motivated worker might become a "paid volunteer", while she provides an effort level which is not distorted at the solutions obtained when motivation is lower. This suggests that, from the principal's viewpoint, very high motivation might not be a desirable worker's characteristic.

The rest of the paper is organized as follows. In the following Subsection we describe the related literature. In Section 2, we set up the model. Section 3 describes the benchmark cases, starting with the first-best (Section 3.1) and considering then asymmetric information on one dimension only, be it ability (Section 3.2) or intrinsic motivation (Section 3.3). In Section 4, we consider the interaction between the two sources of incomplete information. We distinguish between the two polar cases in which: (*i*) motivation has a larger impact than ability on effort provision, or (*ii*) ability has a larger impact than motivation on effort provision. We describe the set of optimal contracts focusing on their qualitative features. Section 5 considers more in detail the two most significant optimal contracts with full separation and full participation of types and offers some interesting comparisons between them. All proofs are relegated to the Appendix as well as the formal analysis of bunching and/or exclusion. Section 6 concludes.

1.1 Related literature

Our work contributes to two different strands of literature: from an economic point of view, it adds to the recent and rapidly growing literature on the selection of workers with intrinsic motivation; from a technical point of view, it explicitly solves the principal-agent problem in a labor market where workers are characterized by two different dimensions of private information.

The problem of the design of optimal incentive schemes for intrinsically motivated workers has been tackled by Murdock (2002), Besley and Gathak (2005) and Ghatak and Mueller (2011), whose attention has primarily been devoted to moral hazard, while we consider the screening problem.

Heyes (2005) and Delfgaauw and Dur (2007) are the first papers that address the issue of the selection of workers who are privately informed about their vocation. They show that, as a worker's motivation increases, the worker's reservation wage decreases. Therefore, as the wage increases, the average motivation of the workers who are willing to accept the job deteriorates. Delfgaauw and Dur (2007) use a directed search framework *à la* Diamond, Mortensen and Pissarides and they show that optimal wage schemes entail a trade-off between the probability of filling a vacancy, the rents left to the workers and the expected motivation of job applicants. Our analysis departs from this work because it includes a second source of asymmetric information (productive ability) and, most importantly, because it resorts to a direct revelation mechanism allowing the principal to infer the workers' true types.

Delfgaauw and Dur (2010) consider a richer framework where workers are heterogeneous with respect to both their intrinsic motivation and their ability. They focus on the issue of managerial self-selection into public vs private sectors under full information on the workers' characteristics: they argue that the return to managerial ability is always lower in the public than in the private sector, provided that the demand for public sector output is not too high and that motivation is unrelated to either effort provision or to the firm's outcome. They conclude that attracting a more able managerial workforce to the public sector by increasing remuneration up to the private sector levels is not efficient. Finally, Barigozzi *et al.* (2013) and Barigozzi and Turati (2012) consider labor supply in a market where workers have private information on both productive ability and motivation. They show that the lemons' problem might be exacerbated by the presence of multidimensional asymmetric information because an increase in the market wage can determine a simultaneous decrease both in average vocation and in average productivity of applicants.

Our paper is also closely related to Handy and Katz (1998) and Delfgaauw and Dur (2008). The first authors argue that non-profits attract motivated managers by offering them compensation packages involving lower money wages and a larger component of institution-specific fringe benefits as compared to the private sector. But their results are driven by an exogenously given ranking of reservation wages for the different types of managers. Delfgaauw and Dur (2008) characterize the optimal incentive schemes offered by a public agency when workers differ in laziness (the opposite of our productive ability) and public service motivation. They show that, when workers' effort is contractible and when the production required by the public institution is sufficiently high, the institution attracts dedicated and productive workers as well as the laziest workers.[3] We depart from the last paper in two main ways: we consider one sector in isolation and our principal is not constrained to hire at most two types of agents.

The literature on the analysis of optimal screening of agents with unknown characteristics has flourished in the last two decades of the twentieth century. Nonetheless, this problem has mainly been examined under the assumption of unidimensional asymmetric information. The interesting and possibly more realistic cases where agents have several unobservable characteristics have been studied by few important exceptions: Armstrong and Rochet (1999), Armstrong (1996), Rochet and Chonè (1998), Armstrong (1999), Basov (2001, 2005) and Deneckere and Severinov (2011). They all show that it is almost impossible to extend to the multidimensional environment the qualitative results and the regularity conditions of the unidimensional case.

Armstrong and Rochet (1999) provide a complete characterization of the optimal contracts when the dimensionality of actions is the same as the dimensionality of private information, the type space is discrete and there are common values (namely the parameters of private information enter directly the objective function of the principal). Our model too is characterized by a discrete type space, but there is one screening instrument only available to the principal (namely the contractible effort level) and there are private values, hence the results in Armstrong and Rochet (1999) cannot be applied to our setting. When the dimensionality of actions is smaller than the dimensionality of private information and the type space is continuous, Laffont et al. (1987) explicitly solve the problem of optimal nonlinear pricing by a regulated monopoly. Again in the continuous setup, Armstrong (1996), Rochet and Chonè (1998), Basov (2001, 2005) and Deneckere and Severinov (2011) present several useful techniques to solve the problem of multidimensional screening, which cannot be applied when the types space is discrete. These

[3] Dedicated workers exert higher effort than in the private, perfectly competitive sector whereas lazy workers' effort is distorted downwards.

papers show that exclusion is generic and full separation of types is impossible. In other words, it is typically optimal for the principal not to serve the lower part of the agent's distribution and to offer the same contract to different (usually intermediate) types of agents.

Our analysis owes much to Armstrong (1999), who considers optimal price regulation of a monopoly that is privately informed about both its cost and demand function. Like ours, his model is discrete and features one instrument and two dimensions of private information, but his analysis has common values while ours has private values. Armstrong (1999) distinguishes two main classes of problems. If cost uncertainty is relatively more important than demand uncertainty, then optimal prices are always weakly above marginal costs. Conversely, if demand uncertainty is more significant than cost uncertainty, then pricing below marginal cost could be optimal. In both classes of solutions some pooling of types always exists, a result which is driven by the fact that "participation constraints do not tie-in nicely with the natural ordering of types".[4] Moreover, the author explicitly ignores the issue of exclusion by restricting the parameter space so that it is never optimal for the regulator to shut down some types of firm. These facts represent a main difference with respect to our model where participation constraints are well-behaved and where we find that, from the principal's viewpoint, full separation of types always dominates pooling and full participation always dominates exclusion.

Finally, some papers analyze the issue of multidimensional screening in insurance markets. Crocker and Snow (2011) consider a perfectly competitive market where consumers possess hidden knowledge of their probability (high or low) of incurring a loss. They show that bundled coverage of different perils and different losses allows insurers to screen applicants in several dimensions thereby reducing the costs that low-risk applicants bear to distinguish themselves from high-risk ones. But their efficient bundling of coverage into a single insurance policy essentially reduces the problem to a one-dimensional screening environment. Olivella and Schroyen (2013) describe a monopolistic insurance company facing a population of individuals who differ both in their risks and in their risk aversion. They solve a discrete bidimensional screening problem that bears many similarities to ours and completely characterize the set of optimal contracts.[5] They find that it is never optimal to fully separate all the types and that exclusion of some high-risk individuals may be optimal.

It is also worth mentioning that in health economics, some research has been conducted on altruistic physicians, whose preferences include both monetary rewards and their patients' health status, which in turn depend on physicians' services. Chonè and Ma (2011) and Liu and Ma (2013) study continuous settings where the physician is the agent characterized by bidimensional private information both on her

[4] See footnote 11 on page 207.

[5] They have a similar double-crossing condition for intermediate types and two possible orderings of co-insurance rates that in turn generate two classes of solutions depending on how the difference in expected losses compares with the difference in the degree of absolute risk aversion.

level of altruism and on the health status of the patient. The insurer is the principal who uses a screening mechanism based only on the physician's degree of altruism. In both papers, the authors apply a limited liability constraint for the physician, implying that his/her participation constraint does not include the psychological benefits due to altruism.

2 The model

We consider a principal-agent model with bidimensional adverse selection. Both the principal and the agent are risk neutral. The principal (he) is willing to hire only one agent (she) to perform a given task.

The production function is such that the only input is labor supplied by the agent. We call e the *observable and measurable* effort (task) level that the agent is asked to provide.[6] The production function displays constant returns to effort in such a way that $q(e) = e$. The principal's payoff function can be written as

$$\pi = e - w,$$

where the price of output is assumed to be exogenous and normalized to 1, and w is the total salary paid to the hired worker. Since the principal's profit only indirectly depends on the type of the agent, we are considering a setting with private values.

Suppose that agents differ in two characteristics, productive ability and intrinsic motivation, that are independently distributed.[7] We interpret workers characterized by high ability as agents incurring in a low cost of providing a given effort level. Workers can have only two possible levels of ability $\theta_i \in \{\theta_L, \theta_H\}$. Employees can have high ability, i.e. they can have a low cost of effort θ_L, with probability ν, or they can have low ability and a high cost of effort θ_H, with probability $1 - \nu$, where $\theta_H > \theta_L > 0$. As for intrinsic motivation, we mainly refer to Delfgaauw and Dur (2008) and assume that workers, to a certain extent, derive utility from exerting effort. Since there exists a one-to-one relationship between effort exerted and output produced by the firm, this interpretation is equivalent to considering intrinsic motivation as the enjoyment of one's personal contribution to the firm's outcome.[8],[9] Paralleling ability, we assume that motivation can take only two possible values $\gamma_j \in \{\gamma_L, \gamma_H\}$. Workers can have either high motivation

[6] In particular, the variable e can be interpreted as a job-specific requirement like the amount of hours of labor the agent is asked to devote to production or the speed at which a production line is run in a factory.

[7] Allowing for more general distribution functions that admit correlation between ability and motivation does not alter our results, since all possible classes of equilibria that we find are still relevant with a more general distribution.

[8] The same interpretation of intrinsic motivation can be found in Besley and Ghatak (2005) and Delfgaauw and Dur (2007, 2008, 2010-only as for Section 5) and traces back to the "warm-glow giving" or impure altruism theory in Andreoni (1990).

[9] A slightly different view of intrinsic motivation (which suits the model as well) is given by Delfgaauw and Dur (2007, page 607), who argue that intrinsic motivation might arise because "the firm has some unique trait that is valued differently by different workers, giving the firm monopsony power". They also add: "Monopsony power arises naturally when intrinsic

γ_H, with probability μ, or low motivation γ_L , with probability $1-\mu$. So there are four types of agents denoted as $ij = \{LH, LL, HH, HL\}$ where the first index represents the cost of effort provision and the second motivation.

Without loss of generality, we normalize the lower bounds of the support of the distribution for both attributes, setting $\theta_L = 1$ and $\gamma_L = 0$. We will thus focus on situations in which agents can be either intrinsically motivated, with motivation parameter taking value $\gamma_H = \gamma$ or not motivated at all. Our results will largely depend on how the difference or heterogeneity in motivation $\Delta\gamma = \gamma_H - \gamma_L = \gamma$ relates to the difference in ability $\Delta\theta = \theta_H - \theta_L = \theta - 1$. Given our normalization, we will refer to the difference in motivation $\Delta\gamma$ and to the level of motivation γ interchangeably. Furthermore, we will impose that $\Delta\gamma \leq 1$ or else that $0 < \gamma \leq 1$ and that $\Delta\theta \leq 1$ or else that $1 < \theta_H = \theta \leq 2$ (the reader is referred to Section 3.1 for the discussion of such assumptions).

The agents' *reservation utility* is normalized to zero for all possible types.

Workers' utility is quasi-linear in income and takes the form

$$u_{ij} = w_{ij} - \frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij},$$

where both productivity θ_i and motivation γ_j are related to effort exertion, although productivity θ_i enters utility with a convex term, while motivation γ_j enters utility with a linear term.[10]

The marginal rate of substitution between effort and wage is given by

$$MRS_{e,w} = -\frac{\partial u_{ij}/\partial e_{ij}}{\partial u_{ij}/\partial w_{ij}} = \theta_i e_{ij} - \gamma_j,$$

which is always positive for non-motivated workers with $\gamma_j = 0$. When the effort required by the principal is sufficiently high, i.e. when $e_{ij} > \frac{\gamma_j}{\theta_i}$, also motivated workers' indifference curves have the standard positive slope in the space (e, w) and effort is a "bad".

Note that providing effort represents a net cost to the agent when

$$-\frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij} < 0.$$

Thus, if the effort required by the principal is sufficiently low, motivated workers could perform their task also when receiving a non-positive reward, in other words they would be ready to volunteer to be hired by the firm.

motivation is firm-specific. When it is related to an occupation rather than to working at a particular firm, monopsony power arises only if there is no other firm (in the neighborhood) offering similar jobs". In turn, the link between workers' motivation and market power justifies our hypothesis concerning profit maximization and wage setting on the part of the principal.

[10] This linear-quadratic specification of the utility function is widely used in the literature on workers' intrinsic motivation (see Besley and Ghatak 2005 and Delfgaauw and Dur 2010). The same objective function for the agent is also considered in the literature on multidimensional screening with a continuum of types (see Laffont et al. 1987, Basov 2005, and Deneckere and Severinov 2011), where solutions are found imposing that the type space be the unit square.

Remark 1 *When* $e_{iH} < \frac{2\gamma}{\theta_i}$, *a motivated worker obtains a net positive utility from effort exertion and is thus willing to receive a non-positive reward. We call such a worker a "volunteer".*

Finally, notice that agents' utility function satisfies the single-crossing property with respect to each parameter of private information at a time.[11]

Remark 2 *The single-crossing property is satisfied with respect to the ability parameter and with respect to motivation. In fact* $MRS_{e,w}$ *is increasing in* θ *(holding motivation constant) and decreasing in* γ *(holding ability constant).*

This means that the indifference curves of types with the same motivation but different ability intersect only once for $e = 0$ **,** and the same is true for the indifference curves of workers endowed with the same ability but different motivation. Nonetheless, the single-crossing property does not hold when both ability and motivation change simultaneously.

Remark 3 *The indifference curves of intermediate types* HH *and* LL *cross twice, the second intersection occurring at* $e = \frac{2\gamma}{\theta - 1}$.

Considering the combined impact of both ability and motivation on the worker's effort and on the firm's output, we can say that the most efficient type is worker LH (with low effort cost and high motivation) who is expected to exert the highest effort, whereas the least efficient type is worker HL (with high effort cost and no motivation) who is expected to provide the lowest effort. Worker types LL and HH are in-between and their effort levels cannot be ordered unambiguously.[12]

In Section 4, we will assume that the principal offers the agent a menu of contracts of the form $\{e, w(e)\}$. Applying the Revelation Principle, we will focus on four contracts such that a worker of type ij exerts effort e_{ij} and receives a wage $w(e_{ij}) = w_{ij}$. Before turning to the second-best, let us examine the benchmark cases in which either there is no private information at all (the first-best) or there is only one characteristic that is the agent's private information.

[11] All the properties of the utility function extend to the more general case in which the cost of effort is still convex while the benefit from exerting effort, due to intrinsic motivation, is concave. Moreover, it is possible to prove that all qualitative results concerning the second-best solutions carry on in this general case.

[12] Notice that the existence of two possible orderings of effort levels is a consequence of the bidimensionality of our problem and of the failure of the single-crossing condition. It could not be generated in a unidimensional set-up with different types of employees characterized by a single summary statistic, like the overall cost of providing effort.

3 Benchmark cases

3.1 Full information

At the first-best, both ability and motivation are observable. For $i = L, H$ and $j = L, H$, the principal solves the problem

$$\max_{(e_{ij}, w_{ij})} \pi = e_{ij} - w_{ij} \tag{FB}$$
$$\text{s.t. } u_{ij} \geq 0$$

which yields a level of effort equal to

$$e_{ij}^{FB} = \frac{1 + \gamma_j}{\theta_i} \tag{1}$$

and where the wage levels are set such that each worker receives her zero reservation utility

$$w_{ij}^{FB} = \frac{\left(1 + \gamma_j\right)\left(1 - \gamma_j\right)}{2\theta_i}.$$

If $\gamma_j \leq 1$ is satisfied, then, at the first-best, all wages are non-negative and motivated workers are not volunteers since they face a net cost from exerting effort.[13]

Assumption 1 *Let $0 < \gamma \leq 1$. Then, motivated workers are not volunteers and always receive a non-negative salary at the first-best.*

The intuition for this requirement is straightforward. Given Program FB and first-order condition (1), we can interpret $1 + \gamma$ as the total marginal productivity of effort. When $\gamma \leq 1$, the contribution of worker's intrinsic motivation on the marginal productivity of effort does not dominate the standard one.

Importantly, at the second-best, Assumption 1 is no longer sufficient to ensure that the low-ability motivated type HH bears a net cost of exerting effort, because effort levels might be distorted downwards (the standard result of distortion for types different from the "top" one holds). This implies that, in the next Sections, it will be necessary to check whether $e_{HH}^{SB} \gtreqless \frac{2\gamma}{\theta}$ and it will be shown that the worker type HH can experience a net utility from effort provision, so that she may become a (paid) volunteer at the second-best.

It is immediate to check that $e_{LH}^{FB} > e_{HH}^{FB} > e_{HL}^{FB}$ and $e_{LH}^{FB} > e_{LL}^{FB} > e_{HL}^{FB}$ both hold. Also note that, for intermediate types, one has

$$e_{LL}^{FB} < e_{HH}^{FB} \text{ if and only if } \gamma > \Delta\theta \equiv \gamma^*, \tag{2}$$

[13] This assumption allows us to exclude situations where, at the first-best, motivated workers receive a negative wage while non motivated employees receive a positive salary. Our analysis can easily be extended to allow for volunteers and standard workers to coexist at the first-best. At the second-best, threshold values obtained when the difference in motivation is more important than the difference in ability would change, whereas the classes of equilibria when ability prevails over motivation would not be affected.

or

$$e_{LL}^{FB} > e_{HH}^{FB} \text{ if and only if } \gamma < \Delta\theta \equiv \gamma^*, \tag{3}$$

while $e_{LL}^{FB} = e_{HH}^{FB}$ whenever $\gamma = \gamma^*$. Hence, $\gamma = \gamma^*$ can be defined as the value of motivation such that the type space corresponds to a square and types LL and HH become equivalent.[14]

Inequality (2) can be restated as[15]

$$e_{LL}^{FB} < e_{HH}^{FB} < \frac{\gamma}{\Delta\theta}, \tag{4}$$

and (3) is equivalent to

$$e_{LL}^{FB} > e_{HH}^{FB} > \frac{\gamma}{\Delta\theta}. \tag{5}$$

Given Assumption 1, a necessary condition for (2) is that $\gamma^* \leq 1$ or else that $\theta \leq 2$.

Remark 4 *The ordering of effort levels at the first-best is as follows:*

1. *If $\theta \leq 2$ and $\gamma > \gamma^*$ both hold, then the ordering of effort levels is $e_{LH}^{FB} > e_{HH}^{FB} > e_{LL}^{FB} > e_{HL}^{FB}$.*

2. *If $\gamma < \gamma^*$, then the ordering of effort levels is $e_{LH}^{FB} > e_{LL}^{FB} > e_{HH}^{FB} > e_{HL}^{FB}$.*

Intuitively, when $\gamma > \gamma^*$, the difference in motivation, or simply γ, is higher than the difference in ability $\Delta\theta$, in which case the effort provided by type HH is higher than that of type LL and the first ranking in the Remark is relevant. Conversely, when $\gamma < \gamma^*$ the difference in ability $\Delta\theta$ is more important than the difference in motivation: now the second ranking in the Remark is relevant and the effort provided by type LL must be higher than the effort level of type HH.

The parameter γ^* will play a crucial role not only under full information but also at the second-best. In particular, we will show that the second-best optimal contracts require pooling between types LL and HH in a whole region around $\gamma = \gamma^*$ and that the ranking of types at the first- and second-best is the same (see Figure 4 in Section 4).

Since both instances in Remark 4 are economically relevant, we impose that $\gamma^* \leq 1$ which is equivalent to $\theta \leq 2$.

Assumption 2 *Let $1 < \theta \leq 2$. Then $0 < \gamma^* \leq 1$ holds and all orderings $e_{HH}^{FB} \gtreqless e_{LL}^{FB}$ are possible.*

Let us consider the ranking of wages with perfect information.

[14] Note that both γ and $\Delta\theta$ may assume different values in our model so that a continuum of possible "square" type spaces exists.

[15] Take $e_{LL}^{FB} > e_{HH}^{FB}$. This amounts to $1 > \frac{1+\gamma}{\theta}$ or else to $\theta > 1 + \gamma$. It follows that $\gamma < \theta - 1 = \Delta\theta$ or else that $\frac{\gamma}{\Delta\theta} < 1 = e_{LL}^{FB}$. Similarly, starting from $\gamma < \Delta\theta$ and adding to both sides of the inequality $\gamma\Delta\theta$ yields $\frac{\gamma}{\Delta\theta} < \frac{1+\gamma}{\theta} = e_{HH}^{FB}$. The same reasoning can be applied to the opposite case in which $e_{HH}^{FB} > e_{LL}^{FB}$.

Remark 5 *The ordering of wage levels at the first-best is as follows:*

$$w_{LL}^{FB} > \max\left\{w_{LH}^{FB}, w_{HL}^{FB}\right\} \geq \min\left\{w_{LH}^{FB}, w_{HL}^{FB}\right\} > w_{HH}^{FB} \geq 0$$

For fixed ability, motivated workers always obtain lower rewards than non-motivated ones. In addition, when $w_{HL}^{FB} > w_{LH}^{FB}$, motivated workers always earn less than non-motivated workers independently of their ability.

3.2 Adverse selection on ability

Suppose that workers' motivation γ_j is observable to the principal but ability θ_i is not, we call this case Benchmark A, or BA. For fixed $j = L, H$ the principal solves

$$\max_{(e_{Hj}, w_{Hj});(e_{Lj}, w_{Lj})} E(\pi) = \nu\left(e_{Lj} - w_{Lj}\right) + (1-\nu)\left(e_{Hj} - w_{Hj}\right) \qquad (BA)$$

subject to the participation constraint of the low-ability type and to the incentive compatibility constraint of the high-ability worker. Solving for the effort levels, we find

$$e_{Lj}^{BA} = 1 + \gamma_j = e_{Lj}^{FB}$$

and

$$e_{Hj}^{BA} = \frac{\left(1+\gamma_j\right)(1-\nu)}{(\theta-\nu)},$$

where the results of *no distortion at the top* and downward distortion in the effort exerted by the low-ability worker both hold. It is easy to show that full participation is always optimal or that it is never in the principal's interest to exclude low-ability workers (types Hj).[16]

As for wages, we have $w_{HH}^{BA} > 0$ if and only if

$$\gamma < \frac{\theta(1-\nu)}{(\theta-\nu)+\nu\Delta\theta} \equiv \gamma^{BA} < 1,$$

meaning that, when ability is workers' private information while motivation is observable, type HH can become a volunteer if motivation is high enough. Moreover, for any given level of employees' motivation, the wage rate is increasing in workers' ability.

3.3 Adverse selection on motivation

Suppose now that workers' ability θ_i is observable to the principal but motivation γ_j is not, we call this case Benchmark M, or BM. For fixed $i = L, H$ the principal solves

$$\max_{(e_{iH}, w_{iH});(e_{iL}, w_{iL})} E(\pi) = \mu\left(e_{iH} - w_{iH}\right) + (1-\mu)\left(e_{iL} - w_{iL}\right) \qquad (BM)$$

[16] In fact, the principal's benefit from keeping worker Hj is the expected profit from this worker $(1-\nu)\left(e_{Hj} - w_{Hj}\right)$, whereas the cost from letting her participate is the information rent $\frac{1}{2}\Delta\theta e_{Hj}^2$ multiplied by the proportion of workers receiving the rent, that is ν. By substituting the expression for the wage in $(1-\nu)\left(e_{Hj} - w_{Hj}\right)$, it can be checked that the principal always offers a non-null contract to low-ability workers, independently of their motivation.

subject to the participation constraint of the non-motivated type and to the incentive compatibility constraint of the motivated worker. In fact, motivated agents have interest in mimicking non-motivated ones whenever the effort they are required to provide is sufficiently high so as to cause a disutility.

Solving for effort levels we find

$$e_{iH}^{BM} = \frac{1+\gamma}{\theta_i} = e_{iH}^{FB}$$

and

$$e_{iL}^{BM} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\,\theta_i}$$

where the results of *no distortion at the top* and downward distortion in the effort exerted by the non-motivated worker both hold. Also, $e_{iL}^{BM} > 0$ for

$$\gamma < \frac{1-\mu}{\mu} \equiv \gamma^{BM}$$

where $\gamma < \gamma^{BM}$ always holds if $\mu < \frac{1}{2}$. In words, when γ is sufficiently high, the information rent that the principal must pay to the motivated types is so costly that he prefers to exclude non-motivated workers. However, the necessary condition for full participation, that is $e_{iL}^{BM} > 0$, is always satisfied if the proportion μ of motivated workers is sufficiently low. Following the same procedure as in Footnote 16, it can be checked that $e_{iL}^{BM} > 0$ is both necessary and sufficient for full participation.

As for wages, they are always increasing in motivation and $w_{iH} > w_{iL}$. Hence, when motivation is workers' private information and ability is observable, the ranking of salaries for workers who are equally skilled but have different vocation is reversed with respect to the first-best. This is because of the information rents that motivated workers receive.

4 Screening on both ability and motivation

The benchmark cases with unidimensional hidden information provide the following predictions: (i) When the principal cannot observe workers' skills (but is perfectly informed about their motivation), he might take advantage of motivated workers and make them work for free; as we will see, this turns out to be impossible at the second-best. (ii) When the principal cannot observe workers' motivation (but is perfectly informed about their skills), he might find in his interest to exclude non-motivated employees, no matter whether they have high- or low-ability; the exclusion of skilled, non-motivated workers will always be dominated at the second-best. Furthermore, given ability, motivated employees are always offered a higher salary than non-motivated ones; this result stands in contrast with the first-best but will be confirmed at the second-best.

Suppose now that both the workers' ability θ_i and motivation γ_j are the agents' private information, we call this situation the second-best. The principal offers the worker a choice of four effort-wage combinations. For $i = L, H$ and $j = L, H$, the principal's program is

$$\max_{(e_{ij},w_{ij})} E(\pi) = \begin{array}{l} \nu\mu(e_{LH} - w_{LH}) + \nu(1-\mu)(e_{LL} - w_{LL}) + \\ (1-\nu)\mu(e_{HH} - w_{HH}) + (1-\nu)(1-\mu)(e_{HL} - w_{HL}) \end{array} \tag{SB}$$

subject to four participation constraints whose generic form is

$$w_{ij} - \frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij} \geq 0 \tag{PC_{ij}}$$

and twelve incentive compatibility constraints that are such that

$$w_{ij} - \frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij} \geq w_{i'j'} - \frac{1}{2}\theta_i e_{i'j'}^2 + \gamma_j e_{i'j'} \tag{$IC_{ijvsi'j'}$}$$

with ij different from $i'j'$. All constraints are listed in Appendix B, where we show that incentive compatibility and participation constraints satisfy some regularity conditions.

When all participation and incentive compatibility constraints are satisfied, the solution to the principal's program SB is characterized by full separation and full participation of types. In such instances, we find that if the participation constraint of the worst type HL holds then all other PCs are also satisfied. Moreover, the following monotonicity (or implementability) condition holds

$$e_{LH} \geq \max\{e_{LL}; e_{HH}\} \geq \min\{e_{LL}; e_{HH}\} \geq e_{HL}. \tag{6}$$

Concerning intermediate types HH and LL, one can add IC_{LLvsHH} and IC_{HHvsLL} and find that either

$$e_{HH}^{SB} > e_{LL}^{SB} \quad \text{and} \quad e_{LL}^{SB} + e_{HH}^{SB} \leq \frac{2\gamma}{\Delta\theta}, \tag{7}$$

or

$$e_{LL}^{SB} > e_{HH}^{SB} \quad \text{and} \quad e_{LL}^{SB} + e_{HH}^{SB} \geq \frac{2\gamma}{\Delta\theta} \tag{8}$$

holds. Otherwise $e_{HH}^{SB} = e_{LL}^{SB}$ and bunching between intermediate types necessarily occurs. Although conditions (7) and (8) are less transparent than the corresponding first-best conditions (4) and (5), we can still observe that $e_{HH} > e_{LL}$ holds at the second-best when the quantity $\frac{\gamma}{\Delta\theta}$ is high: paralleling the first-best, we will say that condition (7) is satisfied when the difference in motivation prevails over the difference in ability in determining effort and output provision. On the contrary, if condition (8) and $e_{LL} > e_{HH}$ hold at the second-best, then it is because the quantity $\frac{\gamma}{\Delta\theta}$ is low and we will say that the difference in ability prevails over motivation in determining effort and output provision. A difference with respect to the first-best is that the term $\frac{\gamma}{\Delta\theta}$, reflecting the relative importance of motivation heterogeneity *vis à vis* ability heterogeneity, is doubled in (7) and (8), since it must now be compared with the sum of the two effort levels exerted by intermediate types.[17] Observations 1 and 2, respectively, will show that

[17] Note that condition (4) is *per se* more restrictive than (7) and that condition (5) is again more restrictive than condition (8). Hence, one can in principle expect some misalignment between first- and second-best effort levels as for intermediate types, but this will not be the case as Proposition 2 points out.

while monotonicity condition (7) implies that the incentive compatibility constraint IC_{HHvsLL} is binding, condition (8) does not rule out situations in which IC_{HHvsLL} still binds or in which the allocations of types HH and LL are envy-free.

When considering solutions characterized by full separation and full participation of types, we will solve a relaxed program in which only PC_{HL} and some (mostly downward) incentive constraints will bind; we will check ex post that the omitted constraints are verified as well. Nonetheless, it might well be that some constraints fail to hold and that the optimal contract calls for the exclusion of some workers' types or for pooling or both. It is possible to show that the solution to program SB entailing full participation and full separation of types always yields the highest profits to the principal, who will then always implement it when possible.[18]

Proposition 1 *Independently of whether motivation or ability prevail, i.e. independently of whether condition (7) or condition (8) holds, the principal's profits are maximal at the solution with full participation and full separation of types.*

Proof. The proof for the situation in which motivation prevails is illustrated in Appendix C.3. It requires all results derived from the full characterization of the different classes of optimal contracts (full screening, pooling and contracts with exclusion) and the conditions for their existence. The procedure for the case in which ability prevails is equivalent and then omitted. ■

The above Proposition is anticipated here for expositional purposes. In what follows, we will present the classes of solutions characterized by full participation and full separation of types, classifying them according to whether condition (7) or condition (8) holds. We are going to explain which incentive constraints are binding in the two alternative situations. The aim of such discussion is mainly descriptive, since we would like to give the reader a complete overview of the results we obtained. The more detailed analysis of the different classes of optimal contracts is given in Sections 5.1 and 5.2 and in the Appendix. At the end of this Section, we will briefly discuss the remaining classes of solutions entailing bunching or exclusion, but their formal analysis can be found in the Appendix.

Observation 1 ***Motivation prevails (Case M)*** *When condition (7) holds, motivation has a higher impact on effort provision than ability, and a solution to program SB with full participation and full separation of types such that $e_{HH} > e_{LL}$ might be attained. The binding downward incentive constraints specific to this case are those of high-ability types mimicking low-ability ones, that is IC_{LHvsHH} and IC_{LLvsHL}. The additional downward incentive constraint is IC_{HHvsLL}, connecting the previous ones.*

If motivation has a higher impact on effort and output provision than ability, then from the principal's viewpoint, types can be ordered as $LH \succ HH \succ LL \succ LH$. Now we have to solve a bidimensional

[18] As mentioned in the Related Literature, the implementability of fully separating and fully participating solutions is unusual in models of multidimensional screening with discrete types and impossible when the types space is continuous.

screening problem which embeds and generalizes the two sub-problems with adverse selection on the workers' ability only (Benchmark BA in Subsection 3.2). The two sub-problems BA are now considered simultaneously and linked by incentive constraint IC_{HHvsLL}. Figure 1 describes this case. On the horizontal axis, we represent effort cost or ability, while on the vertical axis we have motivation. Types are located at the corners of a rectangle whose width is the difference in ability $\Delta\theta$, and whose height is the difference in motivation $\Delta\gamma$, or simply γ.[19] An arrow from one type to another represents that the incentive constraint that the former type does not choose the contract designed for the latter type is binding.

Insert Figure 1 and Figure 2a around here

Intuitively, when the difference in motivation is more relevant than the difference in ability, the rectangle on which types are located has height greater than width. Since types LH and HH on one hand and types LL and HL on the other hand are close to each other, then it is natural that the incentive constraints that bind first are IC_{LHvsHH} and IC_{LLvsHL}. The remaining binding constraint is the one that concerns intermediate types, namely IC_{HHvsLL}. Indeed, Case M occurs when motivation γ is high enough, then type HH is asked to provide a relative high effort in exchange for a relatively low salary and she might find the contract (e_{LL}, w_{LL}) potentially convenient.

Observation 2 ***Ability prevails (Case A)*** *When condition (8) holds, ability has a higher impact on effort provision than motivation, and a solution to program SB with full participation and full separation of types such that $e_{LL} > e_{HH}$ might be attained. The binding downward incentive constraint specific to this case is that of the high-ability motivated type mimicking the high-ability non-motivated agent, that is IC_{LHvsLL}. As for the other relevant binding constraints, three sub-cases must be considered: (1) Case A.1. The binding incentive constraints are the two adjacent ones IC_{LLvsHH} and IC_{HHvsHL}; (2) Case A.2. The binding incentive constraints are IC_{LLvsHL} and IC_{HHvsHL}; (3) Case A.3. The binding incentive constraints are IC_{LLvsHL} and the upward IC_{HHvsLL}.*

If the difference in ability has a higher impact on effort and output provision than the difference in motivation, then, from the principal's viewpoint, types can be ordered as $LH \succ LL \succ HH \succ LH$. Now we have a plurality of situations arising because the principal faces a trade-off between the need to satisfy condition $e_{LL} > e_{HH}$ and the incentive to increase e_{HH} as motivation grows.

Case A.1 is the most natural one and is symmetric to Case M: it requires to solve a bidimensional screening problem that consists of the two sub-programs related to adverse selection on workers' motivation only (as Benchmark BM in Subsection 3.3) together with incentive constraint IC_{LLvsHH} (see

[19] For simplicity, Figures from 1 to 2c are all drawn letting the difference in motivation $\Delta\gamma$ vary while keeping the difference in effort cost $\Delta\theta$ constant. Actually, it might well be that different contracts with full separation and full participation exist for different values of both $\Delta\gamma$ and $\Delta\theta$.

Figure 2a). Now the rectangle on which types are located has height smaller than width, whereby the types that are closest to each other are LH and LL on one hand and HH and HL on the other hand. Then the incentive constraints that bind first are those of the closest pairs IC_{LHvsLL} and IC_{HHvsHL}. The remaining binding constraint is the one that concerns intermediate types, namely IC_{LLvsHH}. Here, the motivation level γ is sufficiently low and then type LL might be induced to mimic type HH because the former can benefit from a lower effort e_{HH} and still enjoy a salary w_{HH} which cannot be too low (given that motivation plays a minor role).

Between Case M and Case $A.1$, that is when motivation is still less important than the difference in ability but is not too low, the two less intuitive situations occur: Case $A.2$ and Case $A.3$. Here a tension realizes since, on the one hand, type LL is asked to provide a higher effort than type HH; on the other hand, when moving from Case $A.1$ to Case M (i.e. when letting motivation γ grow), type HH worker faces a diminishing disutility of effort so that it becomes more and more convenient for the principal to ask her to provide a larger effort and to pay her a lower salary. This tension leads anomalous incentive constraints to bind. In particular, Cases $A.2$ and $A.3$ emerge when type LL prefers to mimic HL rather than HH so that the local downward incentive constraint IC_{LLvsHH} is not binding anymore and the global IC_{LLvsHL} is binding instead. Then, together with the standard no-distortion at the top, the principal will have no interest in distorting the effort required to worker HH, because no other type is willing to mimic HH.

Case $A.2$ represents a bidimensional screening problem consisting of the two sub-programs related to adverse selection on workers' motivation (as in Benchmark BM) which are now connected by incentive constraint IC_{LLvsHL} (see Figure 2b). Interestingly, it is the unique case in which there is no envy between types LL and HH so that neither IC_{LLvsHH} nor IC_{HHvsLL} are binding.

In Case $A.3$, motivation keeps increasing and the disutility from the effort exerted by type HH is even lower than in Case $A.2$. Thus, it turns out that type HH mimics LL rather than HL. Now, not only is the local downward incentive constraint IC_{LLvsHH} slack, so that no-distortion for type HH occurs, but the upward incentive constraint IC_{HHvsLL} is binding instead. This occurs since the motivated type HH values a relatively high wage associated with a high effort (that she would obtain by mimicking LL) more than the combination of low wage and low effort (that she would get by mimicking HL). Case $A.3$ is represented in Figure 2c.

Insert Figure 2b and Figure 2c around here

Figure 3 illustrates the classes of optimal contracts just presented. Which class of solution realizes depends on the relative position of the term $\frac{2\gamma}{\Delta\theta}$ with respect to the sum of different pairs of effort levels. The distinction between Case M and Case A depends on conditions (7) and (8). In addition, in order to discriminate among the different situations arising when ability prevails, we consider which incentive constraints are binding and which are slack, as described in Observation 2.

Insert Figure 3 around here

Finally, Figure 4 describes optimal contracts as a function of motivation, focusing not only on the existence regions for the four fully separating and fully participating equilibria but also considering equilibria with pooling and/or exclusion that arise in-between. All equilibria are mutually exclusive, since, for any given realization of the parameters $\gamma \in (0,1]$ and $\theta \in (1,2]$, a different solution to program SB is obtained. In other words, the different contacts appearing in Figure 4 are the ones that assure the highest profits to the principal among all possible solutions that might coexist in a given parameter region. The dominating contracts for Case M are listed in Appendix C.4, while the dominating contracts for Case A can be derived in the same way from the results contained in Appendix D.[20]

Insert Figure 4 around here

As for exclusion, Figure 4 shows that the occurrence of equilibria with exclusion is really limited and essentially relegated to small regions lying in-between fully participating and fully separating solutions for Case A.1 and Case A.2 and in-between fully participating and fully separating solutions for Case A.2 and Case A.3.

Concerning pooling contracts, note that, when motivation takes the lowest possible values (that is below Case A.1 with full separation and full participation), then a pooling equilibrium where the low-ability types HH and HL are given the same contract emerges (see Appendix D.1.2). At the other extreme, for the highest possible values of motivation (that is above Case M with full separation and full participation), a pooling equilibrium where the non-motivated types HL and LL are given the same contract is attained (see Appendix C.2).

More importantly, there is a wide range of situations in which a solution with bunching of intermediate types HH and LL is attained, especially when motivation and ability have a similar impact on effort provision, that is for values of γ close to γ^*. In a pooling equilibrium with $e_{LL} = e_{HH} = e_p$, we must distinguish two sub-cases: the first one where the binding incentive constraint is IC_{HHvsHL}, which is relevant when $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$ (see Appendix D.4.1) and the second one where the binding incentive constraint is IC_{LLvsHL}, occurring when $e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$ (see Appendix D.4.2). When separation of types LL and HH becomes impossible, Case A.1 turns to the pooling equilibrium with IC_{HHvsHL} binding, whereas Case M, Case A.2 and Case A.3 all turn to the pooling equilibrium with IC_{LLvsHL} binding.

Interestingly, Figure 4 clearly shows that all optimal contracts involving separation of intermediate types HH and LL (including not only full participation and full separation but possibly exclusion or

[20]Moreover, note that for some parameter configurations, some optimal contracts might not exist. Appendix E considers the possible equilibria arising in the particular case in which the probability distribution of types is uniform.

bunching of other types) under Case M hold when $\gamma > \gamma^*$ while all optimal contracts involving separation of intermediate types HH and LL (again, including not only full participation and full separation but possibly exclusion or bunching of other types) under Case A attain when $\gamma < \gamma^*$. Therefore, when effort levels are aligned in a given way at the first-best, then the same ordering of effort levels arises at the second-best.

Proposition 2 *The ranking of second-best effort levels is always the same as the first-best ranking.*

Proof. Looking at Figure 4, in order to prove the above Proposition it is sufficient to show that: (i) the interval of motivation levels for which there exists a solution with full participation and full separation under Case M lies to the right of γ^* (this is done in Appendix C.1); (ii) the interval of motivation levels for which there exists a solution with full participation and full separation under Case $A.3$ lies to the left of γ^* (and this is done in Appendix D.3.1). ■

5 Optimal contracts with full separation and full participation

In light of Proposition 1, in this Section we will focus on the characterization of contracts with full participation and full separation of types, relegating to the Appendix the analysis of all situations with pooling and/or exclusion of types. Instead of presenting all four possible classes of solutions, we will restrict attention to the ones that arise when motivation is sufficiently high, namely Cases M and $A.3$.[21] The rationale behind such a choice is manyfold: (i) since our main contribution is in studying the impact of intrinsic motivation on optimal screening contracts that a principal might offer, it is reasonable to focus on situations where motivation is significant; (ii) Cases M and $A.3$ are almost exhaustive because they belong to the two different states of the world where motivation prevails or ability prevails, respectively: (iii) Cases M and $A.3$ are close to each other in terms of existence conditions (see Figures 3 and 4) and therefore they are more easily comparable than distant cases; (iv) finally and more interestingly, contrasting the two situations, one can show that very high motivation might not be a desirable worker's characteristic for the principal.

To improve readability, in the text we mainly provide a qualitative description of the two different solutions with economic intuitions; we refer the reader to Appendices C.1 and D.3.1 for quantitative results, technical statements concerning existence conditions and proofs concerning Cases M and $A.3$, respectively.

[21] The analysis of Cases $A.1$ and $A.2$ will be carried out in Appendices D.1.1 and D.2.1, respectively.

5.1 The solution when motivation prevails (Case M)

In Case M, a separating equilibrium occurs if condition (7) holds, that is if $e_{HH} > e_{LL}$ and $e_{LL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta}$ both hold. The implementability condition (6) thus becomes $e_{LH} > e_{HH} > e_{LL} > e_{HL}$. The constraints that are expected to bind at the optimum with full participation are the downward local incentive constraints IC_{LHvsHH}, IC_{HHvsLL}, IC_{LLvsHL} and PC_{HL} (see Observation 1 and Figure 1). In this situation, motivation γ is high enough for type HH to be asked to provide a relative high effort in exchange for a salary that is quite low (in fact w_{HH} may also be lower than the salary offered to worker LL, as the inequality in Remark 6 which follows points out). Thus, type HH might find the contract (e_{LL}, w_{LL}) appealing.

Note that Case M (together with Case $A.1$) corresponds to the situation where our bidimensional screening problem is equivalent to a unidimensional screening one with four types, the unidimensional parameter of private information being the workers' overall cost of effort exertion.[22]

Given the binding constraints, we can solve for the wage schedules, which allow us to isolate the information rents received by each type of worker

$$w_{HL} = \frac{1}{2}\theta e_{HL}^2, \tag{9}$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \underbrace{+\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL}, \tag{10}$$

$$w_{HH} = \frac{1}{2}\theta e_{HH}^2 - \gamma e_{HH} \underbrace{-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } HH} \tag{11}$$

and finally

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\frac{1}{2}\Delta\theta e_{HH}^2 - \frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH} \tag{12}$$

All types except HL receive an information rent and information rents cumulate when moving from the worst type HL up to the best type LH. Since information rents are always increasing in the effort exerted by the types that can be mimicked, we obtain the familiar results of *no distortion at the top* and downward distortions in effort levels for all agents' types other than LH. Moreover, all information rents include at least one expression of the form $\frac{1}{2}\Delta\theta e_{ij}^2$, as in Benchmark BA. Only motivated types HH and LH receive an information rent depending also on motivation γ, which comes from the incentive constraint IC_{HHvsLL} linking the two unidimensional screening problems of Benchmark BA. In particular,

[22] Notice that, when motivation prevails over ability, we obtain a screening problem which is equivalent to the one of benchmark BA, where the unidimensional private information is workers' ability. The fact that here the relevant incentive constraints are the ones of benchmark BA might seem counterintuite. Nonetheless, when motivation prevails, the constraints that bind first are precisely the ones connecting types characterized by the same motivation but different ability (see Figure 1a).

the rent received by type HH when mimicking LL is given by $-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL}$ which is always positive and increasing in e_{LL} when motivation prevails.

Substituting the wage schedules into the principal's objective function and maximizing yields the optimal effort levels e_{ij}^{SBM}, whose expressions are provided in Appendix C.1. Interestingly, e_{HH}^{SBM} has the same expression as in Benchmark BA. Nonetheless, effort levels required from workers LL and HL are more distorted than those in program BA, because of the cumulative effect of information rents stemming from the bidimensional nature of adverse selection.

Given the optimal efforts, the optimal wage levels and the informational rents (i.e. indirect utilities) can be ranked as follows.

Remark 6 *When motivation prevails (Case M), at the optimal contract with full separation and full participation, the ranking of wages is*

$$w_{LH}^{SBM} > \max\left\{w_{HH}^{SBM}, w_{LL}^{SBM}\right\} \geq \min\left\{w_{HH}^{SBM}, w_{LL}^{SBM}\right\} > w_{HL}^{SBM} > 0$$

and the ordering of information rents (indirect utilities) is

$$u_{LH}^{SBM} > u_{HH}^{SBM} > u_{LL}^{SBM} > u_{HL}^{SBM} = 0.$$

Information rents have the same ordering as effort levels, while there can be a reversal in the ranking of salaries for intermediate types. In other words, the principal could offer the motivated but high-cost type HH a contract in which effort provision is higher and remuneration is lower than in the contract proposed to type LL. Nonetheless, type HH always enjoys a higher utility than type LL, because her high motivation generates higher informational rents. This result is not trivial and depends on the peculiarity of motivated workers' utility function, which admits voluntary work.[23] Moreover, when $w_{HH}^{SBM} < w_{LL}^{SBM}$ holds, then it is always the case that $e_{HH}^{SBM} < \frac{2\gamma}{\theta}$, implying that motivated high-cost types HH enjoy a net positive utility from effort provision. This has a negative impact on total salary w_{HH} (see the first two terms in expression 11), which is more than compensated by the positive effect of information rents.

Corollary 1 *When motivation prevails (Case M), at the optimal contract with full separation and full participation, worker HH might be a "paid volunteer": she is offered a positive wage, but she enjoys a positive utility from effort exertion.*

Actually, it is also possible (although very rare) that $e_{HH}^{SBM} < \frac{\gamma}{\theta}$, in which case effort required to type HH is in the range in which her utility increases in effort and her indifference curve is downward sloping in

[23] In particular, $w_{HH}^{SBM} < w_{LL}^{SBM}$ holds when the probability of high motivation μ is low relative to the probability of low effort cost ν, when the difference in ability $\Delta\theta$ is high and when the level of motivation is high too.

the space (e, w). Then type HH would be willing to exert more effort than her optimal contract specifies and it would be necessary for the principal to forbid this type to engage in voluntary overtime.[24], [25]

Finally, the interval of motivation levels for which full participation and full separation is attained in Case M always lies to the right of γ^*. Hence Case M realizes at the second-best when condition (2) holds at the first-best: this confirms the result anticipated in Proposition 2, namely that effort levels have the same ranking both at first- and at the second-best.

5.2 The solution when ability prevails (Case A)

When ability prevails, full separation occurs under condition (8), that is if $e_{LL} > e_{HH}$ and $e_{LL}+e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ both hold. The implementability condition (6) thus becomes $e_{LH} > e_{LL} > e_{HH} > e_{HL}$.

Three different solutions characterized by full separation and full participation of types might be found according to which incentive compatibility constraints are binding together with constraints IC_{LHvsLL} and PC_{HL} (see Figure 3). In what follows, we concentrate on Case $A.3$.

5.2.1 Case $A.3$

Suppose that the binding incentive constraints are IC_{LHvsLL}, IC_{LLvsHL} and the *upward* incentive constraint IC_{HHvsLL}, together with participation constraint PC_{HL} (see Figure 2c). This results in inequality $e_{HL} + e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HH} + e_{LL}$. This program bridges Cases A and Case M. Indeed, the incentive constraint that is shared with the other cases in which ability prevails is IC_{LHvsLL}, whereas the other two binding constraints are IC_{HHvsLL} and IC_{LLvsHL} as in Case M.

The relevant wage levels and informational rents are now

$$w_{HL} = \frac{1}{2}\theta e_{HL}^2, \tag{13}$$

$$w_{HH} = \frac{1}{2}\theta e_{HH}^2 - \gamma e_{HH} \underbrace{- \frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } HH}, \tag{14}$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL} \tag{15}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+ \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH}. \tag{16}$$

[24] The same conclusion also holds when the optimal contract requires pooling between types HH and LL (see Appendix D.4.2). Conversely, no such instance occurs when ability prevails.

[25] Nonetheless, being effort perfectly observable and contractible, forbidding voluntary overtime does not invalidate our results.

The information rent of type HH is composed of two terms: one is $\frac{1}{2}\Delta\theta e_{HL}^2$ and represents the rent received through type LL mimicking HL (which accrues to all types except HL); the other one is $-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL}$ and represents the part of the rent specific to type HH mimicking LL. The latter has the same expression as in Case M and is strictly positive if and only if $e_{LL} < \frac{2\gamma}{\Delta\theta}$, a condition which is always satisfied when $e_{HL} + e_{LL} \leq \frac{2\gamma}{\Delta\theta}$, as in this case. Also note that motivated types receive an information rent which depends both on the difference in ability and on motivation, so that this case shares some features both with Benchmark BA and with Benchmark BM.

Substituting the wage functions into the principal's expected profits and maximizing, we obtain the optimal effort levels e_{ij}^{SBA3}, whose expressions are provided in Appendix D.3.1. We observe that e_{HL}^{SBA3} has the same expression as e_{HL}^{SBM} and, more importantly, that both e_{LH}^{SBA3} and e_{HH}^{SBA3} are equal to their first-best levels.

Remark 7 *When ability prevails and constraints IC_{LLvsHL} and IC_{HHvsLL} are binding (Case $A.3$), at the optimal contract with full separation and full participation, the effort of motivated types is never distorted.*

Since type LL is not willing to mimic type HH, the principal has no reasons to distort downwards her effort level. The same result holds in Case $A.2$ (see Appendix D.2.1).

Moreover, the usual downward distortion holds for type LL, despite the upward incentive constraint IC_{HHvsLL} being binding. Nonetheless note that, when the optimal contract calls for exclusion of type HL (that is when motivation falls below the range in which full participation and full separation is guaranteed), then it might well be that effort e_{LL}^{SBA3} is distorted upward with respect to its first-best level. The existence of an upward distortion in second-best effort levels parallels the result of sub-marginal cost pricing in Armstrong (1999). Nonetheless, this result is only found when private and social incentives diverge, that is when first- and second-best allocations are not aligned.

Again, we find that the interval of motivation levels for which full participation and full separation is attained in Case $A.3$ always lies to the left of γ^*. Hence Case $A.3$ realizes at the second-best when condition (3) holds at the first-best: this confirms the result anticipated in Proposition 2, namely that effort levels have the same ranking both at first- and at the second-best.

Given the optimal efforts, the optimal wage levels and the informational rents (i.e. indirect utilities) can be ranked as follows.

Remark 8 *When ability prevails and constraints IC_{LLvsHL} and IC_{HHvsLL} are binding (Case $A.3$), at the optimal contract with full separation and full participation, the ordering of wages is*

$$w_{LH}^{SBA3} > w_{LL}^{SBA3} > w_{HH}^{SBA3} > w_{HL}^{SBA3} > 0$$

while the ranking of information rents is

$$u_{LH}^{SBA3} > u_{HH}^{SBA3} > u_{LL}^{SBA3} > u_{HL}^{SBA3} = 0.$$

While total salaries are monotonic in effort levels, informational rents are not, since there is a switch between intermediate types. In particular, the effort that the motivated unskilled worker is asked to provide is lower than the effort required to the non-motivated skilled worker, even though the former gains higher information rents than the latter. Such a switch of the information rents for intermediate types depends both on the fact that type LL is not willing to mimic type HH (so that LL does not receive any rent depending of the effort exerted by HH) and on the fact that γ is sufficiently high to substantially reduce the disutility of effort provision for type HH.

In the last part of this subsection, we provide a final and important conclusion. In general, it is extremely difficult to make a comparison across the four classes of solutions with full participation and full separation because they exist for different and mutually exclusive parameter ranges. In particular, it is not possible to provide a clear-cut ranking of Case $A.3$ and Case M, neither in terms of the firm's profits nor in terms of social surplus. Nonetheless, considering the trade-off between efficiency and rent extraction faced by the principal, we can state the following result.

Remark 9 *Given the difference in ability $\Delta\theta$, at the optimal contracts with full separation and full participation of types, the principal pays lower informational rents in Case $A.3$ than in Case M.*

Proof. See Appendix D.3.3. ■

The facts highlighted in Remarks 9, 7 and 1 suggest that the principal might be worse-off in Case M than in Case $A.3$. Putting it differently, the high level of motivation supporting Case M might not be a desirable workers' characteristic from the principal's point of view.

This result is reminiscent of Van den Steen (2006), who analyses the consequences of pay-for-performance incentives when principal and agent might disagree on the optimal course of action and concludes that motivation might be too high because it triggers agent's disobedience.

6 Conclusion

It is argued that the efficient selection of workers is more effective, from the principal's point of view, than optimally designing incentives once the worker has been hired. In different words, firms might partially solve their agency problems by hiring agents with specific preferences (see Brehm and Gates 1997, Prendergast 2007, 2008). This seems particularly relevant in a labor market where potential workers can be intrinsically motivated for the job, as in the public sector where employees might be endowed with public service motivation, or in the market for health professionals and for teachers where employees frequently care for their patients and for their students respectively.

The existing literature on intrinsic motivation in the labor market has focused on two major issues: (i) the lemons' problem, mainly investigating adverse (vs propitious) selection effects of workers' private information on the composition of the pool of active workers; (ii) the sorting of different workers' types into different sectors (vocational and non-vocational) of the labor market. We depart from the first strand of literature because we focus our attention at the individual level and examine a principal-agent relationship. We also depart from the second strand of literature because we consider a single sector in isolation. This allows us to fully solve the bidimensional screening problem and to contribute to the existing literature, where the problem of workers' self-selection has either been avoided (because full information on the workers' attributes has been considered, as in Delfgaauw and Dur 2010), or has been modeled in a reduced form (with only a subset of workers being employed, as in Delfgaauw and Dur 2008).

In the paper, we fully characterize the different classes of screening contracts, offered by the firm to the workers, which depend on whether ability or motivation is the main determinant of the workers' overall productivity. Unexpectedly, situations where motivation prevails over ability might not be desirable from the firm's point of view. The intuition for this result is the following: (i) when motivation is much more important (or much less important) than ability, then we obtain standard screening contracts where informational rents and effort distortions cumulate in the usual way; (ii) when instead motivation is less important than ability, but is however sufficiently high, then it interacts with ability on incentives in such a way that non standard contracts emerge with no effort distortions for motivated workers and lower information rents for all worker types.

A possible interpretation of our results is in terms of motivation-based versus skilled-based jobs. Let us consider again the market for nurses. In the case of nurse aides, motivation is the main determinant of workers' performance, while in the case of registered nurses, performance in mainly driven by workers' skills. Similarly in education, standard teachers are asked to perform tasks that are relatively more skilled-based, whereas support teachers are involved in a more motivation-based job. Our results predict that, from the employer's point of view, screening workers comes at a lower cost in the case of a skilled-based profession where, however, motivation is sufficiently important in driving workers' performance. In particular, provided that the impact of motivation on performance is still sufficiently high, screening for highly performing registered nurses and high quality teachers is relatively cheaper than selecting good nurse aides or good support teachers, respectively. Indeed, motivated registered nurses and teachers will be required to exert the efficient level of effort, despite their private information on both ability and motivation. We can conclude that motivation is a desirable workers' characteristic only in the case of skilled-based jobs.

In our future research, we are willing to tackle the problem of sorting of different workers' types into different sectors of the labor market (being one of them vocation-based). In particular, we are going to consider two principals competing for workers who are characterized by different motivation and skill levels. One principal represents the vocational sector and is thus interested in screening potential workers with respect to both motivation and ability (as in the present analysis), while the other principal is only interested in workers' skills.

A Appendix

B Constraints

For type LH the constraints are

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq 0 \qquad (PC_{LH})$$

and

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{LL} - \frac{1}{2}e_{LL}^2 + \gamma e_{LL} \qquad (IC_{LHvsLL})$$

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HH} - \frac{1}{2}e_{HH}^2 + \gamma e_{HH} \qquad (IC_{LHvsHH})$$

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}e_{HL}^2 + \gamma e_{HL}. \qquad (IC_{LHvsHL})$$

For type LL

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq 0 \qquad (PC_{LL})$$

and

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{LH} - \frac{1}{2}e_{LH}^2 \qquad (IC_{LLvsLH})$$

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{HH} - \frac{1}{2}e_{HH}^2 \qquad (IC_{LLvsHH})$$

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{HL} - \frac{1}{2}e_{HL}^2. \qquad (IC_{LLvsHL})$$

For type HH

$$w_{HH} - \frac{1}{2}\theta e_{HH}^2 + \gamma e_{HH} \geq 0 \qquad (PC_{HH})$$

and

$$w_{HH} - \frac{1}{2}\theta e_{HH}^2 + \gamma e_{HH} \geq w_{LH} - \frac{1}{2}\theta e_{LH}^2 + \gamma e_{LH} \qquad (IC_{HHvsLH})$$

$$w_{HH} - \frac{1}{2}\theta e_{HH}^2 + \gamma e_{HH} \geq w_{LL} - \frac{1}{2}\theta e_{LL}^2 + \gamma e_{LL} \qquad (IC_{HHvsLL})$$

$$w_{HH} - \frac{1}{2}\theta e_{HH}^2 + \gamma e_{HH} \geq w_{HL} - \frac{1}{2}\theta e_{HL}^2 + \gamma e_{HL}. \qquad (IC_{HHvsHL})$$

Finally, for type HL one has

$$w_{HL} - \frac{1}{2}\theta e_{HL}^2 \geq 0 \qquad (PC_{HL})$$

and

$$w_{HL} - \frac{1}{2}\theta e_{HL}^2 \geq w_{LH} - \frac{1}{2}\theta e_{LH}^2 \qquad (IC_{HLvsLH})$$

$$w_{HL} - \frac{1}{2}\theta e_{HL}^2 \geq w_{LL} - \frac{1}{2}\theta e_{LL}^2 \qquad (IC_{HLvsLL})$$

$$w_{HL} - \frac{1}{2}\theta e_{HL}^2 \geq w_{HH} - \frac{1}{2}\theta e_{HH}^2. \qquad (IC_{HLvsHH})$$

Considering participation constraints, one can show that participation constraint PC_{HH} is automatically satisfied when PC_{HL} and IC_{HHvsHL} both hold. Also, participation constraint PC_{LH} is automatically satisfied when PC_{LL} and IC_{LHvsLL} are. Finally, once incentive constraint IC_{LLvsHL} and participation constraint PC_{HL} hold, then also participation constraint PC_{LL} is satisfied. So, when all worker types are expected to be hired by the principal and when there is truthful revelation (that is under full participation and full separation of types), it is only necessary to consider the participation constraint of the worst type HL.

As for the incentive compatibility constraints, one can sum them two by two yielding a partial ranking of effort levels. In particular, adding IC_{LLvsHL} to IC_{HLvsLL} and summing IC_{HHvsLH} to IC_{LHvsHH} one has $e_{Lj} \geq e_{Hj}$ $\forall j = L, H$, meaning that, given motivation, effort required must be higher the lower the effort cost. In the same way, adding IC_{HHvsHL} to IC_{HLvsHH} on the one hand and adding IC_{LHvsLL} to IC_{LLvsLH} on the other hand yields $e_{iH} \geq e_{iL}$ $\forall i = L, H$. Namely, for a given effort cost, effort is higher the higher the motivation. Hence the monotonicity condition (6) in the main text holds. Condition (6) also allows us to eliminate some "global" downward incentive constraints and focus on "local" ones. Indeed, adding IC_{LHvsHH} and IC_{HHvsHL} one obtains

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}\theta e_{HL}^2 + \gamma e_{HL} + \frac{1}{2}\Delta\theta e_{HH}^2.$$

But, when $e_{HH} \geq e_{HL}$, the right-hand side of the above inequality is greater than $w_{HL} - \frac{1}{2}e_{HL}^2 + \gamma e_{HL}$, which in turn implies that the global downward incentive constraint IC_{LHvsHL} is satisfied when the two local incentives constraints IC_{LHvsHH} and IC_{HHvsHL} are.[26]

What about intermediate types HH and LL? Adding IC_{LLvsHH} and IC_{HHvsLL} one has

$$\frac{1}{2}\Delta\theta\left(e_{LL} - e_{HH}\right)\left(e_{LL} + e_{HH}\right) - \gamma\left(e_{LL} - e_{HH}\right) \geq 0,$$

which is satisfied for all $e_{HH} = e_{LL}$ or for $e_{HH} \neq e_{LL}$ when either condition (7) or condition (8) in the main text hold.

[26] The same conclusion holds taking the two local incentives IC_{LHvsLL} and IC_{LLvsHL}.

Using the same arguments as before, one can get rid of other global constraints. Suppose that condition (7) is verified: then, the sum of the local constraints IC_{LHvsHH} and IC_{HHvsLL} implies that the global constraint IC_{LHvsLL} is satisfied as well. Moreover, IC_{HHvsLL} and IC_{LLvsHL} imply IC_{HHvsHL}. By the same token, suppose that condition (8) holds: then, one can prove that constraints IC_{LHvsLL} and IC_{LLvsHH} imply constraint IC_{LHvsHH} and also that IC_{LLvsHH} and IC_{HHvsHL} can be used to eliminate IC_{LLvsHL}.

C Motivation prevails (Case M)

C.1 Full separation and full participation

Let us look for a contract such that every type of worker is offered a different contract and such that all types are hired. When all downward local incentive constraints IC_{LHvsHH}, IC_{HHvsLL}, IC_{LLvsHL} and participation constraint PC_{HL} hold with equality, optimal effort levels are given by

$$e_{LH}^{SBM} = 1 + \gamma, \tag{17}$$

$$e_{HH}^{SBM} = \frac{(1-\nu)(1+\gamma)}{(\theta - \nu)}, \tag{18}$$

$$e_{LL}^{SBM} = \frac{\nu(1-\mu) - \mu\gamma}{(1-(1-\nu)(1-\mu)) - \mu\theta} \tag{19}$$

and

$$e_{HL}^{SBM} = \frac{(1-\nu)(1-\mu)}{\theta - (1-(1-\nu)(1-\mu))}, \tag{20}$$

where the superscripts stand for second-best, Case M.

Note that all effort levels are always strictly positive, except for e_{LL}^{SBM}. In order for e_{LL}^{SBM} to be positive and to be a maximum of the principal's expected profits, it is necessary to impose that both the numerator and the denominator in expression (19) be positive;[27] it must be that both

$$\gamma < \frac{\nu(1-\mu)}{\mu} = \gamma_0, \tag{21}$$

where $\gamma_0 > 1$ for $\mu > \frac{\nu}{1+\nu} = \mu_0$ (thus $\mu > \mu_0$ implies that $\gamma < \gamma_0$ is always verified), and

$$\theta < \frac{(1-(1-\nu)(1-\mu))}{\mu} = \overline{\theta}_1^M,$$

with $\overline{\theta}_1^M > 1$, hold.

As far as the monotonicity conditions are concerned, $e_{HH}^{SBM} > e_{LL}^{SBM}$ is satisfied if and only if

$$\gamma > \frac{(\mu(1-\nu) + \nu(1-\mu))\Delta\theta}{\nu\mu\Delta\theta + (1-\nu)(1-(1-\nu)(1-\mu))} = \underline{\gamma}^{SBM},$$

[27] This can easily be seen by collecting e_{LL} in the principal's objective function, once the wage schedules have been substituted, and observing the sign of the coefficient of e_{LL}^2.

where $\underline{\gamma}^{SBM} < 1$ is always the case for $(3\mu\nu - \nu - \mu) \geq 0$, that is for $\nu > \frac{1}{3}$ and $\mu \geq \frac{\nu}{(3\nu-1)}$, whereas, for $(3\mu\nu - \nu - \mu) < 0$, inequality $\underline{\gamma}^{SBM} < 1$ is true when

$$\theta < \frac{\mu + \nu - 3\mu\nu + (1-\nu)(1-(1-\nu)(1-\mu))}{\mu + \nu - 3\mu\nu} = \overline{\theta}_2^M$$

with $\overline{\theta}_2^M > \overline{\theta}_1^M$ if and only if $\mu > \mu_0$ (with $\mu_0 < \frac{1}{2}$). Moreover, $e_{HL}^{SBM} < e_{LL}^{SBM}$ holds for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(\Delta\theta + (1-\mu)(1-\nu))} = \overline{\gamma}^{SBM},$$

with $\overline{\gamma}^{SBM} < 1$ being always the case for $\mu \geq \mu_0$.

Recall that condition (7), which amounts to $e_{LL}^{SBM} + e_{HH}^{SBM} \leq \frac{2\gamma}{\Delta\theta}$, must be satisfied and this is equivalent to

$$\gamma \geq \frac{\Delta\theta(2\nu(1-\mu)(1-\nu) + (\nu-\mu)\Delta\theta)}{2\nu(1-\nu)(1-\mu) + \Delta\theta\nu(2-\mu(\theta+1)) - \Delta\theta(1-\nu)(1-(1-\nu)(1-\mu))} = \gamma_1^{SBM},$$

where $\gamma_1^{SBM} < \underline{\gamma}^{SBM}$ if and only if $\theta < \overline{\theta}_1^M$. Finally, note that the chain of inequalities $\gamma_1^{SBM} < \gamma^* < \underline{\gamma}^{SBM} < \overline{\gamma}^{SBM} < \gamma_0$ holds provided that the denominator of e_{LL}^{SBM} is positive, that is provided that $\theta < \overline{\theta}_1^M$.

Result 1 summarizes what we have found so far.

Result 1 ***Full participation and full separation when motivation prevails.*** *A solution to the principal's program SB which entails full participation and full separation of types, which satisfies the monotonicity condition* $e_{LH} > e_{HH} > e_{LL} > e_{HL} > 0$, *and which is such that effort levels are given by expressions from (17) to (20) exists if and only if* $\theta < \min\left\{\overline{\theta}_1^M, \overline{\theta}_2^M\right\}$ *and* $\underline{\gamma}^{SBM} < \gamma < \overline{\gamma}^{SBM}$ *with*

$$\begin{array}{rcl} \underline{\gamma}^{SBM} & \equiv & \frac{(\mu(1-\nu)+\nu(1-\mu))\Delta\theta}{(\nu\mu\Delta\theta+(1-\nu)(1-(1-\nu)(1-\mu)))} \\ \overline{\gamma}^{SBM} & \equiv & \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(\theta-(1-(1-\nu)(1-\mu)))} \\ \overline{\theta}_1^M & \equiv & \frac{(1-(1-\nu)(1-\mu))}{\mu} \\ \overline{\theta}_2^M & \equiv & \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)} \end{array}.$$

Interestingly, both $\gamma^* < \underline{\gamma}^{SBM}$ and $\min\left\{\overline{\theta}_1^M, \overline{\theta}_2^M\right\} < 2$ hold, so that the alignment of second-best effort levels with the ranking obtained at first-best under condition (4) necessarily holds.

C.2 Pooling and exclusion

The principal can also resort to contracts involving pooling of types and eventually exclusion of some workers' types.

Observe that the fully separating and fully participating solution is characterized by the lower bound $\underline{\gamma}^{SBM}$, which comes from the condition $e_{HH}^{SBM} > e_{LL}^{SBM}$. Therefore, if $\gamma \leq \underline{\gamma}^{SBM}$, the principal is forced to offer the same contract to both types HH and LL. Likewise, the fully separating and fully participating solution is characterized by the upper bound $\overline{\gamma}^{SBM}$, which corresponds to $e_{LL}^{SBM} > e_{HL}^{SBM}$. And if $\gamma \geq \overline{\gamma}^{SBM}$ we always expect a pooling contract where types HL and LL receive the same offer. We refer the reader to Appendix D.4.2 for the detailed analysis of the first situation, while we consider the second one in what follows.

Suppose that there's *pooling* between non motivated types and that $e_{LL} = e_{HL} = e_{\overline{p}}$. Then the ordering of effort levels is $e_{LH} > e_{HH} > e_{LL} = e_{HL} = e_{\overline{p}}$ and the relevant downward incentive constraints that one expects to be binding are IC_{LHvsHH} and IC_{HHvsLL} (or IC_{HHvsHL}, which is equivalent) together with participation constraint PC_{HL}. Since here worker types LL and HL receive the same wage and provide the same effort, $u_{LL} > u_{HL}$ necessarily holds. The wages are

$$w_{LL} = w_{HL} = w_{\overline{p}} = \frac{1}{2}\theta e_{\overline{p}}^2, \tag{22}$$

$$w_{HH} = \frac{1}{2}\theta e_{HH}^2 - \gamma e_{HH} \qquad \underbrace{+\gamma e_{\overline{p}}}_{\text{Info rent worker } HH}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\frac{1}{2}\Delta\theta e_{HH}^2 + \gamma e_{\overline{p}}}_{\text{Info-rent worker } LH}.$$

Substituting the wages into the objective function of the principal and maximizing yields

$$e_{LH} = e_{LH}^{SBM} = e_{LH}^{FB} = 1 + \gamma,$$

$$e_{HH} = e_{HH}^{SBM} = \frac{(1-\nu)(1+\gamma)}{(\theta-\nu)}$$

and

$$e_{LL} = e_{HL} = e_{\overline{p}}^{SBM} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\theta} = e_{HL}^{BM} \tag{23}$$

Note that the expressions for e_{LH} and e_{HH} are the same as in Case M, meaning that *no distortion at the top* is verified and that the effort of individual HH is lower than the corresponding first-best level. Moreover, $e_{LH} > e_{HH}$ still holds. Concerning $e_{\overline{p}}^{SBM}$, it is the same as in Benchmark BM, it is strictly positive for $\gamma < \gamma^{BM}$ and such that $e_{HH} > e_{\overline{p}}^{SBM}$ holds if and only if

$$\gamma > \frac{\nu(1-\mu)\Delta\theta}{\theta(1-\nu) + \mu\nu\Delta\theta} = \gamma_{\overline{p}}$$

where $\gamma_{\overline{p}} < \underline{\gamma}^{SBM}$ always holds.

We are then able to state the following result.

Result 2 ***Full participation and pooling between types*** LL ***and*** HL ***when motivation prevails.*** *The solution to the principal's program SB which entails full participation and pooling between types LL and* HL*, which satisfies the monotonicity condition* $e_{LH} > e_{HH} > e_{LL} = e_{HL} > 0$*, and which is such* *that effort levels are given by expressions (17), (18) and (23) exists if and only if* $\gamma_{\overline{p}} < \gamma < \min\left\{\gamma^{BM}, 1\right\}$ *with*

$$\begin{aligned} \gamma_{\overline{p}} &\equiv \tfrac{\nu(1-\mu)\Delta\theta}{\theta(1-\nu)+\mu\nu\Delta\theta} \\ \gamma^{BM} &\equiv \tfrac{(1-\mu)}{\mu} \end{aligned}$$

being $\gamma^{BM} < 1$ *for* $\mu > \frac{1}{2}$.

The conditions of existence of an equilibrium with full participation and pooling of workers HL and LL are less stringent than the ones we obtained in Result 1 because the requirement $e_{HL}^{SBM} < e_{LL}^{SBM}$ is no longer relevant. Also, the pooled effort $e_{\overline{p}}^{SBM}$ is always in-between expressions (19) and (20): in particular, $e_{HL}^{SBM} > e_{\overline{p}}^{SBM} > e_{LL}^{SBM}$ holds if and only if $\gamma > \overline{\gamma}^{SBM}$.

Note that $\gamma^{BM} \geq 1$ if and only if $\mu \leq \frac{1}{2}$, therefore the principal always proposes a pooling contract to types LL and HL when motivation is sufficiently high (i.e. for $\gamma \geq \overline{\gamma}^{SBM}$) and the probability of being motivated is sufficiently low (i.e. for $\mu \leq \frac{1}{2}$). Conversely, when $\mu > \frac{1}{2}$ and $\gamma^{BM} < 1$, then for $\gamma \geq \gamma^{BM}$ the principal is expected to exclude type HL since the probability of motivated types is high and the productivity loss from type HL is low.

As for *exclusion*, the necessary and sufficient condition for full participation requires in general that, for any type ij, the expected profit from employing type ij be higher than the expected information rents that have to be paid to her mimickers; this condition is satisfied as long as type ij's effort is strictly positive. However, the condition $e_{ij} > 0$ might call for some restrictions on the parameter space, as occurred in Benchmark BM (see footnote 16).

In order to derive the conditions for existence and to characterize the contract with exclusion of type HL, we proceed as in the case with full participation, but we obviously drop worker HL from the principal's maximization program SB and omit the monotonicity condition $e_{LL} > e_{HL}$. Since the upper bound $\overline{\gamma}^{SBM}$ of the existence range for an equilibrium with full participation comes precisely from the constraint $e_{LL}^{SBM} > e_{HL}^{SBM}$, the range for the existence of a separating equilibrium with exclusion of HL is broader on the right side with respect to the interval $\left[\underline{\gamma}^{SBM}, \overline{\gamma}^{SBM}\right]$. Moreover, the optimal effort levels of the remaining types are given by the same expressions from (17) to (19), even with exclusion. Instead, the optimal wages of the remaining types will be lower than expressions from (10) to (12), since the portions of the three information rents that depend on e_{HL} disappear.

Result 3 ***Exclusion of type HL when motivation prevails.*** *The solution to the principal's program SB, which entails separation and exclusion of type HL, which satisfies the monotonicity condition* $e_{LH} > e_{HH} > e_{LL} > e_{HL} = 0$ *and which is such that effort levels are given by expressions from (17) to (19), exists if and only if* $\theta < \min\left\{\overline{\theta}_1^M, \overline{\theta}_2^M\right\}$ *and* $\underline{\gamma}^{SBM} < \gamma < \gamma_0 \equiv \frac{\nu(1-\mu)}{\mu}$.

Up to now we have identified all possible classes of solutions to the principal's program SB, when motivation prevails, and their corresponding (possibly overlapping) existence regions. Actually, there still remains to characterize the solution when there is bunching between intermediate types HH and LL and we refer the reader to Appendix D.4.2 for the analysis of such a situation. In order to single out the optimal contract chosen by the principal, we still have one step to go: when different solutions coexist, we must pick the one that yields the highest profits to the firm and discard the others. This is precisely what we do next.

C.3 Proof of Proposition 1

We want to show that the solution entailing full participation and full separation of types dominates (meaning that it provides higher profits to the principal) both full separation but exclusion of at least worker HL and full participation but pooling of two workers' type. Moreover, we prove that full participation and pooling of two different types dominates full separation and exclusion of (at least) worker HL, whenever the two solutions coexist. We consider the situation in which motivation prevails over ability (Case M); the same line of reasoning applies to Case A as well, which is therefore omitted.

Start with the comparison between full participation and full separation of types and exclusion of at least worker HL. We must evaluate the costs and benefits from participation of the worst worker type HL. The principal's benefit from employing worker HL is the expected profit

$$(1-\mu)(1-\nu)(e_{HL}-w_{HL}), \tag{24}$$

whereas the cost from participation of HL is represented by the information rents paid to the three remaining workers' types, which add up to

$$\frac{1}{2}(1-(1-\mu)(1-\nu))\Delta\theta e_{HL}^2 \tag{25}$$

Thus, the principal prefers full participation to exclusion of type HL if and only if (24) is strictly greater than (25). Taking into account expression (9) for the wage w_{HL} in Case M, the inequality reduces to $2e_{HL}^{SBM} > e_{HL}^{SBM}$, which is obviously satisfied as long as $e_{HL}^{SBM} > 0$. Alternatively, substituting both expression (9) for the wage and expression (20) for e_{HL} in (24) and (25), we obtain that full separation and full participation dominates full separation and exclusion of type HL if and only if $\theta - (1-(1-\mu)(1-\nu)) > 0$. This is the denominator of e_{HL} in expression (20). The previous inequality says that the effort cost of a low ability type θ must be larger than the joint probability of hiring types that are overall more efficient than HL. This condition always holds in our setting, given that $\theta > 1$.

Similar conclusions can be drawn considering exclusion of both workers HL and LL.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers HH and LL (as said, this solution is considered in detail in Appendix D.4.2 but we anticipate some findings here for expositional convenience). Now the trade-off between costs and benefits from full separation becomes less clear, so let us resort directly to the comparison between the principal's profits under the two solutions. The principal's payoffs under full separation and full participation of types are

$$\pi^{SBM}_{FS,FP} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \mu\frac{(1-\nu)^2(1+\gamma)^2}{\theta-\nu} + \frac{(\nu(1-\mu)-\mu\gamma)^2}{(1-(1-\nu)(1-\mu))-\mu\theta} + \frac{(1-\nu)^2(1-\mu)^2}{\theta-(1-(1-\nu)(1-\mu))}\right)$$

while, under full participation but pooling of workers HH and LL, profits amount to

$$\pi^{SBM}_{FP,HH=LL} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \frac{(\nu(1-\mu)+\mu(1-\nu)-\gamma\mu\nu)^2}{\nu(1-\mu)+\mu(1-\nu)} + \frac{(1-\nu)^2(1-\mu)^2}{\theta-(1-(1-\nu)(1-\mu))}\right)$$

It is immediate to check that $\pi^{SBM}_{FS,FP} > \pi^{SBM}_{FP,HH=LL}$ always holds.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers HL and LL (see Appendix C.2). The principal's payoffs under full participation but pooling of workers HL and LL are given by

$$\pi^{SBM}_{FP,HL=LL} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \frac{\mu(1-\nu)^2(1+\gamma)^2}{(\theta-\nu)} + \frac{((1-\mu)-\mu\gamma)^2}{\theta(1-\mu)}\right)$$

and, again, it is straightforward to check that $\pi^{SBM}_{FS,FP} > \pi^{SBM}_{FP,HL=LL}$ always holds.

Finally, consider the comparison between full participation but pooling of workers HL and LL and full separation but exclusion of worker HL. Since both solutions are dominated by full separation and full participation, they can be candidate optimal contracts only above $\overline{\gamma}^{SBM}$. The principal's profits at the latter solution are

$$\pi^{SBM}_{FS,HL=0} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \frac{\mu(1-\nu)^2(1+\gamma)^2}{(\theta-\nu)} + \frac{(\nu(1-\mu)-\mu\gamma)^2}{\nu(1-\mu)-\mu(\theta-1)}\right)$$

and $\pi^{SBM}_{FP,HL=LL} > \pi^{SBM}_{FS,HL=0}$ if and only if

$$\left((1-\mu)-\mu\gamma\right)e^{SBM}_{\overline{P}} > \left(\nu\left(1-\mu\right)-\mu\gamma\right)e^{SBM}_{LL}.$$

The above inequality is always verified since $((1-\mu)-\mu\gamma) > (\nu(1-\mu)-\mu\gamma)$ always holds and $e^{SBM}_{\overline{P}} > e^{SBM}_{LL}$ is true above $\overline{\gamma}^{SBM}$.

Note that the comparison between full participation but pooling of workers HH and LL and full separation but exclusion of worker HL is meaningless because, below $\underline{\gamma}^{SBM}$, it is never feasible to separate types HH and LL. So we are done.

C.4 Optimal contracts when motivation prevails

Considering Proposition 1, Results from 1 to 3 and Result 13 in Appendix D.4.2, it is now possible to characterize the optimal contracts when motivation prevails.

Result 4 *When motivation prevails, the optimal contracts proposed by the principal are as follows:*
(i) Full participation and pooling between types HH and LL and IC_{LLvsLH} binding (characterized in

Result 13) is implemented if and only if $\theta < \overline{\theta}_1^M$ *and* $\gamma^* \leq \gamma < \underline{\gamma}^{SBM}$.

(ii) Full participation and full separation of types (characterized in Result 1) is implemented if and only if $\theta < \min\left\{\overline{\theta}_1^M, \overline{\theta}_2^M\right\}$ *and* $\underline{\gamma}^{SBM} \leq \gamma \leq \overline{\gamma}^{SBM}$.

(iii) Full participation and pooling between types LL and HL (characterized in Result 2) is implemented if and only if $\theta < \overline{\theta}_1^M$ *and* $\overline{\gamma}^{SBM} \leq \gamma \leq \min\left\{\gamma^{BM}, 1\right\}$.

(iv) Full separation and exclusion of type HL (characterized in Result 3) is implemented if and only if $\mu > \frac{1}{2}$, $\theta < \overline{\theta}_1^M$ *and* $\gamma^{BM} < \gamma \leq 1$.

D Ability prevails (Case A)

When ability prevails, condition (8) holds and $e_{LL} > e_{HH}$ together with $e_{LL}+ e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ must be satisfied. In line with Observation 2, in order to find a fully separating and fully participating solution to the principal's problem SB take the participation constraint PC_{HL} and the following incentive constraints: IC_{LHvsLL}, IC_{LLvsHH} or eventually IC_{LLvsHL} (whichever one binds first), IC_{HHvsHL} or IC_{HHvsLL} (again whichever one binds first). Note that all incentive compatibility constraints considered are downward constraints except for IC_{HHvsLL} which points upwards. Since IC_{LLvsHH} and IC_{HHvsLL} cannot be simultaneously binding at a separating equilibrium, then the possible situations are the following: (A.1) all downward local ICs are binding and thus IC_{LHvsLL}, IC_{LLvsHH} and IC_{HHvsHL} hold with equality, as shown in Figure 2a; (A.2) the downward local constraints IC_{LHvsLL} and IC_{HHvsHL} and the global downward constraint IC_{LLvsHL} are all binding, as shown in Figure 2b; (A.3) constraints IC_{LHvsLL}, IC_{LLvsHL} and the upward IC_{HHvsLL} hold with equality, as shown in Figure 2c.

Such three possible cases will be analyzed in detail in what follows.

D.1 Case $A.1$

D.1.1 Full separation and full participation

This represents the most intuitive case where all downward local ICs are binding and occurs when γ is sufficiently low so that worker HH receives a relatively high salary in exchange for a relatively low effort, and such a contract is attracting for type LL. Solving the binding constraints for salaries, one obtains the following wage schedules and informational rents

$$w_{HL} = \frac{1}{2}\theta e_{HL}^2, \tag{26}$$

$$w_{HH} = \frac{1}{2}\theta e_{HH}^2 - \gamma e_{HH} \underbrace{+\gamma e_{HL}}_{\text{Info rent worker } HH}, \tag{27}$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \underbrace{+\frac{1}{2}\Delta\theta e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LL} \tag{28}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LH}. \tag{29}$$

All information rents, except the one of type HL, are positive and have the usual cumulative structure. They all include at least one expression of the form γe_{ij} as in Benchmark BM where asymmetric information concerns motivation only. Only type LL receives an information rent which also depends on the difference in ability $\Delta\theta$: this comes from the fact that this program embeds the two subcases in Benchmark BM and links them through constraint IC_{LLvsHH}. Type LH cumulates this rent too when trying to mimic LL.

This case is peculiar because an additional constraint needs to be satisfied, which comes from expression (28). The rent accruing to type LL when mimicking HH must be positive and this occurs if and only if $e_{HH} > \frac{2\gamma}{\Delta\theta}$ (which is more restrictive than condition 8).[28] In different words, only when γ is sufficiently low, does type LL benefit from mimicking type HH. Otherwise, type LL will rather prefer to mimic type HL as in Case $A.2$ and Case $A.3$ that follow.

Substituting the wage schedules into the objective function and deriving with respect to effort levels we obtain

$$e_{LH}^{SBA1} = 1 + \gamma \tag{30}$$

$$e_{LL}^{SBA1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)} = e_{LL}^{BM}, \tag{31}$$

$$e_{HH}^{SBA1} = \frac{(1-\nu)\,\mu + (1-(1-\nu)(1-\mu))\,\gamma}{(1-(1-\nu)(1-\mu))\,\theta - \nu} \tag{32}$$

and

$$e_{HL}^{SBA1} = \frac{(1-\nu)(1-\mu) - (1-(1-\nu)(1-\mu))\,\gamma}{(1-\nu)(1-\mu)\,\theta}. \tag{33}$$

The standard result of *no distortion at the top* and downward distortion in effort levels for all other agent's types is obtained. Note that e_{LL}^{SBA1} is equal to the effort level we obtained for type LL in Benchmark BM. Instead, the effort levels required from the less productive workers (types HH and HL) are characterized by a larger downward distortion than in program BM because of the cumulative effect of informational rents.

Also observe that e_{LH}^{SBA1} and e_{HH}^{SBA1} are strictly positive, while $e_{LL}^{SBA1} > 0$ if and only if $\gamma < \gamma^{BM}$, and $e_{HL}^{SBA1} > 0$ if and only if

$$\gamma < \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} = \gamma_1^{SBA1}.$$

[28] Note that condition $e_{HH} > \frac{2\gamma}{\Delta\theta}$ implies condition $e_{HH} > \frac{2\gamma}{\theta}$. Hence if LL receives a positive information rent when mimicking HH, then it must be that type HH is not a volunteer and that she is experiencing a net cost from providing effort.

Actually, $e_{LL}^{SBA1} > 0$ always holds when $\mu \leq \frac{1}{2}$ or when e_{HL}^{SBA1} is strictly positive, since $e_{HL}^{SBA1} > 0$ implies $e_{LL}^{SBA1} > 0$ (being $\gamma^{BM} > \gamma_1^{SBA1}$).

As for the monotonicity conditions, it can be checked that $e_{LH}^{SBA1} > e_{LL}^{SBA1}$ always holds, that $e_{LL}^{SBA1} > e_{HH}^{SBA1}$ is true for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(1-(1-\nu)(1-\mu))\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))} = \gamma_2^{SBA1}$$

and that inequalities $e_{LH}^{SBA1} > e_{HH}^{SBA1}$, $e_{LL}^{SBA1} > e_{HL}^{SBA1}$ and $e_{HH}^{SBA1} + e_{LL}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ all hold when $\gamma < \gamma_2^{SBA1}$. Note that $\gamma_2^{SBA1} < \gamma_1^{SBA1}$ if and only if

$$\theta < \frac{\mu\left(1-\nu\left(1-\nu\right)\right)+\nu\left(1-\mu\right)}{\nu\left(1-\left(1-\nu\right)\left(1-\mu\right)\right)} \equiv \theta_3.$$

Finally, $e_{HH}^{SBA1} > e_{HL}^{SBA1}$ for

$$\gamma > \frac{\nu\left(1-\nu\right)\left(1-\mu\right)\Delta\theta}{\left(1-\left(1-\nu\right)\left(1-\mu\right)\right)\left(\theta-\nu\right)} = \underline{\gamma}^{SBA1}$$

where it is always the case that $\underline{\gamma}^{SBA1} < \min\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}\right\}$.

In addition, it must be true that $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$. Such condition is equivalent to

$$\gamma < \frac{\mu\left(1-\nu\right)\Delta\theta}{\nu\Delta\theta+\mu\left(1-\nu\right)\left(\theta+1\right)} = \gamma_3^{SBA1},$$

where $\gamma_3^{SBA1} > \underline{\gamma}^{SBA1}$ holds if and only if $\mu > \frac{\nu}{1+\nu} = \mu_0$. Thus, $\mu > \mu_0$ is a necessary condition ensuring that the two requirements $e_{HH}^{SBA1} > e_{HL}^{SBA1}$ and $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ can both be met.[29] Finally, $\gamma_3^{SBA1} < \gamma_1^{SBA1}$ if and only if

$$\theta < \frac{\mu\left(1+\nu\right)-\nu}{\left(2\mu-1\right)\left(1-\left(1-\mu\right)\left(1-\nu\right)\right)} \equiv \theta_4,$$

which is always the case for $\mu \leq \frac{1}{2}$. Observe that $\gamma_3^{SBA1} < \gamma_2^{SBA1}$ if and only if $\mu < \frac{(1-2\nu)+\sqrt{1+4\nu(1-\nu)}}{4(1-\nu)} \equiv \mu_2$, with $\mu_2 > \frac{1}{2}$ and that $\theta_3 < \theta_4$ if and only if $\mu < \mu_2$.

We are then able to state the following result.

Result 5 ***Full participation and full separation when ability prevails and IC_{LLvsHH} and IC_{HHvsHL} are binding.*** *The solution to the principal's program SB, which entails full participation, full separation of types and constraints IC_{LLvsHH} and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} > 0$ and which is such that effort levels are given by expressions from (30) to (33) exists and represents the optimal contract if and only if $\mu > \frac{\nu}{1+\nu} \equiv \mu_0$ and $\underline{\gamma}^{SBA1} < \gamma < \overline{\gamma}^{SBA1}$ with*

$$\begin{aligned} \underline{\gamma}^{SBA1} &\equiv \frac{\nu(1-\nu)(1-\mu)\Delta\theta}{(1-(1-\nu)(1-\mu))(\theta-\nu)} \\ \overline{\gamma}^{SBA1} &= \min\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}, \gamma_3^{SBA1}\right\} \end{aligned}$$

[29] Full participation and full separation in Case $A.1$ is possible only if $\mu > \mu_0$, or if the probability of motivated workers is sufficiently high, implying that information rents are not too costly.

and

$$\begin{array}{lll} \gamma_1^{SBA1} & \equiv & \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} \\ \gamma_2^{SBA1} & \equiv & \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(1-(1-\nu)(1-\mu))\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))} \\ \gamma_3^{SBA1} & \equiv & \frac{\mu(1-\nu)\Delta\theta}{(\nu\Delta\theta+\mu(1-\nu)(\theta+1))} \end{array}.$$

Finally note that $\gamma^* > \max\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}, \gamma_3^{SBA1}\right\}$ is always true, therefore Case $A.1$ with full participation and full separation is always a subset of the first-best state of the world in which condition (3) holds.

To better characterize the optimal contracts in Case $A.1$, we add the following.

Remark 10 *At the optimal contract with full participation and full separation under Case* $A.1$*, the ordering of wages is*

$$w_{LH}^{SBA1} > w_{LL}^{SBA1} > w_{HH}^{SBA1} > w_{HL}^{SBA1} > 0$$

and the ordering of information rents (indirect utilities) is

$$u_{LH}^{SBA1} > u_{LL}^{SBA1} > u_{HH}^{SBA1} > u_{HL}^{SBA1} = 0.$$

Case $A.1$ represents the unique instance in which wages and information rents always have the same ordering as effort levels. As in Case M, here the bidimensional screening problem is equivalent to the unidimensional screening one with four types, the unidimensional parameter of private information being the workers' overall cost of effort exertion.

D.1.2 Pooling and exclusion

In light of Proposition 1, the optimal contract will always be characterized by full participation and full separation of types, except when the former solution is not viable, in which case pooling and possibly exclusion will also be part of the optimal contract.

First of all consider *pooling.* Observe that the lower bound $\underline{\gamma}^{SBA1}$ corresponds to condition $e_{HH}^{SBA1} > e_{HL}^{SBA1}$. Thus, if $\gamma \leq \underline{\gamma}^{SBA1}$, then we expect a pooling equilibrium where types HH and HL receive the same contract. Suppose that there's pooling between the less productive types and that $e_{HH} = e_{HL} = e_{\underline{p}}$ holds. Then the ordering of effort levels is $e_{LH} > e_{LL} > e_{\underline{p}} > 0$ and the relevant downward incentive constraints that one assumes to be binding are IC_{LHvsLL} and IC_{LLvsHL} (or IC_{LLvsHH}, which is equivalent) with participation constraint PC_{HL}. Since here the incentive constraints IC_{LLvsHH} and IC_{LLvsHL} are both binding by construction, we do not need any condition on the sum of e_{HH} and e_{HL}. Moreover, since the two types of workers receive the same wage and provide the same effort, $u_{HH} > u_{HL}$ necessarily holds. The wages are

$$w_{HH} = w_{HL} = w_{\underline{p}} = \frac{1}{2}\theta e_{\underline{p}}^2,$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } LL}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } LH}.$$

Substituting the salaries into the objective function of the principal and maximizing with respect to effort levels yields

$$e_{LH}^{SBA1} = 1 + \gamma,$$

$$e_{LL}^{SBA1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)} = e_{LL}^{BM}$$

and

$$e_{HH} = e_{HL} = e_{\underline{p}}^{SBA1} = \frac{(1-\nu)}{(\theta-\nu)} = e_{HL}^{BA} \tag{34}$$

Note that the expressions for e_{LH} and e_{LL} are the same as in Case $A.1$ (and $A.2$ that follows) with full separation, meaning that *no distortion at the top* is verified and that the effort of individual LL is lower than the corresponding first-best level. Moreover, $e_{LH} > e_{LL}$ still holds. Concerning $e_{\underline{p}}^{SBA1}$, which is strictly positive, we expect that this effort lies in-between the effort exerted by types HH and HL in Case $A.1$ with full separation. One can easily check that $e_{HH}^{SBA1} < e_{\underline{p}}^{SBA1} < e_{HL}^{SBA1}$ if and only if $\gamma < \underline{\gamma}^{SBA1}$. Finally, $e_{LL}^{SBA1} > e_{\underline{p}}^{SBA1}$ if and only if

$$\gamma < \frac{(1-\mu)\,\Delta\theta}{\mu\,(\theta-\nu)} = \gamma_{\underline{p}}$$

where $\gamma_{\underline{p}} > \underline{\gamma}^{SBA1}$ always holds.

Now consider the upper bounds (recall that condition $\gamma < \gamma_1^{SBA1}$ is equivalent to $e_{HL}^{SBA1} > 0$, that inequality $\gamma < \gamma_2^{SBA1}$ is equivalent to $e_{LL}^{SBA1} > e_{HH}^{SBA1}$ and finally that $\gamma < \gamma_3^{SBA1}$ ensures that requirement $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ holds): if $\gamma \geq \overline{\gamma}^{SBA1}$, we expect an equilibrium in which either types HH and LL are pooled together or exclusion occurs or both.[30]

Result 6 (i) ***Full participation and pooling between types HH and HL when ability prevails.*** *The solution to the principal's program SB which entails full participation, pooling between types HH and HL, which satisfies the monotonicity condition* $e_{LH} > e_{LL} > e_{HH} = e_{HL} = e_{\underline{p}} > 0$ *and which is such that effort levels are given by expressions (30), (31) and (34) exists if and only if* $0 < \gamma \leq \gamma_{\underline{p}} \equiv \frac{(1-\mu)\Delta\theta}{\mu(\theta-\nu)}$ *and represents the optimal contract when* $0 < \gamma \leq \underline{\gamma}^{SBA1}$.

(ii) ***Full participation and pooling between types HH and LL when ability prevails.*** *The solution to the principal's program SB which entails full participation, pooling between types HH and LL*

[30] We refer the reader to Appendix D.4.1 for the detailed analysis of this situation.

and IC_{HHvsHL} *binding, which satisfies the monotonicity condition* $e_{LH} > e_{HH} = e_{LL} = e_{\overline{p}} > e_{HL} > 0$ *and which is such that effort levels are given by expressions (30), (33) and*

$$e_{HH} = e_{LL} \equiv e_{\overline{p}}^{SBA1} = \frac{(\nu(1-\mu) + \mu(1-\nu))(1+\gamma)}{\nu\mu\Delta\theta + (\nu(1-\mu) + \mu(1-\nu))\theta},$$

represents the optimal contract only if $\overline{\gamma}^{SBA1} \neq \gamma_1^{SBA1}$ *and* $\overline{\gamma}^{SBA1} < \gamma < \min\left\{\overline{\gamma}^{SBPa}, \gamma_1^{SBA1}\right\}$ *with*

$$\overline{\gamma}^{SBPa} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))(\theta+1)}.$$

Note that when $\overline{\gamma}^{SBA1} = \gamma_1^{SBA1}$ and $\gamma_1^{SBA1} < \gamma < \min\left\{\gamma_2^{SBA1}, \gamma_3^{SBA1}\right\}$, the principal will necessarily exclude worker HL. This would lead us to consider alternative solutions where either full separation but exclusion of type HL (and where IC_{LLvsHH} and PC_{HH} are binding), or pooling of types HH and LL and exclusion of type HL, or else exclusion of both types HL and HH are implemented.[31]

D.2 Case $A.2$

D.2.1 Full separation and full participation

As in Case $A.3$ which follows, type LL is willing to mimic HL rather than type HH. This occurs since motivation γ is high enough so that type HH is asked to make a relatively high effort in exchange for a relatively low wage and her contract is not appealing to type LL. As a consequence no type is willing to mimic worker HH and her effort is not distorted.

Interestingly, this is the unique case where neither the incentive compatibility constraint of type HH versus type LL nor the one of type LL versus HH are binding.

The salaries of types HH and HL are the same as in Case $A.1$, and given by expressions (27) and (26) respectively, whereas the salary of type LL has the same expression as in Cases M and $A.3$ (see equations 10 and 15); the other relevant wage level is now

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH}.$$

The information rent of worker LL is formed by one term only, $\frac{1}{2}\Delta\theta e_{HL}^2$ (as in Benchmark BA, Case M and Case $A.3$ that follows) which depends on the effort exerted by worker HL, while no rent depending on e_{HH} appears: this is because type LL mimics type HL directly, without "going through" type HH. For the same reason, information rents accruing to both types LH and LL are "shorter" than in Case $A.1$, as the paths of binding incentive constraints in Figure 2b show. Also the information rent of type HH only depends on the effort provided by of worker HL, however in w_{HH} the rent is γe_{HL} (as the one

[31] In the region $\gamma \geq \overline{\gamma}^{SBA1}$, we do not provide the full characterization of the solution (available upon request though) because several different cases might arise and the analysis becomes cumbersome without being very insightful.

Benchmark BM). Thus, we can interpret this specific sub-case as a program that is in-between Case $A.1$ and Case $A.3$.

Substituting the wage functions into the principal's expected profits and deriving with respect to effort levels, we obtain

$$e_{LH}^{SBA2} = 1 + \gamma, \tag{35}$$

$$e_{LL}^{SBA2} = \frac{(1-\mu) - \gamma\mu}{(1-\mu)} = e_{LL}^{SBA1} = e_{LL}^{BM}, \tag{36}$$

$$e_{HH}^{SBA2} = \frac{1+\gamma}{\theta} = e_{HH}^{FB} \tag{37}$$

and

$$e_{HL}^{SBA2} = \frac{(1-\nu)\left((1-\mu) - \gamma\mu\right)}{\nu\Delta\theta + \theta(1-\mu)(1-\nu)}. \tag{38}$$

Note that e_{LL}^{SBA2} has the same expression as e_{LL}^{SBA1} and as e_{LL}^{BM} in Benchmark BM with adverse selection on motivation. As already mentioned, both e_{LH}^{SBA2} and e_{HH}^{SBA2} are equal to their first-best levels, while both e_{LL}^{SBA2} and e_{HL}^{SBA2} are distorted downwards and e_{HL}^{SBA2} has a larger distortion than the corresponding term in program BM.

Also observe that both $e_{HL}^{SBA2} > 0$ and $e_{LL}^{SBA2} > 0$ hold provided that $\gamma < \gamma^{BM}$, that $e_{LH}^{SBA2} > e_{LL}^{SBA2} > e_{HL}^{SBA2}$ and $e_{HH}^{SBA2} > e_{HL}^{SBA2}$ always hold, while $e_{LL}^{SBA2} > e_{HH}^{SBA2}$ if and only if

$$\gamma < \frac{(1-\mu)\Delta\theta}{1+\mu\Delta\theta} = \gamma_1^{SBA2}.$$

It is easy to check that the condition $\gamma < \gamma_1^{SBA2}$ implies both $e_{HL}^{SBA2} > 0$ and $e_{LL}^{SBA2} > 0$, being $\gamma_1^{SBA2} < \gamma^{BM}$, and also that $\gamma_1^{SBA2} < \gamma_2^{SBA1}$ always holds, being the requirement $e_{LL}^{SBA2} > e_{HH}^{SBA2} = e_{HH}^{FB}$ more restrictive than $e_{LL}^{SBA1} > e_{HH}^{SBA1}$, the corresponding requisite in Case $A.1$. Then, all monotonicity conditions are satisfied provided that $\gamma < \gamma_1^{SBA2}$. Moreover, condition $\gamma < \gamma_1^{SBA2}$ suffices for $e_{HH}^{SBA2} + e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$.

There remains to check that incentive constraint IC_{LLvsHL} is binding rather than IC_{LLvsHH} and that IC_{HHvsHL} is binding rather than IC_{HHvsLL}, which amounts to $e_{HL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HL} + e_{LL}$. As for inequality $e_{HL}^{SBA2} + e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$, it holds if and only if

$$\gamma \leq \frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu))+2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu)+\Delta\theta^2\mu(1-\mu(1-\nu))+2\Delta\theta(1-\mu)} = \gamma_2^{SBA2},$$

conversely $e_{HH}^{SBA2} + e_{HL}^{SBA2} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{\Delta\theta(2\theta(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta(1-\mu)(1-\nu))(\theta+1)+\theta\Delta\theta(1-\nu)\mu} = \underline{\gamma}^{SBA2},$$

whereby a solution exists for $\underline{\gamma}^{SBA2} \leq \gamma < \min\left\{\gamma_1^{SBA2}, \gamma_2^{SBA2}\right\} \equiv \overline{\gamma}^{SBA2}$. Now, $\underline{\gamma}^{SBA2} < \gamma_2^{SBA2} < \gamma_1^{SBA2}$ is true if and only if $\mu < \frac{1}{2}$ and

$$\theta > \frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))} = \underline{\theta}^{A2},$$

hence a solution with full separation and full participation under Case $A.2$ does not exist for $\mu \geq \frac{1}{2}$.

We are then able to state the following result.

Result 7 ***Full participation and full separation when ability prevails and IC_{LLvsHL} and IC_{HHvsHL} are binding.*** *A solution to the principal's program SB, which entails full participation, full separation of types and IC_{LLvsHL} and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} > 0$ and which is such that effort levels are given by expressions from (35) to (38) exists and represents the optimal contract if and only if $\mu < \frac{1}{2}$, $\theta > \underline{\theta}^{A2}$ and $\underline{\gamma}^{SBA2} \leq \gamma < \overline{\gamma}^{SBA2}$, with*

$$\begin{aligned} \underline{\gamma}^{SBA2} &\equiv \frac{\Delta\theta(2\theta(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta(1-\mu)(1-\nu))(\theta+1)+\theta\Delta\theta(1-\nu)\mu} \\ \overline{\gamma}^{SBA2} &\equiv \frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu))+2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu)+\Delta\theta^2\mu(1-\mu(1-\nu))+2\Delta\theta(1-\mu)} \\ \underline{\theta}^{A2} &\equiv \frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))} \end{aligned} .$$

Note that a fully separating and fully participating equilibrium in Case $A.2$ only exist if $\mu < \frac{1}{2}$, that is if the probability of motivated workers is sufficiently low. In fact, the information rents of workers HH and LH depend on γ, which is relatively large in Case $A.2$. Thus, this optimal contract exists if the total number of information rents that the principal pays to motivated workers is not too high.

Finally, observe that $\gamma_2^{SBA2} = \overline{\gamma}^{SBA2} < \gamma^*$ always holds, thus implying that this solution is attained when, at the first-best, condition (3) holds.

Looking at optimal wages and information rents we can state the following.

Remark 11 *At the optimal contract with full participation and full separation under Case $A.2$, the ranking of wages is*

$$w_{LH}^{SBA2} > w_{LL}^{SBA2} > w_{HH}^{SBA2} > w_{HL}^{SBA2} > 0$$

and the ordering of information rents is

$$u_{LH}^{SBA2} > u_{HH}^{SBA2} > u_{LL}^{SBA2} > u_{HL}^{SBA2} = 0.$$

Wages have the same ordering as effort levels, while the ranking of information rents of intermediate types is reversed (and is the same as in Case M).

D.2.2 Pooling and exclusion

What happens when full participation and full separation is not viable? Below $\underline{\gamma}^{SBA2}$, one expects the principal to exclude the less efficient types, namely HL and possibly HH too, while above $\overline{\gamma}^{SBA2}$, one expects to have a pooling equilibrium where types LL and HH are given the same contract and, possibly, the worst type HL is excluded. Again, we refer the reader to Appendix D.4.2 for the detailed analysis of the latter situation and we concentrate here on the first one, exclusion.

Suppose that the principal excludes type HL and offers her the null contract. The principal's program must be slightly modified with respect to full participation, the main differences being that monotonicity constraint $e_{HH} > e_{HL}$ is omitted and that PC_{HH} (rather than PC_{HL}) is assumed to be binding. Moreover, the requirement that incentive constraint IC_{LLvsHL} be binding and IC_{LLvsHH} be slack reduces to the need that PC_{LL} binds and that $e_{HH}^{SBA2} \leq \frac{2\gamma}{\Delta\theta}$ holds, which is true if and only if

$$\gamma \geq \frac{\Delta\theta}{\theta + 1} = \underline{\underline{\gamma}}^{SBA2},$$

where $\underline{\underline{\gamma}}^{SBA2} < \underline{\gamma}^{SBA2}$ always holds when $\mu < \frac{1}{2}$. Furthermore, the requirement that incentive constraint IC_{HHvsHL} be binding and IC_{HHvsLL} be slack reduces to PC_{HH} being binding and to $e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$, which is true for

$$\gamma \leq \frac{(1-\mu)\,\Delta\theta}{2\,(1-\mu) + \mu\Delta\theta} = \overline{\overline{\gamma}}^{SBA2},$$

with $\underline{\underline{\gamma}}^{SBA2} < \min\left\{\overline{\overline{\gamma}}^{SBA2}, \underline{\gamma}^{SBA2}\right\}$. Hence a solution characterized by exclusion of type HL, separation of the remaining types and both PC_{LL} and PC_{HH} binding exists for $\underline{\underline{\gamma}}^{SBA2} \leq \gamma < \min\left\{\overline{\overline{\gamma}}^{SBA2}, \underline{\gamma}^{SBA2}\right\}$.

Result 8 (*i*) ***Separation and exclusion of (at least) type HL when ability prevails.*** *The solution to the principal's program SB, which entails separation but exclusion of type HL, both PC_{HH} and PC_{LL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} = 0$ and which is such that effort levels are given by expressions from (35) to (37) represents the optimal contract when $\mu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SBA2} \leq \gamma \leq \min\left\{\underline{\gamma}^{SBA2}, \overline{\overline{\gamma}}^{SBA2}\right\}$, where*

$$\begin{array}{rcc} \underline{\underline{\gamma}}^{SBA2} & \equiv & \frac{\Delta\theta}{(\theta+1)} \\ \overline{\overline{\gamma}}^{SBA2} & \equiv & \frac{(1-\mu)\Delta\theta}{2(1-\mu)+\mu\Delta\theta} \end{array}.$$

The solution characterized by exclusion of both types HL and HH represents the optimal contract either when $\gamma < \underline{\underline{\gamma}}^{SBA2}$ or when $\overline{\overline{\gamma}}^{SBA2} < \gamma < \underline{\gamma}^{SBA2}$.

(*ii*) ***Full participation and Pooling between HH and LL when ability prevails and IC_{LLvsHL} is binding.*** *The solution to the principal's program SB which entails full participation and pooling between types LL and HH and IC_{LLvsHL} binding, which is such that effort levels are given by expressions (35), (38) and*

$$e_{LL} = e_{HH} \equiv e_{\overline{p}}^{SBA2} = \frac{(\nu\,(1-\mu) + \mu\,(1-\nu)) - \gamma\mu\nu}{(\nu\,(1-\mu) + \mu\,(1-\nu))} = e_{\underline{p}}^{SBM} \tag{39}$$

represents the optimal contract when $\gamma \geq \gamma^{SBPb}$, where

$$\gamma^{SBPb} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)} > \overline{\gamma}^{SBA2}.$$

(*iii*) ***Pooling between HH and LL and exclusion of HL when ability prevails.*** *The solution to the principal's program SB which entails pooling between types LL and HH, exclusion of type HL and PC_{LL} binding, which is such that effort levels are given by expressions (35) and (39) represents the optimal contract when $\overline{\gamma}^{SBA2} \leq \gamma < \gamma^{SBPb}$.*

Observe that Result 8 (ii) describes precisely the same pooling equilibrium obtained in Case M for motivation levels below the threshold $\underline{\gamma}^{SBM}$.

D.3 Case $A.3$

D.3.1 Full separation and full participation

Suppose that constraints IC_{LHvsLL}, IC_{HHvsLL}, IC_{LLvsHL} and PC_{HL} are all binding and that inequality $e_{HL} + e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HH} + e_{LL}$ holds. Substituting the expressions for the wage levels from (13) to (16) given in Section 5.2.1 into the principal's profits and maximizing with respect to effort levels, yields

$$e_{LH}^{SBA3} = 1 + \gamma, \tag{40}$$

$$e_{LL}^{SBA3} = \frac{\nu(1-\mu) - \mu\gamma}{(\mu(1-\nu) + \nu(1-\mu)) - \mu(1-\nu)\theta}, \tag{41}$$

$$e_{HH}^{SBA3} = \frac{1+\gamma}{\theta} = e_{HH}^{SBA2} = e_{HH}^{FB} \tag{42}$$

and

$$e_{HL}^{SBA3} = \frac{(1-\mu)(1-\nu)}{\theta - (1-(1-\mu)(1-\nu))} = e_{HL}^{SBM}. \tag{43}$$

All effort levels are always strictly positive, except for e_{LL}^{SBA3}. In order for e_{LL}^{SBA3} to be positive and to be a maximum of the principal's expected profits, it is necessary to impose that both the numerator and the denominator of its expression be positive: the numerator of e_{LL}^{SBA3} is positive for $\gamma < \gamma_0$ (see expression 21) and the denominator of e_{LL}^{SBA3} is positive when

$$\theta < \frac{(\mu(1-\nu) + \nu(1-\mu))}{\mu(1-\nu)} = \overline{\theta}^{A3}.$$

Note that $\overline{\theta}^{A3} > 2$ if and only if $\mu < \nu$, thus under Assumption 2 the requirement $\theta < \overline{\theta}^{A3}$ is always satisfied when $\mu < \nu$.

As for the monotonicity conditions, it must be that $e_{LL}^{SBA3} > e_{HH}^{SBA3}$, which holds if and only if

$$\gamma < \frac{(\mu(1-\nu) + \nu(1-\mu))\Delta\theta}{\mu\nu\theta + (\mu(1-\nu) + \nu(1-\mu))} = \overline{\gamma}^{SBA3}$$

where $\overline{\gamma}^{SBA3} < \gamma^*$ and $\overline{\gamma}^{SBA3} < \gamma_0$ are always true. Moreover, $e_{HH}^{SBA3} > e_{HL}^{SBA3}$ always holds and $e_{LL}^{SBA3} > e_{HL}^{SBA3}$ is always satisfied when $e_{LL}^{SBA3} > e_{HH}^{SBA3}$ is (namely when $\gamma < \overline{\gamma}^{SBA3}$). Notice that e_{LL}^{SBA3} is distorted downwards if and only if

$$\gamma > (1-\nu)\Delta\theta = \gamma_1^{SBA3}$$

where $\gamma_1^{SBA3} < \overline{\gamma}^{SBA3}$. Hence if motivation is not too high, Case $A.3$ could be compatible with an upward distortion in the effort of the productive but non-motivated worker LL.

Consider now the additional constraints $e_{LL}+e_{HL}\leq\frac{2\gamma}{\Delta\theta}\leq e_{LL}+e_{HH}$. As for $\frac{2\gamma}{\Delta\theta}\leq e_{LL}+e_{HH}$, it is always satisfied provided that $\gamma<\overline{\gamma}^{SBA3}$, while $e_{LL}+e_{HL}\leq\frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma\geq\frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu)+\left(\nu-\mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu)-\mu(1-2\nu)\Delta\theta)}=\underline{\gamma}^{SBA3}$$

where $\underline{\gamma}^{SBA3}>\gamma_1^{SBA3}$ (implying that e_{LL}^{SBA3} is always distorted downwards when full participation and full separation is possible) and $\underline{\gamma}^{SBA3}<\overline{\gamma}^{SBA3}$ when

$$\theta>\frac{\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1-(1-\mu)(1-\nu)))}{\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1+(1-\mu)(1-\nu)))}=\underline{\theta}^{A3},$$

with $\underline{\theta}^{A3}<\overline{\theta}^{A3}$ if and only if

$$\mu<\frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)}=\mu_1>\frac{1}{2}$$

(for $\nu\neq\frac{2}{3}$ or if and only if $\mu<\frac{\mu}{4\nu-\nu^2-1}$ for $\nu=\frac{2}{3}$).

We are thus able to provide the conditions under which the optimal contract with full separation and full participation is implemented.

Result 9 ***Full participation and full separation when ability prevails and IC_{HHvsLL} and IC_{LLvsHL} are binding.*** *The solution to the principal's program SB, which entails full participation, full separation of types and IC_{HHvsLL} and IC_{LLvsHL} binding, which satisfies the monotonicity condition $e_{LH}>e_{LL}>e_{HH}>e_{HL}>0$ and which is such that effort levels are given by expressions from (40) to (43), exists and represents the optimal contract if and only if $\mu<\mu_1$, $\underline{\theta}^{A3}<\theta<\overline{\theta}^{A3}$ and $\underline{\gamma}^{SBA3}\leq\gamma<\overline{\gamma}^{SBA3}$, with*

$$\begin{aligned}
\underline{\gamma}^{SBA3} &\equiv \frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu)+\left(\nu-\mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu)-\mu(1-2\nu)\Delta\theta)}\\
\overline{\gamma}^{SBA3} &\equiv \frac{\Delta\theta(\mu(1-\nu)+\nu(1-\mu))}{\mu\nu\theta+(\mu(1-\nu)+\nu(1-\mu))}\\
\mu_1 &\equiv \frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)}>\frac{1}{2}\\
\overline{\theta}^{A3} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))}{\mu(1-\nu)}\\
\underline{\theta}^{A3} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1-(1-\mu)(1-\nu))))}{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1+(1-\mu)(1-\nu))))}
\end{aligned}.$$

D.3.2 Pooling and exclusion

What happens when full participation and full separation is not viable? Above $\overline{\gamma}^{SBA3}$, one expects to have a pooling equilibrium where types LL and HH are given the same contract. And also below $\underline{\gamma}^{SBA3}$ one still finds that this solution is relevant. Again, we refer the reader to Appendix D.4.2 for the conditions of existence of a pooling equilibrium and we focus attention here on optimal contracts.

Result 10 ***Full participation and Pooling between HH and LL when productivity prevails and IC_{LLvsHL} is binding.*** *The solution to the principal's program SB which is characterized by full participation and pooling between types LL and HH and IC_{LLvsHL} binding, by effort levels described* *by expressions (40), (43) and (39) represents the optimal contract when* $\overline{\gamma}^{SBA3} \leq \gamma \leq \gamma^{*}$ *and when* $\gamma^{SBPb} \leq \gamma \leq \underline{\gamma}^{SBA3}$.

Below $\underline{\gamma}^{SBA3}$ one also finds pooling between types HH and LL and exclusion of type HL and (possibly) a solution with separation but exclusion of type HL. Interestingly, in the latter case, it is possible to have an *upward distortion* of the effort required to type LL, but not so important as to allow for a pooling equilibrium where types LH and LL are given the same contract.

Suppose that type HL is left out. In this circumstance, the optimal levels of effort are the same as under full participation, except for $e_{HL} = 0$, and all relevant constraints are satisfied whenever the chain of inequalities $e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{LL} + e_{HH}$ holds.

Now, $\frac{2\gamma}{\Delta\theta} \leq e_{LL}^{SBA3} + e_{HH}^{SBA3}$ is always satisfied when $\gamma < \overline{\gamma}^{SBA3}$, whereas $e_{LL}^{SBA3} \leq \frac{2\gamma}{\Delta\theta}$ is true if and only if

$$\gamma \geq \frac{\nu\left(1-\mu\right)\Delta\theta}{\left(2\nu\left(1-\mu\right)-\mu\Delta\theta\left(1-2\nu\right)\right)} = \underline{\underline{\gamma}}^{SBA3}$$

where $\underline{\underline{\gamma}}^{SBA3} < \underline{\gamma}^{SBA3}$ always holds and where $\underline{\underline{\gamma}}^{SBA3} > \gamma_1^{SBA3}$ if and only if $\nu > \frac{1}{2}$. Hence a solution with exclusion of type HL under Case $A.3$ exists for $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \overline{\gamma}^{SBA3}$ and $\theta < \overline{\theta}^{A3}$. Observe that, when $\nu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \gamma_1^{SBA3}$, the solution entails an upward distortion in the level of effort provided by type LL.

Result 11 (i) ***Pooling between HH and LL and exclusion of type HL when ability prevails and PC_{LL} is binding.*** *A solution to the principal's program SB with pooling between types LL and HH and exclusion of type HL, with PC_{LL} binding and with effort levels described by expressions (40) and (39) represents the optimal contract when* $\underline{\gamma}^{SBPb} < \gamma < \min\left\{\underline{\underline{\gamma}}^{SBA3}, \gamma^{SBPb}\right\}$, *where*

$$\begin{array}{ll} \underline{\underline{\gamma}}^{SBA3} \equiv & \frac{\nu(1-\mu)\Delta\theta}{(2\nu(1-\mu)-\mu\Delta\theta(1-2\nu))} \\ \underline{\gamma}^{SBPb} \equiv & \frac{\Delta\theta(\nu(1-\mu)+\mu(1-\nu))}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+\mu(1-\nu)))} \end{array}.$$

(ii) ***Separation and exclusion of type HL when ability prevails and IC_{HHvsLL} and PC_{LL} are binding.*** *A solution to the principal's program SB with exclusion of type HL and IC_{HHvsLL} and PC_{LL} binding and with effort levels described by expressions from (40) to (42) represents the optimal contract only if* $\underline{\underline{\gamma}}^{SBA3} < \gamma^{SBPb}$ *and* $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \gamma^{SBPb}$.

Result 11(i) describes precisely the same pooling equilibrium obtained in Case M and Case $A.2$.

D.3.3 Proof of Remark 9

Consider the contracts with full separation and full participation in Cases M and $A.3$ and let $\Delta\theta$ be the same in the two situations.[32]

Expected informational rents paid by the principal in Case M are higher than in Case $A.3$ if and only if

$$\mu\nu u_{LH}^{M}+\mu\left(1-\nu\right)u_{HH}^{M}+\nu\left(1-\mu\right)u_{LL}^{M}>\mu\nu u_{LH}^{A3}+\mu\left(1-\nu\right)u_{HH}^{A3}+\nu\left(1-\mu\right)u_{LL}^{A3},$$

where u_{HL} is omitted from both sides because it is equal to zero. A sufficient condition for the above inequality to hold is that $u_{ij}^{M}\geq u_{ij}^{A3}$ for every type of worker ij, with at least one strict inequality, where the actual expressions for informational rents appear in the wages given by (10) to (12) for Case M and by (14) to (16) for Case $A.3$.

Now, $u_{LH}^{M}>u_{LH}^{A3}$ holds if and only if

$$\tfrac{1}{2}\left(\theta-1\right)\left(e_{HH}^{M}\right)^{2}-\tfrac{1}{2}\left(\theta-1\right)\left(e_{LL}^{M}\right)^{2}+\gamma e_{LL}^{M}+\tfrac{1}{2}\left(\theta-1\right)\left(e_{HL}^{M}\right)^{2}\quad>\quad\gamma e_{LL}^{A3}+\tfrac{1}{2}\left(\theta-1\right)\left(e_{HL}^{A3}\right)^{2}\ .$$

Given that $\frac{1}{2}\left(\theta-1\right)\left(e_{HH}^{M}\right)^{2}-\frac{1}{2}\left(\theta-1\right)\left(e_{LL}^{M}\right)^{2}$ is always positive in Case M and that $e_{HL}^{M}=e_{HL}^{A3}$, a sufficient condition for the above inequality to hold is simply that $e_{LL}^{M}>e_{LL}^{A3}$ which is indeed the case. Moreover, $u_{HH}^{M}>u_{HH}^{A3}$ if and only if

$$-\tfrac{1}{2}\left(\theta-1\right)e_{LL}^{2}+\gamma e_{LL}+\tfrac{1}{2}\left(\theta-1\right)e_{HL}^{2}\quad>\quad-\tfrac{1}{2}\left(\theta-1\right)e_{LL}^{2}+\gamma e_{LL}+\tfrac{1}{2}\left(\theta-1\right)e_{HL}^{2}\ .$$

Since $e_{HL}^{M}=e_{HL}^{A3}$ one can simplify the above inequality as

$$-\tfrac{1}{2}\left(\theta-1\right)\left(e_{LL}^{M}+e_{LL}^{A3}\right)\left(e_{LL}^{M}-e_{LL}^{A3}\right)+\gamma\left(e_{LL}^{M}-e_{LL}^{A3}\right)\quad>\quad0$$

and, being $e_{LL}^{M}>e_{LL}^{A3}$, one can further simply it as

$$e_{LL}^{M}+e_{LL}^{A3}<\frac{2\gamma}{\Delta\theta}.$$

Substituting for the expressions of e_{LL}^{M} and e_{LL}^{A3}, the latter condition is equivalent to

$$\gamma>\quad\frac{(\theta-1)(1-\mu)(2\mu+2\nu-2\theta\mu-3\mu\nu+\theta\mu\nu)}{(2\mu+2\nu-2\theta\mu-6\mu\nu+2\theta\mu\nu-3\mu^{2}+4\theta\mu^{2}+4\mu^{2}\nu-2\theta\mu^{2}\nu-\theta^{2}\mu^{2})}\quad\equiv\quad\gamma_{LL}\ .$$

Note that $\gamma_{LL}<\underline{\gamma}^{SBA3}$ always holds so $e_{LL}^{M}+e_{LL}^{A3}<\frac{2\gamma}{\Delta\theta}$ is always satisfied when both Cases M and $A.3$ are relevant.

Finally, $u_{LL}^{M}=u_{LL}^{A3}$ because $u_{LL}=\frac{1}{2}\left(\theta-1\right)e_{HL}^{2}$ and again $e_{HL}^{M}=e_{HL}^{A3}$.

[32] It is always the case that $\min\left\{\overline{\theta}_{1}^{M},\overline{\theta}_{2}^{M}\right\}<\overline{\theta}^{A3}$ but there is a wide range of probabilities $\mu<\frac{1}{2}$ and ν such that $\underline{\theta}^{A3}<\min\left\{\overline{\theta}_{1}^{M},\overline{\theta}_{2}^{M}\right\}$, meaning that the two subsets of θ are at least partially overlapping.

D.4 Pooling between intermediate types HH and LL

Suppose that the principal offers a single contract to both agents LL and HH. Then one has $e_{LL} = e_{HH} = e_p$ and $w_{LL} = w_{HH} = w_p$. The relevant constraints are

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_p - \frac{1}{2}e_p^2 + \gamma e_p$$

for type LH,

$$w_p - \frac{1}{2}e_p^2 \geq w_{HL} - \frac{1}{2}e_{HL}^2 \tag{44}$$

for type LL or

$$w_p - \frac{1}{2}\theta e_p^2 + \gamma e_p \geq w_{HL} - \frac{1}{2}\theta e_{HL}^2 + \gamma e_{HL} \tag{45}$$

for type HH. Finally, for type HL

$$w_{HL} - \frac{1}{2}\theta e_{HL}^2 \geq 0.$$

The binding participation constraint is the one of type HL above, while all other participation constraints are satisfied provided that PC_{HL} is. The monotonicity condition

$$e_{LH} \geq e_p \geq e_{HL}$$

holds; but which incentive compatibility constraint between (44), that is IC_{LLvsHL}, and (45), or else IC_{HHvsHL}, binds first? Taking into account the binding participation constraint of type HL, it must be that

$$w_p \geq \max\left\{\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{HL}; \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2\right\}.$$

Thus, IC_{HHvsHL} is binding first when

$$\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{HL} \geq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2 \Longleftrightarrow e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta},$$

whereas IC_{LLvsHL} is binding when

$$\frac{1}{2}\theta e_p^2 - \gamma e_p + \gamma e_{HL} \leq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2 \Longleftrightarrow e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$$

In what follows we study the two sub-cases separately.

D.4.1 Pooling between intermediate types with IC_{HHvsHL} binding

Suppose that when pooling occurs, IC_{HHvsHL} is binding while IC_{LLvsHL} is slack. We call this situation Case $P(a)$. Then one has $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$w_{HL} = \frac{1}{2}\theta e_{HL}^2, \tag{46}$$

$$w_p = \frac{1}{2}\theta e_p^2 - \gamma e_p \underbrace{+\gamma e_{HL}}_{\text{Info rent worker } HH}, \tag{47}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} + \underbrace{\frac{1}{2}\Delta\theta e_p^2 + \gamma e_{HL}}_{\text{Info rent worker } LH}. \tag{48}$$

The wage w_p has the same expression as w_{HH} in Cases $A.1$ and $A.2$ (see equation 27). Since IC_{HHvsHL} is binding while IC_{LLvsHL} is not, we expect that the information rent of worker LL is higher than the one of worker HH and this occurs for $e_p > \frac{2\gamma}{\Delta\theta}$. This requirement is more restrictive than $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$ and it must be imposed ex-post, as was done in Case $A.1$.

Substituting again the wage schedules into the principal's program we find

$$e_{LH}^{SBPa} = 1 + \gamma,$$

$$e_p^{SBPa} \equiv e_p^{SBA1} = \frac{(\nu(1-\mu) + \mu(1-\nu))(1+\gamma)}{\nu\mu\Delta\theta + (\nu(1-\mu) + \mu(1-\nu))\theta} \tag{49}$$

and

$$e_{HL}^{SBPa} = \frac{(1-\nu)(1-\mu) - \gamma(1-(1-\nu)(1-\mu))}{(1-\nu)(1-\mu)\theta} = e_{HL}^{SBA1}.$$

Note that $e_{LH}^{SBPa} > e_p^{SBPa}$ and $e_{LH}^{SBPa} > e_{HL}^{SBPa}$ always hold. Moreover e_{HL}^{SBPa} is the same as e_{HL}^{SBA1} since in both cases participation constraint of worker HL is binding. Also observe that e_{HL}^{SBPa} is strictly positive if and only if $\gamma < \gamma_1^{SBA1}$, and $e_p^{SBPa} > e_{HL}^{SBPa}$ if and only if

$$\gamma > \frac{\nu\mu(1-\nu)(1-\mu)\Delta\theta}{\nu\mu(1-(1-\nu)(1-\mu))\Delta\theta + \theta(\mu(1-\nu)+\nu(1-\mu))} = \underline{\gamma}^{SBPa},$$

where $\underline{\gamma}^{SBPa} < \underline{\gamma}^{SBA1}$ always holds. Moreover, $e_{LL}^{SBA1} < e_p^{SBPa} < e_{HH}^{SBA1}$ if and only if $\gamma > \gamma_2^{SBA1}$ and the condition $e_p > \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma < \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta + (\nu(1-\mu)+\mu(1-\nu))(\theta+1)} = \overline{\gamma}^{SBPa}$$

where $\overline{\gamma}^{SBPa} > \underline{\gamma}^{SBPa}$ is always true, $\overline{\gamma}^{SBPa} < \gamma_1^{SBA1}$ if and only if

$$\theta < \frac{(\nu(1-\mu)(1-\mu(1-\nu)) + \mu(1-\nu)(1-\nu(1-\mu)))}{\left((2\nu-1)(\nu(1-\mu)+\mu(1-\nu)) + 2(1-\nu)^2\mu^2\right)} = \theta_5$$

(always for $\nu < \frac{1}{2}$ and $\mu < \frac{(1-2\nu)^2 + \sqrt{(1-2\nu)(1+2\nu-4\nu^2)}}{4(1-\nu)^2} \equiv \mu_3 < \frac{1}{2}$), where $\theta_3 < \theta_5 < \theta_4$ if and only if $\mu < \mu_2$, and $\gamma_3^{SBA1} < \overline{\gamma}^{SBPa} < \gamma_2^{SBA1}$ if and only if $\mu < \mu_2$.

It is now possible to state the following result.

Result 12 (i) ***Full participation and Pooling between types HH and LL with IC_{HHvsHL} binding.*** *A solution to the principal's program SB which entails full participation and pooling between types HH and LL and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{HH} = e_{LL} > e_{HL} > 0$, and which is such that effort levels are given by expressions (30), (33) and (49) exists if and* *only if* $\underline{\gamma}^{SBPa} < \gamma < \min\left\{\gamma_1^{SBA1}, \overline{\gamma}^{SBPa}\right\}$ *with*

$$\begin{aligned} \underline{\gamma}^{SBPa} &\equiv \frac{\nu\mu(1-\nu)(1-\mu)\Delta\theta}{\nu\mu(1-(1-\nu)(1-\mu))\Delta\theta+\theta(\mu(1-\nu)+\nu(1-\mu))} \\ \overline{\gamma}^{SBPa} &\equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))(\theta+1)} \\ \gamma_1^{SBA1} &\equiv \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} \end{aligned}$$

(ii) ***Pooling between types HH and LL with*** IC_{HHvsHL} ***binding and exclusion of type*** HL. *A solution to the principal's program SB which entails pooling between types HH and LL and* PC_{HH} *binding, exclusion of type* HL *and which satisfies the monotonicity condition* $e_{LH} > e_{HH} = e_{LL} > 0$, *and which is such that effort levels are given by expressions (30) and (49) exists if and only if* $\gamma < \overline{\gamma}^{SBPa}$.

Note that in this Case $P(a)$ it never happens that type HH is asked to provide an effort which falls in the range where her utility is increasing in effort, namely it is never the case that $e_{HH} = e_{LL} = e_p^{SBPa} < \frac{\gamma}{\theta}$. This might occur in the subsequent Case $P(b)$.

D.4.2 Pooling between intermediate types with IC_{LLvsHL} binding

Suppose now that when pooling occurs, IC_{LLvsHL} is binding while IC_{HHvsHL} is slack. We call this situation Case $P(b)$, in which $e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$\begin{aligned} w_{HL} &= \frac{1}{2}\theta e_{HL}^2, \\ w_p &= \frac{1}{2}e_p^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL} \end{aligned} \tag{50}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_p + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH}.$$

Note that the wage w_p now has the same expression as w_{LL} in Case M (see equation 10), Case $A.3$ (see equation 15) and Case $A.2$.

Substituting the wage schedules into the program and deriving yields

$$\begin{aligned} e_{LH}^{SBPb} &= 1+\gamma, \\ e_p^{SBPb} \equiv e_{\underline{p}}^{SBM} = e_{\overline{p}}^{SBA2} &= \frac{(\nu(1-\mu)+\mu(1-\nu))-\gamma\mu\nu}{(\nu(1-\mu)+\mu(1-\nu))} \end{aligned} \tag{51}$$

and

$$e_{HL}^{SBPb} = \frac{(1-\nu)(1-\mu)}{\theta-(1-(1-\nu)(1-\mu))} = e_{HL}^{SBM} = e_{HL}^{SBA3},$$

where e_{HL}^{SBPb} is equal to e_{HL}^{SBM} and e_{HL}^{SBA3} since in all cases the incentive constraint IC_{LLvsHL} is binding. Note that $e_p^{SBPb} > 0$ if and only if

$$\gamma < \frac{\nu(1-\mu)+\mu(1-\nu)}{\mu\nu} = \overline{\gamma}^{SBPb},$$

which is always the case for $\mu < \frac{\nu}{3\nu-1}$, and which is such that $\overline{\gamma}^{SBPb} > \gamma^{*}$ if and only if $\theta < \overline{\theta}_1^M$ and such that $\overline{\gamma}^{SBPb} > \underline{\gamma}^{SBM}$ and $\overline{\gamma}^{SBPb} > \overline{\gamma}^{SBA2}$ always hold. Furthermore, observe that $e_{LH}^{SBPb} > e_{p}^{SBPb}$ and $e_{LH}^{SBPb} > e_{HL}^{SBPb}$ always hold, while $e_{p}^{SBPb} > e_{HL}^{SBPb}$ holds whenever $e_{p}^{SBPb} > 0$ is true. Finally, the condition $e_{p}^{SBPb} + e_{HL}^{SBPb} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)} = \gamma^{SBPb}$$

where $\gamma^{SBPb} < \min\left\{\gamma^{*}, \overline{\gamma}^{SBPb}\right\}$ is always true and where $\overline{\gamma}^{SBA2} < \gamma^{SBPb}$ and $\underline{\gamma}^{SBA3} < \gamma^{SBPb} < \overline{\gamma}^{SBA3}$ are also true.

Result 13 ***Full participation and Pooling between types HH and LL with IC_{LLvsHL} binding.*** *A solution to the principal's program SB which entails full participation and pooling between types HH and LL and IC_{LLvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{HH} = e_{LL} > e_{HL} > 0$, and which is such that effort levels are given by expressions (17), (20) and (51) exists if and only if $\gamma^{SBPb} \leq \gamma < \overline{\gamma}^{SBPb}$ with*

$$\gamma^{SBPb} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)}$$
$$\overline{\gamma}^{SBPb} \equiv \frac{\nu(1-\mu)+\mu(1-\nu)}{\mu\nu}$$

Concerning exclusion of the worst type, we need to consider a similar program where, instead of having IC_{LLvsHL} binding and IC_{HHvsHL} slack, we need PC_{LL} to be binding and PC_{HH} to be slack. In this case, the requirement $e_{p}^{SBPb} + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$ reduces to the more general condition $e_{p}^{SBPb} \leq \frac{2\gamma}{\Delta\theta}$, which is satisfied if and only if

$$\gamma \geq \frac{\Delta\theta\left(\nu\left(1-\mu\right)+\mu\left(1-\nu\right)\right)}{\left(\nu\mu\Delta\theta+2\left(\nu\left(1-\mu\right)+\mu\left(1-\nu\right)\right)\right)} = \underline{\gamma}^{SBPb}$$

where $\underline{\gamma}^{SBPb} < \gamma^{SBPb}$.

Result 14 ***Pooling between types HH and LL with PC_{LL} binding and exclusion of type HL.*** *A solution to the principal's program SB which entails pooling between types HH and LL with PC_{LL} binding and exclusion of type HL, which satisfies the monotonicity condition $e_{LH} > e_{HH} = e_{LL} > 0$, and which is such that effort levels are given by expressions (17) and (51) exists if and only if $\underline{\gamma}^{SBPb} \leq \gamma < \overline{\gamma}^{SBPb}$ with*

$$\underline{\gamma}^{SBPb} \equiv \frac{\Delta\theta(\nu(1-\mu)+\mu(1-\nu))}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+\mu(1-\nu)))}.$$

For further reference note that $\underline{\gamma}^{SBPb}$ is smaller than $\overline{\gamma}^{SBA2}$ provided that $\theta \leq 2$, namely provided that Assumption 2 holds.

Also note that it might eventually be the case that $e_{HH} = e_{LL} = e_{p}^{SBPb} < \frac{\gamma}{\theta}$ in which situation type HH would have incentive to provide more effort than the one required by her optimal contract since the required effort falls in the range in which her indifference curve is downward sloping in the space (e, w).

E Example

Let $\gamma_L = 0$ and $\gamma_H = \gamma \in (0,1]$ and let $\theta_L = 1$ and $\theta_H = \theta \in (1,2]$. Assume that motivation and skills are uniformly distributed across workers, so that $\mu = \nu = \frac{1}{2}$. Case M is attained for $1 < \theta < \frac{3}{2}$, Case $A.2$ does not exist, while Case $A.3$ holds for $\frac{5}{3} < \theta \leq 2$. Hence one can have three classes of problems: (i) the difference in ability is low and $1 < \theta < \frac{3}{2}$, and either motivation prevails and Case M is attained or ability prevails and Case $A.1$ holds; (ii) the difference in ability is high and $\frac{5}{3} < \theta \leq 2$, ability always prevails and either Case $A.1$ or Case $A.3$ hold depending on the value taken by γ; (iii) the difference in ability is intermediate so that $\frac{3}{2} \leq \theta \leq \frac{5}{3}$, ability prevails and only Case $A.1$ holds.

In situation (i), one observes the following optimal contracts: when $0 < \gamma \leq \frac{\Delta\theta}{3(2\theta-1)} = \underline{\gamma}^{SBA1}$ the principal offers a pooling contract to low-skilled types HH and HL, when $\underline{\gamma}^{SBA1} < \gamma < \overline{\gamma}^{SBA1} = \gamma_3^{SBA1} = \frac{\Delta\theta}{3\theta-1}$ full participation and full separation under Case $A.1$ is implemented, when $\overline{\gamma}^{SBA1} \leq \gamma < \overline{\gamma}^{SBPa} = \frac{\Delta\theta}{2\theta}$ the principal offers a pooling contract to intermediate types HH and LL, which is such that IC_{HHvsHL} is binding, when $\overline{\gamma}^{SBPa} \leq \gamma < \underline{\gamma}^{SBPb} = \frac{2\Delta\theta}{\theta+3}$ there is exclusion of both types HH and HL, when $\underline{\gamma}^{SBPb} < \gamma < \gamma^{SBPb} = \frac{4(2\theta-1)\Delta\theta}{(4\theta-3)(\theta+3)}$ there is pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and exclusion of type HL. Note that $\gamma^{SBPb} < \gamma^*$ so that we still are in the domain in which ability prevails and $e_{LL} > e_{HH}$. When $\gamma^{SBPb} \leq \gamma \leq \underline{\gamma}^{SBM} = \frac{4\Delta\theta}{2\theta+1}$ we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding but full participation is attained, and we cross γ^* so that motivation prevails and $e_{HH} > e_{LL}$. When $\underline{\gamma}^{SBM} < \gamma < \frac{3\Delta\theta}{4\theta-3} = \overline{\gamma}^{SBM} < \frac{1}{2}$, full separation and full participation is attained under Case M. When $\overline{\gamma}^{SBM} \leq \gamma < 1$ the principal offers a pooling contract to non-motivated types LL and HL.

In situation (ii), one observes the following: when $0 < \gamma < \gamma^{SBPb}$ there are the same optimal contracts as in (i), when $\gamma^{SBPb} \leq \gamma < \underline{\gamma}^{SBA3} = \frac{(3\theta-1)\Delta\theta}{2(4\theta-3)}$ we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and full participation, when $\underline{\gamma}^{SBA3} < \gamma < \overline{\gamma}^{SBA3} = \frac{2\Delta\theta}{\theta+2}$ there is full participation and full separation under Case $A.3$, when $\overline{\gamma}^{SBA3} \leq \gamma \leq 1$, we have full participation and pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding.

In situation (iii), one observes the following optimal contracts: when $0 < \gamma < \gamma^{SBPb}$ there are the same solutions as in (i) and (ii), when $\gamma^{SBPb} \leq \gamma < 1$ we have full participation and pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding.

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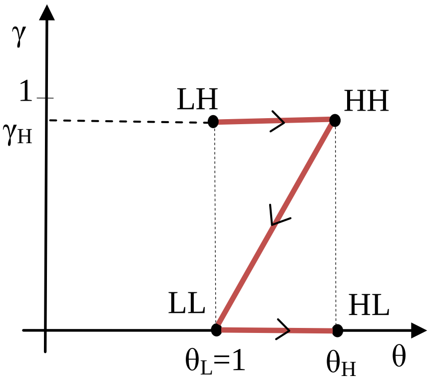

Figure 1. Case M ($e_{HH} > e_{LL}$): $2\gamma/\Delta\theta > e_{HH} + e_{LL}$.

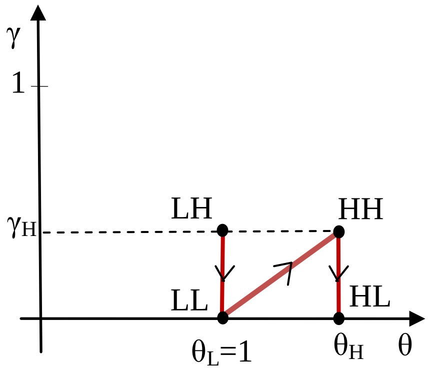

Figure 2a. Case A.1 ($e_{HH} < e_{LL}$): $2\gamma/\Delta\theta < e_{HH} + e_{HL}$.

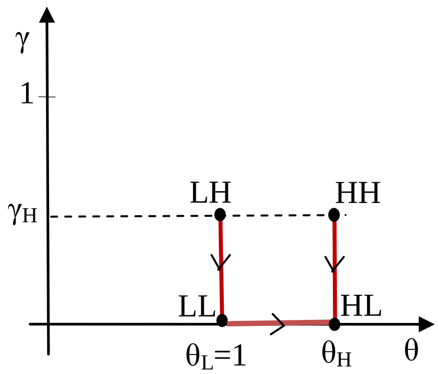

Figure 2b. Case A.2 ($e_{HH} < e_{LL}$): $e_{HH} + e_{HL} < 2\gamma/\Delta\theta < e_{LL} + e_{HL}$.

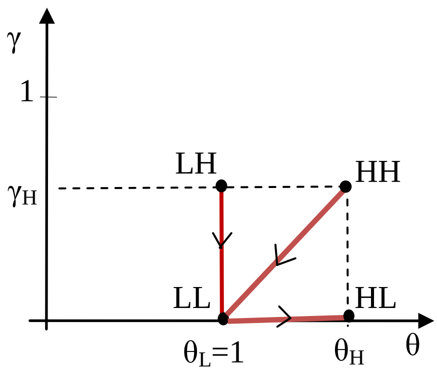

Figure 2c. Case A.3 ($e_{HH} < e_{LL}$): $e_{LL} + e_{HL} < 2\gamma/\Delta\theta < e_{LL} + e_{HH}$

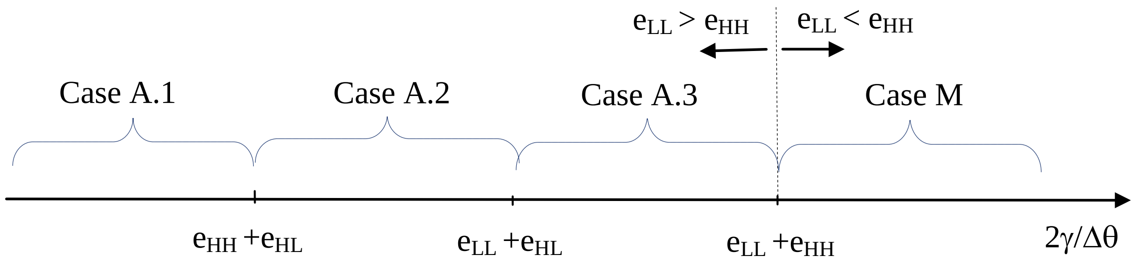

Figure 3. Existing classes of equilibria as a function of $2\gamma/\Delta\theta$.

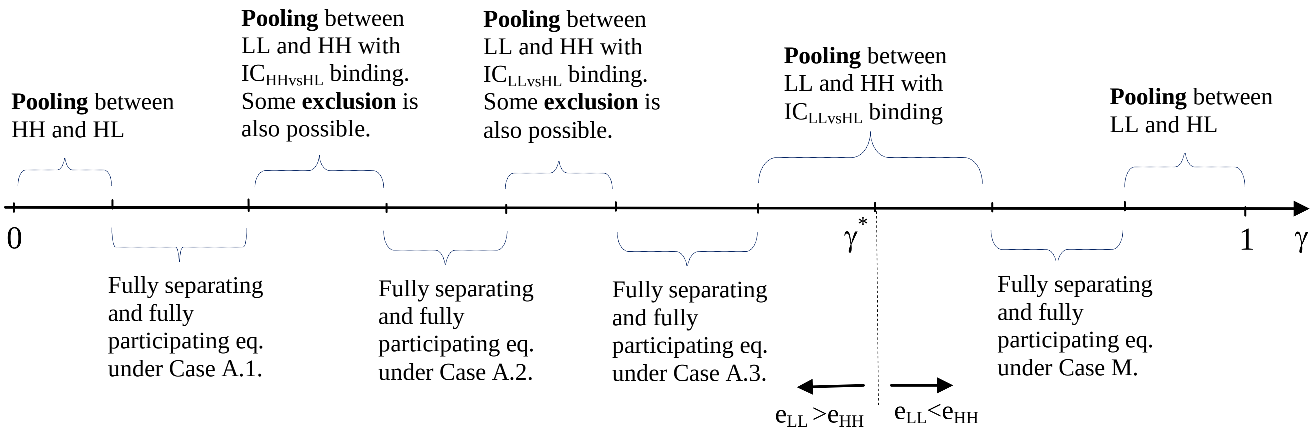

Figure 4. Equilibria with pooling and/or exclusion of some workers' types as a function of γ.

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