ISSN 2282-6483

Alma Mater Studiorum - Università di Bologna

DEPARTMENT OF ECONOMICS

Egalitarianism.
An evolutionary perspective

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Quaderni - Working Paper DSE N°888

Egalitarianism. An evolutionary perspective

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May 20, 2013

Abstract

Two parties bargaining over a pie, the size of which is determined by their previous investment decisions. Investment costs are heterogeneous. The bargaining rule is sensitive to investment behavior. Two games are studied which differ for the considered sociopolitical structure: communal property in one case and private property in the other. We hereby show that in both games when a unique stochastically stable outcome exists a norm of investment and a norm of surplus division must coevolve. While the investment norm always supports the efficient investment profile, the surplus division norm may differ among these games depending on the size of investment cost gap. Under private property only the egalitarian surplus division evolves. Under communal property instead two different surplus division norms may evolve: the egalitarian one and an inegalitarian norm. We show that no cap to payoffs inequality emerges under private property while an inequality payoff cap endogenously evolves under communal property. The games have been proposed to explain the social norms used in modern hunter-gatherer societies.

Key Words: evolution; social norms; stochastically stable equilibrium; egalitarianism; inequality; Rawlsian division; modern hunter-gatherer societies.

JEL Codes. C78, D83, L14, Z13.

1 Introduction

There is consensus among anthropologists that, despite some observed differences, the strong central tendency across contemporary hunter-gatherer societies is to cooperate in the hunt and to share the game. A telling example can be found among the Ache of Paraguay which seem to have developed a rule of thumb for hunted resources of the kind "cooperate frequently and share fully" (Hill, 2002). In some cases the egalitarian distribution of hunted resources is found even when differences in ability are observed (Kaplan and Hill, 1985)

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meaning that the more skilled hunters are taxed to create a common pool of resources that feeds everyone about equally (Bohem, 2004). The widespread observation of cooperation and sharing led some social scientists to suggest that they must be part of the ruling social norms that must have been evolving over time, probably as a way to regulate large-game hunting (Bohem, 2004).

Although egalitarianism seem to characterize these small scale societies, Lee (1990) recommends not to take egalitarianism as synonymous of perfect equality:

> "(...) perfect equality doesn't exist anywhere. It is a fact of life that human beings differ in their abilities (...). What is significant is that some societies take these differences and minimize them, to the point of making them disappear, while other societies take the same basic material and magnify it" (Lee, 1990; 236).

According to Lee (1990) the watershed line between egalitarian and not egalitarian societies seems to lay in the role of property: societies based on communal property are egalitarian since they have developed a leveling device; societies in which property is secured do not have such a device and therefore are not egalitarian.

The goal of this paper is twofold. On one hand we want to explore whether and under what conditions social norms supporting both an efficient outcome and neat distributional rule can endogenously arise through an evolutionary process in societies characterized by skill heterogeneity. On the other hand we want to test whether these evolved social norms support the idea that inequality in societies based on common property is smaller than in societies in which property is secured.

In order to do this we study a two-stage game with two risk-neutral players (A and B). In stage one, the production stage, both players have to simultaneously decide whether to invest or not; in stage two, the bargaining stage, after observing the gross surplus produced, they have to decide how to divide it. We assume that the investment is costly and that the cost incurred by A is greater than the cost incurred by B. This difference among costs can be seen as the result of skill heterogeneity with A being the less efficient agent. The surplus depends on the investment profile; we further assume that when nobody invests no surplus is produced and both agents receive nothing. Thus, a bargaining stage only occurs when at least one agent has invested in the first stage.

Two different extensive games, corresponding to two different structures of political and social organization, are considered. In both extensive games we assume that when the two players have chosen to invest the bargaining stage follows the rule of the Nash Demand Game (NDG). When, on the other hand, an asymmetric investment profile is observed two simple alternatives are considered. Each of these specifies how the unique investing agent can reap the rewards of her own action.

In the first alternative we assume that the sociopolitical organization promotes full right of possession. As a consequence all the bargaining power is assigned to the player who has decided to invest; by allowing her to behave as a dictator, the bargaining stage then collapses into a Dictator Game (DG). In this framework if a sharing occurs it only depends on the free will of the unique investing agent. The anthropological literature suggests that this game can be appropriate in a society in which production is a collective venture and property rights are secured, as it seems to happen among the Mbuti pygmies (Ichikawa, 1983).

In the second alternative we assume that, since the sociopolitical organization promotes communal property, in order to gather the surplus, an agreement has to emerge. However, despite nobody can be excluded from receiving a fair share, this agreement has to emerge in a situation in which only the unique investing player has the right to make a proposal. This suggests that bargaining can take the form of an Ultimatum Game (UG) in which the player who has chosen to invest proposes a distribution which is only realized if the opponent accepts it otherwise the surplus is lost due to conflict. The anthropological literature suggests that this game can be suitable for a society in which production is an individual activity but in which full right of possession is not supported, as it seems to be the case for the !Kung (Woodburn, 1982).

By Γ_{DG} (resp. Γ_{UG}), we denote the whole game in which an NDG occurs when a symmetric investment profile is observed, and a DG (resp. UG) occurs when an asymmetric investment profile is observed. Both games (i.e. Γ_{UG} and Γ_{DG}) have a multiplicity of equilibria; nevertheless the equilibrium selection problem may be solved if adequate social norms are in place (Binmore, 1998, 2007; Binmore and Shaked, 2010). Since in our model agents strategically interact in each stage of the game, the social norm of interest is twofold: it is a norm of cooperation, which dictates how to play the production stage, and a norm of distribution, which dictates how to divide the surplus produced. In order to identify the evolved social norms we use the concept of stochastic stability and we apply the evolutionary framework for extensive games put forward by Noldeke and Samuelson (1993). We claim that a social norm has evolved when the stochastically stable set only supports an homogeneous behavior for at least one population.

Our main result states that in both games when a social norm evolves then not only do we observe an homogeneous behavior along the whole path of play but the prescribed actions are uniform across populations. We can thus claim that in both games norms coevolve.

When a coevolution of norms is observed, the investment norm supports full cooperation in the production stage (meaning that both agents choose to invest), regardless of the sociopolitical organization considered. However the two games differ depending on both the condition needed for the two norms to evolve and the kind of evolved bargaining norm.

When property is secured (i.e Γ_{DG}) we show that norms always coevolve and the bargaining norm supports an equal surplus division. In this case a higher cost gap raises payoffs inequality and no endogenous inequality cap exists. When communal property is considered (i.e Γ_{UG}) we show that norms coevolve only if is the total investment cost sufficiently limited; in this case the evolved bargaining norm depends on the parameters configuration. In particular equal sharing continues to be the observed norm when it gives to both agents the incentive to invest. When instead equal sharing gives the incentive to invest to the most efficient agent only we show that an unequal surplus division norm arises granting to the less efficient agent a larger surplus share. As a consequence payoffs inequality now turns out to be a decreasing function of the cost gap. Given that the size of the cost gap matters in determining which kind of bargaining norm evolves, we claim that an inequality cap now springs from evolution. In addition, for some parameters configuration the unequal bargaining norm gives to each agent a share equals to her investment cost plus half of the remaining net surplus, as predicted by Equity Theory; when this occurs payoffs equality is observed.

For whatever sociopolitical organization considered, the agent with smaller investment cost is the advantaged one. However we claim that the unequal surplus division norm emerging under communal property has a Rawlsian taste: by providing to the disadvantaged agent the largest share of the surplus, it ensures to the advantaged agent the cooperation of the disadvantaged one.

The paper is organized as follows. In Section 2 we present the model. The evolutionary dynamics are studied in Section 3 and the main results presented in Section 4. Section 5 is devoted to the discussion of our results together with the analysis of inequality. In the last Section we compare our results with those of the related existing literature.

2 The model

Two risk neutral players (A and B) are engaged in a two-stage game. In stage one both have to simultaneously decide whether to invest (action H) or not (action L); when a player chooses H she incurs in a cost. We denote by c_A and c_B the cost supported by agent A and B respectively. A surplus is produced and observed at the end of stage one; each player can then correctly estimate his opponent's investment. We denote the surplus arising when both choose H by V_H; the surplus accruing when only one chooses H by V_M; and lastly, when both choose L, by $V_L = 0$. Obviously, $V_H > V_M > 0$.

In stage two, they bargain over the available surplus. The bargaining rule depends on the investment profile. If both have chosen H, they are engaged in a Nash Demand Game (NDG). If they have chosen different investments, two alternatives are conceivable: an Ultimatum Game (UG) and a Dictator Game (DG). We denote Γ_{UG} the whole extensive game in which a NDG occurs when both players have invested while a UG occurs when only one player has invested. Analogously we denote Γ_{DG} the whole extensive game in which a DG occurs when only one agent has invested. Let $D(V_j) = \{\delta, 2\delta, ..., V_j - \delta\}$, $j \in \{H, M\}$ denotes the set of feasible claims.

Throughout the paper we make the following assumptions:

Assumption 1 *(a) $V_H/2$, c_A and c_B are all divisible by δ;*

(b) $c_A \geq c_B > \delta$;

(c) the maximum payoff attainable by playing H when the opponent chooses L is not negative, i.e.

$$c_A < V_M - \delta; \tag{1}$$

(d) the efficient net surplus arises when both players choose H, i.e.

$$V_H - c_A - c_B > \max\{0, V_M - c_A, V_M - c_B\}. \tag{2}$$

(e) the population is sufficiently large, i.e.

$$\frac{V_H}{N} < \delta. \tag{3}$$

When δ is negligible, Points (b), (c) and (d) of Assumption 1 are satisfied for $c_A \geq c_B$, $c_A \leq V_M$ and $c_A < V_H - V_M$. In turn, these conditions are satisfied when:

$$\begin{array}{lll} c_B \leq c_A < V_M & \text{if} & V_M \leq \frac{V_H}{2} \\ c_B \leq c_A < V_H - V_M & \text{if} & V_M \geq \frac{V_H}{2}. \end{array} \tag{4}$$

Figure 1 summarizes the parameters configuration compatible with Points (b), (c) and (d) of Assumption 1.

In NDG players A and B simultaneously make demands y and x, respectively. If these demands are compatible, each receives what she claimed; otherwise they receive nothing. The payoffs are

$$\pi_A = \begin{cases} y - c_A & if \quad y + x \leq V_H \\ -c_A & if \quad y + x > V_H \end{cases}$$

and

$$\pi_B = \begin{cases} x - c_B & if \quad y + x \leq V_H \\ -c_B & if \quad y + x > V_H. \end{cases}$$

In UG the player who has chosen H makes a proposal which the opponent can either accept or reject. Let's suppose HL is observed and A proposes the division $(y, V_M - y)$. If B accepts, the payoffs are $y - c_A$ for A and $V_M - y$ for B; otherwise A gets $-c_A$ and B nothing. An analogous situation occurs when LH

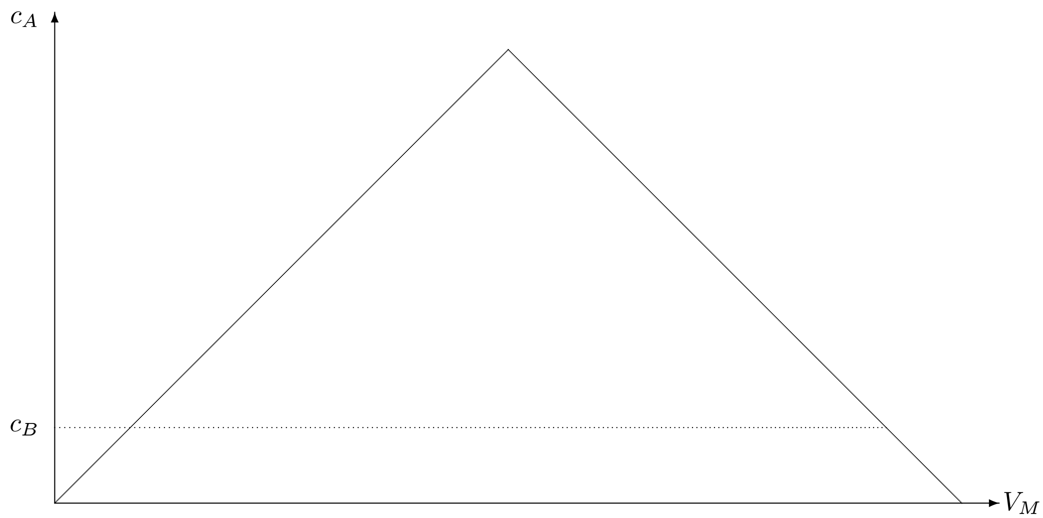

Figure 1: Parameter region compatible with Points (b), (c) and (d) of Assumption 1.

is observed and B proposes the division $(V_M - x, x)$. If A accepts, the payoffs are $V_M - x$ for A and $x - c_B$ for B; otherwise B gets $-c_B$ and A nothing.

In DG the division continues to be advanced by the player who has chosen H; however her opponent now has no choice but to accept. Suppose HL is observed and A demands y. The payoffs are $y - c_A$ for A and $V_M - y$ for B.

It is worth noticing that, under previous Assumption 1, both Γ_{UG} and Γ_{DG} admit a subgame perfect equilibrium which supports investment profile HH. Note, however, that the games admit a great number of subgame perfect equilibria, some of which are inefficient.

3 Evolutionary dynamics

In this Section, we apply to our model the evolutionary dynamics put forward by Noldeke and Samuelson (1993). To this end we postulate a finite population of size N agents for each player, A and B. In each period, every possible match between agents occurs meaning that each agent belonging to population A interacts with each agent of population B, one at a time. An agent is described by a *characteristic* which consists of a detailed plan of action and a set of beliefs about the opponent's behavior.

In Γ_{UG}, the plan of actions for player A must specify: (i) the type of investment; (ii) the demand when both players choose H (the action at HH); (iii) the demand when A chooses H and B chooses L (the action at HL); (iv) whether to accept or reject any demands made by B, when in the first stage B chooses H and A chooses L. Analogously for player B.

In Γ_{DG}, the plan of actions for player A must specify: (i) the type of investment; (ii) the demand when both players choose H (the action at HH); (iii) the division of the surplus when A chooses H and B chooses L (the action at HL). Analogously for player B.

A state, θ, is a profile of characteristics of the overall population and $z(\theta)$ is the probability distribution over terminal nodes generated by θ. The set of possible states, Θ, is finite.

At the end of every period each agent has a probability λ to observe the distribution of outcomes $z(\theta)$ and may change her characteristics. In particular this stream of information allows agents to correctly update their beliefs on opponent's choices at the reached information sets. Given their new beliefs they also update their action profile by choosing a best reply[1] at each information set. With probability $1-\lambda$ the single agent does not observe $z(\theta)$ and her characteristics do not change. This learning mechanism engenders an (unperturbed) Markov process (Θ, P) where P is the transition matrix on Θ.

By Ω we denote a generic limit set[2] of the unperturbed process; this is a minimal subset of states such that, when the process enters, it does not exit. By Σ we denote the union of limit sets of the unperturbed process. Lastly by $\rho(\Omega)$ we denote the set of outcomes that can be observed.

Besides being updated, agents' beliefs and actions can also change by mutation. In every period each agent has a probability ϵ of mutating. When mutating, agents change their characteristics according to a probability distribution assigning positive probability on each possible characteristic. Mutations are independently distributed across agents. Assume that the process is in some limit set Ω and that a single mutation occurs which alters the characteristics of a single agent (the mutant). If this mutation does not alter the action prescribed and/or the beliefs held by the mutant, then the mutation is called drift. Since the expected payoff of the others does not change, their characteristics do not change; we then move from one limit set Ω to another limit set Ω' by drift.

Mutations generate a new (perturbed) Markov process $(\Theta, P(\epsilon))$, which is ergodic. It is well known that, for any fixed $\epsilon > 0$, the perturbed process has a unique invariant distribution μ_ϵ. Let $\mu_* = \lim_{\epsilon \to 0} \mu_\epsilon$ denote the limit distribution. A state θ is stochastically stable if $\mu_*(\theta) > 0$. We denote the set of stochastically stable states by Σ_S; this is the set of states which has a positive probability in limit distribution. Noldeke and Samuelson (1993) proved that the stochastically stable set is contained in the union of the limit sets of the unperturbed process. Only when the set of stochastically stable states (Σ_S) contains equilibria supporting the same unique outcome can we speak of

[1] However, if the learning agent has already played a best reply, her action does not change. Moreover when the best reply contains more than one action, then one of these can be chosen randomly, according to a distribution with full support.

[2] A set $\Omega \subseteq \Theta$ is called a $\omega-$limit set of the process (Θ, P) if: (a) $\forall \theta \in \Omega$, $Prob\{\theta_{t+1} \in \Omega \mid \theta_t = \theta\} = 1$; (b) $\forall (\theta, \theta') \in \Omega^2$, $\exists s > 0$ s.t. $Prob\{\theta_{t+s} = \theta' \mid \theta_t = \theta\} > 0$.

a stochastically stable outcome rather than a stochastically stable set. In order to detect the stochastically stable set, we have first to characterize the limit sets of our model; this is the aim of the following two Propositions.

Proposition 2 *In Γ_{UG}, all the limits sets have one of the following structures: (a) they contain one state only and this is a self-confirming equilibrium of the game; (b) they contain more than one state and all investment profiles are observed. Moreover, only one outcome is realized for each investment profile in which the claims exhaust the surplus.*

Proof. See the Appendix[3].

Proposition 3 *In Γ_{DG}, all the limits sets contain one state only and this is a self-confirming equilibrium. Moreover, at least one agent chooses to invest in every equilibrium.*

Proof. See the Appendix.

From now on when we speak of equilibrium we refer to self-confirming equilibrium.[4]

Propositions 2 and 3 state that the considered evolutionary dynamic gives rise to a large multiplicity of limit sets. However, this dynamic admits limit sets in which both investment and bargaining behavior is uniform in each population. It is thus likely that homogeneous behavior in one or both populations could be molded by evolution. When this happens we say that a norm has evolved. Accordingly, an investment norm has evolved if all agents belonging to the same population make the same investment and the investment behavior is anticipated correctly. Analogously, a bargaining norm has evolved if a pair of demands (y, x) exists at some reached information set which exhausts the gross surplus and the bargaining behavior is anticipated correctly.

Despite the fact that Propositions 2 and 3 do not help to pin down which behavior is more likely to become the conventional one, in the next Section we shall show that a unique stochastically stable outcome can exist in both games.

4 Main results

In this Section we show that when a unique stochastically stable outcome exists it always supports the efficient investment profile and a well-defined distribution rule which is not always the egalitarian one. First and foremost, we provide a characterization of the stochastically stable set for Γ_{UG}. We then briefly consider Γ_{DG}.

[3] Careful reading of the Proposition proof shows that the claims must satisfy a well-defined set of constraints.

[4] According to Noldeke and Samuelson (1993) a state is a self-confirming equilibrium if each agent's strategy is a best response to that agent's conjecture and if each agent's conjecture about opponent's strategies matches the opponent's choices at information sets that are reached in the play of some matches.

4.1 Communal property

Consider Γ_{UG}. By x_B^U (resp. $V_H - x_A^U$) we denote the share going to player B (resp. A), such that she receives an equilibrium payoff equal to $(V_M - \delta)$ when both agents have invested:

$$\begin{aligned} x_B^U &= V_M - \delta + c_B \\ x_A^U &= V_H + \delta - c_A - V_M. \end{aligned} \tag{5}$$

Since c_A and c_B are divisible by δ and $c_B > \delta$, then $x_B^U, x_A^U \in D_\delta(V_H)$. Let

$$\begin{aligned} \widehat{x}_A^U &= \max\left\{ x \in D_\delta(V_H) \middle| (V_H - x)\tfrac{N-1}{N} - c_A \geq V_M - \delta \right\} \\ \widehat{x}_B^U &= \min\left\{ x \in D_\delta(V_H) \middle| x\tfrac{N-1}{N} - c_B \geq V_M - \delta \right\}. \end{aligned} \tag{6}$$

In other terms $\widehat{x}_A^U$ is the largest demand agent B can make at HH such that A does not have any incentive to change action by playing L when she knows that: (i) $N-1$ agents B play H and claim $\widehat{x}_A^U$; (ii) one agent B claims a larger demand. Analogously for $\widehat{x}_B^U$. Point (e) of Assumption 1 implies that $\widehat{x}_A^U = x_A^U - \delta$ and $\widehat{x}_B^U = x_B^U + \delta$. Therefore $\widehat{x}_B^U \leq \widehat{x}_A^U$ if[5]

$$c_A + c_B \leq V_H - 2V_M. \tag{7}$$

When this condition holds, then we can define the following set

$$\Sigma_{IH}^U = \left\{ \theta \in \Sigma_H \mid x \in \left[\widehat{x}_B^U, \widehat{x}_A^U\right] \right\}$$

where Σ_H is the set of equilibria in which both agents choose to invest and the surplus division rule assigns a share $V_H - x$ to player A and x to player B. By definition when $\theta \in \Sigma_{IH}^U$ each agent receives an equilibrium payoff not smaller than the maximum payoff attainable when she deviates by playing L. Then any equilibrium in Σ_{IH}^U dominates all the equilibria supporting other investment profiles. Hence even if at an equilibrium $\theta \in \Sigma_{IH}^U$ the beliefs on the outcome in high-low matches drift, allowing some agents to expect to get almost the whole surplus if they do not invest, this drift does not push the process away from the basin of attraction of θ.

We can always partition the set Σ_H into Σ_{IH}^U and $\Sigma_{CH}^U = \Sigma_H \setminus \Sigma_{IH}^U$ where the latter denotes the set of equilibria in which both agents choose to invest but $x \notin \left[\widehat{x}_B^U, \widehat{x}_A^U\right]$. Obviously when condition (7) does not hold then Σ_{IH}^U is empty and $\Sigma_H = \Sigma_{CH}^U$.

Condition (7) is compatible with Assumption 1 only when[6]

[5] Condition (7) implies that the average total cost of investment is limited, i.e. $\frac{c_A + c_B}{2} < \frac{V_H}{2} - V_M$. This condition can be satisfied only when $V_M < \frac{V_H}{2}$.

[6] To see this observe that condition (7) is compatible with point (b) of Assumption 1 only when $c_B \leq c_A \leq V_H - 2V_M - c_B$; this, in turn, requires $c_B \leq \frac{V_H}{2} - V_M$. On the other hand points (b) and (c) of Assumption 1 imply that $c_B < V_M$.

$$c_B \leq \min\left(\frac{V_H}{2} - V_M, V_M\right). \tag{8}$$

It is easy to see that condition (7) is never satisfied when $c_B \geq \frac{V_H}{4}$; in this case the set Σ_{IH}^U is empty. Condition (7) can be rewritten as:

$$\zeta_B + \zeta_A \geq 0$$

where $\zeta_i = \frac{V_H}{2} - c_i - V_M$ and $i \in \{A, B\}$. Notice that $\frac{V_H}{2} - c_i$ denotes the payoff i gets under equal sharing and full cooperation in the production stage; V_M denotes instead the maximum (when δ is negligible) payoff i could hope to get should she play L and all opponents continue to play H. Broadly speaking ζ_i denotes the minimum payoff gain that has to be granted to agent i to provide her the incentive to stick to the rule "cooperate in the production and equally share the surplus", on condition that all opponents population adheres too**.** Indeed if $\zeta_i < 0$ agent i has the incentive to deviate from the rule by choosing L. Since $\zeta_B \geq \zeta_A$, condition (7) always requires $\zeta_B > 0$; by contrast, under condition (7), ζ_A can take both positive and negative values.

The fact that, under condition (7), $\zeta_B > 0$ implies $\widehat{x}_B^U < \frac{V_H}{2}$. Therefore, under condition (7), the set Σ_{IH}^U is compatible with two different setting: (a) $\widehat{x}_B^U < \frac{V_H}{2} \leq \widehat{x}_A^U$ and (b) $\widehat{x}_B^U \leq \widehat{x}_A^U < \frac{V_H}{2}$. The first case occurs when, in addition to (7), we have $\zeta_A \geq 0$ i.e.

$$c_A \leq \frac{V_H}{2} - V_M; \tag{9}$$

the second one occurs when, in addition to (7), we have $\zeta_A < 0$ i.e.

$$\frac{V_H}{2} - V_M < c_A \leq V_H - 2V_M - c_B. \tag{10}$$

Therefore two are the possible bargaining norms we can expect to arise from the considered evolutionary dynamic: the egalitarian surplus division norm in case (a) and an unequal surplus division norm in case (b). The next Proposition states our main result for Γ_{UG}. In order to derive this Proposition, we make use of both the sufficient condition developed by Ellison (2000) and some of the results for the NDG proved by Young (1993).

Proposition 4 *Consider Γ_{UG} and let Assumption 1 be satisfied. When δ is sufficiently small the following cases are possible:*

(a) if condition (7) holds then $\Sigma_s \subseteq \Sigma_{IH}^U$ and a stochastically stable outcome exists. This supports the efficient investment and a bargaining norm which is either $(V_H/2, V_H/2)$ if $c_A \leq \frac{V_H}{2} - V_M$ or $(V_M + c_A, V_H - V_M - c_A)$ if $\frac{V_H}{2} - V_M \leq c_A \leq V_H - 2V_M - c_B$.

(b) if condition (7) does not hold then no norm evolves for whatever values of c_A and c_B.

Proof. See the Appendix

Proposition 4 says that social norms supporting full cooperation at the production stage (i.e. the efficient investment profile) and a neat division rule at the bargaining stage coevolve provided that the total investment cost is limited and investment are complementary. Investments are complementary if the marginal effect of action H when the opponent plays H is greater than the marginal effect of action H when the opponent plays L, condition satisfied when $V_M < V_H/2$. When the investments are not complementary our result states that no norm evolves; analogously no norms evolve when investment are complementary but total investment cost is too high.

When total investment cost is limited and investments are complementary, Proposition 4 says that the specific division rule observed depends on the investment cost incurred by the less efficient agent. When condition (9) holds, this cost is sufficiently limited and agents A do not have any incentive to deviate from the rule "cooperate in the production and equally share the surplus"; in this case the egalitarian surplus division rule evolves. When instead the investment cost incurred by the less efficient agent is high enough, this agent may be induced to break the above rule. However when rooms of manoeuvre to a different bargaining norm still exist (i.e. when condition (10) holds in lieu of condition (9)) a not egalitarian surplus division rule evolves which still supports full cooperation in the production stage. Lastly when the total cost is such that no efficient bargaining norm can emerge (i.e. when condition 7 does not hold) no norm arises.

It is straightforward to see that all the results stemming from Proposition 4 can be rewritten in terms of cost gap. Indeed, conditions (9) can be translated into $c_A - c_B \leq \zeta_B$ and condition (10) can be translated into $\zeta_B < c_A - c_B \leq 2\zeta_B$. Hence what really matters in triggering a coevolution of norms and in specifying the division rule is the cost gap size. In particular norms evolve only when the cost gap is not higher than the two-fold minimum payoff gain of the most efficient agent. When this condition is satisfied, we can observe two different bargaining norms: (i) the egalitarian surplus division when the cost gap is not higher than ζ_B; (ii) an unequal division when ζ_B is smaller than the cost gap.

Proposition 4 is illustrated in Figure 2. In this Figure the parameters space compatible with Assumption 1 is divided into three regions. In region HH_e the unique stochastically stable state supports full cooperation in the production stage and the egalitarian division of the surplus; in region HH_{ne} the unique stochastically stable state still supports full cooperation in the production stage but a not egalitarian surplus division; lastly region NN corresponds to the parameter configurations in which no norm evolves since several outcomes are stochastically stable and no uniform behavior emerges.

Figure 2 has been drawn for a given c_B. Figure 3 shows that when c_B increases both regions HH_e and HH_{ne} shrinks. When $c_B = 0$, region HH_{ne} is the largest possible and it coincides with the triangle PSS'. When $c_B = V_H/12$ (resp. $V_H/6$), region HH_{ne} shrinks to the triangle PRR' (resp. PQQ') since

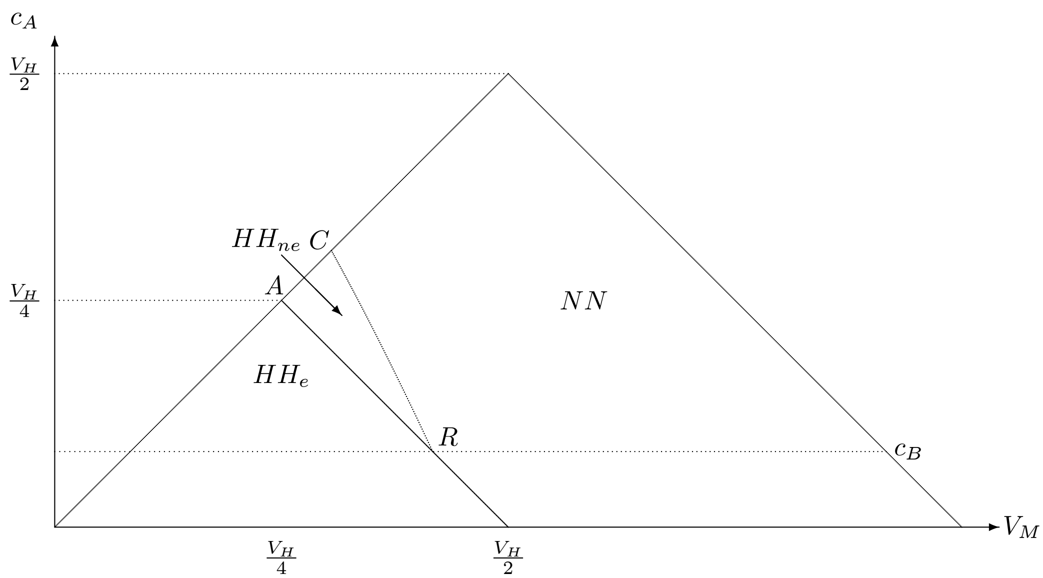

Figure 2: game Γ_{UG}. Parameter regions supporting the unique stochastically stable outcome (HH) compared with the region (NN) where no norm evolves. HH_e is the region in which an equal surplus division norm is observed; HH_{ne} is the region in which an unequal surplus division norm is observed.

only values of $c_A > c_B$ are allowed.When identical investment costs are considered (i.e. $c_A = c_B$), it is easy to check that region HH_{ne} disappears and its place is taken over by region NN.

4.2 Private property

We now turn to the case in which a Dictator Game (instead of an Ultimatum Game) is played when an asymmetric investment profile is reached. In this case we denote by x_B^D (resp. $V_H - x_A^D$) the share going to player B (resp. player A) such that she receives an equilibrium payoff equal to δ when both agents have invested:

$$\begin{aligned} x_B^D &= c_B + \delta \\ x_A^D &= V_H - c_A - \delta. \end{aligned} \tag{11}$$

Since c_A, c_B and V_H are divisible by δ, then $x_B^D, x_A^D \in D_\delta\left(V_H\right)$. Let

$$\widehat{x}_A^D = \max\left\{x \in D_\delta\left(V_H\right) \middle| \left(V_H - x\right)\tfrac{N-1}{N} - c_A \geq \delta\right\}$$

$$\widehat{x}_B^D = \min\left\{x \in D_\delta\left(V_H\right) \middle| \, x\tfrac{N-1}{N} - c_B \geq \delta\right\}.$$

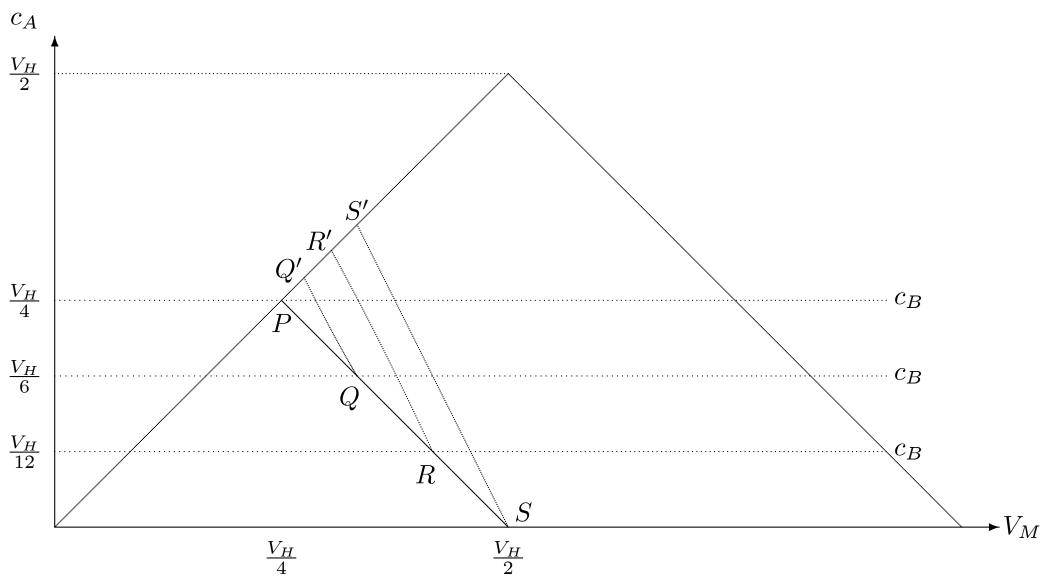

Figure 3: game Γ_{UG}. As c_B decreases the region HH_{ne} enlarges. The Figure shows the cases $c_B = \frac{V_H}{6}$, $c_B = \frac{V_H}{12}$ and $c_B = 0$. For $c_B > \frac{V_H}{4}$ social norms do not evolve.

By keeping in mind Point (e) of Assumption 1 we obtain $\widehat{x}_A^D = x_A^D - \delta$ and $\widehat{x}_B^D = x_B^D + \delta$, i.e.

$$\begin{aligned} \widehat{x}_B^D &= c_B + 2\delta \\ \widehat{x}_A^D &= V_H - c_A - 2\delta. \end{aligned}$$

It is easy to see that $\widehat{x}_B^D \leq \widehat{x}_A^D$ provided:

$$c_A + c_B \leq V_H - 4\delta. \tag{12}$$

When this condition holds, we can define the following set

$$\Sigma_{IH}^D = \left\{\theta \in \Sigma_H \mid x \in \left[\widehat{x}_B^D, \widehat{x}_A^D\right]\right\}.$$

Also for Γ_{DG} we can always partition the set Σ_H into Σ_{IH}^D and Σ_{CH}^D where $\Sigma_{CH}^D = \Sigma_H \diagdown \Sigma_{IH}^D$; the latter denotes the set of equilibria in which both agents choose to invest but $x \notin \left[\widehat{x}_B^D, \widehat{x}_A^D\right]$. However in this case the set Σ_{IH}^D is always well-defined and $\Sigma_{CH}^D = \Sigma_H \backslash \Sigma_{IH}^D$ only supports $\left(x_B^D, x_A^D\right)$ as distributional rule. Indeed when δ is sufficiently small Point (d) of Assumption 1 ensures that the total cost is always smaller than V_H. The following Proposition summarizes our finding concerning Γ_{DG}.

Proposition 5 *Consider Γ_{DG} and let Assumption 1 be satisfied. When δ is sufficiently small, i.e. $\delta < \frac{V_H - 2c_A}{4}$, then $\Sigma_S \subseteq \Sigma_{IH}^D$ and a stochastically stable*

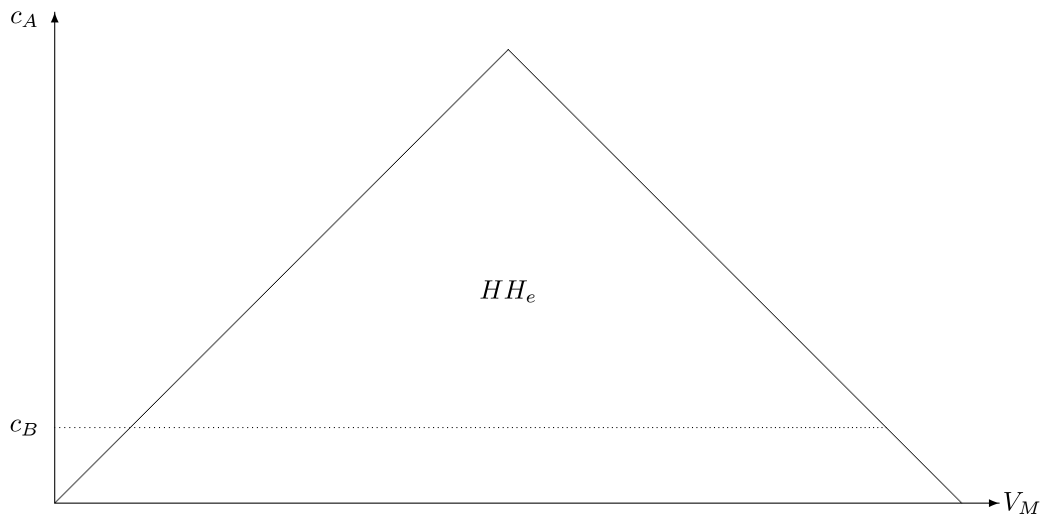

Figure 4: game Γ_{DG}. Parameter regions supporting the unique stochastically stable outcome. HH_e is the region in which an egalitarian norm is observed.

outcome always exist. This supports the efficient investment and the egalitarian rule as bargaining norm, i.e. $(V_H/2, V_H/2)$.

Proof. See the Appendix

Proposition 5 is illustrated in Figure 4. By comparing Figure 4 with Figure 2 we see that when the rules of the game give all the bargaining power to the unique investing agent, under Assumption 1 norms of cooperation and norms of division always coevolve without any further conditions. Moreover, since norms are now insensitive to the degree of heterogeneity among agents (i.e. cost gap), these norms do not change when the investment costs are identical; the only consequence of cost heterogeneity is that region HH_e shrinks as c_B increases. Lastly, while an unique stochastically outcome can arise in Γ_{UG} only if investments are complementary, this is not required in Γ_{DG} : norms coevolve even when investments are not complementary, i.e. when $V_M \geq \frac{V_H}{2}$.

5 Discussion

In order to offer an interpretation of the results so far derived, let us consider again Proposition 4. This suggests that under communal property social norms supporting cooperation in the production stage are compatible with two different distributional rules.

The first is observed in region HH_e and it supports the egalitarian division of the surplus or, in other terms, equality of resources. Equality of resources implies equality of welfare only when $c_A = c_B$. When instead $c_A > c_B$, equality of resources generates inequality of welfare: in this case agent A (the less efficient one) is disadvantaged. All this holds true also for Γ_{DG}.

The second distributional rule is observed in region HH_{ne} and, since it is no longer the egalitarian division, it allows for inequality of resources. In particular, given that in this region $c_A > \frac{V_H}{2} - V_M$, the share of the surplus going to agent A (i.e. the agent with higher investment cost) is greater than the share going to agent B (i.e. the agent with smaller investment cost), i.e. $V_M + c_A > V_H - V_M - c_A$. Nevertheless in this region the payoff granted to A continues to be not greater than that granted to B meaning that agent A is still the disadvantaged one. Indeed, A's payoff is $\pi_A = (V_M + c_A) - c_A = V_M$ and B's payoff is $\pi_B = (V_H - V_M - c_A) - c_B$ and $\pi_B \geq \pi_A$ for $c_A \leq V_H - 2V_M - c_B$. Therefore this distributional norm ensures to player A a payoff which is slightly greater than the maximum payoff this agent can hope to get should she decide not to cooperate in the production stage (i.e. $V_M - \delta$). In this sense we can say that in HH_{ne} the observed norm has a Rawlsian taste: by providing to the less efficient agent the incentive to invest, it ensures to the advantaged agent B the cooperation of the less efficient and disadvantaged A. This inegalitarian surplus division thus represents the maximal inequality compatible with the cooperation in the production stage[7].

Lastly, it is worth noticing that, though this division rule is not egalitarian, it supports equality of payoffs if $c_A = V_H - 2V_M - c_B$. In this particular case the evolved division norm is compatible with a rule suggested by the so-called Equity Theory[8] in which each player gets her investment cost plus an equal share of the remaining surplus[9]:

$$\begin{aligned} y &= c_A + \frac{V_H - c_A - c_B}{2} &= \frac{V_H}{2} + \frac{c_A - c_B}{2} \\ x &= c_B + \frac{V_H - c_A - c_B}{2} &= \frac{V_H}{2} + \frac{c_B - c_A}{2}. \end{aligned} \tag{13}$$

Few computations shows that $V_M + c_A = \frac{V_H}{2} + \frac{c_A - c_B}{2}$ and $V_H - V_M - c_A = \frac{V_H}{2} + \frac{c_B - c_A}{2}$ when $c_A = V_H - 2V_M - c_B$. However when $c_A < V_H - 2V_M - c_B$ agent A (resp. B) gets a share smaller (resp. larger) than the share predicted by Equity Theory.

5.1 Inequality

What we have just said has obvious consequences for the inequality observed in the two games. Figure 5 shows payoff as function of c_A.

[7] A somehow smilar result has been derived by Barling and von Siemens (2010) in the framework of incentivating contracts. They show that equal sharing is the optimal solution to incentive problems in partnership provided partners are inequity averse. This because equal sharing maximizes the incentives of the partner who has the weakest incentive to exert effort. In our case equal sharing provides an adequate incentive to invest to the less efficient agent in region HH_e but not in region HH_{ne}.

[8] Homas (1961).

[9] This rule is known in bargaining literature as "split the difference".

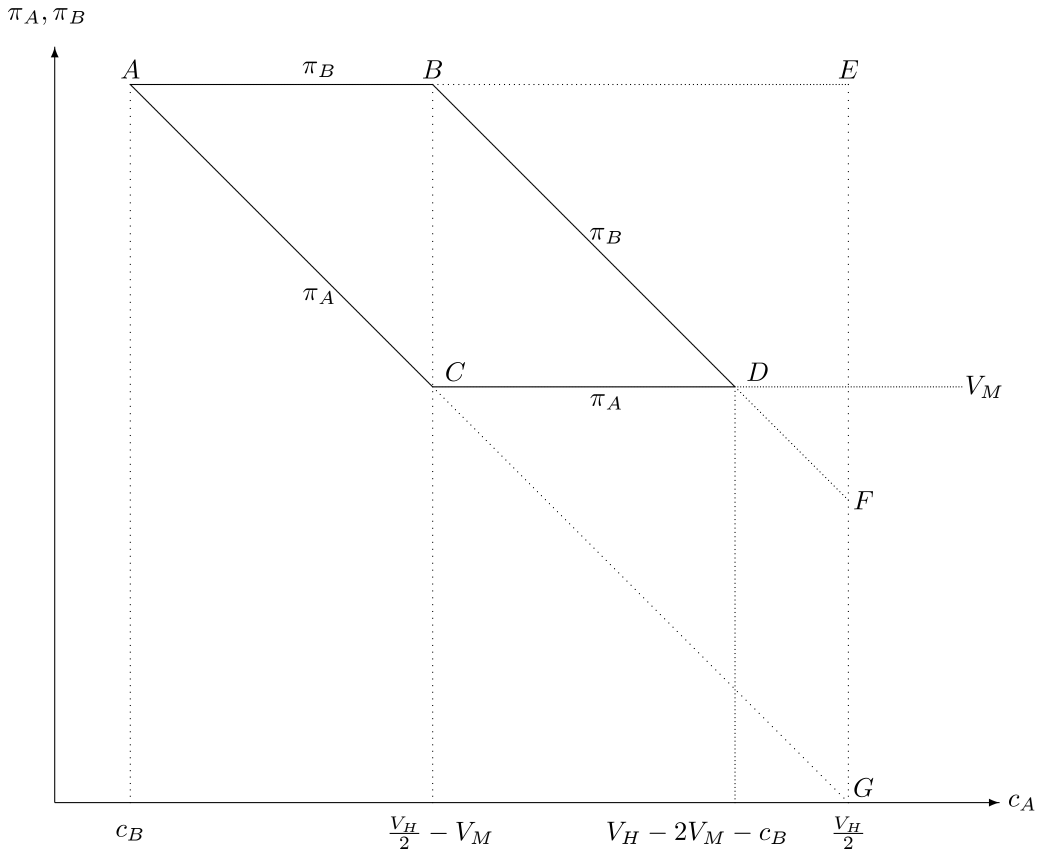

Figure 5: Payoff inequality as functions of c_A. The Figure is drawn for δ negligible.

Consider Γ_{UG} first. We are in region HH_e for $c_A \leq \frac{V_H}{2} - V_M$ and in region HH_{ne} for $\frac{V_H}{2} - V_M < c_A \leq V_H - 2V_M - c_B$. In the first region payoff inequality increases with c_A and it reaches its maximum level when $c_A = \frac{V_H}{2} - V_M$ and $V_M = \frac{V_H}{4}$ (i.e. at point A of Figure 2; in Figure 5 the maximal payoff inequality corresponds to segment BC); in the second region payoff inequality decreases with c_A and, as shown in Figure 5, the less efficient agent A receives a constant payoff V_M (i.e. the line CD of Figure 5). We get payoff equality when $c_A = V_H - 2V_M - c_B$ (i.e. at point D of Figure 5 and along the line CR of Figure 2).

Consider now Γ_{DG}. In this case, since only region HH_e exists, payoff inequality increases with c_A. The maximum observable inequality is found when $c_A = \frac{V_H}{2}$.

Two are the conclusions drawing from the above analysis. First the two games admit a parameters configuration giving rise to the same inequality; this occurs for the parameters configuration for which region HH_e of Γ_{UG} exists. Second the parameter configuration giving rise to region HH_{ne} has the main effect to set a cap to the observable inequality in Γ_{UG}. Since no inequality cap endogenously arises in Γ_{DG} we may conclude that our analysis supports Lee's conjecture (Lee, 1990), namely that only societies based on communal property are egalitarian since only these have developed a leveling device. In terms of our model the leveling device is represented by region HH_{ne}; however our analysis suggests that this leveling device only matters when the investment cost of the less efficient agent is high enough.

Why an upper limit to inequality evolve in Γ_{UG} but no in Γ_{DG} ? We believe this is due to the fact that the considered sociopolitical organizations, giving rise to two different games, accord to the less efficient agent A different incentives to cooperate. Consider Γ_{UG}. In region HH_e we observe $V_M < \frac{1}{2}V_H - c_A$. When δ is negligible the left hand side denotes the maximum payoff A can hope to get, should she decide not to cooperate (i.e. in node LH); the right hand side denotes instead the payoff she gets under full cooperation given the evolved distributional rule. The inequality means that in region HH_e agent A has always the incentive to cooperate in the production stage so that no further incentive is needed. In region HH_{ne} we observe instead $V_M > \frac{1}{2}V_H - c_A$ so that, under the egalitarian distribution of the surplus, A is better off by not cooperating (in terms of Figure 5 the payoff accruing to A under the egalitarian surplus division is found along the line CG). Since all agents B might deem that A will only accept a distribution granting her almost the whole pie at LH profile, then B must grant to A a larger share of the surplus (i.e. $V_M + c_A$) in order to induce this agent to cooperate. This, in turn, implies that A's payoff is equal to V_M; the larger share granted to A reduces payoff inequality as well.[10]

One may wonder why in Γ_{DG} we do not observe an upper limit to inequality. When $V_M < \frac{1}{2}V_H - c_A$, the same argument used above holds: agent A is better off by cooperating in the production stage so that no further incentive is needed. When instead $V_M > \frac{1}{2}V_H - c_A$ agent A should be incentivated to invest; however this is no longer effective in Γ_{DG}. The reason is that when a node LH is reached (and this occurs when A does not invest), the investing agent B is now a dictator. Therefore even if A can hope to get $V_M > \frac{1}{2}V_H - c_A$, this belief soon turns out to be a mistake since the dictator B shall only be willing to offer δ. By realizing that, A is better off by cooperating in the production stage being $\frac{1}{2}V_H - c_A > \delta$. This, in turns, poses no limit to payoff inequality as c_A increases.

The main difference between game Γ_{UG} and game Γ_{DG} is that only the former has a collective action problem at its core. Indeed, by postulating an Ultimatum Game at nodes HL and LH, we have implicitly assumed that nobody can be easily excluded from consuming the produced good even when someone has not cooperated in the production stage. When one agent makes some bread

[10] This is what allows us to qualify region HH_{ne} as Rawlsian: given the rule of play the inequality observed in this region is the maximal inequality needed to give to the less efficient agent A the incentive to invest. As a consequence, the observed surplus division norm ensures to B the cooperation of the less efficient and disadvantaged A. By contrast HH_e is not Rawlsian since B can always count on the cooperation of A.

and the cost of refusing a slice to another is too high to be worth paying, then the bread is not a perfectly private good (Hawkes, 1993). In contrast, by postulating a Dictator Game at nodes LH and HL we have swept aside this collective action problem since the social structure now guarantees full protection to the unique investing agent. Our results suggest that a Rawlsian region evolves under two conditions: (a) the social structure must not offer full cover to the unique investing agent and (b) the investment cost supported by the less efficient agent must be large enough but not too high, i.e. $\zeta_B < c_A - c_B \leq 2\zeta_B$ **.** When one of these two conditions does not hold, our result suggest that either an egalitarian surplus division evolves or no norms evolve.

6 Further discussion

Evolutionary dynamics in models with advance production and successive bargaining have been studied by Troger (2002), Ellingsen and Robles (2002) as well as Dawid and MacLeod (2001, 2008). Broadly speaking this literature has shown that evolution (i.e. stochastic stability) may or may not support an efficient investment profile depending on whether only one (Troger, 2002; Ellingsen and Robles, 2002) or both parties (Dawid and MacLeod, 2001, 2008) make a relation-specific investment in the project, respectively[11] . Despite the differences, all these studies share the same homogeneity assumption since the investment costs are assumed to be the same for all investing agents.

The consequences of investment cost heterogeneity is investigated by Andreozzi (2010, 2011). He retains the same basic structure as in Troger (2002) and Ellingsen and Robles (2002) so that, in each period, the size of the pie is determined by the investment decisions of one agent only (A). However, depending on the investment cost, the population of investing agents can be split into two sub-populations: the more efficient agents A_L (i.e those with a low investment cost, c_L) and the less efficient agents A_H (i.e. those with a high investment, c_H). In any case, all the investing agents get the same share of the resulting surplus. This has an important consequence. In fact, when the evolved bargaining norm gives to the less efficient investing agents A_H the incentive to invest, *a fortiori* it gives the incentive to invest also to the more efficient agent A_L; when this occurs the resulting equilibrium is efficient. When instead the evolved bargaining norm grants the incentive to invest to the more efficient agents only, then the less efficient ones do not invest and the resulting equilibrium is inefficient.[12]

[11] This literature is concerned with (one-sided or two-sided) relation-specific investment; by contrast our main interest is to study a generic setting in which people are not dependent on specific other people for access to basic requirements and in which individuals are not bound to fixed assets or fixed resources. This makes our model more apt for describing simple societies as modern hunter-gatherers (Woodburn, 1982), for instance.

[12] By contrast in our case all the stochastically stable outcomes are supported by efficient equilibria of the considered games.

By comparing payoffs inequality at the efficient equilibrium with payoff inequality at the inefficient equilibrium, Andreozzi (2011) concludes that efficiency calls for greater inequality. This has an obvious explanation: a convention is efficient only if it gives the incentive to invest to the least efficient agent; since all the investing agents receive the same share of the surplus, the more efficient agents get an higher payoff because they bear a smaller investment cost. However, as Andreozzi (2011) correctly suggests, this does not necessarily lend support to the conclusion that, in economies with production, social justice has to be less egalitarian.

Our results support this view. In fact, also in our case at the stochastically stable equilibrium the more efficient agent is granted a larger payoff. In Γ_{DG} the resulting inequality follows from the same mechanism described by Andreozzi (2011), namely the fact that all investing agents receive the same share of the surplus; given the sociopolitical organization considered, this equal share gives to the less efficient agent adequate incentives to invest. In Γ_{UG} the mechanism responsible for the resulting inequality is more subtle. In region HH_e it still follows the same mechanism since the investment cost of the less efficient agent is limited, i.e.$c_A \leq \frac{1}{2}V_H - V_M$, and both investing agents receives the same surplus share. However the mechanism which is responsible for the observed inequality is different in region HH_{ne} since the investment cost of the less efficient agent is sufficiently large, i.e. $\frac{1}{2}V_H - V_M < c_A \leq V_H - 2V_M - c_B$. In this case, in fact, in order to provide to the less efficient agent the incentive to invest, the evolved bargaining norm grants to these agents a share of the surplus which is larger than the share going to the more efficient ones (still preserving for the latter the incentive to invest). As a consequence, although the more efficient agents are still the advantaged ones, the evolved bargaining norm has now the effect of reducing payoffs inequality. For some parameters configuration we have also identified a region in which not only does efficient investment occurs but even payoff equality arises.

The role of fairness in bargaining games with advance production has also received some attention by experimental economics. However, only few of them are concerned with the case in which social output is determined by the investment decisions of all the subjects involved; here we briefly discuss Gantner et al. (2001) and Cappelen et al. (2007) which are the most relevant for our model.

These papers are both concerned with a two-stage game involving two agents. In the first stage agents simultaneously have to choose their investment level; investment is costly and an investment q_i imposes a cost of q_i. Social output is a weighted sum of the individual investment levels where each weight reflects the marginal productivity of the investment chosen by the specific agent; investments are thus perfect substitute. In the second stage, after social output is observed, agents must divide it.

In Gantner et al. (2001) two different games are proposed to study this division; in the first agents play an Ultimatum Game where the proposer is randomly selected (after choosing the bargaining vectors); in the second agents play a Nash Demand Game. In Cappelen et al. (2007) instead the distribution stage is modeled as a Dictator Game; specifically, given informations about both the marginal productivity of opponent's investment and her investment level, each agent decides how to distribute the social output as if it were a dictator; in this case each subject's total earnings is the final outcome plus the amount of the initial endowment not invested.

The purpose of these papers is to test the concepts of fairness which are supported by the experimental evidence.[13] Both Gantner et al. (2001) and Cappelen et al. (2007) show that subjects choose the efficient production in most cases and that, although several concepts of fairness are observed, the best predictor for the bargaining phase is equal shares.[14]

Although there are several important differences between our own models and these aforementioned papers[15], we believe our results do not overtly contrast with the quoted experimental evidence. A point of agreement lies in the importance of equal sharing. In addition this experimental evidence gives some weight to the idea of accountability, i.e. on the notion that a fair distribution of joint output demands that this be allocated in proportion with individual contribution. Accountability also receives some weight in our model, albeit in a loose form. This occurs in region HH_{ne} of Γ_{UG} where in order to give to the less efficient agent the incentive to invest, he must be granted a larger share of the surplus. If investment cost is measured by effort, the condition $c_A > c_B$, coupled with the fact that (in our case) marginal productivity of investment are identical for the two agents, means that in order to provide the same ad-

[13] For both Gantner et al. (2001) and Cappelen et al. (2007), the excercise boils down to find which of the different fairness concepts a priori postulated receives suppport from the experimental results, given the possible distributional situations faced. The distributional situation, in turns, depends on the distribution of marginal productivities (high and low) and of investment decisions (identical or different investment). Cappelen et al. (2007) assume the existence of three "types" of individuals in the population, each with a different idea of fair distribution: the strict egalitarian (who considers fair a distribution in which each receives an equal share of the gross social product), the libertarian (who considers fair a distribution in which each receives an amount equal to her contribution) and the liberal egalitarian (who considers fair a distribution in which each receives a share proportional to her contribution). With some variants, the same list is also studied by Gantner et al. (2001).

[14] This holds true for both games considered by Gantner et al. (2001) for the second stage of the game. The estimates provided by Cappelen et al. (2007) show that 43.5% of subjects are egalitarian, 38.1% are liberal egalitarian and 18.4% are libertarian. Using the same data as Cappelen et al. (2007) but a different econometric technique, Conte and Moffatt (2009) claim that strict egalitarianism is chosen by 51.6% of participants, liberal egalitarianism by 46% and libertarianism by 2.4% only.

[15] In particular, while in our case agents can choose either to invest or not, in Gantner et al. (2001) and Cappelen et al. (2007) the investment set includes more options. Moreover, in our case the marginal productivities of investment are the same across agents and heterogeneity is entirely due to different investment costs; in the quoted papers instead heterogeneity is due to marginal productivities of the investment and investment costs are the same only if agents choose the same investment levels. We also suggest that the bargaining protocol should be sensitive to the observed investment profile; this, coupled with the consideration of two different structures of socio-political organizations, lead us to propose two different games. By contrast, in Gantner et al. (2001) and Cappelen et al. (2007) the same bargaining protocol applies for wathever observed investment profile.

ditional output agent A has to provide more effort than agent B. This more effort is rewarded by a larger share. One may interpret this reward as stemming from responsibility considerations; however, since responsibility does not enter explicitly into our model, we prefer to explain the larger share received by the less efficient agent in Rawlsian terms, i.e. as representing the condition needed to ensure to the more efficient and advantaged B the cooperation of the less efficient and disadvantaged A. Nevertheless this interpretation leads exactly to a form of accountability, as expressed by Equity Theory (Gantner et al., 2001), for the specific parameters configuration giving rise to the distribution specified by (13), i.e. along the line CR of Figure 1. However our results suggest that this concept of fairness can only evolve under quite strict conditions.

7 Appendix

First of all we introduce some notations. We denote by $\lfloor s \rceil$ the least integer number greater than s when s is not an integer and $(s+1)$ is otherwise. Consider a state θ and suppose that all agents observe $z(\theta)$. Consider Γ_{UG}; for an agent $i \in A$, action L is not preferred to action H if

$$p_B(\theta)\left(\widetilde{y}^i_{HH}(\theta) - \widetilde{y}^i_{LH}(\theta) - c_A\right) + (1 - p_B(\theta))\left(\widetilde{y}^i_{HL}(\theta) - c_A\right) \geq 0;$$

for an agent $i \in B$ action L is not preferred to action H if

$$p_A(\theta)\left(\widetilde{x}^i_{HH}(\theta) - \widetilde{x}^i_{HL}(\theta) - c_B\right) + (1 - p_A(\theta))\left(\widetilde{x}^i_{LH}(\theta) - c_B\right) \geq 0.$$

Here we denote by $p_A(\theta)$ (resp. $p_B(\theta)$) the frequency of agent A (resp. B) who played H in θ, and by $\widetilde{y}^i_{HH}(\theta)$ (resp. $\widetilde{x}^i_{HH}(\theta)$) the expected payoffs of agent $i \in A$ (resp. $i \in B$) at the information set HH, given $z(\theta)$. Similar conditions hold for Γ_{DG}. The following result will be used afterwards.

Lemma 6 *Let $x_{HH,1} < x_{HH,2} < ... < x_{HH,k}$ be the demands made by B at HH for some state θ. Then the set of best behavioral demands following HH for agents A is a subset of $\{V_H - x_{HH,l}\}_{l=1}^{k}$.*

Proof. See Lemma A.1 in Ellingsen and Robles (2002)).

Lemma 7 *Consider Γ_{UG} and let Ω be a limit set of (Θ, P). If $(HL, y_{HL}, x_{HL}) \in \rho(\Omega)$ [resp. $(LH, y_{LH}, x_{LH}) \in \rho(\Omega)$] then:*
(i) $x_{HL} = V_M - y_{HL}$ [resp. $y_{LH} = V_M - x_{LH}$];
(ii) (HL, y_{HL}, x_{HL}) [resp. (LH, y_{LH}, x_{LH})] is the only outcome which supports investment profile HL [resp. LH] in $\rho(\Omega)$.

Proof. We only consider profile HL; the same holds true for LH.

Point (i). Let θ be a state such that: (a) $\theta \in \Omega$; (b) (HL, y_{HL}, x_{HL}) belongs to the support of $z(\theta)$ and $x_{HL} \neq V_M - y_{HL}$. Let us suppose that only B agents update their characteristics: they will all accept y_{HL}. For whatever belief on opponents' behavior, this action is always a best reply. It is then impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$.

Point (ii). First we show that Ω cannot include a state θ in which multiple demands are made at HL. Subsequently, we show that Ω cannot include two different states supporting different outcomes following HL.

Let θ be a state such that: (a) $\theta \in \Omega$ and (b) multiple demands are made by agents A at HL. We already know from point (i) that at θ agents B accept all the demands made by their opponents. Suppose only agents A revise their characteristics; then any agent A will make the maximum demand observed at HL. Hence, it is impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$.

Now let θ and θ' be two states such that: (a) both states belong to Ω and (b) HL is observed. A single demand is made by A but $y_{HL}(\theta') > y_{HL}(\theta)$. Since it is impossible to return to θ then assumption $\theta \in \Omega$ is contradicted. ■

Lemma 8 *Let Ω be a limit set of (Θ, P). If $\{(HH, y, x), (HH, y', x')\} \in \rho(\Omega)$ and either $x \neq x'$ or $y \neq y$, then Ω is a singleton and a self-confirming equilibrium.*

Proof. Consider a set Ω and let $\theta \in \Omega$ be a state in which at least two demands have been observed in one population (i.e. B). Suppose that at least one of these demands (x^*) is not a best reply to $z(\theta)$. Suppose also that, after observing $z(\theta)$, all agents who demanded x^* revise; as a consequence x^* disappears. A new state $\overline{\theta} \in \Omega$ is then reached in which profile HH is still observed. Suppose now that all A update; then, by Lemma 6, nobody will make demand $\{V_H - x^*\}$. These two demands have thus disappeared and it is impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$. Therefore, if multiple demands are made, each must be a best reply to $z(\theta)$.

Now consider an agent belonging to population A who has played H in θ and suppose this agent has the incentive to change her investment should she know $z(\theta)$. When this agent updates, the distribution of the demands made by population A in subgame HH differs from the original. This implies that at least one demand made by some opponents (i.e. B) is no longer a best reply when B updates. By applying the argument made in the above paragraph, we conclude that at least one pair of demands has disappeared and cannot reappear. This contradicts the assumption that $\theta \in \Omega$.

By Lemma 7, since the set $\rho(\Omega)$ can include at most one outcome following the profile HL or LH, then state θ must be a self-confirming equilibrium. ■

Proof of Proposition 2. Assume that Ω is not a singleton. We know from Lemmas 7 and 8 that, if a bargaining subgame is reached, only one of its terminal nodes is observed.

First we show that $\rho(\Omega)$ must contain one outcome for every bargaining subgame. Of course $\rho(\Omega)$ must differ from $\{(HH, y_{HH}, V_H - y_{HH}), (LL, 0, 0)\}$. Suppose $\rho(\Omega)$ includes the following outcomes: (a) (HH, y_{HH}, x_{HH}) with $y_{HH} + x_{HH} = V_H$; (b) (HL, y_{HL}, x_{HL}) with $y_{HL} + x_{HL} = V_M$. In Ω a state θ in which both outcomes are observed must exist and it cannot be an equilibrium. We show that, from θ, it is possible to reach either the basin of attraction of one equilibrium or a state in which all bargaining nodes are observed. Suppose some agents B update; if $x_{HH} - c_B > x_{HL}$ they will choose H so that, at the new state θ', the frequency of this action in population B will increase.

Suppose now that at least one agent A has beliefs $\widetilde{y}^i_{LH}$ leading her not to prefer H to L when all agents B play H; then, starting from θ, it is possible to reach a state in which all investment profiles are realized. To see this suppose $y_{HH} - \widetilde{y}^{i^*}_{LH} - c_A < 0$ but $y_{HL} - c_A > 0$. Let $p^{i^*}_B$ be:

$$p^{i^*}_B = \left\lfloor \frac{c_A - y_{HL}}{\left(y_{HH} - \widetilde{y}^{i^*}_{LH} - c_A\right)} \right\rceil.$$

Consider now the case in which $p^{i^*}_B$ agents B have revised at θ and only agent $i^* \in A$ observes the distribution of outcomes $z(\theta')$. Since the specific agent $i^* \in$

A will play L, then all investment profiles are realized afterwards. Therefore this contradicts the assumption that $\rho(\Omega) = \{(HH, y_{HH}, x_{HH}), (HL, y_{HL}, x_{HL})\}$. Otherwise, by letting all agents B to update, from θ it is likely to reach the basin of attraction of one equilibrium of the game supporting the outcome (HH, y_{HH}, x_{HH}). If $x_{HH} - c_B \leq x_{HL}$ the same conclusion obtains by a similar argument. It is simple to see that the same conclusion holds when $\rho(\Omega)$ includes any two different outcomes. Therefore if Ω is not a singleton, all the bargaining nodes are visited meaning that $\rho(\Omega)$ includes four outcomes, each of which is a subgame equilibrium.

We now have to show that the payoffs must satisfy a well-defined set of constraints. Notice that a state $\theta \in \Omega$ in which all the investment profiles are observed must exist. Moreover when we allow all agents to update, all agents A will choose H if

$$p_B(\theta)(y_{HH} - y_{LH} - c_A) + (1 - p_B(\theta))(y_{HL} - c_A) > 0,$$

and all agents B will choose H if

$$p_A(\theta)(x_{HH} - x_{HL} - c_B) + (1 - p_A(\theta))(x_{LH} - c_B) > 0.$$

We can rewrite these conditions as

$$\begin{aligned} p_B(\theta) A_1 + (1 - p_B(\theta)) A_2 &> 0 \\ p_A(\theta) B_1 + (1 - p_A(\theta)) B_2 &> 0. \end{aligned}$$

First of all notice that all A_i and B_i can not be null since this would imply that θ is an equilibrium and Ω a singleton. Furthermore, when — for some population — both expressions are either not negative or not positive, and at least one is not null, then the process can reach a new state which is a self-confirming equilibrium.

Consider the case in which both expressions are null for population A only. When B_1 is strictly positive and B_2 is strictly negative all Bs prefer H if $p_A(\theta) > p_A^*$ where:

$$p_A^* = \frac{c_B - x_{LH}}{x_{HH} - x_{HL} - x_{LH}}. \tag{14}$$

Otherwise when B_1 is strictly negative and B_2 is strictly positive all Bs prefer H if $p_A(\theta) < p_A^*$. In both cases, when all B agents update they will choose H. Hence a state which is an equilibrium of the game can be reached.

When both expressions B_1 and B_2 are null, we get a similar conclusion where the threshold value of $p_B(\theta)$ is:

$$p_B^* = \frac{c_A - y_{HL}}{y_{HH} - y_{LH} - y_{HL}}. \tag{15}$$

We are left with the case in which the product of the corresponding two expressions is strictly negative for each population. However, when A_1 and B_1

have the same sign, a similar argument allows us to reach the same conclusion. Indeed, suppose that both A_1 and B_1 are strictly positive. This implies that all Bs prefer H if $p_A(\theta) > p_A^*$ and all As prefer H if $p_B(\theta) > p_B^*$. Hence, for whatever values of $p_A(\theta)$ and $p_B(\theta)$, starting from θ the process can reach an equilibrium when one population revises at a time. The remaining possible case occurs when $B_1B_2 < 0$ and $A_1A_2 < 0$ but $A_1B_1 < 0$.■

Proof of Proposition 3. It follows by applying the same arguments used in the proof of Proposition 2 above and taking into account that $y_{HL} = x_{LH} = V_M - \delta$ holds at any limit set. In this case the conditions $B_1B_2 < 0$, $A_1A_2 < 0$ and $A_1B_1 < 0$ can not be simultaneously met because both A_2 and B_2 are strictly positive. Consequently all limit sets are singleton. In addition an equilibrium of the game can only support outcome $(LL, 0, 0)$ if it also supports at least one outcome following each investment profile. ■

Several intermediate results are needed before turning to the proof of Propositions 4 and 5. From now on by slightly abusing notation, by (HH, y_{HH}, x_{HH}) we denote a terminal node in which both agents have chosen H, agent A makes a demand y_{HH} and agent B makes a demand x_{HH}. This applies for the other terminal nodes, too.

Definition 9 *Consider a union of limit sets X. This set is mutation connected if for all pairs Ω, $\Omega' \in X$ exists a sequence of limit sets $(\Omega_1 = \Omega, \Omega_2, ..., \Omega_n = \Omega')$ such that (a) for any $k \in \{1, ..., n-1\}$, $\Omega_k \in X$ and (b) every transition from Ω_k to Ω_{k+1} needs no more than one mutation.*

Consider a limit set Ω which does not support all information sets and suppose a single mutation occurs. If this mutation is a drift then the process reaches a new limit set Ω' which differs from Ω only for some beliefs and/or actions at some unreached information sets. Let $\Sigma(\Omega)$ be the set of equilibria which only differ from Ω for some beliefs (and/or actions) held in some unreached information set. Sure enough the set $\Sigma(\Omega)$ is mutation connected. When Ω is singleton, namely $\Omega = \{\theta\}$, we use $\Sigma(\theta)$ instead of $\Sigma(\Omega)$.

The next Lemma states our first preliminary result which holds true for both extensive games considered.

Lemma 10 *Consider a limit set Ω such that $\rho(\Omega)$ is not a singleton. An equilibrium supporting one outcome only can be reached from Ω by a sequence of single-mutation transitions.*

Proof. We give the detailed proof for Γ_{UG}; we then suggest how to adapt it to Γ_{DG}. When multiple demands are observed at HH, we denote by $\{x_{HH,l}(\theta)\}_{l=1}^{k}$ the ordered sets of demands made by B and by $\{y_{HH,l}(\theta)\}_{l=1}^{k}$ the ordered sets of demands made by A. By iterative applications of Lemma 6 we get $\{y_{HH,l}\}_{l=1}^{k} = \{V_H - x_{HH,l}\}_{l=k}^{1}$. Since θ is an equilibrium, the expected payoffs at HH are

$$\widetilde{y}^{i}_{HH}(\theta) = y_{HH,1} = V_H - x_{HH,k};\ \forall i \in A$$

$$\widetilde{x}^{i}_{HH}(\theta) = x_{HH,1} = V_H - y_{HH,k};\ \forall i \in B$$

where $y_{HH,1} = y_{HH,k}\eta_1^B$ and $x_{HH,1} = x_{HH,k}\eta_1^A$ and where η_1^B (resp. η_1^A) is the fraction of B (resp. A) who claim $x_{HH,1}$(resp. $y_{HH,1}$) under θ.

I) Consider an equilibrium θ in which only the investment profile HH is observed and multiple demands are made. Let a single agent B switch from $x_{HH,k}(\theta)$ to $x_{HH,1}(\theta)$. When agents A update they will make a demand $y_{HH,k}(\theta) = V_H - x_{HH,1}(\theta)$. Hence, we reach a new equilibrium θ' in which only HH is observed and only the two demands $(V_H - x_{HH,1}(\theta), x_{HH,1}(\theta))$ occur.

II) Suppose now that two investment profiles are observed at the equilibrium θ. We give the proof only when both HH and HL are observed. The other remaining cases are similar.

II.1) Consider the case in which multiple demands are made following HH. Since θ is an equilibrium, the following conditions must always be met:

$$p_B(\theta)\left(y_{HH,1} - \widetilde{y}^{i}_{LH}(\theta) - c_A\right) + (1 - p_B(\theta))(y_{HL} - c_A) \geq 0,\ \forall i \in A$$

$$(V_H - y_{HH,k}) - c_B = V_M - y_{HL},\ \forall i \in B.$$

Consider an equilibrium $\theta_1 \in \Sigma(\theta)$. When $y_{HH,1} - c_A > \delta$, by a sequence of single mutations the population can get from θ to $\theta_1 \in \Sigma(\theta)$ where $y_{HH,1} - \widetilde{y}^{i}_{LH}(\theta_1) - c_A > 0$ for all As. At θ_1 let a single agent A mutate from $y_{HH,k}(\theta_1)$ to $y_{HH,1}(\theta_1)$ and let all agents B revise; as a consequence they will all choose H and ask $V_H - y_{HH,1}$. Therefore, the process reaches a new equilibrium θ' where $\rho(\theta') = \{HH, y_{HH,1}(\theta), V_H - y_{HH,1}(\theta)\}$. When instead $y_{HH,1} - c_A \leq \delta$, the inequality $y_{HL} - c_A \geq 0$ must hold for all A. Suppose a single A mutates from $y_{HH,k}(\theta_1)$ to $\overline{y}$ where $\overline{y} > y_{HH,k}(\theta_1)$ and let all agents B update: as a consequence they will all choose L. Therefore, the process reaches a new equilibrium θ' where $\rho(\theta') = \{HL, y_{HL}, V_M - y_{HL}\}$.

II.2) Consider now the case in which a single demand is made following HH. Suppose $y_{HL} - c_A \geq 0$; the process can reach a new equilibrium θ' where $\rho(\theta') = \{HL, y_{HL}, V_M - y_{HL}\}$ when a single agent A mutates from $y_{HH,1}(\theta_1)$ to $\overline{y}$ – with $\overline{y} > y_{HH,1}$ – and all B revise. Suppose instead $y_{HL} - c_A < 0$; then: (a) Point (c) of Assumption 1 implies that the subgame $(HL, V_M - \delta)$ at θ is not reached; (b) $y_{HH,1} - \widetilde{y}^{i}_{LH}(\theta_1) - c_A \geq 0$ for every A. By drifting, all agents B are led to accept the maximum feasible demand made by A in HL so that a new equilibrium θ_1 is reached. Sure enough, $\theta_1 \in \Sigma(\theta)$. Suppose now a single agent A changes her demand from y_{HL} to $(V_M - \delta)$. When all agents A update, they observe that all B have accepted the demand $(V_M - \delta)$; therefore, in HL

their best response is $y_{HL} = V_M - \delta$. When all agents B update, they will choose H being $x_{HL} = \delta$. Hence, the process reaches an equilibrium θ' where $\rho(\theta') = \{HH, y_{HH,1}, V_H - y_{H,1}\}$.

III) Suppose now that all investment profiles are observed at θ. Since θ is an equilibrium, the following conditions must be satisfied:

$$p_B(\theta)(y_{HH,1} - y_{LH} - c_A) + (1 - p_B(\theta))(y_{HL} - c_A) = 0$$

$$p_A(\theta)(x_{HH,1} - x_{HL} - c_B) + (1 - p_A(\theta))(x_{LH} - c_B) = 0.$$

where $y_{HH,1} = V_H - x_{HH,k}$, $y_{HL} = V_M - x_{HL}$ and $y_{LH} = V_M - x_{LH}$. We may rewrite these conditions as

$$\begin{aligned} p_B(\theta) A_1' + (1 - p_B(\theta)) A_2' &= 0 \\ p_A(\theta) B_1' + (1 - p_A(\theta)) B_2' &= 0. \end{aligned}$$

We argue that when the second expression (A_2' or B_2') is not positive for at least one population then the process, through a sequence of single mutations, can reach an equilibrium supporting a smaller number of investment profiles. In order to see this suppose, for instance, $A_2' < 0$. In this case Point (c) of Assumption 1 assures that the subgame $(HL, V_M - \delta)$ is not reached at θ. A drift can lead all agents B to accept the opponent's maximum feasible demand at HL. A new $\theta_1 \in \Sigma(\theta)$ is then reached. Suppose now that at this new equilibrium a single agent A mutates her demand from y_{HL} to $V_M - \delta$. When all agents A revise, they will play H and will make a demand $y_{HL} = V_M - \delta$. Let all agents B update. Since each agent B knows that $x_{HL} = \delta$ and that all A have played H, then her best reply depends on the sign of $(x_{HH,1} - \delta - c)$. However, it is simple to see that whatever the value of $x_{HH,1} - \delta - c$ is, the process can reach a new equilibrium in which a smaller number of investment profiles is realized. If, at this new equilibrium, two investment profiles are realized, then the process can reach an equilibrium which supports a single outcome by a further sequence of single transitions (see point II.2 above).

When both A_2' and B_2' are positive, a single mutation occurring in population A is enough to move the process from θ to a new equilibrium θ' where $\rho(\theta') = \{LH, y_{LH}, V_M - y_{LH}\}$. The mutation needed depends on how many demands are observed at HH. In particular:

(i) when multiple demands are made at HH, one mutation from $y_{HH,k}(\theta)$ to $y_{HH,1}$ is enough;

(ii) when only one demand is made at HH, one mutation from H to L is enough.

IV) The remaining case occurs when Ω is not a singleton. Under Point (c) of Assumption 1, at least one of the following two subgames $(LH, V_M - \delta)$ and $(HL, V_M - \delta)$ is never reached. The same argument used above implies that the population can get from Ω to θ' through a sequence of single-mutations.

Let us consider now game Γ_{DG}. The above-mentioned arguments continue to work with minor modifications. In particular notice that, since $y_{HL} = x_{LH} = V_M - \delta$, then: (a) in case II) the set of investment profiles supported by an equilibrium can be either $\{HH, HL\}$ or $\{HH, LH\}$; (b) in case III) both A_2' and B_2' are positive; (c) case IV) does not arise. ■

Lemma 10 tells us that if a limit set underpins several outcomes, then the process can reach an equilibrium sustaining only one outcome by a sequence of single mutations. We now turn our attention to the set of equilibria supporting one outcome only. According to the investment profile observed, we can partition this set of equilibria into four subsets denoted respectively Σ_H, Σ_L, Σ_{HL} and Σ_{LH}. Of course, Σ_H includes all the equilibria supporting the outcome $\{HH, V_H - x_{HH}, x_{HH}\}$ where $x_{HH} \in D_\delta(V_H)$. The same applies for the other subsets. The following Lemma highlights that in both games the process can move from any single-outcome equilibrium outside Σ_H to a new equilibrium $\theta \in \Sigma_H$ through a sequence of single-mutation transitions.

Lemma 11 *Consider an equilibrium θ; then:*

(a) if $\theta \in \Sigma_L$ an equilibrium $\theta' \in \Sigma_H$ can be reached from θ by a sequence of single-mutation transitions provided that $c_B + \delta < x_{HH} < V_H - c_A - \delta$;

(b) if $\theta \in \Sigma_{HL}$ (resp. Σ_{LH}) an equilibrium $\theta' \in \Sigma_H$ can be reached from θ by a sequence of of single-mutation transitions provided that $c_B + \delta < x_{HH} < V_H - c_A - \delta$.

Proof. Since in Γ_{DG} the set Σ_L is empty, the first point of the Lemma holds for Γ_{UG} only**.**

(a) Let θ be an equilibrium belonging to Σ_L. From θ, by a sequence of single mutations, the process reaches a new equilibrium $\theta^* \in \Sigma(\theta)$ in which, for every agent A and B, it is true that: (i) $\widetilde{y}^i_{HH}(\theta^*) = V_H - x_{HH}$ and $V_H - x_{HH} > c_A + \delta$; (ii) at subgame $(LH, V_M - \delta)$ each agent A accepts (i.e. she chooses δ); (iii) $\widetilde{x}^i_{HL}(\theta^*) = \delta$, $\widetilde{x}^i_{HH}(\theta^*) = x_{HH}$ and $x_{HH} - \delta - c_B \geq 0$. Suppose now an agent B mutates by playing H and making a demand $V_M - \delta$ in LH. When agents B update they will choose H since all agents A have accepted $V_M - \delta$. When agents A revise they will all play H since $V_H - x_{HH} > c_A + \delta$. Hence the process reaches a new equilibrium $\theta' \in \Sigma_H$ where $\rho(\theta') = \{HH, (V_H - x_{HH}), x_{HH}\}$.

(b) Consider Γ_{UG} and let θ be an equilibrium belonging to Σ_{HL}. At θ the pair of demands $(y_{HL}, V_M - y_{HL})$ is observed. Suppose $y_{HL} < V_M - \delta$. By drifting, all agents B are led to accept the maximum feasible demand made by A in HL and deem that all A make a demand larger than $V_H - c_B + \delta$ at HH. A new equilibrium $\theta_1 \in \Sigma(\theta)$ is thus reached. Suppose now a single agent A changes her demand from y_{HL} to $V_M - \delta$. When agents A update, they observe that all Bs have accepted $V_M - \delta$; therefore in HL their best response is $y_{HL} = V_M - \delta$. When agents B update they continue to play L since $\widetilde{x}^i_{HH}(\theta_1) < c_B + \delta$ holds for all Bs. Hence, the process reaches a new equilibrium $\theta' \in \Sigma_{HL}$ where $\rho(\theta') = \{HL, (V_M - \delta), \delta\}$. From θ', by a sequence of single mutations, the process can reach an equilibrium $\theta^* \in \Sigma(\theta')$ in which all agents A have beliefs:

(i) $\widetilde{y}^i_{HH}(\theta^*) = y_{HH}$; (ii) $\widetilde{y}^i_{LH}(\theta^*) = \delta$; (iii) $\delta + c_A < y_{HH} < V_H - c_B - \delta$. Suppose now an agent B mutates by playing H and making a demand $V_H - y_{HH}$ in HH. Let all agents B revise; they will choose H and ask $V_H - y_{HH}$. When agents A update, the process reaches a new equilibrium $\theta' \in \Sigma_H$ in which the pair of demands is $(y_{HH}, V_M - y_{HH})$. Of course only the last sequence of mutations is required when $y_{HL} = V_M - \delta$.

The case in which θ is an equilibrium belonging to Σ_{LH} is similar. Moreover the same argument holds true also for Γ_{DG} with the caveat that any equilibrium belonging to Σ_{HL} now supports the outcome $(V_M - \delta, \delta)$ only. ■

Lemma 12 *Consider Γ_{UG}. Let $\theta \in \Sigma_L$ be an equilibrium. An equilibrium $\theta' \in \Sigma_{LH} \cup \Sigma_{HL}$ can be reached from θ by a sequence of single-mutation transitions provided that at θ' the agent who has chosen H is better off;*

Proof. Let $\theta \in \Sigma_L$ be an equilibrium. From θ, by a sequence of single mutations, the process can reach a new equilibrium $\theta^* \in \Sigma(\theta)$ in which: at subgame $(LH, V_M - y_{LH})$ each agent A accepts (i.e. she chooses y_{LH}); for every A, $\widetilde{y}^i_{HH}(\theta^*) - y_{LH} - c_A < 0$ but $V_M - y_{LH} - c_B > 0$ for every B. Suppose an agent B mutates by playing H and making a demand $V_M - y_{LH}$ in LH. When all agents B update, they will choose H since population A has accepted demand $V_M - y_{LH}$. When agents A revise they will continue to play L since $\widetilde{y}^i_{HH}(\theta^*) - y_{LH} - c_A < 0$. Hence the process reaches a new equilibrium $\theta' \in \Sigma_{LH}$ in which the pair of demands is $(y_{LH}, V_M - y_{LH})$. The case in which θ' is an equilibrium belonging to Σ_{HL} is similar. ■

Lemma 10 and Lemma 11 together asset that, in both games, the adaptive process can lead to an equilibrium $\theta \in \Sigma_H$ by a sequence of single mutations, starting from any limit set $\Omega \notin \Sigma_H$. Therefore, according to Proposition 1 of Noldeke and Samuelson (1993), if Σ_S is a strictly subset of Σ then $\Sigma_S \subseteq \Sigma_H$.

Both Proposition 4 and Proposition 5 in the main text stem from a direct application of Theorem 2 of Ellison (2000) which we now briefly recall. Let Σ be a union of limit sets; these sets can be either mutation connected or not. The Radius $R(\Sigma)$ is the minimum number of mutations needed to escape from the basin of attraction of Σ and enter into another one with positive probability. Consider an arbitrary state $\theta \notin \Sigma$ and let $(m_1, m_2, .., m_T)$ be a path from θ to Σ where $\Omega_1, \Omega_2, ..\Omega_r$ is the sequence of limit sets through which the path passes consecutively. Obviously $\Omega_i \notin \Sigma$ for $i < r$ and $\Omega_r \subset \Sigma$. Furthermore, notice that a limit set can appear several times in this sequence but not consecutively. The modified cost of this path is defined by:

$$c^*(m_1, ..., m_T) = c(m_1, .., m_T) - \sum_{i=2}^{r-1} R(\Omega_i)$$

where $c(m_1,..,m_T)$ is the total number of mutations over the path $(\theta, m_1, m_2, .., m_T)$. Let $c^*(\theta, \Sigma)$ be the minimal modified cost among all paths from θ to Σ. The Modified Coradius of the basin of attraction of Σ is then:

$$CR^*(\Sigma) = \max_{\theta \notin \Sigma} c^*(\theta, \Sigma).$$

Theorem 2 of Ellison (2000) shows that every union of limit sets Σ with $R(\Sigma) > CR^*(\Sigma)$ encompasses all the stochastically stable states. In order to compute the minimum number of mutations needed to escape from an equilibrium belonging to Σ_H, both Propositions 4 and 5 make use of the result stated in Lemma 13 below. In what follows we write θ_x as shorthand for an equilibrium belonging to Σ_H with $(V_H - x, x)$ as the distributional rule.

Lemma 13 *For δ sufficiently small, the minimum number of mutations needed to get from $\Sigma(\theta_x)$ to an equilibrium with the same investment profile but different demands is:*

$$\begin{array}{rclcl} r_B^+(x) & = & \left\lfloor N\left(\frac{\delta}{V_H - x}\right)\right\rceil & \text{if} & x < \frac{V_H}{2} \\ r_A^-(x) & = & \left\lfloor N\left(\frac{\delta}{x}\right)\right\rceil & \text{if} & x > \frac{V_H}{2} \end{array} \quad (16)$$

where $r_B^+(x)$ is the number of mutations needed for the transition from θ_x to $\theta_{x+\delta}$ whereas $r_A^-(x)$ is the number of mutations needed for the transition from θ_x to $\theta_{x-\delta}$. Moreover, $r_B^+(x)$ is a strictly increasing function of x and $r_A^-(x)$ is a strictly decreasing function of x.

Proof: By a direct application of Young (1993). ■

Before giving the proof of Proposition 4 two further preliminary results are needed. These are provided by Lemma 14 and Lemma 15 below. The first allows us to argue that a norm of cooperation supporting the efficient investment profile evolves in the long run when condition (7) holds. Given this, the second result allows us to argue that norms of distribution arise when the stochastically stable outcome support the efficient investment profile.

Lemma 14 *Consider Γ_{UG}. Then:*

(a) an equilibrium $\theta' \in \Sigma_L$ can be reached from $\theta \in \Sigma_{CH}^U$ by a sequence of single-mutation transitions;

(b) under condition (7), $CR^*\left(\Sigma_{IH}^U\right) = 1$;

(c) under condition (7), $R\left(\Sigma_{IH}^U\right) > 1$ and, consequently, $\Sigma_S \subseteq \Sigma_{IH}^U$.

Proof. Point (a). Consider some $\theta \in \Sigma_{CH}^U$ and let $\{V_H - x, x\}$ be the observed pair of demands. We show that starting from Σ_{CH}^U it may be possible to enter into the basin of attraction of an equilibrium $\theta' \in \Sigma_L$ through a sequence of single-mutation transitions. In order to describe this transition four cases have to be taken into account: (1) $x > x_A^U$; (2) $x = x_A^U$; (3) $x < x_B^U$; (4) $x = x_B^U$. We give the proof for cases (1) and (2) only; the remaining cases are symmetric.

Case (1): let $x > x_A^U$. At θ the following inequality must hold:

$$\begin{array}{ll} \text{Population } A: & \text{Population } B: \\ \left(V_H - x - c_A\right) - \widetilde{y}^i_{LH}(\theta) \geq 0 & x - c_B - \widetilde{x}^i_{HL}(\theta) \geq 0 \\ V_H - x - c_A < V_M - \delta & x - c_B > V_M - \delta. \end{array} \tag{17}$$

From θ the process can reach a new equilibrium $\theta_1 \in \Sigma(\theta)$ by a sequence of single mutations in which the following is true for every agent: (i) $\widetilde{x}^i_{LH}(\theta_1) = \delta$ and (ii) $\widetilde{y}^i_{HL}(\theta_1) - c_A < 0$. Suppose an agent A mutates by playing L and accepting her opponent's demand at LH. Let all agents A update. Since the mutant receives $V_M - \delta$, all As imitate and play L. When agents B revise they will play L. The process then reaches a new equilibrium $\theta' \in \Sigma_L$.

Case (2): let $x = x_A^U$. At θ, for any agent A it must be true that $V_H - x_A^U - c_A = V_M - \delta$. From θ the process can reach a new equilibrium $\theta_1 \in \Sigma(\theta)$ by a sequence of single mutations in which the following is true for every agent: (i) $\widetilde{x}^i_{LH}(\theta_1) = x_{LH}$; (ii) $x_{LH} - c_B < 0$; (iii) $\widetilde{y}^i_{LH}(\theta_1) = (V_M - \delta)$ and (iv) $\widetilde{y}^i_{HL}(\theta_1) - c_A < 0$. Suppose an agent B mutates by demanding $x' > x_A^U$ at HH. When agents A update they will all choose L since, for whatever best action at HH, the expected payoff by playing H is now smaller than $V_M - \delta$. When all agents B revise they will play L . The process then reaches a new equilibrium $\theta' \in \Sigma_L$.

Point (b). Under condition (7) Σ^U_{IH} is well defined. By a direct application of previous point (b), along with point (a) of Lemma 11, it follows that from $\theta \in \Sigma^U_{CH}$ it is possible to reach $\widehat{\theta} \in \Sigma^U_{IH}$ through a sequence of single mutations. Therefore $CR^*\left(\Sigma^U_{IH}\right) = 1$ for any $\theta \in \Sigma^U_{CH}$.

Besides, from Lemmas 10 and 11, we can deduce that for any $\Omega \notin \Sigma_H$ the minimal modified cost for all paths from θ to Σ_{IH}, is equal to one, whatever the number of limit sets the path goes through may be. Therefore, by putting together these results, we get:

$$CR^*\left(\Sigma^U_{IH}\right) = \max_{\Omega \notin \Sigma^U_{IH}} c^*\left(\Omega, \Sigma^U_{IH}\right) = 1$$

Point (c). We show that, under condition (7), more than one mutation is needed to leave the basin of attraction of Σ^U_{IH} even when the worst-case equilibrium scenario is considered as starting state.

I) First we show that a single mutation from H to L does not enable the process to leave the set Σ^U_{IH} even if at $\theta_x \in \Sigma^U_{IH}$ each agent expects to receive: (i) the maximum payoff when she plays L but the opponent still plays H; (ii) the minimum payoff when she plays H but the opponent shifts to L.

Let us consider this scenario and suppose that an agent B had switched from H to L and all agents A revised. This updating does not cause agents A to play L if

$$\frac{N-1}{N}\left[(V_H - x - c_A) - (V_M - \delta)\right] + \frac{1}{N}(\delta - c_A) > 0 \tag{18}$$

which can be rewritten as

$$\frac{N-1}{N}(V_H - x) - c_A > \frac{N-1}{N} V_M - \delta. \tag{19}$$

Since $x \in \left[\widehat{x}_B^U, \widehat{x}_A^U\right]$, condition (19) holds by definition. Therefore no agent A will change her action after the revision. Similar argument can be applied to population B. Since as soon as the mutant revises the process returns to Σ_{IH}^U, then a single mutation from H to L is not enough to leave the basin of attraction of Σ_{IH}^U.

II) We now show that a single mutation from x to x' (resp. $V_H - x$ to y') does not enable the process to run away from the basin of attraction of Σ_{IH}^U even if at $\theta_x \in \Sigma_{IH}^U$ each agent expects to get the maximum payoff when she plays L and the opponent chooses H. Let one agent B only change her demand to x'. Obviously, no agents B imitate the mutant when revising. When As update we know from Lemma (6) that their best response is either $V_H - x$ or $V_H - x'$.

If $x' > x$, agent A expects to receive $(V_H - x)\frac{N-1}{N} - c_A$ when she demands $V_H - x$ and $V_H - x' - c_A$ when she demands $V_H - x'$. Under Point (e) of Assumption 1, the former payoff is greater than the latter. Hence agents A will not change their demands when updating. Moreover, since $(V_H - x)\frac{N-1}{N} - c_A \geq V_M - \delta$, then updating will not cause agents A to play action L.

If $x' < x$, agent A expects to get $V_H - x - c_A$ when she demands $V_H - x$ and $\frac{1}{N}(V_H - x') - c_A$ when she demands $V_H - x'$. Under Point (e) of Assumption 1 the former payoff is greater than the latter. Hence, agents A will not change their demands when updating. Moreover, since $V_H - x - c_A > V_M - \delta$, then updating will not cause agents A to play action L. The case in which an agent A mutates from $V_H - x$ to y' is symmetric. Since as soon as the mutant revises the process returns to Σ_{IH}^U, then a single mutation from x to x'(resp. from $V_H - x$ to y') is not enough to escape from the basin of attraction of Σ_{IH}^U.

Points I) and II) taken together say that more than one mutation is needed in order to escape from the basin of attraction of Σ_{IH}^U, i.e. $R\left(\Sigma_{IH}^U\right) > 1$. Given that $CR^*\left(\Sigma_{IH}^U\right) = 1$, by Theorem 2 of Ellison (2000) we get $\Sigma_S \subseteq \Sigma_{IH}^U$. ■

We have now to derive the norms of distribution supporting Σ_{IH}^U. In order to do this we have: (i) to compute the radius of $\Sigma(\theta)$, i.e. the minimum number of mutations required to destabilize the outcome supported by θ, $\forall \theta \in \Sigma_{IH}^U$; (ii) to find an equilibrium belonging to Σ_{IH}^U such that $R(\Sigma(\theta)) > CR^*(\Sigma(\theta))$. Lemma 13 provides the minimum number of mutations required to make a transition from $\theta_x \in \Sigma_{IH}^U$ to another equilibrium supporting the same investment profile HH, but a different distributional rule. Lemma 15 below completes all the required details by giving the minimum number of mutations required to make a transition from $\theta_x \in \Sigma_{IH}^U$ to $\theta' \notin \Sigma_{IH}^U$.

Lemma 15 *Consider Γ_{UG}. The minimum number of mutations required to get from $\theta_x \in \Sigma_{IH}^U$ to an equilibrium which supports a different investment profile is:*

$$\begin{array}{rcll} \overline{r}_A(x) & = & \left\lfloor N\left(1 - \frac{V_M-\delta+c_B}{x}\right)\right\rceil & \text{if} \quad x < \frac{V_H}{2} \\ \overline{r}_B(x) & = & \left\lfloor N\left(1 - \frac{V_M-\delta+c_A}{V_H-x}\right)\right\rceil & \text{if} \quad x > \frac{V_H}{2}. \end{array} \tag{20}$$

Proof. Consider $\theta_x \in \Sigma_{IH}^U$. Suppose p_1 agents B mutate by playing L and p_2 agents B mutate by claiming $x' > x_A^U$. For a given pair (p_1, p_2), agents A have the largest incentive to change into L if their beliefs are such that: (i) they expect to get the maximum payoff in an LH match; (ii) they expect to obtain the minimum payoff in an HL match. Consider equilibrium $\theta_x \in \Sigma_{IH}^U$ in which the following holds for all agents: (i); $\widetilde{y}_{LH}^i = V_M - \delta$ and $\widetilde{y}_{HL}^i = \delta$; (ii) $\widetilde{x}_{LH}^i = \delta$ and in the subgame $\{HL, \delta\}$ all agents B accept. At θ_x, when some agents B mutate and these mutations induce all agents A to play L, the process enters into the basin of attraction of equilibrium $\theta' \in \Sigma_L$ with positive probability. Sure enough, after updating, all agents A decide to play L if

$$\frac{N - p_1}{N}(V_M - \delta) > \mu_H\left(\widetilde{\theta}_x, p_1, p_2\right) \tag{21}$$

where LHS is the expected payoff by playing L and RHS is the expected payoff by playing H. However, $\mu_H(\theta_x, p_1, p_2)$ depends on what the best demand in a match HH is. In particular

$$\mu_H(.) = \begin{cases} \frac{N-p_2-p_1}{N}(V_H - x) + \frac{p_1}{N}\delta - c_A & if \quad \frac{N-p_2-p_1}{N-p_1}(V_H - x) \geq V_H - x' \\ \\ \frac{N-p_1}{N}(V_H - x') + \frac{p_1}{N}\delta - c_A & if \quad \frac{N-p_2-p_1}{N-p_1}(V_H - x) < V_H - x'. \end{cases} \tag{22}$$

The minimum number of mutations in population B comes from the comparison between the solutions of two constraint minimization problems ($M1$ and $M2$). In both problems the objective function is $p_1 + p_2$. In the first (resp. second) problem we contemplate the case in which the best action in HH is $V_H - x'$ (resp. $V_H - x$). Both problems require $p_1 = 0$ as a solution. Moreover $p_2^{M1} = N\left(\frac{x'-x}{V_H-x}\right)$ is the solution to the first problem and $p_2^{M2} = N\left(1 - \frac{V_M-\delta+c_A}{V_H-x}\right)$ is the solution to the second. Since $p_2^{M1} > p_2^{M2}$, the minimum number of mutations in population B involves that: (i) mutating agents only change their demands in the HH profile; (ii) these mutations cause agent A to shift to action L when the best action in HH continues to be $V_H - x$. Hence:

$$\overline{r}_B(x) = \left\lfloor N\left(1 - \frac{V_M - \delta + c_A}{V_H - x}\right)\right\rceil \tag{23}$$

and

$$\overline{r}_B = \min_x \overline{r}_B(x) = \overline{r}_B\left(\widehat{x}_A^U\right). \tag{24}$$

Suppose now some agents A mutate. As before, two kinds of mutations must be considered: p_1 agents A mutate by playing L and p_2 agents A mutate by demanding $V_H - x'$ where $x' < x_B^U$. In this case we look for an equilibrium $\theta_x \in \Sigma_{IH}^U$ in which for all agents: (i) $\widetilde{x}_{LH}^i = \delta$ and $\widetilde{x}_{HL}^i = V_M - \delta$; (ii) $\widetilde{y}_{HL}^i = \delta$ and in the subgame $\{LH, \delta\}$ all agents A accept. It is easy to see that if some mutations of agents A occurs at θ_x and these mutations induce all agents B to play L, then with positive probability the process enters into the basin of attraction of equilibrium $\theta' \in \Sigma_L$. After updating all agents B decide to play L if

$$\frac{N-p_1}{N}(V_M - \delta) > \mu_H\left(\widehat{\theta}_x, p_1, p_2\right) \tag{25}$$

where

$$\mu_H(.) = \begin{cases} \frac{N-p_2-p_1}{N}x + \frac{p_1}{N}\delta - c_B & if \quad \frac{N-p_2-p_1}{N-p_1}x \geq x' \\ \\ \frac{N-p_1}{N}x' + \frac{p_1}{N}\delta - c_B & if \quad \frac{N-p_2-p_1}{N-p_1}x < x'. \end{cases}$$

Proceeding as before, the minimum number of mutations in population A is

$$\overline{r}_A(x) = \left\lfloor N\left(1 - \frac{V_M - \delta + c_B}{x}\right)\right\rfloor \tag{26}$$

and

$$\overline{r}_A = \min_x \overline{r}_A(x) = \overline{r}_A\left(\widehat{x}_B^U\right) \tag{27}$$

By comparing (23) and (26) we obtain $\overline{r}_B(x) < \overline{r}_A(x)$ if $x > \frac{V_H}{2}$. ■

Proof of Proposition 4. Point (a). Consider Γ_{UG}. From Point (c) of Lemma 14 we know that $\Sigma_S \subseteq \Sigma_{IH}^U$ when condition (7) holds. Therefore, we are only left with the task of deriving the distributional norm supporting the equilibria belonging to Σ_{IH}^U.

To detect $R(\Sigma(\theta_x))$ for any $\theta_x \in \Sigma_{IH}^U$ we compare the results coming from Lemma 13 with those coming from Lemma 15. Notice that $r_B^+(x) \leq \overline{r}_B(x)$ if $V_M - \delta \leq V_H - (x+\delta) - c_A$ and $r_A^-(x) \leq \overline{r}_A(x)$ if $V_M - \delta \leq x - \delta - c_B$; since these conditions are always satisfied for any $x \in \left[\widehat{x}_B^U, \widehat{x}_A^U\right]$ we conclude that

$$R(\Sigma(\theta_x)) = \begin{cases} r_B^+(x) & if \quad x < \frac{V_H}{2} \\ \\ r_A^-(x) & if \quad x > \frac{V_H}{2}. \end{cases}$$

In order to derive $CR^*(\Sigma(\theta_x))$, two cases must be considered, both compatible with condition (7).

In the first case $\widehat{x}_A^U \geq \frac{V_H}{2}$; this occurs when $c_A \leq \frac{V_H}{2} - V_M$. Let $\overline{x} \equiv \frac{V_H}{2}$ and consider the set of equilibria $\Sigma(\theta_{\overline{x}})$; let $\theta_x \in \Sigma_{IH}^U$ be an equilibrium with $x \neq \overline{x}$. From Lemma 13 we know that

$$c^*(\theta_x, \Sigma(\theta_{\overline{x}})) = \begin{cases} r_B^+(x) & \text{if} \quad x < \frac{V_H}{2} \\ \\ r_A^-(x) & \text{if} \quad x > \frac{V_H}{2}. \end{cases}$$

By the monotonicity of $r_B^+(x)$ and $r_A^-(x)$ we obtain

$$CR^*(\Sigma(\theta_{\overline{x}})) = \max\left(r_B^+(\overline{x} - \delta), r_A^-(\overline{x} + \delta)\right).$$

Of course, when $x = \overline{x} \equiv \frac{V_H}{2}$, then $R(\Sigma(\theta_{\overline{x}})) = r_B^+(\overline{x}) = r_A^-(\overline{x})$. Since

$$R(\Sigma(\theta_{\overline{x}})) = r_B^+(\overline{x}) = r_A^-(\overline{x}) > CR^*(\Sigma(\theta_{\overline{x}})),$$

it follows from Theorem 2 of Ellison (2000) that $\Sigma_S = \Sigma(\theta_{\overline{x}})$; the only stochastically stable outcome is thus $\left\{HH, \frac{V_H}{2}, \frac{V_H}{2}\right\}$ and the distributional norm is $\left(\frac{V_H}{2}, \frac{V_H}{2}\right)$.

In the second case $\widehat{x}_A^U < \frac{V_H}{2}$; this occurs when $\frac{V_H}{2} - V_M < c_A \leq V_H - 2V_M - c_B$. Let $\widetilde{x} \equiv \widehat{x}_A^U$ and consider the set of equilibria $\Sigma(\theta_{\widetilde{x}})$. Let $\theta_x \in \Sigma_{IH}^U$ be an equilibrium with $x \neq \widetilde{x}$. For any $x \in \left[\widehat{x}_B^U; \widehat{x}_A^U\right)$ we know from Lemma 13 that $c^*(\theta_x, \Sigma(\theta_{\widetilde{x}})) = r_B^+(x)$. The monotonicity of $r_B^+(x)$ implies that $CR^*(\Sigma(\theta_{\widetilde{x}})) = r_B^+(\widetilde{x} - \delta)$. Since $R(\Sigma(\theta_{\widetilde{x}})) = r_B^+(\widetilde{x}) > CR^*(\Sigma(\theta_{\widetilde{x}}))$, it follows from Ellison (2000) that $\Sigma_S = \Sigma(\theta_{\widetilde{x}})$; the only stochastically stable outcome is $\left(HH, V_H - \widehat{x}_A^U, \widehat{x}_A^U\right)$ and the distributional norm is $(V_M + c_A, V_H - V_M - c_A)$.

Point (b). Recall that when condition (7) does not hold then $\Sigma_H = \Sigma_{CH}^U$; hence one mutation is enough to exit from the basin of attraction of Σ_H (Point (a) of Lemma 14). In what follow we shall apply Theorem 3 of Ellison (2000). It is noticing that $R(\Omega) = 1$ for any limit set Ω. Indeed let Ω' be a limit set; then it is always possible to reach $\Omega^* \in \Sigma(\Omega)$ with one mutation by letting one agent to drift at some unreached information set. Hence, if for limit sets Ω and Ω' we have $CR^*(\Omega') = 1$, then $\mu_*(\Omega) > 0$ implies that $\mu_*(\Omega') > 0$. Concerning the minimal modified cost among all paths from a generic limit Ω set we already know that:

(i) if $\rho(\Omega)$ is not a singleton, at least one Ω' exists with $\rho(\Omega')$ singleton, such that $CR^*(\Omega') = 1$ (Lemma 10);

(ii) if $\Omega \in \Sigma_L$, at least two limit sets Ω' and Ω^* exist, with different distributional rules but both belonging to either Σ_H or $(\Sigma_{HL} \cup \Sigma_{LH})$ and such that $CR^*(\Omega') = CR^*(\Omega^*) = 1$ (Point (a) of Lemma 11 and Lemma 12);

(iii) if $\Omega \in \Sigma_{HL}$ (resp. Σ_{LH}), at least two limit set Ω' and Ω^* exist, with different distributional rules but both belonging to Σ_H and such that $CR^*(\Omega') = CR^*(\Omega^*) = 1$ (Point (b) of Lemma 11);

(iv) if $\Omega \in \Sigma_H$, at least one limit set $\Omega' \in \Sigma_L$ exists such that $CR^*(\Omega') = 1$ (Point (a) of Lemma 14).

Let Ω be a limit set such that $\mu_*(\Omega) > 0$. By collecting previous information and using Theorem 3 of Ellison (2000) we conclude that: (i) if $\rho(\Omega)$ is not a singleton, then $\mu_*(\Omega') > 0$ where $\rho(\Omega')$ is a singleton; (ii) if $\Omega \in \Sigma_L$, then $\mu_*(\Omega') > 0$ and $\mu_*(\Omega^*) > 0$ where Ω' and Ω^* both belong to either Σ_H or $\Sigma_{HL} \cup \Sigma_{LH}$; (iii) if $\Omega \in \Sigma_{HL}$ (resp. Σ_{LH}) then $\mu_*(\Omega') > 0$ and $\mu_*(\Omega^*) > 0$ where both Ω' and Ω^* belong to Σ_H; (iv) if $\Omega \in \Sigma_H$ then $\mu_*(\Omega') > 0$ where $\Omega' \in \Sigma_L$. Hence in this case an investment norm and a bargaining norm cannot evolve in the long run. ■

We now turn our attention to game Γ_{DG} and to the proof of Proposition 5. As for game Γ_{UG}, also in this case two preliminary results are needed. In Lemma 16 below we show that $\Sigma_S \subseteq \Sigma^D_{IH}$.

Lemma 16 *Consider Γ_{DG}. Then:*
(a) $CR^*\left(\Sigma^D_{IH}\right) = 1$;
(b) $R\left(\Sigma^D_{IH}\right) > 1$ and, consequently, $\Sigma_S \subseteq \Sigma^D_{IH}$.

Proof. Point (a). Firstly notice that the set Σ^D_{IH} is always well-defined since we can always find a sequence of $\{\delta_i\}$ converging to zero compatible with condition (12). Indeed it is enough to consider $\delta_i < \delta_{IH}$ where $0 < \delta_{IH} \leq \frac{V_H - c_A - c_B}{4}$. Secondly observe that Σ^D_{CH} only supports two distributional rules, namely, x^D_B and x^D_A. Consider an equilibrium $\theta \in \Sigma^D_{CH}$ with x^D_B as distributional rule and suppose a single mutation from $V_H - x^D_B$ to $V_H - x' > V_H - x^D_B$ occurs in population A. Suppose all Bs revise. Notice that whatever the best reply at HH is, the expected payoff by playing H is now smaller than δ. Hence updating will now cause agents B to change investment action and to play L. Therefore when $\theta_x \in \Sigma^D_{CH}$ one mutation is enough to enter into the basin of attraction of $\theta' \in (\Sigma_{HL} \cup \Sigma_{LH})$. A similar conclusion holds also when we consider x^D_A as distributional rule. Hence, from Lemma 11 point (b) we can deduce that, for any $\theta \notin \Sigma^D_{IH}$, the minimal modified cost for across all paths from $\theta_x \in \Sigma^D_{CH}$ to Σ^D_{IH} is equal to one, whatever the number of limit sets the path goes through.

In addition, from Lemma 10 and from Point (b) of Lemma 11, we can deduce that, for any $\Omega \notin \Sigma_H$, the minimal modified cost for all paths from θ to Σ^D_{IH} is equal to one, whatever the number of limit sets the path goes through may be. Therefore

$$CR^*\left(\Sigma^D_{IH}\right) = \max_{\Omega \notin \Sigma^U_{IH}} c^*\left(\Omega, \Sigma^D_{IH}\right) = 1.$$

Point (b). Let $\theta_x \in \Sigma^D_{IH}$ and consider the worst-case equilibrium scenario in which $\widetilde{y}^i_{LH}(\theta_x) = V_H - x - c_A$ and $\widetilde{x}^i_{HL}(\theta_x) = x - c_B$. Suppose a single mutation from H to L occurs in population B. This implies that profile HL is reached in which agents A behave as dictators and claim $V_M - \delta$. When agents B revise, their updated beliefs become $\widetilde{x}^i_{HL} = \delta$; as a consequence they choose H. The process then returns to an equilibrium $\theta' \in \Sigma(\theta)$. The same occurs also when a single mutation from H to L occurs in population A.

Suppose now a single agent B mutates her demand from x to x'. Obviously no agent B imitates the mutant when updating. When agents A revise, we know from Lemma 6 that their best response is either $V_H - x$ or $V_H - x'$.

When $x' > x$, agent A expects to receive $(V_H - x)\frac{N-1}{N} - c_A$ by claiming $V_H - x$ and expect to receive $V_H - x' - c_A$ by asking $V_H - x'$. When instead $x' < x$, agent A expects to receive $V_H - x - c_A$ by claiming $V_H - x$ and expect to receive $(V_H - x')\frac{1}{N} - c_A$ by asking $V_H - x'$. It is simple to see that, whatever the relation between x and x' is, Point (e) of Assumption 1 implies that the best response is always $V_H - x$. Hence, updating will not cause agents A to change both claim and investment action. This result allow us to assert that $R\left(\Sigma_{IH}^D\right) > 1$. Therefore, given that $CR^*\left(\Sigma_{IH}^D\right) = 1$, by using Theorem 2 of Ellison (2000) we get $\Sigma_S \subseteq \Sigma_{IH}^D$. ■

We have now to derive the norms of distribution supporting Σ_{IH}^D. As for Γ_{UG}, we have to compute the radius of $\theta \in \Sigma_{IH}^D$ (i.e. the minimum number of mutations required to destabilize the outcome supported by θ, $\forall\theta \in \Sigma_{IH}^D$), and to find an equilibrium belonging to Σ_{IH}^D such that $R(\Sigma(\theta)) > CR^*\Sigma(\theta)$. The relevant informations are provided by Lemma 13, which continues to be true, and by Lemma 17 below.

Lemma 17 *Consider the game Γ_{DG}. The minimum number of mutations required to get from $\Sigma(\theta_x)$ to an equilibrium which supports a different investment profile is:*

$$\begin{aligned}\overline{r}_A(x) &= \left\lfloor N\left(1 - \tfrac{\delta + c_B}{x}\right)\right\rceil \\ \overline{r}_B(x) &= \left\lfloor N\left(1 - \tfrac{\delta + c_A}{V_H - x}\right)\right\rceil.\end{aligned}$$

Proof. We give the proof for $\overline{r}_B(x)$ only; a similar argument can be used for $\overline{r}_A(x)$. Consider $\theta_x \in \Sigma_{IH}^D$. Suppose p_1 agents B mutate by playing L and p_2 agents B mutate by claiming $x' > x_A^D$. Let $\widetilde{\theta}_x$ be the resulting state. Suppose these mutations induce all agents A to play L. Suppose all A believe to receive at LH a payoff $\widetilde{y}_{LH}$; this belief is compatible with the fact that θ_x is an equilibrium only if $\widetilde{y}_{LH} \leq V_H - x - c$. Sure enough, at $\widetilde{\theta}_x$ all agents A decide to play L if, after updating,

$$\frac{N - p_1}{N}\widetilde{y}_{LH} > \mu_H\left(\widetilde{\theta}_x, p_1, p_2\right)$$

where the LHS is the expected payoff by playing L and the RHS is the expected payoff by playing H. However, $\mu_H\left(\widetilde{\theta}_x, p_1, p_2\right)$ depends on the best demand in an HH match. In particular

$$\mu_H(.) = \begin{cases} \frac{(N-p_2-p_1)(V_H-x)}{N} + \frac{p_1(V_M-\delta)}{N} - c_A & if \quad \frac{(N-p_2-p_1)(V_H-x)}{N-p_1} \geq (V_H - x') \\ \\ \frac{(N-p_1)\left(V_H-x'\right)}{N} + \frac{p_1(V_M-\delta)}{N} - c_A & if \quad \frac{(N-p_2-p_1)(V_H-x)}{N-p_1} < (V_H - x'). \end{cases}$$

Given $\widetilde{y}_{LH}$, the minimum number of mutations in population B are obtained by solving two constrained minimization problems ($M1$ and $M2$). In both problems, the objective function is $p_1 + p_2$. In $M1$ (resp. $M2$), we contemplate the case in which the best action at HH is $V_H - x'$ (resp. $V_H - x$). Both problems require $p_1 = 0$ as a solution. Moreover, $p_2^{M1} = N\left(\frac{x'-x}{V_H-x}\right)$ is the solution of the first minimization problem for whatever value of $\widetilde{y}_{LH}$; $p_2^{M2}\left(\widetilde{y}_{LH}\right) = N\left(1 - \frac{\widetilde{y}_{LH}+c_A}{V_H-x}\right)$ is the solution of the second minimization problem. Notice that p_2^{M2} depends on $\widetilde{y}_{LH}$. Suppose $p_2^{M2}\left(\widetilde{y}_{LH}\right)$ agents B claim $x' > x$. By updating, all agents A play L so that only profile LH is observed. Since all agents B claim $V_M - \delta$ then, after updating, all agents A learn that $\widetilde{y}_{LH} = \delta$. This implies that no agent A has the incentive to play H if

$$\frac{N - p_2^{M2}\left(\widetilde{y}_{LH}\right)}{N}\left(V_H - x\right) - c_A \leq \delta,$$

condition weakly satisfied when $\widetilde{y}_{LH} = \delta$.Therefore, in the second minimization problem the minimum number of mutations (concerning B agents) needed to enter ^{into} the basin of attraction of θ' from θ_x is

$$p_2^{M2} = N\left(1 - \frac{\delta + c_A}{V_H - x}\right).$$

Since $p_2^{M1} > p_2^{M2}$, the minimum number of mutations involves that: (i) mutating agents only change their demands in the HH profile; (ii) these mutations cause agent A to shift to action L when at HH the best action continues to be $\left(V_H - x\right)$;(iii) all agents A correctly anticipate the distribution occurring at LH. Hence:

$$\overline{r}_B\left(x\right) = \left\lfloor N\left(1 - \frac{\delta + c_A}{V_H - x}\right)\right\rceil.$$

■

Proof of Proposition 5. Consider Γ_{DG}. From Lemma 16 we know that $\Sigma_S \subseteq \Sigma_{IH}^D$. Thus, we are only left with the task of deriving the distributional norm supporting the equilibria belonging to Σ_{IH}^D.

To detect $R\left(\Sigma\left(\theta_x\right)\right)$ for any $\theta_x \in \Sigma_{IH}^D$ we compare the results coming from Lemma 13 with those coming from Lemma 17. Notice that $r_B^+\left(x\right) \leq \overline{r}_B\left(x\right)$ if $2\delta \leq V_H - x - c_A$ and $r_A^-\left(x\right) \leq \overline{r}_A\left(x\right)$ if $2\delta \leq x - c_B$; since these conditions are always satisfied for any $x \in \left[\widehat{x}_B^D, \widehat{x}_A^D\right]$ we conclude that

$$R\left(\Sigma\left(\theta_x\right)\right) = \begin{cases} r_B^+\left(x\right) & if \quad x < \frac{V_H}{2} \\ \\ r_A^-\left(x\right) & if \quad x > \frac{V_H}{2}. \end{cases}$$

Before deriving $CR^*\left(\Sigma\left(\theta_x\right)\right)$, we observe that: (i) $\widehat{x}_B^D < \frac{V_H}{2}$ if $\delta \leq \delta_B \equiv \frac{V_H - 2c_B}{4}$ condition always satisfied when $\delta \leq \delta_{IH}$; (ii) $\frac{V_H}{2} \leq \widehat{x}_A^D$ if $\delta \leq \delta_A \equiv$

$\frac{V_H - 2c_A}{4} < \delta_{IH}$. Since under Assumption 1 $c_A < \frac{V_H}{2}$, then $\delta_A > 0$. Therefore, for given V_H, V_M and c_A compatible with Assumption 1, it is always possible to detect a sequence of $\{\delta_i\}$ converging to zero such that $\delta_i < \delta_A$. In this case not only Σ_{IH}^D is well defined but we also have $\widehat{x}_B^D < \frac{V_H}{2} < \widehat{x}_A^D$. Let $\overline{x} \equiv \frac{V_H}{2}$ and consider the set of equilibria $\Sigma(\theta_{\overline{x}})$; let $\theta_x \in \Sigma_{IH}^D$ be an equilibrium with $x \neq \overline{x}$. From Lemma 13 we know that

$$c^*(\theta_x, \Sigma(\theta_{\overline{x}})) = \begin{cases} r_B^+(x) & \text{if} \quad x < \frac{V_H}{2} \\ \\ r_A^-(x) & \text{if} \quad x > \frac{V_H}{2}. \end{cases}$$

By the monotonicity of $r_B^+(x)$ and $r_A^-(x)$ we obtain

$$CR^*(\Sigma(\theta_{\overline{x}})) = \max\left(r_B^+(\overline{x} - \delta), r_A^-(\overline{x} + \delta)\right).$$

Of course, when $x = \overline{x} \equiv \frac{V_H}{2}$, then $R(\Sigma(\theta_{\overline{x}})) = r_B^+(\overline{x}) = r_A^-(\overline{x})$. Since

$$R(\Sigma(\theta_{\overline{x}})) = r_B^+(\overline{x}) = r_A^-(\overline{x}) > CR^*(\Sigma(\theta_{\overline{x}}))$$

it follows from Theorem 2 of Ellison (2000) that $\Sigma_S = \Sigma(\theta_{\overline{x}})$; the only stochastically stable outcome is thus$\left\{HH, \frac{V_H}{2}, \frac{V_H}{2}\right\}$ and the distributional norm is $\left(\frac{V_H}{2}, \frac{V_H}{2}\right)$. ■

References

[1] Andreozzi L., 2010. An evolutionary theory of social justice: choosing the right game. European Journal of Political Economy 26, 320-329.

[2] Andreozzi L., 2011.Property rights and investment: an evolutionary approach. Games and Economic Behavior, ???

[3] Barling B., von Siemens F., 2010. Equal sharing rules in partnership. Journal of Institutional and Theoretical Economics 166, 299-320.

[4] Binmore K., 1998. The evolution of fairness norms. Rationality and Society 10, 275-301.

[5] Binmore K., 2007. Economic man or straw man ?. ELSE working paper 262.

[6] Binmore K., Shaked A., 2010. Experimental economics: where next ?. Journal of Economic Behavior and Organization 73, 87-100.

[7] Bohem C., 2004. What makes human economically distinctive ? A three-species evolutionary comparison and historical analysis. Journal of Bioeconomics, 6, 109-135.

[8] Cappelen A., Hole A. D., Sorensen E., Tungodden B., 2007. The pluralism of fairness ideals: an experimental approach. American Economic Review 97, 818-827.

[9] Conte A., Moffatt P., G., 2009. "The pluralism of fairness ideals": a comment. Quaderni DEF 162, Luiss University, Rome.

[10] Dawid H., MacLeod B., 2001. Hold up and the evolution of bargaining conventions. European Journal of Economic and Social Systems 15, 139-169.

[11] Dawid H., MacLeod B., 2008. Hold-up and the evolution of investment and bargaining norms. Games and Economic Behavior 62, 26-52.

[12] Ellingsen T., Robles J., 2002. Does evolution solve the hold-up problem ?. Games and Economic Behavior 39, 28-53.

[13] Ellison G., 2000. Basin of attraction, long-run stochastic stability, and the speed of step-by-step evolution. Review of Economic Studies, 67, 17-45.

[14] Gantner A., Guth W., Konistein M., 2001. Equitable choices in bargaining games with joint production. Journal of Economic Behavior and Organization 46, 209-225.

[15] Hill K., 2002. Altruistic cooperation during foraging by the Ache, and the evolved human predisposition to cooperate. Human Nature 13, 105-128.

[16] Homas G. C., 1961. Social Behavior: Its Elementary Forms. New York: Harcourt, Brace and World.

[17] Hawkes K., 1993. Why hunter-gatherers work. Current Anthropology, 34, 341-361.

[18] Ichikawa M., 1983. An examination of the hunting-dependent life of the Mbuti Pygmies, eastern Zaire. African Study Monographs 5, 55-76.

[19] Kaplan H., Hill K., 1985. Food sharing among the Ache: tests of explanatory hypotheses. Current Anthropology 26, 223-246.

[20] Lee R. B., 1990. Primitive communism and the origin of social inequality. In: Upham S. (Ed.), The Evolution of Political Systems: Sociopolitics in Small-Scale Sedentary Societies, Cambridge University Press, 225-246.

[21] Noldeke G., Samuelson L., 1993. An evolutionary analysis of backward and forward induction. Games and Economic Behavior 5, 425-454.

[22] Troger T., 2002. Why sunk costs matter for bargaining outcomes: an evolutionary approach. Journal of Economic Theory 102, 375-402.

[23] Woodburn J., 1982. Egalitarian Societies. Man 17, 431-451.

[24] Young P., 1993. An evolutionary model of bargaining. Journal of Economic Theory 59, 145-168.

[25] Young P., 1993a. The evolution of conventions. Econometrica 61, 57-84.

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