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Short term trend estimation and turning point prediction

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1.Introduction

The basic approach to the analysis of current economic and business conditions (known as recession and recovery analysis, Moore, 1961) is that of assessing the short-term trend of major economic indicators (leading, coincident and lagging) using percentage changes, based on original units and calculated for months and quarters in chronological sequence. The main goal is to evaluate the behavior of the economic indicators during incomplete phases by comparing current contractions or expansions whith corresponding phases in the past. This is done by measuring changes of single time series from their standing at cyclical turning points with past changes over a series of increasing spans. Most statistical agencies published seasonally adjusted data but, in recent years, there has been an increasing interest in providing smoothed seasonally adjusted data or trend-cycles estimates to facilitate recession and recovery analysis. Among other reasons, this interest originated from major global economic and financial changes that introduced more variablity in the series and consequently, in their seasonally adjusted values, making difficult to determine the short-term trend and an early detection of the turning point.

The estimation of the trend-cycle with the X11ARIMA seasonal adjustment method (Dagum, 1980 and 1988) as well as the U.S. Bureau of Census Method II-X11 variant (Shiskin, Young and Musgrave,1967) is done by the application of linear filters due to Henderson (1916). These Henderson filters are applied to seasonally adjusted data where the irregulars have been modified to take into account the presence of extreme values. The length of the filters is automatically selected on the basis of specific values of noise to signal ratios for the seasonal (I/S) and trend cycle (I/C) components.

The problem of short-trend estimation within the context of seasonal adjustment as well as for current economic analysis has attracted the attention of several authors in the past, among others, Rhoades (1980), Cholette (1981and 1982), Moore et al. (1981), Kenny and Durbin (1982), Castle (1987), Dagum and Laniel (1987), Cleveland et als (1990), Walgren and Wallgren (1990), Gray and Thomson (1990.a, and 1996.b), Findley and Monsell (1990) and Scott (1990).

The 13-term Henderson trend-cycle estimator is the most often applied because of its good property for early turning point detection but, it has the disadvantages of: (1) producing a large number of unwanted ripples (short cycles of 9 and 10 months) that can be interpreted as false turning points and, (2) large revisions for the most recent values (often larger than those of the corresponding seasonally adjusted data). The use of longer Henderson filters is not an alternative since the reduction in false turning points is achieved at the expense of increasing the time lag of turning point detection.

Recently, one of the authors, Dagum (1996) has proposed a new method that enables the use of the 13-term Henderson filter with the advantages of: (1) reducing the number of unwanted ripples, (2) reducing the size of the revisions to most recent trend-cycle estimates and, (3) no increase in time lag of turning point detection. This new method basically consists of producing one year of ARIMA extrapolations from a seasonally adjusted series with extreme values replaced by default; extending the series with its extrapolated values; and then applying the 13-term Henderson filter to the extended seasonally adjusted series imposing stricter sigma limits (not the default) for the replacement of extreme values. The object is to pass through the 13-term Henderson filter, an input with reduced noise.This procedure was applied to the nine Leading Indicator series of the Canadian Composite Leading Index with excellent results.

In another study, Dagum, Chhab and Morry (1996) compared this new method with two model- based trend-cycle estimators: (a) a trend-cycle state-space model and (2) an ARIMA trend-cycle model. Three typical series characterized by small, medium and large signal to noise ratios were chosen and the results indicated the overall superiority of the new approach according to the above three criteria. In a recent study, Gray and Thomson(1996.a) have developed a family of trend local linear filters based on the same criteria of fitting and smoothing as in smoothing spline functions. These authors show that their filters are a generalization of the standard Henderson filters. Smoothing splines, particularly cubic splines with smoothing parameter estimated by generalized cross validation, has been widely applied to smooth noisy data.

The main purpose of our study is to investigate whether cubic spline functions can be used as a substitute of the 13-term Henderson filter and, if so, they can further improve on Dagum (1996) method. We also investigate the extent to which the first and second order differences of cubic spline trend-cycle curves can be useful to predict the upcoming of a turning point by monitoring the most recent values.

Section 2 summarizes the properties of the 13-term Henderson filter for central and end values of the series. Section 3 introduces the cubic spline model and discusses the selection of the smoothing parameter λ. Section 4 compares the results given by the cubic spline and the standard 13-term Henderson applied to the same input, that is, seasonally adjusted data with extreme values replaced by the default option and no ARIMA extrapolations. The comparison is done with a coincident indicator, the Italian Index of Industrial Production and three Canadian Leading Indicator Series. Section 5 analyses the information given by the first and second order differences of the cubic spline trend-cycle to assess the upcoming of turning points. Finally, section 6 gives the conclusions and directions for future research.

2. The 13-term Henderson Trend-Cycle Filter

The properties of the 13-term Henderson filter have long been discussed by authors preoccupied with the problem of trend-cycle estimation. Two important studies on the theoretical properties of the 13-term Henderson filter are found in Gray and Thomson (1996.a, 1996.b) and in Dagum, Chhab and Chiu (1996) where the derivation and properties of all the linear filters, single and combined, available in X11ARIMA and Census X11 is discussed using spectral analysis.

The 13-term Henderson filter as its variants, 9-and 23-term, are based on summation formulas mainly used by actuaries. The basic principle for the summation formula is the combination of operations of differencing and summation in such a manner that when differencing above a certain order is ignored, they will reproduce the functions operated on. The merit of this procedure is that the smoothed values thus obtained are functions of a large number of observations whose errors, to a considerable extent, cancel out. These filters have the properties that, when fitted to second or third degree parabolas, their output will fall exactly on those parabolas and, when fitted to stochastic data, they will give smoother results than can be obtained from weights which give the middle point of a second degree parabola fitted by least squares. Recognition of the fact that the smoothness of the resulting filtering depends on the smoothness of the weight diagram led Henderson (1916) to develop a formula which makes the sum of squares of the third differences of the smoothed series a minimum for any numbers of terms. In other words, the $\sum\left(\Delta^3 y_t\right)^2$ is minimized (where Δ is the difference operator, and y_t is the output or smoothed series) if and only if $\sum\left(\Delta^3 h_k\right)^2$ (where h_k's are the weights) is minimized.

If the span of the average is $2m\text{-}3$, Henderson showed that the general expression for the n-th term of the filter that minimizes $\sum\left(\Delta^3 h_k\right)^2$ is:

$$\frac{315\left[(m-1)^2-n^2\right]\left(m^2-n^2\right)\left[(m+1)^2-n^2\right]\left[3m^2-16-11n^2\right]}{8m\left(m^2-1\right)\left(4m^2-1\right)\left(4m^2-9\right)\left(4m^2-25\right)} \qquad (2.1)$$

To derive a set of 13 weights from (2.1), 8 is substituted for m and the values are obtained for each n from -6 to 6. The Henderson 13-term trend-cycle filter is thus given by,

$$\begin{aligned} H_{13}(B) = & -.019B^{-6} - .028B^{-5} + .00B^{-4} + .065B^{-3} \\ & + .147B^{-2} + .214B^{-1} + .24B^{-0} + .214B \\ & + .147B^{2} + .065B^{3} + .00B^{4} - .028B^{5} - .019B^{6} \end{aligned} \qquad (2.2)$$

where B is the backshift or lag operator defined by $B^m y_t = y_{t-m}$ and $B^0 \equiv 1$.

The calculation of the weights of the asymmetric Henderson filter in the X11ARIMA method is based on the minimization of the mean squared revision (MSR) between the final estimates (obtained by the application of the symmetric filter) and the preliminary estimate (obtained by the application of an asymmetric filter) subject to the constraint that the sum of the weights is equal to one. The assumption made is that at the end of the series the seasonally adjusted values are equal to a linear trend-cycle plus a purely random irregular ε_t such that $NID \sim \left(0,\sigma_a^2\right)$. The equation used is

$$E\left[r_t^{(i,m)}\right]^2 = c_1^2\left(t-\sum_{j=-i}^{m} h_{ij}(t-j)\right)^2 + \sigma_a^2 \sum_{j=-m}^{m}\left(h_{mj}-h_{ij}\right)^2 \qquad (2.3)$$

where h_{mj} and h_{ij} are the weights of the symmetric (central) filter and the asymmetric filters, respectively; $h_{ij}=0$ for $j=-m,\ldots,-i-1$, c_1 is the slope of the line and σ_a^2 denotes the noise variance. There is a relation between c_1 and σ^2 such that the noise to signal ratio, I/C is given by

$$I/C = \left(4\sigma_a^2/\pi\right)^{\frac{1}{2}}/\left|c_1\right| \quad \text{or} \quad \frac{c_1^2}{\sigma^2} = \frac{4}{\pi(I/C)^2} \qquad (2.4)$$

The I/C noise to signal ratio (2.4) determines the length of the Henderson trend-cycle filter to be applied. Thus, setting t=0 and m=6 for the end weights of the 13-term Henderson, we have,

$$\frac{E\left[r_0^{(i,6)}\right]^2}{\sigma_a^2} = \frac{4}{\pi\left(I/C\right)^2}\left(\sum_{j=-i}^{6} h_{ij}\right)^2 + \sum_{j=-6}^{6}\left(h_{6j}-h_{ij}\right)^2 \qquad (2.5)$$

Making I/C = 3.5 (the most noisy situation where the 13-term Henderson is applied), equation (2.5) gives the same set of end weights of Census X11 variant (Shiskin, Young and Musgrave, 1967). The end weights for the remaining monthly Henderson filters are calculated using $I/C = .99$ for the 9-term filter and $I/C = 7$ for the 23-term filter. The estimated final trend-cycle is obtained by cascade filtering that results from the convolution of various linear trend and seasonal filters. In fact, if the output from the filtering operation H is the input to the filtering operation Q, the coefficients of the cascade filter C result from the convolution of $H * Q$. For symmetric filters $H * Q = Q * H$ but this is not valid for asymmetric filters. Assuming an input series x_t, $t = 1,2,\ldots,T$, we can define a matrix $H = \left[h_{kj}\right]$, $k = 1,2,\ldots,m_k$, $j = 1,2,\ldots,2m_k + 1$, where each row is a filter and m_k is the half length of the symmetric filter. $h_{1,.}$ denotes an asymmetric filter where the first m_h coefficient are zeroes and $h_{m_k+1,.}$ denotes the symmetric filter.

Given data up to time T, the $m_k + 1$ most recent values of the output (filtered series) are given by

$$y^h_{T+1-k} = \sum_{j=m_k-k+2}^{2m_k+1} h_{kj} x_{T-k+m_k+2-j} \tag{2.6}$$

The 13-term Henderson filter can then be put in matrix form as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -.092 & -.058 & .012 & .120 & .244 & .353 & .421 \\ 0 & 0 & 0 & 0 & 0 & -.043 & -.038 & .002 & .080 & .174 & .254 & .292 & .279 \\ 0 & 0 & 0 & 0 & -.016 & -.025 & .003 & .068 & .149 & .216 & .241 & .216 & .148 \\ 0 & 0 & 0 & -.009 & -.022 & .004 & .066 & .145 & .208 & .230 & .201 & .131 & .046 \\ 0 & 0 & -.011 & -.022 & .003 & .067 & .145 & .210 & .235 & .205 & .136 & .050 & -.018 \\ 0 & -.017 & -.025 & .001 & .066 & .147 & .213 & .238 & .212 & .144 & .061 & -.006 & -.034 \\ -.019 & -.028 & 0 & .066 & .147 & .214 & .240 & .214 & .147 & .066 & 0 & -.028 & -.019 \end{bmatrix} \tag{2.7}$$

The properties of the cascade filters can be studied by analyzing their frequency response functions. The frequency response function is defined by

$$H(\omega) = \sum_{j=-m}^{m} \alpha_j e^{-i\omega j} \tag{2.8}$$

where α_j are the weights of the filter and ω is the frequency in cycles per unit of time. In general, the frequency response functions can be expressed in polar form as follow,

$$H(\omega) = A(\omega) + iB(\omega) = G(\omega) e^{i\phi(\omega)} \tag{2.9}$$

where $G(\omega) = \left[A^2(\omega) + B^2(\omega)\right]^{\frac{1}{2}}$ is called the gain of the filter and $\phi(\omega) = \tan^{-1}\left(-B(\omega)/A(\omega)\right)$ is called the phase shift of the filter and is usually expressed in radians. The expression (2.9) shows that if the input function is a sinusoidal variation of unit amplitude and constant phase shift $\psi(\omega)$, the output function will also be sinusoidal but of amplitude $G(\omega)$ and phase shift $\psi(\omega) + \phi(\omega)$. The gain and phase shift vary with ω. For symmetric filters the phase shift is 0 or $\pm\pi$, and for asymmetric filters take values between $\pm\pi$ at those frequencies where the gain function is zero. For a better interpretation the phase shifts will be here given in months instead of radians (the phase shift in months is given by $\phi(\omega)/2\pi\omega$ for $\omega \neq 0$). The gain function shown should be interpreted as relating the spectrum of the original series to the spectrum of the output obtained with a linear time-invariant filter. For example, let $Y_t^{(0)}$ be the estimated seasonally adjusted observations for the current period based on data x_t $t = 1,2,\ldots,T$, then the time series $\left\{Y_t^{(0)}\right\}$ is obtained from $\left\{X_t\right\}$ by application of the concurrent linear time-invariant filter $h^{(0)}(B)$. Thus, the gain function shown in Figure 2.a below relates the spectrum of $\left\{x_t\right\}$ to the spectrum of $\left\{y_t^{(0)}\right\}$ and not to the spectrum of the complete seasonally adjusted series produced at time t (which includes $Y_t^{(0)}$, a first revision of time t-1, a secon revision at time t-2, and so on).

The 13-term Henderson filter combined with the standard seasonal filters (5 and 7 term weighted moving averages) produces a symmetric cascade filter for central values (at least four years from each end of the series) with a gain as exibit in Figure 1.

Figure 1 also shows the gain functions of short and long convolutions corresponding to the 9-term Henderson (H-9) and the 23-term Henderson (H-23) respectively. It is apparent that cycles of 9 and 10 months periodicity (in the 0.08-0.18 frequency band) will not be suppressed by any of the cascade filters, particularly those using the 9 and 13-term filters. In fact, 90%, 72% and 21% of the power of these short cycles are passed by the 9, 13 and 23-term Henderson filters, respectively. For the concurrent asymmetric trend-cycle filter which is applied to the last observation, the peak reached at the 0.08-0.16 frequency band is even larger as shown in Figure 2.a. When ARIMA extrapolations are used the gain resembles more the symmetric filter but there is a slight increase in phase shift as shown in Figure 2.b.

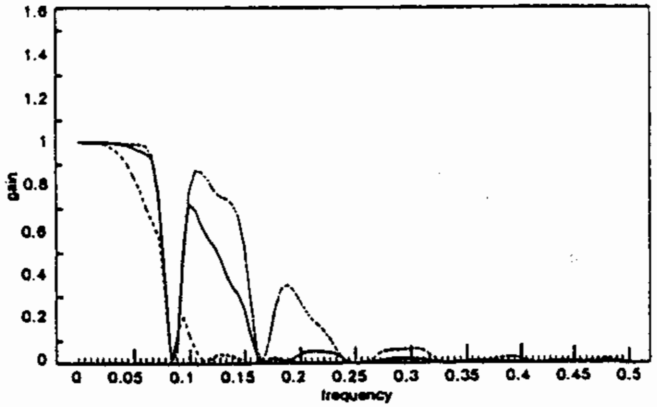
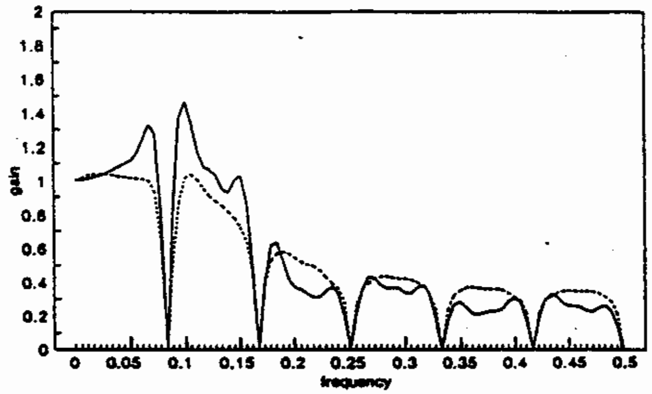
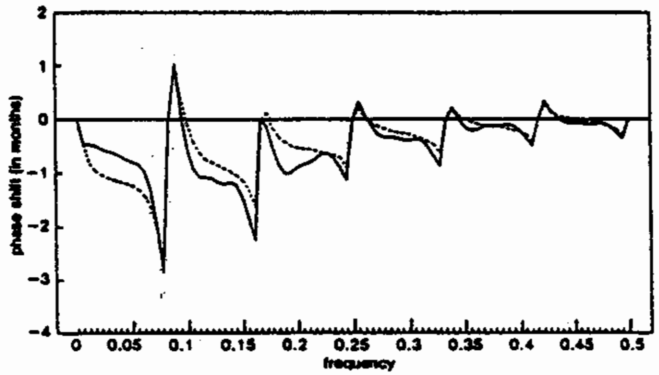


Fig.1 Trend-cycle symmetric cascade filters: gain function.
··· (3×3)(3×3)[H-9] — (3×3)(3×5)[H-13]— (3×3)(3×9)[H-23]



Model: (0,1,1)(0,1,1) $\theta = .4$ $\Theta = .6$

Fig. 2.a Trend-cycle concurrent cascade filters, (3×3)(3×5)[H-13], *with and without ARIMA extrapolations: gain function.*
--- With consistent extrapolation — Without extrapolation



Model: (0,1,1)(0,1,1) $\theta = .4$ $\Theta = .6$

Fig. 2.b Trend-cycle concurrent cascade filters, (3×3)(3×5)[H-13], with and without ARIMA extrapolations: phase-shift.
--- With consistent extrapolation — Without extrapolation

3. Smoothing Spline Functions

The current literature on spline functions, particularly on smoothing splines, is very extense and we refer the reader to Wahba (1989) for an excellent summary of the most important contributions on this topic. The problem of smoothing via spline functions is closely related to that of smoothing priors and signal extraction in time series, where these later are approached from a parametric point of view (see, among others, Akaike 1980.a and 1980.b; Gersch, 1992 and Kitagawa and Gersch, 1996).

Similar to the Henderson filters, the original work on smoothing spline functions was based on the theory of graduation. The first two seminal works related to smoothing splines are due to Whittaker (1923) and Whittaker and Robinson (1924) who proposed a new graduation method that basically consisted of a trade-off between fitting fidelity and smoothing. The problem was that of estimating an unknown "smooth" function f, observed with errors assumed to be white noise. That is, given a set of observations y_i t=1, 2, ...,T such that,

$$y_i = f_i + \varepsilon_i \qquad \varepsilon_i \sim N(0,\sigma^2) \tag{3.1}$$

we want to minimize

$$\sum_{i=1}^{N}(y_i - f_i)^2 + \mu^2 \sum_{i=k+1}^{N}(\Delta^k f_i)^2 \tag{3.2}$$

where $\Delta^k f_i$ denotes the k-th order difference of f_i, e. g. $\Delta f_i = f_i - f_{i-1}$, $\Delta^2 f_i = \Delta(\Delta f_i)$, and so on. The smoothing trade-off parameter μ must be appropriately chosen.

Following this direction, Schoenberg (1964) extended Whittaker smoothing method to the fitting of a continous function to observed data, not necessarily evenly spaced. In this case, the model is

$$y_i = f(x_i) + \varepsilon_i \qquad \varepsilon_i \sim N(0,\sigma^2) \tag{3.3}$$

where the unobserved function f is assumed to be "smooth" on the interval $[a,b]$ and the observations are at the n points $x_1, x_2, \ldots x_n$. The problem is to find

$$f_\lambda = \min_{f \in C^m} \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2 + \lambda \int_a^b \left[f^{(m)}(u)\right]^2 du \tag{3.4}$$

where C^m is the class of functions with m continous derivatives and $\lambda > 0$.

The solution to (3.4) known as a smoothing spline is unique and given by a univariate natural polynomial (unp) or piecewise polynomial function spline of degree 2m-1 with knots at the data points $x_1, x_2, \ldots x_n$. The smoothing trade-off parameter λ controls the balance between the fit to the data as measured by the residual sum of squares and the smoothness as measured by the integrated squared m-th derivative of the function . When m=2, which is the case of a cubic smoothing spline then the integral of the squared second order derivative $f^{(2)}$ is curvature and a small value for the integral corresponds visually to a smooth curve. As $\lambda \to 0$ the solution f_λ tends to the unp spline which interpolates the data, and as $\lambda \to \infty$, the solution tends to the polynomial of degree m best fitting the data in the least squares sense. The smoothing trade-off parameter λ is known as hyperparameter in the Bayesian terminology and it has the interpretation of a noise to signal ratio, the larger the λ the smoother the trend-cycle.

The estimation of λ was first done using ordinary cross-validation (OCV). OCV consisted of deleting one observation and solving the optimization problem with a trial value of λ, computing the difference between the predicted value and the deleted observation, accumulating the sums of squares of these differences as one runs through each of the data points in turn, and finally choosing the λ for which the accumulated sum is the smallest. This procedure was improved by Craven and Wahba

(1979) who developed the generalized cross validation (GCV) method currently available in most computer packages. GCV can be obtained from OCV by rotating the system to a standard coordinate system, doing OCV, and rotating back. The GCV estimate of λ is obtained by minimizing

$$V(\lambda)=\frac{(1/n)\|I-A(\lambda)y\|^2}{\left[(1/n)Tr(I-A(\lambda))\right]^2} \tag{3.5}$$

where $A(\lambda)$ is the influence matrix associated with f, that is, $A(\lambda)$ satisfies

$$\begin{pmatrix} f_\lambda(x_1) \\ \vdots \\ f_\lambda(x_n) \end{pmatrix} = A(\lambda)\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \tag{3.6}$$

Trace $A(\lambda)$ can be viewed as the "degrees of freedom for the signal" and so, (3.5) can be interpreted as minimizing the standardized sum of squares of the residuals. Cross-validated smoothing splines has also given good results for the estimation of derivatives or other local features of maxima and minima and will be the ones used in this study.

4. Comparison between the Cubic Smoothing Spline and the 13-term Henderson Trend-cycle Estimators

The comparison between the Cubic Smoothing Spline (CSS) and the 13-term Henderson (H-13) estimator is done as follows:
(1) The input for both trend-cycle estimators is the seasonally adjusted series modified by extreme values with zero weight. The identification and replacementof extreme values is done with the default option of X11ARIMA which defines as extreme value with zero weight any irregular falling outside $\pm 2.5\sigma$.

(2) To estimate the CSS trend-cycle we use the default option of the S-Plus computer program subroutine which is based on Hastie and Tibshirani (1990). The default option automatically selects the knots and calculate λ using the generalized cross-validation procedure. In those cases where the CSS results differ significantly from those given by H-13 in the sense that the trend-cycle is either too smooth or too bumpy, we search for other values of λ to approximate the H-13 trend as discussed later in this section.
(3) The comparison between both trend-cycle estimators is based on two of the three criteria used by Dagum (1996), namely: (i) number of unwanted ripples and (ii) time lag to detect the turning point. At this stage of the research we have not carried out an analysis of the revisions to the most recent estimates from CSS, however, by visual inspection of the graphs used for criteria (ii) we can get some information on the size of the total revision of the turning point.

Next, we discuss the results for the Italian Index of Industrial Production (IIP), a coincident economic indicator (Klein, 1995), and the three Canadian Leading Indicators, New Orders for Durable Goods (NODG), Average Workweek in Manufacturing (AWM) and House Spending Index (HSI) discussed in Dagum (1996).

Case 1. Italian Index of Industrial Production

The monthly data for the Italian Index of Industrial Production are those published in seasonally adjusted form by the Italian statistical agency ISTAT and cover the period January 1985-December 1994. This series was adjusted for extreme values as described in (1) above and the CSS and H-13 trend-cycles were estimated .

Figure 3 shows the major differences between the two trend-cycle curves, being the CSS too smooth. We then looked at the value of the noise to signal ratio (I/C) given by the X11ARIMA method which would have been used to determine the appropriate length of the Henderson filter. The I/C was equal to 4.11 which correspond to the 23-term Henderson instead of the 13-term. In fact, we found that the automatic option of the S-Plus software for the estimation of the CSS trend-cycle of the IIP gave results close to those from X11ARIMA. These two trend-cycle curves are shown in Figure 4. However, as it is already well known, the 23-term Henderson filter is a poor predictor of turning points and, as shown in Figure 5, it takes 8 months the detect the July 1993 turning point of this series. Similar time lag is observed for the automatic CSS trend-cycle as exhibit in Figure 6. These Figures 5 and 6, resembling "porcupines", give the revision path of the last available point, in this case, July 1993 as we keep adding one observation at a time. Only after 8 months have being added to the series ending in July 1993, the turning point is clearly detected. A crude measure of the size of the revision is given by the distance between the first estimate when the series ends in July 1993 and the final estimate for the same month when the series ends in April 1994. Here, the revisions for the CSS trend-cycle estimate looks slightly larger.

Next, we tried to approximate the results given by the 13-term Henderson which, although introduces 4 false turning points (see Figure 3) it has the advantage of reducing the turning point detection to 4 months (see Figure 7). After experimenting with several values of the smoothing parameter, we found that for a $\lambda = 5.8\times10^{-6}$ the CSS trend cycle ressembles that of the H-13 and reduces the false turning points to three (see Figure 9). The time lag to detect the July 1993 turning point is of four months as with the H-13 with similar revisions (see Figure 8).

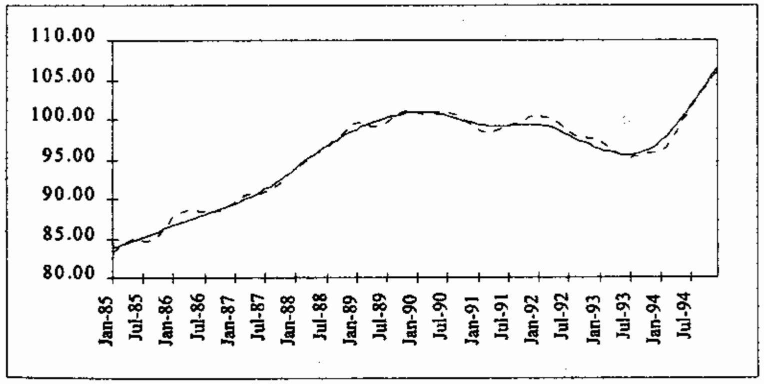


Fig.3. CSS and standard H-13 trend-cycle estimates (IIP).
— *CSS* --- H-13.

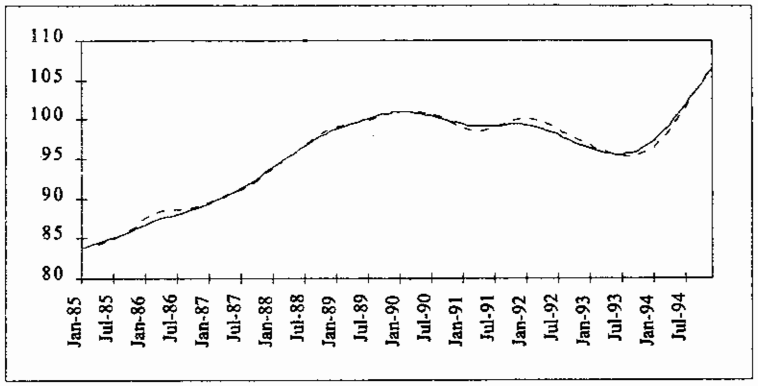


Fig. 4. CSS and standard H-23 trend-cycle estimates (IIP).
— *CSS* --- H-23.

Revision path of July-August 1993 turning point (IIP)

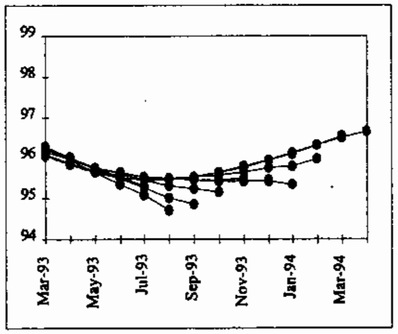


Fig. 5. With standard H-23

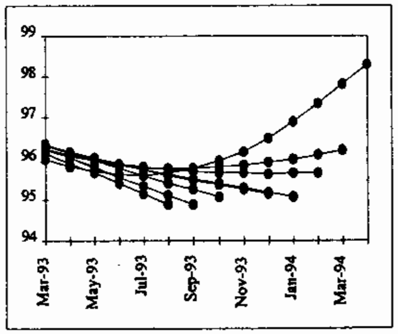


Fig. 6. With CSS(automatic λ)

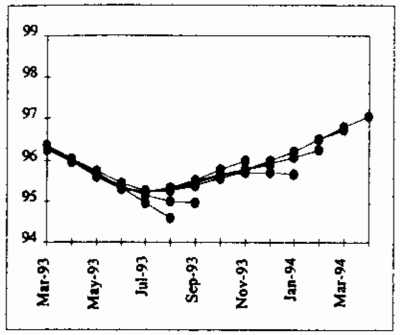


Fig. 7. With standard H-13

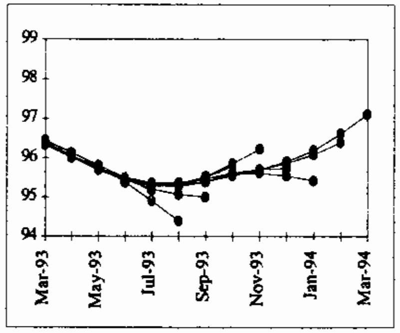


Fig. 8. With CSS($\lambda = 5.8\times10^{-6}$)

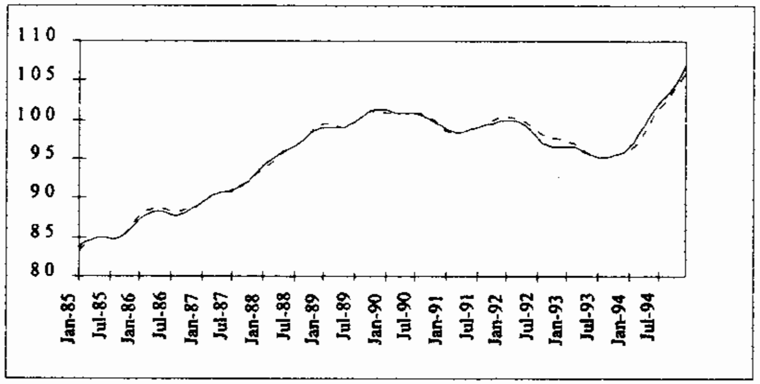


Fig.9. CSS ($\lambda = 5.8\times10^{-6}$) and standard H-13 trend-cycle estimates (IIP).
— *CSS* --- H-13.

Finally, we compare the cubic smoothing spline with $\lambda = 5.8\times10^{-6}$ with the trend-cycle estimate developed by Dagum (1996). The results shown in Figure 10 indicate a close relation between both trend-cycle curves except that the Dagum method based on a modification of the 13-term Henderson reduces two more false turning points. The revision path of the turning point starting in July 1993 with Dagum trend-cycle is given in Figure 11. It is apparent that July-August are very close and that it takes 4 months to detect the turning point.

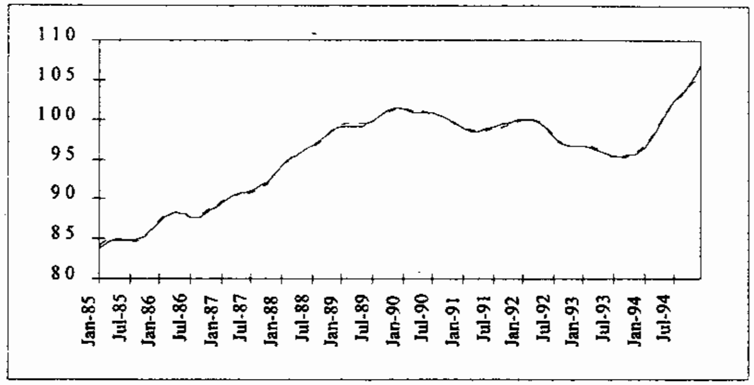


Fig.10. CSS ($\lambda = 5.8\times10^{-6}$) and Dagum trend-cycle estimates (IIP).
— *CSS* --- *Dagum.*

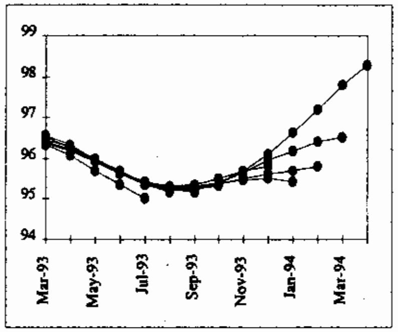


Fig.11. Revision path of July-August 1993 turning point with Dagum method (IIP).

Case 2. Canadian Leading Indicators

The three Canadian Leading Indicator series are New Orders for durable goods (NODG), Average Workweek in Manufacturing (AWM) and the House Spending Index (HSI) used in Dagum (1996). These series, for the period January 1981-December 1993, are seasonally adjusted and published by Statistics Canada.

For NODG and AWM, the automatic CSS trend-cycle estimator gives results similar to those of the standard H-13 with respect to both the number of unwanted ripples and time lag to detect the turning point. These are shown in Figures 12, 13 and 14 for NODG and in Figures 15,16 and 17 for AWM.

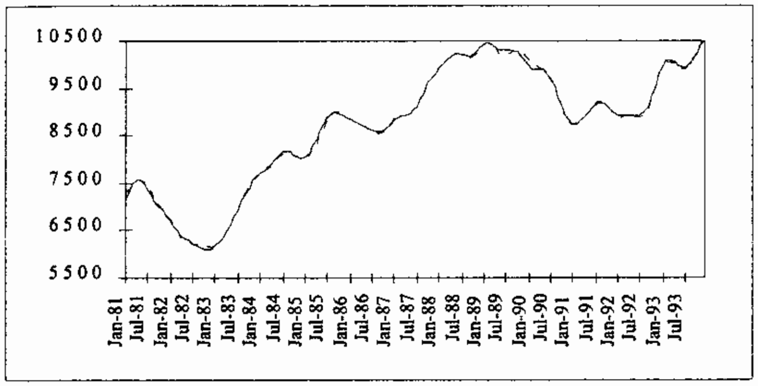


Fig.12. CSS and standard H-13 trend-cycle estimates (NODG).
— CSS --- H-13.

Revision path of February 1991 turning point
(NODG)

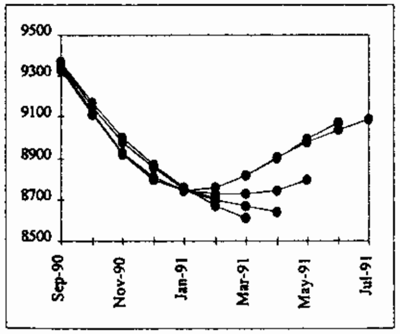


Fig.13. With standard H-13

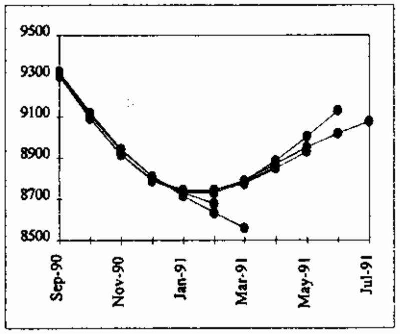


Fig.14. With CSS (automatic λ)

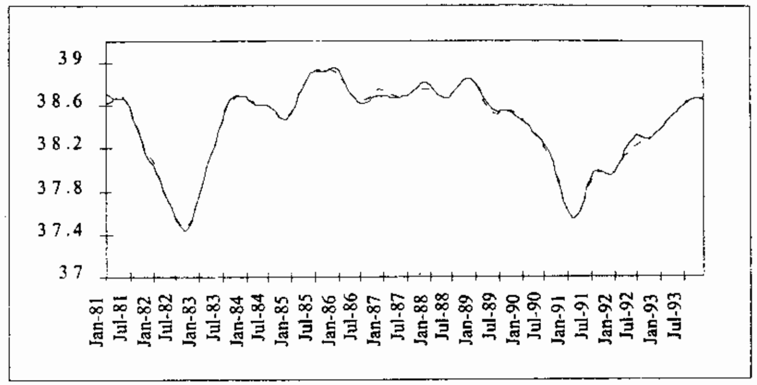


Fig.15 CSS and standard H-13 trend-cycle estimates (AWM).
—CSS --- H-13.

Revision path of March 1991 turning point
(AWM)

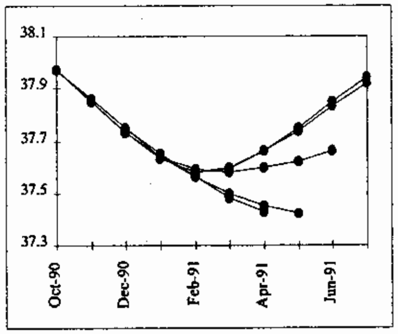


Fig.16. With standard H-13

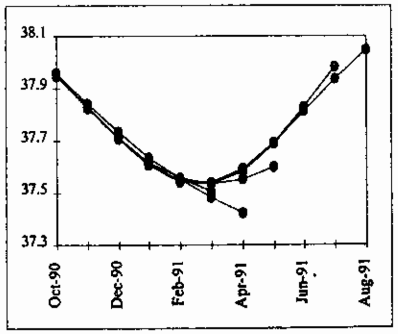


Fig.17. With CSS (automatic λ)

On the other hand, for the House Spending Index (HSI) series, the automatic CSS trend-cycle estimator produces much more false turning points than the standard H-13 as exhibit in Figure 18 but the time lag to detect the December 1990 turning point is of four months in both cases (see Figures 19 and 20).

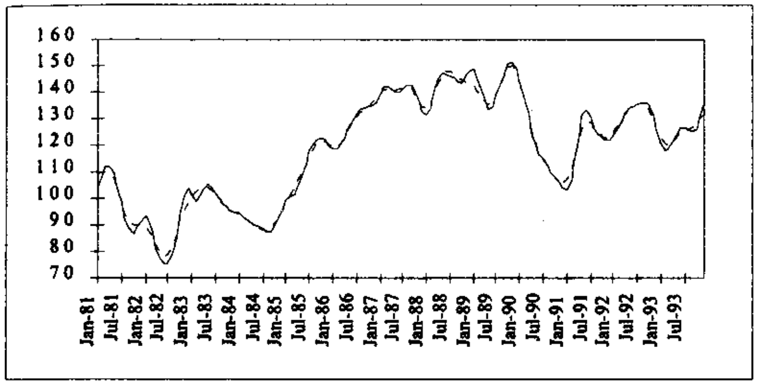


Fig.18. CSS and standard H-13 trend-cycle estimates (HSI). — CSS --- H-13.

Revision path of December 1990 turning point (HSI)

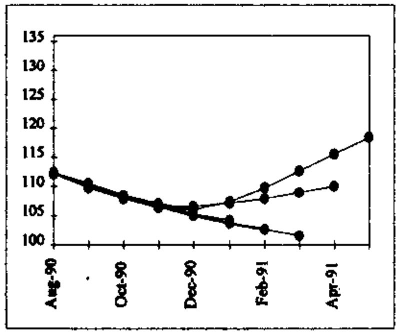


Fig.19. With standard H-13

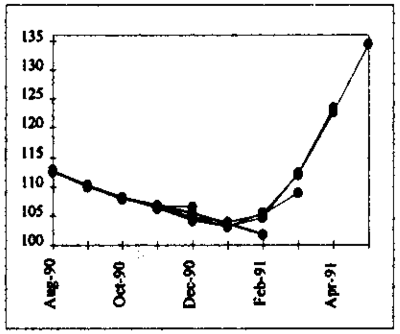


Fig.20. With CSS (automatic λ)

Similar to the Italian Index of Industrial Production, we looked for other values of the smoothing parameter to produce trend-cycle estimates resembling more those of the standard H-13 filter. The CSS with $\lambda = 5.8\times10^{-6}$ produces a trend-cycle curve very close to that of the standard H-13 with no distortions in the revision path of the December 1990 turning point (see Figures 21 and 22). This value of λ was

obtained by fitting the automatic CSS to the middle part of HSI series (1986-89) where the modified seasonally adjusted series changes its directions many times.

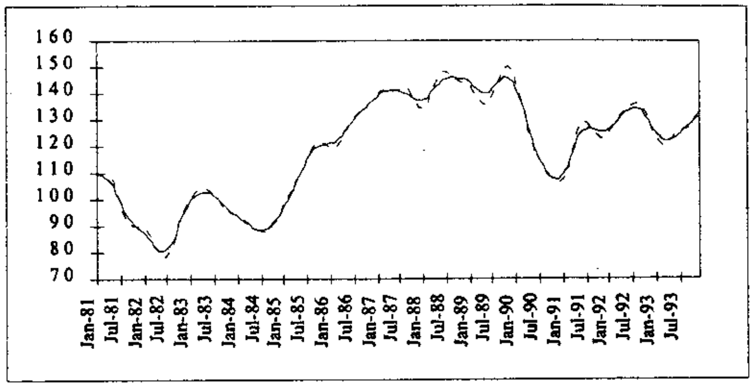


Fig.21. CSS ($\lambda = 5.8\times10^{-6}$) and standard H-13 trend-cycle estimates (HSI). — CSS --- H-13.

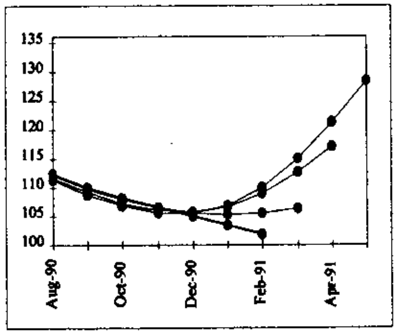


Fig.22. Revision path of December 1990 turning point (HSI) with CSS ($\lambda = 5.8\times10^{-6}$).

5. Turning Point Prediction

One important property of the CSS trend-cycle estimator is its capability for fast detection of turning points as new observations are added to the series. Cubic spline functions have first and second order derivatives and these can be interpreted as giving the trend-cycle rate of change and its acceleration, respectively. Both provide useful information on the upcoming of a turning point which depends on: (1) the presence of a zero point in the first order derivative and (2) the sign of the second order derivative at that point. To predict the upcoming of a turning point the following should be taken into consideration: (1) the presence of an inflection point in the first derivative curve, (2) the distance of the inflection point from the abscissa and (3) the acceleration given by the second order derivative. Instead of using the derivatives of the CSS trend-cycle, we here use the first and second order differences of the most recent months (the last six) to predict the upcoming of a turning point. This procedure can also be used with any other trend-cycle estimator as long as it detects turning points with short time lags and the revisions of the most recent value converge monotonically to the final value. Figure 23.a shows the time path of the first differences of the IIP cubic smoothing spline trend-cycle with $\lambda=5.8\times10^{-6}$ for the most recent months before July-August 1993 turning point. It is apparent that the rate of change of the series is rather constant and close to zero. This information together with the acceleration of the change, given in Figure 23.b, provide an indication of the arrival of a turning point. In fact, the second difference curve is also flat and close to zero, signalling that the acceleration of the change has stopped. It becomes positive with the addition of September, indicating that the turning point will be a trough.

July-August 1993 turning point
(IIP)

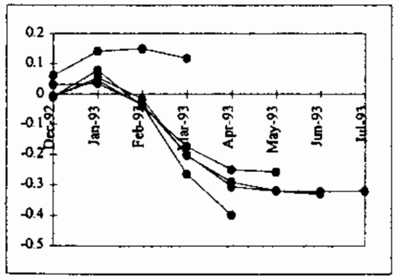


Fig.23a. CSS ($\lambda=5.8\times10^{-6}$) first differences.

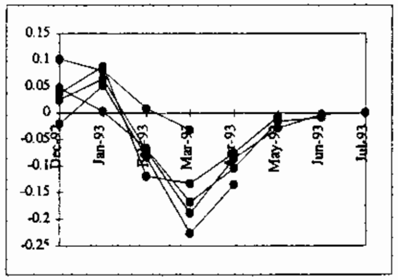


Fig.23b. CSS ($\lambda=5.8\times10^{-6}$) second differences

Figure 24.a shows the time path of the first differences of the NODG automatic CSS trend-cycle for the last six months before February 1991 turning point. It is apparent that the first difference curve is negative but linearly increasing and approaching to zero. The arrival of a turning point is confirmed as a point of maxima or minima by the time path of the second order differences. In this case, it is positive and thus indicating the presence of a trough (see Figure 24.b). Figures 25.a and 25.b provide similar kind of information to predict the arrival of the March 1991 turning point of the Average Workweek in Manufacturing series. In fact, we can see a pattern similar to the NODG series.

February 1991 turning point
(NODG)

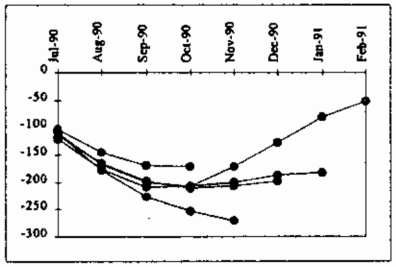


Fig.24a. CSS ($\lambda=5.8\times10^{-6}$) first differences.

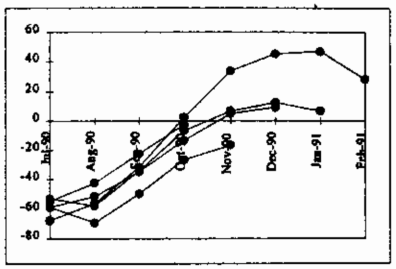


Fig.24b. CSS ($\lambda=5.8\times10^{-6}$) second differences.

March 1991 turning point
(AWM)

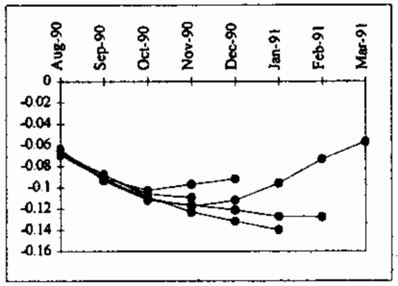


Fig.25a. CSS ($\lambda=5.8\times10^{-6}$) first differences.

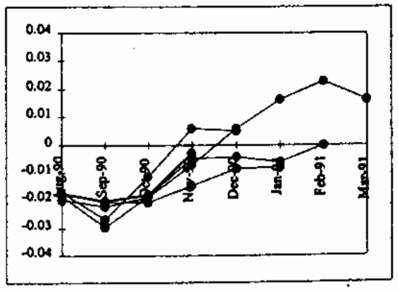


Fig.25b. CSS ($\lambda=5.8\times10^{-6}$) second differences.

Finally, to predict the arrival of the December 1990 turning point of the House Spending Index we look at Figures 26.a and 26.b. We can see from Figure 26.a that since July 1990 till November 1990, the changes are negative and the curves are nearly constant indicating a linear decrease in the original data but with the addition of December, the value is near zero signalling the arrival of a turning point. This information together with the fact that the acceleration is positive is useful to predict the upcoming of the turning point. It is also worthwhile to notice that the acceleration curve changes direction often since June 1990, coinciding with the high variability of this series during that period.

December 1990 turning point
(HSI)

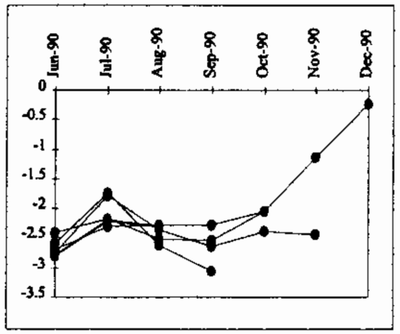


Fig. 26a. CSS ($\lambda = 5.8 \times 10^{-6}$) first differences.

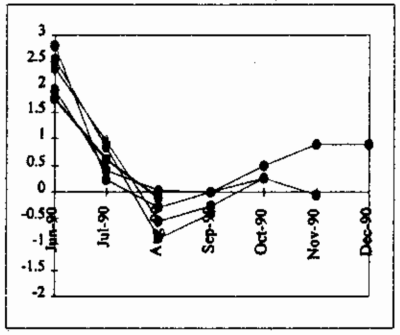


Fig.26b. CSS ($\lambda = 5.8 \times 10^{-6}$) second differences

6. Conclusions and Directions for Future Research.

We have investigated the use of cubic smoothing spline functions as an alternative to the standard 13-term Henderson filter often applied for trend-cycle estimation for current economic analysis. Both trend-cycle estimators are applied to seasonally adjusted data modified by extreme values and the comparison is based on: (1) number of unwanted ripples or false turning points present in the final trend-cycle estimates, and (2) time lag to detect a true turning point.

We used a very important Italian series, the Index of Industrial Production (IIP) which is a coincident indicator and three Canadian Leading Indicators, New Order for Durable Goods (NODG), Average Workeek in Manufacturing (AWM) and House Spending Index (HSI). The cubic smoothing spline (CSS) trend-cycle was first estimated with the default option that automatically selects both the number of knots and smoothing parameter. In this manner, the CSS trend-cycle estimator is as easy to apply as the standard 13-term Henderson filter. To our surprise, the results indicated that the automatic CSS produces estimates equivalent to the kind of Henderson filter chosen by the X11-ARIMA based on the noise-signal ratio (I/C) for the series under question. Hence, for the IIP and HSI series the estimates were very different from the H-13 trend-cycle. For the other two, the automatic CSS trend estimator gave results equivalent to those of the H-13 filter in the sense of criteria (1) and (2) above.

Further experimentation with IIP and HSI to find a smoothing parameter λ (no longer chosen automatically) to approximate the H-13 trend-cycle as imposed on these two series, lead to the value of $\lambda = 5.8 \times 10^{-6}$ as the one giving the closest estimates.

We proposed an approach to predict the upcoming of a turning point, given the availability of first and second order derivatives of cubic smoothing spline functions which can be interpreted as rate of change and acceleration of the trend-cycle, respectively.

In order to extend the possibility of using this approach for any trend-cycle estimator (as long as it detects rapidly true turning points and the revisions of current values converge monotonically to the final) we used first and second order differences instead of first and second order derivatives.

For the series analysed, first and second order differences were calculated for the most recent trend-cycle values and its time path monitored as new observations were added. The proposed procedure gave good results to predict the arrival of the true turning point of each series.

We intend to apply this procedure to a much larger set of coincident and leading Italian and Canadian Indicators in order to evaluate better its capability as a substitute of the standard 13-term Henderson filter. Further research is still needed on:

(1) Application of the CSS in a moving manner but with a fixed window, e.g five years of data .

(2) Using ARIMA extrapolations and a smoother input as in Dagum (1996) to see whether the CSS trend estimates are improved according to this author three main criteria.

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