Equivalent-Voltage Approach for Modeling Low-Frequency Dispersive Effects in Microwave FETs

A. Santarelli, *Member, IEEE*, G. Zucchelli, *Student Member, IEEE*, R. Paganelli, *Student Member, IEEE*, G. Vannini, *Member, IEEE*, and F. Filicori

***Abstract*—In this letter, a simple and efficient approach for the modeling of low-frequency dispersive phenomena in FETs is proposed. The method is based on the definition of a virtual, *nondispersive associated device* controlled by *equivalent port voltages* and it is justified on the basis of a physically-consistent, charge-controlled description of the device. Dispersive effects in FETs are accounted for by means of an intuitive circuit solution in the framework of any existing nonlinear dynamic model.**

The new equivalent-voltage model is identified on the basis of conventional measurements carried out under static and small-signal dynamic operating conditions. Nonlinear experimental tests confirm the validity of the proposed approach.

***Index Terms*—Electron device modeling, empirical modeling, low-frequency dispersive effects.**

I. Introduction

ACCURATE nonlinear modeling of FETs for microwave circuit design must also account for low-frequency dispersive phenomena of the electrical characteristics due to charge "trapping" and device self-heating [1]–[6]. These phenomena cause considerable deviations between static and dynamic (e.g., pulsed) measurements of the I/V characteristics, or, if we think in terms of differential parameters, frequency dependent behavior of the trans-admittance and output impedance even at low frequencies (e.g., lower than 1 MHz).

This letter shows how an intrinsic nondispersive associated device can be defined when dispersive phenomena are taken into account separately by means of "extrinsic" series voltage sources (as shown in Fig. 1), which are linearly controlled by the voltages at the device ports.

The following discussion only considers the presence of dispersion due to trapping phenomena, but future work will be devoted to deal with thermal phenomena in a comprehensive way.

II. Equivalent-Voltage Approach

Let us consider, first, an ideal intrinsic field effect transistor where no low-frequency dispersive phenomena are present, so that a purely algebraic nonlinear relationship can be assumed between charges and voltages. Such a device can be described, with acceptable accuracy, by adopting the charge-controlled quasi-static common-gate model formulation

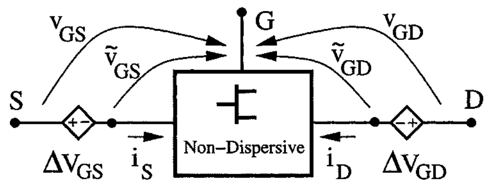

Fig. 1. Intrinsic device circuit schematic describing the equivalent-voltage approach for dispersive effects modeling. The voltage source at the drain port can be neglected by adopting a common source device description.

$$i(t) = \phi\{q(t)\} + \frac{dq(t)}{dt} \quad (1)$$

$$q(t) = \psi\{v(t)\} \quad (2)$$

where $i = [i_s \ i_d]^T, q = [q_{\rm gs} \ q_{\rm gd}]^T, v = [v_{\rm gs} \ v_{\rm gd}]^T$ represent, respectively, the vectors of the source and drain currents, the gate source and gate drain equivalent charges, which are dealt with as state-variables, and the intrinsic port voltages. Moreover, $\phi(\cdot) = [\phi_1(\cdot) \ \phi_2(\cdot)]^T, \psi(\cdot) = [\psi_1(\cdot) \ \psi_2(\cdot)]^T$ are suitable purely algebraic nonlinear functions.

The presence of low-frequency dispersive effects due to traps causes modifications in the charge-based state variables, introducing a dependence with relatively long memory duration of charges on past values of voltages. In such conditions, the stored charge $q(t)$ cannot be correctly predicted by the purely algebraic nondispersive (2), since a charge perturbation $\Delta q(t)$ due to the slow dynamics of dispersive phenomena must also be taken into account. Thus, the dispersive charge versus voltage model equation becomes: $q(t) = \psi\{v(t)\} + \Delta q(t)$.

However, an equivalent result can be obtained by still using the nondispersive (2) (i.e., the associated nondispersive device model) provided that the actual port voltages $v(t)$ are replaced by *equivalent port voltages* $\tilde{v}(t)$. These must clearly satisfy the equivalence condition [1]

$$\tilde{v}(t) = \psi^{-1}\{\psi\{v(t)\} + \Delta q(t)\} \doteq v(t) + \Delta v(t) \quad (3)$$

where dispersive effects are taken into account by the voltage perturbations $\Delta v(t) = \tilde{v}(t) - v(t)$. By adopting the equivalent

[1]We assume that, owing to the typically monotonic shape of the charge/voltage characteristics, the inverse function of ψ can be defined.

Manuscript received December 26, 2001; revised April 19, 2002. The review of this letter was arranged by Associate Editor Dr. Arvind Sharma.

A. Santarelli, G. Zucchelli, R. Paganelli, and F. Filicori are with the Department of Electronics, Computer Science and Systems, University of Bologna, Italy.

G. Vannini is with the Department of Engineering, University of Ferrara, Ferrara, Italy.

Publisher Item Identifier 10.1109/LMWC.2002.803148.

voltage description for the associated nondispersive device, we have

$$i(t) = F\{\tilde{v}(t)\} + C\{\tilde{v}(t)\} \cdot \frac{d\tilde{v}(t)}{dt} \quad (4)$$

where $F\{\tilde{v}\} = \phi\{\psi\{\tilde{v}\}\}$ and $C\{\tilde{v}\} = (d\psi(\tilde{v})/d\tilde{v})$ are purely-algebraic functions.

This shows that any intrinsic field effect transistor affected by dispersive trapping phenomena and excited by port voltages v can be described in terms of a virtual nondispersive associated device[2] excited by equivalent port voltages $\tilde{v} = v + \Delta v$. Thus, when a suitable identification procedure exists for the Δv terms, the nonlinear modeling of a dispersive device is transformed into the modeling of the associated nondispersive device; to this end, any nonlinear dynamic approach can be adopted such as, for example, lumped-element equivalent circuits. The circuit schematic in Fig. 1 is coherent with the model definition outlined above, where the Δv terms correspond to series voltage-controlled voltage sources, yet to be identified. Once the two sources are known, the associated nondispersive device can be obtained from a "de-embedding-like" operation on the intrinsic device, i.e., after parasitic identification.

The voltage perturbation term $\Delta v(t)$ represents a relatively small, yet not negligible, perturbation of a virtually nondispersive device, involving weak dependence [3] on the actual voltages $v(t)$. A simply linear approximation is assumed in this work. The validity of such an assumption is empirically confirmed by comparisons between model prediction and experimental data. The following frequency-domain vector relationship is therefore assumed:

$$\Delta V = A(\omega) \cdot V(\omega) \quad (5)$$

where $\Delta V, V$ are the Fourier transforms of $\Delta v, v$, respectively, and A is a suitable matrix of transfer functions, whose frequency dependence corresponds to the low-pass behavior of dispersive phenomena. In particular, for operation at microwave frequencies, (5) becomes

$$\Delta V = A_0 \cdot V_0 \quad (6)$$

where $A_0 = A(0)$ and $V_0 = V(0)$ are the dc components of $A(\omega)$ and $V(\omega)$, respectively.[4]

III. Model Identification

Simple linear algebraic relations allow for the transformation of the voltage perturbations, expressed by (6) for the common-gate device description, into the corresponding terms valid for the common-source device description denoted by the S subscript. Correspondingly, $F_S\{\cdot\}$ and $C_S\{\cdot\}$ are vectors of purely algebraic functions describing the associated nondispersive device in the $v_{\mathrm{GS}}, v_{\mathrm{DS}}$ domain.

Since all of the dynamic drain current characteristics give $i_D = 0$ for any v_{GS} when $v_{\mathrm{DS}} = 0$, the constraint: $A^S_{21_0} = A^S_{22_0} = 0$ must be satisfied, $A^S_{ij_0} (i, j = 1, 2)$ being the dc components of the $A^S(\omega)$ matrix elements. Thus, the circuit equations become

$$\begin{aligned} \tilde{v}_{\mathrm{GS}}(t) &= v_{\mathrm{GS}}(t) + A^S_{11_0} \cdot V_{G0} + A^S_{12_0} \cdot V_{D0} \\ \tilde{v}_{\mathrm{DS}}(t) &= v_{\mathrm{DS}}(t) \end{aligned} \quad (7)$$

where V_{G0}, V_{D0} are the mean values of $v_{\mathrm{GS}}(t), v_{\mathrm{DS}}(t)$ and $A^S_{11_0}, A^S_{12_0}$ are the only two model parameters to be identified. To this end, let us now consider static device operation, such that: $v_{\mathrm{GS}}(t) = V_{G0}$ and $v_{\mathrm{DS}}(t) = V_{D0}$. Thus

$$\begin{aligned} i(t) = I_0 &= F_S\{\tilde{V}_{G0}, V_{D0}\} = F_S^{\mathrm{DC}}\{V_{G0}, V_{D0}\} \\ &= F_S^{\mathrm{DC}} \left\{ \frac{\tilde{V}_{G0} - A^S_{12_0} \cdot V_{D0}}{1 + A^S_{11_0}}, V_{D0} \right\} \end{aligned} \quad (8)$$

where $F_S^{\mathrm{DC}}\{\cdot\}$ is the common-source dc current characteristic of the intrinsic device and $i = [i_G \; i_D]^T$. By differentiation of the drain current around a generic $\hat{V}_{G0}, \hat{V}_{D0}$ voltage pair, we obtain

$$\begin{cases} \hat{g}_m^{\mathrm{DC}} = \hat{g}_m^{\mathrm{AC}} \cdot \left(1 + A^S_{11_0}\right) \\ \hat{g}_d^{\mathrm{DC}} = \hat{g}_m^{\mathrm{AC}} \cdot A^S_{12_0} + \hat{g}_d^{\mathrm{AC}} \end{cases} \quad (9)$$

where $\hat{g}_m^{\mathrm{DC}}, \hat{g}_d^{\mathrm{DC}}, \hat{g}_m^{\mathrm{AC}}, \hat{g}_d^{\mathrm{AC}}$ are the static and low-frequency-dynamic device trans- and output-conductances, respectively. In the derivation of (9), it has been considered that static and low-frequency dynamic conductances of the nondispersive associated device are equal and coincide with the dynamic conductances of the real device, since the purely dc voltage perturbations Δv do not modify the ac components of electrical variables.

Identification of the two model coefficients $A^S_{11_0}, A^S_{12_0}$ can easily be carried out by means of (9). In fact, by considering a sufficiently large set of bias conditions for the measurement of the dc and ac differential parameters in (9), an overdetermined linear system of equations to be solved for $A^S_{11_0}$ and $A^S_{12_0}$, is obtained. The closed-form, analytical least-squares solution allows for easy and robust model identification.

Once the $A^S_{11_0}, A^S_{12_0}$ parameters are known, (8) can be directly used as the identification formula for the virtual nondispersive device current characteristics $F_S\{\cdot\}$ on the basis of the static measurements $F_S^{\mathrm{DC}}\{\cdot\}$ carried out on the actual device (after parasitic network resistances de-embedding). Note that the proposed approach is fully compatible with any existing high-frequency dynamic nonlinear model for the virtual associated nondispersive device. To this end, a simple preliminary "de-embedding" of the intrinsic device characteristics from the voltage-controlled voltage source $\Delta V_{\mathrm{GS}} = A^S_{11_0} \cdot V_{G0} + A^S_{12_0} \cdot V_{D0}$ is only required before model identification.

IV. Experimental Validation

The proposed equivalent-voltage approach has been applied for the modeling of low-frequency dispersive phenomena of different devices. Fig. 2 shows the comparison, for a

[2]The above-defined nondispersive associated device is not necessarily a device where traps are not present, but rather a device where trapped charges are "frozen" in a particular state.

[3]Instead, the charge perturbation $\Delta q(t)$ would be strongly dependent on the actual voltages. In fact, the Δq term has a relevant amplitude in the on-state device operation, while it becomes vanishingly small in or near to the off-state.

[4]A common case in microwave circuit analysis is that, apart from the dc value, every spectral component of the signals involved is above the cutoff frequency of trapping effects.

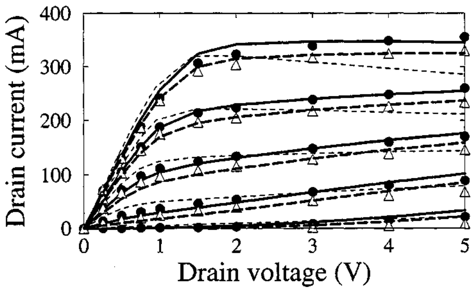

Fig. 2. Pulsed drain current characteristics of a medium-power MESFET for two different quiescent bias conditions (1: $V_{G_0} = -3$ V, $V_{D_0} = 3$ V; 2: $V_{G_0} = -1$ V; $V_{D_0} = 3$ V). Measurements (1: •; 2: △) versus predictions (1: solid lines; 2: dashed lines). Measured dc characteristics are also plotted (thin dashed lines). Model parameters values: $A^S_{11_0} = -0.109, A^S_{12_0} = -0.155$.

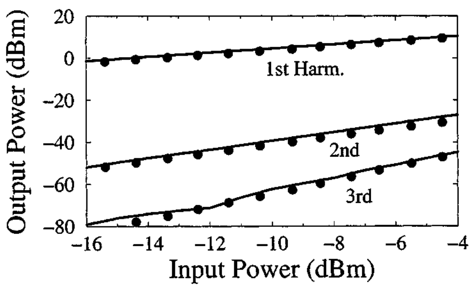

Fig. 3. Harmonic distortion in a 50-Ω-loaded power amplifier. Bias: $V_{G_0} = -0.55$ V, $V_{D_0} = 6$ V. Fundamental frequency: $f_0 = 5$ GHz. Measurements (•) versus predictions (-) by means of a nonlinear dynamic model [4] embedding the proposed approach. Model parameters values: $A^S_{11_0} = -0.019, A^S_{12_0} = -0.046$.

medium-power MESFET, between predicted dynamic drain current characteristics and measurements obtained by applying short, simultaneous voltage pulses at the gate/drain electrodes starting from different quiescent conditions [3]. Since nonnegligible self-heating phenomena occur in this device, as can be seen from dc characteristics also shown in Fig. 2, the identification of the $A^S_{11_0} A^S_{12_0}$ model parameters should be preferably carried out by also taking into account thermal effects in a comprehensive way. In any case, the good agreement shown in Fig. 2 substantially confirms the validity of the proposed approach.

Finally, the equivalent voltage approach has been tested together with a high-frequency nonlinear model for the virtual nondispersive associated device, namely the finite memory model [4]. In particular, a 600-μm PHEMT has been characterized on wafer under static and small-signal dynamic conditions up to 50 GHz. Fig. 3 shows good agreement between measurements and model predictions for harmonic distortion at 5 GHz, confirming the accuracy of the proposed approach also under large-signal high-frequency operation.

V. Conclusion

A new model of low-frequency dispersive phenomena in FETs has been presented. The approach is based on the definition of a nondispersive associated device, which is controlled by equivalent port voltages and can be identified on the basis of conventional dc and small-signal S-parameter measurements. Dispersive effects are described in a very easy way by means of a linearly voltage-controlled voltage source. The associated nondispersive device is suitable for modeling based on conventional nonlinear dynamic approaches in order to take into account also high-frequency dynamic effects. Experimental validation under pulsed and RF large-signal operation confirms the accuracy of the proposed approach.

Acknowledgment

The authors would like to thank Dr. A. Costantini for the harmonic distortion measurements.

References

[1] N. Scheinberg, R. Bayruns, and R. Goyal, "A low frequency GaAs MESFET circuit model," *IEEE J. Solid-State Circuits*, vol. 23, Apr. 1988.
[2] C. Camacho-Penalosa and C. S. Aitchison, "Modeling frequency dependence of output impedance of a microwave MESFET al low frequencies," *Electron. Lett.*, vol. 21, June 1985.
[3] T. Fernandez, Y. Newport, J. M. Zamanillo, A. Mediavilla, and A. Tazon, "High speed automated pulsed I/V measurement system," *23rd EUMC*, 1993.
[4] F. Filicori, A. Santarelli, P. Traverso, and G. Vannini, "Electron device model based on nonlinear discrete convolution for large-signal circuit analysis using commercial CAD packages," *GAAS*, Oct. 1999.
[5] F. Filicori, G. Vannini, A. Santarelli, A. Mediavilla, A. Tazon, and Y. Newport, "Empirical modeling of low-frequency dispersive effects due to traps and thermal phenomena in III-V FETs," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2972–2981, Dec. 1995.
[6] A. Santarelli, F. Filicori, G. Vannini, and P. Rinaldi, "Backgating model including self-heating for low-frequency dispersive effects in III-V FETs," *Electron. Lett.*, vol. 34, Oct. 1998.