# Hardware Implementation of a Broad-Band Vector Spectrum Analyzer Based on Randomized Sampling

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***Abstract*—A hardware prototype of vector spectrum analyzer, which is based on a randomized sampling strategy and associated alias-free digital signal processing (DSP) algorithms, has been fully designed and implemented. The instrument exploits a couple of identical independently programmable digital data acquisition channels, whose synchronization allows to sample the input signal at instants that can be very close along the time axis (tens of picoseconds), against a maximum single-channel average throughput rate of 250 ksamples per second (kSa/s). This architecture has shown to be suitable for the accurate implementation of the randomized periodic sampling with uniform jitter on which the instrument is based. In addition, the design solutions adopted for the sampling command generators, which exploit the interaction between a digital gate signal and a phase-controlled sinusoidal wave, together with the particular analytical definition proposed for the spectral estimators, have allowed to make practically negligible the quantum time according to which the sampling instants are generated. Experimental results are provided, which validate both the alias-free spectrum analysis technique and the capability of the architecture proposed of reliably implementing the analytical benefits deriving from the randomized sampling strategy adopted.**

***Index Terms*—Alias-free digital signal processing, aliasing suppression, digital spectral analysis, nonuniform sampling, randomized sampling, vector spectrum analyzer.**

## I. Introduction

THE experimental characterization of spectral information associated with periodic signals is commonly accomplished in modern instrumentation by applying suitable algorithms [e.g., fast Fourier transform (FFT)-based techniques] to finite sequences of signal samples. The equally spaced (i.e., periodic) scheme is certainly the most common sampling strategy adopted. It guarantees ease of implementation, efficient numerical processing, and a satisfying overall accuracy at the output of the spectrum analysis if well-known constraints on the measurement process are satisfied. Unfortunately, among these requirements, the theoretical minimum value of the sampling rate, which is strictly imposed when this digital acquisition scheme is adopted by the input signal bandwidth in order to avoid unacceptable aliasing effects, represents a critical parameter that can make in practice unreliable, very expensive, or even impossible the measurement in the presence of high-frequency signals or signals characterized by harmonics up to very high orders. The technological problems to be dealt with in the implementation of very high values of sampling rate which preserve, at the same time, an adequate vertical resolution, are in all respects the actual limitation to the instrument input bandwidth, even when the frequency response of the analog circuitry within its digital data acquisition channels [amplifiers /attenuators, filters, transmission lines, and sample/hold (S/H) stages] would allow for much higher dynamics at the input. In the context of spectral investigation, the reduction of the sampling step down to the values which are typical of very fast analog-to-digital (A/D) converters (such as those adopted within modern top-quality oscilloscopes) can be hardly considered, since the low vertical resolution (8 bits) provided by these devices would dramatically reduce the accuracy of the numerical algorithms, which are applied to the signal samples and exploited for the vector spectral analysis.

In order to overcome the bandwidth limitations, which are inherently introduced by the conventional equally spaced sampling strategy, many alternative digital data acquisition schemes have been largely considered in the literature. From a very general standpoint, all of them can be classified as *nonuniform sampling* since, despite the strong difference in terms of sample distribution properties along the time axis, the sequences generated by the approaches proposed are all characterized by a structure, which some way or other differs from the strictly organized scheme of equally spaced sampling. In practice, nonuniform sampling is referred mostly as the result of some kind of nonideality or wrong operation during the digital acquisition, which otherwise would follow a conventional periodic strategy. The target signal can be inaccessible during several time slots within the overall observation interval needed, as in many radio–astronomical and geophysical applications [1]–[4], or strongly perturbed by noise and interference, with the consequent loss of single samples or subsequences [5]. Temporary, recoverable short failures within the operation of the instruments or measurement subsystems can be the reason for similar discontinuities of the acquired digital record. Some examples of deliberately generated irregular sampling (according to deterministic analytical schemes) can be found in the literature, for general-purpose [6], [7], as well

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as special-purpose applications such as tomography [8], optics [9], telecommunications [10], antenna techniques [11], data compression [12], and many others. Minimum-norm optimal solutions for the reconstruction of band-limited function properties from irregular sampling are known since several decades [13]–[15]. More recently, several methods have been proposed to extend the discussion also in the presence of noise [16]. A quite complete survey on the subject can be found in [17].

When the deliberate irregular sampling is devoted to the application of spectral investigation techniques, which allow for the accurate estimation of input signal harmonic characteristics even in the presence of an average sampling rate well below the value imposed by the Shannon criterion if a conventional strategy would have been considered, *statistical* approaches are usually adopted. Techniques for the *randomized* generation of the sampling time instants have been proposed since the 1960s in many pioneering papers [18]–[21]. Since then, the prospect of obtaining new-generation, alias-free instruments with very broad digital bandwidth and high vertical resolution has attracted the scientific community toward in-depth theoretical developments regarding many randomized sampling strategies and associated digital signal processing (DSP) algorithms, together with their performance estimation through both analytical discussion and simulation [22]–[26]. Nevertheless, the theoretical study has not been sufficiently accompanied by design and prototyping activity, which is aimed at the identification of suitable hardware architectures capable of implementing the nonconventional sampling techniques, as well as carrying out the experimental validation and performance testing of the spectral analysis algorithms, which make use of data acquired through them. Some efforts in this sense can be found in [27] and [28].

In recent years, several randomized sampling techniques have been extensively investigated (e.g., in [29]) by the authors and implemented for the design of digital wattmeters [30] and power spectrum analyzers [31], [32]. More recently, one of these techniques (namely, the randomized periodic sampling with uniform jitter) has been considered for *vector* spectral analysis: the measurement procedure has been proposed [33], and the statistical investigation performed [34], which is aimed at the evaluation of the methodology accuracy. In particular, the expected value and the variance associated with the harmonic estimators proposed by this approach at the output of the measurement process have been analytically computed.

In this paper, the design solutions and the hardware architecture are described, which have been exploited for the laboratory implementation of a vector spectrum analyzer based on the above randomized sampling and the associated alias-free DSP algorithms. The prototype practical realization has been aimed both at the validation of the main design idea, which has allowed for the reliable implementation of the randomized sampling technique, and the experimental testing of the spectrum analysis methodology proposed in [33] and [34]. Measurement results showing the performance achieved both in terms of implementation accuracy and method capabilities are provided. The discussion is organized as follows. In Section II, the analytical description of the randomized sampling scheme adopted and the vector spectrum digital estimation algorithms will be shortly recalled. The prototype twin-channel architecture is illustrated in Section III, while Section IV will be devoted to the discussion of experimental tests and results. Conclusions will be provided in Section V.

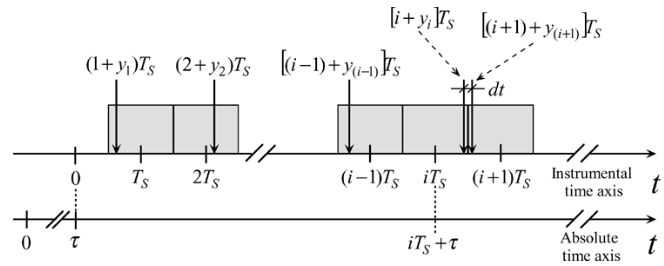


Fig. 1. Representation of a generic sampling sequence realization when the randomized periodic sampling with uniform jitter scheme is adopted. The presence of a couple of successive acquisitions at an infinitesimal distance $dt$ along the time axis, with this event to be considered inherent in the algorithm (1), is pointed out.

## II. Randomized Sampling-Based Measurement Procedure

The randomized sampling technique adopted in [33] and [34] for the alias-free vector spectral analysis of periodic signals is described by the following *random point process*:

$$t_i = \tau + (i + Y_i)\, T_s \tag{1}$$

where $T_S$ is the *mean* sampling step and $\{Y_i\}$ is a set of zero-mean random variables, which describe the statistical distribution $p_i(t)$ of each random sampling instant $t_i$ around the expected value $\bar{t}_i = \tau + iT_S$. For the sake of generality and methodology success, the unknown offset $\tau$ has been considered, between the origin of the sampling instant generator time axis $t'$ (*local* to the instrument) and that pertaining to the *absolute* time axis $t$, which is exploited to analytically describe the input signal [with practically finite spectrum ($2M + 1$ harmonics) and fundamental frequency $f_1$] by means of the function $x(t)$ (Fourier series expansion):

$$x(t) = \sum_{q=-M}^{+M} X_q \mathrm{e}^{j2\pi q f_1 t}. \tag{2}$$

The random variables $\{Y_i\}$ are statistically independent and each *uniformly* distributed within the interval $(-1/2, +1/2)$. This sampling scheme is well known as randomized periodic sampling with *uniform jitter* (see Fig. 1 for the representation of a generic *realization* of the point process (1)).

By adopting this strategy, in which all the sampling instants are statistically uncorrelated, the global *sampling point density function* defined as

$$p(t) = \sum_i p_i(t) \tag{3}$$

is strictly constant within all the observation interval and equal to the value $1/T_S$: this means that each instantaneous value of the input signal has "the same probability" to be sampled during the measurement process. This represents an important feature of the model (1) since, from a general standpoint, any randomized sampling scheme potentially perturbs the input signal spectrum: in fact, the spectrum that is actually evaluated can be described in terms of the convolution between the input spectrum and the density function (3) representation in the frequency domain, independently of the algorithms that are exploited in the DSP analysis. Other randomized periodic sampling strategies, such as those adopting a nonuniform jitter, are not associated with a constant $p(t)$; thus, they inherently introduce a spectral distortion at the output of the analysis. The schemes that can be classified as additive randomized sampling [23] have a sampling point density that tends to a constant value only after a transient from the starting instant of the observation. Although in the past the authors exploited additive randomized sampling [30]–[32], for the design and implementation of alias-free digital instrumentation, in the last few years, the investigation has been focused on the sampling strategy described by model (1). Besides presenting the fundamental property discussed previously, not only in terms of approximation but instead from a rigorous analytical standpoint, the randomized periodic sampling with uniform jitter leads to realizations of the point process, which are characterized by sampling instants placed close to well-defined deterministic points on the time axis, and the random distribution affecting the actual sampling instant around the nominal value is very narrow if compared with the duration of the overall observation of the input signal, with many benefits in terms of implementation practical feasibility for this randomized sampling technique.

Since the sequence of sampling instants resulting from a realization of the point process (1) is not uniformly spaced, conventional algorithms [e.g., the discrete Fourier transform (DFT)] cannot be directly applied, and alternative procedures are needed for a successful processing of the samples obtained. A different analytical expression for the vector estimate of each input signal spectral component is thus introduced in the following:

$$\hat{X}_n\left(p_0,\tau,N,\widetilde{f}_1\right)=\frac{1}{2N+1}\sum_{i=p_0-N}^{i=p_0+N}x\left(t_i\right)\mathrm{e}^{-j2\pi n\widetilde{f}_1 t_i'} \quad (4)$$

which represents a direct approximation, for numerical implementation purposes, of (2)'s analytical solution

$$X_n=\frac{1}{T_1}\int_{-T_1/2}^{+T_1/2}x(t)\mathrm{e}^{-j2\pi nf_1 t}\mathrm{d}t \text{ with } 1\leq n\leq +M,\ T_1=\frac{1}{f_1} \quad (5)$$

where $X_{-n}=X_n^*$, with $x(t)$ being a real signal. It can be easily shown that the conventional DFT operator is a particular case of (4), when a deterministic, equally spaced sampling scheme is considered for the acquisition of experimental information.

The $n$th harmonic estimate described by (4), which is itself a random variable—more precisely, an *estimator*—fully correlated to the random point process, depends on the total number $(2N+1)$ of samples within the subsequence, which is extracted from the global realization of process (1) and used in (4), the index $p_0$ associated with the "central" element of the latter (i.e., the position of the subsequence along the overall observation interval) and the estimate $\widetilde{f}_1$ of the input signal fundamental frequency. In addition, expression (4) is implicitly a function with respect to the time offset $\tau$, which can be interpreted as a random continuous variable uniformly distributed within the interval $(-T', +T')$ with $T'$ tending to infinity.[1] Since this dependence introduces, for each realization of the estimator (4), a perturbing random phase factor that practically would reduce to zero the estimator expected value, the *ratio* between $\hat{X}_n$ and the estimate $\hat{X}_1$ (raised to the $n$th power) of the fundamental harmonic component has been proposed as the actual output of the instrument [33], [34]

$$\hat{\rho}_n=\frac{\hat{X}_n}{\left(\hat{X}_1\right)^n},\qquad 2\leq n\leq +M. \quad (6)$$

The statistical moments (with respect to also both $\tau$ and index $p_0$) of the normalized estimators (6) have been computed. In particular, it results[2] [33]

$$\underset{\rightarrow}{\mathrm{E}}\left\{\hat{\rho}_n\right\}\cong\frac{X_n}{X_1^n}\frac{\mathrm{sinc}\,(n\Delta)}{\mathrm{sinc}^n\,(\Delta)} \text{ with } \Delta=\left(f_1-\widetilde{f}_1\right)(2N+1)\,T_s \quad (7)$$

if a sufficiently large number $(2N+1)$ of signal samples have been taken into account.

Thus, the experimental mean value, computed from a set of normalized estimator realizations (each corresponding to a subsequence of samples extracted from a realization of (1), the latter obtained strictly *asynchronously* with respect both the signal and all the other sequences—i.e., each sampling realization is associated to a different realization of offset $\tau$) is a good estimate for the ratio between the signal $n$th harmonic and the fundamental component (to the $n$th power) of $x(t)$, since the relative amplitude error $(1-\mathrm{sinc}(n\Delta)/\mathrm{sinc}^n(\Delta))$ can be made very small by means of an adequately accurate estimation of $f_1$. It is important to emphasize that this bias effect can be reduced below a given specification independently of the (low) value of the average sampling rate $f_S=1/T_S$, since the quantity $\Delta$ is an infinitesimal with respect to $(f_1-\widetilde{f}_1)$. A suitable technique is proposed in [33] for the run-time estimation of both $f_1$ and the magnitude of the fundamental $X_1$, which is needed in order to derive the estimate of each harmonic $X_n$ from (7).

The asymptotic variance of normalized estimators (6) has been computed as well [34], in order to perform the accuracy analysis of the proposed measurement procedure. Both the exact and approximated (under mild hypotheses, which can be considered satisfied if a good estimation of the signal frequency is achieved) expressions for $\underset{\rightarrow}{\mathrm{Var}}\left\{\hat{\rho}_n\right\}$ have been achieved showing that, among other properties, the statistical dispersion of the instrument output around its expected value is inversely proportional to the number of samples exploited in the computation of estimator (6) experimental realizations.

[1]The assumptions made on the statistical properties of $\tau$ descend from the application of the *Bayesian approach* [35], useful for the analytical treatment of random variables in the presence of a total lack of information about them.

[2]The formalism $\underset{\rightarrow}{E}\{\cdot\}$ indicates that the *asymptotic* expected value (i.e., for $T'\rightarrow\infty$) has been considered.

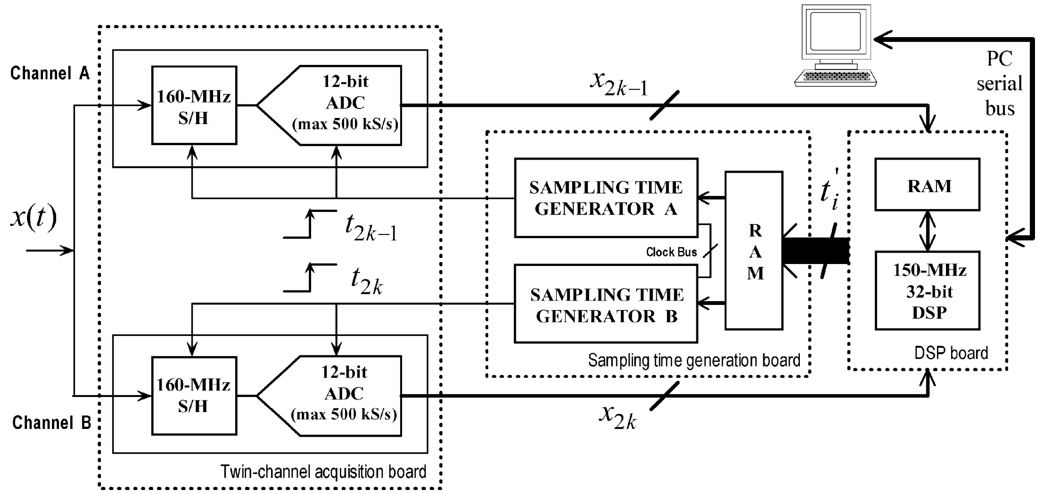


Fig. 2. Twin-channel architecture of the instrument prototype.

## III. Hardware Implementation of the Instrument

In this section, the architecture and the main design solutions are described of a hardware instrument prototype, which has been fully implemented in order to test the proposed alias-free digital spectral analysis procedure based on the randomized sampling strategy described by (1) and the normalized estimators (6). As shown in the following, the hardware prototype architecture is very flexible and allows to accurately implement not only the model (1), but also a wide family of deliberately nonuniform sampling strategies.

### A. Twin-Channel Architecture

As briefly discussed in the previous section, the nature of the probability distribution along the time axis of each sampling instant, which is part of the random point process (1), leads to the important property of the randomized sampling scheme adopted of being *stationary* (i.e., each instantaneous value of the input signal is sampled with equal probability), with many advantages over other randomized techniques in terms of input signal spectrum distortion suppression.

Unfortunately, this condition is achieved only if the statistical distribution of the random variables $\{Y_i\}$ is *strictly* uniform within their variability interval $(-1/2, +1/2)$ up to the boundaries: this means that, from a theoretical standpoint, a couple of successive sampling instants (better, their realizations) could be separated by an *infinitesimal* time interval $dt$ (see again Fig. 1), with evident limitations to the practical implementation of the instrument when a single digital data acquisition channel is considered.

In order to overcome such a drawback of the sampling scheme (1), its implementation has been carried out by adopting a two fully-independent channel architecture. Fig. 2 illustrates the main blocks within the instrument prototype. A motherboard with a 32-b, 150-MHz TI-TMS320C6711 DSP with 12 MB of RAM is in charge of controlling the overall system. As first step of the measurement procedure, the DSP unit computes the $N_P$ realizations of the point process (1) (pseudorandom sequences, each with a different realization of offset $\tau$) according to the algorithm chosen. In this paper, the described scheme (1) has been considered, but the instrument time-base can be actually programmed for the implementation of a wide family of different strategies, randomized or deterministically nonuniform, and the prototype can be operated (in some cases, with limitations in terms of equivalent sampling rate) for the performance experimental test of different state-of-the-art alias-free techniques [22], [23], [26], [28].

The DSP motherboard controls, through its two input–output (I/O) buses, a couple of fully independent sampling time generators, which have been implemented onto a separate board. During the second step, the binary data, which codify the precomputed sampling sequences, are sent to this board, which has by itself RAM storage capabilities. Each couple of two successive sampling time values within a generic sequence is *forked* to the generators (say, the odd-indexed time value $t'_{2k-1}$ to generator A and the even-indexed $t'_{2k}$ to generator B). By means of such an architectural solution, the practical generation of the sampling pulses corresponding to the DSP-computed pseudorandomized time instants is possible even in the presence of couples that are very close along the time axis, allowing to satisfy an accurately uniform statistical distribution of instants $t_i$ around their mean values.

### B. Sampling Time Generators

The generation of the sampling gate transitions is carried out by two identical time generators, each one based on the synchronization between a 150-MHz digital down-counter and a 14-b-phase programmable sinusoidal source. The interaction between the phase-controlled 100-MHz analog sinusoid and the end-of-count digital transition at the output of the counter generates, at the DSP precomputed time value, a sampling clock command at the output of a very fast op-amp comparator. By means of this design solution, which exploits the interaction between a digital and an analog waveform, the time-resolution (*time quantum*) $\Delta T_S$ according to which the sampling instants can be generated is very short (nominally $\Delta T_S < 1$ ps). In addition, the "hidden periodicity," which would be inherently associated with the actual realizations of the random point process (1) due to the residual time resolution $\Delta T_S$ is made further negligible, since the $N_P$ subsequences of $(2N+1)$ samples exploited for the estimation of the harmonic components through expressions (6) are measured, each one on an independent, asynchronous operative time axis with a random origin, and the overall time resolution is strongly reduced by the averaging process performed on the $N_P$ realizations of (6).

This theoretical very high performance in terms of equivalent sampling rate is partially perturbed by nonidealities of different nature, such as the jitter effects at the gate level, the phase noise of the sinusoidal signal and the noise at the input of the comparator (where the two timing signals interact), which lead to an expanded uncertainty $U_S$ of the time value, at which the sampling command is synthesized, of several tens of picoseconds. The overall consequence is an additional random jitter (in the conventional sense) on the precomputed sampling instants. Since the probability density function (pdf) of such jitter effect is not controllable, besides issues of inaccuracy due to the discrepancy between the precomputed and the actual value of the instant at which the signal is sampled, the sampling point density (3) implemented is not strictly uniform and the nondistortion property of the strategy is somehow slightly not verified. Moreover, the DSP algorithm must take into account the above uncertainty during the calculation of pseudorandom instants, by reducing the statistical distribution of them within a shorter interval of width $(T_S - 2U_S)$ around the expected values. Without this precaution, the *time inversion* of two successive sampling instants could affect the physically generated sequences.

### C. Acquisition and A/D Conversion

The gate signal at the output of each sampling time generator controls a separate digital data acquisition channel. Within each channel, an Analog Device AD9101 S/H with 160-MHz analog bandwidth and fast acquisition time receives the input signal $x(t)$ (a front-end conditioning circuitry—amplifiers/attenuators and filters—can be activated on the signal path in order to obtain different instrumental ranges) and transfers the S/H level to an accurate 12-b, $\pm$1.5-LSB INL, Analog Device AD7892 ADC with 1.7-$\mu$s acquisition and conversion time (maximum average throughput rate of the instrument: 500 kSa/s) for the conversion to digital of the signal samples. The adoption of a high-performance S/H device is essential, since the overall bandwidth of the instrument is imposed by the frequency response of this component, the equivalent digital bandwidth being virtually nonlimited. The effect of the relevant droop rate, which usually affects very fast S/H devices, has been minimized by choosing an analog-to-digital converter (ADC) with internal S/H capabilities. It is worth to note the nonconventional matching, within the acquisition channels, between the 160-MHz S/H bandwidth and the theoretical (according to the Shannon criterion in presence of classical periodic sampling) 250-kHz digital bandwidth of the ADC device. Even better performance in terms of input bandwidth and vertical resolution could have been achieved by simply choosing, in the design phase, faster S/Hs and slightly more expensive ADCs, without the need for the increase of the average sampling rate. Nevertheless, the purpose of prototype realization has been that of experimentally validating the alias-free spectral analysis procedure and the main architectural idea at the basis of the reliable implementation of randomized periodic sampling, more than overcoming the performance of present top-level commercial instrumentation.

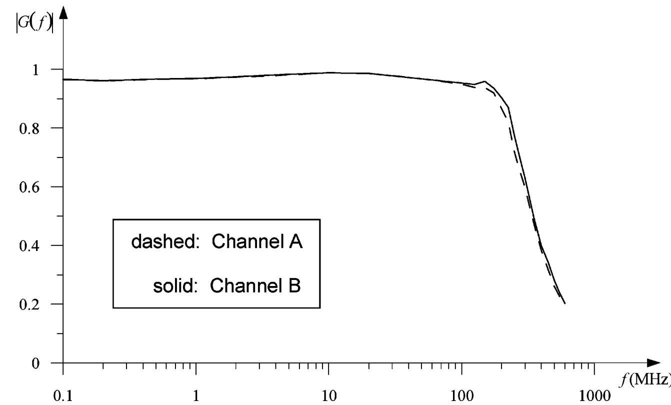

Fig. 3. Experimental characterization of the frequency response of both acquisition channels.

The data alternatively acquired by the two channels are finally stored into the RAM memory and read by the DSP in order to process the sample vectors and obtain the signal harmonic estimates through the algorithm described. Since the DSP device computes *all* the realizations of the point process (1) and stores them within the board memory *before* the acquisition procedure starts [a complex programmable logic device (CPLD) is in charge of passing the coded information from the RAM to the time generators], its floating-point resources are fully available for the processing step while the acquisition proceeds, allowing to consider a high number of records with a quite short processing time.

## IV. Experimental Results

The instrument prototype obtained is characterized by high flexibility, since it can be operated according to many possible sampling strategies of different nature. In particular, it accomplishes an accurate implementation of the randomized periodic sampling with uniform jitter described by the point process (1). While the maximum average throughput rate which is required to each acquisition channel is equal to $1/2T_S$ (up to 250 kSa/s, with a minimum interval of 2 $\mu$s between two successive sampling instants), a couple of adjacent instants within one of the precalculated *overall* sampling sequences can be generated at the distance of a few tens of picoseconds. This feature makes the statistical distribution of the realizations of random variables $\{Y_i\}$ very close to the theoretical uniform window required by the methodology.

Fig. 3 shows the experimentally characterized frequency response of the two channels. The instrument can entirely exploit its wide analog bandwidth ($>$ 150 MHz) even in the presence of a very low average digital throughput rate.

To the aim of testing the statistical properties of the sampling sequences that are generated by the board, it can be observed that the $i$th time interval between two successive acquisitions

$$\Delta T_i = T_S\left[1 + (Y_{i+1} - Y_i)\right] \tag{8}$$

is a random variable defined within the domain (0, $2T_S$) with a pdf that is the convolution between those pertaining to the variables $Y_i$ and $Y_{i+1}$. An accurate time interval analyzer has

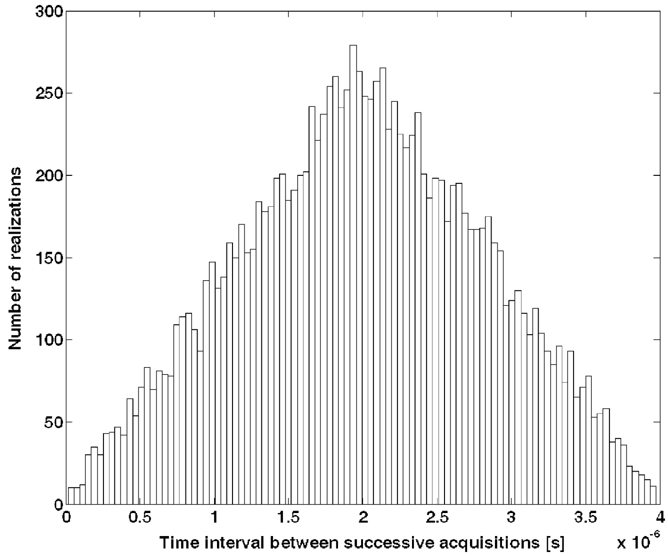


Fig. 4. Experimental statistical distribution of the time interval between successive sampling pulses at the output of the time generators ($T_S = 2\ \mu$s).

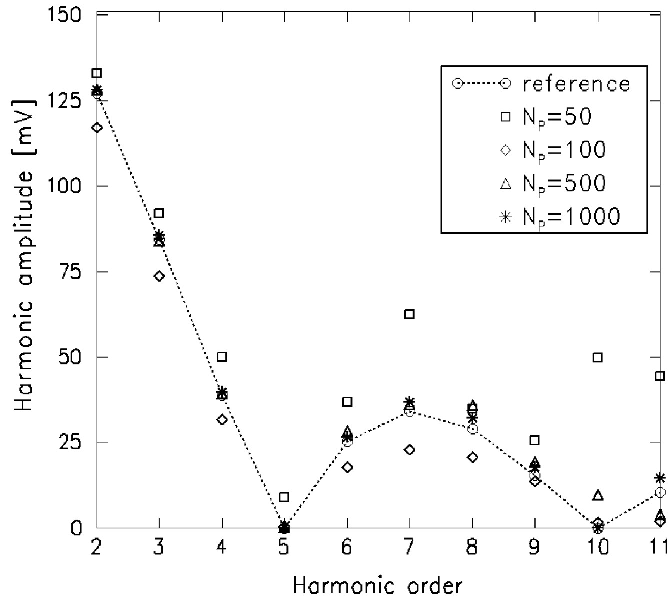


Fig. 5. Magnitude of the estimated harmonic components (second to eleventh orders) at the output of the instrument, for different values $N_p$ of the sampling sequences exploited in the averaging procedure (20%-duty-cycle, 1-MHz rectangular input waveform).

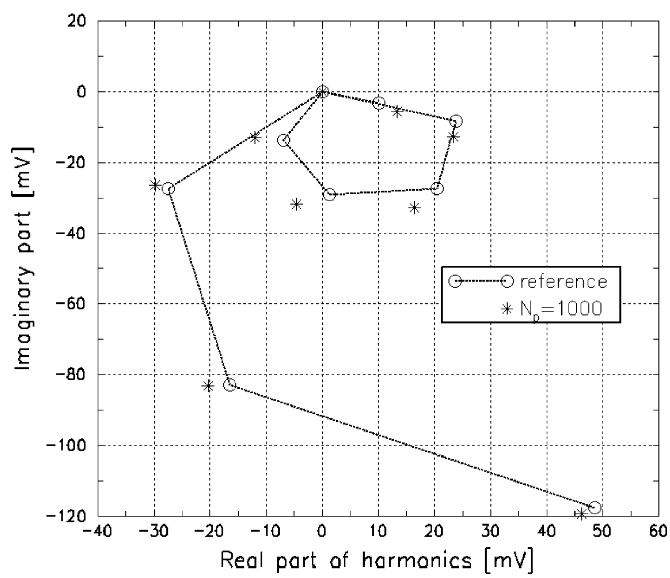


Fig. 6. Representation on the complex plane of the estimated vector harmonics, whose magnitudes are shown in Fig. 5 ($N_p = 1000$).

been exploited in order to characterize the output of the sampling time generators for $T_S = 2\ \mu$s. Fig. 4 shows the experimental distribution measured for $\Delta T_i$, which matches the *triangular* theoretical shape with good accuracy up to the boundaries of the variability domain. A *direct* analysis is in progress on the distribution of experimental instants $t_i$. This is actually a quite difficult task, since the realizations of each sampling instant $t_i$ are nominally generated with a time resolution which is in the field of picoseconds, while the width of the interval in which $t_i$ is distributed is equal to 2 $\mu$s. This means that many millions of realizations for the sampling instant will be needed for an accurate statistical inference with a small, critical value of the time-domain discretization step.

Different tests have been carried out in order to estimate the additive *stochastic* jitter affecting the generation of sampling commands at the output of time generators. In particular, the time interval analyzer has been exploited for the statistical moment estimation on sequences of equal-width rectangular pulses, obtained by considering as raising and falling transitions the gate commands at the output of generator A and B, respectively. The estimation of experimental variance of the pulsewidth leads to a reasonable estimate of the variance of the jitter affecting the single sampling instants, since the two transitions are fully uncorrelated. The $3\sigma$-values obtained for the jitter have shown to be in the range of tens of picoseconds and practically independent of the pulse nominal duration: 23 and 26 ps have been measured for 10 and 20 $\mu$s of pulsewidth, respectively.

In the following, an example of measurement is provided, which shows the capability of the instrument of characterizing the vector spectrum of wide-band input signals. A 20%-duty-cycle, rectangular wave with 1-$\mu$s period and 800-mV peak-to-peak amplitude has been generated and applied to the input of the prototype without any anti-alias filtering. In such conditions, a conventional (i.e., based on equally spaced sampling) digital spectral analysis exploiting 12-b resolution ADCs would require a sampling rate of 100 MSa/s at least, since the harmonics of the input signal should be considered important (in terms of aliasing effect) up to the fiftieth order. The prototype has been operated instead with a 236-kSa/s average throughput rate per channel ($T_S = 2.12\ \mu$s). In order to evaluate the convergence properties of the estimated main value of the normalized harmonic estimators to the theoretical expected one (7), different values for the number $N_p$ of point process (1) realizations (each with $N = 100$) have been considered during the averaging procedure. Fig. 5 shows the output of the prototype (magnitudes) from the second up to the eleventh harmonic component. A number of $N_p = 1000$ sequences allows to achieve good results, even though $N_p = 500$ could be an acceptable tradeoff between accuracy and overall observation time duration. Fig. 6 is the vector representation of the instrument output (second to eleventh orders), compared with the reference. Since the input waveform harmonics are characterized by phases that are distributed very strictly around the two values $\{0, \pi\}$, an arbitrary phase has been associated with the fundamental in order to make the plot on the complex plane more appreciable.

## V. Conclusion

The alias-free digital analysis, which is based on the randomized periodic sampling with uniform jitter strategy (1) and the definition of suitable harmonic estimators (6), has been successfully implemented in a vector spectrum analyzer prototype. The randomized sampling model considered offers attractive analytical properties in terms of input spectrum distortion suppression. The twin-channel architecture adopted has allowed to overcome the main drawback in the theoretical formulation of the randomized sampling technique, which is the presence, within the realizations of the random point process (1), of couples $(t_k, t_{k+1})$ that can be very close along the time axis. In addition, the introduction into the estimator definitions of the dependence with respect to the random asynchronous offset between the absolute and the instrumental time axes has allowed to minimize the time quantum, which is associated with the sampling command generation, thus maximize the equivalent digital bandwidth of the instrument. Low average throughput rate is required for each single acquisition channel (maximum 250 kSa/s) against an overall instrumental bandwidth of 150 MHz.

Experimental results have been provided, which show the capability of the instrument of performing accurate vector spectral analysis with a bandwidth depending only on the frequency response of the S/H devices adopted. The flexibility that characterizes the instrument will allow in the near future for the implementation and performance experimental investigation of other nonconventional sampling schemes, based on algorithms for the generation of both randomized and deterministically nonuniform sampling instant sequences.

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