Workers' Enterprises Are Not Perverse: Differential Oligopoly Games with Sticky Price

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Abstract

We take a differential game approach to study the dynamic behaviour of labour managed (LM) firms, in the presence of price stickiness. We find that the oligopoly market populated by LM firms reaches the same steady state equilibrium allocation as the oligopoly populated by profit-maximising (PM) firms, provided that the LM membership and the PM labour force are set before the market game starts. The conclusion holds under both the open-loop solution and the closed-loop solution. The result confirms the point made by Sertel (1987) in a static framework.

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1 Introduction

The issue of the inefficient behaviour of labour-managed (LM) firms has been long debated in the related literature (see, e.g., Ireland and Law, 1982, and Moretto and Rossini, 2003, for exhaustive overviews). Distortions in the supply of output (Cremer and Crémer, 1992) and quality (Lambertini, 1997), as well as in the investment in productive capacity (Futagami and Okamura, 1996; Lambertini and Rossini, 1998) or in R&D for process innovation (Lambertini, 1998) have been highlighted, in comparison with the corresponding behaviour of profit-maximising (PM) firms. In particular, the so called perverse reaction of LM firms to price changes materialises as a tendency to restrict output (and membership) as compared to what a PM firm would do all else equal, with a view to increase the value added accruing to those who remain members at equilibrium.

A connected stream of literature highlights that such distortions are generated by the assumption that the amount of labour employed within the LM firm is endogenously determined by its output or investment decisions. This may well be unrealistic. If a market for memberships is properly modelled (Dow, 1986; Sertel, 1982, 1987), then the LM firm turns out to behave similar to a PM one. Specifically, if one takes into account that, for any given list of members, the partnership itself is not a choice variable, but constraints to entry and exit are operative, then the perverse behaviour of LM disappears. In this literature (Sertel, 1982, 1987; Fehr and Sertel, 1993), LM firms are relabelled as *workers' enterprises* (WE). Provided that the size of the membership (i.e., the number of participants) is decided upon prior to investment and output decisions, the WE firm ultimately replicates the performance of a PM firm irrespective of the intensity of market competition (Sertel, 1991, 1993).

In this paper we consider a dynamic setting in which oligopolistic firms compete, in the presence of sticky market prices, *à la* Simaan and Takayama (1978). Since the alleged perversity of LM firms is supposed to take the form of a peculiar reaction to price changes, the choice of this setup seems to be particularly appropriate to deal with this issue. We show that, once we have considered the proper constraint to the size of the membership, the alleged perverse behaviour of labour-managed firm, compared to PM firms, does not appear in the long-run equilibrium allocation. Since we are going to consider labour-managed firms in which the membership is given, we call them WE.

In solving the dynamic games at hand, we take into account both the case in which the information structure is of the open-loop type, and the case in which feedback effects are present. In both frameworks, we compare the steady state equilibrium allocations under the cases that all firms are either WE or PM. We find that the objective function of the firm (profits vs individual value added) has no bearings upon the steady state equilibrium. These results closely replicate Murat Sertel's findings concerning several static versions of the problem at hand.

The structure of the paper is as follows. Section 2 presents the setup. Section 3 solves the models under the open-loop information structure while Section 4 provides the solution under a closed-loop information structure. Conclusions are in Section 5.

2 The setup

We investigate an oligopoly game with n firms competing over continuous time $t \in [0, \infty)$ in a market for a homogeneous good. The output produced by any firm i at any time t is $q_i(t)$. At each instant, market demand determines the "notional" level of price, $\widehat{p}(t) = A - B\sum_{i=1}^{N} q_i(t)$. In general, however, $\widehat{p}(t)$ will differ from the current level of market price $p(t)$ due to price stickness, as in Simaan and Takayama (1978) and Fershtman and Kamien (1987).[1] Market price moves according to the following equation:

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s\left\{\widehat{p}(t) - p(t)\right\} \tag{1}$$

Notice that the dynamics described by (1) establishes that the price adjusts proportionately to the difference between its notional level, given by the inverse demand function, and its current level; the speed of adjustment is determined by the constant s, with $s \in [0, \infty)$. The lower is s, the stickier is market price. This amounts to saying that the price mechanism is sticky, that is, firms face menu costs in adjusting price to the demand conditions deriving from consumers' preferences.

Firms use the constant returns to scale technology defined by

$$q_i(t) = \sqrt{\ell_i k_i(t)}, \tag{2}$$

where ℓ_i is a fixed labour input and $k_i(t)$ is a (non-labour) input, labelled as "tangible asset", chosen by firm i at time t.[2]

In particular, the assumption that ℓ_i is exogenously given and remains constant throughout the game amounts to saying that the membership of any

[1] See also Tsutsui and Mino (1990) and Cellini and Lambertini (2004), where open-loop, memoryless closed-loop and feedback equilibria are charaterised for the Cournot oligopoly with profit-seeking firms. The finite horizon case in in Fershtman and Kamien (1990). Trade policy issues are investigated by Dockner and Haugh (1990, 1991) in the same setup.

[2] For instance, $k_i(t)$ may measure the amount of raw materials or intermediate goods used by firm i at time t in producing the final good.

firm is decided upon before the firm itself starts its productive activity. For this reason -as we already mentioned- we label these LM firms as WE firms. This assumption is consistent with the hypothesis that constraints to enter and exit the membership ℓ_i become operative due to binding agreements taken before the firm start producing, as suggested by Sertel (1987).

Total costs correspond to the remuneration of the tangible asset input and the labour input, so that the instantaneous cost function of firm i is:

$$C_i(t) = w\ell_i + ck_i(t) \tag{3}$$

where $w > 0$ is the wage rate and $c > 0$ is the unit price of input k. Of course, function (3) is relevant to a profit-seeking firm, whose instantaneous profits are defined as follows:

$$\pi_i(t) = pq_i(t) - w\ell_i - ck_i(t) = p(t)\sqrt{\ell_i k_i(t)} - w\ell_i - ck_i(t) \tag{4}$$

while the objective of a labour-managed firm is to maximise value-added per worker:

$$v_i(t) = \frac{p(t)\sqrt{\ell_i k_i(t)} - ck_i(t)}{\ell_i} \tag{5}$$

with $v_i(t) \geq w$ in order for the participation into the labour-managed firm to be attractive.

Irrespective of the firm's objective function, the (individual) control variable is $k_i(t)$ while the (common) state variable is $p(t)$. Hence, the problem of firm i, when it is a profit-maximising unit, is:

$$\max_{k_i(t)} J_i = \int_0^\infty e^{-\rho t} \left[p(t)\sqrt{\ell_i k_i(t)} - w\ell_i - ck_i(t)\right] dt \tag{6}$$

subject to (1) and to the conditions $p(0) = p_0$, and $p(t) > 0$ for all $t \in [0, \infty]$.

Otherwise, if the firm is a workers' enterprise, its objective is:

$$\max_{k_i(t)} J_i = \int_0^\infty e^{-\rho t} \frac{\left[p(t)\sqrt{\ell_i k_i(t)} - ck_i(t)\right]}{\ell_i} dt \tag{7}$$

subject to the same set of constraints.

In what follows, we solve the game adopting, in turn, the open-loop solution concept and a closed-loop concept. These two solution concepts can be interpreted as two different choice rules, corresponding to different information sets. Under the open-loop rule, the players choose the optimal plan at the beginning of time, that is, at $t = 0$, and stick to it forever, regardless of the feedback effects generated by rivals during the game. Generally, the resulting equilibrium is only weakly time consistent. Under the closed-loop rule, players do not precommit on any path and their strategies at any instant of time depend on all the preceding history, generally summarised by the current level of the state variable(s). The resulting equilibrium is strongly time consistent.[3]

3 The open-loop Nash solution

Under the open-loop solution concept, firms choose the optimal dynamic plan at $t = 0$ and stick to it forever, regardless of the feedback effects generated by rivals during the game. We consider first the WE oligopoly.

3.1 The WE oligopoly

The Hamiltonian function of firm i is:

$$\mathcal{H}_i(t) = e^{-\rho t}\left\{\frac{\left[p\left(t\right)\sqrt{\ell_i k_i\left(t\right)} - ck_i\left(t\right)\right]}{\ell_i} + \lambda_i(t)s\left[A - B\sum_{j=1}^{n}\sqrt{\ell_j k_j\left(t\right)} - p(t)\right]\right\} \tag{8}$$

[3]In the jargon of differential game theory, strong time consistency is equivalent to subgame perfection. For further details, see Başar and Olsder (1982) and Dockner *et al.* (2000).

where $\lambda_i(t) = \mu_i(t)e^{\rho t}$, and $\mu_i(t)$ is the co-state variable associated to $p(t)$. In the remainder of the paper, superscript OL indicates the *open-loop* equilibrium level of a variable.

The outcome of the game is summarised by:

Proposition 1 *The open-loop game between n WE firms produces a unique steady state equilibrium where:*

$$p^{OL} = \frac{A\left[Bs\ell + 2c\left(s+\rho\right)\right]}{2c\left(s+\rho\right) + B\ell\left[s + n\left(s+\rho\right)\right]},$$
$$k^{OL} = \frac{A^2\ell\left(s+\rho\right)^2}{\left[2c\left(s+\rho\right) + B\ell\left(s + n\left(s+\rho\right)\right)\right]^2},$$

which is a saddle point.

Proof. The first order condition w.r.t. $k_i\left(t\right)$ is:[4]

$$\frac{\partial\mathcal{H}_i(t)}{\partial k_i(t)} = \frac{k_i\left(t\right)\left\{\ell_i\left[p(t) - Bs\ell_i\lambda_i\left(t\right)\right] - 2c\sqrt{\ell_i k_i\left(t\right)}\right\}}{2\sqrt{\left[\ell_i k_i\left(t\right)\right]^3}} = 0 \tag{9}$$

which yields:

$$\lambda_i\left(t\right) = \frac{\ell_i p(t) - 2c\sqrt{\ell_i k_i\left(t\right)}}{Bs\ell_i^2} \tag{10}$$

and

$$k_i\left(t\right) = \frac{\ell_i\left[p(t) - Bs\ell_i\lambda_i\left(t\right)\right]^2}{4c^2}. \tag{11}$$

The above expression can be differentiated to obtain the dynamic equation of the control variable:

$$\frac{dk_i(t)}{dt} \equiv \dot{k}_i(t) = \frac{\ell_i\left[p(t) - Bs\ell_i\lambda_i\left(t\right)\right]\left[\dot{p}(t) - Bs\ell_i\dot{\lambda}_i(t)\right]}{2c^2}. \tag{12}$$

[4]Here, as well as in the remainder of the paper, for brevity we drop the indication of exponential discounting in deriving the first order conditions.

The co-state equation is:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} = \frac{\partial \mu_i(t)}{\partial t} \Rightarrow \dot{\lambda}_i(t) = \lambda_i(t)(s+\rho) - \frac{k_i(t)}{\sqrt{\ell_i k_i(t)}}, \tag{13}$$

and the transversality condition is:

$$\lim_{t\to\infty} \mu_i(t)\, p(t) = 0. \tag{14}$$

Now, using (10) and (13), we can rewrite (12) as follows:

$$\dot{k}_i(t) = \frac{2c(s+\rho)\,k_i(t) - Bs\ell_i(n-1)\,k_i(t) + [As - (2s+\rho)\,p(t)]\sqrt{\ell_i k_i(t)}}{c} \tag{15}$$

which entails that $\dot{k}_i(t) = 0$ in:[5]

$$k_{i1} = 0;\ k_{i2} = \frac{\ell_i [As - (2s+\rho)\,p(t)]^2}{[Bs\ell_i(n-1) - 2c(s+\rho)]^2}. \tag{16}$$

For obvious reasons, we can disregard the solution $k_{i1} = 0$. Then, we can impose the simmetry conditions $\ell_i = \ell$ and $k_i = k$ for all i, and plug k_{i2} into the state equation (1). Hence, imposing the stationarity condition $\dot{p} = 0$, we obtain the steady state level of price:

$$p^{OL} = \frac{A[Bs\ell + 2c(s+\rho)]}{2c(s+\rho) + B\ell[s + n(s+\rho)]}. \tag{17}$$

Therefore, the optimal tangible asset endowment is:

$$k^{OL} = \frac{A^2\ell(s+\rho)^2}{[2c(s+\rho) + B\ell(s+n(s+\rho))]^2}. \tag{18}$$

Of course, the Nash equilibrium output per-firm is $q^{OL} = \sqrt{\ell_i k^{OL}}$. The corresponding steady state equilibrium level of the value-added per worker is:

$$v^{OL} = \frac{A^2(s+\rho)[B\ell s + c(s+\rho)]}{[2c(s+\rho) + B\ell(s+n(s+\rho))]^2} \tag{19}$$

[5]We drop the indication of time in the remainder of the subsection, for the sake of brevity.

which must be higher than w.

Now we can check the stability properties of the dynamic system formed by (1) and (15). Its Jacobian matrix is as follows:

$$J=\left[\begin{array}{cc} \dfrac{\partial \dot{p}}{\partial p}=-s & \dfrac{\partial \dot{p}}{\partial p}=-\dfrac{B\ell ns}{2\sqrt{\ell k}} \\ \dfrac{\partial \dot{k}}{\partial p}=-\dfrac{(2s+\rho)\sqrt{\ell k}}{c} & \dfrac{\partial \dot{k}}{\partial k}=\dfrac{\Psi+4c\,(s+\rho)\,k-p\,(2s+\rho)\sqrt{\ell k}}{2ck} \end{array}\right] \tag{20}$$

where $\Psi \equiv \left[A\sqrt{\ell k}+2B\ell\,(n-1)\,k\right]s$. The trace and determinant of the above 2×2 Jacobian matrix are:

$$T\,(J)=\frac{2k\,[c\,(2s+\rho)-B\ell\,(n-1)]+[As-p\,(2s+\rho)]\sqrt{\ell k}}{2ck}; \tag{21}$$

$$\Delta\,(J)=-\frac{s\left[k\,(4c\,(s+\rho)+B\ell\,(n\rho+2s))+(As-p\,(2s+\rho))\sqrt{\ell k}\right]}{2ck}. \tag{22}$$

In correspondence of $\left\{p^{OL},k^{OL}\right\}$, the determinant is:

$$\Delta\,(J)=-\frac{s\,[2c\,(s+\rho)+B\ell\,(n\,(s+\rho)+s)]}{2c}<0. \tag{23}$$

Therefore, $\left\{p^{OL},k^{OL}\right\}$ is a saddle point. ■

3.2 The PM oligopoly

If the n players are profit maximising firms, then the Hamiltonian function of firm i is:

$$\begin{aligned} \mathcal{H}_i(t) &= e^{-\rho t}\{p\,(t)\sqrt{\ell_i k_i\,(t)}-w\ell_i-ck_i\,(t)+ \\ &\quad \lambda_i\,(t)\,s[A-B\sum_{j=1}^{n}\sqrt{\ell_j k_j\,(t)}-p(t)]\} \end{aligned} \tag{24}$$

The first order condition w.r.t. $k_i\,(t)$ is:

$$\frac{\partial \mathcal{H}_i(t)}{\partial k_i(t)}=\frac{\ell_i\,[p(t)-Bs\lambda_i\,(t)]-2c\sqrt{\ell_i k_i\,(t)}}{2\sqrt{\ell_i k_i\,(t)}}=0 \tag{25}$$

from which we obtain:

$$\lambda_i(t) = \frac{\ell_i p(t) - 2c\sqrt{\ell_i k_i(t)}}{Bs\ell_i} \tag{26}$$

and

$$k_i(t) = \frac{\ell_i \left[p(t) - Bs\lambda_i(t)\right]^2}{4c^2}. \tag{27}$$

Expression (27) can be differentiated w.r.t. t, to obtain the differential equation describing the evolution of the control variable over time:

$$\dot{k}_i(t) = \frac{\ell_i \left[p(t) - Bs\lambda_i(t)\right]\left[\dot{p}(t) - Bs\dot{\lambda}_i(t)\right]}{2c^2}. \tag{28}$$

The co-state equation is:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} = \frac{\partial \mu_i(t)}{\partial t} \Rightarrow \dot{\lambda}_i(t) = \lambda_i(t)(s+\rho) - \sqrt{\ell_i k_i(t)}, \tag{29}$$

and the transversality condition coincides with (14).

Using (26) and (29), and imposing the symmetry conditions by which $k_i(t) = k$ and $\ell_i = \ell$ for all i, we can establish that

$$\dot{k} \propto 2ck(s+\rho) + \left[As - p(2s+\rho)\right]\sqrt{\ell k} - Bs\ell(n-1)k. \tag{30}$$

Accordingly, $\dot{k} = 0$ in

$$k_1 = 0;\ \ k_2 = \frac{\ell\left[As - (2s+\rho)p\right]^2}{\left[Bs\ell(n-1) - 2c(s+\rho)\right]^2}, \tag{31}$$

which clearly coincide with the solutions in (16). Given that the state equation and the demand function are the same in the two games, this suffices to prove the following:

Proposition 2 *The open-loop game between PM firms reaches the same steady state as the corresponding game between WE firms, in terms of optimal input, output and price.*

The steady state profits of any given firm are:

$$\pi^{OL} = \ell \left\{ \frac{A^2 (s+\rho) [B\ell s + c(s+\rho)]}{[2c(s+\rho) + B\ell (s + n(s+\rho))]^2} - w \right\} \tag{32}$$

which is positive iff

$$w < v^{OL} \tag{33}$$

as it can be easily ascertained by comparing (32) with (19). That is, the viability condition for a generic PM firm also ensures that the equilibrium individual value added inside any WE firm be incentive compatible.

3.3 Price stickiness and market allocations

With reference to the amount of the tangible asset used by the individual firm in steady state (see equations (18) and (31)), it is immediate to check that

$$\frac{\partial k^{OL}}{\partial s} = -\frac{A^2 \ell^2 B \rho (s+\rho)}{[2c(s+\rho) + B\ell (s + n(s+\rho))]^3} < 0. \tag{34}$$

Since $q^{OL} = \sqrt{\ell k^{OL}}$, the derivative (34) means that the steady state equilibrium level of the production of any firm decreases (and price increases), else else equal, as s increases. More explicitly, the higher is the level of price stickiness (i.e., the smaller is s), the larger is the steady state production. This result is very well-known in the available literature on profit maximising firms (see, e.g., Fershtman and Kamien (1987); Cellini and Lambertini (2004)). A rough intuition for this result is provided by the following argument: when prices are sticky, the current production levels of firms are weakly effective in moving current prices; this fact leads firms to high levels of current and future production. On the contrary, when prices move largely in response to production decisions, firms choose to shrink the output levels.

Moreover, with reference to the steady state level of individual surplus obtained by any WE firm, the following holds:

$$\frac{\partial v^{OL}}{\partial s} = \frac{A^2 B^2 \ell^2 \rho \left[s(n-1) + n\rho\right]}{\left[2c\left(s+\rho\right) + B\ell\left(s + n\left(s+\rho\right)\right)\right]^3} > 0 \tag{35}$$

which menas that the individual surplus in the steady state equilibrium allocation increases as prices become less and less sticky. Intuition is clear: as prices become elastic, the production level of each individual firm shrinks; this involves a higher price and ultimately leads to a higher level of value added per worker, provided that the size of membership is kept constant.

4 The closed-loop Nash solution

Under a closed-loop solution information structure, firms do not stick to any given plan designed at the outset for the entire time horizon. On the contrary, each of them takes into account the strategic effects exerted by the rivals' behaviour at any point in time. Among the different closed-loop solution concepts, we rely on the so-called memoryless closed-loop, in which the only feedback effect to be taken into account is the effect of the rivals' choices upon the current value of the state variable(s).[6] The Hamiltonian functions for the WE oligopoly case and the PM oligopoly game remain defined as in (8) and (24), respectively. The difference in the solution procedure concerns the adjoint equations pertaining to the dynamics of co-state variables. In such adjoint equations, the feeback effects have to be duly accounted for. In the present case, the feedback effects are non-nil so that the closed-loop

[6]In principle, the proper feedback solution would be given by the Bellman equation. However, since in the present case the problem of the representative firm is not linear quadratic, there exist no obvious functional form for the value function.

solutions indeed differ from the open-loop ones.[7]

4.1 The WE oligopoly

The outcome of the game is summarised by the following:

Proposition 3 *The steady state equilibrium of the game between* n *WE firms, under the memoryless closed-loop information structure, is:*

$$p^{CL} = A - nB\sqrt{k\ell}\,, \tag{36}$$

$$k^{CL} = \frac{A^2\ell[B\ell(n-1)s + 2c\,(s+\rho)]^2}{\left[B^2\ell^2(n-1)ns + 4c^2\,(s+\rho) + 2nBc\ell\,(2s+\rho)\right]^2}, \tag{37}$$

which is a saddle point.

Proof. See the Appendix. ■

Also in this case, the steady state levels of the intangible asset, k^{CL}, and production, $q^{CL} = \sqrt{\ell k^{CL}}$, turn out to be decreasing in the speed of price adjustment, s. The value added per worker in steady state is:

$$v^{CL} = \frac{A^2c\,[B\ell s\,(n-1) + 2c\,(s+\rho)]\,[B\ell s\,(n+1) + 2c\,(s+\rho)]}{\left[4c^2\,(s+\rho) + B\ell n\,(B\ell s\,(n-1) + 2c\,(2s+\rho))\right]^2} \tag{38}$$

which must be higher than the labour market wage w.

4.2 The PM oligopoly

In the light of the foregoing analysis, the closed-loop game among profit-seeking firms can be quickly dealt with, its outcome being summerised by:

[7] There exist special classes of differential games where closed-loop solutions coincide with open-loop solutions. To this regard, see, e.g., Dockner *et al.* (2000, ch. 7).

Proposition 4 *The closed-loop game between PM firms reaches the same steady state as the corresponding game between WE firms, in terms of optimal tanglible asset input, outputs and price.*

Proof. See the Appendix. ■

Once again, it can also be easily shown that the steady state qualifies as a saddle point. The steady state profits associated with the closed-loop equilibrium are:

$$\pi^{CL} = \ell \left\{ \frac{A^2 c \left[B\ell s \left(n-1 \right) + 2c \left(s+\rho \right) \right] \left[B\ell s \left(n+1 \right) + 2c \left(s+\rho \right) \right]}{\left[4c^2 \left(s+\rho \right) + B\ell n \left(B\ell s \left(n-1 \right) + 2c \left(2s+\rho \right) \right) \right]^2} - w \right\} \tag{39}$$

with $\pi^{CL} > 0$ for all $w < v^{CL}$.

Moreover, the steady state level of production, q, decreases as the speed of price adjustment s increases. The reason is the same as in the case of open-loop equilibria.

At this point, it is more interesting to note that the steady state levels of intangible asset (and hence production) turn out to be larger under the closed-loop information structure, than under the open-loop one, as the comparison between equations (31) and (53) makes clear. Both levels are larger than the production of the static Cournot game. This fact is explained on the following grounds. The closed-loop output level is higher than the open-loop output level because, taking into account feedback effects, each firm tries to preempt the rivals. Since this holds for all firms alike, the outcome is that the closed-loop steady state production exceeds the open-loop steady state production. In turn, the open-loop steady state output exceeds the static output because in the static game, by definition, there is no time for adjustment and therefore firms have no way of trying to overproduce in order to preempt the rivals. These considerations hold for all finite values of s, while taking the limit of q^{OL} for $s \to \infty$, one obtains the static equilibrium output level (the Appendix provides the solutions of the static problems). This result is known in the existing literature, though focussing so far only on PM firms (see Fershtman and Kamien, 1987; and Cellini and Lambertini, 2004): the static problems produce the same equilibrium allocation and profits as the steady state of the corresponding open-loop dynamic problem with sticky price, under the proviso that the value of the price adjustment coefficient tends to infinity, that is, price adjustment is immediate.

As a consequence, from the firms' viewpoint, the static situation is the most profitable one. On the contrary, the steady state allocation in the closed-loop equilibrium is socially preferred both to the open-loop steady state and to the static equilibria, given that production levels are higher and market price is lower. It remains true, however, that the open-loop solution is not subgame perfect in this calss of models; therefore firms should consistently choose closed-loop plans in order to produce subgame perfect equilibria.

5 Concluding remarks

In the present paper we have taken a differential game approach to study the behaviour of labour-managed firms over time, when prices are sticky.

Our main interest has been to check whether, provided that the membership of labour managed firms is given (so that these firms can be labelled as *workers' enterprises* as suggested by Murat Sertel) , the steady state equilibrium allocation reached by an oligopoly populated by these firms is the same as in an oligopoly populated by profit-maximising firms. The answer is positive, and the result holds under both the open-loop information structure and the memoryless closed-loop information structure.

This finding closely reflects the point made by Sertel (1987) regarding the static allocation reached by a workers' enterprise: once we take into account that the membership is not endogenously determined when choosing the output level, but it is set before the market game starts, then the alleged perverse beaviour of LM firms, as compared to profit-maximising firms, indeed disappears.

Appendix

Proof of Proposition 3. The Hamiltonian function corresponding to the problem of firm i is the same as in (8). Likewise, the first order condition w.r.t. $k_i(t)$ is the same as (9) which yields (10) and (11); the differentiation of the latter w.r.t. time gives (12).

The co-state equation under the closed-loop information structure is:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} - \sum_{j \neq i} \frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} \frac{\partial k_j(t)}{\partial p(t)} = \frac{\partial \mu_i(t)}{\partial t} \tag{40}$$

Notice that the sum $\sum_{j \neq i} \frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} \frac{\partial k_j(t)}{\partial p(t)}$appearing in the above condition takes into account the feedback effects, that are absent by definition under the open-loop solution. Since $\mu_i(t) = \lambda_i(t)e^{-\rho t}$, condition (40) rewrites as

$$\dot{\lambda}_i(t) = -\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} - \sum_{j \neq i} \frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} \frac{\partial k_j(t)}{\partial p(t)} + \lambda_i(t)(s+\rho). \tag{41}$$

The transversality condition is:

$$\lim_{t \to \infty} \mu_i(t)\, p(t) = 0. \tag{42}$$

To build up the expression describing the feedback effects, consider that[8]

$$\frac{\partial \mathcal{H}_i}{\partial k_j} = -\frac{\lambda_i s B \ell_j}{2\sqrt{\ell_j k_j}} \tag{43}$$

$$\frac{\partial k_j}{\partial p} = \frac{\ell_j (p - B\ell_j \lambda_j s)}{2c^2}. \tag{44}$$

Now, using the above expressions, under the symmetry conditions $\ell_i = \ell$ and $k_i = k$ for all i, we can write the dynamics of k as follows:

$$\dot{k} = \frac{-B\ell(n-1)ps\sqrt{\ell k} + 4c^2 k(s+\rho) + 2c(As - p(2s+\rho))\sqrt{\ell k}}{2c^2} \tag{45}$$

and the price dynamics:

$$\dot{p}(t) = [A - B\sqrt{\ell k} - B(n-1)\sqrt{\ell k}) - p]s \tag{46}$$

so that, apart from the solution $k = 0$, the steady state turns out to be the allocation given by

$$p^{CL} = A - nB\sqrt{k\ell}\,,$$

$$k^{CL} = \frac{A^2 \ell [B\ell(n-1)s + 2c(s+\rho)]^2}{[B^2\ell^2(n-1)ns + 4c^2(s+\rho) + 2nBc\ell(2s+\rho)]^2}.$$

It is easy to check, also in the present case, that the corresponding steady state equilibrium level of the value-added per worker is larger than the wage rate. Moreover, the equilibrium $\{p^{CL}, k^{CL}\}$ is a saddle, since the determinant of the corresponding Jacobian matrix is negative. The details are omitted for brevity, as the prof of stability is technically analogous to that provided in the proof of Proposition 1.■

Proof of Proposition 4. If the n players are profit maximising firms, from the Hamiltonian function (24), one obtains the first order condition (25) and

[8] Again, for brevity we omit the indication of time henceforth.

hence (26) and (27). Expression (27) can be differentiated w.r.t. t, to obtain the differential equation describing the evolution of the control variable over time:

$$\dot{k}_i(t) = \frac{\ell_i \left[p(t) - Bs\lambda_i(t)\right] \left[\dot{p}(t) - Bs\dot{\lambda}_i(t)\right]}{2c^2}. \tag{47}$$

The co-state equation under the closed-loop information structure is:

$$-\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} - \sum_{j \neq i} \frac{\partial \mathcal{H}_i(t)}{\partial k_j(t)} \frac{\partial k_j(t)}{\partial p(t)} = \frac{\partial \mu_i(t)}{\partial t} \tag{48}$$

The usual transversality has to be considered. Each feedback effect can be build using the following expressions:

$$\frac{\partial \mathcal{H}_i}{\partial k_j} = -\frac{\lambda_i s B \ell_j}{2\sqrt{\ell_j k_j}} \tag{49}$$

$$\frac{\partial k_j}{\partial p} = \frac{\ell_j (p - B\ell_j \lambda_j s)}{2c^2}. \tag{50}$$

On these bases, and imposing the symmetry conditions $k_i(t) = k$ and $\ell_i = \ell$ for all i, from (48) we obtain:

$$\dot{\lambda} = -\sqrt{\ell k} + \lambda(\rho + s) + \frac{B\ell^2 \lambda (n-1) \left[p(t) - Bs_i(t)\right] s (p - B\lambda s)}{4c^2 \sqrt{\ell k}}. \tag{51}$$

Hence,

$$\dot{k} = \frac{k[-(n-1)B\ell^2 ps + 4c^2\sqrt{\ell k}(\rho + s) + 2c\ell(As - p(\rho + 2s))]}{2\rho^2\sqrt{\ell k}} \tag{52}$$

Accordingly, $\dot{k} = 0$ in

$$k = 0;\ k = \frac{A^2 \ell [B\ell(n-1)s + 2c(s+\rho)]^2}{\left[B^2\ell^2(n-1)ns + 4c^2(s+\rho) + 2nBc\ell(2s+\rho)\right]^2} \tag{53}$$

which clearly coincide with the solutions in (37). Given that the state equation and the demand function are the same in the two games, this suffices to prove the proposition.■

Solution of the static problems. In a static framework, the $i-th$ firm operating in a WE industry faces the problem:

$$\max_{k_i} \frac{\left[A - B\sum_{j=1}^{n}(\sqrt{\ell_j k_j})\right]\sqrt{\ell_i k_i} - ck_i}{\ell_i} \tag{54}$$

The first order condition is:

$$-\frac{B\ell_i \sum_{j\neq i}(\sqrt{\ell_j k_j}) + 2(B\ell_i + c)\sqrt{\ell_i k_i} - A\ell_i}{2\ell_i\sqrt{\ell_i k_i}} = 0 \tag{55}$$

Under symmetry conditions, the equilibrium value of k is:

$$k = \frac{A^2\ell}{[B\ell(n+1)+2c]^2} \tag{56}$$

The static version of the PM oligopoly is as follows. Firm i solves the problem:

$$\max_{k_i} \left[A - B\sum_{j=1}^{n}(\sqrt{\ell_j k_j})\right]\sqrt{\ell_i k_i} - w\ell_i - ck_i \tag{57}$$

The first order condition is:

$$-\frac{B\ell_i\left[A - B\sum_{j\neq i}(\sqrt{\ell_j k_j})\right]}{2\sqrt{\ell_i k_i}} + \frac{A\ell_i}{2\sqrt{\ell_i k_i}} - B\ell_i - c = 0 \tag{58}$$

Under symmetry conditions, the equilibrium value of k is:

$$k = \frac{A^2\ell}{[B\ell(n+1)+2c]^2} \tag{59}$$

Notice that the equilibrium value of k (and hence the production level) is the same, in the PM oligopoly and in the WE oligopoly alike. It is also immediate to check that this value coincides with the limit of expression (18) as $s \to \infty$. As mentioned in the main text, this means that the static problems provide the same results as the steady state of the dynamic problems with sticky price, under the particular case that the price adjustment coefficient tends to the infinity.■

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