Entry-Exit Timing and Profit Sharing

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Abstract

We analyze the effects of two compound investment options, a shut down and a reopening option, on a Aoki's profit sharing firm organization. Whilst the introduction of a credible threat of shutting down weakens labour's position in the bargaining and favors the shareholders on profit sharing, the option to reopen the plant acts in the opposite direction, reducing the abandoning threat and reinforcing the workers' bargaining power. More specifically, as long as an increase in uncertainty leads to an increase in the benefit from reopening, and hence in the firm's market value, the overall result implies a weakening of the shut down threat and the profit distribution process becomes more favorable to workers.

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1 Introduction

In a recent paper by Moretto and Rossini (1995) Aoki's profit sharing organization of the firm is associated with the recent option valuation model of investment. Following Aoki (1980,1984), the firm is defined as a joint organization of stockholders and employees endowed with *skill and knowledge specific to the firm as a result of quasi permanent association with it.* The organization is run by a manager who acts on behalf of shareholders and mediates between workers and shareholders as to the distribution of profits. However, before mediating with workers on the firm's organization, the manager has to take decisions about the firm's operating policy. More specifically, stressing the irreversibility of most investment decisions and the ongoing uncertainty of the economic environment, the manager must decide the investment timing, that is whether to enter a market now or to postpone entry in order to obtain more information while keeping the opportunity of doing so in the future. Should the firm be already operating the manager must choose the optimal shut down time to prevent extreme losses. In the latter case, if suspension has a direct cost for the firm (for example a considerable severance payment to the workers or the loss of tangible or intangible capital), the shut down option will lead the firm to delay exit while trying to preserve the capital, so that it can be used profitably if environmental conditions improve in the future.

This phenomenon, which was publicised by Dixit (1989) in a model of entry and exit and recently surveyed by Dixit and Pindyck (1994), highlights the role played by the option value of waiting for better information and the analogy with the option theory in financial markets.[1] Because of irre-

[1] A firm with an opportunity to invest holds an option which is analogous to a financial call option. That is, it has the right but not the obligation to buy an asset of some value at a future time at a fixed "exercise price". On the other hand, an operating firm with an opportunity to abandon is holds an option analogous to a financial put option which gives the right but not the obligation to sell an asset at some future time for a fixed price. In both cases, however, when a firm makes an irrreversible investment/abandonment decision, it "kills" its option to invest/quit. McDonald and Siegel (1985) were the first to show that if the price is described by a geometric Brownian motion, a unit-output investment project with fixed operating costs can be valued as the sum of an infinite set of European call options. For a review of the analogy between financial options and real corporate options

versibility there is an opportunity cost of investing or abandoning now rather than waiting. The firm waits and does not enter (undertakes the investment) when the price rises just above the full cost of making the investment, and does not exit (suspend operation) when the price falls just below average variable costs. For a single and discrete project, this implies the existence of two optimal trigger prices p_H and p_L; the investment should be made if the price rises above p_H and should be abandoned if the price falls below p_L.

However, even if the baseline framework is as described above, Moretto and Rossini (1995) assume that an incumbent firm cannot re-enter once it has left the market if future profits turn favourable again. Therefore, they analyse how a viable shut down option influences profit sharing assuming that when the firm closes workers are laid off, getting a bonus which represents the entire sunk cost of shutting down.

They show that workers' participation departs from Aoki's original model. In particular, the introduction of a credible threat to stop production weakens labour's position in burgeoning and the profit distribution process becomes more favourable to shareholders.

However, if for many projects the assumption that it is impossible to restart operation at a later date if economic conditions improve seems reasonable, in other cases it appears to be too severe. Many firms may suspend production temporarily or "mothball" the current project while allowing it to be reactivated in the future, incurring partial or full investment costs.

The present paper deals with this issue. Allowing for re-entering typically weakens the strength of the shut down threat by shareholders during bargaining. We investigate how a viable threat of abandoning may lose its effectiveness when the firm also has a viable benefit to reopen in the future. Moreover, we go a step further with respect to Moretto and Rossini (1995), analysing how the distributive share of profits varies along with variations in uncertainty affecting the future evolution of demand.

The existence, in this case, of two compound options highlights two other important issues. The first one concerns the precise timing of the bargaining process, which must rule out time inconsistent sharing rules (in a extended model this choice should be included in the bargaining). The second regards the relationship between workers and the firm during the period(s) in which the firm suspends production.

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see Mason and Merton (1985), Kulatilaka and Marcus (1988), and Dixit and Pindyck (1994).

While the former can be resolved considering the natural time for bargaining to be the moment in which the firm enters the market (or when the project restarts), the latter calls for some remarks. Since the re-entering option changes the employees' outside opportunities it calls for modelling the firm's labour market. This can be done in many ways, for example when the firm shuts down, it may offer workers a simple menu option of the following type: either a worker accepts an exit bonus and leaves the firm, or she/he accepts an option to be rehired in the future if the firm reopens again. In the first circumstance the worker loses his quasi permanent association with the firm and the contract terminates; he might have a probability of being re-employed in the future at the market wage by another or the same firm.

In the second case rehiring is warranted as well as the right to receive part of the profits once the firm restarts; in turn the worker may or may not receive payments during the temporary lay-off period. However, by a seniority arrangement and/or by avoidance of permanent quits causing loss of job-specific human capital accumulated on workers, it is commonly observed that in many firms rehiring is warranted and workers also receive a "mothballed" bonus during the temporary lay-off period(s).[2] Such a *mothballed* bonus plays the role of an unemployment insurance for keeping workers close to the firm and/or a cost to maintain their firm-specific efficiency during the temporary suspension.[3]

Although both examples are plausible we have chosen the latter to model the firm's labour market.

As well as departing from Aoki's model the presence of an optimal entry and exit timing policy by the firm also leads us away from Moretto and Rossini (1995)'s result. In particular, we show that their result may be reversed if reopening the plant is allowed and the effect of an increase in uncertainty is taken into account. Indeed, whilst the option to reopen weakens the shut down threat on the part of the shareholders, the increase in uncertainty raises the value of future expected profits reducing their weight in the bargaining process.

The paper is organized as follows. The next section presents the general

[2]Feldstein (1978) and Lilien (1980) show that in the USA temporary lay-offs account for 50% of all unemployment, and were an even larger proportion of cyclical changes in the number of job losers during the seventies. For a review of the theoretical and empirical literature on firms' cycle and employment decisions see Moretto (1988).

[3]Temporary suspension only makes sense if the "mothballed" bonus is lower than the cost that the firm would have incurred by continuing operation.

features of the basic model. Section 3 exposes the bargaining set of the two actors using the option valuation approach. Section 4 deals with the bargaining process as well as with the general characteristics and comparative studies of the bargaining equilibrium. Finally, the conclusions are drawn in section 5.

2 The model

As in Moretto and Rossini (1995), we consider a firm endowed with a given capital stock. In each period if active the firm produces one unit of output and lasts forever until shut down. Moreover, marginal and average operating costs c are known and constant. The number of workers employed is, for the sake of simplicity, normalized to one.

The firm's revenue, expressed by the market price, is driven by a geometric Brownian motion

$$dp_t = \mu p_t dt + \sigma p_t dzw_t \quad \text{with } \; p_0 = p; \mu, \sigma \geq 0 \tag{1}$$

where z_t is a standard Wiener process, that is a nondifferentiable continuous Gaussian process with independent increments, satisfying the conditions that $E(dz_t) = 0$, and $E(dz_t^2) = dt$.

The firm has an option to shut down in the future by paying a lump-sum cost l which represents a total statutory severance. However, the firm must incur a lump-sum cost k to start production again which will be completely lost when it stops. If k goes to infinite the re-entry option becomes worthless and no more investment is allowed after shut down has occurred, which is the case analysed by Moretto and Rossini (1995).

For a firm that already exists operating profits at time t are expressed by $\pi(p_t) = p_t - c$ when the firm is working and zero in the idle state.

Since, by (1) the firm knows that the market price can go up or down with non zero probability, it stays in the market even if demand conditions are adverse hoping, by doing so, to avoid the exit sunk cost l; for the same reason an idle firm will invest when demand conditions become sufficiently favourable to cover the entry cost k. As shown by Dixit (1989), the optimal strategy for entry and exit, or for holding or exercising the two options will take the form of two threshold prices, p_L and p_H, with $p_L < c < p_H$. An active firm will find it optimal to remain active as long as the price continues to stay above p_L, but it will quit if p falls down to p_L. A firm which has suspended production will find it optimal to remain idle as long as p remains below p_H, and will invest as soon as p reaches p_H.

This flexibility has no counterpart on the workers' side. Even if they are associated with the firm they do not control the entry-exit strategy which pertains only to shareholders. However, to avoid the loss of human capital accumulated by the firm on workers, they receive part of the profits as an extra above the market wage. The share of profit they get is the result of bargaining with shareholders.

Therefore, payments to employees consist of two parts: a market wage component w which is constant over time, and a premium earning Δw_t which represents the employees' share of profits accruing to workers. Following Aoki (1980, 1984) and letting θ be the share of profits going to shareholders, the premium per employee can be expressed as,

$$\Delta w(p_t) = (1-\theta)\pi(p_t) = (1-\theta)(p_t - c), \qquad 0 < \theta < 1. \tag{2}$$

Equation (2), is also crucial to interpret the corporate policy of employees. Since the exit threshold p_L is lower than c, the firm may stay in the market even in the case of operating losses. That is, (2) may become negative, which means that workers and shareholders also share the firm's losses before exit[4].

2.1 The shareholders' objective

If we assume that the shareholders are homogenous in all respects, the problem of a risk neutral firm is one of an optimal operating policy, deciding when to enter and when to exit in order to maximize the expected sum of discounted profits.

Defining $S_1(p;\theta)$ as the firm's value starting with price p in the active state, the Bellman equation is:

$$S_1(p;\theta) = E\left\{ \int_0^{T_L} e^{-\rho t}\theta(p_t - c)dt \mid p_0 = p \right\} + \tag{3}$$

[4]Moretto and Rossini (1995) allow for a more flexible employees' corporate policy setting the premium per employee as $\Delta w_t = (1-\theta)\max[-m, \pi(p_t)]$, with $0 \leq m \leq c - p_L$. The parameter m represents the workers' willingness to share losses. However, allowing for a different sharing rule for losses by workers complicates the model without adding any new insight to the firm's entry-exit performance.

$$E\left\{e^{-\rho T_L}\left[S_0(p_L;\theta)-l\right] \mid p_0=p\right\}, \quad for\ p\in[p_L,\infty),$$

where $\rho > \mu$ is the cost of capital. Moreover, $S_0(p_L;\theta)$ represents the firm's valuation in the idle state at time $T(p_L) = \inf(t \geq 0 \mid p_t < p_L)$ when the firm shuts down, and p_L is the trigger exit price.

In the idle state the optimal policy implies restarting whenever the price rises above the critical level p_H . Since in the idle state the firm does not produce, the Bellman value function reduces:

$$S_0(p;\theta) = E\left\{e^{-\rho T_H} S_1(p_H;\theta) - k \mid p_0 = p\right\}, \quad for\ p \in (0, p_H], \tag{4}$$

where $T(p_H) = \inf(t \geq 0 \mid p_t \geq p_H)$ represents the entry time.

2.2 The workers' objective

Incumbent employees are interested in the amount of lifetime earning they can get by taking part in the firm's production. Under the assumptions that the employees' relative share $1-\theta$ remains constant over time, that the workers are remunerated equally and that rehiring is allowed and warranted when the firm suspends production, the level of a worker's lifetime well-being up to the shut down is given by:

$$L_1(p;\theta) = E\left\{\int_0^{T_L} e^{-\rho t}\left[w+(1-\theta)(p_t-c)\right]dt \mid p_0=p\right\} + \tag{5}$$

$$E\left\{e^{-\rho T_L}\left[l+L_0(p_L;\theta)\right] \mid p_0=p\right\}, \quad for\ p\in[p_L,\infty),$$

$L_0(p_L;\theta)$ represents the worker's valuation of lifetime well-being in the idle state at time $T(p_L)$. We get a similar result considering a worker in the idle state who evaluates her/his lifetime well-being taking account of the option of being rehired if the firm re-enters in the future:

$$L_0(p;\theta) = E\left\{e^{-\rho T_H} L_1(p_H;\theta) \mid p_0 = p\right\}, \quad for\ \ p \in (0, p_H]. \tag{6}$$

Finally, both $L_1(p;\theta)$ and $L_0(p;\theta)$ must be positive to induce participation.[5]

[5] If rehiring is allowed but not warranted the firm's optimal operating policy is altered.

3 The efficient bargaining set

To identify the efficient bargaining set of the two actors let us start with the shareholders, who independently decide the operating policy of entry and exit. For (3) and (4) we look for tentative solutions S_1 and S_0 to solve the following free boundary dynamic programming problem[6]:

$$\Gamma S_1(p;\theta) = -\theta(p-c), \qquad for \;\; p \in [p_L, \infty), \tag{7}$$

$$\Gamma S_0(p;\theta) = 0, \qquad for \; p \in (0, p_H], \tag{8}$$

where Γ is the operator:

$$\Gamma = -\rho + \mu p \frac{\partial}{\partial p} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2}{\partial p^2} \; .$$

For example, if the firm offers workers, as was mentioned in the introduction, the menu of receiving l and being laid-off completely or being temporarily laid-off without a *mothballed* bonus but with the option of being rehired once the firm reopens, the firm's value becomes:

$$S_1 = E\left\{\int_0^{T_L} e^{-\rho t}\theta\pi(p_t)dt + e^{-\rho T_L}\left[S_0(p_L) - \delta l\right]\right\}.$$

In the same way the lifetime well-being of a worker becomes:

$$L_1 = E\left\{\int_0^{T_L} e^{-\rho t}\left[w + \Delta w_t\right]dt + e^{-\rho T_L}\max\left[l + W_0, L_0(p_L)\right],\right\}$$

$$L_0(\delta) = (1-\delta)E\left(e^{-\rho T_H}L_1\right) + \delta\left(l + W_0\right),$$

where $\delta = \begin{cases} 1 & \text{if} \quad l + W_0 > L_0(p_L) \\ 0 & \quad \text{otherwise} \end{cases}$, is an indicator function, and W_0 is the worker's discounted value of future earnings associated with an alternative job, starting from unemployment. In a previous numerical simulation we used this formulation with a couple of Poissons processes to model the dynamics between the state of employed and the state of unemployed workers after the shut down. Apart from the analytical complication of the model we got no new qualitative insight about the firm's operating policy with respect to the case in which rehiring is warranted.

[6]Alternatively, we could have assumed ρ as the competitive risk-adjusted discount rate for an asset or portfolio perfectly correlated with dz_t, and solve the firm's valuation using contingent claims methods. For more on this approach and the analogy with dynamic programing in the context of corporate real options see Dixit and Pindyck (1994).

The boundary conditions are:

$$S_1(p_L;\theta) = S_0(p_L;\theta) - l, \tag{9}$$

$$S_0(p_H;\theta) = S_1(p_H;\theta) - k, \tag{10}$$

$$S_1^{'}(p_L;\theta) = S_0^{'}(p_L;\theta), \tag{11}$$

$$S_0^{'}(p_H;\theta) = S_1^{'}(p_H;\theta). \tag{12}$$

The above four equations (9)-(12) stand for the usual value matching conditions and smooth pasting conditions for optimal exercise. These conditions are easy to interpret: the value matching conditions are just the zero profit conditions at exit and entry respectively, while the smooth pasting conditions are equivalent to a marginal-cost-equal-to-marginal-revenue condition. The boundary conditions also require the limits:

$$\lim_{p\to\infty}\left\{S_1(p;\theta) - \theta\left(\frac{p}{\rho-\mu} - \frac{c}{\rho}\right)\right\} = 0 \ , \qquad \lim_{p\to 0} S_0(p;\theta) = 0.$$

The second term in the first limit represents the discounted present value of the profit flows over an infinite horizon starting from a price level p going to shareholders (Harrison 1985, p.44).

By the linearity of differential equations (7) and (8), and using the above limits, the optimal policy of shareholders can be expressed as:

$$S_1(p;\theta) = Ap^{-\alpha} + \theta\left(\frac{p}{\rho-\mu} - \frac{c}{\rho}\right) \ , \qquad for \ \ p \in [p_L, \infty), \tag{13}$$

and:

$$S_0(p;\theta) = Bp^{\beta} \ , \qquad for \ \ p \in (0, p_H], \tag{14}$$

where $-\alpha < 0$ and $\beta > 1$, are respectively the negative and the positive roots of the quadratic equation associated with the Γ operator. As usual, since the terms $Ap^{-\alpha}$ and Bp^{β} represent the option value to suspend and restart production respectively, the constants A and B must be positive.

The constants A and B as well as the trigger points p_L and p_H are determined by using the boundary conditions (9)-(12). Since they are nonlinear in p_L and p_H we cannot get any closed form solution for the firm's value as well as for the optimal shareholders policy, and hence numerical simulations are needed to get a quantitative idea of the properties of the sharing policy on the firm's entry and exit timing of the firm. This is the subject of the next section.

Let us now turn to the workers. They get their extra wage in the form of new shares coming from the dividends which are not distributed to shareholders. Moreover, the employees do not have voting rights and in particular they do not participate in the choice of the entry-exit operating policy. When the firm is in the idle state they receive an exit bonus and have the right to be hired again once the firm re-enters the market. Therefore, referring to (5) and (6) the workers lifetime well-being L_1 and L_0 are the solutions for the following free boundary dynamic programming problem:

$$\Gamma L_1(p;\theta) = -(1-\theta)(p-c) \qquad , for\ p \in [p_L, \infty), \tag{15}$$

$$\Gamma L_0(p;\theta) = 0 \qquad , for\ p \in (0, p_H]. \tag{16}$$

The boundary conditions are:

$$L_1(p_L;\theta) = L_0(p_L;\theta) + l, \tag{17}$$

$$L_0(p_H;\theta) = L_1(p_H;\theta), \tag{18}$$

whilst the limit conditions are:

$$\lim_{p\to\infty} L_1(p;\theta) - (1-\theta)\left(\frac{p}{\rho-\mu} - \frac{c}{\rho}\right) - \frac{w}{\rho} = 0\ , \qquad \lim_{p\to 0} L_0(p;\theta) = 0.$$

Again the first limit stands for the discounted present value of the profits plus wage flows over an infinite horizon for workers who continue to stay with the same firm during their lifetime.

Even for the workers the differential equations (15) and (16) are linear in L_1 and L_0 respectively. Then, taking account of the above limits, the optimal policy is:

$$L_1(p;\theta) = Fp^{-\alpha} + (1-\theta)\left(\frac{p}{\rho-\mu} - \frac{c}{\rho}\right) + \frac{w}{\rho}\ , \qquad for\ \ p \in [p_L, \infty), \tag{19}$$

and

$$L_0(p;\theta) = Dp^{\beta}\ , \qquad for\ \ p \in (0, p_H]. \tag{20}$$

The term $Fp^{-\alpha}$ accounts for the difference between the extra earnings the workers lose if the firm suspends production and the per-capita transfer l. On the contrary, the term Dp^{β} stands for the value of the option to re-enter by a temporary laid-off worker.

Given the trigger levels p_L and p_H chosen by the firm, the constants F and D are determined by using the boundary conditions (17) and (18).

Focusing on the relationship between p_L and the distributive parameter θ it is immediate to understand that it could be the case that p_L becomes negative as long as θ decreases. Therefore, there exists a reservation distributive parameter $\hat{\theta} = \inf\left(\theta \geq 0 \mid p_L > 0\right)$, above which the firm keeps the shut down option alive (see appendix). That is:

$$p_L > 0 \qquad iff \qquad \theta > \hat{\theta} \equiv \frac{lc}{\rho}, \tag{21}$$

$$\text{i.e.} \qquad \frac{\alpha}{1+\alpha}\frac{\rho-\mu}{\rho}\left(c - \frac{\rho}{\theta}l\right) \leq p_L \leq \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c - \frac{\rho}{\theta}l\right).$$

On the other hand, for $\theta \leq \hat{\theta}$, p_L is set at zero and the option to exit becomes worthless which, for an incumbent firm, implies $A = B = 0$. Similarly for the workers we have $F = D = 0$.

In this last case the system expressed by (9)-(12) with (17) and (18) admits a closed solution for p_H and the constants B and D. In particular for the former we obtain (see appendix)[7] ,

$$p_H = \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c + \frac{\rho}{\theta}k\right) \tag{22}$$

As $\frac{\beta}{\beta-1} > 1$, the upper price level which triggers entry is greater than the usual flow-equivalent, per unit of time, full cost of investment $\frac{\rho-\mu}{\rho}\left(c + \frac{\rho}{\theta}k\right)$. The option value multiple $\frac{\beta}{\beta-1}$ accounts for the difference the firm will require

[7] It is worth noting that the expression for p_H given in (22) is different from Dixit's simple option criterion when the project is never abandoned. They coincide only when $\theta = 1$.

before being willing to make irreversible investment. The firm should wait to obtain more information on profit evolution before entering.[8]

4 The bargaining

In the spirit of Aoki (1980,1984), the bargaining process is carried out between a representative employee and the firm manager. The manager mediates between shareholders and employees to find an explicit agreement on the internal distribution of the profits before actual production starts. This is equivalent to finding a Nash Bargaining Solution (NBS), as formulated by Harsanyi (1956, 1977).

Both players share the same information about future profits and are adverse to the risk of opening internal conflicts. Such aversion is represented by concave Von Neumann Morgenstern utility functions defined on the domain of the firm's value $v(S_1)$ for shareholders, and the total lifetime well-being for incumbent workers $u(L_1)$.

The NBS can be characterized as the result of maximising with respect to θ of the joint objective function:

$$\nabla = \left\{u\left[L_1(p_H;\theta)\right] - u(\hat{L}_1)\right\}\left\{v\left[S_1(p_H;\theta)\right] - v(\hat{S}_1)\right\}, \tag{23}$$

subject to the relevant constraints, (3) and (5).

If cooperation fails, the bargaining has no solution and the players get the utility levels $u(\hat{L}_1) \geq 0$ and $v(\hat{S}_1) \geq 0$ which are assumed to be known and given. $u(\hat{L}_1)$ represents the workers' wage utility of alternative jobs available in the labour market, while $v(\hat{S}_1)$ is the utility shareholders can get by investing the lump-sum k elsewhere. As in Aoki (1980, 1984), utilities from cooperative bargaining are higher than the reservation values, which is the reason for both parties' interest in reaching an agreement.

Although the time dependence of the firm's entry and exit policy makes the NBS not renegotiation proof, the distributive policy is time consistent. That is, as the firm always enters at the same level of profits, $p_H - c$, rehires the workers who are temporarily laid-off, and the information contained in the current state of the underlying stochastic variable does not provide any

[8] In addition, as $\frac{\partial\beta}{\partial\sigma} < 0$ an increase in σ raises the option multiple $\frac{\beta}{\beta-1}$. That is, the greater the uncertainty over the future realization of p, the larger the wedge between p_H and the full cost of investment.

further insight on future evolution of profits, at each reopening both parties would choose the same distributive parameter θ.

4.1 Equilibrium distribution

From the previous sections we should distinguish between two cooperative objective functions for the Nash bargaining game. That is, the case in which $\theta > \hat{\theta}$ from the case in which $\theta \leq \hat{\theta}$.

Moreover, as the rehiring of temporary laid-offs is warranted and the workers' wage from alternative jobs is simply $\hat{L}_1 = \frac{w}{\rho}$, we can simplify (23) setting $u(\hat{L}_1) = 0$ and also dropping the term $\frac{w}{\rho}$ from $L_1(p;\theta)$.

We further assume that the two parties have CRRA utility functions.

I) for $\theta > \frac{l\rho}{c}$

$$\max_{\theta} \nabla_{\mathrm{I}} = \frac{\left(L_1^{\mathrm{I}}(p_H;\theta)\right)^{\gamma_u}}{\gamma_u} \left\{ \frac{\left(S_1^{\mathrm{I}}(p_H;\theta)\right)^{\gamma_v}}{\gamma_v} - \frac{\left(\hat{S}_1^{\mathrm{I}}\right)^{\gamma_v}}{\gamma_v} \right\}$$

where $L_1^{\mathrm{I}}(p_H;\theta) = Fp_H^{-\alpha} + (1-\theta)\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right)$, and $S_1^{\mathrm{I}}(p_H;\theta) = Ap_H^{-\alpha} + \theta\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right)$

II) for $\theta \leq \frac{l\rho}{c}$

$$\max_{\theta} \nabla_{\mathrm{II}} = \frac{\left(L_1^{\mathrm{II}}(p_H;\theta)\right)^{\gamma_u}}{\gamma_u} \left\{ \frac{\left(S_1^{\mathrm{II}}(p_H;\theta)\right)^{\gamma_v}}{\gamma_v} - \frac{\left(\hat{S}_1^{\mathrm{II}}\right)^{\gamma_v}}{\gamma_v} \right\}$$

where $L_1^{\mathrm{II}}(p_H;\theta) = (1-\theta)\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right)$, $S_1^{\mathrm{II}}(p_H;\theta) = \theta\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right)$ and $p_H = \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c + \frac{\rho}{\theta}k\right)$

Finally, we make the following assumptions about $\hat{S}_1$. If the shareholders do not invest in the productive activity, they dispose of an alternative asset, characterized by a riskless instantaneous interest rate λ. This asset provides an income flow, discounted at the rate ρ, equal to $\frac{K\rho}{\lambda}$, which we assume equals $\widehat{S}_1$.

4.2 Numerical results

As already mentioned, the high degree of nonlinearity of the relationships associated with the equilibrium of both types of individuals does not allow for derivation of closed form formulae for the unknowns p_H, p_L, A, B, F and D, apart from the somewhat marginal case of negative p_L, for which exact solutions are available for the remaining three variables. In the general case an insight into the empirical implications of the model outlined above can only be obtained by numerically solving the system. Let us first consider the general case, where p_L is positive. The six equations (two value matching conditions and two smooth pasting conditions for the shareholders, plus two value matching conditions for the employees) are highly nonlinear in the two price unknowns, but are linear in the four constants A, B, F and D. We therefore started by solving the value matching and the smooth pasting conditions of the shareholders in p_H with respect to A and B, expressing them as functions of p_H. Secondly, these formulae were plugged into the value matching and the smooth pasting conditions of shareholders in p_L, thus obtaining two nonlinear relationships in the price variables. The high complexity of the two equations and the explosive behaviour of their solutions for certain values of some critical parameters (most notably θ) require an ad hoc solution procedure which disregards derivatives, implemented in a GAUSS program. The price levels solution of the previous step can be substituted in the value matching conditions for the employees to get the equilibrium values of F and D.

To choose the values of the relevant parameters we started from the work of Dixit (1989). We set $c = 1$ (simply a normalization) representing the operative costs of the productive activity. Labour costs amount to 10% of operating costs: $w = 0.1$. The subjective discount rate is set at $\rho = 0.025$, while the instantaneous riskless rate is $\lambda = 0.005$. The lump sum costs k and l are chosen to equal 4 and 1, respectively. To avoid paradoxical results due to the presence of inflation in the price process but not on the side of costs we set $\mu = 0$ (the instantaneous rate of growth of the price of output). σ (the price instantaneous volatility) takes on a variety of values ranging from 0.05 (the certainty case) to 0.45. Finally, both shareholders and employees have a parameter of risk aversion (with respect to the emergence of internal conflicts) equal to $\frac{1}{2}$.

Let us reformulate the joint objective function characterizing the Nash Bargaining Solution:

I) for $\theta > 0.025$:

$$\max_{\theta} \nabla_I = 2\sqrt{L_1^I(p_H;\theta)}\left\{2\sqrt{S_1^I(p_H;\theta)} - 4\sqrt{5}\right\}$$

where $L_1^I(p_H;\theta) = Fp_H^{-\alpha} + (1-\theta)\left(\frac{p_H-1}{0.025}\right)$, and $S_1^I(p_H;\theta) = Ap_H^{-\alpha} + \theta\left(\frac{p_H-1}{0.025}\right)$.

II) for $\theta \leq 0.025$:

$$\max_{\theta} \nabla_{II} = 2\sqrt{L_1^{II}(p_H;\theta)}\left\{2\sqrt{S_1^{II}(p_H;\theta)} - 4\sqrt{5}\right\}$$

where $L_1^{II}(p_H;\theta) = (1-\theta)\left(\frac{p_H-1}{0.025}\right)$and $S_1^{II}(p_H;\theta) = \theta\left(\frac{p_H-1}{0.025}\right)$. The entry trigger price is given by $p_H = \frac{\beta}{\beta-1}\left(1+\frac{1}{100\theta}\right)$, where $\beta = \frac{1+\sqrt{1+\frac{0.2}{\sigma^2}}}{2}$. The above two cases define the objective function on two non overlapping subintervals of the domain of θ, that is $[0,1]$.

Figure 1 and 2 plot the ratios of the trigger prices on the respective Marshallian costs of entry/exit, $W_H = \frac{p_H}{c+\rho k}$ and $W_L = \frac{p_L}{c-\rho l}$, as functions of θ and σ. They generalize the graphs in Dixit (1988), who did not consider the issue of profit sharing. It is already known that an increase in uncertainty widens the wedge between the two triggers, but the two plots highlight the fact that the widening is much more pronounced for small values of θ than for large ones. This is of course especially true for the W_H side, since p_L is constrained between 0 and $c = 1$. We interpret this result as a reaction by shareholders to an adverse profit sharing rule: even if they can only get a small fraction of operating profits, they are required to pay the lump sum costs of reopening or reclosing the firm without any help on the workers' side. Thus, in order to payback the initial investment k, they postpone entry until price conditions are extremely favourable. In the same way, when production has begun and price conditions deteriorate, they wait much longer before closing the plant, because they sustain only a small fraction of operating losses, while they are totally hit by the lump sum closing cost l.

Figure 3 plots the objective function ∇ (the composition of ∇_I and ∇_{II} on the respective subdomains) as a function of θ and σ. For moderate to intermediate values of the uncertainty parameter σ and small values of θ the shareholders utility associated with the production activity, $2\sqrt{S_1^I}$ or $2\sqrt{S_1^{II}}$, may be smaller than the utility associated with the alternative riskless investment. Since the only utility argument on the workers' side is made

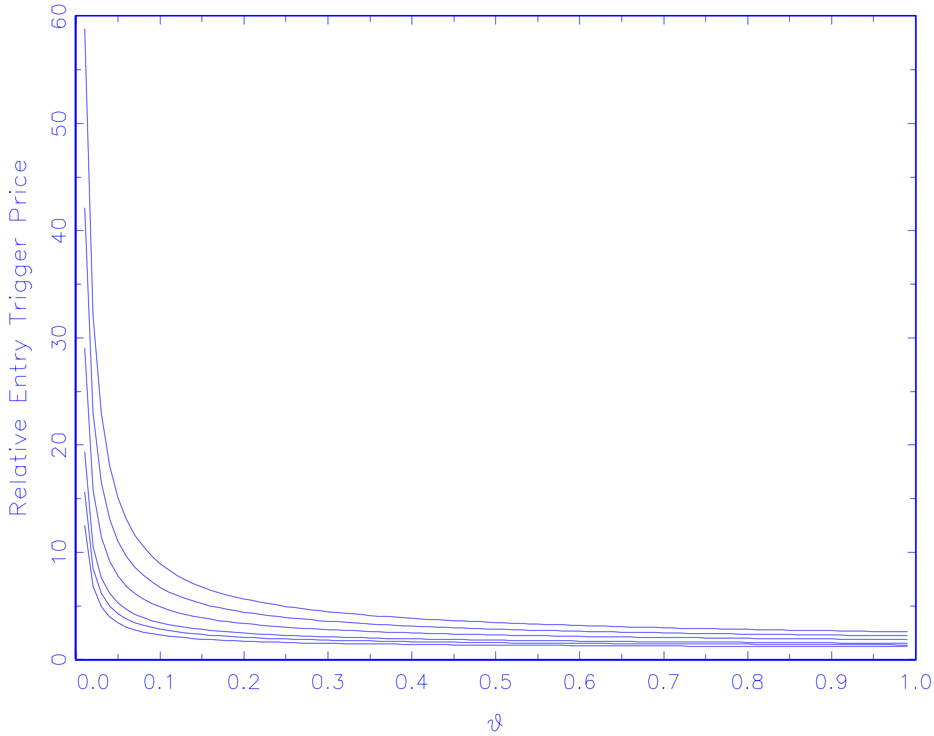

Figure 1: Relative Entry Trigger Prices, as Functions of θ and σ. The lines correspond to six different values of σ: .05, .1, .15, .25, .35 and .45. For each value of θ, a smaller value of σ lowers the corresponding curve: the firm reopens later (the entry trigger price is higher) when volatility is higher, especially for small values of θ..

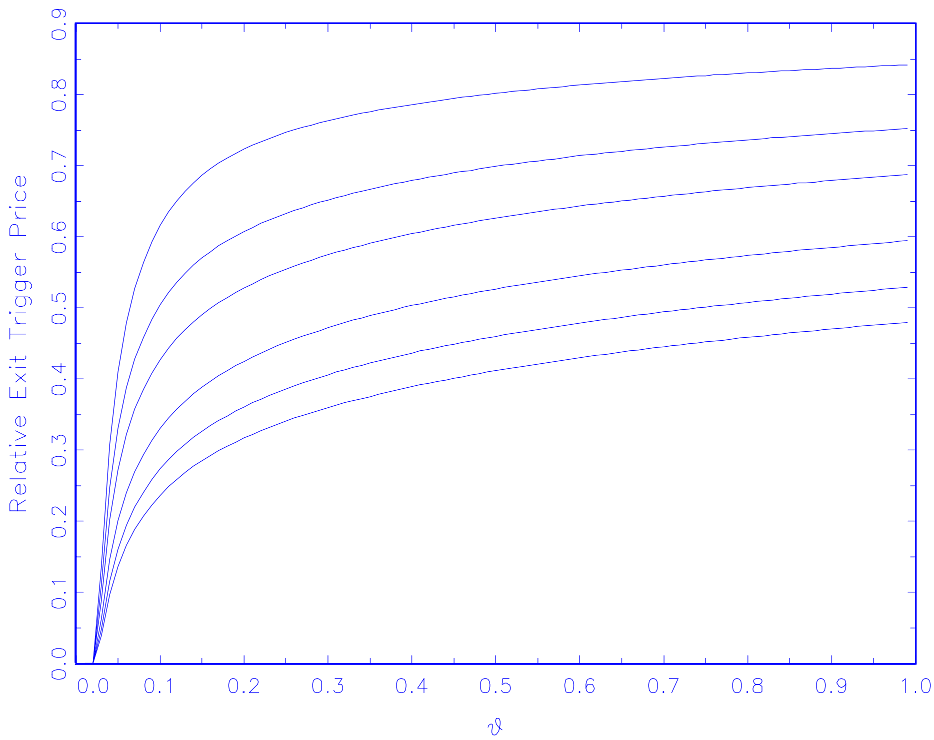

Figure 2: Relative Exit Trigger Prices as Functions of θ and σ. The lines correspond to six different values of σ: .05, .1, .15, .25, .35 and .45. For each value of θ, a higher value of σ lowers the corresponding curve: the firms shuts down later (the exit trigger price is lower) when volatility is higher.

up of extra wages, which are always positive, and there is no reservation value for such a variable, the objective function appears to be negative for the parameters configuration mentioned above. This does not exclude the possibility of a feasible equilibrium profit sharing, since for this it is sufficient that ∇ be positive for some value of θ. For the six volatility values that we have considered, this is true as far as σ is (roughly) not smaller than 0.1.

There is a variety of other aspects to be noticed about this plot. Except for the case of the largest σ, it is apparent that the reduced profits accruing to shareholders for small θ are sufficient to avoid a corner solution to the bargaining problem, since the left side of the plot is consistently negative. This is no longer true for the largest value of σ that we have considered (0.45). In this case the huge volatility increases the value of the shut down and reopening options in shareholders' hands at a point where the alternative investment is no longer attractive, even when θ is extremely small. We thus observe a ∇ function which is positive on the whole domain. As we have already seen from the two preceding plots, the wedge between the triggers widens as θ decreases, which means that if production begins, the price is extremely high, and the profit shares accruing to workers are very large. The combination of positive (even if small) net utility to shareholders and huge utility of extra wages to workers make it optimal to adopt a corner sharing rule, where $\theta = 0$: the firm tends to be transform itself from a profit sharing one into a labour-managed one. Of course this conclusion is no more than intuitive, since if this were true, we would have abandoned our initial framework. It is clear that this conclusion should be seen as nothing more than a feature of the model for large values of uncertainty.

It should however be noticed that a lighter conclusion holds concerning the optimal sharing rule, i.e. the value of θ maximizing the bargaining objective function. As is apparent from Figure 3[9] the optimal θ^* decreases with σ. This fact should be added and contrasted with the one reached in Moretto and Rossini (1995). In their model the impossibility of reopening after plant closure strengthens the shut down threat on the part of shareholders, and reinforces their position in the bargaining process. As a consequence, θ is higher than Aoki's simple weighting rule $\frac{\gamma_u}{\gamma_u+\gamma_v} = \frac{1}{2}$. However, they had nothing to say on the influence of uncertainty on the bargained profit sharing param-

[9]For the six values of σ considered in Figure 3, the optimal sharing rules θ^* are the following:

σ	.05	.1	.15	.25	.35	.45
θ^*	1	.91	.76	.6	.47	.3

eter. The plot shows that their result can be reversed if we allow for plant reopening and account for the effect of uncertainty. In such a case, indeed, as an increase in σ raises the option to reopen and hence the firm's value, it weakens the shut down threat by shareholders and lowers their contractual strength in the bargaining process. We then have the *paradoxical* result that high uncertainty makes the alternative risk free investment unattractive for shareholders, and leads to the firm being transformed in the way outlined above.

5 Conclusions

Following Aoki's contribution the paper presents a model of the firm defined as a joint organization of stockholders and employees, where the latters are considered to embody firm-specific skills and knowledge which give them explicit bargaining power over the firm's profits. Employees in cooperation with the management, which acts on behalf of the shareholders, decide the profit distribution policy, whilst the flexible operating policy of entry and exit is in the management's hands alone.

Although the introduction of a credible threat of shutting down weakens the labour's position in the bargaining process, favouring the shareholders on profit sharing, the option to reopen the plant in the future acts in the opposite direction, reducing the threat on the part of shareholders and reinforcing the workers' bargaining power. Moreover, as long as an increase in uncertainty leads to an increase in the benefit from reopening and hence the firm's market value, the threat of abandoning loses its effectiveness and the profit distribution becomes more favourable to workers.

There seem to be many lines of research open to deeper examination regarding the firm's operating flexibility and labour participation. The Achilles' heel in the above framework is the contrast between the intrinsically dynamic policy of entry and exit and the static cooperative bargaining process. An important direction in future research would be to relax this drastic assumption, positing that employees and management can cooperatively renegotiate profit sharing over time, although the firm maintains flexibility of operation. Such a direction seems to be more in line with a model of the firm as a stockholders-employees cooperative game.

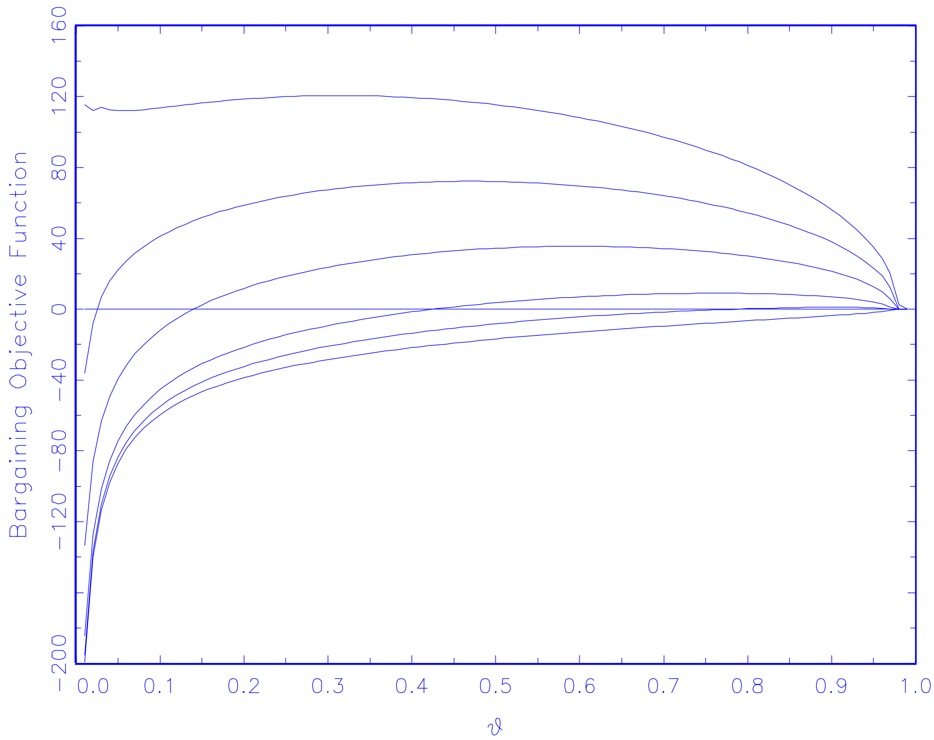

Figure 3: Bargaining Objective Function for Various Values of θ and σ. The lines correspond to six different values of σ: .05, .1, .15, .25, .35 and .45. For each value of θ, a smaller value of σ lowers the corresponding curve (the objective bargaining function moves upwards with increasing σ).

A Appendix: Closed form solution for θ sufficiently small

Substituting equations (13) and (14) into the four conditions (9)-(12), and equations (19) and (20) into the two conditions (17) and (18), we obtain a six equations system:

$$Ap_L^{-\alpha} + \theta\left(\frac{p_L}{\rho-\mu} - \frac{c}{\rho}\right) = Bp_L^{\beta} - l, \tag{24}$$

$$-A\alpha p_L^{-\alpha-1} + \frac{\theta}{\rho-\mu} = B\beta p_L^{\beta-1}, \tag{25}$$

$$Ap_H^{-\alpha} + \theta\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right) = Bp_H^{\beta} + k, \tag{26}$$

$$-A\alpha p_H^{-\alpha-1} + \frac{\theta}{\rho-\mu} = B\beta p_H^{\beta-1}, \tag{27}$$

$$Fp_L^{-\alpha} + (1-\theta)\left(\frac{p_L}{\rho-\mu} - \frac{c}{\rho}\right) + \frac{w}{\rho} = Dp_L^{\beta} + l, \tag{28}$$

$$Fp_H^{-\alpha} + (1-\theta)\left(\frac{p_H}{\rho-\mu} - \frac{c}{\rho}\right) + \frac{w}{\rho} = Dp_H^{\beta}. \tag{29}$$

The first four equations determine the constants A and B, and the trigger levels p_L and p_H while the last two determine the constants F and D. Moreover, as the first four equations are linear in A and B, substituting (24) into (25) we get:

$$Ap_L^{-\alpha} = \left[\frac{1-\beta}{\alpha+\beta}(\frac{\theta}{\rho-\mu}p_L) + \frac{\beta}{\alpha+\beta}\left(\frac{\theta}{\rho}c - l\right)\right], \tag{30}$$

$$Bp_L^{\beta} = \left[\frac{1+\alpha}{\alpha+\beta}(\frac{\theta}{\rho-\mu}p_L) - \frac{\alpha}{\alpha+\beta}\left(\frac{\theta}{\rho}c - l\right)\right]. \tag{31}$$

Equation (30) is the option value of shut down evaluated at the exit time, when the price is at p_L. For such an option to be positive, the r.h.s. must be positive. That is:

$$p_L \leq \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c-\frac{\rho}{\theta}l\right). \tag{32}$$

On the other hand, equation (31) refers to the option value of becoming active evaluated at the exit trigger p_L. For this option value to be positive it must be the case that:

$$p_L \geq \frac{\alpha}{1+\alpha}\frac{\rho-\mu}{\rho}\left(c-\frac{\rho}{\theta}l\right). \tag{33}$$

Recalling that $\beta-1>0$, and that $\frac{\beta}{\beta-1}>\frac{\alpha}{1+\alpha}$, the solution of the above system yields:

$$p_L \leq 0 \qquad iff \qquad \theta \leq \frac{lc}{\rho}, \qquad \text{i.e.} \qquad p_L \leq \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c-\frac{\rho}{\theta}l\right), \tag{34}$$

$$\begin{aligned} p_L \quad &> \quad 0 \qquad iff \qquad \theta > \frac{lc}{\rho} \\ \text{i.e. } \frac{\alpha}{1+\alpha}\frac{\rho-\mu}{\rho}\left(c-\frac{\rho}{\theta}l\right) \quad &\leq \quad p_L \leq \frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\left(c-\frac{\rho}{\theta}l\right) \end{aligned} \tag{35}$$

The same line of reasoning could be applied to the threshold p_H, which is always positive and greater than p_L.

Therefore, there exists a reservation distributive parameter $\hat{\theta} \equiv \inf(\theta \geq 0 \mid p_L > 0) = \frac{lc}{\rho}$, above which the shut down option is kept alive. For $\theta \leq \hat{\theta}$, p_L is set at zero and the option to exit becomes worthless, which implies $A = B = 0$ for the firm, and $F = D = 0$ for the workers. Then, the above system reduces to:

$$\theta\left(\frac{p_H}{\rho-\mu}-\frac{c}{\rho}\right) = Bp_H^{\beta}+k, \tag{36}$$

$$\frac{\theta}{\rho-\mu} = B\beta p_H^{\beta-1}, \tag{37}$$

$$(1-\theta)\left(\frac{p_H}{\rho-\mu}-\frac{c}{\rho}\right)+\frac{w}{\rho} = Dp_H^{\beta}, \tag{38}$$

from which we get the closed solution for p_H and for the two remaining constants B and D given in the text.

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