Bargaining with Noisy Communication

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Abstract

In this paper we show that in a bargaining situation the seller may not necessarily want to fully exploit communication possibilities. In the standard two-period bargaining model with one-sided incomplete information, the seller, who owns an indivisible good, makes offers which the buyer can either accept or reject. We ask whether the seller can profit from manipulating the communication mechanism by sending offers that reach the buyer with probability less than one (noisy communication). Noisy communication is a way to improve the seller's second period beliefs about the buyer's willingness to pay for the good and is therefore a way to "buy" commitment. We study the case of a discrete distribution of buyer's types and show that there exist equilibria with noisy communication when there are at least three different types of buyers.

Keywords: Bargaining, Communication, Incomplete Information.

1 Introduction

In this paper we show that in a bargaining situation the seller may not necessarily want to fully exploit communication possibilities, even if they are available at zero cost. We assume that the seller has no private information; nevertheless, strategic considerations induce him to refrain from communicating to the buyer with maximum effectiveness.

More specifically, we consider the Fudenberg and Tirole (1983) model of bargaining with one-sided incomplete information, where one seller and one buyer bargain over one indivisible object. There are two periods. The seller's valuation is common knowledge and the buyer's valuation is private information. In the original model, the seller makes an offer in period 1, which the buyer can either accept or reject. If the buyer accepts, the game is over; if he rejects, the seller can make another offer in period 2. Again, the buyer can either accept or reject the second offer; if he rejects, the game ends and the object is left unsold. We alter this basic structure by assuming that the messages sent by the seller may not hit the buyer with a probability that is controlled by the seller. We show that the seller may gain from sending messages in the first period that are received by the buyer with a probability strictly lower than 1 (noisy communication).

It is not clear to us what could prevent a seller from introducing some frictions into the communication process, and therefore we feel that a proper modeling of the bargaining process should allow for such a strategy. Our theoretical analysis is consistent with the observation that communication possibilities are sometimes voluntarily underexploited. For instance, some shops do not post prices. To make an example which is closer to the subject of this paper, our analysis may help explain why the owner of a house (in a small village, say) may somewhat conceal his willingness to sell, informing only a few people and letting the information circulate by word of mouth, instead of resorting to more effective ways of communication, even if the cost of more intense advertising would be negligible compared to the value of the transaction. This kind of behavior seems to be frequent in bargaining situations[1].

[1] At first, one may think that incomplete information about the seller's valuation could provide an alternative explanation; that is, this behavior may be used as a signal. However, on reflection, while it is reasonable for the seller to signal through an abnormally high price, it seems difficult to understand why he would not contact all potential buyers. Thus incomplete information by itself is not sufficient to explain noisy communication. Our analysis, that assumes that the seller has no private information, shows that it is not necessary either. Whether noisy communication can be optimal under two-sided incomplete information is an issue that we leave for further research.

The intuition for our result is as follows. By assumption, the seller cannot commit to a fixed sequence of offers. As a consequence, if the first offer is rejected, he will have an incentive to reduce the second period price, as shown by Fudenberg and Tirole (1983). Anticipating this behavior, the buyer will reject some first period offers that he would otherwise have accepted. But if the seller sends messages that do not hit the buyer with probability one, his second period beliefs about the buyer's willingness to pay will be more optimistic than in the case of deterministic messages. This translates into a higher second period price. In short, the seller uses noisy communication to 'buy' some commitment. Of course, noisy communication is costly as some opportunities for immediate agreement are foregone, and showing the optimality of noisy communication requires proving that the strategic advantage illustrated above may outweigh the cost.

Another interpretation of the model is that the seller is a durable-goods monopolist selling to a number of consumers whose willingness to pay for the good is distributed according to a known distribution function. In this context, the counterpart of noisy communication is that the seller rations randomly a fraction of consumers at the first period. This interpretation is developed in a companion paper (Denicolò and Garella, 1996), where the case of a continuous distribution of consumers' types is analyzed. In this paper, we focus on the case of a discrete distribution, which allows a more transparent analysis of the comparative advantages and costs of noisy communication.

To the best of our knowledge, the behavior analyzed in this paper has not been discussed in the literature so far. However, related problems have been treated in the bargaining literature and in the game-theoretic literature on communication.

In the bargaining literature, the basic communication structure of Fudenberg and Tirole (1983) has been changed in a number of ways. For instance, Muthoo (1994) considers the possibility that the offer is not irrevocable. In his model, the seller can choose to withdraw his offer after the buyer has accepted it. Admati and Perry (1987) study a model with two-sided incomplete information where the players can choose the time between offers, and use this instrument to signal their bargaining strength[2]. The general message of these papers is that the structure of the bargaining process can be manipulated by the players in order to gain a strategic advantage. The present

[2]See also Ma and Manove (1993) and, for a survey, Kennan and Wilson (1993).

paper adds to this literature.

Noisy communication between players has been explicitly modelled in the game-theoretic literature. Rubinstein (1989), in a paper on common-knowledge where no bargaining problem is involved, assumes that messages sent by players have an exogenously given probability of getting lost. Communication has also been studied in the literature on cheap-talk in sender-receiver games (see Forges, 1986 and Farrell, 1993). Myerson (1991) discusses an example due to Farrell where if the informed player sends a noisy message with a probability 1/2 of being received, an equilibrium is reached in which both players get a payoff exceeding the one they can get in any equilibrium with noiseless communication. But obviously sender-receiver games are quite different than bargaining models.

The rest of the paper is organized as follows. We present the model in section 2. In section 3, we provide a characterization of equilibria where the seller does not fully exploit the possibilities of communication (Equilibria with Noisy Communication, ENC). In section 4 we analyze the case where the buyer can be one of two types, showing that no ENC exists in this case. The three types case is studied in sections 5 and 6. It is shown that ENC's exist in the three type case. We also briefly discuss, in section 7, the efficiency properties of ENC. Section 8 concludes.

2 The Model

Assume that a bargaining involving an indivisible good can last for two trading periods. The seller's valuation of the good is public information, and is strictly lower than any possible valuation of the buyer; thus, without any further loss of generality, it can be normalized to zero. The buyer's valuation, v, is instead private information. The seller has a probability distribution $F(v)$ over some support $\left[\underline{v}, \bar{v}\right]$, with $\bar{v} > \underline{v} > 0$, which is common knowledge.

Both players are risk neutral. The seller's utility is simply equal to the discounted price $\delta^{t-1}p_t$ ($t = 1, 2$) if the object is sold, and zero otherwise; the buyer's utility from buying at date t is given by

$$u = \delta^{t-1}(v - p_t), \tag{1}$$

where δ is a discount factor common to both players.

In a one-period model, the seller would simply communicate with probability 1 an offer p^* where $p^* = \arg\max_p p[1-F(p)]$. We shall refer to p^* as the static optimal price. To simplify the exposition, we assume that $p[1-F(p)]$ is strictly quasi-concave so that p^* is unique.

At each period $t = 1, 2$, the seller sends a message which is an irrevocable offer to sell the good at a specified price p_t. The seller chooses also the probability $1 - \mu_t$ in $(0, 1)$ that the message hits the potential buyer. There is no cost of increasing this probability. If the buyer receives the message and accepts the offer, the object is sold. Finally, we assume that if the buyer receives the message he can also observe μ_t, but the seller cannot observe whether the buyer has received the message or not.

It is clear that in the second period the seller has nothing to gain from choosing $\mu_t > 0$, for this could only lower the probability that an agreement is reached without increasing the equilibrium price. Without loss of generality therefore we can define a strategy of the seller as a triplet $(p_1, \mu, p_{2,})$, where p_1 is the first period price, $\mu = \mu_1$ is the noise in the first period message, and p_2 is the second period offer which is made with zero noise, conditional on disagreement in the first period.

In the second period, the buyer's decision problem is trivial and he will accept the offer if and only if $p_2 \leq v$. If the buyer does not receive the message in the first period he has no decision to take. Thus, a strategy for the buyer can be described simply as a first period reservation price, b.

One may wonder whether the buyer who has not received any offer could tell the seller that the message went lost, or whether the buyer who has not accepted an offer could send a negative reply to the seller. Since this behavior will make noisy communication ineffective thus destroying the ENC, a seller who would gain from noisy communication, would want a buyer to reply only if he accepts a received offer. In this paper, we assume that the seller is empowered to choose the communication structure and therefore we assume that the buyer cannot send unwanted messages. For instance, the seller could use an appropriate electronic device, or instruct a middleman to transmit only acceptances.

If the seller could commit to a prescribed price sequence, it would be optimal to set $p_1 = p_2 = p^*$ and the solution to the seller's problem would be trivial. We assume, however, that the seller cannot commit to a sequence of offers. This implies that the second period price will be set in order to maximize second period profits, given first period choices. We look for the perfect Bayesian equilibria of the game.

In a perfect Bayesian equilibrium, conditional on no agreement in the first period, p_2 will be set so as to maximize $p_2 \Pr(v \geq p_2)$, where $\Pr(v \geq p_2)$ is the revised belief that the buyer's willingness to pay is higher than p_2 conditional on disagreement in the first period. On the other hand, the buyer's reservation price $b(v)$ will satisfy the condition that the buyer is indifferent between accepting and rejecting a first period offer $p_1 = b(v)$. Since the buyer can anticipate the second period price in case of a disagreement, $b(v)$ will satisfy $(v - b) = \delta(v - p_2)$, so that:

$$b(v) = (1 - \delta)v + \delta p_2. \tag{2}$$

A first period offer p_1 will therefore be accepted by a buyer whose valuation is at least

$$v_1 = \frac{p_1 - \delta p_2}{1 - \delta}, \tag{3}$$

and will be rejected if the buyer's valuation is lower than v_1. This holds both with perfect and noisy communication; all that changes is the value of p_2 (which in a perfect Bayesian equilibrium is anticipated by the buyer).

3 A Characterization of Equilibria with Noisy Communication

An equilibrium with noisy communication (ENC) is an equilibrium where the seller sets $\mu > 0$.

In this section we prove that any ENC necessarily entails a rising pattern of named prices. Moreover, the second period price is always equal to the static optimal price (i.e. the price if bargaining lasted one period only).

proposition 1 *At any ENC, $p_2 > p_1$.*

All proofs are in the Appendix.

The intuition is that if, contrary to the Proposition, $p_2 < p_1$ and $\mu > 0$, then the seller could increase profits by decreasing μ and increasing p_1 in such a way as to leave $\Pr(v \geq p_2)$ unaltered over the relevant range. This would increase first period profits, while leaving expected second period revenue unchanged, thus showing that the initial strategy is sub-optimal.

Proposition 1 has a simple corollary:

corollary 1 *At any ENC,* $p_2 = p^* > p_1$.

The reason is that when $p_2 \geq p_1$ the optimal strategy to the buyer is to set $b = v$ (he cannot gain by waiting for the second period offer) and therefore at any ENC the second period price will solve the maximization problem

$$\max_p (1-\mu)p\,[1 - F(p)] \tag{4}$$

provided the solution is greater than p_1. It follows immediately that $p_2 = p^*$ if $p^* > p_1$. If instead $p_1 \geq p^*$, then it would be optimal to set $p_2 = p_1$, but by Proposition 1 this cannot be an ENC.

This completes our general characterization of ENC's. In order to address existence, we further specialize our model assuming a discrete distribution function. We first consider the case of two types of consumers and show that there cannot be equilibria with $\mu > 0$ in that case. Then we turn to the case of three types, where ENC's exist.

4 Two types

In this section we assume that the buyer's valuation is v_H with probability x_H and $v_L < v_H$ with probability $x_L = 1 - x_H$. Letting $\mu = 0$ by assumption, one obtains exactly the model analyzed by Fudenberg and Tirole (1983). They show that equilibrium takes one of the following forms:

(i) $p_1 = v_L$, all buyers accept immediately;

(ii) $p_1 = (1-\delta)v_H + \delta v_L$ and $p_2 = v_L$, a high valuation buyer accepts the first offer and a low valuation buyer accepts the second offer;

(iii) $p_1 = v_H$, this offer is rejected by a high valuation buyer with probability y; if the buyer does not accept, the seller makes a second offer which is again v_H. Here y is determined so as to make the seller indifferent between setting $p_2 = v_H$ and $p_2 = v_L$. This requires that the probability that the buyer be of type H in case the first period offer is rejected, updated by the Bayes rule, is v_L/v_H, that is:

$$\frac{yx_H}{yx_H + (1 - x_H)} = \frac{v_L}{v_H} \tag{5}$$

Fudenberg and Tirole's (1983) characterization of the equilibrium does not consider the possibility of noisy communication. We show that this does not involve any loss of generality since noisy communication cannot be optimal in the two type case.

proposition 2 *With two types of consumers only there can be no ENC.*

Intuitively, in the two type case the results of the previous section imply that any ENC must involve $p_1 = v_L$. But then the only effect of setting $\mu > 0$ is to reduce the probability of agreement in the first period without affecting the seller's incentives in the second period, because the seller's posterior beliefs about v will coincide with the *ex-ante* distribution. As a consequence, at equilibrium it must be $\mu = 0$.

5 Three Types: Necessary Conditions for an ENC

We now assume that the buyer can be one of three types. Let $v_H > v_M > v_L$ denote the valuations of the three types, which are in proportions x_H, x_M and $x_L = 1 - x_H - x_M$.

In this section we describe situations where ENC's cannot emerge. This analysis serves two purposes. First, it shows circumstances where the traditional analysis, that assumes away noisy communication, is valid. Second, it provides necessary condition for an ENC.

We begin with a useful lemma.

lemma 1 *In the three type case, setting $p_1 = v_L$ cannot be part of a ENC.*

The intuition for this result is similar to that behind Proposition 2. An immediate implication of this lemma, combined with Proposition 1 and Corollary 1 is that a ENC can exist only if the static optimal price is v_H.

proposition 3 *A necessary condition for a ENC is that $p^* = v_H$.*

That is, it must be $v_H x_H > v_M(x_H + x_M)$ (so that pricing at v_M is suboptimal in the one-period set-up) and $v_H x_H > v_L$ (so that pricing at v_L is not optimal).

Assume that $p^* = v_H$. By Proposition 1 and Lemma 1 it then follows that the candidate ENC involves setting $p_1 = v_M$ and $p_2 = v_H$.

corollary 2 *In the three type case, at any ENC,* $p_1 = v_M$ *and* $p_2 = v_H$.

Now suppose $p_1 = v_M$ and consider the seller's problem at the beginning of period 2. If $v_M x_M > v_L(x_L + x_M)$, then, irrespective of μ, it is optimal to price at v_M in the second period. This case is in fact equivalent to the two type case considered in the previous section because the presence of the lowest valuation type does not alter the seller's incentives in the second period. Like in the two type case, it follows that noisy communication cannot be optimal.

proposition 4 *If* $v_M x_M > v_L(x_L + x_M)$*, there cannot exist any ENC.*

To show existence of ENC's, hereafter we confine our attention to the case where the above inequality is reversed.

6 Existence of an ENC in the Three Type Case

In this section we show that ENC's exist in the three type case. Let us denote $h = v_H x_H$, $m = v_M(x_H + x_M)$, and $\ell = v_L$. The rest of our analysis will be based on the following restrictions on the parameters:

A1: $h > m > \ell$.

A2: $v_M x_M < v_L(x_L + x_M)$.

A1 guarantees that the static optimal price is v_H, and at the same time ensures quasi-concavity of the function $p[1 - F(p)]$. Under A2, the three type case is genuinely different from the two type one. Indeed, the strategy of pricing so as to induce only the high valuation type to accept the first period offer and then setting $p_2 = v_M$ without noisy communication (namely, Fudenberg and Tirole's (1983) equilibrium of type (ii) restricted to high and medium valuation buyers only)) under A2 is no longer subgame perfect. It may still be optimal to set p_1 so as to satisfy $v_H - p_1 = \delta(v_H - v_M)$, so that a high valuation buyer, being indifferent between accepting or refusing the first period offer, randomizes with an appropriate probability of rejecting. But this equilibrium has now a cost to the seller because the revenue from the high valuation buyer is delayed with a positive probability. Thus under

A2 the presence of the lowest valuation type erodes the profitability of this pricing strategy (which does not involve noisy communication) and creates a comparative advantage for noisy communication.

To proceed, we first describe the subgame perfect equilibria that do not involve noisy communication and that can emerge given assumptions A1-A2. Then, we calculate the candidate ENC, and the corresponding expected profit to the seller, using the results of section 3. Finally, we compare the profit under the equilibria without noisy communication to the ENC profit and show that the latter may be the highest for some parameter values.

6.1 Full communication subgame perfect equilibria

The equilibria with full communication that can emerge given A1-A2 are described in the following proposition.

proposition 5 *With three types of buyers, if A1-A2 hold, only three types of full communication equilibria can emerge, namely:*
H) $p_1 = p_2 = v_H$,
M) $p_1 = (1-\delta)v_H + \delta v_M$ *and* $p_2 = v_M$,
L) $p_1 = (1-\delta)v_H + \delta v_L$ *and* $p_2 = v_L$.
Equilibria H and M involve the use of mixed strategies on the part of the buyer, with the highest valuation buyer randomizing appropriately between buying soon or waiting with probabilities

$$y^H = \max\left[\frac{v_M x_M}{x_H(v_H - v_M)}, \frac{v_L(x_M + x_L)}{x_H(v_H - v_L)}\right].$$

and

$$y^M = \frac{v_L(x_L + x_M) - v_M x_M}{x_H(v_M - v_L)}$$

of waiting, respectively.

All other possible pricing strategies either are not subgame perfect or yield lower profits. On the other hand, it can be shown that each one of equilibria H, M, and L may occur for certain parameters values.

6.2 The candidate ENC

The candidate ENC can be easily characterized using the results of the previous sections. It must be $p_1 = v_M$ and $p_2 = v_H$. The necessary amount of noise μ in the first period message is given by the condition that the seller does not find it profitable to lower the second period price. It is clear that $p_2 = v_M$ cannot be optimal for any μ given that v_H is the optimal static price. Hence, it must be $\mu x_H v_H \geq (x_L + \mu x_H + \mu x_M)v_L$, that is

$$\mu \geq \frac{v_L x_L}{x_H(v_H - v_L) - v_L x_M} \equiv \tilde{\mu}. \tag{6}$$

Conditions A1 and A2 imply $\tilde{\mu} < 1$.

The corresponding expected profit is:

$$\pi^{NC} = (1 - \mu)m + \delta\mu h \tag{7}$$

and is therefore linear in μ. Thus we can restrict our attention to the points $\mu = \tilde{\mu}$ and $\mu = 1$.

In particular, if $m < \delta h$ the optimal strategy with noisy communication would involve $\mu = 1$, so that bargaining is effectively delayed to the second period. However, setting $p_1 = v_M$ and $p_2 = v_H$ with $\mu = 1$ cannot be the globally optimal strategy. Indeed, the seller could do better at the equilibrium where the price is equal to v_H in both periods with $\mu = 0$ and the high valuation buyer randomizes, like in strategy H described in Proposition 5 above. This would yield profits higher than δh. Therefore, only the ENC with $\mu = \tilde{\mu}$ can dominate the full communication strategies and thus it will be our unique candidate ENC[3]. The expected discounted profit at the candidate ENC is therefore:

$$\pi^{NC} = (1 - \tilde{\mu})m + \delta\tilde{\mu}h. \tag{8}$$

Obviously π^{NC} is increasing in δ.

[3]This implies that $\delta < m/h$ is a necessary condition for noisy communication since it guarantees that $\mu = \tilde{\mu}$ is superior to $\mu = 1$. Actually, even stronger conditions must be satisfied to obtain an ENC, as we shall show presently.

6.3 Comparison

In order to understand the advantages of noisy communication, we now compare the candidate ENC to the equilibria described in Proposition 5. It is also instructive to compare these equilibria with the full commitment optimum.

Let us begin with equilibrium L, which is the only full communication equilibrium that does not involve mixed strategies. Equilibrium L indeed illustrates the typical Coasian dynamics: an agreement is reached with certainty and in the first period the seller is unable to extract all the rent from the high valuation buyer who can wait for the price discount in the second period. Expected profit is:

$$\pi^L = (1-\delta)h + \delta\ell \tag{9}$$

It is obvious that if players are impatient (δ is close to zero), then the seller retains much of its bargaining power. Indeed, for $\delta = 0$ the solution coincides with the full commitment optimum. As players get more patient, however, expected profit falls, and achieves a minimum of v_L at $\delta = 1$.

Noisy communication is a way to limit the scope for strategic rejection by the high valuation buyer because the possibility that the buyer has not been reached by the message eliminates the seller's incentive to cut the second period price. Thus, at the candidate ENC, we have $p_2 = v_H$ like under full commitment. However, as compared to the full commitment strategy $p_1 = p_2 = v_H$ that yields the static optimal profit, noisy communication involves two types of costs. First, in the first period the price is lower than v_H which involves a loss of expected revenue. Second, the agreement is delayed to the second period with positive probability. While the former type of cost is independent of δ, the latter becomes less and less important as δ increases. As a consequence, for low values of δ, equilibrium L is superior to noisy communication but when δ approaches 1 it can be easily checked from (8) and (9) that the candidate ENC gives higher profits:

lemma 2 $\pi^{NC} < \pi^L$ *for*

$$0 \leq \delta \leq \frac{h-(1-\mu)m}{h(1+\mu)-l},$$

and $\pi^{NC} > \pi^L$ *for*

$$\frac{h-(1-\mu)m}{h(1+\mu)-l} \leq \delta \leq 1.$$

Equilibria H and M can be subgame perfect only if the buyer uses mixed strategies. Equilibrium H mimics the equilibrium with full commitment in that the price sticks to the static optimal level in both periods. However, subgame perfection requires that the buyer randomize with a sufficiently large probability of rejecting the offer in the first period (since a high valuation buyer gets zero surplus, he is indifferent between accepting the first period offer or waiting and so can randomize). The cost of subgame perfection is therefore given by the discounting of a fraction of static optimal revenue. Expected profit is:

$$\pi^H = \left[1 - (1 - \delta) y^H\right] h \tag{10}$$

where y^H is determined by the subgame perfection constraint (see the proof of Proposition 5 in the appendix for details). Clearly, equilibrium H is equivalent to the full commitment strategy at $\delta = 1$ but its profitability decreases as the discount factor δ decreases.

The candidate ENC is similar to equilibrium H in that in both cases $p_2 = v_H$. Moreover, the candidate ENC also involves delaying the agreement with a positive probability. However, noisy communication not only leads to a positive belief that the buyer be of the highest type in period 2 but it also lowers the belief that the buyer has medium valuation because a medium valuation buyer would have accepted the first period offer. As a consequence, the probability 1-$\tilde{\mu}$ that a high valuation buyer is hit by the message in the first period at the candidate ENC is higher than the probability 1-y^H that a high valuation buyer accepts the offer in the mixed strategy equilibrium H. This means that the share of expected revenue from the high valuation buyer that is delayed is lower at the candidate ENC than at equilibrium H. As a consequence, for low values of δ the candidate ENC may dominate equilibrium H as can be checked comparing (6.3) and (6.5):

lemma 3 $\pi^{NC} < \pi^H$ *for*

$$\frac{(1-\tilde{\mu})m - (1 - y^H)h}{h(y^H - \tilde{\mu})} \leq \delta \leq 1,$$

and $\pi^{NC} > \pi^H$ *for*

$$0 \leq \delta \leq \frac{(1-\tilde{\mu})m - (1 - y^H)h}{h(y^H - \tilde{\mu})},$$

provided $(1-\mu)m > (1-y^H)h$. *If, instead,* $(1-\mu)m \leq (1-y^H)h$, *then strategy H dominates the candidate ENC for all values of* δ.

Let us finally consider equilibrium M. This involves both the cost of Coasian dynamics (like equilibrium L) and of delayed revenue (like equilibrium H). Indeed, if A2 did not hold, the high valuation buyer could accept with probability 1 the first period offer without upsetting equilibrium M; in this case, as shown by Proposition 4, noisy communication cannot be optimal. But when A2 holds the possibility that the buyer's valuation be v_L would destroy perfection of equilibrium M, unless a high valuation buyer reject the offer with a sufficiently high probability. Expected profit is:

$$\pi^M = (1-\delta)y^M h + \delta(1-y^M)m \tag{11}$$

where y^M is the probability that a high valuation buyer rejects the first period offer. The higher the incentive to cut the second period price to v_L, the higher is y^M, and therefore the higher is the cost in terms of delayed expected revenue. The cost of delayed revenue may be so high that equilibrium M becomes likely dominated by the candidate ENC.

To summarize, our informal discussion suggests that equilibrium H will occur when δ is close to 1, equilibrium L will occur when δ is close to 0, and the candidate ENC can be optimal for intermediate values of δ. We now confirm this conjecture.

proposition 6 *A necessary and sufficient condition for an ENC to exist in the three type case under A1-A2 is*

$$\frac{(1-\mu)m - (1-y^H)h}{h(y^H-\mu)} > \frac{h-(1-\mu)m}{h(1+\mu)-l}. \tag{12}$$

In fact, if the above inequality holds, then there is a non empty interval $(\underline{\delta},\bar{\delta})$ such that ENC's exist if and only if $\delta \in (\underline{\delta},\bar{\delta})$.

A numerical example will show that condition (6.7) is consistent with A1-A2, and will also help to illustrate the above discussion. Let $v_H = 700$, $v_M = 420$, and $v_L = 140$, and let $x_H = 2/7$, $x_M = 1/7$, and $x_L = 4/7$. The optimal static price is then 700 and the corresponding (full commitment) profit to the seller is 200.

Then, under strategy H, a high valuation buyer accepts the first period offer with probability 1/4 so that expected discounted profits to the seller are $\pi^H = 50 + 150\delta$.

Under strategy M, a high valuation buyer randomizes between accepting the first period offer or not with probability 1/2; total discounted expected profits are $\pi^M = 100 + 80\delta$.

Strategy L yields profits $\pi^L = 200 - 60\delta$.

Finally, in the candidate ENC the noise in the first period offer is μ =4/7 and total discounted profits are $\pi^{NC} = (540 + 800\delta)/7$.

Figure 1 illustrates π^{NC}, π^H, π^M and π^Las functions of δ. Strategy M is never optimal. The candidate ENC yields the highest discounted profit for $43/61 \leq \delta \leq 19/25$ (approximately, $0.705 \leq \delta \leq 0.76$).

7 Efficiency Properties of ENC's

As is well known, bargaining equilibria may be *ex-post* Pareto inefficient as an agreement may fail to occur even when it would be in both parties interest, or it may be delayed.

Can noisy communication improve the efficiency of the bargaining process? Surprisingly, it turns out that it can. However, there are also cases where noisy communication results in an even more inefficient outcome than the standard equilibrium without noisy communication.

The example discussed at the end of the previous section may help illustrating the welfare consequences of noisy communication. They obviously depend on which type of equilibrium without noisy communication is displaced by the ENC. In the example, the ENC displaces equilibrium of type L for $43/61 < \delta < 5/7$ and it displaces equilibrium of type H for $5/7 < \delta < 19/25$.

When the ENC displaces equilibrium H, efficiency clearly increases, for two reasons. First, there is a positive probability that an agreement is reached with a type M buyer in the first period. Second, there is larger probability that an agreement with a type H buyer is reached in the first period. Another way to confirm that expected social welfare (i.e., the sum of expected seller's and buyer's surpluses) is increased is to note that in the ENC the buyer obtains a positive expected surplus, and the seller obtains a larger profit than in the displaced equilibrium H. Thus noisy communication need not be socially inefficient.

When the ENC displaces equilibrium L, things are more complex. In equilibrium L, an agreement is always reached. Moreover, a buyer of type H will always reach an agreement in the first period. These two effects tend to make equilibrium L more efficient than the ENC. However, in the ENC there is a positive probability that a type M buyer will reach an agreement in the first period, which cannot happen in equilibrium L. In our example, one can easily verify that equilibrium L is superior to the ENC on efficiency grounds.

To sum up, we have shown that there are circumstances where noisy communication, in addition to being profitable to the seller, also leads to greater social welfare. In fact, even the buyer may gain from an ENC displacing a type-H equilibrium, in which case the ENC Pareto dominates the equilibrium without noisy communication.

8 Concluding remarks

In this paper we have shown that a seller may find it profitable to introduce some frictions into the communication mechanism in a bargaining situation. The reason is that this allows him to buy some commitment, which is valuable in bargaining.

Our analysis assumes that the seller makes all offers, and also decides the communication structure. In the bargaining literature, models have been studied where the buyer plays a more active role, for example making counteroffers. In the present framework, allowing the buyer to play more actively would lead to the question of whether he could also gain from manipulating the communication mechanism. For instance, the buyer may wish to use noisy communication himself, or he may try to make it ineffective the use of noisy communication on the part of the seller. This opens many interesting issues which are left for future research.

9 Appendix

Proof of Proposition 1. Suppose to the contrary that $p_1 > p_2$ and $\mu > 0$ at the equilibrium. If $p_1 > p_2$, then the probability of an agreement at date 2 is $\mu[1 - F(v_1)] + F(v_1) - F(p_2)$ where v_1 is given by (2.4). Consider now a strategy where $\mu' = 0$ and p_1 is increased to p_1', with a corresponding value v_1', in such a way that $\mu[1 - F(v_1)] + F(v_1) = F(v_1')$. Clearly, the choice of p_2 in the second period will not change, and hence the probability of an agreement in the second period will be unaltered. Since the overall probability of agreement $1 - F(p_2)$ also remains unchanged, the probability of an agreement in the first period is unaffected. But p_1 is increased, so that expected profits must be higher. □

Proof of Proposition 2. Our characterization of ENC's in section 3 implies immediately that there can be no noisy communication if the optimal static price is v_L. Thus suppose that $v_H x_H > v_L$, so that the optimal static price is v_H. Proposition 1 and Corollary 1 imply that at any ENC it must be $p_2 = v_H$ and $p_1 < v_H$. In this context the latter inequality implies $p_1 = v_L$, since lowering the price below v_L, as well as offering any price strictly between v_H and v_L, is clearly suboptimal.

The expected profit at the candidate ENC is then

$$\pi^{NC} = (1 - \mu)v_L + \delta\mu v_H x_H. \tag{13}$$

Now notice that when $p_1 = v_L$, setting $p_2 = v_H$ is optimal independently of the value of μ because v_H is by assumption the optimal static price. Since the choice of μ is unconstrained and π^{NC} is linear in μ, at the optimum either $\mu = 0$ (implying that there is no noisy communication), or $\mu = 1$ (if $\delta v_H x_H > v_L$). However, setting $\mu = 1$ effectively means that bargaining is deferred to the second period. The seller's expected profit with this "deferring" strategy is $\delta x_H v_H$.

But the Fudenberg and Tirole (1983) strategy (iii) described in section 4 above dominates this deferring strategy. Indeed, let $p_1 = p_2 = v_H$. A high valuation consumer will then be indifferent between accepting the first or the second period offer. Consider then the mixed strategy equilibrium where a high valuation consumer randomizes between accepting soon or waiting with probabilities $1-y$ and y respectively, such that

$$yx_Hv_H = (x_L + yx_H)v_L. \tag{14}$$

This implies that in the second period the seller will still find it optimal to offer v_H. The profit at this equilibrium is $(1-y)x_Hv_H + \delta yx_Hv_H > \delta v_Hx_H$. Hence noisy communication is not optimal in this case. □

Proof of Lemma 1. When $p_1 = v_L$, the optimal second period pricing strategy is independent of μ since the value of μ does not alter the seller's second period beliefs. It follows that in the case of noisy communication expected profit is linear in μ, so that either $\mu = 0$ or $\mu = 1$ at the optimum. Thus noisy communication can occur only if it is optimal to defer all agreements to the second period. However, deferring all agreements to the second period is dominated (for $\delta > 0$) by the strategy of offering v_H in both periods, with a positive probability of an agreement in the first period, like in the proof of Proposition 2 above. □

Proof of Proposition 4. Consider the following subgame perfect equilibria, which correspond to those considered by Fudenberg and Tirole (1983) in their analysis of the two type case:

(i) $p_1 = (1-\delta)v_H + \delta v_M$, $p_2 = v_M$;

(ii) $p_1 = p_2 = v_H$.

At equilibrium (i), a high valuation consumer accepts the first period offer and a medium valuation consumer accepts the second period offer. At equilibrium (ii), only high valuation consumers accepts the seller's offers and they are indifferent between reaching an agreement in the first or in the second period. In the resulting mixed strategy equilibrium, a high valuation consumer randomizes: he accepts the first period offer with probability $1-y$ and wait for the second period offer with probability y, where y is determined so as to make the seller indifferent between setting $p_2 = v_H$ and $p_2 = v_M$. This requires $yv_Hx_H = v_M(x_M + yx_H)$ or

$$y = \frac{x_Mv_M}{x_H(v_H - v_M)} \tag{15}$$

where $y \leq 1$ by condition $v_Hx_H > v_M(x_H + x_M)$,which must hold by Proposition 3.

Since $x_M v_M > v_L(x_L + x_M)$ implies that when a high valuation consumer accepts the first period offer with probability 1 it is optimal to price at v_M in the second period (thus aiming at reaching an agreement with the medium valuation consumer only), equilibrium (i) is clearly subgame perfect. To check that equilibrium (ii) is also subgame perfect, it remains to be shown that in period 2 it is not optimal to set $p_2 = v_L$, that is:

$$y v_H x_H \geq v_L(x_L + x_M + y x_H) \tag{16}$$

This follows easily from the condition $v_H x_H > v_M(x_H + x_M)$. Thus (i) and (ii), with y given by (7.3), are candidate subgame perfect equilibria and they do not involve noisy communication.

In the mixed strategy equilibrium (ii) expected profit is:

$$\pi^{(ii)} = \left[1 - (1-\delta)\frac{x_L v_L}{x_H(v_H - v_L)}\right] x_H v_H. \tag{17}$$

whereas at equilibrium (i) expected profit is:

$$\pi^{(i)} = (1-\delta)v_H x_H + \delta v_L. \tag{18}$$

By Corollary 2, on the other hand, we know that at any ENC $p_1 = v_M$ and $p_2 = v_H$. It follows that at the candidate ENC expected profit is:

$$\pi^{NC} = (1-\mu)v_M + \delta\mu x_H v_H. \tag{19}$$

Now it is easy to check that:

$$\max[\pi^{(i)}, \pi^{(ii)}] > \pi^R, \tag{20}$$

for all $\delta < 1$ and for all $\mu > 0$. This means that noisy communication cannot be used at equilibrium. □

Proof of Corollary 2. By Proposition 2 and Corollary 1, p_1 must be lower than the optimal static price, whereas by Lemma 1 it must be higher than v_L. Now suppose the optimal static price is v_M. It follows that $v_L < p_1 < v_M$, but this is clearly dominated by $p_1 = v_M$. □

Proof of Proposition 5. Clearly, in the second period the optimal price offer must be one of $\{v_H, v_M, v_L\}$. We have therefore three possible types of equilibria without noisy communication, labelled equilibrium L, equilibrium M, and equilibrium H, according to whether the second period price is v_L, v_M or v_H, respectively.

Equilibrium H

For $p_2 = v_H$ to be part of a subgame perfect equilibrium, the first period offer cannot be lower than v_H. The probability y that a high valuation buyer defers agreement to the second period must be such that pricing at v_H in the second period is optimal to the seller. This requires $y^H v_H x_H \geq v_M(x_M + y^H x_H)$, so that pricing at v_M is dominated, and $y^H v_H x_H \geq v_L(x_L + x_M + y^H x_H)$, so that pricing at v_L is dominated as well. Both conditions must hold and this implies

$$y^H = \max[\frac{v_M x_M}{x_H(v_H - v_M)}, \frac{v_L(x_M + x_L)}{x_H(v_H - v_L)}].$$

Expected profit is:

$$\pi^H = \left[1 - (1 - \delta) y^H\right] h.$$

Equilibrium M

For $p_2 = v_M$ to be part of a subgame perfect equilibrium,it is necessary that $p_1 \neq v_M$. The reason is as follows. If $p_1 = p_2 = v_M$, then a high valuation buyer will strictly prefer to accept the first period offer, whereas a medium valuation buyer will be indifferent between accepting the first or the second period offer. However, even if a medium valuation buyer waits for the second period with probability 1, by condition A2 it would then be optimal to set $p_2 = v_L$. On the other hand, setting p_1 at a level such that a high valuation buyer strictly prefers to wait for the second period offer implies that in the second period $p_2 = v_H$ is the optimal price. Thus the only possibility is that of setting p_1 at the highest level which makes a high valuation buyer willing to accept the first period offer instead of waiting, i.e. $p_1 = (1 - \delta) v_H + \delta v_M$. Being indifferent as to when to accept, a high valuation buyer at equilibrium randomizes and accepts the first period offer with a probability $(1 - y^M)$ such that:

$$v_M(x_M + y^M x_H) = v_L(x_L + x_M + y^M x_H) \tag{21}$$

that is, such that in the second period the seller does not find it convenient to lower the price in order to reach an agreement with low valuation buyers.

This implies:

$$y^M = \frac{v_L(x_L + x_M) - v_M x_M}{x_H(v_M - v_L)}. \tag{22}$$

Note that $y^M \geq 0$ by A1. It must also be $y^M \leq 1$, that is $\ell < m$, which is satisfied by A1. Expected profit is:

$$\pi^M = (1-\delta)(1-y^M)h + \delta m. \tag{23}$$

Equilibrium L

When $p_2 = v_L$, an agreement is always reached. Clearly, $p_2 = v_L$ can be part of a subgame perfect equilibrium only if the first period price takes one of the following values: a) $p_1 = v_L$, b) $p_1 = (1-\delta)v_H + \delta v_L$, c) $p_1 = (1-\delta)v_M + \delta v_L$. With $p_1 = v_L$ the agreement is reached immediately and profit is ℓ. With $p_1 = (1-\delta)v_M + \delta v_L$ both a high and a medium valuation buyer will accept the first period offer and profit is $(1-\delta)m + \delta\ell$. Finally, setting $p_1 = (1-\delta)v_H + \delta v_L$ implies that only a high valuation buyer will accept the first period offer and yields profit $(1-\delta)h + \delta\ell$. By condition A1, it follows immediately that this latter equilibrium dominates the other ones. □

Proof of Proposition 6. Inequality (6.9) implies that there is a non empty interval of values of δ where $\pi^{NC} > \pi^L$ and $\pi^{NC} > \pi^H$. Thus we must show that $\pi^{NC} > \pi^M$ over at least a subset of this interval. To show this, it suffices to compare the values of π^{NC} and π^M at the upper bound of that interval, i.e. at point

$$\bar{\delta} = \frac{(1-\tilde{\mu})m - (1-y^H)h}{h(y^H - \tilde{\mu})}. \tag{24}$$

At that point, by construction $\pi^{NC} = \pi^H$. Thus, we must show that

$$(1-y^H)h + \bar{\delta} y^H h > (1-\bar{\delta})(1-y^M)h + \bar{\delta} m, \tag{25}$$

that is

$$(y^M - y^H)h > \bar{\delta}\left[(m - h + h(y^M - y^H)\right]. \tag{26}$$

Substituting, we get

$$(y^H - \tilde{\mu})(y^M - y^H)h^2 > \left[(1-\tilde{\mu})m - (1-y^H)h\right]\left[(m - h + h(y^M - y^H)\right]. \tag{27}$$

A sufficient condition for (27) to hold is:

$$(y^H - \tilde{\mu})(y^M - y^H)h^2 > \Big[(1-\tilde{\mu})h - (1-y^H)h\Big]\Big[(m - h + h(y^M - y^H)\Big] \quad (28)$$

which is always true given A1. □

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Figure 1

Values for δ are reported on the horizontal axes, values for profits on the vertical axes. The decreasing function is π^L, the profit under equilibrium L; the increasing function with highest value equal to 200 is π^H, the profit under equilibrium H; the increasing function which starts above π^H ,and terminates just below it, is π^{NC}, the profit with noisy communication. The value of π^M, the profit under equilibrium of type M does not exceed that of π^{NC} for any value of δ, and therefore it has not been reported.