# With a Little Help from my Enemy: Comparative Advertising

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## Abstract

Comparative advertising content differs from generic. We discover that dissipative advertising has consequences depending upon content and cost. Comparison advertising may trigger legal action by rival firms that are named. In the model an entrant signals its product quality. By a comparative ad the entrant refers to the incumbent's product. We show that comparison can enhance the signaling potential of dissipative advertising. From the viewpoint of the entrant, the choice of comparative advertising empowers the rival with the right to sue; generic does not. Consumers therefore infer that if the entrant uses comparative instead of generic ads it has a strong case.

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# 1. Introduction

Comparative advertising, in which the advertised brand is explicitly compared with one or more competing brands, has become popular in recent years.[1] For the United States, Muehling *et al.* (1990) note that around 40 percent of all advertising is comparative. According to Pechmann and Steward (1990) around 80 percent of all ads are comparative: 60 percent are indirectly comparative ads and 20 percent are directly comparative ads, whereas the remaining 20 percent are non-comparative. Sectors in which comparative advertising is frequently used include food, retail and motoring, which all rely heavily on aggressive marketing strategies. The upsurge of comparative advertising contrasts with previous attitudes of advertisers. In the United States, no law has prevented the use of comparative advertising. However, due to concerns about possible misidentification of the sponsoring brand and fear of consumers' mistrust, advertisers were reluctant to use it. They also believed that some long-term consequences of comparison advertising may be seriously detrimental to all advertising (Wilkie and Farris 1975, Prasad 1976). Indeed, also some recent marketing literature on comparative advertising is skeptical about its effectiveness (see Pechmann and Ratneshwar 1991, Jain *et al.* 1998, Barone and Miniard 1999).

The relatively sudden increase, during the 1970s, in the use of explicit comparisons in advertising, was in part a result of a Federal Trade Commission (FTC) sanction of such a technique as a means of improving competition. Although this was not publicized as an official position, it was argued that direct comparison ads would encourage consumers to make more informed purchasing decisions—moreover, comparison ads would ease the consumer's task of evaluating the performance of particular brands against other brands.

A well known explanation of advertising as a rational phenomenon hinges upon the idea that spending money is a way to signal a high quality of a brand (Nelson 1976). The signaling explanation, as advanced by Nelson, filled a gap in the understanding of a controversial economic phenomenon like apparently wasteful advertising campaigns. The argument, that was meant to apply to the case of *generic* advertising, was that the cost and not the content of an ad is what really matters (see also Milgrom and Roberts 1986). However, the argument so far cannot answer the question why a firm may want to choose to spend money in comparison rather than generic advertising. We note in the first place that generic and comparative advertising do differ in their content. More importantly, they open up different strategic opportunities for rivals to the launch of an ad.

Indeed, the law in many countries erects barriers against unfair use of comparison

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[1] A distinction is made between *direct* and *indirect* comparative advertising. In direct comparative claims the advertised brands is described as superior to *named competitors* on specific attributes or benefits, including price. (A 1999 General Motors Corp. ad claimed that the Cadillac Seville STS outperforms the BMW 540 in a slalom course. Other well-known examples of direct comparative ads concern the following brands: Coke and Pepsi, Burger King and McDonald's, Unilever and Procter and Gamble, Avis and Hertz (see David Teather in *Marketing*; London; May 11, 2000), Oracle and IBM). On the contrary, in indirect comparative claims, competitors are not named and the advertised brand is described as being superior on specific attributes or benefits.

The distinction between direct and indirect comparative ads is not relevant in the present model because only two firms (the advertising firm and its competitor) exist.

advertising. Today, in the U.S., federal advertising legislation is found in two major laws: the Federal Trade Commission Act and the Trademark (Lanham) Act (which prohibits the use of false designations of origin and false or misleading descriptions of facts). (See Shy 1995). Moreover, when advertising is comparative, lawsuit may be followed by significant fines and damages.[2] Frequently, the court simply imposes to the advertising firm to cease a campaign.

The European Union began to address the issue of comparison advertising in the late 1970s. The position was that comparison advertising should be legal if it provides verifiable details and is neither misleading nor unfair. However, practice was markedly different among member states and laws on comparative advertising were harmonized only in April 2000. According to current European legislation "comparative advertising is allowed only if it is not misleading, compares like with like, does not create confusion, discredit or take unfair advantage of a rival's trademark or present goods as imitations of those bearing a protected trade name".

It appears, therefore, important that rivals can legally persecute a firm using misleading comparisons, especially if they are explicitly named. For the advertising firm the danger to be persecuted is much lower when generic advertising is used.[3]

Also, we note that a general agreement exists in the advertising literature concerning the fact that a low-share or unfamiliar brand can enhance the relevance of an ad by naming a leading brand that consumers regularly purchase (Muehling *et al.* 1990, Pechmann and Stewart 1990). For example, citing the words of the dealer marketing manager at Mitsubishi, the company used comparative advertising when entering the European market, "to guide the consumer by making associations with top brand names" (*Director*; London; June, 2000; Alison Coleman). In line with this widespread idea, we base our theory on a model of a market where an incumbent competes against an entrant. The incumbent sells a product of known quality, while the entrant's quality is yet unknown to consumers and dissipative advertising is used as a signal of quality. Moreover, as we shall motivate further, in our model both firms know more about the entrant's quality than consumers do.[4]

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[2]In 1999 U-Haul International claimed the comparative advertising campaign undertaken by the Jartran do-it-yourself moving company was false and sued Jartran for violating the Lanham Act. The court sustained U-Haul's complaint and awarded $40 million in damages. Notice that it is generally difficult to prove that consumers are really misled by an ad. This problem was solved by the court ruling that if a marketer spends "substantial funds" in an advertising campaign, the court will presume that consumers were misled if the ad is determined to be false. In 2000 a federal judge ruled that Papa John's must pay over $468,000 in damages to Pizza Hut and cease and desist from using its tag line "Better ingredients. Better pizza." The judge ruled in favor of Pizza Hut, as the ingredient comparison was misleading. In fact the claim cannot be scientifically substantiated nor taste tests exist that prove a statistically significant preference for Papa John's product.

[3]An example that shows in which sense a comparative ads makes the advertiser more vulnerable, is what the US law defines as "puffery". An ad can be considered a puffery as long as "the customer believes the commercial statement is so vague, ridiculous or opinionated that it could not possibly be taken serious" (for a clear explanation see *Brandweek*, New York; May 2001, Jim Edwards). Puffery is accepted in generic advertising but it becomes illegal whenever it tags a competitor, exactly as in the case of Papa John's ad "Better ingredients better pizza".

[4]We could extend our model to situations in which the entrant is uncertain whether its product fits well the consumers' tastes while consumers have this information. In this situation firms have initially different information than consumers (not more). Our insights concerning the role of comparative

We assume that the entrant's profits are positively related to the quality of its product as perceived by consumers. Then a low quality entrant is interested in mimicking a high quality one.

In the model, to signal quality, the entrant can choose among generic or comparative advertising. If advertising is comparative, the entrant says explicitly that its quality is not lower than the incumbent's. The latter has the possibility to react to a comparative ad by applying to court. Thus, from the viewpoint of the entrant, choice of comparative advertising empowers the rival with the right to sue; generic does not. This implies that money spent in misleading comparative advertising is more risky for the entrant than money spent in generic advertising: if a lawsuit follows the court may impose damages. In addition, when an adverse court ruling becomes publicly known, consumers will revise their perceived quality of the advertised product. As a first insight, therefore, if the entrant uses comparative instead of generic ads it chooses to give the incumbent the right to take action, and therefore consumers infer that it must think to have a strong case. Note that, this is consistent with practitioners's point of view on the use of comparative advertising. On the web site of the advertising agency Kaye&Company (http:\\www.kayeco.com, consulted May 2001) the use of comparative advertising is encouraged because it increases credibility for the advertised product: "Side-by-side or 'A-B' comparisons can provide prospective customers with compelling reasons to buy from you. They can also help build credibility for your product. Subconsciously, the prospective customer says: -Who would risk making a direct comparison if they didn't have something truly superior?-".

To capture different features of the real world, we use a simple Bayesian game, in two versions. In both versions of the model we assume that both firms observe the same imperfect signal about the entrant's quality that can take only two values: "good" or "bad". This allows us to analyze in which way the entrant's strategy and the incumbent's reactions depend on the signal precision. (The assumption that the two signals to the firms are perfectly correlated is relaxed in section 6). The two versions of the model differ with respect to the consumers' information acquisition. In the first model, *model A*, (in section 4 below), consumers observe what type of advertising has been chosen and how much is spent on it, but they *do not* observe the incumbent's action and any judgement by the court. In the second model, or *model B*, (in section 5 below), consumers also observe the incumbent's action and the judgement by the court; consumers then can make their purchasing decision depending on the court's verdict, knowing that the court's ruling may be wrong.

Model A fits reality when consumers are not perfectly aware of firms' legal actions; for instance because these are not broadly discussed by the media or because purchasing decisions are made before any legal action is taken. Model B applies where consumers not only observe the incumbent's reaction but also the court's verdict. There, if the incumbent sues the entrant, consumers can update their beliefs on quality after the court's ruling. Note that in contrast to the first model, in this case the court's precision is important in that the court's findings affect consumer beliefs.

The results in both versions of the model point that comparative advertising can be a more powerful signal than generic advertising. In particular, the first can be

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advertising appear to be robust to this extension.

used as a signal in situations in which the second cannot - for instance, when the high quality entrant also has higher production costs than the low quality. In that case, indeed, existence of equilibria with generic advertising is ruled out.

We study in detail also the case when both type of advertising can arise at a subgame perfect Bayesian equilibrium of the game. We explicitly show that if the legal costs to be paid by the incumbent are not in excess of the damages it may receive, both generic and comparison ads can arise. However, in particular if the correlation between the good signal and high quality is large enough, comparative advertising is easily singled out as the preferred type of advertising in the unique equilibrium. We make use of the dominance selection criterion. Basically, dominance applies here because the minimum amount of "money burning" necessary to separate under comparative advertising is lower than the corresponding amount under generic advertising, due to the possible damage payments.

In this last respect some features of the equilibria with comparative advertising are worth emphasizing. Comparative advertising appears in two types of equilibria: the first where the incumbent sues the entrant also when it receives the good signal about the entrant's quality. Damages can be paid along this equilibrium path. The second where the incumbent only sues the entrant if the signal it receives is bad. This accommodating behavior occurs when the good signal is highly correlated with high quality. Damages are not paid along this equilibrium path. Therefore, the incumbent's right to file for damages reduces the cost of advertising for the entrant, even if this right is not exercised along the equilibrium path.

It is noteworthy that the first model shows that comparative advertising can be more efficient (less costly) than generic advertising even if consumers are not informed on the incumbent's actual strategy choice, and on the court's verdict. In the second model results show that comparative advertising not only can be less costly than generic advertising, but it can also be more informative, thanks to the revelation of the court's (imprecise) findings to the public. Moreover the higher signaling potential of comparative advertising is compatible with a damage level equal to zero. This is in line with empirical evidence that courts often rule that an advertising be stopped, without imposing damages.

Within the framework of model B, we also analyze the case where the court does not make mistakes, and the firms receive imperfectly correlated signals. There we see that the observation of the incumbent's choice of strategy, whatever it is, reveals additional information to consumers.

To our knowledge the economics literature has rather neglected the analysis of comparative advertising as providing explicit information about quality. Shy (1992) and (1995) focus on the matching of heterogeneous consumers with differentiated brands, where the brand producing firms dynamically compete on market shares. In his model the two firms can use either non-comparative or comparative advertising. In Shy's terms, a non-comparative advertising is 'persuasive' since it is aimed to attract new users. In contrast a comparative advertising is 'informative' and is targeted to experienced users: it is used to inform those consumers who have already purchased the product before, about the difference between their ideal brand and what they have purchased in the past. Garella and Peitz (2000b) focus upon exclusionary practices

and briefly mention that their model may also apply to comparative advertising. In that context, when the entrant can choose to produce either low or high quality *the incumbent* can use comparison ads to vehicle useful information to consumers. However, as we said, it has been claimed that comparative advertising is used by and useful to entrants more than to incumbents.

In Matthews and Fertig (1990) an incumbent and an entrant can use dissipative generic advertising to signal the entrant's quality. Similar to our model, their model allows for a reaction by the incumbent to the entrant's advertising decision in the form of a counteradvertising campaign. However, their model is markedly different from our model of comparative advertising mainly because in their model the incumbent's strategy space is not affected by the entrant's advertising decision.

The present paper is also related to the literature on advertising and market entry (Bagwell and Ramey (1988, 1990) and Linnemer (1998)). In these models the advertising strategy is undertaken by the incumbent to deter entry.

The plan of the paper is as follows: in section 2, we show by example the signaling potential of comparative advertising. In section 3 we present the model. In section 4 we analyze in detail model A. We obtain unique equilibria using standard equilibrium refinements. In section 5 we analyze model B. In section 6 we analyze a variant of model B in which the information of the two firms is not perfectly correlated. Section 7 concludes.

## 2. An illustration of the signaling potential of comparative advertising

In this section we provide a simple example of the signaling potential of comparative advertising.

An established and a new firm operate in a market. Consumers do not know the product quality of the new firm, whereas both firms do. The entrant's quality is either high ($H$) or low ($L$). Producing high quality leads to fixed costs $F$, producing low quality leads to zero fixed costs, while variable costs are zero for both qualities. It is commonly known which quality the established firm offers. Without loss of generality, it is assumed that this quality is high. Profits of the established firm depend on its own quality (always $H$) and consumer beliefs about the product quality of the new firm $q^e$. Given high quality of the established firm its reduced profits are written as $\Pi_I(q^e)$. Given the quality of the established firm profits of the new firm depend on its true quality $q_E$ and its perceived quality $q^e$. Reduced profits are written as $\Pi_E(q^e, q)$. Note in particular that $\Pi_E(q^e, H) = \Pi_E(q^e, L) - F$. The established firm's profits are decreasing in the competitor's perceived quality. In particular, $\Pi_I(L) > \Pi_I(H)$. The new firm makes higher profits the higher its perceived quality. In particular, $\Pi_E(H, q) > \Pi_E(L, q)$.

The two firms play the following game in extensive form: Stage 1: Nature chooses the product quality of the new firm (for illustration: quality $H$ with probability $1/2$, and $L$ with probability $1/2$). Both firms observe the quality of the new firm, consumers do not. Stage 2: The entrant decides among the set of advertising types $\{c, g, n\}$, namely comparative, generic, or no advertising respectively. Associated costs

are $A_c$, $A_g$ and 0, respectively. We assume, for simplicity, that after choosing $g$ or $c$ the advertising cost is unavoidable and can only take a given value $A_c = A_g = \tilde{A}$; this emphasizes that technically the two types of advertising differ only in the "wording" of the message. Stage 3: If the new firm uses comparative advertising the established firm decides whether to make a complaint to court, paying legal costs $C$. The court verifies the quality of the new firm and thus whether its claim was justified. If it was not justified the new firm has to pay damages $D$. Stage 4: Firms set prices (or quantities). Stage 5: Consumers observe the decisions on stages 1 to 4 (including the court verdict) and update their beliefs concerning the product quality of the new firm based on the actions in stages 2 and 3. Then they make their purchasing decisions.

We make the assumption that the new firm gains from generic advertising if this makes consumers believe in high quality, $\Pi_E(H, q) - \tilde{A} > \Pi_E(L, q)$. While this makes generic ads possibly attractive, it also implies that there does not exist a perfect Bayesian equilibrium in which the new firm uses generic advertising[5].

Does comparative advertising suffer the same fate? Not necessarily. Suppose that using comparative advertising (choosing $c$ at stage 2) makes consumers believe in high quality unless the court verdict contradicts the advertising claims. The separating constraint for a high type entrant is $\Pi_E(H, H) - \tilde{A} > \Pi_E(L, H)$. Suppose furthermore that the established firm makes higher profits unmasking its competitor to be of low quality and receiving damages than it would make under high quality beliefs, namely $\Pi_I(L) - C + D > \Pi_I(H)$. When the incumbent reacts to a false claim, the separating constraint for an entrant of type $L$ is $\Pi_E(L, L) - \tilde{A} - D \leq \Pi_E(L, L)$, which is trivially verified. Since $\Pi_I(L) > \Pi_I(H)$ holds by assumption, a separating equilibrium with comparative advertising exists if $D > C$ and for sufficiently low cost of advertising, namely for $\Pi_E(H, H) - \Pi_E(L, H) > \tilde{A}$. This is the only possible separating perfect Bayesian equilibrium.

The general idea behind the example is that comparative advertising triggers strategic interaction between informed parties. This interaction allows the uniformed party (consumers) to infer about the realization of an unobservable variable (product quality). There are two channels through which the incumbent's strategy may help the entrant. First, the choice of comparative advertising is interpreted as a stronger signal than generic because it would lead to legal action and payment of damages in case of cheating. Further, the observation of the informed incumbent not reacting to comparative claims is interpretable as good news about the entrant's quality. The first channel may operate with or without the second as it shall be clear from the analysis in the following sections.

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[5]At a separating equilibrium type $L$ uses $n$ and type $H$ chooses $g$ and forcibly pays $\tilde{A}$. The separation constraint for the low type is $\Pi_E(L, L) \geq \Pi_E(H, L) - \tilde{A}$. For the high type it is: $\Pi_E(L, H) \leq \Pi_E(H, H) - \tilde{A}$. Therefore, a separating equilibrium could exist only if the interval $\Pi_E(H, L) - \Pi_E(L, L) \leq \tilde{A} \leq \Pi_E(H, H) - \Pi_E(L, H)$ was not empty, which is impossible considering that the assumption on the cost of quality implies $\Pi_E(q^e, H) = \Pi_E(q^e, L) - F$.

## 3. The model

In the introductory example, by generic advertising the entrant cannot signal its quality and comparative advertising prevails. This seems to be an extreme result. We shall now consider a model that gives more realistic predictions and explain the choice between generic and comparative advertising when both can potentially provide a signal of quality.

As in the example, we only make assumptions on reduced profit functions and we shall maintain the assumption that the incumbent $(I)$ is known to be of high quality.[6] The entrant's $(E)$ quality can be either high $(q = H)$ or low $(q = L)$. We denote $\Pi_E(q^e, q)$ the gross profits of an entrant with true quality $q$ whose perceived quality is $q^e$. Therefore, for instance, $\Pi_E(H, L)$ and $\Pi_E(L, H)$ respectively denote gross profits of a low and high quality entrant whose quality is wrongly perceived. In the same way $\Pi_I(q^e, q)$ denote profits of the incumbent dependent on the entrant's perceived and true quality. For a discussion and an example see Subsection 4.3.

We assume that the entrant's sales and profits are increasing, while the incumbent ones are decreasing, in the entrant *perceived* quality, $q^e$. Thus, the following inequalities are assumed:

- A.1  $\Pi_E(q^e, q) > \Pi_E(q^{e\prime}, q)$ if $q^e > q^{e\prime}$

- A.2  $\Pi_I(q^e, q) < \Pi_I(q^{e\prime}, q)$ if $q^e > q^{e\prime}$

Assumptions (A.1) and (A.2) are standard properties of many oligopoly models where products have some degree of both, horizontal and vertical differentiation (for a discussion see Garella and Peitz, 2000a).

Concerning the way product quality affects the entrant's profits, we will consider the case where production costs are increasing and the case they are decreasing in quality＿in the introductory example of section 2 they were constant. If the fixed cost is independent of quality, the dependence of marginal costs on true quality translates into the dependence of the entrant's profits for given perceived quality on true quality: profits are increasing or decreasing in true quality.

According to Nelson (1974), and as formalized by Milgrom and Roberts (1986), advertising has no direct impact on utility, demand, or profits. Its only possible influence is through consumers' perception of quality. The new feature of our model is that the content of an ad is important: when the entrant uses generic advertising it cannot be sued, because claims on product's attributes are generic. This is clearly a limit case, nevertheless we do not loose in generality because the probability of the incumbent's legal action is surely higher when advertising is comparative than when it is generic.

The content of advertising and its cost are publicly observable. We assume that advertising is not needed to inform consumers of the marketed products' existence; therefore, if consumers were fully informed concerning the entrant's quality, the entrant would not advertise at all.

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[6] Our results can be extended to the case in which the incumbent is of any known quality.

In our model incentives to cheat on quality are straightforward: $\Pi_E\left(q^e, L\right)$ is an increasing function of $q^e$.

The entrant's quality is high with probability $\alpha_0$ or low with probability $1 - \alpha_0$. Hence, the *ex ante* expected quality of the entrant is

$$M_0 \equiv \alpha_0 H + (1 - \alpha_0)L. \tag{3.1}$$

Both the incumbent and the entrant observe a signal $s_i \in \{s_L, s_M\}$ and update their priors on quality. If the observed signal is $s_L$, both firms exactly know that the entrant quality is low. Whereas, if the observed signal is $s_M$, both firms know the entrant's quality is high with probability $\alpha_1$ and low with probability $1 - \alpha_1$; where $\alpha_1$ and $1 - \alpha_1$ represent updated beliefs (see figure 1). This particular way to introduce a noise in the knowledge of firms can be interpreted as follows. Testing a product allows a firm to find out that it is of low quality if the test is not successful. However, even after a series of successful tests there remains a doubt whether the product is indeed of high quality. We assume for simplicity that the signals received by the two firms are perfectly correlated (an analysis of the case with imperfect correlation of signals is deferred to section 6).

Let $\lambda$ be the probability that the signal $s_M$ is observed when the type is low: $\lambda = prob(s_M \mid L)$. By applying Bayes' rule we find:

$$prob(H \mid s_M) \equiv \alpha_1 = \frac{\alpha_0}{\alpha_0 + \lambda\left(1 - \alpha_0\right)} > \alpha_0$$

and $1 - \alpha_1 = \left[\lambda\left(1 - \alpha_0\right)\right] / \left[\lambda\left(1 - \alpha_0\right) + \alpha_0\right] < 1 - \alpha_0$. Notice that, for $\lambda = 0$, observing the signal $s_i$, $i = L, M$, both firms perfectly infer the type; while, for $\lambda = 1$, there is no updating and the signal is useless. In other words, the lower is $\lambda$, the higher is the signal precision. After observing the signal $s_M$ the expected quality is

$$M_1 \equiv \alpha_1 H + (1 - \alpha_1)L. \tag{3.2}$$

Consumers do not observe the signal and their priors remain $\alpha_0$ and $1 - \alpha_0$.[7]

insert figure 1 here

*Advertising.* After observing the signal $s_i$, the entrant chooses between comparative, generic or no advertising to signal its quality to consumers. Formally, the choice consists of picking an element in the set $\{c, g, n\}$, where $c$ corresponds to comparison advertising, $g$ to generic, and $n$ to no advertising. After choosing $n$ the entrant has no cost and no further option to claim a high quality. After choosing $c$ or $g$ the entrant decides the level of advertising expenditure, denoted respectively $A_c$ or $A_g$. For ease of exposition we impose that having chosen $c$, or $g$, advertising expenditures must exceed a nonnegative threshold, $A_j \geq \underline{A} \geq 0$, $j = g, c$, where $\underline{A}$ can be set to zero

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[7]The assumption that an entrant's quality may not be known to consumers is not new (Farrell 1986) and seems natural for many markets. The assumption that the incumbent has access to the same information about the competitor's quality than the entrant itself, while consumers have not, is certainly admissible for a wide range of cases (see Garella and Peitz 2000b).

eventually, and is interpreted as a diffusion cost of advertising which is necessary to reach consumers.

*Lawsuits threats.* In the case comparative advertising is undertaken, the incumbent's strategy space, $\{\ell, a\}$, includes the option to go to court; i.e. the decision whether to file for a lawsuit, choosing $\ell$, or not (accommodate the quality claim), choosing $a$. If the entrant chooses $g$, instead of $c$, the incumbent cannot file and its strategy space is trivially $\{a\}$.

To simplify the presentation, prices or quantities are not used by consumers to extract information concerning product quality. Firms observe the true quality (and related costs) before setting prices or quantities[8] (see the example in section 4.3 below). Competition takes place only once so that no repeated purchases occur. This allows us to concentrate on the signaling role of advertising strategies.

We assume that, by applying to court, the incumbent pays legal costs $C$, it obtains verification of the entrant's quality and receives an indemnity, $D$, if the claim was found to be false. No payments are made by the entrant if the entrant is really a high quality producer.[9] The sequence of moves can be summarized as follows:

Stage 1. Nature chooses the type of the entrant ($H$ or $L$). Firms receive a signal $s_i \in \{s_L, s_M\}$ about the entrant's product quality.

Stage 2. At stage 2.a, the entrant chooses among $\{c, g, n\}$ and, at stage 2.b, advertising expenditures $A_j \geq \underline{A}$, for $j = c, g$.

Stage 3: The incumbent can choose among $\{\ell, a\}$ if $j = c$ where choice of $\ell$ implies cost $C$. If the incumbent chooses $\ell$ the court becomes active. In case the court finds the comparative claim to be wrong the entrant has to pay damages $D$ to the incumbent.

Stage 4: Competition is resolved (firms simultaneously choose prices or quantities).

Stage 5: Consumers make purchasing decisions.

*Consumers' information.* In *model A*, (section 4) consumers only observe the entrant's advertising strategy — neither do they observe incumbent's choice at the third stage among $\ell$ and $a$, nor do they observe the court's verdict. Thus consumers update their priors on the entrant's quality based only upon the observed advertising. In *model B*, in Section 5 below, consumes observe the entrant's and the incumbent's strategy and the verdict of the court in a lawsuit. The updating of beliefs is based upon the firms' strategies and upon the verdict.

Using the two models has the advantage of representing two situations that could arise. In the first one, consumers are not able to learn about the existence of a legal dispute before their purchase is made, or the incumbent's decision to go to

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[8]A similar simplifying assumption is used in the limit-price models in the vein of Milgrom and Roberts (1982) where the entrant comes to know the incumbent's cost after entry and before setting prices.

[9]Note that the special case $D = C$ is equivalent to the situation in which the loosing party has to pay the legal costs.

court happens with a delay. In this first model court ruling cannot have an effect on consumers' *information* by construction; nevertheless, as we shall show, comparative advertising can affect consumers' perceptions. The second model may be relevant when consumers information about legal procedures spreads quickly and courts are fast in reaching a verdict. Model B, moreover, allows for studying the role of the court's precision, interpreted as decreasing with the likelihood of errors in judgements. Finally model B is also used to describe a situation where the court's verdict is always correct but the firms' signals are imperfectly correlated.

We shall use the solution concept of Subgame Perfect Bayesian Equilibrium (SPBE). A SPBE of the game is defined as a strategy profile such that, given beliefs and equilibrium strategies of the other players, profits or utilities are maximized at each stage (in each subgame) and firms' and consumers' beliefs are updated according to Bayes' rule. An equilibrium is said to be separating when consumers beliefs at equilibrium allow them to infer the signal received by the two firms. In particular, we are interested in separating equilibria where no advertising ($n$) is used when the signal is $s_L$, while advertising ($c$ or $g$) is chosen when signal is $s_M$.

# 4. Belief revision and the type of advertising

## 4.1. Separating equilibria

In model A consumers only observe the entrant's advertising decision, update their priors on the entrant's quality, and make the purchasing decision. They do not observe the firms' legal dispute. This means that in terms of the information received by consumers we consider here the most unfavorable situation for comparative ads. As we shall see, the risk of actually paying damages when cheating on quality is crucial for the results in the present section.

**Definition 1.** *We distinguish three types of separating equilibria, where no advertising, $n$, is used if the signal is $s_L$:*

1. *equilibria of type 1, with comparative advertising, $c$, when the signal is $s_M$, in which the incumbent files for a lawsuits whenever comparative advertising is used;*

2. *equilibria of type 2, with comparative advertising, $c$, when the signal is $s_M$, in which the incumbent files only if comparative advertising was used in state $s_L$;*

3. *equilibria of type 3, in which the entrant uses generic advertising, $g$, when the signal is $s_M$.*

Notice that in both equilibria of type 1 and 2 the incumbent claims damages if comparative advertising was used in state $s_L$: such reaction represents the incumbent's strategy off the equilibrium path. What distinguish these two types of equilibria is the incumbent' reaction in state $s_M$. Equilibria of type 2 are particularly interesting because the incumbent does not file for a lawsuit along the equilibrium path: the

threat of a legal action in state $s_L$ is sufficient to support comparative advertising as a signaling device.

At a separating equilibrium, if the entrant undertakes an advertising campaign $A_j \geq \underline{A}$, with $j = c, g$, consumers learn that the signal $s_M$ was observed, they update their beliefs and form expectations about quality $M_1$ as defined by (3.2). Both with comparative and generic advertising the entrant gains $\Pi_E(M_1, H)$ with probability $\alpha_1$ and $\Pi_E(M_1, L)$ with probability $1 - \alpha_1$, gross of advertising expenditures (and damages when advertising is comparative and the equilibrium is of type 1).[10] Let

$$\bar{\Pi}_E(M_1) \equiv \alpha_1 \Pi_E(M_1, H) + (1 - \alpha_1)\Pi_E(M_1, L)$$

denote the expected profit when it is common knowledge among firms that the realized state is $s_M$ (recall that this uses the simplifying assumption that firms know the true quality when taking price or quantity decisions) and consumers' up-dated beliefs are $M_1$. In the same way, let

$$\bar{\Pi}_E(L) \equiv \alpha_1 \Pi_E(L, H) + (1 - \alpha_1)\Pi_E(L, L)$$

denote expected profit when the realized state is $s_M$ and no advertising is made such that the entrant is perceived as a low type producer. When it does not advertise, the entrant obtains the profit $\Pi_E(L, H)$ if it is a high type, and the profit $\Pi_E(L, L)$ if it is a low type.

Let us focus first on *generic advertising,* that is on *equilibrium of type 3.* In case generic advertising is used, there cannot be any legal claim by the incumbent and the separating constraint for an entrant who observes the signal $s_M$ is:

$$\bar{\Pi}_E(M_1) - A_g \geq \bar{\Pi}_E(L). \tag{4.1}$$

(4.1) ensures that, when the signal is $s_M$, the entrant has no interest in mimicking the state of nature $s_L$ by using no advertising.

The separation constraint for an entrant who observes the signal $s_L$ is:

$$\Pi_E(L, L) \geq \Pi_E(M_1, L) - A_g \tag{4.2}$$

(4.2) ensures that, when the signal is $s_L$, the entrant has no interest in mimicking the state of nature $s_M$ by using generic advertising. From (4.2) we find the lower bound for generic advertising, which corresponds to the mimicking gain, denoted $\widehat{A}_g$:

$$A_g \geq \Pi_E(M_1, L) - \Pi_E(L, L) \equiv \widehat{A}_g \tag{4.3}$$

Putting together (4.1) and (4.2), the condition for the existence of a separating SPBE with generic advertising expenditure, that is an equilibrium of type 1, is:

$$\Pi_E(M_1, L) - \Pi_E(L, L) \leq A_g \leq \bar{\Pi}_E(M_1) - \bar{\Pi}_E(L) \tag{4.4}$$

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[10]This follows from the hypothesis on the timing of the incumbent's reaction so that consumers do not know the quality of the entrant before they make their purchase.

Furthermore note that, substituting for $\bar{\Pi}_E(M_1)$ and $\bar{\Pi}_E(L)$ in (4.4) the interval for $A_g$ given by (4.4) is non-empty if and only if:

$$\Pi_E(M_1, L) - \Pi_E(M_1, H) \le \Pi_E(M_1, H) - \Pi_E(L, H). \tag{4.5}$$

If the cross derivative of $\Pi_E$, $\partial^2 \Pi_E / \partial q_E^e \partial q_E$, exists for all qualities between $L$ and $H$ and does not change in sign on this range then condition (4.5) is equivalent to condition:

C.1
$$\frac{\partial^2 \Pi_E(q_E^e, q_E)}{\partial q_E^e \partial q_E} \ge 0$$

Condition C.1 says that profit changes due to changes in the expected quality are non-decreasing in the true quality of the entrant's product.[11] This condition limits the scope for generic advertising. Notice that condition C.1 can hold only if the entrant's unit costs are non-increasing in quality (assuming that competition is one-shot).[12] If, on the contrary, the entrant's unit costs are increasing in quality, then C.1 is never satisfied and no separating equilibrium with *generic* advertising can exist. Notice also that condition C.1 holds with equality if product quality only affects fixed costs or does not affect costs at all. (For further discussion see the example in section 4.3 below).

Focus now on *comparative advertising* and *equilibria of type 1*. Recall that a type 1 equilibrium has the property that the entrant chooses $c$ if the signal is $s_M$ and, observing $c$, the incumbent chooses $\ell$. The incumbent's expected profit after a comparative claim in state $s_M$ is, if it goes to court:

$$\alpha_1 \Pi_I(M_1, H) + (1 - \alpha_1)\left[\Pi_I(M_1, L) + D\right] - C.$$

Let $\alpha_1 \Pi_I(M_1, H) + (1 - \alpha_1)\Pi_I(M_1, L) \equiv \bar{\Pi}_I(M_1)$. If the signal $s_M$ has been observed, the incumbent files for damages when observing a comparative ad if the following condition is verified:

$$\bar{\Pi}_I(M_1) - C + (1 - \alpha_1) D \ge \bar{\Pi}_I(M_1) \tag{4.6}$$

which can be rewritten as:

$$\alpha_1 \le \hat{\alpha} \equiv (D - C)/D \tag{4.7}$$

that is, according to intuition, reaction always occurs when the probability that the entrant is a high quality producer is low enough. Recall that $\alpha_1$ is decreasing in $\lambda$, the "signal precision". While, if $\alpha_1 > \hat{\alpha}$, the incumbent does not react in state $s_M$.

If the signal $s_L$ has been observed, the incumbent chooses $\ell$ when observing a comparative ad if:

$$\Pi_I(M_1, L) - C + D \ge \Pi_I(M_1, L) \tag{4.8}$$

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[11] When $C.1$ is verified the so-called "single crossing condition" is automatically verified.

[12] In some markets mandatory warranties exist. Then a low quality can be interpreted as a high probability of replacing defective items. This translates in higher costs.

that is if the indemnity, $D$, is higher than legal costs, $C$. If $D < C$, instead, the content in advertising cannot play a signaling role because the incumbent never goes to court and advertising with a comparative claim is formally equivalent to generic advertising.

Given that the incumbent reacts and the penalty $D$ must be payed if quality is low, that is with probability $1 - \alpha_1$, the expected profit of an entrant who observes $s_M$ and uses comparative advertising, is

$$\bar{\bar{\Pi}}_E(M_1) - (1 - \alpha_1)D - A_c. \tag{4.9}$$

>From (4.9), the separating constraint for an entrant who observes $s_M$ is

$$\bar{\bar{\Pi}}_E(M_1) - (1 - \alpha_1)D - A_c \geq \bar{\Pi}_E(L) \tag{4.10}$$

where, again, the r.h.s. indicates expected profit when no advertising is made and, thus, the entrant is perceived as a low type producer. Inequality (4.10) ensures that, when the signal is $s_M$, the entrant has no interest in mimicking the state of nature $s_L$ by using no advertising.

Given that the incumbent always files for damages if comparative advertising is used, the separating constraint for an entrant who observes the signal $s_L$ is

$$\Pi_E(L, L) \geq \Pi_E(M_1, L) - D - A_c. \tag{4.11}$$

The r.h.s. of (4.11) shows that the mimic is sanctioned by the court's verdict and is obliged to pay the penalty $D$. Inequality (4.11) gives the lower bound for comparative advertising, which corresponds to the mimicking gain with this type of advertising:

$$A_c \geq \Pi_E(M_1, L) - \Pi_E(L, L) - D \tag{4.12}$$

(4.12) shows that, if the damages are large enough, the cost of signaling high quality can be zero. Whereas, with generic advertising, the lower bound of advertising expenditure is always positive (see (4.3)).

When the incumbent always chooses $\ell$ after comparative advertising (both inequalities (4.7) and (4.8) hold), putting together (4.10) and (4.11) we find that a separating equilibrium of type 1 entails:

$$\Pi_E(M_1, L) - \Pi_E(L, L) - D \leq A_c \leq \bar{\bar{\Pi}}_E(M_1) - \bar{\Pi}_E(L) - (1 - \alpha_1)D \tag{4.13}$$

Turn now to *type 2* equilibria. Recall that a type 2 equilibrium has the property that the entrant only uses comparative advertising if the signal is $s_M$ and the incumbent only goes to court as a response to comparative advertising if the signal is $s_L$. Looking for type 2 equilibrium existence conditions, then, we must replace the constraint (4.10) by (4.1). From the above considerations (see inequality (4.6)) we know that the constraint (4.1) is relevant only if $\alpha_1 > \hat{\alpha}$, that is if the probability of a high quality entrant is high enough. Putting together (4.11) and (4.1) we find that a separating equilibrium of the *type 2* entails:

$$\Pi_E(M_1, L) - \Pi_E(L, L) - D \leq A_c \leq \bar{\bar{\Pi}}_E(M_1) - \bar{\Pi}_E(L) \tag{4.14}$$

Notice that the lower bound for $A_c$ is the same in the equilibria of type 1 and of type 2.

**Remark 1.** *(a) A separating equilibrium with generic advertising (type 3) can only exist if C.1 holds. (b) If $D > C$, the existence of the equilibrium of type 3 is a sufficient condition for the existence of equilibria of types 1 and 2. (c) If $D < C$, only equilibria of type 3 can exist (under C.1), because the incumbent does not file for damages.*

We denote by $A_g^*$, and $A_c^*$ the minimal value for $A_g$ and $A_c$ respectively, as given by the maximum among $\underline{A}$ and the lower bound defined by (4.3) or (4.12) as it applies. Hence, $A_g^* = \max\{\underline{A}, \widehat{A}_g\}$ and $A_c^* = \max\{\underline{A}, \widehat{A}_g - D\}$.

**Remark 2.** *Comparative advertising can signal at a lower cost than generic advertising, in particular this holds when $A_g^* > \underline{A}$ such that $A_g^* = \widehat{A}_g$.*

**Proposition 1.** *Suppose condition C.1 holds.*
   *(a) If $D > C$ and $\alpha_1 \leq \widehat{\alpha}$, there exist SPBE of type 1 with no advertising in state $s_L$, $A_c \geq A_c^*$ in state $s_M$, and where the incumbent claims for damages if comparative advertising is used.*
   *(b) If $D > C$ and $\alpha_1 > \widehat{\alpha}$, there exist SPBE of type 2 with no advertising in state $s_L$, $A_c \geq A_c^*$ in state $s_M$, and where the incumbent claims for damages only if comparative advertising is used in state $s_L$.*
   *(c) There exist SPBE of type 3 with no advertising in state $s_L$ and $A_g \geq A_g^*$ in state $s_M$*

Proof: As stated above C.1 implies that (4.5) is satisfied. Furthermore, there exist advertising expenditures which satisfy (4.13) and (4.14). The result then follows from above. ∎

Figure 2 illustrates Proposition 1 in the plane $(D, \alpha_1)$.

<div align="center">insert figure 2 here</div>

Note that equilibria of type 1 and 2 are mutually exclusive. Moreover, as we have seen, *if C.1 does not hold*, there cannot exist a signaling role for generic advertising. Nevertheless, there possibly exist SPBE of type 1 or 2. The next proposition provides such existence results.

**Proposition 2.** *Suppose condition C.1 does not hold.*

*(a) If $D > C$ and $\alpha_1 \leq \widehat{\alpha}$, there exist SPBE of type 1, provided that*

$$[\Pi_E(M_1, H) - \Pi_E(L, H)] - [\Pi_E(M_1, L) - \Pi_E(L, L)] \geq -D.$$

*(b) If $D > C$ and $\alpha_1 > \widehat{\alpha}$, there exist SPBE of type 2, provided that*

$$[\Pi_E(M_1, H) - \Pi_E(L, H)] - [\Pi_E(M_1, L) - \Pi_E(L, L)] \geq -\frac{D}{\alpha_1}.$$

*(c) There do not exist SPBE with generic advertising.*

Proof: The conditions in (a) and (b) follow from rewriting (4.13) and (4.14), respectively. The result then follows from above. ∎

This means that if condition C.1 does not hold, separating equilibria with comparative advertising exist for damages $D$ sufficiently large.

Proposition 2 shows that comparative advertising can prevail over generic based on existence results. However, when condition C.1 holds this prevalence can only be found by use of standard equilibrium selection criteria, as shown in the following subsection 4.2.

## 4.2. Reasonable beliefs and selection among separating equilibria

In this subsection we select among the different separating equilibria that can exist. Since we are interested in knowing when generic advertising prevails over comparative advertising (of types 1 and 2), and vice versa, we restrict attention to the case in which condition C.1 holds (selection of pooling equilibria is analyzed in the appendix 8.1).

We base our analysis on the dominance criterion. In a given state, an action by the entrant is dominated *for a particular belief system* if a different action, given the incumbent's (belief-based) equilibrium response to that action, leads to an increase in the entrant's payoff. Roughly speaking, an equilibrium action is *dominated* for a given entrant type (e.g. type $L$) if it is dominated in the sense just specified for *all possible belief systems*. According to the dominance criterion, if an equilibrium entails beliefs attaching strictly positive probability to a strategy which is dominated for all possible beliefs, then that equilibrium can be eliminated.

Formally, for some action (advertising) $A_j$ by an entrant of type $s_t$, $t = M, L$, denote by $x$ the equilibrium responses by the incumbent for given beliefs attaching strictly positive posterior probabilities to $s_t$; the set $X^*(s_t, A_j)$ being defined as the collection of all such elements $x$. Similarly let $x'$ denote the best responses to action $A'_k$, with $k = c, g$. Net profits of the entrant are denoted by $\pi_E$.

**Definition 2.** *An action $A_j$, is strictly dominated for type $s_t$ if there exists an action $A'_k$, such that*

$$\min_{x' \in X^*(s_t, A'_k)} \pi_E(A'_k, x', s_t) > \max_{x \in X^*(s_t, A_j)} \pi_E(A_j, x, s_t) \tag{4.15}$$

Denote $S^*(A_j) = \{s_t | \nexists A'_k \text{ that satisfies (4.15)}\}$. A SPBE then satisfies the dominance criterion if for all actions $A_j$ with $S^*(A_j) \neq \emptyset$ the posterior belief of $s_t$ upon observing $A_j$ is strictly positive only if $s_t \in S^*(A_j)$.

**Lemma 1.** *(a) Equilibria of type 1 or type 2 with $A_c > A_c^*$ in state $s_M$ do not satisfy the dominance criterion.*
*(b) Equilibria of type 3 with $A_g > A_g^*$ in state $s_1$ do not satisfy the dominance criterion.*

Loosely speaking this means that only the least-cost signals can be undominated. Note that result (a) alone is sufficient to ensure that there is at most one separating SPBE that satisfies the dominance criterion if condition C.1 is violated.

**Lemma 2.** *Suppose $\underline{A} < A_g^*$. If a SPBE of type 2 exists, SPBE of type 3 do not satisfy the dominance criterion.*

**Proof:** Posterior beliefs which are consistent with the dominance criterion have the property that upon observing $A_c^*$ or $A_g^*$ the entrant is believed to be of quality $M_1$. Gross of advertising expenditures, the equilibrium profits following these two advertising strategies are the same. Since $A_c^* - A_g^* = -\min\{A_g^* - \underline{A}, D\} < 0$ the entrant obtains higher profits with comparative advertising. Therefore, SPBE of type 3 do not satisfy the dominance criterion. ∎

As a comment on the last result, even if consumers *are not* informed about the incumbent's legal actions and about the court's verdict, comparative advertising is more efficient in terms of advertising expenditures than generic advertising. This always holds when the conditions for the existence of equilibria of type 2 are verified.

The selection between equilibria of type 1 and 3 is less straightforward. The reason is that damages affect the expected payoff of the entrant. In the dominant equilibrium with comparative advertising of type 1 the entrant's expected profits are:

$$\alpha_1 \Pi_E(M_1, H) + (1 - \alpha_1)(\Pi_E(M_1, L) - D) - A_c^*.$$

These profits have to be compared to expected profits using generic advertising (taking the least-cost equilibrium of type 3):

$$\alpha_1 \Pi_E(M_1, H) + (1 - \alpha_1)\Pi_E(M_1, L) - A_g^*$$

The difference between these two terms is positive if

$$D < (A_g^* - \underline{A})/(1 - \alpha). \tag{4.16}$$

If damages to be paid are lower than the threshold given above, then the entrant's expected profits under generic are lower than under comparative advertising of type 1. Low damages therefore make comparative advertising of type 1 more attractive than generic. In particular, if the advertising needed to satisfy the entrant's incentive constraint, $\Pi_E(M_1, L) - \Pi_E(L, L) - D$ exceeds the technological constraint $\underline{A}$ the least-cost equilibrium of type 1 is more attractive for the entrant than the corresponding equilibrium of type 3, independent of $\alpha_1$.[13]

**Lemma 3.** *Whenever SPBE of type 1 and type 3 coexist, type 3 equilibria do not satisfy the dominance criterion if (4.16) holds. Type 1 equilibria do not satisfy the dominance criterion if the reverse of (4.16) holds.*

Putting the above results together, we always find a unique SPBE among all separating SPBE that satisfies the dominance criterion provided that C.1 holds (and that non-negativity of profits constraints are satisfied).

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[13]Notice that, in this latter case, the difference between the entrant's expected profit with type 1 equilibrium and expected profit with type 3 equilibrium reduces to $\alpha_1 D$; thus it is maximal when the signal is perfectly informative ($\lambda = 0$ that implies $\alpha_1 = 1$) and falls to zero when the signal is non-informative ($\lambda = 1$ that implies $\alpha_1 = \alpha_0$).

**Proposition 3.** *Suppose C.1 holds, then in the parameter regions such that expected profits are non-negative, there exists a unique SPBE within the set of separating SPBE that satisfies the dominance criterion:*
*(1) If $D > C$ and $(D - A_g^* + \underline{A})/D < \alpha_1 < \widehat{\alpha}$ it is of type 1;*
*(2) If $D > C$ and $\alpha_1 > \widehat{\alpha}$ it is of type 2;*
*(3) If $D \leq C$ or $\alpha_1 < (D - A_g^* + \underline{A})/D$ it is of type 3.*

Figure 3 and 4 illustrate the proposition 3 respectively when $A_g^* - D > \underline{A}$ and $A_g^* - D < \underline{A}$.

<center>insert figure 3 and 4 here</center>

## 4.3. An example with horizontal differentiation

The analysis rests on some assumptions. The purpose of this subsection is to present an example where they are all met. Assume that the incumbent is a firm located at point 0 in the Hotelling linear city of unit length, with the consumer population distributed according to the uniform distribution of unit mass. Further, assume that each consumer buys one unit at most of the indivisible, differentiated good. The transportation costs are linear, and the unit transport cost is $t$. The entrant can only enter at point 1. Therefore after entry the firms play a noncooperative and one-shot price game. A consumer located at point $x$ in $[0, 1]$ obtains utility $u_I^x = r + \theta H - tx - p_I$ if she purchases the good from the incumbent at price $p_I$. The sum $r + \theta H$ is the maximum reservation price (surplus) when $p_I = x = 0$ for the high quality good produced by the incumbent. Similarly, the expected utility from buying at price $p_E$ from the entrant is $u_E^x = r + \theta q_E^e - t(1 - x) - p_E$, where $q_E^e$ is the expected surplus obtained from purchase from the entrant given beliefs on quality. Suppose that $r$ is sufficiently high so that the whole market is covered in equilibrium. After entry, and having observed the two prices, consumer $\tilde{x}$ is indifferent between buying from the incumbent or the entrant, and

$$\tilde{x} = \frac{2t + p_E - p_I + \theta(H - q_E^e)}{2t}.$$

Note that $\theta(H - q_E^e)$ is the additional surplus of the incumbent's product due to the perceived quality difference. Let the cost function be $c_j(q_j)x$, for $j = E, I$ where $q_j$ is the true quality. Suppose that costs are known by the firms. Then the solution for the incumbent profits, $\pi_I$, and for the entrant's, $\pi_E$ gives

$$\pi_I(q_E^e, q_E) = \frac{(3t + c_E(q_E) - c_I(q_I) + \theta(H - q_E^e))^2}{18t},$$

$$\pi_E(q_E^e, q_E) = \frac{(3t - c_E(q_E) + c_I(q_I) - \theta(H - q_E^e))^2}{18t}.$$

It is easy to check that A.1 and A.2 are met. Note that, in general, also the incumbent's profits depend on the entrant's quality and not only on the consumers' beliefs with respect to this quality.

<center>18</center>

Furthermore, C.1 is met if $dc_E(q_E)/dq_E \leq 0$ because $\partial^2 \pi_E/\partial q_E^e \partial q_E = -(\theta/9t)(dc_E(q_E)/dq_E)$. This means that costs are non-increasing in quality. Hence, in this adverse selection world higher, correctly perceived quality is good news for two reasons: the consumers willingness to pay is higher and production costs are lower. In terms of the R&D vocabulary, a higher consumer willingness to pay due to higher quality stems from product innovation. Lower costs because of higher quality stem from process innovation. Hence, in the present example condition C.1 holds if process and product innovations go hand-in-hand.

## 5. Belief revision after the legal outcome

To focus on the role of consumer expectations, we assume in this section that consumers observe (i) the incumbent's reaction and (ii) the court's verdict. Type 1 and type 2 equilibria exist in model B too. As opposed to model A, note that the incumbent's strategy choice here is an explicit argument of the consumers' belief revision function. At an equilibrium of type 1 this does not imply that the action of the incumbent is informative per se: it leads to more information being diffused only because the court intervenes, as it shall be described in the sequel. By contrast, at an equilibrium of type 2 (where the court shall not intervene), the observation that the incumbent does not react against comparison claims confirms that the entrant has not tried to cheat consumers. If the incumbent reacted, that would mean that there has been a deviation from the equilibrium path by the entrant. As for point (ii) we assume that the court does not take into account the signal received by the firms, since they both have incentives to manipulate their reports. It collects (incomplete) evidence and declares whether the advertising was misleading making use of this evidence. Firms cannot take any action after the verdict. The evidence in court is also observed by the firms, who do not gain more information after that. It is common knowledge that the court can make mistakes due to imperfect evidence gathering; however, as a net result the verdict improves consumers' information about the entrant's quality at a separating equilibrium of type 1, as we shall see in the appendix 8.2. In this setup damages needed to make comparison advertising viable is reduced. Indeed, it is even possible to have $D = 0$.

Introducing the court's imprecise verification process adds an additional dimension to the problem, as explained below. We assume that with probability $1 - \mu$ the court does not discover a false claim, i.e. a claim coming from type-$L$ in either state $s_L$ or $s_M$. A claim from an entrant of underlying quality $H$ is never falsified by the court. In short, the court does not always uncover verifiable evidence against a low quality entrant, while it accepts verifiable evidence. Therefore, by assumption the entrant does not pay damages if its true, yet unknown, quality is high.[14]

Consumers know that with probability $1 - \mu$ a low quality entrant who uses comparative advertising is not unmasked by the court. As a consequence, when observing

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[14]This is in line with the information received by the firms. Firms and the court recognize certain failures. One could also analyze the model under the assumption that the court can take as wrong a true claim. Our main results remain valid under this different assumption. Important is that the court's action reveals additional information.

a verdict in favor of the entrant, consumers update their beliefs on the entrant's quality taking the possible mistake by the court into account.

As before, at a separating equilibrium comparative advertising is chosen only if the signal received is $s_M$ and not $s_L$. We call $\alpha_2$ the updated probability to face a high quality entrant when the court's verdict, denoted $\widehat{q}$, claims that quality is high:[15]

$$\alpha_2 = \text{prob}(q = H | \widehat{q} = H) = \alpha_1 / [1 - \mu(1 - \alpha_1)].$$

Obviously $\alpha_2 > \alpha_1$. Correspondingly, $1 - \alpha_2 = \text{prob}(q = L | \widehat{q} = H)$ is such that $1 - \alpha_2 < 1 - \alpha_1$. Accordingly, uncertainty about quality is reduced if the court is called to act after the firms receive signal $s_M$. This contrasts with the case in which the court is called to intervene when the entrant cheats (out-of-equilibrium) in state $s_L$: in that case the court can wrongly rule in favor of the entrant. The latter pays damages with a probability that is less than 1 and is increasing in the court's precision.

At a separating perfect Bayesian equilibrium in which the entrant signals $s_M$ with a non-negative amount of comparative advertising and the incumbent reacts by suing the entrant consumers revise beliefs concerning quality to

$$M_2 = \alpha_2 H + (1 - \alpha_2)L > M_1.$$

It is important to recall at every stage that $M_2$ is the up-dated quality when consumers observe comparative advertising *only if* followed by a verdict (namely only in equilibria of type 1). The analysis of generic advertising is unchanged with respect to model A.

We will analyze how parameters $\alpha_1$ and $\mu$ affect equilibrium outcomes. We shall concentrate in the main text on the analysis of equilibrium of type 2, while a detailed analysis of equilibrium of type 1 is performed in appendix 8.2.[16] In our view the difference between model A and model B is more apparent in type 2 equilibria than in type 1. In the context of model B, in type 2 equilibria the incumbent uses different strategies against comparison ads in state $s_M$ (no-reaction) and $s_L$ (reaction) and therefore consumers rely upon both firms' actions in their interpretation of comparison advertising as a signal. In type 1 equilibria, the incumbent uses the same strategy (reacts) against comparison ads whether the signal is $s_M$ or $s_L$, consumers beliefs only hinge upon the entrant's strategy (and, in case of comparison ads, upon the court's verdict).

At an equilibrium of type 2 the incumbent does not file for damages in state $s_M$ and it does in state $s_L$.

**Lemma 4.** *(a) The incumbent files for damages after receiving signal $s_L$ if*

$$\mu \geq \mu_{I_1} \equiv \frac{\Pi_I(M_1, L) - \Pi_I(M_2, L) + C}{\Pi_I(L, L) - \Pi_I(M_2, L) + D}. \tag{5.1}$$

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[15]Indeed $\text{prob}(\widehat{q} = L | s_M) = \mu \text{prob}(L | s_M) = \mu(1 - \alpha_1)$ and $\text{prob}(\widehat{q} = H | s_M) = 1 - \text{prob}(\widehat{q} = L | s_M) = 1 - \mu(1 - \alpha_1)$.

[16]Like in the first model, to support an equilibrium of type 1 rather than 2 requires that the probability that the product is of high quality has to be low.

*(b) The incumbent does not file for damages after receiving signal $s_M$ if*

$$\mu \leq \mu_{I_2}(\alpha_1) \equiv \left(\frac{1}{1-\alpha_1}\right)\left(\frac{\bar{\Pi}_I(M_1) - \bar{\Pi}_I(M_2) + C}{\Pi_I(L,L) - \Pi_I(M_2,L) + D}\right).$$

**Proof:** Note that the beliefs are such that if the incumbent *does not* sue the entrant after comparison ads, the entrant is believed to be of quality $M_1$. This corresponds to what was established in model A where there is no further updating after observing the advertising policy. The condition for the incumbent to react to comparative advertising in state $s_L$ is:

$$\mu\left[\Pi_I(L,L) + D\right] + (1-\mu)\Pi_I(M_2,L) - C \geq \Pi_I(M_1,L) \tag{5.2}$$

Which is equivalent to $\mu \geq \mu_{I_1}$ in the lemma.

(b) Consider now state $s_M$. The incumbent benefits from winning the case because it receives damages and it enjoys a stronger position in the market in case the court unmasks a low quality entrant. However, the incumbent obtains a decrease in profits if the court's verdict is favorable to the entrant. In fact, in that case, consumers up-dated beliefs are $M_2$ instead of $M_1$. Formally the constraint for the incumbent *not* to react to comparative advertising in state $s_M$ reads:

$$\bar{\Pi}_I(M_1) \geq \alpha_1\Pi_I(M_2,H) + (1-\alpha_1)\left[\mu(\Pi_I(L,L)+D) + (1-\mu)\Pi_I(M_2,L)\right] - C \tag{5.3}$$

Condition (5.3) reduces to $\mu \leq \mu_{I_2}(\alpha_1)$. ∎

Note that $\mu_{I_1} > 0$ so that if the court's precision is too low ($\mu < \mu_{I_1}$) the incumbent's reaction in state $s_L$ cannot be supported. Note also that $\mu_{I_1} < 1$ is implied by $D > C$. In words, if damages paid at least cover legal costs the incumbent reacts in state $s_L$ if the court's precision is sufficiently high ($\mu > \mu_{I_1}$).

Note also that, if damages paid exceed legal costs one can always find values for $\alpha_1$ such that the critical $\mu_{I_2}(\alpha_1)$ is between zero and 1 (note incidentally that $\mu_{I_2}(0) = \mu_{I_1}$ and that a sufficient condition for $\mu_{I_2}(\alpha_1) > \mu_{I_1}$ is that C.1 holds also for the incumbent's profits).

Reasoning along the same lines as in section 4, the separation constraint for an entrant that receives signal $s_L$ is identical in type 1 and in type 2 equilibria:[17] $\Pi_E(L,L) \geq \mu\left[\Pi_E(L,L) - D\right] + (1-\mu)\Pi_E(M_2,L) - A_c$. As it appears, in the separation constraint a mimic gets $\Pi_E(L,L)$ if the court uncovers the truth and $\Pi_E(M_2,L)$ if it does not. This can be rewritten as

$$A_c \geq (1-\mu)\left[\Pi_E(M_2,L) - \Pi_E(L,L)\right] - \mu D \tag{5.4}$$

so that the minimum amount of advertising expenditure necessary to separate is:[18]

$$A_c^{**} \equiv \max\{\underline{A}, (1-\mu)\left[\Pi_E(M_2,L) - \Pi_E(L,L)\right] - \mu D\} \tag{5.5}$$

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[17]Without the assumption that the court can make mistakes, that is when $\mu = 1$, the separation constraint for a low type would be: $\Pi_E(L,L) \geq \Pi_E(L,L) - D - A_c$. The constraint reads $A_c \geq -D$, so that it does not give any restriction on the amount of comparative advertising required.

[18]>From the analysis in the appendix 8.2 it follows that the minimum amount of advertising needed is the same as for an equilibrium of type 1.

**Lemma 5.** *For the interval of admissible comparative advertising expenditures to be non-empty with separating SPBE of type 2 one must have*

$$(1 - \mu)[\Pi_E(M_2, L) - \Pi_E(L, L)] - \mu D \leq \bar{\Pi}_E(M_1) - \bar{\Pi}_E(L) \qquad (5.6)$$

**Proof:** The entrant after receiving the signal $s_M$, uses comparative advertising when the incumbent chooses $a$, if $\alpha_1 \Pi_E(M_1, H) + (1 - \alpha_1) \Pi_E(M_1, L) - A_c$ is larger than $\alpha_1 \Pi_E(L, H) + (1 - \alpha_1) \Pi_E(L, L)$, which can be rewritten as

$$A_c \leq \bar{\Pi}_E(M_1) - \bar{\Pi}_E(L) \qquad (5.7)$$

Note that the above condition is equivalent to (4.1) and that, under condition (C.1), the r.h.s. is strictly larger than $\Pi_E(M_1, L) - \Pi_E(L, L)$. Combining conditions (5.4) and (5.7) gives comparative advertising expenditures that are compatible with the incentive constraints of the entrant for equilibria of type 2. ∎

For a given $\alpha_1$, the condition in the lemma is satisfied if the precision of the court's verdict is sufficiently high. Let $\Delta = [\Pi_E(M_1, H) - \Pi_E(L, H)] + [\Pi_E(M_1, L) - \Pi_E(L, L)]$. Then the inequality $\mu \geq \mu_E(\alpha_1)$ must hold, where

$$\mu_E(\alpha_1) \equiv \frac{\Pi_E(M_2, L) - \Pi_E(M_1, L) - \alpha_1 \Delta}{\Pi_E(M_2, L) - \Pi_E(L, L) + D}.$$

Under condition (C.1) we could use a stronger condition which is independent of $\alpha_1$,

$$\mu \geq \frac{\Pi_E(M_2, L) - \Pi_E(M_1, L)}{\Pi_E(M_2, L) - \Pi_E(L, L) + D}$$

Note that the critical $\mu_E(\alpha_1) \in (0, 1)$. Also, higher damages lower the value of $\mu_E(\alpha_1)$.

**Proposition 4.** *Suppose $\underline{A} < A_c^{**}$. If $\alpha_1$ and $\mu$ are such that $\mu_{I_2}(\alpha_1) \geq \mu \geq \max\{\mu_{I_1}, \mu_E(\alpha_1)\}$ then there exists a perfect Bayesian equilibrium of type 2 with $A_c = A_c^{**}$.*

**Proof** follows from the analysis above. ∎

Like in the analysis performed in section 4, here too the minimum amount of expenditure necessary to separate in the case of generic advertising in state $s_L$ corresponds to: $A_g^* = \max\{\underline{A}, \widehat{A}_g\}$.

Comparing $A_g^*$ and $A_c^{**}$ it is clear that comparative advertising (in the payoff-dominant SPBE of type 1 or 2) is less costly than generic advertising provided that $\underline{A} \leq A_c^*$ and $\mu$ is sufficiently high. As in the previous section we can use the dominance criterion to select the comparative advertising separating equilibrium of type 2 when also equilibria of type 3 exist, based on the value of $\mu$ among other things.

Note that in model A of section 4, comparative advertising can cost less than generic only if $D > 0$. In the present model this can happen if

$$(1 - \mu)[\Pi_E(M_2, L) - \Pi_E(L, L)] - \mu D < \Pi_E(M_1, L) - \Pi_E(L, L) \qquad (5.8)$$

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Two effects arise in this model: the first is the "double upgrading" effect by which in the l.h.s. of the inequality $M_2$ appears in place of $M_1$. This increases the mimicking gain for the entrant in state $s_L$ and raises the cost of comparative advertising. The second effect is due to the presence of the court, which can unmask a false claim, and is reflected in the term $\mu$. If $\mu$ increases the cost of comparative advertising decreases. If this effect is strong enough comparative advertising may cost less than generic even if $D = 0$. When $D = 0$ the minimum amount of advertising in a type 2 equilibrium is

$$A_{c0}^* \equiv (1 - \mu)[\Pi_E(M_2, L) - \Pi_E(L, L)].$$

When $A_g^*$ exceeds $\underline{A}$ the payoff dominant equilibrium of type 2 payoff dominates the equilibrium of type 3 if it entails a lower cost of advertising, i.e. if $A_{c0}^* < \widehat{A}_g$ , or

$$\mu > \frac{\Pi_E(M_2, L) - \Pi_E(M_1, L)}{\Pi_E(M_2, L) - \Pi_E(L, L)}.$$

This shows the role of court's verdict, and clarifies the difference between the present model and the first model.

## 6. Imperfectly correlated information

So far we have assumed that the information of the two firms was perfectly correlated. However, suppose that this is not the case. For instance, each firm independently undertakes a single or a series of tests providing some imprecise knowledge about the entrant's product quality. The result of the testing is private information of the firm carrying it, so that the total information in the market is richer than the individual information initially obtained by the firms. We show here that in such an environment, the general insight that comparative advertising can be a more efficient way to signal product quality than generic dissipative advertising retains its power. Note that the analysis of generic dissipative advertising remains unchanged. To analyze comparative advertising we maintain that consumers observe all firms' actions and the court verdict (model B). We shall make two additional observations. First, we show here that there exist equilibria in which the entrant signals its information to the incumbent and both firms signal information that is favorable for the entrant's product to the consumers. This implies that consumers obtain more than the information initially available to either firm, but less than the pooled information. Second, we show that there exist separating equilibria, like those of type 2 above, where claiming for damages occurs only if the incumbent receives signal $s_L$ so that it knows that the quality of the entrant is low.[19]

We will first specify the changes in the model and then present the analysis. To clarify the exposition we consider the special case that the court perfectly verifies the entrant's quality (i.e. $\mu = 1$ in our notation). Hence, if the incumbent applies for damages no uncertainty with respect to the entrant's quality remains. The modification of the model concerns the way in which firms initially receive information about

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[19] After signal $s_M$, and comparative advertising, the incumbent does not react and by doing so reveals to consumers that the entrant is "strong".

the entrant's product quality. A test carried out by firm $i$ can show the failure of the entrant's product. In this case the signal for firm $i$ is $s_L^i$ and it reveals that the product is of low quality. If the test does not show a failure, the signal for firm $i$ is $s_M^i$. It reveals that it is more likely that the product is of high quality. Hence if the true state is $H$ firms receive signals $s_M^I$ and $s_M^E$, respectively. On the other hand, if the true state is $L$ firms receive signals $s_M^I$ and $s_M^E$ with probability $\lambda^2$, signals $s_M^I$ and $s_L^E$ with probability $\lambda(1-\lambda)$, signals $s_L^I$ and $s_M^E$ with probability $\lambda(1-\lambda)$, and signals $s_L^I$ and $s_L^E$ with probability $(1-\lambda)^2$. This information structure implies that the information that the two firms receive is positively but not perfectly correlated. Furthermore, the probability a firm assigns to the entrant being a high type if the private signal is $s_M$ is again $\alpha_1 = \dfrac{\alpha_0}{\alpha_0 + \lambda(1-\alpha_0)}$, as in the previous sections.

We focus on the following type of perfect Bayesian equilibrium (type 2-bis):

**Definition 3.** *equilibrium of type 2-bis*

*- If the entrant receives $s_M^E$ it chooses $c$; if the entrant receives $s_L^E$ it chooses $n$ (does not advertise).*

*- If the incumbent receives $s_M^I$ it chooses $a$, while if it receives $s_L^I$ it chooses $\ell$, claiming for damages, in case the entrant has used $c$.*

*- The separating beliefs must be consistent with strategies, of a type 2 equilibrium, such that: $prob(c \mid (s_j^I, s_L^E)) = 0$, $prob(c \mid (s_j^I, s_M^E)) = 1$; while $prob(\ell \mid (s_M^I, s_j^E)) = 0$ and $prob(\ell \mid (s_L^I, s_j^E)) = 1$, for $j = M, L$.*

In this type of equilibrium the belief updating is as follows: observing comparative advertising by the entrant the incumbent learns that the entrant has received signal $s_M^E$. If the incumbent claims for damages the true quality is then revealed by the court to consumers.[20] This only happens if the incumbent has received signal $s_L^I$ in which case the true state is $L$. If the entrant uses comparative advertising and the incumbent *does not* sue the entrant, consumers interpret this as both firms having received $s_M^i$ and update their beliefs to

$$\alpha_1' = \frac{\alpha_0}{\alpha_0 + \lambda^2(1-\alpha_0)} > \alpha_1.$$

The expected product quality then is $M_1' = \alpha_1' H + (1 - \alpha_1')L$.

With these preliminaries we are now in the position to write down the incumbent's and entrant's separating constraints that have to be satisfied to support a type 2-bis equilibrium. If the entrant receives signal $s_L$ the incumbent receives signal $s_M$ and does not react with probability $\lambda$. The incentive constraint of the entrant that receives

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[20]The fact that consumers know the final verdict of the court is not essential for the argument. If consumers only observed the firms' actions but not the final vedict of the court we would obtain the same insights although under modified conditions that are derived from the separating constraints. In that case at a separating equilibrium it is also the case that consumers' beliefs are such that $prob(s_M \mid \ell) = prob(s_M \mid (s_L^I, s_M^E)) = 0$. In particular, the following point still holds: under some parameter restriction there exist equilibria such that consumers when making the purchasing decision are better informed than the entrant at the point of making its advertising expenditure.

signal $s_L^E$ is $\Pi_E(L,L) \geq (1-\lambda)[\Pi_E(L,L) - D] + \lambda\Pi_E\left(M_1', L\right) - A_c$. Which provides the lower bound for $A_c$ as

$$A_c \geq \lambda[\Pi_E\left(M_1', L\right) - \Pi_E(L,L)] - (1-\lambda)D \tag{6.1}$$

Note that this inequality is similar to (5.4); only the probability of the court to make a mistake, $1 - \mu$, has been replaced by the probability $\lambda$, and the updated value for $M$ is now $M_1'$.

When the entrant receives the signal $s_M$ it expects the incumbent's signal to be $s_M$ with probability $\alpha_1 + (1 - \alpha_1)\lambda$. Therefore it expects a comparative claim to trigger a lawsuit with probability $(1 - \lambda)(1 - \alpha_1)$. Meanwhile, consumers update quality to $M_1'$, based upon $\alpha_1'$ defined above, if the incumbent does not react to a comparison ad, while the information of the entrant is still only based upon $\alpha_1$. Accordingly, the separation constraint for the entrant after it receives signal $s_M^E$ is:[21]

$$\alpha_1\Pi_E\left(M_1', H\right) + (1-\alpha_1)\left\{(1-\lambda)[\Pi_E(L,L) - D] + \lambda\Pi_E\left(M_1', L\right)\right\} - A_c \geq \alpha_1\bar{\Pi}_E(L)$$

This gives the upper bound for $A_c$ as equal to

$$A_c \leq \alpha_1[\Pi_E(M_1', H) - \Pi_E(L,H)] + \lambda(1-\alpha_1)[\Pi_E\left(M_1', L\right) - \Pi_E(L,L)] - (1-\lambda)(1-\alpha_1)D. \tag{6.2}$$

Hence from (6.1) and (6.2), there exist comparative advertising expenditures that are consistent with the entrant's incentive constraints if

$$[\Pi_E(M_1', H) - \Pi_E(L,H)] - \lambda[\Pi_E\left(M_1', L\right) - \Pi_E(L,L)] \geq -(1-\lambda)D. \tag{6.3}$$

This condition has the same structure as condition (5.4) in the previous subsection and is always satisfied if condition (C.1) holds—although C.1 is not necessary. Now let us turn to the incumbent's separating constraints. Suppose the entrant uses comparative advertising. If the incumbent receives signal $s_L^I$ it claims for damages if

$$\Pi_I(L,L) + D - C \geq \Pi_I(M_1', L). \tag{6.4}$$

This inequality is automatically satisfied if $D \geq C$. If the incumbent receives signal $s_M^I$ it can update according to $\alpha_1'$, as for consumers who observe that the incumbent does not file for damages. Therefore the incumbent does not claim for damages if

$$\alpha_1'\Pi_I(M_1', H) + (1 - \alpha_1')\Pi_I(M_{1'}, L) \geq \alpha_1'\Pi_I(H,H) + (1 - \alpha_1')(\Pi_I(L,L) + D) - C.$$

This inequality can be rewritten as

$$\alpha_1'[\Pi_I(M_1', H) - \Pi_I(H,H)] + (1 - \alpha_1')[\Pi_I(M_1', L) - \Pi_I(L,L) - D] + C \geq 0. \tag{6.5}$$

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[21]Note that this inequality is similar to (8.2) in appendix 8.2. (8.2) indicates the separation constraint for an entrant which observed signal $s_M$ when equilibrium is of type 1. Again the probability of the court to make a mistake, $1 - \mu$, has been replaced by the probability $\lambda$.

For $C > 0$ this inequality is satisfied if $\alpha_1'$ is sufficiently large which is implied by $\lambda$ sufficiently small. (Note that for a given $\alpha_0$ the inequality is satisfied for all $\lambda$ if $\alpha_0[\Pi_I(M_0, H) - \Pi_I(H, H)] + (1 - \alpha_0)[\Pi_I(M_0, L) - \Pi_I(L, L) - D] + C \geq 0$.) Summarizing, to support a type 2-bis equilibrium conditions (6.3), (6.4), and (6.5) have to be satisfied. The minimum amount of advertising necessary to separate is

$$A_c^{***} \equiv \max\{\underline{A}, \lambda[\Pi_E\left(M_1', L\right) - \Pi_E\left(L, L\right)] - (1 - \lambda)D\}.$$

**Proposition 5.** *Suppose $\underline{A} \leq A_c^{***}$. If $D \geq C$ and parameters $\lambda$ and $\alpha_0$ are such that (6.5) and (6.3) holds then there exists a perfect Bayesian equilibrium of type 2-bis with $A_c = A_c^{***}$.*

Note that the expression for $A_c^{***}$ is just the same as that for $A_c^{**}$ in the case of the court making mistakes, with $\lambda = (1 - \mu)$.

Clearly type 2-bis is not the only type of equilibrium that can arise. There exist parameter constellations such that the incumbent claims for damages whenever the entrant uses comparative advertising (type 1-bis equilibrium). In this case, since the court verifies the entrant's quality, consumers learn the entrant's true quality when the entrant received signal $s_M^E$, that is comparative advertising always allows perfect information disclosure.

# 7. Conclusion

We have shown in this paper that comparative advertising has a different signaling potential than generic advertising. In particular, comparison ads may trigger a reaction (possibly only off the equilibrium path) by the firm that is the target of the comparison. This may be especially useful for firms wishing to enter a market in which an incumbent with a renown brand already operates.

We use two different versions of a model. These versions represent different possible legal and technical set-ups. In the first version there is no problem of ex-post verifiability of comparison claims (the court does not make mistakes), but consumers are not warned off if a misleading advertising is made, so that they are exposed to the risk of a wrong purchase. In the second model the court does not always discover a false claim. Consumers, however, observe whether there is a legal action against the entrant's comparison ads and what the court's (imperfect) findings are. Here, under some parameter constellations the observed action by the incumbent reveals the information that the firms have received and therefore consumers update their beliefs upon observing the incumbent's action.

In the two models comparison ads and generic ads can be used in equilibrium. Our results show that comparison ads are often preferred because they enable a legal action against the entrant so that the cost of cheating through a comparative ad is higher. This implies that the entrant's incentive constraint is relaxed. Thus lower advertising expenditures support the signaling mechanism.

The model is also extended to allow for imperfect correlation between the signals received by the two firms. In this extension the role of the observability of the actions

taken by the incumbent is further emphasized. There, when the incumbent does not react to a comparison advertisement reveals that he too, and not only the entrant, has received a good signal about the entrant's quality.

Not to take an action can reveal important information, as readers of detective stories may know. In Conan Doyle's "Silver Blaze" inspector Gregory and Sherlock Holmes hold the following conversation: *"Is there any point to which you would wish to draw my attention?", "to the curious incident of the dog in the night-time". "The dog did nothing in the night-time." "That was the curious incident", remarked Sherlock Holmes.*

# 8. Appendix

## 8.1. Pooling equilibria

To characterize all pure strategy SPBE we also have to consider pooling equilibria. These can be of several types. We refer here to the case where consumers do not observe the incumbent's action and the court's decision (model A). We check wether pooling equilibria can survive the application of selection criteria, like dominance or the intuitive criterion. The analysis shall show the relative weakness of pooling equilibria. Equilibrium beliefs may be characterized by the following two statements taken together: (i) if the entrant spends at least $P_g \geq 0$ in generic, or $P_c \geq 0$ in comparative advertising then the quality is believed to be $\alpha_0 H + (1 - \alpha_0)L$, namely consumers do not revise their beliefs; (ii) if the expenditure is lower than the thresholds $P_g$ or $P_c$ then the quality is believed to be $L$.[22] The pooling equilibrium with minimum cost for the entrant is one where $P_g = P_c = 0$, the entrant never advertises, and consumers beliefs are never up-dated. We shall limit the analysis to this minimum cost equilibrium, which is also the more resistant to selection. In a pooling equilibrium of this type the incumbent (respectively: the entrant) receives profits $\Pi_I(M_0, L)$ in state $s_L$ (respectively $\Pi_E(M_0, L)$) and $\alpha_1 \Pi_j(M_0, H) + (1 - \alpha_1)\Pi_J(M_0, L)$ in state $s_M$, for $j \in \{E, I\}$.

For comparative advertising with expenditure $A_c^*$ consumer beliefs are such that expected quality is $M_1$.

If we accept the stronger refinement of the intuitive criterion (see Cho and Kreps, 1987) we can select the unique SPBE characterized in Proposition 3. Indeed, the entrant that receives signal $s_L$ cannot profit from a deviation to $A_c^*$, given that it receives profits $\Pi_E(M_1, L) - A_c^* - D$ instead of the equilibrium profits given by $\Pi_E(M_0, L)$. Substituting for $A_c^*$ this reduces to the inequality $\Pi_E(L, L) < \Pi_E(M_0, L)$, which is true by assumption A.1. For generic advertising the condition writes as $\Pi_E(M_1, L) - A_g^* \geq \Pi_E(M_0, L)$, which again can be reduced to $\Pi_E(L, L) < \Pi_E(M_0, L)$. It follows:

---

[22]More generally, one could further refine the belief system adding to point (i) and (ii) the thresholds $P_g' > P_g$ and $P_c' > P_c$ leading to a perceived quality, $q'$, with $q' > \alpha_0 H + (1 - \alpha_0)L$, but sufficiently low so as to ensure that separation between state $s_L$ and $s_M$ is less profitable for the entrant than pooling using $P_g$ or $P_c$.

**Lemma 6.** *Whether C.1 holds or does not hold, pooling equilibria do not satisfy the intuitive criterion.*

Consider now the application of dominance. First, consider parameter constellations such that separating SPBE of type 2 exist. Pooling equilibria do not satisfy the dominance criterion if the profits to the entrant receiving signal $s_M$ satisfy

$$\bar{\Pi}_E(M_1) - A_c^* \geq \alpha_1 \Pi_E(M_0, H) + (1 - \alpha_1)\Pi_E(M_0, L).$$

Suppose that $\underline{A} < A_c^*$. Then the above condition becomes $D \geq \alpha_1 \Pi_E(M_0, H) + (1 - \alpha_1)\Pi_E(M_0, L) - \bar{\Pi}_E(M_1) + \Pi_E(M_1, L) - \Pi_E(L, L)$. This can be rewritten as

$$D \geq \alpha_1\{\Pi_E(M_1, L) - \Pi_E(M_0, L) - \Pi_E(M_1, H) + \Pi_E(M_0, H)\} + \Pi_E(M_0, L) - \Pi_E(L, L).$$

Under C.1 one has $\Pi_E(M_1, H) - \Pi_E(M_0, H) \geq \Pi_E(M_1, L) - \Pi_E(M_0, L)$. Then it follows:

**Lemma 7.** *Suppose C.1 is met and $\underline{A} < A_c^*$. Then pooling equilibria do not satisfy the dominance criterion for parameters such that SPBE of type 2 exist under the condition that*

$$D \geq \Pi_E(M_0, L) - \Pi_E(L, L).$$

Second, consider parameter constellations such that separating SPBE of type 1 exist. Pooling equilibria do not satisfy the intuitive criterion if

$$\bar{\Pi}_E(M_1) - A_c^* - (1 - \alpha_1)D \geq \alpha_1 \Pi_E(M_0, H) + (1 - \alpha_1)\Pi_E(M_0, L)$$

We rewrite the above inequality as

$$
\begin{aligned}
\alpha_1 D \quad \geq \quad & (\Pi_E(M_1, L) - \Pi_E(L, L)) - \alpha_1(\Pi_E(M_1, H) - \Pi_E(M_0, H)) \\
& -(1 - \alpha_1)(\Pi_E(M_1, L) - \Pi_E(M_0, L)) \tag{8.1}
\end{aligned}
$$

**Lemma 8.** *Suppose C.1 holds and $\underline{A} < A_c^*$. Then pooling equilibria do not satisfy the dominance criterion for parameters such that SPBE of type 1 exist under the condition that*

$$D \geq \frac{\Pi_E(M_0, L) - \Pi_E(L, L)}{\alpha_1}.$$

The above results show that for certain parameter values pooling equilibria do not satisfy the dominance criterion. Note that the conditions derived in Proposition 1 and the above Lemmas are not mutually exclusive.

## 8.2. Analysis of equilibria of type 1 when consumers observe the court's verdict

The effect on consumers' belief revision at an equilibrium of type 1 is stronger than in model A. In this setup, the court's verdict after signal $s_M$ works as a certification, albeit as an imperfect one.

Consider the incumbent incentive to react in state $s_M$. Recalling discussion in section 5 and inequality (5.3), the constraint for the incumbent to react to comparative advertising in state $s_M$ reduces to $\mu \geq \mu_{I_2}(\alpha_1)$.

Consider now the separating constraint for the entrant. Given that the incumbent reacts if the signal is $s_M$, an entrant who learns that the signal is $s_M$ uses comparative advertising if:

$$\alpha_1 \Pi_E (M_2, H) + (1 - \alpha_1) \{\mu [\Pi_E (L, L) - D] + (1 - \mu)\Pi_E (M_2, L)\} - A_c \geq \bar{\Pi}_E(L) \tag{8.2}$$

In the case of no damages (namely $D = 0$) constraint (8.2) reduces to impose as an upper bound for $A_c$ the value $\alpha_1 [\Pi_E (M_2, H) - \Pi_E (L, H)] + (1 - \alpha_1)(1 - \mu) [\Pi_E (M_2, L) - \Pi_E (L, L)]$. Hence, provided that condition (C.1) holds, even if no damages are paid, as in equilibrium of type 2, there exists a non-empty interval for comparative advertising expenditures that satisfies the incentive constraints of the entrant.

The condition for comparative advertising expenditure which is compatible with the entrant's separating constraints (combining (5.4) and (8.2)) is

$$(1 - \mu) [\Pi_E (M_2, L) - \Pi_E (L, L)] - \mu D \leq A_c$$
$$\leq \alpha_1 [\Pi_E (M_2, H) - \Pi_E (L, H)] + (1 - \alpha_1) (1 - \mu) [\Pi_E (M_2, L) - \Pi_E (L, L)] - \mu(1 - \alpha_1)D \tag{8.3}$$

Hence, the condition that there exists comparative advertising expenditures which are compatible with the entrant's separating constraints is

$$\Pi_E (M_2, H) - \Pi_E (L, H) - (1 - \mu) [\Pi_E (M_2, L) - \Pi_E (L, L)] + \mu D \geq 0. \tag{8.4}$$

This condition is automatically satisfied if condition (C.1) holds. Again, we observe that (C.1) is not necessary to satisfy the separating constraints of the entrant. Only in the limit, as $\mu$ turns to zero condition (8.4) becomes equivalent to C.1. This is the limit case, in which the court system is useless in that it does not uncover any evidence so that advertising that contains content is equivalent to generic advertising. In the opposite case when $\mu = 1$ the court perfectly verifies and condition (8.4) is always satisfied. We rewrite this condition as a lower bound on $\mu$:

$$\mu \geq \mu_E \equiv \frac{[\Pi_E(M_2, L) - \Pi_E(L, L)] - [\Pi_E(M_2, H) - \Pi_E(L, H)]}{\Pi_E(M_2, L) - \Pi_E(L, L) + D}$$

Summarizing, to support a type 1 equilibrium the conditions (8.4), (5.1), and $\mu \geq \mu_{I_2}(\alpha_1)$ have to be satisfied. The minimum amount of advertising expenditure necessary to separate is: $A_c^{**} \equiv \max\{\underline{A}, (1 - \mu) [\Pi_E (M_2, L) - \Pi_E (L, L)] - \mu D\}$

**Proposition 6.** *Suppose $\underline{A} < A_c^{**}$. If $\alpha_1$ and $\mu$ are such that $\mu \geq \max\{\mu_E, \mu_{I_1}, \mu_{I_2}(\alpha_1)\}$ then there exists a perfect Bayesian equilibrium of type 1 with $A_c = A_c^{**}$.*

**Proof** The existence of a SPBE follows from above. ∎

Note that the equilibrium of type 1 allows more information disclosure than that of type 2. In fact, if quality is signaled with generic advertising or with comparative

advertising in equilibrium of type 2, consumers only learn that average (updated) quality is $M_1$. While, if quality is signaled with comparative advertising in equilibrium of type 1, when the court says that the claim is true ($\widehat{q} = H$), consumers updated beliefs on quality are $M_2 > M_1$. Thus, comparative claims followed by a court decision transmit more information to consumers than generic ads or comparative ads without the incumbent's reaction.

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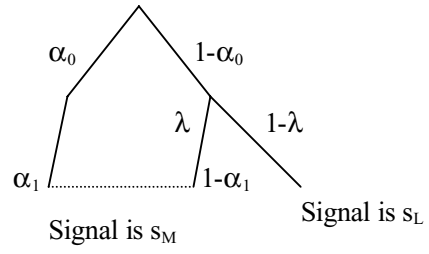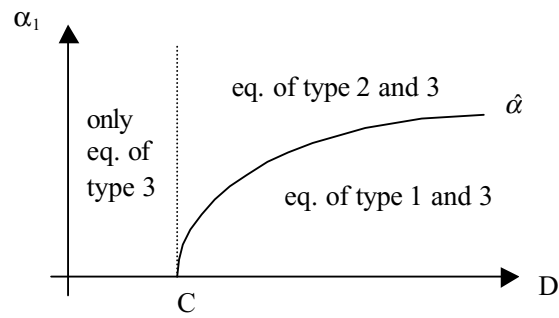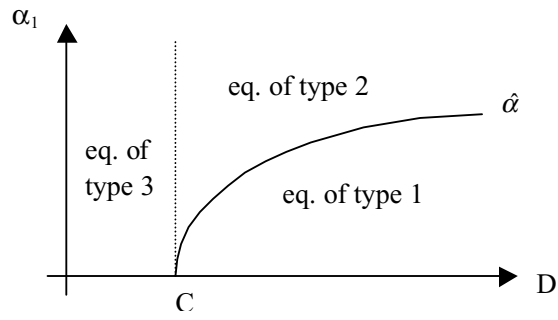
Figure 1



Figure 2

Figure 3: selected equilibria when $A_g^* - D > \underline{A}$
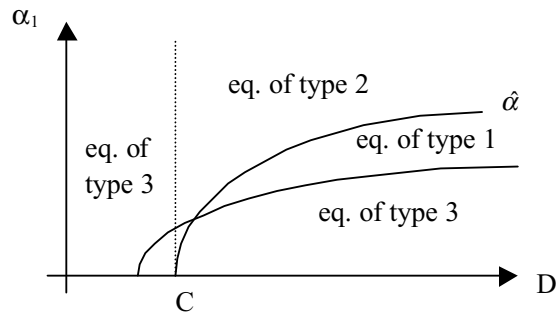


Figure 4: selected equilibria when $A_g^* - D \leq \underline{A}$

and $C > \Pi_E(M_1, L) - \Pi_E(L, L) - \underline{A}$