# A timed method for the estimation of aeroplane takeoff and landing distances. 

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#### Abstract

This paper describes a method by which, without the use of external personnel or equipment, take-off and landing distances of an aeroplane may be estimated. An error analysis for the method, allowing determination of outcome accuracy, is also shown. The method is validated through use of flight test results from two certification programmes: one on a light aeroplane, and one on a microlight aeroplane.


## NOMENCLATURE

\(\left.$$
\begin{array}{ll}\mathrm{a}_{1} & \begin{array}{l}\text { (During take-off) Acceleration from brakes off to rotation (Must be } \\
\text { positive). }\end{array} \\
\mathrm{a}_{1} & \begin{array}{l}\text { (During landing) Acceleration along flightpath from point at which aircraft } \\
\text { descends through screen height to touchdown point. }\end{array} \\
\mathrm{a}_{2} & \text { (During take-off) Acceleration from rotation to unstick } \\
\mathrm{a}_{2} & \begin{array}{l}\text { (During landing) Acceleration along ground for segment of ground roll } \\
\text { whilst aircraft is running on two wheels. }\end{array} \\
\mathrm{a}_{3} & \begin{array}{l}\text { (During take-off) In flightpath acceleration from unstick to achieving screen } \\
\text { height }\end{array}
$$ <br>
Note: all accelerations described above are positive forwards (i.e. during the <br>

take-off) and negative rearwards (i.e. during the landing).\end{array}\right\}\)| (During landing) Acceleration along ground from point at which all three |
| :--- |
| wheels touch the ground until aircraft stops. |


| CAA | (United Kingdom) Civil Aviation Authority |
| :---: | :---: |
| CAS | Calibrated Air Speed |
| CofA | Certificate of Airworthiness (the term normally implies an ICAO compliant document) |
| GPS | Global Positioning System (satellite navigation) |
| IAS | Indicated Air Speed |
| ISA | International Standard Atmosphere (also sometimes known as US Standard Atmosphere). |
| MTOW | Maximum Take-Off Weight |
| OAT | Outside Air Temperature |
| QFE | Altimeter setting giving an indication of zero height on the ground at a destination aerodrome. Given in hPa (heptopascals) or mb (millibars) the units being identical, ISA sea level value being 1013.25 |
| S | Total take-off or landing distance |
| $\mathrm{S}_{1}$ | (During take-off) Distance from brakes-off to rotation |
| $\mathrm{S}_{1}$ | (Landing) Distance along ground from directly below point at which aircraft descends through screen height, until touchdown |
| $S_{1}^{\prime}$ | (Landing) Straight line distance from point at which aircraft descends through screen height, until touchdown. |
| $\mathrm{S}_{2}$ | (During take-off) Distance from rotation to unstick |
| $\mathrm{S}_{2}$ | (Landing) Distance aircraft is on two wheels during ground roll |
| $\mathrm{S}_{3}$ | (During take-off) Straight line distance measured along the ground from unstick point to directly below point at which screen height is achieved |
| $\mathrm{S}_{3}$ | (Landing) Distance from all three wheels being on the ground until aircraft is stopped |
| $S_{3}^{\prime}$ | (During take-off) Straight line distance from unstick point to point at which screen height is achieved |
| sHp | Standard Pressure Altitude (altimeter reading with 1013.25 hPa set on subscale) |
| $\mathrm{t}_{1}$ | (During take-off) Time from brakes off to rotation |
| $\mathrm{t}_{1}$ | (During landing) Time from screen height to touchdown |
| $\mathrm{t}_{2}$ | (During take-off) Time from rotation to unstick |
| $\mathrm{t}_{2}$ | (During landing) Time spent on two wheels during ground roll |
| $\mathrm{t}_{3}$ | (During take-off) Time from unstick to achieving screen height |
| $\mathrm{t}_{3}$ | (During landing) Time spent from all three wheels touching down until aircraft stops. |
| TAS | True Air Speed |
| TODR | Take-Off Distance Required (to clear screen height) |
| $\mathrm{V}_{1}$ | (During take-off) True airspeed at rotation. |
| $\mathrm{V}_{1}$ | (During landing) True airspeed at screen height |


| $\mathrm{V}_{2}$ | (During take-off) True airspeed at unstick |
| :--- | :--- |
| $\mathrm{V}_{2}$ | (During landing) True airspeed at touchdown |
| $\mathrm{V}_{3}$ | (During take-off) True airspeed at screen height. <br> $\mathrm{V}_{3}$ |
| $\mathrm{~V}_{\text {REF }}$ | (During landing) True airspeed at point when all three wheels touch the <br> ground |
| Recommended final approach speed (normally given in IAS) |  |

## 1. INTRODUCTION

Many methods have been used in order to estimate the take-off and landing distances required by an aeroplane. These have included high speed photography, markers adjacent to a runway, inertial methods, and increasingly recording conventional or differential GPS. Often multiple methods will be used, in order to provide crossverification and enhance confidence in results.

A common factor to most available methods is the necessity of personnel and/or measuring equipment external to the aeroplane whose performance is being measured. This inevitably introduces cost and complexity which any test programme would wish to minimise. In order to achieve this, a method has been developed by which take-off and landing distances can be estimated by recording speeds and times, applying a form of integration, and then using an error analysis method to ensure that results are conservative. To date, this method has only been used for light and microlight aeroplanes, and it is likely that this will remain its main application; however, the author is also currently researching its use for validation of flight simulator models.

## 2. ESTIMATING TAKE-OFF DISTANCE

### 2.1 The Method

It is conventional to divide the take-off into three distinct segments, the initial ground roll, the post-rotation ground roll, and the climb to screen height.

Complying with normal certification practice, which requires at least 6 data points ${ }^{1}$, a minimum of 7 take-offs are carried out, and the results (times, speeds) tabulated. The least favourable 6 results (or most consistent, as judged by the flight test team) will be
taken, a mean of each time and speed value used, then distances calculated as shown below.

The method makes following assumptions:-

- During each segment, the aircraft's acceleration / deceleration is constant.
- Surface wind velocity and direction are constant between ground and screen height.
- During the air segment, the aircraft climbs in a straight line between the unstick point and screen height: for microlight aircraft and smaller light aeroplanes particularly this is a reasonable assumption, since the initial climb condition for light and microlight aeroplanes is normally established within $5-10 \mathrm{ft}$ of the ground, which is small within the 50 ft climb to screen height. Where an aircraft's curved transition from rotation to steady climbing flight comprises a larger part of the climb to screen height (particularly likely to be the case for an airliner where screen height is normally 35 ft , pitch rates are lower, and inertia greater) then it is likely that this assumption will require reconsideration, probably by comparison with aircraft measured flightpath using GPS, kinetheodolite or video analysis methods. To date however, this has not been found necessary .

The following notation is used (see Figure 1 below): speeds at start, rotate, unstick and screen height are $0, \mathrm{~V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}$ respectively. These are known in TAS by reduction to CAS from IAS values using determined $\mathrm{PEC}^{2,3}$, then if required smaller corrections are made for density altitude. Surface wind is $\mathrm{V}_{\mathrm{W}}$ and is positive when a headwind is encountered. Times of each segment are $t_{1}, t_{2}, t_{3}$ respectively. Note that if the aircraft has a very short distance from rotation to the unstick point (such as a taildragger taking off in a 3-point attitude), then it is taken that $\mathrm{t}_{2}=0, \mathrm{~V}_{1}=\mathrm{V}_{2}$ and the method is reduced to 2 -segments (this is the case for all of the examples shown within this paper). The lengths of each segment, measured along the ground, are $S_{1}, S_{2}, S_{3}$. Straight line distance from unstick to top of screen $=S^{\prime}{ }_{3}$. Accelerations during each segment are $a_{1}, a_{2}, a_{3}$ ( $a_{3}$ is acceleration along flightpath, not along the ground). Screen height is h . For calculation, all the above will be in SI units ( $\mathrm{m}, \mathrm{ms}^{-1}$, s ).


Figure 1, Illustration of take-off segments

To determine the length of the first (pre-rotation) ground roll segment:-
Assuming that the aircraft is initially stationary, $S_{1}=1 / 2 a_{1} t_{1}^{2}$

$$
\begin{equation*}
V_{1}-V_{W}=a_{1} t_{1} \therefore a_{1}=\frac{V_{1}-V_{W}}{t_{1}} \tag{2}
\end{equation*}
$$

(Noting that the aircraft translational velocity used in calculation is $\mathrm{V}_{1}-\mathrm{V}_{\mathrm{W}}$.)
Inserting (2) into (1) gives: $S_{1}=\frac{t_{1}}{2}\left(V_{1}-V_{W}\right)$

To determine the length of the second (post rotation) ground roll segment we use:

$$
\begin{align*}
& S_{2}=\left(V_{1}-V_{W}\right) \cdot t_{2}+1 / 2 a_{2} t_{2}^{2}  \tag{4}\\
& V_{2}-V_{W}=\left(V_{1}-V_{W}\right)+a_{2} t_{2} \\
& \therefore a_{2}=\frac{V_{2}-V_{1}}{t_{2}} \tag{5}
\end{align*}
$$

$$
\text { Inserting (5) into (4) gives: } \begin{align*}
S_{2} & =\left(V_{1}-V_{W}\right) \cdot t_{2}+1 / 2\left(\frac{V_{2}-V_{1}}{t_{2}}\right) t_{2}^{2} \\
& =\left(V_{1}-V_{W}\right) \cdot t_{2}+\left(\frac{V_{2}-V_{1}}{2}\right) t_{2} \\
& =t_{2}\left(\frac{V_{2}+V_{1}}{2}-V_{W}\right) \tag{6}
\end{align*}
$$

To determine actual length of the air segment, we obtain:

$$
\begin{equation*}
S_{3}^{\prime}=t_{3}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right) \tag{7}
\end{equation*}
$$

But by Pythagoras:

$$
\begin{equation*}
S_{3}=\sqrt{S_{3}^{\prime 2}-h^{2}} \tag{8}
\end{equation*}
$$

And inserting (7) into (8)

$$
\begin{equation*}
S_{3}=\sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}} \tag{9}
\end{equation*}
$$

Total take-off distance to screen height, in actual conditions, is then $\mathrm{S}_{1}+\mathrm{S}_{2}+\mathrm{S}_{3}$, as determined above. Adjustments to standard conditions may be made using the usual variance factors ${ }^{4}$.

However, this method does not take account of the errors which normally will exist in the variables. It is assumed that each of these factors are accurate to within the precision of recording (which is normally done manually, preferably by a Flight Test Observer (FTO) or exceptionally by the Test Pilot themselves). These precisions are usually taken to be $\pm 1$ second for all time measurements, and $\pm 1 \mathrm{~ms}^{-1}$ (about 2 knots) for all speed values including the headwind component. It is assumed that the height is correct, and that any errors in determining time to height are time errors alone. Using this, it is possible to conduct an error analysis starting with the following equation, which sums (3), (6) and (8) above.

$$
\begin{equation*}
S=\frac{t_{1}}{2}\left(V_{1}-V_{W}\right)+t_{2}\left(\frac{V_{2}+V_{1}}{2}-V_{W}\right)+\sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}} \tag{10}
\end{equation*}
$$

Or in slightly modified form;

$$
\begin{equation*}
S=\frac{t_{1}}{2}\left(V_{1}-V_{W}\right)+\frac{t_{2}}{2}\left(V_{2}+V_{1}-2 V_{W}\right)+\frac{1}{4} \sqrt{t_{3}^{2}\left(V_{3}+V_{2}-2 V_{W}\right)^{2}-2 h^{2}} \tag{10a}
\end{equation*}
$$

Taking partial derivatives with respect to each component of (10) in turn, the following series of factors are obtained.

$$
\begin{align*}
& \frac{\partial S}{\partial t_{1}}=\frac{V_{1}-V_{W}}{2}  \tag{11}\\
& \frac{\partial S}{\partial t_{2}}=\frac{V_{2}+V_{1}}{2}-V_{W}  \tag{12}\\
& \frac{\partial S}{\partial t_{3}}=\frac{t_{3}\left(\left(V_{2}+V_{3}\right)^{2}+4\left(V_{W}^{2}-V_{W}\left(V_{2}+V_{3}\right)\right)\right)}{4 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}  \tag{13}\\
& \frac{\partial S}{\partial V_{1}}=\frac{t_{1}+t_{2}}{2}  \tag{14}\\
& \frac{\partial S}{\partial V_{2}}=\frac{t_{2}}{2}+\frac{t_{3}^{2}\left(V_{3}+V_{2}-2 V_{W}\right)}{4 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}  \tag{15}\\
& \frac{\partial S}{\partial V_{3}}=\frac{t_{3}^{2}\left(V_{3}+V_{2}-2 V_{W}\right)}{4 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}  \tag{16}\\
& \frac{\partial S}{\partial V_{W}}=-\left(\frac{t_{3}^{2}\left(V_{3}+V_{2}-2 V_{W}\right)}{2 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}+\frac{\left(t_{1}+t_{2}\right)}{2}\right) \tag{17}
\end{align*}
$$

The estimated error due to each individual component will be the factor of that component's assumed error and the take-off distance's partial derivative with respect to that component. However it is also normal practice, based upon the assumption that errors are normally distributed, that the total error may be taken to be the square
root of the sum of the squares of errors. Thus, in any individual test, the estimated maximum error may be taken as.

$$
\begin{equation*}
\sqrt{\left(t_{1} \frac{\partial S}{\partial t_{1}}\right)^{2}+\left(t_{2} \frac{\partial S}{\partial t_{2}}\right)^{2}+\left(t_{3} \frac{\partial S}{\partial t_{3}}\right)^{2}+\left(V_{1} \frac{\partial S}{\partial V_{1}}\right)^{2}+\left(V_{2} \frac{\partial S}{\partial V_{2}}\right)^{2}+\left(V_{3} \frac{\partial S}{\partial V_{3}}\right)^{2}+\left(V_{W} \frac{\partial S}{\partial V_{W}}\right)^{2}} \tag{18}
\end{equation*}
$$

(Note: combination of errors. Justifying the approach taken above, it is assumed that all errors, e are independent and follow a normal (Gaussian) distribution, with a mean of zero and variance $\sigma^{2}$, then the sum of errors $\sum e=e_{1}+e_{2} \ldots e_{n}$ is itself normally distributed with a mean of zero and a variance of $\sum \sigma_{n}^{2}$. This means that the standard deviation, which is proportional to the total error is defined by $\sigma^{2}=\sum \sigma_{n}^{2}$. Written otherwise, this may be stated as Total error $=\sqrt{e_{1}^{2}+e_{2}^{2} \ldots+e_{n}^{2}}$ which has an identical form to (18) above. Further examples of this method of combination of errors may be found in reference ${ }^{5}$ )

So, for a conservative analysis, the take-off distance should be calculated as shown in (10) above. Then, the maximum error should be calculated, using (18) and estimates of the accuracy to which each value was measured, and this added to the estimate for take-off distance. This sum, may then be used as a planning take-off distance value, with high confidence that the actual distance required is no greater than that.

### 2.2 Validation using a Mignet HM1000 Balerit

Use of this may be demonstrated using the following worst 6 results (i.e. the six sets of values giving the greatest distances) for flight tests carried out for an increase in MTOW for the HM1000 Balerit aircraft ${ }^{6,7}$ similar to that shown in Figure 2 below. The results (using a 2 -segment method) were as follows:-

Table 1, Take-off test data for HM1000 Balerit at 420kg.

| No. $\rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{1}$ | 11 | 15 | 15 | 12.5 | 14.5 | 13.5 |
| $\mathrm{t}_{3}$ | 7.5 | 9 | 5 | 6 | 4.5 | 4.5 |
| $\mathrm{V}_{2}$, mph IAS | 55 | 55 | 51 | 52 | 58 | 53 |
| $\mathrm{V}_{3}$, mph IAS | 47 | 47 | 50 | 50 | 50 | 50 |
| $\begin{aligned} & \hline \mathrm{V}_{2}, \mathrm{kn} \text { CAS } \\ & \left(\mathrm{ms}^{-1} \mathrm{CAS}\right) \end{aligned}$ | $\begin{gathered} 47 \\ (24.2) \end{gathered}$ | $\begin{gathered} 47 \\ (24.2) \end{gathered}$ | $\begin{gathered} 46 \\ (23.6) \end{gathered}$ | $\begin{gathered} 46 \\ (23.6) \end{gathered}$ | $\begin{gathered} 49 \\ (25.2) \end{gathered}$ | $\begin{gathered} 48 \\ (24.7) \end{gathered}$ |
| $\begin{array}{\|l} \hline \mathrm{V}_{3}, \mathrm{kn} \text { CAS } \\ \left(\mathrm{ms}^{-1} \mathrm{CAS}\right) \end{array}$ | $\begin{gathered} 43 \\ (27.2) \end{gathered}$ | $\begin{gathered} 43 \\ (27.2) \end{gathered}$ | $\begin{gathered} 45 \\ (23.1) \end{gathered}$ | $\begin{gathered} 45 \\ (23.1) \end{gathered}$ | $\begin{gathered} 45 \\ (23.1) \end{gathered}$ | $\begin{gathered} 45 \\ (23.1) \end{gathered}$ |

Mean time to unstick= $\quad 13.6 \mathrm{~s}$
Mean climb time $=\quad 6.1 \mathrm{~s}$
Mean unstuck speed =
$24.3 \mathrm{~ms}^{-1}$
Mean screen speed =
Screen Height =
15m
Surface wind =
negligible.
(Determination of the relationship between IAS and CAS was determined for all tests in this paper prior to field performance testing using the racetrack method as detailed in reference ${ }^{2}$. CAS was taken to equal TAS since conditions were very close to ISA.)

From (10), the take-off distance is calculated (normally using a spreadsheet programme such as Microsoft Excel ${ }^{\mathrm{TM}}$ ) as in Table $\mathbf{2}$ below:

Table 2, Results for Balerit take-off analysis

| Take-off distance $=$ | 313 m, of which; |
| :--- | :--- |
| Ground Roll= | 165 m, and it may be determine that: |
| Maximum error $=$ | $\pm 40 \mathrm{~m}( \pm 13 \%)$, from (18) |

Assuming that the worst case error applies, the conservatively estimated take-off distance may then be taken as 343 m .
(In practice, the certification standard ${ }^{8}$ and normal working practice ${ }^{9}$ both require the use of a 1.3 safety factor $(+30 \%)$ in any case, which is clearly greater than the greatest predicted error from this test (and from most others); but, were the predicted error greater than that then it would be sensible and conservative to use this in place of a 1.3 factor). It is most conservative to use both the estimated maximum error and the certification standard's 1.3 safety factor, which is what has become the most common practice, at-least for aeroplanes such as microlights with short take-off and landing distances. It is cautioned however that for aeroplanes which already require larger field lengths, such conservatism may cause operational difficulties and thus be inappropriate.


Figure 2, Mignet HM1000 Balerit microlight aeroplane

A simple "reality check" upon this data may be obtained from the runway length and an external observer / camera. In this case the take-off tests were flown from Chilbolton (Stonefield Park) airfield in Hampshire ${ }^{\mathbf{1 0 , 1 1}}$ - also Figure 3 below, which has a runway length of 411 m . A coarse check upon the results was provided by external observers and the pilot who estimated that about $75 \%$ of the runway was required to reach the 15 m screen height, an observation which is consistent with the estimated distance.


Figure 3, Illustration of Chilbolton Airfield (from reference ${ }^{\mathbf{1 0}}$, © Robert Pooley)

### 2.3 Validation using a Naval Aircraft Factory N3N-3

A further check was made when this method was used during flight testing for issue of a Public Transport CofA of a Naval Aircraft Factory N3N-3 Aeroplane ${ }^{12}$ as shown in Figure 4 below.


Figure 4, Naval Aircraft Factory N3N-3, G-ONAF ((c) Keith Tomlin)

Nine take-offs were flown from Isle of Wight (Sandown) airport10, 13, at the conditions shown in

Table 3 below Using the 2-part segmented method (the rotation phase being extremely short, justifying this), the results shown in the following Table 4 were obtained:-

Table 3, Test Conditions for N3N take-off trials

| Weight: | $2870 \mathrm{lb}(1300 \mathrm{~kg})$, |
| :--- | :--- |
| Headwind: | 8 kn headwind $(4.1 \mathrm{~m} / \mathrm{s})$ |
| Crosswind: | Nil |
| OAT: | $15^{\circ} \mathrm{C}$ |
| QFE: | 1023 hPa |
| Runway: | 60 ft amsl, short-dry grass. |

Table 4, Take-off test data for N3N-3 Aeroplane

| $\underline{\text { No. }}$ | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | $\underline{5}$ | $\underline{6}$ | $\underline{7}$ | $\underline{8}$ | $\underline{9}$ | $\underline{\text { Mean }}$ | $\frac{\text { Mean }}{}$ <br> $\mathrm{t}_{1}, \mathrm{~s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14.5 | 15 | 13.5 | 13. <br> 5 | 13 | 13 | 14 | 13.5 | 12.5 | 13.6 | $\underline{\mathrm{~m} / \mathrm{s}}$ |  |
| $\mathrm{t}_{1}+\mathrm{t}_{3}$, <br> s | NR | 22 | 19.5 | 18. <br> 5 | 20 | 17.5 | 19.5 | 20 | 19.5 | 19.6 | - |
| $\mathrm{t}_{3}, \mathrm{~s}$ | - | 7 | 6 | 5 | 7 | 4.5 | 5.5 | 6.5 | 7 | 6.1 | - |
| $\mathrm{V}_{1}$, <br> mph <br> IAS | 52 | 52 | 52 | 52 | 52 | 55 | 53 | 52 | 52 | 52.4 | 23.6 |
| $\mathrm{V}_{2}$, <br> mph <br> IAS | 30 | 60 | 60 | 60 | 58 | 60 | 58 | 58 | 57 | 55.7 | 25.3 |

(Headwind component 8 kn)

These values were reduced to calibrated SI units and input to a segmented method analysis model, and gave the following results at the tested conditions:

Table 5, Analysis of N3N take-off performance: segment method

| Ground roll $=$ | 104.7 m |
| :--- | :---: |
| Air segment distance along the ground $=$ | 123.2 m |
| Estimated maximum total error in calculation $=39.6 \mathrm{~m}$ | $(17.4 \%)$ |
| Total conservative calculated take-off distance $=$ | 268 m |

Verification of this data was performed using video analysis. A fixed video camera was used adjacent to the control tower, and a relationship established between aircraft position and height as seen in the camera, and relative to the runway, by comparing the geometry of 4 points in the field of view (two runway markers, a hangar, and a mid point) with that determined using an airfield plan. Figure 5, Table 6 and Figure 7 below show the geometry of this, and this relationship was then used to relate from the video monitor to estimated values for take-off distance. Not all take-off ground-
segments were recorded, due to an misunderstanding between the pilot and cameraman concerning the available field of view, nonetheless, sufficient data was obtained for reasonable verification purposes (as with the timed method, the assumption that the aircraft maintained the runway centreline was essential). From this, the following data were obtained:

Table 6, Analysis of N3N take-off performance: video method

| Ground roll, mean of 5 data points $=$ | 127 m |
| :--- | :---: |
| Air segment distance, along the ground mean of 9 data points |  |
|  | $=$108 m  <br> Estimated maximum error $=$ $\pm 14 \mathrm{~m}$ |
| $\qquad$(based upon $\pm 10 \mathrm{~m}$ accuracy for each data point) |  |
| Total estimated take-off distance therefore $=$ | 249 m |

This gives a slightly reduced take-off distance than that from the timed method, thus the timed method seems to be slightly more conservative. Correlation is good, in that (before addition of error margins) the timed method gives 228 m and the video method gives 235 m . The more elaborate error analysis of the timed method however ultimately results in it being the more conservative method


Figure 5, Geometry of Sandown airport as used for N3N field performance estimation (not to scale)


Figure 6, Chart of distance along runway centreline versus distance across video monitor screen, showing quadratic best fit curve.

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Figure 7, Excerpt from airfield guide for Sandown (from reference ${ }^{10} ®$ Robert Pooley)

## 3. ESTIMATING LANDING DISTANCE

### 3.1 The method

Similarly to the method used for take-off distance estimation above, landing distances may also be estimated. In this case, the following notation is used: Speeds at screen height, touchdown, 3-wheels down, and stop are $\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}$ and 0 respectively. Surface wind is $\mathrm{V}_{\mathrm{W}}$ and is positive when a headwind. The difference in $\mathrm{V}_{\mathrm{W}}$ effect due to flightpath angle is assumed to be small. Times of each segment are $t_{1}, t_{2}, t_{3}$. Note that in the case of a taildragger making a 3-point landing or other aircraft with an insignificantly short 2 -wheel roll, then $\mathrm{t}_{2}=0$ and the method is reduced to 2 -segments.

Lengths of each segment, measured along the ground, are $S_{1}, S_{2}, S_{3}$. Straight line distance from top of screen to touchdown $=S_{1}{ }^{1}$. Accelerations during each segment are $a_{1}, a_{2}, a_{3} . a_{1}$ is acceleration along flightpath, not along the ground.

The total landing distance may be given by:-

$$
\begin{equation*}
S=\sqrt{t_{1}^{2}\left(\frac{V_{2}+V_{1}}{2}-V_{W}\right)^{2}-h^{2}}+t_{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)+t_{3}\left(\frac{V_{3}-V_{W}}{2}\right) \tag{19}
\end{equation*}
$$

Similarly to the take-off case, it is essential to conduct an error analysis. This gives the following results:-

$$
\begin{align*}
& \frac{\partial S}{\partial t_{1}}=\frac{t_{1}\left(\frac{V_{2}+V_{1}}{2}-V_{W}\right)^{2}}{\left(\sqrt{t_{1}^{2}\left(\frac{V_{2}+V_{1}}{2}-V_{W}\right)^{2}-h^{2}}\right)}  \tag{20}\\
& \frac{\partial S}{\partial t_{2}}=\frac{V_{3}+V_{2}}{2}-V_{W}  \tag{21}\\
& \frac{\partial S}{\partial t_{3}}=\frac{V_{3}-V_{W}}{2}  \tag{22}\\
& \frac{\partial S}{\partial V_{1}}=\frac{t_{1}^{2}\left(V_{1}+V_{2}-2 V_{W}\right)}{4 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}  \tag{23}\\
& \frac{\partial S}{\partial V_{2}}=\frac{t_{1}^{2}\left(V_{1}+V_{2}-2 V_{W}\right)}{4 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}+\frac{t_{2}}{2}  \tag{24}\\
& \frac{\partial S}{\partial V_{3}}=\frac{t_{2}+t_{3}}{2} \tag{25}
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial S}{\partial V_{W}}=-\left(\frac{t_{1}^{2}\left(V_{1}+V_{2}-2 V_{W}\right)}{2 \cdot \sqrt{t_{3}^{2}\left(\frac{V_{3}+V_{2}}{2}-V_{W}\right)^{2}-h^{2}}}+t_{2}+\frac{t_{3}}{2}\right) \tag{26}
\end{equation*}
$$

### 3.2 Validation using Mignet HM1000 Balerit

Below is actual test data for an HM1000 Balerit aircraft.
Table 7, Landing test data for HM1000 Balerit at 420kg

| No. $\rightarrow$ | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | $\underline{5}$ | $\underline{6}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{1}$ | 7 | 11.5 | 10 | 13.5 | 12 | 13.5 |
| $\mathrm{t}_{3}$ | 43 | 20.5 | 28 | 26.5 | 25 | 29 |
| $\mathrm{~V}_{1}, \mathrm{mph}$ IAS | 50 | 53 | 48 | 50 | 50 | 55 |
| $\mathrm{~V}_{2}$ mph IAS | 46 | 37 | 41 | 45 | 37 | 37 |
| $\mathrm{~V}_{1}, \mathrm{kn}$ CAS |  |  |  |  |  |  |
| $\left(\mathrm{ms}^{-1}\right.$ CAS $)$ | 45 | 48 | 44 | 45 | 45 | 47 |
| $\mathrm{V}_{2}, \mathrm{kn}^{-1}$ CAS <br> $\left(\mathrm{ms}^{-1}\right.$ CAS $)$ | $43.1)$ | $(24.7)$ | $(22.6)$ | $(23.1)$ | $(23.1)$ | $(24.2)$ |
| $(19.5)$ | 41 | 42 | 38 | 38 |  |  |
| $(21.1)$ | $(21.6)$ | $(19.5)$ | $(19.5)$ |  |  |  |

[The aircraft was stopped on the ground using moderate braking once at a fast walking pace).

Mean time to from screen height to touchdown: 11.25
Mean time to stop $=28.7 \mathrm{~s}$
Mean speed at screen height $=23.5 \mathrm{~ms}^{-1}$
Mean touchdown speed $=20.6 \mathrm{~ms}^{-1}$

Using this data, analysis was carried out as shown in Table 8 below. Again this test was flown at Chilbolton with a 411 m runway, and the pilot estimated that the aircraft was stopped in about the full length of the runway - having descended through screen height before the threshold. In this case, the conservative estimate using this method matches well the visual estimate.

| Landing distance $=$ | 364 m , of which |
| :--- | :--- |
| Ground Roll= | 132 m, and it may be determined that |
| Maximum error $=$ | $\pm 34 \mathrm{~m}( \pm 9.2 \%)$, from (18) |

The conservative planning landing distance was therefore $364+34=398 \mathrm{~m}$.

### 3.2 Validation using a Naval Aircraft Factory N3N-3

As for take-off distances above, an opportunity also arose to use this method, and verify data using an external video source during testing of a Naval Aircraft Factory N3N-3 Aeroplane. The following test data were obtained:

Table 9, Landing test data for N3N-3 Aeroplane

| No. | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ | $\underline{4}$ | $\underline{5}$ | $\underline{6}$ | $\underline{7}$ | $\underline{8}$ | $\underline{\text { Mean }}$ | $\underline{\text { Mean, }}$ <br> $\underline{m} / \mathrm{cAS}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{1}, \mathrm{~s}$ | 15 | 13 | 10 | 10 | 10.5 | 8 | 8 | 10 | 10.9 |  |
| $\mathrm{t}_{1} \mathrm{t}_{3}, \mathrm{~s}$ | 33 | 34 | 28 | 26.5 | 27.5 | 24.5 | 25.5 | 25.5 | 28.6 |  |
| $\mathrm{t} 3, \mathrm{~s}$ | 18 | 21 | 18 | 16.5 | 17 | 16.5 | 17.5 | 15.5 | 17.6 |  |
| $\mathrm{V}_{1}$, <br> mph <br> IAS | 65 | 65 | 65 | 67 | 65 | 70 | 65 | 65 | 65.3 | 30.2 |
| $\mathrm{V}_{2}$, <br> mph <br> IAS | 50 | 52 | 52 | 52 | 53 | 55 | 53 | 53 | 52.1 | 23.5 |

(Headwind component 8 kn )
These values were input to a segmented method analysis model, which gave the results shown in Table 10 below

Table 10, Results for N3N-3 landing analysis: segment method

| Landing distance $=$ | 331 m, of which; |
| :--- | :--- |
| Ground Roll $=$ | 120 m, and it may be determined that: |
| Maximum error $=$ | $\pm 30 \mathrm{~m}( \pm 8.9 \%)$, from (18) |

The conservative planning landing distance was therefore $331+30=361 \mathrm{~m}$.

Verification of this data was again performed using video analysis and sufficient data was obtained for verification purposes. From this, the data in Table 11 below were obtained:

## Table 11, Analysis of N3N landing performance: video method

| Air segment distance, 1 data point only $=162 \mathrm{~m}$ (over ground); |  |
| :--- | :---: |
| Ground segment distance, mean of 5 data points $=160 \mathrm{~m}$ |  |
| Estimated maximum error $=$ | $\pm 14 \mathrm{~m}$ |
|  |  |
|  |  |
| (based upon $\pm 10 \mathrm{~m}$ accuracy for each data point) |  |
| Conservatively estimated landing distance therefore $=336 \mathrm{~m}$. |  |

Thus the timed method (including error analysis) is more conservative than the method of video analysis and may be accepted for safety purposes. There is an apparent mismatch between the ground and air segment distances - ground roll is somewhat longer on the video analysis compared to air segment, which is longer on the timed method. This is attributed to the difficulty in accurately identifying the touchdown point from video analysis, nonetheless the total distance before addition of estimated errors (which effectively does not take into account this point) is extremely
close and the timed method is made more conservative primarily by the larger value determined by the error analysis for that method.

## 4. DETERMINING SCREEN HEIGHT

Part of the problem faced in developing this method has been the determination of screen height; barometric altimeters suffer sufficient lag, combined potentially with static errors as airspeed increases ${ }^{2}$ that in virtually any aeroplane they are an inappropriate method to measure height immediately after take off. The use of a radio altimeter (RadAlt) would be desirable but, generally, the complexity and mass penalty associated with installing one into a test aeroplane is unjustifiable except where such a device would have been fitted any case. Following some discussion and experimentation, it was however found that screen height could be measured by an observer in the aircraft using a sighting device to the edge of the runway - this relies upon the pilot holding the centreline accurately, and accurate knowledge of the height of the observer above the wheels, location relative to aircraft lateral centreline, and runway width. Since the only height required is the screen height, the device can be as straightforward as a single mark upon a strut or canopy, although two marks in-line or a wire frame have proven most useful.

## 5. CONCLUSIONS

This paper has shown how an alternative method, which makes use of speeds, times, and a geometric method of height determination, may be used to estimate take-off and landing distances. The method offers the substantial advantage compared to classical methods such as a kinetheodolite or video analysis in that it does not require any observer or equipment external to the aircraft, although potentially does offer lower accuracy. A method of error analysis has also been shown, which allows the user to determine whether the degree of accuracy achieved is acceptable.

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