Linear unknown input functional observer for thermal estimation in power electronic modules

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***Abstract*—High integrated power electronic modules are more and more designed with the emergence of new semi-conductor technologies. Thus, increase of reliability of power modules induces the precise knowledge of the local temperature, even if it can not be measured at any location. Moreover, some external variables of having an effect on the system may be unknown. In this paper, the application of an unknown input observer is proposed. It allows us to estimate the temperature at any location using measurements provided from thermal sensors located at a few precise points without measuring all inputs of the system. The aim is then to estimate internal temperature of a system in order to prevent over-temperature operations and then fault of the system. Consequently, a linear unknown input functional observer (LUIFO) of minimal order observer is design for thermal estimation of silicone gel used in power electronic modules.**

***Keywords*-Unknown input observer, functional observer, thermal estimation, power electronic modules monitoring.**

I. Introduction

The joint emergence of Wide Band Gap materials (SiC, GaN, C) and new generation hybrid integration techniques significantly enhance performances of power electronic modules. Such modules should operate in severe environment and constraints: high temperature and high power density, fast switching, etc. Consequently of high temperature, new constrains appear and become critical for power electronics assemblies. Several studies aim at identifying failure modes or critical interfaces [1], [2]. Thus, estimation of local temperatures becomes a real challenge in new generation of power modules to monitor their behavior and to increase their lifetime. Indeed, it has been shown in [3], [4] that the evolution of local constraints in a power electronic module, which can be thermal or thermo-mechanical, have a negative effect on the lifetime of the module. These constraints increase the occurrence of potentially critical defects and failures on the module. Consequently, it becomes necessary to have a precise knowledge of the temperatures at specific locations in the module, such as the temperature of semi-conductor chips or wire bondings. However, due to the size of sensors and possible electromagnetic field disturbances close to measurement points, the use of thermal sensors may be difficult at some locations inside of the power module. Moreover, some external variables that affects thermal response of the module, such as heat dissipation to environment may not be precisely measured. For these reasons the objective of the following work is to estimate internal temperature in a specific non measured location, using measured data by few sensors and without knowledge of some inputs of the system.

As a case study, a simple one-dimension ($1D$) thermal system is considered in this paper and then modeled. Equations of thermal evolution of the system with respect to time and space can be rewritten using a linear state-space representation with unknown inputs. Using this representation, the temperature can be estimated at any location with a LUIFO or an unknown input partial state observer.

The first section deals with the construction of a thermal model of the proposed system and its representation in state space. In this work, the thermal behavior of a $(160mm)$ $1D$ bar of silicone gel which may represent the thermal behavior of one of the materials used in power electronic modules is considered as a test benchmark of our technics. The matrix representation of previous model is established and aims to design a LUIFO. We propose in the third section a way to design such an observer, based on the use of successive derivatives of the measured outputs. The interest of the observer design lies in the possibility to observe the temperature at any location in the system. Finally, through comparison with experimental data, the application of the proposed observer is validated in the last section.

II. System modeling

As this paper deals with the feasability of the design of a LUIFO for thermal phenomenon, it is not necessary to take into account a whole power electronic module. In order to simplify the problem and highlight the proposed estimation approach, this study deals with the case of a bar of silicone gel of length much more larger than radius. Thermal phenomena occurring along different directions of the main dimension of the bar will be neglected. Thus, the bar will be considered thermally insulated outside of its main dimension. Thermal expansion induced by thermal behavior will also be neglected. Indeed, thermal expansion coefficient of the material is about $10^{-4}K^{-1}$. This means that thermal resistance and capacitance of the material, that depends on geometrical parameters, are considered as constant whatever the considered temperatures.

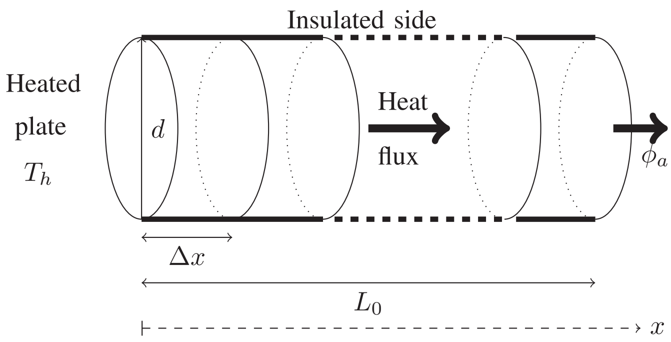

Fig. 1. Studied system

The modeling methodology is based on a discretization of the heat equation described in [5].

A. Description of the experimental system

The proposed system is composed of an encapsulating gel bar dedicated to power electronic modules submitted to thermal stress. The material is a silicone gel (Silgel 616), with constant thermal properties that have been experimentally measured or deduced from its datasheet:

Thermal properties:

- conductivity: $\lambda = 0,1\,\text{W.(m.K)}^{-1}$,
- heat capacity: $C_p = 80\,\text{W.K}^{-1}.\text{kg}^{-1}$,
- density: $\rho = 970\,\text{kg.m}^{-3}$,
- convection coefficient: $h = 70\,\text{W.m}^{-2}$.

The material is cast and crosslinked in a glass tube of internal diameter $d = 1.10^{-2}\,m$ over a length $L_0 = 16.10^{-2}\,m$. The tube is then placed vertically on a temperature-controlled heating plate of temperature $T_h(t)$ that is measured by a thermocouple. Heat dissipation to environment $\phi_a(t)$ is not measured. Thermocouples are inserted into the silicone gel at different positions along length of the bar to measure the local temperature. All thermocouples are placed at the center of the bar along its radius. Finally, a thermal insulator composed of extruded polystyrene is positioned around the tube. It has a thermal conductivity approximately ten times lower than the silicone gel, avoiding heat transfer along the radius axis of the cylinder. Consequently, the experimental setup allows us to maximize the mono-dimensional nature of thermal phenomena.

B. Thermal model

To establish the model, the system is sampled into $n = 16$ elementary volumes of length Δx (see Figure 1).

The thermal boundary conditions are therefore defined on the two orthogonal bases to the main length of the bar. One of the bases is in convection with environment. The thermal boundary condition on the opposite base is defined as a heating temperature $T_h(t)$. This temperature corresponds to the operating temperature of a semiconductor component in a power module. It is chosen so as to be lower than the maximum limit temperature of use of the silicone gel. Finally, the gel is submitted to temperature included between its glass transition temperature and its destruction temperature.

Within the framework of the considered system, thermal transfers are governed by the heat equation [6].

Using spatial sampling, we get equations n equations for n elementary volumes that can then be easily written in a matrix form (1) where $\mathbf{x}(t)$ is the state vector of the local temperatures, $\mathbf{u}(t) = T_h(t)$ and $\mathbf{f}(t) = \phi_a(t)$ are the vectors of measured and unknown inputs respectively. Note that the thermal dynamic matrix A is of size $n \times n$, B and E are of size $n \times 1$.

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + E\mathbf{f}(t) \tag{1}$$

The thermal properties of the material allows us to calculate thermal resistance and capacity used in the model and matrices A, B and E express respectively as (2) and (3).

$$A = \begin{pmatrix} \alpha & \gamma & & & \\ \gamma & \beta & \ddots & 0 & \\ & \ddots & \ddots & \ddots & \\ & 0 & \ddots & \beta & \gamma \\ & & & \gamma & \delta \end{pmatrix} \tag{2}$$

$$B = \begin{pmatrix} -\beta \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \xi \end{pmatrix} \tag{3}$$

with: $\alpha = -0.0387$, $\beta = -0.0258$, $\gamma = 0.0129$, $\delta = -0.0329$ and $\xi = 1$.

Depending on measured temperature locations, an equation for output measured temperatures is established (4) with C the measurement matrix of size $(m \times n)$.

$$T_{mes}(t) = C\mathbf{x}(t) \tag{4}$$

The temperature of the 8^{th} node $x_8(t)$ is measured by a thermocouple. This means that the measurement matrix C is of size (1×16):

$$C = [0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0]$$

III. Estimation of non-measured state variables

A. Linear unknown input functional observer

Let us consider a system described by the linear state space equations:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + E\mathbf{f}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) \end{cases} \tag{5}$$

where, $\forall t \in \mathbb{R}^+$, $x(t)$ is the n-dimensional state vector, $u(t)$ is a p-dimensional control vector supposed to be measured, $y(t)$ is a m-dimensional measured output vector, and, $f(t)$ is a r-dimensional unknown input vector. $A(n \times n)$, $B(n \times p)$, $C(m \times n)$ and $E(n \times r)$ are constant matrices. Without loss of generality C and E are respectively of full row and of full column ranks.

To design a functional observer, the triplet (A, C, L) has to be functionally observable (6) where $\mathcal{O}_{A,C,n}$ is the observation matrix of the system [9].

$$\operatorname{rank}\left(\left[\begin{array}{c} \mathcal{O}_{A,C,n} \\ L \end{array}\right]\right) = \operatorname{rank}\left(\mathcal{O}_{A,C,n}\right) \tag{6}$$

Moreover, in order to avoid a trivial algebraic part in the observer (where observed functional can be estimated through linear combination of measured outputs), it is supposed without loss of generality that:

$$\operatorname{rank}\left(\left[\begin{array}{c} C \\ L \end{array}\right]\right) = m + l.$$

The aim of a functional observer is to estimate state variables, at least asymptotically, from the measurements on the system. Estimated state variables are defined by :

$$\mathbf{v}(t) = L\mathbf{x}(t) \tag{7}$$

where L is a constant full row rank $(l \times n)$ matrix selecting estimated components.

The observation of $\mathbf{v}(t)$ can be carried out by a linear unknown input functional observer which is a Luenberger observer, [10], [11], described by the state equations:

$$\begin{cases} \dot{\mathbf{z}}(t) = F\mathbf{z}(t) + G\mathbf{u}(t) + H\mathbf{y}(t) \\ \hat{\mathbf{v}}(t) = P\mathbf{z}(t) + V\mathbf{y}(t) \end{cases} \tag{8}$$

where $\mathbf{z}(t)$ is a q-dimensional state vector and $\hat{\mathbf{v}}(t)$ is a l-dimensional vector. The constants matrices $F(q \times q)$, $G(q \times p)$, $H(q \times m)$, $P(l \times n)$, $V(l \times m)$ and the order q are determined such that $\lim_{t \to +\infty} (\mathbf{v}(t) - \hat{\mathbf{v}}(t)) = 0$.

The necessary and sufficient conditions for the existence of an asymptotic observer (8) for the system (5) if and only if F is Hurwitz and there exists a matrix $T(q \times n)$ such that [12], [13]:

$$FT + HC - TA = 0, \tag{9}$$

$$L - PT - VC = 0, \tag{10}$$

$$G - TB = 0, \tag{11}$$

$$TE = 0. \tag{12}$$

Moreover, a LUIFO cannot be designed if there are unstable transmission zeros from the unknown input to the output [14]. Figure 2 expresses the structure of unknown input observer.

B. Design of a LUIFO observer

This section deals with the search for a minimal order LUIFO. Let us define recursively the matrices K_ν and Σ_ν, $\nu \in \mathbb{N}$:

- $K_0 = I_n$ and for $\nu \geq 1$, $K_\nu = \left[\begin{array}{cc} AK_{\nu-1} & D \end{array}\right]$;
- $\Sigma_0 = C$ and for $\nu \geq 1$,

$$\Sigma_\nu = \left[\begin{array}{c|c} \begin{array}{c} \Sigma_{\nu-1} \\ LK_{\nu-1} \end{array} & 0(\nu(m+l) \times r) \\ \hline \multicolumn{2}{c}{CK_\nu} \end{array}\right].$$

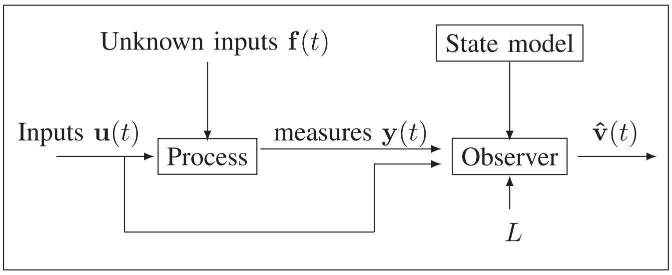

Fig. 2. Unknown input observer principle

More explicitly, we get:

$$\Sigma_\nu = \left[\begin{array}{cccccc} C & 0 & \cdots & 0 & 0 & 0 \\ L & 0 & \cdots & 0 & 0 & 0 \\ CA & CD & \cdots & 0 & 0 & 0 \\ LA & LD & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \vdots & \vdots & & 0 & 0 & 0 \\ CA^{\nu-1} & CA^{\nu-2}D & \cdots & CAD & CD & 0 \\ LA^{\nu-1} & LA^{\nu-2}D & \cdots & LAD & LD & 0 \\ CA^\nu & CA^{\nu-1}D & \cdots & CA^2D & CAD & CD \end{array}\right].$$

where the "0" blocks are of adapted dimensions.

In the following we use the notation:

$$\Sigma_\nu = \left[\begin{array}{c} C_{\nu,0} \\ L_{\nu,0} \\ C_{\nu,1} \\ L_{\nu,1} \\ \vdots \\ C_{\nu,\nu-1} \\ L_{\nu,\nu-1} \\ C_{\nu,\nu} \end{array}\right], \tag{13}$$

where the matrices $C_{\nu,i}$, for $i \in [\![0;\nu]\!]$, and $L_{\nu,i}$, for $i \in [\![0;\nu-1]\!]$, are respectively of dimensions $(m \times (n + r\nu))$ and $(l \times (n + r\nu))$.

Moreover, let q the smallest integer such that (14) is satisfied.

$$\operatorname{rank}\left(\Sigma_q\right) = \operatorname{rank}\left(\left[\begin{array}{c} \Sigma_q \\ LK_q \end{array}\right]\right) \tag{14}$$

1) First step: The design of the observer uses the successive derivations of $\mathbf{v}(t)$. After q derivations of $\mathbf{v}(t) = L\mathbf{x}(t)$, we obtain:

$$\begin{aligned} \mathbf{v}^{(q)}(t) = \quad & LA^q\mathbf{x}(t) + \sum_{i=0}^{q-1} LA^iB\mathbf{u}^{(q-i-1)}(t) \\ & + \sum_{i=0}^{q-1} LA^iE\mathbf{f}^{(q-i-1)}(t) \end{aligned} \tag{15}$$

It can be noticed from (14) that it exists matrices, Γ_i, $i \in [\![0\ ;\ q]\!]$ and Λ_i, $i \in [\![0\ ;\ q-1]\!]$ such that:

$$LK_q = \sum_{i=0}^{q} \Gamma_i C_{q,\,i} + \sum_{i=0}^{q-1} \Lambda_i L_{q,\,i}, \tag{16}$$

Note that Γ_i and Λ_i matrices are derived from partitioning of Σ_q in (13) and the unique solution of the equation $X = LK_q\Sigma_q^\dagger$ where $\Sigma_q^\dagger$ is the pseudo-inverse of Σ_q. Moreover, (16) can be explicitly written as:

$$\begin{array}{rcl} LA^q & = & \sum_{i=0}^{q} \Gamma_i CA^i + \sum_{i=0}^{q-1} \Lambda_i LA^i, \\ LA^{q-1}E & = & \sum_{i=1}^{q} \Gamma_i CA^{i-1}E + \sum_{i=1}^{q-1} \Lambda_i LA^{i-1}E, \\ & \vdots & \\ LA^{q-k}E & = & \sum_{i=k}^{q} \Gamma_i CA^{i-k}E + \sum_{i=k}^{q-1} \Lambda_i LA^{i-k}E, \\ & \vdots & \\ LAE & = & \Gamma_q CAE + \Gamma_{q-1}CE + \Lambda_{q-1}LE, \\ LE & = & \Gamma_q CE. \end{array} \tag{17}$$

Using expression of LA^q in (17), (15) can be written as:

$$\begin{array}{rl} \mathbf{v}^{(q)}(t) = & \sum_{i=0}^{q} \Gamma_i CA^i \mathbf{x}(t) + \sum_{i=0}^{q-1} \Lambda_i LA^i \mathbf{x}(t) \\ & + \sum_{i=0}^{q-1} LA^i B \mathbf{u}^{(q-i-1)}(t) + \sum_{i=0}^{q-1} LA^i E \mathbf{f}^{(q-i-1)}(t) \end{array} \tag{18}$$

2) Second step: The second step is to eliminate the state $\mathbf{x}(t)$ and unknown input $\mathbf{f}(t)$ from (18) so that $\mathbf{v}^{(q)}(t)$ will be expressed only with $\mathbf{v}(t)$, $\mathbf{y}(t)$, $\mathbf{u}(t)$ and their successive derivatives. Indeed, from $\mathbf{y}(t) = C\mathbf{x}(t)$ and the expression of the matrices LA^iE from (17), for $i \in [\![0; q-1]\!]$, it leads to:

$$\mathbf{v}^{(q)}(t) = \sum_{i=0}^{q} \Gamma_i \mathbf{y}^{(i)}(t) + \sum_{i=0}^{q-1} \Lambda_i \mathbf{v}^{(i)}(t) + \sum_{i=0}^{q-1} \Phi_i \mathbf{u}^{(i)}(t), \tag{19}$$

where, for $i \in [\![0; q-2]\!]$:

$$\begin{array}{rl} \Phi_i = & \Big[LA^{q-1-i} - \sum_{j=i+1}^{q} \Gamma_j CA^{j-i-1} \\ & - \sum_{j=i+1}^{q-1} \Lambda_j LA^{j-i-1} \Big] B, \end{array} \tag{20}$$

and $\Phi_{q-1} = [L - \Gamma_q C]\, B$.

3) Third step: The third step consists in realizing the input-output differential equation (19) [15], [16], as:

$$\left\{ \begin{array}{rl} \dot{z}(t) = & Fz(t) + \begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_{q-1} \end{bmatrix} u(t) \\ & + \begin{bmatrix} \Gamma_0 + \Lambda_0\Gamma_q \\ \Gamma_1 + \Lambda_1\Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1}\Gamma_q \end{bmatrix} y(t), \\ \hat{v}(t) = & \begin{bmatrix} 0_l & \cdots & 0_l & I_l \end{bmatrix} z(t) + \Gamma_q y(t), \end{array} \right. \tag{21}$$

with:

$$F = \begin{bmatrix} 0_l & \cdots & \cdots & 0_l & \Lambda_0 \\ I_l & \ddots & & \vdots & \Lambda_1 \\ 0_l & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0_l & \Lambda_{q-2} \\ 0_l & \cdots & 0_l & I_l & \Lambda_{q-1} \end{bmatrix},$$

and the observer design is complete with I_l the identity matrix of size l.

When F is a Hurwitz matrix, it is demonstrated that (21) is an asymptotic observer of the functional linear $L\mathbf{x}(t)$. Otherwise, it becomes necessary to increase the order q and to do again the building procedure with a higher order, [17], [18].

Moreover, it is demonstrated in [19] that for $l = 1$, if q is the smallest integer satisfying (14), if F is a Hurwitz matrix, if conditions (9)-(12) are verified, then the obtained observer is of minimal order. Finally, it has to be noticed that the matrix T is determined through the proposed recursive procedure and its properties are verified *a posteriori*.

IV. Application to the Thermal System

First of all, it has to be remarked that a classical full-state unknown input observer cannot be designed considering that its design criteria is not satisfied (22) [12], [20].

$$\text{rank}(CE) \neq \text{rank} \begin{pmatrix} CE \\ E \end{pmatrix} \tag{22}$$

A. Preliminary checks

The observer will be designed to estimate the temperature at a distance of $1cm$ from the heated plate. The estimation point corresponds to the average temperature of the first and the second node in the model i.e. the functional matrix L is of size (1×16) with $l = 1$:

$$L = [0.5\,0.5\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0]$$

A thermocouple is inserted at the estimation position for the validation of the results of the observer. Moreover, as $l = 1$, the integer q defined in the procedure corresponds directly to the order of the resulting observer. Note that other locations have been considered for temperature estimation. The same results have been observed. The presented case mainly correspond to the estimation of chip temperature in power module i.e. temperature estimated as close as possible to heating source.

First of all, it has to be verified that the triplet (A, C, L) is functionally observable (6) and we get:

$$\text{rank}\left(\begin{bmatrix} \mathcal{O}_{A,C,n} \\ L \end{bmatrix}\right) = \text{rank}\left(\mathcal{O}_{A,C,n}\right) = 16.$$

In a second step, it is verified that the system has no unstable transmission zeros from unknown input to output. To do that, transfer function is obtained with:

$$G(s) = C(sI_n - A)^{-1}E$$

Numerator of $G(s)$ is of seventh order and all its roots are of strictly negative real parts.

Considering preliminary checks, it is concluded that a LUIFO can be designed. Moreover, as the system has 7 detectable invariant zeros, the LUIFO will be at least of seventh order and these zeros will appear as poles of the observer [21].

B. Design of a minimal-order observer

First of all, the minimum integer g that verifies the condition in (14) is looked for. All positive integers from 1 are iteratively tested. For $q = 1$, we get $\text{rank}(\Sigma_1) = 3$ and $\text{rank}\left(\begin{bmatrix} \Sigma_1 \\ LA \quad LD \end{bmatrix}\right) = 4$ and a first-order minimum observer cannot be designed. For $q = 2$, we get $\text{rank}(\Sigma_2) = 5$ and $\text{rank}\left(\begin{bmatrix} \Sigma_2 \\ LK_2 \end{bmatrix}\right) = 6$ and a second-order minimum observer cannot be designed.

For $q = 7$, we get $\text{rank}(\Sigma_7) = 15$ and $\text{rank}\left(\begin{bmatrix} \Sigma_7 \\ LK_7 \end{bmatrix}\right) = 15$. Thus the seventh order observer is a candidate as LUIFO.

From the unique solution $X = LK_7\Sigma_7^{\dagger}$, we obtain Λ_i and Γ_i parameters. That yields to:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -8,85.10^{-13} \\ 1 & 0 & 0 & 0 & 0 & 0 & -6,41.10^{-10} \\ 0 & 1 & 0 & 0 & 0 & 0 & -1,34.10^{-7} \\ 0 & 0 & 1 & 0 & 0 & 0 & -1,24.10^{-5} \\ 0 & 0 & 0 & 1 & 0 & 0 & -5,88.10^{-4} \\ 0 & 0 & 0 & 0 & 1 & 0 & -1,49.10^{-2} \\ 0 & 0 & 0 & 0 & 0 & 1 & -0,193 \end{bmatrix}.$$

As expected the eigenvalues of F are the detectable zeros of the systems. Consequently, all eigenvalues of F are of strictly negative real parts and F is a Hurwitz matrix.

Moreover, to design the seventh-order observer we get using (21):

$$G = \begin{bmatrix} 7,67.10^{-13} \\ 4,17.10^{-10} \\ 6,47.10^{-8} \\ 4,3.10^{-6} \\ 1,39.10^{-4} \\ 2,16.10^{-3} \\ 0,0129 \end{bmatrix}, \; H = \begin{bmatrix} 1,18.10^{-13} \\ 2,29.10^{-12} \\ 4,96.10^{-20} \\ 1,02.10^{-18} \\ 9,39.10^{-18} \\ 2,35.10^{-16} \\ 1,99.10^{-15} \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, V = -1.91.10^{-16}.$$

The observer design is complete. Matrix T is computed and conditions (9) to (12) are verified. It is then concluded that the candidate is the minimal order LUIFO for the system.

As it can be seen, the poles of the observer are completely determined and its dynamic cannot be set. In order to modify the dynamic of the observer, it is necessary to increase the integer q and design a new observer with some degrees of freedom that allows us to tune its eigenvalues [19].

V. Experimental results

A. Experimental setup

The experimental setup is shown in Fig. 3. Only thermal insulator is not depicted.

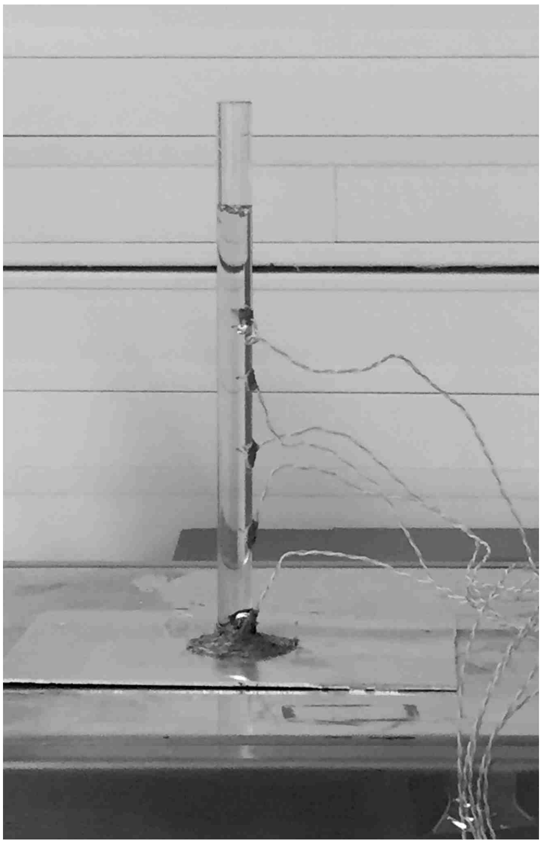

Fig. 3. Experimental setup

B. Experimental results

As the estimated temperature is obtained by integration of (21), initial conditions of the state vector $z(t)$ may have an influence on the the results. Consequently, identical initial conditions between the system and the observer are considered to check convergence properties and different initial conditions are considered to study the dynamic of the observer.

1) Identical initial conditions: First of all, experimental measured temperature at a distance of $1cm$ from the heated plate is compared in figure 4 with the simulated temperature at the same position using thermal model in (**??**). This simulation shows a good accuracy of the model regarding the experimental system. Note that the dissipation heat flux is known for simulation through its measure in experimental environment. Moreover, figure 4 shows that the designed LUIFO allows us to asymptotically estimate the temperature with identical initial conditions.

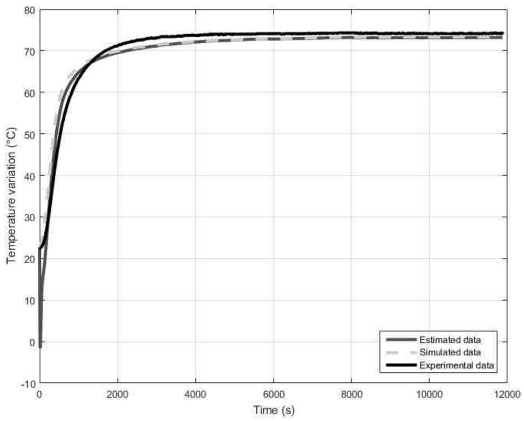

Fig. 4. Estimation results with identical initial conditions

2) Different initial conditions: In this section, the given local temperature is estimated using previously designed observer with arbitrary initial conditions. First of all, it has to be noticed that the decay rate of the estimation error in figure 5 is consistent with the greater time constant of the observer. Moreover, an erratic behavior of estimated temperature can be seen at the very beginning of the curves. This is due to unstable transmission zeros in the transfer function of the observer from the measure $\mathbf{y}(t)$ to the estimated output $\hat{\mathbf{v}}(t)$. In order to avoid this behavior, unstable zeros must be compensated with the poles of the observer. To do that, poles of the observer must be tuned using degrees of freedom obtained by increasing the order of the observer.

VI. Conclusion

In this paper, the design of LUIFO has been presented and the corresponding implementation procedure has been given. This kind of observer induces a relevant reduction in the observer order comparing to the initial system dimension. It has been demonstrated, using experimental data, that the observer was able to accurately estimate the temperature evolution of a desired location in the considered system.

Finally, with this study, it is demonstrated that in a power module, knowing heating sources related to Joule losses, the temperature and then the thermal constraint on materials and elements such as power chips can be accurately estimated using few sensors and without knowledge of the environment such as dissipation heat flux.

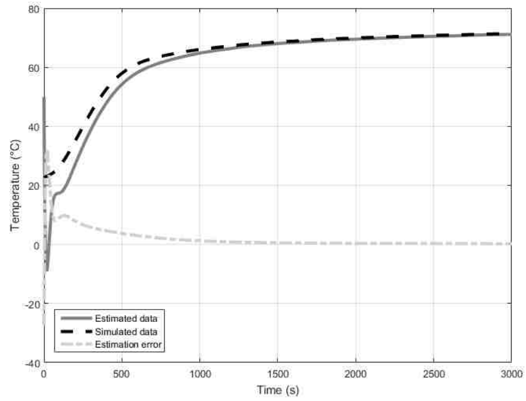

Fig. 5. Estimation results with different initial conditions

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