An exploration of learners' autonomous learning of mathematics by using selected Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) video clips: A case study

A full thesis in fulfilment of the requirements for degree of

MASTERS OF EDUCATION

(MATHEMATICS EDUCATION)

of

RHODES UNIVERSITY

by

THOMAS HAYWOOD

December 2015

ABSTRACT

One of the major problems in the achievements of learners in mathematics is the difficulty they experience in performing tasks involving higher level thinking skills which are developed through autonomous learning behaviours (Karp, 1991). Thus, to engage meaningfully in high level mathematical tasks, one should be able to work independently (Karp, 1991). Teachers therefore should support learners in developing the skills that will afford them the opportunity to manage their own learning outside the sheltered surroundings of the classroom, when the teacher is no longer there for support (St. Louis, 2003).

A study was undertaken with 11 Grade-10 learners to ascertain how their engagement with the VITALmaths video clips support and improve the learners' understanding of the Pythagorean Theorem and the addition and subtraction of fractions autonomously. The VITALmaths database of video clips, which consists of short video clips (1 - 3 minutes long) was developed collaboratively by students and researchers at the School of Teacher Education at the University of Applied Sciences North-Western Switzerland and Rhodes University in South Africa (Linneweber-Lammerskitten, Schäfer & Samson, 2010). The video clips, which are freely available, can be downloaded on mobile phones.

The study was structured into four different phases during which data was collected and analysed using both quantitative and qualitative methods. I specifically looked at the learners' use of manipulatives during their learning of the Pythagorean Theorem and the addition and subtraction of fractions, whether there was a growth of a discourse-for-oneself and whether or not their engagement with the video clips enhanced the learners' understanding of the Pythagorean Theorem and the addition and subtraction of fractions.

While the theoretical framework provided a sound basis for researching autonomous learning, it required a considerable effort to determine whether the participants showed growth in terms of moving from a discourse-for-others to a discourse-for-oneself.

The study revealed that the learners' engagement with the VITALmaths video clip encouraged the use of manipulatives in their learning of the Pythagorean Theorem and the addition and subtraction of fractions. The majority of the learners involved in the study showed a growth of a discourse-for-oneself. A number of the learners showed an

enhancement in their understanding of the Pythagorean Theorem and the knowledge involved in the addition and subtraction of fractions.

The overall findings showed that mobile technology can easily be incorporated in the learners' learning of mathematics. The VITALmaths video clips can play a significant role in the learners' autonomous learning and understanding of certain mathematical concepts.

ACKNOWLEDGEMENTS

There are a number of people without whose support I would never have been able to successfully complete this project. To the following people I extend my heartfelt gratitude:

My supervisor, Professor Marc Schäfer, whose guidance, patience and dedication to the VITALmaths project made me persevere.

Professor Helmut Linneweber-Lammerskitten from the North-Western University of Switzerland and his wife Anne for hosting me during my memorable trip to Biel/Bienne in Switzerland to learn more about the VITALmaths project.

Dr Duncan Samson, whose initial involvement with the project played an important role in my interest in getting involved with the VITALmaths project. He was also involved in supporting me during the writing of my research proposal and uploaded the VITALmaths video clip on the mobile phones for my research participants.

My colleagues at Rhodes University Mathematics Education Project (RUMEP) for their unwavering support. I could approach them anytime if I was uncertain about aspects of my research project.

Percy Brooks whose computer skills I could tap into whenever my computer skills let me down.

My wife Marilyn and children Tammy-Lynn and Simonè who gave their time and support in order to allow me the freedom to sit and work on my research project.

DECLARATION OF ORIGINALITY

I, Thomas Haywood (Student number 00h0003) declare that this thesis entitled: "An Exploration of Learners' Autonomous Learning of Mathematics by using Selected Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) Video Clips: A Case Study", is my own work and written in my own words. Where I have drawn upon the words or ideas of others, I have acknowledged the author/s by using the reference practices as set out by Rhodes University Educational Guide to referencing.

……Haywood…… 01/12/2015

Thomas Haywood (Signature) Date

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION AND CONTEXT

1.1 INTRODUCTION

I am a lecturer at Rhodes University Mathematics Education Project (RUMEP: www.ru.ac.za/rumep) a non-governmental organisation, that runs Rhodes University accredited Bachelor of Education (In-service) courses for teachers from rural areas of South Africa. Most of the teachers that attend the courses are either under-qualified or they do not have a qualification to teach mathematics. RUMEP's mission is to develop innovative mathematics teachers and is committed to developing learning materials to support this mission. I was thus particularly interested in the VITALmaths (www.ru.ac.za/vitalmaths/) video clips as a medium of teaching and learning. The VITALmaths video clips are a databank of videos which are developed by the FRF Mathematics Chair (Rhodes University) and the North-Western University of Switzerland. The video clips unpack visually a variety of mathematical concepts and include topics such as the Pythagorean Theorem, addition and subtraction of fractions, patterns, symmetry that is generated through tiling activities, various results from elementary number theory, interior angles of polygons, equivalence of different are formulae, probability activities and the distributive law to name but a few (Linneweber-Lammerskitten, Schafer & Samson (2010). New video clips are also added to the database on a regular basis. The video clips are very short (1 - 3 minutes long) and they are developed by using natural materials. The Rhodes University website is used to house the growing databank of video clips. Some of the video clips have also already been made available through the You Tube platform. Linneweber-Lammerskitten et al (2010) are especially interested in making use of mobile phone technology as the primary distribution platform. Although the text in the video clips is in English, German and isiXhosa, the video clips are self-explanatory and require minimal instruction. The video clips are also aesthetically delightful and after they have been uploaded onto a mobile phone, they are ready to be observed and used in the learners' learning of the mathematical concepts. As stated in the Curriculum Assessment Policy Statement (CAPS, 2010), it provides opportunities for learners to communicate their mathematical ideas effectively by using visual, symbolic and/or language skills in various modes. According to Linneweber- Lammerskitten et al. (2010) the use of the video clips does not require learners to have much mathematical knowledge, nor do they make unnecessary intellectual demands that could lead to frustration. Nonetheless, "the video clips encourage genuine mathematical exploration that transcends the mere mathematical content of the film by encouraging a desire to experiment, use trial-and-error, formulate conjectures, and generalize results" (Linneweber-Lammerskitten et al., 2010 p. 355). The above mentioned statement ties in well with the general principles in the South African Curriculum and Assessment Policy Statement (CAPS) (2010), which suggests that investigations should provide opportunities to develop the learner's ability to be methodical, to generalize, make conjectures and try to justify or prove them. The CAPS principles (2010) also asserts that learners need to reflect on the processes and not only be concerned with getting the answers.

I found through my mathematics teaching experiences that learning procedures and proofs without a good understanding of why they are important leave learners ill-equipped to use their knowledge later in life. Mathematical modelling, as suggested in the CAPS (2010) should thus be an important focal point of the learners' learning of mathematics. The VITALmaths video clips provide unique opportunities to model mathematical concepts such as the proof of the Pythagorean Theorem and the addition and subtraction of fractions, which is the focus of my study. Although these topics are covered extensively in the Intermediate Phase and Senior Phase in South African schools, I found it necessary to revisit the topics in Grade-10 by using a different approach. In order to consider an alternative approach to teaching these topics, it was necessary to examine the theories underpinning autonomous learning, social constructivism and the use of mobile technology in the learning and teaching of mathematics. This resulted in me not only having to engage in what it entails to be an autonomous learner and how social constructivism influences the autonomous learning of mathematics, but also how mobile technology can be introduced in the teaching and learning of mathematics.

Reading for this Degree has created an appreciation for the amount of research that has gone into education, but it has also made me realise how little seems to translate to importance of autonomous learning of mathematics by using mobile technology. This is due mainly, I believe, to a lack of either awareness of the potential of mobile technology as a teaching and learning resource for the teaching of mathematics, or a lack of resources to apply such research to practice. It was this lack of understanding on my part with regard to how research can be used to inform practice using mobile technology, coupled with my interest in the VITALmaths video clips that resulted in the idea of how the VITALmaths video clips combined with mobile technology could be used in the autonomous learning of mathematics.

The research orientation of my project is underpinned mainly by an interpretivist paradigm whereby I am committed to understanding the phenomena that I am researching and interpreting within the social and cultural context of the participants. This implies a mostly qualitative research approach in which I employed an in-depth case study research design. Although my case study shed light on my specific experiences in the learners' engagement with the VITALmaths video clips, it forms the basis for a broader understanding of learning with the VITALmaths video clips using mobile technology. My contribution provides rich evidence as to why it is important to consider the use of the VITALmaths video clips in conjunction with mobile technology in the autonomous learning of mathematics.

Quantitative approaches were also employed in the data that required statistical analyses. In my research projects a pre- and post-test design has been employed. Furthermore, the research design contains elements of action research, whereby the findings of the research continuously fed into the refinement of the design of newly developed video material. The resulting databank of videos that can be uploaded onto mobile phones thus continues to grow on the basis of this research thereby ensuring sustained relevance.

1.2 CONTEXT OF RESEARCH

1.2.1 The South African Curriculum

Stenhouse (1975, p.4) defines the curriculum as "an attempt to communicate the essential principles and features of an educational proposal in such a form that it is open to critical scrutiny and capable of effective translation into practice". Curriculum is the foundation of the teaching and learning process. Furthermore, the development of study programs, teaching and learning resources, lesson plans, assessment of learners and teacher education are all grounded in the curriculum (De Pree, 1987). Curriculum and curriculum development are thus of critical concern to educators, government and parents, and they have relevance and impact on the development of communities and their prosperity (De Pree, 1987).

In the past sixteen years the South African curriculum has gone through four curriculum changes. The implementation of the first post-apartheid curriculum was introduced in 1998 (Curriculum 2005). According to the South African Department of Education (2003), the curriculum was intended to rebuild a fragmented society and thus included new reform terminology. This curriculum leaned heavily on the constructivist learning theories developed by theorists such as Jean Piaget, Ernest von Glasersfield and Lev Vygotsky (Brodie, 2010). Learning according to these theories implies that new experiences are linked to and integrated into existing knowledge (Bodner, 1986). Teachers thus needed to set learning experiences that would guide learners in their active development of understanding (Bruner, 1990). This implementation of the curriculum was unfortunately poorly communicated to teachers. The misunderstandings and incorrect application of policies resulted in teachers not knowing what to teach and how to teach in order to effectively achieve the aims of the curriculum, which resulted in the failure of the curriculum.

In 2004 the National Curriculum Statement (NCS) was implemented in Grades 0 – 9. This curriculum however did not provide for a new plan for Grades 10 – 12, which forced the Department of Education (DOE) to revisit the curriculum in order to provide continuity (National Curriculum Statement, DOE, 2008). A lack of clarity and a lack of teacher training for the effective implementation of the curriculum led to a Revised National Curriculum Statement being issued in 2009.

This curriculum provided a comprehensive view of education from Grade-R to Grade-12. The problem with the implementation of the new Revised National Curriculum Statement however seemed to continue. Teachers were focussing on learning the new terminology and categories into which the content had been organised in order to report on the assessment of the learners' learning. As part of RUMEP's teacher training, we spent hours teaching the new terminology and categories into which the content had to be organised. During classroom support visits it was clear that the majority of teachers was still struggling with the organization of the categories. Another problem with the curriculum was that the Grade-12 Mathematics examination consisted of three papers instead of the normal two core papers. The third paper, which was optional, consisted of Euclidian Geometry and probability. Most of the teachers thus did not bother to teach these two topics. I found during my Further Education and Training (FET) classroom support visits that these two topics were not only neglected in Grade-12, but also in Grades-10 and 11.

In 2009, after reviewing the Revised National Curriculum statement, the DOE recommended yet another curriculum change, namely the Curriculum and Assessment Policy Statement (CAPS) (DOE, 2009b). The CAPS document was published for comment in 2010 and was implemented in Grade-10 in 2012. In this curriculum the DOE reverted back to subjects instead of learning areas. The terms, outcomes and assessment standards were no longer used. Instead, the CAPS document specified content topics (knowledge and skills). The intention was for the first Grade-12 examination for CAPS to be written in 2014. In this curriculum Euclidian Geometry and probability were again compulsory at FET level and formed part of the core examination papers.

1.2.2 Grade-10 Mathematics and the Curriculum

The main aim of the mathematics curriculum at the FET level is to ensure that learners acquire and apply knowledge and skills in ways that are meaningful to their own lives (DOE, 2011). The curriculum also encourages active and critical learning, which means that CAPS is underpinned by constructivist learning theory. Teachers should promote accessibility of mathematical content to all learners.

Teachers should thus employ different strategies to allow learners access to mathematical knowledge and skills (DOE, 2011).

I found through my interaction with Grade-10 mathematics teachers at RUMEP that a good curriculum policy or regular curriculum changes do not necessarily guarantee the improvement of teaching and learning. Mathematics teachers have become change-weary and are struggling to cope with the demands of all the curriculum changes, which have an effect on their teaching practice and the learners' learning of mathematics. The teachers that I dealt with have stopped being innovative and depend solely on text books. My study will thus support teachers in finding innovative ways of affording learners opportunities to access mathematical content.

1.2.3 The Northern Cape Province and Mathematics Education

According to Census South Africa 2011 the South African population stood at 51.77 million people. Although the Northern Cape Province (NCP) is the largest province in South Africa, it has the country's smallest population that is 1.15 million people which equates to 2.2% of the country's total population. The population density is three people per square kilometre. Just over 50% of the population in the NCP speaks Afrikaans, with other languages being Setswana, IsiXhosa and English (SAinfo, 2012).

The John Taolo Gaetsewe District Municipality, the geographical location of this research, one of five district municipalities in the NCP, consists of 15 towns and villages (Northern Cape Department of Cooperative Governance and Traditional affairs, 2012). The majority of the people in this district municipality live in rural areas that have backlogs with regard to basic infrastructure (Northern Cape Department of Cooperative Governance and Traditional affairs, 2012).

Herselman (2003) writes that many of South Africa's rural areas exist below subsistence levels and remain impoverished because they have little access to basic infrastructure essentials such as water, proper sanitation and learning resources for economic growth and development. This was evident during the Rhodes University Mathematics Education Project (RUMEP)'s visit to schools in the John Taolo Gaetsewe District Municipality areas of the NCP. (Thirty-three teachers from the NCP registered for a B.Ed. (In-service) in mathematics course through RUMEP. The course is fully funded by SISHEN Iron Ore Company in the NCP). It was thus not surprising that violent service delivery protests erupted in the Johan Taolo Gaetsewe District Municipality during 2012. According to Lesufi (spokesperson for the Department of Basic Education) (2012), three schools were burnt down, 64 schools were closed and 16 000 learners were unable to attend school.

Although the NCP's pass rate increased from 68.8% in 2011 to 74.6% in 2012 (Gernetzky & Magubane, 2013), Northern Cape Department of Education officials from both rural and urban schools have identified mathematics as a problem subject during interviews with RUMEP. This has been confirmed by the National Education Department's Examiners' Report (2012), which states that mathematics and science are still the subjects that raise concerns among educators from all provinces.

1.3 OBJECTIVES OF THE RESEARCH

1.3.1 Research Purpose and Goals

The main purpose of this study was to ascertain how eleven Grade-10 learners experience the autonomous use of selected VITALmaths video clips, which incorporated animated manipulatives, in their study of the Pythagorean Theorem and the addition and subtraction of fractions.

The underlying goals of this research is to explore the following:

- Does the use of the VITALmaths video clips in conjunction with specially prepared worksheets specifically encourage: a) the use of manipulatives in their learning of the Pythagorean Theorem and the addition and subtraction of fractions, b) the growth of a discourse-for-oneself?

- Does the use of the video clips enhance the learning of the Pythagorean Theorem and the addition and subtraction of fractions?

1.3.2 Research Participants and Site

The study involved eleven Grade-10 learners from a high school in the Northern Cape Province, South Africa. The Mathematics teacher of these learners was doing a B.Ed. (In-service) course through RUMEP. The collection of the data was done after school hours at the learners' school.

Permission was sought from the Headmaster of the school and the Northern Cape Department of Education to conduct the research at the school (Appendix 1B and 1C). Consent was also obtained from the parents of the learners that were involved in the study (Appendix 1D). The consent forms for the Headmaster, Northern Cape Department of Education and the parents of the participating learners also included an information sheet that explained the structure of the study (Appendix 1A).

1.4 THE STRUCTURE OF THE THESIS

The remaining chapters of the thesis are presented as follows:

Chapter 2: The literature review, which discusses the educational theories informing the theoretical framework and design of the study undertaken. Different ideas informing autonomous learning, social constructivism, the use of manipulatives and mobile phones to enhance the learners' understanding of mathematics are explored.

Chapter 3: The focus of this chapter is the methodology used to collect data during the participants' engagement with the VITALmaths video clips and their subsequent presentations after their engagement with the video clips.

Chapter 4: The analysis chapter where I collate, present and discuss the results that were obtained from the methods described in the methodology chapter. These results are analysed by looking at the participants' experiences in engaging with the VITALmaths video clips that were downloaded on mobile phones and whether this engagement had an influence on their understanding of the Pythagorean Theorem and the addition and subtraction of fractions.

Chapter 5: Concludes the thesis by giving the most important findings, exploring some of the possible implications of these findings and making recommendations based on these findings.

CHAPTER 2: LITERATURE REVIEW

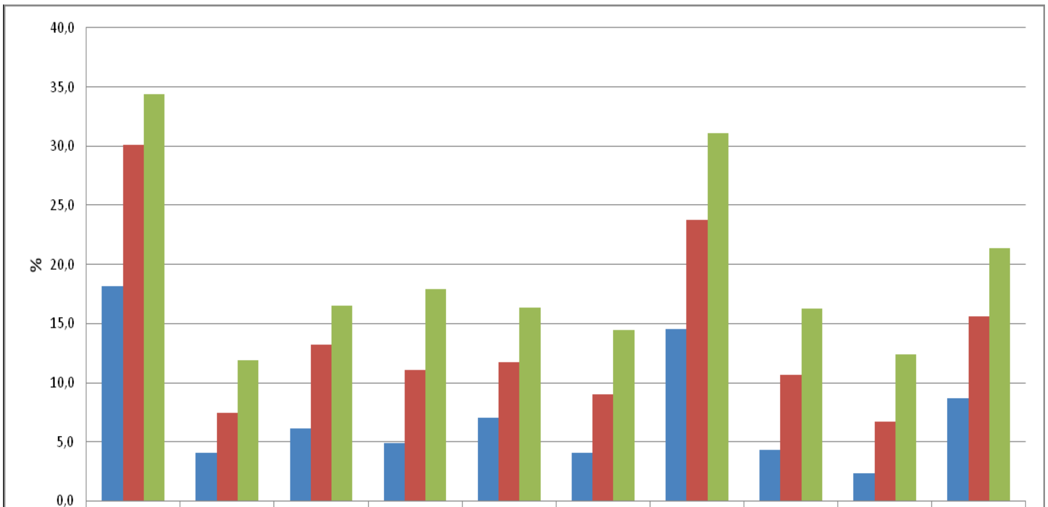
2.1 INTRODUCTION

It is self-evident that what children learn is heavily dependent on what teachers know and do in their classrooms. This is particularly true for children who receive little support for schoolwork at home and little intellectual stimulation in their broader social environments (Taylor, 2008). This is particularly evident in poor areas. I found through my interaction in schools that in successful institutions not only is punctuality observed during the school day, but additional teaching time is created outside of normal hours to support learners who are struggling with certain subjects. Added to this is that two factors are commonly associated with improved performance in schools, that is, reading and homework (Taylor, 2008). However, homes in rural areas are often ill-equipped to meet the educational demands of learners due to the lack of basic facilities, like electricity. Furthermore, parents in rural areas are less likely to be educated themselves and thus might have less ability to support their children with education at home (Mulkeen, 2005). I thus concur with Elmore & Fuhrman (2001) who write that in order to improve school performance, schools, especially in rural areas, "must do different things and not do the same things differently" (p. 6). One way of doing things differently and potentially lessen the above difficulties would be to introduce learners to technology in and out of school. This is the digital age and technology allows learners to engage positively in subjects in which they lack confidence (Isaac, 2002).

2.2 MOBILE TECHNOLOGY IN EDUCATION

Although half of the more than 50 million people in South Africa live below the poverty line, more than 75% of the low income group that is above 15 years old, own a mobile phone (Peyper, 2013). Furthermore, 98.5% of this low income group have a pre-paid SIM card with only a small percentage having contracts. Mobile phone usage is the main form of voice and data communication among low income users and for informal businesses. There is however a clear difference between urban and rural mobile phone users in the low income category (Peyper, 2013). I concur with Peyper (2013) who writes that urban mobile phone users seem to be more knowledgeable about applications on their mobile phones and use these applications to communicate with friends, browse the internet, watch and download videos and download music. Rural mobile phone users, on the other hand, seem more sceptical and in some cases even suspicious about the value of these mobile applications. They rather rely on traditional media, such as radio, television and newspapers. They do however use SMS text to communicate via their mobile phones (Peyper, 2013).

Figure 2.1 shows that although there has been an increase in the number of people with access to computers from 2007 to 2011 in South Africa, only 25.9% of households in the Northern Cape Province have access to computers. The graph below shows the percentage of households per province that have access to computers (Statistics South Africa, 2011).



	Western Cape	Eastern Cape	Northern Cape	Free State	KwaZulu-Natal	North West	Gauteng	Mpumalanga	Limpopo	South Africa
2001	18,2	4,1	6,1	4,9	7,0	4,1	14,5	4,3	2,4	8,6
2007	30,1	7,5	13,2	11,1	11,7	9,1	23,8	10,7	6,7	15,6
2011	34,4	11,9	16,5	17,9	16,4	14,4	31,1	16,3	12,4	21,4

Figure 2.1: Percentage of households that have a computer in working order by province.

Source: Census 2001, 2011 and Community Survey 2007 (Statistics South Africa, 2012)

Furthermore, only 26% of households in the Northern Cape Province had access to the internet in 2011. Figure 2.2 below shows the percentage of households with access to the internet in South Africa per province (Statistics South Africa, 2012).

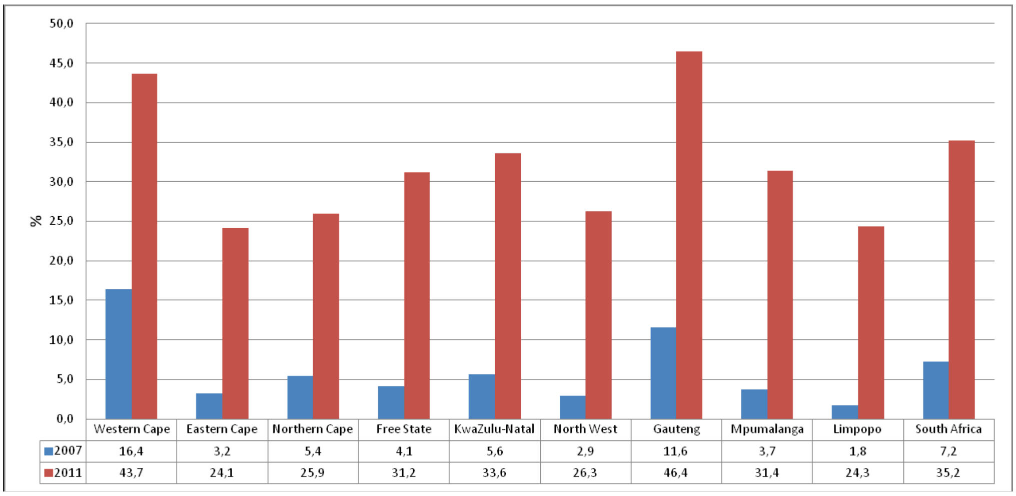


Figure 2.2: Percentage of households with access to the internet:

Source: Community Survey 2007and Census 2011 (Statistics South Africa, 2012)

Mobile phone usage in the Northern Cape Province, however, has increased from 24.5% of households in 2007 to 81.1% in 2011. Figure 2.3 below shows the percentage of households per province that have mobile phones according to Census South Africa 2011 (Statistics South Africa, 2012).

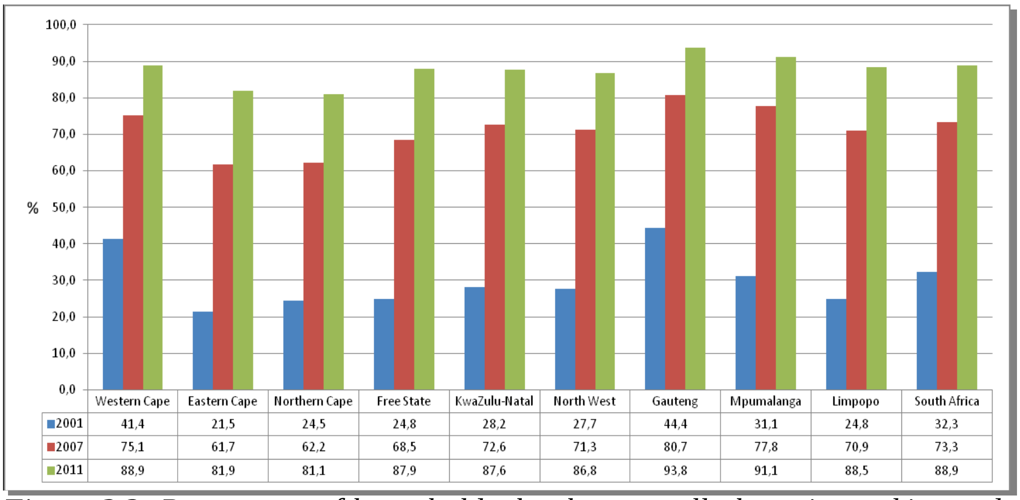


Figure 2.3: Percentage of households that have a cell phone in working order by province:

Source: Census 2001, 2011 and Community Survey 2007 (Statistics South Africa, 2012)

Aljohani, Davis and Tiropanis (2011) write that the swift growth in mobile communication technologies has allowed people to no longer be restricted in their interactions and communication through geographical positioning. They are also able to access and download endless varieties of data on mobile devices from any location (Hyde, 2011). The use of mobile phones thus may provide an alternative for engaging in mathematical concepts and ideas, especially out of normal school hour learning in the rural areas of the NCP.

According to Lenhart, Ling, Campbell & Purcell (2010) 24% of learners in South Africa attend schools where mobile phones are banned, while 62% were permitted to have their mobile phones at school but were not allowed to use them in their classrooms. Hyde (2011) further writes that only 12% of learners were allowed to have their mobile phones at school without restrictions. Ormiston (2013) argues that regardless of the school's mobile phone policy, the reality is that all students carry mobile phones with them, so why not use these tools for "good rather than evil". Koebler (2011) asserts that schools are supposed to prepare learners for real life and in real life people use mobile phones. It thus makes sense to use mobile phones for teaching and learning especially where there is a lack of the latest technology such as computers with internet connectivity. This lack is evident in the Northern Cape Province if you consider Figure 2 above (only 26% of households in the Northern Cape Province have access to the internet). Another reason to rethink the mobile phone debate is that mobile phone usage can be extended beyond the walls of the school or the confines of the classroom period and promote autonomous learning (Ormiston, 2013).

2.2.1 Mobile Technology in General

Figure 2.4 below shows a comparison of household goods in working order from 2001 to 2011. Mobile phones in working order in South African households increased from 31.9% in 2001 to 88.9% in 2011 compared to computers that increased from 8.5% in 2001 to 21.4% in 2011. The majority of South Africans do not have the advantages of internet access and with the increase in mobile phone usage, landline usage decreased from 23.9% in 2001 to 14.5% in 2011 (Statistic South Africa, 2011).

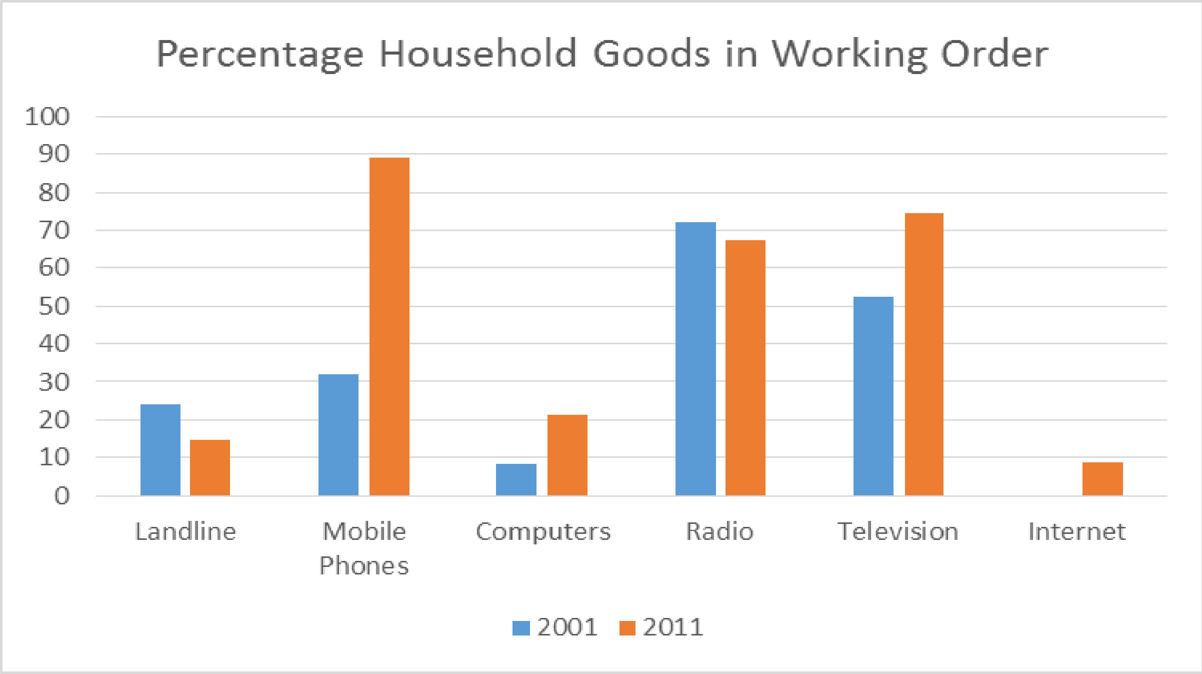


Figure 2.4: Household goods in South African homes

Source: Statistics South Africa – Census (2011)

More Africans have access to mobile phones than to clean drinking water. In South Africa, the continent's strongest economy, mobile phone usage has increased from 17% of adults in 2000 to 76% in 2010 (Hutton, 2011). Hutton's (2011) mobile insight study that was conducted in South Africa, which examined consumers' usage of and attitudes towards mobile phones, found that most South Africans are loyal to their mobile networks because 95% of mobile subscribers have been with their mobile networks for more than 4 years. Between 82% and 85% of mobile users are on pre-paid plans rather than contracts. However, 25% of mobile users said that they would switch from pre-paid to contract packages within the next year. Mobile phones as an internet device are also on the increase in South Africa and the most popular social media platforms in the country are Facebook and Mxit (Hutton, 2011). Access to these social media platform services, however, is not free on any of the mobile networks. The use of mobile phones in South Africa thus is not cheap.

Calandro, Gillwald & Stork (2012) write that mobile prices are cheaper in more than 30 countries than what it is in South Africa. Furthermore, South African pre-paid services are three times more expensive than the pre-paid services of

Namibia. South Africa is also only ranked 32nd out of 46 countries for which mobile pricing data was available on the web for pre-paid mobile users. The South African mobile data pricing is however 3.6 times more expensive than the cheapest product in its smaller neighbouring country, Namibia (Calandro et al, 2012).

2.2.2 Mobile Technology in Mathematics Education

Although there is an increasing dependence of mobile phones in most of the people's daily lives, the use of mobile phones in education in South Africa is still in its infancy. This is not only true for education in general, but it is especially true for the use of mobile phones in mathematics education (Baya'a & Daher, 2009). This was confirmed during interviews conducted with the participants of my study to ascertain whether anyone of them had ever used mobile phones in their study of Mathematics. Although 9 of the 11 participants owned a mobile phone and the other two had access to a mobile phone, none of them had used it for their study of Mathematics.

The increasing usages of mobile phones, especially amongst the young generation, offer new possibilities and opportunities for education. The beauty of a mobile phone is that it is always there, because people seldom leave their homes without their mobile phones. Cooney (2014) lists five benefits for using mobile technology in the classroom. That is:

- Mobile devices encourage learners to learn anywhere and anytime because they can process information inside and outside the classroom;
- Mobile technologies are fairly inexpensive and can reach underserved learners with limited incomes. In the case of the VITALmaths video clips, learners do not need airtime or a SIM card to access the video clips once the clips have been downloaded on their mobile phones;
- Mobile devices teach learners social skills that are necessary for success in the twenty-first century;
- Mobile devices are small which make it easy for use within the learning environment and;

- Mobile devices can be customized in many different ways. This provides learners with personalized educational experiences.

Mobile phone integration with its diversified mobile features can thus also be used to build mathematical knowledge. Furthermore, mobile phones do not only make a contribution to dynamic mathematical applications, but they can support the execution of mathematical tasks that are closer to the learner's experiences. This has the potential to improve experiential learning (Baya'a & Daher, 2009). I concur with Baya'a & Daher (2009) who assert that in order to help educators know what factors influence the learner's learning of mathematics using mobile phones, they need to understand the learner's perceptions of learning mathematics using mobile phones. This will go a long way in motivating learners to do the mathematical learning successfully and with enjoyment. Eble (1994) cited in Baya'a & Daher (2009) argues that learners understand and apply studied materials better when they are engaged in real-life issues and situations. The nature of mobile technology ensures that real-life contexts are immediately accessible. Mobile phones can thus facilitate the learning of mathematics by providing simulations of real-life context in order to simplify and illustrate it for the learners who have to solve complex authentic mathematical problems. Mobile phones extend the learning environment in which learners work and integrate it in real-life situations, through modelling, where learning can occur in real-life context (Baya'a & Daher, 2009).

Considering the above mentioned advantages in using mobile technologies in education, Cooney (2014) also lists challenges that both teachers and learners might face in using mobile technologies. That is:

- Mobile devices may contribute to unethical behaviour in learners;
- It can be a distraction in the classroom;
- It may compromise the physical health and privacy of learners;
- Most parents and teachers consider mobile phones to be a distraction in schools;

- The diverse nature of mobile technologies presents a major challenge for both teachers and learners in education and;
- Some mobile technologies' poor design and usage limitations may affect learning adversely.

Barring all the above-mentioned challenges, people need to realize that mobile technologies are unlikely to depart from the learners' lives any time soon. The educational possibilities of these devices can thus not be ignored (Cooney, 2014).

2.3 MANIPULATIVES AND ANIMATIONS

My experiences in visiting in-service teachers in their classrooms is consistent with Morris' (2013) view that mathematical concepts should not simply be taught as a concept within the classroom environment, but should be connected with authentic, real-life experiences that will support the learner in acquiring the conceptual knowledge and skills. One way of exposing learners to real-life situations within mathematics is through the modelling of the concepts. This will encourage learners to use models so that solutions to problems are scaffolded, visualized and reflected upon (Morris, 2013). According to Durmus & Karakirk (2006) cited in Morris (2013) "mathematical modelling is used to understand, explain, to describe and to predict the different aspects of the real world" (p. 118). Learners should thus be exposed to different forms of modelling. One way of exposing learners to different forms of modelling is through the use of concrete or virtual manipulatives. The knowledge acquisition process and an authentic learning context can be attained by incorporating virtual manipulatives with closely related mathematical information on the handheld mobile device (Baya'a & Daher, 2009).

Boggan, Harper & Whitmire (2010) describe manipulatives as physical objects that teachers use to support learners in their active learning of mathematics. Many different civilizations, since ancient times, have used manipulatives to support them in solving mathematical problems. Manipulatives are either bought from stores, brought from home by the teacher or learner, or made by the teacher or the learner. A good manipulative should bridge the gap between formal mathematics and informal mathematics (Boggan et al. 2010). Manipulatives enable learners to interact and touch the problem, which in turn allows the learner to construct his/her own mathematical knowledge and make connections with the real world. Furthermore, manipulatives allow the learner to be engaged in his/her own learning, which creates opportunities for the learner to develop a deeper understanding of the mathematical content that is taught Morris (2013). McNeil & Jarvin (2007) write that although some researchers suggest that manipulatives facilitate learning by activating real-world knowledge and improving knowledge through physical action, others argue that manipulatives might lead learners to having fun using the manipulatives at the expense of proper learning and its use might make learning more difficult because of dual representation. Although the efficacy of manipulative use is debated from two contrasting angles, I found through my research project that the use of manipulatives provides an additional way for conveying mathematical information and is thus facilitating learning.

According to Morris (2013) there are two types of manipulatives, which are concrete manipulatives and virtual manipulatives. Concrete manipulatives (also called physical manipulatives) are concrete objects that learners use to support their exploration of mathematical concepts by using their visual and tactile senses. Virtual manipulatives, to which my research participants were exposed to during their engagement with the VITALmaths video clips, on the other hand, are interactive tools which are visual representations of a dynamic object (McNeil & Jarvin, 2007). The visual representations in the VITALmaths video clips are animated representations of the Pythagorean Theorem and the addition and subtraction of fractions. The video clips offer learners, especially those with language difficulties, the opportunity to express their thinking about the topics with which they engaged in the video clips. The video clips, for example, afford learners the opportunity to experience a step-by-step visual representation of the Pythagorean Theorem. Hunt, Nipper & Nash (2011) write that there are many perceived advantages and disadvantages to the use of both concrete and virtual manipulatives. Some of the advantages of concrete and virtual manipulative use are illustrated in table 2.3.1 below.

Table 2.3.1: The difference between concrete- and virtual manipulatives

Concrete manipulatives	**Virtual manipulatives**
• It is simpler and more moveable than virtual manipulatives. • Tactile experience adds a dimension of learning. • Learners have more control than with virtual manipulatives. • It allows for trial and error. • It easier to relate to real-world applications. • Learners can be more creative. • It allows information to be received visually and kinaesthetically. • It clarifies misconceptions and build connections between mathematical concepts and representations, encouraging more precise and richer understanding	• Learners get immediate feedback because the learner will know when it is right or wrong. • It is a lot quicker to grasp the concept. • It offers a larger variety of experiences. • It allows more complex operations to be learned. • It catches the attention of the technology generation. • It is more accessible at home than the concrete manipulatives. • It gives step-by-step instructions, allowing the learner to see what he/she was really doing. • It keeps the learner's attention. • It often provides explicit connections between visual and symbolic representations

Source: Hunt et al, 2011 p.4

Westenskow (2012) writes that combining concrete and virtual manipulative use may bring advantages to a learner's achievement in Mathematics. The VITALMaths video clips, which consist of animated virtual manipulatives can be incorporated with concrete manipulatives to enhance the learning of certain mathematics topics. For example, during the learners' study of the addition and subtraction of fractions using the VITALmaths video clips, learners could use the dynamic virtual animated models of fractions to identify the need for equivalent fractions during fraction addition and subtraction and represent them accordingly. The learners could visualize $\frac{1}{3}+\frac{1}{4}$ and $\frac{1}{3}-\frac{1}{5}$, which according to Moyer-Packenham, Ulmer & Anderson (2012) are difficult concepts for most learners to understand. Bruner (1990) promotes the use of manipulatives in his writings, because it enables the learners to build mathematical knowledge as they progress from concrete experiences to abstract thinking in social context (McNeil & Jarvin, 2007). Social constructivism construes learning as an interpretive and a recursive building process by learners' active interactions with their physical and social world (Fosnot, 1996). As my project is anchored in the type of learning environment and conditions articulated above, it is appropriate that social constructivism informs the theoretical underpinnings of my project.

2.4 SOCIAL CONSTRUCTIVISM

Social constructivism emphasizes the significance of culture and context in understanding what occurs in society and the construction of knowledge based on this understanding (Derry, 1999). Powell & Kalina (2004) writes that Lev Vygotsky, one of the founding fathers of social constructivism, asserts that social interaction is an integral part of learning. Vygotsky developed principles and concepts for his social constructivist theories, namely:

- Learners construct their own knowledge, which means that knowledge is not transferred passively, but learners personally construct their knowledge;
- Learning is mediated, which asserts that cognitive development is not a direct result of activity. Other people must interact with the learner and use mediatory tools to facilitate the learning process in order for cognitive development to occur;

- Language plays an important role in cognitive development because it is the most significant socio-cultural tool. It can be used as a teaching tool and also a tool to express the learner's learnt understanding and knowledge;

- Development cannot be separated from its social context. The context that is needed for learning is that where the learners can interact with each other in an authentic environment. Learning should thus be extended to out of school environments (Vykotsky, 1986).

Learners thus learn more effectively when others are involved. It is common when assessing the learners' knowledge construction skills in schools, teachers try to ascertain what learners **cannot yet do**, what they **can do with help** and what they **can do alone** without the help or support from others**.** Vygotsky (1986) called the zone where learning takes place with the help from others the zone of proximal development (ZPD). When a learner for example works on an assignment with others, the achievement of the initial goal of the activity allows the growth of this zone so that he/she can eventually do more on his/her own (Powell & Kalina, 2004). The participants' engagement with the interactive VITALmaths video clips played a major role in the participants' establishment of a ZPD, because the video clips supported them in their understanding of the Pythagorean Theorem, and the addition and subtraction of fractions. The participants were then able to use what they had learned from the video clips, in problems that involved the concepts in the video clips. They were thus able to use their learnt understandings to do more on their own. Vygotsky (1986) further asserts that learning must occur in a meaningful context and it should not be separated from learning and knowledge that the learner developed in the real world. In the real world learners are exposed to current technology. This current technology is provided by their engagement with VITALmaths video clips.

Vygotsky believed that social learning preceded development. The cultural development of a learner firstly appears on a social level with other people (inter-psychological level) and then inside the learner (intra-psychological level) (Vygotsky, 1978). On the inter-psychological level, the learner interacts with the **more knowledgeable other** that might include a teacher, a coach, an older adult or the learner's peers (Vygotsky, 1978). In the case of my study the more knowledgeable other is represented by the VITALmaths video clips. Vygotsky (1978) also believed that the internalization of human tools such as speech and writing could lead to higher level thinking skills. The learner's use of speech and writing in conjunction with modern technology (VITALmaths video clips) can assist learners in expressing their mathematical skills and thinking at a higher level. The use of the VITALmaths video clips can be extended to home and other out-of-school environments, which ties in with one of Vygotsky's principles of social constructivism that states that learning should be extended to out-of-school environments (Vygotsky, 1978).

According to Gredler (1997) aspects of social context that largely influence the nature and extent of the learner's learning are: the mathematical symbol systems that the learner inherits as a member of a particular culture; how these symbol systems have an impact on the learner's learning of mathematics; and the nature of the learner's social interaction with more knowledgeable members of the society. Gredler (1997) argues that without the social interaction of the more knowledgeable others, it is difficult to acquire social meaning to these mathematical symbol systems and learn how to use them.

Gredler (1997) further explains that there are four general perspectives that inform the facilitation of learning within a social constructivism framework. That is:

- Idea-based social constructivism, where learners interpret and conceptualize ideas and meanings in mathematics by using representations instead of using rules and algorithms to solve mathematical problems;
- Pragmatic or emergent approach, which asserts that knowledge, meaning and understanding of mathematical concepts can be addressed in the classroom from both the view of the learner and the collective view of the entire class, including the educator;
- Transactional or situated cognitive perspective, which is concerned with the relationship between the people and their environment. This perspective asserts that learning should not take place in isolation from his/her environment;

- The fourth perspective that falls in the realm of my research is cognitive tools perspective, which focuses on the learning of cognitive skills and strategies through the learner's engagement in social learning activities and hands-on cognitive tools.

Bruner (1990), who was strongly influenced by Vygotsky's writings on social constructivism, theory of constructivism aligns with the cognitive domain. He viewed learners as creators and thinkers who use enquiry and the role of learning experiences in their learning. His theoretical framework is based on the notion that learners construct new ideas or concepts based on existing knowledge. Opportunities thus need to be created for learners to construct new knowledge from authentic experiences (Bruner, 1990). Bruner (1990) also introduced the idea of spiral curriculum, which refers to the revisiting of basic ideas time and time again and building on them to a level where the learner has a full understanding of the concept. This ties in well with the participants' engagement with the VITALmaths video clips, which are uploaded on the learners' mobile phones to provide them with opportunities to engage in the uploaded concepts on the Pythagorean Theorem and the addition and subtraction of fractions on a continuous basis in order to get a full understanding of the concepts. Furthermore, although these concepts are covered in the lower grades, I found through my interaction with Grade-10 and Grade-11 learners that their conceptual understanding was fundamentally poor. Revisiting the concepts in Grade-10 could develop and encourage the learners' intuitive and analytical thinking, enabling them to apply the learnt concepts in problem-solving activities that involve the concepts (Bruner, 1960).

Bruner's theory emphasized four features of instruction:

- **Predisposition to learn**, which includes the experiences that move the learner to a love for learning in general. He writes that learning and problem solving skills emerge from exploration.

- **Structure of knowledge**, which asserts that knowledge should be structured in such a way that learners are able to readily grasp the information. In order for a learner to learn a concept, he/she needs to understand the fundamental structure of concept. Bruner argued that details are better retained when placed in a structured pattern. The VITALmaths video clips lend themselves to this fundamental structure which learners can readily follow in order to grasp the information. This generated knowledge is transferable to other contexts that involve the learnt concepts from the video clips.

- **Effective sequencing**, which asserts that no one sequence of representing mathematical concepts will fit every learner. However, sequencing, or a lack of it, can make learning easier or more difficult. Categorization, which falls under sequencing, refers to conceptualization, learning, decision making and the making of inferences. Categorization is also closely related to scaffolding, which is discussed later.

- The fourth feature of instruction is **mode of representation**, which refers to the way knowledge is stored and encoded in the memory of the learner. It includes visual, word and symbolic. These three modes are closely related to Bruner's three stages of intellectual development (Bruner, 1990).

2.4.1 Bruner's three stages of intellectual development

Bruner (1990) developed three stages of intellectual development, which are, the **enactive stage**, the **iconic stage** and the **symbolic stage**. He asserts that none of these stages is age specific (Bruner, 1990). In the enactive stage learners may be able to perform a physical task better than verbally describing the very same task. During this stage knowledge is mainly in the form of motor responses (Overbaugh, 2004). Learners represent past events through motor responses. Learners are able to perform a variety of motor tasks, like operating a mobile phone, which they might find difficult to describe in iconic (picture) or symbolic (word) form. Learners should thus be allowed to play with the materials in order to fully understand how it works.

In the iconic stage the learners are able to visualize concrete information. The learners are capable of making mental images of the material and no longer need to manipulate them directly. The learner is thus able move from engaging with virtual manipulatives (the VITALmaths video clips) to developing their own concrete manipulatives to show their understanding of the mathematical concepts. It is thus often helpful to have diagrams or illustrations to accompany verbal information when learners learn a new topic in Mathematics or any other subject. In the symbolic stage, the learners can use abstract ideas to represent the world. They are able to evaluate, judge and think critically (Bruner, 1990).

The symbolic stage is the most sophisticated stage. During this stage knowledge is stored primarily as words, mathematical symbols or other symbol systems. Language thus plays an important role for the increased ability of the learner to deal with abstract concepts. Words help with the development of the concepts they represent (Bruner, 1996). Learners who operate at this stage will be able to describe the Pythagorean Theorem in words, which ties in with Vygotsky's (1986) argument that word meaning is both thought and speech. The learner, during this stage, is thus able to verbalize his/her thoughts on how the Pythagorean Theorem works.

Bruner in contrast with Piaget's stages of development argues that learners, even at a very young age, are able to learn any material so long as the instruction is organised appropriately. The instruction will dictate the stage that the learner utilizes when constructing the meaning of the concept (Bruner, 1990). Bruner (1996) further writes that educators must provide guidance and assistance throughout the stages through a process he called **scaffolding**. Scaffolding involves helpful structured interaction between an adult and a child with the aim of supporting the child in achieving a specific goal (Bruner, 1978). Bruner's thinking articulates well with a social constructivist view of learning which suggests that learning is not a result of development, but "learning is development" (Fosnot, 1996, p. 29). This requires creativity and self-organization on the part of the learner (Fosnot, 1996). The teacher thus needs to create opportunities for learners to raise their **own** questions, create their **own** hypotheses and models and test them for viability (Fosnot, 1996). Social constructivism has driven the development of educational situations which emphasise the need to encourage greater participation by learners in their requisition of learned knowledge (Larochelle, Bednarz & Garrison, 1998). Candy (1991) cited in Thanasoulas (2000) asserts that social constructivism leads to the proposition that knowledge cannot be taught but only learned because knowledge is something built up by the learner in a social milieu. Social constructivist approaches to learning thus encourage and support self-directed learning as a crucial condition for learner autonomy (Thanasoulas, 2000).

2.5 AUTONOMOUS LEARNING

2.5.1 What is Autonomous Learning?

Holec cited in Benson & Voller (1997) writes that autonomy is the ability to take charge of your own learning. They use five ways to describe autonomy, that is:

- for situations in which learners study entirely on their own;
- for a set of skills which can be learned and applied in self-directed learning; for an inborn capacity which is expressed by institutional education;
- for the exercise of learners' responsibility for their own learning and;
- for the right of learners to determine the direction of their own learning. (Benson & Voller, 1997, p1).

Autonomous learners should be willing to learn and develop a meta-cognitive capacity, which refers to the learners' automatic awareness of their own knowledge and their ability to understand, control and manipulate their own mental processes. This enables them to handle change, negotiate with others and use their learning environment strategically (St Louis, 2003). Meta-cognition falls within the following categories:

- Meta-memory, which refers to the learners' awareness of their knowledge about their own memory systems and strategies for using their memories effectively. This includes the learners' awareness of the different memory strategies, the knowledge of which strategy to use for a specific memory task and knowledge of how to use memory strategies most effectively.
- Meta-comprehension, which refers to the learners' ability to monitor the effective understanding of information being communicated to them, identify failures in comprehending the information and the employment of

repair strategies when failures are identified. A learner would for example show the proof of the Pythagorean Theorem without knowing that he/she does not have an understanding of the theorem. Learners who are able to adapt to meta-comprehension on the other hand would look for inconsistencies in the proof and would undertake corrective strategies by relating current information to prior knowledge.

- Self-regulation refers to the learners' ability to make changes to their own learning processes in response to feedback regarding their status of learning. This concept overlaps greatly with meta-memory and meta-comprehension. It focusses on the learners' ability to monitor their own learning without the influences of external interventions. The concept asserts that to learn most effectively learners should not only understand the available strategies and the purpose these strategies will serve, but should be able to select, employ, monitor and evaluate their use of the strategies. The VITALmaths for example proposes different strategies to solve the Pythagorean Theorem. During the assessment of the proof of the Pythagorean Theorem in the pre- and post-tests, my research participants should have been able to adequately select a strategy, employ the selected strategy, monitor if the strategy works and evaluate their use of the strategy to show whether or not they have learned the proof of the theorem effectively. The use of the process of proving the theorem will eventually become natural without them being aware that they are doing so, which is a prerequisite for learner autonomy (Sindhwani & Sharma, 2013).

Little cited in Thanasoulas (2000) sees autonomy as the learners' psychological relation to the learning process which includes the learning content, the learners' capacity for detachment, the critical reflection process, decision making and actions without the involvement of the knowledgeable others. Thanasoulas (2000) further asserts that autonomous learning is not simply just another teaching method, but is the learners' capacity and willingness to take charge of his/her own learning. For a learner to qualify as an autonomous learner, he/she must be able to decide on his/her own on aims, purposes and goals for learning, select learning materials, choose tasks and methods to complete the tasks, and finally decide on criteria for evaluating the tasks (Holec, 1981). The autonomous learner thus takes a pro-active role in the learning process and does not merely react to the different stimuli of the teacher (Thanasoulas, 2000). Mobile technology and the VITALmaths video clips lend themselves to supporting the learner in this pro-active learning process because it incorporates easy accessible virtual manipulatives with closely related information on their handheld devices.

Candy (1991) regards an autonomous learner as a learner who is compliant to a law that he/she prescribes to him/herself. Wenden cited in Thanatoulas (2000) explains that the main attributes characterising autonomous learners are:

- Autonomous learners have insights into their learning styles and strategies;
- autonomous learners take an active approach to the learning task at hand;
- they are willing to take risks;
- they are good guessers;
- And they attend to form as well as to content, that is, place importance on accuracy as well as appropriacy (p. 4).

Thanasoulas (2000) however argues that although these attributes are necessary, they are not sufficient conditions for the development of an autonomous learner and that factors such as learner needs, motivation and learning strategies should also be considered. Thanasoulas further argues that a person does not become autonomous but rather works towards autonomy. Autonomy thus is a "**process not a product**" (Thanasoulas, 2000, p. 4).

I thus concur with St Louis (2003) who argues that the paradigm shifts from teacher-dependent to teacher-independent is difficult for learners who have been engrossed in an education system that has largely been controlled by the teacher, who now have to let go of the control to support learners in becoming autonomous and self-sufficient. Autonomous learning however does not mean that the learner does not need the teacher's input and support. The teacher's role will only change from being the proprietor of knowledge, who transfers information, to a facilitator that supports learners in becoming autonomous (St Louis, 2003). Learner autonomy emphasizes the role of the learner rather than the role of the teacher. Learner autonomy thus transfers the focus from teaching to learning (Turloiu & Stefánsdóttir, 2011). For this reason, Bruner (1961) argues that the purpose of education is not to impart knowledge, instead it should facilitate the learner's thinking and problem solving skills which the learner can ultimately use in a range of other situations.

2.5.2 Autonomous Learning in Mathematics Education

Piaget (1948) writes that autonomy should be the goal of education. He explained this idea in the context of mathematics learning. Kamii (1994) elaborated on Piaget's definition of autonomy by explaining it as the ability to think for oneself and to decide between truth and untruth. Wood's (2008) definition of autonomous learning, in the realm of mathematics education, focuses on the learner's desire to understand experiences including both physical experiences and interactions with others. For her autonomous learning is a constellation of mathematizing and identifying activities, reflecting curiosity about how things are and what others think and say about what seems to be true (Wood, 2008). Sfard (2007) writes that mathematizing refers to an individual's participation in a mathematical discourse whether that participation is mathematically appropriate or not. The Oxford dictionary defines curiosity as "the desire on inclination to know or to learn about anything especially what is novel or strange; a feeling of interest leading one to enquire about anything". Wood's (2008) purposeful use of curiosity captures the ways in which autonomous learners compare their thinking with their observations of experiences. Wood (2008) connects curiosity with novel and strange, which she describes as the autonomous learner's curiosity to make sense of ideas that he/she does not understand. Dewey (1910) cited in Wood (2008) writes that curiosity is when the learner's interest in problems is provoked by the observation of things. Observation, however, is not enough, because the curious learner, through active participation, explores and seeks material for thought by formulating questions about those observations (Dewey, 1910 cited in Wood, 2008). He/she is not satisfied with memorization of materials or the answers to the mathematical discourse for the sake of others approval. The autonomous learner's curiosity thus allows him/her to explore problems arising from observations or the mathematical discourse (Wood, 2008). The learner investigates a mathematical discourse and ascertains whether his/her communication about the discourse is attuned with the communication of others who are proficient in the specific discourse. This corroboration ensures that the learner's interpretation of the discourse is consistent with others' (a teacher, the text book or another learner) use of the discourse (Wood, 2008). Wood (2008) further asserts that autonomous learning also involves the pursuit for mathematical truth. The autonomous learner works through a mathematical discourse and looks for contradictions within the mathematical discourse or between different mathematical discourses. The learner then tries to resolve any contradictions by changing their thinking or by proposing alterations or additions to the discourses (Wood, 2008).

This notion of autonomous learning is consistent with that of Sfard (2008) who writes that autonomous learners explore the discourse of others to make the discourse-for-others into a discourse-for-oneself. A discourse-for-oneself is that which a learner would spontaneously turn to whenever it may assist the learner in solving his/her own problems. For example, a teacher might explain to the learner that in the theorem of Pythagoras, the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse. In order to make this into a discourse-for-oneself, the learner will investigate this discourse by constructing squares of the same length of the sides on each side of the right-angled triangle. By cutting out the two squares, which fit onto the sides of the right-angled triangle and altering them to fit onto the square on the hypotenuse, the learner examines the discourse and incorporates the discourse into his/her own thinking. The discourse thus becomes a discourse for the learner (a discourse-for-oneself). As the learner transforms this into a discourse-for-oneself, he/she becomes able to use the discourse to solve problems that involves the discourse (Wood, 2008). The discourse thus is not merely recited to get the approval of other people such as a teacher, but because of the learner's ownership of the meaning of the discourse, the learner is able to use the discourse as tool to solve problems (Wood, 2008).

However, ownership of meaning acknowledges the social nature of a discourse. If a learner's use of a discourse is not consistent with the way the discourse is used by others, then the learner does not own the meaning in a way that it is valued within the community (Ben-Svi and Sfard, 2007 cited in Wood, 2008). Discourse-for-oneself thus does not necessarily mean that the discourse is used to communicate with oneself. Instead it is used to communicate with the learner him/herself and with others using the same discourse (Ben-Svi and Sfard, 2007 cited in Wood, 2008).

The VITALmaths video clips with their animated virtual manipulatives provide the learner with opportunities to evoke his/her curiosity to actively engage in a discourse-for-others. According to Wood (2008) “the process of autonomous learning results in a discourse-for-oneself and ownership of meaning as the learner mathematizing and identifies in ways that reflects curiosity about others’ discourses and mathematical truth” (p. 43). The learner’s curiosity about a mathematical discourse is supported by the learner’s engagement in autonomous learning (Wood, 2008). The autonomous learner interacts in ways that suggest that his/her audience (educator or co-learners) should provide support, evaluation or verification of his/her investigation (Wood, 2008). There are two ways in which the autonomous learner can position his/her audience. Namely, as experts in the discourse that can provide feedback on the authenticity of the learner’s interpretation of the mathematical discourse, or as co-learners who show interest in what the autonomous learner has to say and asking questions to clarify meaning. The learner is however also an important member of his/her audience because he/she communicates with him/herself as he/she tries to make sense of the discourse for him/herself (Wood, 2008).

Wood (2008) argues that an autonomous learner will loyally assume the mathematical discourse of those whom he/she regards as experts in the discourse. This does not necessarily mean that the learner is not critical of the discourse. However, before he/she can critique the discourse of others, he/she needs to be convinced that his/her interpretation and use of the discourse is consistent with others’ use of the same discourse.

There are four distinct features for discourse: word use, visual mediators, endorsed narratives and discursive routines. If a discourse is loyally adopted it means that the learner’s use of the discourse is consistent with others’ use of the discourse across the four above-mentioned features (Wood, 2008). Sfard (2008) writes that the use of specific words or expressions in certain ways indicates that we have a mathematical discourse. For example, one third plus a quarter is equal to seven-twelfths indicates that we have a mathematical discourse because when we use the word a quarter or one third we mean exactly a quarter and exactly one third respectively. Although these words appear in every day discourse, its use in the mathematical discourse is implicit (Sfard, 2008). Word usage thus is very important because it constitutes meaning (Sfard, 2007).

Visual mediators on the other hand, are visible objects which the autonomous learner uses to show his/her thinking or communication about the mathematical discourse (Berger, 2013). Sfard cited in Berger (2013) distinguishes between iconic mediators (graphs and pictures), symbolic mediators (symbols that are used in mathematical discourse) and concrete mediators (card that the research participants use in their demonstration of the Pythagorean Theorem and the addition and subtraction of fractions). The use of visual mediators shows the autonomous learner's thinking about the specific mathematical discourse (Berger, 2013). For example, the VITALmaths video clips act as visual mediators to support the learner in his/her understanding of the Pythagorean Theorem and the use of colour cards to demonstrate the Pythagorean Theorem supports the learner's understanding and thinking about the Pythagorean Theorem.

"Endorsed narratives are any text (spoken or written) that are framed as a description of objects, of relations between objects, or processes with or by objects" (Sfard, 2008, p. 300). Examples of endorsed narratives include statements such as in the Pythagorean Theorem the two squares on the two sides of the right-angled triangle make the square of the hypotenuse side of the same triangle. Endorsed narratives can also include statements such as two-sixths is the same as one-third because it can be labelled as true or false. What also needs to be determined during the autonomous learner's mathematizing is his/her substantiation of endorsed narratives (Wood, 2008). According to Sfard (2008) the substantiation of mathematical discourses is directed by meta-rules, which are in turn guided by mathematical proof.

Just as with endorsed narratives, discursive routines are guided by certain rules. These rules may include rules about the objects in the discourse (object-level rules) or rules that constitutes an acceptable mathematical proof (meta-rules) (Berger, 2013). Sfard (2008) writes that meta-rules are the rules that guide someone to determine the truth of a statement. Sfard (2008) further argues that meta-rules are reflections of social principles and are not necessarily imposed by external reality. Discursive routines are thus used to generalise, justify, endorse or reject discursive narratives (Berger, 2013). For example, if a learner needs to describe the Pythagorean Theorem and the learner describes it as the sum of two sides of a right-angled triangle is equal to the hypotenuse side, it will be rejected because it is not consistent with the meta-rules that determine the truth of the statement, which states that the sum of the squares of two sides of a right-angled triangle is equal to the square of the hypotenuse side. Sfard (2008) cited in Berger (2013) also distinguishes between the **how** of a discursive routine and the **when** of a discursive routine. The **how** has to do with a set of meta-rules that constrain the course of action. During the proof of the Pythagorean Theorem as demonstrated in the VITALmaths video clips, two methods are shown for proving the Pythagorean Theorem. If learners for example struggle to demonstrate the proof of the theorem by using either of the methods, the learner's perception of theorem could be restricted to the idea that the proof only works for certain right-angled triangles. The **when** has to do with a set of meta-rules that determine when it is necessary to use a specific discursive routine (Sfard, 2008 cited in Berger, 2013). Furthermore, Sfard (2008) distinguishes between three discursive routines, namely, explorations, rituals and deeds. The purpose of explorations is to verify endorsed narratives. For example, if a learner generates a mathematical investigation to prove a mathematical result, the learner may embark on an exploration to substantiate the mathematical result (Sfard, 2008). When the learner for example constructs a squares or another shape on each side of a right-angled triangle and the learner cuts out these squares or other shapes to show that the two squares or the other shapes on the two sides fit perfectly on the square or other similar shape of the hypotenuse side, the learner embarks on an exploration to proof a mathematical result. According to Sfard (2008) a ritual is when the learner aligns his/her mathematical activity with the mathematical activity of other people's routines. Rituals thus are routines that seek social approval through the imitation of other people's routines (Sfard, 2008). If the learner is able to correctly imitate the proof of the Pythagorean Theorem, he/she would able to align his mathematical proof to other people's routines, which is a ritual.

Deeds' main focus is a change in objects and that is not just in narratives as it is in the case of explorations (Sfard, 2008). For example, a learner might be able to practically show that the two squares on the two sides of a right-angled-triangle fit perfectly on the square on the hypotenuse side (the deed), but might not be able calculate the length of the hypotenuse when given the lengths of the other two sides of a right-angled triangle.

Learners who engaged in autonomous learning may produce new discursive features that can emanate from the adopted features of a discourse (Wood, 2008). For example, the discourse in the theorem of Pythagoras which states that the sum of the squares of the lengths of the sides of a right-angled triangle is equal to the square of the length of the hypotenuse, the learner may realize that the difference between the square that forms the hypotenuse side of the right-angled triangle and the square on the side of another side of the same triangle is equal to the square of the side on the third side of the same triangle. The learner built from the original discourse and thus produce a new discursive feature which arises from and contributes to the investigation of the original discourse (Wood, 2008). The autonomous learner's curiosity about other people's use of a specific mathematical discourse and how this discourse relates to mathematical truth, helps the learner in his/her explanation about what can be communicated using this specific mathematical discourse (Wood, 2008).

Two aspects of substantiation must be considered when examining autonomous learning, namely, whether the autonomous learner depends on his/her own judgement and how well the autonomous learner uses the meta-rules of the mathematical discourse (Wood, 2008). When the research participants were asked to describe how they will use the Pythagorean Theorem to find the length of a side when given the hypotenuse and any other side, one participant responded during their presentation "Like maybe I'm looking for this adjacent (participant showing the square on the one side). I think I am going to use the very same way but as the steps go down, there will be a point where by I have to minus." The participant thus depended on her own judgement, by using what she had learned from her engagement with the video clips on the Pythagorean Theorem, to describe how she would solve a problem involving the theorem, which is different from the proof that is described in the video clips. Different individuals may also use different meta-rules for substantiating a mathematical discourse because their verification of the mathematical truth may differ (Wood, 2008). A learner might for example use another shape other than a square to demonstrate the proof of the Pythagorean Theorem. A learner's substantiation of a discourse must thus be evaluated by the rules the learner uses to guide his/her substantiation (Wood, 2008).

I found that in most school contexts learners may not use their own discursive resources to substantiate a narrative, but rather rely on others such as the teacher or a mathematics text book (Wood, 2008). Wood argued that this dependence on others for the correct answer relates to heteronomy rather than a feature of autonomy, where a learner might co-learn with others but does not necessarily rely on their authority to substantiate narratives. Furthermore, if a learner is not involved in investigating or exploring a discourse or showing curiosity and the learner relies entirely upon others to verify a statement, the learner is heteronomous not autonomous (Wood, 2008). A learner, however, is more autonomous when he/she explores the substantiations given by others or uses the substantiation of others to work through his/her own substantiations (Wood, 2008). Although autonomy does not necessarily mean that the learner does not interact with other people at all, it still requires from the learner to engage in their own examination of a discourse even if that examination is supported or initiated by another. Autonomous learners thus need to demonstrate the ability to use the meta-rules of a discourse to substantiate its narratives by accessing the support from others (Wood, 2008). If a person is familiar with a few basic mathematical narratives, he/she cannot reason from those narratives on how the truth in a mathematical discourse is established (Wood, 2008). The person learns the meta-rules of a mathematical discourse through his/her interaction with others who are more proficient in the specific mathematical discourse. Through this interaction with the more proficient other the learner is able to better evaluate the narratives he/she produced in terms of the mathematical discourse and become more autonomous in his/her ability in communicating about and using a mathematical discourse (Wood, 2008).

The autonomous learner phenomenon ties in well with the social phenomena such as language, negotiation, conversation and group acceptance of mathematical truth (Ernest, 1998).

2.5.3 Autonomous Learning and Influence of Social Interaction

Autonomous learning propagates a change in focus in the classroom from the teacher to the learner or from the teaching to the learning (Taylor, 2000). This is based on a constructivist theory of learning whereby each individual learner constructs their own understanding based on their prior knowledge and current learning experiences (Kember, 1997). The level of intellectual development of a learner is the extent to which the learner has been given appropriate instruction, together with practice and experience (Bruner, 1996). These learning experiences are gained through social interaction with more knowledgeable individuals (teacher or peers) (Vygotsky, 1978). The context in which learning takes place and the social contexts that the learner brings to their learning environment are thus crucial in order for learning to take place (Gredler, 1997).

Yackel and Cobb (1996) argue that "the development of an individual's reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings." Learners thus develop their mathematical understanding as they participate in negotiating environmental norms. Yackel et al further asserts that socio-mathematical norms are deduced from the identification of regularities in patterns of social interaction. They describe socio-mathematical norms as those understandings that are mathematically different, mathematically sophisticated, mathematically efficient and mathematically elegant. Furthermore, acceptable and justifiable mathematical explanations are also categorized as socio-mathematical norms. Socio-mathematical norms are important for the development of mathematical beliefs and values that will ultimately help the learner to become intellectually autonomous in mathematics (Yackel et al., 1996).

Explaining, on the other hand, which is viewed as an act of communication, clarifies aspects of the learner's mathematical thinking that might not be apparent to others. This clears the learner's understanding for others of what is perceived as an acceptable explanation or justification (Yackel, 1992 cited in Yackel et al., 1996).

According to Roth and Radford (2010) Vygotsky's zone of proximal development has been widely used to theorize learning and learning opportunities. The zone of proximal development is "the distance between the actual developmental levels as determined by autonomous problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In this case the adult (teacher) or peer is more capable than the learner. Rituals are the forms that routines take in the zone of proximal development (Vygotsky, 1978 and Sfard, 2008 cited in Berger, 2013). The learner and the teacher or peer engages in an inter-mental plane from where the learner constructs knowledge for him/herself on an intra-mental plane (Roth and Radford, 2010). Vygotsky thus sees learning as the internalization of social interactions, in a specific context, in which communication is central. Internalization is used to describe how shared thinking (inter mental functioning) leads to the thinking of the individual (intra mental functioning). Through shared knowledge the learner thus becomes an autonomous learner (Jane & Robbins, 2007).

2.6 RATIONALE

2.6.1 Teaching of Fractions

During my interaction with both teachers' teaching of fractions and the learners' learning and understanding of fractions I found that learners do not perceive fractions as numbers but as symbols that need to be manipulated to produce answers that would satisfy the teacher. The learners also see numerators and denominators as separate numbers rather than thinking of a fraction as a single number. For this reason, learners often view a fraction with the bigger denominator as being larger than the fraction with the smaller denominator. For example $\frac{2}{5}$ is larger than $\frac{2}{3}$. Another common misconception is that learners confuse whole numbers and fractions and would for example think that because there is no whole number between 5 and 6 there is not any type of number between $\frac{7}{9}$ and $\frac{8}{9}$.

Although the above mentioned misconceptions can interfere with the learners' understanding of computational procedure (addition and subtraction of fractions), Mack (1990) argued that studies that are concerned with the learners' understanding of fractions have concentrated more on the learners' misconceptions than on their informal knowledge of fractions. The learners' operational understanding of fractions is thus characterised by knowledge of rote procedures (which are often incorrect), rather than the mathematical concepts that are underlying the procedures (Mack, 1990). Behr et al cited in Mack (1990) argued that learners come to a mathematics class with informal knowledge about fraction parts, equivalence and estimating quantities that involve fractions. The learners are thus able to successfully draw on this informal knowledge when they have to perform operations that involve fractions. The VITALmaths video clips provide learners with opportunities to draw on their informal knowledge about fraction parts, equivalence and estimation of quantities in a practical way in order to do operations that involve the addition and subtraction of fractions.

2.6.2 Geometry and the Pythagorean Theorem

My own observations and experiences in rural secondary schools through my work with RUMEP, confirm Mahassey and Perrodin's (1973, p. 15) assertion that "geometry tends to arouse fear in most courageous of elementary teachers." In my view this also applies to secondary school teachers. Many rural schoolteachers teach geometry straight from the textbook and then give the learners problems, which they themselves cannot solve. This causes learners to lose interest and confidence in geometry.

I found through my interaction with teachers that the Pythagorean Theorem, which falls under geometry, is also taught straight from the textbook. The teachers simply teach the theorem by using the Pythagorean equation ($a^2 = b^2 + c^2$). This equation is simply given to the learners without explaining the reasoning involved. Learners are thus able to rattle off the equation without having a clear conceptual understanding of the nature of area and the squares that are involved in a^2, b^2 and c^2. I concur with Newton (2010) who writes that the Pythagorean Theorem and its proof are too important to mathematics to be taught procedurally. Butler, Miller, Crehan, Babbitt, & Pierce cited in Newton (2010) asserted that not teaching mathematical concepts such as the Pythagorean Theorem concretely, can have a detrimental effect on the learners' overall understanding of other topics such as trigonometry. The VITALmaths video clips thus offer the learners concrete proofs of how the squares and areas are in involved in the proof of the Pythagorean Theorem by using animated virtual manipulatives.

2.6.3 Mathematics Teaching and Technology

My interaction with rural schools showed that due to a lack of modern technology, such as, computers in schools and at the homes of learners, teachers struggle to find innovative methods to support the learning of mathematics. The VITALmaths databank of video clips can help teachers in achieving this goal. Research done by Hyde (2011) found that during the implementation of the video clips in the teaching of mathematics, the different teachers used the same video clip innovatively to teach different concepts in mathematics. In her study, teacher A, for example, used the video clip on hubcaps to investigate the calculation of shaded areas, while teacher B saw the possibility of using the same video clip for multiplying binomials. This clearly shows the multitude of possibilities these videos present in the teaching of mathematics. My experiences with both teachers and learners have shown that learners adapt more easily to technology. My interest in modern technology, coupled with the desire to have learners do mathematics in out-of-school context, convinced me to get involved in the VITALmaths project.

Hyde (2011) writes that research has shown that video animations can both facilitate and enhance the learning of mathematics, if used effectively, because it could help learners to develop abstract mathematical knowledge from their own concrete experiences. The short (approximately three minutes long) video clips afford learners the opportunity to experience concrete animated video clips of mathematical concepts that they might not have fully understood during the teaching of the concepts in classroom context. Furthermore, learners can engage with the video clips at their own leisure to enhance their understanding of the specific mathematical concept. The enhancement of mobile technology, especially in the rural areas of South Africa, seems to show that the VITALmaths video clips have a place in the teaching and especially the autonomous learning of mathematics across the country.

The purpose of my research thus is to explore how 11 selected Grade-10 learners experience the autonomous learning of the Pythagorean Theorem and the addition and subtraction of fractions by using the VITALmaths video clips.

2.7 CONCLUSION

Although this research project seeks to understand how 11 Grade-10 participants experience the learning of certain mathematical concepts autonomously by using VITALmaths video clips, I found it important to mention **teaching practice**, because there is no learning without teaching (Jaworski, 2006). Improvement in the learning of mathematics is fundamentally related to innovative teaching practices (Jaworski, 2006). Theoretical considerations such as the nature of mathematical knowledge and what it means to know mathematics, is widely related to social interaction. Theories, such as constructivism and socio-cultural theory, were thus highly influential in the acquisition of mathematical knowledge and the learning of mathematics (Jaworski, 2006). However, although these theories provide teachers with lenses for analysing mathematical learning, they do not directly offer guidance for teaching practice. Theories, thus, may not show us what teaching should involve, but teachers can get a clearer picture of what teaching might involve. Theories in a sense provide teachers with methods of learning with the possibility to develop teaching (Jaworski, 2006). Teachers thus need to devise innovative mathematical models and methods to promote the learning of Mathematics Jaworski (2006). The VITALmaths video clips, with its animated modelling of different mathematical concepts that can be uploaded onto mobile phones, can go a long way in bridging the gap between learning, teaching practice and theory. It provides learners with innovative autonomous learning experiences in mathematics in out-of-school as well as in-school contexts. I concur with Linneweber-Lammerskitten, et al. (2010) who assert that the combination of the VITALmaths video clips with mobile phone technology will be advantages to a broad spectrum of teachers and learners across the entire South Africa, especially in the rural areas where little or no mathematical resources are available and access to modern technology, such as computers and the internet, is barely available. The VITALmaths video clips also offer learners the opportunity to engage with the video clips on their own or engage with it in a social context, where learners share ideas.

Kim (2012) writes that when two or more people look at something together, they do not see the same thing in the same way. This means that each individual has his/her own unique constructed version of reality that he/she carries around with him/her on his/her day-to-day experiences (Kim, 2012). However, unique constructed versions of mathematical discourses are bound by certain meta-rules which determines whether the individual's interpretations of his/her constructed versions of learnt experiences are true or false (Wood, 2008). Although my research participants' engagement with the VITALmaths video clips offered them opportunities to construct unique versions of the Pythagorean Theorem and the addition and subtraction of fractions, the participants' interpretations of these concepts were bound by meta-rules that guided me on whether these interpretations could be construed as true or false. Categorizing these interpretations ultimately showed whether the participants were able to make a discourse-for-others into a discourse-for-themselves, thus demonstrating an enhancement in their understanding of the concepts. This indicates whether or not they have shown signs of being an autonomous learner.

Assessing the literature available on the use of mobile technology and animated video clips for the autonomous learning of mathematics, I have come to realize that the VITALmaths project with its ever growing database of video clips can make a massive contribution to the learners' autonomous learning of Mathematics.

CHAPTER 3: METHODOLOGY

3.1 RESEARCH ORIENTATION

Schwartz-Shea & Yanow (2012) write that meanings are not expressed directly, but are embedded by the person in the physical, linguistic and enacted artifacts that they create. Researchers need to know how participants understand and interpret their situation (Pring, 2000). For this reason, researchers talk of the subjective meanings of those whom they are researching: that is, the different understandings and interpretations which the participants bring with them to the situation (Pring, 2000). For my research I explored the subjective understandings and interpretations that are the experiences, of the participants concerning the use of VITALmaths video clips to encourage the use of physical manipulatives to enhance learning autonomously. My research therefore was conducted within the interpretive paradigm.

A research paradigm is an all-inclusive system of interconnected practices that define the nature of enquiry along with the three dimensions mentioned below (Terreblanch, 1999).

The research process has three main dimensions, namely:

- **Ontology**, which refers to the way we construct reality. That is, how things are and how they work.

- **Epistemology**, referring to the different forms of knowledge of that reality. It seeks to understand the nature of the relationship that exist between the inquirer (researcher) and the inquired (the nature of human knowledge and understanding that can be learnt through different types of inquiry).

- **Methodology**, refers to the tools that we use to know that reality and how the researcher goes about to find what he/she believes can be known. (Terreblanche, 1999).

Researchers working in an interpretivist paradigm believe that reality consists of people’s subjective experiences of the external world. Their ontological belief is that reality is socially constructed (Walsham, 1995). Myers (2009, p. 39) argued that, “the premise of interpretivist researchers is that access to reality (whether given or socially constructed) is only through social constructions such as language, consciousness and shared meanings" The interpretive paradigm is thus underpinned by observations and interpretations. Myers (2009) wrote that to observe is to collect information about researched events, while to interpret is to make sense of the observed interpretations by drawing inferences or judging the match between the information and an abstract pattern. This ties in well with my study where I observed the participants' engagement with the VITALmaths video clips. By drawing inferences and judging the participants' shared meaning of information, I could interpret whether the participants' engagement with the video clips supported me in answering my research questions.

3.2 RESEARCH QUESTIONS

3.2.1 Main Question

How do selected Grade-10 Mathematics learners experience the autonomous use of selected VITALmaths video clips, which incorporate animated manipulatives, in their learning of Mathematics?

3.2.2 Sub Questions

- Does the use of the video clips in conjunction with specially prepared worksheets specifically encourage: a) the use of manipulatives in their learning of Mathematics, and b) the growth of a discourse-for-oneself?
- Does the use of the video clips enhance the learning of the Pythagorean Theorem and the addition and subtraction of fractions?

3.3 RESEARCH METHODS

A case study was undertaken with eleven Grade-10 Mathematics learners from a rural secondary school. The school is located in the John Taolo Gaetsewe District Municipality of the NCP, South Africa. Stake (1995) wrote that a case study is the study of particularity and complexity of a single case and involves an interpreter that observes the workings of the case. "The interpreter records objectively what is happening but simultaneously examines its meaning and redirects observation to refine or substantiate those meanings" (Stake, 1995, p. 9). The main unit of analysis was the eleven participants' experiences of using the VITALmaths video clips in their learning of mathematics. These included their thoughts and experiences in using manipulatives in their learning of mathematics. The usage of animated video clips that can be downloaded onto mobile phones to support the learners' autonomous learning of Mathematics is a fairly new research area.

One of the advantages of case study research is that the researcher can focus on current and interesting cases (Shuttleworth, 2008). The study should however be relevant. It is thus important to properly plan and design how the researcher is going to address the study and to ensure that the collected data is relevant (Shuttleworth, 2008). Eisenhardt (1989, p. 548) wrote that "case studies are particularly well suited to new research areas or research areas for which existing theory seems inadequate." Yin (1994) wrote that case studies are useful when a how or when question is being asked about a set of events over which the researcher has little or no control. In the main question of my research, I wanted to ascertain how selected participants experienced the autonomous learning of Mathematics by using VITALmaths video clips. I had little control over how the participants would react during and after their engagement with the video clips. I thus had less control over the variables involved in my research. Furthermore, case study research can be based on any mix of quantitative and qualitative approaches and it can use multiple data sources such as observations, interviews and documents (Rowley, 2002). The above-mentioned is typical of my case study research, which is based on a mix of qualitative and quantitative approaches where I used multiple data sources for my data collection.

The study contributed and built on the research done by Hyde (2011) and Ndafenongo (2011) on the use of visual technology for autonomous learning of Mathematics by using VITALmaths video clips. While Hyde and Ndafenongo's studies were based on the teaching of Mathematics by using the VITALmaths video clips, my study is based on the autonomous learning of Mathematics by using the VITALmaths video clips. Although teachers in both studies had differing ideas on how they would incorporate the video clips in their teaching, all participating teachers were enthusiastic about using the video clips during their teaching.

In my study I attempted to obtain as complete a picture of the participants' experiences in the autonomous use of the VITALmaths video clips as possible by using a variety of research instruments in gathering appropriate data. These included worksheets, questionnaires, interviews using an audio recorder, observations using a video recorder and tests.

3.4 RESEARCH DESIGN

3.4.1 Sampling and Participants

Oliver (2003) writes when research participants are selected, the researcher should not select them in isolation from all the other thoughts about the research project. The researcher thus needs to consider his/her research goals, research question, research design, data collection strategies that were employed, the sampling strategies and reflect on the study population when selecting participants (Oliver, 2003). The eleven Grade-10 participants were taken from one of the RUMEP project schools in the John Taolo Gaetsewe District Municipality areas of the NCP. The school was selected in consultation with the NCP Department of Education and was one of the well-functioning schools where there is a high teacher effort with time-on-task. Van der Berg, Taylor, Gustafson, Spaull & Armstrong (2011) write that low teacher effort is considered to be one of the most serious problems in South African education even more serious than having teachers with weak content and pedagogic knowledge and skills.

The eleven participants in the study were purposefully selected from the Grade-10 Mathematics class of the selected school. According to Patton (2002) purposeful sampling focuses on selecting information-rich cases whose study will clarify the research questions. The participants included both males and females taken from the bottom, average and top learners in the Mathematics class. The selection was done in consultation with the Grade-10 Mathematics teacher of the participants. The research was done out of normal school time. All the learners stay in close proximity to the school.

The study is divided into four phases:

Phase 1: The aim of this phase was to explore what selected learners do with selected VITALmaths video clips in their free-time. I conducted a workshop with my selected learners and introduced them to some of the VITALmaths video clips found on the VITALmaths website. I did not use any of the video clips that were used later in phase 2, 3 and 4. I engaged the participants with some of the clips and created an awareness amongst the group about doing Mathematics with a mobile phone. The participating learners also completed a questionnaire on their own on use of mobile phones. The questionnaire probed how learners use mobile phones. I downloaded three selected VITALmaths video clips onto the mobile phones of the participants from the selected school. I briefed the participants on the video clips and asked them to go and explore. I purposefully did not provide them with any prescribed guidelines or activities, as I wanted to obtain an initial sense of how the learners would use the video clips autonomously. The learners returned after two days and explained their experiences in using the video clips. I asked them to describe their experiences and asked questions such as: did they show any of the video clips to their friends or family? I also asked them what they learnt from the video clips and whether they thought the video clips were a good idea. I also probed them on what they thought about the manipulatives used in the video clips – were they appropriate, did they try out the mathematical activities themselves using the manipulatives? I asked them whether they thought that they could use the video clips in their own study of Mathematics. Our conversations were audio recorded and transcribed for analysis.

Phase 2: After the completion of phase 1, I engaged the participants more formally. I selected six video clips based on the Pythagorean Theorem and fractions. I found during schools visits (as part of my work for RUMEP) that learners struggle with the conceptual understanding of the Pythagorean Theorem and fractions. These topics are consistently used in the teaching of Mathematics, in particular the teaching and learning of trigonometry in Grade-10. Participants wrote a pre-test based on the Pythagorean Theorem and fractions with the rest of their Grade-10 Mathematics class. After writing the pre-test, I conducted a workshop using one of the VITALmaths video clips (not on Pythagoras' theorem or fractions) with the eleven participants to once again familiarize them with the use of the VITALmaths video clips. The three selected Pythagoras video clips were then downloaded on the mobile phones of the eleven participants. Mobile phones were provided to the learners. They were also provided with worksheets that scaffolded the prompts in the video clips based on the Pythagorean Theorem.

Participants had two weeks to complete the mathematics exercises based on the three video clips.

Phase 3: After the two weeks we got together again as a group and a post-test on the Pythagorean Theorem similar to the pre-test was written by the participants and the rest of their Grade-10 Mathematics class. All eleven participants were asked to do a presentation on the work that they had done with the video clips. All eleven presentations were video recorded and transcribed for analysis. Eight of the eleven participants that presented were interviewed individually on their experiences in using the video clips and the associated manipulatives in their understanding of the Pythagorean Theorem. The interviews were audio recorded and transcribed for analysis. I then download the three newly selected VITALmaths video clips, which were based on fractions, on the mobile phones of the eleven participants. Participants were also provided with worksheets that scaffolded the prompts in the video clips based on the addition and subtraction of fractions.

Phase 4: The entire process as phase 3 was repeated. The eleven participants including the rest of the Grade-10 Mathematics class wrote a post-test based on both the Pythagorean Theorem and the fraction video clips. All eleven participants were again interviewed individually on their experiences in using the video clips and manipulatives in their understanding of fractions. The interviews were audio recorded and transcribed for analysis.

3.4.2 Techniques/ Tools

The **questionnaires** gave me detailed information on the learners’ experiences and perceptions in using mobile phones in general and whether they had ever used them for study purposes. Hannan & Anderson (2007) write that when designing a questionnaire, you should have a clear reason and understanding why you want to use a questionnaire rather than any other tool. The fundamental question is: what is the researcher trying to find out by using a questionnaire? (Hannan & Anderson, 2007). Eiselen & Uys (2005) write that before one starts formulating questions to include in a questionnaire, it is very important to have a clear understanding of the research question and the intended goals. I will thus link each question in the questionnaire to the research question and research goals.

The **interviews** were semi-structured (Arksey & Knight, 1999). I drew up initial questions, and then followed them up with probing questions, where I was either rephrasing the original question to clarify meaning or follow through with further different questions, which were suggested by the answers to the original questions (Keats, 2000). For the interview process, I followed Seidman's (1991) suggestions by first completing all the interviews and then analysing the transcripts.

All eleven **presentations** that the participants made on their experiences in using the VITALmaths video clips were video recorded. As most cameras superimpose a time-code, it made transcription and analysis easier. I also took copious field notes to supplement the information provided by the video, because the field notes helped capture whispers and asides not picked up by the microphones (Plowman, 2004).

The Mathematics **worksheets** that accompanied the Pythagoras' theorem video clips were an extension of the video clips and thus scaffolded the prompts used in the video clips based on Pythagoras' theorem. The second set of mathematics worksheets that accompanied the video clips on fractions were an extension of the fractions used in the video clips and scaffolded the fraction prompts in the video clips. These were analysed qualitatively for emerging themes. I was particularly looking for evidence on how the video clips encourage learning aspects about Pythagoras' theorem and fractions. Of particular interest was evidence pertaining to the manipulatives used.

The **pre- and post-tests** tested the participants' conceptual understanding of Pythagorean Theorem and the addition and subtraction of fractions. The test did not only comprise of pen-and-paper exercises, but consisted of practical hands-on activities that seek to explore the participants' conceptual understanding of the two topics. The pre- and post-tests of the whole Grade-10 Mathematics class were analysed quantitatively. Graphs were produced to show the comparative results. The test results of the eleven participants were also analysed qualitatively.

3.5 DATA ANALYSIS

My research project involved both quantitative and qualitative analysis. Although I only used quantitative analysis during the pre- and post-test, it is necessary to explain the difference between the two data analysis approaches.

3.5.1 Quantitative Approaches

According to Creswell and Clark (2007) quantitative analysis approaches include closed-ended information such as information found in attitudes, behaviours and performance instruments. The analysis consists of statistically analysed scores, such as the scores of the participants' pre- and post-tests scores that were analysed by using bar graphs to support me in answering my research questions.

3.5.2 Qualitative Approaches

Qualitative analysis approaches on the other hand consist of closed-ended information that is gathered through interviews, observation of participants or collecting audio-visual materials (Creswell and Clarck, 2007). During the qualitative analysis words, text or images are combined into categories of information that are presented to show the diversity of ideas that were gathered during the data collection process (Cresswell and Clark, 2007). During my research analysis I combined the participants' presentations, interviews and pre- and post-test results into categories to show the diversity of ideas and to answer my research questions

3.6 ETHICS AND VALIDITY

Ruane (2005) writes that the principle of informed consent is about the right of any individual to determine for themselves whether they want to participate in a research project. To enable my participants to make these decisions, I informed them fully about all aspects of the research project (Ruane, 2005). Consequently, freedom of choice and self-determination were at the heart of informed consent (Ruane, 2005). My consent letter, which was disseminated to all the different stakeholders, that is the parents of the learners (Appendix 1A and 1B), the School Governing Council and Principal (Appendix 1A and 1C), the National Department of Education, the Provincial Department of Education, the District

Director of Education (Appendix 1A and 1D) contained the following points of information as set out by Sieber (1992):

"*Identification of the researcher; Explanation of the purpose of the study; Request for participation, mentioning right to withdraw at any time without impunity; Explanation of research method; Duration of research participation; A description of how confidentiality will be maintained; Mention of subject's right of refusal without penalty; Mention of right to withdraw own data at end of session; Explanation of any risk; Description of any feedback and benefits to subjects; Information on how to contact the person designated to answer questions about the subjects' right or injuries; and Indication that subjects may keep a copy of the consent."* (p. 35).

According to Diener & Crandall cited in Cohen & Manion (2000) privacy has been considered from three different perspectives, namely the sensitivity of the information being given, the setting being observed, and the dissemination of information. I did not refer to sensible information without the knowledge of the participants: I respected the participants' privacy and I always consulted the participants during the dissemination process. The essence of anonymity is that information provided by participants should in no way reveal their identity (Cohen & Manion, 1994). I thus refrained from revealing the real names of my participants or the name of the research sites. Although I knew who the providers of the information were and was able to identify participants from the given information, I will in no way make the connection known publicly. The boundaries surrounding the shared secret will be protected (Cohen & Manion, 1994). I will also in no way betray the participants by revealing data that was disclosed in confidence in such a way as to cause embarrassment, anxiety, or perhaps suffering to the subject or participants who disclosed the information (Cohen & Manion, 1994). I used Learner 1 - 11 instead of the participants' real names. Further, I cropped the photographs in Chapter 4 so as not to reveal the faces of my participants.

The main threat to valid description, in the sense of describing what is seen and heard, is the inaccuracy or incompleteness of the data (Maxwell, 1996). The audio or video recordings of observations and interviews, and the verbatim transcriptions of these recordings, largely solves this problem; if you are not doing this, it poses a potentially serious threat to the validity of a study (Maxwell, 1996). Two other important threats to the validity of qualitative conclusions are the selection of data that fit the researcher's existing theory or preconceptions and the selection of data that might stand out to the researcher (Miles & Huberman, 1994). I therefore tried to ensure that my design decisions and data analysis are not based on personal desires but on careful assessments of the implications of these for my methods and conclusions. Soliciting feedback from others, for example my supervisor, was an extremely useful strategy for identifying validity threats, my own biases and assumptions and flaws in my logic or methods (Maxwell, 1996). Presenting some of my initial findings and two international conferences was also extremely useful in soliciting feedback. To rule out misinterpretations of the meaning of what my participants say and the perspective they have on what was going on, I did random member checks as described by Guba & Lincoln (1989). I gave the pre- and post-test to colleagues and my supervisor to ensure that there were no misinterpretations of meanings or ambiguities in the test questions. I also pilot tested the Pythagorean Theorem test with ten Grade-10 learners from a school in Grahamstown, Eastern Cape Province to ascertain whether there were misinterpretations of meanings or ambiguities.

3.7 CONCLUSION

In this chapter I discussed the research paradigm that underpinned my research study and guided my design and process. I also described how the four phases that I used in my research designed supported me in answering my research questions. I then discussed how I used quantitative and qualitative approaches in my data analysis. Although it might seem that the sampling procedure was purposive, since the participants are generally better than their classmates, they were selected without knowing their abilities. Finally, I elaborated on how I addressed possible validation threats and how I adhered to appropriate ethical practices in order to maintain the trust of my participants and avoid embarrassment and anxiety.

CHAPTER 4: DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

Shuttleworth (2008) writes that the analysis of data results for a case study tends to be more opinion based than relying on statistical methods. Although my research included some form of statistical analysis, my data was collated into a manageable form from which I have constructed a narrative. The data analysis started immediately after phase one of the research design and continued throughout the other phases. In phase one the participants completed a questionnaire on their experiences with mobile phones. The responses of the participants' experiences in using a mobile phone were captured in a table. Only the responses that were relevant to my study were captured. During my analysis of the other data, I firstly analysed their engagement with the Pythagorean Theorem and then I did the analysis of the addition and subtraction of fractions. The analysis started with a general quantitative analysis of the Pythagorean Theorem pre- and post-test of the entire Grade-10 Mathematics class, which included a comparison between the test results of the participants and the rest of the Grade-10 Mathematics class. This was followed by qualitative analysis of the Pythagorean pre- and post-test per question for each participant. The Pythagorean Theorem presentations were analysed according to how the participants demonstrated the Pythagorean Theorem using manipulatives and how they described the Pythagorean Theorem. The same process as with the Pythagorean Theorem was followed for the analysis of the participants' engagement with the addition and subtraction of fractions video clips. The video clips of the presentations of the participants were uploaded on a DVD, which accompanied the analysis. Full transcripts of the video recorded presentations are added as appendices.

During the analysis of the participants' pre- and post-test results of the Pythagorean Theorem, and the addition and subtraction of fractions; and the presentations of the Pythagorean Theorem and the addition and subtraction of fractions, I ascertained how the participants experienced the autonomous use of the selected VITALmaths video clips, which incorporated animated manipulatives, in their learning of Mathematics. I also looked at how the use of the video clips encouraged the participants' use of manipulatives in their learning of Mathematics and if the use of the video clips encouraged the growth of a discourse-for-oneself. Finally, I discussed whether the use of the VITALmaths video clips enhanced the participants' learning of the Pythagorean Theorem, and the addition and subtraction of fractions.

4.2 THE QUESTIONNAIRE

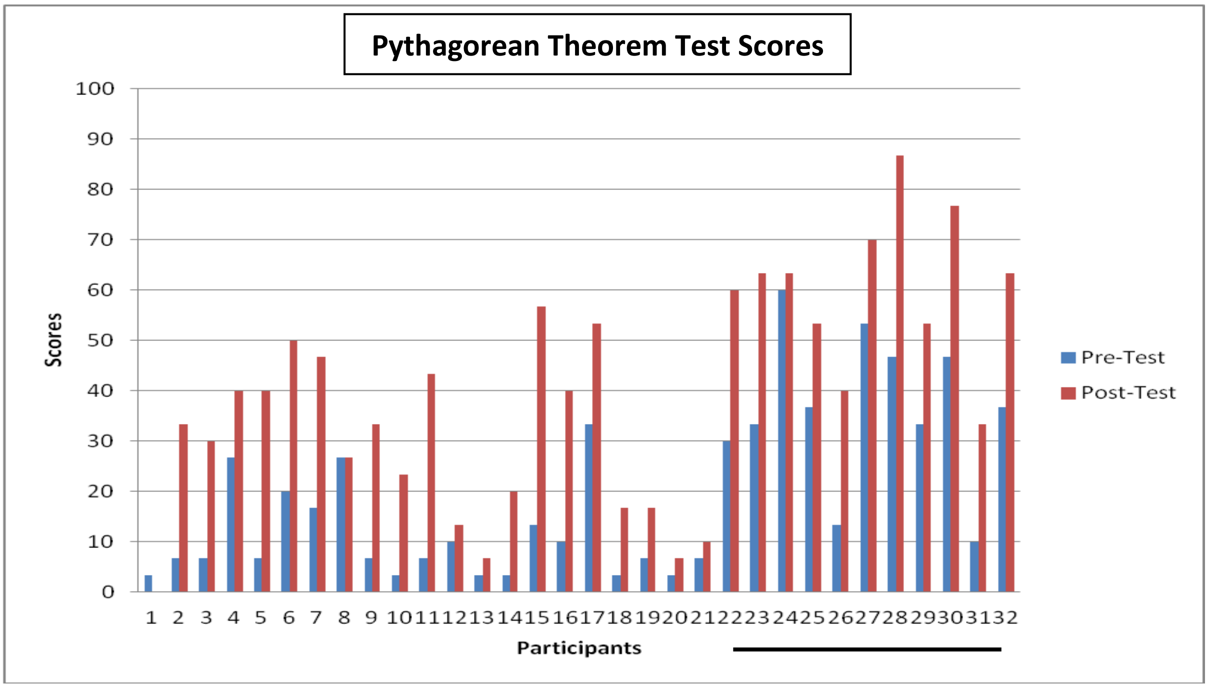
In Table 4.2.1 below the data indicates that although all the participants have their own mobile phones and that nine of the ten that completed the questionnaire have their own SIM cards, only one of the participants used the mobile phone for activities other than for calls and SMS's. Six of the ten participants have access to the internet on their mobile phones. Only one of the ten participants has ever used their mobile phones to do Mathematics.

Table 4.2.1: The participants' access to mobile phones and purpose for which their mobile phones were used.

Question1	Number of participants that have a mobile phone	Number of participants without mobile phones
Response	10	1
Question 2	Number of participants who have their own SIM card	Number of participants who do not have their own SIM card
Response	9	2
Question 3	Participants who use their mobile phones for calls and SMS's only	Participants who use their mobile phones for other activities other than calls and SMS's. For example research and social media
Response	7	4
Question 4	Number of participants that have access to internet on their mobile phones	Number of participants who do not have access to the internet on their mobile phones
Response	9	2
Question 5	Number of participants that have used their mobile phones to do mathematics	Number of participants that have never used their mobile phones to do mathematics
	1	10

4.3 THE PYTHAGOREAN THEOREM

Figure 4.2.1 below shows a comparison between the pre- and post-test results of the whole Grade-10 Mathematics class. Only one of the learners in the Grade-10 class did not show an improvement from the pre-test to the post-test in the Pythagorean Theorem test. The results of the learners who participated in the research study are represented on the graph from participant 23 to 32. All these participants showed an increase in their results from the pre-test to the post-test. Only two of the 11 participants had a test score of above 50% in the pre-test, while only two of the 11 participants had a test score below 50% in the post-test. Six learners had test scores above 60% in the post test. Only participant 28 had a test score above 80% in the Pythagorean Theorem test.



11 Participants

Figure 4.2.1: Pythagorean pre- and post-test results. (22 – 32 indicates the research participants)

Table 4.2.2 below shows the analysis of Pythagorean pre-and post-tests by using a grading scale, from poor to excellent, for how the 11 participants faired in every question of the test (Appendix 5A). For example, in Question 1 of both tests:

seven of the participants' answers to the question were poor in the pre-test, while none of the participants gave poor answers to the question in the post-test. Three of the participants gave fair answers to the question in the pre- and post-test. One of the participants gave good answers to the question in the pre-test, while three gave good answers to the question in the post-test. None of the learners gave excellent answers to the question in the pre-test, while five gave excellent answers to the question in the post-test. A comparison of the grading from pre-test to post-test thus shows that the 11 participants' understanding of the Pythagorean Theorem improved from being mostly poor-to-fair in the pre-test to mostly good-to-excellent in the post-test.

Table 4.2.2: The grading of the pre- and post-test of the participants.

Question Number	Total Number of participants		Grading
	Pre-test	Post-test	
Question1	7	0	Poor
	3	3	Fair
	1	3	Good
	0	5	Excellent
Question 2	4	0	Poor
	6	4	Fair
	1	6	Good
	0	1	Excellent
Question 3a	2	0	Poor
	6	5	Fair
	3	6	Good
	0	0	Excellent
Question 3b	1	0	Poor
	2	1	Fair
	1	3	Good
	7	7	Excellent
Question 4	1	1	Poor
	0	1	Fair
	10	6	Good
	0	3	Excellent
Question 5	4	0	Poor
	0	1	Fair
	0	0	Good
	7	10	Excellent
Question 6	3	1	Poor
	3	3	Fair
	1	0	Good
	4	7	Excellent
Question 7	0	0	Poor
	10	1	Fair
	1	3	Good
	0	7	Excellent

Table 4.2.3 and table 4.2.4 below show the analysis of the participants' responses to the questions in the pre- and post-test, based on the Pythagorean Theorem. The marks ranged from poor (1) where the participant did not attempt the question at all to excellent (4) where the participant gave correct or excellent response to the question.

Table 4.2.3: Marks per question for the Pythagorean Theorem pre-test

Participant	**Pythagorean Theorem Questions**							
	Q1	**Q2**	**Q3**	**Q3b**	**Q4**	**Q5**	**Q6**	**Q7**
1	1	3	2	4	3	4	4	3
2	2	1	3	4	3	1	2	2
3	3	2	2	3	1	4	3	2
4	1	2	3	4	3	4	4	2
5	2	1	3	4	3	1	1	2
6	1	2	2	4	3	4	4	2
7	1	2	2	4	3	4	2	2
8	1	2	1	4	3	4	4	2
9	1	1	2	1	3	1	1	2
10	2	2	1	2	3	1	2	2
11	1	1	2	2	3	4	1	2

1= Poor 2 = Fair 3 = Good 4 = Excellent

Table 4.2.4: Marks per question for Pythagorean post-test

Participant	**Pythagorean Theorem Questions**							
	Q1	**Q2**	**Q3**	**Q3b**	**Q4**	**Q5**	**Q6**	**Q7**
1	2	3	3	3	2	4	4	4
2	4	2	3	4	3	4	2	3
3	3	2	2	4	1	4	4	4
4	4	3	2	4	4	4	4	4
5	3	2	3	4	3	2	4	3
6	3	3	2	4	3	4	4	4
7	4	3	3	3	3	4	1	4
8	2	3	2	4	4	4	4	4
9	2	1	2	1	3	4	2	2
10	2	2	3	3	3	4	2	2
11	4	4	3	2	3	4	2	3

4.3.1 Analysis of Participants' Pythagorean Theorem Results

Before I present the analysis of each participant's data, I provide a short introduction of every participant. The analysis of the data started off with the pre- and post-test that the participants wrote. During the analysis of the tests, I looked at whether the participants were able to translate what they had learned from their engagement with the VITALmaths video clips to the post-test (Tables 4.2.3 and 4.2.4). I looked at the participants' use of endorsed narratives, the use of meta-rules that guide the discursive routines of the mathematical discourse and whether the participants were able to develop a discourse-for-others into a discourse-for-oneself.

In the presentation I have looked at how the participants use the virtual manipulatives from the VITALmaths video clips to develop their own concrete manipulatives to demonstrate their understanding of the Pythagorean Theorem. I finally used the analysed information above to ascertain whether the participant showed an enhancement in his/her learning of the Pythagorean Theorem and whether the participant showed features of an autonomous learner.

4.3.2 Learner 1

Learner1 is 15-year-old girl who comes from a middle class family. She is the most confident of the eleven participants and speaks English very well. Her mother is an English language teacher at a high school in Mothibistad, which is a rural town five kilometres from Kuruman in the Northern Cape Province. She stays within walking distance from the school. Learner 1 has her own mobile phone, which is on contract. She has access to the internet on her mobile phone and she was the only one who indicated that she had used her mobile phone to do Mathematics.

Pre-and post-test

In Question 1 in the pre-test, Learner 1 was unable to explain the meaning of area at all while in the post-test she gave a vague explanation of area by using the given information. In Question 2 in the pre- and post-test, she was able to demonstrate some idea of what the Pythagorean Theorem entails. In Question 3a in the pre-test, the participant attempted to find the area of the missing squares but did it incorrectly, while in the post-test the participant was only able to find the area of one of the missing squares. She attempted to find the other missing square but did it incorrectly. In Question 3b in the pre- and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles but was only able to find the length of the first side of the right-angled triangle by using the Pythagorean Theorem in the post-test. In Question 4 in the pre-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In the post-test she chose the incorrect triangle but provided reasons for her choice. In Question 5 in the pre- and post-test, Learner 1 chose the correct letter for the correct solution to the question. In Question 6 in the pre- and post-test, she was able to apply correctly the Pythagorean Theorem to solve a problem involving a right-angled triangle; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives with minor errors. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

Learner 1 demonstrated the Pythagorean Theorem by using different coloured cards. She was able to show how the two squares that she got from the other two sides fit onto the square that represents the hypotenuse. She was able to demonstrate both methods for proving the Pythagorean Theorem using manipulatives. Although she mentioned the sides when she described the theorem of Pythagoras, she pointed to the squares that were stuck onto the sides of her right-angled triangle (Figure 4.3.1).

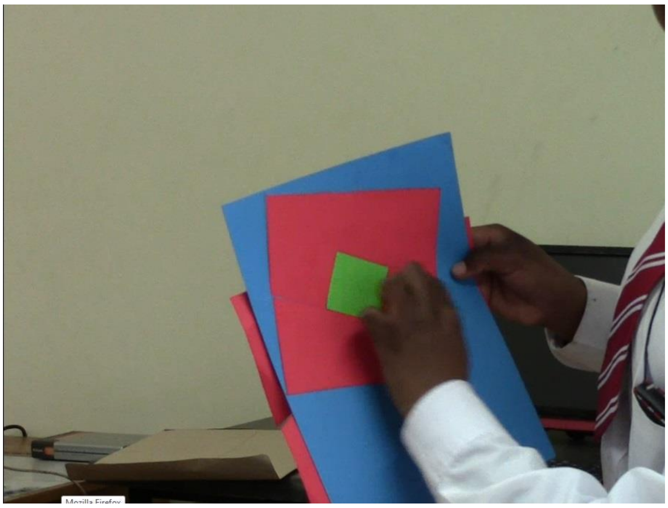


Figure 4.3.1: Learner 1 pointing to the squares

In her response to the question on the worksheet on what she understood by the theorem of Pythagoras, she said (*The theorem of Pythagoras is (pause) two squares, not of the same size, could form. I don't know really; I don't know really how to phrase it but (pause). What I know is the bigger square is going to be formed by the hypotenuse. If you draw, if you have a square, three squares, one is to go on the down side of the right-angled triangle, one is that. The theorem of Pythagoras is just that (pause) a-the opposite of the right angled triangle, the opposite and the adjacent make the hypotenuse. If you do squares like that then you see that the biggest square is where the hypotenuse is of a right angled triangle).*

When she described the method, she pointed to the sides of her model and naming the different sides the adjacent side, the opposite side and the hypotenuse. She described how to find the hypotenuse.

General Findings

Learner 1's use of keywords was consistent with the words used by others to describe the Pythagorean Theorem in her post-test and presentation, thus showing that she was talking about a mathematical discourse. She was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking about the Pythagorean Theorem. Learner 1 was able to solve problems that involved the Pythagorean Theorem correctly in her post-test. She was also able to use endorsed narratives about the Pythagorean Theorem in her presentation and partially in the post-test (*the opposite and the adjacent make the hypotenuse)*. However, the endorsed narratives she used in the pre-test were was not consistent with those that are used to describe the Pythagorean Theorem. Learner 1's description of the Pythagorean Theorem in both her presentation and partially in her post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She thus demonstrated through her exploration of and interaction with the discourse-for-others that she was able to make the discourse-for-other into a discourse-for-herself. Learner 1 thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. This also indicates that there was an enhancement in her understanding of the Pythagorean Theorem after her engagement with the VITALmaths video clips.

4.3.3 Learner 2

Learner 2 is a 15-year-old boy who stays in one of the villages in Mothibistad, which is not far from his school. He walks to school every day. Learner 2 comes from a poor family where his brother is the only permanent employed member in the family. He seems to have a lot of respect for his elder brother. When I asked Learner 2 during an interview whom he showed the VITALmaths video clips to and why he showed it to the person he said: *I showed this video to, I show this video my brother. My brother knows maths very well. He helped me to form this thing (gestures towards the square). Mmm, because why, because why I show my brother is because he is a person who understands me when I have a problem with maths. He knows maths very well.* He does not have his own mobile phone, but has access to the mobile phone of his brother and mother. He was very happy to have access to his own mobile phone when the mobile phones were given to the participants at the conclusion of the project.

Pre-and post-test

In Question 1 in the pre-test, Learner 2 gave a vague explanation of the meaning of area by using the given information, while in the post-test he was able to give a clear and concise meaning of area by using the given information. In Question 2 in the pre-test, he was unable to explain what the Pythagorean Theorem entailed, while he had a vague idea of what the Pythagorean Theorem entailed in the post-test. In Question 3a in the pre-and post-test, the participant was only able to find the area of one missing square but attempted to find the area of the second missing square and did it incorrectly. In Question 3b in the pre-and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-and post-test, he chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-test, Learner 2 chose the incorrect letter for the correct solution to the question. He chose the correct letter for the correct solution to the question in the post test. In Question 6 in the pre-and post-test, he attempted the problem but did it incorrectly; and in Question 7 in the pre-test, he attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test he used manipulatives to demonstrate the Pythagorean Theorem with minor mistakes.

Presentation

During the demonstration of the Pythagorean Theorem. Learner 2 showed the theorem by using different coloured cards for the squares that he stuck onto another coloured card. He was able to demonstrate both methods for proving the Pythagorean Theorem using manipulatives. He showed that the two squares combined form the hypotenuse. He was not able to describe the Pythagorean Theorem in the conventional way but he used the squares from his manipulatives to show some understanding of the theorem (figure 4.3.2).

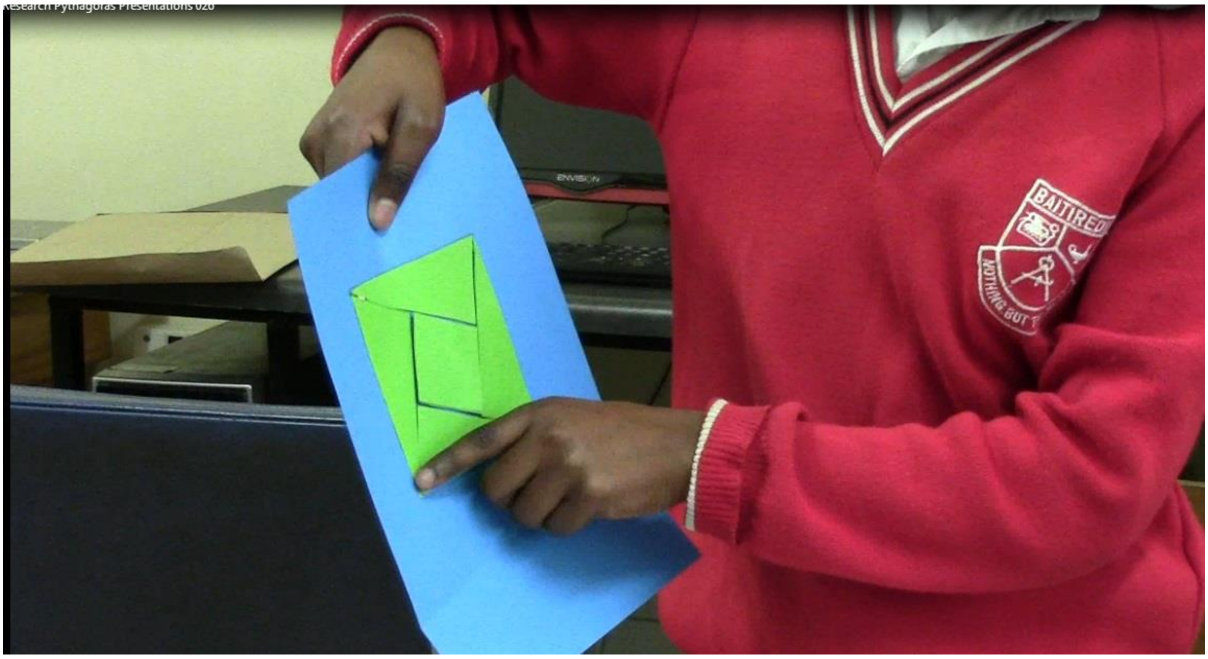


Figure 4.3.2: Learner 2 showing what the Pythagorean Theorem entails

He mentioned that the big square forms the hypotenuse. He did not clearly demonstrate his understanding of the Pythagorean Theorem. This is how he explained the Pythagorean Theorem: *I have learnt a lot from this, from this thing (gestures to the square formed). If you want to construct the third square, you must have a first and a second square. My first square is this one (points to the small square in the center) and my second is this. I used a ruler to draw my second one. After a ruler, I measured 10 and then I measure 5, and I used a ruler to, to (pause…..)*

General Findings

Learner 2's use of words was partially consistent with the words used by others to describe the Pythagorean Theorem in his post-test and presentation (*If you want to construct the third square, you must have a first and a second square).* He was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In his post-test he was able to use manipulatives correctly to show his thinking of the Pythagorean Theorem. He was not, however, able to clearly and accurately describe what the Pythagorean Theorem entails in the post-test and presentation. Learner 2 was unable to apply the Pythagorean Theorem in solving problems that involved the Pythagorean Theorem in both the pre- and post-test. His use of endorsed narratives was not fully consistent with the endorsed narratives that are used to describe the Pythagorean Theorem in his presentation and the post-test. He did not relate the squares to the sides of his right angled triangle. Learner 2's description of the Pythagorean Theorem in both his presentation and post-test was partially consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 2 thus demonstrated that he was partially able to make the discourse-for-other into a discourse-for-himself. He thus did not demonstrate features of an autonomous learner after his engagement with the VITALmaths video clips. There was also not an enhancement of his understanding of the Pythagorean Theorem after his engagement with the VITALmaths video clips.

4.3.4 Learner 3

Learner 3 is a 17-year-old boy who had repeated Grade-8 and moved from a FET science school to his current school. He seemed to be struggling with his school work in Grade-10. Although the language of instruction of the school is English, Learner 3 struggled to express himself in English. He stays in a village in Mothibistad and both his parents are unemployed. They are thus dependant on a government grant. He was the only participant that did not have access to a mobile phone. I found out during post-interviews in 2014 that he also failed Grade-10 and was not attending school regularly.

Pre- and post-test

In Question 1 in the pre-and post-test, Learner 3 could partially explain the meaning of area by using the given information. In Question 2 he had a vague idea of what the Pythagorean Theorem entails in both the pre-and post-test. In Question 3a in the pre-and post-test, the participant attempted to find the areas of the missing squares but did it incorrectly. In Question 3b in the pre-test, the participant was only able to find the length of the missing side of the first right-angled triangle and attempted to find the length of the side of the second right-angled triangle but did it incorrectly. In the post-test the participant was able to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-and post-test, he was unable to identify the right-angled triangle by using the given information. In Question 5 Learner 3 chose the correct letter for the correct solution to the question in the pre-and post-test. In Question 6 in the pre-test, he attempted to solve the problem involving the Pythagorean Theorem with minor errors, while in the post-test he was able to solve the problem correctly; and in Question 7 in the pre-test, he attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test he used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

During the demonstration of the Pythagorean Theorem, Learner 3 showed the Pythagorean Theorem by using different coloured cards for the squares that he stuck onto another coloured card. He was partially able to demonstrate both methods for proving the Pythagorean Theorem using manipulatives, because he concentrated more on the construction of the squares and how they combined to form a new square. In Figure 4.3.3 he shows how he produced the squares.

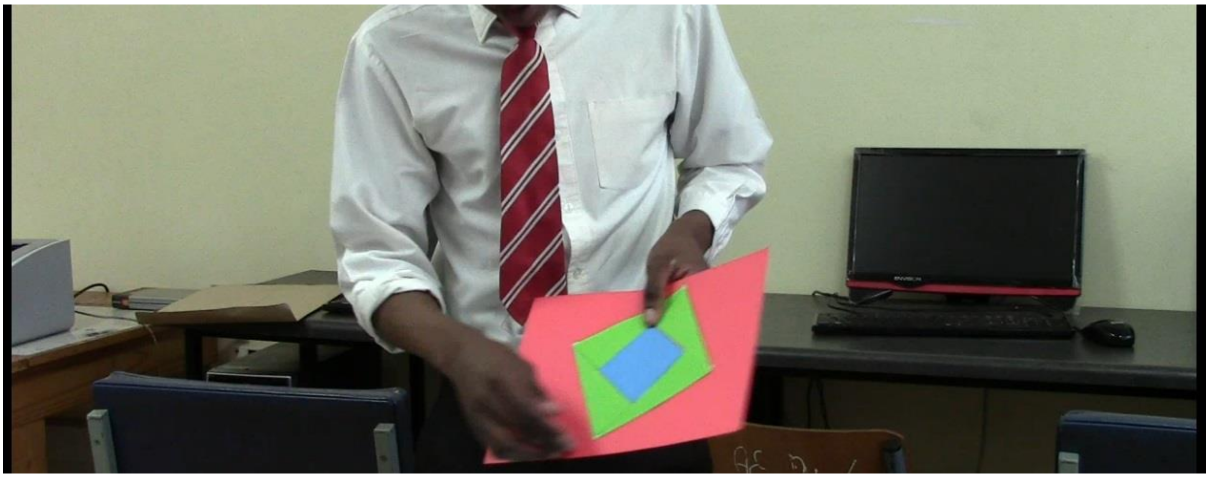


Figure 4.3.3: Learner 3 shows his work on the Pythagorean Theorem

He did not use this demonstration to describe the Pythagorean Theorem. It was only when he was probed for a description of the Pythagorean Theorem that he used words like adjacent, opposite and hypotenuse. (*When I have a, maybe when I have adjacent and opposite, adjacent and hypotenuse I can get the opposite).* He thus demonstrated that he only has a partial understanding of what the Pythagorean Theorem entails. He was unable to respond to the question on the worksheet on what he understood by the theorem of Pythagoras.

General Findings

Learner 3's use of words was partially consistent with the words used by others to describe the Pythagorean Theorem in his post-test and presentation. He was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In both his presentation and post-test he was able to use manipulatives to demonstrate the Pythagorean Theorem. He was able to solve problems that involved the Pythagorean Theorem in his post-test, which he was partially able to do in the pre-test. His use of endorsed narratives was not fully consistent with the endorsed narratives that are used to describe the Pythagorean Theorem in his presentation and the pre- and post-test (*When I have a, maybe when I have a adjacent and opposite, erh adjacent and hypotenuse I can get the opposite).* Learner 3's description of the Pythagorean Theorem in both his presentation and post-test was partially consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 3 thus partially demonstrated the ability to make the discourse-for-other into a discourse-for-himself. He thus partially demonstrated features of an autonomous learner after his engagement with the VITALmaths video clips. The use of the VITALmaths also partially enhanced Learner 3's understanding of the Pythagorean Theorem.

4.3.5 Learner 4

Learner 4 is a 15-year-old girl who comes from a struggling family. She is however always clean and neatly dressed and seemed to look well after herself. Although she is very shy, she seems to be a bright girl. This was evident during her presentation of her work on the Pythagorean Theorem and the addition and subtraction of fractions. She has her own mobile phone and SIM card but has never used it to do Mathematics. Learner 4 stays in a village close to the school with her parents and elder siblings. She walks to school with her friend who stays in the same village and is in the same class.

Pre-and post-test

In Question 1 in the pre-test, Learner 4 was unable to explain the meaning of area at all, while in the post-test she could partially explain the meaning of area by using the given information. In Question 2 in the pre-test, she was unable to explain what the Pythagorean Theorem entails, while she could partially explain the Pythagorean Theorem in her own words. In Question 3b in the pre-and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle, in the post test she was able to choose the correct triangle and gave a good reason for her choice. In Question 5 in the pre-and post-test, Learner 4 chose the correct letter for the correct solution to the question. In Question 6 she was able to apply correctly the Pythagorean Theorem to solve a problem involving a right-angled triangle in both the pre-and post-test; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

During the demonstration of the Pythagorean Theorem, Learner 4 showed the Pythagorean Theorem by using different coloured cards for the squares that she stuck onto the three sides of her right-angled triangle. She was able to demonstrate both methods for proving the Pythagorean Theorem using manipulatives. Although she mentioned the sides when she described the theorem of Pythagoras, she pointed to the squares that were stuck onto the sides of her right-angled triangle (Figure 4.3.4).

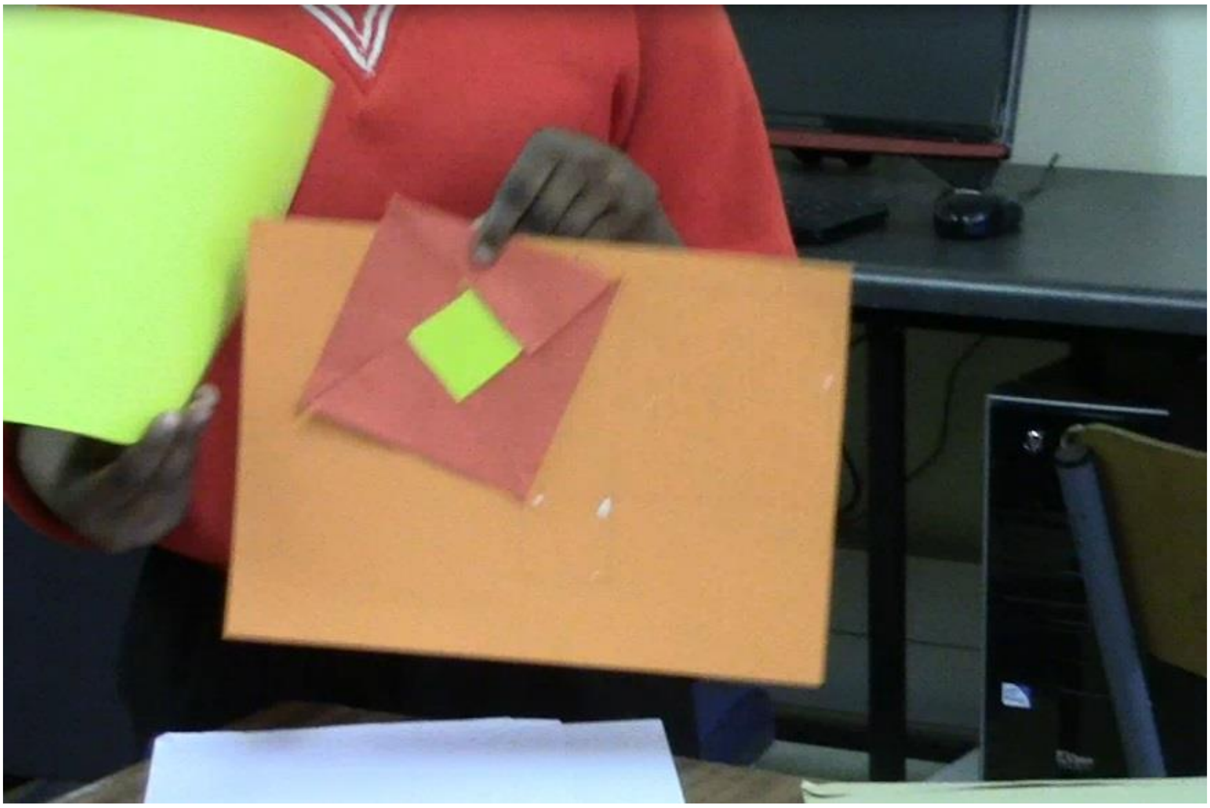


Figure 4.3.4: Learner 4 shows her work on the Pythagorean Theorem

In her response to the question on the worksheet on what she understood by the theorem of Pythagoras, she described it as I will say it’s *the sum of two squares, the adjacent and hypote-the adjacent side. They equal to the hypothe-the hypotenuse.* When she described the theorem, she pointed to the sides of her model and naming the different sides the adjacent side, the opposite side and the hypotenuse.

General Findings

Learner 4’s use of words was consistent with the words used by others (teachers or textbooks) to describe the Pythagorean Theorem in her post-test and presentation. She was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking about the Pythagorean Theorem. This shows that she has a conceptual understanding of what the Pythagorean Theorem entails. She was also able to use endorsed narratives about the Pythagorean Theorem in her presentation and the post-test (*the sum of two squares, the adjacent and hypote-the* *adjacent side. They equal to the hypothe-the hypotenuse).* However, the endorsed narratives she used in the pre-test were not consistent with the endorsed narratives used by others to describe the Pythagorean Theorem. Learner 4's description of the Pythagorean Theorem in both her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She was also able to apply the meta-rules that guide the discursive routines in solving problems that involved the Pythagorean Theorem in the post-test. She correctly applied the Pythagorean Theorem to find the one side after she was given the other two sides of a right-angled triangle. Learner 4 thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-other into a discourse-for-herself. She thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. The use of the VITALmaths video clips thus supported the enhancement of her understanding of the Pythagorean Theorem.

4.3.6 Learner 5

Learner 5 is a 15-year-old girl who stays in the same village as Learner 4. Both her parents are employed. She has a younger sister who is in Grade-7 whom she needs to look after when they get home after school. She is always neatly dressed. Learner 5 has her own mobile phone and SIM card but has never used it to do Mathematics. Although she expressed during her interview that she loves Mathematics, her results in Grade-10 Mathematics showed that she was an average student in Mathematics.

Pre-and post-test

In Question 1 in the pre-test, Learner 5 gave an unclear meaning of area, while in the post-test she was able to give a clear and concise meaning of area. In Question 2 in the pre-and post-test, she had a vague idea of what the Pythagorean Theorem entailed. In Question 3a she was only able to find the area of one missing square in the pre- and post-test. In Question 3b in the pre-and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles.

In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-and post-test, Learner 5 chose the incorrect letter for the correct solution to the question. In Question 6 in the pre-test, she did not attempt the question at all, she was however able to solve the problem correctly by using the Pythagorean Theorem in the post-test; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly, in the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

Although Learner 5 was able to show both Alex's and Ben's method for proving the Pythagorean Theorem, she was unsure what the theorem entailed. She concentrated more on the construction of the squares and how she manoeuvred them to fit onto the hypotenuse. When she was asked to describe the Pythagorean Theorem, she said *I will explain the theorem of Pythagoras, when you have a right-angle triangle, you can make it because this side of the rec-right-angle triangle I'm going to make it. (Pause).*

And a square and the hypotenuse of hmm, the right angled triangle to make this side this side is this side.

She pointed to the squares that she constructed when she explained her description of the theorem (Figure 4.3.5).

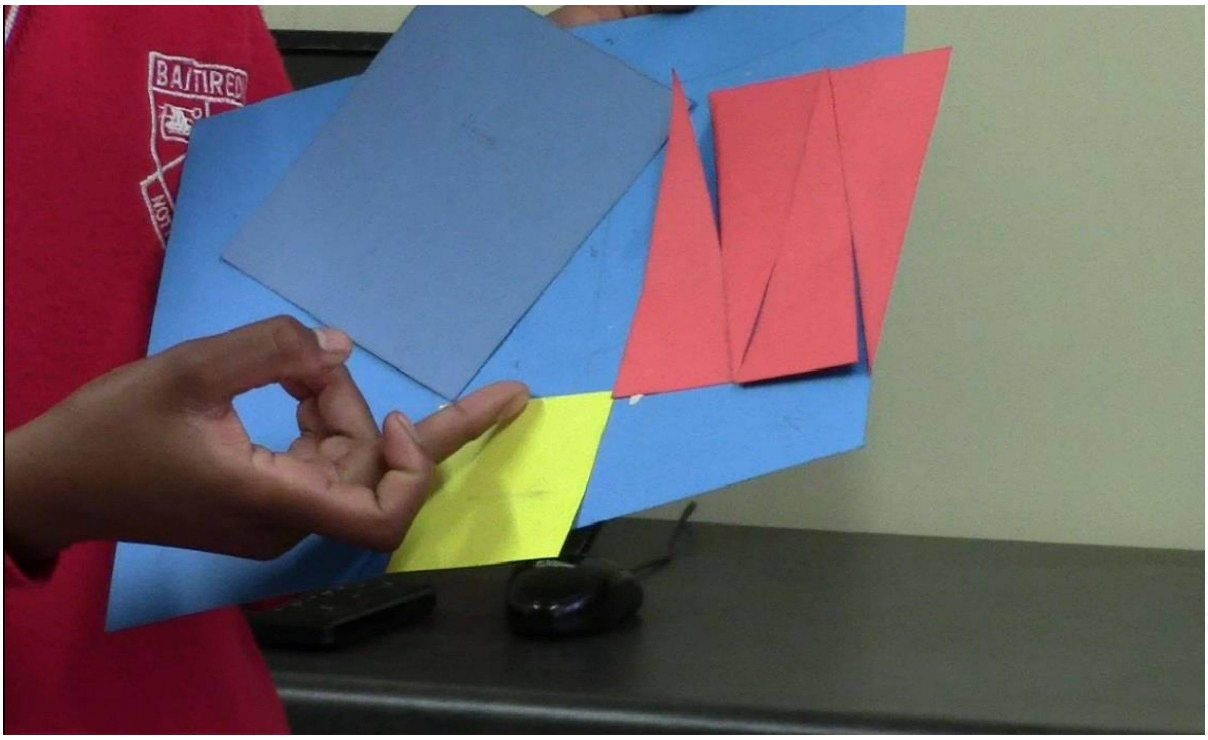


Figure 4.3.5: Learner 5 points to the sides of her right-angled triangle

When probed further, she said this can fit onto this pointing the squares in an unconvincing manner. The unknown side was the hypotenuse.

General Findings

Learner 5's use of words to describe the Pythagorean Theorem was partially consistent with the words used by others. During her presentation, she tried to explain the Pythagorean Theorem by using the different sides of her right-angled triangle. She struggled to demonstrate her understanding of the Pythagorean Theorem in the pre-tests. In the post-test she was partially able to describe what the Pythagorean Theorem entails. She however was able to use manipulatives to construct the Pythagorean Theorem with minor errors in the post-test. She was unable to fully use the endorsed narratives of the mathematical discourse for the Pythagorean Theorem in her post-test and presentation, Learner 5 description of the Pythagorean Theorem was partially consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 5 also struggled to apply the meta-rules that guide the discursive routines in solving problems involving the Pythagorean Theorem in both the pre- and post-test. She thus was partially able to make the discourse-for-others into a discourse-for-herself.

Although she was able to apply the Pythagorean Theorem in solving a problem involving the theorem in the post-test, she did not show all the features of an autonomous learner after her engagement with the VITALmaths video clips. Learner 5's engagement with the VITALmaths video clips partially enhanced her understanding of the Pythagorean Theorem.

4.3.7 Learner 6

Learner 6 is a 15-year-old girl. She comes from a family where both parents are employed. She also has an elder brother who is employed in the mines near Kuruman, a town approximately 15kilometers from Mothibistad. Learner 6 is always neatly dressed and seemed to look well after herself. She has her own mobile phone and SIM card. Although she has never used her mobile phone to do Mathematics, she has used it to do a research project.

Pre-and post-test

In Question 1 in the pre-test, Learner 6 was unable to explain the meaning of area at all, while in the post-test she could partially explain the meaning of area by using the given information. In Question 2 in the pre-test, she had a vague idea of what the Pythagorean Theorem entailed, while she could partially explain the Pythagorean Theorem in her own words. In Question 3a in the pre-and post-test, the participant attempted to find the areas of the missing squares but did it incorrectly. In Question 3b in the pre-and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre- and post-test, Learner 6 chose the correct letter for the correct solution to the question. In Question 6 in the pre-and post-test, she was able to apply correctly the Pythagorean Theorem to solve a problem involving a right-angled triangle; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

During the demonstration of the Pythagorean Theorem, Learner 6 showed the Pythagorean Theorem by using different coloured cards for the squares that she stuck onto the three sides of her right-angled triangle. She was only able to demonstrate the one method for proving the Pythagorean Theorem using manipulatives. Although she mentioned the sides when she described the theorem of Pythagoras, she pointed to the squares that were stuck onto the sides of her right-angled triangle. In her response to the question on the worksheet on what she understood by the theorem of Pythagoras, she described the theorem and said*: I will tell them, I describe the sum of uhm two sides can give you the unknown side. For example, the adjacent and the (Pause). Ja the adjacent and opposite can give you the hypotenuse.*

When she described the theorem, she pointed to the sides of her model and name the different sides the adjacent side, the opposite side and the hypotenuse. She described how to find the hypotenuse by pointing to the two squares that will form the hypotenuse (Figure 4.3.6).

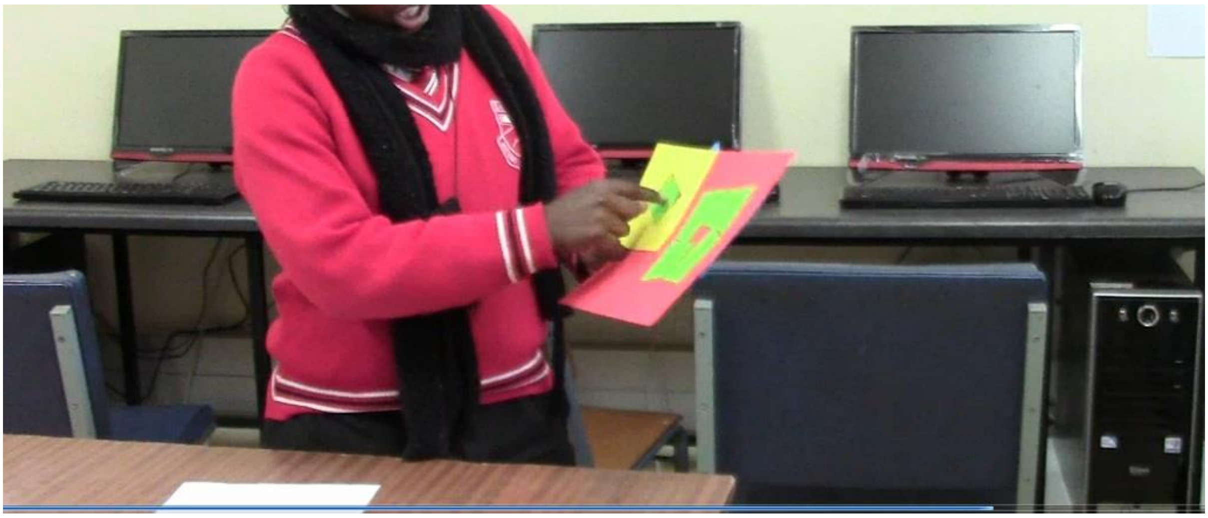


Figure 4.3.6: Learner 6 shows her work on the Pythagorean Theorem

General Findings

Learner 6's use of words was consistent with the words used by others to describe the Pythagorean Theorem in her post-test and presentation. She had a vague idea of what the Pythagorean Theorem entailed in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking of the Pythagorean Theorem. Learner 6 was able to apply the Pythagorean Theorem in solving problems that involved the Pythagorean Theorem in her post-test. She was also able to use endorsed narratives about the Pythagorean Theorem in her presentation and the post-test (*the adjacent and opposite can give you the hypotenuse)*. However, the endorsed narratives she used in the pre-test were not consistent with the endorsed narratives used by others to describe the Pythagorean Theorem. Learner 6's description of the Pythagorean Theorem in both her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 6 thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-others into a discourse-for-herself. She thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. Her engagement with the VITALmaths video clips also enhanced her understanding of the Pythagorean Theorem.

4.3.8 Learner 7

Learner 7 is a 16-year-old girl that stays in Mothibistad close to her school. She comes from middle-class home and both her parents are employed. She always seems to be in a good mood. She is always neatly dressed and was always the first one in class during the data collection process. According to her class teacher she is hard-working and is one of the top students in her class. Her enthusiasm was evident during the presentations of the Pythagorean Theorem and the addition and subtraction of fractions. She has her own mobile phone and SIM card but has never used her mobile phone to do Mathematics. She often uses her mobile phone for chats on social media.

Pre-and post-test

In Question 1 in the pre-test, Learner 7 was unable to explain the meaning of area at all, while in the post-test she was able to give a clear and concise meaning of area by using the given information. In Question 2 in the pre-test, she had a vague idea of what the Pythagorean Theorem entails, while she could partially explain the Pythagorean Theorem in her own words in the post-test. In Question 3a in the pre-test, the participant attempted to find the area of one of the missing squares but did it incorrectly, while in the post-test the participant was able to find the area of one of the missing squares and attempted to find the area of the second missing square but did it incorrectly. In Question 3b in the pre-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles, but was only able to find the length of the side of the first right-angled triangle by using the Pythagorean Theorem in the post-test. She attempted to find the length of the side of the second right-triangle but did it incorrectly. In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-and post-test, Learner 7 chose the correct letter for the correct solution to the question. In Question 6 in the pre-test, she attempted the problem but did it incorrectly. She did not attempt the problem at all in the post-test; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

During the demonstration of the Pythagorean Theorem Learner 7 showed the Pythagorean Theorem by using different coloured cards for the squares that she stuck onto the hypotenuse of her right-angled triangle. Although she struggled with the second method for proving the Pythagorean Theorem, she was able to demonstrate the proof of the Pythagorean Theorem using manipulatives. When she mentioned the sides when describing the theorem of Pythagoras, she pointed to the squares that were stuck onto the sides of her right-angled triangle. In Figure 4.3.7a she shows how the two squares fit onto the hypotenuse.

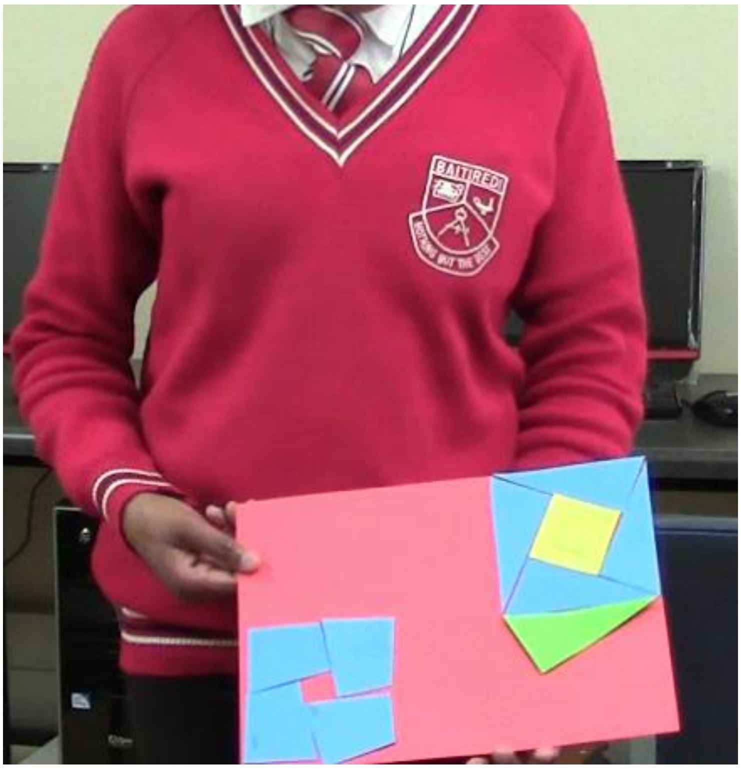


Figure 4.3.7a: Learner 7 shows her work on the Pythagorean Theorem

In her response to the question on the worksheet on what she understood by the theorem of Pythagoras, she said: *I think the theorem of Pythagoras is to find the- let's say maybe I have this side (adjacent) and I have this side (opposite), but I don't have this side (hypotenuse), I'm going to use the theorem of Pythagoras to find this side (hypotenuse).*

When she described the method, she pointed to the sides of her model and naming the different sides the adjacent side, the opposite side and the hypotenuse. She described how to find the hypotenuse (Figure 4.3.7b).

Figure 4.3.7b: Learner 7 explains the Pythagorean Theorem using sides of the triangle

When further probed on finding the adjacent side when given the hypotenuse and the opposite side of her right-angled triangle, she said: *Like maybe I'm looking for this one (adjacent). I think I am going to use the very same way but as the steps go down, there will be a point where by I have to minus.*

General Findings

Learner 7's use of words was consistent with the words used by others (teachers or textbooks) to describe the Pythagorean Theorem in her post-test and presentation. This shows that she has a conceptual understanding of what the Pythagorean Theorem entails. She was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking about the Pythagorean Theorem. Learner 7 was however unable to apply the Pythagorean Theorem to solve problems that involved the application of the theorem in her pre- and post-test. She was able to use endorsed narratives about the Pythagorean Theorem in her presentation and the post-test (*let's say maybe I have this side (adjacent) and I have this side (opposite), but I don't have this side (hypotenuse), I'm going to use the theorem of Pythagoras to find this side (hypotenuse.*

However, she was unable to use endorsed narratives in the pre-test that are consistent with those that are used to describe the Pythagorean Theorem. Learner 7 description of the Pythagorean Theorem in both her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 7 thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-others into a discourse-for-herself. She thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips in the presentation and description of the Pythagorean Theorem. She, however, did not demonstrate these features during the application of the Pythagorean Theorem. Her engagement with the VITALmaths video clips partially enhanced her understanding of the Pythagorean Theorem.

4.3.9 Learner 8

Learner 8 is a 16-year-old girl who stays in a village in Mothibistad. She comes from a poor family with her mother as a single parent. During the data collection process she was always sick, complaining of headaches. When I asked her why she did not go and see a doctor, she said that she did want to miss school and that the clinic was full in the afternoons. Learner 8 seemed to be a bright girl and was doing well in school especially in Mathematics. She has her own mobile phone and SIM card but has never used it to do Mathematics.

Pre-and post-test

In Question 1 in the pre-test, Learner 8 was unable to explain the meaning of area at all, while in the post-test she gave a vague explanation of area by using the given information. In Question 2 in the pre-test, she had a vague idea of what the Pythagorean Theorem entails, while she could partially explain the Pythagorean Theorem in her own words. In Question 3a in the pre-test, she was unable to find the areas of the missing squares, while in the post-test she attempted to find the areas of the missing squares but did it incorrectly. In Question 3b in the pre-and post-test, the participant was able to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle, in the post-test she was able to choose the correct triangle and gave a good reason for her choice. In Question 5 in the pre-and post-test, Learner 8 chose the correct letter for the correct solution to the question. In Question 6 in the pre- and post-test, she was able to apply correctly the Pythagorean Theorem to solve a problem involving a right-angled triangle; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem correctly.

Presentation

During the demonstration of the Pythagorean Theorem. Learner 8 showed the Pythagorean Theorem by using different coloured cards for the squares that she stuck onto the three sides of her right-angled triangle. She said: *So, this is A^2 this is B and this is C. so if you take A^2 and B^2 they fit exactly on this (the square on the hypotenuse side.) And this is what we do.* In Figure 4.3.8a she shows how the squares fit onto the different sides of her right-angled triangle.

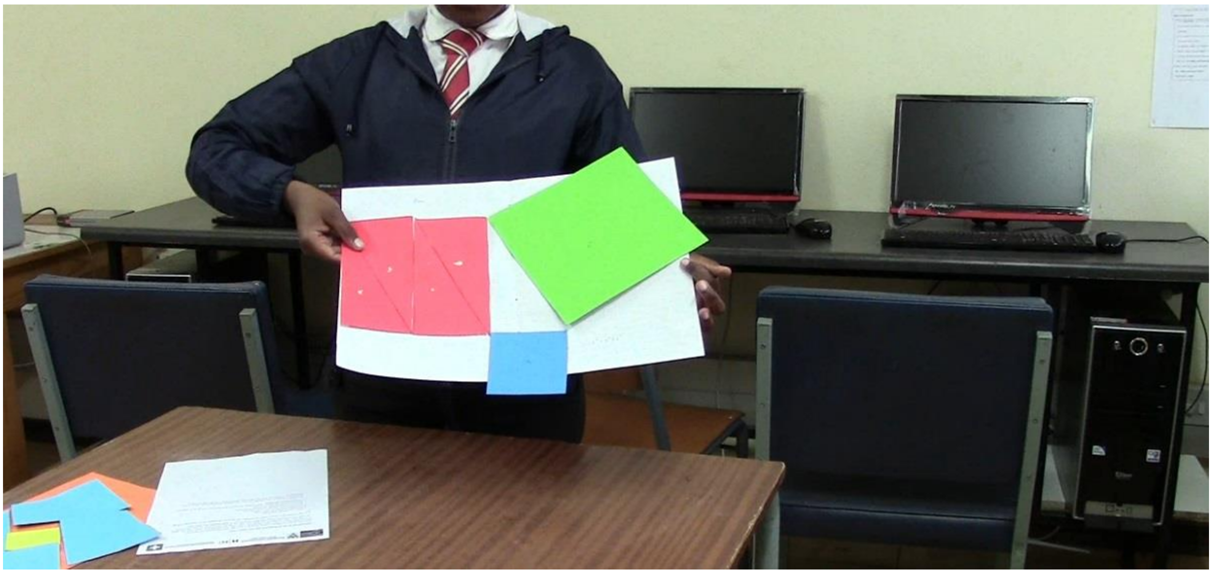


Figure 4.3.8a: Learner 8 shows her work on the Pythagorean Theorem

She then dislodged the two squares and showed how they fit onto the square on the hypotenuse (Figure 4.3.8b).

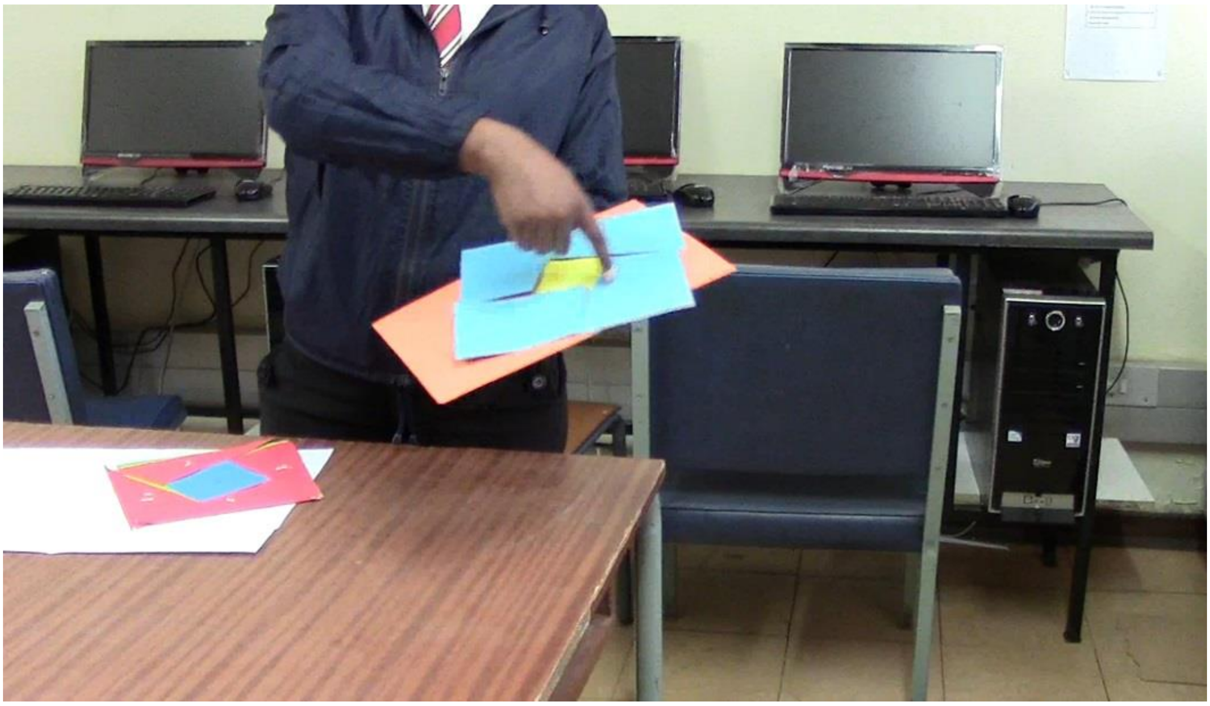


Figure 4.3.8b: Learner 8 shows how the two squares fit onto the square at the hypotenuse

When she needed to describe the Pythagorean Theorem, she said *the adjacent side and the opposite side make the hypotenuse.* She pointed to the squares that were stuck on the different sides of her right-angled triangle. Learner 8 was able to use manipulatives to demonstrate both methods to prove the Pythagorean Theorem during her presentation.

General Findings

Learner 8's was partially able to explain the Pythagorean Theorem in her post-test and used manipulatives to describe the theorem during her presentation. She was unable to demonstrate or describe the Pythagorean Theorem in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking about the Pythagorean Theorem. Learner 8 was also able to apply the Pythagorean Theorem in solving problems that involved the Pythagorean Theorem correctly in the post test. She was able to use endorsed narratives about the Pythagorean Theorem in her presentation (*the adjacent side and the opposite side* *make the hypotenuse)*. However, the endorsed narratives she used in the pre-test were vague. Learner 8's use of words during the demonstration of the Pythagorean Theorem, was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 8 thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-other into a discourse-for-herself. She thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. Her engagement with the VITALmaths video clips enhanced her understanding of the Pythagorean Theorem.

4.3.10 Learner 9

Learner 9 is a 16-year-old boy who comes from a family where both parents are employed. He stays in town in Mothibistad. He is a class representative for his Grade-10 class. Learner 9, however is struggling a lot with his school work, especially Mathematics. This was evident during his presentations of the Pythagorean Theorem. He also struggled to express himself during the post presentation interviews. Learner 9 has his own mobile phone and SIM card. He has however never used it to do Mathematics.

Pre-and post-test

In Question 1 in the pre-test, Learner 9 was unable to explain the meaning of area at all, while in the post-test he gave a vague explanation of area by using the given information. In Question 2 in the pre-and post-test, he was unable to explain what the Pythagorean Theorem entails. In Question 3a in the pre-and post-test, the participant attempted to find the areas of the missing squares, but did it incorrectly. In Question 3b in the pre-and post-test, the participant was unable to find the lengths of the two missing sides of the two right-angled triangles. In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-test, Learner 9 chose the incorrect letter for the correct solution to the question, while he chose the correct letter for the correct solution in the post-test. In Question 6 in the pre-test, he did not attempt the problem. He attempted the problem in the post-test but did it incorrectly; and in Question 7 in the pre-and post-test, he attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly.

Presentation

During the demonstration of the Pythagorean Theorem, Learner 9 showed the Pythagorean Theorem by using different coloured cards for the squares that she stuck onto the two sides of her right-angled triangle. He was able to demonstrate both methods for proving the Pythagorean Theorem using manipulatives. *The adjacent (small red square in center) and the opposite (blue triangle shapes) can give us the hypotenuse. So all of this (shape formed by adjacent and opposite sides) I think it is the hypotenuse.* He pointed to squares in Figure 4.3.9a to show the two sides that will form the hypotenuse.

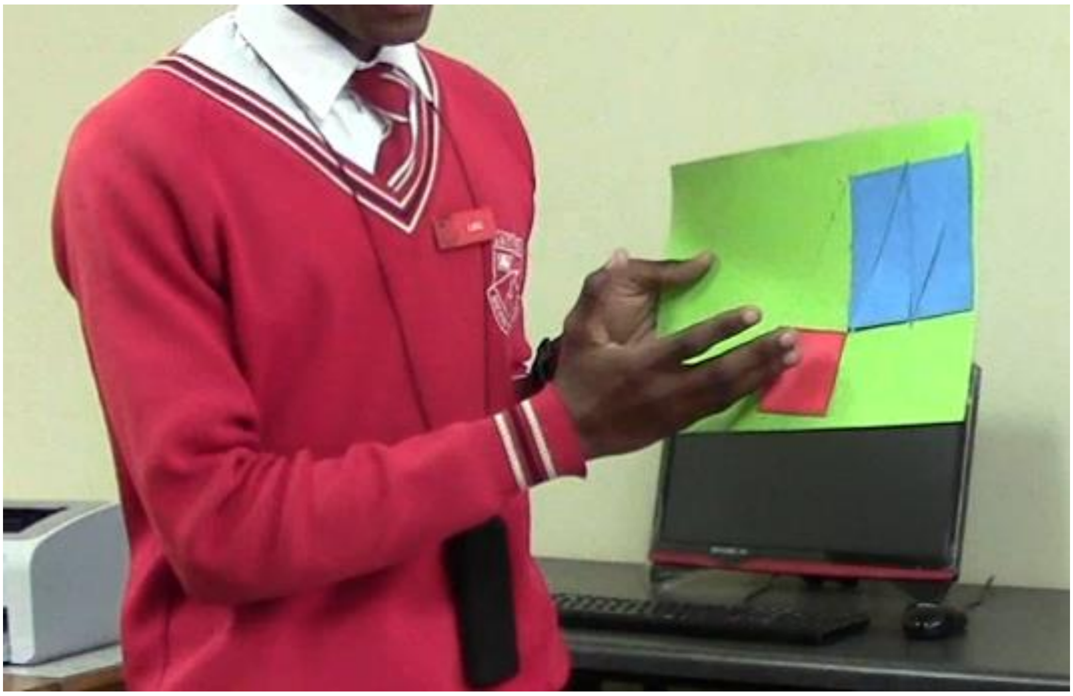


Figure 4.3.9a: Learner 9 shows his work on the Pythagorean Theorem

He detached the two squares that were stuck on the two sides of his right-angled triangle and rearranged them to fit onto the hypotenuse (figure 4.3.9b).

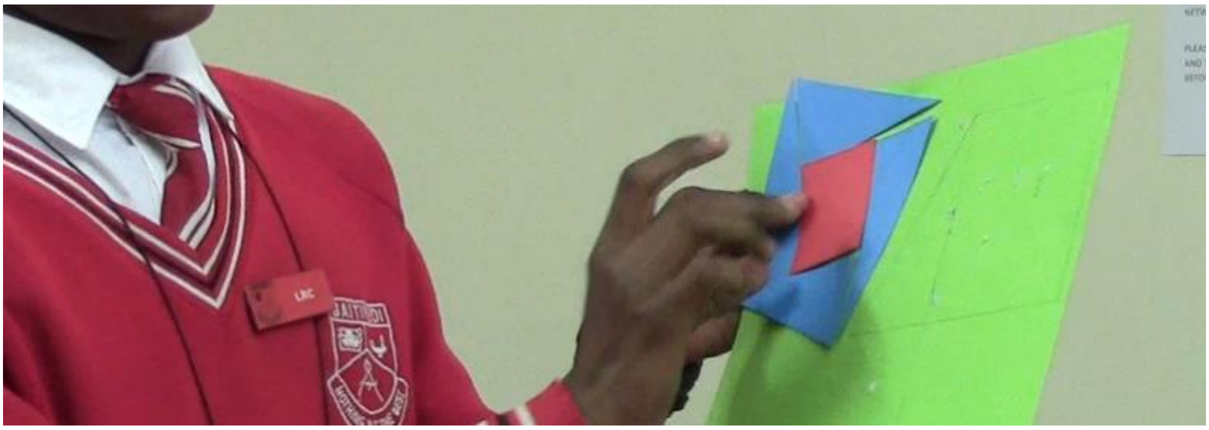


Figure 4.3.9b: Learner 9 struggles to explain what the Pythagorean Theorem entails

When I further probed him on finding the adjacent side when given the hypotenuse and the opposite side of his right-angled triangle, he said: *You going to take the adjacent, then measure it, measure the height of it and then you draw the opposite- the opposite of the hypotenuse.*

General Findings

Learner 9's use of words was consistent with the words used by others to describe the Pythagorean Theorem in his presentation. He was unable to demonstrate or describe the Pythagorean Theorem in the pre-and post-test. In both his presentation and post-test he was able to use manipulatives to show his thinking of the Pythagorean Theorem. He however did not do it correctly in the pre- and post-test. He could not correctly apply the Pythagorean Theorem in solving problems that involved right-angled triangles in her pre- and post-test. He was also unable to use endorsed narratives about the Pythagorean Theorem in his pre-and post-test. However, the endorsed narratives he used in his presentation was consistent with the endorsed narratives used by others (teachers or textbooks) (*the adjacent (small red square in center) and the opposite (blue triangle shapes) can give us the hypotenuse)*. Learner 9's description of the Pythagorean Theorem in his presentation was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Learner 9 thus partially demonstrated that, through his exploration of and interaction with the discourse-for-others, he was able to make the discourse-for-other into a discourse-for-himself. He thus did not fully demonstrated features of an autonomous learner after his engagement with the VITALmaths video clips. Learner 9 also did not fully show an enhancement of his understanding of Pythagorean Theorem after his engagement with the VITALmaths video clips.

4.3.11 Learner 10

Learner 10 is a 16-year-old girl who comes from a family where both parents are employed. She has a twin sister who is in the same class as her. Learner 10 seemed to be the one who was struggling more than what her sister did in their school work. Although her twin sister was not part of the research project, Learner 10 shared all the information she received on the research project with her. Learner 10's twin sister thus did better from pre- to post-test in both the Pythagorean Theorem tests and the addition and subtraction of fraction tests. In Figure 4.2.1 Learner 10 is participant 26 and her twin sister is participant 15.

Pre-and post-test

In Question 1 in the pre-and post-test, Learner 10 gave a vague meaning of area by using the given information. In Question 2 in the pre-and post-test, had a vague idea of what the Pythagorean Theorem entails. In Question 3a in the pre-test, the participant was unable to find the areas of the missing squares, while in the post-test she was able to find the area of one of the missing squares. She attempted to find the area of the second missing square but did it incorrectly. In Question 3b in the pre-test, the participant attempted to find the lengths of the two sides of the two right-angled triangles by using the Pythagorean Theorem but did it incorrectly. In the post-test was able to find the lengths of the missing side of the first right-angled triangle. She attempted to find the missing side of the second right-angled triangle but did it incorrectly. In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-and post-test, Learner 10 chose the correct letter for the correct solution to the question. In Question 6 in the pre-and post-test, the participant attempted the question but did it incorrectly; and in Question 7 in the pre-and post-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly.

Presentation

Although Learner 10 was able to show both Alex's and Ben's method for proving the Pythagorean Theorem, she was unsure what the theorem entails. She mentioned that she tried out what she saw in the video clips, but struggled to put the model together to demonstrate the theorem. When she was asked to describe the Pythagorean Theorem, she described it as: *The theorem of Pythagoras is the relation in Euclidean geometry. The hypotenuse is the side opposite the right angle. Here is our hypotenuse (points to a side on her example), so our hypotenuse is the side opposite to the right angle triangle- right angled. This is our-our-our angle and this angle add up to 90 degrees. (Brief pause), yes.*

When probed further about the Euclidean relationship, she said: *Okay, I will tell them that, uhm, you can-you can use the theorem of Pythagoras in the right angle triangle. Let me say this is our A, this is our B and this is our C (points on the triangle). So you-you-you-you are given this side (opposite) and this side (adjacent)* (She pointed to the squares, Figure 4.3.10)*, so this side (hypotenuse) is the missing side. So you need to-to work-work them out, you need to find the-this missing side. So that the- the formula that you are going to use is A (squared); this is our A, this is our B, and this is our C.* When probed about the formula, she said that: *the formula is* $A^2 + B^2 = C^2$*. I get the formula on the books, Mathematics books.*

Figure 4.3.10: Learner 10 tries to explain her work on the Pythagorean Theorem

She pointed to the different sides of her right-angled triangle when she mentioned A^2, B^2 and C^2. She could not explain what was meant by squared.

General Findings

Learner 10's use of words to describe the Pythagorean Theorem was partially consistent with the words used by others. She, for example, described the Pythagorean Theorem as a relationship in Euclidean geometry. During her presentation, she later tried to explain the Pythagorean Theorem by using the adjacent side, opposite side and hypotenuse. However, her description of the theorem was still correct. She also struggled to demonstrate her understanding of the Pythagorean Theorem in the pre- and post-tests. Learner 10 was unable to apply the Pythagorean Theorem in solving problems that involved the Pythagorean Theorem in the pre- and post-test. Her use of endorsed narratives was not consistent with the endorsed narratives of the mathematical discourse for the Pythagorean Theorem in her presentation. She was also unable to use the endorsed narratives that are used during the description of the Pythagorean Theorem in her pre- and post-test. Learner 10 description of the Pythagorean Theorem was partially consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She thus was unable to fully make the discourse-for-others into a discourse-for-herself. Learner 10 did not show all the features of an autonomous learner after her engagement with the VITALmaths video clips. Her engagement with the VITALmaths video clips did not fully enhance her understanding of the Pythagorean Theorem.

4.3.12 Learner 11

Learner 11 is a 16-year-old girl that comes from a middle-class family. She is always neatly dressed and seems to look well after herself. 2013 was her first year at the school and it seemed as if she was struggling to settle in at the school. According to her class teacher she was struggling to cope with her school work, especially with Mathematics. She had repeated Grade-9 at her previous school. Learner 11 has her own mobile phone and SIM card but has never used it to do Mathematics. She has however used it to chat on social media.

Pre-and post-test

In Question 1 in the pre-test, Learner 11 was unable to explain the meaning of area at all, while in the post-test she was able to give a clear and concise meaning of area by using the given information. In Question 2 in the pre-test, she was unable to explain what the Pythagorean Theorem entails, while she was able to give a good explanation of the Pythagorean in her own words. In Question 3a in the pre-test, Learner 11 attempted to find the areas of the missing squares but did it incorrectly, while in the post-test she was able to find the area of one missing square. She attempted to find the area of the other missing square but did it incorrectly. In Question 3b in the pre-and post-test, the participant attempted to find the lengths of the two sides of the two right-angled triangles by using the Pythagorean Theorem, but did it incorrectly. In Question 4 in the pre-and post-test, she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle. In Question 5 in the pre-and post-test, Learner 11 chose the correct letter for the correct solution to the question. In Question 6 in the pre-test, she did not attempt the problem at all, while in the post-test she attempted the problem but did it incorrectly; and in Question 7 in the pre-test, she attempted to demonstrate the Pythagorean Theorem with the aid of manipulatives but did it incorrectly. In the post-test she used manipulatives to demonstrate the Pythagorean Theorem with minor errors.

Presentation

During the demonstration of the Pythagorean Theorem, Learner 11 used different colour cards which she stuck onto the three sides of her right-angled triangle. She then mentioned that the squares form the right-angled triangle. She points to the squares that will form the right-angled triangle in Figure 4.3.11

Figure 4.3.11: Learner 11 shows her work on the Pythagorean Theorem

She also had a separate set of squares that was combined to form another square. She mentioned that: *So, when we are looking to, erh-uhm, when we are looking to the hypothesis-hypotenuse, maybe let's say you were having an adjacent and you are having a hypothesis, you are going to- I mean opposite. This is our adjacent, opposite and hypotenuse. So when you are looking for the adjacent and hypothesis, you are going to add these two together (the adjacent and opposite sides of the triangle) we of hypothesis.* She pointed to the squares on the sides when she mentioned the adjacent side, the opposite side and the hypotenuse as shown in the image above. She then said: *So you going to square them, ja w-we going to square them to get the hypotenuse.*

General Findings

Learner 11's use of words was consistent with the words used by others (teachers or textbooks) to describe the Pythagorean Theorem in her presentation. Her use of words to describe the Pythagorean Theorem in the pre- and post-test was however inconsistent with words used by others to describe the theorem. She was unable to demonstrate or describe the Pythagorean Theorem in the pre- and post-test. In both her presentation and post-test she was able to use manipulatives to show her thinking about the Pythagorean Theorem. There were minor errors in her demonstration during the post-test. Learner 11 was unable to apply the Pythagorean Theorem in solving problems that involved the Pythagorean Theorem in the pre- and post-test. She was able to use endorsed narratives about the Pythagorean Theorem in her presentation (*This is our adjacent, opposite and hypotenuse. So when-when-when erh, when you are looking for the adjacent and hypothesis, you are going to add these two together (the adjacent and opposite sides of the triangle) we of hypothesis)*. However, the endorsed narratives she used in the pre- and post-test were not consistent with the endorsed narratives that are used to describe the Pythagorean Theorem. Learner 11's description of the Pythagorean Theorem in her presentation was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. The meta-rules that guide the discursive routines of this mathematical discourse were however inconsistent with her description of the Pythagorean Theorem in the pre- and post-test. Learner 11 thus demonstrated that she was only partially able to make the discourse-for-other into a discourse-for-herself. She thus demonstrated features of an autonomous learner in her presentation after her engagement with the VITALmaths video clips, but was unable to translate these features into her post-test. Her engagement with the VITALmaths video clips thus did not fully enhance her understanding of the Pythagorean Theorem.

4.3.12 Consolidation of Findings

All the participants struggled to describe area and to correctly demonstrate the Pythagorean Theorem in the pre-test. It was also clear from both the pre- and post-test results that the participants did not understand that the Pythagorean Theorem was about the areas of the squares that are formed by the sides of a right-angled triangle rather than the sides squared, for example side $(AB)^2$. Although the majority of the participants used squares to show their understanding of the Pythagorean Theorem during the presentations, this knowledge was not translated to their understanding of the Pythagorean Theorem in Question 3a of the pre- and post-tests (Appendix 4a). The participants squared the given values although the values were already in squared form.

Learner 1, Learner 4, Learner 6, Learner 7, Learner 8 and Learner 9 use of key words was consistent with the words used by others to describe the Pythagorean Theorem. These six participants were also able to use the endorsed narratives for the Pythagorean Theorem during their presentations and post-tests. They were able to make a discourse-for-others into a discourse-for-themselves and each of them thus demonstrated features of an autonomous learner. The other five participants were partially able to use key words that are used by others to show their understanding of the Pythagorean Theorem. Due to the five participants' inability to use endorsed narratives in either their presentations or post-tests, they were not able to fully make the discourse-for-others into a discourse-for-themselves. They thus partially demonstrated features of autonomous learners after their engagement with the VITALmaths video clips.

4.4 ADDITION AND SUBTRACTION OF FRACTIONS

The participants' data is analysed in the same order as with the Pythagorean Theorem. I started off with a general comparison of the fractions pre- and post-test by using a bar graph. I then did a qualitative analysis of each participants' pre- and post-test and the participants' presentations. I used the test analysis with the analysis of the presentations to write general findings for each participant.

4.4.1 General

Figure 4.1 below shows a comparison between the pre- and post-test results of the whole Grade-10 mathematics class. Only three of the learners in the Grade-10 class did not show an improvement from the pre-test to the post-test in the addition and subtraction of fractions test. The results of the learners who participated in the research study are represented on the graph from participant 23 to 32. All these participants showed an increase in their results from the pre-test to the post test. None of the 11 participants had a test score of above 50% in the pre-test. Four of the 11 participants had a test score above 50% in the post-test and only one learner had a test score below 30% in the post test. Only participant 25 had a test score above 60%. Six of the 11 participants showed an increase of 10% and above from pre- to post-test.

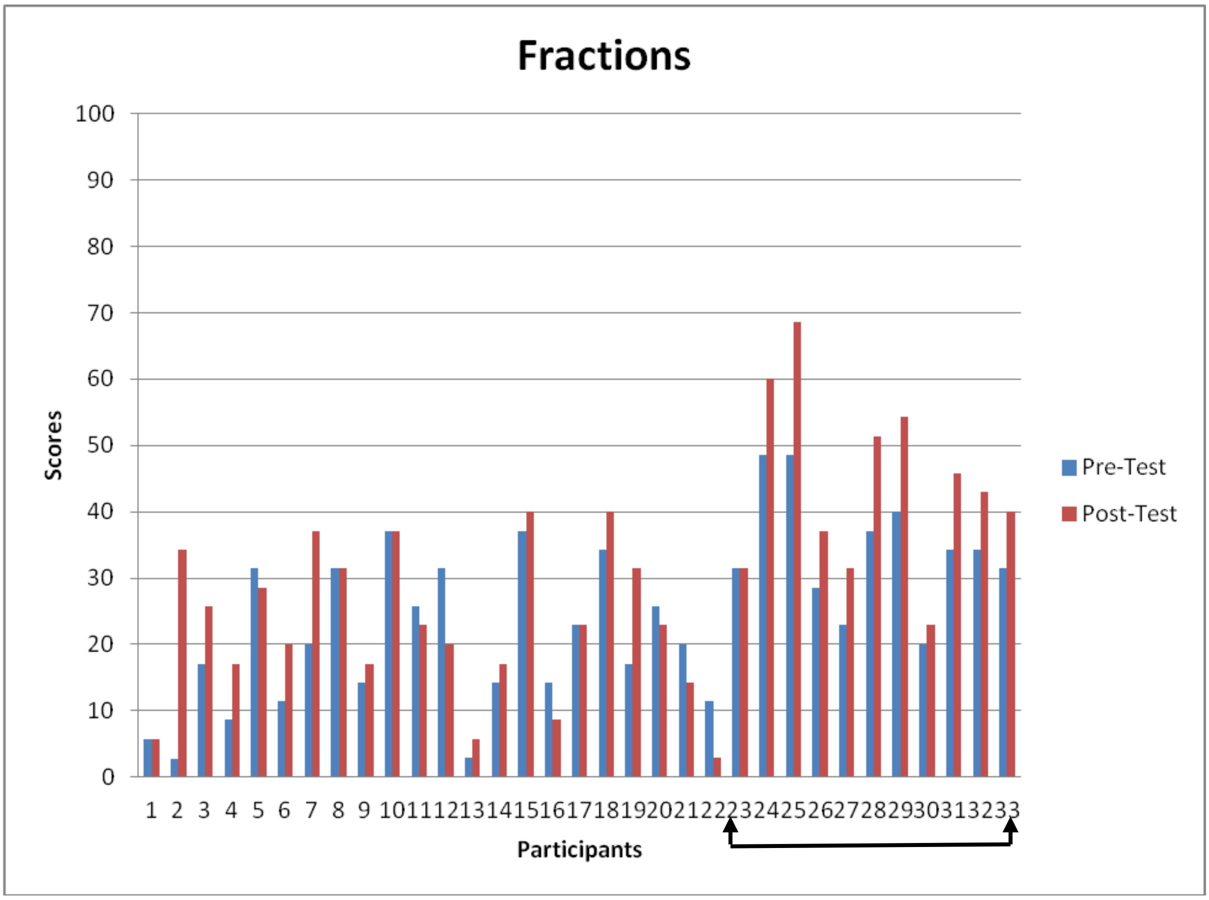


Figure 4.1: Pre- and post-test marks on fractions. (23 – 33 indicates the research participants).

Table 4.2 below shows the analysis of the fractions pre- and post-test by using a grading scale, from poor to excellent, for the participants' performance per question (Appendix 5B). For example, in Question 1, of the pre-test, nine of the participants did poorly, none did fairly, one had a good response to the question and one responded excellently to the question. On the other hand, in Question 1 of the post-test, no one had a poor or fair response to the question, one had a good response and 10 participants responded excellently to the question.

In Question 8, in the pre-test, where the participants needed to apply what they have learned from the fractions video clips on the addition of fractions, six responded poorly or fairly, while four had good response and one responded excellently to the question. In the same question, in the post-test, no one responded poorly, three participants had fair responses, four had good responses and four responded excellently.

In Question 9, in the pre-test, where the participants needed to apply what they have learned from the fractions video clips on the subtraction of fractions, eight responded poorly or fairly, while two had good response and one responded excellently to the question. In the same question, in the post-test, three responded poorly, one participants had fair responses, seven had good responses and no one responded excellently. This shows that although the participants grasped the addition of fractions well, they were still struggling with the subtraction of fractions after their engagement of the video clips. Appendices 5A and 5B contains the definitions of the ratings that were captured in Figure 4.2.

Table 4.2: The fractions pre- and post-test using a grading scale

Question Number	Total Number of participants		Grading
	Pre-test	Post-test	
Question1	9	0	Poor
	0	0	Fair
	1	1	Good
	1	10	Excellent
Question 2	7	3	Poor
	0	1	Fair
	0	1	Good
	4	6	Excellent
Question 3a	3	1	Poor
	4	3	Fair
	0	2	Good
	4	5	Excellent
Question 4	0	0	Poor
	6	2	Fair
	0	1	Good
	5	8	Excellent
Question 5	1	1	Poor
	1	4	Fair
	1	0	Good
	8	6	Excellent
Question 6	2	0	Poor
	0	0	Fair
	1	2	Good
	8	9	Excellent
Question 7	2	3	Poor
	0	0	Fair
	0	0	Good
	9	8	Excellent
Question 8	2	0	Poor
	4	3	Fair
	4	4	Good
	1	4	Excellent
Question 9	1	3	Poor
	7	1	Fair
	2	7	Good
	1	0	Excellent
Question 10	4	2	Poor
	4	5	Fair
	1	0	Good
	2	4	Excellent
Question 11	4	5	Poor
	5	6	Fair
	1	0	Good
	1	0	Excellent

1= Poor 2 = Fair 3 = Good 4 = Excellent

Tables 4.3 and 4.4 below are the analysis of the pre- and post-test of the fractions tests by using a grading scale where 1 represents a poor response to a question, 2 represents a fair response to the question, 3 represents a good response to the question and 4 an excellent response to the question.

Table 4.3: Fractions pre-test grading for participants

	Question Number										
Participants	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**
1	1	1	1	2	3	4	1	2	2	2	2
2	1	1	2	4	4	1	4	2	2	2	2
3	1	4	4	2	4	4	4	3	3	4	2
4	1	1	1	4	4	4	1	1	2	1	1
5	1	1	1	2	1	4	4	2	2	2	1
6	1	4	4	4	4	4	4	1	1	1	1
7	1	1	4	4	4	4	4	2	2	1	2
8	1	1	2	2	4	3	4	3	2	2	2
9	3	4	2	4	4	4	4	4	4	4	4
10	1	1	4	2	4	4	4	3	3	3	3
11	4	4	2	2	2	1	4	3	2	1	1

Table 4.4: Fractions post-test grading for participants

	Question Number										
Participants	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**
1	4	1	1	2	4	4	4	2	1	2	1
2	4	3	4	4	1	4	1	3	3	2	2
3	4	4	4	4	2	4	4	4	3	4	1
4	3	2	2	4	2	4	1	3	1	1	1
5	4	1	3	3	4	4	4	2	1	1	1
6	4	4	3	4	4	3	4	4	3	4	1
7	4	4	4	4	4	4	4	2	2	2	2
8	4	1	2	4	2	3	1	3	3	2	2
9	4	4	4	4	4	4	4	4	3	4	2
10	4	4	4	4	4	4	4	4	3	4	2
11	4	4	2	2	2	4	4	3	3	2	2

4.4.2 Learner 1

Pre-and post-test

In Question 1 in the pre-test, Learner 1 could correctly identify only two of the three rectangles' parts that were shaded, while in the post-test she could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was able to name the fraction part correctly. In Question 3 in the pre-and post-test, the participant gave good reasons for choosing the specific part of the chocolate. In Question 4 in the pre-and post-test, she was able to give a good reason why she chose the specific part of the chocolate. In Question 5 in the pre-and post-test, Learner1 was able to mention the whole and the fraction part correctly. In Question 6 in the pre-and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre-and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre-and post-test, Learner 1 shaded the fraction parts correctly to show how the two fractions could be added and gave the correct answer for the addition of the two fractions. In Question 9 in the pre-test, she shaded the fraction parts correctly to show how the two fractions could be subtracted and gave the correct answer for the subtraction of the two fractions. In the post-test, she attempted to shade the fraction parts but did it incorrectly. She, however, was able to subtract the fractions by using the conventional method. In Question 10, in the pre-and post-test, the participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions could be added correctly. In Question 11 in the pre-test, the participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions could be subtracted correctly. In the post-test, the participant attempted to divide the rectangle into different equal parts but did it incorrectly. She was able to subtract the fractions using a conventional method.

Presentation

Learner 1 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. In Figure 4.4.2a Learner 1 showed which fraction part overlapped.

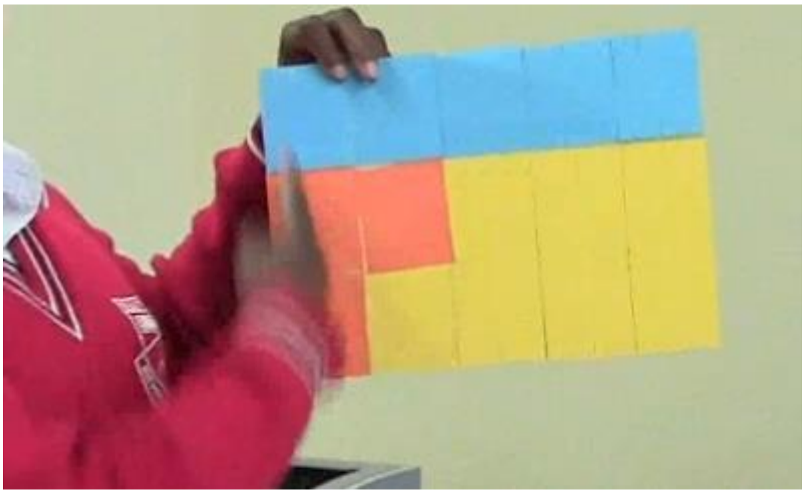


Figure 4.4.2a: Learner 1 shows the fraction parts that overlap

She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. She was also able to give the correct answer for the addition and the subtraction of the two fractions. *Uhm, from the videos I learnt that the one where you minus the 1/5, you see, this is the 1/5 (actually points towards the 1/3) and this is the 1/3 (actually points towards the 1/5), ja. If we subtract this, then these three are going to die out with these, with the three of these, so we are going to be have 2/15. Because these blocks all in all are 15 and if we subtract a 1/5 from a 1/3 then we are left with 2/15 blocks. We have 2 left over all of these 15 blocks, because these 3 (from 1/5) die out with these three (from 1/3). And then the one about- the one of here (the addition example), I didn't put this one (the overlapping block) over here, because it's easy. I just count this one double. So it's 1, 2, 3, 4 (counts the one block with two overlapping blocks twice), 5, 6, 7. You see there's a 7 out of a 12. Because it 4 times 3 and 3 times 4, which is a 12.*

When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions. *A 1/5 and a 1/6, I'm going to just increase this one, maybe divide this one into a half, a half, a half, and then I have 6. And this-this- these are 5 so I don't need to add or subtract. And if it's a 1/6 and a 1/10, then I just divide it (the fifths), these into two and then I get a 1/10* (She shows how she will divide her card to demonstrate the subtraction of fractions in Figure 4.4.2 b).

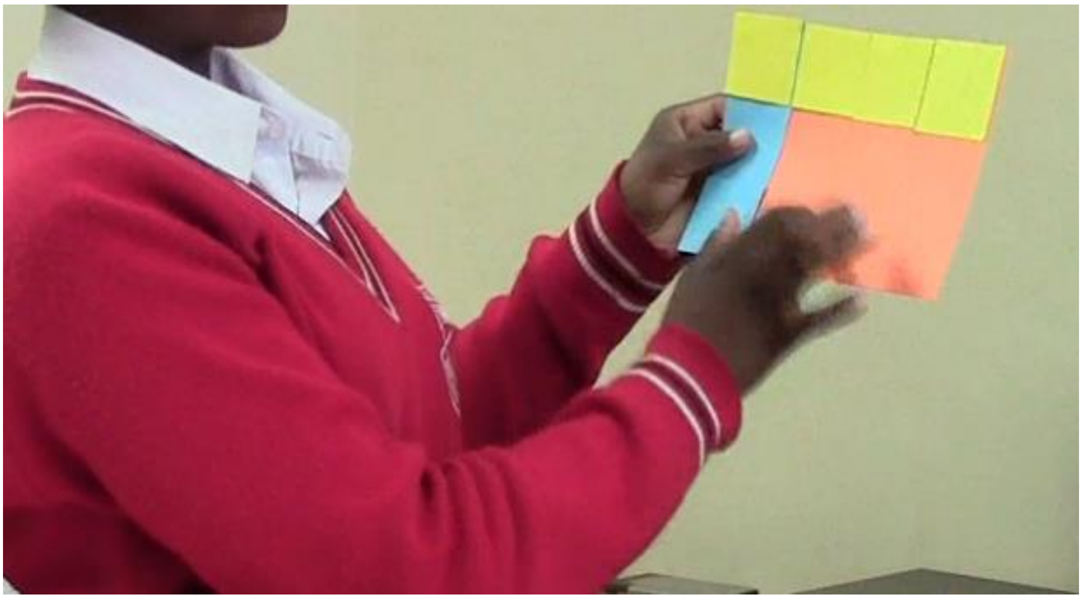


Figure 4.4.2b: Learner 1 shows how she used manipulatives to demonstrate the subtraction of fractions

When probed on what the number of equal parts represent in a fraction, she could not explain.

General Findings

Learner 1's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking on how to add and subtract fractions. Learner 1 was able to solve problems correctly that involved fractions in her post-test. She was also able to use endorsed narratives about fractions in her presentation. Learner 1's description of the addition and subtraction of fractions in her presentation and partially in her post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-other into a discourse-for-herself. Learner 1 thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. This also indicates that there was an enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions after her engagement with the VITALmaths video clips.

4.4.3 Learner 2

Pre-and post-test

In Question 1 in the pre-test, Learner 2 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test he could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre-test, he was unable to name the fraction part correctly at all, while in the post-test he had some idea of how to name the fraction part. In Question 3 in the pre-test, the participant chose part of the chocolate, but his reason for choosing this part did not match the answer that he gave. In the post-test the participant gave a good reason for choosing the part of the chocolate. In Question 4 in the pre-and post-test, he was able to give a good reason for choosing the specific part of the chocolate. In Question 5 in the pre-test, Learner 2 was able to mention the whole and the fraction part correctly. He, however, was not able to mention any of the fraction parts correctly in the post-test. In Question 6 in the pre-test, he did not use a drawing and could not mention the fraction part that each person gets. In the post-test had an accurate drawing of the different equal parts and could accurately mention the fraction part that each person gets. In Question 7 in the pre-test, the participant gave the correct answer with a good reason for choosing the specific answer. In the post-test the participant gave an incorrect answer and reason. In Question 8 in the pre-test, Learner 2 did not use the shading of the fraction parts to show how the two fractions could be added but used a conventional method correctly to add the fractions. In the post-test he attempted to shade the fraction parts but did it incorrectly. He was however able to correctly add the fractions by using a conventional method. In Question 9 in the pre-test, he did not use the shading of the fraction parts to show how the two fractions could be subtracted but used a conventional method correctly to subtract the fractions. In the post-test he attempted to shade the fraction parts but did it incorrectly. He was however able to correctly subtract the fractions by using a conventional method. In Question 10, in the pre-and post-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. He was able to add the fractions correctly by using a conventional method in the pre- and post-test. In Question 11 in the pre-and post-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. He was able to subtract the fractions correctly by using a conventional method in the pre- and post-test.

Presentation

Learner 2 demonstrated the addition and subtraction of fractions by using different coloured cards. He was able to divide his card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. In Figure 4.4.3 Learner 2 showed how he will divide his paper if he needed to subtract other fractions.

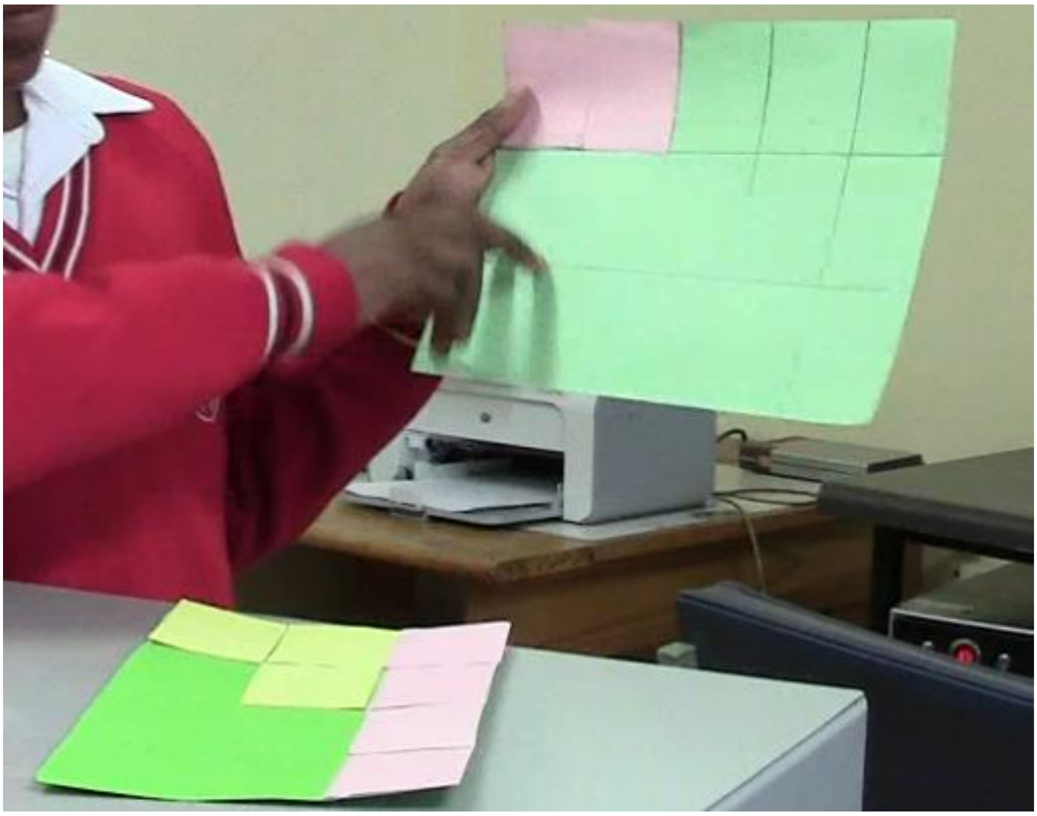


Figure 4.4.3: Learner 2 shows how he will use his card if he needs to add other fractions

He was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. Learner 2, however, talked about 7 is to 12 instead of $\frac{7}{12}$ (seven-twelfths), when he presented his answer to the addition of the fractions *(I* *take 4 plus 3 and I get 7 is to 12).* When he was asked how he would go about adding other fractions using manipulatives, he was able to explain how he would divide his card to add the fractions.

General Findings

Learner 2's use of words was partially consistent with the words used by others when explaining how to add or subtract fractions in his post-test and presentation. He named the fraction as 7 is to twelve instead of seven-twelfths. In his presentation he was able to use manipulatives correctly to show his thinking on how to add and subtract fractions. He, however, could not translate the knowledge that he gained from his engagement with the video clips to his post-test. Although he attempted to use shading to explain the addition and subtraction of fractions, he did it incorrectly in his post-test. Learner 2 was able to solve some of the problems correctly that involved fractions in his post-test. He was unable to use endorsed narratives about fractions consistently during his presentation. Learner 2's description of the addition and subtraction of fractions in his presentation and in his post-test was inconsistent with the meta-rules that guide the discursive routines of this mathematical discourse. He did not fully demonstrate that, through his exploration of and interaction with the discourse-for-others, he was able to make the discourse-for-others into a discourse-for-himself. Learner 2 thus did not fully demonstrate features of an autonomous learner after his engagement with the VITALmaths video clips. There was also only a partial enhancement in his understanding of how to use manipulatives in the addition and subtraction of fractions after his engagement with the VITALmaths video clips.

4.4.4 Learner 3

Pre-and post-test

In Question 1 in the pre-test, Learner 3 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test he could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre-and post-test, he was unable to name the fraction part correctly at all. In Question 3 in the pre-and post-test, the participant chose part of the chocolate, but his reason for choosing this part did not match the answer that he gave. In Question 4 in the pre-test, the participant chose part of the chocolate but his reason for choosing this part did not match the answer that he gave. He was able to give a good reason for choosing the specific part of the chocolate in post-test. In Question 5 in the pre-test, Learner 3 was able to mention the whole and the fraction part correctly. He, however, was only able to mention one of the two fraction parts correctly in the post-test. In Question 6 in the pre-and post-test, he used a drawing but the equal parts were inaccurate but he was able to mention the fraction part that each one gets. In Question 7 in the pre-test, the participant gave the correct answer with a good reason for choosing the specific answer. In the post-test the participant gave an incorrect answer and reason. In Question 8 in the pre- and post-test, Learner 3 attempted to shade the fraction parts but did it incorrectly. He was however able to correctly add the fractions by using a conventional method. In Question 9 in the pre-test, he did not use the shading of the fraction parts to show how the two fractions could be subtracted but used a conventional method correctly to subtract the fractions. In the post-test he attempted to shade the fraction parts but did it incorrectly. He was however able to correctly subtract the fractions by using a conventional method. In Question 10, in the pre- and post-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. He was able to add the fractions correctly by using a conventional method in the pre- and post-test. In Question 11 in the pre- and post-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. He was able to subtract the fractions correctly by using a conventional method in the pre- and post-test.

Presentation

Learner 3 only demonstrated the addition of fractions by using different coloured cards. He was able to divide his card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of fractions. In Figure 4.4.4a Learner 3 showed how he counted the fraction pieces to get the solution for the addition of fractions by pointing to the fraction pieces

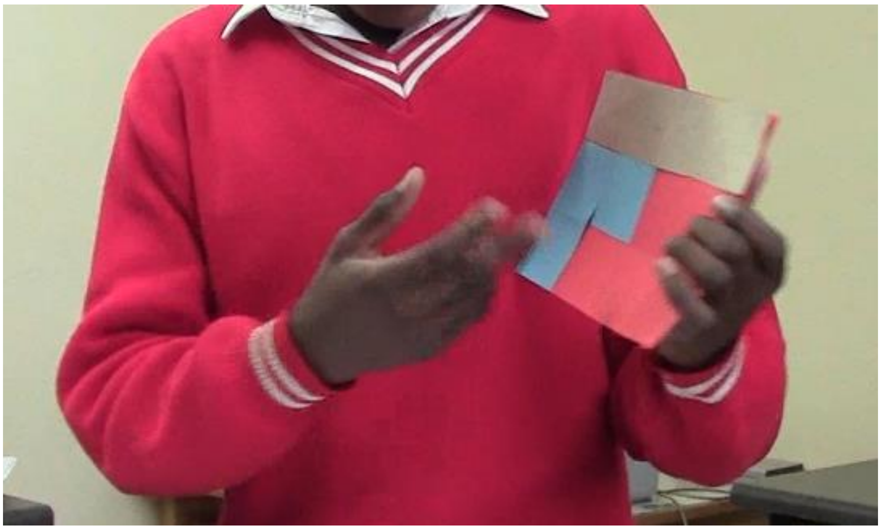


Figure 4.4.4a: Learner 3 shows how he used the card to demonstrate the addition of fractions

He was thus able to demonstrate the addition of fractions using manipulatives. Learner 3, however, seemed unsure of how to explain the addition because he stuttered and was looking for support when he explained his workings *(I didn't- I didn't understand to- I didn't understand to fold it. Because when I fold this fold this paper it give me (brief pause) 16-16 blocks. So that I cut it to get to (mumbles)- I cut it to-to have 12 blocks).* When he was asked how he would go about adding other fractions using manipulatives, he was able to explain how he would divide his card to add the fractions (*I will increase at the top and I will increase* (gestures with his hand to the bottom). In figure 4.4.4b Learner 3 showed how the fraction pieces cancelled out when fractions are subtracted.

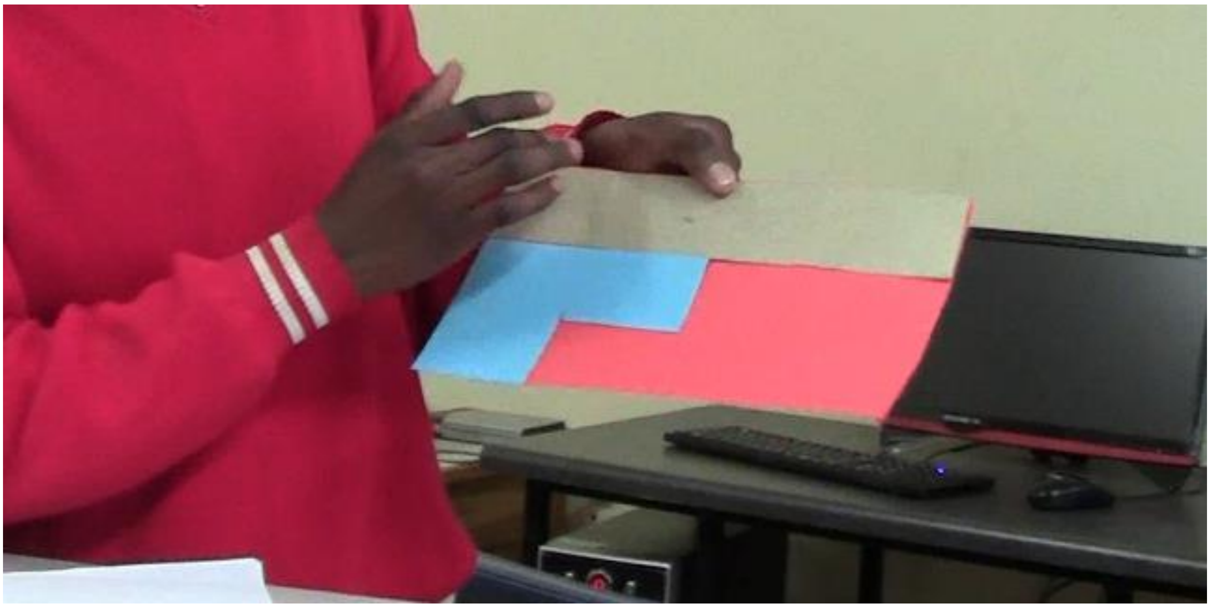


Figure 4.4.4b: Learner 3 shows how he will increase at the top if he has to add other fractions

General Findings

Learner 3's use of words was partially consistent with the words used by others when explaining how to add fractions in his post-test and presentation. In his presentation he was able to use manipulatives correctly to show his thinking on how to add fractions. He could not translate the knowledge that he gained from his engagement with the video clips to his post-test. Although he attempted to use shading to explain the addition and subtraction of fractions, he did it incorrectly in his post-test. Learner 3 was only able to solve some of the problems correctly that involved fractions in his post-test. He was unable to use endorsed narratives about fractions correctly during his presentation. Learner 3's description of the addition and subtraction of fractions in his presentation and in his post-test was inconsistent with the meta-rules that guide the discursive routines of this mathematical discourse. He did not fully demonstrate that, through his exploration of and interaction with the discourse-for-others, he was able to make the discourse-for-other into a discourse-for-himself. Learner 3 thus did not fully demonstrate features of an autonomous learner after his engagement with the VITALmaths video clips. This indicates that there was only a partial enhancement in his understanding of how to use manipulatives in the addition and subtraction of fractions after his engagement with the VITALmaths video clips.

4.4.5 Learner 4

Pre-and post-test

In Question 1 in the pre-test, Learner 4 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test she could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre-test, she was unable to name the fraction part correctly at all. In the post-test she was able to name fraction part correctly. In Question 3 in the pre- and post-test, the participant gave good reasons for choosing the specific part of the chocolate. In Question 4 in the pre- and post-test, she was able to give a good reason why she chose the specific part of the chocolate. In Question 5 in the pre- and post-test, Learner 4 was able to mention the whole and the fraction part correctly. In Question 6 in the pre- and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre- and post-test, Learner 4 did not use shading of the fraction parts to show how the two fractions are added. She was however able to correctly add the fractions by using a conventional method in the pre- and post-test. In Question 9 in the pre- and post-test, she did not use the shading of the fraction parts to show how the two fractions could be subtracted but used a conventional method correctly to subtract the fractions. In Question 10, in the pre-test, the participant did not attempt to divide the rectangle into different equal parts or attempt to add the fractions by using a conventional method. She attempted to divide the rectangle into different equal parts, but did it incorrectly in the post-test. However, she was able to add the fractions correctly by using a conventional method in the post-test. In Question 11 in the pre- and post-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. However, she was able to subtract the fractions correctly by using a conventional method in the pre- and post-test.

Presentation

Learner 4 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. She was also able to give the correct answer for the addition and the subtraction of the two fractions *(When I tried to add the fifth, erh the third and quarter, I drew the third and the quarter. (Demonstrates on colour card by showing what she had done.) This one is my third and this one is my quarter. When I, I take this one here so that it doesn't overlap. So a third plus a quarter is 1, 2, 3, 4, 5, 6, 7, 7/12).* She showed how she did the addition of fractions in Figure 4.4.5a.

Figure 4.4.5a: Learner 4 shows how she used the fraction parts to add fractions

When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions (as shown in Figure 4.4.5b).

Figure 4.4.5b: Learner 4 shows how she will fold the card to add other fraction

General Findings

Learner 4's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She was only able to use manipulatives correctly to show her thinking on how to add and subtract fractions in her presentation. She, however, was not able to translate the knowledge gained from the video clips or her presentation to her post-test. Learner 4 was able to solve problems correctly that involved fractions in her post-test. She was able to use endorsed narratives about fractions in her presentation. Learner 4's description of the addition and subtraction of fractions in her presentation and partially in her post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She partially demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-other into a discourse-for-herself. Learner 4 thus did not fully demonstrate features of an autonomous learner after her engagement with the VITALmaths video clips. This indicates that there was a partial enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions after her engagement with the VITALmaths video clips.

4.4.6 Learner 5

Pre-and post-test

In Question 1 in the pre-test, Learner 5 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test she could identify two of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was unable to name the fraction part correctly at all. In Question 3 in the pre- and post-test, the participant did not attempt the question at all. In Question 4 in the pre- and post-test, she was able to give a good reason why she chose the specific part of the chocolate. In Question 5 in the pre-test, Learner 5 was able to mention the whole and the fraction part correctly. In the post-test she was only able to mention one of the two parts that were represented. In Question 6 in the pre- and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre- and post-test, the participant gave an incorrect answer and reason. In Question 8 in the pre-test, Learner 5 did not use shading of the fraction parts to show how the two fractions are added or add the fractions by using a conventional method. In the post-test she attempted to shade the fraction parts but did it incorrectly. She was however able to add the fractions by using a conventional method. In Question 9 in the pre- and post-test, she did not use the shading of the fraction parts to show how the two fractions could be subtracted. She only used a conventional method to subtract the fractions in her pre-test. In Question 10, in the pre- and post-test, the participant did not attempt to divide the rectangle into different equal parts or attempt to add the fractions by using a conventional method. In Question 11 in the pre- and post-test, the participant did not attempt to divide the rectangle into different equal parts or attempt to subtract the fractions by using a conventional method.

Presentation

Learner 5 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. In Figure 4.4.6a she showed where she put the fraction piece that overlapped.

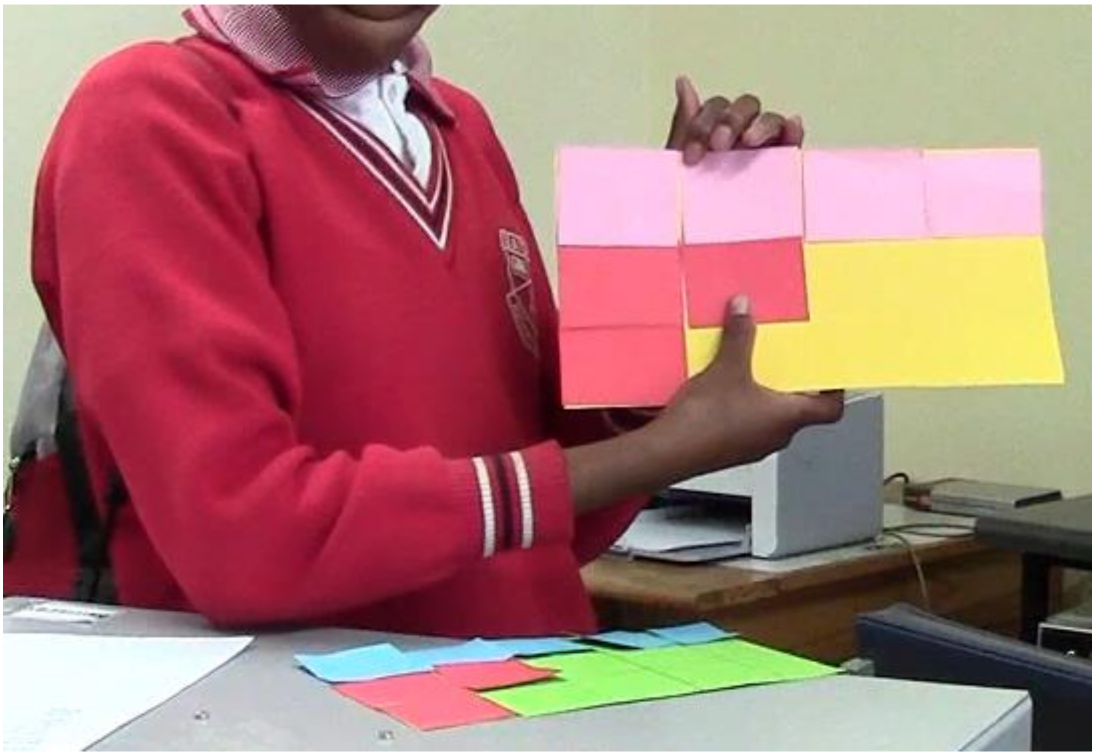


Figure 4.4.6a: Learner 5 shows where she put the fraction part that overlapped

Although she made a minor error when she gave the answer for the subtraction of the fractions as 1 over 15 instead of 2 over 15, she was able to give the correct answer for the addition of the fractions. When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions (Figure 4.4.6b).

Figure 4.4.6b: Learner 5 shows how she will fold the card if she has to add other fractions

General Findings

Learner 5's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She was only able to use manipulatives correctly to show her thinking on how to add and subtract fractions in her presentation. She, however, was not able to translate the knowledge gained from the video clips or her presentation to her post-test. Learner 5 was able to solve three problems correctly that involved fractions in her post-test. She was able to use endorsed narratives about fractions in her presentation. Learner 5's description of the addition and subtraction of fractions in her presentation was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She partially demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-others into a discourse-for-herself. Learner 5 thus did not fully demonstrate features of an autonomous learner after her engagement with the VITALmaths video clips. This indicates that there was a partial enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions after her engagement with the VITALmaths video clips.

4.4.7 Learner 6

Pre-and post-test

In Question 1 in the pre-test, Learner 6 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test she could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre-test, she was unable to name the fraction part correctly and in the post-test she was able to name the fraction part correctly. In Question 3 in the pre- and post-test, the participant gave good reasons for choosing the specific part of the chocolate. In Question 4 in the pre-test, the participant chose part of the chocolate but her reason for choosing this part did not match the answer that she gave. She was able to give a good reason for choosing the specific part of the chocolate in post-test. In Question 5 in the pre- and post-test, Learner 6 was able to mention the whole and the fraction part correctly. In Question 6 in the pre- and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre-test, Learner 6 did not use shading of the fraction parts to show how the two fractions are added. She was however able to correctly add the fractions by using a conventional method. In the post-test Learner 6 shaded the fraction parts correctly to show how the two fractions could be added and gave the correct answer for the addition of the two fractions. In Question 9 in the pre-test, Learner 6 did not use shading of the fraction parts to show how the two fractions are subtracted. She was however able to correctly subtract the fractions by using a conventional method. In the post-test, she attempted to shade the fraction parts but did it incorrectly. She, however, was able to subtract the fractions by using the conventional method. In Question 10, in the pre-test, the participant attempted to divide the rectangle into different equal parts but did incorrectly. She, however, was able to add the fractions correctly by using a conventional method in the pre-test. In the post-test the participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions could be added correctly. In Question 11 in the pre- and post-test, the participant attempted to divide the rectangle into different equal parts but did it incorrectly. She was able to subtract the fractions using a conventional method in the pre- and post-test.

Presentation

Learner 6 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. This is how she explained: *What I did is that I took my A4 paper, and I covered my 1/3, this is my third (shows on card), so I covered my other part, so then this is my 1/3. Then I put it this way, I had five parts. So I covered my first part, so this is 1/5. So because this other part is covered twice, so it's overlapping. So I removed it here so that it does not overlap. I put it here, and put it here and prove that 1/3 minus 1/5 is equal to 2/15. Because this cancels this, this one this, this this and I was left with 2 of the, of the, I was left with 2 of the 15 cards, because all in all there are 15. So my 1/3 minus 1/5 is equals to 2/15.* In Figure 4.4.7a Learner 6 pointed to the fraction pieces that she added.

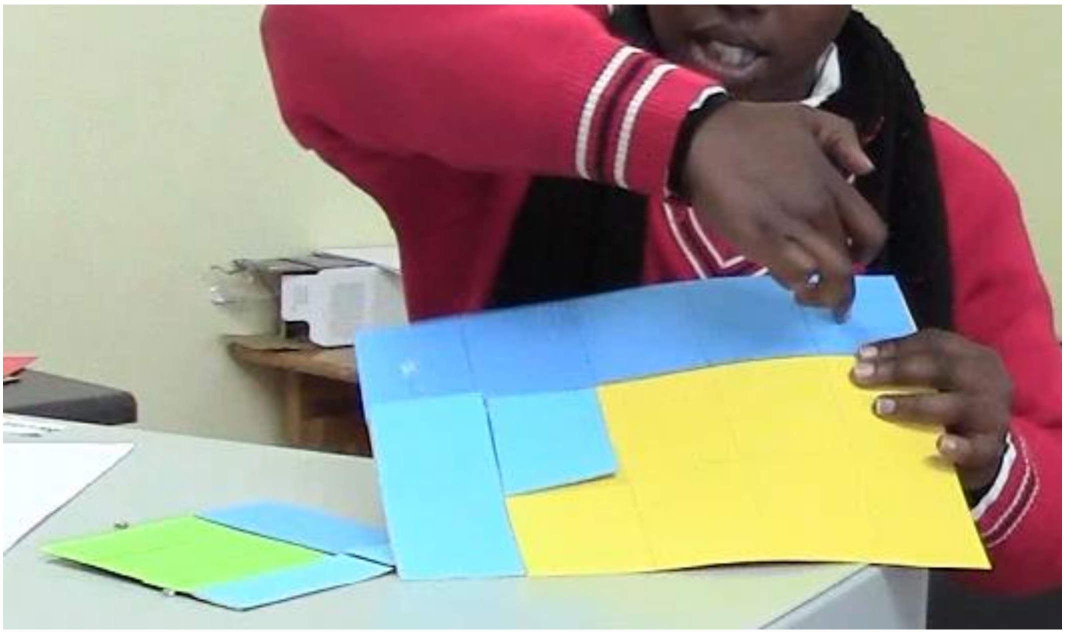


Figure 4.4.7a: Learner 6 shows how the fraction parts cancel out when subtracting fractions

She was able to give the correct answer for the addition and subtraction of the fractions. When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions *(If I had to add the fifth and the sixth, I think I would, let me use this one, because this one is a fifth I would add the another two and the other one I would add this way. (*In Figure 4.4.7b). She shows where she would add the extra row and column on the colour card).

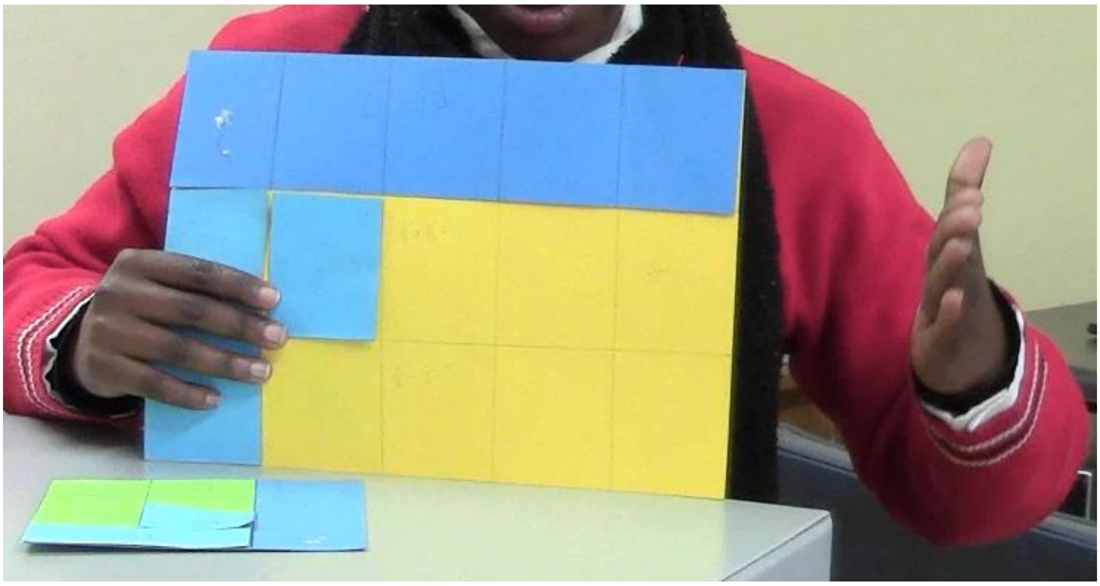


Figure 4.4.7b: Learner 6 explains how she will extend her cut if she needs to add other fractions

General Findings

Learner 6's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She was only able to use manipulatives correctly to show her thinking on how to add and subtract fractions in her presentation. In the post-test she was able to show her thinking about the addition of fractions by shading the given rectangles. She attempted to shade the rectangles in her post-test to show her thinking on the subtraction of fractions, but did it incorrectly. She was thus partially able to translate the knowledge gained from the video clips or her presentation to her post-test. Learner 6 was able to solve all the problems correctly that involved fractions in her post-test. She was able to use endorsed narratives about fractions in her presentation and post-test. Learner 6's description of the addition and subtraction of fractions in her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Although her attempt to demonstrate her thinking about the subtraction of fractions was incorrect, Learner 6, through her exploration of and interaction with the discourse-for-others, was able to make the discourse-for-others into a discourse-for-herself. Learner 6 thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. This indicates that there was an enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions, after her engagement with the VITALmaths video clips.

4.4.8 Learner 7

Pre-and post-test

In Question 1 in the pre-test, Learner 7 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test she could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was able to name the fraction part correctly. In Question 3 in the pre- and post-test, the participant gave good reasons for choosing the specific part of the chocolate. In Question 4 in the pre-test, the participant chose part of the chocolate but her reason for choosing this part did not match the answer that she gave. She was able to give a good reason for choosing the specific part of the chocolate in post-test. In Question 5 in the pre-test, Learner 7 was able to mention the whole and the fraction part correctly. She was only able to mention one of the two parts that were presented in the post-test. In Question 6 in the pre- and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre-test, Learner 7 attempted to shade the fraction parts but did it incorrectly. She was however able to correctly add the fractions by using a conventional method. In the post-test Learner 7shaded the fraction parts correctly to show how the two fractions could be added and gave the correct answer for the addition of the two fractions. In Question 9 in the pre- and post-test, Learner 7 attempted to shade the fraction parts but did it incorrectly. She was however able to correctly subtract the fractions by using a conventional method. In Question 10 in the pre- and post-test, the participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions could be added correctly. In Question 11 in the pre-test, the participant attempted to divide the rectangle into different equal parts but did it incorrectly. She was able to subtract the fractions using a conventional method in the pre-test. In the post-test she did not attempt to divide the rectangle into different equal parts or subtract the fractions by using a conventional method.

Presentation

Learner 7 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. This was how she explained the addition of the fractions: *First of all I started with the one that says 1/3 plus ¼. This is how I did it: I folded my into my thirds, this is my thirds (points at thirds) and this is my fourths or can I say my quarters (points at fourths). And then, for me to show you that I can add them- I can add them all. I add yes, I putted this one that are my fourth over here and then I put my third over here. Now you this one, it overlaps. Because I don't want it to overlap, I decided to take this part and put it here, so that I can count all. This is 1, 2, 3, 4, 5, 6, 7. This cards that I put here, they add up to 7. But all in all if I count my-my small squares they are 12. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So 1/3 plus ¼ gave me this 7 (colour blocks stuck on bigger card) over the whole 12. This is how I did mine.* She showed in Figure 4.4.8a how she added the two fractions by using manipulatives.

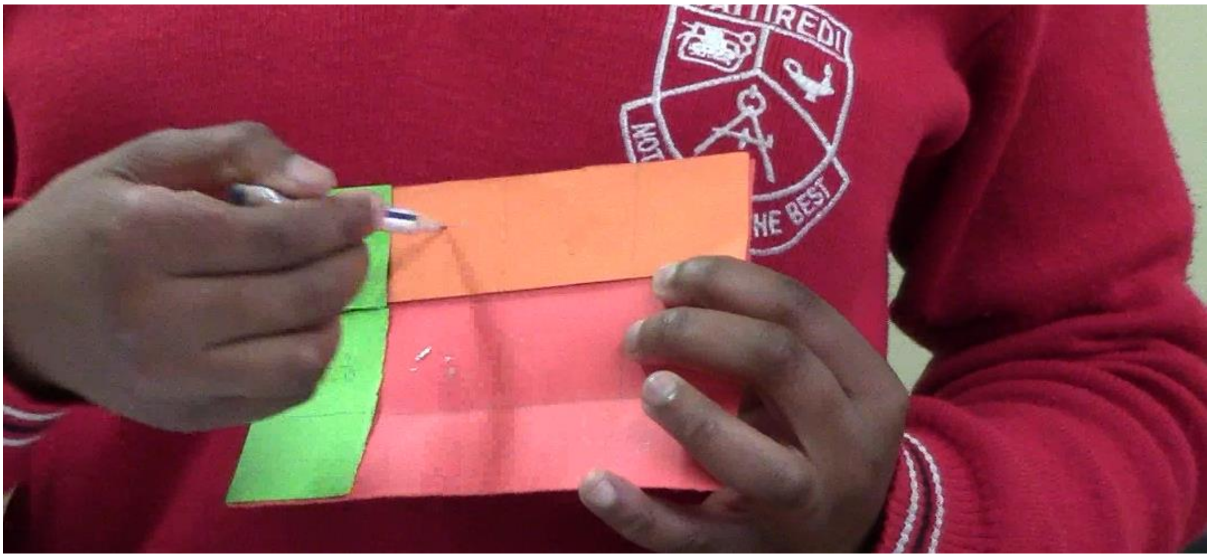


Figure 4.4.8a: Learner 7 indicates the different fraction parts

She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. She was able to give the correct answer for the addition and subtraction of the fractions. When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions. In Figure 4.4.8b she showed how she would extend her card to add other fractions.

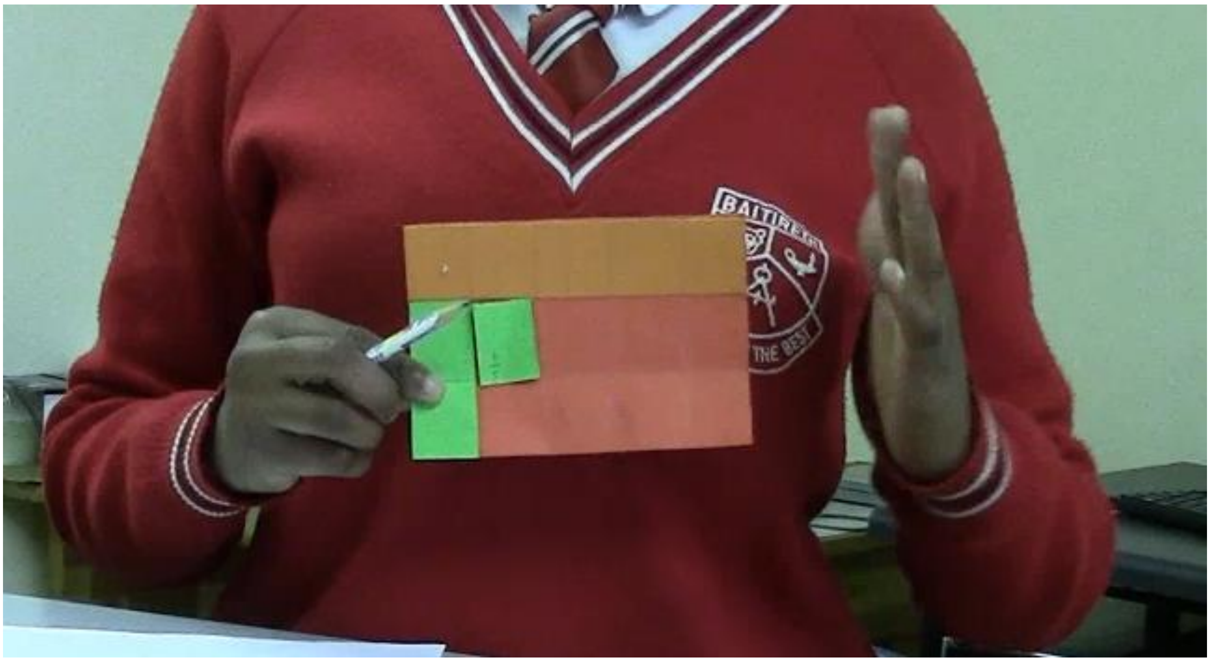


Figure 4.4.8b: Learner 7 shows how she will extend her card to add other fractions

General Findings

Learner 7's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She was only able to use manipulatives correctly to show her thinking on how to add and subtract fractions in her presentation. In the post-test she was able to show her thinking about the addition of fractions by shading the given rectangles. She attempted to shade the rectangles in her post-test to show her thinking on the subtraction of fractions, but did it incorrectly. She was thus partially able to translate the knowledge gained from the video clips or her presentation to her post-test. Learner 7 was able to solve all the problems correctly that involved fractions in her post-test. She was able to use endorsed narratives about fractions in her presentation and post-test. Learner 7's description of the addition and subtraction of fractions in her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Although her attempt to demonstrate her thinking about the subtraction of fractions was incorrect, Learner 7, through her exploration of and interaction with the discourse-for-others, was able to make the discourse-for-other into a discourse-for-herself. Learner 7 thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. This indicates that there was an enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions, after her engagement with the VITALmaths video clips.

4.4.9 Learner 8

Pre-and post-test

In Question 1 in the pre-test, Learner 8 could correctly identify two of the three rectangles' parts that were shaded, while in the post-test she could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was able to name the fraction part correctly. In Question 3 in the pre- and post-test, the participant gave good reasons for choosing the specific part of the chocolate. In Question 4 in the pre- and post-test, the participant was able to give a good reason for choosing the specific part of the chocolate. In Question 5 in the pre- and post-test, Learner 8 was able to mention the whole and the fraction part correctly. In Question 6 in the pre-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In the post-test her drawing was inaccurate but she was able to mention correctly the part that each person will get. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre-test, Learner 8 did not attempt to shade the fraction parts. She was also unable to correctly add the fractions by using a conventional method. In the post-test Learner 8 shaded the fraction parts correctly to show how the two fractions could be added and gave the correct answer for the addition of the two fractions. In Question 9 in the pre-test, Learner 8 did not attempt to shade the fraction parts to show how the two fractions could be subtracted and she did not try a conventional method to subtract the fractions. In the post-test Learner 8 attempted to shade the fraction parts but did it incorrectly. She was however able to correctly subtract the fractions by using a conventional method. In Question 10 in the pre-test, the participant did not attempt to divide the rectangle into different equal parts to show how the two fractions could be added or use a conventional method to add the two fractions. In the post-test the participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions could be added correctly. In Question 11 in the pre- and post-test, she did not attempt to divide the rectangle into different equal parts or subtract the fractions by using a conventional method to subtract the two fractions.

Presentation

Learner 8 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. She was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. This is how she explained*: I have a paper here and I'm demonstrating uhm, 1/3 minus 1/5 and I'm going to show you how you get the answer. And, this is my paper and I'm going to fold it to show 1/3. You fold it like this and then fold it like this (demonstrates by folding the card), and this is our three eighths, 1, 2 and 3. And I am going to use a blue paper* *to demonstrate 1/3. Yes, it's 1/3. And 1/5, I'm using the same paper (demonstrates how she folded the paper to represent 1/5). And I fold 1 and 2 and 3 and 4 and 1, 2, 3, 4. So I'm going to use this yellow one,* uhm, *this one will represent 1/5. And 1/5 (whispers). Participant shows in the image below how she folded her card to show* $^{1}\!/_{3}$ *and* $^{1}\!/_{5}$. In Figure 4.4.9a she showed how she folded her card to get 1/3.

Figure 4.4.9a: Learner 8 shows how she divided her card into thirds

She was able to give the correct answer for the addition and subtraction of the fractions. In Figure 4.4.9b she showed how the fraction pieces cancelled out during the subtraction of fractions.

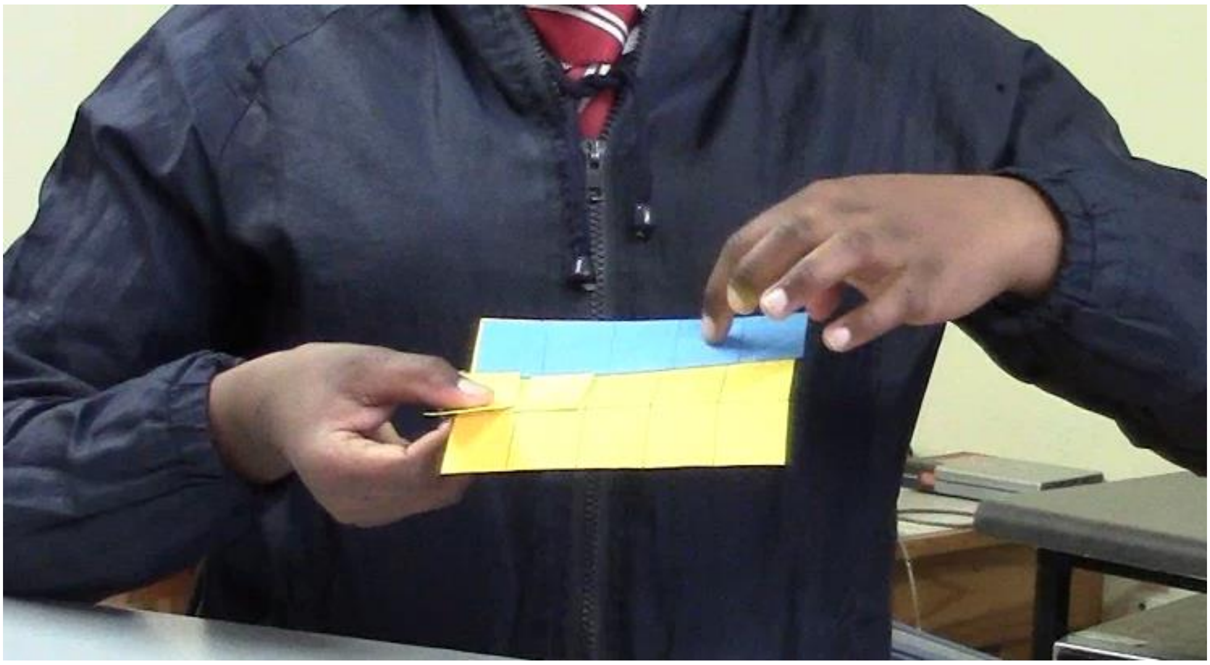


Figure 4.4.9b: Learner 8 shows how the fraction parts will cancel out when subtracting fractions

When she was asked how she would go about adding other fractions using manipulatives, she was able to explain how she would divide her card to add the fractions.

General Findings

Learner 8's use of words was consistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She was only able to use manipulatives correctly to show her thinking on how to add and subtract fractions in her presentation. In the post-test she was able to show her thinking about the addition of one of the fractions by shading the given rectangles. She attempted to shade the rectangles in her post-test to show her thinking on the subtraction of fractions, but did it incorrectly. She was thus partially able to translate the knowledge gained from the video clips or her presentation to her post-test. Learner 8 was able to solve most of the problems correctly that involved fractions in her post-test. She was able to use endorsed narratives about fractions in her presentation and post-test. Learner 8's description of the addition and subtraction of fractions in her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse.

Although her attempt to demonstrate her thinking about the subtraction of fractions was incorrect, Learner 8, through her exploration of and interaction with the discourse-for-others, was able to make the discourse-for-others into a discourse-for-herself. Learner 8 thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips. This indicates that there was an enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions, after her engagement with the VITALmaths video clips.

4.4.10 Learner 9

Pre-and post-test

In Question 1 in the pre-test, Learner 9 could not correctly identify any of the three rectangles' parts that were shaded, while in the post-test he could identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, he was unable to name the fraction part correctly. In Question 3 in the pre-test, the participant chose part of the chocolate but his reason for choosing this part did not match the answer that she gave. In the post-test the participant gave and interesting reason for choosing the specific part of the chocolate, for example he likes chocolate. In Question 4 in the pre-test, the participant chose part of the chocolate but his reason for choosing this part did not match the answer that he gave. In the post-test the participant gave and interesting reason for choosing the specific part of the chocolate, for example he likes chocolate. In Question 5 in the pre-test, Learner 9 was unable to mention any of the two parts correctly. In the post-test he was able to mention the whole and the fraction part correctly. In Question 6 in the pre- and post-test, he was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre- and post-test, Learner 9 did not use shading of the fraction parts to show how the two fractions are added. He was however able to correctly add the fractions by using a conventional method in the pre- and post-test. In Question 9 in the pre-test, Learner 9 did not use shading of the fraction parts to show how the two fractions are subtracted. He was however able to correctly subtract the fractions by using a conventional method. In the post-test he did not attempt to shade the fraction parts to show how the two fractions could be subtracted and did not use a conventional method to subtract the two fractions. In Question 10 in the pre-test, the participant attempted to divide the rectangle into different equal parts to show how the two fractions could be added but did it incorrectly. He was however able to add the fractions by using a conventional method. In the post-test the participant did not attempt to divide the rectangle into different equal parts or use a conventional method to add the two fractions. In Question 11 in the pre- and post-test, he did not attempt to divide the rectangle into different equal parts or subtract the fractions by using a conventional method.

Presentation

Learner 9 demonstrated the addition and subtraction of fractions by using different coloured cards. He was able to divide his card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. This is how he explained the subtraction and addition of fractions: *As you can see, we have five rectangles here (demonstrates using a sheet of colour card). Then they say 1/3 minus 1/5. We need an answer here. So we have five rectangles. Then these are representing 1/5- 1/3 (corrects himself). So I say this rectangle will cancel this one, this will cancel that one, and this will cancel that one. So we have two remaining rectangles. I should have taken then out but I pritted them. So in these two (the remaining rectangles) they are going to re-repa-ja. So we have two over this fifteen (the total number of blocks on the large sheet).*

(Starts with a new example). As for this one, we have 1/3 plus ¼. We need the answer for those two. So I said, this yellow paper is for ¼ and this orange one is for ½. Because when I put it here (the orange card representing 1/3) it is 1, 2, and here representing the one (the orange card), and this (the yellow card), will also be representing the one. So I said- I said, erh, 1, 2, 3, 4, 5, 6, 7. This should be out, but its posted (the overlapping block). It's going to be like this, 1, 2, 3, 4, 5, 6, 7. Then this rectangle going to overlap, to here. So it should be 1, 2, 3, 4, 5, 6, 7
In Figure 4.4.10a he showed where the fraction pieces overlapped.

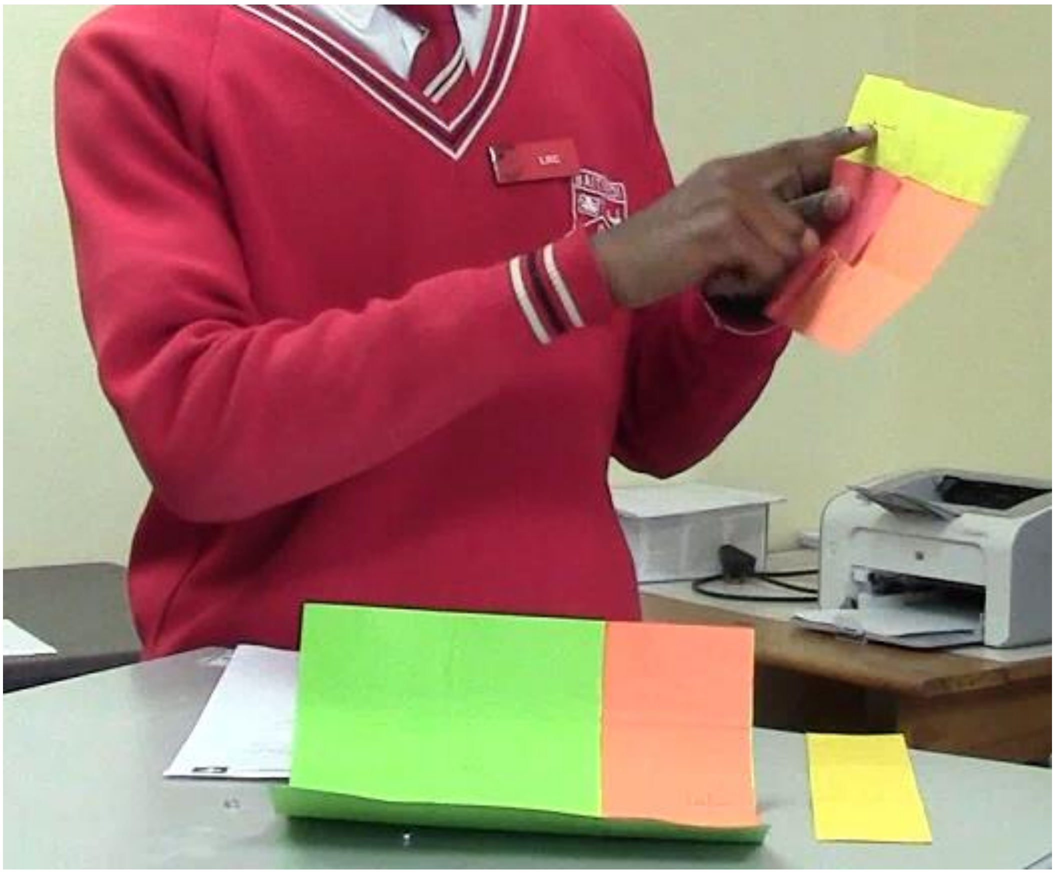


Figure 4.4.10a: Learner 9 shows how fractions are added

He was thus able to demonstrate both the addition and the subtraction of fractions using manipulatives. He was able to give the correct answer for the addition and subtraction of the fractions. When he was asked how he would go about adding other fractions using manipulatives, he was able to explain how he would divide his card to add the fractions. He said: *I think I will add 8 this side (columns) I mean this side (rows), I will make it longer this side and 6 this side (columns)* *(*Figure 4.4.10b).

Figure 4.4.10b: Learner 9 shows how he will extend his card if he needs to add other fractions

General Findings

Learner 9's use of words was partially consistent with the words used by others when explaining how to add fractions in his post-test and presentation. In his presentation he was able to use manipulatives correctly to show his thinking on how to add fractions. He could not translate the knowledge that he gained from his engagement with the video clips to his post-test. He did not attempt to use shading to explain the addition and subtraction of fractions in his post-test. Learner 9 was only able to solve some of the problems correctly that involved fractions in his post-test. He was able to use endorsed narratives about fractions in his presentation, but unable to use endorsed narratives consistently in his pre- and post-test. Learner 9's description of the addition and subtraction of fractions in his post-test was inconsistent with the meta-rules that guide the discursive routines of this mathematical discourse. He did not fully demonstrate that, through his exploration of and interaction with the discourse-for-others, he was thus partially able to make the discourse-for-other into a discourse-for-himself. Learner 9 thus did not fully demonstrate features of an autonomous learner after his engagement with the VITALmaths video clips. This indicates that there was only a partial enhancement in his understanding of how to use manipulatives in the addition and subtraction of fractions after his engagement with the VITALmaths video clips.

4.4.11 Learner 10

Pre-and post-test

In Question 1 in the pre- and post-test, Learner 10 was able to identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was unable to name the fraction part correctly. In Question 3 in the pre- and post-test, the participant chose part of the chocolate but her reason for choosing this part did not match the answer that she gave. In Question 4 in the pre- and post-test, the participant chose part of the chocolate but his reason for choosing this part did not match the answer that she gave. In Question 5 in the pre- and post-test, Learner 10 was able to mention the whole correctly but not the fraction part. In Question 6 in the pre-test, she did not use a drawing and did not mention the fraction part that each person will get. She was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets in the post-test. In Question 7 in the pre- and post-test, the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre- and post-test Learner 10 attempted to use shading of the fraction parts to show how the two fractions are added but did it incorrectly. She was however able to correctly add the fractions by using a conventional method in the pre- and post-test. In Question 9 in the pre-test, Learner 10 did not use shading of the fraction parts to show how the two fractions are subtracted. She was however able to correctly subtract the fractions by using a conventional method. In the post-test she attempted to shade the fraction parts to show how the two fractions could be subtracted but did it incorrectly. She was able to use a conventional method to subtract the two fractions in the post-test. In Question 10 in the pre-test, she did not attempt to divide the rectangle into different equal parts or add the fractions by using a conventional method post-test. In the post-test the participant attempted to divide the rectangle into different equal parts to show how the two fractions could be added but did it incorrectly. She was however able to add the fractions by using a conventional method. In Question 11 in the pre-test, she did not attempt to divide the rectangle into different equal parts or subtract the fractions by using a conventional method. In the post-test she attempted to divide the rectangle into different equal parts to show how the two fractions could be subtracted but did it incorrectly. She was however able to subtract the fractions by using a conventional method.

Presentation

Learner 10 only demonstrated the subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of fractions and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. She could only explain *how the fractions could be subtracted by using manipulatives. She explained: Uhm, it's like this. This one goes with this one, this one goes with this one, and this one goes with this one (shows how the blocks from 1/3 and 1/5 cancel each other out and then points to the two that are left).* In Figure 4.4.11 she showed where the two fraction pieces overlapped.

Figure 4.4.11: Learner 10 shows how she added two fractions

Learner 10, however, seemed unsure of how to explain the addition of fractions when probed. She showed how she would fold the card to find the different fraction parts. She explained: *Plus, one over four, okay, uhm. Okay, let me do this (folds a sheet of paper). Okay, this will be 1, 2, 3, 4. I know that my shapes they* *are not (brief pause) equal. This is it, 1, 2, 3, this is one over third and then 1, 2, 3, 4, I will shade one and then it will give us one over four.*

General Findings

Learner 10's use of words was partially consistent with the words used by others when explaining how to add and subtract fractions in her post-test and presentation. In her presentation she was able to use manipulatives correctly to show her thinking on how the subtraction of fractions. She could not translate the knowledge that he gained from her engagement with the video clips to her post-test. She attempted to use shading to explain the addition and subtraction of fractions in her post-test. Learner 10 was only able to solve some of the problems correctly that involved fractions in her post-test. She was only partially able to use endorsed narratives about fractions in her presentation, pre- and post-test. Learner 10's description of the addition and subtraction of fractions in her post-test was inconsistent with the meta-rules that guide the discursive routines of this mathematical discourse. She did not fully demonstrate that, through her exploration of and interaction with the discourse-for-others, that she was able to make the discourse-for-others into a discourse-for-herself. Learner 10 thus did not fully demonstrate features of an autonomous learner after her engagement with the VITALmaths video clips. There was only a partial enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions after her engagement with the VITALmaths video clips.

4.4.12 Learner 11

Pre-and post-test

In Question 1 in the pre-test, Learner 11 was unable to identify any of the three rectangles' parts that were shaded. In the post-test she was able to identify all three of the three rectangles' parts that were shaded. In Question 2 in the pre- and post-test, she was unable to name the fraction part correctly. In Question 3 in the pre- and post-test, the participant did not attempt the question. In Question 4 in the pre- and post-test, the participant chose part of the chocolate but her reason for choosing this part did not match the answer that she gave. In Question 5 in the pre-test, Learner 11 was able to mention the whole correctly but not the fraction part.

In the post-test she was able to mention the whole and the fraction part correctly. In Question 6 in the pre- and post-test, she was able to make an accurate drawing of the different equal parts and could accurately mention the part that each person gets. In Question 7 in the pre-test, the participant gave an incorrect answer and reason. In the post-test the participant gave the correct answer with a good reason for choosing the specific answer. In Question 8 in the pre- and post-test, Learner 11 did not use shading of the fraction parts to show how the two fractions are added. She was however able to correctly add the fractions by using a conventional method in the pre- and post-test. In Question 9 in the pre-test, Learner 11 did not use shading of the fraction parts to show how the two fractions are subtracted. She was however able to correctly subtract the fractions by using a conventional method. In the post-test she did attempted to shade the fraction parts to show how the two fractions could be subtracted. She was also unable to use a conventional method to subtract the two fractions in the post-test. In Question 10 in the pre- and post-test, she attempted to divide the rectangle into different equal parts but did it incorrectly. She, however, was able to add the fractions by using a conventional method. In Question 11 in the pre-test, she attempted to divide the rectangle into different equal parts but did it incorrectly. She, however, was able to subtract the fractions by using a conventional method. In the post-test she did not attempt to divide the rectangle into different equal parts or subtract the fractions by using a conventional method.

Presentation

Learner 11 demonstrated the addition and subtraction of fractions by using different coloured cards. She was able to divide her card into the different equal parts that represented $\frac{1}{3}$ and $\frac{1}{4}$ for the addition of the fractions, and $\frac{1}{5}$ and $\frac{1}{3}$ for the subtraction of fractions. In Figure 4.4.12a she showed where she put the fraction piece that overlapped.

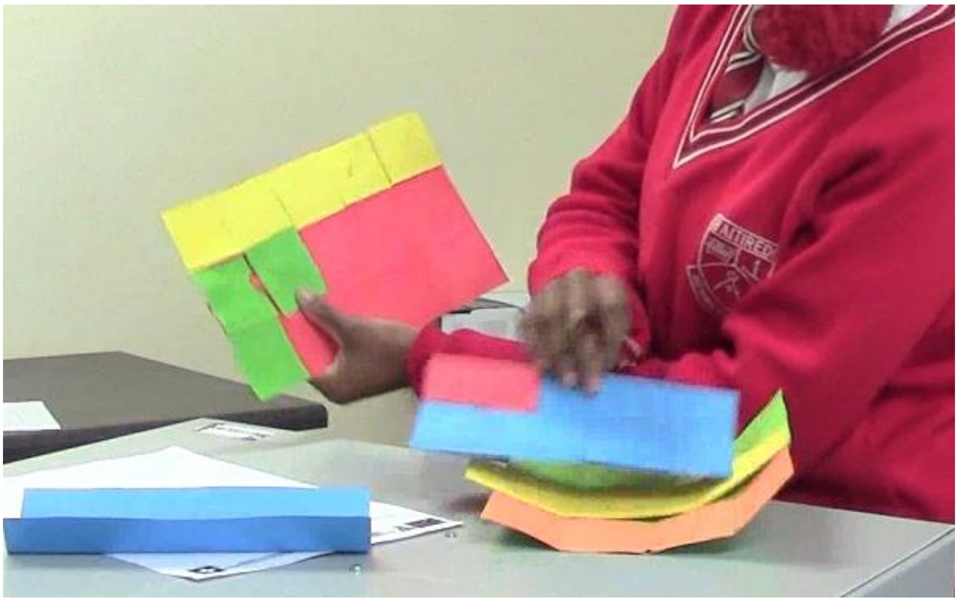


Figure 4.4.12a: Learner 11 shows where she put the fraction part that overlapped

She was however unable to demonstrate the addition and the subtraction of fractions using manipulatives. She explained: *Uhm, firstly I folded this colour card- this colour card into three parts. And then I take the- I took the other colour card to- to represent this card here (holds up a different sheet of card). Uhm, this- this- this square will go out with this one, and this one will go out with this one, and the-this one will go out with this one (shows how the blocks representing 1/5 and 1/3 cancel each other out). And then you are going to remain with this two. So here they are (shows them represented on a different colour card). If we, uhm, let me see- (pause) this three, I mean this part and this part and the last one, we call them, they are perfect squares. All of them here, they are perfect squares. Ja.*

Mmm, so here's my other colour card. This is 1, 2, 3, 4. If we fold this, this square, then we will remain with- with three squares. There's going to be one in between. This one, it goes with that square that we've folded. And then, if we put this colour card here, uhm, into four parts, 1, 2, 3, 4. And then we fold, we fold this one and it will be 1/4. Uhm, I think I'm done hey.

When she was asked how she would go about adding other fractions using manipulatives, she was unable to explain. From the image below it was clear that she did not know where to put the different fraction parts. In Figure 4.4.12b it is clear that Learner 11 did not know that fractions needed to be divided into equal parts.

Figure 4.4.12b: Learner 11 try to show how the fractions were added

General Findings

Learner 11's use of words was inconsistent with the words used by others when explaining how to add or subtract fractions in her post-test and presentation. She could not translate the knowledge that she had gained from her engagement with the video clips to her presentation or post-test. She did not attempt to use shading to explain the addition and subtraction of fractions in her post-test. Learner 11 was only able to solve some of the problems correctly that involved fractions in her post-test. She was unable to use endorsed narratives about fractions in her presentation, pre- and post-test. Learner 11's description of the addition and subtraction of fractions in her presentation and post-test was inconsistent with the meta-rules that guide the discursive routines of this mathematical discourse. She did not fully demonstrate that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-others into a discourse-for-herself. Learner 11 thus did not fully demonstrate features of an autonomous learner after her engagement with the VITALmaths video clips. There was also not an enhancement in her understanding of how to use manipulatives in the addition and subtraction of fractions after her engagement with the VITALmaths video clips.

4.4.13 Consolidation of findings

The majority of the participants could not translate what they had learned during the presentations and their engagement with the video clips to the shading exercises in the post-test. Learner 1, Learner 6, Learner 7 and Learner 8 were able to use words that were consistent with words used by others when dealing with fractions. They could also solve most of the fraction problems in the post-test even though they struggled with some of the same problems in the pre-test. Their use of endorsed narratives in this mathematical discourse and the use of meta-rules for the discourse were consistent with endorsed narratives used by others. The four participants thus showed clear features of autonomous learners after their engagement with the VITALmaths video clips.

Although Learner 2, Learner 3, Learner 4, Learner 5, Learner 9 and Learner 10 were either consistent or partially consistent in their use of words that involved fractions, they were not able to fully use endorsed narratives for this mathematical discourse. Their use of the meta-rules for the discourse were also only partially consistent with the meta-rules that are used by others. The thus did not fully show features of autonomous learners. Learner 11 struggled with the whole fraction exercise. She was the only one who could not correctly use manipulatives to demonstrate her understanding of the addition and subtraction of fractions.

4.5 CONCLUSION

This chapter, which was the analysis chapter was divided into two sections. The first section provided an analysis of the Pythagorean Theorem, while the second section provided the analysis of the addition and subtraction of fractions. In the two sections I analysed the eleven participants' work on the Pythagorean Theorem and the addition and subtraction of fractions individually. Each section was concluded with a consolidation of the findings. An extensive summary of the findings will follow in chapter 5.

CHAPTER 5: CONCLUSION

5.1 INTRODUCTION

When I visited a school in the rural areas of the Eastern Cape Province the principal of the school had a poster on the wall in his office with the following words written on it "Learning occurs when we are able to make sense of a subject, event or feeling by interpreting it into our own words or actions." These words encompass what I feel this research was supposed to highlight. The participants' engagement with the VITALmaths video clips was intended to specifically allow them to interpret the Pythagorean Theorem and the addition and subtraction of fractions, and express these in their own words or actions. My research as a whole showed that for many of my research participants' engagement with the selected VITALmaths video clips resulted in a better understanding of the use of squares in the Pythagorean Theorem and how fractions can be added or subtracted by using a different method. Although some of the participants struggled to express themselves adequately due to language barriers, the majority could clearly explain what they had learnt from their engagement with the video clips. I, however need to mention that the majority of the participants was not able to translate their engagement with the video clips and the presentations of their work into the post-test. Although they have shown some form of learning through their actions during their presentations, the enhancement of learning was not necessarily evident in their test scores.

This chapter serves as a conclusion of my research project where I will attempt to pull all the strings together by offering: a summary of the findings, the significance of the study, recommendations, some limitations of the study, suggestions for further research and personal reflections.

5.2 SUMMARY OF FINDINGS

I found during Phase 1 of the study (from the questionnaires that were completed by the participants) that none of them had ever done Mathematics using mobile phones. They were thus very enthusiastic to explore the video clips that were uploaded on their mobile phones. During their engagement with the video clips, most of the participants showed the video clips to either a classmate or a family member. They were also eager to explain what they had learnt from the video clips. None of the participants have however tried out any of the mathematical activities for themselves that were introduced in the video clips. All the participants thought that using mobile phones in their study of Mathematics was a good idea and that they would be able to use the mathematical activities in their study of Mathematics.

In Phases 2 and 3 the participants wrote the pre-test on the Pythagorean Theorem before the Pythagorean Theorem video clips were uploaded on their mobile phones. The participants did not do particularly well in the pre-test especially in the question on the formal proof of the Pythagorean Theorem. Only one of the participants was able to partially demonstrate the Pythagorean Theorem. After their engagement with the VITALmaths video clips on the Pythagorean Theorem all the participants were able to use manipulatives to show their understanding of the Pythagorean Theorem. Although English is not the participants' mother tongue, the majority of them were able to confidently present the work that they did on the Pythagorean Theorem. After the presentations six of the eleven participants were able to correctly show, by using manipulatives, the proof of the Pythagorean Theorem in the post-test. Three of the remaining five were able to demonstrate the proof with minor errors. The Grade-10 curriculum demands that learners should be able to use the Pythagorean Theorem in solving problems that involve right-angled triangles in trigonometry. Six of the eleven participants were able to solve problems that involved the Pythagorean Theorem in the post-test.

Phase 4 involved the addition and subtraction of fractions. The participants performed marginally better in the fractions pre-test than what they did in the Pythagorean pre-test. Participants, however, still struggled with most of the questions. After the participants' engagement with the video clips on fractions, all the participants were able to demonstrate their understanding of either the addition or subtraction of fractions by using manipulatives. In the post-test 10 of the eleven participants were able to identify the shaded parts and name the fractions. Only four of the eleven participants were able to accurately divide a rectangle into equal parts to show how fractions could be added. None of the participants were however able to divide the rectangles to show the subtraction of fractions. Although there was an increase in the results from the pre-test to the post-test, the increase was not significant. During post-presentation interviews when seven of the eleven learners were in Grade-11, all seven learners were able to divide the rectangles to show how they would add and subtract fractions. When I asked them why they could not show it during the post-test, all seven responded that they went back to look at the video clips over and over again and that gave them a better understanding of the concepts (*I watched the videos almost every day after I wrote the test*). The participants' revisiting of the video clips ties in well with Bruner's idea of spiral curriculum, which refers to the revisiting of basic ideas time and time again and building on them to a level where the learner has a full understanding of the concept.

Table 5.1: Classification of participants according to their discourse

Comparison of how the learners were classified according to their mathematical discourse and hence as an autonomous learner						
Pythagorean Theorem			Learner	Fraction work		
Completely	Partially	Not at all		Completely	Partially	Not at all
			1			
			2			
			3			
			4			
			5			
			6			
			7			
			8			
			9			
			10			
			11			

(Source: the idea of including this table originated from the report of my examiner, Dr. Piet van Jaarsveld, after the examination of this thesis)

From the information in Table 5.1 above only 4 of the learners were able to completely make the discourse for others into a discourse for themselves in both the Pythagorean Theorem and the addition and subtraction of fractions. These four learners thus showed features of autonomous learning. All the other participants were partially able to make the discourse of others into a discourse for themselves. The majority of the participants however came to understand why squares are used when applying the Pythagorean Theorem during the calculation of the sides in right-angled triangles. More than 50% of the participants could solve Question 7, of the Pythagorean Theorem post-test (Appendix 5A) properly, while no one attempted Question 7 during the pre-test. The majority of the participants were also able to do sums that involved the addition of fractions (Appendix 5B). However, I found that a number of the participants still struggled with the subtraction of fractions.

During the presentations, the majority of the participants' descriptions of the work that they did on the Pythagorean Theorem and the addition and subtraction of fractions were consistent with the meta-rules that guide the discursive routines of the two mathematical discourses. The majority of the participants were able to demonstrate the proof of the Pythagorean Theorem and how to add fractions, by using concrete manipulatives, during their presentations and/or the post-test. The participants' engagement with the VITALmaths video clips thus encouraged the use of manipulatives in their learning of Mathematics.

5.3 SIGNIFICANCE OF THE STUDY

I agree with Hyde (2011) who writes that the VITALmaths project is unique in the South African education context. The study dealt with the learning of Mathematics by using animated video clips that were disseminated through mobile technology. This study was also one of the first such studies that was carried out in the Mothibistad district of the Northern Cape Province. Furthermore, the study is an extension of the studies that were done by Hyde (2011) and Ndafenongo (2011) in the Easter Cape Province, which dealt with the teaching of Mathematics by using VITALmaths video clips.

The study firstly attempted to explore how the VITALmaths video clips could be incorporated into the learners' autonomous learning of Mathematics. Secondly, I found through my study that although most learners have access to mobile phones, there are not many schools that have explored the use of mobile phones during the learners' learning of Mathematics. It also appears that not much research has been done on the use of mobile technology or the use of animated video clips that are incorporated into mobile technology in the learning of Mathematics. It was thus appropriate to explore how mobile technology in conjunction with the VITALmaths video clips could be used to support learners in their autonomous learning of Mathematics. It is hoped that this study will enhance the research thereof.

5.4 ASSUMPTIONS AND LIMITATIONS OF THE STUDY

5.4.1 Assumptions.

When I started the project, I had limited experience in how video clips could be used in mobile technology for the learning of Mathematics. After I explained to the participants how to use the video clips that had been uploaded on a mobile phone, I conducted a workshop on how to open the video clips on the participants' mobile phones and how to view the video clips. I travelled back to Grahamstown and left the participants on their own to view and explore the video clips. I thus had to make a number of assumptions such as:

- That the participants would be able to open and view the video clips while I was away.
- That there would be sufficient time for the participants to view the video clips.
- That there would be sufficient time for the participants to prepare for their presentations.
- That language barriers, which could cause a total breakdown in communication, would not impede the presentations of the participants' work.
- That the practicality of working with the participants after school with all the extra mural activities that schools have would not impede in the data collection processes.

- That the participants would not lose interest after they had committed to participate in the research project.

- That none of the participants would drop out of school during the data collection process.

5.4.2 Limitations

In a small research project such as this one, limitations are bound to exist which constrain the generalization of the findings. Some of these limitations were:

- The research project was conducted in one school and only eleven learners were involved.

- Although there is a large and ever growing data base of VITALmaths video clips, only six video clips, which cover three topics, were used during the research project.

- The research was done in a school in Mothibistad in the Northern Cape Province, which is approximately 960 kilometers from Grahamstown in the Eastern Cape Province where I am located. I thus had to reduce the timeframes for data collection to only three weeks.

- Some of the interviews with the participants were done a year later. Four of the eleven participants left the school. I could thus only interview seven of the eleven participants.

- The participants had seven days to view and explore the Pythagorean Theorem videos before they did the presentations and wrote the post-test on the Pythagorean Theorem. They, however, only had three days to explore the fractions video clips before they did the presentations and wrote the post-test on fractions.

- Both these topics are however part of the curriculum in the primary school. Learners might have been able to remember how to do work based on the two topics, which might have had significant implications on the data collection and analysis of the data, especially the participants' pre- and post-test scores.

5.5 SUGGESTIONS FOR FURTHER RESEARCH

The study was done on a very small scale. There are thus prospective avenues to expand the study, such as:

- Include more schools and a larger number of learners to avoid generalizations that only included a small sample.
- Include other video clips that are appropriate for the Grade-10 curriculum. Although the topics that were covered in the video clips can be incorporated into other topics of the Grade-10 curriculum, the learners are introduced to these topics in the primary school.
- Give participants ample time to explore the video clips at their own leisure. I found during later interviews that participants were able to explain the work, which was covered in the video clips, better after they had revisited the video clips once they had written the post-tests.
- Explore research possibilities in the incorporation of mobile technology during the teaching and learners' autonomous learning of Mathematics.
- How to change the perceptions of South African schools on the use of mobile technology in their teaching. The majority of these schools only recognize the negative influences of mobile technology use in their classrooms.

5.6 PERSONAL REFLECTIONS

I had done numerous small research projects that I presented at national or international conferences before I conducted this research. Although the small projects gave me glimpses of what research entails, the VITALmaths research project made me discover the wonderful world of real intense research where I needed to spend hours and hours on sifting and reading through numerous books, journals and texts to find connections between other researchers and my own research study. These readings did not only broaden my knowledge on the work that was related to my study, it also broadened my knowledge and understanding of aspects in education that were totally unrelated to my study.

One of the areas of difficulty that stimulated reflection was autonomous learning and how autonomous learning influences the learners learning of Mathematics. The numerous readings that I pored over to find out when a learner can be classified as being an autonomous learner were more than just confusing until I came across Sfard's (2008) interpretation of discourses-for-others and discourses-for-oneself and how these two discourses relate to autonomous learning. It gave me new insights on how and when a learner could be classified as being an autonomous learner.

This research project also allowed me to rethink my own uninformed perceptions on the use of mobile technology in the classroom for the teaching and learning of Mathematics. I will definitely be an advocate for the use of mobile technology for the teaching and learning of Mathematics.

The VITALmaths project afforded me my first opportunity abroad. I was able to visit Switzerland in 2014 where I worked with Professor Helmut Linneweber-Lammerskitten on the redevelopment of VITALmaths video clips. I endeavour to use the experiences gained from the visit to get involved in increasing the VITALmaths database of video clips.

The demands that the research put on timeframe management put a lot of pressure on my work as a lecturer and on my family. I have thus not only come to realize the importance of colleagues and family relationships, but also how to manage my time to avoid conflict between myself and the other two entities.

I must admit that I fell short when good time management was required. According to my scheduled time frames, I was supposed to complete my thesis in 2014. I would thus advise fellow researchers to take sabbatical leave in order to focus on the writing up of a thesis.

5.7 CONCLUSION

I concur with Martin (2006) who writes that in order to meet the needs of all learners, alternative methods to the traditional Mathematics teaching and learning should be explored. There is no doubt that the VITALmaths project can and already has made huge strides in the participating learners' positive attitudes towards the learning of Mathematics by using short video clips that can be downloaded on mobile phones. Furthermore, the video clips afforded the participants opportunities to move away from the traditional classroom teaching and learning to the autonomous learning of Mathematics in out of school context.

Although my research has shown that the use of the VITALmaths video clips can be advantageous for the learners' learning of Mathematics, ongoing research on the use of the VITALmaths video clips as well as feedback from those who are using the video clips is essential for the development and growth of the VITALmaths project.

In conclusion Richard Bach (1977, p. 21) writes that "Learning is finding out what you already know. Doing is demonstrating that you know it and teaching is just reminding others that they know just as well as you". This resonates well with what my research and the VITAmaths project encompass.

REFERENCES

Aljohani, N.R., Davis, H., & Tiropanis, T. (2011). Comparisons between mobile and ubiquitous learning from the perspective of human computer interaction. *Proceedings of mlearn 2011*.Retrieved 14 March 2013 from http://mlearn.bnu.edu.cn/source/Conference_Procedings.pdf

Almeida, D. (2013). Misconceptions in mathematics and diagnostic teaching. Retrieved 13 March 2013 from http://www.academia.edu/1677587/Misconceptions_in_mathematics_and_diagnostic_teaching

Arksey, H. & Knight, P. (1999). *Interviewing for social scientists*. London: Sage Publications.

Bach, R. (1977). Illusions: The adventures of the reluctant Messiah. Illinois: Dell Publishers.

Bailey, J. (2008). First steps in qualitative data analysis: transcribing. Retrieved 4 March 2013 from http://fampra.oxfordjournals.org/content/25/2/127.full.

Baya'a, N., & Daher, W. (2009). Students' perceptions of mathematics learning using mobile phones. Retrieved 20 February 2014 from http://staff.najah.edu/sites/default/files/Students%20Perceptions%20of%20Mathematics%20Learning%20Using%20Mobile%20Phones.pdf.

Benson, P., & Voller, P (1997). Autonomy and independence in language learning. Essex: Longman.

Berger, M. (2013). Examining mathematical discourse to understand in-service teachers' mathematical activities. Pythagoras 34 (1), 1-10.

Bodner, G. M. (1986). Constructivism: A theory of knowledge. Journal of Chemical Education, 63(10), 873-877.

Boggan, M., Harper, S., & Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. Retrieved 9 September 2014 from http://www.aabri.com/manuscripts/10451.pdf

Bowman, M. (1994). Using video in research. Retrieved 30 July 2004 from http://www.scre.ac.uk/spotlight/spotlight45.html

Brodie, K. (2010). *Teaching mathematical reasoning in secondary school classrooms.* New York: Springer.

Bruner, J (1990). *Acts of meaning.* Cambridge: Harvard University Press.

Bruner, J. (1996). *The culture of education*. Cambridge: University Press.

Bruner, J. S. (1960). *The Process of education.* Cambridge, Mass.: Harvard University Press.

Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, 31, 21-32

Calandro, E. Gillwald,A., & Stork, C. (2012). Internet going mobile: Internet access and usage in eleven African countries. Retrieved 15 March 2014 from http://www.researchictafrica.net/presentations/Presentations/2012%20Calandro%20Stork%20Gillwald%20-%20Internet%20Going%20Mobile

Candy, P. C. 1991. Self-direction for Lifelong Learning: A comprehensive guide to theory and practice. California: Jossey-Bass.

Cohen, L., & Manion, L. (1994). *Research methods in education (*4th ed.). North Yorkshire: J & L Composition Ltd.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education (*5th ed.). London: Routledge Falmer.

Cooney, J.G. (2014). Utelizing mobile technologies for classroom learning. Retrieved 5 May 2015 from http://www.qrcodescanning.com/mobile-technology-classroom-learning.html.

Creswell, J.W., & Clark, V.L.P., (2007). *Designing and conducting mixed methods research*. Thousand Oaks: Sage Publications.

Department: Basic Education Republic of South Africa. (2011). Curriculum and assessment policy statement: Grade10 – 12: Mathematics. Retrieved 10 August 2015 from http://www.education.gov.za/LinkClick.aspx?fileticket=uXLZcIa67rE%3D&tabid=570&mid=1558

Department: Basic Education Republic of South Africa. (2003). Retrieved 10 August 2011 from http://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwiEk9GrqKzLAhVCyRQKHWZmCqcQFggdMAA&url=http%3A%2F%2Fwww.hsrc.ac.za%

Department: Basic Education Republic of South Africa. (2011). Retrieved 11 August 2011 from http://www.education.gov.za/LinkClick.aspx?fileticket=nlYYGPv1/94%3D

De Pree, M. (1987). Curriculum development processes. Retrieved 10 March 2015 from https://education.alberta.ca/media/6809242/d_chapter1.pdf

Derry, S. J. (1999). A fish called peer learning: Searching for common themes. Retrieved 20 September 2013) from http://www.citeulike.org/pdf_options/user/iasminaermalai/article/4045402?fmt=pdf

Elmore, R. F. & Fuhrman, S. H. (2001). Holding schools accountable. Is it working? Retrieved 7 March, 2013, from http://repository.upenn.edu/cgi/viewcontent.cgi?article=1007&context=gse_pubs

Eiselen, R. J. & Uys, T. (2005). Questionnaire design. Retrieved 7 March 2013 fromhttp://www.uj.ac.za/EN/Research/Statkon/Documents/Statkon%20Questionaire%20Design.pdf

Eisenhardt, K. M. (1989). Building theories from case study research. The Academy of Management Review. 14(4), 532-550

Ernst, M. (1998). Pairing Learners in the Reciprocal Style of Teaching: Influence on Student Skill, Knowledge and Socialization. Physical Educator, 55, 24-37.

Fosnot, C.T. (1996). *Constructivism: Theory, perspectives, and practice* New York: Teachers College Press.

Gernetzky, K. & Magubane, K. (2013). Matric pass rate edges up. Retrieved 12 February 2013 from http://www.bdlive.co.za/national/education/2013/01/02/

Glaser, B.G. (2002). Constructivist grounded theory. Retrieved 5 March 2013 from http://www.qualitative-research.net/index.php/fqs/article/view/825/1792

Gordon, A. (1999). Rural Schools: Are the new policies bearing fruit, Education Africa Forum, Johannesburg.

Gredler, M. E. (1997). Learning and instruction: Theory into practice. (3rd Ed) New Jersey: Prentice-Hall Press.

Guba, E. G., & Lincoln, Y. S. (1989). *Fourth generation evaluation.* Newbury Park, CA: Sage Publications.

Heath, C. (1997). *Qualitative Research: Theory, Method and Practice*. London: Sage Publications.

Herselman, M. E. (2003). ICT in rural areas in South Africa: Various case studies. Retrieved 20 March 2013 from https://www.researchgate.net/publication/250569904_ICT_in_Rural_Areas_in_South_Africa_Various_Case_Studies

Hofmeyr, A. (1997). *Teacher education under the spotlight.* Johannesburg: Education Africa.

Holec, H. (1981). *Autonomy and foreign language learning.* Oxford: Pergamon Press.

Hunt, A. W., Nipper, K. L., Nash, L. E. (2011). Virtual vs concrete manipulatives in mathematics teacher education: Is one type more effective than the other? Retrieved 9 September 2014 from http://www.napomle.org/cimle

Hutton, J. (2011). Mobile phones dominate in South Africa. Retrieved 12 June 2013 from http://www.nielsen.com/us/en/insights/news/2011/mobile-phones-dominate-in-south-africa.html

Hyde, J.C. (2011). An investigation into the use of Visual Technology for the Autonomous Learning of Mathematics (VITALMaths) video clips in the teaching of mathematics in selected South African Grade 9 classes: A case study. Master of Education Thesis. Rhodes University. Grahamstown.

Janse van Rensburg, E. (2001). An orientation to research. Research methods short course, Environmental Education Unit, Rhodes University, Grahamstown.

Jaworski, B. (2006). Teaching better mathematics: What, how and why. Retrieved 15 July 2013 from http://tapir.pdc.no/pdf/FOU/2010/2010-03-2.pdf

Jenkin, T. (1995). The rural development strategy of the government of national unity. Retrieved 4 June 2002 from http://www.polity.org.za/govdocs/rdp/rural17.html.

Kamii, C. (1994). Autonomy: The aim of education as envisioned by Piaget. Retrieved 9 October 2013 from http://www.thepatchworkschool.com/documents/autonomy-piaget.pdf

Karp, K.S. (2010). Elementary school teachers' attitudes toward mathematics: The impact on students' autonomous learning skills. Retrieved 4 February 2013 from http://onlinelibrary.wiley.com/doiml/10.1111/j.1949-8594.1991.tb12095.x/abstract.

Keats, D. **M.** (2000). *Interviewing: A practical guide for students and professionals*. Philadelphia: Open University Press.

Kim, B. (2012). Social constructivism. Retrieved 12 February 2013 from http://epltt.coe.uga.edu/index.php?title=Social_Constructivism

Koebler, J. (2011). Teachers use cell phones in the classroom. Retrieved 4 February 2013 from http://www.usnews.com/education/blogs/high-school-notes.

Lenhart, A., Ling, R., Campbell, S. & Purcell, K. (2010). Teens and mobile phones. *Pew Internet and American Life Project.* Retrieved 13 March 2013 from http://www.pewinternet.org/Reports/2010/Teens-and-Mobile-Phones.aspx

Linneweber-Lammerskitten, H., Schäfer, M., Samson, D. (2010) Visual technology for the autonomous learning of mathematics. Pythagoras 72,27-35.

Larochelle, M., Bednarz, N., & Garrison, J. (1998). *Constructivism and education.* New York: Cambridge University Press.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. Journal for Research in Mathematics Education. 21(1), 16032.

Mahassey, M. L. & Peroddin, A. (1973). *Teaching elementary school mathematics.* Illinois: S.C. Peacock.

Markle, D.T., West, R.E. & Rich, P.J. (2011). Beyond transcription: Technology, change and refinement of method. Retrieved 4 March 2013 from http://www.qualitative-research.net/index.php/fqs/article/view/1564/3249

Martin, H. (2006). Differential instruction for mathematics: Instructions and activities for the diverse classroom. Retrieved 17 September 2015 from https://www.google.co.uk/search?sourceid=navclient&aq=&oq=Differential+instruction+for+mathematics&ie=UTF-8&rlz=1T4VRHB_enZA641ZA641&q

Mayer, R.E. & Moreno, R. (2002). Animation as an aid to multimedia learning. Retrieved 14 March 2013 from http://ydraw.com/wp-content/uploads/2012/04/Stop-Motion-Aids-Multimedia-Learning.pdf

Maxwell, J.A. (1996). *Qualitative research design: An interactive approach.* London: International Educational and Professional Publishers.

McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulative debate. Theory into practice 46(4), 309-316. Retrieved 10 September 2014 from http://eric.ed.gov

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: A source of new methods.* Beverley Hills, CA: Sage Publications.

Morris, J. (2013). The use of virtual manipulatives in fourth grade to improve mathematics performance. Retrieved 9 September 2014 from http://dspace.sunyconnect.suny.edu

Moyer-Pachenham, P., **Ulmer, L. A., & Anderson**, **K. L**. (2012) Examining pictorial models and virtual manipulatives for third grade faction instruction. Retrieved 10 September 2014 from http://www.ncolr.org/jiol/issues/pdf/11.3.2.pdf

Mulkeen, A. (2005). Teachers for rural schools a challenge for Africa. Retrieved 15 March 2013 from http://people.umass.edu/educ870/teacher_education/Documents/Tchrs%20for%20Rural%20Schools%20-%20Africa.%20Mulkeen%20WB.pdf

Myers, M. D. (2009). *Interpretive research in information systems*. London: McGraw-Hill

Ndafenongo, G. (2011). An investigation into how cell phones can be used in the teaching of mathematics using VITALmaths video clips: A caser study of two schools in Grahamstown, South Africa. Unpublished master's thesis. Rhodes University Grahamstown

Newton, A. (2010). The Pythagorean Theorem: Its importance in mathematics and a teaching approach. Retrieved 14 July 2013 from http://jwilson.coe.uga.edu/EMAT6680Su10/Newton/emat6690/Pythagorean%20Theorem/Pythagorean%20Theorem.html

Oliver, P. (2003). *The student's guide to research ethics*. Berkshire: Open University Press.

Ormiston, M. (2013). How to use cell phones as learning tools. Retrieved 4 February 2013 from http://www.teachhub.com/how-use-cell-phones-learning-tools

Overbaugh, R. (2004). An overview of Jerome Bruner his theory of constructivism. Retrieved 21 January 2013 from http://ww2.odu.edu/educ/roverbau/Class_Websites/761_Spring_04/Assets/course_docs/ID_Theory_Reps_Sp04/Bruner-Cherry.pdf

Patton, M. Q. (2002). Qualitative research and evaluation methods. Retrieved 5 March 2013 from http://books.google.co.za/books?

Peyper, L. (2013). Mobile phone usage in South Africa. Retrieved 3 April 2014 from http:finweek.com/2013/01/22/mobile-phone-usage-in-sa

Piaget, J. (1968). *The development of memory and identity.* Barre, MA: Clarke University Press.

Pitler, H., Hubbel, E. R., & Kuhn, H. (2012). *Using technology with classroom instruction that works.* Denver. McRel Press.

Plowman, L. (2004). Using video for observation interaction in the classroom. Retrieved 7 June 2004 from http://www.scre.ac.uk/spotlight/spotlight72.html.

Powell, K. C., & Kalina, C. J. (2009). Cognitive and social constructivism: Developing tools for an effective classroom. Retrieved 15 August 2013 from http://eds.b.ebscohost.com/eds/pdfviewer/pdfviewer?sid=1e39331b-3ff3-4f0f-b320-9a12b2e30559%40sessionmgr113&vid=0&hid=119

Pring, R. (2000). *Philosophy of educational research.* London: Continuum.

Roth, W.-M. & Radford, L. (2010). Re/thinking the Zone of Proximal Development (Symmetrically). Mind, culture and activity, 17(4).

Roulston, K. (2004). *Analysing the interviewer's work in generating research data*. Oxford: Blackwell Publishing.

Rowley, J. (2002). Using case studies in research. Management Research News. 25(1), 16-27.

Ruane, J.M. (2005). *Essentials of research methods: a guide to social science research*. Brisbane: Queensland University Press.

SAinfo (2012). Northern Cape Province, South Africa. Retrieved 20 February 2013 from http://www.southafrica.info/about/geography/northern-cape.htm.

Schwartz-Shea, P., & Yanow, D. (2012). *Interpretive research design: Concepts and processes.* New York: Routledge.

Seidman, I. E. (1991). *Interviewing as quality research: A guide for researchers in education and the social sciences.* New York: Teachers College Press.

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. New York: Cambridge University Press.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. Retrieved 28 July 2014 from http://lchc.ucsd.edu/mca/Mail/xmcamail.2009_04.dir/pdfLBjRfSfwm2.pdf.

Shuttleworth, M. (2008). Case Study Research Design. Retrieved May 07, 2015 from https://explorable.com/case-study-research-design

Sieber, J.E. (1992). *Planning ethically responsible research: A guide for students and internal review boards*. California: Sage Publications.

Shaw, J.M. (2002) Manipulatives enhance the learning of mathematics. Retrieved 12 February 2013 from http://www.eduplace.com/state/author/shaw.pdf.

Sindhwani, A., & Sharma, M. K. (2013). Metacognitive learning skills. Retrieved 10 October 2014 from http://www.confabjournals.com/confabjournals/images/652013823499.pdf.

Shaw, J.M. (2002) Manipulatives enhance the learning of mathematics. Retrieved 12 February, 2013, from http://www.eduplace.com/state/author/shaw.pdf.

South African National Education Examiner's Report. (2012). Retrieved 13 August 2013 from http://www.education.gov.za/LinkClick.aspx?fileticket=nlYYGPv1/94%3D

South African National Curriculum Statement. (2003) Retrieved 13 August 2013 from

Spear-Swerling, L. (2006). The use of manipulatives in mathematics instruction. Retrieved11 February 2013 from http://www.education.gov.za/LinkClick.aspx?fileticket=WBtQtdS0Azk=http://www.Idonline.org/spearswerling/The_use_of_manipulatives_in_mathematics_Inst

Stake, R.E. (1995). *The art of case study research.* London: Sage Publications.

Stenhouse, L (1975). An introduction to curriculum research and development. Retrieved 10 March 2015 from https://books.google.co.uk/books/about/An_introduction_to_curriculum_research_a.html?id=LQsNAQAAIAAJ&hl=en

Statistics South Africa. (2012). Census 2011 provinces at a glance. Retrieved 4 February 2013 from http://www.statssa.gov.za/Census2011/Products/Provinces.

St Louis, R. (2003). Helping students become autonomous learners: Can technology help? Retrieved 11 February 2013 from http://home.learningtimes.net/learningtimes?go=1042165.

Taylor, N. (2008). What's wrong with South African schools? Retrieved 5 February 2013 from http://www.jet.org.za/events/conferences/What%20works%20in%20school%20development/Papers/Taylor%20Whats%20wrong%20with%20SA%20schools%20JET%20Schools%20Conf%20final.pdf

Terre Blanche, M., Durrheim, K., & Painter, D. (2006). Research in practice: Applied methods for the social sciences. Cape Town: University of Cape Town Press.

Thanasoulas, D. (2000). What is learner autonomy and how can it be fostered? Retrieved 11 February 2011 from: http://iteslj.org/Articles/Thanasoulas-Autonomy.html

Turloiu, A & Sif Stefánsdóttir, I (2011). Learner autonomy: Theoretical and practical information for language teachers. Retrieved 14 May 2013 from http://skemman.is/stream/get/1946/7668/19634/1/

Urquhart, C. (2013). *Grounded theory for qualitative research: practical guide*. Los Angeles: Sage Publications.

Van der Berg, S. et al. (2011). Improving education quality in South Africa. Retrieved 11 March 2013 from http://resep.sun.ac.za/wpcontent/uploads/2012/10/2011-Report-for-NPC.pdf.

Vygotsky, L. S. (1986). *Thought and language.* Cambridge. MIT Press.

Vygotsky, L. S. (1978). *Mind in society.* Cambridge. Harvard University Press.

Walsham, G. (1995). Interpretive case studies in IS research: Nature and method. Retrieved 21 February 2014 from http://www.palgrave-journals.com/ejis/journal/v4/n2/pdf/ejis19959a.pdf``

Wetenskow, A. (2012). Equivalent fraction learning trajectories for students. Retrieved 15 April 2013 from https://www.google.co.uk/search?sourceid=navclient&ie=UTF-8&rlz=1T4VRHB_enZA641ZA641&q=Wetenskow+%282012%29+virtual+manipulatives

Whiteley, W. (2004). Visual form is mathematics: Thinking, communicating, learning. Retrieved 14 March 2013 from http://link.springer.com/chapter/10.1007%2F1-4020-7910-9_56?LI=true#page-1

Wood, S. B (2008). Mathematizing, identifying and autonomous learning. Retrieved 30 April 2014 from http://books.google.co.za/books?id=-0L9wJuHFnIC&pg=PR3&lpg=PR3&dq=Mathematizing,+Identifying+and+autonomous+learning&source=bl&ots

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematical Education 27(4), 458-477.

Yin, R. K. (2012). *Apllications of case study research*. Tousand Oaks: SAGE.

APPENDICES

Summary of data generation process and tools used

Tools	Purpose	Data generated	Analysis
Questionnaires	To ascertain the learners experiences in using mobile phones in general and whether they ever use them for study purposes	Qualitative data. Transcripts.	Qualitative emerging themes such as participant's perceptions in using mobile phones for their studies with specific reference to the study of mathematics by using colour code categorization. .
Interviews	To obtain the story behind the participants' experiences in using the VITALmaths video clips. Also to pursue in-depth information around the use of manipulatives after the presentations.	Qualitative data Transcripts	Qualitative emerging themes, such as the meanings, insights and clarifications of the participants' responses to the interview questions. The participants' responses will be categorised by using colour codes for analysis. The categories will include themes depicting the autonomous learning of mathematics.
Participants' Presentations	To obtain further in-depth information about the participants' experiences in using mobile phones	Qualitative data Transcripts	Qualitative emerging themes such as common elements among the different presentations

	autonomously. The presentations will also show the participants' understanding in using the video clips and the manipulatives to complete the worksheet activities.		observed and the consistencies and inconsistencies in the use of the manipulatives of the video clips. I will also analyse the different concepts that will emerge from the categorizations of the codes used in the analysis.
Worksheet	To scaffold the VITALmaths video clips on Pythagoras' theorem on addition and subtraction of fractions.	Qualitative data. Transcripts	Qualitative emerging themes for analysis that will be used in conjunction with the presentations of the participants.
Pre- and Post-Tests	To explore whether the use of the VITALmaths video clips enhances the participants' learning of the Pythagoras' theorem and addition and subtraction of fractions.	Both quantitative and qualitative data. Descriptive statistics and transcripts.	Quantitative emerging themes such as the test scores that will be analysed using descriptive statistics. The qualitative emerging themes such as the conceptual understandings of the two topics including the errors and misconceptions. These errors and misconceptions will be categorised for analysis. The categories will include themes depicting the autonomous learning of mathematics.

APPENDIX 1A

SIOC-cdt and Rhodes University Teacher Education Project

Information sheet

I, Thomas Haywood, am a Rhodes University Mathematics Education (RUMEP) staff member involved in a teacher education project in the Northern Cape Province (NCP). The primary aim of the project is to provide teachers with a formal Bachelor of Education (B.Ed) qualification. A secondary aim of the project is to undertake research in order to better understand the educational context of the NCP. As part of the second aim I would like to:

Explore ten grade-10 learners' autonomous learning of mathematics by using selected Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) video clips. The VITALmaths database of video clips, which consists of very short video clips (1-3 minutes long) was developed by students and researchers at the School of Teacher Education at the University of Applied Sciences Northwestern Switzerland and Rhodes University. The video clips can be freely downloaded on mobile phones which learners and teachers can then use for the learning and teaching of mathematics.

The ten grade-10 participants will be taken from one of the RUMEP project schools in the John Taolo Gaetsewe District Municipality areas of the NCP. The school will be selected in consultation with the NCP Department of Education and will preferably be a well-functioning school where there is a high teacher effort with time-on-task. The 10 participating participants will include both males and females taken from the bottom, average and top learners in the mathematics class. The selection will be done in consultation with the grade-10 mathematics teacher of the learners. The research will be done out of normal school time. The learners will preferably stay in close proximity to the school. The participating learners will be provided with mobile phones with the downloaded video clips.

I intend to conduct workshops on the video clips and interview learners individually to explore their experiences in using the video clips. The learners will also do presentations to give me a clearer picture of their learning experiences in using the video clips. I will audio record and transcribe the interviews and video record and transcribe the presentations of the learners.

The learners' participation is entirely voluntary, and they may withdraw from the project at any point. In carrying out the research I promise to acknowledge the help of those who participate, respect their confidentiality and guarantee their anonymity and the anonymity of the school.

As part of the research towards a Masters of Education degree, I will write conference papers and publishable articles. I undertake to provide the school, Northern Cape Department of Education and the participants concerned with a copy of these papers as work in progress so that they can check the accuracy of the information.

Signature: *Haywood*

Date: 19 April 2013

APPENDIX 1B

Consent form for parent/guardian

I, ……………………………………………………………., a parent/guardian of …………………………………..(Name of student) understand the research project and am willing to allow him/her to participate in the research.

Signature

Parent/Guardian: …………………………….. Date: ………………………

APPENDIX 1 C

Consent form for the principal of school

I, ………………………………………………………………., the principal of ………………………………………………………………(Name of school) understand the research project and am willing to allow the learners from the school to participate in the research.

Signature

Principal: …………………………….. Date: ………………………

APPENDIX 1D

Consent form for the Northern Cape Province Department of Education

I, …………………………………………………………….., the director of Northern Cape Province Department of Education, understand the research project and am willing to allow the learners from the school to participate in the research.

Signature

Director: …………………………….. Date: ………………………

APPENDIX 2

VITALmaths Questionnaire for Learner Participants

Please read and answer the following the following questions:

1. Name:____

   ____

2. Age:____

   ____

3. Do you own a mobile

   phone?____

4. If you answered **YES** to question 2 what make and

   model?______________________

   OR

   If you answered **NO**, do you have access to a mobile

   phone?____________________

   If so what make and

   model?____

   If you have answered **NO** to question 4, who does the mobile phone

   belong to?_____

   ____

   ____

5. Do you have your own SIM card?_______________________________________

6. Which network do you use?____

7. What do you use your mobile phone for?__________________________________

____

____

8. Can you access the Internet via your mobile phone?_________________________

9. If you have answered **YES** to question 8 what did you use the internet for?_________

____

____

10. Do you enjoy doing mathematics?______________________________________

11. How good do you think you are at mathematics? Circle the answer that you have chosen.

A Excellent **B Good** **C Fairly Good** **D Poor**

12. Is there anything you like about mathematics?________________________________

____

____

13. Is there anything you dislike about mathematics?______________________________

____

____

14. What do you find difficult in mathematics?__________________________________

____

____

15. Have you ever used your mobile phone to do mathematics?_____________________

16. If you answered **YES** in question 13, What did you use it for in mathematics?_______

APPENDIX 3A

Information on the Addition and Subtraction of Fractions video clips

You were given a ruler, scissors, rectangular colour card and a mobile phone with two video clips on addition and subtraction of fractions. Please look at the video clips and carefully study what they show. You may view them as often as you wish. You may use the ruler, scissors and rectangular colour card that will assist you in understanding the video clips.

You will be required to do a 5 minute presentation on Wednesday 4 September 2013 where you will tell us:

- How you used the video clips;
- How often you used the video clips;
- If you showed the video clips to anyone else;
- Why you showed it to the specific person/persons;
- What you learnt from the video clips;
- How you used the ruler, scissors and rectangular colour cards that you were given;

  You will be required to demonstrate to us how you used the ruler, scissors and rectangular colour card in conjunction with the video clips on the addition and subtraction of fractions during the presentation.

APPENDIX 3B

Information on the Pythagorean Theorem Video Clips

You were given a protractor, a ruler, scissors, colour card and a mobile phone with three Pythagorean Theorem video clips. Please look at the video clips and carefully study what they show. You may view them as often as you wish. You may use the protractor, ruler, scissors and colour card that you were given in any way that will assist you in understanding the video clips.

You will be required to do a 5 minute presentation on Thursday 29 August 2013 where you will tell us:

- How you used the video clips;
- How often you used the video clips;
- I you showed the video clips to anyone else;
- Why you showed it to the specific person/persons;
- What you learnt from the video clips;
- How you used the protractor, ruler, scissors and colour cards that you were given to help you understand the video clips;

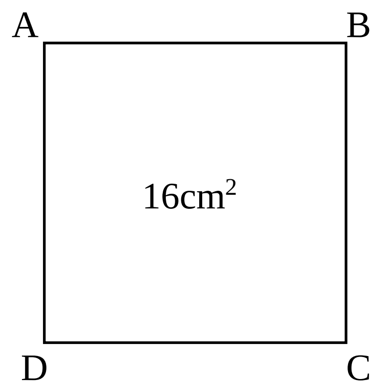
You will be required to demonstrate to us how you used the protractor, ruler, scissors and colour card in conjunction with the video clips on the Pythagorean theorem during the presentation.

APPENDIX 4A

Pythagorean Theorem Test

Name:______________________________
Grade:_______________

1. The area of the square ABCD is $16cm^2$. Explain what this means.



____

____

____

____

____

____

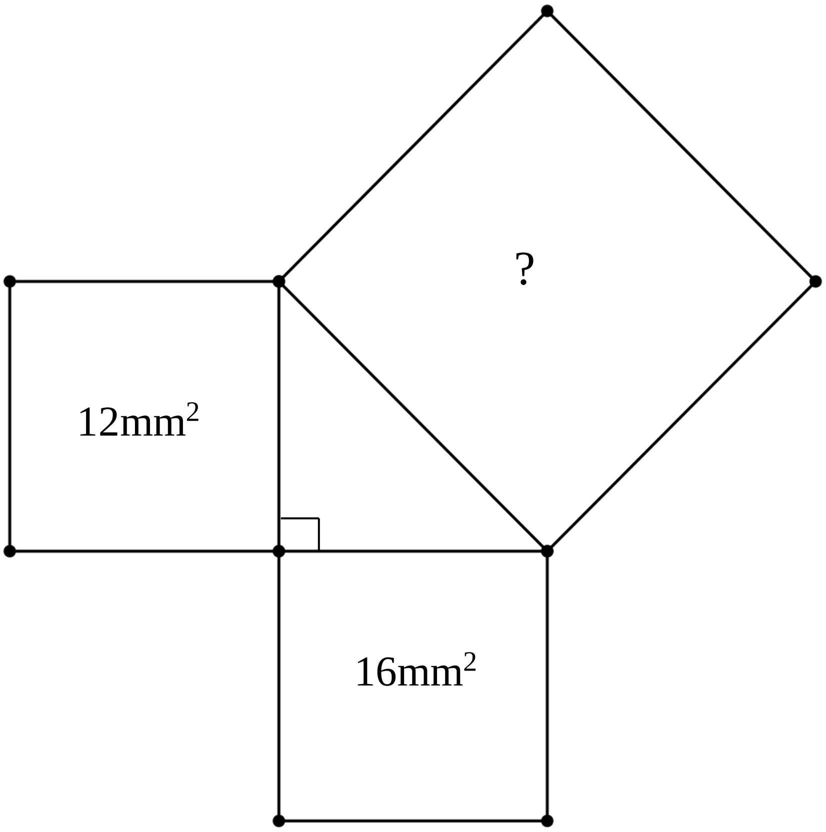
____

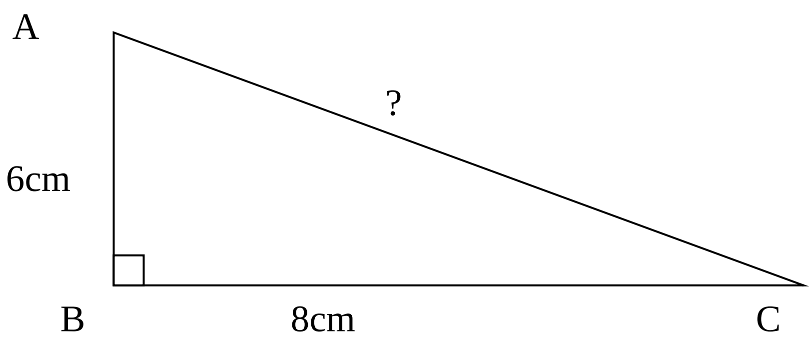
____

2. Write down the theorem of Pythagoras in your own words:

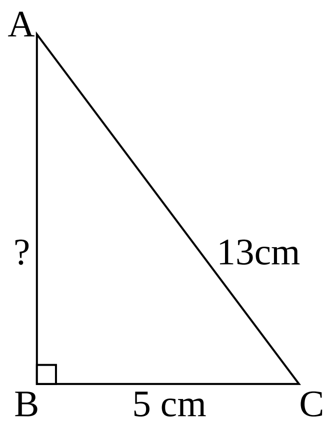
3. Find the area of the missing square in each. The sketches have not been drawn to scale. SHOW YOUR WORK.

a)





b)



4. Thandi and Thando each have a triangle. Thandi's triangle's sides

have the lengths 7cm, 14cm and 21cm. Thando's triangle's sides have the lengths 9cm, 40cm and 41cm. Which of the two triangles is a right-angled triangle? Explain.

____

____

____

____

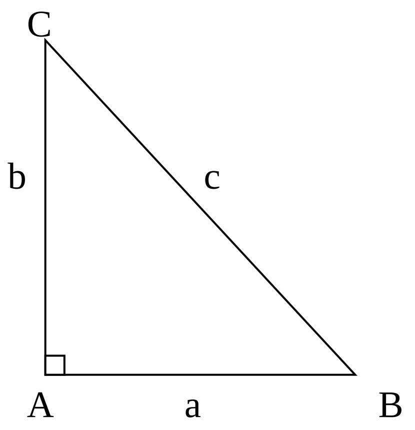
____

____

____

6. The Pythagorean result for triangle ABC with the right angle A is:

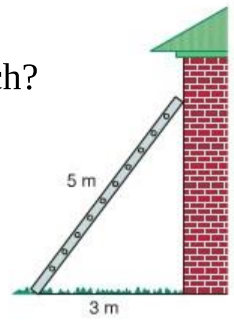
A $a^2 = b^2 + c^2$ B $b^2 = a^2 + c^2$ C $c^2 = a^2 + b^2$ D none of these



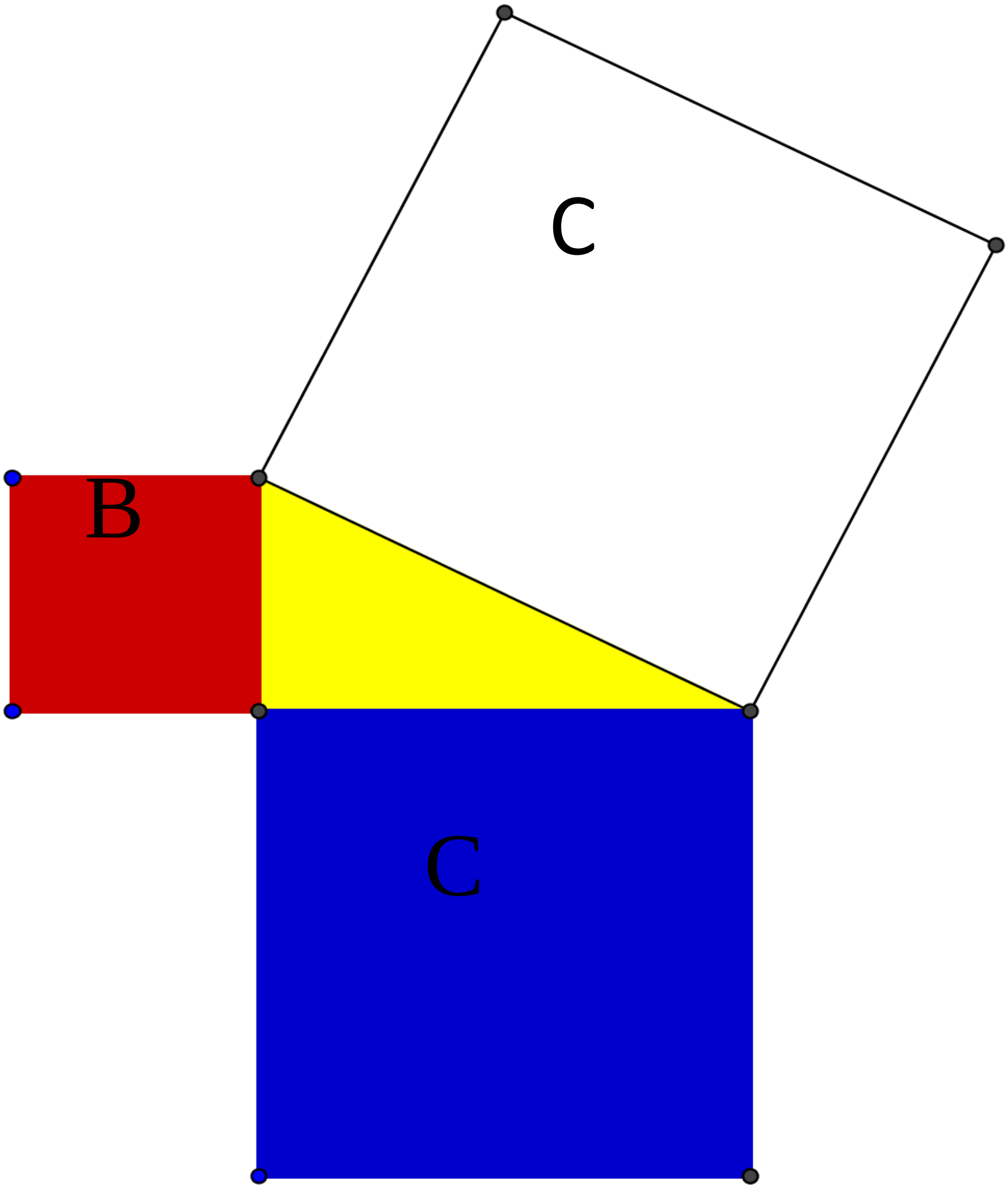
Circle the correct answer.

7. A 5m ladder leans against a house. It is 3m from the base of the wall.

How high does the ladder reach?



8. Using a pair of scissors, glue and the sketch below demonstrate the theorem of Pythagoras



APPENDIX 4B

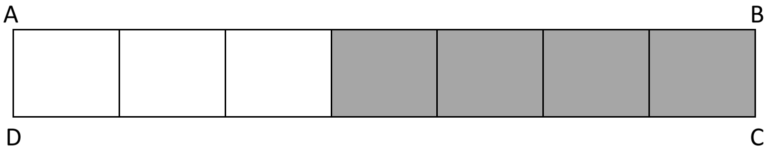
Fractions Test

Name:______________________________

Grade:_______________

1. How much of the rectangles ABCD and EFGH are shaded?

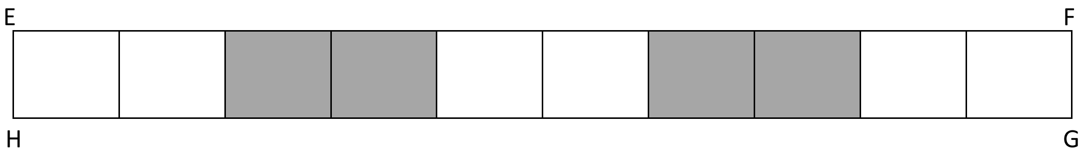
a)



____

____

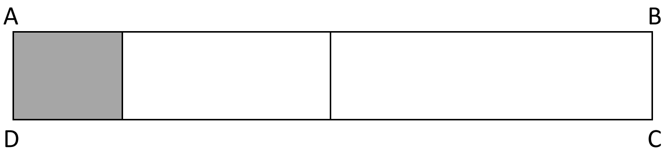
b)



____

____

c)



____

____

2. When we divide something into 10 equal parts, we call these parts ______________

3. What would you rather have: a sixth of a chocolate bar or a quarter of a chocolate bar? Explain your decision.

____

____

____

4. What would rather have: $\frac{1}{7}$ of a chocolate bar or a $\frac{1}{3}$ of a chocolate bar? Explain your decision.

____

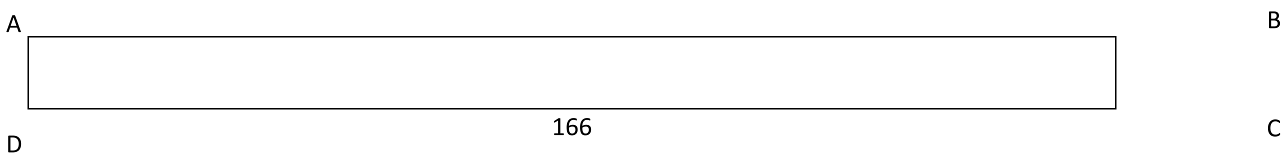
____

____

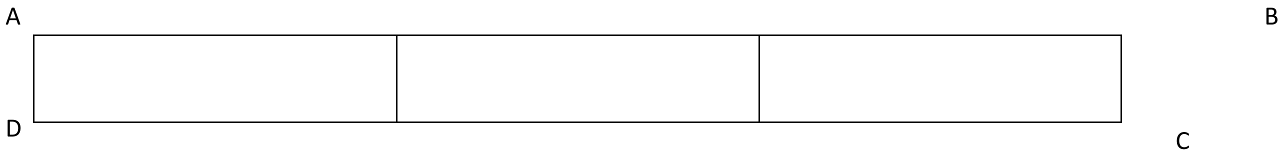
____

____

5. Look at the rectangles ABCD below. For each question, ABCD has been divided into different equal parts. For each question, what fraction of the whole does each part represent:



a) ______________________________________



b) ______________________________________

6. Nine friends want to share a rectangular chocolate bar equally. Show by using a drawing how they can do that. What fraction of the chocolate bar will each one of the friends get.

____

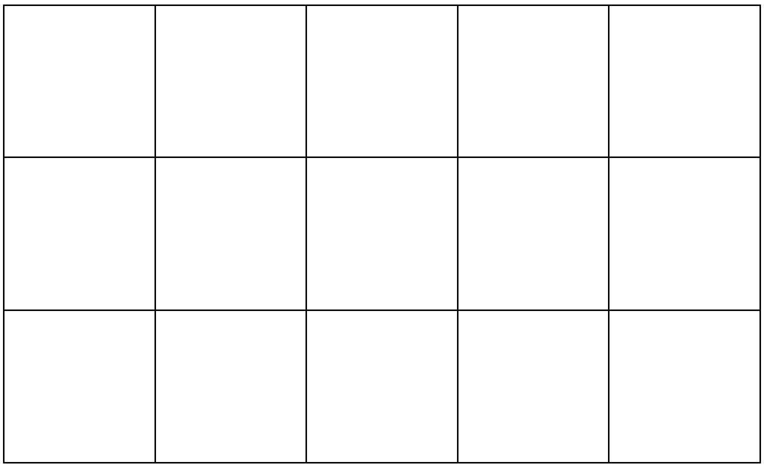
____

____

____

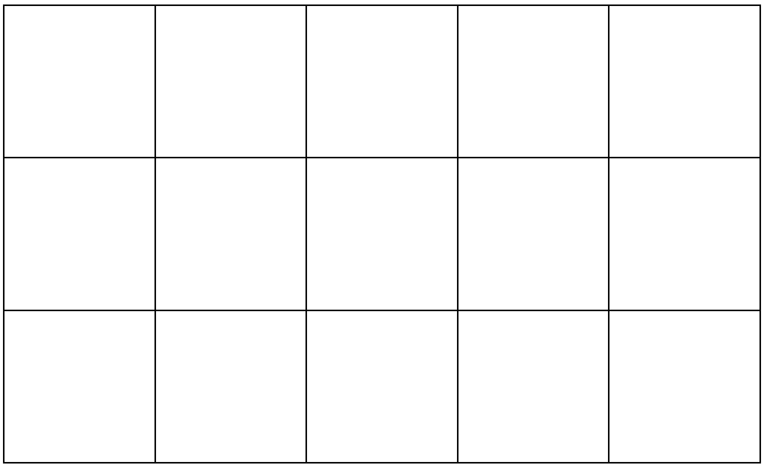
7. Tom fills $\frac{3}{4}$ of a bottle with water. Peter fills a same sized bottle with $\frac{4}{5}$ of water. Mary says that Peter has more water in his bottle. Do you agree?
Explain.

8. Use the rectangle below to help calculate:



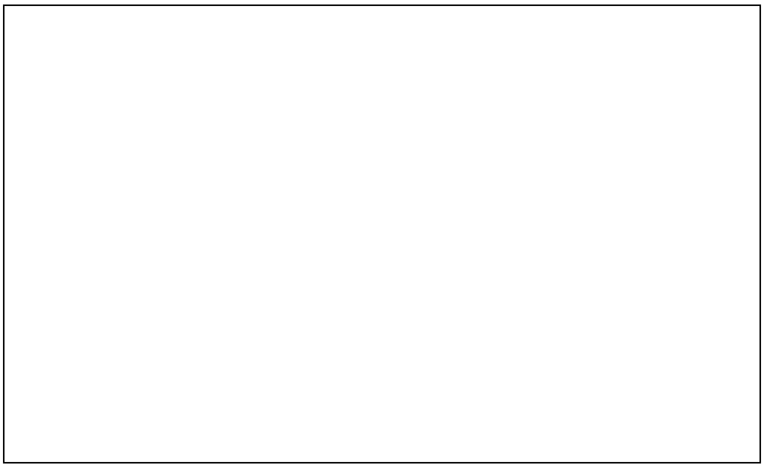
$$\frac{3}{5}+\frac{1}{3}$$

9. Use the rectangle below to help calculate:



$$\frac{2}{3}-\frac{1}{5}$$

10. Use the rectangle below to help calculate:



$$\frac{1}{3}+\frac{3}{4}$$

11. Use the rectangle below to help calculate:

$$\frac{4}{5} - \frac{1}{4}$$

APPENDIX 5A

Grading assessment for Pythagorean Theorem	Poor	Fair	Good	Excellent
Question 1 Meaning of area of a square.	Participant was not able to explain the meaning of area at all.	Participant gave a vague meaning of area by using the given information.	Participant could partially explain what area means by using the given information	Participant was able to give a clear and concise meaning of area using the given information
Question 2 Explaining the theorem of Pythagoras in own words	Participant was not able to explain what the theorem of Pythagoras means.	Participant has a vague idea of what the theorem of Pythagoras entails.	Participant has some idea of how to explain the theorem of Pythagoras in his/her own words.	Participant was able to give a good explanation in his/her own words of the meaning of the theorem of Pythagoras
Question 3 Finding the areas of the missing squares.	Participant was unable to find the areas of the missing squares	Participant attempted to find the areas of one of the missing squares but did it incorrectly	Participant was only able to find the area of one of the missing squares. He/She attempted to find the area of the second missing square but did it incorrectly	Participant was able to find the areas of both missing squares correctly.
Question 3 Applying the theorem of Pythagoras in right-angled triangles.	Participant was not able to find the lengths of any of the two sides of the two right-angled triangles using the theorem of Pythagoras	Participant attempted to find the lengths of the two sides of the two right-angled triangles by using the theorem of Pythagoras but did it incorrectly.	Participant was only able to find the length of the first side of the first triangle by using the theorem of Pythagoras. He/She attempted to find the length of the side of the second triangle but did it incorrectly	Participant was able to find the lengths of the two sides of the two right-angled triangles correctly using the theorem of Pythagoras.
Question 4 Identifying right-angled triangles using the theorem of Pythagoras	Participant was unable to identify the right-angled triangle by using the given information	Participant gave the incorrect right-angled triangle but provide reasons for choosing the specific triangle	Participant chose the correct triangle but the reason for his/her choice was vague.	Participant chose the correct triangle and gave good reasons for choosing the specific triangle.

Question 5 Choosing Pythagorean sides from given triangle	Participant did not choose any letter or chose D	Participant chose A or B	Participant chose A or B	Participant Chose C
Question 6 Application of the Pythagorean theorem	Participant did not attempt the problem at all.	Participant attempted the problem but did it incorrectly	Participant attempted the problem but with minor mistakes.	Participant was able the solve the problem correctly
Question 7 Demonstrating the theorem of Pythagoras	Participant did not attempt to demonstrate the theorem of Pythagoras at all.	Participant attempted to demonstrate the theorem of Pythagoras but did it incorrectly.	Participant attempted to demonstrate the theorem of Pythagoras with minor mistakes.	Participant was able to demonstrate the theorem of Pythagoras correctly.

APPENDIX 5B

Grading assessment for fractions	Poor	Fair	Good	Excellent
Question 1 Identifying shaded parts of a rectangle	Participant could not identify any of the 3 rectangles' parts that were shaded	Participant could correctly identify only one or less of the 3 rectangles' parts that were shaded	Participant could correctly identify only 2 of the 3 rectangles' parts that were shaded	Participant could correctly identify all three of the 3 rectangles' parts that were shaded
Question 2 Naming the divided parts	Participant was not able to name these parts at all	Participant has a vague idea of what the parts were	Participant has some idea of how to name the parts	Participant was able to name the parts correctly
Question 3 Sharing a chocolate bar	Participant did not attempt the question.	Participant chose a part of the chocolate but the reason for choosing the specific part of the chocolate is vague	Participant chose part of the chocolate and gives interesting reasons for choosing the specific part of the chocolate. For example, I do not like chocolate or I like chocolate.	Participant chose part of the chocolate and gives a good reason for choosing the specific part of the chocolate.
Question 4 Choosing a fraction of a chocolate bar.	Participant did not attempt the question.	Participant chose a part of the chocolate but the reason for choosing the specific part of the chocolate is vague	Participant chose part of the chocolate and gives interesting reasons for choosing the specific part of the chocolate. For example, I do not like chocolate or I like chocolate.	Participant chose part of the chocolate and gives a good reason for choosing the specific part of the chocolate.
Question 5 What fraction of the whole does each represent	Participant was not able to mention any of the two parts.	Participant was able to mention the whole but not the fraction part	Participant was able to mention the whole but has a vague idea of the fraction part.	Participant was able to mention the whole and the fraction part correctly.
Question 6 9 friends sharing a chocolate bar equally.	Participant did not use a drawing and could not mention the fraction part that each person gets.	Participant did not use a drawing at all but was able to mentioned the fraction part that each person gets.	Participant used a drawing but the equal parts were inaccurate, but was able to mention the fraction part that each person gets.	Participant has an accurate drawing of the different equal parts and could accurately mention the part that each person gets.
Question 7 Filling a bottle with water	Participant gives an incorrect	Participant gives an answer that does not match the	Participant gives a correct answer but the reason for	Participant gives a correct answer with a

	answer and reason	reason for choosing the specific answer	choosing the specific answer is vague.	good reason for choosing the specific answer
Question 8 Shading fractions to show addition of two fractions	Participant was not able to shade fraction parts to show the addition of two fractions or add the two fractions using a conventional method.	Participant did not use the shading of fraction parts to show how the two fractions are added but was able to add the fractions using a conventional method.	Participant attempted to shade the fraction parts to show how the two fractions can be added but added the fractions incorrectly.	Participant shaded the fractions parts correctly to show how the two fractions can be added and gave the correct answer to the addition of the two fractions.
Question 9 Shading fractions to show the subtraction of two fractions	Participant did not attempt to shade fraction parts to show the subtraction of two fractions or subtract the two fractions using a conventional method.	Participant did not use the shading of fraction parts to show how the two fractions are subtracted but was able to subtract the fractions using a conventional method.	Participant attempted to shade the fraction parts to show how the two fractions can be subtracted but subtracted the fractions incorrectly.	Participant shaded the fractions parts correctly to show how the two fractions can be subtracted and gave the correct answer to the subtraction of the two fractions.
Question 10 Using a rectangle, dividing it into equal parts and shading the parts to show how two fractions can be added.	Participant did not attempt to divide the rectangle into different equal parts or add the fractions using a conventional method.	Participant attempted to divide the rectangle into different equal parts, but did it incorrectly. He/She was unable to add the two fractions using a conventional method	Participant was able to divide the rectangle into different equal parts but was unable to show how to add the fractions using shading. He/She was able to add the fractions using a conventional method.	Participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions can be added correctly.
Question 11 Using a rectangle, dividing it into equal parts and shading the parts to show how two fractions can be subtracted.	Participant did not attempt to divide the rectangle into different equal parts or subtract the two fractions using a conventional method	Participant attempted to divide the rectangle into different equal parts but did it incorrectly. He/She was able to subtract the two fractions using a conventional method	Participant was able to divide the rectangle into different equal parts but was unable to show how to subtract the two fractions using shading. He/She was able to subtract the two fractions	Participant was able to divide the rectangle correctly into different equal parts and shade the parts to show how the two fractions can be s correctly.

			using a conventional method.	