MICROECONOMIC FOUNDATIONS OF RESEARCH AND DEVELOPMENT

Thesis by

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PREFACE

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ABSTRACT

Numerous studies of the economics of technological change have appeared since the seminal work of Abramovitz and Solow. Most are empirical studies that are without a formal theoretical basis. Scherer was the pioneer of theoretical work on the problem of R&D rivalry.

This thesis revisits the issues in the literature on R&D. In Chapter I, sources of R&D allocative failures are identified and suggestions to remedy the situation are covered. In Chapter II, a selective critique of theoretical R&D models is provided. This completes Part I of the thesis. Part II constitutes the thesis proper. In Chapter III, I develop a nonsequential R&D search model and examine the economic determinants of R&D decisions. Predictions based on comparative statics results are given. The Reservation Technology concept is introduced. In Chapter IV, welfare implications of market structure on industrial R&D are investigated. It is shown that a monopolist may be less persistent in R&D search than a social decision maker. Sufficient conditions for noncooperative duopolists to be more persistent in R&D search than a monopolist are provided. A discussion on R&D economies of scale and a treatment of product and process innovation are also provided. Chapter V presents a new approach to the theory of R&D. A sequential R&D model with a two dimensional search space is developed and a Reswitching Property of R&D is established.

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Chapter I

Sources of R&D Allocation Failures

A Paradox

A substantial amount of empirical evidence suggests that there is a close and positive relationship between productivity change and aggregate R&D expenditure. Earlier work by Griliches (1957, 1958), Kendrick (1961), Mansfield (1968) and Salter (1969), all support the hypothesis at the industrial level. Results from individual industry studies are confirmed by Brown and Conrad (1967), Raines (1968), Terleckyj (1974), and Mansfield and others (1977) at an inter-industry level.

A central theme of economic research on R&D has been the role of government intervention to assure a more efficient rate of innovation. Four sources of market failures associated with R&D are summarized by Noll (1975) in a state of the art review. These sources are:

(1) Indivisibility: the minimum efficient scale of R&D operations can be sufficiently large that the market for a particular class of ideas is not competitive;

(2) Inappropriability: innovators are unable to capture the full economic gains made possible by their innovations;

(3) Indirect failures: if a good must be produced outside a competitive market, the institutions created to bring this

about may lead to inefficiencies in the advancement of knowledge with respect to production and distribution of the good;

(4) Uncertainty: the economic uses of the technical ideas that will emanate from R&D activities are not known in advance, so that the search for innovation is a gamble. (Noll 1975, p. 3).

The first two sources of failures are due to the public good nature of R&D, but all four sources cause deviations from an efficient economic outcome. Thus, a paradox is apparent; while R&D may lead to substantial gains for society through reducing costs of production, an individual user may be motivated to let someone else develop cost-saving technologies.

Because R&D has positive costs, other economic opportunities compete with R&D projects. However, the social benefits of R&D appear to be large enough to warrant an effort to clarify the issues and evaluate mechanisms which might solve the incentive problem. This thesis will critique some of the debate surrounding the theory of innovation as initiated by Schumpeter's two books, Business Cycles, and Capitalism, Socialism, and Democracy. In particular, I will focus on the motivation behind innovative activities.

Based on this critique, I will construct a model of industrial R&D. In that model, industrial R&D will be restricted to the applied type. A clear distinction will be drawn between exogenous and endogenous variables. This in turn should help to explain the inconclusive and inconsistent results obtained in some empirical studies purporting to test Schumpeter's hypothesis. Welfare implications of the relationships between market structure and R&D activity will also be derived.

Two causes of inappropriability are explored. The first is the indivisibility of R&D output. Once R&D comes to fruition, the resulting output can be used an unlimited number of times without any form of depletion in quantity or quality. Hence, if the results of R&D can be applied to production processes or products not under the control of the innovator, the innovator will have little incentive to innovate in the first place. The reason is that even if the innovator can charge the first user for his R&D results, that buyer can sell them to someone else. But, if the innovator restricts the result to his own use, the gain from developing the innovation will be less.

Despite the theoretical arguments just outlined, the problem of indivisibility of R&D activity itself is not crucial, because large scale R&D is closely associated with large scale technology, which may limit the possibility of decentralized market structures. Since large scale technology limits the technical feasibility of market structure choices, there may not be a great loss to society if the technology is not developed. Further, there may be a systematic bias against the development of small scale technology. Because the objective of a firm is to maximize profit, an innovation that limits the technical feasibility of market structures has value to an innovator beyond its direct effects on production costs.

The second cause of inappropriability is due to information dissemination from the success of an innovation. R&D work is filled with uncertainty. A successful breakthrough on a particular R&D project tells others that the area of research is workable. Even if the original breakthrough is patented, a substantial amount of research opportunity may remain in related areas. To visualize this point, treat research as exploration in an Euclidean space with dimensions representing same technological characteristics, and a convex body representing the existing stock of knowledge. Success in pinpointing a point outside the current state of knowledge will lead latecomers to "convexify" the newly identified point with the set of prior knowledge. If benefits can be generated from this secondary activity, the innovator may not be able to capture all the benefits. Alternatively, if the innovator hides the breakthrough, and carries out the "convexifying" work himself, he suffers two losses. Someone else may discover the breakthrough and file a patent before he does, and in any case postponing development of the original breakthrough sacrifices present projects.

An Approach

The study of technological change is one departure point in moving from static to dynamic analysis in economics. The area was first seriously explored by Joseph A. Schumpeter (1939, 1947). Schumpeter proposed a theory of innovation and later adapted it to the concept of creative destruction.[1] He argued that a monopolistic market structure may be more conducive to technological change than any other market structure, especially a competitive market. This implication rests heavily on the presence of potential rivalry, i.e., entry. The theory is further restricted to "big" innovations:

> "...we shall impose a restriction on our concept of innovation and henceforth understand by an innovation a change in some production function which is of the first and not of the second or a still higher order of magnitude."
>
> (Schumpeter, 1939, p. 94)

Galbraith (1956), through a different route, arrived at the same conclusion. He relied on indivisibility of R&D processes as the main explanation for concentrated industry. Thus he wrote:

> "There is no more pleasant fiction than that technical change is the product of the matchless ingenuity of the small man forced by competition to employ his wits to better his neighbor. Unhappily, it is a fiction. Technical development has long since become the preserve of the scientist and engineer. Most of the cheap and simple inventions have, to put it bluntly and unpersuasively, been made... Because development is costly, it follows that it can be carried

only by a firm that has the resources which are associated with considerable size."

(Galbraith, 1956, pp. 86-87)

As the School of Neo-Schumpeterians gradually evolved, Schumpeter's arguments were reinterpreted and much of the original flavor of his work was lost. However, most of the work by that school was empirical. This caused a tendency to reinterpret Schumpeter's theory as needed to make use of the available data. Muller and Tilton (1969) were particularly worried about the trend of misconception. They pointed out that a theoretical disparity existed between Schumpeter's original arguments and those of Neo-Schumpeterians. Fisher and Temin (1973) wrote a critique of the whole body of empirical literature purporting to test Schumpeter's Hypothesis. Grether (1974), reviewing the empirical literature,pointed out the presence of a simultaneous equation problem in most of the work. Kamien and Schwartz (1975) provided a more comprehensive review, covering both empirical and theoretical studies. Their conclusion with respect to the empirical literature is that the results are inconclusive and sometimes inconsistent. A possible explanation of this observation is that the equations used for regression are not based on sound theoretical models. This motivates the present study of the microeconomic foundations of production and R&D.

A line of studies distinct from the neo-Schumpeterians' empirical analysis was generated from Schumpeter's work. Arrow (1962) developed a model to show that "the incentive to invest is less under monopolistic than under competitive conditions but even in the latter case it will be less than is socially desirable." (Arrow, 1962, p. 619). Whether Arrow's result is in fact a counterexample to Schumpeter's Hypothesis is questioned. Demsetz (1969) argued against Arrow's implications. Hirshleifer (1971) investigated "pecuniary gains" as a counter argument to Arrow's result. Needham (1975) summarized the debate. However, Montgomery and Quirk (1974) questioned the validity of Hirshleifer's argument in a general equilibrium framework.

A third independent line of theoretical research was also generated out of Schumpeter's thought-provoking work. Scherer (1967) first outlined a duopoly model with rivals competing to be the first in introducing a new technology. Extensions of Scherer's work were provided by Barzel (1968), Baldwin and Childs (1969), Kamien and Schwartz (1972), and Flaherty (1977). Ruff (1969) posed the problem in a Cournot Economy. He used an optimal control approach and allowed a variable degree of appropriability. His results reinforced Schumpeter's Hypothesis that "the rate of technological progress decreases as the number of firms increase." (Ruff, p. 398) Certainty is assumed throughout Ruff's model. Further work in line with Scherer's tradition but introducing uncertainty formally can be found in Loury (1976), Lee (1977), and Lee and Wilde (1978). Loury's work is of special interest. He adapted the rivalry model to evaluate the welfare implications of market structures. The results he obtained rejected Arrow's results across a spectrum of market structures. The implausibility of the restrictive assumptions in these modeling efforts motivates Part II of this thesis. In Chapter III, I introduce a

Treatment of public goods in general equilibrium models was first tackled with the Lindahl solution concept.[3] A literature review of this approach can be found in Roberts (1974). The difficulty with this solution concept is the test of stability. Individual incentive compatibility is not satisfied, and the free-rider problem is implicity ignored. Another approach incorporating incentive compatibility into public input allocation mechanisms is given by Groves and Loeb (1975). Their results are derived from a partial equilibrium model and based on competitive assumptions. Groves and Ledyard (1974, 1978) extended this to a general equilibrium model. Hurwicz (1972), and Ledyard and Roberts (1974) demonstrated that if strategic behavior is allowed, it is impossible to find a resource allocation mechanism that yields 'individually rational' Pareto-optima and which is also 'individually incentive compatible' for all agents. Ledyard (1977) further investigated the effect of allowing incomplete information of the allocation mechanism and of the response of other individuals in the model. The conclusion is that "for most differentiable mechanisms and environments, incentive compatibility will usually not be obtained even if information is incomplete." (Ledyard, p. 26.)

A recently identified problem of R&D in terms of policy issues is indirect failure. Demsetz (1969) was concerned with the proposal of government involvement. Capron and Noll (1971) summarized the effect of regulation on technological change in regulated industries. Montgomery and Noll (1974) documented two case examples, namely, environment and transportation. Montgomery and Quirk (1974) investigated tax issues and their effects on technological change. Eads (1974, 1977) clarified the misnomer of "unregulated" industries. He urged a more thorough study of the effects of laws and regulations, subsidies and other forms of financial incentives, and the many forms of "externalities" generated from government interventions in the economy on the speed and direction of technological change in both regulated and "unregulated" industries.

Chapter I: Footnotes

1. Creative destruction is a dynamic economic process through which economically obsolete production units are replaced by economically more efficient production units.

2. The reswitching property of research and development is derived in Chapter V. It states the choice between research or development is an economic decision such that "research and development" is not a linear sequence. One may do a little bit of research, then a little bit of development and some more research.

3. A Lindahl solution is a feasible allocation of private and public goods, a price system of private and public goods, and individuals' contributions to the public goods such that profits are maximized by producers and each consumer prefers this allocation to any other allocation within his budget constraint.

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Chapter II

A Critique of Theoretic R&D Models

Literature Review

The economics of R&D has grown extensively in the last two decades, so this critique will be selective. The interested readers are referred to more comprehensive surveys by Nelson (1959), Blaug (1963), Kennedy and Thirlwall (1972), Kamien and Schwartz (1975), and Noll (1975). Both deterministic and stochastic models are developed in the literature. In this critique, I shall concentrate on theoretical R&D models incorporating uncertainty explicitly.

The blossoming of theoretical R&D literature is due to Scherer (1967). Scherer considers R&D as a class of investment projects characterized by a high level of uncertainty. The objective of a decision maker is to maximize expected net benefit. Scherer is interested in the timing of innovation and uses new technology introduction time as a control variable. Unfortunately, uncertainty is not treated explicitly. He assumes an exogenous expected reward to the first innovator. Uncertainty is then reduced to a premium on the discount rate. Subsequent revisions and extensions of Scherer's model by Barzel (1968), Baldwin and Childs (1969), and Flaherty (1977) suffer the same shortcoming. They assume away the problem of uncertainty and make the assumption that at least one rival firm chooses to do R&D. The question whether a firm will innovate at all under a greater or lesser degree of rivalry is essentially left open.

Kamien and Schwartz (1972) reformulate Scherer's model and introduce subjective probability distributions over technology introduction time, which remains as the control variable. Lee (1977) investigates a mirror image of Kamien and Schwartz's model by treating R&D costs as the control. It is demonstrated that the R&D decision is highly sensitive to assumptions on the relative payoffs to innovator and imitator. Kamien and Schwartz (1976) revisit the problem. They derive the result that there is generally some intermediate degree of rivalry at which a firm's innovative activity is maximized.

Loury (1976) criticizes the work by Scherer and Kamien and Schwartz, identifying the partial equilibrium nature of their models. They and previous model builders do not consider the interrelations of rivals' decisions. Granted technological uncertainty is incorporated in Kamien and Schwartz's model, market uncertainty is left out. Loury instead follows Scherer's suggestion of looking into R&D rivalry as a Nash non-cooperative game. An equilibrium model emerges. An interesting aspect of Loury's model is that he treats R&D costs as one-period expenditures. The assumption is criticized by Lee and Wilde (1978). They formulate a variant of the model by treating R&D costs as continuous expenditures up to the time when one of the firms introduces a new technology. The differences and similarity of the conclusions of these two models are detailed in Section I.

Ruff (1969) introduces a unique deterministic model which investigates the appropriability of R&D results. Allowing the final results of a firm's R&D investment to be determined by its own R&D effort and those of its rivals , Ruff arrives at the same conclusion as Loury, namely, the rate of technological progress decreases as the number of firms increases. But Ruff assumes a fixed market structure. This assumption, together with the formulation of the appropriability problem, make one wonder why potential entrants are not benefited by existing firms' R&D effort if R&D results are not appropriable.

A classificatory summary of the models in Table II-1 helps to clarify the different approaches.

Treatment of Uncertainty	Types of Model: Partial Equilibrium Model	Interactive Equilibrium Model
Deterministic Model	Scherer (1967) Barzel (1968) Baldwin and Childs (1969) Flaherty (1977)	Ruff (1969)
Stochastic Model	Kamien and Schwartz (1972,1976) Lee (1977)	Loury (1976) Lee and Wilde (1978)

TABLE II-1: Classificatory Summary of R&D Models

All attempts except Ruff's consider static R&D decisions. Once a decision is made, the R&D manager abides by it and no changes are made until one of the other firms introduces a new technology. This feature is forced on the structure of the models when the authors use fixed reward and "winner gets all" assumptions. Such unrealistic assumptions are first discarded by Ruff when he integrates production with R&D.

Evenson and Kislev (1976) introduce a dynamic progamming model of R&D. Their model follows the footsteps of market search models. For an overview of the search theory development, see Kohn and Shavell (1974), Lippman and McCall (1976a, 1976b), Landsberger (1977), Karni and Schwartz (1977), Wilde (1976, 1977) and Burdett (1978). In Evenson and Kislev's model, parallel research is considered. Unfortunately, the discrete nature of their model creates several analytical difficulties and only a few interesting results are derived. In Chapter III of this thesis, a modification of the model is developed and analyzed; in particular, a <u>continuous</u> control variable is allowed. These two R&D models will be reviewed in Section II of this chapter.

In contrast to previous models, Spulber (1977) introduces a two dimensional search space. This model incorporates <u>non-sequential</u> search strategies. Two cases are investigated, one for "once and for all" innovation and one for "innovation in each period." In Chapter V of this thesis, I formulate a <u>sequential</u> R&D search model with a two dimensional search space. A comparison of these two models is given in Section III of this chapter. A classificatory summary of the sophisticated models is provided by Table II-2:

<u>Degree of Freedom for R&D Search</u>

Nature of R&D Decision	One Dimensional Search Space	Two Dimensional Search Space
Sequential		Lee (Chapter V)
Nonsequential	Evenson and Kislev (1976) Lee (Chapter III)	Spulber (1977)

TABLE II-2: Classification Summary of Sophisticated R&D Models

A recent breakthrough in the literature on R&D was due to Futia (1977). Futia proves the existence of a long run stochastic industry equilibrium under R&D rivalry. A short run industry equilibrium is defined as a pair of positive integers (n,j), where n denotes the number of identical firms and j the product produced or the production technique employed by all firms. Assume that if innovation takes place in any given time only one firm can be successful in innovation and all other firms lose the race. If innovation does not occur, all existing firms survive. Let $\lambda(\ell,k)$ be the probability that the number of R&D rivals in any given period is ℓ given the number of survivors at the beginning of that period is k. Futia assumes no exit of survivor firms, i.e. $k \leq \ell$. With respect to entry, the conditional expectation of the number of firms in the industry at any time period is assumed to be an increasing function of the number of survivors at the beginning of that period, i.e. for any fixed m, $\sum_{k=1}^{m} \lambda(\ell,k)$ is a decreasing function of ℓ. With this set of assumptions Futia proves the existence of a unique stationary probability density over industry sizes, i.e. a long run stochastic industry equilibrium. An important implication of Futia's model is that even though market structure determines the intensity of R&D rivalry in the short run, in the long run, market structure itself becomes endogenous. R&D rivalry and market structure are stochastically determined by the demand situation, R&D costs and ease of entry in the long run.

Section I

Loury and Lee & Wilde

Two defects in earlier models are corrected in Loury's model. First, technological uncertainty is considered explicitly. Second, market uncertainty is introduced. Entry and exit of firms are allowed and welfare results are derived. A Nash noncooperative solution concept is employed.

The structure of the model is as follows. Firms can choose the level of a one time cost to purchase an exponential distribution of technology introduction time, the higher the R&D cost the earlier the expected technology introduction time. Taking all rivals' choices as given, a firm maximizes expected return net of R&D cost. A first order condition for expected profit maximization yields a solution to a firm. A symmetry assumption then leads to the determination of the equilibrium R&D fixed cost investment. Loury proves two important results:

(i) As the number of firms in the industry increases, the equilibrium level of firm investment declines, and

(ii) increasing the number of firms always increases the expected industry technology introduction date.

The driving forces behind these conclusions are the assumptions of a one-time R&D expenditure and full appropriability. Lee and Wilde (1978) reformulate the model allowing variable R&D expenditure. Cost is incurred in each period up to the time when one of the firms in the industry introduces a new technology. Surprisingly, opposite results are obtained:

(i) As the number of firms in the industry increases, the equilibrium level of firm R&D investment increases, and

(ii) an increase in the number of firms in the industry leads to an earlier technology introduction time.

Thus, conclusions in this respect are highly sensitive to the cost assumption, which leads to a different reaction pattern to rivals' decision.

The two models are not left without coincidence of conclusions. In particular, welfare results on market structure are the same. They are

(i) The equilibrium expected profits of a representative firm decreases as additional firms enter the industry.

(ii) Given the same number of projects, an industry with noncooperative rivals will invest more in R&D than a monopolistic one.

(iii) A zero profit equilibrium industry will always incur more R&D investment in the aggregate than a monopolistic one.

Welfare results from these models, while interesting, should not be taken without a grain of salt. The fixed reward and "winner gets all" assumptions are clearly unrealistic for two reasons. First, R&D benefits are derived from production cost reductions. The amount of cost savings or, more important, the profit increment depends on the demand elasticity of the final good. Market opportunities are not considered. Second, R&D is discussed with no reference to production. For a given demand elasticity, benefits from cost reducing innovation are sensitive to the current production level. Market power in price setting is not incorporated in the models.

Section II

Evenson and Kislev and Lee

Evenson and Kislev were the first to formulate R&D into a search model. They assume that nature defines a probability distribution of the technology level $F(y)$, $y \varepsilon (0,\infty)$. A decision maker can choose an integer number of observations n from the distribution by paying a cost, $c(n)$, increasing with an increase in the number of observations, i.e. $c'(n) > o$. After obtaining a sample of realized technology levels, the decision maker rank-orders them and picks the best to compare with the current technology level. If there is improvement, the new technology is adopted; if not, sampling continues with technology maintained at the previous level. Analytical difficulties of the model arise out of the discrete nature of the control variable. First, even though an optimal functional of the Bellman equation can be established, the optimal sampling policy may not be unique. Second, the effects of parametric changes of exogenous variables on the optimal sample size can not be derived without imposing restrictive assumptions, such as allowing only one time sampling. Hence the "optimal stopping technology level" is not characterized. All these defects are corrected in Chapter III of this thesis. A continuous control variable, viz. R&D cost, is introduced. An optimal stopping rule is characterized by a Reservation Technology level[1] and the optimal R&D search intensity is derived. Comparative statics results on these two variables are established.

In passing, a misleading benefit function is defined by Evenson and Kislev. Let V(y) be the optimal expected discounted return net of search cost. The Bellman equation of their model is restated:

$$V(y) = \max_{n} [y-c(n)+\alpha \int_{y}^{\infty} V(z) + \alpha F^{n}(y)V(y)]$$

Where y stands for the net return, and $F^{n}(y)$ is the probability that all n observations are less than or equal to y. Evenson and Kislev defined a benefit function B(y,n) as the last two terms on the right hand side of the equation. Consider the choice of doing nothing in the current period. The expected discounted return is given by $y-c(a)+\alpha V(y)$. Thus, the incremental benefit due to R&D is $B(y,n)-\alpha V(y)$. It is the incremental benefit that should be interpreted as the benefit function of R&D. The former definition puts a "subsidy" on the true benefits of R&D. The definition implies greater R&D benefits at higher technology levels, which in turn implies no stopping if R&D is currently an optimal choice. The latter definition shows declining R&D benefits with increases in technology level. Stopping will eventually be optimal.

Section III

Spulber and Lee

In Spulber's model, a two dimensional R&D search space is defined. Denote w_t as the research performance level. Let the sequence $\{w_t\}$ be a submartingale[2] and a stationary Markov process with a given transition probability. Denote x_t as the quality level corresponding to the outcomes of a development process. Assume x_{t+1} is drawn from the distribution given by $F_{w_t}(\cdot)$ and $F_{w_t}{}'(\cdot) \leq F_{w_t}(\cdot)$ if and only if $w_t{}' \geq w_t$. Spulber proves the existence of a unique 'switch-point' level of quality x^*, with the property that if $x \geq x^*$, the R&D process is stopped, and if $x < x^*$, the R & D process is continued. Spulber considers two cases. For the "one time innovation" case, the two dimen-mensional search space is essentially treated as one dimensional. In his own words, "the stopping rule can be put in terms of either w^* or x^*", where w^* is the switch-point research performance level. In the "innovation in each period" case, only sketchy results are discussed. In his model, Spulber never explicitly considers the economics of R&D. R&D are treated exogenously in a stylized growth model with the R&D decision determined by the trade off between immediate and future benefits. Optimal choice of reasearch, development or stopping under different state points in the two dimensional search space is not discussed. These shortcomings are corrected in Chapter V of this thesis.

Conclusion

Although this survey is selective, it has covered the mainstream developments in one aspect of the R&D literature. Over the recent past, theoretical R&D models have flourished. The rapid improvements in conceptualizing R&D problems transcend the deterministic partial equilibrium models leading to stochastic equilibrium models and from static decision models to dynamic search models of sequential and non-sequential strategies. More promising is Futia's breakthrough in proving a stochastic equilibrium industry size. A synthesis of the models by Lee and Futia is clearly an approach which has the potential of formalizing one aspect of the theory of innovation, verbally sketched out by Schumpeter some forty years ago.

Chapter II: Footnotes

1. A Reservation Technology Level is that below which R&D is continued and above which R&D is stopped.

2. Let $\{W_t\}$ be a sequence of random variables and I_t be the information in the past history just before the observation of W_{t+1} and such that $I_1 \subset I_2 \subset \ldots.$ The sequence $\{W_t\}$ in a submartingale with respect to I_t if

$$E\,(W_{t+1} \mid I_t) \geq W_t$$

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Chapter III

Non-Sequential R&D Search Model

The Problem

The number of studies of technological change increased ever since the seminal work of Solow (1957) and Abramovitz (1962). Lacking a theoretical model, Solow used residual analysis[1] and discovered that about 80% of the increase in productivity[2] in the U.S. economy over the period 1909-1949, cannot be explained by capital investment alone. This large residual was attributed as "technological change." Most subsequent work in this area has been empirically oriented. The results in general confirm Solow's conclusions. For example, Denison (1962) found that 42% of the rise in output per worker[3] between 1929 and 1957 is caused by improved worker's education, 36% by technological change, and only 9% by capital accumulation.

The scattered amount of effort invested in developing theoretical bases for econometric models has yielded little insight into the process of technological change. It is conceived as a parameter in most models. Results are obtained by shifting this parameter to generate comparative statics results.

The basic notion of the theory of technological change can be summarized by Hicks neutral technical progress.[4] The equation representing this concept is given by

$$Y = aF(K, L)$$

where

Y = quantity output,

a = a(t)

= shift parameter of the production function, F,

K = capital input to the production process, and

L = labor input to the production process.

If the production function F is linear and homogeneous in K and L, it can be rewritten as:

Y = aF(K, L)

= F(aK, aL), if F is linear and homogeneous.

Now, if a(t) is expressed as an exponential growth function, say

$$a = e^{-mt}$$

where

m = the rate of improvement,

t = time elapsed from initial period,

and, if the rate of improvement for capital and labor differs, one has a generalized Hicks-neutral technical change. Special cases are m being constant for k, i.e., Harrod-neutral technical process; and m being constant for ℓ, i.e., Solow-neutral technical progess. These are the three basic notions of technical change and equations used by econometricians for regression purposes. They are usually referred to by the expression "manna from heaven." Embodiment theory[5] was proposed requiring that new capital or labor will only incorporate knowledge at that moment of time. Unfortunately, according to a comment by Griliches (1965),

> "The fact that standard economics had no theory of technical change explains, I think, why we got around to trying to measure it as the 'residual.' Because it was an empty box, we proceeded to define it as everything that cannot be explained by standard theory... The hypothesis of embodiment, while potentially very fruitful, in practice turned out to be nothing more than a relabeling of an already empty box." [Griliches, p. 344.]

If progress is to be made in understanding technological change, economists must study the production function itself, and how a larger set of production functions can be provided to extend society's choice options. With this orientation in mind, an immediate observation is that any talk of shifts in a production function (not necessarily systematic changes) implicitly assumes a cost (perhaps of research and development) of bringing a new production function (or technology) into existence. There exists a "production function" of production functions. Society's choice space is extended to a higher dimension including not just the combinations of capital and labor but the production processes as well.

There are studies that try to explain this second "production function." Organization theorists, for example, have made some progress on the problem. They suggest that more effective research output can be generated by a better organizational set-up, which in turn depends on the size of the research unit. Cooper (1964) in a study of the electronics and chemical industries concluded that large firms are plagued with bureaucracy, which causes a disharmony for creative activities. This in turn leads to the observation that a given product can cost three to ten times as much if developed by a large firm instead of a small one. Blair (1972) supported this conclusion and stated that bureaucracy and creativity are incompatible. For a review of studies of technological changes in the organizationalist paradigm, see Noll (1976).

A third production function may be asked, and an infinite regress problem of the following sort arises:

> We wish to minimize some production costs through the production of new production processes, but then we also want to minimize the cost of this second production. A third production function (distilled from organization studies perhaps) is then postulated. So, we get into research on research...

Somehow a stop to this infinite problem is necessary to reveal which "research" is studied. The existence of other "research" is obvious, but only one move can be made at a time.

In this chapter, the type of research under consideration will be made explicit. The distinction between endogenous and exogenous variables is also clarified. Comparative statics results are derived in order to explain anomalies in controversial econometrics predictions.

I propose to study technological change, considering one kind of research at a time. Uncertainty and imperfect information are incorporated in applied research. Technological change is studied only at the firm level. The rational actor approach[6] is used. I assume the objective of a firm is to maximize its discounted expected stream of profits. Two sources of technological change are considered. They are:

(1) "Exogenous" Technological Change

Technological opportunities in any area of research are influenced by many factors. In particular, research in other sectors of the economy may yield useful information. However, such activities are not within the control of the firm. The study of this aspect of technological change (from the individual firm's point of view) is much in line with the traditional way of formulating the R&D problem. Nevertheless, any study of exogenous technological change must include consideration of the interrelation between different technologies. For example, basic research results from non-profit organizations such as government research facilities and universities may have a significant impact on industrial R&D.

(2) Research Oriented Active Technological Search

This second aspect of technological change is very important, and in fact complementary to the first. Obviously, one cannot talk about new technology without asking how the new alternatives are generated. Unfortunately, by its very nature theoretical modeling of this activity is very difficult. Only an initial attempt is made in this chapter, where a basic model is formulated. A more detailed study, integrating production and R&D, is attempted in the next chapter, where welfare ranking of market structures will be considered.

Since R&D is a very complex process, a number of interesting aspects must necessarily be ignored. Rosenberg (1975) correctly pointed out that research in a broad sense may be as simple (but yet significant) as ascertaining the production function associated with different input mixes. Technological change may come about in the form of process and product innovation,[7] the two being difficult to separate. A new product may be an input to an existing on-the-shelf production process which is not used because of a missing input. Alternatively, the successful commercialization of a laboratory technology may lower the price of a product so much that the product can be used as an input to produce a once expensive product. These are very interesting research topics but will not be taken up here. Product and process innovation will be discussed in the next chapter. The inter-relatedness of research in different sectors of an economy is more difficult and ultimately calls for a general equilibrium model. As a start, I offer a non-sequential R & D search model (at the firm level), which will provide a first step toward a microeconomic foundation for technological change.

The layout of this chapter is as follows. In Section I, the basic concepts of the model will be outlined. I assume that the present state of knowledge of the firm defines a distribution of potential technologies for it. The researcher knows the distribution. The objective of the decision maker is to maximize the discounted expected sum of profits. The existence of a solution to the stochastic dynamic programming problem facing the decision maker is proved. In Section II, comparative statics results are derived. The concept of a Reservation Technology Level is introduced. This helps to explain why technology search may be stopped in a research area for some time, and later resumed. In Section III, I consider the effects of certain shifts in the distribution of potential technologies. This work substantiates the complementarity of basic and applied research. The chapter concludes with a summary of the results obtained and some speculation on other potential areas of research to be discussed in the next chapter.

Section I

Introduction

In this section, I introduce the basic concepts of the model which will be developed and generalized in this and the next chapter. The basic structure of the model will be outlined here. Mathematical complications are presented in an appendix whenever it will not affect the continuity of the discussion. I also wish to emphasize at the beginning that the model will be structured in such a way that the firm-relevant R&D variables are endogenous, while R&D in other sectors are treated as exogenous. Exogenous technological change is considered briefly in Section III.

The basic model is an extension of a model developed by Evensen and Kislev (1976), using agricultural research as an example. Their model uses stochastic dynamic programming with a discrete control variable. But discreteness in the control variable causes technical difficulties in their formulation. A few interesting results were obtained by imposing very restrictive assumptions, e.g., the researcher can perform only one experiment, or a steady state prevails. I shall modify their model by introducing a continuous control variable, the R&D search intensity. This brings my model closely in line with labor market search models as first investigated by McCall (1970). Lippman and McCall (1976) provided a survey of this branch of research. Wilde (1977) extended McCall's sequential labor search model to nonsequential search, and established the existence of an equilibrium distribution of wage offers. I shall use a non-sequential search approach as well. A justification is that the firm can decide on a R&D budget, but cannot dictate the precise nature of the results. This may be due to the researchers doing some irrelevant projects, either consciously for professional interest or unintentionally due to wrong set-up and blind alley search. The number of actual observations from the distribution of potential technologies is therefore random.

A number of interesting results are obtained from the new model: The learning concept as explored by cost engineers is formulated as statistical learning;[8] the formal results are stronger than those derived by Evensen and Kislev; a corrected benefit concept emerges; and the complementarity of basic and applied research is emphasized.

We turn now to a description of the concepts used in the formal model.

Concepts of the Model

A number of empirical studies will help in conceptualizing the formal model. Technological opportunity was defined by Phillips (1966) as the environment to which possible technological advances are constrained. Thus, associated with any state of basic scientific knowledge is a distribution of technological opportunities. The driving force in shaping this environment for applied research is basic research. Basic research is considered to be exogenous in my model of applied research. As technological progress pushes to the frontier of the environment, the prospect of getting any further advance lessens. Thus, technological progress will slacken over time unless exogenous changes reshape the research possibility more favorably. Technological opportunities, therefore, enter into the firm's objective function.

Scherer (1965), Kelly (1970), and Baily (1972) found evidences to support this conjecture. Baily, for example, found that the number of new drugs introduced in any year was positively related to R&D spending in preceding years, and negatively related to a seven-year moving average of past total new drug introductions. The premise also explains Schmookler's observation (1966) that there is no support to the hypothesis that inventions in a field beget further invention. Indeed, within a given research environment, my model predicts inventions, when viewed as applied to technological progress, may have a negative effect on further investment in inventive activities. However, some inventions or discoveries can contribute to basic technological progress, thus promoting more progress. Moreover, such inventions are often exogenous to the research environment for which they are considered to be basic technological progress.

While Schmookler (1962) emphasized the importance of the demand side of innovative activities, the decision to engage in applied R&D can be viewed as largely an economic decision, in which the benefits and costs of the endeavor are balanced against each other. Thus, the importance of the state of knowledge can be attributed to two factors. First, it makes possible higher levels of technology. Second, it may allow the achievement of a given level of technology at a lower cost.

The Formal Model

In the formal model, technology opportunities are represented by a probability distribution of potential technology level, F(z), defined on the closed interval $[y' + \theta, y'' + \theta]$, where z is an index of potential technology level, and θ is the parameter representing other related R&D activities. Delaying the consideration of exogenous technological change, we suppress the θ notation. For the present, the relationship of R&D to production will be ignored, although it will be examined in the next chapter. The following assumptions are made:

Assumption I: A research manager has a subjective probability assessment of the potential results of R&D. Assume this subjective distribution coincides with the true distribution as defined by the state of knowledge, F(z). The distribution is defined on the closed interval $[y', y'']$ with the current technology level practiced by the firm given by y.

Assumption II: The manager can form a research team, which conducts research at some intensity, λ. The cost of this effort is given by $K(\lambda)$, where $K'(\lambda) > 0$, and $K''(\lambda) \geq 0$.

Assumption III: R&D is conceived as a process of acquiring observations from the distribution of potential technologies. At R&D search intensity, λ, the number of observations is random, but controlled by a Poisson distribution with mean λ. Hence, an alternative interpretation of λ is the expected number of observations of new technologies, or the mean time for observing a new technology by the inverse of λ. Thus, the manager can choose the expected number of observations, but not the realized number of observations.

Assumption IV: Let the realized number of observations be n. The task of the manager is then to rank order these n observations, and compare the highest value of these observations with the current technology level. Assume that the higher the technology level, y, the higher the net benefit $R(y)$ to the firm, i.e., $R'(y) > 0$. If the best observed technology is higher than the current one, the new technology is adopted without further cost. If not, the current technology is retained until a better technology shows up. Hence, the relevant distribution to the decision maker is the distribution of best observed technology, defined as $H(z, \lambda)$.

Assumption V: R&D is nontransferable.

A number of results can be readily derived from this set of assumptions.

Proposition III-1: $H(z, \lambda) = \exp\{-\lambda[1 - F(z)]\}$

Proof: See Appendix.

Several properties of the best observed technology function, $H(z, \lambda)$, are easily obtained:

(i) $H(z, 0) = 1, \forall z \varepsilon [y', y'']$

(ii) $H(z, \lambda) = 0, \forall z < y''$,

$= 1, z = y''$,

(iii) $H(y', \lambda) = \exp(-\lambda) > 0$,

(iv) $H(y'', \lambda) = 1, \lambda > 0$,

(v) $\frac{\partial H(z,\lambda)}{\partial \lambda} = -H(z, \lambda)\,[1 - F(z)] < 0$

(vi) $\frac{\partial^2 H(z,\lambda)}{\partial \lambda^2} = H(z, \lambda)\,[1 - F(z)]^2 > 0$.

The figure below will make clear the stochastic dominance of the best observed technology distributive function with respect to R&D search intensity, λ.

Thus, the following proposition is established:

Proposition III-2: The best observed technology distributuion is stochastically ranked by R&D search intensity. In particular, it is convex downwards with respect to λ.

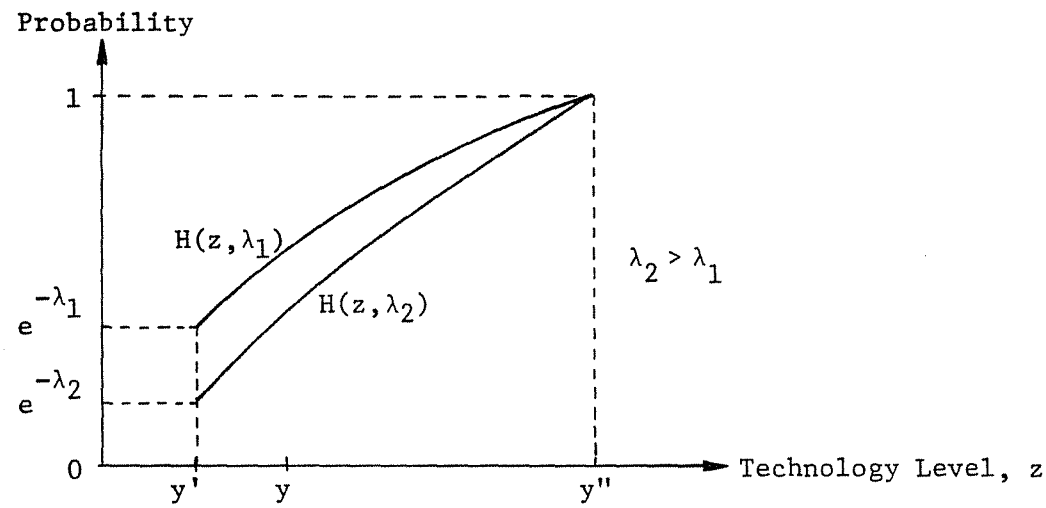

Figure III-1. Stochastic Dominance of Best Observed Distributive Functions

Define the following terminology:

$E(\Delta y) \equiv$ expected technological improvement with R&D search intensity λ

$$= \int_y^{y''} (z - y)\, dH(z;\lambda)$$

$$= y'' - y - \int_y^{y''} H(z;\lambda)dz, \text{ integration by parts}$$

Two further propositions can be derived:

<u>Proposition III-3:</u> There is a diminishing increment in technological improvement for a given state of knowledge.

Proof:

$$\frac{\partial E(\Delta y)}{\partial \lambda} = \int_y^{y''} H(z;\lambda)[1 - F(z)]dz > 0$$

$$\frac{\partial^2 E(\Delta y)}{\partial \lambda^2} = -\int_y^{y''} H(z;\lambda)[1 - F(z)]^2 dz > 0$$

Q.E.D.

Proposition III-4: (i) For a given state of knowledge the higher the current level of technology the less the expected technological improvement with a fix R&D search intensity.

(ii) The expected technological improvement decreases at a decreasing rate as the current level of technology is improved.

Proof: $$\frac{\partial E(\Delta y)}{\partial y} = -1 + H(y;\lambda) < 0 \ ,$$

$$\frac{\partial^2 E(\Delta y)}{\partial y^2} = H(y;\lambda)\ \lambda f(y) > 0 \ ,$$

where

$$f(y) = \frac{\partial F(y)}{\partial y} \ .$$

Q.E.D.

Hence, if the technology level, y, is interpreted as the negative of the average cost of production with constant marginal cost, one has a statistical explanation of the learning curve phenomenon, i.e., average cost decreases over time at a decreasing rate in an expected sense.

The decision problem to be solved by the R&D manager is to find a level of R&D search intensity so as to maximize the expected discounted stream of net income. Let V(y,N) be the optimal expected present value of the system of production and technological search when there are N decision periods left. If the R&D manager chooses a R&D search intensity λ, the discounted expected value of the system starting from the next period is given by:

$$\alpha \int_{y}^{y''} V(z,N-1)\,dH(z,\lambda_N)$$

if the best observed technology is at a higher level than the current one; α stands for the discount factor. The corresponding term is given by:

$$\alpha V(y,N-1)\; H(y,\lambda_N)$$

if the best observed technology is no better than the current one. Combining these two terms with the current net income from production and the current cost incurred for the R&D program, gives:

$$V(y,N) = \max_{\lambda_N} \left\{ R(y) - K(\lambda_N) + \alpha \int_{y}^{y''} V(z,N-1)\; dH(z,\lambda_N) + \alpha V(y,N-1)\; H(y,\lambda_N) \right\}$$

$$\forall N > 0$$

where as above, $R(y)$ is the net revenue to the firm at the technology level y and $K(\lambda_N)$ is the cost of searching at the level λ_N. This is a recursive functional as developed by Bellman (1957). Define $V(y,0) \equiv 0$. Further assumptions on the R&D cost functions are:

$$\lim_{\lambda_N \to 0} K'(\lambda_N) = \bar{K} > 0\;, \quad \text{and} \quad K(0) = 0\;, \quad \forall N > 0$$

For the infinite horizon case, the analogous functional is:

$$V(y) = \max_{\lambda} \left\{ R(y) - K(\lambda) + \alpha \int_{y}^{y''} V(z)\; dH(z;\lambda) + \alpha V(y)\; H(y;\lambda) \right\}$$

$$= \max_{\lambda} \left\{ [R(y) + \alpha V(y)] - K(\lambda) + \alpha \int_{y}^{y''} [V(z) - V(y)]\; dH(z;\lambda) \right\}$$

An explanation is required for the second manipulation. Consider the following interpretation:

$R(y) + \alpha V(y)$ = value of the system if the R&D manager does not engage in any form of R&D at the present period, but behaves optimally from the next period onwards;

$K(\lambda)$ = cost of R&D search at intensity level λ;

$$B(\lambda, y) = \alpha \int_{y}^{y''} [V(z) - V(y)]\, dH(z;\lambda)$$

= discounted expected optimal benefit if research is pursued in the current period at intensity λ.

Clearly, this definition of a benefit function differs from that commonly used in dynamic programming literature.[9] The latter was used by Evensen and Kislev (op. cit., p. 271). I claim that my definition is the appropriate one for economic analysis. Indeed, it can be shown that the definition used by Evensen and Kislev has some contradictory implications.[10]

I shall now establish some properties of the benefit function:

Proposition III-5: $B(y,\lambda)$ is concave w.r.t. λ, and $B(y,0) = 0, \forall y$.

Proof: $$B(y;\lambda) = \alpha \int_{y}^{y''} [V(z) - V(y)]\, dH(z;\lambda)$$

$$= \alpha \left\{ V(y'') - V(y) - \int_{y}^{y''} H(z,\lambda) \frac{\partial V(z)}{\partial z}\, dz \right\},$$ integration by parts.

$B(y;\lambda)$ is clearly non-negative, and $B(y;0) = 0$,

since $H(z;0) = 1, \forall z$.

Proof: (contd)

Concavity of the benefit function is straightforward:

$$\frac{\partial B(y;\lambda)}{\partial \lambda} = \alpha \int_{y}^{y''} H(z,\lambda)\ [1-F(z)]\ \frac{\partial V(z)}{\partial z}\, dz > 0 \quad ,$$

and

$$\frac{\partial^2 B(y;\lambda)}{\partial \lambda^2} = -\alpha \int_{y}^{y''} H(z,\lambda)\ [1-F(z)]^2\ \frac{\partial V(z)}{\partial z}\, dz < 0 \quad .$$

Q.E.D.

Note that concavity of the benefit function implies diminishing returns to the intensity of R&D search. Hence, with a convexity assumption on the cost function of R&D, the optimal level of R&D intensity must be unique. A second property of the benefit function is given by the following:

<u>Proposition III-6:</u> For a given level of knowledge, the higher the current level of technology, the less the benefit of R&D search.

Proof:

$$\frac{\partial B(y,\lambda)}{\partial y} = -\alpha \left\{ \frac{\partial V(y)}{\partial y} - H(y;\lambda)\ \frac{\partial V(y)}{\partial y} \right\}$$

$$= -\alpha[1-H(y;\lambda)]\ \frac{\partial V(y)}{\partial y}$$

$$< 0 \ .$$

Q.E.D.

<u>Existence</u>

I shall now prove the existence of a solution to the infinite horizon problem. The proof is a straightforward application of Denardo's Existence Theorem (1967). Two prerequisites are necessary before one can use the theorem, namely, monotonicity and contraction assumptions. I state the result in the form of a theorem, leaving the proof in an appendix.

Theorem III-1: Given the assumptions of the model, there exists a unique, continuous, bounded solution to the functional equation.

Proof: See appendix.

This completes the preliminary analysis of the model. The next task is to derive implications from the model.

Section II

Optimal R&D Search Intensity

Theorem III-1 establishes the existence of a functional, such that the following is satisfied:

$$V(y) = \max_{\lambda}\left\{R(y) + \alpha V(y) - K(\lambda) + \alpha\int_{y}^{y''} [V(z) - V(y)]\, dH(z;\lambda)\right\}$$

$$= \max_{\lambda}\left\{R(y) - K(\lambda) + \alpha V(y'') - \alpha\int_{y}^{y''} H(z,\lambda)\frac{\partial V(z)}{\partial z}dz\right\}.$$

Thus, one can get the first order condition for maximization of the problem on the right-hand side by differentiating the expression in the bracket. Comparative statics results are then derived. Let

$$W(\lambda;y) = R(y) - K(\lambda) + \alpha V(y'') - \alpha\int_{y}^{y''} H(z;\lambda)\,\frac{\partial V(z)}{\partial z}\,dz \quad .$$

Assume $W(\lambda;y)$ is twice differentiable w.r.t. λ.

The first order condition for maximization of $W(\lambda;y)$ λ is given by:

$$\frac{\partial W(\lambda;y)}{\partial\lambda} = -K'(\lambda) + \alpha\int_{y}^{y''} H(z;\lambda)\;[1 - F(z)]\;\frac{\partial V(z)}{\partial z}\,dz = 0 \;.$$

This condition requires that the marginal cost of R&D, $K'(\lambda)$, should be equal to the discounted expected marginal benefit,

$$\alpha \int_{y}^{y''} H(z;\lambda) \; [1 - F(z)] \frac{\partial V(z)}{\partial z} \, dz \quad .$$

Note that the expression takes into account all future benefits and costs, assumed to be optimally balanced. The second-order condition is obtained by showing a negative sign for the second derivative of $W(\lambda;y)$ λ:

$$\frac{\partial^2 W(\lambda;y)}{\partial \lambda^2} = -K''(\lambda) - \alpha \int_{y}^{y''} H(z;\lambda) \; [1 - F(z)]^2 \frac{\partial V(z)}{\partial z} \, dz < 0 \quad .$$

Assume that $(\partial W(\lambda;y))/(\partial \lambda)$ is differentiable w.r.t. α and y. Applying the Implicit Function Theorem on the first-order condition, and denoting the optional search intensity by λ^*, the following comparative statics results can be derived:

<u>Theorem III-2</u>: $\frac{d\lambda^*}{d\alpha} > 0$, and $\frac{d\lambda^*}{dy} < 0$, if $\frac{\partial^2 V(z)}{\partial z \partial \alpha} > 0, \forall \; z \varepsilon [y',y'']$.

Proof:

$$\frac{d\lambda^*}{d\alpha} = - \frac{\int_{y}^{y''} H(z;\lambda)[1-F(z)]\frac{\partial V(z)}{\partial z} dz + \alpha \int_{y}^{y''} H(z;\lambda)[1-F(z)]\frac{\partial^2 V(z)}{\partial z \partial \alpha} dz}{-K''(\lambda) - \alpha \int_{y}^{y''} H(z;\lambda)[1-F(z)]^2 \frac{\partial V(z)}{\partial z} dz} > 0 \quad .$$

$$\frac{d\lambda^*}{dy} = - \frac{-\alpha H(y,\lambda)[1-F(y)] \frac{\partial v(y)}{\partial y}}{-K''(\lambda) - \alpha \int_{y}^{y''} H(z;\lambda)[1-F(z)]^2 \frac{\partial V(z)}{\partial z} dz} < 0 \quad .$$

Q.E.D.

The comparative statics results have the following interpretations. An increase in the value of the discount factor (i.e., the less one discounts the future) will lead to more intensive search for a better technology because *ceteris paribus* the future stream of profits is valued more. At the same time the opportunity cost of investment is lower than before, and leads to more investment. Similarly, the higher the current technology level, the less intensive is the search for a better technology because, *ceteris paribus*, the lower the probability of finding a better technology.

Given the definition of V(y), and the first-order condition, it follows that an increase in the current level of technology will lead to an increase in the optimal discounted expected stream of profit.

Corollary III-1: $\frac{\partial V(y)}{\partial y} > 0$.

Proof:

$$\frac{\partial V(y)}{\partial y} = R'(y) - K'(\lambda)\frac{d\lambda}{dy} + \alpha H(y;\lambda)\frac{\partial V(y)}{\partial y} + \alpha \int_y^{y''} \frac{\partial V(z)}{\partial z} H(z;\lambda)[1-F(z)] \frac{d\lambda}{dy} dz$$

$$= R'(y) + \alpha H(y,\lambda)\frac{\partial V(y)}{\partial y} \text{ , by the first order condition .}$$

Therefore,

$$\frac{\partial V(y)}{\partial y} = \frac{R'(y)}{1 - \alpha H(y;\lambda)} > 0, \text{ since } \alpha H(y;\lambda) < 1 \quad .$$

Q.E.D.

Reservation Technology Level

As noted earlier, this model is closely related to labor market search models. It is therefore not surprising to find a natural analogue to the reservation wage concept. Call this a reservation technology. Define the following marginal benefit function:

$$\Psi(\lambda;y) = \alpha \int_{y}^{y''} H(z;\lambda)\ [1-F(z)]\ \frac{\partial V(z)}{\partial z}\, dz \quad .$$

<u>Theorem III-3:</u> Given the assumptions of the model, and

$$\exists\, \tilde{y} \in [y',y''] \ni : \psi(0,\tilde{y}) = \alpha \int_{\tilde{y}}^{y''} [1-F(z)]\ \frac{\partial V(z)}{\partial z} dz > \overline{K} \ ,$$

then

$$\exists\, y^{*} \in (\tilde{y},y'') \ni : \psi(0,y^{*}) = \alpha \int_{y^{*}}^{y''} [1-F(z)]\ \frac{\partial V(z)}{\partial z} dz = \overline{K} \ .$$

Proof:

Claim: $\psi(\lambda;y)$ is convex and decreasing λ.

First and second derivatives of the $\psi(\lambda;y)$ λ are given by,

$$\frac{\partial \psi(\lambda;y)}{\partial \lambda} = -\alpha \int_{y}^{y''} H(z;\lambda)\ [1-F(z)]^2\ \frac{\partial V(z)}{\partial z} dz < 0 \text{ , and}$$

$$\frac{\partial^2 \psi(\lambda;y)}{\partial \lambda^2} = \alpha \int_{y}^{y''} H(z;\lambda)\ [1-F(z)]^3\ \frac{\partial V(z)}{\partial z} dz > 0 \ .$$

Four additional properties of $\psi(\lambda;y)$ are:

(i) $\lim_{\lambda \to 0} \psi(\lambda;y) = \alpha \int_{y}^{y''} [1-F(z)]\ \frac{\partial V(z)}{\partial z} dz > 0$,

(ii) $\lim_{\lambda \to \infty} \psi(\lambda;y) = 0 \ , \ \forall\, y < y''$,

(iii) $\frac{\partial\psi(\lambda;y)}{\partial y} = -\alpha H(y;\lambda)\ [1-F(y)]\frac{\partial V(y)}{\partial y} < 0\ , \forall\lambda \geq 0$, and

(iv) $\psi(\lambda;y'') = 0\ , \forall\lambda \geq 0$.

Hence, if $\exists\tilde{y} \varepsilon [y',y''] \ni:\psi(0,\tilde{y}) > \overline{K}$

and since $\psi(0,y'') = 0$, and $\frac{\partial\psi(0,y)}{\partial y} < 0$,

$\exists y^* \varepsilon (\tilde{y},y'') \ni:\psi(0,y^*) = \overline{K}$ by the Mean Value Theorem.

A simple diagram will help to illustrate the theorem:

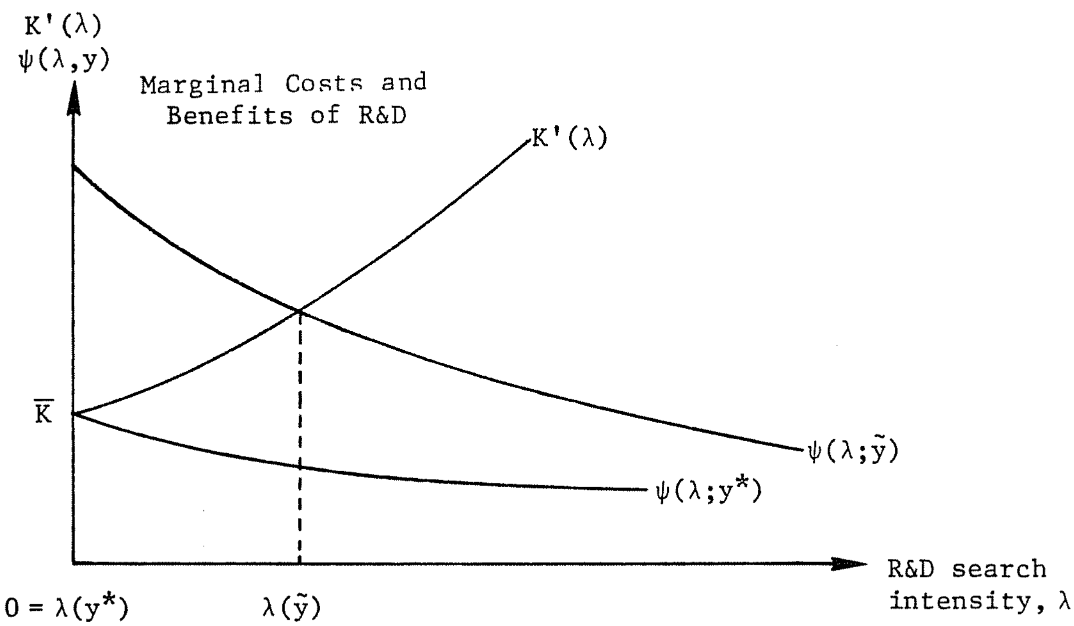

Figure III-3: Marginal Costs and Benefits of R&D

The above diagram also indicates the uniqueness of an optimal choice of R&D intensity. Q.E.D.

y* is the reservation technology. One property of the reservation technology is that, whenever a technology level greater than or equal to y* is observed from R & D, the search process stops unless exogenous changes modify the underlying search environment. If the current and best observed technology is less than the reservation technology level, the R&D search effort continues. Hence, the reservation technology is a stopping rule for R&D. The importance of this concept will be made clear in the next chapter. Comparative statics results on this concept are also of interest.

Theorem III-4: $\frac{dy^*}{d\bar{K}} < 0$, and $\frac{dy^*}{d\alpha} > 0$, if $\frac{\partial^2 V(z)}{\partial z \partial \alpha} > 0$, $\forall z \in [y', y'']$.

Proof: By the definition of y^*,

$$\alpha \int_{y^*}^{y''} [1-F(z)] \frac{\partial V(z)}{\partial z} dz - \bar{K} = 0 \quad .$$

Using the Implicit Function Theorem, the desired results are obtained:

$$\frac{dy^*}{d\bar{K}} = \frac{1}{-\alpha[1-F(y^*)] \frac{\partial V(y^*)}{\partial y^*}} < 0 ,$$

$$\frac{dy^*}{d\alpha} = - \frac{\int_{y^*}^{y''} [1-F(z)] \frac{\partial V(z)}{\partial z} dz + \alpha \int_{y^*}^{y''} [1-F(z)]^2 \frac{\partial^2 V(z)}{\partial z \partial \alpha} dz}{-\alpha[1-F(y^*)] \frac{\partial V(y^*)}{\partial y^*}} > 0 .$$

Q.E.D.

Observe that a sufficient shift in $\bar{K}$, the fixed cost of R&D, may stop the R&D search process. On the other hand, if $\bar{K}$ is lowered due to better organization of research, search opportunities which may have been economically unprofitable, and hence abandoned previously, might be resumed. This clearly indicates the importance of the market for research manpower. An increase in the supply price of research input, whether fixed or variable, will affect R&D search activities.

One also observes that the less the future is discounted, the higher will be the reservation technology level. Thus, the economic value of a research unit is not constant. It changes with changes in economic variables of the economy. In particular, endogenous changes in the applied technology of a particular research area will affect the value of further exploitation of the area. Exogenous changes in other research areas may have positive or negative effects on a given area of research. For example, better understanding of the theory behind the research area may improve the distribution of potential technologies by shifting the mean of the distribution. On the other hand, discovery of a new research area may increase the opportunity cost of exploiting the old research area.

The preceeding model clearly indentifies a number of determinants of R&D. The R&D search intensity (and the monetary reward associated with it) in a given research area is dependent on its total and marginal cost (the former determines whether R&D is worthwhile, and the latter determines the efficient amount, given that it is worthwhile), the underlying state of knowledge, the discount factor, the current level of technology, and the number of decision periods taken into account by the decision maker. This last factor remains to be analyzed.

The Finite Horizon Case

Consider a N period version of the optimal functional, V(y,N), as stated in the previous discussion.

$$V(y,N) = \max_{\lambda_N} \left\{ R(y) - K(\lambda_N) + \alpha \int_{y}^{y''} V(z,N-1) d\, H(z,\lambda_N) + \alpha V(y,N-1)\, H(y,\lambda_N) \right\} .$$

By definition, $V(y,0) \equiv 0$, and $\lambda_0^* \equiv 0$.

For a one period decision,

$$V(y,1) = \max_{\lambda_1} \left\{ R(y) - K(\lambda_1) \right\} .$$

Clearly, the optimal R&D search intensity for a one period decision problem is zero. Hence,

$$V(y,1) = R(y), \quad \text{and} \quad \frac{\partial V(y,1)}{\partial y} = R'(y) ,$$

and

$$\frac{d\lambda_1^*}{dy} = 0, \quad \text{and} \quad \frac{d\lambda_1^*}{d\alpha} = 0 .$$

Results for the R&D problem with decision periods greater than one are stated in the following theorem:

Theorem III-5: Given the assumptions of the model, suppose the following also hold:

(i) $\dfrac{\partial^2 V(y,M)}{\partial\alpha\,\partial y} > 0, \ \forall y \in [y',y''], \ \forall M \geq 2 ,$

(ii) $\dfrac{\partial V(y,M)}{\partial y} > \dfrac{\partial V(y,M-1)}{\partial y}, \ \forall y \in [y',y''], \ \forall M \geq 2 .$

Then the following comparative statics results hold for the optimal R&D search intensity, λ_M^*, and the reservation technology y^*:

(i) $\frac{d\lambda_M^*}{dy} < 0$,

(ii) $\frac{d\lambda_M^*}{d\alpha} > 0$,

(iii) $\frac{dy_M^*}{dK} < 0$,

(iv) $\frac{dy_M^*}{d\alpha} > 0$,

(v) $\lambda_{M+1}^*(y) > \lambda_M^*(y) > \cdots > \lambda_2^*(y) > \lambda_1^*(y) = 0$, and

(vi) $\lim_{M\to\infty} \lambda_M^*(y) = \overset{*}{\lambda}(y)$.

Proof: See Appendix.

Results similar to (i) to (iv) of the theorem have already been proved for the infinite horizon case. Result (v) indicates the importance of the number of decision periods considered by a decision maker. The more periods in which the R&D results can be used profitably, the more intensively one searches for a better technology. This variable may vary within an industry, e.g., according to the expected longevity of the decision maker; or it may vary across industries, e.g., according to some planning rules adopted for financial planning. The sixth result provides an upper bound on the sequence $\{\lambda_M^*\}$. It is complementary to result (v). It also indicates the sensitivity of decision to the length of the decision horizon, the importance of which is most significant for a short horizon.

Section III

Exogenous Technological Change

In the above discussion of reservation technology levels, the possibility that exogenous technological change might affect the optimal R&D decision was raised. Recall that R&D in other sectors of the economy may change the cost structure of the R&D search process (e.g., better organization); or the technology opportunity (e.g., better instrumentation to cut off the possibility of poor technology). This type of technological change is introduced in the following discussion.

The state of knowledge is defined as a distribution of potential new technologies, F(z). Clearly, the state of knowledge need not be static since knowledge can be enriched by basic research, or less fundamentally, by other applied research. It is probable that basic and applied knowledge may emerge from applied research projects as joint products. For the moment, I simplify the problem by considering basic research as the sole driving force of technology opportunity.

Success in basic research will change the distribution of technology within which R&D advances are confined. This can happen in a variety of ways. One special case, a shift in mean, is evaluated here. Suppose the distribution is defined on the closed interval $[y'+\theta,\ y''+\theta]$, instead of $[y',\ y'']$, where $y-y' \geq \theta \geq 0$ and θ represents a shift parameter of the technology distribution. It shifts the mean of the distribution without affecting the shape (and in particular, the variance). With this simple form of a distribution shift, it is simple to modify the original best observed technology distributive function. For all $z \varepsilon [y'+\theta,\ y''+\theta]$, we have $H(z-\theta,\lambda)$ as the probability of the best observed technology being less than or equal to z. This is sensible because one has not changed the basic mechanism describing how observations are generated. The basic model for an infinite horizon case may be restated as follows:

$$V(y) = \max_{\lambda} \left\{ [R(y) + \alpha V(y) - K(\lambda) + \alpha \int_{y}^{y''+\theta} [V(z) - V(y)] d\, H(z-\theta,\lambda) \right\} .$$

Redefining variables, let $x = z - \theta$, and obtain the modification:

$$V(y) = \max_{\lambda} \left\{ [R(y) + \alpha V(y)] - K(\lambda) + \alpha \int_{y-\theta}^{y''} [V(x+\theta) - V(y)] d\, H(x,\lambda) \right\}$$

$$= \max_{\lambda} \left\{ R(y) - K(\lambda) + \alpha V(y''+\theta) - \alpha \int_{y-\theta}^{y''} H(x,\lambda) \frac{\partial V(x+\theta)}{\partial (x+\theta)} dx \right\} .$$

Make the additional assumption that

$$\frac{\partial^2 V(y)}{\partial y \partial \theta} > 0, \forall y \varepsilon [y'+\theta,\ y''+\theta] ,$$

and proceed as before by first defining two familiar terms:

i. Expected change in technology

$$= E(\Delta y | \lambda > 0)$$

$$= \int_{y}^{y''+\theta} (z-y)\ dH(z-\theta,\lambda)$$

$$= \int_{y-\theta}^{y''} (x+\theta-y)\ dH(x,\lambda),\ \text{where } x = z-\theta$$

$$= y'' + \theta - y - \int_{y-\theta}^{y''} H(x,\lambda)dx,\ \text{integration by parts, and}$$

ii. R&D derived-benefit function

$$= B(\lambda;y,\theta)$$

$$= \alpha \int_{y-\theta}^{y''} [V(x+\theta) - V(y)]\ dH(x;\lambda)$$

$$= \alpha[V(y''+\theta) - V(y)] - \alpha \int_{y-\theta}^{y''} H(x,\lambda) \frac{\partial V(x+\theta)}{\partial (x+\theta)} dx\ .$$

The effects of a change in the basic research parameter can be obtained.

<u>Proposition III-7</u>: (i) $\frac{\partial E(\Delta y)}{\partial \theta} > 0 \quad , \forall y < y'' + \theta ,$

(ii) $\frac{\partial^2 E(\Delta y)}{\partial \theta^2} > 0 \quad , \forall y < y'' ,$

(iii) $\frac{\partial^2 E(\Delta y)}{\partial \theta \partial \lambda} > 0$, and

(iv) $\frac{\partial B(\lambda;y,\theta)}{\partial \theta} > 0$, if $\frac{\partial^2 V(y)}{\partial y \partial \theta} > 0, \forall y \varepsilon [y', y'']$.

Proof: Using the definition of the two terms, we obtain the desired results.

(i) $\frac{\partial E(\Delta y)}{\partial \theta} = 1 - H(y-\theta,\lambda) > 0 \ , \ \forall y < y'' + \theta \ ,$

(ii) $\frac{\partial^2 E(\Delta y)}{\partial \theta^2} = H(y-\theta,\lambda) \ \lambda f(y-\theta) > 0 \ , \quad y < y'' + \theta \ ,$

(iii) $\frac{\partial^2 E(\Delta y)}{\partial \theta \partial \lambda} = H(y-\theta,\lambda) \ [1-F(y-\theta)] > 0$, and

(iv) $$\frac{\partial B(\lambda;y,\lambda)}{\partial \theta} = \alpha \int_{y-\theta}^{y''} \frac{\partial V(x+\theta)}{\partial (x+\theta)} \, dH(x,\lambda) + \alpha \int_{y-\theta}^{y''} \left[\frac{\partial V(x+\theta)}{\partial \theta} - \frac{\partial V(y)}{\partial y} \right] dH(x;\lambda) > 0.$$

Q.E.D.

Now, let

$$W(\lambda,y,\theta) = R(y) + \alpha V(y''+\theta) - K(\lambda) - \alpha \int_{y-\theta}^{y''} H(z,\lambda) \frac{\partial V(x+\theta)}{\partial (x+\theta)} dx \quad .$$

Assume $W(\lambda,y,\theta)$ is twice differentiable λ.

The first order condition for maximizing $W(\lambda,y,\theta)$ is given by:

$$\frac{\partial W(\lambda,y,\theta)}{\partial \lambda} = -K'(\lambda) + \alpha \int_{y-\theta}^{y''} \frac{\partial V(x+\theta)}{\partial (x+\theta)} H(x,\lambda) \ [1-F(x)]dx = 0 \quad .$$

The second order condition is satisfied by showing that the second derivative of $W(\lambda,y,\theta)$ is negative:

$$\frac{\partial^2 W(\lambda,y,\theta)}{\partial \lambda^2} = -K''(\lambda) - \alpha \int_{y-\theta}^{y''} \frac{\partial V(x+\theta)}{\partial (x+\theta)} H(x,\lambda) \ [1-F(x)]^2 dx < 0 \quad .$$

Assume $(\partial W(\lambda,y,\theta))/(\partial \lambda)$ is differentiable θ.

Applying the Implicit Function Theorem, the comparative statics result on the optimal R&D search intensity is obtained:

$$\frac{d\lambda}{d\theta} = - \frac{\alpha \int_{y-\theta}^{y''} \left[\frac{\partial^2 V(x+\theta)}{\partial (x+\theta)^2} + \frac{\partial^2 V(x+\theta)}{\partial (x+\theta) \partial \theta} \right] H(x,\lambda) \; [1-F(x)] dx + \alpha \frac{\partial V(y)}{\partial y} \; H(y-\theta,\lambda) \; [1-F(y-\theta)]}{-K''(\lambda) - \alpha \int_{y-\theta}^{y''} \frac{\partial V(x+\theta)}{\partial (x+\theta)} \; H(x,\lambda) \; [1-F(x)]^2 \; dx} \quad .$$

Unfortunately, even with the additional assumption this last value cannot be signed. The hint from this is clear. An improvement in basic research need not lead to more intensive applied research, because the optimal applied R&D depends on the discounted expected marginal optimal benefit, and not on the absolute amount of benefit. For example, subsidization of R&D by lumpsum payment may merely raise the return to R&D investment without changing the optimal R&D search intensity. However, a shift in the basic research parameter can have an effect on the stopping rule for R&D search, i.e., the reservation technology level. Again, define the reservation technology level as y^*, such that the following is satisfied:

$$\overline{K} = \alpha \int_{y^*}^{y''+\theta} H(z-\theta,\lambda) \; [1-F(z-\theta)] \; \frac{\partial V(z)}{\partial z} dz$$

$$= \alpha \int_{y^*-\theta}^{y''} H(x,\lambda) \; [1-F(x)] \; \frac{\partial V(x+\theta)}{\partial (x+\theta)} dx \quad , \quad \text{where } x = z - \theta \quad .$$

Using the Implicit Function Theorem,

$$\frac{dy^*}{d\theta} = - \frac{\alpha H(y^*-\theta,\lambda)\ [1-F(y^*-\theta)]\ \frac{\partial V(y^*)}{\partial y^*} + \alpha \int_{y''-\theta}^{y''} H(x,\lambda)\ [1-F(x)]\ \frac{\partial^2 V(x+\theta)}{\partial (x+\theta)\partial\theta} dx}{-\alpha H(y^*-\theta,\lambda)\ [1-F(y^*-\theta)]\ \frac{\partial V(y^*)}{\partial y^*}} > 0 .$$

Shifting the distribution of potential technologies in such a way that higher technology levels are made possible will yield a higher expected gain by raising the probability of getting a better technology level. A more important implication is that the claim that a higher benefit will lead to more R&D needs to be modified. The corrected claim should be:

> "The higher the benefit of R&D from derived demand, the higher is the technology level one has to reach before further search is unprofitable; but it need not imply a higher intensity of R&D search. It implies more persistent search."

Summary and Conclusion

Two things have been demonstrated in this chapter. First, dependent and independent variables in R&D decision at the firm level have been explicitly identified. The dependent variables are the future technology level, and current R&D search intensity. The independent variables are:

(i) Discount factor (hence interest rate),

(ii) Current technology level,

(iii) Basic R&D search intensity,

(iv) State of Knowledge, and

(v) Decision horizon in R&D planning, N.

Second, a concept borrowed from labor market search literature has been introduced, which is denoted by the Reservation Technology Level (RTL). The property of RTL is straightforward. If the current technology level is less than RTL, R&D search continues. Once the best observed technology level is not less than the RTL, R&D search is terminated. Furthermore, RTL < y", the upper limit of technology possibilities, given the state of knowledge. Thus, ex ante applied research does not push to the frontier. Ex post, however, one may have a technology level exceeding the RTL. If the current technology level lies between RTL and y", it indicates unprofitable R&D, even though progress is still possible. On the other hand, if external conditions change, economic or technical R&D may be resumed in an abandomed research area.

So far, it has been assumed that the measure of technology level, and net income to the firm have a straightforward relation. In the next chapter, production will be made explicit in the decision process. In particular, I investigate the interaction of production and R&D decisions. The importance of demand for the product output will be emphazied together with the cost of R&D.

Appendix

<u>Proposition III-1:</u> $H(z;\lambda) = \exp\{-\lambda[1-F(z)]\}$

Proof (Wilde, 1977):

$$H(z;\lambda) = \text{Prob (best observation} \leq z)$$

$$= \sum_{n=0}^{\infty} \text{Prob (n observation)} \times \text{Prob (best of n} \leq z)$$

$$= \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^{n}}{n!} F^{n}(z)$$

$$= e^{-\lambda} \sum_{n=0}^{\infty} \frac{[\lambda F(z)]^{n}}{n!}$$

$$= e^{-\lambda} e^{\lambda F(z)}$$

$$= e^{-\lambda[1-F(z)]}$$

Q.E.D.

<u>Theorem III-1:</u> Given the assumptions of the model, there exists a unique, continuous, bounded solution to the functional equation.

Proof: The proof is a straightforward application of Denardo's Theorem. Elaboration of the Monotonicity and Contraction assumptions is necessary before applying the theorem.

(i) Monotonicity Assumption

Let $h(y,\lambda,V) \equiv R(y) + \alpha V(y) - K(\lambda) + \alpha \int_{y}^{y''} [V(z) - V(y)] \; dH(z;\lambda)$

where $y \varepsilon [y',y'']$

$V : [y',y''] \to \mathbb{R}^{+}$

Similarly, $h(y,\lambda,U) \equiv R(y) + \alpha U(y) - K(\lambda) + \alpha \int_{y}^{y''} [U(z) - U(y)] \; dH(z;\lambda)$.

If $V(x) \geq U(x) \; \forall x \varepsilon [y',y'']$, and $\lambda = \lambda^{*}(y)$, the optimal choice of λ given y, then

$$h(y,\lambda^{*}(y),V) - h(y,\lambda^{*}(y),U)$$

$$= \alpha[V(y) - U(y)] + \alpha \int_{y}^{y''} [V(z) - U(z)] \; dH(z,\lambda^{*}(y))$$

$$-\alpha \int_{y}^{y''} [V(y) - U(y)] \; dH(z,\lambda^{*}(y))$$

$$= \alpha \int_{y'}^{y} [V(y) - U(y)] \; dH(z,\lambda^{*}(y)) +$$

$$\alpha \int_{y}^{y''} [V(z) - U(z)] \; dH(z,\lambda^{*}(y)) \geq 0$$

$\therefore \; h(y,\lambda^{*}(y),V) \geq h(y,\lambda^{*}(y),U)$ if $V(x) \geq U(x) \; \forall x \varepsilon [y',y'']$

(ii) Contraction Assumption

$$|h(y,\lambda^{*}(y),V) - h(y,\lambda^{*}(y),U)|$$

$$= \left|\alpha \int_{y'}^{y} [V(y) - U(y)] \; dH(z,\lambda^{*}(y)) + \alpha \int_{y}^{y''} [V(z) - U(z)] \; dH(z,\lambda^{*}(y))\right|$$

$$\leq \alpha \int_{y'}^{y} \max_x |V(x) - U(x)| dH(z,\lambda^*(y)) + \alpha \int_{y}^{y''} \max_x |V(x) - U(x)| dH(z,\lambda^*(y))$$

$$= \alpha \max_x |V(x) - U(x)|$$

$$\equiv \alpha \rho [V,U] \text{ , where } \rho \text{ is the metric so defined.}$$

Since $1 > \alpha > 0$, one has the contraction assumption, which embeds a contraction mapping. Invoking Denardo's Theorem, one has the result.

Q.E.D.

Theorem III-5: Given the assumptions of the model, if the following holds:

(i) $\frac{\partial^2 V(y,M)}{\partial\alpha\partial y} > 0, \forall y \in [y',y''], \forall M \geq 2$,

and (ii) $\frac{\partial V(y,M)}{\partial y} > \frac{\partial V(y,M-1)}{\partial y}$, $\forall y \in [y',y''], \forall M \geq 2$.

Then, for all $M \geq 2$, the following comparative statics results hold for the optimal R&D search intensity λ_M^*, and the reservation technology level, y_M^*:

(i) $\frac{d\lambda_M^*}{dy} < 0$,

(ii) $\frac{d\lambda_M^*}{d\alpha} > 0$,

(iii) $\frac{dy_M^*}{d\bar{K}} < 0$,

(iv) $\frac{dy_M^*}{d\alpha} > 0$,

(v) $\lambda_{M+1}^*(y) > \lambda_M^*(y) > \cdots \lambda_2^*(y) > \lambda_1^*(y) = 0$,

(vi) $\lim_{M \to \infty} \lambda_M^*(y) \to \lambda^*(y)$.

Proof: $V(y,0) \equiv 0$.

$$V(y,1) = \max_{\lambda_1} \{R(y) - K(\lambda_1)\} .$$

$\therefore \quad \lambda_1^* = 0$, and

$$V(y,1) = y \quad .$$

For $M \geq 2$,

$$V(y,M) = \max_{\lambda_M} \left\{ R(y) + \alpha V(y,M-1) - K(\lambda_M) + \alpha \int_y^{y''} [V(z,M-1) - V(y,M-1)] \, dH(y,\lambda_M) \right\}$$

$$= \max_{\lambda_M} \left\{ R(y) + \alpha V(y'',M-1) - K(\lambda_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M) dz \right\} .$$

Let

$$W(\lambda_M,y,M) = R(y) + \alpha V(y'',M-1) - K(\lambda_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M) dz .$$

The first order condition for maximization of $W(\lambda_M, y, M)$ λ_M is:

$$\frac{\partial W(\lambda_M,y,M)}{\partial \lambda_M} = -K'(\lambda_M) + \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M)\ [1-F(z)]dz = 0 .$$

The second order condition for maximization of $W(\lambda_M, y, M)$ is:

$$\frac{\partial^2 W(\lambda_M,y,M)}{\partial \lambda_M^2} = -K''(\lambda_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M)\ [1-F(z)]^2 dz < 0 .$$

Using the Implicit Function Theorem on the first order condition, one has:

$$\frac{d\lambda_M^*}{d\alpha} = - \frac{\int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M)\ [1-F(z)]\ dz + \alpha \int_y^{y''} \frac{\partial^2 V(z,M-1)}{\partial z \partial \alpha} H(z,\lambda_M)\ [1-F(z)]dz}{-K''(\lambda_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M)\ [1-F(z)]^2 dz} > 0 ,$$

$$\frac{d\lambda_M^*}{dy} = - \frac{-\alpha \frac{\partial V(y,M-1)}{\partial y} H(y,\lambda_M)\ [1-F(y)]}{-K''(\lambda_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda_M)[1-F(z)]^2 dz} < 0 .$$

Let y_M^* be defined by:

$$\alpha \int_{y_M^*}^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,0)\ [1-F(z)]dz = \overline{K} \equiv K'(0) .$$

Using the Implicit Function Theorem, one has

$$\frac{dy_M^*}{d\alpha} = - \frac{\int_{y_M^*}^{y''} \left[\frac{\partial V(z,M-1)}{\partial z} + \alpha \frac{\partial^2 V(z,M-1)}{\partial z \partial \alpha} \right] H(z,0) \;\; [1-F(z)]dz}{- \alpha \frac{\partial V(y_M^*,M-1)}{\partial y_M^*} \; H(y_M^*,0) \;\; [1-F(y_M^*)]} < 0 \; ,$$

$$\frac{dy_M^*}{d\bar{K}} = \frac{1}{- \alpha \frac{\partial V(y_M,M-1)}{\partial y_M^*} \; H(y_M^*,0) \;\; [1-F(y_M^*)]} < 0 \quad .$$

Thus, proofs of (i) to (iv) are completed.

By assumption,

$$\frac{\partial V(y,M)}{\partial y} > \frac{\partial V(y,M-1)}{\partial y} \quad \forall y \varepsilon [y',y''] \;\; \forall M \geq 2$$

$$\therefore \;\; \alpha \int_{y}^{y''} \frac{\partial V(z,M-1)}{\partial z} \; H(z,\lambda) \;\; [1-F(z)]dz > \alpha \int_{y}^{y''} \frac{\partial V(z,M-2)}{\partial z} \; H(z,\lambda) \;\; [1-F(z)]dz, \; \forall \lambda \quad .$$

But

$$K'(\lambda_M^*) = \alpha \int_{y}^{y''} \frac{\partial V(z,M-1)}{\partial z} \; H(z,\lambda_M^*) \;\; [1-F(z)]dz \quad ,$$

$$K'(\lambda_{M-1}^*) = \alpha \int_{y}^{y''} \frac{\partial V(z,M-2)}{\partial z} \; H(z,\lambda_{M-1}^*) \;\; [1-F(z)]dz \quad ,$$

and

$$K'(\lambda) > 0 \ , \ K''(\lambda) \geq 0 \ .$$

$$\lambda^*_M(y) > \lambda^*_{M-1}(y) \ , \ \forall M \geq 2 \ .$$

This completes the proof of (v). A diagram (Figure IV-3) will help to clarify the proof.

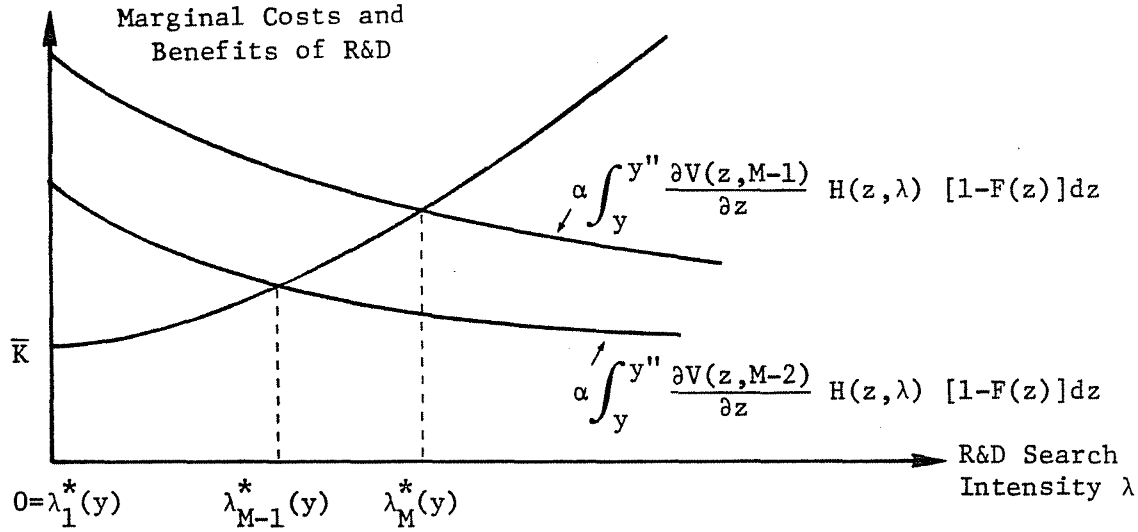

Figure III-3: Marginal Costs and Benefits of R&D

To prove (vi), the following are used:

$$V(y,M) = R(y) + \alpha V(y'',M-1) - K(\lambda^*_M) - \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} \ H(z,\lambda^*_M)dz$$

$$\frac{\partial V(y,M)}{\partial y} = R'(y) + \alpha \frac{\partial V(y,M-1)}{\partial y} \ H(y,\lambda^*_M)$$

$$+ \frac{d\lambda^*_M}{dy} \left\{ -K'(\lambda^*_M) + \alpha \int_y^{y''} \frac{\partial V(z,M-1)}{\partial z} H(z,\lambda^*_M) \ [1-F(z)]dz \right\}$$

$$= R'(y) + \alpha \frac{\partial V(y,M-1)}{\partial y} H(y,\lambda_M^*) \quad ,$$

by the first order condition for maximization of $W(\lambda_M,y,M)$.

Let $\beta_M \equiv \alpha H(y,\lambda_M^*)$. $1 > \beta_M > 0$, and

$$\frac{\partial V(y,M)}{\partial y} = R'(y) + \beta_M \frac{\partial V(y,M-1)}{\partial y} \quad .$$

From (v) $\lambda_M^*(y) > \lambda_{M-1}^*(y) > \cdots > \lambda_2^*(y) > \lambda_1^*(y) = 0$,

$$\therefore \beta_M < \beta_{M-1} < \cdots < \beta_2 < \beta_1 = \alpha < 1 \quad .$$

Hence,

$$\begin{aligned}
\frac{\partial V(y,M)}{\partial y} &= R'(y) + \beta_M \frac{\partial V(y,M-1)}{\partial y} \\
&< R'(y) + \beta_{M-1} \frac{\partial V(y,M-1)}{\partial y} \\
&= R'(y) + \beta_{M-1}\left[R'(y) + \beta_{M-1} \frac{\partial V(y,M-2)}{\partial y}\right] \\
&< R'(y) + \beta_{M-1}\left[R'(y) + \beta_{M-2} \frac{\partial V(y,M-2)}{\partial y}\right] \\
&= R'(y)\left[1 + \beta_{M-1}\right] + \beta_{M-1}\beta_{M-2} \frac{\partial V(y,M-2)}{\partial y} \\
&\ \ \vdots \\
&< R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots\right] \\
&\quad + \beta_{M-1}\beta_{M-2} \cdots \beta_3\beta_2 \frac{\partial V(y,2)}{\partial y} \\
&= R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots\right] \\
&\quad + \beta_{M-1}\beta_{M-2} \cdots \beta_3\beta_2\left[R'(y) + \beta_2 \frac{\partial V(y,1)}{\partial y}\right]
\end{aligned}$$

$$< R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots\right]$$

$$+ \beta_{M-1}\beta_{M-2} \cdots \beta_3\beta_2\left[R'(y) + \beta_1\frac{\partial V(y,1)}{\partial y}\right]$$

$$= R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots + \prod_{j=1}^{M-1} \beta_j\right],$$

since $$\frac{\partial V(y,1)}{\partial y} = R'(y) \quad .$$

But $$\frac{\partial V(y,M-1)}{\partial y} = R'(y) + \beta_{M-1}\frac{\partial V(y,M-2)}{\partial y}$$

$$= R'(y) + \beta_{M-1}\left[R'(y) + \beta_{M-2}\frac{\partial V(y,M-3)}{\partial y}\right]$$

$$= R'(y)\left[1 + \beta_{M-1}\right] + \beta_{M-1\ M-2}\frac{\partial V(y,M-3)}{\partial y}$$

$$\vdots$$

$$= R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots\right] + \prod_{j=2}^{M-1}\beta_j \frac{\partial V(y,1)}{\partial y}$$

$$= R'(y)\left[1 + \beta_{M-1} + \beta_{M-1}\beta_{M-2} + \cdots + \prod_{j=2}^{M-1}\beta_j\right]$$

$$\therefore \quad \frac{\partial V(y,M)}{\partial y} < \frac{\partial V(y,M-1)}{\partial y} + R'(y)\prod_{j=1}^{M-1} \beta_j$$

or $$\frac{\partial V(y,M)}{\partial y} - \frac{\partial V(y,M-1)}{\partial y} < R'(y)\prod_{j=1}^{M-1} \beta_j \quad .$$

$$\therefore \lim_{M\to\infty}\left\{\frac{\partial V(y,M)}{\partial y} - \frac{\partial V(y,M-1)}{\partial y}\right\} \le \lim_{M\to\infty} R'(y) \prod_{j=1}^{M-1} \beta_j \quad .$$

But
$$\lim_{M\to\infty} \prod_{j=1}^{M-1} \beta_j = 0 \ ,$$

since
$$0 < \beta_M < \beta_{M-1} < \cdots < \beta_2 < \beta_1 = \alpha < 1 \ ,$$

$$\therefore \lim_{M\to\infty}\left\{\frac{\partial V(y,M)}{\partial y} - \frac{\partial V(y,M-1)}{\partial y}\right\} \le 0 \quad .$$

But
$$\frac{\partial V(y,M)}{\partial y} > \frac{\partial V(y,M-1)}{\partial y} \ , \forall y \ , \forall M \ge 2 \quad \text{by assumption,}$$

$$\therefore \lim_{M\to\infty}\left\{\frac{\partial V(y,M)}{\partial y} - \frac{\partial V(y,M-1)}{\partial y}\right\} = 0 \ , \quad \text{and}$$

$$\lim_{M\to\infty} \lambda_M^*(y) = \lambda^*(y) \quad .$$

Q.E.D.

Chapter III: Footnotes

1. In 1957, Robert M. Solow found that increased capital intensity accounted for only 12.5% of the increase in U.S. nonfarm output per manhour between 1909 and 1949. The remaining unexplained increase in output per manhour was attributed to change in labor quality and technological improvements. His method was to attribute to new technology the increase that could not be explained by changes in factors of production in a time-series required, and is therefore known as residual analysis.

2. Productivity is measured by several methods. The simplest one is the rate of change of output per manhour. This is a partial productivity index since labor is the only input factor taken into account. A total productivity index is the ratio of the change in output to the sum of changes in labor and capital costs, holding factor prices constant. Salter (1969) defined a third measure of productivity change as the relative change in total unit costs when the techniques in each period are those which would minimize unit costs holding factor prices constant. None of these measures are satisfactory when one considers product quality improvements.

3. Denison's work suffers from the same shortcomings as Solow's.

4. Hicks neutral technical progress occurs when the production function of a technology shifts over time by a uniform upward displacement. Harrod (Solow)-neutral technical change assumes capital (labor) input being constant over time.

5. Embodiment theory postulates that technical progress, like manna from heaven, falls on certain types of capital equipment and on certain sections of the labor force.

6. Rational actor approach is one when individuals are assumed to be rational and set out to maximize some well defined objective function. Utility maximization by consumers and profit maximization by producers are examples of the rational actor approach.

7. The distinction between process and product innovations is arbitrary. A process innovation is one in which a new production technique is introduced. A product innovation is one in which a new product is introduced to the market. In general, however, an innovation will involve changes in both products and processes.

8. There are two types of statistical learning. First, there is the learning by which one updates an order statistic from sampling. Second, there is the learning by which one obtains a new sample and

updates one's prior subjective probability distribution. The latter is known as Bayesian learning. However, I shall use the former. An example of the latter is exhibited by Grossman, Kihlstrom and Merman (1977).

10. There is a vast literature on dynamic programming. For example, see Bellman (1957), Blackwell (1962), Ross (1970) and Kohn and Shavell (1974).

11. See Chapter II for details.

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Chapter IV

Market Structure and Industrial R&D

The Problem

Several studies of industrial R&D and market structure were discussed and reviewed in Chapter II. It was argued there that the derived demand nature of R&D is not explicitly recognized in most studies. This has led some students of R&D to formulate models with a fixed reward for successful R&D, the entire amount of which is received by the first successful firm. However, the assumption of a fixed reward is not a good approximation to reality. The magnitude of R&D benefits depends on the degree of R&D success, that is, how much improvement is made possible by the new technology. It also depends on how much improvement one's rival has achieved. In addition, a "winner gets all" argument assumes the presence of a perfect, infinite patent system. An innovative firm must incur costs to enforce an effective patent system and undoubtedly patents do not work perfectly in any case. Even if the patent system were costless, further inventions may push technology ahead of present patentable technology, and hence lower the value of the latter. In fact, an efficient patent system should not be a barrier to further genuine progress. One way to bypass this problem is to introduce a continual innovation process with success in each stage rewarded by some fixed amount. Futia (1977) uses this formulation, and assumes a Markovian process to establish the existence of a distribution of firm sizes in an industry.[1] I shall study continuous innovations with endogenous payoffs. To this end, I integrate final goods production and R&D, and consider R&D benefits from a derived demand point of view.

Schumpeter and some of his followers argued that because a monpolist comes closest to fully appropriating R&D benefits, imperfect competition might sacrifice static efficienty but promote dynamic efficiency.[2] Static efficiency refers to production with a given technology. It requires that the price to marginal cost be equal for all products. Dynamic efficiency refers to an optimal rate of technological progress, made possible by R&D. It requires cost reductions over time with a changing technology. Schumpeterians further argue that the gains in dynamic efficiency might outweigh static efficiency losses so much that monopoly would be the socially desirable choice of market structure. This conclusion is based on the premise that a monopolist has a "natural" tendency to invest more in R&D, and share the fruit of its success with consumers. I intend to investigate this set of conclusions in a formal model. In particular, I am interested in the welfare ranking of market structures with respect to static and dynamic efficiency. Standard economic theory suggests that a monopolist will produce at a lower level of output than is socially optimal. A related and important question is whether a monopolist will perform more R&D than a competitive firm. If so, this would partially compensate for the loss in static efficiency associated with monopolies.

The last issue of interest here is whether we should treat product and process innovations separately.[3] Are they theoretically identical? Indeed, about 75% of industrial R&D expenditures are allocated to new products or product improvement, while only 25% are allocated to process innovations. A production-R&D model needs to reconcile these two types of R&D, otherwise a substantial part of innovative activities will be left unexplained. The theoretical similarities of product and process innovations will be investigated.

Discussion

In this study, I wish to investigate the issues identified above. First, the importance of studying production and R&D decisions simultaneously is examined. Modeling production without considering R&D sacrifices elements of dynamic competition over time. For example, the results of R&D cause firms (and markets) to move from old equilibria to new ones, typically with gains accruing to the innovators during the transition. At the same time, evaluating R&D decisions without tying in the benefits derived from production ignores the demand side of R&D.[4] Studies of the effects of rivalry on R&D with this shortcoming give an incomplete picture of the problem. But by introducing production, a benefit function for R&D can be generated and employed as derived demand. Given a cost function of R&D, the equilibrium level of R&D can thus be calculated.

In addition, I wish to show that a monopolist may not perform R&D in a socially optimal fashion. In fact, since a monopolist produces at a lower level of output than a competitive industry, the gain in profit from any cost reduction can be less than the gain in social welfare for the same reduction in average cost that would accrue at competitive equilibrium. A monopolist applies the cost saving technology to a lesser quantity of output than a social planner would. Hence, a monopolist may perform R&D at a level lower than the social optimum. In particular, he may stop R&D search at too low a level of reservation technology.[5]

If the above is indeed true, it will be worthwhile to investigate whether a monopolist will always search more intensively for better technology than an oligopolist. If so, it makes sense to include monopoly in the social menu of market structures. If a monopolist always performs less R&D, a monopoly situation may be excluded from society's menu of market structures because both static and dynamic efficiency can be gained by moving from monopoly to ologopoly. If the answer is indefinite, one needs to investigate the specific market in question before making a definite choice. However, I shall provide a sufficient condition for a noncooperative duopolistic market structure to have a higher reservation technology level than that for a monopolist. Thus, the claim that a monopolistic market structure has innate merit will be seriously questioned.

At this point, it is useful to mention the possibility of technology choice bias in R&D.[6] A detailed discussion will be postponed until Section III, but note here that a transient monopolist may spend less in developing a small scale efficient technology in favor of a large scale efficient technology. Referring to Figure IV-1, I claim that a monopolist will develop technology A rather than technology B, if both technologies are under consideration by the monopolist alone, (i.e., there is no R&D rivalry). The reason is that an antitrust suit

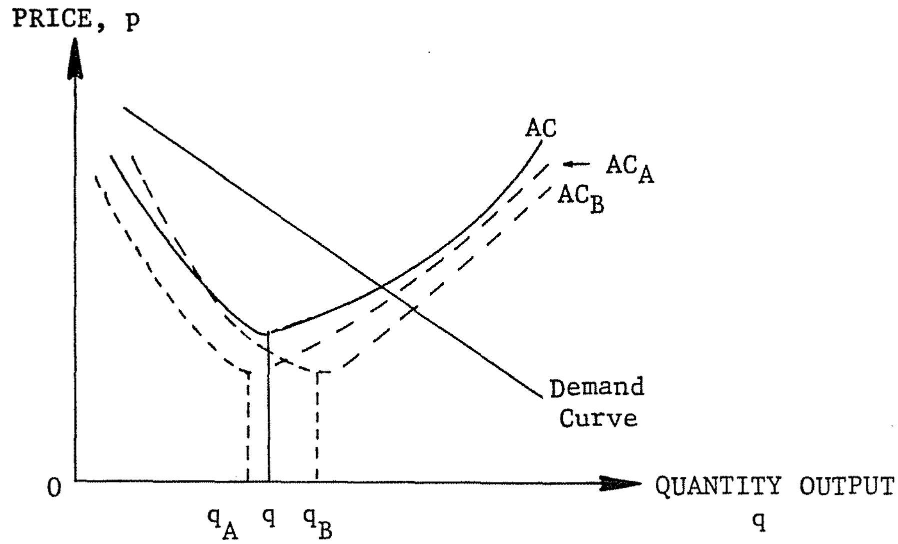

Figure IV-1: Technology choice bias of a monopolist.

may break up a monopoly using the former type of technology (assuming external economies of scale are not pertinent), while the latter will yield a natural monopoly argument against antitrust, and will provide a justification to prevent entry through regulatory protection. Indeed, if planning becomes more sophisticated in the future, one may expect a tendency toward large scale production technologies in monopolistic markets. As time elapses, it may even make sense to develop large scale technology alone, since most readily available knowledge is for that intent. The opportunity cost of developing small scale technology may become very high.

For the case of oligopolists, if there is a collusive agreement about market shares, the same kind of technology choice bias occurs. However, with sufficient rivalry, the bias may be in the opposite direction, i.e., oligopolists may choose smaller scale technologies. The reason is that smaller scale technology gives more flexibility in the choice of output and, therefore, is less risky. This second type of bias is socially preferable to the former. In both cases, the biases are limited by technology opportunities. With R&D being endogenous, technology opportunities may be endogenous to some extent for the economy as a whole. Thus the bias may be significant. Since technology opportunity is exogenous to this model, this kind of technology choice bias is not considered.

Furthermore, to avoid this issue, it is assumed that the number of firms in the industry under study remains unchanged within the relevant periods. With this restrictive assumption, it is equally likely that a monopolist will develop an efficient, small scale technology and apply it to large number of production units, even though such a technology can sustain a number of firms, i.e., it is assumed that there is no entry threat. Thus, endogenous changes in the market structure are not covered within the present discussion. Future consideration in this direction may generate insights to the dynamic process of growth and structural changes in an industry.[7]

There is still another problem. If a technology exhibits increasing returns to scale for all relevant output levels, a natural monopoly prevails and a model of oligopolistic market structure is useless. The relevant model for this situation can either be that of regulator-monopolist interaction or product differentiation and monopolistic competition. I shall save them for future research.

For the present, I assume that the currently used technology shows initial increasing returns to scale but changes to decreasing returns to scale at a relatively low level of output. This implies that the market can sustain the concurrent operation of several firms.

Furthermore, I assume that both production and R&D cost functions exhibit this property. Hence, it is relevant to study the welfare ranking of different market structures since all of them are compatible with given technologies of production and R&D. An empirical study by Bain (1954) showed that even for highly concentrated industries the minimum optimal scale plant may be a small percentage of an industry's output. See Table IV-1.[8] For future consideration, a R&D cost function may be treated as having increasing return to scale, so that R&D effort is separated from the industry, e.g., in the electric utility industry, equipment suppliers are doing most of the R&D work. One may then evaluate the nature of technology adoption under different market structures.[9]

Finally, it is assumed that the results of any individual firm's R&D efforts are not automatically disseminated, i.e., the exclusion principle of public good is applicable. Indeed, when Schumpeter argues that a monopolist has a strong incentive to invest in R&D, he implicitly assumes that the results of R&D must not be disseminated instantaneously and without cost. Otherwise, no matter how hard a monopolist tries to be technically progressive, he will suffer from entry at any instant of time. The feasibility of exclusion with respect to R&D output is assumed throughout this chapter. Thus, the producer of R&D is a monopolist with respect to R&D.

Section I

The Model

To evaluate the welfare implications of market structure with respect to R&D, I will use the basic model developed in the last

Table IV-1: Estimates of Minimum Optimal Plant Scale Ranges for 20 U.S. Manufacturing Industries, 1951

(Source: Bain, J. S. "Economies of Scale, Concentration, and the Condition of Entry in Twenty Manufacturing Industries," AER, March 1954, pp. 15-39)

Industry	Percentage of National Capacity Provided by One Plant Complex of Minimum Optimal Scale
Flour Milling	0.1 to 0.5
Shoe Manufacturing	0.14 to 0.5
Canned Fruits and Vegetables	0.25 to 0.5
Cement Manufacturing	1
Distilled Liquors	1.25 to 1.75
Farm Machinery, excluding Tractors	1 to 1.5
Petroleum Refining	1.75
Integrated Steel Mills	1 to 2.5
Tin Can Manufacturing	0.3 to 2.0
Diversified Meat Packing	2 to 2.5
Rubber Tires and Tubes	1.4 to 2.75
Gypsum Plaster and Plasterboard	2 to 3
Rayon Yarn and Fibers	4 to 6
Soap and Detergents	4 to 6
Cigarettes	5 to 6
Integrated Auto Production	5 to 10
Fountain Pen Production	5 to 10
Primary Copper Refining	10
Tractor Manufacturing	10 to 15
Typewriter Production	10 to 30

chapter. That model is extended here by making production explicit. Assume that there is a well defined demand function for a final product, $p(q)$, where $p: [0,\bar{q}] \to [0,\bar{p}]$ with $p(0) = \bar{p}$, $p'(q) < 0$, for all $q \varepsilon [0,\bar{q}]$, and $p(q) \equiv 0$, for all $q \geq \bar{q}$, $\bar{q} < \infty$ (p stands for the price of the final good, and q the quantity of output). The total cost of producing a given output q with a given technology level denoted by y, is represented by $nC(q/n,y)$, where C is the total cost function of one production unit (e.g., a production plant), and n is the number of production units; $C : [0,\infty) \times [y',y''] \to [0,\infty]$, $C_1 \equiv \partial C/\partial(q/n) > 0$, $C_{11} \equiv \partial^2 C/(\partial(q/n)^2) > 0$, $C_2 \equiv \partial C/\partial y < 0$. A total cost function allows R&D output to be interpreted as either lowering the variable cost or the fixed cost. Since R&D involves long term decisions, and its effect is to alter the production function, it is necessary to treat fixed cost as endogenous. A decision maker can improve his current technology by paying the costs of R&D, $mK(\lambda/m)$, where K is the total cost function of one R&D search unit (e.g., project team), m is the number of search units, and λ is the intensity of R&D search; $K : [0,\infty] \to [0,\infty]$, $K_1 \equiv \partial K/\partial(\lambda/m) > 0$, and $K_{11} \equiv \partial^2 K/\partial(\lambda/m)^2 > 0$.

Recall that the probability of the best observed technology of research unit i being less than or equal to z is denoted by $H^i(z,\lambda/m) = e^{-\lambda/m[1 - F(z)]}$, where λ/m is the R&D search intensity, $F(z)$ is the probability that an observed new technology level is less than or equal to z, and $i=1,\ldots,m$. Hence, the distribution function of the best observed technology for a firm operating these m research units is defined by $\tilde{H}(z;\lambda,m)$, where

$$\tilde{H}(z;\lambda,m) = \text{Prob } \{\text{observations from all of the m research units are less than or equal to } z\}$$

$$= \prod_{i=1}^{m} H^i(z;\lambda/m)$$

$$= \left[e^{-\lambda/m[1-F(z)]}\right]^m$$

$$= e^{-\lambda[1-F(z)]}$$

But $H(z,\lambda) = e^{-\lambda[1-F(z)]}$, where $H(z;\lambda)$ is the best observed technology distribution for a single research unit operating at R&D search intensity λ. Thus, breaking up research into several units will not affect overall performance. It will, however, lower the total cost up to a point where fixed costs prohibit more decentralization.

Let $V^s(y)$ be the stream of social benefits, given a current technology level $y \varepsilon [y',y'']$ and an optimal program of production and R&D, $V^s : [y',y''] \rightarrow \mathbb{R}$. Denote the discount factor by α. The discounted expected optimal value of the production/R&D system in the next period is given by the expression

$$\alpha \int_y^{y''} V^s(z) \; dH(z;\lambda)$$

given R&D investment succeeds in generating a better technology level, and by the expression

$$\alpha V^s(y) \; H(y;\lambda)$$

given R&D investment fails to yield a better technology level. The objective of a social decision maker is assumed to be the maximizing of social surplus, that is, the sum of consumer surplus and producer surplus. The objective functional for the infinite horizon production/ R&D system, given current technology level y, can therefore be stated as

$$V^s(y) = \max_{m,n,q,\lambda}\left\{\int_0^q p(x)\,dx - n\,C\left(\frac{q}{n},y\right) - mK\left(\frac{\lambda}{m}\right) + \int_y^{y''} V^s(y)\,dH(z,\lambda) + \alpha V^s(y)\,H(y;\lambda)\right\} .$$

Let $W^s(m,n,q,\lambda;y)$ be the function in the bracket to be maximized.

$$W^s(m,n,q,\lambda;y) = \int_0^q p(x)\,dx - nC\left(\frac{q}{n},y\right) - mK\left(\frac{\lambda}{m}\right) + \alpha\int_y^{y''} V^s(z)\,dH(z;\lambda) + \alpha V^s(y)\,H(y;\lambda) .$$

Integrating by parts,

$$W^s(m,n,q,\lambda;y) = \int_0^q p(x)\,dx - nC\left(\frac{q}{n},y\right) - mK\left(\frac{\lambda}{m}\right) + \alpha V(y'') - \alpha\int_y^{y''} \frac{\partial V^s(z)}{\partial z}\,H(z;\lambda)dz .$$

First order conditions for maximizing $W(m,n,q,\lambda;y)$ are given by:

$$(1) \quad \frac{\partial W^s}{\partial q} = p(q) - C_1\left(\frac{q}{n},y\right) = 0 ,$$

$$(2)\quad \frac{\partial W^s}{\partial \lambda} = -K_1\left(\frac{\lambda}{m}\right) + \alpha \int_{y}^{y''} \frac{\partial V^s(z)}{\partial z} \; H(z;\lambda)\;[1-F(z)]dz = 0 \;,$$

$$(3)\quad W^s(m,n+1,q,\lambda;y) - W^s(m,n,q,\lambda;y) \leq -C\left(\frac{q}{n},y\right) + \frac{q}{n}\,C_1\left(\frac{q}{n},y\right) = 0,$$

$$(4)\quad W^s(m+1,\; n,q,\lambda;y) - W^s(m,n,q,\lambda;y) \leq -K\left(\frac{\lambda}{m}\right) + \frac{\lambda}{m}\,K_1\left(\frac{\lambda}{m}\right) = 0 \;.$$

The first equation requires that output should be produced up to the point where marginal cost of production equals price. This is a production efficiency criterion. Equation (3) can be restated as $n/q\ C(q/n,y) = C_1(q/n,y)$: the average cost of production should equal the marginal cost of production. This is a criterion for the optimal choice of the number of production plants. The second equation states that the marginal cost of R&D is just compensated by the discounted expected marginal benefit of R&D. The last equation is a criterion for optimal choice of the number of R&D units. It expresses the equality of marginal cost and average cost of R&D.

Thus, (1) and (3) together yield the conditons that

Price of output = Marginal Cost of Production

= Average Cost of Production.

Similarly, Eqs. (2) and (4) imply that

Discounted Expected Marginal Benefit of R&D

= Marginal Cost of R&D

= Average Cost of R&D.

Note that a social welfare maximizer will equate the marginal benefit of R&D with its marginal cost, but will set price equal to the marginal cost of production.

From the definition of $V^S(y)$

$$\frac{\partial V^S(y)}{\partial y} = -n^S\, C_2\!\left(\frac{q^S}{n^S}, y\right) + \alpha\frac{\partial V^S(y)}{\partial y}\, H(y,\lambda^S) + \left[p(q) - C_1\!\left(\frac{q}{n}, y\right)\right]\frac{dq}{dy}$$

$$+ \left[-K_1\!\left(\frac{\lambda}{m}\right) + \alpha\int_y^{y''}\frac{\partial V^S(z)}{\partial z}\, H(z;\lambda)\ [1 - F(z)]dz\right]\frac{d\lambda}{dy}$$

$$+ \left[-C\!\left(\frac{q}{n}, y\right) + \frac{q}{n}\, C_1\!\left(\frac{q}{n}, y\right)\right]\frac{dn}{dy} + \left[-K\!\left(\frac{\lambda}{m}\right) + \frac{\lambda}{m}\, K_1\!\left(\frac{\lambda}{m}\right)\right]\frac{dm}{dy}\ .$$

By the first order condition

$$\frac{\partial V^S(y)}{\partial y} = -n^S\, C_2\!\left(\frac{q^S}{n^S}, y\right) + \alpha\frac{\partial V^S(y)}{\partial y}\, H(y,\lambda^S)\ ,$$

or

$$\frac{\partial V^S(y)}{\partial y} = \frac{-n^S C_2(q^S/n^S, y)}{1 - \alpha H(y,\lambda^S)} > 0\ .$$

For comparison, consider a monopolist. The objective of a monopolist is to maximize a discounted stream of profit. Thus, his objective functional is given by

$$V^M(y) = \max_{m,n,q,\lambda}\left\{p(q)\ q - n\ C\!\left(\frac{q}{n}, y\right) - mK\!\left(\frac{\lambda}{m}\right) + \alpha\int_y^{y''} V^M(z)\ dH(z;\lambda) + \alpha V^M(y)\ H(y;\lambda)\right\}\ .$$

Let $W^M(m,n,q,\lambda)$ be the function in the bracket to be maximized.

$$W^M(m,n,q,\lambda) = p(q)\ q - nC\left(\frac{q}{n},y\right) - mK\left(\frac{\lambda}{m}\right) + \alpha\int_y^{y''} V^M(z)\ dH(z;\lambda)$$

$$+ \alpha V^M(y)\ H(y;\lambda)$$

$$= p(q)\ q - nC\left(\frac{q}{n},y\right) - mK\left(\frac{\lambda}{m}\right) + \alpha V^M(y'')$$

$$- \alpha\int_y^{y''} \frac{\partial V^M(z)}{\partial z}\ H(z;\lambda)dz \quad .$$

First order conditions for maximization are given by

$$(1')\quad \frac{\partial W^M}{\partial q} = \frac{\partial[p(q)q]}{\partial q} - C_1\left(\frac{q}{n},y\right) = 0 \quad ,$$

$$(2')\quad \frac{\partial W^M}{\partial \lambda} = -K_1\left(\frac{\lambda}{m}\right) + \alpha\int_y^{y''} \frac{\partial V^M(z)}{\partial z}\ H(z;\lambda)[1 - F(z)]dz = 0 \quad ,$$

$$(3')\quad W^M(m,n+1,q,\lambda) - W^M(m,n,q,\lambda) \leq -C\left(\frac{q}{n},y\right) + \frac{q}{n}\ C_1\left(\frac{q}{n},y\right) = 0 \quad ,$$

$$(4')\quad W^M(m+1,n.q,\lambda) - W^M(m,n,q,\lambda) \leq -K\left(\frac{\lambda}{m}\right) + \frac{\lambda}{m}\ K_1\left(\frac{\lambda}{m}\right) = 0 \quad .$$

Assume that demand is elastic:

$$\frac{\partial[p(q)q]}{\partial q} > 0 \qquad \text{and} \qquad \frac{\partial^2[p(q)q]}{\partial q^2} < 0 \quad .$$

The differences between this set of first order conditions and that for a social welfare maximizer are provided by Eqs. (1) and (1'), and (2) and (2'). Equation (1') requires that the marginal revenue of output be equal to the marginal cost of production, while Eq. (1) requires price equals marginal cost of production. The implication of this difference is shown in Figure IV-2.

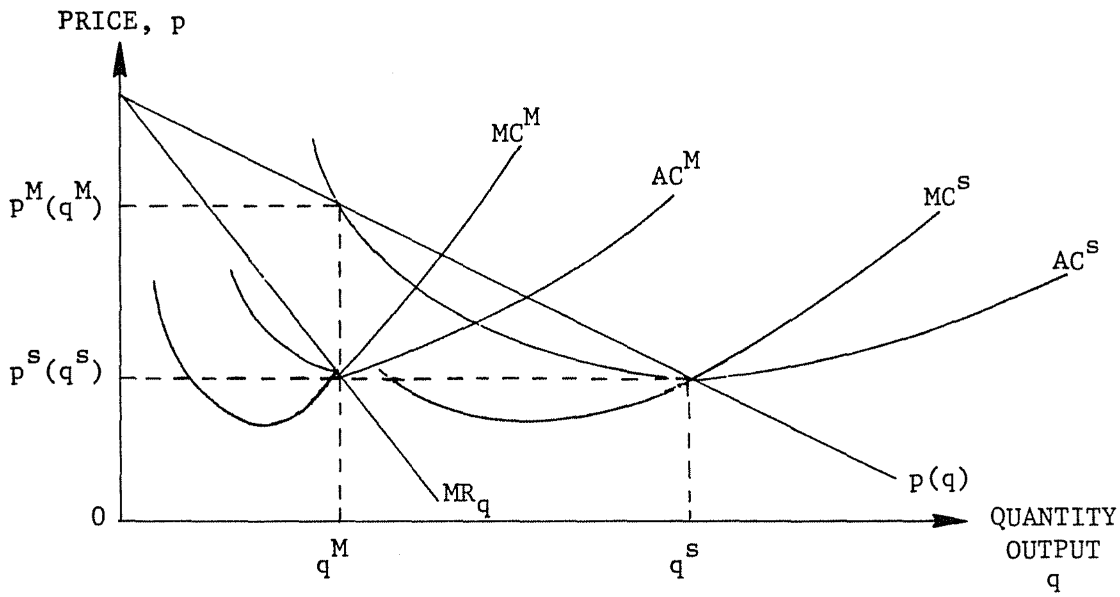

Figure IV-2: Price-Output Decision: Monopolist vs. Social Decision Maker

The assumption that the minimum efficient scale for each production unit is unique implies that there is a unique value of minimum average cost, which equals marginal cost at that output. For a social welfare maximizer, this equals the price of output. For a monopolist, this equals marginal revenue of output. With downward sloping demand, the demand curve lies to the right of the marginal revenue curve. Thus, the corresponding quantities of output for a social welfare maximizer and for a monopolist differ. In fact, a social welfare maxmizer produces more output than a monopolist. This is a classical result. It is recalled here because it is crucial to show the welfare implications of market structures. Conditions (2) and (2') are similar in form. However, there is a difference in the definitions of the optimal functionals V^s and V^m. V^s is expressed in terms of social welfare maximization, while V^m is derived from profit maximization. This is also crucial for a proof of the main theorem in this chapter.

The concept of Reservation Technology Level (RTL) is used here. Recall that at a technology level greater than or equal to RTL, a decision maker will stop R&D search. At any technology level less than that, the decision maker will continue R&D search. Let y^* be the RTL for the social welfare maximizer, and y^{**} be that for a profit maximizing monopolist,

where $$\alpha \int_{y^*}^{y''} \frac{\partial V^S(z)}{\partial z} H(z,0)[1-F(z)]dz = K_1(0) > 0 \text{ , and}$$

$$\alpha \int_{y^{**}}^{y''} \frac{\partial V^M(z)}{\partial z} H(z,0)[1-F(z)]dz = K_1(0) > 0 \text{ .}$$

Clearly, the optimal R&D search intensity should not be different between a social welfare maximizer and a monopolist for all levels of technology. In particular, one has

$$\lambda^S(y) = \lambda^M(y) = 0 \text{ for } y \geq \max\{y^*, y^{**}\} \quad .$$

A necessary (but not sufficient) condition for a social welfare maximizer to invest more in R&D search is that $y^* \nleq y^{**}$. In fact, $y^* > y^{**}$.

Theorem IV-1: $y^* > y^{**}$

Proof: The definition of a reservation technology level requires that

$$\alpha \int_{y^*}^{y''} \frac{\partial V^S(z)}{\partial z} H(z,0)[1-F(z)]dz = \alpha \int_{y^{**}}^{y''} \frac{\partial V^M(z)}{\partial z} H(z,0)[1-F(z)]dz \text{ .}$$

Now,

$$\frac{\partial V^S(z)}{\partial z} = \frac{-n^S C_2(q^S/n^S,z)}{1-\alpha H(z,\lambda^S)}$$

$$= \frac{-n^S C_2(q^S/n^S,z)}{1-\alpha}, \text{ for all } z\varepsilon[y^*,y''].$$

Similarly,

$$\frac{\partial V^M(z)}{\partial z} = \frac{-n^M C_2(q^M/n^M,z)}{1-\alpha H(z,\lambda^M)}$$

$$= \frac{-n^M C_2(q^M/n^M,z)}{1-\alpha}, \text{ for all } z\varepsilon[y^{**},y''] .$$

Therefore

$$\frac{\partial V^S(z)}{\partial z} > \frac{\partial V^M(z)}{\partial z}, \text{ for all } y \geq \max\{y^*,y^{**}\}.$$

Suppose

$$y^* \leq y^{**}.$$

This implies

$$\alpha\int_{y^{**}}^{y''} \frac{\partial V^S(z)}{\partial z} H(z,0)[1-F(z)]dz \leq \alpha\int_{y^*}^{y''} \frac{\partial V^S(z)}{\partial z} H(z,0)[1-F(z)]dz$$

$$= \alpha\int_{y^{**}}^{y''} \frac{\partial V^M(z)}{\partial z} H(z,0)[1-F(z)]dz .$$

But,

$$\frac{\partial V^S(z)}{\partial z} > \frac{\partial V^M(z)}{\partial z} \text{ , for all } z\varepsilon[y^{**},y''] \text{ , a contradiction.}$$

Therefore,

$$y^* > y^{**} \text{ .} \qquad \text{Q.E.D.}$$

The implication of this theorem is important. It states that a monopolist will stop short of further R&D search at a level less than that for a social welfare maximizer. With exogenous conditions fixed, a social welfare maximizer may end up with a higher level of technology than that of a monopolist on the average. The driving force behind this theorem can be found from the observation that a social welfare maximizer has a higher output level than a monopolist. The cost saving of having better technology is greater for a social welfare maximizer than a monopolist. This provides a stronger incentive to persist in R&D. Since a monopolist pick output so that marginal avenue and marginal cost are equated, he has less incentive to do R&D. However, it is not necessarily true that greater R&D benefits will lead to greater R&D investment. It is marginal benefit and cost that count. There is no a priori reason why the marginal benefit to a social welfare maximizer will always be greater than that to a monopolist at each given level of R&D investment. Hence, it is not necessarily true that a social welfare maximizer will always invest more in R&D at all technology levels less than the reservation technology level. Since the R&D cost function is arbitrary other than it is convex, more assumptions are required to show that the marginal benefit to a social welfare maximizer is greater than that to a monopolist for all levels of R&D search intensities. See Figure IV-3.

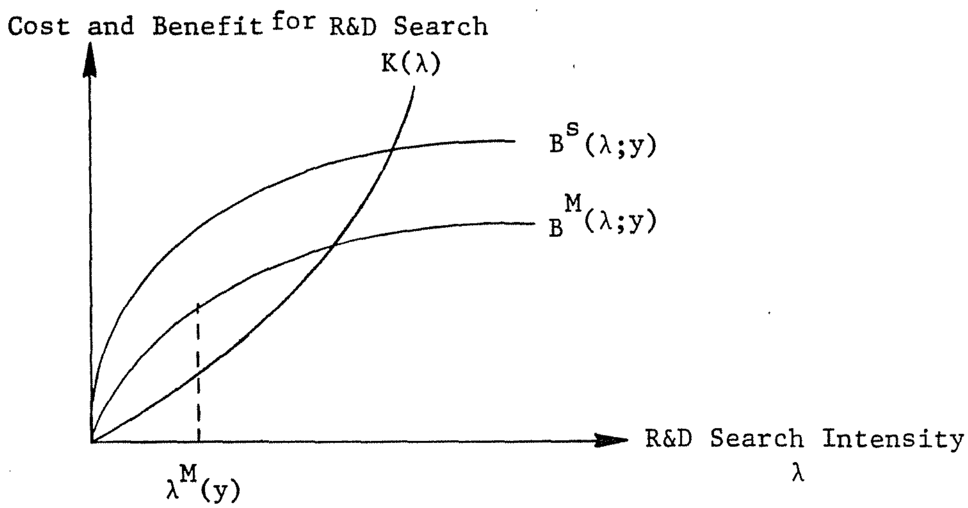

Figure IV-3: Optimal R&D Search Intensity.

The benefit function for a welfare maximizer is clearly greater than that for a monopolist. However, there is no reason to believe that the marginal benefit will also be greater. In particular, if the benefit function for a welfare maximizer rises steeply at low levels of R&D search and then climbs slowly, while the benefit to a monopolist continues to increase, then it may be the case that a monopolist will have a higher marginal benefit at high levels of R&D search intensity.

Section II

Comparative Statics

It remains to show the effects of changes in the parameters of the model on the endogenous variables. These results should be useful for predictions. Recall that there are great similarities between the first order conditions for maximizing social welfare and those for maximizing profit. The comparative statics expressions should have the same signs for both cases. Unfortunately, some of the comparative statics expressions cannot be signed. The following special assumptions are added to tentatively sign the rest of the comparative statics expressions. They are:

(i) $p(q,\gamma)$ is the inverse demand function, where γ is a shift paramter and $p_2 \equiv \frac{\partial p}{\partial \gamma} > 0$.

(ii) $MR_{q\gamma} \equiv \frac{\partial^2 [p(q,\gamma)q]}{\partial q \partial \gamma} > 0$,

(iii) $\frac{\partial^2 V(z)}{\partial z \partial \alpha} \geq 0 \quad \forall z \varepsilon [y',y'']$,

(iv) $\frac{\partial^2 V(z)}{\partial z \partial \gamma} \geq 0 \quad \forall z \varepsilon [y',y'']$,

(v) $\frac{\partial^2 V(z)}{\partial z \partial S} \leq 0 \quad \forall z \varepsilon [y',y'']$, where S is the fixed cost of a R&D unit, and $K(\lambda/m)$ represents the variable cost of R&D.

Assumptions (i) and (ii) are standard for regular demand functions. Assumptions (iii) and (iv) postulate that the discount factor and demand shift have a reinforcing property with technological advances on the optimal return function. Assumption (v) postulates that a unit increase in the fixed cost of R&D tends to lower the marginal benefit of technological advance. The comparative statics results are summarized by a theorem.

Theorem IV-2: **Given the assumptions of the model, the following comparative statics are obtained:**

Exogenous Variable \ Endogenous Variable	Output q	Number of Production Unit n	Output per Production Unit q/n	R&D Search Intensity λ	Number of R&D Units m	R&D Intensity per Unit λ/m	R&D Intensity per Unit Sale λ/pq
Current Technology Level, y	+	?	?	-	-	0	-
Discount Factor, α	0	0	0	(+)	(+)	0	(+)
Fixed Cost per R&D Unit, S	0	0	0	(-)	(-)	+	(-)
Demand Shift Parameters, γ	(+)	(+)	0	(+)	(+)	0	?

N.B. Signs within brackets are true with the additional assumptions (i) to (v).

Proof: See Appendix.

The results of this theorem are interesting. An increase in the current level of technology will lower the optimal level of R&D search intensity. However, R&D search intensity per R&D unit is unaffected. The reason is that the optimal number of R&D units is decreased by an increase in the technology level. The overall effects of the two endogenous adjustments compensate one another, so that R&D search intensity per R&D unit remains unchanged. The effect of a change in the interest rate has similar consequences on the optimal level of R&D, the optimal number of R&D search units, and the R&D search intensity per R&D search unit. An increase in the fixed cost of R&D will lead to a decrease in the optimal level of R&D. The optimal number of R&D units is also decreased, but the extent of its change is large enough that the net result causes the R&D search per unit to increase. In the present model, there are no inventories, so that a change in the discount factor has no effect on production. Furthermore, current production is independent of current R&D activities. Thus, a change in the fixed cost of R&D will have no effect on current production. Observing that a change in the discount factor affects the optimal choice of R&D, it is expected that the technology level in future periods will be affected. This in turn implies that future production levels will be affected. Even though an increase in the current technology level will lower the optimal R&D search intensity, it has a positive effect on the current production level. More interesting is the result that an upward shift in the demand function will increase current production, the optimal R&D search intensity, the optimal number of R&D search units, and the optimal number of production units. These results confirm the claim by many economists that one policy which would promote R&D is to improve the general state of the economy. For example, with ample time for adjustment, a decrease in the tax rate may increase disposable income, and if the relevant markets are producing normal goods, it will increase both the production level and the intensity of R&D search. A further observation is that the effect of any change in the exogenous variables has quite an opposite result on the R&D search intensity per unit of sale as against the R&D search intensity per R&D search unit. While an increase in the technology level, the interest rate, and the demand for the final product have no effect on the R&D search intensity per R&D unit, there are negative effects on the R&D search per unit of sale for the first two changes but ambiguous results from the last change. The ambiguous results are due to the positive effect of a change in the demand on both the production level and the optimal R&D search.

Noncooperative Duopolists

For the rest of this section, a non-cooperative duopolistic market is considered. The model used is similar to that described in the previous section. Assume that the objective of a duopolist is to maximize discounted profits over time. Consider Duopolist I. There are four choice variables at his disposal. They are:

q^1 = Duopolist I's current output,

λ^1 = Duopolist I's R&D search intensity,

n^1 = Duopolist I's number of production units, and

m^1 = Duopolist I's number of R&D units.

Similar notations are used for Duopolist II, except for superscript differences. The current technology level of Duopolist I is given by y.[1] The demand function for the product of the industry is defined by:

$$p(q^1 + q^2),$$

where p is the price of the industry output.

The Nash equilibrium concept is employed in this model. The reason is that R&D implies great uncertainty with respect to the competitive positions of the rivals. With collusion ruled out, calculating the indirect effect of one's behavior on one's rival is difficult. Thus, Duopolist I takes the output level q^2 and R&D search intensity λ^2 of his rival as given. Assume that both firms know the set of distributions of potential technology levels for each level of R&D search intensity. If Duopolist II's technology level is z^2 in the next period, Duopolist I's expected discounted profit stream from the next period onwards is given by

$$\alpha \int_{y'}^{y''} V^1(z^1, z^2)\ dH(z^1, \lambda^1) \ ,$$

conditional on finding a higher technology level, and

$$\alpha V^1(y^1, z^2)\ H(y^1, \lambda^1) \ ,$$

conditional on failing to develop a better technology. $V^1(z^1, z^2)$ stands for the optimal expected discounted profit stream for Duopolist I given his technology level is z^1, and his rival's is z^2. Now, Duopolist I knows that his rival is searching for a new technology at intensity λ^2. Hence, the expected discounted profit stream including the expectation of his rival's technology in the next period is given by

$$\int_{y^2}^{y''} \left[\alpha \int_{y^1}^{y''} V^1(z^1, z^2)\ dH(z^1; \lambda^1) + \alpha V^1(y^1, z^2)\ H(y^1; \lambda^1) \right] dH(z^2; \lambda^2) \ ,$$

conditional on Duopolist II getting a better technology, and

$$\left[\alpha \int_{y^1}^{y''} V^1(z^1,z^2)\ dH(z^1;\lambda^1) + \alpha V^1(y^1,z^2)\ H(y^1;\lambda^1)\right] H(y^2;\lambda^2)\ ,$$

if Duopolist II fails to get a better technology in the next period. The full expression of this optimal functional may be written as

$$V^1(y^1,y^2) = \max_{q^1,\lambda^1,n^1,m^1} \Bigg\{ p(q^1+q^2)\ q^1 - n^1\ C\left(\frac{q^1}{n^1}, y^1\right) - m^1\ K\left(\frac{\lambda^1}{m^1}\right)$$

$$+ \int_{y^2}^{y''} \left[\alpha \int_{y^1}^{y''} V^1(z^1,z^2)\ dH(z^1;\lambda^1) + \alpha V^1(y^1,z^1)\ H(y^1;\lambda^1)\right] dH(z^2;\lambda^2)$$

$$+ \left[\alpha \int_{y^1}^{y''} V^1(z^1,y^2)\ dH(z^1;\lambda^1) + \alpha V^1(y^1,y^2)\ H(y^1;\lambda^1)\right] H(y^2;\lambda^2) \Bigg\}\ .$$

Applying integration by parts twice, it is restated as

$$V^1(y^1,y^2) = \max_{q^1,\lambda^1,n^1,m^1} \Bigg\{ p(q^1+q^2)\ q^1 - n^1\ C\left(\frac{q^1}{n^1}, y^1\right) - m^1\ K\left(\frac{\lambda^1}{m^1}\right) + \alpha V^1(y'',y'')$$

$$- \alpha \int_{y^1}^{y''} \frac{\partial V^1(z^1,y'')}{\partial z^1}\ H(z^1;\lambda^1) dz^1 - \alpha \int_{y^2}^{y''} \frac{\partial V^1(y'',z^2)}{\partial z^2}\ H(z^2;\lambda^2) dz^2$$

$$+ \alpha \int_{y^2}^{y''} \int_{y^1}^{y''} \frac{\partial^2 V^1(z^1,z^2)}{\partial z^1 \partial z^2} H(z^1;\lambda^1)\; H(z^2;\lambda^2) dz^1 dz^2 \Big\} \quad .$$

Let $W^1(q^1,\lambda^1,n^1,m^1)$ be the function in the bracket. First order conditions for maximizing $W^1(q^1,\lambda^1,n^1,m^1)$ are given by

$$\frac{\partial W^1}{\partial q^1} = p_1 q^1 + p - C_1\left(\frac{q^1}{n^1}, y^1\right) = 0 \quad , \tag{5}$$

$$\frac{\partial W^1}{\partial \lambda^1} = -K_1\left(\frac{\lambda^1}{m^1}\right) + \alpha \int_{y^1}^{y''} \frac{\partial V^1(z^1,y'')}{\partial z^1} H(z^1;\lambda^1)[1-F(z^1)]dz^1$$

$$- \alpha \int_{y^2}^{y''} \left[\int_{y^1}^{y''} \frac{\partial^2 V(z^1,z^2)}{\partial z^1 \partial z^2} H(z^1;\lambda^1)[1-F(z^1)]dz^1 \right] H(z^2;\lambda^2)dz^2 = 0 \quad , \tag{6}$$

$$W^1(q^1,\lambda^1,n^1+1,m^1) - W^1(q^1,\lambda^1,n^1,m^1) \le -C\left(\frac{q^1}{n^1}, y^1\right) + \frac{q^1}{n^1} C_1\left(\frac{q^1}{n^1}, y^1\right) = 0 \quad , \tag{7}$$

$$W^1(q^1,\lambda^1,n^1,m^1+1) - W^1(q^1,\lambda^1,n^1,m^1) \le K\left(\frac{\lambda^1}{m^1}\right) + \frac{\lambda^1}{m^1} K_1\left(\frac{\lambda^1}{m^1}\right) = 0 \quad , \tag{8}$$

where

$$p_1 \equiv \frac{\partial p(q^1+q^2)}{\partial (q^1+q^2)} < 0 \ , \ C_1 \equiv \frac{\partial C}{\partial q^1} \ , \text{ and } K_1 \equiv \frac{\partial K}{\partial \lambda^1} \ .$$

Equation (5) requires that for equilibrium output, marginal revenue to a duopolist equals marginal cost of production. Equation (6) is more complicated. It requires that the marginal cost of R&D search intensity equal the marginal benefits of R&D. The latter is the sum of two terms. The first term is the marginal benefit derived from an increase in the optimal return holding his rival's R&D search intensity constant. The second term is a result of interaction between the two rivals. It states the effect of a change in a duopolist's technology on the optimal return to another duopolist. The sign of this term is ambiguous. Equations (7) and (8) are familiar. They require marginal cost equals average cost of production and for R&D.

A symmetry condition is imposed on the duopolistic market. Assume that technology levels of the two duopolists are the same. With this further assumption, the equilibrium output and R&D search intensity should be the same. Hence, letting $2q^D$ be the industry's output, and $2\lambda^D$ be the industry's R&D search intensity, the first order conditions may be rewritten as:

$$p_1 q^D + p - C_1\left(\frac{q^D}{n^D}, y\right) = 0 \ , \qquad (5')$$

$$-K_1\left(\frac{\lambda^D}{m^D}\right) + \alpha\int_y^{y''}\left\{\frac{\partial V(z^1, y'')}{\partial z^1} - \int_y^{y''}\frac{\partial^2 V(z^1, z^2)}{\partial z^1 \partial z^2} H(z^2;\lambda^D)dz^2\right\} H(z^1;\lambda^D)[1 - F(z^1)]dz^1 = 0 \ , \qquad (6')$$

$$-C\left(\frac{q^D}{n^D}, y\right) + \frac{q^D}{n^D} C_1\left(\frac{q^D}{n^D}, y\right) = 0 \ , \qquad (7')$$

$$-K\left(\frac{\lambda^D}{m^D}\right) + \frac{\lambda^D}{m^D} K_1\left(\frac{\lambda^D}{m^D}\right) = 0 \ . \qquad (8')$$

It is possible to proceed to derive the analytical solutions necessary for comparative statics. However, it is difficult to sign the results, because they involve the third derivative of the optimal return functional, such as

$$\frac{\partial^3 V(z^1,z^2)}{\partial z^1 \partial z^2 \partial \alpha} .$$

It was shown in the first section that a monopolist may invest less in R&D search than a social welfare maximizer. It was further demonstrated that the Reservation Technology Level (RTL) of a monopolist is lower than that of a social welfare maximizer. The welfare result in this section is weaker. It will be shown in the following theorem that a non-cooperative duopolistic market may have more R&D investment than that of a monopolistic one, but the result requires more assumptions. It is proved by showing that the RTL of a non-cooperative duopolist y^{***} may be greater than that of a monopolist y^{**}. Thus, the innate advantage of monopolistic market as upheld by Schumpeterians is questioned, but not disproved. A conclusive judgment requires more detailed knowledge of the specific demand situation and the interactive effects among rivals' behavior.

Theorem IV-3: If $n^M(z) \le n^D(z) \ \forall z \varepsilon [y^{***}, y'']$, and

$$\frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} < 0 \quad \forall z^1 \varepsilon [y^{***}, y''] \ , \ \forall z^2 \varepsilon [y^{***}, y'']$$

then $y^{**} < y^{***}$.

Proof: By the definition of a reservation technology level, one obtains

$$\alpha \int_{y^{**}}^{y''} \frac{\partial V^M(z)}{\partial z} H(z,0)[1-F(z)]dz$$

$$= \alpha \int_{y^{***}}^{y''} \left\{ \frac{\partial V^D(z^1,y'')}{\partial z^1} - \int_{y^{***}}^{y''} \frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} H(z^2,0)dz^2 \right\} H(z^1,0)[1-F(z^1)]dz^1 .$$

Now, $\frac{\partial V^M(z)}{\partial z} = \frac{-n^M C_2(q^M/n^M,z)}{1-\alpha} \quad \forall z \varepsilon [y^{**},y^*]$,

and $\frac{\partial V^D(z^1,y'')}{\partial z^1} = \frac{-n^D C_2(q^D/n^D,z)}{1-\alpha} \quad \forall z \varepsilon [y^{***},y'']$.

If $n^M(z) \le n^D(z)$, $\frac{\partial V^M(z)}{\partial z} \le \frac{\partial V^D(z,y'')}{\partial z}$.

Suppose $y^{**} \ge y^{***}$.

Then $\alpha \int_{y^{**}}^{y''} \frac{\partial V^M(z)}{\partial z} H(z,0)[1-F(z)]dz$

$$= \alpha \int_{y^{***}}^{y''} \left\{ \frac{\partial V^D(z^1,y'')}{\partial z^i} - \int_{y^{***}}^{y''} \frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} H(z^2,0)dz^2 \right\} H(z^1,0)[1-F(z^1)]dz^1$$

$$\geq \alpha \int_{y**}^{y''} \left\{ \frac{\partial V^D(z^1,y'')}{\partial z^1} - \int_{y**}^{y''} \frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} H(z^2,0)dz^2 \right\} H(z^1,0)[1-F(z^1)]dz^1 .$$

But $$\frac{\partial V^M(z)}{\partial z} < \frac{\partial V^D(z^1,y'')}{\partial z^1} - \int_{y**}^{y''} \frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} H(z^2,0)dz^2 ,$$

if $$\frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} < 0 \text{ , a contradiction.}$$

Therefore,

$$y^{**} < y^{***} .$$ Q.E.D.

It is possible that $n^M(z) \leq n^D(z)$ $\forall z \varepsilon [y^{***},y'']$. For example, see Figure IV-4. Here the demand as perceived by the duopolist is more elastic than the actual demand of the industry. The marginal revenue as perceived by the duopolist is also more elastic. Thus, market opportunities for the duopolist are larger than those for a monopolist. Rivalry may create a better environment for R&D. The assumptions in the theorem are sufficient conditions for the theorem. As long as $\partial^2 V^D(z^1,z)/(\partial z^1 \partial z^2) < 0$ $\forall z^1 \varepsilon [y^{***},y'']$ $\forall z^2 \varepsilon [y^{***},y'']$, it is possible that even if $n^M(z) > n^D(z)$, $y^{**} < y^{***}$. A sufficient condition then is that:

$$\frac{\partial V^M(z)}{\partial z} < \frac{\partial V^D(z,y'')}{\partial z} - \int_{y**}^{y''} \frac{\partial^2 V^D(z^1,z^2)}{\partial z^1 \partial z^2} H(z^2,0)dz^2 \text{ , } \forall z \varepsilon [y^{**},y''] .$$

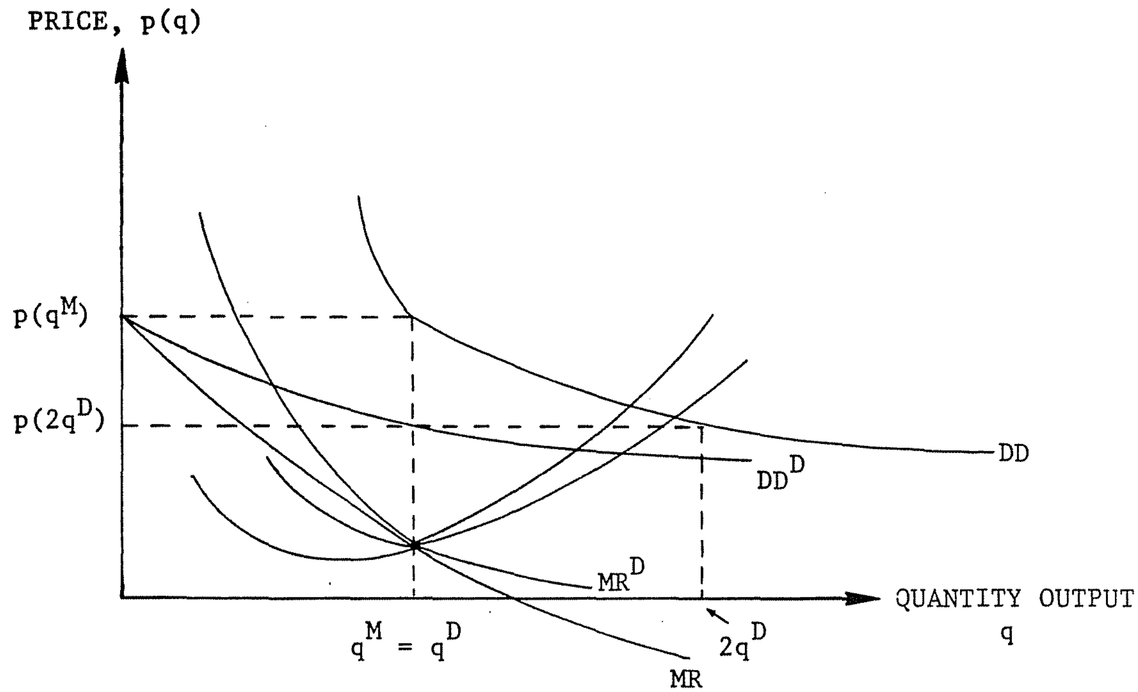

Figure IV-4: Nash Equilibrium of Duopolists.

Section III

Two things need to be resolved in this section. The first is related to an assumption made in the previous sections. It has been assumed that the R&D cost function in the model exhibits increasing returns to scale, and changes to decreasing returns to scale at a low level of R&D search intensity. Schumpeterians usually assume the contrary, i.e., there are increasing returns to scale throughout the relevant range of R&D search intensity.

It has been presumed so far that technological change can be modeled as a reduction of the cost of production alone. No distinction has been made between product and process innovations. Indeed, it will be argued here that product innovation is a special case of process innovation in the sense that the former does not have a positive current output level due to the current high cost of production, and hence is an unprofitable activity. Theoretical similarities of the two types of innovations will be discussed.

R&D Economies of Scale

Consider the evolution of an industry. At the early stage of development, firms in the industry are striving for survival. Each is trying to capture a market share. Changes in relative market shares are significant. As the market size ceases to increase rapidly, and a small group of dominant firms emerge out of fortuitous technical improvement and management superiority, the relative market shares begin to stabilize. If price fixing occurs, it would only reinforce the stability of relative market shares.

If these firms are faced with the choice of developing a small scale technology versus a large scale technology, both with the same potential minimum average cost, though at different levels of output, there is a tendency to invest in the large scale technology. There are two reasons for such a bias. The first is an employment-control rationale. The entrepreneur acting to lower the probability of new entrants, will avoid spinning off experts from his firm to form new ones. With a large scale technology, employees of the firm will find it difficult to understand the complete production process. An example is the amazingly complex assembly process for automobiles. There are plants that assemble engines alone. Furthermore, the employees in production and those in R&D are separated. The direct motive is to take advantage of specialization, but a consequence is the further compartmentalization of experts. It has been a paradox that economists emphasize the importance of better relations between the production and R&D departments of a firm, and yet the two remain isolated to a large extent for many firms. Nevertheless, a small group of ingenious employees may be able to master the whole relation of processes in the firm. Granted this is true, they will still have difficulty finding financing. Their knowledge is limited to large scale technology, which in most cases implies a need for a large start-up investment. Finally, they will have to compete with a group of large and collusive firms. Overall, there would appear to be little incentive to start a new venture.

A second motivation for an entrepreneur to favor large scale technology is the potential of future antitrust suits. As long as the market grows small firms can survive. When the growth slows down, dominant firms will carve up the market. Small firms may not be in the "club" of collusive firms. Antitrust suits are prone to happen in the face of political maneuvering generated by marginal firms, which are now struggling for survival. A possible defense against potential antitrust will be to argue along natural monopoly lines. Technical sustainability of market structures may foreclose economic arguments against the group of dominant firms.[10]

As more and more R&D investment is spent on large scale technology development, only a selective type of knowledge is generated, namely, the design and control of large scale systems. Over time, potential competitors will be forestalled from developing small scale technology, because the technical knowledge relevant for that scale is nonexistent due to historical large scale technology bias, and the large investment necessary to develop a large scale technology is prohibitive in the face of collusive firms. Thus, large scale R&D for the development of large scale technology may be motivated by profit seeking behavior, and hence it is an endogenous choice. It may not necessarily be socially preferable.

As far as policy choices are concerned, there is a dilemma. The historical technology scale bias has resulted in a stock of knowledge for developing large scale technology. The cost of developing small scale technology is high. However, continuing the trend will preclude competition. The alternative is to correct the situation over time, i.e. a reverse in technology bias at the margin that works to restore competition. But existing firms in the industry will have no incentive to develop small scale technology. Small firms need to be encouraged to develop small scale technology whenever technical opportunity exists. However, if large subsidies are used, small firms may have the same bias towards large scale technology discussed above. If small scale technology is cost competitive in the normal economic sense, a preferable method may be to guarantee that the technology will not be foreclosed by large firms using cross subsidization.

Treatment of Product and Process Innovation

For theoretical purposes, I treat product innovation as a situation in which the good had such a high cost of production in the past that equilibrium output quantity was zero. Hence, a product innovation can be considered as a special case of process innovation.

This may not be true as some critics may say. They offer the reason that with process innovation, one knows the number of producers. However, a new product innovation implies that there is no producer in the past, although there may be an unknown number of potential producers. Hence, a monopoly situation can be temporarily established for the first innovator. Indeed, this is true. But, in process innovation there is an unknown number of potential producers too, granted that the number of current producers is known. Thus, in either case the same element of uncertainty occurs, namely, the number of potential producers. An example will be the watch industry. No one in the industry would have expected Texas Instrument to enter the watch market a decade ago. Whether a model can be applied to product or process innovation will depend on the level of uncertainty considered.

Again, one may argue that for product innovation, there is no existing market, and hence no information on the demand for the product nor the supply capability of potential producers. Rosenberg (1975) indirectly provides a rebuttal to the argument. He points out that if R&D is defined as information buying, assessment of demand at prices other than the current one is also a form of R&D. This observation provides the basis for a counter argument. The demand structure and the supply capability of the current and potential producers are not known for situations where prices are substantially lower than the current one. Since process innovation will eventually lead to lower real costs of production, product and process innovations may be treated as essentially equivalent for theoretical purposes. The difference again lies in the level of uncertainty.

A critic may present a third argument as follows: For a process innovation, any improvement by one producer affects the production level of all his rivals while for the case of a product innovation, the successful results of R&D by one producer has no production effect on his rivals since none of them were producing in the past (by the definition of a new product). One may produce an immediate counter argument. For a product innovation, the success of an innovator has no actual production effect on his rivals, yet it has a potential production effect on them. The reason is that his rivals, in the event they also lower the production cost substantially in the future, will not be producing as many units as in the case that the first producer had not succeeded in entering the market previously. Hence, I conclude that integrating production with product or process innovative R&D should lead to no theoretical disparity. The only difference is the level of uncertainty involved.

Summary and Conclusion

In this chapter, it has been shown that a monopolist is less persistent in R&D search than a social welfare maximizer is. Since a monopolist produces at less than the socially optimal level, his benefit derived from cost reduction is less. Therefore, he will stop searching at a lower level of technology. However, the fact that a monopolist has a benefit function dominated by that of a social welfare maximizer does not imply he will always invest less in R & D at all levels of technology. Marginal benefits and marginal costs determine the level of R&D investment. In fact, the benefits to a social welfare maximizer might rise so rapidly that the increase in benefits at high levels of R&D search intensity may taper off gradually. The marginal benefits to a social welfare maximizer may be less than those to a monopolist at such levels of R&D search intensity. This point is not recognized by some economists when they model R&D with a fixed reward. They ignore the continuous nature of R&D.

It is further demonstrated that the welfare ranking of market structures is highly sensitive to the demand elasticity as well as the cross effect of one's rivals' behavior on one's objective. If demand is sufficiently elastic below the monopolistic price, a duopolistic market may produce more than twice the output of a monopolist. In that event, the returns to each duopolist on R&D investment may be sufficient to motivate more R&D than that invested by a monopolist. It remains to evaluate other situations with different oligopolistic market structures, and with different solution concepts. There are a number of group behavior theories which have different implications. Thus, the conclusions with respect to the welfare ranking of market structures is only tentative. What is achieved here is to demonstrate some important determinants to such judgements.

Last of all, it is resolved that there need not be any special treatment for product as contrasted with process innovation. The problem is a matter of degree of uncertainty. It is shown that there may be technological bias in developing potential technologies. A descriptive theory of such bias is provided. The motivation of such behavior is also identified. Breaking down R&D decision on a microeconomic level has provided many insights to the problem as a trade-off to the rapid increase in complexity. Research in this direction will definitely give more insights on the growth and behavior of an industry. It may also explain some interactive reactions in firms' investment decisions, both for production and R&D. Formal modeling of these aspects is left for future research.

APPENDIX

Proof of Theorem IV-2: Totally differentiate (1′) to (4′) gives

$$\left(MR_{qq} - \frac{1}{n} C_{11}\right) dq - C_{12}\, dy + \frac{q}{n^2} C_{11}\left(\frac{q}{n}, y\right) dn + MR_{q\gamma}\, d\gamma = 0$$

$$-\left(\frac{1}{m} K_{11} + \alpha \int_y^{y''} \frac{\partial V(z)}{\partial z} H(z,\lambda)\left[1 - F(z)\right]^2 dz\right) d\lambda + K_{11} \frac{\lambda}{m^2}\, dm$$

$$+\left(\int_y^{y''} \frac{\partial V(z)}{\partial z} H(z,\lambda)\left[1 - F(z)\right] dz + \alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial \alpha} H(z,\lambda)\left[1 - F(z)\right] dz\right) d\alpha$$

$$+ \alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial \gamma} H(z,\lambda)\left[1 - F(z)\right] dz\, d\gamma + \alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial S} H(z,\lambda)\left[1 - F(z)\right] dz\, dS$$

$$- \alpha \frac{\partial V(y)}{\partial y} H(y,\lambda)\left[1 - F(y)\right] dy = 0$$

$$\frac{q}{n^2} C_{11}\, dq - \frac{q^2}{n^3} C_{11}\, dn + \left(\frac{q}{n} C_{12} - C_2\right) dy = 0$$

$$\frac{\lambda}{m^2} K_{11}\, d\lambda - \frac{\lambda^2}{m^3} K_{11}\, dm - dS = 0$$

In matrix-vector form, the following holds:

$$\begin{bmatrix} MR_{qq} - \frac{1}{n} C_{11} & 0 & 0 & \frac{q}{n^2} C_{11} \\ 0 & -\left(\frac{1}{m} K_{11} + \alpha \int_y^{y''} \frac{\partial V(z)}{\partial z} H(z,\lambda)\left[1 - F(z)\right]^2 dz\right) & \frac{\lambda}{m^2} K_{11} & 0 \\ \frac{q}{n^2} C_{11} & 0 & 0 & -\frac{q^2}{n^3} C_{11} \\ 0 & \frac{\lambda}{m^2} K_{11} & -\frac{\lambda^2}{m^3} K_{11} & 0 \end{bmatrix} \begin{bmatrix} dq \\ d\lambda \\ dm \\ dn \end{bmatrix}$$

$$= \begin{bmatrix} C_{12}\, dy & 0 & -MR_{q\gamma}\, d\gamma \\ \alpha \frac{\partial V(y)}{\partial y} H(y,\lambda)\left[1 - F(y)\right] dy - \left(\int_y^{y''} \frac{\partial V(z)}{\partial z} H(z,\lambda)\left[1 - F(z)\right] dz + \alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial \alpha} H(z,\lambda)\left[1 - F(z)\right] dz\right) d\alpha & -\alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial S} H(z,\lambda)\left[1 - F(z)\right]^2 dz\, dS & -\alpha \int_y^{y''} \frac{\partial^2 V(z)}{\partial z\, \partial \gamma} H(z,\lambda)\left[1 - F(z)\right] dz\, d\gamma \\ -\frac{q}{n} C_{12} + C_2\ dy & 0 & 0 \\ 0 & dS & 0 \end{bmatrix}$$

Let A be the coefficient matrix

$$|A| = MR_{qq} \frac{q^2}{n^3} \frac{\lambda^2}{m^3} C_{11} K_{11}\, \alpha \int_y^{y''} \frac{\partial V(z)}{\partial z} H(z,\lambda)\left[1 - F(z)\right]^2 dz \cdot 0$$

Applying Cramer's Rule, the following are obtained:

$$\frac{dq}{dy} = \frac{C_2}{MR_{qq}\,\frac{q}{n}} > 0$$

$$\frac{dq}{d\alpha} = 0$$

$$\frac{dq}{dS} = 0$$

$$\frac{dq}{d\gamma} = \frac{-\frac{q^2}{n^3} C_{11}\, MR_{q\gamma}\, \frac{\lambda^2}{m^3} K_{11}\, \alpha \int_y^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\cdot)\left[1 - F(z)\right]^2 dz}{|A|} > 0$$

$$\frac{d\lambda}{dy} = \frac{-\frac{\lambda^2}{m^3} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\, \alpha\, \frac{\partial V(y)}{\partial y} H(y,\cdot)\left[1 - F(y)\right]}{|A|} < 0$$

$$\frac{d\lambda}{d\alpha} = \frac{\frac{\lambda^2}{m^3} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\left(\int_y^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\cdot)\left[1 - F(z)\right] dz + \alpha \int_y^{y^{**}} \frac{\partial^2 V(z)}{\partial z\,\partial \alpha} H(z,\cdot)\left[1 - F(z)\right] dz\right)}{|A|} \quad 0$$

$$\frac{d\lambda}{dS} = \frac{MR_{qq}\left(-\frac{\lambda}{m^2} K_{11}\, \frac{q^2}{n^3} C_{11} + \frac{\lambda^2}{m^3} K_{11}\, \frac{q^2}{n^3} C_{1}\, \alpha \int_y^{y^{**}} \frac{\partial^2 V(z)}{\partial z\,\partial S} H(z,\cdot)\left[1 - F(z)\right]^2 dz\right)}{|A|} < 0$$

$$\frac{d\lambda}{d\gamma} = \frac{\frac{\lambda^2}{m^3} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\, \alpha \int_y^{y^{**}} \frac{\partial^2 V(z)}{\partial z\,\partial \gamma} H(z,\cdot)\left[1 - F(z)\right] dz}{|A|} \quad 0$$

$$\frac{dm}{dy} = \frac{-\frac{\lambda}{m^2} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\, \alpha\, \frac{\partial V(y)}{\partial y} H(y,\cdot)\left[1 - F(y)\right]}{|A|} < 0$$

$$\frac{dm}{d\alpha} = \frac{\frac{\lambda}{m^2} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\left\{\int_y^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\cdot)\left[1 - F(z)\right] dz + \alpha \int_y^{y^{**}} \frac{\partial^2 V(z)}{\partial z\,\partial \alpha} H(z,\cdot)\left[1 - F(z)\right] dz\right\}}{|A|} \quad 0$$

$$\frac{dm}{dS} = \frac{MR_{qq}\left\{\frac{q^2}{n^3} C_{11}\, \frac{\lambda}{m^2} K_{11}\, \alpha \int_y^{y^{**}} \frac{\partial^2 V(z)}{\partial z\,\partial S} H(z,\cdot)\left[1 - F(z)\right]^2 dz - \frac{q^2}{n^3} C_{11}\left(\frac{1}{m} K_{11} + \alpha \int_y^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\cdot)\left[1 - F(z)\right]^2 dz\right)\right\}}{|A|}$$

$$< 0$$

$$\frac{dm}{dS} = \frac{\frac{\lambda}{m^2} K_{11}\, MR_{qq}\, \frac{q^2}{n^3} C_{11}\, \alpha \int_y^{y^{**}} \frac{\partial^2 V(\cdot)}{\partial z\,\partial S} H(z,\cdot)\left[1 - F(z)\right] dz}{|A|} \quad 0$$

$$\frac{dn}{dy} = \frac{\left(-C_2 + \frac{q}{n} C_{12}\right)}{C_{11}\,\frac{q^2}{n^3}} + \frac{\frac{1}{n} C_2}{MR_{qq}\,\frac{q^2}{n^3}}$$

$$\frac{dn}{d\alpha} = 0$$

$$\frac{dn}{dS} = 0$$

$$\frac{dn}{d\gamma} = \frac{-\frac{q}{n^2} C_{11} \frac{\lambda^2}{m^3} K_{11} MR_{q\gamma}\, \alpha \int_{y}^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\lambda) \left[1 - F(z)\right]^2 dz}{|A|} > 0$$

$$\frac{d\left(\frac{q}{n}\right)}{dy} = \frac{n \frac{dq}{dy} - q \frac{dn}{dy}}{n^2} = \frac{\frac{n}{q} C_2 - C_{12}}{C_{11}}$$

$$\frac{d\left(\frac{q}{n}\right)}{d\alpha} = 0$$

$$\frac{d\left(\frac{q}{n}\right)}{dS} = 0$$

$$\frac{d\left(\frac{S}{n}\right)}{d\gamma} = 0$$

$$\frac{d\left(\frac{\lambda}{pq}\right)}{dy} = \frac{pq \frac{d\lambda}{dy} - \lambda MR_q \frac{dq}{dy}}{(pq)^2} \cdot 0$$

$$\frac{d\left(\frac{\lambda}{pq}\right)}{d\alpha} = \frac{1}{pq} \frac{d\lambda}{d\alpha} \quad 0$$

$$\frac{d\left(\frac{\lambda}{pq}\right)}{dS} = \frac{1}{pq} \frac{d\lambda}{dS} \cdot 0$$

$$\frac{d\left(\frac{\lambda}{pq}\right)}{d\gamma} = \frac{pq \frac{d\lambda}{d\gamma} - \lambda MR_q \frac{dq}{d\gamma}}{(pq)^2}$$

$$\frac{d\left(\frac{\lambda}{m}\right)}{dy} = 0$$

$$\frac{d\left(\frac{\lambda}{m}\right)}{d\alpha} = 0$$

$$\frac{d\left(\frac{\lambda}{m}\right)}{dS} = \frac{MR_{qq} \frac{q^2}{n^3} C_{11}\, \alpha \int_{y}^{y^{**}} \frac{\partial V(z)}{\partial z} H(z,\lambda) \left[1 - F(z)\right]^2 dz}{m^2\, |A|} \cdot 0$$

$$\frac{d\left(\frac{\lambda}{m}\right)}{d\gamma} = 0$$

Chapter IV: Footnotes

1. See Chapter II for a review of Futia's model.

2. Static efficiency is achieved when marginal benefits and costs of production are equal holding technology constant. Dynamic efficiency is achieved when marginal benefits and costs of technology development are equal. The former is a short run efficiency criterion while the latter is a long-run efficiency criterion.

3. See section III of this chapter.

4. The demand side of R&D is actually a derived demand. R&D results are usually not for final consumption. It is the cost reduction that benefits consumers by allowing more quantity purchases for a given income.

5. Reservation technology level is that below which R&D is continued and above which R&D is stopped.

6. Technology choice bias occurs when a R&D decision maker chooses to develop a technology that requires a larger output level to achieve minimum average cost of production while there is another technology that yields the same minimum average cost of production but at a lower output level and the expected costs of developing the two technologies are the same.

7. Futia (1977) has some breakthroughs in this direction. See chapter II for a more detailed discussion of his model.

8. The implication from this table is biased in the sense that some industries are highly regionalized, i.e., within their geographical region.

9. The diffusion of technology is not investigated in this paper. Its importance will only urge for future research.

10. This is a too strong a statement when one considers the A&P grocery chain store case and the ALCOA aluminum industry case. In both cases, economic argument fails to revert antitrust decisions.

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Chapter V

Sequential R&D Search Model

Properties of Technological Change

It has long been emphasized that technological change has been both continuous and discontinuous. There is continuity in minor innovations, and discontinuity in major breakthroughs involving radical overhauls of manufacturing processes and/or the introduction of new products. In spite of this duality, technological change follows the Marshallian principle of supply and demand, with the addition of uncertainty. The supply side is governed by technical feasibility (hence costs), while the demand side is ruled by the market value of the final product. Attempts to model these aspects of technological change remain a challenge to economists.

Stages of Technological Change

Technological change is commonly delineated into stages. One dichotomy is between research and development.

For example, Ames (1961) defines research as "a flow of new statements about the natural world" and development as "a flow of instructions (blueprints, diagrams etc.) which enable the construction and equipment industries to build fixed plants of kinds never used before; and also enable the personnel of these plants to operate them when finished." Mansfield (1968) defines research as "original investigation directed to the discovery of new scientific knowledge" and development as "technical activity concerned with non-routine problems encountered in translating research findings into products and processes." More recently, Spulber (1977) defines research as a "stage where discoveries are made and the technical feasibility of their application is ascertained," and development as a "stage, where the results obtained in the research stage are made 'operational' and the utility of implementing the outcome is determined."

Admittedly, these definitions of research and development lack economic content. One needs to introduce the costs and benefits relevant to each stage and investigate the allocative process of expending effort between the two stages. Furthermore, one needs to postulate linkages between research and development to avoid separating technological change into two unrelated stages. To this end, economists approach the problem from different directions. Ames investigates the capital market and the equilibrium interest rate. Mansfield tests empirical relations between productivity changes and economic variables, such as level of research and development funding. Spulber theorizes on the microeconomic foundations of research and development.

I postulate that development is a costly process through which positive economic return may be the reward. One may interpret development as commercial testing of a technology from a given technology field. Secondly, I postulate research is also a costly process through which improvements in the likelihood of positive economic returns may occur in the development process. One may interpret research as searching for a new technology field with a better prospect of development success. Thus, development is identified with direct economic gains and research with indirect economic gains. The purpose of the latter is to make the former economically more attractive. I shall assume away the possibility that research may be a consumption good. Though researchers may view research as a consumption good, individuals funding the research activities will have in mind an expectation of economic gains.

The "Switching" Property of R&D

Standard results from search theory with a one dimensional search space indicate that there is a reservation technology level, below which research is continued and above which research is stopped. This constitutes Spulber's "switchpoint" property of research and development, namely, once research is discontinued, it will never be resumed. This is contrary to casual observation. Research and development occur continually with changes in emphasis over time, but neither one is discontinued forever. The only way to introduce this "reswitching" property of research and development to the standard search model of one dimensional search space is to allow exogenous changes in the probability distribution over the search space. I propose to provide a model in which the "reswitching" property of research and development is endogenous. To achieve this end, a two dimensional search space will be introduced into the standard search model. Wilde (1977) and Burdett (1978) introduced this extension. The structure of the model will closely follow Burdett's. The next section will describe the model. A following section will analyze it in detail. The last section will deal with a number of comparative statics results.

Section I

The Model

Following Spulber's (1977) conception of the microeconomic processes of research and development, I introduce the concept of a technological knowledge index, $z \varepsilon [0,1]$, which can be improved stochastically by engaging in research activities. Based on a given technological knowledge index, there is a probability distribution of an economic index, $G(n|z)$ which can be achieved by a development process. Associated with a given economic index, $n \varepsilon [0,1]$, there is an economic return function $R(n)$. A higher economic index implies higher economic benefits per time period, i.e. $R'(n) > 0$. The distribution of the economic index is rank ordered by the technological knowledge index, based on the first order stochastic dominance concept introduced by Quirk and Saposnik (1962), i.e., $\partial G(n|z)/\partial z < 0$ for all $n \varepsilon [0,1)$ and $z \varepsilon [0,1)$. Additional properties imposed on the distributive function G are:

$$
\begin{aligned}
G(0|y) &= 0 \,, \\
G(1|y) &= 1 \text{ for all } y \varepsilon [0,1], \text{ and} \\
\frac{\partial G(n|y)}{\partial n} &> 0 \text{ for all } n \varepsilon [0,1] \,.
\end{aligned}
$$

Thus the objective of research is to seek a higher technological knowledge index and to improve the likelihood of achieving higher than current economic return through the development process. Let $H(z)$, $z \varepsilon [0,1]$, be the distribution function of technological knowledge index, and assume

$$H(0) = 0\ ,$$

$$H(1) = 1,\ \text{and}$$

$$H'(z) > 0 \text{ for all } z\varepsilon[0,1]\ .$$

To take an observation from this distribution, a decision maker has to pay a constant cost of K. Similarly, independent of the technological knowledge index, a decision maker has to pay a constant cost of C in order to take an observation from the distribution of the economic index.

Suppose the current state of the world is defined by the pair (m,y). A decision maker has three options. Option I is doing nothing other than collecting whatever the existing economic index, m, allows in the current period, and behaving optimally in all future periods. A case for Option I is that research and development costs are prohibitively high and/or the probability of further net return is very low. The second option is to search for a higher value of the technological knowledge index in the current period and then behave optimally thereafter. The remaining option is to exploit existing technological knowledge and search for a higher economic return. Mutual exclusiveness of the three options is assumed. See Figure V-1.

Assume the decision maker attempts to maximize the expected discounted economic return net of R&D search costs. Let V(m,y) be the optimal expected discounted return net of search costs if the current technological knowledge index is y and the current economic index is m. Independent of which option is chosen, a decision maker receives R(m) in the current period. If option I is chosen, the decision maker will receive the optimal expected discounted return net of search costs V(m,y) in the next period. Expressed in present value form, the last expression is given by $\alpha V(m,y)$, where α is the discount factor. Define $u_1(m,y) = \alpha V(m,y)$. If option II is chosen, the decision maker has to pay a fixed search cost K in the current period, and receive the present value of an expected sum

$$\alpha \int_y^1 V(m,z)\ dH(z) + \alpha V(m,y)\ H(y)$$

in the next period. The first term in the expression is the payoff if the research activity is successful. The second term is the payoff if the research activity yields no improvement. Define

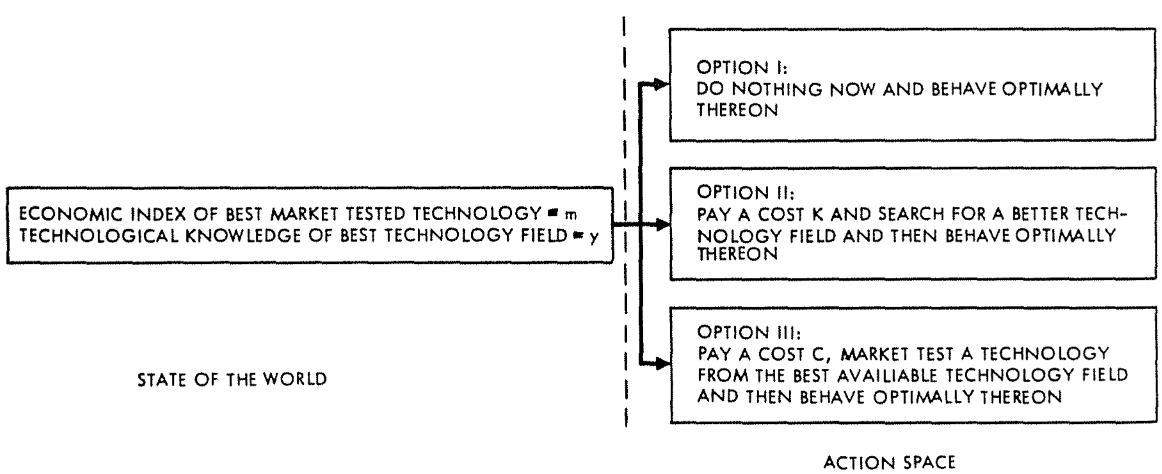

Figure V-1. Mutually Exclusive R&D Action Choices for a Given State of the World

$$u_2(m,y) = -K + \alpha \int_y^1 V(m,z) \; dH(z) + \alpha V(m,y) \; H(y).$$

Last of all, if option III is chosen, the decision maker has to pay a fixed search cost C in the current period and receive a present value of an expected discounted sum

$$\alpha \int_m^1 V(n,y) \; dG(n|y) + \alpha V(m,y) \; G(m|y) \; .$$

The first term is the payoff if the development activity is successful. The second term is the payoff if the development activity yields no improvement. Define

$$u_3(m,y) = -C + \alpha \int_m^1 V(n,y) \; dG(n|y) + \alpha V(m,y) \; G(m|y) \; .$$

Thus the optimal expected discounted return net of search costs can be restated as

$$V(m,y) = R(m) + \max \{ u_1(m,y), \; u_2(m,y), u_3(m,y) \} \; . \qquad \text{(eq. V-1)}$$

<u>Existence</u>

The existence of a functional V that satisfies the equation can be proved by a straightforward application of Denardo (1967). The result is stated as a theorem while the proof is left in an appendix.

<u>Theorem V-1</u>: Given the assumptions of the sequential research and development model, there exists a unique, continuous, bounded solution to the functional equation V-1.

Proof: see appendix

The Existence Theorem itself does not require the differentiability of the optimal functional. Assume that the optimal functional V is twice differentiable. Additional assumptions are made as follows:

(1) $\frac{\partial V(m,y)}{\partial m} \geq 0$ for all $(m,y) \varepsilon S$, and strictly positive for some $(m,y) \varepsilon S$,

(2) $\frac{\partial V(m,y)}{\partial y} \geq 0$ for all $(m,y) \varepsilon S$, and strictly positive for some $(m,y) \varepsilon S$,

(3) $\frac{\partial^2 V(m,y)}{\partial m \partial y} \leq 0$ for all $(m,y) \varepsilon S$, and strictly negative for some $(m,y) \varepsilon S$,

where

$$S = [0,1] \times [0,1] .$$

Preliminary examination of the expected discounted return will be useful.

<u>Proposition V-1:</u>

$$\frac{\partial u_1}{\partial y} \geq \frac{\partial u_2}{\partial y} \geq 0 .$$

Proof: Recall the definitions of u_1 and u_2.

$$\frac{\partial u_1}{\partial y} = \alpha \frac{\partial V(m,y)}{\partial y} .$$

$$\frac{\partial u_2}{\partial y} = \alpha \frac{\partial V(m,y)}{\partial y} H(y) .$$

Therefore

$$\frac{\partial u_1}{\partial y} \geq \frac{\partial u_2}{\partial y} \geq 0 .$$

Q.E.D.

Proposition V-1 states that as the technological knowledge index is increased by research activities, Option II (choosing research) becomes less attractive relative to Option I thus, stopping may eventually be preferable over research. For a given distribution of the technological knowledge index, the probability of further research success decreases with increases in the technological knowledge index. On the other hand, the search cost of research remains constant. Thus the economic attractiveness of Option II decreases with an increase in the technological knowledge index.

Proposition V-2:

(1) $\frac{\partial u_1}{\partial m} \geq \frac{\partial u_2}{\partial m} \geq 0$, and

(2) $\frac{\partial u_1}{\partial m} \geq \frac{\partial u_3}{\partial m} \geq 0$.

Proof: Recall the definitions of u_1, u_2, and u_3.

$$\frac{\partial u_1}{\partial m} = \alpha \frac{\partial V(m,y)}{\partial m} .$$

1) $$\frac{\partial u_2}{\partial m} = \alpha \frac{\partial V(m,y)}{\partial m} + \alpha \int_y^1 \left[\frac{\partial V(m,z)}{\partial m} - \frac{\partial V(m,y)}{\partial m}\right] dH(z) \geq 0$$

Therefore

$$\frac{\partial u_1}{\partial m} \geq \frac{\partial u_2}{\partial m} \geq 0.$$

2) $$\frac{\partial u_3}{\partial m} = \alpha \frac{\partial V(m,y)}{\partial m} G(m|y) \geq 0 .$$

Therefore

$$\frac{\partial u_1}{\partial m} \geq \frac{\partial u_3}{\partial m} \geq 0 . \qquad \text{Q.E.D.}$$

Proposition V-2 states that as the economic index is increased by development activities over time, Options II and III became an inferior choice compared to Option I. The attractiveness of Option II is that research activities may raise the technological knowledge index which in turn raises the probability of improving the current economic index. However, as the current economic index increases, the probability of future development success is lowered. Thus stopping may eventually be preferable to research. The second part of Proposition V-2 is more straightforward. The higher the current economic index, the lower the probability of development successes. With the search cost of development remaining constant, the economic attractiveness of development decreases relative to stopping. It remains to identify criteria under which research is preferable to development.

Section II

Optimal Choice Regions

Define the following sets

$$S_{ij} = \left\{ (m,y) : u_i(m,y) = u_j(m,y),\ (m,y) \in S \right\} . \qquad \text{(def V-1)}$$

For example, if the current state $(m,y)\varepsilon S$ is also an element of S_{12}, it implies that the decision maker is indifferent between Option I and Option II. Note that neither one is claimed to be the optimal choice. Consider S_{12}.

$$u_1(m,y) = \alpha V(m,y) .$$

$$u_2(m,y) = -K + \alpha \int_y^1 V(m,z)\ dH(z) + \alpha V(m,y)\ H(y)$$

$$= -K + \alpha V(m,y) + \alpha \int_y^1 \left[V(m,z) - V(m,y) \right] dH(z) .$$

Thus, $(m,y)\varepsilon S_{12}$ implies that $u_1(m,y) = u_2(m,y)$ or

$$0 = -K + \alpha \int_y^1 \left[V(m,z) - V(m,y) \right] dH(z) . \qquad \text{(eq V-2)}$$

Note that the second term on the right hand-side is decreasing with respect to y. Thus for each $m\varepsilon[0,1]$, there is only one $y\varepsilon[0,1]$ such that the above equation is satisfied.

Applying the Implicit Function Theorem, one obtains

$$\frac{dm}{dy} = \frac{\int_y^1 \frac{\partial V(m,y)}{\partial y}\, dH(z)}{\int_y^1 \left[\frac{\partial V(m,z)}{\partial m} - \frac{\partial V(m,y)}{\partial m}\right] dH(z)} ,$$

which is nonpositive if the denominator is negative.
This result is stated as a lemma.

Lemma IV-1:

If $(m,y)\ \varepsilon\ S_{12}$ and $\partial V(m,z)/\partial m > \partial V(m,y)/\partial m$ for some $z > y$ and nonnegative for all $(m,y)\varepsilon\ S_{12}$ then

$$\frac{dm}{dy} \leq 0 \quad .$$

Let m* be defined by

$$0 = -K + \alpha \int_0^1 \left[V(m^*,z) - V(m^*,0)\right] dH(z) , \qquad \text{(def V-2)}$$

and y* by

$$0 = -K + \alpha \int_{y^*}^1 \left[V(0,z) - V(0,y^*)\right] dH(z) . \qquad \text{(def V-3)}$$

Note that $y^* < 1$ for $K > 0$.

Uniqueness of y* is established by noting that the second term of the definition of y* is decreasing with respect to y*. Uniqueness of m* can be established if one assumes $\partial^2 V(m,z)/\partial m \partial z < 0$ for all $(m,z) \varepsilon S_{12}$. Since $\partial u_1/\partial m \geq \partial u_2/\partial m$, $u_1(m,y) = u_2(m,y)$ implies $u_1(n,y) \geq u_2(n,y)$ for $(n,y) \varepsilon S$ and $n \geq m$. Using the notations "$\succ$" and "$\sim$" to represent "is strictly preferred to" and "is indifferent to" respectively, I illustrate the results in Figure V-2.

Based on the results stated above, the state space, S, can be partitioned into three sets. One represents a subset of the state space with the property that Option I (stopping) is strictly preferred to Option II (research). Another represents the set S_{12}, and the third represents a subset of S with the property that Option II (research)

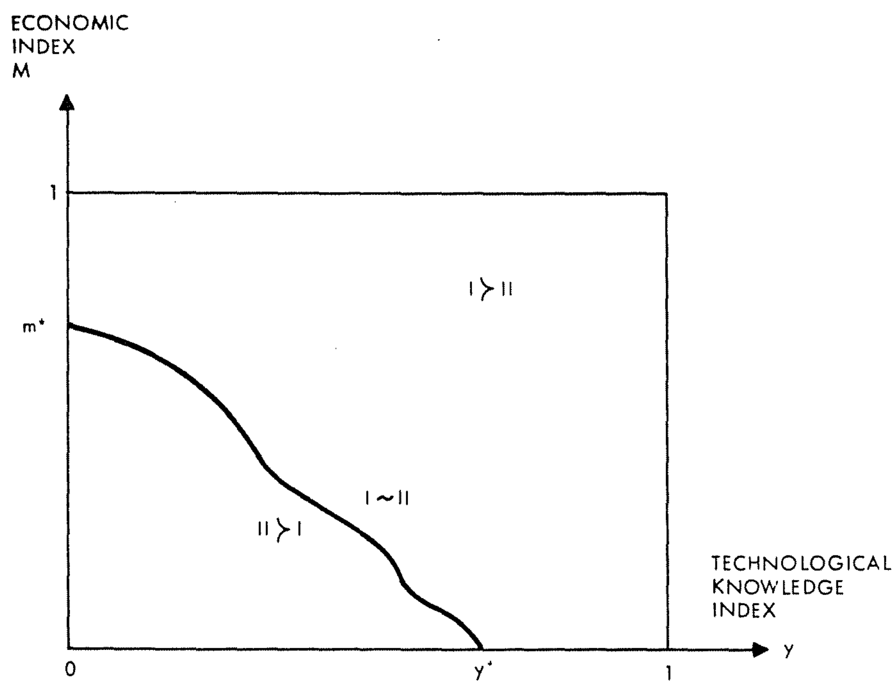

Figure V-2. Rank Ordering of Option I of Stopping vs. Option II of Research is strictly preferred to Option I (stopping). Thus Option II is an optimal choice only in the last two sets.

Next, Options I and III are rank ordered. Consider S_{13}.

$$u_1(m,y) = \alpha V(m,y) .$$

$$u_3(m,y) = -C + \alpha \int_m^1 V(n,y)\, dG(n|y) + \alpha V(m,y)\, G(m|y)$$

$$= -C + \alpha V(m,y) + \alpha \int_m^1 \left[V(n,y) - V(m,y) \right] dG(n|y).$$

Thus $(m,y) \varepsilon S_{13}$ implies that $u_1(m,y) = u_3(m,y)$, or

$$0 = -C + \alpha \int_m^1 \left[V(n,y) - V(m,y) \right] dG(n|y). \qquad \text{(def V-4)}$$

Impose the condition that Option I is indeed the optimal choice of the three options. This implies that

$$V(m,y) = R(m) + u_1(m,y)$$
$$= R(m) + \alpha V(m,y) \quad ,$$

or

$$V(m,y) = \frac{R(m)}{1 - \alpha} .$$

From previous results, $\partial u_1/\partial m > \partial u_3/\partial m$, and $\partial u_1/\partial m > \partial u_2/\partial m$. Thus, if Option I is the optimal choice for $(m,y) \varepsilon S$, Option I is also the optimal choice for $(n,y) \varepsilon S$ and $n \geq m$. Stated alternatively, $V(m,y) = R(m)/(1 - \alpha)$ implies $V(n,y) = R(n)/(1 - \alpha)$ for all $n \geq m$.

Define

$$S_{ij}^{\ell} = \Big\{ (m,y) : u_i(m,y) = u_j(m,y),$$

$$V(m,y) = R(m) + u_{\ell}(m,y), \ (m,y) \varepsilon S \Big\} . \qquad \text{(def V-5)}$$

Note that S_{ij}^{ℓ} may be empty.

For all $(m,y) \varepsilon S_{13}^{1}$, the following equation must be satisfied:

$$0 = -C + \alpha \int_m^1 \left[\frac{R(n)}{1-\alpha} - \frac{R(m)}{1-\alpha} \right] dG(n|y) . \qquad \text{(eq V-3)}$$

Note that the second term on the right-hand side is decreasing with respect to m. Thus for each $y \varepsilon [0,1]$, there is only one $m \varepsilon [0,1]$ such that the above equation is satisfied. Integrating by parts, I have

$$0 = -C + \frac{\alpha}{1-\alpha} \Big[R(1) - R(m) \Big] \qquad \text{(eq V-4)}$$

$$- \frac{\alpha}{1-\alpha} \int_m^1 R'(n) \, G(n|y) \, dn .$$

Note that the third term on the right-hand side is increasing with respect to y. Thus, for each $m \varepsilon [0,1]$, there is only one $y \varepsilon [0,1]$, such that the above equation is satisfied.

Applying the Implicit Function Theorem, one obtains

$$\frac{dm}{dy} = - \frac{-\frac{\alpha}{1-\alpha}\int_m^1 R'(n)\,\frac{\partial G(n|y)}{\partial y}\,dn}{-\frac{\alpha}{1-\alpha}R'(m) + \frac{\alpha}{1-\alpha}R'(m)\,G(m|y)}$$

$$= \frac{\int_m^1 R'(n)\,\frac{\partial G(n|y)}{\partial y}\,dn}{R'(m)\left[G(m|y) - 1\right]}$$

$$> 0 \;.$$

The result is stated as a lemma.

Lemma V-2: If $(m,y) \in S^1_{12}$, $dm/dy > 0$.

Since S^1_{12} represents the set of optimal stopping states, $dm/dy > 0$ implies that the development "switchpoint" is increasing with respect to the technological knowledge index. A similar result is obtained by Spulber [1977]. Let $\bar{m}$ be defined by

$$0 = -C + \alpha \int_{\bar{m}}^1 \left[\frac{R(n)}{1-\alpha} - \frac{R(\bar{m})}{1-\alpha}\right] dG(n|0)\;, \qquad \text{(def V-6)}$$

and $\bar{\bar{m}}$ by

$$0 = -C + \alpha \int_{\bar{\bar{m}}}^1 \left[\frac{R(n)}{1-\alpha} - \frac{R(\bar{\bar{m}})}{1-\alpha}\right] dG(n|1)\;. \qquad \text{(def V-7)}$$

Note that $\bar{m} < \bar{\bar{m}} < 1$ for $C > 0$.

Uniqueness of $\bar{m}$ and $\bar{\bar{m}}$ can be established by noting that the second term on the right-hand side of each equation is decreasing with respect to $\bar{m}$ and $\bar{\bar{m}}$ respectively. Since $\partial u_1/\partial m \geq \partial u_3/\partial m$, $u_1(m,y) = u_2(m,y)$ implies $u_1(n,y) \geq u_3(n,y)$ for all $(n,y) \varepsilon S$ and $n \geq m$. Figure V-3 serves to illustrate the result. It remains to show that $\bar{m} \geq m^*$.

Theorem V-2: $\bar{m} \geq m^*$

Proof: Assume the contrary i.e. $\bar{m} < m^*$

The assumption implies that there is a $\tilde{y} > 0$ such that

(1) $u_1(m^*,\tilde{y}) > u_2(m^*,\tilde{y})$, and

(2) for all $y \ \varepsilon[0,\tilde{y}]$, $V(m^*,y) = V(m^*,0)$.

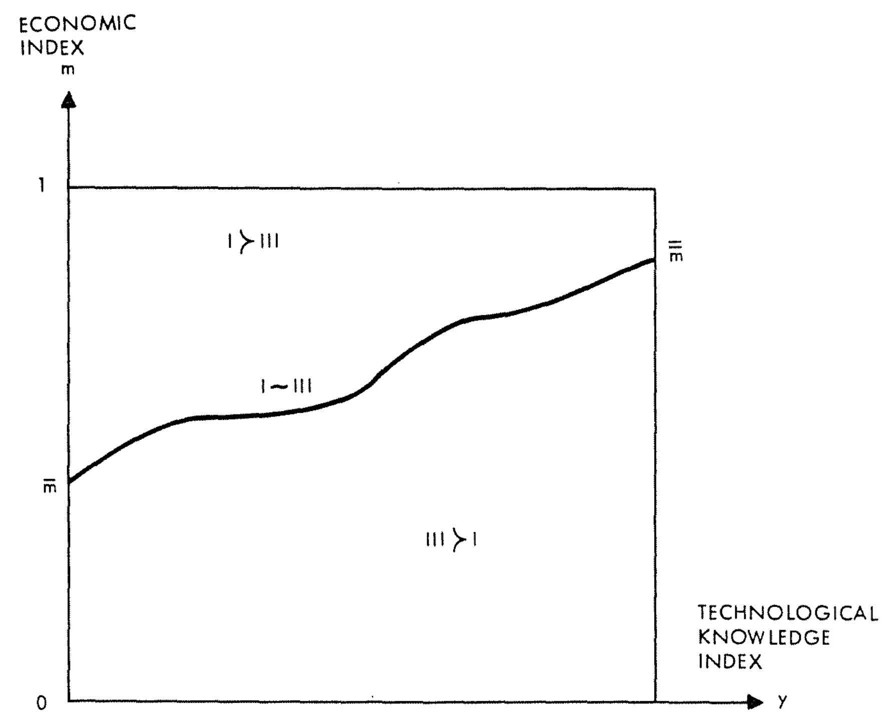

Figure V-3. Rank Ordering of Option I of Stopping and Option III of Development

From (1), I have

$$0 > -K + \alpha \int_{\tilde{y}}^{1} \left[V(m^*,z) - V(m^*,\tilde{y}) \right] dH(z).$$

But $u_1(m^*,0) = u_2(m^*,0)$ since $(m^*,0) \varepsilon S_{12}$.

Therefore, I have

$$\begin{aligned} 0 &= -K + \alpha \int_{o}^{1} \left[V(m^*,z) - V(m^*,0) \right] dH(z) \\ &= -K + \alpha \int_{\tilde{y}}^{1} \left[V(m^*,z) - V(m^*,0) \right] dH(z) \\ &\quad + \alpha \int_{o}^{\tilde{y}} \left[V(m^*,z) - V(m^*,0) \right] dH(z) \\ &= -K + \alpha \int_{\tilde{y}}^{1} \left[V(m^*,z) - V(m^*,0) \right] dH(z), \end{aligned}$$

which yields an obvious contradiction.

Therefore.

$\overline{m} \geq m^*$. Q.E.D.

<u>Corollary V-1:</u> $1 > m^*$.

Proof: Recall $1 > \overline{m}$. Q.E.D.

Figure V-4 combines the last two figures.

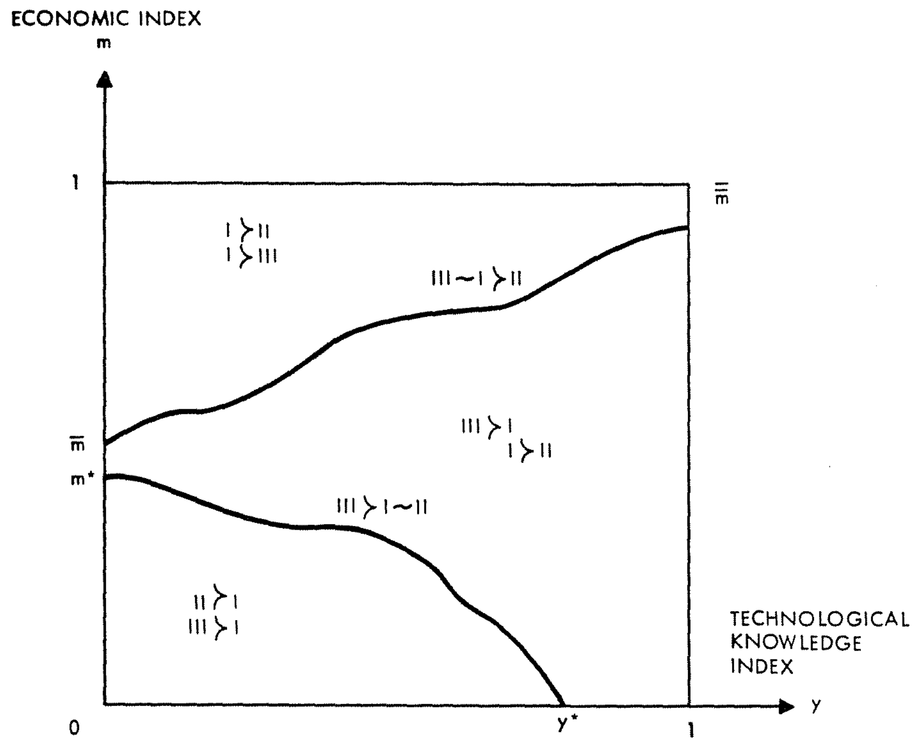

Figure V-4. Rank Ordering of Option I of Stopping, Option II of Research and Option III of Development.

Referring to Figure V-4, there is a set at the lower left hand corner that does not indicate an optimal choice of an option. Consider rank ordering of Options II and III in this set. Define the following two sets:

$$E = \{(m,y) \varepsilon\ S : H(y) = G(m|y)\} , \qquad \text{(def V-8)}$$

and

$$R = \{(m,y) \varepsilon\ S : u_2(m,y) = u_3(m,y) > u_1(m,y)\} . \qquad \text{(def V-9)}$$

Consider the set E. Recall that $H'(y) > 0$ and $\partial G(m|y)/\partial y < 0$. Thus E is a graph. For all $(m,y) \varepsilon$ E,

$$\frac{dm}{dy} = \frac{H'(y) - \frac{\partial G(m|y)}{\partial y}}{\frac{\partial G(m|y)}{\partial m}}$$

$$> 0 .$$

Note that $(0,0) \in E$.

Consider the set R. By definition,

$$u_2(m,y) = u_3(m,y) ,$$

or

$$- K + \alpha V(m,y) + \alpha \int_y^1 \Big[V(m,z) - V(m,y) \Big] dH(z) = -C + \alpha V(1,y)$$

$$- \alpha \int_m^1 \frac{\partial V(n,y)}{\partial n} G(n|y) \, dn . \qquad \text{(eq V-4)}$$

Assuming the Implicit Function Theorem requirements are satisified, the following is derived

$$\frac{dm}{dy} = \frac{\alpha \frac{\partial V(m,y)}{\partial y} - \alpha \int_y^1 \frac{\partial V(m,y)}{\partial y} dH(z) - \alpha \frac{\partial V(1,y)}{\partial y} + \alpha \int_m^1 \left[\frac{\partial V(n,y)}{\partial n} \frac{\partial G(n|y)}{\partial y} + \frac{\partial^2 V(n,y)}{\partial n \partial y} G(n|y) \right] dn}{\alpha \frac{\partial V(m,y)}{\partial m} + \alpha \int_y^1 \left[\frac{\partial V(m,z)}{\partial m} - \frac{\partial V(m,y)}{\partial m} \right] dH(z) - \alpha \frac{\partial V(m,y)}{\partial m} G(m|y)}$$

$$= \frac{\left\{ \alpha \frac{\partial V(m,y)}{\partial y} \Big[H(y) - G(m|y) \Big] \right\} - \left\{ \alpha \int_m^1 \frac{\partial V(n,y)}{\partial y} dG(n|y) - \alpha \int_m^1 \frac{\partial V(n,y)}{\partial n} \frac{\partial G(n|y)}{\partial y} dn \right\}}{\alpha \frac{\partial V(m,y)}{\partial m} \Big[H(y) - G(m|y) \Big] + \alpha \int_y^1 \frac{\partial V(m,z)}{\partial m} dH(z)} .$$

Suppose $R \cap E \neq \phi$.

Let $(m,y) \varepsilon R \cap E$. The above equation is reduced to

$$\frac{dm}{dy} = \frac{\left\{ \alpha \int_m^1 \frac{\partial V(n,y)}{\partial y} \, dG(n|y) - \alpha \int_m^1 \frac{\partial V(n,y)}{n} \frac{\partial G(n|y)}{\partial y} \, dn \right\}}{\alpha \int_y^1 \frac{\partial V(m,z)}{\partial m} \, dH(z)}$$

$$> 0 \, .$$

Hence, at point(s) of intersection, R will cut E with a positive slope. Furthermore, if $\partial V(m,y)/\partial y$, $\partial V(m,y)/\partial m$ and $\partial G(n|y)/\partial y$ are all bounded between $-\infty$ and $+\infty$, then $(m,y)\varepsilon R$ and $H(y) - G(m|y) > 0$ imply that dm/dy is bounded between $-\infty$ and $+\infty$. A similar conclusion may be drawn for $H(y) - G(m|y) < 0$. With these additional assumptions one may conclude that $R \cap E$ has only one element. See Figure V-5.

The Reswitching Property of Research and Development

Referring to Figure V-6, an immediate observation is the possibility of path A. Suppose the initial state is the pair (y_1,m_1). The optimal choice is to do research and improve the level of technological knowledge. Suppose the observed improvement is y_2. The state in the second period is given by the pair (y_2,m_1). The optimal choice switches to development. With some luck, the economic index is improved to m_2. The new state is (y_2,m_2) which switches back to the region where research is the optimal choice. Thus, one might observe any positive number of switches between research and development. Suppose technological knowledge is limited in the sense that if the state enters into a point

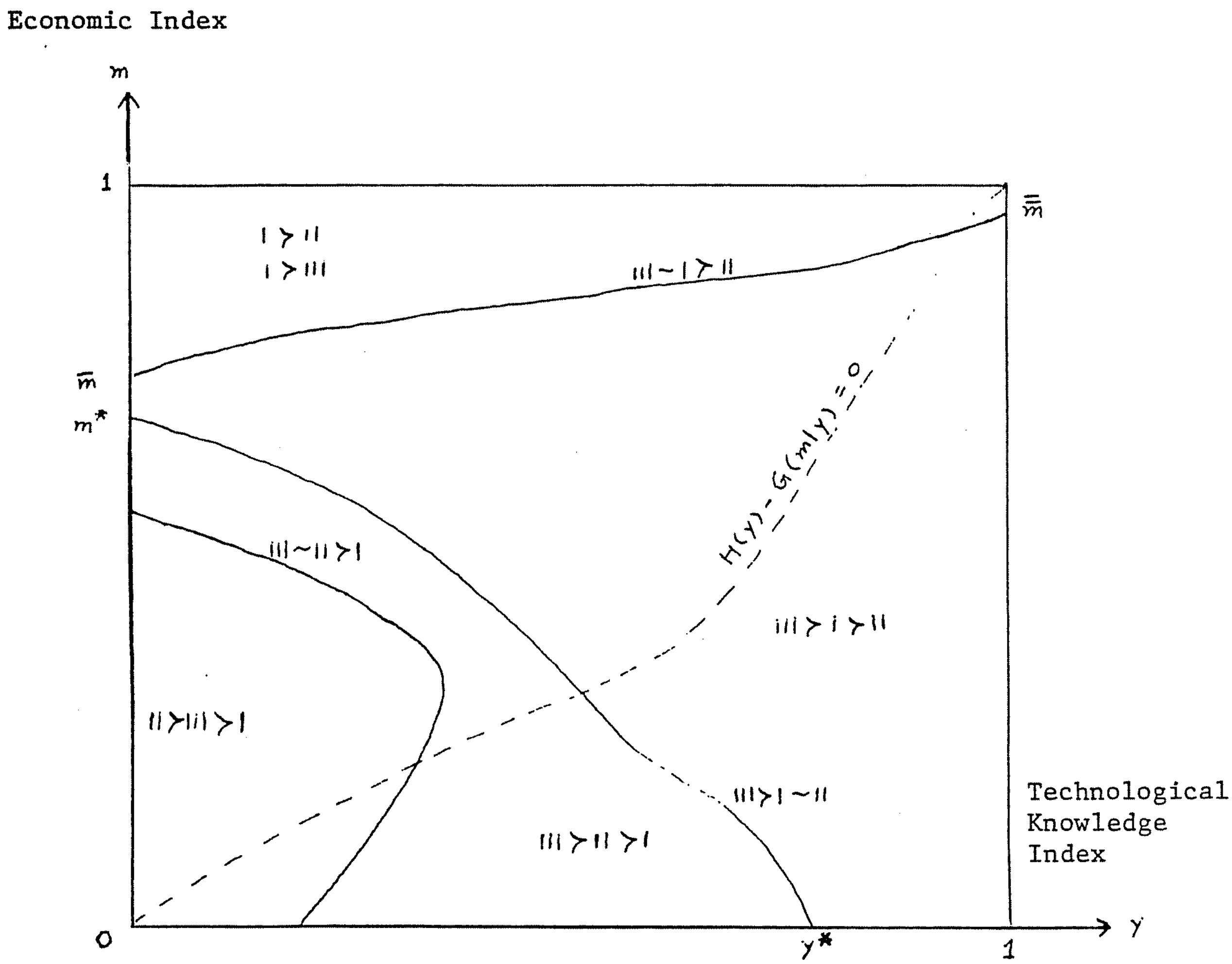

Figure V-5: Rank Ordering of Option I of Stopping, Option II of Research and Option III of Development--a final analysis.

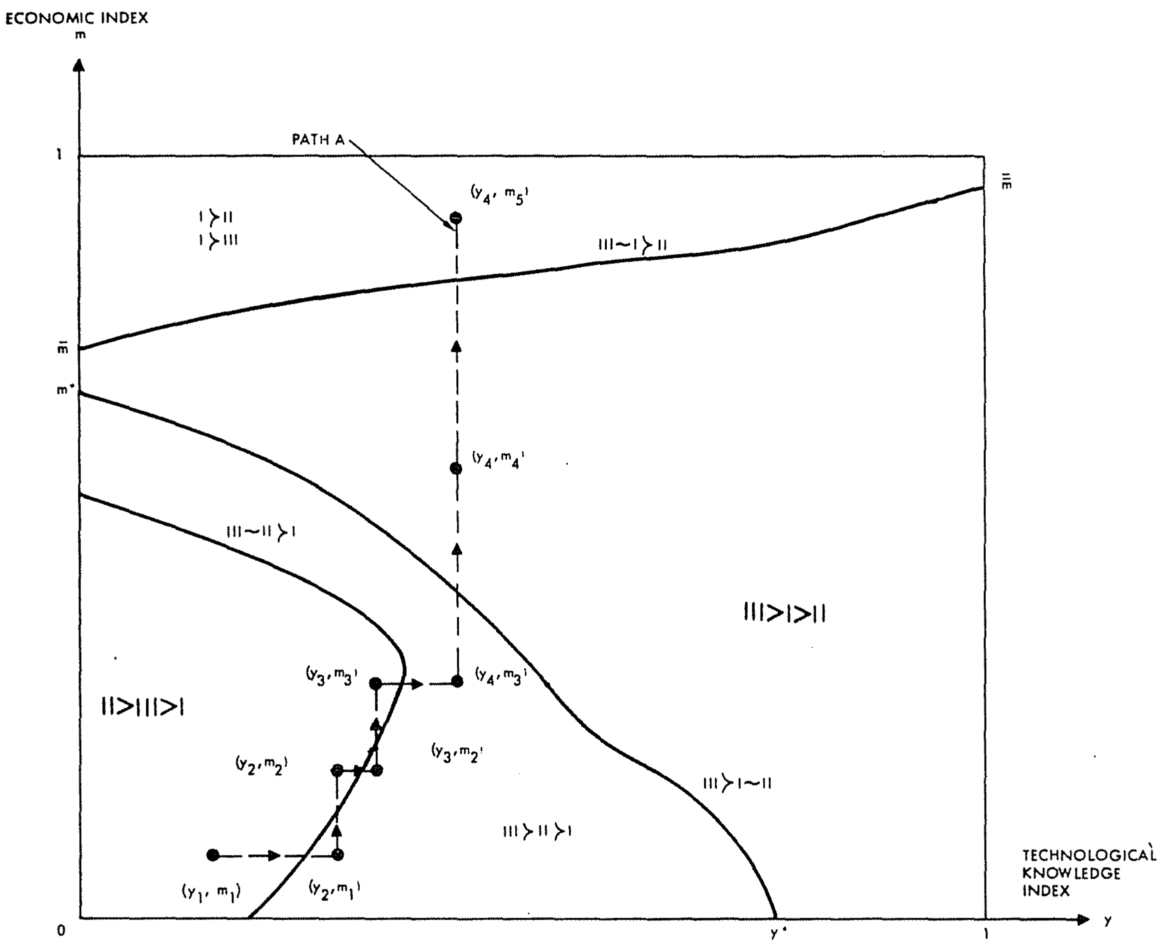

Figure V-6. The Reswitching Property of Research and Development say (y_4,m_3) then for all $m \geq m_3$, the optimal choice at a state (y_4,m) excludes research. In that event, continual development will eventually move the state into an absorbing region, say at point (y_4, m_5) where stopping is optimal. A static state of the world may then be observed.

Section III

Comparative Statics

Two tasks remain to be shown in this section. First, further properties of optimal choice regions will be characterized. Second, comparative statics results will be investigated.

Consider the set S_{13}. It is informative to know the conditions under which S_{13} is empty. Recall the definition of $\bar{\bar{m}}$, i.e.

$$0 = -C + \frac{\alpha}{1-\alpha} \int_{\bar{\bar{m}}}^{1} [R(n) - R(\bar{\bar{m}})] \, dG(n|1).$$

Note that

$$\frac{\partial}{\partial \bar{\bar{m}}} \left\{ \frac{\alpha}{1-\alpha} \int_{\bar{\bar{m}}}^{1} [R(n) - R(\bar{\bar{m}})] \, dG(n|1) \right\} = -\frac{\alpha}{1-\alpha} \int_{\bar{\bar{m}}}^{1} R'(\bar{\bar{m}}) \, dG(n|1) < 0 \, ,$$

i.e. the second term on the right-hand side of the equation is decreasing with respect to an increase in $\bar{\bar{m}}$. One can solve C in terms of $\bar{\bar{m}}$, i.e. $C = C(\bar{\bar{m}})$. Alternatively, one can solve $\bar{\bar{m}}$ in terms of C, i.e. $\bar{\bar{m}} = \bar{\bar{m}}(C)$. Define $C_{max} = C(0)$. Therefore,

$$0 = -C_{max} + \frac{\alpha}{1-\alpha} \int_{0}^{1} [R(n) - R(0)] \, dG(n|1). \qquad \text{(def V-9)}$$

Thus, for all $C > C_{max}$, no solution exists for $\bar{\bar{m}}$. This implies that the set S_{13} is empty. By Theorem V-2, $\bar{m} \geq m^*$. Since $\bar{\bar{m}} > \bar{m}$, S_{12} is also empty. The result is stated as a theorem.

Theorem V-3: (i) $C > C_{max}$ implies that $S_{13} = \phi$,

(ii) $S_{13} = \phi$ implies that $S_{12} = \phi$.

Theorem V-3 (ii) implies that if development is a poor investment at all points in the search space, so, too, are research activities. The purpose of research is to improve the likelihood of economic gains in development. If development is not profitable at all levels of the technological knowledge index, success in research activities will be fruitless since no development work will be carried out to realize economic returns. Thus, research activities are not pursued either. The following will show that the converse is not necessarily true.

Suppose $0 < C < C_{max}$. One can solve for $\bar{\bar{m}} \varepsilon (0, 1)$, i.e. $S_{13} \neq \phi$. Consider the set S_{12}. Recall the definition of m^*, i.e.

$$0 = -K + \alpha \int_0^1 [V(m^*, z) - V(m^*, 0)] \, dH(z) .$$

Note that

$$\frac{\partial}{\partial m^*} \left\{ \alpha \int_0^1 [V(m^*, z) - V(m^*, 0)] \, dH(z) \right\} = \alpha \int_0^1 \left[\frac{\partial V(m^*, z)}{\partial m^*} - \frac{\partial V(m^*, 0)}{\partial m^*} \right] dH(z)$$

$$< 0 \; ;$$

i.e. the second term on the right-hand side of the above equation is decreasing with respect to an increase in m*. One can solve K in terms of m*, i.e. K = K(m*), or alternatively, m* in terms of K, i.e. m* = m*(K). Let K_{max} = K(0). Therefore,

$$0 = -K_{max} + \alpha \int_0^1 [V(0, z) - V(0, 0)]\, dH(z). \qquad \text{(def V-10)}$$

For all $K > K_{max}$, no solution exists for m*. This implies that S_{12} is empty. Recall the definitions of $\bar{\bar{m}}$ and $\bar{m}$. It is obvious that $\frac{d\bar{\bar{m}}}{dK} = 0$ and $\frac{d\bar{m}}{dK} = 0$. A change in research costs does not affect development. Thus, the fact that S_{12} is empty does not imply S_{13} is empty. Even though research is a poor investment development may still be profitable.

Comparative statics results on C_{max} and K_{max} are stated in the following theorem.

<u>Theorem V-4</u>: (i) $\frac{dC_{max}}{d\alpha} > 0$,

(ii) If $\frac{\partial V(0, z)}{\partial\alpha\partial z} \geq 0$ and is strictly positive for some $z \in [0,1]$,

then $\frac{dK_{max}}{d\alpha} > 0$.

Proof: Applying the Implicit Function Theorem on the definitions C_{max} and K_{max}, one obtains

$$\frac{dC_{max}}{d\alpha} = \frac{1}{(1-\alpha)^2} \int_0^1 [R(n) - R(0)] \; dG(n|1)$$

$$> 0,$$

and

$$\frac{dK_{max}}{d\alpha} = \int_0^1 [V(0, z) - V(0, 0] \; dH(z) + \alpha \int_0^1 \left[\frac{\partial V(0, z)}{\partial \alpha} - \frac{\partial V(0, 0)}{\partial \alpha}\right] dH(z)$$

$$> 0 \; .$$

Interpretation of the results is straightforward. If one discounts future returns less, one would discontinue research (development) only at higher research (development) costs.

Other comparative statics results are stated in the following three theorems.

<u>Theorem V-5</u>: (i) $\frac{d\bar{m}}{d\alpha} > 0$,

(ii) $\frac{d\bar{\bar{m}}}{d\alpha} > 0$,

If $\frac{\partial^2 V(m, y)}{\partial \alpha \partial y} \geq 0$ and strictly positive for some $(m, y) \varepsilon S$

(iii) $\frac{dm^*}{d\alpha} > 0$,

(iv) $\frac{dy^*}{d\alpha} > 0$.

Proof: To prove (i) and (ii), recall the definition of S_{13}, i.e.

$$0 = -C + \frac{\alpha}{1-\alpha}\int_m^1 [R(n) - R(m)]\, dG(n|y)$$

$$= -C + \frac{\alpha}{1-\alpha}[R(1) - R(m)] - \frac{\alpha}{1-\alpha}\int_m^1 R'(n)\, G(n|y)\, dn \,.$$

Applying the Implicit Function Theorem, one obtains

$$\frac{\partial m}{\partial \alpha} = \frac{\dfrac{1}{(1-\alpha)}\displaystyle\int_m^1 [R(n) - R(m)]\, dG(n|y)}{\alpha \displaystyle\int_m^1 R'(m)\, dG(n|y)}$$

$$> 0 \,,$$

and

$$\frac{\partial y}{\partial \alpha} = \frac{-\dfrac{1}{(1-\alpha)^2}\displaystyle\int_m^1 [R(n) - R(m)]\, dG(n|y)}{-\dfrac{\alpha}{1-\alpha}\displaystyle\int_m^1 R'(n)\, \frac{\partial G(n|y)}{\partial y}\, dn}$$

$$< 0 \,.$$

To prove (iii) and (iv), recall the definition of S_{12}, i.e.

$$0 = -K + \alpha \int_y^1 [V(m, z) - V(m, y)]\, dH(z) \,.$$

Applying the Implicit Function Theorem, one obtains

$$\frac{\partial m}{\partial \alpha} = - \frac{\int_y^1 [V(m, z) - V(m, y)]\, dH(z) + \alpha \int_y^1 \left[\frac{\partial V(m, z)}{\partial \alpha} - \frac{\partial V(m, y)}{\partial \alpha}\right] dH(z)}{\alpha \int_y^1 \left[\frac{\partial V(m, z)}{\partial m} - \frac{\partial V(m, y)}{\partial m}\right] dH(z)}$$

$$> 0 ,$$

and

$$\frac{\partial y}{\partial \alpha} = - \frac{\int_y^1 [V(m, z) - V(m, y)]\, dH(z) + \alpha \int_y^1 \left[\frac{\partial V(m, z)}{\partial \alpha} - \frac{\partial V(m, y)}{\partial \alpha}\right] dH(z)}{- \alpha \int_y^1 \frac{\partial V(m, y)}{\partial y}\, dH(z)}$$

$$> 0 .$$

Q.E.D.

Since the decision to engage in research and development is aimed for future returns, the more one values future returns the more one persists in the research and development mode. However, it is not clear whether one will persist more in research or in development.

The next two theorems will consider the effect of a parametric change in the research and development costs. First, consider an increase in the research cost.

<u>Theorem V-6</u>: (i) $\frac{d\bar{m}}{dK} = 0$, and

(ii) $\frac{d\bar{\bar{m}}}{dK} = 0$.

If $\frac{\partial^2 V(m, y)}{\partial K \partial y} \leq 0$ for all $(m, y) \varepsilon S$, then

(iii) $\frac{dm^*}{dK} < 0$, and

(iv) $\frac{dy^*}{dK} < 0$.

Proof: By the definition of S_{13}, one immediately obtains results (i) and (ii). Applying the Implicit Function Theorem on the definition of S_{12}, one obtains

$$\text{(iii)} \quad \frac{dm}{dK} = - \frac{-1 + \alpha \int_y^1 \left[\frac{\partial V(m, z)}{\partial K} - \frac{\partial V(m, y)}{\partial K} \right] dH(z)}{\alpha \int_y^1 \left[\frac{\partial V(m, z)}{\partial m} - \frac{\partial V(m, y)}{\partial m} \right] dH(z)}$$

$$< 0 ,$$

and

$$\text{(iv)} \quad \frac{dy}{dK} = - \frac{-1 + \alpha \int_y^1 \left[\frac{\partial V(m,z)}{\partial K} - \frac{\partial V(m, y)}{\partial K} \right] dH(z)}{-\alpha \int_y^1 \frac{\partial V(m, y)}{\partial y} dH(z)}$$

$$< 0 .$$

Q.E.D.

Holding everything else constant, an increase in the research cost will not affect the overall persistency in the research and development mode. However, one will choose to do development work more often than to do research. This result is stronger than a previous result of a parametric change of the discount factor in the sense that it indicates a clear trade off between research and development. The next theorem will show that an increase in the development cost will again lead to ambiguous trade off between research and development even though the overall picture points towards less persistency in the research and development mode.

<u>Theorem V-7</u>: (i) $\frac{d\bar{m}}{dC} < 0$, and

(ii) $\frac{d\bar{\bar{m}}}{dC} < 0$.

If $\frac{\partial^2 V(m, y)}{\partial C \partial y} \leq 0$ and strictly negatively for some $(m, y) \varepsilon S$, then

(iii) $\frac{dm^*}{dC} < 0$, and

(iv) $\frac{dy^*}{dC} < 0$.

Proof: Applying the Implicit Function Theorem on the definition of S_{13}, one obtains

$$\frac{\partial m}{\partial C} = \frac{1}{-\frac{\alpha}{1-\alpha}\int_m^1 R'(m)\, dG(n|y)}$$

$$< 0,$$

and

$$\frac{\partial y}{\partial C} = \frac{1}{-\frac{\alpha}{1-\alpha}\int_m^1 R'(n)\,\frac{\partial G(n\ y)}{\partial y}\,dn}$$

$$> 0 .$$

Applying the Implicit Function Theorem to the definition of S_{12}, one obtains results (iii) and (iv):

$$\frac{\partial m}{\partial C} = -\frac{\alpha \int_y^1 \left[\frac{\partial V(m,\ z)}{\partial C} - \frac{\partial V(m,\ y)}{\partial C}\right] dH(z)}{\alpha \int_y^1 \left[\frac{\partial V(m,\ z)}{\partial m} - \frac{\partial V(m,\ y)}{\partial m}\right] dH(z)}$$

$$< 0 ,$$

and

$$\frac{\partial y}{\partial C} = -\frac{\alpha \int_y^1 \left[\frac{\partial V(m,\ z)}{\partial C} - \frac{\partial V(m,\ y)}{\partial C}\right] dH(z)}{-\alpha \int_y^1 \frac{\partial V(m,\ y)}{\partial y}\, dH(z)}$$

$$< 0 .$$

Not only does an increase in development costs lower the profitability of development work, it also affects the financial picture of research.

Summary and Conclusion

An analysis of a sequential research and development model has been accomplished in this paper. By allowing a two dimensional search space, I have demonstrated a "reswitching" phenomenon of research and development. The decision to do developmental work depends on the direct economic gains from successful development and its related costs, which are incurred irrespective of success or failure. The decision to perform research hinges upon the indirect economic gains from successful research through which prospect for future development success is improved. Thus economic factors specific to development decision (e.g., development cost) will affect both research and development decisions, but those specific to research decision (e.g., research cost) will not affect development decisions. Even with this unilateral relationship between research and development, a "reswitching" property of the model emerges. Success in development may bring research back as the next optimal choice of action. However, research is never an absorbing state. Eventually, a state may be entered such that no research is undertaken. Note just prior to optimal stopping, the state is always for development and not for research. The model rejects the idea of research for the sake of research. It is the final application (development) of research results that counts.

Results from the model also lead to the conclusion that the optimal choice to do development work is consistent with the presence of a high prospect of research success. The decision to develop or to do research is an economic decision. It depends on expected cost and benefit calculations. Although there is a great opportunity for further research, research may still be temporarily abandoned because of immediately available economic gains from development. Indeed, development may be so successful that the economic environment may be changed drastically enough to render further research uneconomical. An obvious inference from this model is that, when non-sequential search strategy is allowed, research and development may occur simultaneously within some regions in the search space.

Last of all, the model draws implications as to the importance of the "right" choice of discount rate for research and development. The choice will affect the overall preference for research and development vs. stopping. However, the decision to do research or to develop remains to be determined by the choice of discount rate and the relevant economic factors such as costs and benefits of research and development.

APPENDIX

Theorem V-1: Given the assumptions of the sequential research and development model, there exists a unique, continuous, bounded solution to the functional equation V-1.

Proof: The proof is a straightforward application of Denardo's Theorem. Two prerequisites are necessary, namely Monotonicity and Contraction Assumptions.

Monotonicity

Let

$$h(m,y,V) = R(m) + \max \{u_1(m, y), u_2(m, y), u_3(m, y)\}$$

$$= R(m) + \max \Big\{ \alpha V(m, y), -K + \alpha \int_y^1 V(m, z)\, dH(z) + \alpha V(m, y)\, H(y), -C + \alpha \int_m^1 V(n, z)\, dG(n|y) + \alpha V(m, y)\, G(n|y) \Big\} .$$

Let

$$h(m,y,U) = R(m) + \max \Big\{ \alpha U(m, y), -K + \alpha \int_y^1 U(m, z)\, dH(z) + \alpha U(m, y)\, H(y), -C + \alpha \int_m^1 U(n, z)\, dG(n|y) + \alpha U(m, y)\, G(m|y) \Big\} .$$

If $V(m, y) \geq U(m, y)$ for all $(m, y) \varepsilon S \equiv [0, 1] \times [0, 1]$, then

(i) $\alpha V(m, y) \geq \alpha U(m, y)$,

(ii) $$-K + \alpha \int_y^1 V(m, z)\, dH(z) + \alpha V(m, y)\, H(y)$$

$$\geq -K + \alpha \int_y^1 U(m, z)\, dH(z) + \alpha U(m, y)\, H(y), \text{ and}$$

(iii) $$-C + \alpha \int_m^1 V(n, z)\, dG(n|y) + \alpha V(m, y)\, G(m|y)$$

$$\geq -C + \alpha \int_m^1 U(n, z)\, dG(n|y) + \alpha U(m, y)\, G(m|y).$$

Therefore,

$h(m,y,V) \geq h(m,y,U)$ for all $(m, y) \varepsilon S$.

Contraction

Sufficient conditions that $h(m,y,V)$ is a contraction mapping are:

(i) $$|\alpha V(m, y) - \alpha U(m, y)| = \alpha |V(m, y) - U(m, y)|$$

$$\leq \alpha \sup_{(n,y)\varepsilon S} |V(m, y) - U(m, y)| ,$$

(ii) $$|\alpha \int_y^1 [V(m, z) - U(m, z)]\, dH(z) + \alpha H(y)[V(m, y) - U(m, y]|$$

$$\leq \alpha \int_y^1 |V(m, z) - U(m, z)|\, dH(z)$$

$$+ \alpha H(y)\, |V(m, y) - U(m, y)|$$

$$\leq \alpha \int_y^1 \sup_{(m,y)\varepsilon S} |V(m, y) - U(m, y)| \; dH(z)$$

$$+ \alpha H(y) \sup_{(m,y)\varepsilon S} |V(m, y) - U(m, y)|$$

$$= \alpha \sup_{(m,y)\varepsilon S} |V(m, y) - U(m, y)| \text{ , and}$$

(iii) $$|\alpha \int_m^1 [V(n, y) - U(n, y)] \; dG(n|y)$$

$$- \alpha G(m|y) \; [V(m, y) - U(m, y)]|$$

$$\leq \alpha \int_m^1 |V(n, y) - U(n, y)| \; dG(n|y)$$

$$+ \alpha G(m|y) \; |V(m, y) - U(m, y)|$$

$$\leq \alpha \int_m^1 \sup_{(m,y)\varepsilon S} |V(m, y) - U(m, y)| \; dG(n|y)$$

$$+ \alpha G(m|y) \sup_{(m,y)\varepsilon S} |V(n, y) - U(m, y)|$$

$$= \alpha \sup_{(m,y)\varepsilon S} |V(m, y) - U(m, y)| \text{ .} \qquad \text{Q.E.D.}$$

Chapter V: References

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Chapter VI: Conclusion

Several objectives have been accomplished in this thesis. First, on the supply side, technology opportunities and R&D costs are considered. Technology opportunities are conceptualized as a probablity distribution of technology level which can be realized by sampling. R&D as a tool for sampling is costly. For a given R&D cost, the degree of technology improvement is uncertain, but an expected improvement can be defined. There is no a priori reason that technology opportunities and R&D cost structure are the same for different industries. Chances are they are not. Thus, different industries facing the same market situations may have different rates of technology change.

On the demand side, R&D is not treated as final consumption. Its potential for lowering production costs generates its value. In economic jargon, the demand for R&D is a derived demand. By integrating R&D and production decisions, market opportunities and market power are assimilated in the thesis. Economic determinants of R&D decisions are clarified.

Second, a Reservation Technology Level (RTL) concept is introduced. A RTL is one above which R&D is stopped and below which R&D is continued. It is argued that the R&D decision is based on costs and benefits. The possibility of further technical advance is not a sufficient condition for R&D. In fact, it is shown that the RTL is strictly less than the highest achievable technology level.

Furthermore, the concept of RTL is useful for a welfare ranking of market structures. Since the optimal R&D intensity is determined by marginal costs and benefits of R&D, there is no a priori reason to believe a single market structure will be most conducive for R&D throughout the history of an industry. However, it is shown that the RTL of a social decision maker who maximizes social surplus is higher than that of a monopolist. Sufficient conditions also exist for noncooperative duopolists to have a higher RTL than a monopolist.

Thirdly, uncertainty remains as a theme throughout the thesis. An extensive treatment of uncertainty is exemplified by a discussion of product and process innovations. It is argued that the two processes are theoretically the same other than the level of uncertainty involved. Another contribution to the study of uncertainty of R&D is the introduction of a two dimensional search space. It is postulated that a technology is characterized by two indices, namely, a technological knowledge index and an economic index. The two indices are related. Consider a probability distribution over the two indices. It is postulated that the conditional probability of achieving a better than current economic index is greater for higher levels of the technological knowledge index. Searches for a higher technological knowledge index and a higher economic index are defined as research and development, respectively. Characterizations of an optimal sequential strategy yield a disequilibrium-like phenomenon, defined as the Reswitching Property of R&D. An optimal path may lead one to do some research and some development and then some more research, etc. Thus, those economists who are interested in disequilibrium studies of R&D will have to characterize what they mean by disequilibrium dynamics. The moving back and forth from research to development and vice versa need not be a disequilibrium phenomenon per se.

Chapter VI: Footnote

1. The term was used by Nelson and Winter (1976). They postulate a evolutionary theory of innovation, a concept closely related to Darwinism. Selection force may be loosely stated as a survival test.

Chapter VI: Reference

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