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Risk Management in the Financial Services Sector—Applicability and Performance of VaR Models in Pakistan

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I. INTRODUCTION

Financial services sector has become a major driver of economic growth in the developing countries through innovation in response to the forces of globalisation and technology. Sound risk management practices by financial institution are critical to the stability of the institutions and to the sustainability of economic growth. Therefore, measurement of market risk is important to all market participants for devising risk management strategies. Value-at-Risk (VaR) is the most widely used measure of market risk, which is defined as the maximum possible loss to the value of financial assets with a given probability over a certain time horizon. However, the task of implementing the VaR approach still remains a challenge as the empirical return distributions are found to be fat tailed and skewed in contrast to the normal distribution as assumed in the theoretical models. An extensive literature in finance (e.g., Nassim Taleb's *The Black Swan*) underscores the importance of rare events in asset pricing and portfolio choice. These rare events may materialise in the shape of a large positive or negative investment returns, a stock market crash, major defaults, or the collapse of risky asset prices.

In order to address the problems of heavy tails, VaR measures based on the *Extreme Value Theory* (EVT) have been developed which allows us to model the tails of distributions, and to estimate the probabilities of the extreme movements that can be expected in financial markets. The basic idea behind EVT is that in applications where one is concerned about risk of extreme loss, it is more appropriate to separately model the tails of the return distribution. At a more fundamental level, the issue is whether or not the return distributions remain stable over time. EVT's usage to model risk, however, still assumes that the probability distribution parameters extracted from the historical data are stable.

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Application of the EVT to the developing countries' financial markets poses special challenges. In particular, these offer only limited data histories while EVT uses extreme observations which typical are rather rare. In addition, the return generating processes may not be stable due to the evolving institutional and regulatory environment. Global Financial Crisis of 2007-09 (GFC) has raised further questions as to the efficacy of the financial models in risk management. The backdrop of the GFC, however, also provides us with an historical experiment to examine the tails of stock return distributions. During the GFC period stock market volatility increased many folds and large swings in the stock prices were observed with an unprecedented frequency, thus, providing us with a rich data set for applying EVT. Pakistan offers an instructive case study since due to its turbulent political and economic environment its equity market has experienced very high volatility and incidence of extreme returns, thereby, providing a richer dataset. Yet the country has one of the oldest stock markets among the developing countries with well-established institutions and regulatory structure.

Up till now only a few recent studies have examined the impact of GFC on the stock market behaviour. Among these, Uppal and Mangla (2013) compare the tail distributions of stock returns for the pre- and post-Global Financial Crises periods for ten countries, and find that the distribution tails had different characteristics in the two periods. Uppal (2013) tests the EVT-VaR model and reports that the model does not describe the tail-risk in the US and the UK market well during the crisis period, though it performed better in case of emerging markets. There have been other studies using EVT following previous stock markets crashes and periods of high volatility in the developed as well as the emerging markets. For example, Gencay and Selcuk (2004) employ VaR models using EVT to a sample of emerging markets after the Asian financial crisis of 1998. Onour (2010) presents estimation of extreme risk in three stock markets in the Gulf Cooperation Council (GCC) countries, Saudi Arabia, Kuwait, and United Arab Emirates, in addition to the S& P 500 stock index, using the Generalised Pareto Distribution (GPD). Djakovic, Andjelic and Borocki (2011) investigates the performance of the extreme value theory for four emerging markets, the Serbian, Croatian, Slovenian and Hungarian stock indices. Bhattacharyya and Ritolia (2008) suggest a Value-at-Risk (VaR) measure for the Indian stock market based on the Extreme Value Theory for determining margin requirements for traders. In the context of Pakistan, Iqbal, Azher, and Ijaz (2010) compare the accuracy of six different VaR models using Karachi Stock Exchange 100 Index (KSE 100) over the 1992–2008 period. They find that that VaR measures are more accurate when return volatility is estimated by GARCH (1,1) model. Qayyum and Nawaz (2010) compare two methods of applying the extreme value theory to compute VaR using return series for KSE 100 over the 1993-2009 period. Nawaz and Afzal (2011) compare the margin requirements based on KSE 100 index under different margin systems, and conclude that a system based on VaR is most effective.

Compared to the earlier studies, this study examines the applicability and performance of the risk models over the Global Financial Crisis period, employing back-testing procedures for the equity market of Pakistan, the Karachi Stock Exchange (KSE). Various techniques of measuring market risk based on the VaR and EVT approaches are evaluated for the tail of the conditional distribution of KSE index return series over the period January 2001–June 2012. In order to study the performance of the risk models over the GFC, we explicitly incorporate a GFC-dummy variable in our econometric model, and also separately analyse the pre- and post-GFC periods. Motivated by the approach suggested by McNeil and Frey (2000), we model the conditional quantiles of the loss distribution under the dynamic framework. Our back-testing results show that the procedure based on the EVT is applicable for modelling market risk, and seems to perform better than methods which ignore the heavy tails of the innovations or the hetero-skadasticity in returns. First part of our work considered the whole sample period, whereas in second part we modelled the dynamic VaR measure for pre-crisis and post crisis periods separately.

Our study addresses, firstly, the issue whether EVT can help in measuring and managing tail risk in the emerging markets. Secondly, it addresses the issue of the stability of parameters. Even if the EVT does adequately describe the extreme return distribution, its applicability would be much restricted if the parameters of the distribution were not stable. Our study finds that the KSE is characterised by tail-distributions well described by EVT but with significantly different characteristics in different periods, and suggests that the VaR models based on EVT and dynamic volatility may be helpful in assessing market risk in the emerging markets.

II. EVT MODELS OF DISTRIBUTION TAILS

Value at Risk (VaR) is a high quantile (typically the 95th or 99th percentile) of the distribution of negative returns and provides an upper bound on losses with a specified probability. However, classical VaR measures based on the assumption of normal distribution of the stock returns underestimate risk as the actual return distributions exhibit heavier tails. One alternative to deal with the non-normality of the financial asset distributions has been to employ historical simulation methodology which does not make any distributional assumptions, and the risk measures are calculated directly from the past observed returns. However, the historical approach sill assumes that the distribution of past stock prices will be stable in the future.

Another approach is to use the Extreme Value Theory to construct models which account for such thick tails as are empirically observed. According to EVT, the form of the distribution of extreme returns is precisely known and independent of the process generating returns [Fisher and Tippet (1928); Also see Longin (1996); Longin and Solnik (2001) and Chou (2005); and Diebold, *et al.* (2000)] for a note of caution. The family of extreme value distributions can be presented under a single parameterisation, known as the Generalised Extreme Value (GEV) distribution [Embrechts, *et al*. (1997)].

There are two ways of modelling extremes of a stochastic variable. One approach is to subdivide the sample into m blocks and then obtain the maximum from each block, the *block maxima method*. The distribution of block maxima can be modelled by fitting the GEV to the set of block maxima. An alternative approach takes large values of the sample which exceed a certain threshold u, the peak-over-threshold (POT) approach. The distribution function of these *exceedances* is then obtained employing fat-tailed distributions models such as the Generalised Pareto Distribution (GPD). However, the POT approach is the preferred approach in modelling financial time series.

In this paper, we use a semi-parametric approach based on the Hill estimator [Hill (1975)] for the tail index. We assume that the distribution function underlying the data satisfies, for some positive constant *C*,

$$1 - F(x) \sim \left(\frac{x}{C}\right)^{-1/\gamma}, \text{as } x \to \infty \quad \text{with} \quad \gamma > 0.$$

Weissman (1978) proposed the following estimator of a high quantile (i.e., the Value-at-Risk):

$$\widehat{z_q} = X_{n-k+1}\left(\frac{k}{n(1-q)}\right)^{\hat{\xi}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

Where $X_{n-k+1}$is the *k-th* top order statistic of the *n* number of observations, $\widehat{\xi}$ be any consistent estimator for ξ and $\widehat{z_q}$ stands for quantile function at a given confidence level *q*.

Among various choices, for heavy tails, the classical semi-parametric Hill tail index estimator used in Equation (1) is given by the functional expression

$$\hat{\xi} = \frac{1}{k}\sum_{i=1}^{k} ln\left(\frac{X_{n-i+1}}{X_{n-k}}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

The important step in this procedure is to determine the threshold (i.e., X_{n-k+1}) for identifying the tail region. It involves a trade-off: a very high threshold level may provide too few points for estimation, while a low threshold level may render a poor approximation. Several researchers, [e.g., McNeil (1997, 1999)] suggest employing a high enough percentile as the threshold. We consider 95th percentile as the threshold, as is typically recommended.

III. DYNAMICS OF VOLATILITY AND CONDITIONAL MEAN

Although EVT is an appropriate approach for modelling the tail behaviour of stock returns, the assumption of constant volatility is contradicted by the well documented phenomenon of volatility clustering i.e., large changes in assets values are followed by large changes in either direction. Hence, a VaR calculated in a period of relative calm may seriously underestimate risk in a period of higher volatility.[1] The time varying volatility was first modelled as an ARCH (*q*) process [Bollerslev, *et al.* (1992)] which relates time *t* volatility to past squared returns up to *q* lags. The ARCH (*q*) model was expanded to include dependencies up to *p* lags of the past volatility. The expanded models, GARCH (*p*,*q*) have become the standard methodology to incorporate dynamic volatility in financial time series [see Poon and Granger (2003)]. Similarly the auto-correlation of returns is significant in many situations and there is a need to also incorporate the ARMA(*m*,*n*) structure in the model. Let $(X_t, t \in \mathbb{Z})$ be a strictly stationary time series representing daily observations of the negative log returns on a financial asset price. The dynamics of the model has the following specification:

$$X_t = \mu_t + \sigma_t Z_t,$$

[1]See Hull and White (1998) Acknowledging the need to incorporate time varying volatility VaR models employ various dynamic risk measures such as the Random Walk model, the GARCH, and the Exponentially Weighted Moving Average (EWMA). The Riskmetrics model uses the EWMA.

Where $\mu_t = \varphi X_{t-1}$ and $\sigma^2_t = w + \alpha(X_{t-1} - \mu_{t-1})^2 + \beta\ \sigma^2_{t-1}$ with w, α, β>0, and α + β <1,whereσ_t is the volatility of the return on day *t*, μ_t is the expected return and (X_t) is the actual return. We assume μ_t and σ_t are measurable with respect to $\mathcal{G}_{t-1}$, the information set about the return process available up to time *t*–1. The stochastic variable, Z_t represents the residuals or the innovations of the process, and is assumed to be independently and identically distributed.

In order to specifically capture any structural break the Global Financial Crisis, we also fit an alternative AR(1)-GARCH(1,1) model with a dummy variable that captures the effect of the onset of the GFC. The mean returns and the volatility of the process are as follows:

$$\mu_t = \varphi\ X_{t-1} + \gamma\ D_{GFC} \quad \text{and}$$

$$\sigma^2_t = w + \gamma\ D_{GFC} + \alpha(X_{t-1} - \mu_{t-1})^2 + \beta\ \sigma^2_{t-1},$$

where D_{GFC} equals zero for the period 1st January 2001 to 30th June 2008 and equals one for the period after 1st July 2008.

Let $F_X(x)$ denote the marginal distribution of (X_t) and let $F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x)$ denote the 1-step predictive distribution of the return over the next day, given knowledge of returns upto and including day *t*. We're interested in estimating quantiles in the tails of these distributions. For $0 < q < 1$, an unconditional quantile is a quantile of the marginal distribution denoted by

$$x_q = \inf\{x \in \mathbb{R}: F_X(x) \geq q\}$$

and a conditional quantile that is a quantile of the predictive distribution for the return over next day denoted by

$$x_q^t = \inf\{x \in \mathbb{R}: F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) \geq q\}, \quad \text{where}$$

$$F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) = P\{\sigma_{t+1} Z_{t+1} + \mu_{t+1} \leq x |\ \mathcal{G}_t\} = F_Z((x - \mu_{t+1})/\sigma_{t+1}),$$

$$\text{and thus} \quad x_q^t = \mu_{t+1} + \sigma_{t+1} z_q \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

wherez_q is the upper *q*th quantile of the marginal distribution of Z_t, which does not depend on *t*.

IV. HYPOTHESIS, DATA AND METHODOLOGY

In this paper, we focus on the extreme returns experienced on the Karachi Stock Exchange's KSE100 index for the period January 1, 2001 to June, 2012 or 2972 observations for over 10 years. During the period the market experienced a number of political and economic shocks, including the 9/11 terrorist attack, and the Global Financial Crisis. The stock returns r_t are measured as the first negative log differences of the stock index; $r_t = -ln\ (\text{Index}_t / \text{Index}_{t-1})$, since we are interested in the upper tail of loss distribution.

Following the approach suggested by McNeil and Frey (2000), we apply EVT to the residuals extracted from a GARCH model. Our estimation can be summarised as a two-step procedure: (i) An AR(1)-GARCH(1,1) model is fitted to the historical return data by pseudo maximum likelihood method. The residuals of this model are extracted;

(ii) Hill estimation procedure is employed on the right tail of the standardised residuals to compute VaR $(Z)_q = \widehat{z_q}$. Finally the estimated dynamic or conditional VaR using Equation (3) is:

$$\hat{x}_q^t = \widehat{\mu_{t+1}} + \widehat{z}_q \widehat{\sigma_{t+1}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

We also apply various tests and report test values and the achieved p-values to verify our estimation procedure. First we apply the Augmented Ducky Filler test to see whether the negative return series is stationary or not. After the stage (i), we apply ARCH-LM test with the null hypothesis that there is no autocorrelation among the residuals and squared residuals. Similarly after the stage (ii) of our estimation, we consider the Cramer-von Mises (W^2), Watson (U^2) and Anderson-Darling (A^2) criteria for judging the goodness of fit of the cumulative distribution function for the estimated parameters.

We further back-test the method on historical series of negative log-returns$\{x_1, x_2, \ldots, x_n\}$ from January 2001–June 2012. We calculate $\hat{x}_q^t$ on day *t* in the set $T=\{m, m+1, \ldots, n-1\}$ using a time window of *m* days each time. Similar to McNeil and Frey (2000), we set *m*=1000, but we consider 50 extreme observations from the upper tail of the innovation distribution i.e., we fix *k*=50 each time. On each day $t \in T$, we fit a new AR(1)-GARCH(1,1) model and determine a new Hill tail estimate. We compare $\hat{x}_q^t$ with x_{t+1} for $q \in \{0.95, 0.97, 0.99, 0.995\}$. A violation is said to occur whenever $x_{t+1} > \hat{x}_q^t$. We then apply a one-sided binomial test based on the number of violations for the model adequacy.

We also compare the method with four other well-known parametric methods of estimation. First one is the Static Normal method in which returns are assumed to be normally distributed and the VaR is calculated as the *qth* upper quantile from the normal distribution. Second one is the Dynamic or Conditional Normal in which AR(1)-GARCH(1,1) model with normal innovations is fitted by the method of maximum likelihood to the return data and $\hat{x}_q^t$ is estimated. Third one is the Conditional *t* in which innovations are assumed to have a leptokurtic distribution and the AR(1)-GARCH(1,1) model with *t*-innovations is fitted to the return data. Last one is the Static EVT method in which returns are assumed to have fat-tailed distribution and extreme value theory is applied to the upper tail of the returns.

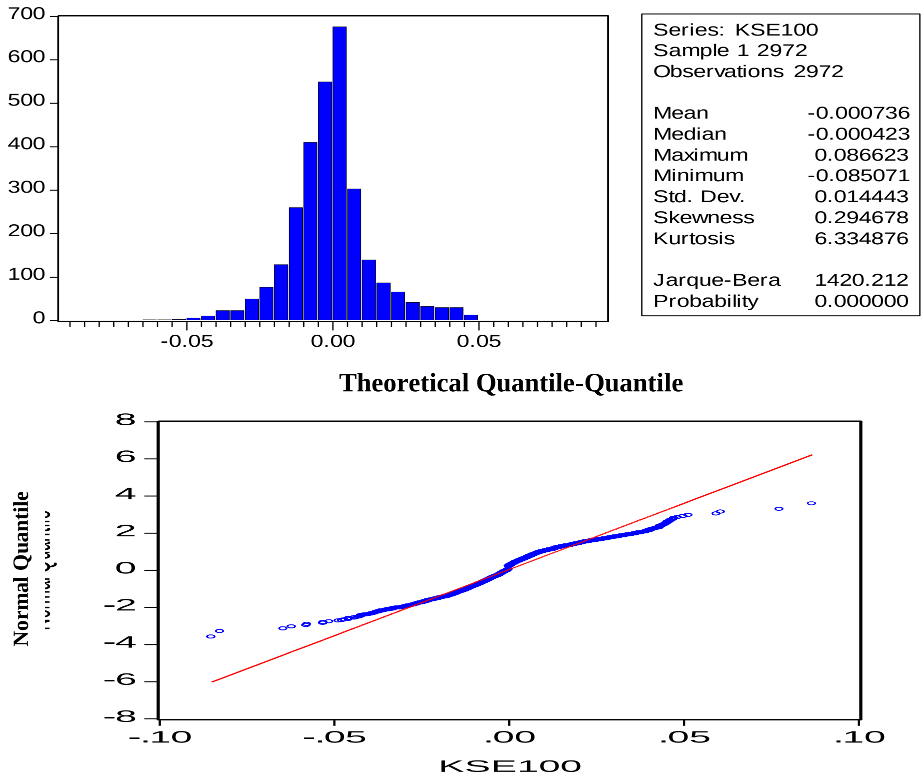
After studying the time-line of the progression of the GFC, we mark the onset of the down turn in the stock markets as the first of July, 2007. This serves to divide our study in two sub-periods of 1158 observations each and model the dynamic Risk measure for two periods separately.

V. RESULTS

(i) Modelling

We use EVIEWS 5.0 and R 2.15.1 for the analysis. The table in the following exhibit provides descriptive statistics for the KSE 100 for the period covered in the study, computing market returns as negative first log differences in the index values; $Rt = -ln$ $(\text{Index}_t / \text{Index}_{t-1})$. The exhibit also shows the distribution of the returns and a QQ-plot against normal distribution.

Exhibit 1: Descriptive Statistics

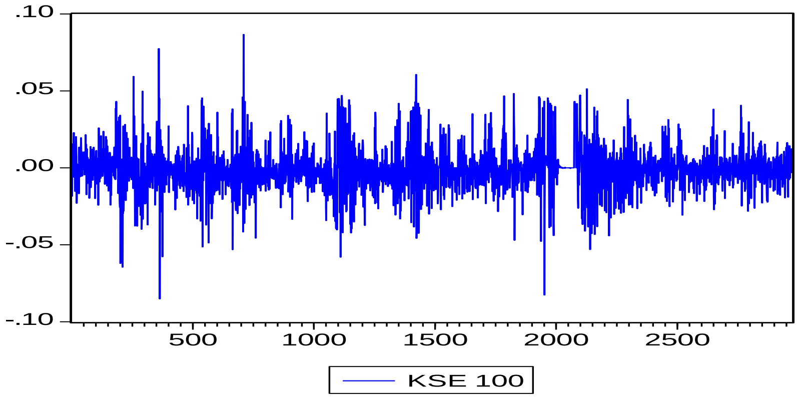


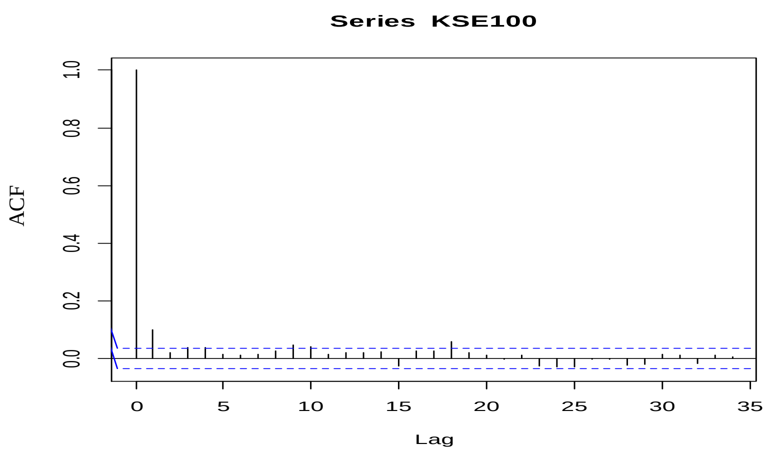
Source: Author calculation.

The descriptive statistics of the stock returns clearly show that the return distributions have heavier tails than of a normal distribution. The Jarque-Bera statistic is significant even at very low levels. Hence, we reject the null hypothesis that the stock returns follow a normal distribution. High values of the Kurtosis statistics indicate that the distributions have fat tails. The positive value of skewness indicates that the upper tail of loss distribution (i.e., the tail of interest for VaR calculation) is particularly thick. The QQ-plot also indicates the departure from normality. Therefore, the nature of distributions provides support for modelling the tails of the distribution using EVT.

Figure 1 clearly depicts that the large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes. This indicates that returns are not independent identically distributed and the volatility clustering phenomenon is present in the data. This suggests the need to employ a GARCH model to estimate dynamic volitality. The figure also shows that there is a structural break in the data after June 2008 and indicates that a dummy variable should be incorporated in the model. Similarly the ACF plot indicates that auto-correlation of returns is significant up to lag 1 and there is a need to incorporate the AR(1) component in the model to capture the effect of conditional mean. The Augmented Dickey Fuller test (given in Appendix A) strongly rejects the null hypothesis which implies that the return series is stationary.

Fig. 1. KSE 100 Return Series and Correlogram of Returns





The next step is to estimate the parameters of the both AR(1)-GARCH(1,1) models i.e., one with dummy variable and the other without it. The models are fitted using the pseudo-likelihood method. This means that the likelihood for a GARCH(1,1) model with normal innovations is maximised to obtain the parameter estimates $\hat{\theta} = (\hat{\varphi}, \hat{\omega}, \hat{\alpha}, \hat{\beta})$. The results of GARCH estimation procedure are given in Table 1. All the coefficients of the volatility and mean equations are significant. The model with Dummy variable indicates that although the coefficient DUM1 is significant in mean and variance equations, but the conditional mean and variance coefficients are not different. It implies that a simpler model without dummy may be satisfactory for applying EVT models, and the structural break does not impact the incidence of extreme returns. The Durbin Watson Statistics are within the acceptable range implying that the model’s specification is tenable.

We ran the ARCH-LM residual test (given in Appendix A) for the standardised residuals extracted from both AR(1)-GARCH(1,1) model and found no evidence against the independent identically distributed (iid) hypothesis for the residuals. We conclude that the fitted model is tenable. The descriptive statistics and QQ-plot of standardised residuals again indicate the departure from normality and a fat right tail.

Table 1

Results from AR-GARCH Estimation

Depend. Variable: Negative Returns
Method: ML - ARCH (Marquardt) - Normal Distribution
Included observations: 2971 after Adjustments

	Without Dummy				With Dummy			
	Coef.	Std. Error	z-Stat	Prob.	Coef.	Std. Error	z-Stat	Prob.
Mean Eq.								
AR(1)	0.0641	0.0190	3.3650	0.0008	0.0631	0.0193	3.2635	0.0011
DUM1*Variance Eq.*					0.0008	0.0004	2.2125	0.0269
C	9.33E-06	6.89E-07	13.5508	0.0000	1.12 E-05	8.67 E-07	12.8661	0.0000
RESID(–1)^2	0.1486	0.0102	14.5711	0.0000	0.14688	0.01049	13.9996	0.0000
GARCH(–1)	0.8073	0.0104	77.4652	0.0000	0.8059	0.0104	77.5187	0.0000
DUM1					–3.83E-06	6.45E-07	–5.9339	0.0000
Durbin-Watson	1.9263				1.9228			

Source: Author calculation.

Next the ordered residuals are used to estimate the right tail index using Equation (2). Table 2 provides results for the estimation of parameters on the right tail of the distribution. We fix the threshold by rounding off 95th percentile value.

We ran the Goodness-of-Fit test (given in Appendix A) to see whether the fitted model to the right tail of the innovation distribution (which represents losses) is appropriate and we found that the p-values for all three different tests are insignificant. It implies that the parameter estimates obtained in Table 4 are tenable.

Table 2

Results from Tail-index Estimation

Method: Maximum Likelihood (Exact Solution)				
Parameter	Value	Std. Error	z-Statistic	Prob.
Threshold, β	2.1597			
Tail Index (1/ξ)	3.4709	0.4548	7.6317	0.0000
Log Likelihood	–52.8886	Mean dependent var.		2.9976
No. of Coefficients	1	S.D. dependent var.		0.9595

Source: Author calculation.

After specifying our models completely by estimating the parameters, we can now calculate the dynamic VaR estimates by using Equation (3). We first calculate the 95th percentile of innovations. The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.95}$ is found out be 2.16346. Using Equation (3), our dynamic VaR specification for the *t+1* day is:

$$VaR_{0.95}^{t} = \widehat{\mu_{t+1}} + 2.16346\ \widehat{\sigma_{t+1}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

where $\widehat{\mu_{t+1}}$ and $\widehat{\sigma_{t+1}}$ are conditional GARCH estimates of mean and volatility respectively.

(ii) Back-testing

We then proceed to conduct back-tests using methodology explained in the Section IV. Table 3 provides the back testing results with theoretically expected number of violations and the number of violations using five different models: (i) Dynamic EVT or GARCH-EVT model, (ii) Static EVT model, (iii) a GARCH model with Student t innovations, (iv) a GARCH-model with normally distributed innovations; and (v) a Static normal model in which returns are assumed to be normally distributed.

Considering a 5 percent level of statistical significance, we may consider a p-value<.05 as a failure of the model. It is found that Dynamic EVT correctly estimates all the conditional quantiles, since the p-value is greater than 5 percent at all quantile levels. The method is very close to the mark in 3 out of 4 cases. Static EVT method fails at 95 percent, 99 percent and 99.5 percent. Dynamic-t fails at 95 percent and 97.5 percent, but performs well at higher levels i.e., it is closest to the mark at 99 percent and 99.5 percent. This indicates that GARCH model with t-innovations can also provide a good fit at higher levels. Dynamic normal fails at all but the 95 percent, whereas Static normal fails at all levels. The results show that the Value-at-Risk models based on the time-varying volatility work better than the Static models. The distribution of the tails of innovations is better modelled using the Extreme Value Theory or t-distribution instead of normal distribution.

Table 3

Back-testing Results

Quantile	95%	97.5%	99%	99.5%
Length of Test	1972	1972	1972	1972
Expected # Violations	99	49	20	10
DYNAMIC-EVT				
Observed # Violations	96	49	18	6
p-value	(0.4199)	(1.0000)	(0.4046)	(0.1386)
STATIC-EVT				
Observed # Violations	78	46	6	1
p-value	(0.0164)	(0.3503)	(0.0003)	(0.0001)
DYNAMIC-t				
Observed # Violations	122	63	22	10
p-value	(0.0107)	(0.0321)	(0.3323)	(0.8730)
DYNAMIC-NORMAL				
Observed # Violations	86	55	25	21
p-value	(0.1782)	(0.0236)	(0.0021)	(0.0006)
STATIC NORMAL				
Observed # Violations	111	79	51	30
p-value	(0.1108)	(0.0000)	(0.0000)	(0.0000)

Source: Author calculation.

We also repeated the back-testing procedure for the Dynamic models after including structural break dummies (results not reported here), and found that it did not materially alter the results. This attests the robustness of our analysis.

(iii) Pre- and Post-Crisis VaR Estimation

Next we subdivided the total study period from January, 2003 to June, 2012, into two even sub-periods of 1158 observations each, as follows:

1. Pre Crisis Period: 01/22/2003 to 06/29/2007
2. Crisis Period: 07/02/2007 to 06/12/2012

Table 4 provides the descriptive statistics and tests for equality of means and variances of negative log returns in the Pre-crisis and the Crisis (or Post-crisis) periods. The mean, extreme values and standard deviation seems to be the same in both periods. However, the test for equality of variances in Pre-Crisis and Post-Crisis periods indicates that the structure of dispersion is different in both periods. This underscores the need to study the distributional characteristics of both periods separately. In both periods returns series have a positive skewness, but the distribution in Pre-Crisis period is skewed more towards right than in the Post-Crisis period. However, the measure of kurtosis indicates that the tails of Post-Crisis return distribution is heavier than the tails of Pre-Crisis return distribution. The high values of Jarque-Bera statistics indicate the rejection of null hypothesis that returns follow the normal distribution in both cases.

Table 4

Summary Statistics for Pre- and Post-Crisis Returns

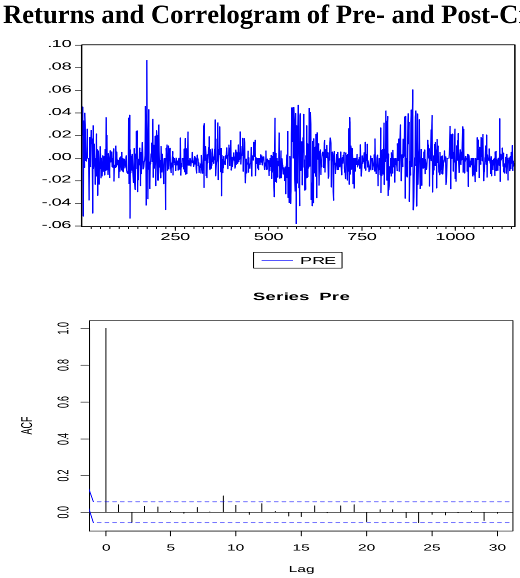
Series	Mean	Maximum	Minimum	Std.Dev.	Skewness	Kurtosis	Jarque-Bera
Pre Crisis	–0.0014	0.0866	–0.0580	0.0152	0.5649	5.5996	387.6502
Crisis	–0.0005	0.0513	–0.0825	0.0131	0.0141	6.3465	540.4040
T-Test for Equality of Means		Value	Probability	F-test for equality of Variances		Value	Probability
		1.4841	0.1379			1.3255	0.0000

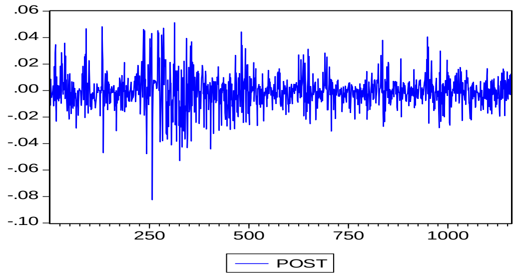
Source: Author calculation.

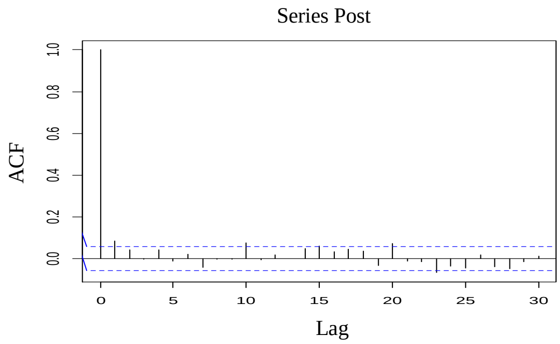
As a preliminary check, the Augmented Ducky Fuller test (given in Appendix B) indicates that the returns series has no unit roots, which implies that pre- and post-crisis series are stationary.

Interestingly the correlograms given in Figure 2, indicates that the auto-correlation of Pre-Crisis period returns is insignificant, whereas Pre-Crisis period indicates the presence of AR(1) component. This implies it is necessary to introduce the ARMA structure for modelling the Post-Crisis period. However, both the return series clearly indicate the presence of the GARCH effects.

Fig. 2. Returns and Correlogram of Pre- and Post-Crisis Series







The behaviour of returns and the correlogram lead to the GARCH models given in Table 5. We again ran the ARCH-LM residual tests (given in Appendix B) for the standardised residuals extracted from the fitted models and found no evidence against the independent identically distributed (iid) hypothesis for the residuals. However, the descriptive statistics and QQ-plot of standardised residuals again indicate the departure from normality and fat right tails.

Table 5

Results from AR-GARCH Estimation

	Pre-Crisis Period			Crisis Period		
	Coeff	z-Stat	Prob.	Coeff	z-Stat	Prob.
Mean Equation						
C	–0.0022	–6.7527	0.0000			
AR(1)				0.0663	2.1512	0.0315
Variance Equation						
C	1.04E-05	5.6549	0.0000	5.36E-06	5.0192	0.0000
RESID(-1)^2	0.1752	8.4535	0.0000	0.0932	8.0005	0.0000
GARCH(-1)	0.7754	35.0254	0.0000	0.8749	60.6717	0.0000
Durbin-Watson stat		1.9048			1.9618	

Source: Author calculation.

The next step is to model the right tails of the innovations of Pre- and Post-Crisis period. We consider ordered residuals and the Hill method given in Equation (2) to estimate the tail index. Table 6 provides results for the estimation of parameters on the right tails of the distribution with threshold being set by rounding off 95th percentile value. It also provides results for various goodness-of-fit tests which indicates that fitted models are appropriate.

Table 6

Results from Tail-index Estimation and Goodness-of-Fit Tests

KSE 100	Estimation of Empirical Distribution					
	Pre-crisis Period			Crisis Period		
Test of Distribution Fit	*Value*		*Prob*	*Value*		*Prob*
Cramer-von Mises (W2)	0.125642		0.2116	0.07799		0.4628
Watson (U2)	0.105444		0.1710	0.057925		0.5048
Anderson-Darling (A2)	1.019529		0.1069	0.634751		0.3235
Parameter Estimate	*Value*	*z-Stat*	*Prob.*	*Value*	*z-Stat*	*Prob.*
Threshold, β	2.2301			2.194		
Tail Index (1/ξ)	4.2475	4.8497	0.0000	3.9022	4.5361	0.0000

Source: Author calculation.

After specifying our models completely by estimating the parameters, we can now calculate the dynamic VaR estimates by using Equation (3) for the Pre-Crisis and the Post-Crisis periods separately.

We first calculate the 95 percentile VaR for Pre-Crisis period. The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.975}$ is found out be 2.242052. Using Equation (3), our dynamic VaR specification for Pre-Crisis returns is:

$$VaR^{t}_{(PRE)0.95} = \hat{C} + 2.242052\ \widehat{\sigma_{t+1}}, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Where $\hat{C}$ is the estimate of constant.

We now report the 95 percentile VaR for Post-Crisis period. The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.975}$ is found out be 2.256842. Using Equation (3), our dynamic VaR specification for Post-Crisis returns is:

$$VaR^{t}_{(POST)0.95} = \widehat{\mu_{t+1}} + 2.256842\ \widehat{\sigma_{t+1}}, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

A comparison of (5) and (6) shows that there is not much difference in the coefficients on the conditional volatility. Since, the mean of the returns is very small, the number of violations under the VaR models will be mostly determined by the conditional volatility. In case the coefficients on the conditional volatility are almost equal, the number of violations is also going to be about the same. This is in conformity with the results of our back-testing using models with structural dummy variables.

VI. CONCLUSIONS AND POLICY IMPLICATIONS

A major shortcoming of various VaR measures has been that the actual return distributions exhibit fatter tails than the normal distribution would specify. As a remedy the *Extreme Value Theory* (EVT) has been employed to explicitly incorporate extreme values, and to modify VaR accordingly. Typically, there would be limited number of extreme observations during a given period, which makes it hard to test and apply EVT as parameters are estimated with low levels of confidence. The equity market in Pakistan provides an excellent case to study the applicability of the EVT theory in a developing country since it exhibits a high degree of volatility reflecting a risky political and economic environment. The recent global financial crisis has been another source of extreme returns. The confluence of the two sources of volatility provides us with an historic experiment to test the EVT more rigorously.

We apply the EVT to the KSE100 index over an 11 years period, 2001-2012. We find that the returns distributions are fat tailed and the General Pareto Distribution (GPD) model fits the observed distribution of extreme values quite well. Our back-testing exercise shows that VaR measures with dynamic adjustment for volatility clustering perform better than measures which are based on normal distribution assumption, or do not take the dynamics of volatility into account. The main theoretical contribution of our work is to validate EVT for a small frontier market like Pakistan. In practical applications it implies that risk management systems based on the Dynamic VaR with tail estimation by EVT may be more helpful than simpler VaR models. For example, a Dynamic VaR based system that increased the securities traders' margin requirements with changes in the market volatility may help in containing market risk, curbing speculation and stabilising the markets. However, we find that the estimated tail-indices of the GPD distribution vary significantly over time. The implication is that the static extreme loss estimates based on one period may not be reliable guide to the risk of actual losses during the subsequent periods, and may need to be updated using a dynamic framework. Since the dynamics of the market may change from one period to another, the findings underscore the need to update the risk models in a timely fashion while considering possible structural shifts. Finally, as the EVT is shown to be applicable with respect to Pakistan, by extension, it may also hold promise with respect to other emerging markets.

APPENDIX – A

Table A1

Augmented Ducky Fuller Unit Root test

t-Statistics	–49.31500
Probability	0.0001

Exhibit A1: Descriptive Statistics of Standardised Residuals

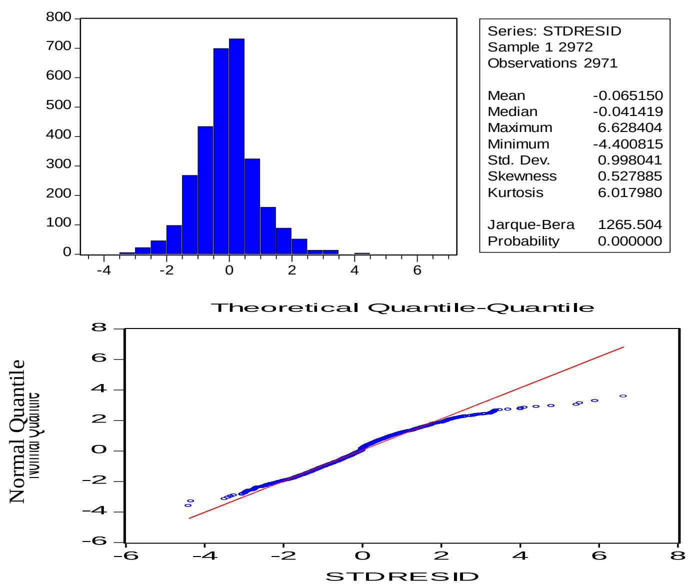


Table A2

ARCH LM Residual Test Results	
F-Statistics	0.596986
Probability	0.439792

Table A3

Goodness of Fit Test Results for Tail Index Estimation

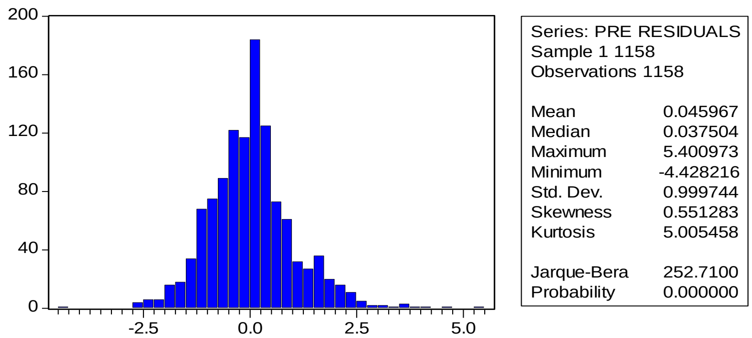
Method	Value	Probability
Cramer-von Mises (W2)	0.05709	0.63644
Watson (U2)	0.05680	0.51569
Anderson-Darling (A2)	0.43121	0.58047

APPENDIX – B

Table B1

Augmented Dickey Fuller Unit Root Test for Pre-Crisis Returns	
t-Statistics	–32.56522
Probability	0.0000

Exhibit B1: Descriptive Statistics for Pre-Crisis Standardised Residuals



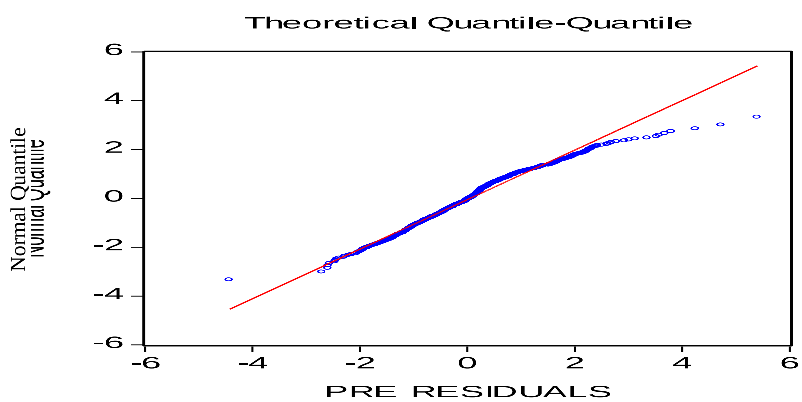


Table B2

ARCH LM Residual test Pre-Crisis Negative Returns	
F-Statistic	0.324707
Probability	0.568903

Table B3

Augmented Dickey Fuller Unit Root Test for Post-Crisis Returns	
t-Statistics	–31.9345
Probability	0.0000

Exhibit B2: Descriptive Statistics for Post-Crisis Standardised Residuals

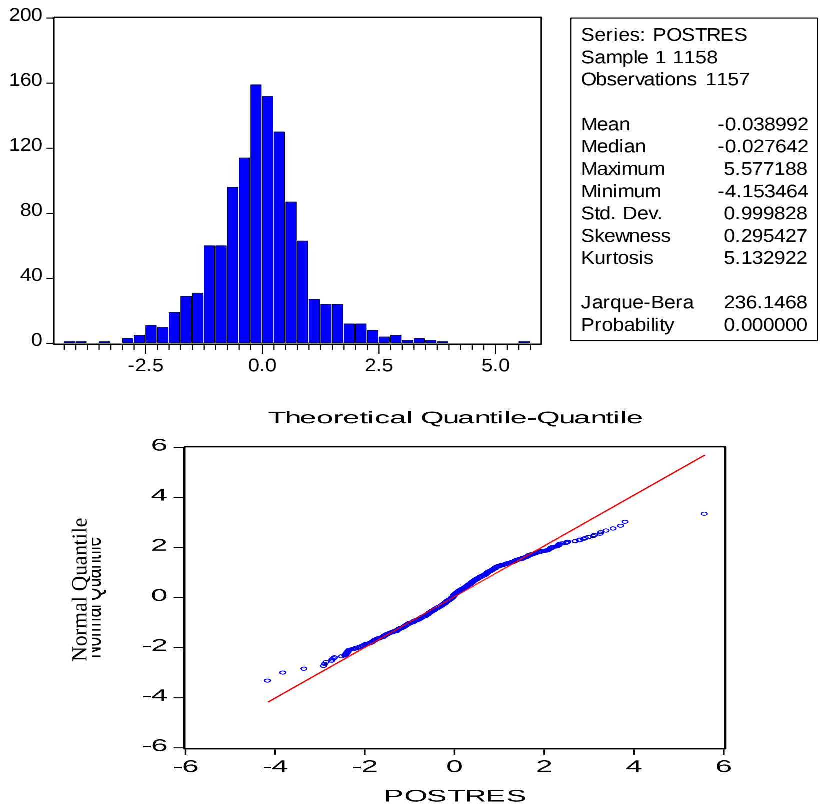


Table B4

ARCH LM Residual Test for Post-Crisis Negative Returns	
F-Statistic	0.000263
Probability	0.987065

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