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Chapter

Dynamic Stiffness Method for Vibrations of Ship Structures

Xuewen Yin, Kuikui Zhong, Zitian Wei and Wenwei Wu

Abstract

Initiated by the objective to address the dynamics of ship structures other than conventional finite element method, a dynamic stiffness method (DSM) is proposed systematically including that for three types of element models. A DSM element accounting for both in-plane and bending vibrations in flat rectangular plates is developed, which makes it possible for modeling wave conversion across junctions in builtup plates. In addition, a DSM element for stiffened plates is formulated, which considers all possible vibrations in plates and beams, i.e., bending, torsion, and extension motions. The third type of DSM plate element takes fluid loading into account, which is induced by vibrating plate. Finally, the proposed DSM method is extended to address vibration transmission in a built-up plate structure, which demonstrates the great potentials of DSM in application to more practical and more general engineering fields.

Keywords: dynamic stiffness method, FEM, power flow, beam-stiffened, ship structures

1. Introduction

The vibrational and acoustic characteristics of ship structures are likely to be one of a number of practical concerns not only to mechanical designers and research scientists, but sometimes even to military defense officers. The reasons lie in the following facts. Firstly, excessive vibration levels induced by operating machines or incident waves can inevitably lead to structural fatigue, failure, or even unexpected disasters. Besides, onboard vibration and noise are one of the most important indexes on ship habitability. Too much exposure to such vibrational and noisy environments can make ship crew members uncomfortable, fatigue, or even unhealthy, which has been convinced from a lot of experimental data, and even witnessed from many ship collision accidents. Last but not least, for naval ships, their vibration and acoustic signals make them as attack targets during war time, which also challenge the performances of onboard acoustic instruments.

Up to now, many numerical methods are developed and then utilized in addressing the vibrational and acoustic characteristics of ship structures, which can be found in numerous literatures. Among them are finite element method (FEM), boundary element method (BEM), statistical energy method (SEA), and mesh free methods, etc. Through intense academic efforts from engineers and scientists, and also due to commercial operations from software developer, most them are coded into commercial software, and comprehensively influent the way we design our products almost covering all the engineering fields such as civil engineering, ship and ocean engineering, chemical engineering, and etc. To some extent, we must confess that we, not only engineers, scientists, but product managers, or even government officers, have underestimate the power of novel numerical methods and how much they forge the manufacturing process in modern industries.

As for finite element method, it is one of the most successful numerical methods in high fidelity modeling of the dynamic behaviors of complex structures. To the best of our knowledge, SAP is the first commercial software. Soon after, other software like ADINA, ANSYS, ABAQUS, NASTRAN, and DYNTRAN have been developed and scattered in worldwide universities and industries.

However, like any other numerical methods, FEM has many inherent drawbacks due to the way it discretizes the structures. For instance, to address the vibrational responses in high frequencies, the mesh size must be as tiny as 1/6, or less, of the structural waves so that it can accurately reproduce the dynamics of the structures. However, such a meshing strategy is not always successful since too much finer meshes need not only excessive computational costs, but also lead to unexpected numerical uncertainties.

As for ship structures, the vibration of fluid-loaded plates or shells composes as a very important part in the studies of many engineering structures [1–3]. One of the major reasons lies in the fact that the dynamics of these structures depends on the structures and the fluid simultaneously. The vibrating structures can induce pressure disturbances in their surrounding fluid, and, in return, the resonance frequencies and vibrational responses of the structures can be altered [4, 5].

Recently, dynamic stiffness method (DSM) has won great interests and received intense studies [6–10] from research and design engineers because it can overcome the above issues without too much geometrical discretization requirements. Various DSM elements have been developed for transverse or in-plane vibrations of plates. In the beginning, more research works were mainly focused on transverse vibrations since bending modes are easily excited, especially in low frequencies. Dozens of investigator [6–15] made comprehensive contributions on DSM that only accounts for transverse vibrations of a plate with two opposite edges simply supported. Later, Bercin and Langley [8, 9] proposed a DSM that incorporates both in-plane and bending vibrations. It is reasonably expected that all these works are only applicable to few specified cases due to oversimplified modeling assumptions. To address the vibrations of more practical engineering structures, Casimir et al. [7] developed DSM elements for a plate with completely free boundary conditions, in which Gorman's superposition method was employed to obtain the exact transverse displacements. Banerjee and his colleagues [10–12] proposed the dynamic stiffness matrix for a rectangular plate with arbitrary boundary conditions. Similarly to DSM for bending plates with arbitrary boundary conditions, the dynamic stiffness matrix for in-plane vibrations of plates is developed by Ghorbel et al. [15, 16], Nefovska-Danilovic and Petronijevic [17, 18] in which all the four edges can be prescribed with any arbitrary conditions by adopting Gorman's superposition method.

Since the year 2016, Yin and his associates [19–21] have conducted comprehensive studies on developing dynamic stiffness method and its application to the dynamics of ship structures. Li et al. [19] proposed a dynamic stiffness formulation accounting for both in-plane and bending vibrations of plates with two opposite edges simply supported. This method was then employed for modeling vibration transmission with built-up plate structures [22] and a ship cabin with complex hulls. To consider the dynamics of stiffened plates, Yin et al. [21] extended Li's formulations and developed a dynamic stiffness method that considers torsion, bending, and extension vibrations in beams with eccentric cross-sections.

The main objective of this work is to formulate the vibration analysis of ship structures based on dynamic stiffness method that accounts for both in-plane and bending vibrations within plate itself, all possible motions in stiffened beams, fluid-loading, respectively. The present paper is organized as follows. In Section 2, this dynamic stiffness method is briefly summarized, which present the development of the three types of models. In Section 3, our proposed method is demonstrated by investigating the characteristics of representative plate structures.

2. Development of dynamic stiffness formulations

2.1 Model description

Figure 1 shows multiple rectangular plates in global coordinates OXYZ, which are rigidly joined along their common edges. Each plate has dimension of $L_x \times L_y$ and thickness of h. Its two opposite edges marked by the symbol 'S-S' denote simply supported boundary conditions while the other two edges are arbitrary. In addition, each plate is reinforced by uniform eccentric beams, and in contact with acoustic fluid on its one side.

2.2 Development of plate element

Consider a vibrating flat plate in contact with acoustic fluid on its lower side, which is made of isotropic material with Young's modulus E, bulk density ρ , Poisson's ratio μ , and damping ratio η . Its governing equations for both in-plane and bending vibrations can be written as,

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + a_1 \frac{\partial^2 u}{\partial y^2} + a_2 \frac{\partial^2 v}{\partial x \partial y} + \frac{m\omega^2}{B} u = 0 \\ \frac{\partial^2 v}{\partial y^2} + a_1 \frac{\partial^2 v}{\partial x^2} + a_2 \frac{\partial^2 u}{\partial x \partial y} + \frac{m\omega^2}{B} v = 0 \\ D\nabla^4 w - m\omega^2 w = -p_a(x, y, 0) \end{cases}$$
(1)



Figure 1. A built-up plate structure with beam stiffeners.

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 p_a is the induced acoustic pressure due to the bending vibration of the plate. u, v and w are the displacements in x-, y- and z-directions. *m* and ω are mass per unit area of the plate and circular frequency, respectively. The parameters a_1 and a_2 in Eq. (1) are defined as

$$a_1 = \frac{1-\mu}{2}, a_2 = \frac{1+\mu}{2} \tag{2}$$

The extension rigidity B and flexural rigidity D can be found in Ref. [19]. According to Bercin and Langley [9], the displacements for the plate, which is simply supported along its two opposite edges, can be expressed as N truncation terms,

$$\begin{cases} u(x,y) = \sum_{n=1}^{N} \left(C_{1n}\lambda_{1n}e^{\lambda_{1n}x} + C_{2n}\lambda_{2n}e^{\lambda_{2n}x} + C_{3n}k_{n}e^{\lambda_{3n}x} + C_{4n}k_{n}e^{\lambda_{4n}x} \right) \sin(k_{n}y) \\ v(x,y) = \sum_{n=1}^{N} \left(C_{1n}k_{n}e^{\lambda_{1n}x} + C_{2n}k_{n}e^{\lambda_{2n}x} + C_{3n}\lambda_{3n}e^{\lambda_{3n}x} + C_{4n}\lambda_{4n}e^{\lambda_{4n}x} \right) \cos(k_{n}y) \\ w(x,y) = \sum_{n=1}^{N} \left(\cos(\alpha_{1n}x)A_{1n} + \sin(\alpha_{1n}x)A_{2n} + \cosh(\alpha_{2n}x)A_{3n} + \sinh(\alpha_{2n}x)A_{4n} \right) \sin(k_{n}y) \\ if \quad k^{2} \ge k_{n}^{2}, \end{cases}$$

$$(3)$$

And, *if* $k^2 < k_n^2$, the bending vibrations can be expanded as near-field disturbance,

$$w(x,y) = \sum_{n=1}^{N} (\cosh(\alpha_{1n}x)A_{1n} + \sinh(\alpha_{1n}x)A_{2n} + \cosh(\alpha_{2n}x)A_{3n} + \sinh(\alpha_{2n}x)A_{4n})\sin(k_ny)$$
(4)

where $k^2 = \sqrt{\rho \omega^2/D}$ and $k_n = n\pi/L_y$. C_{mn} , m = 1, 2, 3, 4 and A_{mn} , m = 1, 2, 3, 4 are the unknown constants. Wavenumbers for in-plane and out-of-plane waves take the following forms:

$$\begin{cases} \lambda_{1n,2n} = \pm \sqrt{k_n^2 - k_L^2}, \lambda_{3n,4n} = \pm \sqrt{k_n^2 - k_T^2} \\ k^2 \ge k_n^2, \alpha_{1n} = \sqrt{k^2 - k_n^2}, \alpha_{2n} = \sqrt{k^2 + k_n^2} \\ k^2 < k_n^2, \alpha_{1n} = \sqrt{k_n^2 - k^2}, \alpha_{2n} = \sqrt{k_n^2 + k^2} \end{cases}$$
(5)
where $k_L^2 = \rho \omega^2 (1 - \mu^2) / E, k_T^2 = 2\rho \omega^2 (1 + \mu) / E.$

Accordingly, the transverse shear force Q_x perpendicular to xy plane, the bending moment M_{xx} , longitudinal force N_{xx} , and in-plane shear force N_{xy} along the plate junctions can be derived as follows,

$$\begin{cases} Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + (2-\mu)\frac{\partial^3 w}{\partial x \partial y^2}\right) \\ M_{xx} = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \\ N_{xx} = -B\left(\frac{\partial u}{\partial x} + \mu\frac{\partial v}{\partial y}\right) \\ N_{xy} = -Ba_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{cases}$$
(6)

Based on Eqs. (3) and (6), for any *n*th mode, the generalized displacement vector \overline{q}_n and force vector \overline{Q}_n are written as,

$$\overline{Q}_{n} = \begin{cases} \overline{W}_{n}|_{x=0} \\ \overline{v}_{n}|_{x=0} \\ \overline{v}_{n}|_{x=0} \\ \overline{v}_{n}|_{x=0} \\ \overline{v}_{n}|_{x=0} \\ \overline{v}_{n}|_{x=1,x} \\ \overline$$

Hence, the relationship between generalized displacements \overline{q}_n and generalized forces \overline{Q}_n at any n_{th} mode can be developed after simple matrix algorithm, which is generally known as dynamic stiffness matrix \overline{K}_n . Once the dynamic stiffness matrix

is obtained, the dynamic responses resulted from excitations can be readily achieved after solving linear equations like those in conventional finite element methods [19].

2.3 Development of beam element

As shown in **Figure 2**, a beam with an eccentric cross section is located with geometric center O and the shear center G. Based on classical beam theory, the governing equations for the forced vibrations at line G - G' are expressed as,

$$\begin{cases} \frac{\partial^2}{\partial y^2} \left(E_r I_z \frac{\partial^2 u_r}{\partial y^2} \right) - m_r \omega^2 u_r + m_r \omega^2 z_G \phi_r = P_r \\ E_r \frac{\partial^2 v_r}{\partial y^2} + \rho_r \omega^2 v_r = N_r \\ \frac{\partial^2}{\partial y^2} \left(E_r I_x \frac{\partial^2 w_r}{\partial y^2} \right) - m_r \omega^2 w_r - m_r \omega^2 x_G \phi_r = Q_r \\ GI_t \frac{\partial^2 \phi_r}{\partial y^2} + I_0 \omega^2 \phi_r - m_r \omega^2 x_G w_r + m_r \omega^2 z_G u_r = T_r \end{cases}$$
(9)

where u_r , v_r and w_r are the displacements in x_r -, y_r - and z_r -directions, and ϕ_r is the rotation about y_r axis. P_r , N_r , Q_r are the forces acting line G - G' in x_r -, y_r - and z_r -directions, and T_r is the torsion moment about y_r axis. I_x and I_z are the principle moments of the beam's cross-section about x_r - and z_r -axes. E_r and ρ_r are Young's modulus and density of the material. m_r is mass per unit length of the beam, i.e., $\rho_r A_r$, where A_r is the cross-sectional area. G and I_0 are shear modulus of the material, polar moment of mass inertia with respect to shear center, respectively, and I_t is cross-sectional factor in torsion.

Since the beam is attached to one edge of the plate, its motions are in the similar forms as that expressed in Eq. (3) and can be readily written as,



Figure 2.

Schematic illustration of a beam: Geometric center G, shear center O; x_G and z_G are the offset between G and O in x_r -, and z_r -directions, respectively.

$$\begin{cases} u_r(y_r) = \sum_{n=1}^N u_{rn} \sin(k_n y_r) \\ v_r(y_r) = \sum_{n=1}^N v_{rn} \cos(k_n y_r) \\ w_r(y_r) = \sum_{n=1}^N w_{rn} \sin(k_n y_r) \\ \phi_r(y_r) = \sum_{n=1}^N \phi_{rn} \sin(k_n y_r) \end{cases}$$
(10)

Substituting Eq. (10) into Eq. (9), and utilizing the orthogonality relationship of the modes, the vibration motions at the $n_{\rm th}$ mode for the beam can be readily derived,

$$\begin{cases} (E_{r}I_{z}k_{n}^{4} - m_{r}\omega^{2})u_{rn} + m_{r}\omega^{2}z_{G}\phi_{rn} = P_{rn} \\ (-E_{r}Ak_{n}^{2} + m_{r}\omega^{2})v_{rn} = N_{rn} \\ (E_{r}I_{x}k_{n}^{4} - m_{r}\omega^{2})w_{rn} - m_{r}\omega^{2}x_{G}\phi_{rn} = Q_{rn} \\ (-GI_{t}k_{n}^{2} + I_{0}\omega^{2})\phi_{rn} - m_{r}\omega^{2}x_{G}w_{rn} + m_{r}\omega^{2}z_{G}u_{rn} = T_{rn} \end{cases}$$
(11)

Without complex derivation procedure, Eq. (11) can be rewritten in a more compact matrix form,

$$\overline{F}_{rn} = \overline{K}_{rn}(\omega)\overline{q}_{rn},\tag{12}$$

where the dynamic stiffness matrix has the following expressions:

$$\overline{K}_{rn} = \begin{bmatrix} E_r I_z k_n^4 - m_r \omega^2 & 0 & 0 & m_r \omega^2 z_G \\ 0 & -E_r A k_n^2 + m_r \omega^2 & 0 \\ 0 & 0 & E_r I_x k_n^4 - m_r \omega^2 & -m_r \omega^2 x_G \\ m_r \omega^2 z_G & 0 & -m_r \omega^2 x_G & -G I_t k_n^2 + I_0 \omega^2 \end{bmatrix}.$$
(13)

2.4 Development of fluid-loaded element: acoustic pressure

The acoustic pressure satisfies the Helmholtz equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) p_a + k_0^2 p_a = 0, \qquad (14)$$

where k_0 is the acoustic wavenumber. The boundary condition at the interface between the plate and the fluid is expressed as

$$\left. \left(\frac{\partial p_a}{\partial z} \right) \right|_{z=0} = \rho_0 \omega^2 w, \tag{15}$$

where ρ_0 is the density of the acoustic fluid. Since the acoustic pressure p_a has the following form:

$$p_a(x,y,z) = \left| p_a \right| \exp\left[-j \left(k_x x + k_y y + k_z z \right) \right], \tag{16}$$

where $|p_a|$ is the amplitude of the acoustic pressure, and k_x , k_y , and k_z are wavenumbers for the acoustic waves. It is ready to obtain the expression for the acoustic pressure at the plate-fluid interface,

$$p_{a}(x,y,0) = \begin{cases} \frac{j\rho_{0}\omega^{2}w}{\left(k_{0}^{2}-k_{b}^{2}\right)^{\frac{1}{2}}}, & \text{if } k_{b} < k_{0}, \\ \frac{-\rho_{0}\omega^{2}w}{\left(k_{b}^{2}-k_{0}^{2}\right)^{\frac{1}{2}}}, & \text{if } k_{b} > k_{0}. \end{cases}$$
(17)

It is noted that we have the expression $k_b^2 = k_x^2 + k_y^2$, where k_b is the wavenumbers for the structural waves propagating within the plates. For sake of brevity, the relationship between the acoustic pressure at the fluid–structure interface and the inertia terms due to the vibration of the plate, which is referred to as fluid-loading parameter, can be rewritten as,

$$\varepsilon_{f} = p_{a}(x, y, 0) / [m\omega^{2}w] = \begin{cases} \frac{j\rho_{0}}{m(k_{0}^{2} - k_{b}^{2})^{\frac{1}{2}}}, & \text{if } k_{b} < k_{0}, \\ \frac{-\rho_{0}}{m(k_{b}^{2} - k_{0}^{2})^{\frac{1}{2}}}, & \text{if } k_{b} > k_{0}. \end{cases}$$
(18)

2.5 Dynamic responses of built-up plate structures

The dynamic stiffness matrices for the plate and the beam (in Sections 2.2 and 2.3) are expressed in local coordinates, which can be termed as local dynamic stiffness matrices. With reference to the conventional finite element technique, the dynamic stiffness matrix for each plate element and each beam element can be readily assembled into overall global dynamic stiffness matrix. Hence, the dynamic responses of a built-up structure composed of plates and beams can be solved through novel numerical methods.

3. Numerical results and discussion

Without loss of generality, we only focus on the vibration transmission in a built-up plate structure that is reinforced by stiffeners or plates. Numerical results for the dynamics of plates with beam stiffeners based on our method can found in [21].

3.1 Transmission modes within a plate stiffened by stiffeners

To demonstrate our method in addressing the vibration transmission within complex built-up structures, a horizontal plate reinforced by a vertical plate, i.e., plate 2 is employed in this subsection. The detailed parameters of the plates are listed in **Table 1**. The two opposite long edges of plate 1 is simply supported. One of the free end of the plate, namely, left edge, is subjected to uniformly distributed vertical forces of 1 N/m.

Yin et al. [22] identify that there are three representative transmission modes in a stiffened plate. As the plate structures get more complex, similar phenomena can be also found, in which a plate is stiffened by 9 identical plates. When the left side of the plate is enforced with transverse force, three representative transmission

	E (Gpa)	$\rho(\text{Kg/m}^3)$	μ	η	$L_x(\mathbf{m})$	$L_y(\mathbf{m})$	<i>h</i> (m)
Plate 1	200	7800	0.3	0.01	6.0	1.0	0.008
Plate 2	200	7800	0.3	0.01	0.5	1.0	0.008
No beam stiffeners are considered in case 1.							

Table 1.

Geometry and material parameters of the plates (case 1).

modes can be clearly identified. In **Figure 3(a)**, only the left local portion of the plates is excited that implies bending waves cannot propagate effectively forward due the presence of the stiffening plates. However, in some frequency regimes as shown in **Figure 3(b)** and (c), bending waves can pass the stiffening members freely. As frequency increases, the stiffening members act more like a barrier that prevent structural waves propagate.

From **Figure 3(a)**–(**d**), we can convince that the vibration transmission modes do exist in even more complex plate structures. In addition, we suggest to explore the underlying mechanisms, if any, between these transmission modes and the well-known pass band and stop band since vibration transmission is probably one of the most important characteristics in complex plate structures, e.g., ship structures, etc.



Figure 3. *Representative vibrational transmission modes of a stiffened plate: (a) 270 Hz, (b) 345 Hz, (c) 395 Hz, and (d) 445 Hz.*



Figure 4. A ship hull reinforced with eight stiffeners.



Figure 5. *Vertical displacement at the middle point in the bottom plate.*

3.2 Vibrations of a ship hull in contact with water

Figure 4 shows a ship hull that is reinforced by eight beams with dimension $0.02m \times 0.02m$ along the junctions of their neural planes. The ship hull has the dimension of $6m \times 4m \times 2.4m$ and with thickness of 0.008 m. The bottom of the ship hull is in contact with water. About 1 N concentrated force is applied at the middle point in upper plate and the response gauge is set at middle point in the bottom plate.

Figure 5 shows the curves for the vertical displacement obtained by FEM and DSM, respectively. The truncation term N is set to 6 in DSM and the mesh size in the FEM is $0.2m \times 0.2m$. It is indicated that satisfactory agreement can be found between the results from DSM and those from FEM, which implies that our proposed method can provide excellent numerical results for ship structures.

4. Conclusion

A DSM is proposed to address the dynamics of ship structures, which include three types of elements. First, a DSM formulation for both in-plane and bending

vibrations in flat rectangular plates is developed. Then, a DSM for stiffening beams is addressed, which accounts for all possible vibrations in plates and beams, i.e., bending, torsion, and extension motions. Finally, a DS plate element with fluid loadings included is formulated. The numerical results for the vibrations for a ship hull based on the proposed DSM have excellent agreement with those results obtained from FEM, which demonstrate its potential in addressing the dynamics of ship structures. In addition, vibration transmission modes of a stiffened plate are also addressed using this method.

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Author details

Xuewen Yin^{*}, Kuikui Zhong, Zitian Wei and Wenwei Wu National Key Laboratory on Ship Vibration and Noise, China Ship Scientific Research Center, Wuxi, China

*Address all correspondence to: x.w.yin@cssrc.com.cn

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References

[1] Mace BR. Sound radiation form fluidloaded orthogonally stiffened plates. Journal of Sound and Vibration. 1981; **79**(3):439-452

[2] Mead DJ. Plates with regular stiffening in acoustic media: Vibration and radiation. Journal of the Acoustical Society of America. 1990;**88**(1):391-401

[3] Missaoui J, Cheng L, Richard MJ. Free and forced vibration of a cylindrical shell with a floor partition. Journal of Sound and Vibration. 1996; **190**(1):21-40

[4] Yin XW, Gu XJ, Cui HF, et al. Acoustic radiation from a laminated composite plate reinforced by doubly periodic parallel stiffeners. Journal of Sound and Vibration. 2007;**306**:877-889

[5] Yin XW, Liu LJ, Hua HX, Shen RY. Acoustic radiation from an infinite laminated composite cylindrical shell with doubly periodic rings. ASME Journal of Vibration and Acoustics.
2009;131(1):011005

[6] Kevorkian S, Pascal M. An accurate method for free vibration analysis of structures with application to plates. Journal of Sound and Vibration. 2001; **246**(5):795-814

[7] Casimir JB, Kevorkian K, Vinh T. The dynamic stiffness matrix of twodimensional elements: Application to Kirchhoff's plate continuous elements. Journal of Sound and Vibration. 2005; **287**:571-589

[8] Langlegy RS. Application of the dynamic stiffness method to the free and forced vibrations of aircraft panels. Journal of Sound and Vibration. 1989; **135**(2):319-331

[9] Bercin AN, Langley RS. Application of the dynamic stiffness technique to the in-plane vibrations of plate structures. Computers and Structures. 1996;**59**(5):869-875

[10] Boscolo M, Banerjee JR. Dynamic stiffness method for exact inplane free vibration analysis of plates and plate assemblies. Journal of Sound and Vibration. 2011;**330**:2928-2936

[11] Boscolo M, Banerjee JR. Dynamic stiffness formulation for composite Mindlin plates for exact modal analysis of structures. Part I: Theory. Computers and Structures. 2012;**96**:61-73

[12] Boscolo M, Banerjee JR. Layer-wise dynamic stiffness solution for free vibration analysis of laminated composite plates. Journal of Sound and Vibration. 2014;**333**:200-227

[13] Danial AN, Doyle JF, Rizzi SA. Dynamic analysis of folded plate structures. Journal of Vibration and Acoustics. 1996;**118**:591-598

[14] Fazzolari FA. A refined dynamic stiffness element for free vibration analysis of cross-ply laminated composite cylindrical and spherical shallow shells. Composites Part B: Engineering. 2014;**62**:143-158

[15] Ghorbel O, Casimir JB, Hammami L, et al. Dynamic stiffness formulation for free orthotropic plates. Journal of Sound and Vibration. 2015;**346**:361-375

[16] Ghorbel O, Casimir JB, Hammami L, et al. In-plane dynamic stiffness matrix for a free orthotropic plate. Journal of Sound and Vibration. 2016;**364**:234-246

[17] Kolarevic N, Nefovska-Danilovic N, Petronijevic M. Dynamic stiffness elements for free vibration analysis of rectangular Mindlin plate assemblies. Journal of Sound and Vibration. 2015;
359:84-106

[18] Nefovska-Danilovic M, Petronijevic M. In-plane free vibration and response analysis of isotropic rectangular plates using the dynamic stiffness method. Computers and Structures. 2015;**152**:82-95

[19] Li H, Yin XW, Wu WW. Dynamic stiffness formulation for in-plane and bending vibrations of plates with two opposite edges simply supported.
Journal of Vibration and Control. 2018;
24(9):1652-1669

[20] Wu WW, Li H, Yin XW. Power flow analysis of built-up plate structures using the dynamic stiffness method. Journal of Vibration and Control. 2018; **24**(13):2815-2831

[21] Yin XW, Wu WW, Li H. Dynamic stiffness formulation for the vibrations of stiffened plates with consideration of in-plane deformations. Journal of Vibration and Control. 2018;**24**(20): 4825-4838

[22] Yin X, Wu W, Li H. Vibration transmission within beam-stiffened plate structures using dynamic stiffness method. Procedia Engineering. 2018; **199**:411-416