MAKING MULTI-MODAL MATHEMATICAL MEANING IN MULTILINGUAL CLASSROOMS

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ABSTRACT:

This thesis investigates communication (verbal and nonverbal) in a bilingual (Farsi-English) complementary school mathematics' classroom. The study examines gestures were used as a resource for teaching mathematics in a bilingual setting, enabling intercolutors to construct meaning and mediate understanding. That is, the ways in which language and gesture can be seen as resources in supporting and conveying mathematical ideas is described. I investigated a number of verbal and nonverbal resources and show how these are culturally and socially shaped. I also explored how modes of communication are employed in creating mathematical meaning in a bilingual classroom context. A multimodality framework was adopted to analyse data which included audio and video recordings, observations and interviews with teachers and pupils. I found that gestures were employed to convey aspects of the mathematical register and how these were used to amplify what interlocutors were expressing verbally. Furthermore, I identified that different languages activated a different conceptual understanding of the same mathematical concept which was reflected through the students' and teachers' gestures.

DEDICATION

To my father Foroud Farsani **ب ه پ هدرم ف رود ف ار سان**

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1. Chapter one – Introduction to this thesis

1.1 Introduction to the chapter

I will introduce this thesis by giving an account to the background of this study in section 1.2, where I will be describing how the ideas for my research have evolved through the origins of my personal and professional experience. I will briefly bring to light my very first exposure to *language* (verbal and visual) as a bilingual learner, which led me to realise how aspects of mathematics can be done differently in different languages. Having identified conceptual tools provided by the language (which varies cross-culturally), in section 1.3, I will describe my rationale for being interested in examining a set of linguistic resources that are available in bilingual mathematics classrooms. In section 1.4, I will then move on to talk about the context of my study. Finally, in section 1.5, I will highlight the structure of this thesis.

1.2 Background to the study

In this section I would like to describe myself briefly as a researcher in order to create a better picture of how the ideas for my research have evolved through the origins of my personal and professional experience. I spent the last 13 years using two languages (Farsi and English) at school, college and universities in the UK as a student, teacher assistant and a researcher of mathematics education. I use these two languages interdependently (Barwell & Setati, 2005), to the extent to know how Farsi/English might shape mathematical thinking for Farsi/English speakers. At the age of fourteen I went to an international school in the UK to study GCSEs and later International Baccalaureate where over 50 different languages were officially recognised. Entering a UK school with no English was very frustrating. This was primarily because it was very hard trying to be engaged with discursive practices that took place in lessons even with visual subjects such as mathematics. At a very early stage of my arrival at the school, I realised that school was an '*English-only zone*' and bilingualism was left at the school gate (Cummins, 2005:590). As an EAL (English as an additional language) learner, I looked out for other ways of trying to communicate with students and teachers. Those resources for meaning making were nonverbal. I looked at other students and teachers' use of body language to decipher not only what my classmates were trying to communicate to me, but also how they felt about me. Although some of these resources for meaning making did not help in understanding the subject knowledge, but, enabled me to realise cues about aspects of social interaction. For example, I realised that often people squinted their eyes (regularly accompanied with arms crossing) if I said something that did not make much sense or they did not like. Squinting or blocking eyes is a nonverbal behaviour which often occurs when one does not like what they see (Navarro, 2008; Driver, 2010). In contrast, in situations where my ideas were appreciated, often listeners' eyebrows were raised followed by three slow repetitive head nods (see Figure 1). Unfortunately, at that time, I did not know how to incorporate certain nonverbal resources to convey mathematical meaning.

Figure 1: Nonverbal cues

Later, I worked as a teacher assistant in an inner city secondary school where the proportion of bi/multilingual students was equal to monolingual Anglophones. Whilst observing students in the classroom I became aware of certain language practices where bilingual students kept referring back to their ‘native tongue’ when they were solving aspects of mathematics, in particular when they were counting fast or under pressure. Furthermore there appeared to be some cultural differences in doing aspects of mathematics which enabled me to realise new perspectives about the ways in which mathematics can be seen/done in different cultural contexts. For example within the context of multiplication, I became aware of how a large population of Polish students used their fingers in multiplying one digit numbers e.g. 7×8 (see also Menninger, 1969:217). For example, in multiplying seven and eight, an open hand with two fingers curled inside, would represent seven (see the left hand in Figure 2) and another open hand with three fingers curled inside was used to signify number eight (the right hand in Figure 2). The sum of curled fingers represented tens, which, in this case, there were five tens, two from one hand (representing seven) and three from the other hand (representing eight). The product of remained open fingers would have represented units, six. There were three remained open fingers in one hand and two in the other.

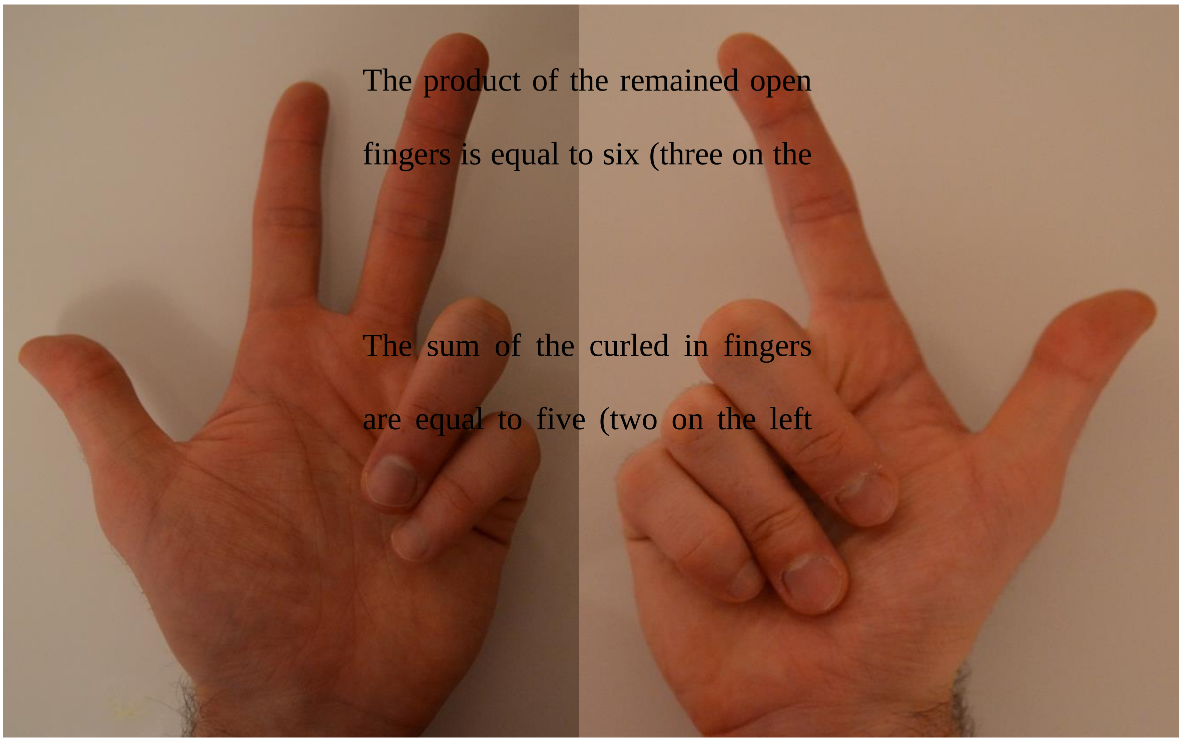


Figure 2: Using fingers to multiply

Many Polish students used their already given biological semiotic resources (fingers), to do aspects of mathematics where less arithmetic was involved but centred more on visual artefacts. Interestingly enough, students from a Japanese heritage background solved multiplication in a way which I had never encountered in Iran or in the UK. The Japanese students used the intersection of lines (see Figure 3) to determine an exact numerical value for any digit number multiplication.

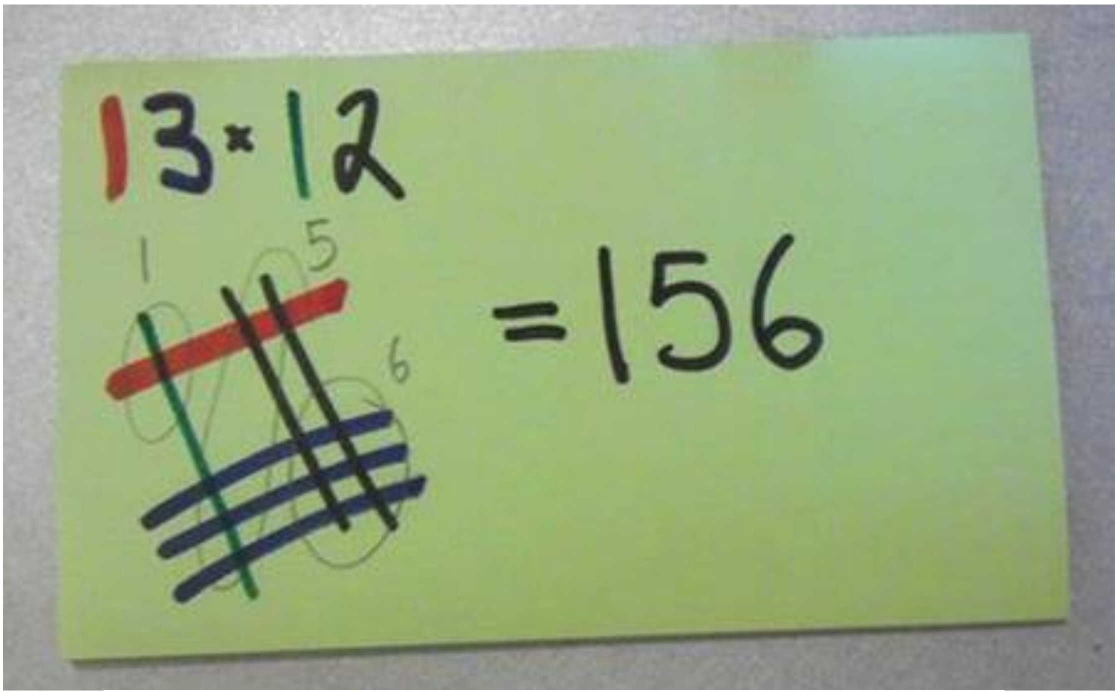


Figure 3: How Japanese kids learn to multiply in primary school

The single red line on the top and the three blue lines at the bottom are representing 13. The single green line of the left and two black lines on the right hand side represent 12. The number one over the circle on the top left hand side represents the number of line intersection (red line crossing the green line) which is one. Similarly the number six over the circle on the bottom right corner represents the number of intersection of the lines blue and black which is six. Number five on top of the top right hand circle is the sum of the diagonal line intersections, which are two (top right hand corner) and three (bottom left hand corner). Therefore the numerical solution to the product of thirteen multiplied by twelve is the number of intersection on the top left hand corner first (1), followed by the sum of diagonal intersection (5) and finally followed by the number of intersection on the bottom right hand corner (6) which creates 156. Being present at such a diverse community of practice led me to think about the ways in which mathematics can present itself differently in different socio-cultural contexts.

I found it fascinating how different languages (and cultures) provide particular epistemological access offering different views on how aspects of mathematics can be seen or even done differently. It has been suggested that "[i]f Aristotle had spoken Chinese or Dakota, his logic and categories would have been different" (cf Bolinger & Sears, 1981:139). There is substantial evidence to support that the language of 'doing mathematics within the classroom' can vary cross-culturally (Gorgorió & Planas, 2001; Ervynck, 1992; Cocking & Mestre, 1988). For example it is commonly believed doing arithmetic is the same regardless of whether one is performing arithmetic in Chinese, Farsi or in English. However although the result is the same, the linguistic support that is behind the arithmetic process is not necessarily identical. Fuson and Kwon (1991) have observed that a language like Chinese offers a linguistic support in basic tasks such as addition or subtraction. Chinese number words are regular and indicate the value of each digit, unlike 'twelve' in English. Therefore the conceptual tools provided by the language are not the same, even for something as basic as counting. Therefore the language in which mathematical operations are embedded is far from being universal (Farsani, in press).

I became interested to look at language and cultural artefacts as a set of resources that bi/multilingual often draw upon to solve and express their mathematical meanings. For example, exploring the influences that a particular culture plays in multiplication, or, how multiplication can be learned in a parrot fashion containing musical rhythm in a particular cultural context. Since I am a bilingual (Farsi/English) speaker and I had histories, practices and experiences of mathematics in both languages across different settings and tasks, I started to think about how aspects of mathematics in each language (as a result of social and cultural construct) can be manifested differently. For example, consider the following:

$$\frac{a}{b} = \frac{c}{d}$$

In the context of mathematics in English in the UK it is not uncommon to refer to the phrase ‘cross multiplication’ to mean the product of ‘a’ and ‘d’ over the product of ‘b’ and ‘c’. ‘Cross multiplication’ encapsulates the process of arithmetic and helps to memorise and recollect the overall procedure. There is not an equivalent in Farsi mathematics register for ‘cross multiplication’. Cross multiplication can be translated in Farsi as: ‘the product of top left hand number and bottom right hand number over the product of bottom left hand number and top right hand number’. However in the context of complex fractions (see Figure 4), there exists a particular phrase in Farsi that is seldom used in other social contexts. In Farsi it is referred to as ‘*door dar door, nazdik dar nazdik*’. The idiomatic English translation of this Farsi statement is ‘far by far, near by near’ where ‘by’ is the preposition for multiplication[1]. Far by far, near by near in Farsi is an idiom which refers to the process of simplifying a division of two fractions. It means the product of the two numbers furthest apart over the product of the two numbers closest to the main division line in the centre.

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

Figure 4: Far by far, near by near

‘*Door dar door, nazdik dar nazdik*’ contains poetic imagery and musical weight and due to its prosodic form of verbal expression it helps a Farsi speaker to be engaged with the pedagogic possibilities in simplifying the division of two fractions. It has been observed that the use of proverbs, idioms and expression in classrooms by teachers can provide a bridging pedagogy to connect abstract mathematical topics and everyday practices (Farsani, 2012b).

[1] Much attention has been given to the potential error and confusion caused as the result of the employment of the prepositions for multiplication and division (see Zagorianakos & Farsani, 2012).

1.3 Aims of research and rationale for study

Throughout my bilingual experience, I became interested and adept in recognizing and identifying resources which are available in bilingual mathematics classrooms and are exploited between learners and teacher. I have observed that learners draw across languages and the additional value and resources that bilingualism brings to their performance in doing mathematics which differs from doing it monolingually. I came to believe that there appeared to be an advantage of being a member of multilingual groups as they have access to two (or more) cultures and can operate in two (or more) different systems that may provide a space for a wider range of linguistic resources than those who belong to a monolingual community. Coming from a Persian[2] heritage background, I decided to compare and contrast the experiences that British-Iranian bilingual learners have in learning mathematics in different kinds of settings: in their complementary school (weekend school) where they can draw on more than one language (namely Farsi and English) when they are engaged in talking about mathematics with their teacher or with other students followed by their experiences in learning mathematics in their mainstream (Monday to Friday) school where they are experiencing learning mathematics solely through the medium of English. In terms of my 'original' research questions, my research interest fell in the following areas:

1) What prompts code-switching in a bilingual (English-Farsi) mathematics classroom in the UK? And
2) Does a complementary school 'complement' the work that goes on in mainstream school? And if so, how?

Throughout the process of data collection, there appeared to be an imbalanced ratio of interactional data collected in the two different institutional settings. Due to constraints of the

[2] Geographically people of Iranian background are people of Persian origin. Even linguistically when one is speaking of the Farsi language, one is talking about Persian.

research journey and being only partially successful in following each student into their corresponding mainstream school, I was left with an insufficient amount of data (from the mainstream schools) for the purpose of comparing between the two institutional datasets. For this reason, I decided to change the focus of my study on gestures as a resource for teaching mathematics in a bilingual complementary school. I will talk more about the change of the focus of my study in section 4.2.1. As well as changing the focus of my study, I decided to alter my research questions as a consequence:

1. What is the pedagogic nature of gesture (forms and functions) and other body-based resources in a British-Iranian complementary mathematics classroom in the UK?

To the best of my knowledge, there has not been any research focusing on British-Iranian bilingual learners' experiences in learning mathematics in their complementary schools. This fact stresses the nature and the significance of the study. I am aware that Parvanehnezhad and Clarkson (2008) and Fardinpour (2004; 2011) have carried out research in Australia and in North-West Iran (respectively) focusing on bilingual Farsi-English and Azari-Farsi mathematics learners.

1.4 Complementary schools

Complementary schools are community education institutions in which both learners and teachers have a greater access to a range of linguistic resources which "seem to offer a window onto a multilingual England [which is] often hidden from the view of policy makers in mainstream education" (Blackledge & Creese, 2010:11). Complementary schools provide a space for performance of alternative languages, heritages and histories (Creese *et al.*, 2006). Complementary schools usually run outside the hours of mainstream schools and are often based in a variety of settings including private home or a community centre. These voluntary institutional educations are "non-statutory schools, run by their local communities, which students attend in order to learn the language normally associated with their ethnic heritage" (Blackledge & Creese, 2009:459). What seems to be at the heart of complementary schools is creating multilingual spaces (Creese *et al*., 2006), about using language flexibly (Blackledge & Creese, 2010), and using a full range of young learners' linguistic repertoires (Creese & Blackledge, 2010). Learners often attend complementary schools in order to learn the language that is associated with their heritage group.

In the United Kingdom, the term 'complementary school' is used for these voluntary educational settings that teach community languages and draw upon cultural/religious practices that are acceptable and valued to that specific community. Previously and also concurrently, these schools were also known as supplementary schools. These independent institutions are commonly called 'heritage language schools' in the Unites States and Canada (Hornberger, 2005a) but in the early part of the twentieth century, these schools were commonly referred to as "ethnic-community mother-tongue schools" (García, 1988). In Australia, these schools are recognised by the name 'community language schools' or 'ethic schools' (c.f. Blackledge & Creese, 2010). In the recent studies (Martin, *et al*., 2006; Creese *et al*., 2008; Blackledge & Creese, 2010:47), the term 'complementary schools' is used which emphasises "the positive complementary function of these teaching and learning environments in relation to mainstream schools". I will settle on the same term because these 'community education institutions' complement the work that goes in the mainstream school (see Farsani, in press).

1.5 The structure of the thesis

In chapter one, I have set the background context about my bilingual experiences of learning mathematics and I have stated my 'initial' aims and objectives of this project. Chapter two constitutes the literature about the *verbal* resources for meaning making where I will be looking at different variations of language in discourse. For example, I will examine the intrinsic relation between mathematics and language in terms of the linguistic notion of register. Furthermore I will raise attention to culturally and socially constructed use of language in discourse in terms of code-switching. Chapter three follows on from chapter two where I will examine nonverbal (visual) and other semiotic aspects of language. Particular attention will be paid to the role that gesture plays in mathematical discourse as a resource in meaning making. More specifically I present aspects of the literature to show how learners and teachers synchronously draw upon and interchange between their verbal and nonverbal repertoires to convey mathematical meaning.

Chapters four and five address methodology. They concern the discussion of data collection and discussion of data analysis respectively. In chapter four, I discuss a shift between my original and current research design which had changed not only my original focus of the study, but also my initial research questions. Furthermore details about the rationale and process of data collection are discussed. Leading on from the discussion of data collection in chapter four, chapter five highlights the process of data analysis. Furthermore, in this chapter I establish a framework under which my data will be analysed. This framework encounters both verbal and nonverbal aspects of language that are used in a bilingual mathematics classroom.

Chapters six to nine are my analysis chapters. Chapters six to nine examine the nonverbal aspect of language in communication. In chapter six, seven and eight I will present an in-depth analysis of the classroom teachers' and students' use of gesture and how gestures conveyed different aspects of the instructional information. Chapter six primarily focuses on '*iconic gestures*' that were enacted by interlocutors. Chapter seven examines how '*metaphoric gestures*' conveyed pertinent mathematical information. Chapter eight scrutinises forms and functions of '*deictic gestures*' in communication around mathematics.

Chapter nine encapsulates and offers another perspective on the nature of gestures that are produced to convey mathematical meaning in a British-Iranian complementary school. Moreover, I will show the cross-linguistic variation of gestures; that is how different languages (Farsi and English) may generate two different gestural representation of the same mathematical concept.

Finally chapter ten is my conclusion where I will encapsulate issues to do with mathematics and communication in a particular British-Iranian complementary school mathematics classroom. At the end I will suggest a way forward for future research and implications.

2. Chapter two – Verbal language: resources for meaning making

2.1 Introduction to the chapter

I am interested in examining different varieties of semiosis that are used in human interaction. These varieties or semiotic resources can be verbal, vocal or visual (see Mehrabian & Ferris, 1967; Mehrabian & Wiener, 1967). In this chapter, I will be focusing on linguistic (verbal) resources for meaning making, and in chapter three; I will examine nonverbal (visual) resources, such as gestures in communication.

This chapter examines how language as a social and cultural construct is used as a resource in meaning making. This literature is relevant to the analysis of data in later chapters. In particular, I will address different variations of language use in discourse. These include the linguistic notion of register and code-switching. These two areas of variation are important to this thesis because of their intertwining nature in a Farsi-English complementary school mathematics classroom. In section 2.2 I will consider different variations of register in a mathematics classroom. For instance, I will examine how the interplay between the technical (mathematical) register and nontechnical (vernacular) can be used as a resource to keep a learning task moving forward.

In section 2.3, I will draw attention to the linguistic notion of code-switching. I will summarise the recent discussion on code-switching and translanguaging to depict language fluidity and movement across/between languages. I will later come to acknowledge that, code-switching, just like register, is another linguistic resource. Code-switching not only increases participation amongst speakers but mediates understanding across languages in the subject of mathematics. The notion of code-switching is of particular interest in this thesis especially of its relevance to the analysis of later chapters, where I will be examining the unbounded and fluid nature of languages used in discourse.

2.2 Register

In this section I will be talking about register from a sociolinguistic perspective. I will extend this notion to the concept of mathematics register. Having defined register, I will then examine the employment of mathematics (also known as technical or formal) register and non-mathematical (also known as non-technical or informal) usage of language to make mathematical meanings. Towards the end of this section I will draw attention to how the non-mathematical talk (or the vernacular) can be seen as a resource in mathematical discussion when learners are unable to access the technical register.

2.2.1 Mathematics register

'Register' is the use of particular language variations that are appropriate in specific situational contexts. The characterisation of the term 'register' refers to the work by Halliday, McIntosh and Strevens (1964) and is significantly and comprehensively referenced by mathematics educators (Cuevas, 1984; Pimm, 1987; Zepp, 1989; Winsløw, 1998a) who have taken up Halliday's work. The social semiotic theory of communication was first proposed by Halliday (1975; 1978) where he specifically speaks about mathematics. He defined mathematics register as the way in which language, symbols and other visual representations are used to create a particular meaning found in mathematics. This is referred to as the 'mathematics register'. He summarised mathematics register as follows:

> "A set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (Halliday, 1975:65; 1978:195).

In the literature 'mathematics register' (Halliday, 1975; Pimm, 1987) is often referred to as 'conventional language' (Miller, 1993); 'mathematical English' (Kane, 1970); 'academic register' (Castellón, 2007) and through my classroom observations, mathematics teachers often refer to ‘academic language’; ‘academic English’; ‘academic register’; ‘technical discourse’ and ‘subject specific language’. In this thesis in order to comply with Halliday’s and Pimm’s definition, I will settle on the term ‘mathematics register’. Linguists use the term register to refer to the meanings that serve a particular function in the language, as well as the words and structures that convey those meanings. With regard to Halliday’s definition, mathematical register could be regarded as a set of vocabulary and meanings that belong to the language of mathematics with respect to the particular language that is being used e.g. English mathematics register, Farsi mathematics register etc. For example the term ‘hypotenuse’ within a mathematical context refers to the largest side of a right angled triangle. Hypotenuse is a specialised word that falls within the English mathematics register which is predominantly used in mathematics, as opposed to other social or academic disciplines. The language of mathematics can be classified as a register which requires a different understanding of word use and forms of argument, explanation, proof and so on. The English mathematics register has its own specialized vocabulary, and sometimes uses many English words in non-standard ways (Gibbs & Orton, 1994). For instance, “a group in the mathematical sense is not the same as a group of corporations or a discussion group” (Zepp, 1989:11). Other examples I have observed include terms such as ‘let’ and ‘property’. The meaning of these terms in a mathematical context is highly specialized and can be distinguished clearly from their meaning in the housing market. The use of different registers reflects the fact that language has many functions. The notion of ‘register’ refers to meanings that serve a particular function in the language, which relate one area of social activity to another.

Within the context of mathematics education, others have viewed mathematics register ‘as an extension of language’ (Weinzweig, 1982) that facilitates communication in mathematics classrooms. The meanings that arise from English mathematics register are much narrower in scope and mastering the mathematics register (in any language) helps to describe and express properties and relationships specifically and to ‘point’ with words (Pimm, 1987). However, it is worth noting that any language serves as a tool of communication, a special encoding of the human competence for expressing ideas, feelings, and thoughts (Schweiger, 1994) and it would be as foolish to attempt to write a love poem in the language of mathematics as to prove the Cauchy Riemann Theorem using the English language (Schwarzenberger, 2000). Mathematics register is a variety of language in discourse which restricts expressing feelings, emotions and opinions that can inevitably arise and be expressed in any language (Winsløw, 1998b).

Mathematics register has developed a set of ‘specialised symbols’ (Pimm, 1995). The specialised symbols that are employed in mathematics vary cross-culturally due to social convention. For example, Hirigoyen (1997:165) differentiates the usage of mathematical symbolism between different countries (e.g. using billion to mean 10 to the power of 9 or 10 to the power of 12). In Iran and Germany, million, milliard and billiard respectively represent $10^6, 10^9$ and 10^{12}. Another example to illustrate the difference in writing mathematics which is culturally based is when looking at the thousands separator (or digit group separator) English uses commas (,) to divide large numbers into groups of three figures, by separating off the thousands and the millions 79,521,989. Sometimes dots (.) are used instead of commas (,) in different countries for instance in Germany. In Iran, ‘subscript slashes’ are used instead $1_/277_/452$. Spaces are also possible to denote the same meaning for instance, there are 1 000 metres in a kilometre. Mathematical registers can also be expressed in terms of certain symbolic notations as well as terms of grammatical structure, such as ‘nominalisation’, ‘passive structure’, and the use of inclusive ‘we’ rather than the personal ‘I’ (Morgan, 1998).

It is interesting to note that the mathematics register does also include vernacular terms to convey mathematical meanings/properties. Mesthrie et al., (2000) define vernacular forms of language as informal types of speech involving relaxed conversation between friends and peers that are often ignored in the classroom. For example, the same word can be used in the context of mathematics to convey a mathematical meaning and used in a social context to convey a non-mathematical meaning. For example, mean. The technical terms that are primarily employed in the context of mathematics are often formed from a combination of elements of Greek and Latin words, such as polynomial, hyperbola, asymptotic, etc. (Halliday, 1975). Most of the English mathematical register used today do not have any linguistic support since "the history of English mathematical vocabulary is so long that in many cases, the original metaphor is no longer apparent to the contemporary users" (Núñez, 2006:173). For example, the etymology of the term polynomial has its roots in Greek and Latin. In Greek 'polus' means 'many' and in Latin, 'nomen' cognates to the English 'name', "so that a polynomial is an expression involving *many names*, i.e. terms" (Schwartzman, 1994:168). Therefore an integrated part of the English mathematics register consists of calque or loan translation from other languages, mostly from Greek and Latin (Schweiger, 1994).

Having discussed what constitutes a mathematics register, in the next section I will consider the employment of both mathematical and non-mathematical terms and their fluid interchangeable nature in classroom discourse.

2.2.2 Movements between mathematical and vernacular register

Often movements between mathematics register to vernacular, or in other words the interplay of vernacular terms within mathematical discourse, is regarded as bad practice. This section offers two different views on the interplay between the mathematical and vernacular words in a mathematics discourse. First I will identify how such movements between different varieties of language use (mathematical versus vernacular) can be seen as a source of ambiguity but later I will describe how such variation in language use may provide access to additional resources to assist in understanding mathematical concepts.

Research in the mathematics classroom has highlighted that the shift between the mathematics register and everyday language is a particular and problematic source of ambiguity for students (see Monaghan, 1999; Rowland, 2000). Farrugia (2007: 23) speaks of "one of the difficulties that might arise for children when moving from everyday language to mathematical language is the problem of ambiguity of meanings". It appears that learners have problems with words that have one meaning in mathematics and another in ordinary English (Monaghan, 1991; Durkin & Shire, 1991). In many mathematics classrooms, there is a mix of ordinary English with the mathematics register (Verschaffel *et al*, 2000; Kane, 1968). According to Pimm (1987:75), teachers can employ a mathematical term in a mathematical context with a pupil trying to interpret what they have heard as ordinary English "thus trying to use non-mathematical meaning in a mathematical context". I would like to offer a couple of examples where students have failed to distinguish the terms 'volume' and 'difference' in a mathematical context that carry mathematical meaning. The following quote is between an interviewer and a secondary school learner who fails to or wilfully choses to interpret 'volume' in a mathematical sense:

> "Interviewer: Do you know what volume means?
>
> Child: Yes
>
> Interviewer: Could you explain to me what that means?
>
> Child: Yes, it's what is on the knob on the TV set" (Hart, 1981: 21).

Letting learners talk enables understanding to be clarified and any misconceptions to be addressed (Griffiths & Clyne, 1994). One interpretation is that the boy is just being silly.

Another interpretation leads to the realisation that the employment of the term ‘volume’ in context appeared to convey a meaning that was not mathematical. The employment of the vernacular ‘volume’ in the context of mathematics served as an immediate noticeable indicator of an error in communication between the interviewer and the learner. This example sits very well with Berenson and Vidakovic’s (1996) observation. They observed that although some learners might employ the same technical mathematical terms as the teacher, the meanings for learners might be very different from that of the teacher. It is therefore possible to believe that in a mathematics lesson, a learner could learn and develop a term’s meanings in a non-mathematical context. Furthermore just because a learner may employ a specific vocabulary, there is no guarantee that there will be mathematical understanding (Cuevas, 1984; Mestre, 1988; Moschkovich, 2000; 2002; Secada, 1992).

Pimm (1987:8) has presented an example of when the term ‘difference’ is used by a teacher in a mathematics context, whose meaning involves the notion of subtraction, and a learner’s failure of interpretation.

> In response to the written question, ‘What is the difference between 24 and 9?’, one nine-year-old replied, ‘One’s even and the other’s odd’, whereas another said, ‘One has two numbers in it and the other has one’.

The term ‘difference’ in English is used not only in the social context but also it serves as a technical means of expression in the English mathematics register. Furthermore the term ‘difference’ is employed to encompass both ‘quantitative’ and ‘qualitative’ characteristics. Interestingly, there are two different terms in the Farsi mathematics register to denote either quantitative (*ekhtelaf*) or qualitative (*tafaavot*) characteristics. In this example, in the absence of the technical mathematical meaning, the everyday meaning (which focuses more on the *qualitative* difference between 24 and 9) takes over to make sense of the task, and the failure to distinguish between English mathematics register and everyday English can cause "incongruous errors and breakdowns in communication" (Pimm, 1987:88).

It is not surprising that learners feel insecure when they talk about a particular topic as they are engaged to use a discourse that they have not yet made fully their own (Lee, 2006). Often learners of mathematics try to describe mathematical relationships with their non-mathematical language which is vague and imprecise. Nevertheless, the non-mathematical code, at least initially for students, has a greater power to communicate their intended meaning (Zazkis, 2000). Castellón (2007:163) has observed that learners in their mathematics classrooms "often substituted common speech for the mathematical terminology" but Gibbons (1998:99) argues that "children's current understandings of a curriculum topic, and their use of familiar 'everyday' language to express these understandings, should be seen as the basis for the development of the unfamiliar registers of school." When the question of moving between the everyday and mathematics register in negotiating mathematical meaning in classrooms is addressed, the vernacular usage of language is often seen as a resource to aid participation in mathematical talk. For example rather than saying the term hypotenuse, learners can convey meaning by expressing 'the longest side of a triangle' or 'the side opposite the right angle'. This can be seen as a switch between different styles, vernacular versus mathematical. To 'style-switch' (García, 2009) is a way to achieve a full range of expression in a mathematics lesson by switching from 'the formal written mathematics to informal spoken English'; 'technical to vernacular' and 'by demystifying a noun phrase to an ordinary verb phrase' (Farsani, in press). Some mathematics educators would emphasize that all learning is analogical, so thus is necessary step towards fluency in mathematical register.

This notion can be extended to a multilingual context. Teaching and learning mathematics in multilingual classrooms can occur where the mathematics register is only available in the language of instruction. In some cases, the medium of instruction can be learner's second or even third language (Setati & Adler, 2000); therefore everyday talk could be seen as a resource to engage students in the mathematical practices taught in the classrooms (Moschkovich, 2010).

This section dealt with how register can be seen as a resource for meaning making which serves a particular function in language. Furthermore I have talked about different variations in register in mathematical discourse, such as technical and non-technical and how they are intertwined. There are two contradictory views about different varieties of language use in a mathematical discourse. I have shown that just because a learner picks up and uses a technical phrase in mathematics (e.g. 'volume') does not necessarily mean that the child has understood the idea, or if there is not a strong connection between the picked up term and the underlying mathematical idea (Zepp, 1989). On the other hand the employment of vernacular language in a mathematics context can be seen as a way in which a learner is engaged with the task by overcoming the unfamiliar registers which results in an increase in participation in mathematical talk. In the next section, I will talk about code-switching and how language alternation between the medium of instruction and learners' home language can be seen as a resource in bi/multilingual mathematics classrooms.

2.3 Code-switching

In this section, I will draw on how different languages or variations of languages are seen as resources that help construct meaning. This section not only defines what code-switching is from a socio-linguistic perspective, but highlights how code-switching can be perceived from a bilingual perspective to depict language fluidity and movement. I will raise awareness to the essential differences between the term 'code-switching' and 'translanguaging' and provide a review of the historical development of the term translanguaging. Later, I will draw attention to research in global contexts where subject knowledge is taught in bilingual settings, (in particular in bilingual mathematics classrooms) and how code-switching can be seen as a resource which keeps a learning task moving forward and aids in accomplishing the lesson objectives (see Heller, 2001). In other words, I will show how code-switching can be seen as an instrument of communication which can facilitate learning by bridging the learner's home language with the language used in teaching and learning.

2.3.1 Code-switching from a bilingual perspective

To interchange between two or more languages within a single communicative event in the field of socio-linguistics is often referred to as code-switching (Gumperz & Hymes, 1972; Gumperz, 1982; Heller, 1999). In the literature, 'code-switching' or 'language alternation' is also referred to as 'language mixing' (Redlinger & Park, 1980), and 'code-alternation' (Auer, 1984). In this section I would like to raise awareness to the linguistic notion of code-switching that has traditionally been studied to the definition of the term that I will intend to use in this thesis. I would like to stress how this notion has changed from its original format which examined the complexity of hybrid languages to flexible competencies of bilingual practices.

Code-switching has traditionally been studied as the complexity of hybrid languages which often forgrounded the notion of language or codes as independent autonomous systems. Languages were assigned to separate territories even with separate functions that could co-exist in the same communicative event. Only in the recent years attention has been given to the importance of what someone does with his/her linguistic repertoires rather than what languages s/he uses. That is how languages can be perceived as resources in meaning making practices rather than just as codes with solid boundaries (Pennycook, 2010). Jørgensen (2010) also pays more attention to the nuanced description of the interplay of practices of bilingual speakers. He asserted the notion of 'polylingualism' to depict language users' employment of every feature of linguistic repertoire at their disposal in order to achieve their communicative aims as best they can. These views are captured by the recent use of terms “metrolingualism” (Otsuji & Pennycook, 2011) and “code-meshing” (Canagarajah, 2011) to reflect bilingualism as fluid resources in meaning making practices. García (2009) and Creese and Blackledge (2010) have employed the term ‘translanguaging’ to depict language fluidity and movement rather than linearity. Creese et al (2011) focused on the flexible rearrengments of young bilingual learners’ range of linguistic repertoirs in UK complementary schools. Translanguaging includes code-switching but pays more attention to the choices that are made through the agents (speaker’s perspective) and not from language perspective as code-switching has often been studied. Often bilingual discursive practices are discussed from a monolingual perspective or a monoglossic lens that views each language as a separate autonomous system. Translanguaging looks at the discursive practices and language choices from a bilingual’s perspective *not* a monoglossic perspective.

	Code-switching	Translanguaging
Emphasis on	Code	Voice
Emphasis on	Language	speaker

García (2009:7) argues for a need to move away from ‘monoglossic’ ideologies of bilingualism which “view[s] the two languages as bounded autonomous systems”. Therefore, translanguaging places its emphasis on the fluidity and dynamicity of languages that are used simultaneously through the user’s choices. In other words, to stress and emphasise the voice/speaker rather than the code or language is at the heart of this argument, and as Blommaert (2010:196) has observed, “it is a matter of voice, not of language”.

Translanguaging is explored in research in classroom contexts revealing all the linguistic resources that a learner draws up on to mazimise understanding and achievement. Both (or more) languages are flexibly used “in a dynamic and functionally integrated manner to organize and mediate mental processes in understanding, speaking, literacy, and, not least, learning” (Lewis et al., 2012: 655). According to Li Wei (2011:1123) “Translanguaging is both going between different linguistic structures and systems, including different modalities (speaking, writing, listening, reading, remembering) and going beyond them”. I will use the conceptual ideas that are discussed in the literature to support the analysis of data in later chapters. I will show several cases in the analysis chapters where British-Iranian bilingual learners with different linguistic profile are involved in group discussions and often ignore the language norms of the classroom, using language flexibly to convey their message. Furthermore, teachers not only draw flexibly across different linguistic but include different modalities such as gestures and writing (mathematical symbolism) as a resource for teaching mathematics to maximize understanding for learners with different linguistic profiles.

Seeing language not as static codes with solid boundaries but rather as fluid resources in meaning making practices therefore, in this thesis, I am interested examining what a speaker does with his/her resoures to convey meaning and contribute to the on-going flow of discourse. Thus, the focus of the attention will be on the ‘voice’ and ‘the speaker’ rather than ‘the code’ or ‘the language’. I am interesting examining how British-Iranian bilingual learners construct meaning utilising their diverse linguistic resources to maximise understanding, engagement and performance.

It is important to illuminate that in both the socio-linguistic and the mathematics education literature, there is an extensive use of the traditional term ‘code-switching’. Therefore in order to comply and to avoid confusion, I will employ the term ‘code-switching’ to refer to the choices that a speaker makes from a bilingual and speaker’s perspective.

Code-switching is distinguished from code-mixing and code-borrowing in language literature. Code-borrowing is often referred to as an insertion of single words or short phrases within a sentence in another language, where code-mixing is regarded as transferring linguistic units from one code to another. For example, Rasul (2009) divides the notion of code-switching into two different sub-groups which are inter-sentential (between sentences) and intra-sentential (within a sentence). She defines code-switching as an umbrella term that covers inter-sentential switching and intra-sentential switching and code borrowing. However, generally speaking the term is only used for inter-sentential switching and distinguished from intra-sentential switching which is called code-mixing. Inter-sentential switches often occur at the lexical level, but can also occur at the phonological levels (Moulton, 1966; McLaughlin, 1984).

Within the context of bilingualism and mathematics education, Moschkovich (2007) describes the complexity of how bilingual learners combine languages and use multiple forms of code switching, code mixing, and code borrowing to convey their meanings. It is interesting to note that sometimes we do not know if we are in one language or another as a result of code-borrowing (foreign terms), code-mixing etc. (Usiskin, 1996) but sometimes the languages we draw upon are precise and function effectively and make perfect sense. For example, within the context of learning mathematics in a British-Iranian setting, the term 'equal-e'[3] is not in English, nor is it in Farsi. Sometimes a term or a phrase, for example, 'equal-e' cannot be quantified or even qualified in a language or languages. This falls very well with the literature about fluidity and dynamic notion of languages. Languages are not fixed or static with solid boundaries but rather are fluid resources in meaning making practices. Sometimes it would be hard to discern whether a term is 'code-borrowed' from Farsi to English or from English to Farsi. For this reason, in this thesis I will employ the

[3] 'equal-e' means both 'Is it equal?' or 'It is equal'.

broader term code-switching to refer to any switches within and in between sentences as well as code-borrowing. My rationale behind this act is because sometimes as a result of employing an extensive amount of technical vocabulary from a different code within a sentence where it constitutes majority of its lexical composition, makes it hard to determine as to which code is the dominant one and which is mixed/borrowed in. For example it makes perfect sense for a British-Iranian to say “Seven factor *e* twenty-four *e*?” [Is seven a factor of twenty-four?], but, it would be hard to distinguish whether Farsi was the dominant code or English, and therefore, which code was mixed in. Very often these young bilingual learners ignore the language norms of the classroom, using language flexibly to convey mathematical meaning. I will talk later about how learners with different linguistic profile are involved in group discussion employing every feature of linguistic repertoire (verbal and nonverbal) at their disposal in order to achieve their communicative aims.

2.3.2 Code-switching and bilingual education

Multilingualism and multiculturalism is now seen as one of the main social factors of 21st century. As a result of migration[4], marriage, education, temporary residence or being the offspring of couples who are themselves members of a bilingual community, education in many countries of the world takes place in multilingual contexts and “between half and two-thirds of the world population is bilingual” (Baker, 2001:8). Nowadays everyone is experiencing multiculturalism and multilingualism in classrooms, playgrounds and in most public places (Hoffmann, 1991). Education in many countries of the world takes place in multilingual contexts. Many students are experiencing learning mathematics through two or more languages in bilingual classrooms in United States (Khisty & Chval, 2002; Moschkovich, 1999, 2002, 2007), in South Africa (Adler, 1998, 2001; Setati, 1998; Setati & Adler, 2000), in Wales (Jones & Martin-Jones, 2004), in Australia (Ellerton & Clements,

[4] E.g. due to wars, political factors and economical issues

1991), in Papua New Guinea (Clarkson & Galbraith, 1992; Souviney, 1983), in Iran (Fardinpour, 2004; 2011), in Italy (Gajo & Serra, 2002) and in the UK (Farsani, 2011). Exploring research in bilingual educational context not only will enhance our understanding of linguistically (Gorgorió & Planas, 2001; Barwell & Setati, 2005, Parvanehnezhad & Clarkson, 2008) but cultural diverse mathematics classrooms (Barton, 1996, 2008; Barton *et al*., 2006). Code-switching appears to be an integral part of research in bilingual educational contexts. Code-switching is more visible in bilingual or multilingual settings where learners have the opportunity to switch between their linguistic resources (Adler, 1998; 2001). Martin (2005:88) describes code-switching in Malaysia: "the use of a local language alongside the 'official' language of the lesson". The literature on the nature of code-switching is of particular interest in this thesis as it helps to understand flexible rearrangements of young bilingual learners' practices in a bilingual mathematics classroom. A great deal of attention is paid to code-switching in multilingual mathematics communities where the medium of instruction differs extensively from the native language of the students and their teacher (Setati, 1996; Adler, 1995; Setati *et al*., 2002). I include here research on mathematics teachers who are code-switching in classrooms; for example, see Setati (1998), Setati and Adler (2000). Adler's (2001) study of code-switching in mathematical conversations is a good example of research that addresses language choice. For example, in urban townships and among many other indigenous provinces in South Africa, there is a great deal of code-switching in learner-learner conversations in mathematics classrooms. This is where the learners' most dominant language is not English and English language happens to be learner's second, third or even fourth language (Moschkovich, 2002). Learners did the mathematics in their most proficient language and English was used as a medium of transmission; to transfer their already formed ideas to the class. The discussion of this literature is relevant to the analysis later chapters where bilingual learners draw across full range of their linguistic repertoire in order to negotiate meaning in their lessons.

In this thesis I would like to pay attention to the term ‘bilingual’ as it is a generic way that describes very complex phenomena. Bilingual learners often switch and mix between all the languages they have at their disposal in different situations and with different people for different purposes (Baker, 2001; Moschkovich, 2010) to exclude or include others (García, 2009) and to be engage with a wider audience (Blackledge & Creese, 2010). “In England the term [bilingual] is used to refer to pupils who live in two languages, who have access to, or need to use, two or more languages at home and at school. It does not mean they have fluency in both languages or that they are competent and literate in both languages” (Hall, 1995:10). Cummins (2008: XIII) defines bilingual education as “the use of two (or more) languages of instruction at some point in a student’s school career”. In the literature bilingualism is defined as “the practice of alternately using two languages will be called bilingualism, and the person involved, bilingual (Weinreich, 1968:1). Others have defined bilingualism “as the alternate use of two or more languages by the same individual” (Mackey, 1970:555) or “[t]he ability of a person to use here and now two or more languages as a means of communication in most situations and to switch from one language to the other if necessary” (Oksaar, 1983:19). Within the context of mathematics education, the term bilingual is referred to someone if they can draw upon more than one of his/her linguistic repertoires during mathematical conversations or when carrying out arithmetic computation (Moschkovich, 2007; 2010). Drawing up on García’s (2009) definition of bilingualism, in this thesis I will use the term bilingual as terms such as second-language learner or EAL learners are problematic from a bilingual perspective. Problems encountered as a result of employing the terms EAL or second-language learners often refer to either the matter of definition (e.g. is a second-language learner someone who speaks with an accent?) or with regard to time direction (at what stage does one stop being a second-language speaker?). In this thesis I use the term bilingual to emphasise the fluid and flexible nature of the linguistic features that interlocutors employ to achieve their communicative aims.

2.3.3 Code-switching as a resource in meaning making

In this section I will report on the benefits of code-switching in multilingual contexts and, in particular, how it bridges the medium of instruction and learners' home language(s). Much of the literature on code-switching within the context of bilingual mathematics classrooms has focused on the way in which the use of two or more languages may assist and facilitate learning (Clarkson, 2005; Barwell, 2005b; Adler & Setati, 2000) and in particular, the way in which learners' 'most dominant language' can help them gain knowledge through an additional language. In this section I will report on the literature around learners' choices of language/s to gain epistemological access rather than socio-political access (see Setati, 2005).

Code-switching is believed to be a pedagogic means of expression which occurs "to bridge the gap between the knowledge acquired by the students through the medium of their first language(s) and the knowledge of the school mediated through . . . the language of instruction" (Martin-Jones & Heller, 1996:9). This finding is in line with Martin (2005:89) who speaks of code-switching as offering classroom participants "creative pragmatic and 'safe' practices ... between the official language of the lesson and a language which the classroom participants have a greater access to". Arthur and Martin (2006) speak of code-switching as a tool that provides greater curriculum access for bilingual learners. Code-switching enables exploring aspects of the lesson content in additional depth and promotes understanding (Canagarajah, 2001). For example the negotiation of meaning through the use of both languages [French and Italian] induces deep processing, as it enables learners to establish links between unfamiliar items in the input and their previous knowledge (Gajo & Serra, 2002:81).

I am interested in scrutinizing how a variety of languages (and other semiotic resources) can be a resource that offers a different window on learning mathematics. Language diversity in multilingual contexts is often seen as a resource that offers new learning techniques (Barton, 2008:174) and "multilingual classrooms are potentially fertile mathematical learning environments because of their linguistic richness". The relevance of this quote will be discussed in chapter nine, where a concept in one language indexes one thing and the same mathematical concept in another language indexes a different concern. Overall, the employments of two languages work together, to convey a point more richly. Switching between languages appears to be one of the sources of empowering mathematical reasoning and learning (Clarkson, 2005; Barwell, 2002, 2005a, 2005b). By switching between languages during a mathematical task it can be easier for learners to understand a problem (when it is perceived to be difficult) (Clarkson, 2005). In addition, Clarkson (2007) reported that code-switching in mathematics lessons served a metacognitive function which facilitates bilingual learners to gain access to supplementary or alternative meanings and relationships. In Pakistan for example; code-switching is seen as a realistic practice in multilingual mathematics classrooms (Halai, 2007; 2009) and swapping between the learners' dominant language and English appears to be a fundamental resource for enabling mathematics learning for learners for whom the medium of instruction is not their dominant language.

Other than how code-switching is perceived to be a bridging pedagogy that facilitates learning, its role in aiding participation has also been acknowledged. Code-switching is seen as a way of engaging with a wider audience and to increase participation (Creese & Blackledge, 2010; Clarkson, 2006; Martin-Jones, 2000; Arthur, 2001). García (2009:307-8) speaks of code-switching as a "powerful mechanism to construct understandings, to include others, and to mediate understandings across language groups". Moreover, it has been reported that teachers often code-switch to ensure motivation (Lin, 1999) and to develop a less formal relationship amongst the interlocutors (Arthur & Martin, 2006). Furthermore Zentella (1997:19) has reported that often teachers' code-switching is to imitate the language the learners have used, and by doing so the teacher is "following the child". From a bilingual perspective, code-switching continually appears to be used as a resource that helps to construct meanings, mediate understanding and increase participation.

Code-switching through the bilingual perspective plays a role in communication, both in facilitating the speakers' language production and in promoting learners' comprehension. Indeed Hornberger (2005b:607) has observed that "bi/multilinguals' learning is maximized when they are allowed and enabled to draw from across all their existing language skills (in two+ languages), rather than being constrained and inhibited from doing so by monolingual instructional assumptions and practices". Again, code-switching through the bilingual perspective offers a wide variety of opportunities to use bilingual instructional strategies that not only acknowledge the linguistic resources that bilingual learners have at their disposal but also to promote 'cross-language transfer' (Cummins, 2008). Through the use of two (or more) languages meaning is negotiated and by integrating both languages "learners … establish links between unfamiliar items in the input and their previous knowledge" (Gajo & Serra, 2002:81). In other words, code-switching can be seen as an approach that can support content and language learning itself (Creese & Blackledge, 2010).

In this section I have discussed the difference between code-switching and translanguaging in terms of 'the agent's perspective', that is the representational choices that a speaker makes to convey meaning. Unlike code-switching, the representational choices that a speaker makes between languages are regarded as being fluid and dynamic, and are not viewed as bounded autonomous communicative system. Furthermore I have talked about what could prompt code-switching in a mathematical context. And finally, I have illustrated how code-switching can be seen as a communicative strategy that bridges learners' experiences in home, community and school which offers a greater linguistic dynamism. Code-switching not only helps recognise diversity and social cohesion, it also offers learners intellectual, cognitive and social advantages (Genesee *et al*., 2006).

2.4 Conclusion

Verbal language offers different resources for conveying ideas. Different variations of language use, such as registers or code-switching are culturally given and socially shape meaning making materials. These linguistic resources are used in conveying meaning making by language users. Both monolingual and bilingual learners have access to a range of ways of using language, depending on who they are speaking with, or what they are writing about. However, code-switching can be seen as the most salient difference between bilingual and monolingual learners, and one of the social functions of code-switching is the negotiation of language choice, which is also unique to bilingual learners (Hoffman, 1991). Code-switching is not simply a combination and mixture of two languages, but it is believed to be a creative strategy used by the language users (Li Wei, 2011).

3. Chapter three – Nonverbal language: resources for meaning making

3.1 Introduction to the chapter

As we have seen in the previous chapter, I am interested in examining different varieties of language that are used in human interaction. These language varieties or linguistic resources can be verbal, or nonverbal. In this chapter, I will examine nonverbal (visual) resources for meaning making, such as the role of gestures in communication. One of the reasons why I am so interested in nonverbal communication is because of its importance in teaching and learning as often research does not address the role of nonverbal communication in teaching and learning environments (Lemke, 1998). Moreover, Roth (2002:365) has observed that, "[t]here exists very little educational research concerned with the role of gestures in learning and teaching, particularly in the subject areas that have been characterized as dealing with abstract matters such as science and mathematics". Of course, this is now a dated comment as it is no longer true.

I will start off this chapter by drawing attention to the general literature on nonverbal language in section 3.2. This literature entails a substantial amount of information about the nature of gesture categorisation and, the role of finger pointing in discourse. The relevance of this literature and its connection to my study will be explored.

In section 3.3, I will be scrutinising the role of gesture in human communication. Particular attention will be paid to the symbiotic nature of gesture and speech in human communication. Furthermore, I will discuss the nature of the information that is conveyed in gestures and in speech, and, how these two modalities (verbal and visual) complement one another in discourse.

Section 3.4 is taken as a bridge in this thesis which connects the literature on nonverbal communication to the literature on mathematics education. In this section I will be reporting how gestures are taken to function as visual amplifiers to what interlocutors express in words around mathematics. Section 3.5 takes an account of cross-cultural variation of meaning generated through gestures, and I will end this chapter by a conclusion in section 3.6.

3.2 Kinesics

3.2.1 What is non-verbal language?

In brief, the nonverbal language "refers to the communicational functioning of body activity, gesture, facial expression and orientation, posture and spacing, touch and smell, and of those aspects of utterance that can be considered apart from the referential content of what is said" (Kendon, 1981:3). In the literature the term kinesics is used to encounter any bodily movements such as gestures, facial expression etc., as a means of communication. For example, Hockings (1995:509) define kinesics as "a science of human gesturing communication". On a similar ground, Collier (1995:235) defines kinesics by means of "the significance of body expression". In particular, my interest falls within the communicative function of gestures. The term 'gesture' is used in a wide sense, as a physical movement of a part of the body (e.g., hands, arms, eyes and face) (Kendon, 1983; Maschietto & Bussi, 2005; 2009) in communicative situations (Kendon, 1997; Streeck, 1988). McNeill (1992:11) used the term gesture to mean "movements of the arm and hands … closely synchronized with the flow of speech". Kendon (1972; 1980) refers to the spontaneous hand movements produced while talking as 'gesticulation'. In this thesis, gestures are defined as movements of hands or arms that are synchronously produced in the act of speaking. Along similar lines, Sfard (2009:194) defines gesture as a "body movement fulfilling communicational function" (Sfard, 2009:194) that are co-produced with speech. In the literature, the spontaneous movements of the hands and arms that accompany speech is referred to as 'speech-accompanying gesture' (Kita, 2009); 'speech-associated movement' (Kendon, 1980); 'speech and discourse-oriented gestures' (McNeill, 1985); 'speech-associated gestures' (Kendon, 1988; McNeill, 1992); 'speech-synchronized gestures' (McNeill, 2005) and speech-gesture co-production (Núñez & Cornejo, 2012). Speech-accompanying gesture is synchronously timed with respect to the speech unit it accompanies and often is semantically coherent with the speech (Butcher & Goldin-Meadow, 2000).

In this thesis I will primarily focus on examining gestures and finger pointing that are co-produced with its accompanying speech unit, but this does not minimize the importance of other body-based resources (e.g. nod, gaze, postures) in a mathematical discourse. Gestures and pointing are of particular interest because those are easily seen and realised on an ordinary audiovisual recording in comparison to other modalities such as the direction of eye gaze (where particular close-up recording equipment is needed to detect eye movement). Moreover, I want to pay particular attention to how gestures complement the verbal message as they are produced in synchrony with the flow of speech. For further discussion about my rationale see section 5.2 of the methodology chapter.

There are different variations in hand gestures and finger pointing as there are with a language (as discussed in the previous chapter). This, however, does not mean that there are formal and informal ways of expressing ideas using hand gestures or finger pointing as there are with language. In the next section by drawing upon McNeill's (1992) gesture categories, I will discuss different forms and functions of hand gestures. Following discussion about different variations of hand gestures, I will discuss how different variations of pointing with index finger correlates with certain discourse factors.

3.2.2 McNeill's gesture categories

This section discusses different forms of gestures that are employed globally in human communication. There are gestures that are produced independently to their corresponding speech units (emblems) versus gestures that are produced in coordination with speech (speech-accompanying gestures). The first categories of gestures are often referred to as 'speech-independent gestures' (Knapp & Hall, 2006) are also known as 'emblems' (Ekman, 1976) or 'autonomous gestures' (Kendon, 1983; 1985). Emblems are "nonverbal acts that have a direct verbal translation … usually consisting of a word or two or a phrase" (Knapp & Hall, 2006:226). These are conventionalised gestures that have a common meaning in a particular cultural context. For example the 'thumbs up' gesture means 'good', 'ok' to most European countries: it also means *one* to an Italian, *five* to Japanese and *sixty* to an Iranian. I will later describe the cross-cultural variation of meaning in section 3.5 Cross-cultural variation in meaning. Therefore, emblems are gestures that are consciously sent and consciously received with specific meaning varying from culture to culture (Ekman & Friesen, 1969).

While emblems may be delivered in utter silence, the spontaneous gestures are produced in synchrony with the flow of speech or what I refer to in this thesis as speech-accompanying gestures. McNeill's (1992) basic gesture category is based on speech-accompanying gestures that are manifested by spatio-temporal bodily movements. His gesture category takes an account of gestures that are dependently produced with their corresponding speech units. He categorised four different forms of hand gestures where each form has a different function in human communication. There are 'deictic', 'beat', 'iconic' and 'metaphorical' gestures.

Deictic gestures consist of indicative or pointing movements which are often produced with an extended index finger but sometimes with the other fingers or the entire hand that serve to indicate locations, directions persons or objects that are either present or non-present in the environment. Deictic gestures do not convey perceptual or action information. They are just used to point to objects and locations. Just like deictic gestures, beat gestures carry no meaning at all. Beat gestures are rhythmically coordinated with the speech and the movement is often short and quick. Beat gestures do not convey the semantic meaning of speech but are "aligned with the prosodic prominence patterns in speech" (Gullberg, 1998:51). This kind of gesture is often more noticeable when produced by musicians or politicians.

The third gesture type is iconic. Iconic gestures resemble concrete objects or action which "depict the semantic content directly via the shape or motion trajectory of the hand(s) (e.g. tracing a triangle in the air to mean triangle)" (Alibali & Nathan, 2012:251). Iconic gestures convey the semantic meaning through the images created by the speaker. Finally the last gesture classification is metaphoric gestures. Metaphoric gestures are similar to iconic gestures in a sense that they convey the semantic content via metaphor. Metaphoric gestures "are like iconic gestures in that they are pictorial, but the pictorial content presents an abstract idea rather than a concrete object or event. The gesture presents an image of the invisible – an image of an abstraction" (McNeill, 1992:14). Iconic and metaphoric gestures are imagistic, they present an image that is either none/present in the environment.

I will use McNeill's gesture category to differentiate different forms and functions of hand gestures that are produced in coordination to speech. Chapters six, seven, and eight will be analysed based on McNeill's framework. Now I will turn to discuss the literature about different variations of finger pointing in a greater depth.

3.2.3 Pointing

"Pointing in sign languages is equivalent to, and used as frequently as, pronouns in spoken languages" (Kita, 2003:1). Pointing (or deictic) gestures are defined as gestures that are used to indicate objects, locations, inscriptions that are either present or non-present in the environment. In the literature 'abstract deixis' are pointing gestures that point to no concrete target. Deictic gestures in a conversation are often employed to reorient the attention of another person so that an object becomes the shared focus for attention (Butterworth, 2003). Deictic gestures enable the speaker to move from verbal to visual and call attention to an object they are indicating (Goodwin, 1986; Clark, 2003).

Pointing can be carried out in different forms and using different materials. For example most often pointing gestures are produced with an extended index finger, and sometimes with an extensible object e.g. a pen or a laser pointer. Interestingly enough, there are different body parts that can conventionally be used to point, with the lips (Enfield, 2001) and eyes (Wilkins, 1999). Unlike pointing in Western cultures (which is often regarded as a bad practice), Arrante culture has developed an interesting repertoire for pointing depending on the nature of object/referent. The functional differentiations in pointing is oriented depending on whether the object (or the referent) is singular (or non-singular), visible in space (or nonvisible), the distance (between the pointer and the object), and most interestingly, the pointer's position as a frame of reference to the object (see Wilkins, 1999).

Finger pointing may seem to be such a natural and trivial form of gesture. Pointing gestures are ubiquitous but they are not redundant in discourse. Pointing like gestures serve a communicative function and maximizes the communicative effectiveness when vocabulary is limited (Volterra & Iversen, 1995). In a conversation between two or more people, the addresser's deictic gesture serves

- for the listener's benefit (so that the location and the identity of the object can be clarified visually) and
- for the purpose of meaning making and transmit visual information (Butterworth, 2003).

Index-finger pointing is a visual part of human communication which often co- occurs with speech and aims to draw the interlocutor's attention to an object or event of interest (Masataka, 2003). There are different variations in index-finger pointing with

- 'the palm vertical'

- 'the palm down'.

Open hand pointing with the palm vertical, and thumb pointing are also common and each has a distinctive meaning in discourse (Kendon & Versante, 2003). Palm down index-finger pointing foregrounds the referred referent into the centre of discourse focus. It is referred to as 'object individuation' (Kendon & Versante, 2003:115) where it individuates a referent as being distinct from other objects. On the other hand palm vertical index-finger pointing "indicates a referent that is relevant to the current discourse but not in the centre of focus" (Kita, 2009:148). Often thumb is used in pointing when the location or the identity of the object indicated is not in the foreground of discussion, perhaps because the location or the identity of the object has been previously established. "This may be linked to what could be called its 'anaphoric' use, where something referred to a second time is indicated by the thumb, where it had been indicated by an index finger the first time" (Kendon & Versante, 2003:134).

What I find of particular interest is the degree of finger closure whilst pointing with an index finger. The degree of finger closure or openness among the little finger, ring finger and mid-finger while pointing with the index finger is correlated with certain discourse factors. It has been observed that emphatic or first mentions of events tend to be “regularly accompanied by the canonical (tightly bunched) index-finger point (which is often held in place). In follow up anaphoric mentions, or the mentioning of nonimportant participants, a looser hand is used (and the action is executed more quickly)” (Wilkins, 2003:193). The forms and functions of index pointing with a looser/tighter degree of closure and its corresponding pedagogical implications within a mathematics lesson will be discussed in section 9.3.

Figure 5: Pointing with a looser/tighter degree of finger closure

Figure 5 illustrates index finger pointing with two different variation of the degree of finger closure. In the top photograph George Bush is displaying an index finger pointing with an open degree of finger closure. The other two photographs are showing the North Korean official Son Kwang Ho. These two photographs were taken from the London 2012 Olympics (CNN). The Olympic organisers incorrectly displayed South Korean flags alongside North Korean women football players in one of the opening Olympic games. The tight degree of finger closure in these two pictures stresses and emphasises a strong message that conveys anger and hatred.

3.3 Role of gesture in human communication

This section discusses the role of our hands in communication. Our hands are always with us but they are often ignored. This section emphasises about the role of our hands, not as tools used to build things, but rather as instruments used to represent information. First in section 3.3.1, I will show that there is a strong symbiotic relation between gesturing and language production and how the two modalities are engrained in communication. Later in section 3.3.2 I will discuss the ways in which gesture convey substantial information that is different to speech. Furthermore the complementary meaning that is conveyed through both speech and gesture will be discussed.

3.3.1 The symbiotic nature of gesture and speech in communication

When we talk, we gesture. Gestures and pointing are ubiquitous whenever speakers express ideas verbally (Kita, 2003). Because gestures and pointing are so ubiquitous and we interpret them with such ease, pointing and gestures can come into view as trivial phenomena. In this section I will discuss the universal nature of gestures in human communication and later, the ways in which these two modalities (gesturing and language production) are intertwined.

People use gesture to point to places or to indicate size and shape of objects, and most importantly, to convey their meanings. Gesturing appears to be global; this is due to the fact that to date, there has been no report of a culture that lacks gestures. Each cultural group has developed repertoires of gestures (known as emblems) and very often are employed in conversation. Of course, the frequency of gesturing and the underlying meaning varies cross-culturally (see Barakat, 1973, and Sparhawk, 1976, for Arabic and Persian emblems respectively).

Although emblems are socially constructed resources for conveying meaning, the prime focus of this thesis is more on gestures that are temporally and semantically coordinated with the flow of speech. It is interestingly to note that, speakers are often unaware that they are gesturing at all. Gestures are often produced when the listener does not have visual access to the gesture (e.g. on the telephone) or in the absence of visual contact for example in news casting (Chapanis, 1971) or even in monologue (Bavelas *et al.*, 2008). Intriguingly, conversations among congenitally blind children involved gestures but the frequency of gestures is not as high as in sighted children (Iverson & Goldin-Meadow, 1998; Iverson, *et al*., 2000).

In one study (Alibali & Kita, 2010), children's hands were restrained inside a furry cloth muff in order to prohibit them from gesturing. Although this manipulation was effective at reducing gesture production but, it did not eliminate gesture entirely. They have noted that occasionally some children "used their muffed hands to indicate or characterize the objects, and some children occasionally tilted their heads or used their elbows to indicate objects" (*ibid*, p.9). This symbiotic relation between gesture and speech shows how deep-rooted these two modalities are in relation to the language production. Interestingly enough, research has presented evidence that the production of speech is not as smooth when gesturing is inhibited. Prohibiting speakers from gesturing influences the nature of the information conveyed in speech. There is suggestive evidence that reveal the prohibition of gesturing affects not only the fluency of speech, but also changes the *content* of speech and increases the time spent pausing (Rauscher *et al*., 1996; Rimé *et al*., 1984). This shows how deeply speech-accompanying gestures are engrained in the thinking process (Kita, 2009). Moreover, research has presented evidence that not only the production of speech is not as smooth when gesturing is inhibited, but learning itself. Cook et al. (2008) compared two groups of learners: first those who were required to gesture as they learned a mathematical concept versus learners who were not required to gesture. The first group of learners retained their new mathematical knowledge significantly more than learners who were not required to gesture while carrying out mathematical tasks. Along similar lines, Broaders et al. (2007) found that learners who were required to gesture during explanations of mathematics problems were later more receptive to instructions about the problems than those who were not required to gesture.

Further micro analysis illustrates the temporal relation between gesture production with its corresponding speech-like vocalization (Butterworth & Beattie, 1978). There appears to be a precise synchronicity between these two modalities; verbal and visual (Bates & Dick, 2002; Goldin-Meadow & Butcher, 2003). Mayberry *et al* (1998; 2000) provided evidence that stutters stutter in gesture too. They claimed that there appears to be a strong synchronic relation between gesture and speech appears even when, as in stuttering, the speech production process goes awry. It is believed that gesture and speech are temporally and semantically intricately coordinated, and synchrony between the two modalities is as an indication of sympathy, rapport, co-operation or conversational involvement (Wallbott, 1995). This synchronic relation also appears between pointing and its co-expressive verbal counterpart. Masataka (2003) claimed that the index-finger extension also is synchronized with speech like vocalisation.

Now that symbiotic nature of speech and gesture in language production has been addressed, in the next section I will focus on the semantic relation between these two modalities.

3.3.2 Complementary function of gesture in relation to its speech unit

In the previous section I discussed how gesture is mediated in coordination with speech and it is an integral part of multi-modal resources used in communication (Kendon, 2004). In other words, speech and spontaneous speech-synchronized gestures are an integrated part of language production and jointly present the same idea unit (McNeill, 2005). In this section I will stress how gesture and speech deliver a coherent message, in particular when the verbal message is ambiguous, abstract or complex (Alibali *et al*., 1997). Furthermore, I will discuss the way in which these two modalities semantically produce different but complementary representations (McNeill, 1985; Goldin-Meadow & Butcher, 2003).

The temporal relation between gesture and speech can be synchronous, but the semantic meaning conveyed in each modality may not be identical because they employ a different form of presentation. Gesture and speech not only encode meaning differently, but also express the information in a different format. The meaning that is conveyed through gesture is visual; it is a representational imagery of an idea or a concrete object. In contrast, speech conveys meaning through arbitrary words and relies heavily on the rules of grammar. Therefore, it is very difficult for the verbal and visual modes of expression to convey identical information to a message.

I am interested in considering to what extent the information that is conveyed in speech and gesture overlap (which I will now turn to). Later, in this section, I will describe the complementary functions of a message that is sent in gesture in relation to its verbal channel. Gestures are served as a visual representations, and often words and gesture can convey information that overlaps a lot. Church and Goldin-Meadow (1986) have called instances in which speech and gesture convey overlapping information 'gesture-speech matches'. On the other hand, they have called instances where the two modalities convey non-overlapping information 'mismatches'. For example, let us consider a child shouting out 'chair' whilst pointing to the chair. The word 'chair' indicates what it is (the object) but does not locate the object. In contrast, pointing clarifies the location of the object but does not classify the object. The two modalities do not convey *identical* information, but overlapping information. They work together to more richly convey the child's intention. Goldin-Meadow (2003) illustrated an example where there is a *mismatch* between the verbal and visual. This is where pointing and speech convey very little overlapping information. For example, a child can indicate an object by pointing that is not referred to in speech. While pointing to the chair, the child can utter 'daddy'. The information that is conveyed in gesture and speech do not overlap. This is a case of 'gesture-speech mismatch'. Interestingly, the information that is expressed in gesture cannot be found in speech (see Ericsson & Simon, 1993; Garber & Goldin-Meadow, 2002). However, the two modalities together convey a simple proposition – "the chair is daddy's" – that neither modality conveys on its own (Goldin-Meadow, 2003:25). Therefore, even in gesture-speech mismatches, although gesture and speech do not convey overlapping information, they work together to more richly specify the object. This literature is relevant to the analysis of data in chapter nine where there is a mismatch between the speech (expressing the underlying mathematical idea) and its accompanying gesture.

There is extensive evidence that gesture plays a functional role in speaking (Kendon, 2004) but it is unclear as to what extent or how much of the information is carried through gesture. I would like to draw attention to another example of gesture-speech mismatch and how the speech and gesture work together in harmony in conveying the underlying idea. Imagine someone utters 'I ran all the way upstairs' in words while spiralling his/her hand upwards. Again, the speaker has produced information in gesture that cannot be found in speech. It is only through the speaker's gesture that conveys not only the nature of the staircase was spiral but the way s/he went up was a spiral. What I find particularly interesting is when gestures convey information that is not articulated in speech. For example, let's consider the statement "it is raining" with fingers pitter-patter downwards. It is interesting to note that the meaning that is conveyed through pitter-pattering fingers downwards provides fundamental information about the physical properties of rain. For example, fingers pitter-patter downwards, would convey not just the concept of rain, but also additional information about how fast the rain falls in and in what direction. Such information about the physical properties of the rain, such as 'how fast' and 'in which direction' (vertical versus slanting) rain falls in was not articulated in the verbal utterance. This literature is relevant to the analysis of data in chapter seven where I will be examining how a classroom teacher's gesture conveys additional information about the *process* of ‘adding’ and ‘subtracting’ numbers.

It appears whether there is a match or mismatch between the verbal and visual channel, the two modalities in combination offer more in conveying the underlying idea than they do alone. In other words, meaning is made using (verbal and nonverbal) signs flexibly. Gesture and speech form a single unified system and “present a single cognitive representation” (McNeill, 1985:353). “Gestures, together with language, help constitute thought” (McNeill, 1992:245) and by gesturing, the speaker has access to a wide range of possible resources to convey ‘meaning’ (Kendon, 1980). A balance combination of gesture and speech, that is without being restricted to verbal language solely can; support learners to construct meanings (Alibali, 1999), assist learners cognitively in providing an avenue to communicate and convey thoughts (Kendon, 1997), interpret knowledge (Kendon, 1995) and help in turn taking (Goodwin & Goodwin 1986). Gestures can specify how a spoken utterance should be interpreted and in chapters six, seven and eight, I will demonstrate how gestures function as visual amplifiers to what interlocutors were expressing verbally conveying aspects of mathematics registers.

This section gave an account of the symbiotic nature of gesture and speech and how these two modalities are temporally and semantically coordinated. I have discussed how information is represented differently in each of these two modalities; hence the meaning that is conveyed in gesture may not be identically reflected in speech. Furthermore I have addressed the overall meaning that emerges from the two modalities is greater than the sum of the two modalities on their own. In the next section, by drawing on the relevant literature I will discuss the complementary functions of gestures in relation to their verbal counterpart in bilingual mathematics classrooms.

3.4 Gesture in mathematical discourse

I am interested in investigating the nonverbal resources that are used within interaction between the teacher-students and student-students to negotiate mathematical meaning in a classroom, and how these resources are used flexibly to convey meaning. In this section I will consider the effectiveness of gestures in mathematical discourse. I will be discussing the complementary function of gestures in relation to its accompanying verbal message in educational contexts. Then, I will extend this notion to examine bilingual mathematics classrooms. I will draw on the relevant literature to show how gestures aid communication by overcoming mathematics register, in specific, for those learners with limited proficiency in English.

3.4.1 Gestures as a communicative resource in bilingual mathematical talk

The phenomenon of nonverbal communication such as gestures has been explored in ordinary communicational contexts by linguists and psychologists (e.g., Alibali, *et al*., 1999; Goldin-Meadow, 2003; McNeill, 1979, 1987, 2000, 2005). Only in the recent years the effectiveness of 'speech-accompanying gestures' within the mathematical communication have attracted the attention of mathematics educators (Maschietto & Bussi, 2005; Radford *et al*., 2005; Morgan & Alshwaikh, 2012). Gestures and other semiotic resources are now considered as part of the resources drawn upon by students and teachers activated in the mathematics classroom (Arzarello *et al*., 2009:97). It is interesting to note that gestures not only have communicative function for listeners but also have a function for speakers. Gestures appear to play a role in communication, both in facilitating language production and in promoting learners' comprehension. Some researchers have focused on the effects of speakers' gestures on listeners' comprehension (Alibali & Nathan, 2007), whereas others have focused on how gestures function for speakers themselves (Alibali & Kita, 2010). In this thesis, I will pay particular attention to show not only how gestures play a role in communication but the ways in which gestures complement the accompanying verbal message in mathematics lessons.

Gesture has a long curious relationship with classroom communication and is often considered as facilitators of verbal expression (Freedman, 1977; Galloway, 1979), especially gestures that convey the spatial content (Hadar & Butterworth, 1997). For example learners are found to produce more gestures when describing mathematical patterns that are difficult to parse than in describing patterns that are straightforward (Hostetter, Alibali & Kita, 2007). Gestures are visual resources that can assist students in resolving register confusion and technical words, as well as in constructing meaning (Alibali, 1999; Alibali *et al*., 1997). For example, Lazaraton (2004) focused on the gesture of language learners as they were becoming proficient (or communicative competent) in English. She reported that ESL (English as a second language) speakers effectively 'stretched' their nonverbal resources into speech in order to mask their verbal shortcomings. A different study focused on a physics classroom where students' use of (deictic and iconic) gestures enabled them to convey certain English physics registers that was not found in speech such as 'velocity', 'momentum' and 'acceleration' (Roth, 2001). For example, the notion of acceleration was carried in gesture by 'a sudden forward movement of hands with an immediate stop'. Students repeated and align their gestures to sustain discourse. A different study (Rasmussen *et al*., 2009) takes an account of students' gestures to illustrate increase and decrease. The notion of decrease was visually conveyed in gestures when students brought together their index fingers and thumbs. The converse process was executed to represent increase. This literature is relevant to the analysis of data in chapter seven, where I will be examining how a classroom teacher's gesture conveyed additional information about the *process* of 'adding' and 'subtracting'.

Over a series of ethnographical observations in mathematical classrooms, Castellón (2007) illustrated how students' gestures functioned as aids in overcoming mathematical registers that became a roadblock (such as the term perpendicular). According to Castellón (2007), the learners' use of gestures in mathematics classrooms were meaningful. Not only gestures served as visual representation to their accompanying verbal discourse, but it facilitated communication when language was unclear or when the mathematical concept was abstract. She focused on bilingual Hispanic learners in a mathematics lesson and she illustrated how students' gestures functioned as aids in overcoming mathematics registers that became a roadblock. This illustrates the flexible use of (verbal and nonverbal) resources bilingual learners drew upon to convey mathematical meaning. A gesture was often used to accentuate, and sometimes to replace talk. She reported that "gesture served as a visual tool that enabled students to communicate their thought processes without being restricted to terminology such as vertical, horizontal, parallel or perpendicular" (Castellón, 2007:163). Gesticulation enabled bilingual learners to participate and communicate mathematically by drawing upon part of their entire repertoire of their semiotic resources which was available to them. I will use this argument in the literature to support the analysis of data in later chapters.

In general the frequency of gestures that are coordinated with the speech is higher when speakers are talking in their second (or additional) language that it is in the first language (Gullberg, 1998). For example, she found that French-Swedish bilingual learners produced more gestures in their second language than in their first language. Similarly, Japanese learners of English have been found to increase their use of gesture in second language learning production (Kita, 1993). These reports illustrate that bilingual learners and in particular learners who are conversing in an additional language do not always rely on their verbal language to convey meaning, but, stretch upon their nonverbal resources to aid maintaining the discourse (Castellón & Enyedy, 2006). Bilingual learners use whatever means they have available to overcome their linguistic obstacles such as code-switching (Parodi, 2008) or by drawing upon other semiotic resources such as movements of hands and feet which have come to be known as a "communication strategy" (Gullberg, 1998:9). Gesturing enables the speakers to access a wide range of possible resources to convey their message and, when learners are limited to what they can say, they gesture (Bates, 1976).

It is interesting to note that although the frequency of gesture production is higher when someone is conversing through a second (or an additional) language, gesturing also occurs significantly when a speaker is fluent in the medium of instruction. More interestingly, there is suggestive evidence that gesturing can also occur when the speaker is fluent and very competent in the register itself. For example, LeBaron and Steeck (2000) and, Corts and Pollio (1999) have observed that professors gesture to convey meaning and mediate understanding without emphasising much on the terminology. Gestures found to have complementary functions in communication and often aid teachers and lecturers to express their instructional information to the class. Along the same lines, Alibali and Nathan (2007) reported that a mathematics teacher's employment of spontaneous hand and arm gestures along with the speech channel in a mathematics lesson served as a scaffolding function which mediated students' understanding. Teachers' gestures often facilitate listeners' comprehension of the accompanying speech, particularly in classroom settings where learners' comprehension is often challenged by instructional discourse which may not be the learners' most dominant language (Castellón, 2007). Learners' comprehension of the lesson content may be aided by the teachers' gestures in particular when the teacher presents new concepts and uses unfamiliar registers (Edwards, 2003; Alibali & Nathan, 2007). Moreover, classrooms are often noisy, with multiple individuals speaking at once and under such circumstances, gestures may play a particularly important role in comprehension. I will use this discussion in the literature to support the analysis of data in chapters six, seven and eight.

I would like to conclude this section by raising awareness to a different role that gesture plays in communication and in particular in mathematical discourse. There is, of course, an increasing interest on the communicative and cognitive functions of gesture in the mathematics education community (Radford *et al.*, 2009; Williams, 2009), and the employment of gesture, if not seen as a bad practice, is regarded as a mode of communication in mathematics classrooms. As Edwards (2009:128) has recently observed, "gesture constitutes a particular modality of embodied cognition, and, along with oral speech, written inscriptions, drawings and graphing, it can serve as a window on how learners think and talk about mathematics". Gesturing is an integral part of mathematical thinking and according to Radford (2009) mathematical cognition can be mediated through gestures as well as conventional mathematical symbols. However, gesturing, unlike a diagram or a written symbol, is transitory. Gestures are physically realised through "movement in different directions and at different speeds" (Condon, 1980:51) in space. It disappears in the air just as quickly as speech, hence ephemeral. Therefore, there is a "need to listen just as carefully to what learners say with their hands as we do to other forms of expression" (Healy & Fernandes, 2011:171). If special attention is not paid to gestures, meaning can be lost, or the message may not be conveyed as comprehensive as intended.

3.5 Cross-cultural variation in meaning

Just as verbal language differs from culture to culture, so many body language signals can also differ (see Pease & Pease, 2006:20). Each culture has a distinct set of 'conventionalised gestures' or 'emblems' that are consciously sent and consciously received with specific meaning varying from culture to culture. As briefly discussed in section 3.2.3, emblems are often used in the absence of speech and convey semantic information. Often the form and meaning of emblems are lexicalized and have specific verbal definition, hence conventional and meaning varies cross culturally (Kita, 2009). I will draw upon three basic culture specific convention emblems and discuss their meaning which varies cross-culturally.

1. Ok sign: the Ok sign is a conventional gesture sign which is formed by a ring by the thumb and the index finger, with other fingers straight up. Most European cultures interpret this emblem as ‘Ok/good’ (Morris *et al*., 1979); ‘zero/worthless’ in France, Tunisia and Belgium; ‘bodily orifice’ (especially the anus) ‘sexual insult’ or a ‘gay man’ in Greece, Turkey, Russia, and Brazil; and ‘money/coin’ in Japan (Pease & Pease, 2006; Morris *et al*., 1979).

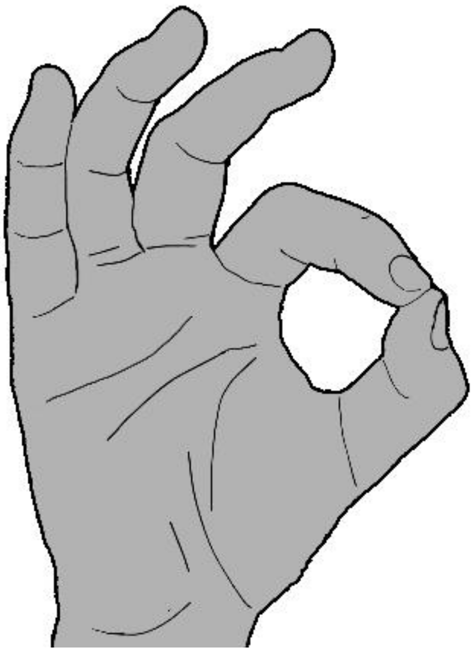


Figure 6: OK sign with the thumb and index finger

2. Another Ok Sign is the ‘thumb-up’ or the ‘Facebook like’ sign, and whilst in France and most westerners ‘thumbs-up’ sign corresponds to the statement ‘Champion’, ‘Super’, the same gesture means ‘Go to hell!’; ‘up yours’ or a ‘crude sexual invitation’ in the Iranian cultural area (Galanti, 2004). It also means *one* to Italians, *five* to Japanese (Koechlin, 1992), and interestingly, in mathematics educational context means *sixty* to Iranians. Another example is the “V” made with the index and middle fingers (used to signify either piece or victory) but the “V”, “held palm in, in

South Africa are insulting gestures similar to the raised middle finger in U.S. culture" (Galanti, 2004:36) but means 'gun' in Iran (Sparhawk, 1981:430).

Figure 7: OK sign 'thumb up'

3. Yes. Yes is normally associated in most Western cultures with a head shake from back to front. Yes is signified in Bulgarian by left-to-right head shake and in Abyssinian cultural region by upward head motion and raised brows (Poyatos, 1992). "In India the head is rocked from side to side, called the Head Wobble to sign yes. This is confusing for Westerners and Europeans, who use this gesture to communicate 'maybe yes –maybe no'. interestingly enough, in Japan, head nodding does not necessarily mean 'yes, I agree' – it usually means 'yes I hear you' (Pease & Pease, 2006:230).

It appears that nonverbal language does not always provide a reliable bridge for intercultural understanding (Schneller, 1992). A gesture, just like any other modes of communication is a cultural artefact. It is culturally given and socially shaped. Therefore there is no reason to assume that the employment of a particular gesture in a culture is for the same purposes and concerns as in another culture. Furthermore, the meaning that is generated through gestures can vary considerably across cultures. What educational implications can cross-cultural variation in meaning have in multi-cultural classroom settings? I will use the ideas that have emerged in this section to justify my claims about vagueness and ambiguity in meanings that can be conveyed in gesture (without and even with their accompanying verbal message) in chapter nine.

3.6 Conclusion

Gestures are an integral part of communication. Gesture and speech are meaning making resources that are not only materially different, but their realisation also differs. The meaning that is generated through gesture varies cross-culturally.

Gestures and speech not only encode information differently, but express the message in a different representational format. As a result, it is relatively easier to convey visuo-spatial information in gestures than it is by speech alone. Therefore, certain information can be presented in gesture that cannot be found in speech alone.

Gestures are salient visual means of expression and serve a number of vital meaning-making functions when accompanying the relevant spoken counterpart; it often complements and intensifies verbalisation (see Gullberg, 1998:227; Kendon, 1970; Crystal, 1969). For example, in mathematics lessons, often learners' comprehension may be challenged by the instructional discourse that presents new and unfamiliar registers. When learners are limited to what they can express using a register that they may or may not be familiar with, they tend to gesture to convey their point. Gestures appear to potentiate and complement speakers' talk, and even sometimes replace their talk (Roth & Lawless, 2002; Roth, 1999; 2000). Such nonverbal communication plays a critical role in conversation as they aid the discourse in a special way. Gestures can act as visual amplifiers to the verbal component of a message; hence both modes of transmission (verbal and visual) have complementary function in relation to one another. Moreover, the overall meaning that emerges through the sum of both modalities, more richly conveys the information than each modality on its own.

I would like to conclude this chapter by raising awareness to the tempo-visual aspects of gestures. The nature of gesture is transitory, as the result, the realisation of the meaning that is generated through gestures is short and ephemeral. Therefore particular attention must be paid to '*hearing gestures*' (Goldin-Meadow, 2003) as much as it is important with hearing speech.

4. Chapter four – Discussion of data collection

4.1 Introduction to the chapter

As evident from the title, this chapter deals with the process of data collection. I will be talking not only about the methodology and methods of data collection but I will also describe the data collection process and the challenges I have encountered. I will start this chapter by giving an overview of the data collection process and the change in my research questions and design in section 4.2. In section 4.3 I will draw on the literature concerning the fluid and dynamic nature of research design in qualitative research and how it is likely to change. Having established the nature of my research design, in sections 4.4, 4.5 and 4.6 I will discuss three different methods of data collection followed by my rationale for justifying why I have used 'participant observation', 'video recording' and 'interviews' for the purpose of data collection.

Having justified my methods of data collection, I will then draw attention to the role of subjectivity in qualitative research in section 4.7. I will emphasise how a researcher can influence what is researched. In section 4.8 I will describe the ethical considerations that were needed for my study. Finally I will summarise this chapter in section 4.9.

4.2 An overview of the data collection process

The complete section 4.2 takes an account of the research journey and the nature of the dataset that was collected. In section 4.2.1 I will talk about not only how my original research questions changed but how the focus of my study also evolved.

4.2.1 The change of the focus of my study

The initial focus of my interdisciplinary research project involved examining the experiences of Farsi/English bilingual learners have whilst learning mathematics in different settings. I was interested in comparing and contrasting British-Iranian bilingual learners' experiences in their local complementary schools in the UK, where they can draw on more than one language when talking about mathematics with their teacher or with other students, and in their mainstream schools, where their learning experience of mathematics occurs solely through the medium of English. One of the ways in which I wanted to explore my initial research questions was to compare young learners' bilingual experiences in a complementary school followed by their experiences of learning mathematics in their mainstream classrooms.

My focus was on bilingual, Farsi/English speaking students of Persian origin. This included a wide range of pupils from newly arrived Iranian learners to the UK, and those who were born and were well established here in the UK. These British-Iranian learners were not only diverse in relation to the time they had spent in the UK, but in the number of languages they had at their disposal. Table 1 gives an overview for each of my research participants. Pseudonyms have been used throughout this thesis.

Table 1: Research participants

Research participants	Year	Number of languages	Citizenship	Years living in the UK
Boy1 (B1)	11	2	British citizen	6
B2	10	3	British citizen	5
B3	10	4	Iranian citizen	4
B4	10	5	British citizen	6
B5	10	3	British citizen	Born in the UK
B6	7	3	British citizen	Born in the UK
B7	7	2	British citizen	Born in the UK
Girl 1 (G1)	9	2	British citizen	10
G2	8	3	British citizen	8
G3	9	2	British citizen	12
T1		4	British citizen	15
T2		3	British citizen	10

My original research questions were:

1) What prompts code-switching in a bilingual (English-Farsi) mathematics classroom in the UK? And

2) Does a complementary school 'complement' the work that goes on in mainstream school? And if so, how?

Code switching was central to my research and was the main force that drove my interest to observe institutional settings where mathematics is taught in multilingual environments (such as complementary schools) and in other environments where code-switching does not happen often (such as in mainstream schools). I followed three of my research participants' learning experiences from their complementary school to their corresponding mainstream school and made fieldnotes. I was only allowed to follow up B1, G1 and B7 in their mainstream school. I did not gain institutional access to the mainstream school of the other seven research participants. I was not able to get permission to carry out research in these schools. I video recorded B1 in his mainstream school but I was not allowed to video record G1 and B7 at the request of the institutions. Below is a list of what I obtained in relation to my dataset:

1) Video-recorded interactional data in the complementary school 16 lessons (approximately 12 hours) and three hours of video-recorded interactional data in one mainstream school took place in the period of December 2010 to March 2011.
2) Ethnographically informed observation and fieldnotes in both the complementary school (eight visits and approximately 16 hours of observation) and three mainstream schools (12 visits and approximately 12 hours of observation) from August 2010 to March 2011.
3) Recorded interviews, two audiovisual recorded interviews from two male research participants, two audio recorded interviews from mainstream school teachers and one audiovisual recorded interview from the complementary school teacher. All interviews were conducted during the period of March 2011 to April 2011.
4) Documentary evidence and other secondary forms of data which were in the form of copies of students' work including any comments that they had made on their

worksheets. I also collected copies of notebooks, coursework, and sample sheets of the teaching materials used by the teacher. These materials were collected from August 2010 to April 2011.

There appeared to be an imbalanced ratio of data collected in the two different institutional settings. Due to constraints of the research journey and being only partially successful in following each student into their corresponding mainstream school, I was left with an insufficient amount of data (from the mainstream schools) for the purpose of comparing between the two institutional datasets. Moreover, due to the substantial number of video recordings needed for the project, I had only obtained three hours of interactional recordings from a mainstream school versus 12 hours in a complementary school. I believed if I continued comparing and contrasting the two institutional settings, I would have ended up drawing conclusions and making claims based on insufficient data. Therefore my findings would not have truly reflected a cross-comparison study. For this reason, I decided to change the focus of my study to bilingual pedagogy in the complementary school. This simply meant that I had to discard all the data I had gained from mainstream schools. As well as changing the focus of my study, I decided to alter my research questions as a consequence:

1) What is the pedagogic nature of gesture (forms and functions) and other body-based resources in a British-Iranian complementary mathematics classroom in the UK?

In section 4.5.2, I will address how the origin of gesture became an area of interest and in particular why I decided to look at the pedagogic nature of gestures in communication.

This section indicated the significance of not gaining sufficient access to the original research field and also the consequent change of focus of my study. Being unable to provide sufficient data to scrutinise my original research questions resulted in a change of research questions. In the next section I will draw attention to how the focus of qualitative research is also subject to change.

4.3 The fluid nature of research questions and research design in qualitative research

In section 4.3.1, I will attend to the fluid and dynamic nature of research design in qualitative research. Later in section 4.3.2 I will review general literature on qualitative and quantitative research methodologies and discuss which approach was best suited to answer my new research questions.

4.3.1 Research design

A research design places a particular emphasis on the focus of the study and research questions (Blaikie, 2000; Tuckman, 1972). A research design creates a disciplined starting point and usually it is established at the beginning of research. However some approaches cannot be planned or arranged at the start because "much of the information that is needed to make these decisions will not be known until the research has been in progress for some time" (Blaikie, 2000:37).

It is interesting to note that, in qualitative research "the focus of the study may change" (Lincoln & Guba, 1985:224). Focus of the study influences the research questions and the research design. In my study, not only did my research questions and unit of analysis change due to lack of sufficient data, but certain analytical ideas evolved throughout my research journey. In qualitative research ideas "evolve, develop and unfold as the researcher proceeds" (Robson, 2002:5). Referring back to my research interests, the role of gestures appeared as an area of interest at a later stage in my research. For example, I decided to focus on nonverbal communication and aspects of body language and how they complement the accompanying speech. Therefore I became interested in paying particular attention to the analysis of gestures through the data that was collected using an audiovisual recorder.

4.3.2 Taking a qualitative approach to research

In this section before I describe the rationale as to why I adopted a qualitative approach to research, I would first like to draw attention to the literature around the nature of both qualitative and quantitative approaches to social research.

Both qualitative and quantitative approaches to research deal with discussions about "how research is done, or should be done, and to the critical analysis of methods of research" (Blaikie, 2000:8). In general, qualitative methods tend to focus on small-scale (micro) aspects of social life which generate rich descriptions about specific phenomena. Quantitative methods on the other hand are often concerned with more large-scale, macro aspects (Robson, 2002). A quantitative approach seeks to convince the reader through credible interpretation of ratios, percentages and statistical significance (Mason, 2002). Using a quantitative approach, someone could find, for example, a significant correlation between two variables, but that correlation cannot be interpreted as indicative of causation (as a correlation is not the causation). Since all quantitative data is ultimately based on qualitative judgment (Trochim, 2006), numbers in and of themselves cannot be interpreted, hence there always needs to be a qualitative hermeneutic dimension to explain the quantitative findings.

A qualitative approach seeks to convince through detailed description, to probe more deeply the details of a situation. The flexibility of a qualitative research design provides a space to take account of the social and historical context/background of the process of collecting data. For example, knowing where a researcher is coming from and how their past experience and family background can influence his/her actions (Miles & Huberman, 1994). Since I am interested in examining the experiences that British-Iranian bilingual learners have in their complementary school mathematics classrooms, I would like to pay particular attention to how they draw upon verbal and nonverbal resources to communicate mathematically. Moreover to be able to answer my research questions, a significant amount of observation is needed in order to describe what happens in the field when young bilingual learners interact with each other in their mathematics classroom. Therefore my study takes a qualitative approach to research. In the subsequent sections I will talk about the methods I have used to obtain my dataset.

The first part of this section has highlighted the dynamic nature of qualitative research and the importance of a flexible research design. Later I stressed how the nature of my research questions suits a qualitative approach. In the next three sections I will be expanding on the three different methods of data collection which I have used: participant observation, video recordings and interviews.

4.4 An ethnographic style approach to participant observation

In this section I will elaborate on one of my three methods of data collection used to obtain my dataset. First in section 4.4.1 I will highlight the literature around participant observation and linguistic ethnography. In section 4.4.2 I will explain why I have combined participant observation and linguistic ethnography in order to employ an ethnographic style approach to participant observation as a method of data collection. Moreover, I would like to discuss how an ethnographic approach to participant observation enabled me to pay particular attention to participant selection at the complementary school.

4.4.1 Literature on participant observation and linguistic ethnography

In this section there are two separate approaches to research I would like to address. One is based on participant observation and the later part is on linguistic ethnography. The link between the two methods or approaches to research will not now be elucidated, but in the next section I will explain why I have taken an ethnographic style approach to participant observation. Now I will turn briefly to draw on the literature on participant observation and linguistic ethnography (or ethnography of communication).

In general, observation methods are powerful tools for gaining insight into situations (Cohen, *et al*., 2000:315) and a major advantage of observation as a technique is its directness (Robson, 2002). Observation methods divide into two subcategories, participant observation and direct observation. A direct observer does not attempt to become a participant in the context s/he is watching (Trochim, 2006). Unlike direct observation, in participant observation an observer needs to be present physically in the field to ask participants about their views, feelings or attitudes. Most often, a participant observer maintains a relatively close distance to his/her participants, whereas there might be a bigger distance in direct observation. An observer in either participant or direct observation observes what participants do and listens to what they say. On the whole, "observational data are attractive as they afford the researcher the opportunity to gather 'live' data from 'live' situations" (Cohen *et al*., 2000:305), and it seems to be an "appropriate technique for getting at 'real life' in the real world" (Robson, 2002:310).

Both direct and participant observations can have a 'structured' organisation or an 'open' form. In a structured observation, an observer goes through a set of 'predetermined' specifications which does not take account of anything new that might occur in the course of an observation. In an open observation the observer can look for unexpected events that occur during the course of the event. In my case unexpected events could encounter, for example 'bilingual pedagogy' and the ways in which teachers and students are engaged in using gestures to convey their meanings during the course of an event. Therefore an 'open' form was most appropriate for my study.

Now I would like to draw attention to ethnography and in particular linguistic ethnography. Ritchie and Lewis (2003:12) define ethnography as understanding "people being studied through immersion in their community to produce detailed description of people, their culture and beliefs". Ethnography refers to highly descriptive writing about particular groups of people in natural settings or the 'field' which enables one to "collect data in a systematic manner" (Brewer, 2000:6). Ethnography usually involves a great "emphasis on exploring the nature of particular social phenomena" (Atkinson & Hammersley, 1994:248) to "particular individuals, groups or organisations" (Hammersley & Atkinson, 1983; 1995). I would also like to draw attention to 'linguistic ethnography' or 'ethnography of communication' as an approach to classroom discourse. Some studies have considered ethnography of communication to focus on language practices in particular micro-cultures, such as schools (Erickson & Mohatt, 1982; Roberts *et al*., 1992). Taking a slightly different approach to ethnography of communication, 'linguistic ethnographers' have focused on multi-ethnic classrooms (Creese, 2003). The slight change is with regard to foregrounding ethnography in ethnography of communication whereas in linguistic ethnography, the value of discourse is foregrounded. "Ethnography is said to be enhanced by the detailed technical analysis which linguistic brings, while linguistic is said to be enhanced by the attention to context" (Creese, 2010: 139). Linguistic ethnography provides a strong emphasis on the role of language in social and cultural production with a common view that "language and the social world are mutually shaping" (Rampton *et al*., 2004:2).

Taking a participant observation approach to my study, I was interested to foreground the linguistic practices to understand classroom discourse. This was because I was primarily interested in how code-switching occurred between teacher and students and students themselves as a way to negotiate meaning and mediate understanding. Having described a general overview about participant observation and linguistic ethnography in social research, in the next section I will describe why I opted to combine both approaches as a method of data collection.

4.4.2 Combining participant observation and linguistic ethnography

In this section I will show why I have chosen participant observation as a method of data collection. Then I will describe my rationale for the complementary functions of combining participant observation and linguistic ethnography.

I started researching in an Iranian complementary school mathematics classroom as a participant observer. Being a participant observer enabled me not only to observe the classroom activities but to be able to move around the classroom and ask students about a point which I found of particular interest. Zevenbergen (1998:21) observed that "being an active participant in the environment, the researcher can ask spontaneous questions as they arise without being seen as threatening". For example, I often asked questions and made a note of each student's approach to a given mathematical problem. Not long into my stay in the complementary school, I became aware of code-switching taking place. Code-switching was at the heart of my study and I wanted to learn from the research participants whether mathematics was carried out differently in different languages. I wanted to foreground language practices as they occurred in the lesson and see whether they were correlated with a change of mathematical topic. For example, whether algebra was done in Farsi and trigonometry was done in English. In other words, an attempt to go in the field and to make the strange familiar and the familiar strange (Erickson, 1973). Therefore to foreground the language choices students and teacher made around mathematical activities was an area of interest.

Now I would like to justify how a linguistic ethnographic style approach to participant observation complemented my observation in particular on discursive practices. When I was present at the research field, I was interested to look at the ways in which British-Iranian learners were engaged in activities in their natural settings, namely their complementary school mathematics classroom. In particular I was interested to note any language practices and code-switching between learners-learners and teacher-learners and how each language could offer a new window on how aspects of mathematical ideas can be presented. In a multilingual mathematics classroom I wanted to document and report not only information about learners' proficiency in each language but also their experiences with listening to and using different languages whilst engaged in different mathematical activities and tasks. Incorporating linguistic ethnography as an approach to participant observation, enabled me to focus more on certain discursive practices to the extent that was needed for the study. For instance, the extent to which each of my participants used Farsi (or English) for certain aspects of mathematics. Combining participation observation and highly descriptive fieldnotes worked together to provide a rich account about particular ways in which students and teacher drew upon their linguistic repertoires to interact with each other. Taking a linguistic ethnographic style approach to participant observation was a great starting point to centre attention on the discursive practices in a mathematics classroom and to achieve a deep understanding of issues surrounding the nature of bilingual interaction.

I would like to reiterate that I have adopted a linguistic ethnographic 'style' approach to participant observation, but I have employed participant observation. Moreover I like to emphasise that my study was not ethnographic, but an ethnographic style primarily because of the extent to which I was involved with the research participants.

In this section I have described the first method of data collection which I incorporated in my study. It was a linguistic ethnographic style approach to participant observation. Incorporating this approach enabled me to understand the classroom discourse by foregrounding the language practices that were made around different mathematical tasks. In the next section I will shift attention to the second method of data collection which was video recording.

4.5 Video recordings

This section discusses video recording both as an instrument and a method of data collection in social research. I will first introduce the relevant literature on video recording as a method of data collection. Later, in section 4.5.2 I justify my rationale for why I have opted to choose video recording as a method of data collection in my study.

4.5.1 Video recording as a tool and a method of data collection

I will start this section by considering some of the literature on the role of visual recording in research. Later I will extend this notion to how video recording is used both as a tool and a method of data collection in mathematics classrooms.

In the past, generally researchers have focused on words and numbers to make sense of society and have adopted methods to meet their needs. Over the last three decades there has been an increase in the number of video recordings in educational research. Hartas (2010:202) observed that "we plan, think and create in terms of imagery, therefore we need appropriate visual methods to aid our understanding of these phenomena". Although the employment of video recording in research is relatively new, visual methods have had a long tradition in research. Darwin (1872) was among the very first researchers to incorporate visual methods to explore areas of nonverbal communication. He used a still camera as a tool and a method to record facial expressions in 'man and animals'. A video (which is a collection of 'moving images') is an extension of still images. The data captured by video recording gives the researcher a unique opportunity to understand dynamic events within their spatio-temporal context. Replaying what has been captured in a video recording, has the advantage of slowing down observation, and enhances focus on a variety of dynamic events

(Webber, 2008). These can include the study of 'proxemics' (the study of personal space) (Collier, 1983; 2001), 'kinesics' (the study of communication and body language) (Hockings, 1995) and 'conversation analysis' (Goodwin, 1981; 2001). Similarly, Collier (1973) examined a classroom interaction and focused on proxemics and kinesics cues that led him to analyse the emotional state and learning intensity of students and teachers that data in a format of video recordings made available.

I would like to narrow down attention to the use of video recording in mathematics classrooms. Only in recent years have mathematics educators begun to include gesture and body movement as either potential sources of information on how students think about mathematics, or as contributors to mathematical thinking and communication itself (Morgan & Alshwaikh, 2012; Healy & Fernandes, 2011). One of the purposes of audiovisual research is to capture and examine the ways that spontaneous production of physical gesture (as one modality), is used in communicating mathematical ideas and problem solving (Edwards, 2009; Núñez, 2009) and to see whether gesture bridges personal, internal imagery and shared, external speech and inscriptions about mathematics (Roth, 2002; 2009). More recently, Cheeseman and Clarke (2007) used video stimuli to understand children's mathematical thinking of a given task. They videoed mathematics classrooms and then asked their participants to reconstruct what they were doing as the video was played back to them. The visual (in the form of objects, drawings etc,) provided a reaction that evoked ideas and memories in respondents (Webber, 2008) and essentially a 'think aloud' approach (Newby, 2009).

Video recordings provide a space for the analysis of: classroom interactions and multi-modal analysis (Radford *et al*., 2009; Maheux *et al*., 2009), gestures (Whitacre *et al*., 2009), pitch and intonation (Roth, 2009), and other multi-modal and non-verbal aspects such as facial expressions, eye motion, body poise, gaze, and other visually observable events. One of the greatest strengths of incorporating video recording is its analysis. In terms of analysis, the researcher can play back the video footage so that the multi-modal levels of analysis can be checked against each other (e.g. gesture versus intonation) to see how each modality complements each other (see chapter nine). The harmony between verbal and visual, between what was said and how it was said, can only be captured through the lens of a video recorder. A similar approach can be suited to record 'some' bilingual interaction between teacher-students and student-student and other non-verbal contextualisation cues (e.g. pointing to relevant graphs/diagrams, gestures, algebraic notations, etc.) whilst working on a mathematical task.

It is worth noting that video recordings (just like any other methods), have advantages and disadvantages. In general, one who uses video recording will have access to a replay-able audio-visual record of an event (e.g. of a classroom interaction). On the other hand, however, there is a chance participants will be uncomfortable when they know their remarks will be recorded word-for-word (Trochim, 2006). Therefore, having a replay-able account of video recording is likely to be distorted by the very process of obtaining it (*ibid*). I will now turn to justify my rationale for opting to record interactional data in the format of audiovisual recording.

4.5.2 Justifying video recording as a method of data collection in my study

Part of my data consists of audio-visual recordings of students working together. The reason as to why I decided to video record mathematics lessons was because whilst adopting an ethnographic style approach to participant observation, maximized my immersion in local activities in the classroom. This means I had less time to write fieldnotes in the site (Emerson *et al*., 1995). Being an active participant observer restricted my opportunities in the field to sit down, observe classroom practices and note the classroom interaction to the degree that was needed for this research project. I decided that by incorporating video recording as a tool I would be provided not only with an extra lens but also with an opportunity to examine closely the classroom discourse by reviewing or 'revisiting' (Erickson, 1990:159). The potential for repetitive viewing allows "us to comprehend complex details that might otherwise remain unseen or fleeting. This ability to look at exactly the same circumstances again and again is at the foundation of all direct visual analysis" (Collier, 2001:49). If I missed an event in my role as participant observer, the recorded data would provide "the opportunity to revisit events vicariously through playback at later times" (Erickson, 1990:148). Although the video did not capture everything, I could always go back to the parts that I missed out and pay particular attention to the parts I did not notice during my observation. By revisiting a recording we "have more time to notice how participants misspeak or make mistakes that we would have if we were observing in real time" (Moschkovich, 2010: 17). Moreover, it allows us more time to notice and really think about what participants said and did (Moschkovich, 2010:17).

Whilst I was waiting for the university ethic approval to video record classrooms, I became interested in the role of gestures in communication. I became interested in multi-modality and the ways in which the nonverbal forms of expression can compensate for the verbal counterpart (Freedman, 1977; Garber *et al*., 1998). Gestures, postures and other bodily movements would be visible in video recording. Therefore, to video record was to capture some of the other nonverbal cues (e.g. gestures) used in conveying mathematical meanings.

Video recording allowed me to observe the effectiveness of various circumstances and styles of classroom culture. I also made notes whilst recording which created a secondary back up record of the session. For example if I noticed a particular event, I wrote a brief description of the event along with the time of the recording e.g. 20 minutes 24 seconds so that it would be easier for me to correspond it to the event. The video recorder was mostly held on a tripod and at times it was hand-held. When the video recorder was on a tripod, it was facing the whiteboard and it was angled in such a position that it could record what the classroom teacher was writing on the whiteboard. It also enabled me to go around the classroom and ask students questions. If I became aware of particular events, for example if I saw something of particular interest in a student's book, the recorder was then hand-held to make a record of the student's work.

In this section I have discussed the second method of data collection which I incorporated in my study. I shifted attention onto the literature on video recording as a tool and method of data collection which captures classroom practices and how it can be used for the analysis of classroom discourse. That is not only encountering the verbal and vocal elements of language but also visual. In the next section I will shift attention onto the third and final method of data collection: interviews.

4.6 Interviews

This section starts off with an overview of literature on interviews in social research in order to consider the purpose of interviews as a method of data collection. At a later stage (in section 4.6.2), I will describe why I chose to do interviews and also how I implemented them.

4.6.1 Interviews as a method of data collection

The purpose of the interview is to probe the ideas of the interviewees about the phenomenon of interest. Interviews facilitate a greater understanding of a particular phenomenon based on the interviewee's point of view. In general, interviews are carried out to complement the information obtained from various methods of data collection such as, observation or, at times, interview data can be the major source of data (Legard *et al*., 2003; Oppenheim, 1992; Seale *et al*., 2004).

Using interviews as a method of data collection allowed me to understand "the meaning of particular phenomena to the participants" (King, 1994:16) including participants' beliefs, experience, attitudes, emotion, their memory of knowledge, etc. Interviewing allows different perspectives to be brought to the research process (Erickson, 2004) because respondents have different 'epistemological positioning' with respect to one another (Silverman, 1993; 2006), that includes different social and historical accounts that can only be revealed by giving them the opportunity to talk about their experiences.

One of the greatest advantages of interviews is flexibility. Interviews "offer the possibility of modifying one's line of enquiry, following up interesting responses and investigating underlying motives in a way that postal and other self-administered questionnaires cannot" (Robson, 2002:273). In essence interviews offer a possibility of modifying someone's line of inquiry, following up by probing interviewees to expand upon their interesting responses and investigating underlying motives (Yin, 2003) in a way that, for example, questionnaires cannot.

There are, of course, advantages and disadvantages in doing interviews. For example the success of the interview to a large extent depends on the professional and personal qualities of the individual interviewer (Legard *et al*., 2003). I have come to understand the importance of acknowledging the way in which our own subjectivity as researchers shapes the way we see activities and events in the social world. From an interpretivist perspective, social reality is perceived through the lens of our experience and our world view and our interpretation is therefore highly subjective. I will talk more about the role of subjectivity and its influence in data collection in chapter five. Now I will turn to justify why I have chosen to use interviews as the third method of data collection

4.6.2 Why and how I have chosen to do interviews

As I mentioned in the previous section, interviews are often implemented to either complement the information obtained from observation or, at times, interview data can be the major source of data. There are two reasons why I used interviews.

Firstly interviews were used to complement instances that I observed during observations. Whilst working in small groups as a participant observer, I came across interesting ways in which students were engaged in tackling mathematical problems. Although I used fieldnotes to 'record' what happened at the time, those instances were transitory. What was written in fieldnotes was a reflection of what happened at that specific time and place. Therefore interviewing provided access to a greater understanding of a particular phenomenon which complemented my initial observation.

Secondly, later in the analysis of the video recordings, I became aware of particular moments in the video where code-switching occurred in the complementary school by both students and teachers. I decided to follow up some of these moments by probing those particular students and teachers to allow their perspectives to be brought to the research process (Bryman, 2004; 2008). In the analysis of video recordings, I replayed back particular excerpts to students with regard to code-switching and asked their opinions about why they code-switched. It was possible that learners and teachers had different perspectives not only from each other, but also from the researcher. Therefore "the role of interviews is to bring forward these different perspectives in order to develop a better understanding of the mathematics classroom" (Zevenbergen, 1998:22).

If I observed a particular event in the classroom, for example, the ways in which a particular student approached a given mathematical problem, soon after the lesson (either at break or at lunch time) I carried out an interview with that student. Carrying out an interview immediately after a particular event was more likely to lead to remembering the event in details. On the other hand, some interviews took place a few months after an event had actually happened. This is because, through the analysis of video recordings, if I became aware of particular moments in the video that needed further clarification, I approached those particular students who were in the recorded event. Whilst replaying back parts of the recorded video to refresh their memories, I carried out interviews with them.

Some interviews were recorded in the format of audio or audiovisual, with the permission of the interviewee. Audio recordings were made for those who wanted their visual identity to be anonymous. Interviews were recorded to maintain a level of accuracy and richness of data. Furthermore, even with the recorded interviews, I also made notes of incidents as they occurred so that they were available later for further reflection. The interviews that were in a recorded format were then transcribed and I made fieldnotes for the ones which were not recorded. I will talk about why some interviews were not recorded in chapter five section 3.3. The data gained from these interviews were then compared and contrasted with my other datasets (video recordings or other ethnographic fieldnotes). To identify any possible relationship between them I used thematic analysis across datasets. In the next chapter, I will describe more about the analysis of dataset.

Other than one occasion, where I interviewed T2 at his home, other interviews were in the complementary school during break time, lunch time and sometimes after school. I was aware of the fact that the question as to where interviews were held was important because as Hammersley and Atkinson (1983: 25) have observed, “different settings are likely to induce and constrain talk of particular kinds”. All interviews except one (interview with T2 as mentioned above) were conducted at the interviewees’ ‘normal’ setting, namely their mathematics classrooms as it was their familiar learning environment. The interview with T2 was conducted for the purpose of following up what I noticed in analysing a video recording (see chapter seven, section 4.2).

In the next section I will focus on the concept of subjectivity and how my role as a 'researcher' could potentially influence what was being 'researched'.

This complete section paid attention not only to the literature on interviews as a method of data collection, but the nature of data it produces. Having discussed some of the literature I then justified my rationale for using interviews as a method that both generates data and as a way to complement my other datasets (video recordings and fieldnotes).

4.7 Subjectivity

In this section I will talk about the role of subjectivity in social research. I focus on how my ontological stand and epistemological positioning shapes what is perceived and how something is constructed. In particular I will emphasise my role as a researcher and how it affected the nature of the datasets I obtained through participant observation, video recording and interviews in sections 4.7.2, 4.7.3 and 4.7.4 respectively.

4.7.1 Ontological stand and epistemological positioning

> We cannot escape questions of epistemology (the study of knowing and how it comes about), or ontology (the study of how ideas come into existence) even though they may at first seem abstract and unrelated to professional practice (Mason, 2002:153)

I will consider how my ontological stand and epistemological positioning as a *researcher* can influence what is being *researched*. In other words I will acknowledge my role as a researcher and how it can potentially be part of what is researched or as Mason (2002) observed, the describer is part of the description and the observer part of the observation. In the first image of figure eight a rhino is depicting what s/he is observing from his/her own ontological stand. Therefore this makes the presenter (rhino) part of the presentation. In the second image we observe two men discussing what they are seeing in accordance with their epistemological positioning. They are both reporting what each is observing with the only difference being that the man on the left sees a six whereas the man on the right views what he sees as a nine. However, they are both correct from their own particular standpoints. Therefore observing and describing involves interpretation which reflects our ontological stands and how we come to know and perceive events.

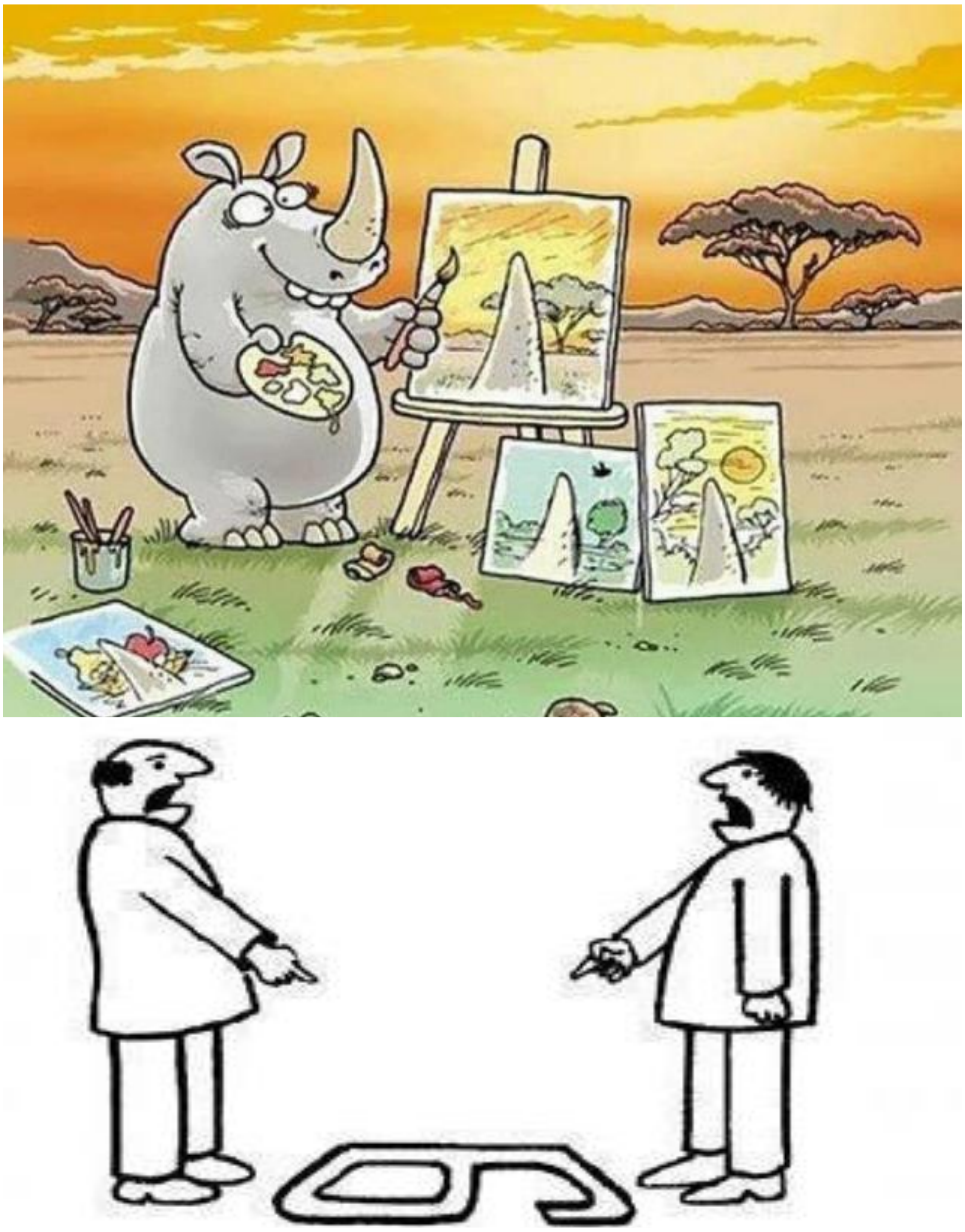


Figure 8: Ontological stands and epistemological positioning

Now I will describe myself (qua researcher) in order to offer a fuller picture of how the ideas for my research have evolved through my professional and past experience. In brief, I have spent the last 13 years using two languages at school, college and universities in the UK as a student, teaching assistant and a researcher of mathematics education. I use these two languages interdependently (Barwell & Setati, 2005), enough to be able to think about how Farsi/English might shape mathematical thinking for Farsi/English speakers. Since I am bilingual, I became interested in examining the experiences that Farsi/English bilingual learners have in learning mathematics in a local complementary school where they can draw on more than one language with their teacher or with other students. This locates my role as an 'insider' (Martin *et al*., 1997) someone who is familiar not only with British-Iranian culture (and languages) but having the knowledge of mathematics. This sits well with Adler's (2001) observation; she observed that sometimes researchers go into "research projects with their own interests and agendas, and exercise a level of agency and thus influence over the direction of the research" (Adler, 2001:45). For example, as a bilingual, I paid a great deal of attention to bilingual students, and I have come to realise that my own lived experience as a bilingual shaped my interest in the mathematical learning of bilingual students. This fact clearly makes my findings a personal-interest-driven research. Therefore there is a strong tie between my journey of growing up learning mathematics bilingually to research in bilingual mathematics contexts. This is how my ontological stand has shaped and influenced my research interests.

I acknowledge a weakness in my thesis which emerges from my epistemological positioning; that is coming to know about the relevant literature or methodological approach through published media. I have encountered the relevant literature and methodological approaches primarily through English and Farsi. Due to the fact that I am only communicative competent in Farsi and English, I have not taken account of (or am not even aware of) the literature or methodological approaches that are published/known through the medium of another language. This sits well with Alain Badiou's concept of 'mathematics is ontology' in his book 'Being and Event' which was written in French in 1987. His book wasn't translated into English and recognised to the rest of English speaking researchers until the year 2005. Another example is Vygotsky's work that only came to be known to the world of English speaking countries through translation in early 1962. Therefore not only does this make knowledge partial but just like the men in figure eight, I come to view and interpret events through a particular standpoint and may not be aware of other existing perspectives.

Having explained the gist of how subjectivity can play a role in what is being researched, in the next three subsections I would like to highlight specifically how my role as a researcher could have affected the data collection process. First I will talk about how by simply being present in the research field, the researcher could affect what is being researched.

4.7.2 The presence of an observer in an observation

I carried out observation as a method of data collection in my study. In this section I will briefly raise awareness of how an observer's presence impacts on what is observed. There is a major issue concerning the extent to which an observer affects the situation under observation (see Emerson *et al*., 1995:3). Blommaert and Dong (2010:27) speak of the role of a research observer in a community of practice:

> "When a researcher enters a classroom, the whole classroom changes, and a lot of what the researcher will witness are reactions, adjustments and adaptations to this change. As a fieldworker, you never belong 'naturally or 'normally' to the field you investigate, you are always a foreign body which causes ripples on the surface of smooth routinised processes. ***There is always an observer's effect***, and it is essential to realise that: you are never observing an event as if you were not there.

You are there, and that makes it a different event" (their emphasis in bold and italic).

Furthermore, observing and describing necessarily involve interpretation by the "interpreter who has desires and prejudices, sensitivities and propensities" (Mason, 2002:237). This makes the describer part of the description and the observer part of the observation (Mason, 2002). Indeed, in participant observation, there is no objective place to stand and all observation involves standing somewhere and indeed, that influences what is seen. A participant observer makes 'choices' and 'decisions' (Erickson, 1990:147) concerning where to be in time and in the setting. Moreover, we observe what we are prepared to observe and we notice what we are sensitised to notice (Mason, 2002). Therefore I acknowledge that the events I observed under observation could have been different had I not been present in that situation or had someone else been observing.

4.7.3 The choices of a recorder in recordings

I am aware of certain disadvantages of audio-visual recording as it seems especially important to consider how video recording might be used to examine language, mathematical activity, and mathematics learning with bilingual students' populations, particularly for evaluative analysis of students' activity. When we use the camera to make an audio-visual record, we make choices influenced by our "identites and intentions, choices that are also affected by our relationship with the subject" (Collier, 2001:35). "The focus of the video camera is selective" (Bezemer & Mavers, 2011:196) and "[e]very camera position excludes other views of what is happening" (Goodwin, 2001:160). Moreover, video recordings produce rich data but only capture a partial view of the social interaction (Flewitt, 2006; Flewitt *et al*., 2009) and the visual analysis is only based on what is visible within the image or collection of moving images. Therefore through analytical insights certain information can be gained and certain details are lost or in other words, certain types of data are stressed and ignored (Mason, 2002). Moreover, in practice recordings that are generated through the lens of a single camera do not capture the whole classroom interaction. The data that is obtained from a single camera has a single focus of attention whereas people are capable of attending to multiple aspects of a complex setting (Pimm, 1993). Therefore, I am aware of the fact that even with video recording, I can still not record the whole bilingual interaction which takes place in a mathematics classroom among teacher/students, students/students and I can only record the rich bilingual interaction only from a particular perspective as video recording represents a single perspective representation of reality.

The video recording production process can be problematic because there are choices which influence when and whom to record. These choices to a large extent are influenced by the agent's interests. There are also choices around *selecting* interactional data from the dataset for the purpose of analysis which I will talk about in the next chapter.

4.7.4 The influences of an interviewer in an interview

In this part I will show how subjectivity plays a part in interviews and how problematic an agent's role can be when probes are employed as an interview technique.

The role of agent or an interviewer has a substantial impact on the interview data. This is because there are differences in the professional qualities and personal interests of individual interviewers. Likewise, personal interests in an interview should not be disregarded (Silverman, 1993). Temperament, prior experience, all sorts of personal idiosyncrasies enters into the research process and we should not pretend otherwise. We often describe what has been observed from our own point of view. This fact makes the describer part of the description and the observer an integral part of observation.

I will now shift attention to the way the interviewer poses a question, his/her tone of voice, the body posture, and the context in which the probe arises. All of these can influence the response (Blommaert & Dong, 2010). Blommaert and Dong (2010:46) highlight certain cultural issues that could be exhibited on the interviewee during the course of an interview prompt. For example, "raising your eyebrows when you sit there silent may be a particularly powerful prompt, but it often triggers an embarrassed reaction from the interviewee". Moreover, a period of silence is acknowledged to be a useful tactic during an interview probe (see Zeisel, 1984:140-54) but it is a natural feature of conversational structure in many cultures that long silences are to be avoided (Jaworski, 1993). Therefore some interview probe techniques could be taken as 'rude' or even 'offensive' by the interviewee if done in a manner that is not appropriate to that specific cultural group of people.

I conducted an interview with T2 in order to capture some of his gestures as he talked about a particular topic. During my interviews with T2 (see section 5.3.3), in order to provoke a gestural representation for a particular mathematical concept, 'power', I repeatedly asked the same question again and again after T2's reply to each question. At the end of the interview I had captured T2's gestural representation for the mathematical concept 'power'. The repeated nature of asking T2 the same question served as a probe to achieve the main purpose of the interview. It is worth iterating on the fact that I went to the interview knowing what I wanted to capture T2's gestures and how to make it happen. I will expand on this notion later in the next chapter. This is where personal idiosyncrasies enter into the research process and influences the data collection and we should not be pretending otherwise (Erickson, 2004).

Section 4.7 paid attention to the role of subjectivity in social research. I have described how the role of a researcher in the field influences what is researched. Furthermore I have looked at the role of the observer in observation and describer in description. It appears that in qualitative research the researcher is an integral part of what is researched, and what is being researched is to a large extent a personal interest.

4.8 Ethics

This section deals with concerns about ethics that arose in the course of conducting my research project. This section deals with literature about ethical consideration in social research, consent forms, confidentiality and pseudonyms. Furthermore, I will discuss my ethical considerations for my study.

4.8.1 Ethical considerations

Ethical considerations at every stage of the research process, from study design to data gathering to data analysis and report writing is crucial (Bryman, 2004; Silverman, 2006). Ethical standards are required so that I, as a researcher, do not put my prospective participants in a situation where they might be at 'risk of harm' as a result of their participation. Harm can be defined as both physical and psychological (Trochim, 2006) and it can entail a number of facets for example 'physical harm, harm to participants' development; loss of self-esteem; stress' (Bryman, 2004:509). My research did not put my research participants in a situation where they were at risk of harm of any kind. For example, I did not ask my participants anything different to what they were doing in their classrooms. Or during my interviews with them, not only a door was open with a third person (e.g. a teacher) always involved, but also the content of the interviews were mathematics related. Most importantly, they freely consented to take part in my interviews.

Ethical guidelines are usually available from the professional associations, for example, the ethical guidelines provided by the Economic and Social Research Council (UK), are available on: www.esrc.ac.uk or BAAL (British Association for Applied Linguistics) *Code of Practice* which I have used. The ethical considerations for my PhD project set out certain conditions to be aware of including consent forms and 'voluntary participation' (which I shall explain later). Ensuring that my research participants participated voluntarily and were protected from harm of any kind and establishing mutual trust between myself as a researcher and research participants who were studied (Silverman, 2006) were my ethical goals in my PhD research. All the ethical information was presented and given to my participants at the start of the research. Now I will turn to talk about consent forms.

I presented my participants with consent forms which provided information about the purpose of the study, for example, what was the time frame, what would their participation in the study involve, how the data would be protected and presented, and the fact that their participation was voluntary (Ritchie and Lewis, 2003). I presented participants' parents/carers with the information related to the study's procedure and purpose and they were provided with consent forms. Both the consent forms and the information related to the study were available in both English and Farsi. The reason for this was that not many new arrival parents/carers (first generation parents) know English and they would feel more comfortable and appreciate reading it in a language that was more familiar to them. In the consent forms (see Appendix C1-5), both student participants and parents/carers were provided with all the information I felt they needed to decide whether to participate. It included the 'focus of the research'; 'reasons for doing this research'; 'what 'your' participation would involve'; 'right of refusal to participate and withdrawal'; 'aspects of confidentiality' and 'what would happen to any information that you provide'.

Although consent was gained from school staff and parents, informed consent was also obtained from the students themselves for their participation in the research. I found it necessary to explain the purposes of the study to each new participant (see Erickson, 1990:142) so that the purpose of my research was clear. In the consent form, there was information about voluntary participation and withdrawal so that my participants knew exactly what they were consenting to and were aware that they had a right to withdraw from my study at any time (Israel & Hay, 2004).

An important aspect was confidentiality; the question as to whether the data obtained from the participants was anonymous and confidential. Due to the nature of data I obtained (video recordings) and the way I presented my findings using still images, participants' faces were exposed. Therefore the data could be traced back to an individual participant and therefore was not anonymous. All the participants (and parents of participants) in my study consented that there were no issue with students' visual identity to be shown in my study, so that the data obtained from the video recordings could be used in my thesis and presented at professional conferences. Furthermore there were no issues concerning anonymity by the University of Birmingham Ethical Review. Pseudonyms were used throughout the study both for individuals and for the school. I kept a record of pseudonyms used so that I could relate data obtained from one source with data from a different source. I was, however, the only one who could track back pseudonyms to the original participants.

This section focused on issues around ethical consideration. I have acknowledged ethical issues by making sure my participants were fully aware of the aims and objectives of my study.

4.9 Conclusion

This chapter has gone through the choices made regarding my research methods and some of the issues which related to these choices. I have talked about how my research design and research questions changed which led me to discard a substantial amount of data gathered across three different mainstream schools. Having new research questions I opted to justify three different methods of data collection which I thought were best suited to obtain interactional data to explore the nature of bilingual interaction.

In the next chapter I will focus on the process of data analysis, that is, ‘how’ I went on analysing my dataset. I will take an account of the analytical process of my dataset which were fieldnotes, video recordings and interview data.

5. Chapter five – Discussion of data analysis

5.1 Introduction to the chapter

This chapter deals with the process of data analysis. I will be talking about the ways in which my dataset were analysed and the challenges that I encountered. I will start this chapter by talking about a particular framework that I have adopted for the analysis of the language in a mathematics classroom. This particular framework focuses on the verbal and non-verbal aspects of language in communication. Having established a framework under which my data will be analysed, in section 5.3, I will discuss how I am going to analyse my dataset which consists of fieldnotes, video recordings and interviews. Later, I will address the role of subjectivity in the analysis of qualitative data in section 5.4. I will also discuss the choices that I made as a researcher around selecting data from an already obtained dataset. Finally I will summarise this chapter in section 5.5.

5.2 Framework under which the data will be analysed

This section focused on the importance of 'multi-modality' as a framework in my thesis. By adopting multi-modality as a framework, I will describe how eclectic approaches can be incorporated in order to analyse the *language* that is employed in a bilingual mathematics classroom. The two different approaches that I have adopted in this section examine the verbal and nonverbal aspects of language. Specifically, I will be looking at code-switching as a means of analysing the verbal language. Furthermore, I will draw attention to examine the non-verbal features of language, such as gestures, that are used to convey mathematical meaning.

5.2.1 Multi-modality

The term 'multi-modality' refers to the complex repertoire of semiotic resources that people draw on in different social spaces. The use of a multi-modal approach involves looking at language and other means of making meanings such as images, text, graphic symbols, three-dimensional forms, speech and gesture (Jewitt, 2008). As such, multi-modality pays particular attention to the relationships between the visual and the written mode and between speech, intonation and the accompanying gesture within the communicative event. In this sense multi-modality treats language as an incomplete system of meaning making whose full meaning can only be derived from the relation of each of its parts to one another. Multi-modality simply means taking an eclectic approach to understand how different modes are used to form a unified message. Now I turn to explain the approaches that I have taken to analyse classroom interaction. First I will focus on the verbal modes of analysis and later on the nonverbal means of expressions.

I was interested to look at the discursive practices that occur between teacher and students, and students themselves. Specifically, I was interested to examine how code-switching was used as a communicative function and in particular what prompts code-switching between

Farsi and English in a complementary school mathematics classroom. In this thesis I have established Classroom Discourse as an *approach* to analyse bilingual discursive practices. I will now expand and discuss the relevant literature on Classroom Discourse as an approach to the analysis of language. In general, research into verbal interaction builds on a tradition of work by researchers such as McHoul (1978), Mehan (1979; 1982), Erickson and Mohatt (1982) and Cazden (1988). The first major contribution to research in Classroom Discourse (or conversation analysis) was that by Sinclair and Coulthard (1975). Conversation analysis is a field that focuses heavily on issues of meaning and context in interaction. It classifies interaction in terms of key linguistic systems such as turn-taking (McHoul, 1978).

I will now turn to describe the approach that I have taken to analyse gestures in a bilingual mathematics classroom. Within the educational contexts, researchers have looked at how gestures interact and interplay in both science (Kress *et al*., 2001) and English classrooms (Kress *et al*., 2005). Particular attention has been paid to the role of gestures in mathematical discourse (Núñez, 2009; Morgan & Alshwaikh, 2012). Specifically, how gestures are used in facilitating language production as well as promoting learners' comprehension (Alibali & Nathan, 2012). The basic gesture categorisation refers back to the work of David McNeill (1992) where he classified gestures into four different subgroups. In this thesis I have established McNeill's (1992) basic gesture categorisation as an approach to analyse the non-verbal communication that takes place around presenting mathematical information.

It is worth considering that communication in mathematics is not just restricted to words and gestures. I would like to shift attention to the multi-modal nature of mathematics classroom in that making mathematical meaning is not just mediated by written symbols, but actions, gestures and other types of signs. In essence a mathematics classroom is multi-modal because it involves the use of visual signs (e.g. bar charts, histograms, etc.), algebraic notations and the mathematics register (O'Halloran, 2005). In a mathematics lesson the teacher and learners are often engaged in exchanging ideas through spatial arrangements, visual displays (such as gestures) and mathematical texts. I am interested to examine how different resources (such as gesture, writing and speech) are combined together creating unity from hybrid modalities. Due to the multi-modal nature of mathematics classrooms it is appropriate to use a multi-modal approach to help understand the complexity of meaning making in different classroom environments.

I would like to stress the co-existence of different modes that occur in communication. These modes are materially different (e.g. verbal and visual) but can be temporally coordinated. In the field of socio-linguistics these modes of meaning making are often referred to as contextualisation cues (Gumperz, 1982), that can accompany the actual words spoken by the interlocutors. Contextualisation cues are the ways in which a speaker signals and conveys his/her message through the choices of vocabulary, intonation and body language. This concept has been influenced in the analysis of bilingual classroom discourse, where code-switching serves as a contextualisation cue (Martin-Jones, 2000). Other kinds of non-spoken contextualisation cues are also used in classrooms where just one language is used. These include: non-verbal communicative resources (or multi-modal communicative resources) like body movements, gestures, facial expression as well as prosodic features (like stress and intonation) and other paralinguistic features like changes in the volume of speech (speaking louder or softer) or changes in the pitch (speaking higher or lower). So, for example, teachers can give negative feedback in ways that 'softens' its effect for instance, by smiling or changing their intonation. Based on the nature of my research question, 'What is the pedagogic nature of gesture (forms and functions) and other body-based resources in a British-Iranian complementary mathematics classroom in the UK?', I was interested in examining the ways in which bilingual learners employ gestures and other non-verbal contextualisation cues to convey their message in a mathematics classrooms. Moreover, I am interested to look at whether the employment of non-verbal modalities such as gestures, supports bilingual learners whilst learning mathematics.

The use of multi-modal analysis in multilingual classrooms is important because it is advantageous in understanding meaning making. Often bilingual learners who are not proficient enough in the medium of instruction make use of other ‘visual’ resources to make sense of the task. Monaghan (2009:25) observed that a “limited English proficiency student plotted the given coordinates on the graph who did not know what the terms ‘plot’, ‘point’ and ‘axis’ meant.” In this example the particular student had been able to make sense of the task, and complete it because the visual illustrations on the card had made it obvious to her what needed to be done. With this in mind, a multi-modal analytical approach investigates the communication that takes place in classroom discourse, beyond just the use of words. With these ideas in mind, I decided that multi-modality would be a useful framework through which to analyse my data.

It is worth noting that multi-modality is a relatively new approach in research in comparison to conversation analysis which is nearly 40 years old. As a result, the tools and mechanisms for analysis of ‘visual’ forms of text (e.g. writing on the board, diagrams, gestures etc.) are less fully developed in comparison to those of the ‘verbal’ forms of text. Much is known about the use of language resources in the classroom, but considerably less is understood about the semiotic potential of gesture, sound, image and so on (Jewitt, 2009). Moreover, it is also important to bear in mind that multi-modal studies are usually conducted on a small scale (like this study), and, as such, do not seek to make generalisations beyond specific cases. Multi-modal approaches can, however, provide depth of understanding into the complex processes of meaning making in learning environments.

This section focused on eclectic approaches that can be incorporated to analyse the language that is employed in a classroom. Multi-modality as an overarching framework was used to understand the verbal and nonverbal aspects of language in interaction in a bilingual mathematics classroom. These two approaches into analysing language are encountered under the framework of multi-modality, which pays particular attention to the relationship between the verbal and visual modes of expression. A multi-modal framework allows for an alternative interpretation between the visual and the written mode (Kress & van Leeuwen, 1990; 1996), or the relationship between speech, intonation (Brazil, 1997; Halliday & Greaves, 2008) and gesture within a single communicative encounter.

The next section focuses on the analysis of my dataset. It discusses the use of multi-modal approaches which were employed in the analyses of bilingual interactions in the mathematics classroom.

5.3 Analysing dataset

This section focuses on the analysis of my dataset. My complete dataset consisted of fieldnotes, video recordings and interviews. The process of analysing the dataset will structurally look at fieldnotes first, followed by video recordings and finally interview data.

5.3.1 Fieldnotes

The analysis of my ethnographic fieldnotes had two chronological stages. The first stage involved writing up fieldnotes and the second stage was a thematic analysis. I shall talk about each in detail.

In the first stage, in support of my observations in the research field (complementary school mathematics classroom), I started taking fieldnotes. My very first fieldnotes were almost in the format of bullet points due to the dynamic and fluid nature of the classroom environment. I tried to capture the events using as few words as possible so that I could keep observing interactions as they occurred. By doing so, I would arguably be less likely to miss further interactions during the process of recording observations. As a consequence, my fieldnotes did not convey a rich description of what exactly happened at a specific moment in time. I decided to write fieldnotes in greater detail following each classroom observation. 'Writing up' fieldnotes in detail after leaving the research site was an integral part of stage one. It often took as much time writing as I had spent observing.

The ethnographic fieldnotes were composed in chronological order and sometimes concerned 'high point' events (Emerson *et al*, 1995). High point events were specific topics of interests or incidents that seemed to stand out considerably due to their importance to the focus of the study. The fieldnotes were then word-processed, allowing me opportunity for greater reflection in that more detailed description could be made, and to allow for the "modification of words, phrases, and sentences in the midst of writing without producing messy, hard-to-read pages" (Emerson *et al*, 1995:41). The process of word-processing facilitated the later coding that was to take place in the second stage and sorting of the data. This process enabled the transformation of the data from general bullet points, to rich, detailed descriptions.

The second stage was the further analysis of the fully detailed fieldnotes. This began when the detailed fieldnotes were word-processed. After reading and re-reading the fieldnotes I became aware of some possible patterns. These patterns involved certain words and register that triggered code-switching from Farsi into English. The patterns of code-switching, which concerned aspects of my research question, were then thematised to match the relevant themes within the same fieldnotes from the same lesson. Later on, those themes were then compared and contrasted with other themes that were obtained across previous observations. Fieldnotes were traces of evidence and in order to show consistency, as Erickson suggested, they "must be pieced together into mosaic representations" (Erickson, 1990:155). Colour coding was used to elucidate all the patterns (or themes) across my fieldnotes. For example, particular patterns of code-switching were highlighted in green and examples of how gestures were used to convey meaning were captured in red. The same colours were used consistently across all my fieldnotes in order to make the analysis as straightforward as possible.

Although ethnographic fieldnotes can provide an account of the 'real' (situation) moments in time and space, Creese *et al* (2008:212) make the following observation: "it is important not to overinflate fieldnotes as interpretive resources; like other data in qualitative research, they are 'ephemeral' 'partial and incomplete', and need 'to be contested' and 'further analysed in relation to other data sets". The fieldnotes constituted one of the three different modes of data analysis in the research study (the others being video recording and interviews). Now, I will talk about the analysis of video recording and how it complemented the analysis of my fieldnotes.

5.3.2 Video recordings

Not unlike the analysis of my ethnographic fieldnotes, there were two chronological stages that were involved in analysing video recordings. The first stage involved the transcription of the recorded data and the second stage was the analysis. I shall discuss each of these chronological stages in further detail.

In the following discussion of video recordings, I will investigate the methodological issues that concern the transcription. Understanding transcription of visual phenomena deals with many different modes that cannot be transcribed simultaneously (Norris, 2004) and research of visual transcription is relatively at the very beginning of such a process (Goodwin, 2001).

Because classroom interaction, for example, contains implicit paralinguistic, kinesic, proxemics and multi-modal features such as writing, graphical presentations, posters, and gestures, transcribing only the verbal interactions would fail to highlight the graphical or visual phenomena. The transcription of classroom discourse often disregards the multi-semiotic nature of mathematics. Traditionally, transcripts fail to acknowledge resources for meaning making such as shifting between the 'language' (written and spoken) and 'the mathematical representation' (diagrammatic and symbolic) that occurs with the constant movement between language, mathematical symbolism, visual display (O'Halloran, 2005) and gestures. In addition, spoken text such as patterns of stress, pause and other paralinguistic features such as inflection and pitch are absent in an ordinary written transcript. The following extract from Wiener *et al* (1980:281) highlights why I am interested in incorporating and embedding the 'vocal' and 'visual' into transcription: "The significance of 'I love you' can vary considerably depending on whether the 'who', the 'what', or the 'whom' is stressed, as in '*I* love you' versus 'I *love* you' versus 'I love *you*'". In English language, pitch movement [tone] is predominantly used for grammatical purposes, for forming questions or statements. It is often used for lexical purposes too. As Kress (2009:56) suggests "try saying 'yes' to mean 'are you really sure?' or 'maybe' or to express sceptical, half-agreement". Emerson et al (1995:75-76) add to this by suggesting that "[p]eople talk in spurts and fragments. They accentuate or even complete a phrase with a gesture, facial expression, or posture". Other subtle types of inflection such as pauses are also often ignored in transcription. Jaworski (1993) refers to this as the 'power of silence'. Mason (2002:132) also noted that "there are pauses whose import is usually constructed from gesture and posture, from tones and undertones, and from being present in the moment oneself". By adopting a multi-modal approach to transcription, I was able to draw on not only the verbal transcripts, but also the vocal and non-verbal contextualisation cues (Gumperz, 1982) that were highlighted in the interaction. Analysing these together enabled a deeper understanding of the relationship between vocal and visual modes and how they supported one another in discourse (Mehrabian, 1972).

It has only been relatively recently that researchers have paid attention to the multi-semiotic nature of mathematics classrooms (O'Halloran, 2005). Radford (2009:63), for example, has attempted a multi-modal approach to transcribing data in a mathematics classroom whilst students were gesturing and talking about a graph. Along with multi-modal transcription, the use of snapshots (Rasmussen *et al*., 2009), drawings (Maheux, *et al*., 2009) and computer-generated images (Whitacre *et al*., 2009; Radford, 2009; Roth, 2009) are used for clarifying non-verbal activities such as gestures that are synchronously performed by both students and teachers in mathematics classrooms. My interest in this thesis lies "in developing precise tools for understanding the inter-relation of resources of representation and forms of knowledge" (Kress, 2010:119). Whilst some studies have focused on multi-modality (e.g. Goodwin & Goodwin, 1986), other researchers have focused on describing multi-modality in multi-linguistic contexts (Li Wei, 2011; Bezemer, 2008; Gullberg, 1998). Research has yet to be conducted, using multi-modality within the context of a *multi-lingual mathematics* classroom. This is one aspect of my research which is original. As such I was interested in what methodological approaches were useful in recognising a range of embodied and multi-semiotic resources that were presented within a bilingual mathematics classroom which could be captured by transcription. By drawing on a multi-modal transcription, this thesis captures and reveals a wider range of mathematical activities that are mediated verbally, vocally and visually.

I will now turn to explain the development of my transcription structure. As part of my data analysis, I presented parts of my transcript in a working group (see Farsani, 2012a). The aim of this working group session was to triangulate (Gorard & Taylor, 2004) classroom discourse analysis (or conversation analysis) with multi-modal analysis (Jewitt, 2009) through the use multi-modal transcripts (Bezemer & Mavers, 2011). I wanted to offer the participants of the working group an experience of trying out two approaches to the analysis of the same transcript and then to discuss the process. I therefore had two versions of the same transcript, one with text only (which was presented without my own analysis in the right hand column) and one with multi-modal additions. When discussing the transcript, the participants reported that a conversation analysis approach (in which they were given the text only) was almost impossible to make sense of. A comment was also made that researchers using such an approach will generally have had access to the rich video or audio data from which the text is created and that this is perhaps highly significant in terms of being able to make sense of the language.

In this transcription, the left hand column illustrates the verbal interactions only. The middle and the right hand column signify the multi-modal nature of the classroom and add richness to the verbal discourse. For example, the middle and right column include mathematical notations, symbols and visual representations. Based on the idea of what is 'Given' and what is 'New', Kress and van Leeuwen (1996:187) believe that "[t]he elements placed on the left are presented as Given, the elements placed on the right as New. What is given is what the viewers are already familiar and agree upon but what is New, on the other hand, is not yet known and viewers must pay special attention to it". Therefore unlike many of the ordinary transcripts, the middle and right hand column are 'new', in that they add clarity and complement the left hand column by bringing visual elements into the verbal transcription. It is important that Kress and van Leeuwen have also acknowledged 'culture' and 'social constructs' which can attach different values to reading directions (for example reading from right to left or from top to bottom). I have used the left hand column to represent 'speech' alone. That is both the verbal and vocal elements of communication. That is, not only what is something said but how it is said. The middle column takes an account of the visual elements of communication. Often in this column snapshots are evident to represent the activity that was in place. Having encountered verbal, vocal and visual elements of meaning making, I have used the right hand column to add clarity by describing the event in a greater detail in a neutral third person's voice.

As a means of an example of this multi-modal transcript, I am going to present part of one of the multi-modal transcripts where the classroom teacher (T2) and learners in this classroom were engaged in solving arithmetic questions from a textbook on the whiteboard. It is worth noting that multi-modal transcripts are not only 'multi-modal' in that they represent multi-modal interaction, they are also multi-modal in that multiple modes operate in the transcript, usually a combination of writing, typography, image, mathematical symbolism, algebraic notations and diagrammatic representations (Bezemer, 2013 personal communication).

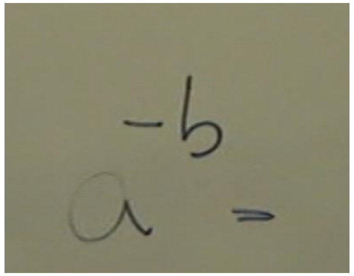
The transcript which follows focuses on a particular task to simplify $\frac{2}{\left(\frac{1}{3}\right)^{-2}}$. This task exhibited a challenge to many bilingual learners in the classroom. The classroom teacher (T2) recommended students just to focus on $\left(\frac{1}{3}\right)^{-2}$ and forget about the numerator for the time being. He then demonstrated that any number 'a' to the power of a minus integer '-b' can be rewritten as 'one over a' to the power of the positive integer 'b'. T2 offered this method of procedure as he wanted to avoid negative powers.

The multi-modal transcript convention I have used is as follows:

T	Teacher
B	Boy
G	Girl

[ ]	Non-verbal communication
{ }	My Translation
Italics	Farsi transliterated into English
Normal font	English language
Dots	Each dot represents one second of silence
Change in font size	Change in volume of an utterance: the bigger the font is, the louder the pronunciation. The smaller the font is, the quieter the pronunciation of the term.

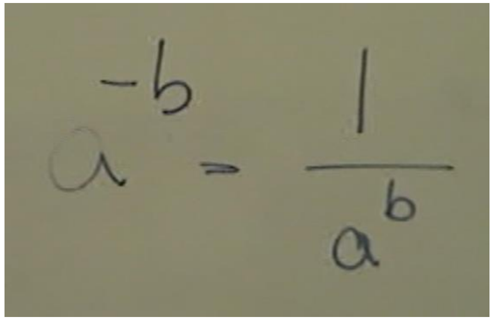
T2: *chi migoftam, migoftam agar darim* {What was I saying, I said if we have} ‘a’ *be* power of minus ‘b’, it’s equal to what? {‘a’ to the power of minus ‘b’, it’s equal to what?}



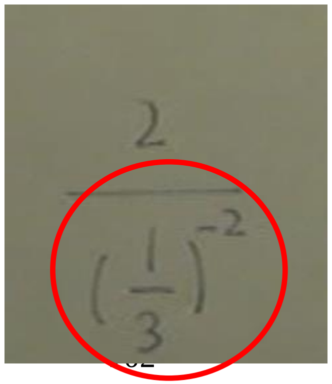
[T2 writes a^{-b} on the whiteboard as he is completing his utterance]

B1: ‘a’ over ‘b’

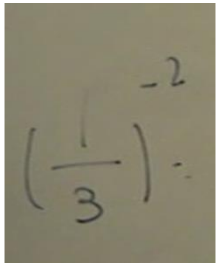
T2: . . No, . . it’s one over ‘a’ *be* power of . . ‘b’ . . {. . No, . . it’s one over ‘a’ to the power of . ‘b’. .}



[T2 writes $\frac{1}{a^b}$ on the whiteboard]

	chera, chon really . *man be* *tore mamooly doost nadaram* *ke che kar bekonam?* {why, . because really, . normally I don't like to have what?}		
Bs:	Negative		[A few boys said negative at the same time]
T2:	Negative as a power *dashte* *basham.* {I don't like to have negative as a power}		[Intonation, emphasis on the term 'power' as well as a hand gesture to indicate the position of the power]
	pas {so} I just take this one, that bit, I just take it out.		[T2 refers back to the original question and indexes the denominator of the complex fraction by producing a deictic gesture]

Man daram chi, one over three *be* power of minus two. {So I have one over three to the power of minus two}



[Teacher writes $(\frac{1}{3})^{-2}$ on the whiteboard as he makes his utterance]

B3: oh, *ye* one *balash mizari* {oh, put a one at the top}

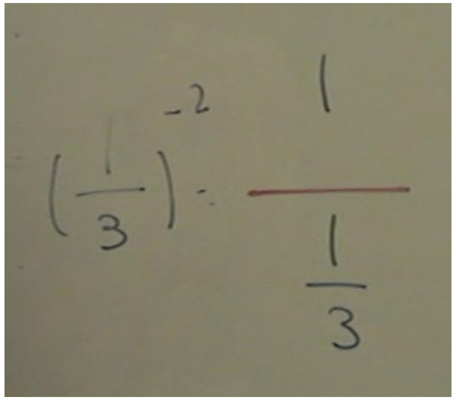
[B3 points with his index finger illustrating one at the top. He is holding his index finger up with the rest of his fingers closely curled, which could possibly indicate one. The location of his index finger is not horizontal as most pointing gestures are, but semi-vertical, more vertical than ordinary pointing gestures tend to be]

T2: *pas migam chi?* {So}

XXX Inaudible

T2: I am gonna use a different colour so you know which
one is which, one big
fraction line,

one in the top, one third in the bottom.



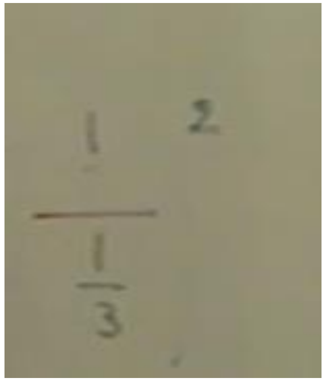
[T2 draws a red horizontal line which appears to coordinate with his utterance timing]

Khob hala mikham ino
reverse *bekonam.* {Ok, now
I want to reverse it.}

[T2 has extended his index and mid fingers facing outwards with the rest of the fingers closed. He then turns his wrist inwards and the palm faces inside]

Khob, ino bekham reverse *bekonam chetor bayad bekonam*? {Ok, how can I reverse it?}

Ok, *in alan injoorie* {this is how it looks like at the moment} the whole thing to the power of two now.



[T2 has raised $\frac{1}{\frac{1}{3}}$ to the power of two now.]

but that is not good, we want to change it.

[There is now a fraction in the top as well as in the bottom. T2 divided the one (in the numerator) by one. So the complex fraction now looks like $\frac{\frac{1}{1}}{\frac{1}{3}}$]

B1: *nazdik be nazdik, door be door* {near by near, far by far}

T2: *door dar door*, {far by far,}

$$\frac{\frac{1}{1}}{\frac{1}{3}}$$

[T2 indexes the two numbers furthest apart]

nazdik dar nazdik. {near by near.}

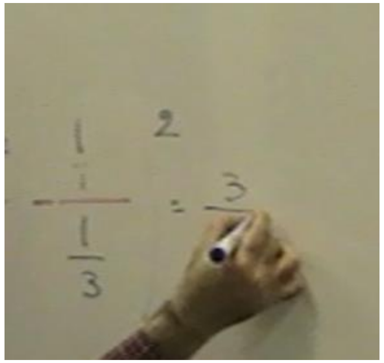
$$\frac{\frac{1}{1}}{\frac{1}{3}}$$

[T2 indexes the two numbers closest to the main division line in the centre]

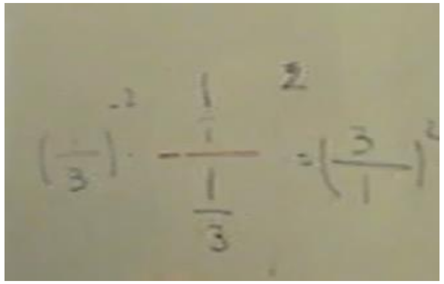
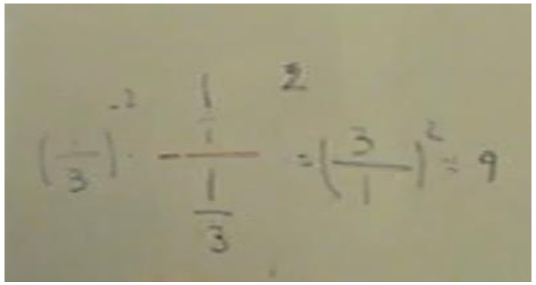
So, *door dar door mire koja*? {So where does far by far go to?}

B2: *dar door* {It goes far!}

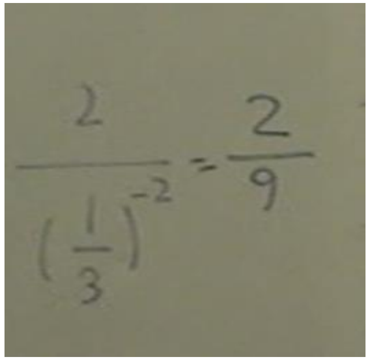
T2: *soorat, door dar door soorat* {Numerator, far by far goes to numerator} or numerator.



[Synchronically to his verbal message, T2 writes three and a division line underneath]

		Nazdik dar nazdik? {Near by near goes to?}		
	Bs:	*posht, jolo* {Behind, in front}		[T2 writes one as the denominator. He also raises the fraction to the power of two.]
70	T2:	*makhraj*. {Denominator.}		
		And, three to the power of two is?		
	B1:	Nine		

T2: So that equals two over nine.



[At this time, T2 goes back to the original question and writes two over nine as the answer]

75 B2: I get it now.

The teacher’s gestures minimized the use of the spoken mathematics register whilst pointing, and drawings substituted certain technical words (see lines 23-4) and sometimes acted as an amplifier to certain mathematics registers such as ‘reverse’ (see lines 39-41). Moreover, the teacher’s uses of demonstrative pronouns (e.g. ‘here’) were often accompanied by demonstrative or pointing gestures to indicate objects, locations or inscriptions even though ‘here’ identifies specific locations close to the speaker (Hanks, 1992). Moreover, within a classroom context, often teachers’ use of demonstrative pronouns (‘this/that’, ‘here/there’, ‘these/those’) relies heavily upon additional visual interaction to compensate the lack of clarity of words alone, and as such indicate with gestures ‘what’ or ‘which one’ they mean.

All of the video recordings were transcribed but only certain excerpts of video were transcribed multi-modally. The parts that were transcribed multi-modally were the parts I wanted to focus particular attention on. These parts often contained gestures or other aspects of visual representation (such as use of different colours) that would not have been recognised in the transcription if only the verbal aspect of language was taken into account. Adopting a multi-modal approach to transcription meant that particular attention could be paid to understanding how different modes were used to form a unified message.

Another interest I have within any research lies in the pedagogic nature of gestures. Therefore in the analysis of video recordings, I paid particular attention to gestures that conveyed mathematical meanings. Regulatory and disciplinary gestures such as these for “Ok/Good”, “wait”, “carry on” or “quiet” were not included for analysis. The analysis involved choices about what to and what not to include from a larger source of data. Based on the nature of my research questions, it was more important to stress the gestures that suggested or implied mathematical meaning rather than gestures that were just used for disciplinary purposes. Later, I will discuss the notion of subjectivity in relation to the representational choices that I have made in research. In conclusion, the transcription is indeed part of the analysis and how the data is presented informs how the data will and can be analysed. The next section (5.3.3) discusses the analysis of the interview data in this research study.

5.3.3 Interview data

Section 5.3.1 and 5.3.2 have described the process of analysis of fieldnotes and video recording respectively. This section moves on to discuss the analysis of interview data. There were two sources for the interview data. Firstly, I wanted to collect bibliographical information about each student. Secondly, some other interviews were conducted to follow up particular moments that I had noticed in the analysis of video recordings (which I shall explain towards the end of this section). I will explain each of these points in the subsequent paragraphs.

Firstly, I intended to interview all the students concerning their bibliographical details, for example, not only information about how many languages they spoke, but their experiences with each language across a different range of tasks and settings. Moreover, I wanted to see whether there were possible patterns or triggers into code-switching as they were talking about themselves. For pragmatic reasons, only two interviews were video recorded. This is because the Sunday school ran between 11am until 3pm. Most of students could not make it earlier into school for a formal video recorded interview. Moreover, all of the students were expected to finish sharply at 3pm so that they were collected straight after school. Due to these pragmatic reasons, I conducted more informal interviews with the rest of my research participants during their lesson times. Notes were collected during these informal interviews with them at lesson times. I believed the ways in which they code-switched to convey their message would have been of particular interest to answer part of my research question. The analysis of the students' use of language in their responses was carried out in the exact same way that I analysed the fieldnotes. For example, informal interview data were later word processed and then thematised.

Secondly, there were also other interviews that I carried out. These interviews were specific to what I had observed during the analysis of some of the video recordings. In the analysis of the video recordings, I became aware of particular moments in the video where I noticed particular instances of gesticulation. For example, in one occasion, I conducted an interview with T2 in order to follow up a particular gesture that was made during his instructional information. Whilst revisiting a particular video excerpt, I realise that T2 generated a particular gesture for the mathematical concept 'power' as he was conveying information in English. I wondered whether T2 would have generated the exact same gesture for the same concept in Farsi. It was important to note that a particular interviewing technique was used to encourage T2 to make a gesture in the interview when discussing the concept of 'power'. In order to achieve a gestural representation for the mathematical concept 'power' in both English and Farsi, I decided to code-switch at frequent intervals as often the language of initiation tends to elicit a response in that specific socio-linguistic register. Moreover, in order to provoke a gestural representation by T2, I decided to ask the same question again and again after T2's reply to each question. I believed that by asking repeated questions, T2 would have thought what he said was not sufficient enough to convey his message, therefore, T2 would have to try to draw upon other resources at his disposal (including semiotics) to express his point. This would eventually lead him to present the concept of 'power' in gestures. I will describe the analysis of this particular interview in chapter seven. Implementing a follow-up interview with T2 enabled me to understand some aspects of the role of gestures in mathematical discourse. As a result, it became clear how particular gestures were used in relation to their referential verbal content. This interview also enabled me to explore part of my research question which examines the role of gesture in mathematical talk. The data that was generated from this interview was then transcribed using a multi-modal approach in the same way that I analysed other gestures in the mathematical talk.

Now that I described the process of data analysis, I would like to bring into light the nature of my dataset. What I believe is particularly important is to raise an awareness to the *chronotope* nature of my dataset. Issues of time and space. For example, fieldnotes and interview notes constituted two of the three different modes of data analysis in the research study (the other being video recording). The three different modes of analysis were not only materially different, but also had different *temporal* presentation. For example, the fieldnotes only provided a *reflection* of what happened at a particular point; therefore it is *not in time*. The second mode of data analysis is the video recording which captures the spatio-temporal relation of modes intact. For example, the hand movements and their accompanying speech can be viewed as it occurred at that particular time in space. As such, the use of video recording captures what happened at a particular moment which is *in time*. In this sense, some modes of data analysis may or may not have been temporally coordinated.

In the next section I will talk about subjectivity and specifically how my role as a researcher influenced certain choices concerned with the selection of and presentation of the available data for further analysis.

5.4 Subjectivity

This section addresses the choices that a researcher makes and the ways in which data is selected and presented. First I will discuss the choices I made concerning the selection of data from an already obtained dataset. Later in section 5.4.2 I will be raising awareness as to my role as a researcher in translation and consequently the interpretations that it generated.

5.4.1 Choices of 'data selection' from the dataset

In the previous chapter (section 4.7), I addressed the influences of a researcher in research, focusing on how the describer is part of the description and the observer part of the observation. I have also talked about the complexities around recording interactional data in the research site. However, what I would like to address here are the choices that a researcher makes regarding the selection of data for analysis from a larger volume of dataset.

Table 2: Choices around recording and selecting interactional data

	In the research site	Outside of the research site
Researcher's choices	What/when/whom to record in the classroom	What to select from the already obtained dataset for further analysis. There are of course choices of presentation, e.g. what to present in the transcription.

The choices surrounding the selection of particular video excerpts for further analysis are heavily influenced by the role of the researcher. For example, in my thesis I have transcribed, using a multi-modal approach, just five minutes of video excerpts from a total of 12 hours of video recordings. This concept fits well with the role of the researcher in choices s/he makes in transcription. Bezemer and Mavers (2011) recently began developing a social semiotic framework to account for multi-modality with the aim of being able to understanding meaning-making practices better. They suggested that the *agency* of transcribers is foregrounded in that they make significant *representational choices*. Examples of these representational choices may include 'What do I select for transcription?' or 'What do I highlight in the transcript?'. These choices are clearly important as the transcripts often highlight the 'original' observed activity through the professional perspective of the researcher which is inevitably different through the participants' perspective in which the ‘original’ activity was constructed. But also Blommaert and Dong (2010) suggest this is not always acknowledged by the transcriber. In addition Mondada (2007:819) highlights that the work of transcribing itself never ends because “[e]ach printed version is the result of a selective process of displaying, foregrounding, highlighting particular details for a specifically recipient-oriented analysis or demonstration”.

The following section (5.4.2) addresses issues surrounding the role of a researcher in translation and interpretation.

5.4.2 Translation

Translating requires interpreting. Much of my data required translating into English due to the dynamic of code-switching, not an uncommon phenomenon in multilingual classrooms. In the classroom, I observed a high frequency of code-switching between teacher-students and students-students in their communication. In order to present these type of data that emerges in multi-lingual contexts, the translator should go back and forth in languages to find equivalent statements in a foreign language. However, sometimes equivalent statements or even words may not exist in another language. For example, within the context of mathematics, there is not a word equivalent for ‘cross-multiplication’ in Farsi. Similarly, there is not a term in English for ‘*door dar door, nazdik dar nazdik*’ (see Figure 4). With this in mind, instead of a single word to describe a property of a certain kind, it may be necessary to describe the whole concept in a few words (or even a few sentences) to convey the message rather than the use of a single word. As Winsløw (1998a:21) suggests, translators “will also experience how cultural environment shapes a language, how subtle differences make some ideas more easily, or even exclusively, expressible in one of the two [languages]”. In addition, Moschkovich (2010:19) reflects on how knowing the subject knowledge (e.g. mathematics register) impacts on translation as they “need to have mathematical knowledge and know mathematical terms and expressions in both languages”.

The translation of a bilingual mathematics lesson highlights challenges as language skills, mathematical knowledge and knowing the mathematical registers and expressions in both languages are required. Moreover translation also presents another layer as it involves interpretation. Moschkovich (2010:18) sounds a note of caution about interpretation in that she acknowledges that it is impossible to translate without putting some piece of ourselves in the new utterance. What also adds to the complexity working within a rich multilingual classroom is the fact that I am not communicative competent in all the languages that interlocutors drew upon as a resource to communicate with each other. Languages such as Turkish, Armenian, Azerbaijani and Kurdish were also present as a medium of transmission among learners. There have been moments when students talked with each other in Azerbaijani which unfortunately I was not able to comprehend.

Interestingly enough, it is not just the verbal translation that is complex, there are non-verbal issues that exhibit many challenges in translation. Interpreting gestures and nonverbal body language is ambiguous especially in the absence of the accompanying speech (Feyereisen *et al*., 1998). It is crucial to remind ourselves about the fact that body language can't be considered in isolation; it is usually an amplifier to what a person is already saying (Witchel, 2013 personal communication). Most body movements can have various different meanings (Driver, 2010). For example 'the evaluation gesture' is often referred to when the hand is moved to the chin, and the hand may also be stroking the chin. This action could be taken in account that the person is evaluating the proposition and drawing conclusions simultaneously. It is also possible to acknowledge 'load bearing' here; when the listener begins to lose interest in the speaker, the head begins to rest on the hand with the head supported by the thumb as the listener becomes uninterested (see Pease & Pease, 2006:160). Even from the multi-modal approach it is difficult to suggest whether this gesture means this or this image means that (Jewitt, 2009) and often the interpretation of gesture is not well understood.

Clearly, the researchers have no direct access to the 'mental representation' that underlies a gesture, but must use the linguistic, social, and cultural contexts, as well as the activity in which the speaker is engaged, in order to construct a plausible interpretation of a gesture (Edwards, 2009:129). Gestures are symbols, and just like words within a language, their "meanings become specified only in the context is being used" (Steeck & Kapp, 1992:12). The gestures are actions that their meaning is given in interaction (McNeill, 1985). The interpretation of gesture is also cross-cultural. As Kress (2009:57) suggests "what is done by speech in one culture may be done by gesture in another … so we cannot assume that translations from one mode to the 'same' mode in another culture can draw on the same resources". This is simply because each mode is culturally given and socially shaped. As such Kress (2010:11) suggests that "there is no reason to assume that the mode of gesture in culture 1 covers the same 'area' or the same concerns, or is used for the same purposes and meanings as the mode of gesture in culture 2". Elaborating upon his point, Kress (2010:11) raises awareness of the following question:

> "the translation of movement, an action, a gesture that is entirely understood in one society and either entirely misunderstood or not understood in any way in another? We simply can no longer assume that the reach of modes is the same across different societies and their cultures".

Given that there are individual, material and cultural differences, Bezemer and Mavers (2011:196) also highlight that "there can never be a perfect 'translation' from one mode to another".

In this section, I have discussed issues and complexities around both *recording* interactional data from the research site and *selecting* interactional data from an already obtained dataset. They both required exclusive choices which are influenced by the researcher. Furthermore there are issues around translation as it involves interpretation. Translation not only covers verbal means of expressions but non-verbal modes of representations. There are non-verbal resources for meaning making that may not be transferable from one language to another.

5.5 Conclusion

In this chapter I have discussed the choices that were made regarding the process of analysing data from an already obtained dataset. These choices involved looking at alternative ways of analysing recorded and non-recorded data to explore the nature of bilingual interaction. There were also important questions in relation to the *selection* and *representation*. That is, not only how to select a particular data from an already obtained dataset but how to represent those information for further analysis which reflected the influences of the role of researcher in research.

In the next following three chapters, I will focus on analysing gestures and how they complemented talk around mathematics. In other words how visual modes of expression acted as amplifiers to what was said verbally.

6. Chapter six – Iconic gestures

6.1. Introduction to the chapter

As well as introducing this chapter, this section gives an overview of chapters seven and eight. In this chapter, as well as chapters seven and eight, I have adopted McNeill's (1992) 'basic gesture types' as a framework to identify and classify gestures that co-occur with other modes of communication (e.g. speech) in a complementary school bilingual mathematics classroom. McNeill's (1992) 'basic gesture types' category has been used as a generic framework extensively in the field of psychology (e.g. Goldin-Meadow *et al*., 1999) and recently in mathematics education (e.g. Zurina & Williams, 2009; Radford *et al*., 2009). Throughout the analysis of chapters six, seven, and eight, I will be discussing the realisation of three different gesture types that were used as a resource in the classroom discourse in a British-Iranian bilingual mathematics classroom. In this chapter, by provide evidence from my interactional recording dataset, I will be primarily looking at the *iconic* gestures that were produced and how they conveyed pertinent mathematical information. In chapter seven I will primary look at the *metaphoric* gestures that were enacted by the interlocutors and finally in chapter eight, I will look at the enactment of *deictic* gestures. I hope this and the subsequent chapters raise awareness to the types of gestures that are produced in this specific community of practice.

The structure of this chapter consists of a brief literature on McNeill's (1992) gesture type categories in section 6.2, data analysis in section 6.3, and, conclusion in section 6.4. In section 6.2, I will bring into light different categories of McNeill's basic gesture type and what defines each classification. Section 6.3 divides into subsections 6.3.1 and 6.3.2 where I will present two different examples of gestures conveying the mathematical concepts parallel and perpendicular respectively. These two examples are *iconic* gestures that were incorporated by the classroom teacher, T1 during the instructional talk. Finally in section 6.4, I will discuss the communicative functions of iconic gestures and other body-based resources in a mathematics classroom.

6.2. A brief summary of McNeill's literature on types of gestures and the communicative effectiveness of gestures and body-based resources

The structure of this section consists of a brief literature on McNeill's (1992) framework of gesture classification. McNeill (1992) has classified gestures into four different categories which consist of: 'iconic' gestures, 'metaphorical' gestures, 'deictic' and 'beat' gestures. Iconic gestures are gestures that resemble concrete objects or actions which "depict the semantic content directly via the shape or motion trajectory of the hand(s) (e.g. tracing a triangle in the air to mean triangle)" (Alibali & Nathan, 2012:251). Iconic gestures are illustrators which "bear a close formal relationship to the semantic content of speech" (McNeill, 1992:14). Therefore iconic gestures direct attention to physical objects in the environment and convey the semantic content visually.

Metaphoric gestures constitute one of McNeill's (1992) basic gesture types which are similar to iconic gestures in a sense that they convey the semantic content via metaphor. McNeill (1992:14) highlights the description of metaphoric gestures by talking about the difference between metaphoric and iconic gestures. "Other gestures are 'metaphoric'. These are like iconic gestures in that they are pictorial, but the pictorial content presents an abstract idea rather than a concrete object or event. The gesture presents an image of the invisible – an image of an abstraction." For example pantomiming a spherical shape to represent a ball is an iconic gesture that resembles a concrete object in the environment but pantomiming a spherical shape to represent the idea of wholeness is a metaphorical gesture as it represents something abstract (Casasanto and Lozano, 2006). In gesturing metaphorically, people use physical space to represent abstract ideas that have no spatial instantiation, for example pointing up in the air to indicate ‘high intelligence’.

Deictic gestures are pointing gestures which often occur with an extended index finger but sometimes with the other fingers or the entire hand that serve to indicate locations, directions persons or objects that are either present or non-present in the environment. There exist different variations of index-finger pointing and each type is correlated to certain discourse factors which I shall draw attention to in section 8.3.

The last classification of gesture is the ‘beat’ gestures. Beat gestures constitute one of McNeill’s (1992) basic gesture types which are repetitive movements of the hands which are rhythmically coordinated and aligned with the speech prosody. For example a teacher can talk about ‘x’ multiplied by itself three times and makes three repetitive hand gestures in the air as he utters ‘three times’. Beat gestures do not express the semantic content of the speech they accompany (McNeill, 1992). Moreover, due to the semantic content of each language, speakers of different languages are likely to manifest different body motion rhythm (Condon, 1980). The reason that beat gestures are not included in the analysis of this thesis, is due to the fact that they were not produced in this complementary school bilingual mathematics classroom to convey mathematical meaning.

In the next section, by providing evidence from my interactional recordings, I will show how a particular classroom teacher (T1) expresses his knowledge of some particular mathematical concept in iconic gestures.

6.3. Iconic gestures:

This section reveals the enactment of two iconic gestures performed by a particular mathematics teacher (T1), as he was conveying the instructional information. Iconic gestures will appear to be crucial to the effectiveness of mathematical communication as not only do they convey the semantic meaning of some specific mathematics register as they visually support the concept but also they can promote a better understanding of some mathematical concepts. The two iconic gestures that are presented in this chapter, each reveal the semantic content for the mathematical concept 'parallel' and 'perpendicular'. The structure of this section therefore falls into two subsections where in section 6.3.1, I will present evidence from my interactional data to describe and analyse the gestural representation for the mathematical concept 'parallel'. Then in section, 6.3.2, not only the enactment of the mathematical concept 'perpendicular' in gestures by T1 is discussed but how the gestural enactment conveys its mathematical definition.

In the following section, by presenting evidence from a particular interactional video recording through multi-modal transcript, I am aiming to show a gestural representation for the mathematical concept parallel as T1 was engaged in conveying instructional information.

6.3.1. ‘Parallel’

The following multi-modal transcript reflects on part of the fluidity and dynamicity of a bilingual mathematics lesson where T1 and students are engaged in working on a number of questions which are projected on the whiteboard. In response to many students’ requests, T1 decides to go through a question and explain the mathematics by working it out to the class. The question was on geometry and was asking to find the length of two sides of two different triangles, BC and EC (see Figure 9). The question had already stated that the angle ABD and BDE are right angles and AB is parallel to DE. Moreover some of lengths of the sides were given in the question. For example, AB= 6cm, AC=8 cm, CD= 10cm and DE= 8cm. I will now turn to present my multi-modal transcripts.

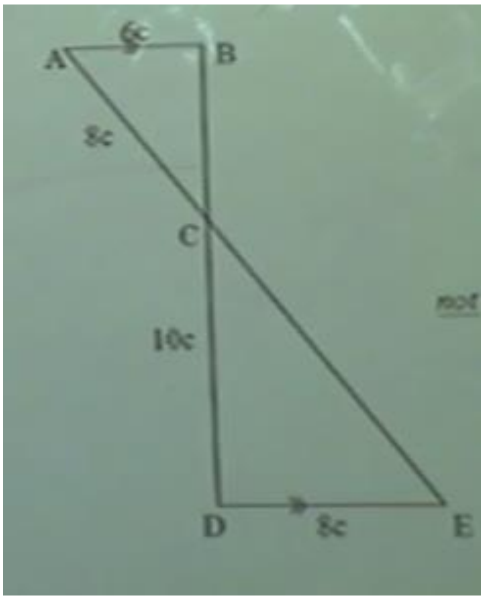


Figure 9: Lines AB and DE

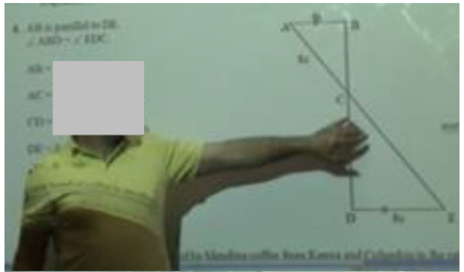
The multi-modal transcript convention is as follows:

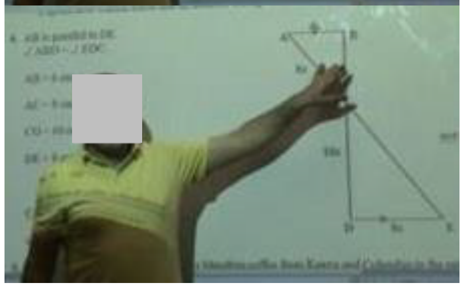
T	Teacher
B	Boy
G	Girl
[ ]	Non-verbal communication
{ }	My Translation
Italics	*Farsi transliterated into English*
Normal font	English language
Dots	Each dot represents one second of silence
Change in font size	Change in volume of an utterance: the bigger the font is, the louder the pronunciation. The smaller the font is, the quieter the pronunciation of the term.

Transcript number 1

1 T1: These two are similar, why?

B1: *chon*, {because}

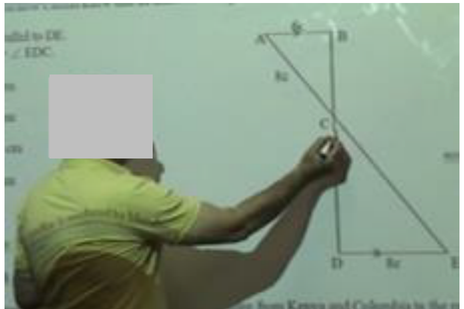


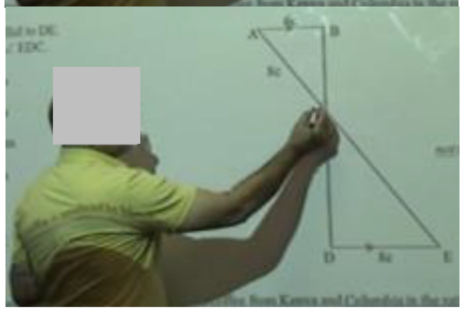


[T1 indexes with his left hand to two triangles which are projected on the whiteboard as he said "These two are similar"]

T1: This equals this one, yeah?

B1: *are* {yeah}

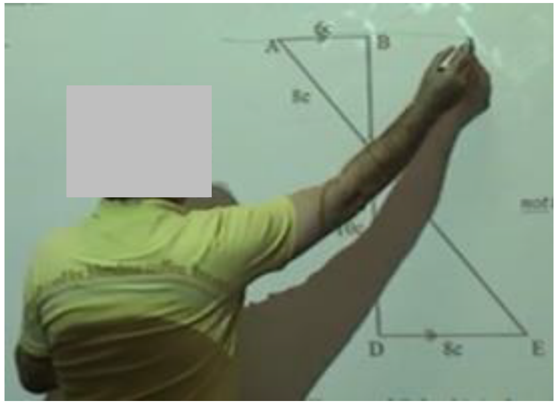




[As T1 utters the first 'this' in the phrase "This equals this one", he uses a board marker and marks one angle and then another angle as he says 'this one'. His actions (what he draws) appear to be in synchrony to what he says]

T1: *in do tah ham gofte chi-an*?

…{It says these two are what? …}



[As T1 raises a question and asks "It says these two are what?", he draws two lines on the projected diagram and extends the length of AB and DE]

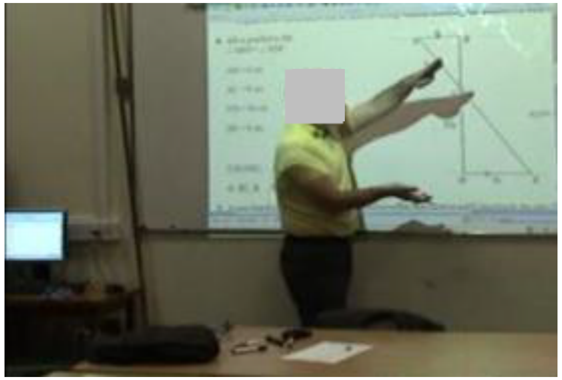
Bs: parallel

T1: *in do ta ham ba ham chi-an,*

parallel *an*. *Doroste?*

{These two are parallel.

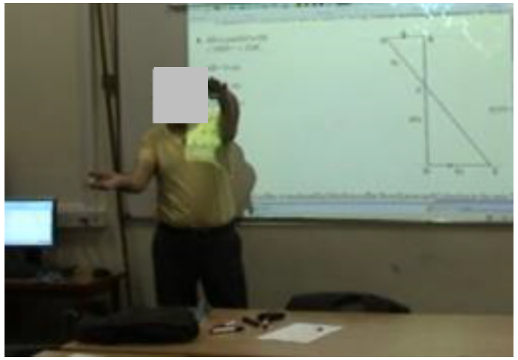
Right?}



[T1 places one hand above the other and approximately one second later he said "These two are parallel. Right?" which simultaneously he turns approximately 90 degrees to the class at the same trajectory.]

B1: *bale bale.* {Yes, correct}

T1: *vaghti in do ta ba ham*
parallel-*an*, {When these
two are parallel,

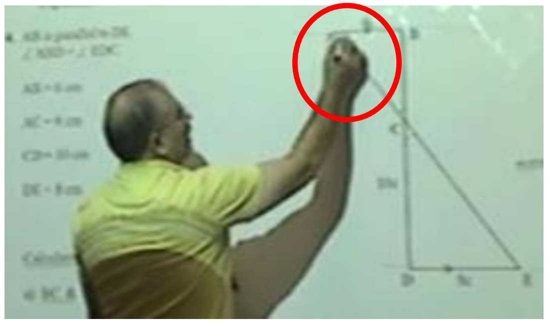


[T1 turns back (approximately) 180 degrees (now facing the whiteboard) with his hands maintaining the same motion and trajectory as he said “When these two are parallel”]

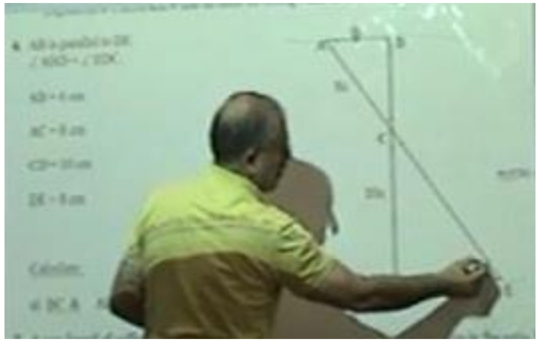
iin . {this .}
B4: the same-*an* {are the same}
B2: *jedan*? {Really?}
Bs: same-*an* {they’re the same}

[As T1 utters ‘this’ in line 17, he marks the angle at B as 90 degrees on the whiteboard. Then one second later without him saying anything, he marks the angle at D as 90 degrees too]

T1: *iin ham* . {and this .}



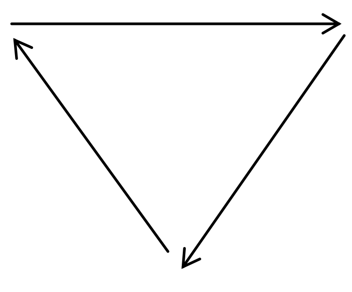
[T1 marks angle A with two semi-brackets as he said '*iin*' {this} in line 21. Then he moves to mark angle E with the same notation in silence.]



B4: same-*an* {they're the same}

I will now turn to one example from the given transcript to show how T1 incorporated his body-based resources as he conveyed his instruction verbally. Specifically I will show a case where T1's 'iconic' gesture revealed the semantic content of a specific mathematics register; 'parallel'. In lines '9-12' T1 makes a statement and at the same time raises a question "These two are parallel. Right?". Before T1 mentions the speech-accompanying gesture 'parallel' in his statement, he holds his hand one above other which seem to maintain the same distance apart. T1 then turns 90 degrees and faces the class as he said "These two are parallel. Right?". By doing so he creates a representational gesture that emphasises the mathematical notion 'parallel' and his hands maintained the same distance apart as he turns 90 degrees. In terms of semantic content, T1's gesture appeared to be approximately one second in advance to its corresponding verbal reference. T1's gesture had already started its trajectory when he uttered the corresponding verb 'parallel'. As I mentioned briefly in the literature often gestures tend to appear a little in advance of their associated speech units and "their preparation begins sometime well in advance" (Wiener, *et al*., 1980: 220).

An interesting issue with geometry is that often a geometric symbol is its own representation. Hence it can be argued that a gestural representation for a triangle is iconic and also conveys its mathematical definition. For example, one can produce a hand gesture in space (in this pattern) depicting a triangle.



The same principle applied to the concept of parallel. The mathematical concept 'parallel' was intrinsically linked to T1's gestural representation as he conveyed the concept of ‘parallel’ in discourse (see Figure 10). T1’s gestural representation manifested the mathematical concept of parallel through visual mode of expression (or gesticulation). T1’s gesture classification for the concept parallel falls under McNeill’s iconic classification because it stressed and emphasised the fact that the two lines (AB and DE) shown on the whiteboard have the same continuously distance between them.

It is interesting to note that although language is a tool for communication, gestures are actual communicational actions. Not all the relevant instructional information comes in the form of spoken words. T1’s gestural representation for the concept parallel, highlighted aspects of the related task that could be taken to emphasise/convey the verbal content; in this case “parallel”. T1’s gestural representation for parallel could have emphasised terminology (parallel) and even complemented his verbal utterance as he incorporated both verbal and nonverbal means of expression to convey the concept of parallel. The ways in which T1’s gesture was enacted links tightly to the conceptual understanding of the mathematical concept ‘parallel’. In a mathematical discourse, body-based gestures for example ‘parallel’, provide evidence that mathematical thinking is embodied. T1’s gesture could therefore be taken into account to reveal the nature of mathematical concept ‘parallel’ as often “gestures ‘reflect’ speakers’ embodied thinking about mathematical concepts” (Alibali & Nathan, 2012:274).

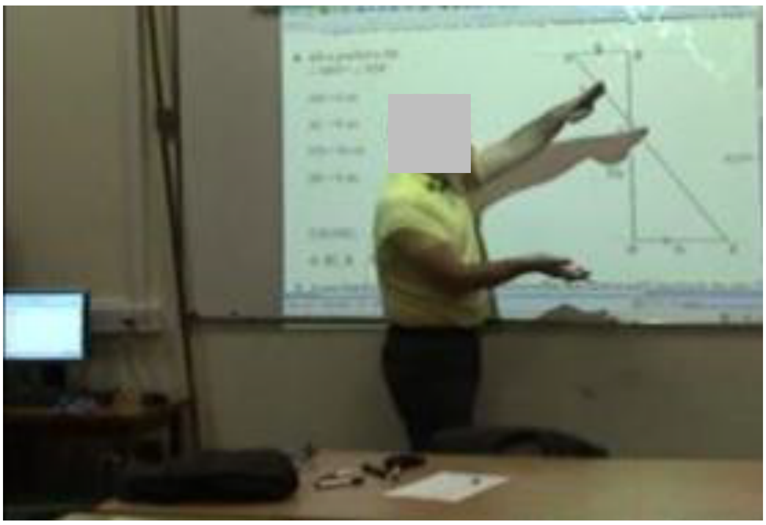


Figure 10: Parallel (I)

T1’s use of hand gesture to convey the mathematical concept ‘parallel’ is also evident in a number of different lessons (e.g. see Figure 11).

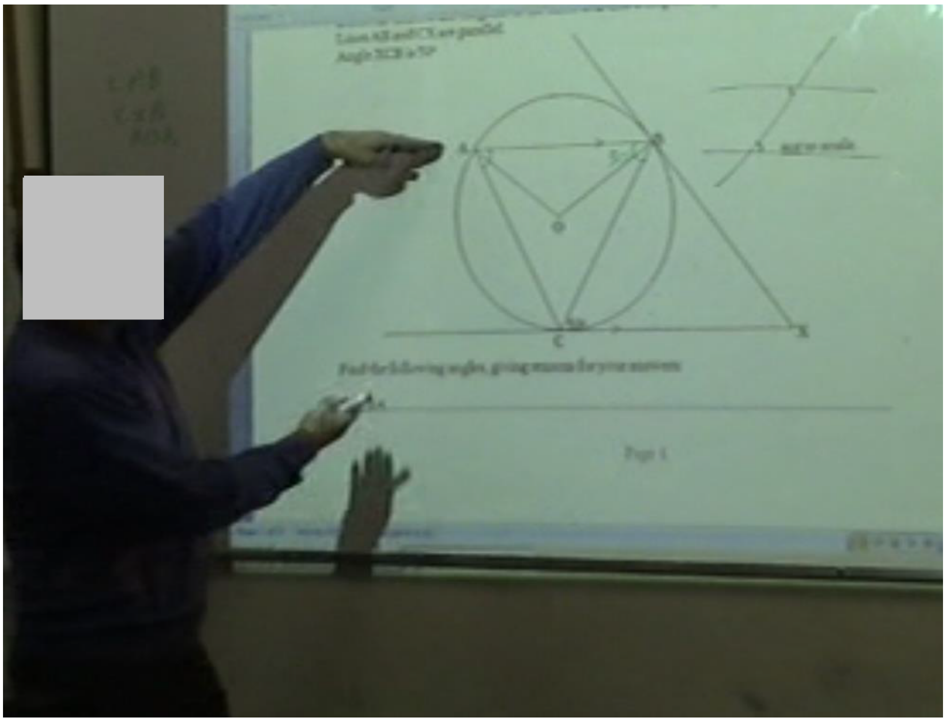


Figure 11: Parallel (II)

From the selected data that I have presented in this section, what appears across iconic gestures is the fact that various forms of visible representations have a complementary function in conveying the instructional information. It appears that T1’s gesture emerged as a powerful mechanism that he uses along with procedural talk to support a mathematical concept visually. In the next section, I will show the pedagogic nature of T1’s gestural representation that supports the mathematical concept ‘perpendicular’.

6.3.2. 'Perpendicular'

In this part, by presenting evidence from my multi-modal transcripts, I will show an example where the classroom teacher's gestural representation revealed the semantic content of the mathematical concept 'perpendicular'. In this example T1 combines a number of different modalities (verbal and nonverbal) to communicate mathematically in order to solve a question on a whiteboard in a different mathematics lesson. A question has been projected on the whiteboard and in the question it has been given that O is the centre of the circle and the line AT is a tangent to the circle at the point A (see Figure 12). In this transcription, the general classroom discussion is on the nature of the angle OAT.

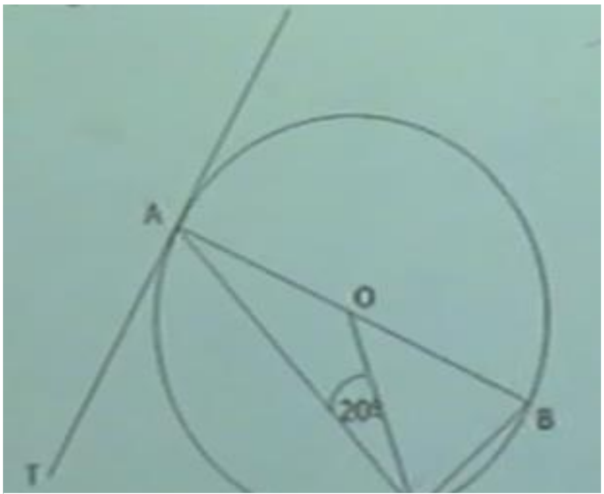
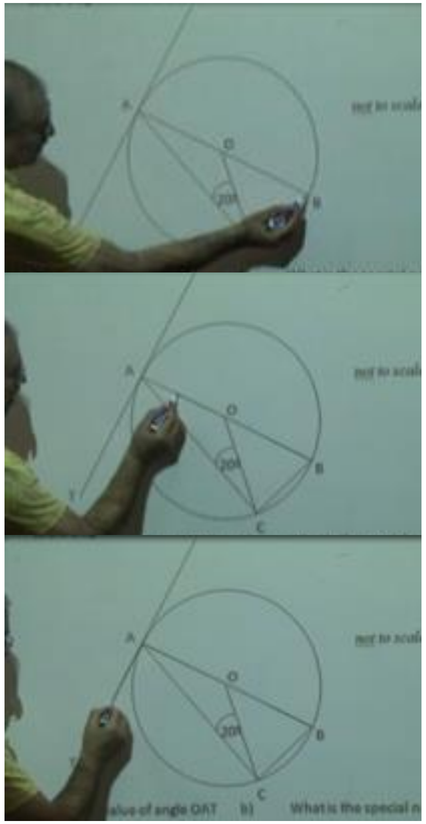


Figure 12: Lines AT and BA

I will now turn to present my multi-modal transcripts to show how T1 is engaged with his body-based resources as he is conveying the instructional information verbally. My focus is particularly on the ways in which T1 emphasises his verbal utterances by drawing upon his nonverbal resources such as writing on the whiteboard. Furthermore, I am also interested to look at how T1's gestural representation conceptualises and conveys an understanding for the mathematical concept 'perpendicular'.

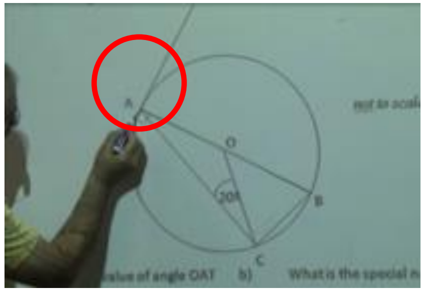
Transcript number 2

1 T1 This one, ….

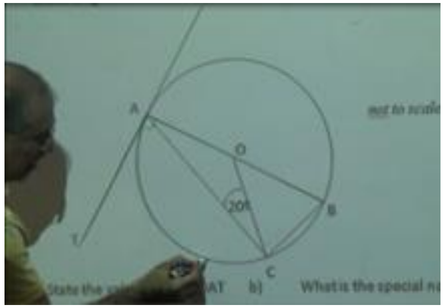


[As T1 utters ‘this one’, he uses a board marker and draws on the projected lines BA and then AT]

this one is



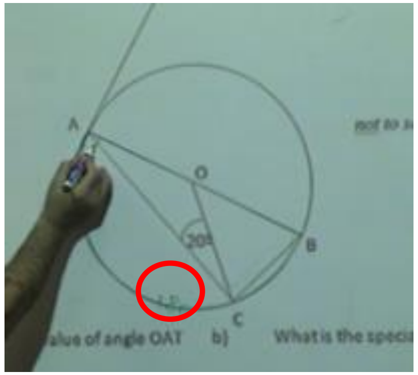
[T1 marks BAT as a right angle. He appears to emphasis his point as he raises intonation] half of .. this circle. What is this circle?



[As T1 says “half of .. this circle”, he highlights lower the semi-circle which starts from the point A to point B on the graph.]

5 Bs: One eighty degrees.

T1: One hundred and eighty, yeah? ..

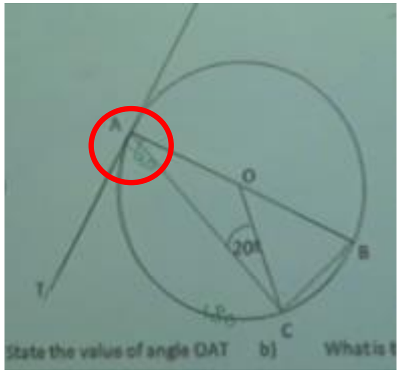


[T1 writes 180 degrees on the arc of the semi-circle as he verbalises “one hundred and eighty”]

This one?

[T1 points to the angle BAT simultaneously.]

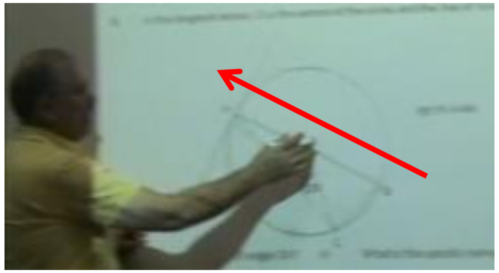
Bs: ninety

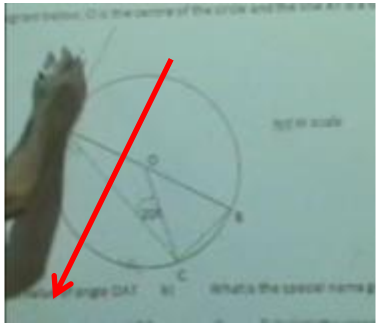


10 T1: ninety.

[T1 writes 90 degrees on angle A as he synchronically utters ninety]

Pas in dota chi mishan be hamdige?.. {So these two are?..}

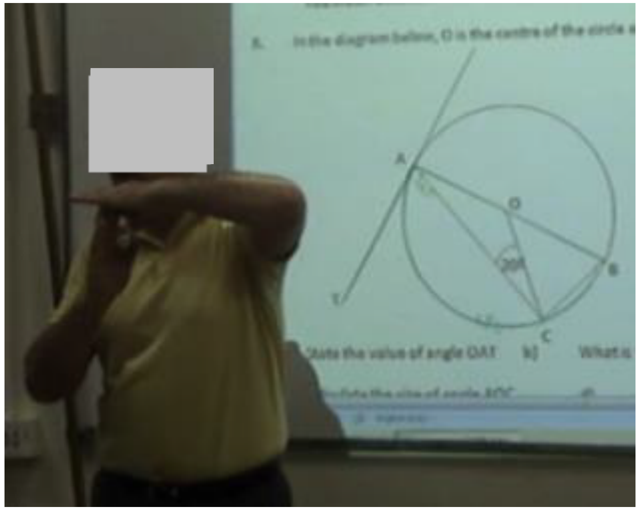




[At this time T1 points/refer to lines BA and then to line AT. The direction of the red arrow indicates the direction of T1's hand motion]

These two ninety..

Ina hamdigaro vertically .. cut *kardan, ha?* {They have vertically .. cut each another, yeah?}



[In this specific moment in time, T1 places his hands in such a format that his left hand is horizontal and his right hand vertical and there appears to be a point of interaction between his left palm and his right hand finger tips. What is also significant is that they appear to be at right angles. T1 maintained his hand gesticulation for approximately five seconds]

In this part, I present evidence from the interactional recordings where some of T1's gestural representations revealed the semantic content of some specific mathematics registers such as 'perpendicular'.

In lines 11-13 T1 raised a question "*pas in dota chi mishan be hamdige?*" {So these two are what?}. What T1 asked by this question could have meant "what is the nature/relation between lines BA and AT" as he referred on the two lines synchronously (see Figure 13). Once the question was asked, there did not appear to be any kind of interactional feedback from the students. Approximately two seconds later when no one offered an answer, T1 gesticulated as appears in Figure 14 to convey the relation/nature of lines BA and AT which he had already attended to in lines 11-13. T1 gesticulation appears in such a way that his left hand is horizontal and his right hand vertical and there appears to be a point of interaction between his left palm and his right hand finger tips. What is also significant is that they appear to be at right angles. T1 maintained this form of gesticulation for approximately five seconds and at the same time in lines 14-18 he said "These two are ninety. *Ina hamdigaro vertically* cut *kardan, ha*?" {They have vertically[5] cut each another, yeah?}. T1 conveyed the instructional information synchronically and co-expressively with nonverbal means of communication. Furthermore T1's gesticulation of the concept perpendicular is intimately connected to the speech channel ('These two are ninety' in line 14) in terms of both semantic and pragmatic function. Therefore T1's gesticulation was served as a visually-based means of expression as he was expressing the perpendicular nature of lines BA and AT. Under McNeill's gesture classification, T1's employment of this particular gesticulation falls under iconic category. It is iconic as it bears a close relation to the appointed two lines (BA and AT) that are projected on the whiteboard.

[5] It is interesting that T1 said 'they have vertically cut another' as they can also cut each other horizontally at the same time.

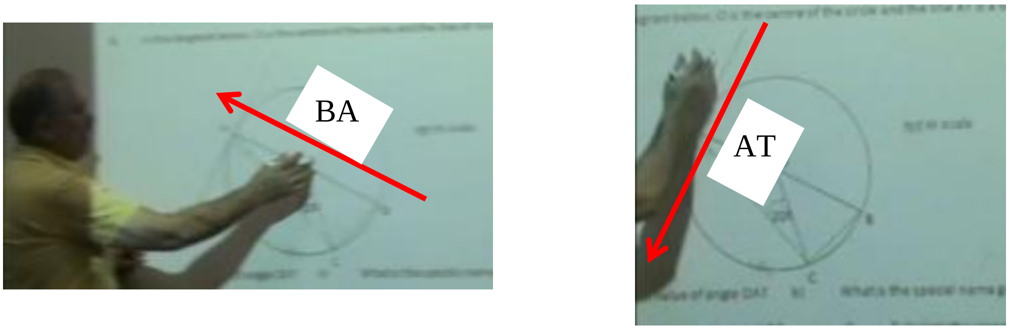


Figure 13: Single handed gesture for 'perpendicular'

T1's employment of this particular type of gesture in his instructional information appeared to give an image of his verbal message. For example, by considering and examining T1's verbal message only "They have vertically cut each another", did not make much sense in relation to what else he was saying. Whether T1's intention was to state the perpendicular nature of lines BA and AT, or to establish 'vertically opposite angles' was not clear from listening to the words only. The meaning comes in the end from the representational gesture which reflects the nature of two lines cutting each other at 90 degrees.

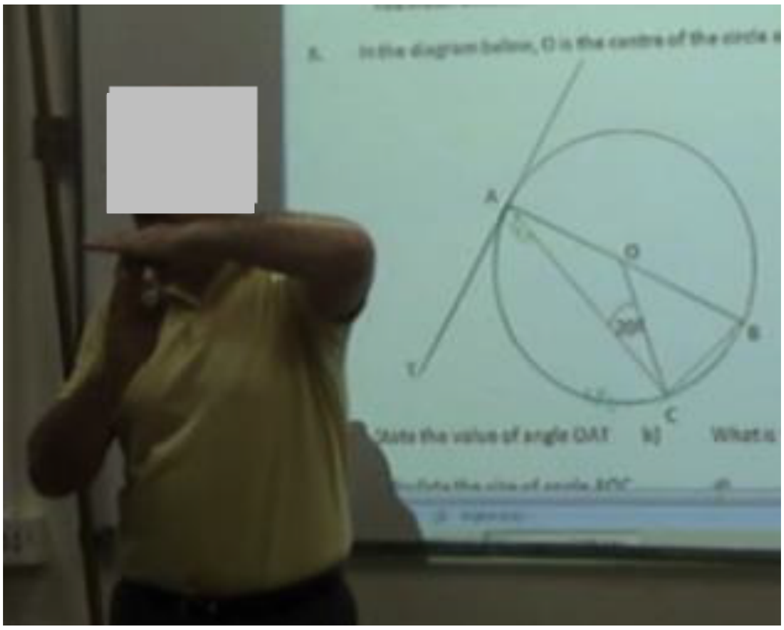


Figure 14: Perpendicular

From the examples that I have provided in this chapter, it appears that the gestural representation for ‘parallel’ and ‘perpendicular’ are intrinsically linked to their corresponding mathematical concept. Iconic gestures reveal the semantic content of the speech. Not only do iconic gestures show another avenue to the teachers’ mathematical representation when they are engaged in certain mathematical tasks but they also enrich the teachers’ verbal instructional information. The employment of iconic gestures appeared to provide a visual counterpart to T1’s verbal message as he was conveying the instructional information. In other words iconic gestures appeared to support the mathematical concept through visual modes of expression; a mode that is culturally given and socially shaped. In the next section, I would like to offer a different perspective on the enactment and interpretation of these gestures.

6.4. Discussion and conclusion

Only in the last decade or so have researchers focused on the communicative functions of gestures in classroom communication. In the literature it is noted that nonverbal and gestural representations add both clarification and richness to the spoken discourse (Goldin-Meadow, 1999). For example it has been reported that a combination of gesture and speech deliver a coherent message to the listener (Alibali, Flevares & Goldin-Meadow 1997) especially in a mathematics lesson when the language is abstract and general (Castellón, 2007) or ambiguous (Flevares & Perry, 2001).

Research in mathematics education has recently acknowledged the special pedagogic nature of gesture that is used as a modality in mathematics classrooms. Research on nonverbal communication emphasizes not only the importance of gestures as a visual counterpart to the spoken words but also the communicative nature of body-based resources in teaching mathematical concepts (e.g., Flevares & Perry, 2001). Gestures are considered as part of the rich resources that are activated in the mathematics (see Roth, 2009) or any other classrooms.

There are a number of different resources which can be activated in a mathematics classroom and generally, the same person exploits many of them simultaneously (Arzarello *et al*., 2009). Various forms of visible representations, such as gestures, appear to have a complementary function in conveying the instructional information. For example, gestures appeared to play a role in communication, both in facilitating language production and in promoting learners' comprehension. It is also been noted elsewhere that teachers' gestures influence students' comprehension of lesson content and leads to greater learning (Flevares & Perry, 2001; Alibali & Nathan, 2007).

From the selected data I presented in this chapter, T1 appears to draw across using different modalities in a flexible manner to enhance students' learning and understanding of certain mathematical concepts and representations. T1 combined different modes to express the instructional information and to bridge between the verbal and visual forms of meaning making. For example, T1 conveyed the concept of perpendicular or 90 degrees in gesture and across different languages. I find the pedagogic nature of iconic gestures of particular interest as they resemble concrete objects or action directly via the shape or motion trajectory of the hands. For example T1's gesticulation for the mathematical concepts 'perpendicular' and 'parallel' emerged as a powerful mechanism that he used along with procedural talk to support the relevant mathematical concepts visually.

Analysing T1's use of iconic gestures gave an index to the richness of his instruction beyond speech. Therefore, I believe it is also important to acknowledge the particular pedagogical nature of communication by classroom teacher, T1, as he drew across his resources that he had at his disposal. Thus it is necessary that students attend to vocal and visual means of expression in order to be engaged not only with the task but also to gain access to the information that is presented nonverbally by the mathematics teacher. This idea is in line with Flevares and Perry (2001:342) who have observed that "just as spoken words cannot teach if they are not heard, the nonspoken forms can only convey an instructional message to students if the students attend to them".

7. Chapter seven – Metaphoric gestures

7.1. Introduction to this chapter

In this chapter, by presenting evidence from teachers' and students' gestures, I will show that certain mathematical concepts are embodied. This embodiment is reflected through gestures which provide an avenue in how certain mathematical concepts can be seen/visualised. In this chapter I will offer four different metaphoric gestures which consist of gestural representation for the following mathematical concepts 1) subtraction, 2) addition, 3) minimum and 4) '*tavan*'. 'T*avan*' is the Farsi mathematics register for 'power' in English. After presenting the metaphorical gestures for each of the four mathematical concepts in the transcription, I will then describe the use and function of these metaphoric gestures in a complementary school bilingual mathematics classroom.

In section 7.2, I will elaborate how the gestural representation for the mathematical concepts 'takeaway' and 'add' was incorporated in the classroom teacher's instruction. Section 7.3 reveals a particular metaphoric gesture that was used to convey the mathematical concept 'minimum'. Section 7.4 will be divided into two subsections where I will be offering two different gestural representations by two different gesticulators (performed by B1 and T2) for the mathematical concept '*tavan*'. Finally in section 7.5, I will discuss and evaluate the implications of gestures, and in particular, metaphoric gestures in a bilingual mathematics context.

7.2. 'Takeaway' and 'add'

The multi-modal transcript that I have drawn from is a selection from one lesson. I will demonstrate instances where the classroom teacher (T1) draws upon his verbal utterance that accompanies physical gestures to convey mathematical meaning in a bilingual mathematics classroom. My focus is particularly on the ways in which T1 gesticulated, providing an additional avenue in parallel to what he expressed in his verbal message (stress and emphasis). In this lesson, T1 projected a number of questions on the board which is about the difference of two squares. T1 asked students to simplify this particular question. T1 wrote an example on the whiteboard to the class which was $a^2 - b^2 = (a - b)(a + b)$ and explicitly asked students to pay attention to this example.

Transcript number 3

T1: *alan* B, *iin dar morede iin*, .

this one, . *khob G2, in* formula-*ro yaddasht kardi?*{Now B, this one, .

this one, . Ok, G2 did you

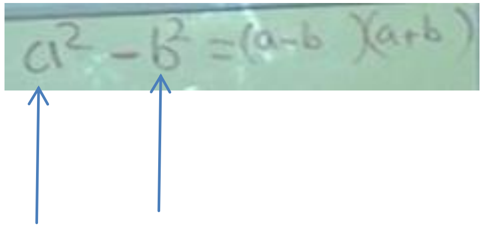
make a note of this formula}

G2: *bale* {Yes}

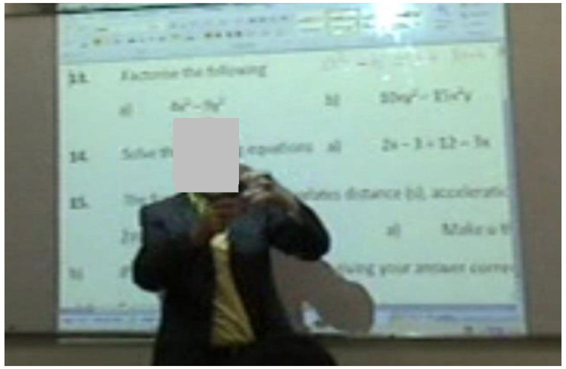
[As T1 expresses a demonstrative pronoun, *iin*, meaning 'this one', simultaneously he points to the question that is projected onto the white board and taps twice on it with his left hand]

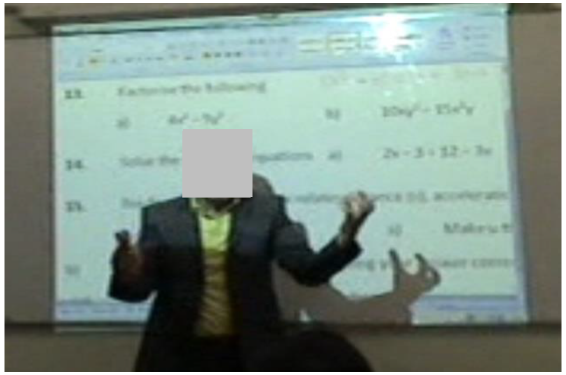
T1: *harja do ta* squares

{Wherever two squares



[T1 creates a pointing gesture with his left hand, indexing first to a^2 and then immediately to b^2 on the whiteboard as he synchronically expresses 'two squares' in words. His gesture aligns with the prosody of his speech 'two squares'] *az ham* takeaway *beshe,* {are taken-away from each other,}





[T1 establishes a speech-accompanying gesture where he put his hands close together in front of his chest and just after approximately one third of a second he utters ‘take-away’ which appears to be in synchrony with his hands taken apart.]

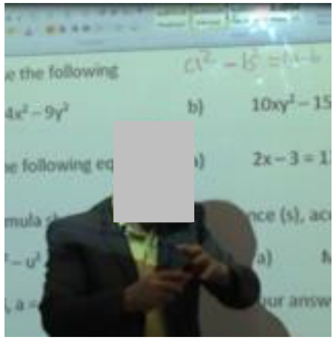
chi mishe, {what will happen,}

Square-root-*eshoon*, {their square root,}

[T1 put his hands on a and then b which are the square root of a^2 and b^2 respectively]

take-away, times-*eshoon*, {takeaway, multiplied by each other,}

ba ham jam mishe. {add up}



[T1's hands are apart and then he put his hands close together in front of his chest and approximately a second later he utters 'add']

Inro bayad tooye zehnet bere {You should memorise this}

By revisiting this multi-modal transcript, gestures appear to be an integral part of T1's communication about mathematical ideas. Elsewhere teachers in mathematics classrooms have been observed in producing gestures routinely along with their corresponding speech units (e.g. Alibali & Nathan, 2007; Flevares & Perry, 2001). For example in lines 11-13 T1 gesticulates as he speaks of the notion of subtraction (see Figure 16). T1 put his hands close together in front of his chest and just after approximately one third of a second he verbalised 'take-away' which appeared to be in synchrony with his hands taken apart. Moreover in line 21-22, T1's gesticulation serves as a visual counterpart to the Farsi mathematics register '*jam*' {add} (see Figure 15). By drawing upon a wider range of body-based resources; T1 indicated how addition can be brought into being as the inverse of subtraction. Not only from the mathematical logic can addition be seen as the inverse of subtraction but this fact is also apparent and enacted through the use of T1's gestures. The gesture for addition is the exact reverse process for subtraction.

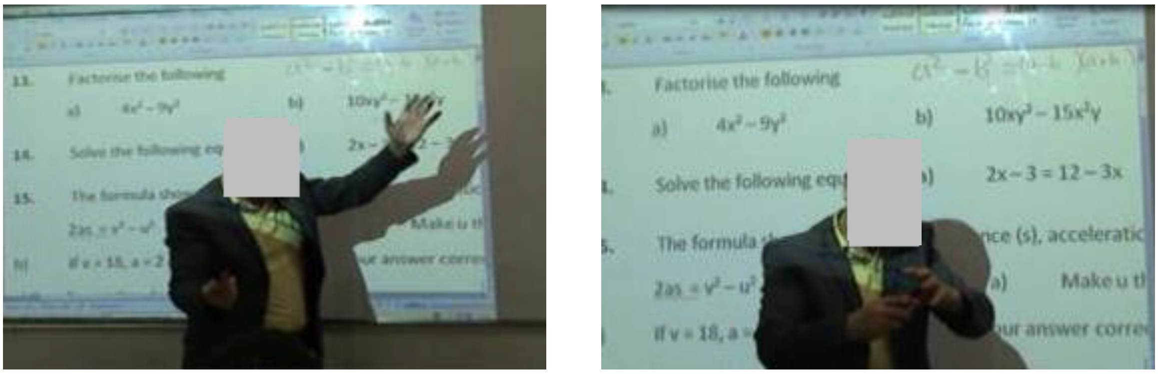


Figure 15: 'jam' {Addition}

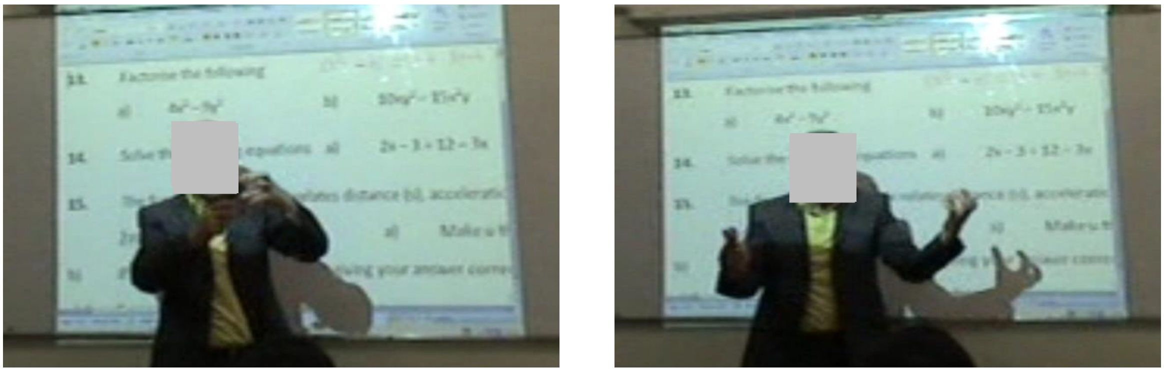


Figure 16: Takeaway

T1’s use of hand gesture to convey the mathematical concept ‘addition’ is also evident in a number of different lessons (e.g. see Figure 17 and Figure 18). The motion and trajectory of T1’s hand gesture that co-occurs with the mathematics register ‘addition’, appears to consist of two phases:

1. both hands wide open in front of T1’s upper body
2. both hands ended up close together in front of T1’s chest

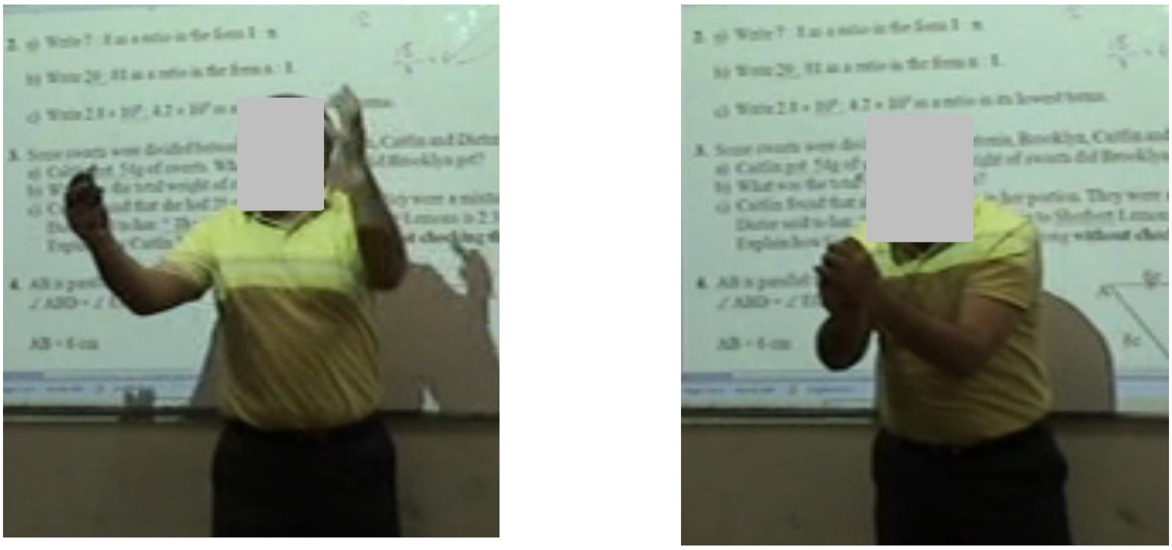


Figure 17: Addition (I)

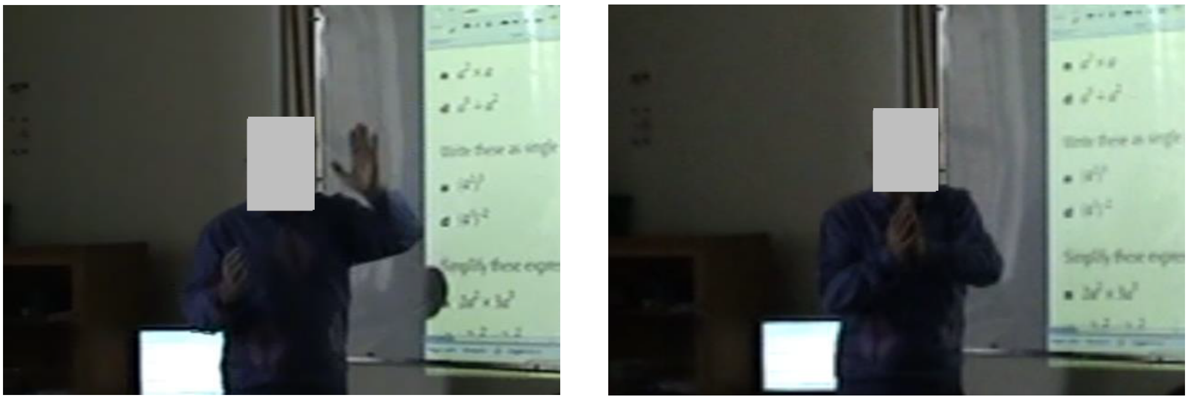


Figure 18: Addition (II)

The motion of T1's hand gesture can signify collection. T1's gesture of adding sits well with the notion of 'addition' within a mathematical context which includes a sense of collecting elements into a particular set. T1 uses his body-based resources to convey information critical to the explanation of mathematical concept addition/subtraction. It is interesting to note that the gestural representation for '*jam*' and addition through two different languages (Farsi and English) is exactly identical (see Figure 15 and Figure 17 respectively). Similarly in another study, the nature of gesture that undergraduate learners produced for the mathematical concept ‘addition’ reflected a metaphor of ‘collecting’ (Marghetis *et al*., 2012) with the same manner and path as T1 had produced.

Due to the abstract meaning that can be emerged from gestures, I am only interested to examine the gestural representations of mathematical concepts in relation to the speech they accompany. In lines 11-13 T1’s speech-gesture-coproduction, serve a number of vital meaning-making functions when accompanying the relevant spoken counterpart; it complemented verbalisation, which intensifies an image of detract and separateness. In both ‘add’ and ‘takeaway’ T1 created a gesture that not only is lexically affiliated to the notion of addition and subtraction but convey the semantic content via metaphor. T1’s gesture for ‘add’ and ‘takeaway’ is metaphorical which falls under McNeill’s basic gesture classification as it resembles an abstract idea for a concept or an action.

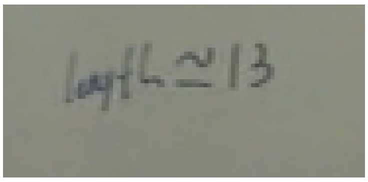
Whilst the gesture for ‘add’ began well before its speech affiliate, its trajectory was completed before the production of the affiliated lexical item. In terms of semantic content, T1’s gesture was approximately one second in advance of the verbal reference. However, in lines 16-17, T1 synchronically and co-expressively gestures ‘takeaway’. It is synchronous at the exact point where that is also co-expressive (McNeill, 2005). In other words gesticulation and talk are thereby aligned. The question as to whether T1’s gesticulation was planned or spontaneous is not clear. Therefore whether T1’s gesticulation served as a pedagogical tool as he was engaged in a mathematical activity or an enactment of mathematical awareness is not clear. T1’s employment of a metaphorical gesture for mathematics register for example ‘takeaway’, also addressed how he supported speech with visual means of expression. T1’s production of ‘body-based resources’ in conjunction with speech can serve as a window on how he thinks and talks about mathematics (Edwards, 2009). Moreover it is possible to talk about the relationship between body-based resources and speech in terms of embodiment. This embodiment is reflected through T1’s employment of metaphoric gestures that creates a bodily counterpart of what is being talked about. I assume that the embodied nature of T1's gestures can help to realise the 'concept' by its accompanying gesture.

I will now draw on my multi-modal transcripts to provide evidence for another metaphoric gesture which was enacted by a different teacher, T2, to express the mathematical notion of 'minimum' as he was engaged in conveying the instructional information in this specific bilingual mathematics context.

7.3. 'Minimum'

In this part, I will present evidence drawn from the classroom teacher's (T2) gesture to show how T2 expressed his conceptual knowledge of the mathematical concept 'minimum' in gesture. The transcription that I will draw on emerges from an interactional recording of a case where T2's gestural representation revealed the semantic content of the mathematics concept 'minimum'. T2 and students are engaged in a question which appears in a written material in their text books. The context in which they are working concerns rounding up/down to the closest whole number. T2 read out a question which stated that 'the length of a pencil is thirteen centimetres. Correct to the nearest centimetre'.

T2 then writes on the whiteboard:



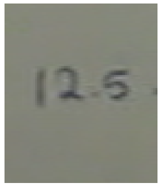
The question is now asking what are the possible lower and upper bound lengths for this pencil. In the following transcripts, my interest lies in showing the pedagogic nature of T2's gestural representation as means of communication to illustrate the mathematical concept 'minimum'.

Transcript number 4

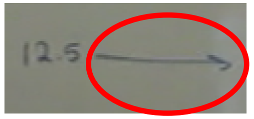
T2: Question two. The length of a pencil is corrected to thirteen centimetre, ok. *Khob, yanni vaghti ke* correct-*esh kardim shode chand?*. . . {Ok, so when it has been corrected, it became? . . .} Thirteen centimetre. *Khob, che* value *haai mitooneste dashte bashe?* {Ok, what are the possible values it could have?}

B1: twelve point five

T2: *khob,* value-*sh mitoone* between twelve point five *bashe* {Ok, its values could be between twelve point five}



[T2 writes 12.5 on the whiteboard as he synchronically says twelve point five] to



[T2 draws an arrow to the right hand side of 12.5 as he synchronically says to in line 20]

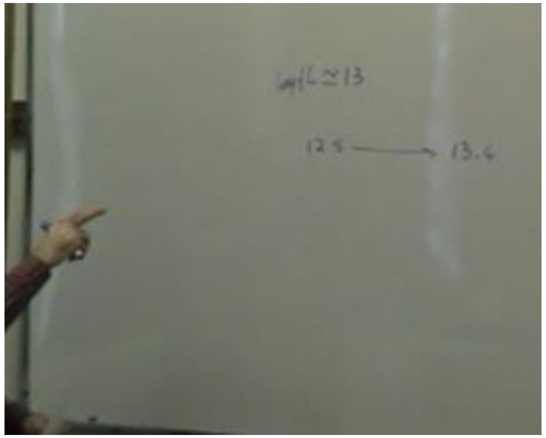
Bs: thirteen point five

T2: thirteen point four! .

[T2 writes 13.4 to the right hand side of the arrow]

Because thirteen point five is fourteen.

Ok, something in between.



[As T2 utters 'something', in his statement, he simultaneously gives an index to what appears on the whiteboard. Immediately after T2 utters 'something' he then utters 'in between'. T2's pointing with his index finger could refer to the numbers 'in between' 12.5 and 13.4. these hidden numbers were represented by an arrow from 12.5 to 13.4]

Soaale chio khaste? {what is the question asking}

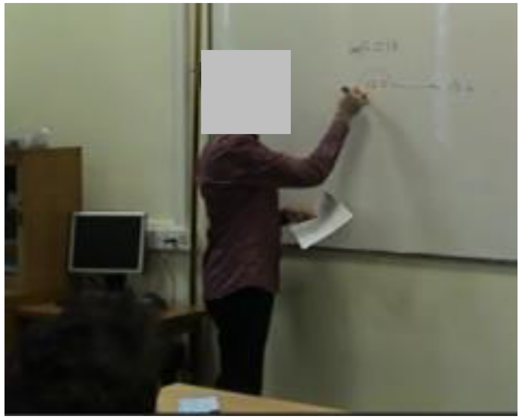
Gofte minimum {it says minimum}

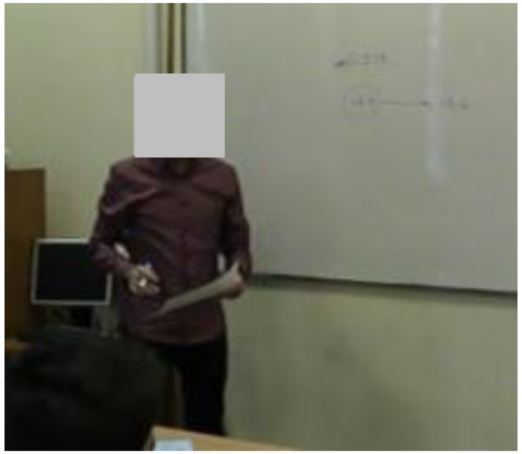
[T2 raises his right hand slightly and sharply brings it down and holds it down for approximately one third of a second]

gofte minimum-*esh*. {find the minimum.} Ok basically the minimum is?

B1: twelve point five.

T2: Twelve point five.





[T2 circles around 12.5 with a board marker which appears to be in a synchronous relation to his verbal utterance]

One of the purposes of this transcript is to show and to examine the nature of T2's spontaneous production of figurative language as one modality which was used as a tool to communicate the notion of 'minimum'. T2 gesticulates and moves in between verbal and visual whilst he was conveying the instructional information using a number of semiotic resources that were accessible to him. Gesture and speech serve a number of vital meaning-making functions when accompanying the relevant spoken counterpart; it often complements verbalisation, which intensifies the image of minimum in this case. In Figure 19, T2's metaphoric gesture for the concept minimum along with its co-expressive verbal counterpart, jointly conveyed related, if not identical, information about rounding down to the lowest whole number. Moreover the two modalities were temporally and semantically coordinated and the synchrony between the two modalities is an indication of sympathy.

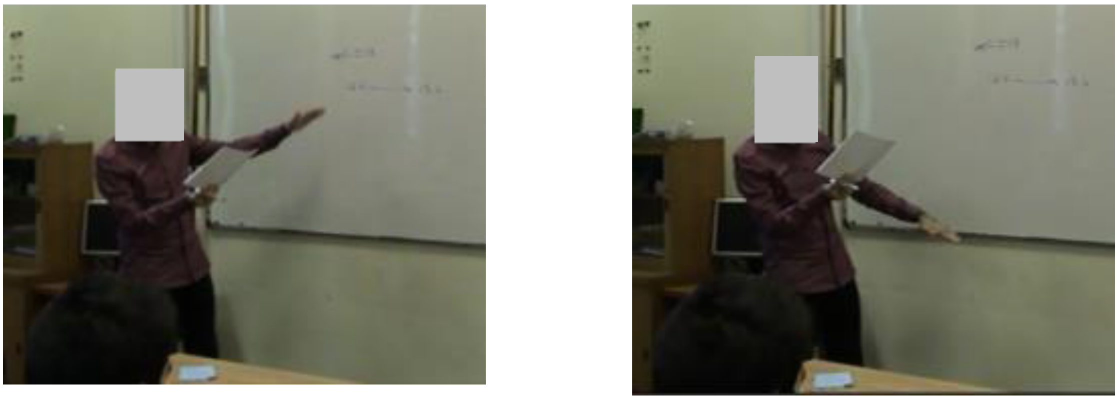


Figure 19: Minimum

T2's gesture and speech appeared to be in a synchronous relation as he drew across verbal and visual. T2's gestural representation for minimum presented "an image of invisible, an image of an abstraction" (See McNeill, 1992:14). T2 created a metaphoric hand gesture to represent an abstract idea that had no spatial instantiation as he was teaching the lesson content. His hand gesture presented an idea of minimum as the term 'minimum' was synchronised with his speech. His hand gesticulation moves from up (his shoulder level) to down and was held there for approximately one third of a second. Although one third of a second seems like a very short time, but it was enough for a pause (see Jarowski, 1993) which appeared to create an emphasis. His hands function as a visual means of representation that is intimately connected to the speech channel in terms of temporal, semantic and pragmatic function. Moreover, just as the metaphorical gestural representation for the mathematical concepts 'addition' and 'takeaway', the classroom teacher visually conveyed the abstract mathematical concept minimum 'which exhibited a close formal relationship to its affiliated semantic content of speech' (McNeill, 1992). T2's gesticulation for minimum is metaphorical which falls under McNeill's basic gesture classification as it resembles an abstract idea for the concept 'minimum'.

In the next section, I will provide an example of a different metaphoric gesture which depicts the Farsi mathematical concept '*tavan*'.

7.4. ‘*Tavan*’

In this complete section I will present evidence drawn from B1’s and T2’s gestures to show how interlocutors express their knowledge of the mathematical concept ‘exponentials’ or ‘power’ in gestures and how different languages activated a different understanding of the term ‘power’ (or *tavan*). Before I present my transcripts, I would like to clarify what I mean by the concept of ‘exponentials or powers’ in mathematics. If a number has been multiplied by itself seven times, we can say that particular number is to the power of seven. For example, if three is multiplied by itself five times 3×3×3×3×3, we can write three to the power of five 3^5. Therefore 3^5 is another way of expressing 3×3×3×3×3.

This concept is known as ‘exponential’ in mathematics. In this thesis I will use the term 'exponential' as a *neutral* term to refer to this mathematical concept. In the subsequent transcripts I will also aim to reveal that when the English word ‘power’ was used to refer to the mathematical concept ‘exponential’, a different gestural representation was exhibited compared to when Farsi was used to refer to the concept of ‘*tavan*’ {power}.

7.4.1. A gestural representation for ‘*tavan*’ performed by B1

In this part I will show the transcripts of a video excerpt that was coming from an interview which was conducted with B1. The purpose of this interview was to know more about B1’s general information and background, for example B1’s proficiency and experiences with each language across a range of settings and tasks. More specifically I asked him about the topics that he liked and disliked in mathematics. In this transcript I will depict a metaphoric gesture that was produced by B1 as he was conveying his mathematical ideas.

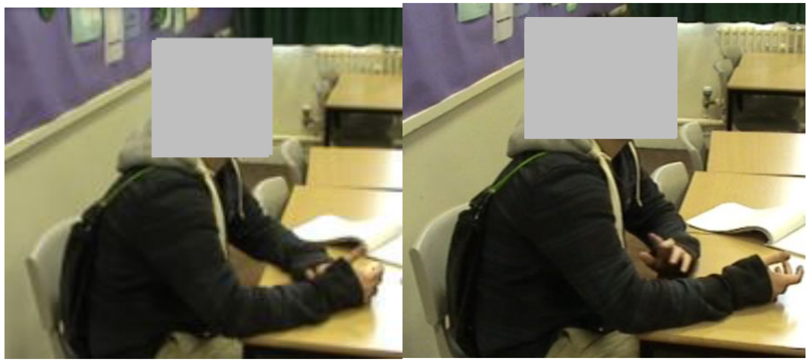
Transcript number 5

D: What is it in particular about mathematics that fascinates you?

B1: Mainly … different subjects like finding the unknown in maths.

D: *mitooni ye mesaal baram bezani* {Can you give me an example?}

B1: *masalan tavan*, {for example Power}



[B1 synchrony creates a hand gesture as he co-expressively utters '*tavan*' {power}. His gesture appears to be in a single stroke where initially his left hand is placed next to his right hand in front of him on the desk and then on his right bicep. The term '*tavan*' appears to trigger his hand to move across and touch his right bicep.]

tavan-e riazi {power

in mathematics}

At first, my focus was on the analysis of B1's verbal discourse working towards an understanding of what could prompt code-switching between B1's intra/inter sentential. Whilst replaying the video I became aware of B1's gesticulation. I then became interested in analysing this video excerpt that consisted of a communicative situation in which B1 used gestures in addition to language to articulate his opinion (Figure 20). At first it was a bit unclear as to why B1 touched his bicep and what possible meaning could be assigned to his gesticulation as he moved from verbal to visual synchronously. Most body movements can have various different meanings in different contexts and cultures. Coincidently, within the specific setting of Persian gestural context, the upper arm takes an account of the notion of strength or being "strong" (Sparhawk, 1981:445). It was a bit unclear as to what possible interpretation could be given when B1 touched his upper arm (bicep) as he was engaged in an activity that was centred on mathematics. At this specific time two things were unclear: 1) What possible meaning did B1's gesticulation convey when he touched his bicep as he was talking about the notion of '*tavan*' in Farsi? 2) What possible explanation could have been given to suggest the nature of what has triggered B1's gesticulation?

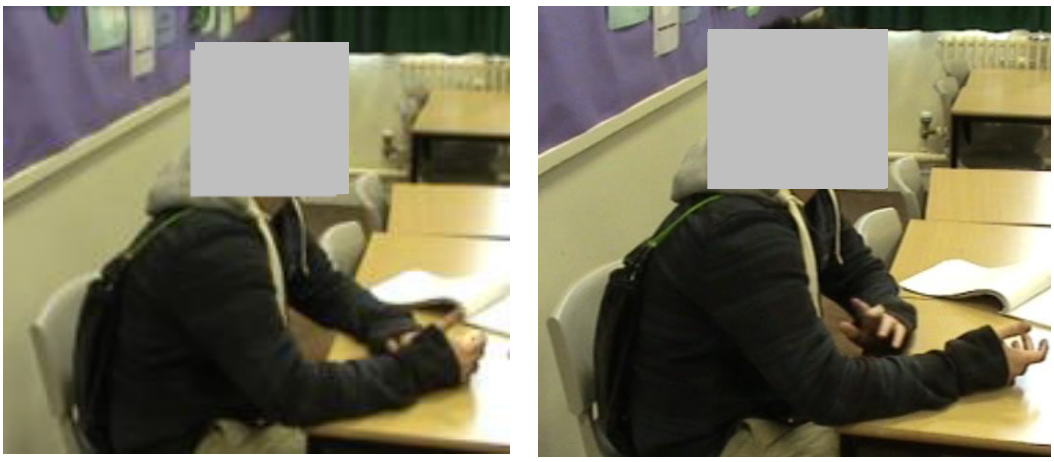


Figure 20: A gestural representation for '*tavan*' performed by T1

Whilst referring to the literature on Persian emblems, the potential interpretation of gesture for '*tavan*' could be taken into account as strength and power. I have then decided to look for the vernacular meaning of the term '*tavan*' which is used outside of the institution (and possibly where '*tavan*' was first learnt). The term '*tavan*' has only one meaning outside of a mathematical domain whose meaning conveys the notion 'potency' and 'strength'. However it is worth considering the meaning of the term 'power' used outside of a mathematical space. In the English language, it is possible to talk of power in terms of: e.g. 'power cut', 'political power', 'power and strength'. Therefore the meanings that are associated with the term 'power' outside of a mathematical context appear to be abstract as there is no direct link between the mathematical meaning for power and vernacular concept. But in Farsi the meaning for '*tavan*' is concrete as it directly suggests *only one* vernacular concept that is the notion of 'power and strength'.

By taking into consideration not only the verbal but also nonverbal and the context in which '*tavan*' was used, I assume that the vernacular concept '*tavan*' activated a different understanding for the notion of exponential whose meaning relates the mathematical concept to the vernacular. Thus the link between the everyday concept and the mathematical concept is not simply depicted in gesture but is actually enacted.

From the one video transcript that I have presented it appears that Farsi activated a vernacular understanding for the mathematical concept exponential which was reflected through gestures. The gesture for '*tavan*' appear to constitute McNeill's (1992) basic gesture type for metaphoric. This is because although the gestural representation for '*tavan*' did not convey a mathematical meaning, it still signified vernacular meaning which is abstract (e.g. powerful) with no real or physical concrete referent.

Now I will turn to show a similar metaphorical gesture that was enacted by T2 as he was conveying the notion of '*tavan*' in discourse.

7.4.2. A gestural representation for '*tavan*' performed by T2

I became interested about the ways in which different languages could generate different gestural representation of the same mathematical concept. Unfortunately I did not have substantial interactional data to the extent that was needed for the purpose of cross-linguistic examination. Whilst researching an on-going study, I decided to conduct a semi-structured interview with T2 centring the discourse on the notion of 'power' in hope of capturing more gestural representation. The interview was conducted at T2's house. In order to achieve balanced gestural representations of 'power' and '*tavan*' I decided that I wanted to code-switch at frequent intervals as often the language of initiation tends to elicit a response in that specific socio-linguistic register (however what appears in the transcript shows the major bulk of discussions in Farsi). Moreover, in order to provoke a gestural representation by T2, I decided to ask the same question again and again after T2's reply to each question. I believed that by asking repeated questions, T2 would have thought what he said was not sufficient enough to convey his message, therefore T2 would have to try to draw upon other resources at his disposal (including semiotics) to express his point. Moreover, I believed it was important that T2 was not aware of the main aims and objectives of this specific interview which was on capturing the correspondent gestures for '*tavan*' and 'power'. Having an account of the main purpose of the interview was likely to distort obtaining the corresponding gestures. When the interview was over, the full aim of the interview was then explained in greater detail to T2. I then asked T2 for his permission and consent as to whether I could use the data that emerged from this interview in my PhD project.

I will now turn to show video excerpts emerging from the interview conducted with T2 where his gestural representations revealed the mathematical concept '*tavan*'. Before I present my transcripts it is worth noting that in the past (about thirty years ago) the term '*ghove*' in Farsi was employed in the Farsi mathematics register to denote the concept of power, for example three to the '*ghove*' of five. The term *ghove* also meant battery in the olden days. *Ghove* is more or less an extinct term and is now seldom used in Farsi mathematics register. The idiomatic translation of *ghove* into English is both 'potency' and 'battery'.

In the analysis of this video excerpt, I will also focus on gestures in communication with other modes such as prosodic features (like stress and intonation) and other paralinguistic features like change in volume of speech (speaking louder or softer) or change of pitch (speaking higher or lower). I will present selected parts of the interview where the focus is on T2's gestural representation of the concept *tavan* only.

Transcript number 6

T2: *Tavan ro, man yadame*

avalin kalame-I ke moalem-
emoon be maa goft esmesh
arzam be shoma ***ghove***

bood. {I remember the first

time my teacher taught us
powers, it was called
potency}.

[In line four, there appeared to be a sudden change in T2's intonation as he uttered '*ghove*' {potency} along with a sharp rise in his pitch. Moreover, T2 makes a tight and clenched fist with his right hand. The degree of closure of his left hand does not appear to be as tight or as intense as his right hand. T2's right hand appears to move up and down again in approximately half a second. His gesture appears to indicate strength]

D: *doroste* {Ok}

T2: *Iin halatike kalameye ghove hala khodet mani-sho be Farsi midooni* {In a sense the term potency as you know in Farsi}

D: xxx

T2: *be Ingilisi mishe e erm mishe* exactly identical to power. {In English is exactly identical to power}

Something very strong.

D: *doroste, masalan, mesle masalan eer .. tavan masalan, Ali tavan, niroomandtare* {Ok, It is like, .. Ali is more powerful than Hassan}

T2: *bale masalan ali tavan-mand tar az masalan*

Hassan-e. masalan kasi ke ghodratesh bishtare. {Yes, for example Ali is stronger than Hassan. Something that has more strength.} *yani har moghe az kalameye tavan sohbat mikonim az kalameye ghodrat estefadeh mikonim.* {In a way every time we were talking of the word ‘power’, we were talking along the lines of potency.}

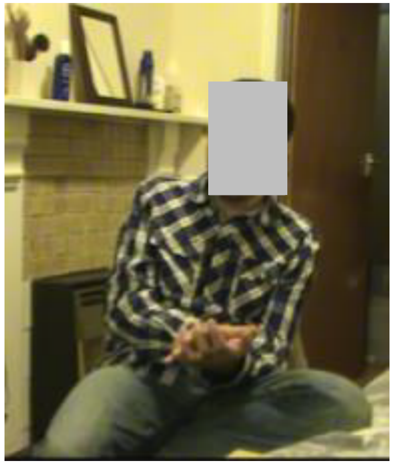
Man yadame avalin bar ke be maa yad dadan va ba bacheha shookhi mikardim

ba ham-dige az kalameye

ghove estefadeh mikardim, {I remember when I was first taught the concept of potency, me and other kids

were joking and making fun

of it}

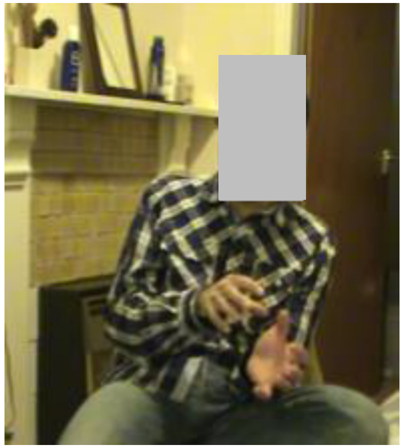


[In lines 42-51 T2 places his both hand together in front of him]

va hata khode moalem ham az kalameye ghove sohbat mikard {even our teacher

used the term ‘potency’}

[He raises his right hand and rotated his wrist slightly inwards to the direction of his back] xxx *vali, .. ma tooye ketabemoon az kalameye tavan estefadeh kardim.* {But in the books, it was referred to as power}.



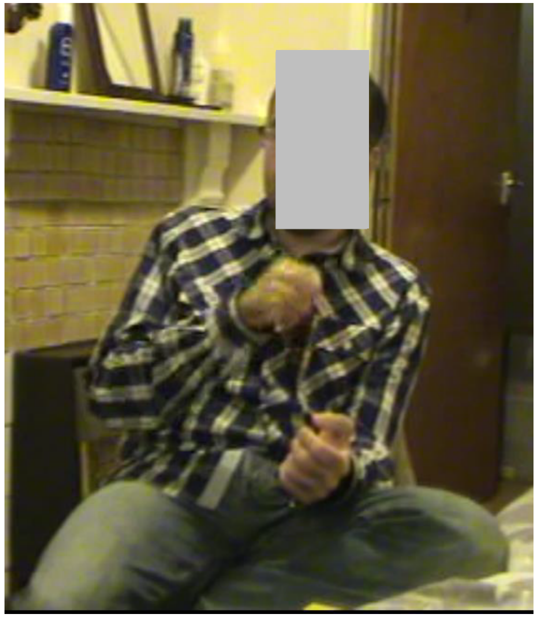
[T2 shifts both his hands together slightly to the left as he said ‘*ma tooye ketabemoon*’ {But in the books}].

D: *doroste* {Ok}

T2: *va iin halate ghove ke maa sohbat mikardim e . e . kheili jaleb bood ke baramoon ke mitoonestim zarb-ro be ghove sahbat Bekonim* {And at the time it was really interesting as we could ‘talk about’/‘do’ the concept of multiplication in relation to ‘potency’.

D: doroste {Ok}

T2: *Va arzam be hozoore shoma, panj be ghove-ye shish.* {For example five to the potency of six}



[T2 synchronises a hand gesture with the translated statement “five to the potency of six”. T2 makes a tight fist with his right hand and maintains it in front of his chest as he utters his sentence.]

Ya masalan two be power, . two *masalan* . *be* . *be* {Or for instance two to the power, . two for example to the . to the}

D: *ghove-ye* {potency of}

T2: power of *tavan-e* *haft*. {power of, power of seven}

[T2 again synchronises a hand gesture with the translated statement “power of seven”. This time he makes a tight and clenched fist with his right hand facing upwards and makes a sharp move downwards with a sudden stop. Note the hand only moved down slightly in this sudden movement. The red line is drawn as a fixed frame of reference on T2’s shoulder to measure this slight movement. T2’s gesticulation ended with an intense stop which was in synchrony with the high pitch ‘seven’]

In in chizaro ma vaghti bache boodim kheili estefadeh mikardim ke kelass-e sevom-emoon bood. {When we were in Year three, we were taught this concept}

In this multi-modal transcription of a video excerpt, I am also interested to show more gestural representations of the term '*tavan/ghove'*. Furthermore by encountering different modalities which were synchronised by T2, I wanted to emphasise the relation between e.g. 1) high pitch (vocal, see lines 1-8), 2) manner and motion of gesture (visual), 3) and the semantic content of the terminology '*tavan'* (verbal). Furtheremore, I wanted to describe a possible interpretation of T2's intent as he moved across his communicative repertoires (verbal, vocal and visual). I will make a claim that the T2's content of the mathematical concept of '*tavan*' was to denote the concept of 'strength' rather than 'exponentials'.

Now I will draw on gestural representation of the concept '*tavan*' which was enacted by T2 concurrent with the relation between the high pitch '*ghove*' {potency}, the ways in which the gesture was performed and the semantic content of the term in order to suggest a possible interpretation of T2's intent. When the term '*ghove*' was first introduced in line 4 (with its idiomatic English translation 'potency' in line 8), it was accompanied by a rise in intonation. Therefore vocally, not only the term '*ghove*' was accentuated with a high pitch, but it was also elongated. Verbally, in terms of the semantic content for '*ghove*' can be referred to as 'strength' and 'potency'. Visually, T2's gestural representation for '*ghove*' was carried out by a tight fist, moved slightly up and came down (within half a second) with an intense stop (Figure 21). The manner of the representational gesture for '*ghove*' was executed fast and forcefully. The path the gesture took also appeared to move slightly up and down forcefully.

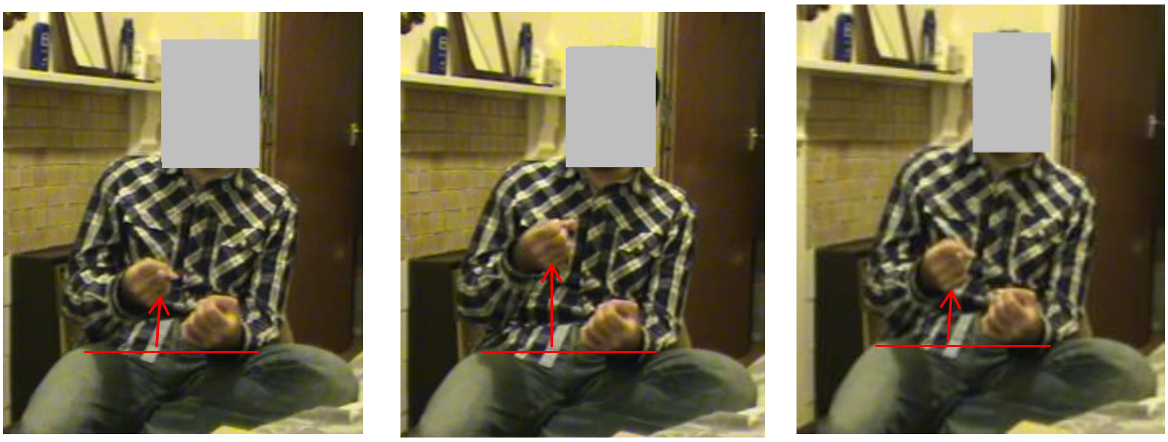


Figure 21: Gestural representation for '*tavan*' performed by T2 (I)

Considering all the three synchronised elements of verbal, vocal and visual, it seems that T2's thinking is involved in a dynamic construction that reflects a series of unconscious mental mappings based on his experiences and existing understandings. Moreover T2 has expressed, stressed and conveyed his mental image (of the notion of *tavan*) by some means of verbal, vocal and visual forms of language representations. But what appears to be common among the three modalities is the notion of 'strength' that was manifested verbally, vocally and visually whilst T2 was making a reference to a mathematical sense. Nevertheless a closer look at the vocal and gestural representation of the mathematical concept '*tavan*' leads to a realisation that a little mathematical understanding or meaning has occurred! What appears is that the informal and vernacular means of expression has dominated and activated a conceptual understanding for the notion of powers. Thus, the gesture T2 produces during explanation reflects aspects of his knowledge about the task being explained. I assume that T2's gestures appeared to play a role in accompanying the inner speech or embodied thought whose meaning involved the notion of 'strength'. It could be argued that the lexical and syntax structure of the term '*tavan/ghove*' in Farsi and its inherent meaning in everyday social context, has led to the mathematical concept '*tavan*' being intrinsically linked to the concept of 'strength'.

Frequently, although not always, T2's verbal message of the notion of '*tavan*' was accompanied by a clenched fist gesture. Other gestural representations for '*tavan*' in the same transcription can be seen in lines 73-76. Moreover in lines 83-85 T2's gestural representation depicts the same manner and trajectory as in lines 1-8. He made a tight and clenched fist with his right hand facing upwards and makes an intense move, shifting downwards slightly (Figure 22). This gesticulation also ended with an intense stop which was in synchrony with the high pitch '*tavan-e haft*' {power of seven}.

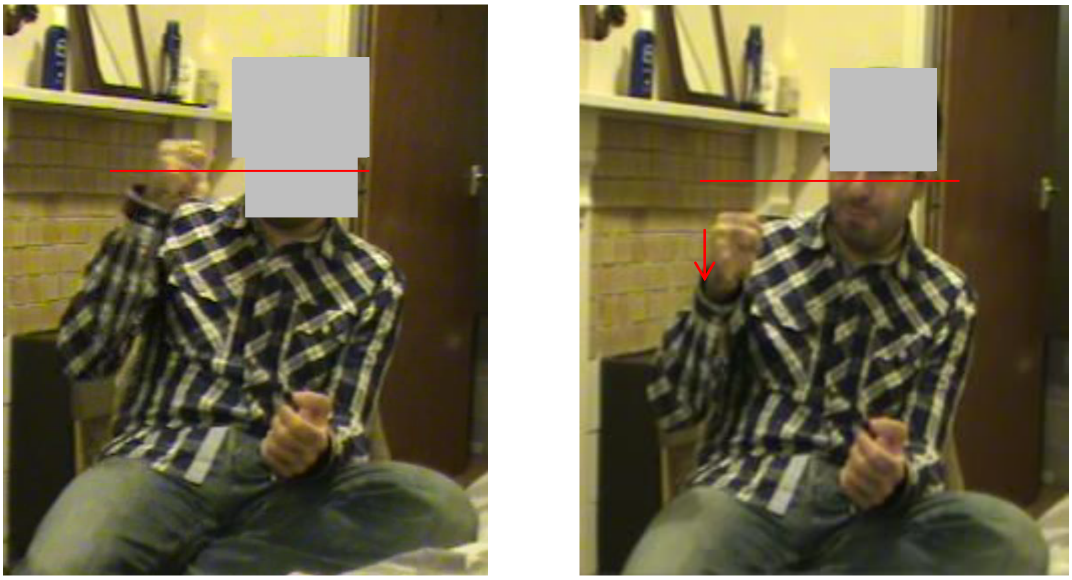


Figure 22: Gestural representation for '*tavan*' performed by T2 (II)

From the selected examples that I have offered in this section, it appears that the gestural representation of the mathematical concept '*tavan*' is influenced by its vernacular syntax and socio-linguistic counterpart; where it depicts the notion of strength. The notion of strength is in itself metaphoric, just like for example, the concept of 'language' or 'knowledge' (McNeill, 1992). Metaphoric gestures present imagery, an image of an abstract concept such as 'strength'. T2's gesticulation is a pictorial illustrator that reveals the semantic content of his speech. Arguably the accompanying gesture for '*tavan*' performed by both B1 and T2, could be defined to be a ‘metaphoric’ gesture due to the fact that aspects of the meaning raised through gesture seemed to concern the social vernacular meaning of ‘strength’.

Metaphoric gestures appeared to play a part in communication both in teachers’ instructional talk (e.g. add, takeaway and minimum where they convey aspect of the instructional information visually) and the talk that takes place outside of a teaching context (e.g. B1’s reflection on the notion of ‘*tavan*’). In the next section, I will address the pedagogic nature of metaphoric gestures when teachers or students communicate about certain mathematical ideas or concepts.

7.5. Discussion and conclusion

One of the purposes of this chapter was to explore the relation between different modes of meaning making such as the verbal discourse and visual representation that are used as a communicative web in a complementary school bilingual mathematics classroom. My focus was particularly on the ways in which the teachers’ metaphoric gesticulation provided an additional avenue in parallel to what they expressed in their verbal message in order to stress and emphasise. I am also interested to pay attention to whether the metaphoric gestures teachers produce have bilingual pedagogical value? Do these gestures promote bilingual learning?

In this chapter it is evident that metaphoric gestures appear to be crucial to the effectiveness of mathematical understanding as they convey the semantic meaning of some abstract mathematical concepts (such as ‘minimum’) through gestures. I am interested in highlighting the pedagogic nature of spontaneous production of figurative language as a resource that conveyed pertinent mathematical information. Often the classroom teachers (T1 and T2) conveyed many of these resources in a number of different modalities and across different languages. Therefore both T1 and T2 were engaged in making different kinds of switches such as code-switching, mode-switching (from verbal to visual) flexibly to convey their instructional information.

This chapter offered a perspective to the visual counterpart of certain mathematics registers that were employed in a bilingual lesson. Students and teachers not only relied on speech alone to convey mathematical meaning, but have incorporated gestures as visual amplifiers which assisted to keep the task moving forward. A combination of gesture and speech not only allow the speaker to have access to a flexible and a wide range of possible resources (both ephemeral and material) to convey their meanings (Alibali & Nathan, 2007) but also to present ideas that are not fully developed in speech, and expressing developing ideas (Goldin-Meadow, 1999). From the examples I have provided in this chapter, T1's and T2's gesticulation and talk were aligned. It is important to note that both T1 and T2 were both proficient in English and their employment of gestures during their instructional talk was to convey their ideas more richly. Their employment of such metaphorical gestures for the mathematical concepts 'add', 'takeaway' and 'minimum' emerged as a powerful mechanism along with procedural talk to visually support the relevant mathematical concept. Therefore the classroom teachers reinforced the concept and the content flexibly through the use of verbal and visual resources.

8. Chapter eight – Deictic gestures

8.1 Introduction

This chapter aims to raise awareness of a different gesture type category, deictic gestures, that have different forms and functions. Deictic gestures mostly occur in synchrony with speech. They are employed when interlocutors shift from verbal to visual, in order to index objects, locations, inscriptions that are either present or non-present in the environment. Index-finger pointing is the most common deictic gesture but there are different variations in index-finger pointing which I shall elaborate in this section.

The structure of this chapter divides into five sections where the introduction is section one. In section two, I will scrutinise B1's and T2's deictic gestures as they conveyed their articulated information about the mathematical concept 'power'. In part three of this chapter, by providing evidence from the interactional recordings, I will examine different variations of index-finger pointing and how they are discourse oriented. In section four, I will address how the classroom teachers' use of deictic gesture served as key to stress and emphasise a particular mathematics register: 'isosceles'. Later, I will look at aspects of bilingual pedagogy in a particular lesson where the classroom teacher not only shows awareness of other languages in the class but develops bilingual and culturally accessible resources through mathematical exemplifications in both languages. In section five, I will discuss and evaluate the implications of deictic gestures in this bilingual mathematics context.

8.2 'Power'

I have divided this section into two subsections where in each subsection I present evidence for the enactment of different gestural representations that fall under deictic gesture type category. The aim of this section is a twofold: 1) to provide more examples of deictic gestures that occur in a bilingual mathematics context, 2) to illustrate how a gestural classification of a particular mathematical concept can vary if it is produced in a different language.

In section 8.2.1 I will use the extended transcription number five (that I have already offered in the previous chapter, section 7.4.1) for further analysis of deictic gestural representation for the concept 'power' performed by B1. In section 8.2.2, I will show the deictic gestural representation for the concept 'power' enacted by T2 as he was conveying the instructional information.

8.2.1 A gestural representation for 'power' performed by B1

The transcription that I am going to present in this chapter is the continuation of what has already appeared in transcription number five (section 7.4). The focus of this section will be primarily on where B1 produces a gesture which appeared to be in a symbiotic relation as he uttered 'power' in English. The reason I have chosen this particular video excerpt for analysis and further discussion is not only to show an example of deictic gesture that is produced by B1 as he is conveying his mathematical ideas, but to explore the ways in which the gestural representation for the concept 'power' differed as B1 switched between his languages.

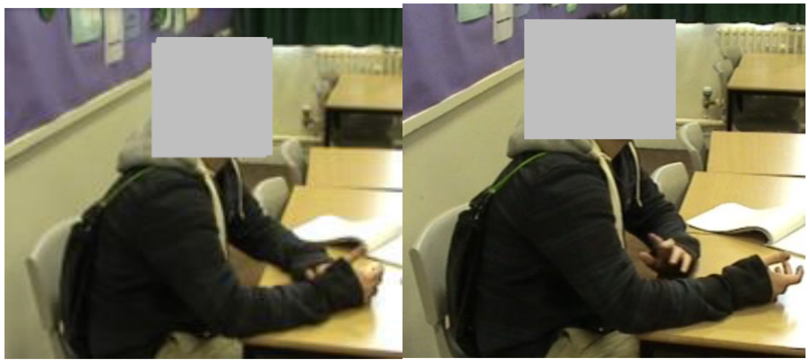
Transcript of extract number 7

D: What is it in particular about mathematics that fascinates you?

B1: Mainly … different subjects like finding the unknown in maths.

D: *mitooni ye mesaal baram bezani* {Can you give me an example?}

B1: *masalan tavan*, {for example Power}

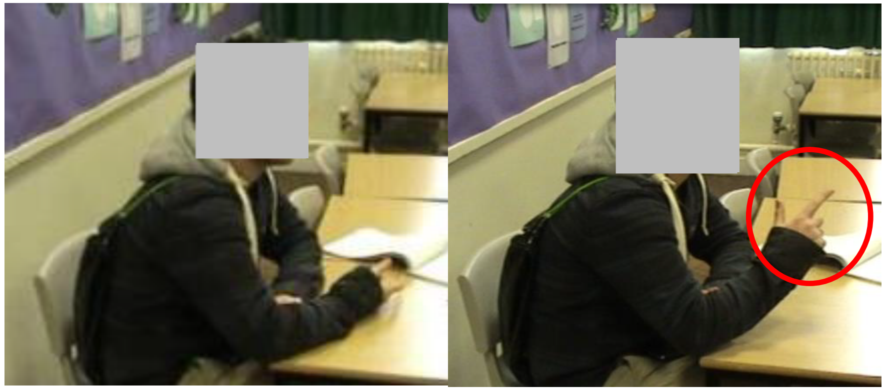


[B1 synchrony creates a hand gesture as he co-expressively utters '*tavan'* {power}. His gesture appears to be in a single stroke where initially his left hand is placed next to his right hand in front of him on the desk and then on his right bicep. The term '*tavan'* appears to trigger his hand to move across and touch his right hand bicep.]

tavan-e riazi {power

in mathematics}

Power,



[B1 code-switches and says power in English. At the same time he moves his right hand from the rest on the table in to the air.]

tavan {power}

D: *Tavan*, {power} oh ok

B1: *Tavan*, {power} equations,

D: Oh ok, *doroste* {oh ok,
right}

By revisiting the video, I became aware that (in line 14) B1 also made a hand gesture which was in synchrony with the term 'power' as he code-switched into English. Not only does B1 use a different socio-linguistic register but also he appears to perform a different gesture to what he had already established for '*tavan'* {power} in lines 10-11. It is interesting to note that B1 used visual forms of representations to convey the concept of 'power' and '*tavan*' in a single sentence. My focus in this transcript will only be in lines 10-14 where B1 gesticulated differently for '*tavan'* (as he was talking in Farsi) and 'power' as he shifted to English to reinforce and convey his point.

I also noticed that both in lines 10-11 and line 15, B1's hand gesture was synchronous with the speech unit '*tavan*' and 'power'. Both '*tavan*' and 'power' appeared to be the speech-accompanying gestures which were used within a mathematical context and both convey meanings that were specific to a particular function of language. The gesture for 'power' (Figure 23) suggested a 'higher up' position as B1 displayed a hand gesture from the 'rest' (with his right hand being on the table) moved up towards his shoulder level.

Figure 23: Gestural representation for 'power' produced by B1

It could be possible that the position of ‘power’ which in writing and literacy practices is placed diagonally (placed at the top right hand corner of the base, e.g. 3^5) activated B1’s gesture for power. In other words B1’s gesticulation of ‘power’ could resemble the mathematical notation of the concept power.

It is interesting to note that vernacular meaning for ‘*tavan*’ activated a non-mathematical meaning through B1 and T2’s gestures. However, due to the abstract concept of ‘power’ outside of a mathematical domain (e.g. ‘power cut’, ‘political power’, ‘power and strength’), its vernacular meaning did not invoke a non-mathematical meaning. Instead the gestural representation for ‘power’ appeared to underlie the notation of the mathematical concept ‘power’ as it possibly signifies the superscript position of power. It underlies the social convention and the literacy practices of, for example, the 5 in 3^5.

It is appealing that when English was used, the gestural representation for ‘power’ served a different category of McNeill’s (1992) gesture classification; it is deictic. I will discuss issue about the cross-linguistic variation of gestures in chapter nine. The gesture B1 produced is deictic because it is used as a pointer to index the notation. Although the gestural representation for ‘power’ did not convey the semantic content of the mathematical concept, it pointed to the position of power. In the definition of deictic gestures, any pointing gestures that serve to indicate locations, directions, persons or objects that are either present or non-present in the environment are deictic (see Alibali & Nathan, 2012). B1’s pointing gesture implied the notation that was not physicaly present during his explanation. Now I will turn to show more examples of deictic gestures by providing more examples.

8.2.2 A gestural representation for 'power' performed by T2

During a lesson, T2 was engaged in a question which asked to simplify and solve an expression. T2 started to expand the powers of 10, for example, ten to the power of seven was expanded to 10,000,000. Many students found the transition from the original format (which was in the standard form) to the expanded version confusing. The original expression was this:

$$(9 \times 10^7) \div (3 \times 10^2)$$

The focus of this part is specifically on T2's gestural representation as he conveyed the instructional information. Therefore in this transcript I am more interested to look at T2's gestural representation of the concept power than any other of his modalities. In the subsequent paragraphs I will present part of my multi-modal transcripts along with snapshots from an interactional video recording to take an account of T2's gestural representation for the concept power through the medium of English.

Transcript number 8

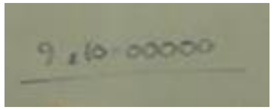
T2: *migam* {So} nine multiplied by ten

[T2 writes 9 × 10 on the whiteboard as he is uttering his sentence]

be {to the} power of seven. What do I mean by ten *be* {to the} power of seven?

One with seven zeros in front of it.

One two, three, four, five, six, seven.



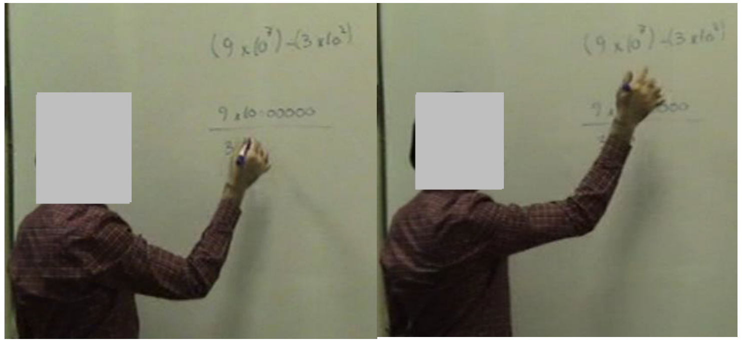
[T2 writes seven zeros as he verbally counts from one to seven.]

Divide is basically, . that’s a division line, or fraction,

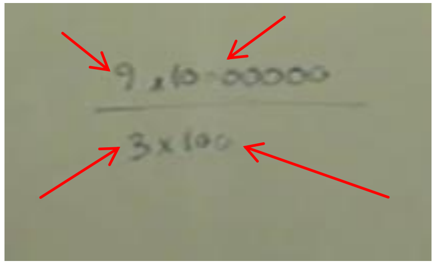
[T2 draws a division line on the whiteboard. His writing appears to be in absolute synchrony as he uttered ‘is basically’] three multiplied by

[T2 writes three followed by the symbol for multiplication]

ten *be* {to the} power of two,



[T2 creates a hand gesture, by moving his right hand to a relatively a higher position] two zeros. *Khob* sign-*e* *beine in do ta* number, {Ok, what is the sign between these two numbers} operation *beine iin do ta* number *chie?* {What is the operation between these two numbers? }



[T2 pointed to the two numbers in the numerator and two in the denominator in lines 15-22.]

In this part, my focus is primarily on lines 13 and 14 where T2 gesticulates for the mathematical concept ‘power’. T2 made a hand gesture in which the manner of T2’s motion (how the motion was carried out) and the path of his motion (the directionality of the motion) of gesture for power was very similar to B1’s gesture for ‘power’ (in the previous section). In other words T2’s gesture trajectory appeared to have a similar form and function to B1’s gesture for ‘power’. This is possibly because both B1’s and T2’s gesture ‘referred to’ or even ‘suggested’ something high/up and one of the plausible interpretations within the context could denote the mathematical position within the notation of the concept power (see Figure 24). It also seems that the gestural representation for the mathematics concept ‘power’ through the medium of English stressed the notation of power. For example the position of power was intrinsically linked to the gestural representation for ‘power’ performed by both B1 and T2. Therefore the gestural representation for ‘power’ can be considered to be deictic as it indexes the notation.

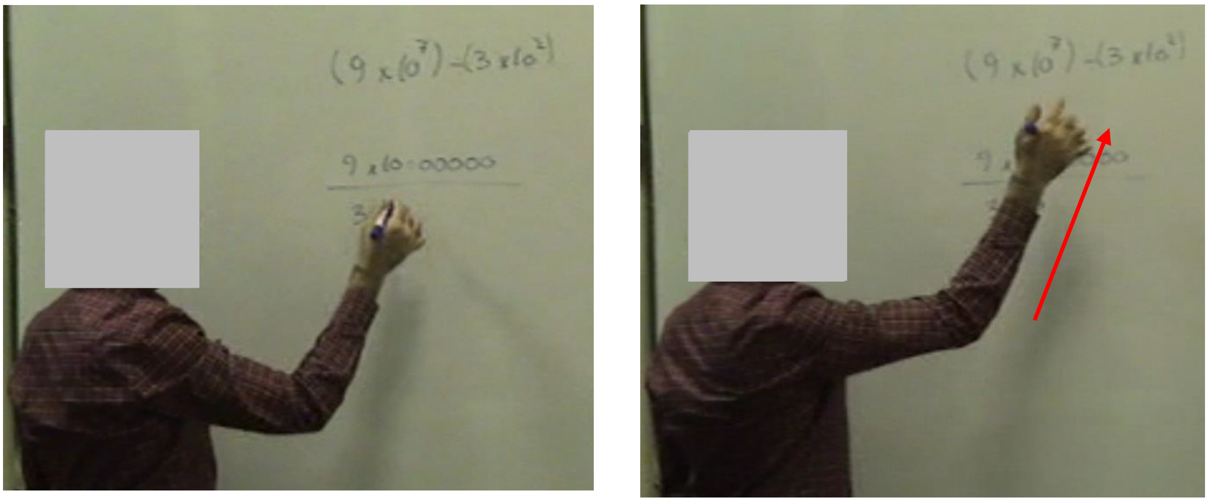


Figure 24: Gestural representation for power produced by T2 (I)

On the other hand it is also possible to interpret that T2's sudden gesticulation had nothing to do with the notion of 'power' but it was served as an index to what he was referring to on the whiteboard. T2 could have signified the location of 10^2 on the whiteboard as it appeared to be in a higher up position in reference to where he was expanding ten to the power of two, therefore deictic. The meaning of gestures is not clear but ambiguous. But the meaning of a particular gesture can become clearer in context if there are more examples. Within the next chapter, I will provide more evidence from my multi-modal transcripts to support the gestural representations for power and '*tavan*'.

In chapter nine, I will turn to draw on another interactional transcript emerging from transcript number six (section 7.4.2) interview that was conducted with T2 not only to provide and analyse more examples of the gestural representation for 'power' and '*tavan*' but to make a comparison between the two gestures to show how speakers of different languages can activate two different gesture type ('*tavan*' = metaphoric versus 'power' = deictic).

8.3 Pointing

In this section other than providing selected examples of pointing gestures that have emerged from my interactional dataset, I will also show how students made an emphasis and stressed upon what they were saying in words with indexing and pointing to objects, shapes and inscription. Furthermore, I will reveal how the degree of finger closure among the little, ring and mid finger was correlated with certain discourse factors. Before presenting my interactional recording I would like to set the background by recapping the literature on pointing gestures.

In general people point to show places or to indicate size and shape of objects. Due to the fact that pointing gestures are so ubiquitous and we interpret them with such ease, pointing can come into view as a trivial phenomena (Kita, 2003). Deictic (pointing) gestures are part of conversation that indexes the speaker’s speech content to objects, locations or inscriptions and the frequency of pointing does vary cross-culturally. Pointing often draws on different modalities; for example, a speaker can shift from auditory to visual representation as they point to objects or inscription. Similarly in a classroom when a teacher can point to objects or inscriptions as s/he speaks, those pointing gestures link his/her verbal stream to its physical referents in the environment (Alibali & Nathan, 2012).

In this section, I would like to draw attention on pointing, and different variations of pointing made by the students in this particular complementary school bilingual mathematics classroom. It is evident in my video recording when students were explaining their mathematical ideas that they often made a reference by pointing to index objects, locations or inscriptions. Most pointing gestures were produced with the index finger whilst others were produced with a pen or a board-marker as a ‘pointer’. The degree of finger closure among the three (little, ring and mid) fingers varied. By providing evidence from the interactional recording, I will show examples where the degree of finger closure appeared to be directly proportional with certain discourse factors. Often students' pointing gestures carried a tightly bunched index finger pointing when they were indexing new ideas or with the first mention of a topic. However, if a topic and its importance was already established but students found the need to direct attention to the topic by pointing, their index finger pointing often accompanied a looser degree of closure. Therefore the tighter degree of finger closure served as an indication to how important/new the topic would have been. Now I will draw on the literature to highlight different forms and variation of index finger pointing and their corresponding function in discourse.

There are two forms of index finger pointing which are with the palm vertical and with the palm down (Wilkins, 2003). Different variations can be seen within these two forms of index finger pointing. For example: open hand pointing with the palm vertical and open hand pointing with the palm down; thumb pointing and lip pointing is also common across different cultures. Each of these variations of pointing has a distinctive meaning in discourse. For example, palm down index-finger pointing foregrounds the referred referent into the centre of discourse focus. This is referred in the literature as "object individuation" (Kendon & Versante, 2003:115) where it individuates a referent as being distinct from other objects. On the other hand palm vertical index-finger pointing "indicates a referent that is relevant to the current discourse but not in the centre of focus" (Kita, 2009:148). Often the thumb is used in pointing when the location or the identity of the object indicated is not in the foreground of discussion, perhaps because the location or the identity of the object has been previously established. "This may be linked to what could be called its 'anaphoric' use, where something referred to a second time is indicated by the thumb, where it had been indicated by an index finger the first time" (Kendon & Versante, 2003:134).

More interestingly the degree of finger closure or openness among the little finger, ring finger and mid-finger while pointing with the index finger is correlated with certain discourse factors. It has been observed that emphatic or first mentions of events tend to be “regularly accompanied by the canonical (tightly bunched) index-finger point (which is often held in place). In follow up anaphoric mentions, or the mentioning of nonimportant participants, a looser hand is used (and the action is executed more quickly)” (Wilkins, 2003:193). It appears that speaking and pointing are a tightly linked system. Whenever speakers discuss their ideas about physical (and sometimes non-physical) objects in speech, they often refer to the location of those objects by pointing. Index-finger pointing is considered to be a part of visual human communication which often co-occurs with its accompanying speech. In a conversation between two or more people, the addresser’s pointing gesture is often for the listener’s benefit as the location can be visually identified. In pointing, speakers try to draw the interlocutor’s attention to an object or event of interest they are indicating (Masataka, 2003; Clark, 2003). Pointing gestures therefore provide a joint visual attention as often the addressee seeks to look to where the speaker is pointing. Therefore in pointing, the speakers direct their addressees’ attention to the objects, locations or inscriptions they are indicating (Clark, 2003).

One of the problems interpreting ‘only’ the pointing gestures is that no meaning can be extracted. If I point to an object, which aspect of the object is indicated? For example, when pointing to a car it maybe unclear as to whether the reference is to the car itself, to the colour of the car or to the person who is sitting inside the car (Kita, 2003). It is therefore essential to take the verbal and the vocal aspect of language that accompanies the visual into consideration. Pointing gestures along with the verbal message it accompanies often manifest speakers’ indexing of speech content to objects, locations, or inscriptions in the physical environment (Alibali & Nathan, 2012).

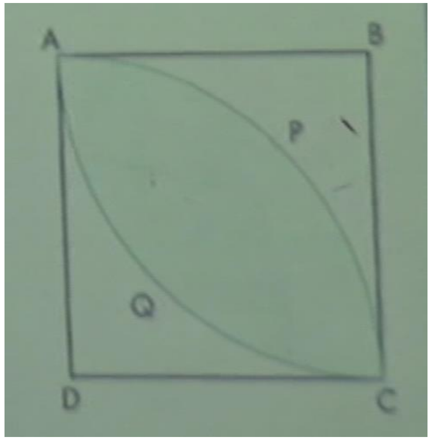
In the subsequent multi-modal transcripts (sections 8.3.1 and 8.3.2), by providing snapshots from the interactional video recordings, I will show examples where the index-finger pointing with a tighter degree of finger closure accompanied by a straight arm was used for first-time emphasis on a topic/idea. Conversely, a looser degree of finger openness along with a bent arm was employed for the anaphoric mentions where the topic/concept was already addressed previously by a tighter degree of finger closure and a straighter arm. In another words the tighter degree of finger closure and straighter arm was proportional to how 'new' the gesticulators' ideas were.

8.3.1 An example of deictic gesture and the degree of arm straightness

In transcript number 9, I will specifically focus on the relation between B4's deictic gestural representation as he points to the whiteboard whilst explaining his reasoning. B4 was engaged in an activity that involved finding the shaded area created within two circular arcs. This geometrical shape was projected onto the whiteboard.

Transcript number 9

T1: K*hob, hala in yeki.*{ok, now this one} Shaded area *ine. chetoori mitoonim be dast biarim?* {how can we find the shaded area for this?}



[As T1 employed a demonstrative pronoun in his utterance, synchronically he outlined the perimeter of the shaded area with his index finger.]

Khob, G1, honey, mikhaim ino hesab bekonim {ok, G1, honey, we want to calculate this area}

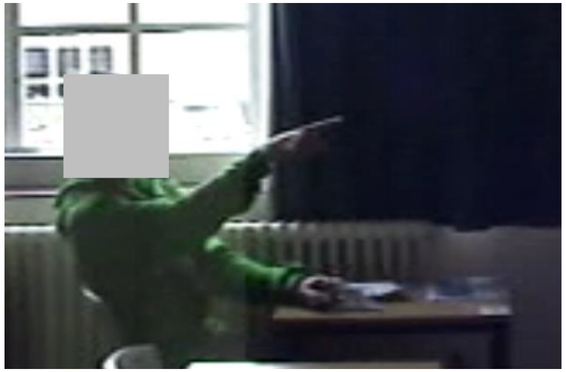
[There appeared to be approximately 11 seconds of silence before T1 addressed G1 directly]

Gs: hehe

T1: *chetor mitoonim in-ro hesab bekonim va dar biarim?in area-ro.* {How can we find this area? This area}

[T1 again highlights the shaded area that is projected on the whiteboard as he points to its perimeter with a board marker in his hand]

B4: *aha, fahmidam, chize, in ham, bayad oono A o C-ro yek line bekeshim.* {Aha, I got it, erm, I mean, you must draw a line through A and C}



[What B4 was referring to was the diagonal which passes through the angles at A and C. He created an index-finger pointing synchronous to his verbal utterance. The arm later returns to the body.]

T1: *kodoom ro?* {Through which one?}

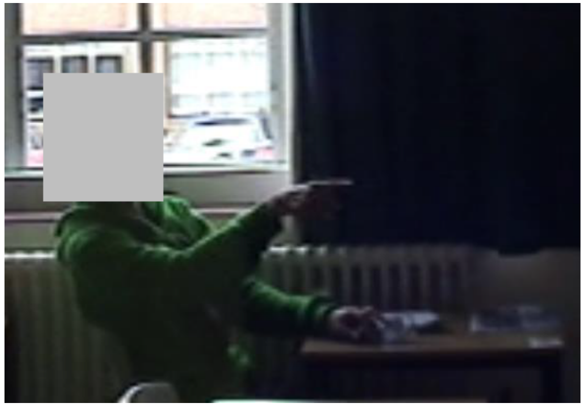
B4: A to C.

T1: *A C-ro migi yek line bekeshim.* {He says to draw a line from A to C}

B4: *bad oon triangle-aro dar miarim bad chiz mishe dige.*

{then take that triangle, then

Erm, you know}



[Having the arm returned to the body, B4 again produces another deictic gesture for the second time. This time B4 produces his pointing gesture as he co-expressively says '*oon'* {that/those}. The degree of finger closure is not clear in this picture, but the arm is more bent. After his pointing gesture was executed, the arm returned back to the body again.]

T1: *iin* triangle-*ro dar miarim* {we take this triangle}

B4: *Bad oon do ta ham chize*

circle-and dige, quarter-*e*

circle. {Then those two becomes erm, the quarter of the circle}

[B4 created a pointing gesture within his on-going flow of his discourse. The term '*oon*' appeared to trigger his pointing gesture to the whiteboard. His arm is even more bent than the previous time and he appears to hold a looser index-finger pointing gesture]

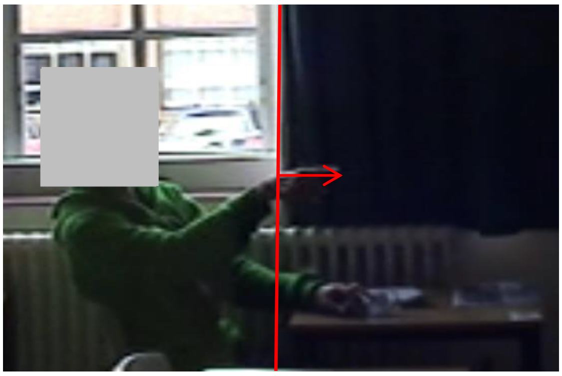
T1: *Ha*, {What?}

Bs: hehe

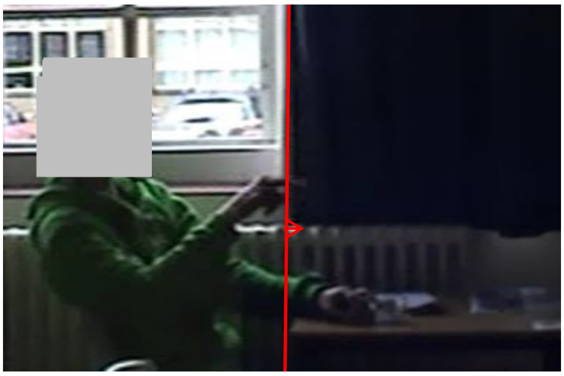
In the analysis of this particular transcript, I am really interested in analysing B4's index-finger pointing aperture (the degree of closure or openness of fingers while pointing with the index finger) and the degree of arm straightness and how these two features of non-verbal language are associated to the verbal discourse. I am primarily focusing on the first, second and third deictic gestures that are made by B4 in lines 16-21, 28-31 and 34-38 respectively. It is worth noting that these were three different pointing gestures which were produced. Unfortunately due to the dark background where B4's right hand is placed, the degree of finger closure/openness is not clear in the first two snapshots. Therefore I would only examine B4's degree of arm straightness.

By considering the static position of the curtains as a frame of reference in Figure 25 (see the vertical red line), the degree of B4's arm bendiness/straightness can be judged whilst pointing. The horizontal red arrow illustrates how far away B4's pointing gesture is from the frame of reference. The longer the length of the arrow, the straighter the arm is.

In first time pointing, B4 is pointing to an imaginary diagonal that can be drawn from A to C that creates two triangles: ACD and ACB. The length of the red arrow is approximately 1.1 cm.



Now the location and the identity of the two triangles have been established in discourse. In second time pointing, the length of the red arrow is approximately 0.8 cm.



In third time pointing, B4 appears to hold a looser index-finger pointing gesture. Moreover, the length of the red arrow is approximately 0.2 cm.

Figure 25: Degrees of arm straightness

From the example that I have offered, the degree of arm straightness appeared to correlate with certain discourse factors. First time mention of a topic/idea accompanies a straight arm index-finger pointing. In the anaphoric (second and third) mentions, when the location and the identity of the object/inscription was established, a bent arm accompanied the index-finger pointing. In the subsequent transcription I will provide evidence to illustrate the degree of finger closure that accompanies the index-finger pointing.

8.3.2. Examples of deictic gestures and the degree of finger closure

In a different lesson, T1 had projected a geometrical question on the whiteboard and asked students to find a numerical value for the requested angles. The video recordings of the projected activity on the whiteboard is blurred therefore not much information can be obtained about the nature of the question. However, I became interested in this particular excerpt to examine B3's deictic gestural representation as he synchronously made a point in speech. B3's deictic gesture indexed aspect of the task that was projected on the whiteboard. B3's comment consisted of two parts. First he said: *na*, C-*e bayad si bashe* {No, C should be thirty} and then approximately two seconds later he uttered: *oonvar* {over there}. As B3 was offering a numerical value for C, he created an index-finger pointing gesture. Approximately two seconds later, he created another index-finger pointing as he said 'over there' in Farsi. But what I find of particular interest is B3's index-finger pointing and the degree of openness/closure and how it relates to certain discourse factors.

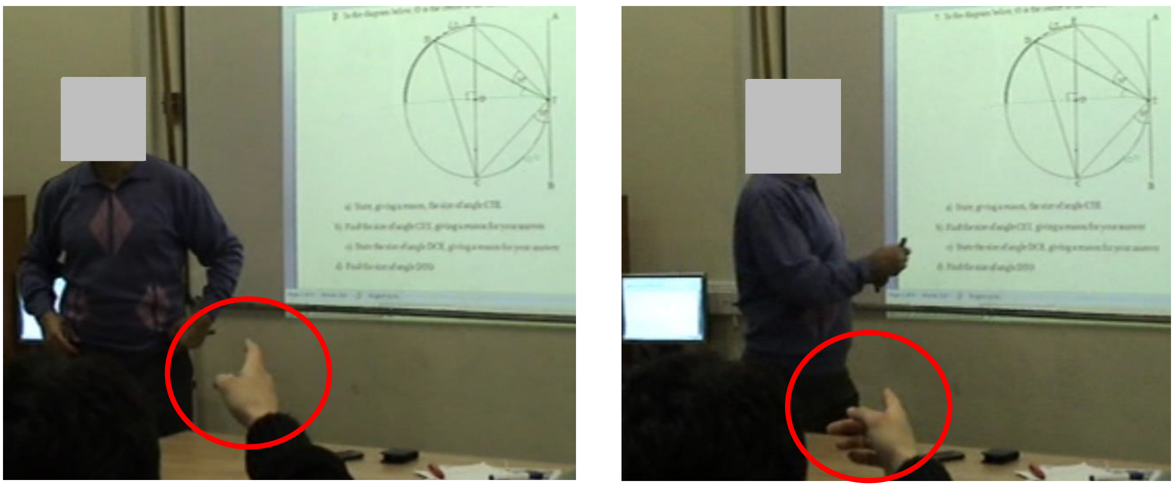


Figure 26: Degree of finger closure

B3's nonverbal behaviour typically consists of pointing, to call attention to inscriptions on the whiteboard. Note the degree of finger closure and openness in Figure 26. In the first image (on the left) B3 foregrounded the referent (C) into the centre of discourse as he said 'No, C should be thirty'. B3 synchronously carried a tight bunched index-finger pointing. In the second image B3 reiterates the location of C 'over there' which appears to be relevant to the current discourse but it is not the centre of focus which is on the numerical value of C. B3's second index-finger pointing carried a looser degree of finger closure among the little, ring and mid fingers.

I will now turn to describe part of a video excerpt where the main lesson agenda was to identify any angle in a circle by its opposite arc. Due to the overlap talk between students-teacher and students and students in this part of the video excerpt, I will not present the whole verbal discourse (as there are at times four different discussions happening simultaneously) but a selected part. I will present each part of the selected discourse integrated with snapshots and computer generated images for the purpose of clarification. In this lesson T1 has taught students that the angle 'a' is half of the angle of the opposite arc. For example in Figure 27, if the arc is eighty degrees, the angle 'a' is forty degrees. This was the typical language that was used to denote "the angle the arc creates at the centre of the circle is 80^{o}".

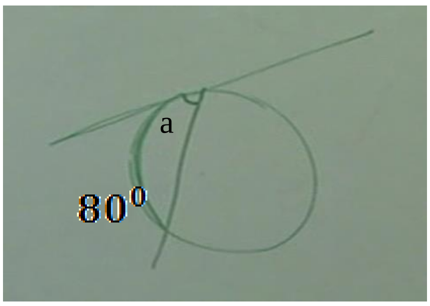


Figure 27: Rule of opposite arc (I)

Figure 28 was projected onto the whiteboard and students were asked to find all the angles subtended by those arcs at the centre of the circle. Just a little information was given in the question which was: O is the centre of the circle and EC is a diameter; the line AB is tangent to the circle; and only two angles were given.

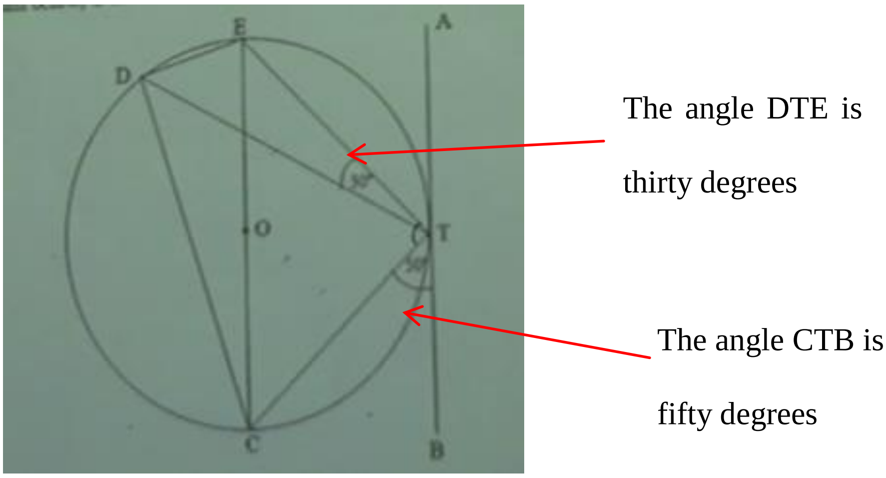


Figure 28: Rule of opposite arc (II)

Students together with T1 worked collaboratively to find the numerical values of some angles. For example in Figure 28, CTE was said to be ninety degrees because its opposite arc is 180 degrees.

Students found a number of unknown angles using this method which involved the symbiotic relation between an angle and its opposite arc. Although in the question it was stated that this figure is not to scale, there appeared to be a mismatch between the geometrical visualisation and the mathematical logic. As shown in Figure 29, there is an arc opposite the angle 30 degrees. According to the rule of opposite arc that was explained by T1, 'the arc opposite a 30 degree angle will be 60'. When T1 wrote 60 on the whiteboard (on the arc that lies between the points D and E), he paused six seconds and then asked 'what is the remaining quarter [of the perimeter] of the circle?'. Mathematically the remaining arc of the quarter of the perimeter of the circle should be 30. That is a quarter of a circle (90 degrees) takeaway 60 degrees which is equal to 30 degrees. *Visually*, the arc that lies in between points E and D (which was 60 degrees) was in contradiction to the arc that is shown by a red arrow which is 30 degrees.

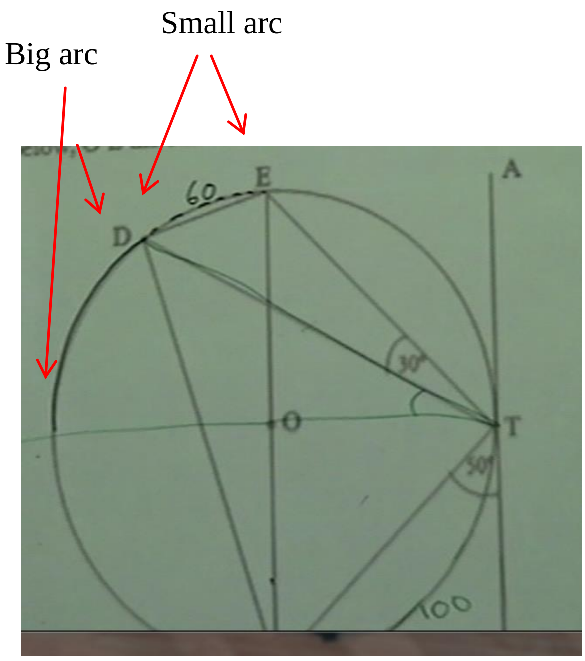


Figure 29: Mismatch between the geometrical visualisation and the mathematical logic

B3 said, ‘how come the small arc became 60 and the big one 30?’ with a prompt response from T1 saying: ‘that is exactly my point’. At this specific moment in time, a number of students talk over each other at the same time with each giving a rationale for different answers they offered. There appeared to be a minute of confusion for students as they kept changing their answers. For example, students offered that the ‘top arc’ (the arc between the points D and E on the perimeter of the circle) is 30 and the ‘one below’ (the arc shown by the red arrow) is 60 and moments later students changed their answers based on the mathematical logic. Suddenly B1 said, *fahmidam; oon nesfeshe dige, . . na nemitoonim nesfesh bekonim?*. {I have got it; that is half of it, . . can’t it be the half of the other one?} although it was a bit unclear as to what was B1 was verbally referring to as ‘half of it’ but I am interested to examine the deictic gesture that B1 created first in comparison to the pointing gesture he created approximately two seconds later. B1’s deictic gesture and speech appeared to be in a synchronous relation as he moved between verbal and visual. B1 carried a tight bunched index-finger pointing as he said ‘I have got it; that is half of it’. His statement appeared to be the centre of focus and by pointing he established a joint visual attention; that is looking where someone else is looking. As apparent in the following snapshots (see Figure 30), T1 is looking at B1’s extension of where he is indexing on the whiteboard. It is common that in pointing, speakers try to direct their addressees’ attention to objects or inscriptions they are indicating (see Clark, 2003). What I find of particular interest is the shift in formation of pointing moments later. Approximately two seconds later, B1 doubted his rationale and uttered ‘can’t it be the half of the other one?’ which was used as some hedging device in his discourse. For the second time B1 created a pointing hand gesture with a looser degree of finger closure. The top left and bottom left images are the first and second pointing gestures respectively. The images on the right are the enlargement of B1’s deictic gestures.

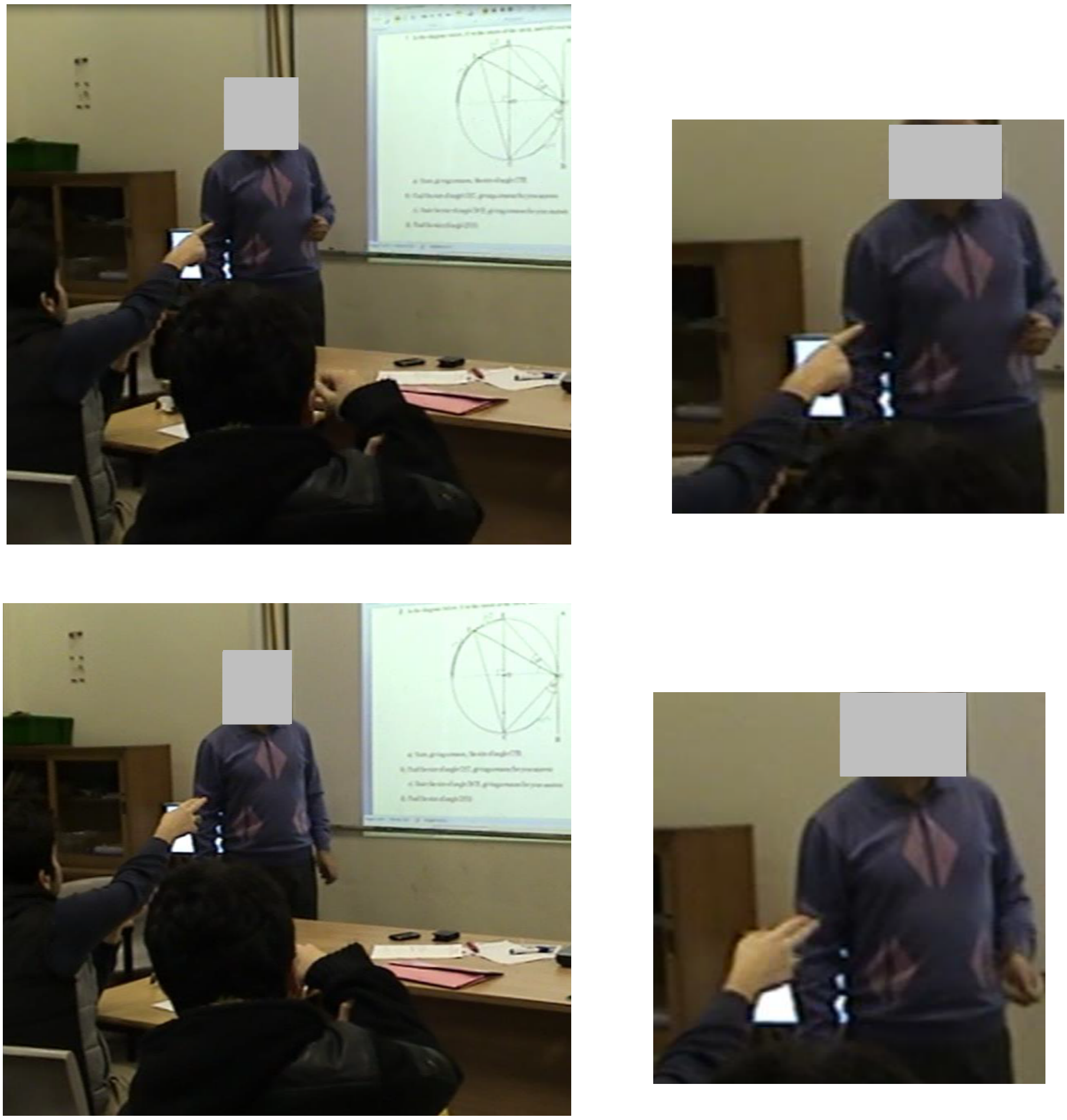


Figure 30: Degree of finger closure (II)

Communities have entire repertoires of pointing gestures they use in conversation to indicate location of objects in space. To date, there has been no report of a culture that lacks from pointing gestures (see Kita, 2003). In this specific community of practice, it appears that bilingual learners often conveyed their ideas expressed through different codes (Farsi and English); styles (vernacular and mathematical) and modes (verbal and nonverbal). It is worth noting that the same principle applies to non-bilingual learners, in a sense that they can convey their ideas through different styles (e.g. ‘everyday’ and technical) and modes (e.g. symbolic, diagrammatic). As Arzarello and colleagues (2009:97) have recently observed “[m]ost often the same person exploits many of these resources simultaneously”. Students draw upon their semiotic resources and use pointing gestures to index objects, locations or inscriptions during their explanation. In other words by pointing students associate their verbal message to evoke objects or inscriptions in the physical environment. Their pointing and speech appeared to be in synchrony and during this process; students drew on different modalities, which often involves a shift from auditory to visual objects, locations or inscriptions. Furthermore, my finding confirms the already existing literature which verifies that the degree of finger closure appeared to be correlated with certain discourse factors in a particular bilingual mathematics classroom. First mentions of events tended to accompany a tightly bunched index-finger pointing but the successive mentions (second or thirds) of the event was escorted with a looser degree of finger closure among the little, ring and mid finger.

In the next section, I will report on the employment of a different form of deictic gesture which carried a different pedagogic function whilst the classroom teachers were conveying the instructional information.

8.4 Isosceles

I will present part of my multi-modal transcripts along with snapshots to show a different example of deictic gesture which appear to have a different form and function. The analysis of this section centres on how the classroom teachers' use of deictic gesture helps to stress and emphasise a particular English mathematics register 'isosceles triangles'. The Farsi translation for isosceles triangle is *motasaavi-al saaghain* which I will use frequently in this thesis. A unique property of an isosceles triangle consists of the possession of having two equal sides and two equal angles within a triangle (see Figure 31).

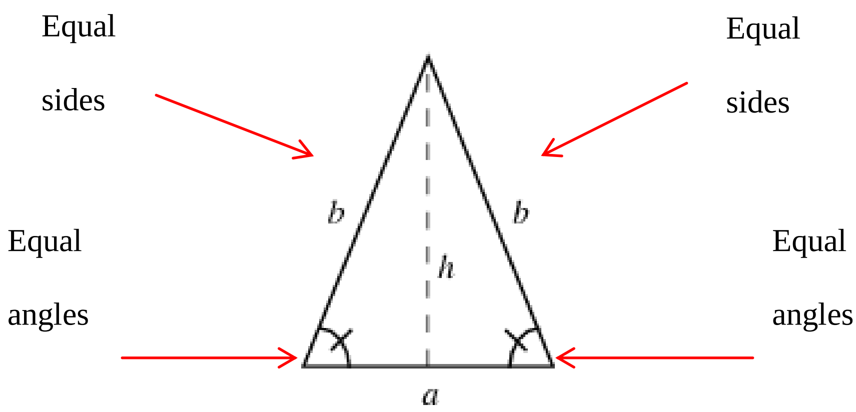


Figure 31: Property of an isosceles triangle

The structure of this section consists of three different parts. Each part consists of a transcript of a different video excerpt followed by an immediate analysis of the interactional data. In sections 8.4.1 and 8.4.2 I am interested in paying attention to different styles in pointing (or the variation in pointing) that are produced by different classroom teachers (T2 and T1 respectively). Then in section 8.4.3, I will focus on the teacher's medium of instruction and how different languages conveyed different instructional information suggested through gestures.

8.4.1 T2’s gesticulation for isosceles

This particular interactional recording emerged from a lesson where T2 engaged in solving an unknown angle in a regular pentagon. T2 was explaining the solution to how he finds an angle of x which lies inside of a regular pentagon. T2 has divided the regular pentagon into five equal isosceles triangles as a starting point (see Figure 32). I find this extract particularly interesting because T2’s instructional talk, gesture and speech convey overlapping information in lines 14-17.

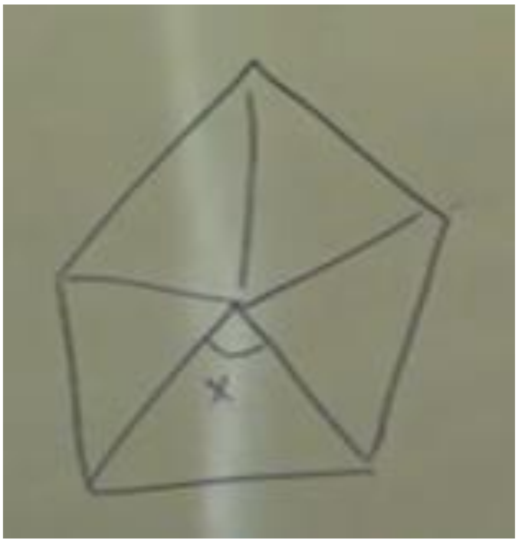
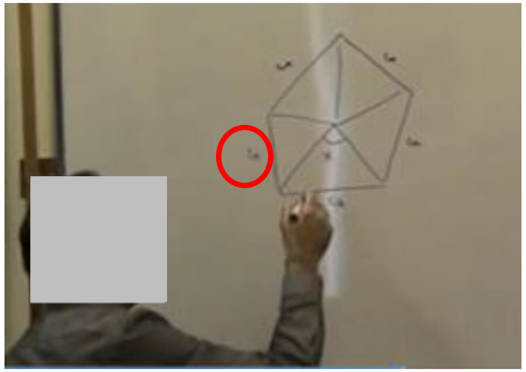


Figure 32: Pentagon

Transcript number 10

T2: *khob, baraye iinke iino perda bekonim*, {In order to solve this} that's a regular pentagon obviously, and each side is four ok.



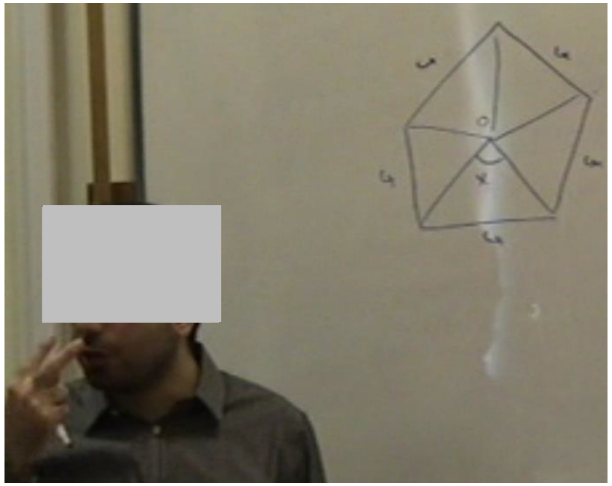
[T2 writes 4 on each side of the drawing of a regular pentagon. He has already connected the centre of the pentagon to every vertex in the pentagon]

Chon regular pentagon-*e* centre-*esh age maa be behesh vasl bekonim mitoonim hamash* {Because this is a regular pentagon, if we connect the vertices to the centre, it will all become}[6]

[T2 writes O in the middle of the pentagon which represents the centre.]

[6] The literal translation of T2's statement is {because it is a regular pentagon, its centre, if we connect to it, it all becomes}

isosceles triangle *peida* *bekonim, dorost bekonim,* *khob* {we will find isosceles triangles, ok}

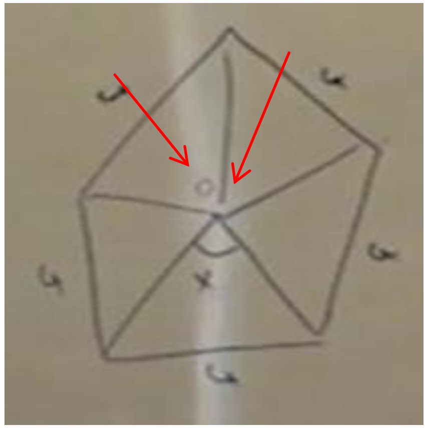


[T2 employs a gesture that incorporates deictic information. He appears to point to his eyes with his index and mid finger as he uttered isosceles in his speech]

B1: *motasavi-al-saghain* {isosceles triangles}

T2: *motasavi-al-saghain* {isosceles triangles}

khob, iino ke peida mikonim, {ok, when we find this angle} angle of x is equal to angle of x equal to angle of x and so on.



[T2 is referring to angle x. He taps twice on the angle x and moves to point to the other remaining four inner angles as he utters his statement "is equal to angle of x equal to angle of x and so on"]

Khob, how many angle of x *darim?* {So, how many angles of x do we have?}

Bs: five/*panj*

T2: five-*ta*. *Khob*, {five,

ok} three hundred and sixty which is xxx the full thing [xxx is inaudible]

B1: divided by five

T2: divided by five

I find this extract of particular interest firstly because of the geometrical transformation that has been made to the regular pentagon; it has converted into five equal isosceles triangles as a way to proceed. It emphasises the fact that a particular geometrical shape with a specific property can be turned into a number of a different shape with a different identity (see Pimm, 1995).

There are different modalities that play a role in this short transcription of a video recording. T2 conveys his instructional message not only in speech but also in the gestures and a number of different modes that he uses as resources in teaching. For example, notice the way in which T2 directs attention by drawing and tapping (see lines 21-5) on the whiteboard. He taps twice on the angle x and moves on pointing to the other remaining four inner angles as he utters his statement “is equal to angle of x equal to angle of x and so on”. Furthermore, code-switching is evident within the technical forms of register. Both B1 and T2 draw upon their linguistic resources in Farsi (see lines 18-21) to emphasis key terminology ‘*motasavi-al-saghain*’ in Farsi.

What I am more interested to examine in this particular transcript lies on 14-17 where T2 incorporated a form of deictic gesture (with his index and mid finger) indexing his eyes as he uttered ‘isosceles’ in his speech (see Figure 33). At this stage there could be two possible interpretations of T2’s gesticulation; depending on whether the focus of attention is on his fingers or to the eyes which I will now discuss. It is possible that T2’s gesticulation could be read as a visual similarity of the two fingers representing two equal sides. Although the index and the mid finger are not exactly the same size in length, but could serve as a primitive tool to convey the idea of two equal sides of an isosceles triangle. Hence T2’s gesticulation would be iconic as it represents not only the geometric representation but also conveys its mathematical definition visually.

Alternatively, if T2's gesticulation was indicating his eyes, then the gesture and the accompanying speech (isosceles) did not appear to have/convey any shared semantic meaning. Semantically there is no overlapping information but phonologically there is a strong connection. The relation between T2's verbal message and his deictic gesture is of homophony. Homophonous words are terms that have the same pronunciation as another but different in meaning, origin or spelling[7]. The way isosceles is pronounced is very similar to what can be thought of as 'eyesosceles'. The pronunciation of the term 'eyesosceles' possibly explains the reason why T2's gestural representation was directed at his own eyes. Isosceles generated a deictic gesture, by means of indexing an object that was phonologically similar to the accompanying speech.

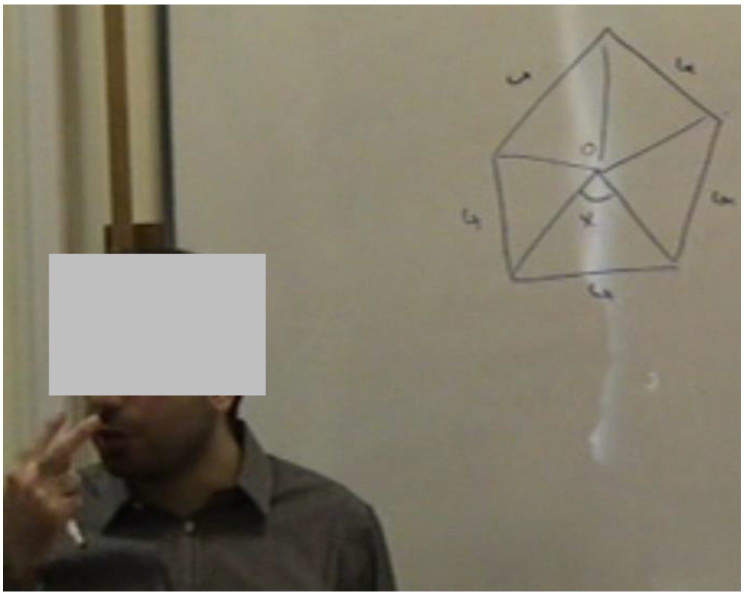


Figure 33: The gestural representation for isosceles triangle performed by T2

[7] For the case of homonymy and homophony and their potential confusion in a bilingual mathematics lesson, see Zagorianakos and Farsani (2012)

I find it interesting to see how a mathematics register activated a particular gestural representation in teacher's instructional talk. Considering gestures as a visual tool in conveying the inner thought (McNeill, 1992), T2's gestural representation gives a sense of his mathematical thinking especially when engaged in conveying the instructional information. An interpretation of T2's gestural representation of the term 'eyesosceles' reveals that there was no mathematical understanding or meaning assigned to property of isoscelesness. At the same time, T2's gestural enactment shows a great linguistic awareness that helps the remembrance and recollection of an English mathematics register easier. The enactment of such gesture also increases the emphasis of its verbal counterpart.

Once again, it is worth noting that by definition, if T2 attended to the shape of his two fingers, his gestural representation for the concept 'isosceles' would have reflect McNeill's (1992) iconic gesture type. However if the foci of attention was directed to his eyes, then T2's gestural representation for the mathematical concept isosceles falls under McNeill's 'deictic' gesture type, because deictic are pointing gestures that index to locations, directions, people and objects. Arguably in this specific context, T2's gestural representation for 'isosceles' was beyond deictic because T2's gesture served more than just indexing as there were also phonological connections to his gestural enactment. There are issues of phonetic and prosodic aspects of language involved in gestural enactment. In chapter nine I will be talking about the enactment of these kinds of gestures as 'linguistic-based gestures'.

In the next section, in order to validate the gestural representation of the mathematical term 'isosceles', and to clarify whether the gestural attention was given to the 'shape of the fingers' or to the 'eyes', I will provide another example to show how the same gesture was generated by a different teacher, T1.

8.4.2. T1's gesticulation for isosceles

In this particular lesson, T1 is going through a number of questions which are projected onto the whiteboard. Just like the previous example, the classroom teacher is engaged in explaining the instruction in speech and gestures. The question has asked to find the angles of x and v (see Figure 34). In the question, O is given as the centre of the circle with AO, BO and PO radii of the circle. T1 uses all the information that is available in the question and in the diagrammatic representation. I find this extract particularly interesting because of T1's gestural representation (and his reasoning for such a gesture) of the concept 'isosceles triangle' as he conveys his instruction talk in speech in lines 24-32.

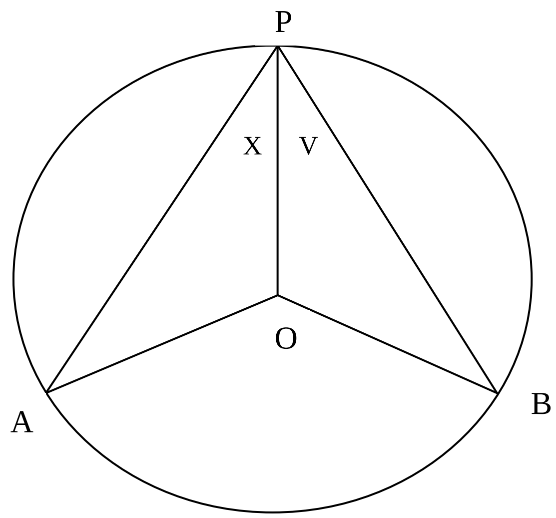
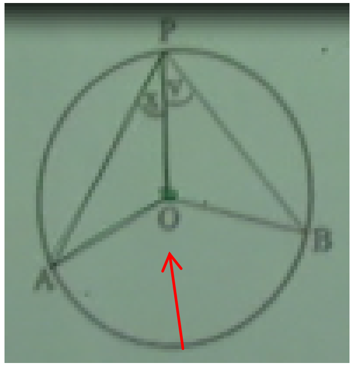


Figure 34:Isosceles

Transcript number 11

T1: *iin markaze dayeran.* {This is the centre of the circle} O is centre, O is centre. O centre-*e,* {O is the centre}



[T2 points to O which is located at the centre of the circle. He directs attention to O as he kept repeating the fact that O is the centre of the circle several times]

in mishe chi?{what is this?}

R*, in yeki ham mishe chi?.* {R, and what is this one?.} *in ham mishe chi?* {This is also what?}Radius-*e dige,* {it is the radius} *shoaa-e dayerast.* {The radius of the circle} *Do ta shoaa-e dayerast. Mosalas mishe chi?* {What is the name of a triangle that contains two radii of a circle?}

[T2 points to the line OA which is the radius of the circle. Synchronically to his pointing action, he expresses '*iin mishe chi?*' {What is this?}]

[At this point T2 turns (180 degrees) to the class and asks the translated question "What is the name of a triangle that contains two radii of a circle?"]

B4: *mishe* squared, {that is a squared}, squared.

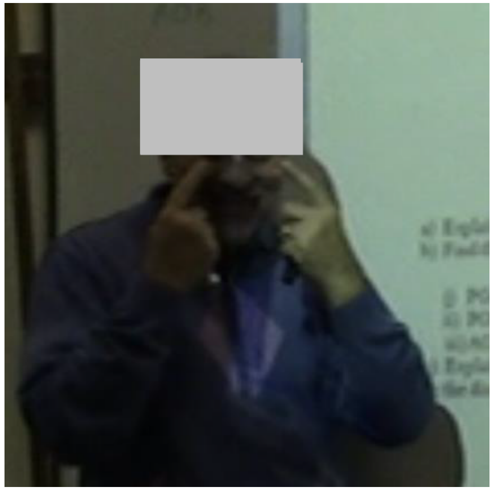
G2: isosceles

T1: *ha, chera besh migim* isosceles? {Why do we call them isosceles?

315 Bs/Gs: xxx

[xxx is inaudible speech]

T1: *baraye inke eine cheshamoone na?* {Because they are like our eyes} Hehe *mesle haman*. {they are identical} *Khob*, {ok} the triangle is isosceles. The reason *ro motavaje shodid,* {Did you understand the reason why we call it that} *G1, shoma* reason *ro motavaje shodid?* {G1, did you understand the reason?}



[T2 explains his answer to the question not only in speech but also in the gesture that accompany that speech. He points with both index fingers to his eyes]

B2: Reason-*esh chie?* Reason-*esh chie?* {What is the reason? What is the reason?}

T1: Reason-*esh chie?* {What is the reason?}

G1: *har jofteshoon* radius-*an* {they are both radius}

B2: *yanni chi har jofteshoon?* {what do you mean by ‘they both’?

Bs/Gs: xxx hehe

I would like to shift attention to lines 21-32 as a way to analyse T1's gestural representation for isosceles. In lines 21-32 T1 is justifying a reason as to why 'we' call those specific types of triangles 'isosceles'. The reason is "Because they are like our eyes". This could be taken to mean that our eyes are identical to one another in the same sense that the two lengths and the two angles within any isosceles triangles should be the same too. T1's gestural representation in Figure 35 could have also tried to reinforce the concept of 'isoscelesness' as there are two identical eyes. It is interesting to note that the etymology of the term isosceles is from the Greek '*isis*' (meaning equal) and '*skelos*' (meaning leg). In geometry, an isosceles triangle has two equal legs (and angles). It makes sense to speak of isosceles triangles in a context of having two equal legs which is given by analogy with the two legs of human body (see Schwartzman, 1994).

Figure 35: T1's gestural representation for isosceles (I)

In general pointing gestures occur when speakers create a visual attention to index objects, inscription, etc., that is a shift from auditory to visual representation in order to convey a concept. But what possible explanation could be given to the enactment of T1's deictic gesture indexing his own eyes in a context that is based on isosceles triangles? To answer this interesting question that has risen from my multi-modal transcripts, I will repeat my rationale for this particular gestural enactment. I assume that due to the fact that isosceles sounds very similar to 'eyesoceles', this could explain why T1 has indexed attention to his eyes; in order to convey a verbal and a visual form of representation. Therefore the *realisation* of this particular gestural representation for the mathematical concept 'isosceles' is phonetically based. In chapter nine, I have classified the realisation and enactment of this particular gesture as 'linguistic-based gestures'. In the next section, by presenting evidence from my multi-modal transcripts, not only will I provide another example for deictic gesture as T1 was conveying the instructional information, but I will also raise awareness as to how T1's gestural representation for the concept isosceles differed through his bilingual instruction.

8.4.3. T1's gesticulation for '*motasavi-al-saghain*': a variation of gesture across English and Farsi

The proceeding transcript contains a short communicative event of a lesson where bilingual learners and T1 are engaged in participating through flexible bilingual practices (code-switching). In this specific extract, T1 provides opportunities by means of bilingual instruction to encourage bilingual learners to draw upon their linguistic repertoire. In this specific section I will focus primarily on deictic gestures that are employed in interaction to address one issue: to show how does the mathematics concept of 'isosceles' activate a different gestural representation when the same concept is expressed in two languages. In other words, I will illustrate the cross-linguistic variation of the mathematical concept 'isosceles'.

In this lesson T1 is explaining an approach to how he can find the numerical value for the angle ‘a’ in the triangle OPQ (Figure 36). In the question it is given that O is at the centre of a regular octagon with P and Q two vertices of this octagon.

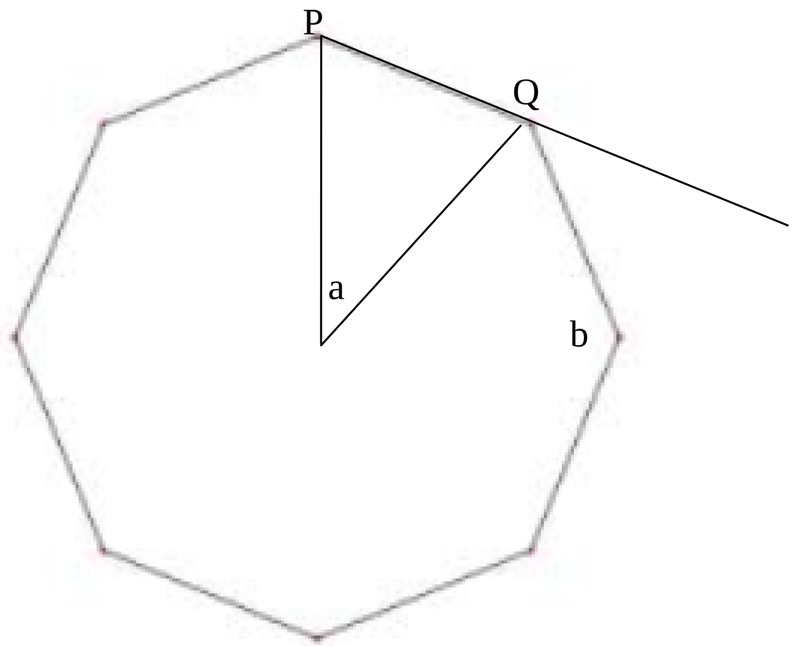


Figure 36: Octagon

I find this specific transcript of particular interest for further analysis primarily because T1’s instructional information through the medium of Farsi and English activate a different gestural representation for the concept ‘isosceles’.

Transcript number 12

T1: Ok , this one is isosceles, this one is the same, they are equal, yeah? *Ha, B1 motavaje mishi? G1?* {B1, G1, do you understand?}

G1: aha

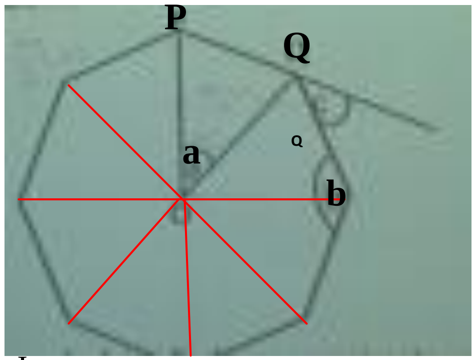
T1: *motavaje-i ino, ha? B1 chi migim ino? In mosalas ro chi migim?* {Did you understand it? B1, what do we say the name of this triangle?}

B2: isosceles

B1: isosceles

T1: *Farsi chi migan?*{how do we say it in Farsi}

B1: *motasaavi-al-saghain* {isosceles}



J

[T1 points to the triangle OPQ as he uttered “this one is isosceles”. Although the picture is very blurred but T1 has drawn redlines in the middle of the regular octagon and connected each vertex to the centre, O, which simply divides the octagon to eight equal isosceles triangles.]

T1: *aha, Farsi sho baladin bacheha?* {aha, do you know the Farsi for it guys?}

Bs: *are motasavi-ASDA* {yeah, isosc-ASDA} hehehe

T1: *saagh yanni chi?*{what does '*saagh*' mean?}

B1: *paa* {leg}

B3: *saagh yanni paaiinesh* {*saagh* means the lower part of the leg}[8]

B4: *chii bood dobare?*{What was it again?}

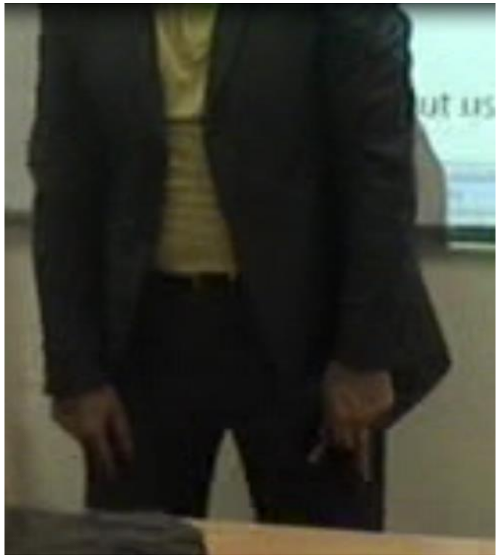
T1: *motasavi-alsaghain.* {Isosceles.} *Motasavi yanni chi?* {What does '*motasavi*' mean?} *Yanni* equal. {It means equal}.

[8] 'saagh' means shin.

Motasavi-al-saghain yanni do ta saghash chi-an, do ta paahash chi-an. {Hence

40 ‘*Motasavi-al-saghain*’

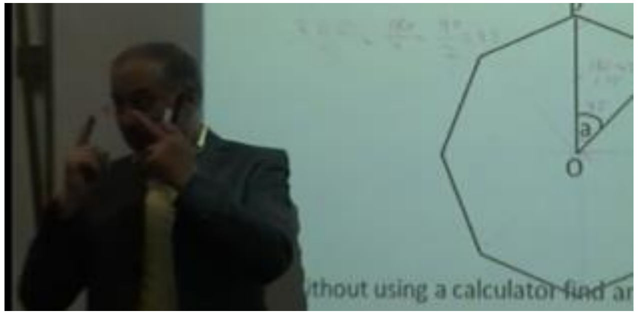
means their legs are what?}



[T1 synchronically touches his thighs with his hands as he made his verbal statement. As he touched his upper part of his thighs, he then started to move his hands lower towards his knees.]

Bs: *mosavi.* {Is equal}

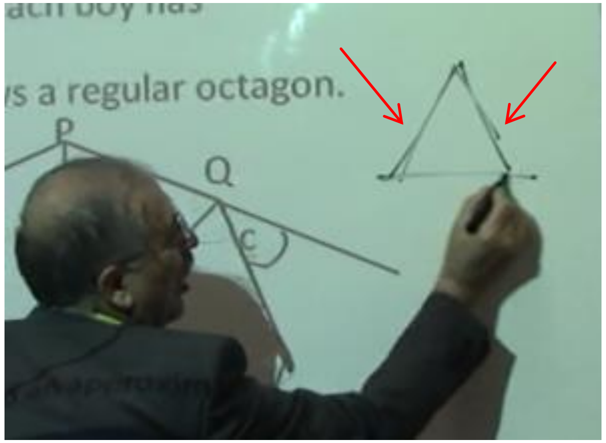
T1: *mosavi.* {Is equal} *Englisi besh migan* isosceles *yanni dota chesha shabihe haman, Irania migan motasavi-al-saghain.* {In English *motasavi-al-saghain* is referred to as isosceles, just like our two eyes. In Farsi it is referred to as 'equal shin'}



[T1 points to his eyes with his two index fingers as he uttered isosceles in his instructional talk]

B4: *arabi-e oon*[9] {is it Arabic}

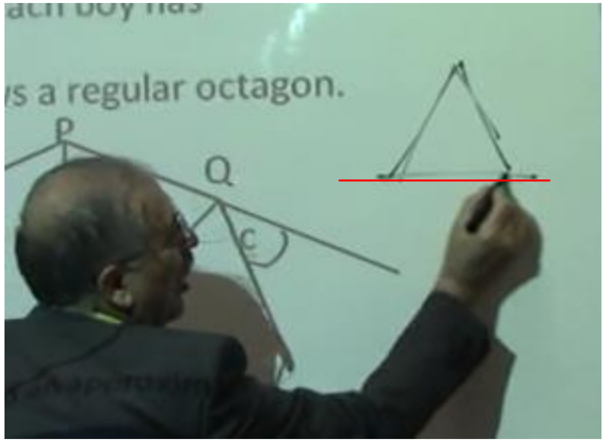
T1: *bebin goosh konid, be iin migan saagh be in migan saagh, sagh,* {listen carefully, they call this *saagh, saagh*}



[T1 draws a triangle and emphasises on the legs of an isosceles triangle]

[9] B4 is referring to *motasavi-al-saghain*

be in migan paaye. {They call this *paaye*[10]}



[T1 points to the '*paaye*' (or the base)]

iin dotaii ke shabihe hame mesle adamie ke, motasavi-al saghain, do ta paahash andazeye hame. {That is like a person who has two equal length legs} *Ha, too Irani injoori migan.* {This is how it is referred to in Iran}

[T1 then draws a person on the whiteboard as an emphasis to refer to someone/something with two equal legs in length]

[10] '*Paaye'* means base/foundation

In the analysis of the above transcript I will primarily focus on how the gestural representation for/of a specific mathematical concept (isosceles triangles) differs conceptually when different languages are employed in a bilingual complementary school classroom.

In the previous two sections, I have provided examples where both T2 and T1 synchronously pointed/referred to their eyes as they said ‘isosceles’ in their utterance. In this transcript, in lines 43-49, T1 conveys the instructional information both in speech (isosceles) and in gesture (see Figure 37and Figure 38).

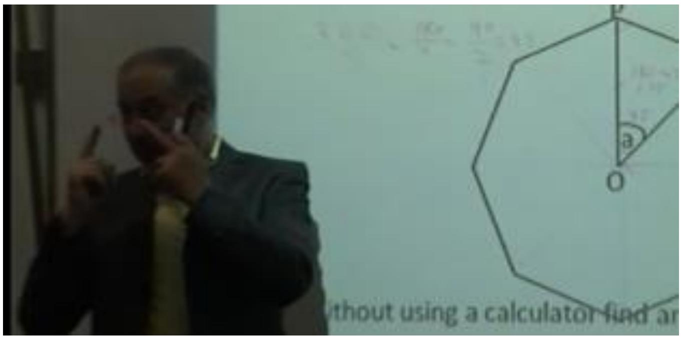


Figure 37: T1's gestural representation for isosceles (II)

Similarly, T1 constructed a gestural representation whilst expressing instructional information about the mathematical notion ‘*motasavi-alsaghain*’. However the location in which the pointing referred to was different. When T1 conveyed his instructional information about the notion of ‘*motasavi-alsaghain*’ in Farsi (see Figure 38), he synchronically pointed to his thighs with his hands and then started to move his hands lower towards his knees.

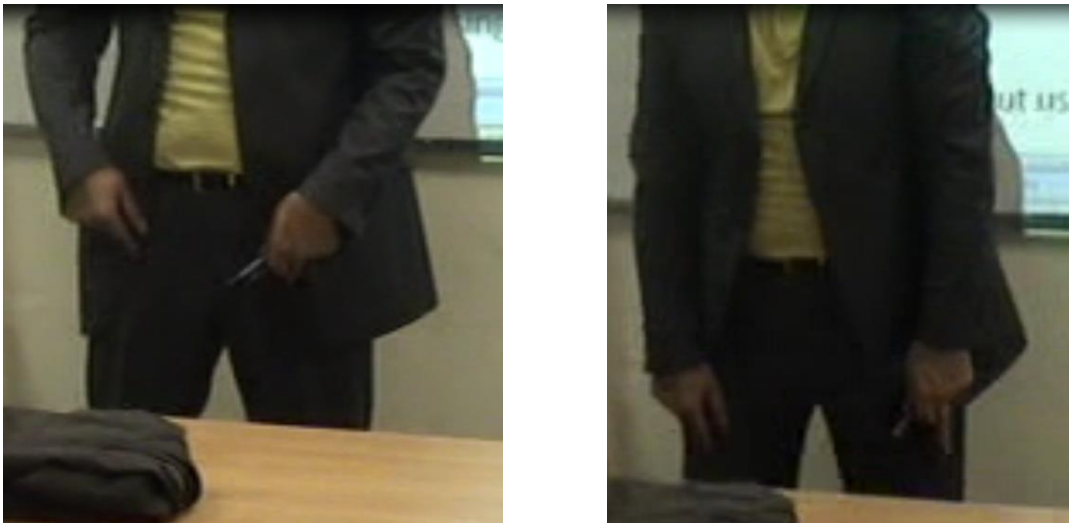


Figure 38: A gestural representation for *'motasavi- alsaghain'*

A question might arise as to what extent a gestural representation has been influenced by the semantic meaning of its verbal counterpart? For example, in Farsi '*motasavi-alsaghain*' is a compound term that is constructed of two components. '*motasavi*' constitutes the first component of the compound meaning 'equal' and the second component '*saghain*' meaning legs. So the semantic meaning that is verbally extracted means equal legs and, by analogy, the two legs of a human body can represent the two equal sides of an isosceles triangle.

It appears that the semantic meaning for '*motasavi-alsaghain*' (equal legs) is inherent and apparent as it was reflected in T1's gesture. However, a different meaning was emerged from the term isosceles, which was of the phonological issue where 'eyesosceles' activated a pointing gesture to the eyes. It appears that Farsi and English, due to their phonological and lexical structure activated different understanding of the concept isosceles in mathematics.

The gestural representation for both '*motasavi-alsaghain*' and 'eyesosceles' outlined some specific properties of isosceles triangles. Arguably, the gestural representation for '*motasavi-alsaghain*' was served to index its semantic meaning in Farsi (equal legs). It is worth nothing that '*motasavi-alsaghain*' is a metaphor that expresses 'equal legs' and unlike many languages, such as English, its semantic meaning is apparent to contemporary users. Therefore it makes sense to talk about how the semantic meaning of the term '*motasavi-alsaghain*' activated an index to T1's own legs, therefore deictic. On the other hand it can be argued that the gesture T1 produced whilst referring to his legs conveyed aspects of the conceptual understanding of isosceles, therefore it is iconic. It is a grey area. It can be argued that the gestural representation for '*motasavi-alsaghain*' and 'isosceles' are both deictic gesture type as they were employed to 'point to' some linguistic features (semantic and phonological respectively) of the concept.

In this section I have shown that both T2 and T1's deictic gestures served to emphasise the verbal language by indexing a visual object that carried a similar sound. Therefore both T1 and T2's deictic gestures served as a pedagogic tool to help memorising/remembering 'technical' mathematical words. In other words, teachers' nonverbal message served as a mnemonic device to help remember the terminology and concept. Now, I will explore aspects of pedagogy further. I will focus on how T1's employment of deictic gestures in instructional talk not only added clarification and richness to the spoken discourse but promoted bilingual pedagogy.

Based on transcript number 12 which was presented, I will reveal aspects of the pedagogic nature of deictic gestures that the classroom teacher (T1) employed. In this particular lesson, T1 not only showed awareness to other languages in the class but developed bilingual and culturally accessible resources through mathematical exemplifications in both languages. The reason I have chosen this particular transcript for the purpose of pedagogy is because it entails a number of instances where T1 supports the content by providing cultural exemplifications in both languages. My focus in this part is primarily on T1's strategy that provides an opportunity to build on both the content and language at the same time. T1 is making sure that students know the mathematics register 'isosceles triangles' in both Farsi and English. I would like to start the analysis of the verbal discourse by T1's question addressing a particular student B1, {'what do we say the name of this triangle?'}

T1: *B1 chi migim ino? In mosalas ro chi migim?* {B1, what do we say the name of this triangle?}

B2: isosceles

B1: isosceles

T1: *Farsi chi migan?*{how do we say it in Farsi}

B1: *motasaavi-al-saghain* {isosceles}

T1: *aha, Farsi sho baladin bacheha?* {aha, do you know the Farsi for it guys?}

Bs: *are motasavi-ASDA* {yeah, isosc-ASDA} hehehe

T1: *saagh yanni chi?*{what does '*saagh*'[11] mean?}

B1: *paa* {leg}

B3: *saagh yanni paaiinesh* {*saagh* means the lower part of the leg}

B4: *chii bood dobare?*{What was it again?}

T1: *motasavi-alsaghain.* {Isosceles.} *Motasavi yanni chi?* {What does '*motasavi*' mean?} *Yanni* equal. {It means equal}. *Motasavi-al-saghain yanni do ta saghash chi-an, do ta paahash chi-an.* {Hence '*Motasavi-al-saghain*' means their legs are what?}

Bs: *mosavi.* {Is equal}

[11] '*saagh'* is the first component of the compound term '*saaghain*'. '*ain*' is a suffix which in this case creates a dual form. A literal translation means two '*saagh*' or two legs.

It is interesting to note that although T1 addressed B1 in particular to answer his question, but B2 first responded and said “isosceles”. This is an example of the British-Iranian classroom culture where there can be multiple discourses at the same time even when only one student is addressed to respond to her/his teacher’s question. In this specific cultural context, ‘interruptions’ or ‘voluntary participations’ were not seen as rude, but as a sign of enthusiastic involvement. T1 then reinforces the content and the concept in Farsi by asking “how do we say [isosceles triangle] in Farsi?” in order to extend students’ repertoire and ensuring that the equivalent Farsi mathematics register is known and recognised by all students. By doing so, T1 provided an opportunity that acknowledges the linguistic resources that bilingual learners have at their disposal by demonstrating their knowledge and understanding of mathematics in Farsi. It appears that T1 ensures that bilingualism is foregrounded and is at the centre of the teaching and learning that takes place as he develops awareness of the Farsi mathematics register.

What I find of particular interest is when T1 mediates learning as he moves to teach aspects of the etymology for the Farsi mathematics register ‘*motasavi-alsaghain*’. Just as the compound term ‘isosceles’ that has its roots in Greek meaning ‘equal legs’, ‘*motasavi-alsaghain*’ is also a compound that is constructed of two components. The first component ‘*motasavi*’ means equal and the second component ‘*saghain*’ meaning legs. It is interesting that the root for the concept isosceles in both Greek and Persian shares the same conceptual meaning, ‘equal legs’. T1 draws on learners’ linguistic repertoire by asking them “what does ‘*saagh*’ mean?” and then “What does ‘*motasavi*’ mean?”. By looking at each component of the compound, T1 is breaking down the task and making it simpler to understand the meaning of ‘*motasavi-alsaghain*’. Furthermore, towards the end of the transcript, T1 mode-switched and used a written mode for demonstrating ‘equal legs’. He first drew an isosceles triangle and pointed to its equal legs. By doing so he centred the concept of isosceles triangle at the heart of the attention. Then, T1 developed and transformed the isosceles triangle into a drawing of a human with a football (see Figure 39). T1 expanded this notion using multi-modality and then extended how it is culturally done by saying “That is like a person who has two equal length legs. This is how it is referred to in Iran” (see lines 64-68).

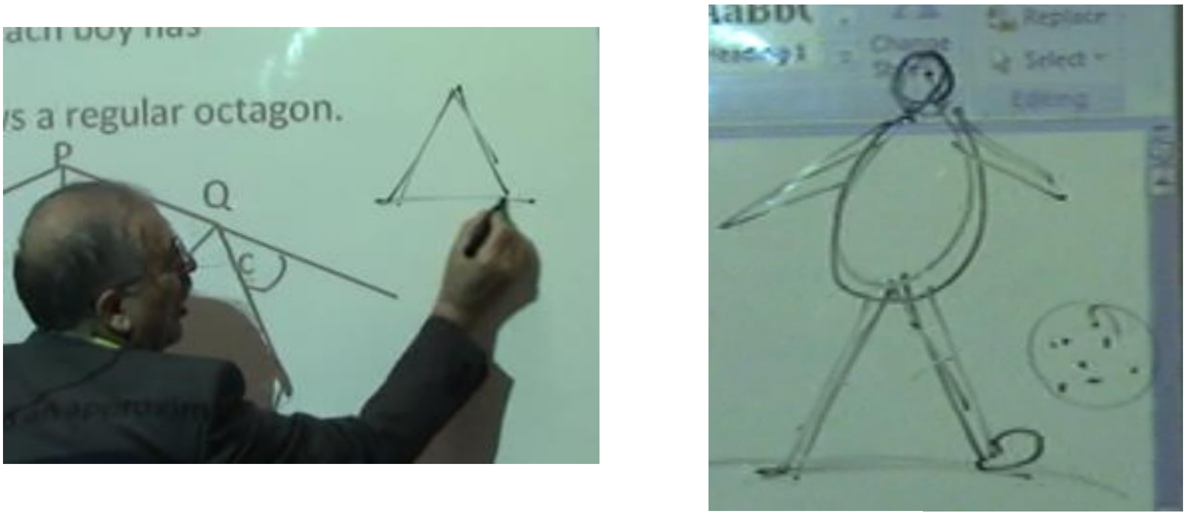


Figure 39: Where does *'motasavi-alsaghain'* come from?

Not only did T1 mode-switch and move across verbal and visual resources as he was conveying the instructional information, but also T1 reinforced the mathematical concept bilingually by providing culturally relevant examples that promoted bilingual learning. T1 employed a teaching strategy that acknowledged the linguistic resources that bilingual learners have at their disposal to co-construct meaning and mediate understanding. T1 reinforced concepts and content through bilingual pedagogy as a way to check understanding and consolidate learning. And where possible, he provided culturally relevant exemplification in supporting learning the content (having two equal legs) and language (teaching the Farsi mathematics register ‘*motasavi-alsaghain*’); in particular for those students who did not know what isosceles triangle was in Farsi and why it is called that. I find it fascinating where in lines 47-52 T1 makes a comparison between the nature of isosceles and ‘*motasavi-al-saghain*’. That is because “In English *motasavi-al-saghain* is referred to as isosceles, just like our two eyes. In Farsi it is referred to as equal shin”. As he uttered isosceles in his instructional talk, he synchronically pointed to his eyes “just like our two eyes”. Therefore T1’s bilingual instructional strategy not only emphasises on code-switching as a resourceful communicative pedagogy but also bridges between how the concept could be seen in the two national languages, in English (just like our two eyes) and in Farsi (just like our two shins/legs) which offers a greater linguistic and cultural dynamism. T1 has taken a bilingual approach in pedagogy that not only mediated the learning, but also enabled and *encouraged* learners to draw on all their resources for learning.

8.5 Discussion and conclusion

This chapter raised awareness to a different gesture type category, deictic gestures. Deictic gestures appeared to have different forms and functions to both iconic and metaphoric gestures. Deictic gestures, unlike iconic and metaphoric gestures, did not convey the semantic meaning of mathematical concepts but appeared to be in synchrony with speech and they shifted attention from verbal to visual. They were employed to index objects, locations, and inscriptions.

Index-finger pointing appeared to be the most common form of deictic gesture but there were different variations in the degree of finger closure among the little, ring and mid finger. The degree of finger closure appeared to be correlated with certain discourse factors. Often students’ pointing gestures carried a tightly bunched index finger pointing when they were indexing new ideas or with the first mention of a topic. However, if a topic and its importance was already established but students found the need to direct attention to the topic by pointing, their index finger pointing often accompanied a looser degree of closure. In other words, the tighter degree of finger closure served as an indication to how important/new the topic of conversation was in discourse.

9. Chapter nine – Discussion on cross-linguistic variation of gestures

9.1. Introduction to this chapter

In this chapter I will address and summarise two issues based on the nature of gestures that were incorporated in teachers' and students' talk. First in section 9.2, I will offer an alternative perspective where I will be discussing issues around the nature of gestures that were employed to carry a mathematical meaning in a bilingual context. Later in section 9.3, I will discuss how different languages (in this case Farsi and English) generate two different gestural representations of the same mathematical concept. Finally, in section 9.4, I will discuss and evaluate the forms and functions of gestures that conveyed mathematical meaning in a bilingual complementary school mathematics classroom.

9.2. Gestures that highlight the mathematical 'concept' versus 'terminology'

One of the purposes of this chapter was to examine the nature of spontaneous production of figurative language which was used as a tool in communicating about mathematical ideas and problem solving. Moreover, my interest lies not only on the pedagogic nature of communication by the classroom teachers as they move between their languages and semiotic resources, but I am also interested to look at what are other possible factors that shaped T1/T2's gestural representations for mathematical concepts. In other words how is a particular gesture generated (and used) to convey mathematical information. More attention needs to be paid to the ways in which gestures are generated and enacted in mathematics classrooms as they convey pertinent mathematical information. For example, in what manner do these mathematics teachers use various forms of visible representations and what prompts their speech-accompanying gesticulation. Having encountered McNeill's (1992) basic gesture type category, in this section I will offer a different perspective to how gestures were enacted in this bilingual complementary school mathematics classroom.

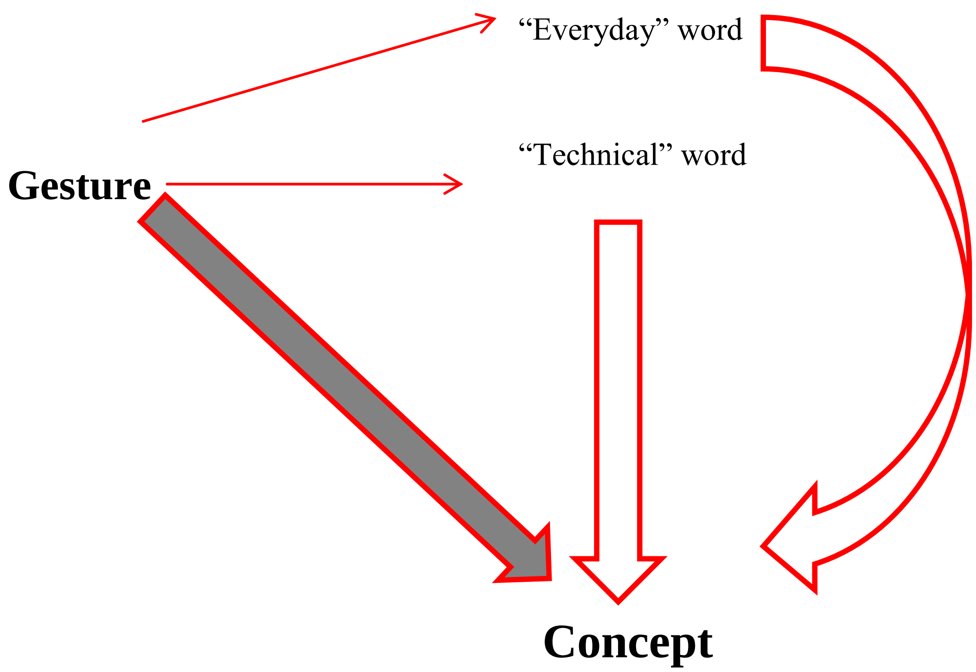


Figure 40: Gestures conveying mathematical 'concept' versus terminology

The gestures conveying mathematical content that were produced by T1 and T2 appeared to emphasis either the 'concept' directly or the 'terminology' (which can be either technical or vernacular). Certain type of gestures which I will refer to as 'conceptual-based gestures' have directly conveyed some aspects of mathematical concepts such as the gestural representation for 'addition', 'subtraction', 'minimum', 'parallel' and 'perpendicular'. These gestures reveal some aspects of the semantic content directly via shape or motion trajectory. Conceptual-based gestures appeared to be crucial to the effectiveness of mathematical communication as not only do they convey the semantic meaning of some specific mathematics register as they visually support the concept but also they can promote a better understanding of some mathematical concepts. Conceptual-based gestures may fall under a combination of McNeill's 'iconic' and 'metaphoric' gesture type categories.

Another type of gesture appeared to concern primarily the terminology and not the mathematical concept such as 'eyesoscelese'. These types of gestures, which I shall call the 'linguistic-based gestures', are influenced by the phonic of the speech but are not semantically affiliated with the on-going flow of utterance. Linguistic-based gestures, due to their lexical and syntax structure, do not reveal or support the semantic content of some mathematical concept. Examples of linguistic-based gestures are the gestural representation for the mathematical concepts 'eyesosceles' and '*tavan*'. Linguistic-based gestures may convey aspects of the prosody (or the prosodic prominence patterns) of the message which helps memorise the terminology. For example, the gestural representation for isosceles stressed and emphasised the technical terminology. However gestural representation for '*tavan*' appeared to be influenced by its vernacular syntax and socio-linguistic counterpart where it emphasised the 'everyday' terminology.

Both conceptual-based and linguistic-based gestures are umbrella terms to reinforce the nature (both forms and functions) of gestures that are enacted in a bilingual complementary school mathematical context.

9.3. Cross-linguistic variation of the mathematical concepts 'power' and 'isosceles triangles' reflected through gestures

In this section, I will primarily look at the functional role that language and gesture co-express to convey mathematical meaning in a bilingual context. I am interested to look at the ways in which gestural representation for/of a specific mathematical concept differs when different languages are employed. In the previous chapter, I have shown how different languages activate a different understanding of a particular mathematical concept. For example, 'power' versus '*tavan*' and 'isosceles' versus '*motasavi-al-saghain*' which offered an alternative interpretation that there was (or was not any) mathematical meaning or understanding. Now I will expand on the cross-linguistic variation of mathematical gestures.

Research outside of mathematics education has suggested that variation in gestures can be due to linguistic diversity across cultures. For example Kita (2009:156) has observed that "words and constructions that are available or commonly used in a given language shape the way the speaker organises information for speaking". Moreover, gestural representation can vary between speakers of different languages simply because languages have different lexical and syntactic resources. He also added that different languages "have different lexical and syntactic resources to express spatial information. This linguistic difference is reflected in how gestures express spatial information" (Kita, 2009:145). It is therefore not surprising that due to the linguistic diversity across cultures, even when describing the same event, gestural representation can vary cross-culturally because language varies cross-culturally.

Although gesturing is universal (due to the fact that to date there is no report of a culture that lacks gestures) (Kita, 2003) the way gestures are produced can vary across cultures and their realisation is also influenced by the social and cultural factors (Núñez & Sweetser, 2006). Although there are a number of reports supporting the embodied nature of mathematical thinking as speakers are engaged in speaking mathematically, to date there has been no evidence which suggests how different languages may generate two different gestural representation of the same mathematical concept. This is another aspect of my research which is original. Now by drawing on my multi-modal transcript, I will provide an example of the cross-linguistic variation of the mathematical concept 'exponential' reflected through gestures.

9.3.1. The gestural representation for the mathematical concept 'exponential': a variation of gesture across English and Farsi

In this section I will present a part of the interview which was conducted with T2 where the focus is on T2's gestural representation of the concept of exponential in both English and Farsi within the same sentence or the same communicative event. I have already presented the first part of this transcript in section 7.4.2, where the focus of analysis was on metaphoric gestures. However, since the focus of this section is on the cross-linguistic variation of gestures, therefore I will present the full transcript to show T2's enactment of his gestural representations for the mathematical concept power and '*tavan*'. In this particular section, I aim to address primarily one issue: to show the ways in which the gestural representation for/of a specific mathematical concept differs when different languages are employed in expressing it. In this case, I will show when the same speaker (T2) talks about the mathematical concept of 'exponential' in English he created a deictic gesture, but when he expresses the same idea in Farsi he produced a metaphoric gesture.

This specific part of interview transcript which I am about to present consists of a number of open ended questions that I asked about T2's opinion on his discursive practices as he conveys the instructional information. For example, 'do you prefer to teach students in Farsi or in English and why?' Although T2's opinion about which medium of instruction he preferred to use and 'why' was of interest in my research, I was more interested about his gestural representation for '*tavan*' and power. T2's gestural representation for '*tavan*' and power eventually appeared together in one sentence as T2 was justifying his rationale for which language he prefers to use in teaching and why. I will now present my multi-modal transcripts followed by my analysis on the ways in which different languages activate a different understanding for the mathematical concept '*tavan*' versus power.

Transcript number 13

D: *be onvane yek moalem, agar shoma bekhain, age masalan farad mikhai beri, .* {as a teacher, if tomorrow you are going to, . } *hala man gir dadam be chize tavan,* {haha, I am keep on about the notion of power} *age shoma farad bekhay beri madrese va tavan dars bedi be bacheha, va ghablesh emshab mikhain ye moroori rooye iin chizhayi ke mikhain dars bedin bedin,* {If tomorrow you are going to teach the concept of powers and tonight you are preparing for it} *aya* concept-*e tavan ro, farz kon yek soal hast* *dar morede tavan, mikhain farad baraye avalin dafe cheshetoon besh va nashe, ke masalan ghablesh yek*

practice-*e dashte bashin,*

ino be farsi hal mikonin ya be ingilisi hal mikoni? {would you do the preparation and the

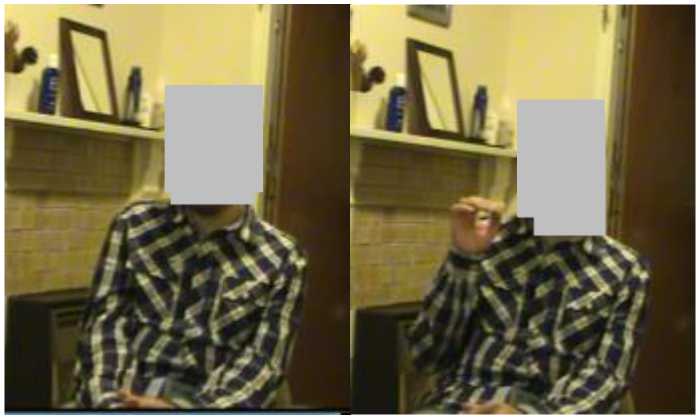
teaching of the concept

power in English or in Farsi?}

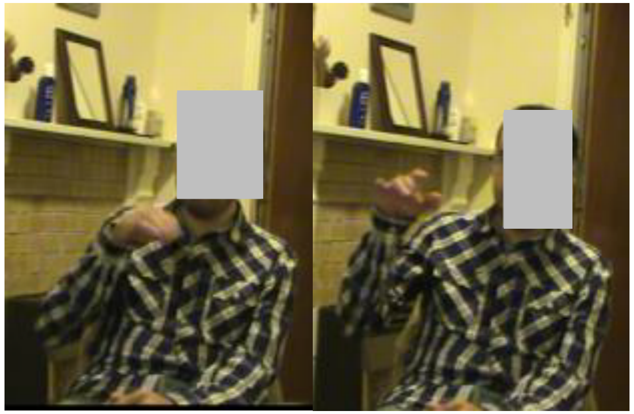
T2: *na, man e e* power *ro hamishe be ingilisi yad*

midam {no, I erm always

teach power in English}



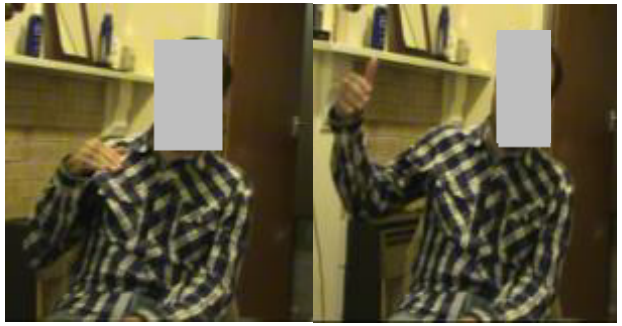
[33: T1 raises a sudden right hand as he code-borrowed and said ‘power’] *masalan* two *be* {to the} power of three, *masalan* {for example} fifteen be {to the} power of something,



[In lines 37-8 T2 creates a hand gesture as he verbalises “two *be* {to the} power of three”. The direction of his one stoke hand gesture appears to be diagonal and it moves up-right. His hand gesture initially begins from his chest and then starts to move towards outside of his shoulder.]

D: uhum

T2: *man hamishe* power-*ro be ingilisi kar mikonam chon ke maa e e . . vaghti be bacheha yad midi bayad be ingilisi yad bedi va masalan vaghti migi* {I personally work with a concept of power in English for example something to the} power of something *in ro be ingilisi yad bedi* {I teach that in English}. *maa tavan-ro hamoon jori ke, er . iinja madrese va* university *ke raftim, khodam didam ke bacheha chetori kar mikonan ba ham, khub nist ke beheshon be farsi yad bedi yani*, {I teach for example power through the same language that they teach/learn here at schools or at universities and it is not good to give instructions in Farsi, I mean} man to kelassi ke dars midadam, yeki az, .. {in the classes that I teach here, ..} ta mogheike bacheha moshkel nadashtan talash mikardam ingilisi beheshoon dars bedam



[In line 51, T1 creates another gesticulation in synchrony to his verbal message as he uttered ‘power of something’ in English] bekhatere iinke {as long as they understand the point and do not have a problem with, I try to use English as the medium of instruction because} at the end of the day they go to a a English school and they learn everything in English. So *e e vaghti ke moshkel, vaghti ke nemifahman motovaje nemishe, iin etefagh kei miofte, vaghti ke bacheha taze oomadan* {but if there is a problem and the students do not understand what is going on due to the fact that they are new arrivals or} when they have just came here or if they have been here for one or

two years, *bachehayi ke*
nemitoonan befahman ke masalan chera power *mishe tavan* {it is only then when those students can't see the
relation between for
example 'power' and '*tavan'*}

D: *doroste* {Ok}

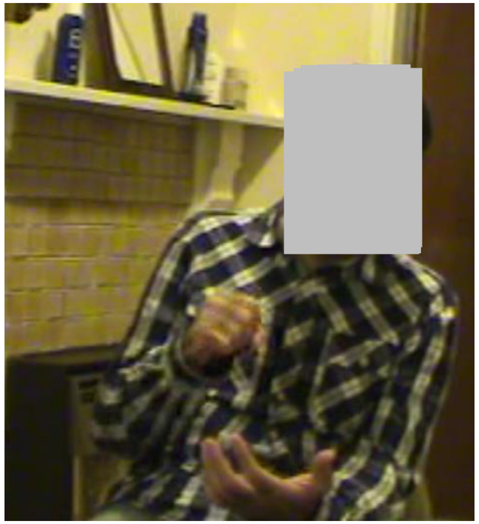
T2: *vaghti inro motavaje*
nemishan, man be farsi inro
beheshoon migam {when they can't comprehend this, then I say it in Farsi}

D: *doroste,* {ok}

T2: *tozih midam* {I'll explain it}

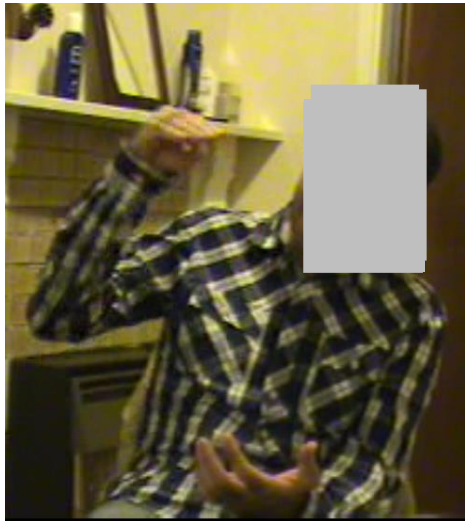
D: *doroste* {ok}

T2: *bad beheshoon migam ke manzooremoon az tavan chi eke* {Then I will tell them what do I mean by '*tavan'*}



[T2 makes a tight and clenched fist with his right hand as he verbalised the term '*tavan'* in Farsi]

for example two *be* {to the} power of something



[T2 raises his right hand relatively above his shoulder level as he synchronically verbalises "power of something" in English] *masalan.ke maa masalan inro mizarim balaye*

125 *shomare va chejoori*

estefadeh bekonim {And we put this on top of a number and the ways in which we use it}

From the above transcripts, I will present evidence and analyse T2's body language with the mathematics register that accompanies it. I will show some of the gestural representations that triggered as a result of accompanying a term or a phrase that was code-borrowed or code-switched in discourse. First I will focus on one case when a term (power) was code-borrowed in T2's utterance and then later, when T2 makes an intra-sentential switch from Farsi to English and creates a gestural representation for both '*tavan*' and power in his utterance.

One of the reasons why I have chosen this transcript is because it consists of a number of body-based resources that T2 uses as he conveys the instructional information. For example, in lines 33, 37-8 (Figure 41) and 51 (Figure 42) T2 creates a representational gesture which accompanies the mathematical register 'power' as it was uttered in English. The direction of T2's hand gesture appears to be diagonal and it moves up to his right but the placement origin of his gesticulation differs. For example in line 33, T2 utters:

{no, I erm always teach} power {in English}

And as he code-borrowed the term power in English, T2 synchronically creates a hand gesture which initially begins from the rest (hands on his lap) and then moves towards up-right direction.

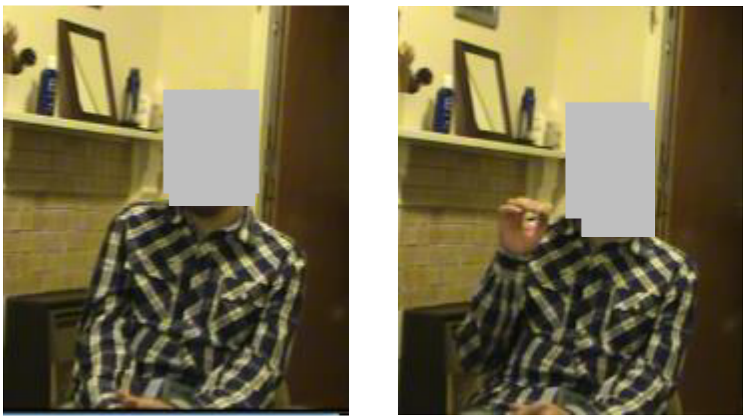


Figure 41: Gestural representation for power produced by T2 (II)

Also in line 51, T2's hand gesture path initially starts from his chest level and then starts to move upward and sharply away from his shoulder (to his right hand side).

Figure 42: Gestural representation for power produced by T2 (III)

The second reason why I have chosen this transcript is due to the fact that T2 expresses a different gestural representation for '*tavan*' and 'power' within the same sentence (lines 117-122). But first to set the context, I would like to draw attention to its predecessor lines. In lines 95-113 T2 is engaged in explaining his rationale for when he believes it is best to teach students through the medium of Farsi. He believes that there are a number of students who are new arrivals to the UK and may not be communicative competent in English. Therefore those students cannot distinguish or realise the similarity between '*tavan*' and power 'therefore I will explain to them in Farsi' (see line 113). T2 then moves on to provide an example to show how being a bilingual teacher can draw across his linguistic resources in a flexible manner to help his students to 'unpack' and 'repack' the English mathematics register by switching between his languages. This may be seen as a resource, affording the learner deeper understanding and remembering the mathematics register in that particular language.

> {Then I will tell them what do I mean by '*tavan'*} for example two {to the} power of something (lines 117-122)

T2 created an intensive clenched fist gesture as he was conveying the idea of '*tavan*' in Farsi followed by an immediate intra-sentential code-switching into English saying 'power' (see Figure 43). As T2 moved in between his linguistic resources, different languages activated a different gestural representation. The gestural representations that I am focusing on are emerging from the lines 117-122. T2 gesticulated a right hand tight and clenched fist with a tight and intense degree of closure as he said '*tavan*' in his utterance. On the other hand, T2 gesticulated a 'higher up' position with his hand as he was expressing the corresponding verbal message 'power' in English.

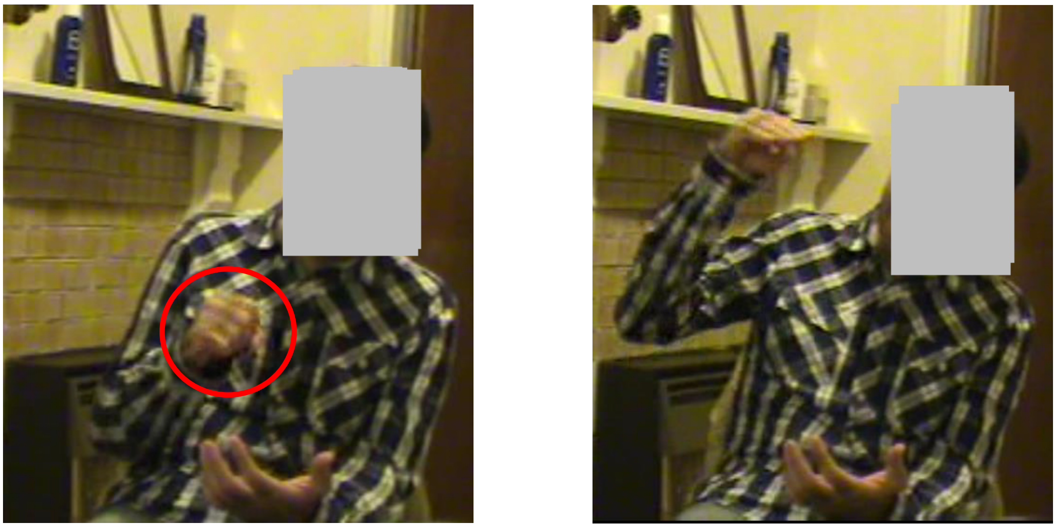


Figure 43: The gestural representation for '*tavan*' and power

Very similar to B1's gestural representation for the concept power (see Figure 23 on page 182), T2's gestural representation for power also suggested a 'higher up' position as he displayed a hand gesture from his chest moving up towards his shoulder level. This is possibly because both B1's and T2's gesture served to index an imaginary high/up location and one of the plausible interpretations within the context could denote the 'notation' or the mathematical position of the concept power. Therefore the accompanying gesture for 'power' performed by both B1 and T2 appeared to constitute McNeill's (1992) deictic gesture type as it points to and indexes the superscript position of power (the notation). Therefore, the evidence from both B1 and T2's gestural representation lead me to believe that it could be possible that the position of 'power' that is placed at the top right hand corner activated both B1's and T2's gestural representation for the mathematical concept 'exponential'.

Although both T2 and B1 express the mathematical concept through the verbal and formal means of mathematics register, their gestures show the dynamism involved in the mathematical ideas. The gestural representation for the mathematical concept 'exponential' appeared to have different forms and functions in the two languages. The gestural representation of the mathematical concept 'exponential' varied depending on which language was used because the lexical and syntactic resources of languages vary. Different languages (in this case Farsi and English) activated different conceptual understanding of the mathematical concept 'exponential' which is reflected through the student's and the teacher's gestures in a number of different events. The gestural representation for power revealed the spatial positioning of the mathematical notation (which is a social convention) but the gestural representation for '*tavan*' offered an alternative interpretation that there was not any mathematical meaning or understanding. The gesture for '*tavan*' revealed the vernacular power and strength that was used within the context of mathematics. It is interesting to note that under McNeill's (1992) basic gesture type category; the gesture classification of the same mathematical concept varies depending on which language is used. The gesture type classification for 'power' is deictic as it indexes the position of the notation whereas for '*tavan*' is metaphoric, as it resembles an abstract idea for the concept strength.

9.3.2. Discussion

In this section I will discuss and summarise the results on the cross-linguistic variation of gestures on a particular mathematical concept. By revisiting already established transcripts, I will first represent different gestural representations of the same mathematical concept that were produced by the same speaker. First I would like to draw attention on the gestural representation for ‘*motasavi-al-saghain*’ versus ‘‘isosceles’ by T1 (see Figure 44) and then I will look at the gestural representation for ‘*tavan*’ versus ‘power’ performed by B1 and T2 (Figure 45).

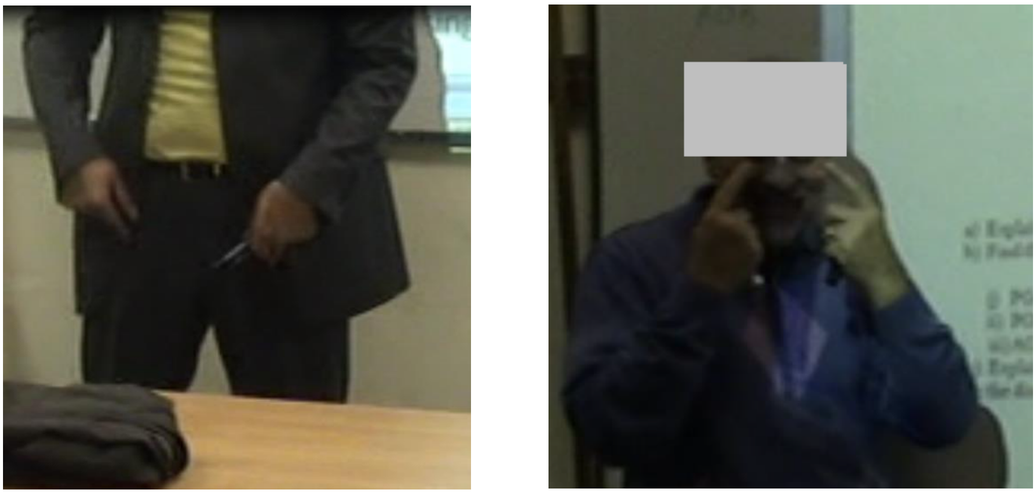


Figure 44: Cross-linguistic variation of the mathematical concept ‘isosceles triangles’ reflected through T1’s gestures

Figure 44 reveals how different languages conveyed different instructional information suggested through T1’s gestures. Although the gestural representation for isosceles is linguistic-based, ‘*motasavi-al-saghain*’ {equal-legs} conveys the semantic meaning of the verbal speech in gesture whereas isosceles conveys aspects of the phonology (‘eyesosceles’). It appears that different languages revealed different aspects of the mathematical concept ‘*motasavi-al-saghain*’ versus ‘isosceles’ in T1’s instructional talk. Moreover, the gestural representations for the mathematical concept 'isosceles triangles' was made due to the linguistic variation among bilingual speakers of Farsi and English.

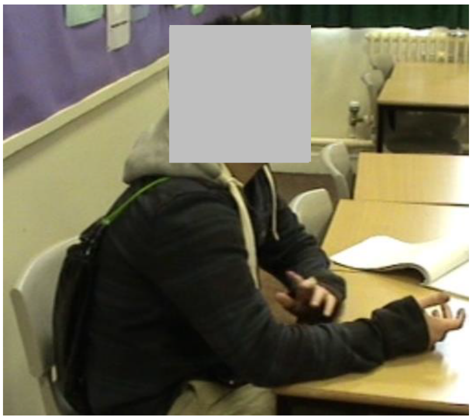
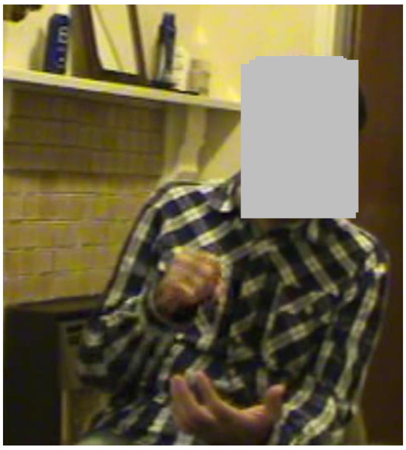
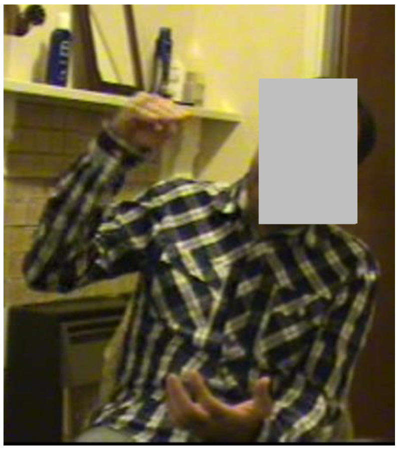
	Tavan	Power
B1		
T2		

Figure 45: Cross-linguistic variation of the mathematical concept 'exponential' reflected through B1's and T2's gestures

In Figure 45, the gestural representation for '*tavan*' reflects the notion of strength whereas the same concept described in English signals the position of the mathematical notation. From the examples I have offered in this chapter, it appears that the gestural representation of the mathematical concept 'exponential' varies between speakers of different languages because the lexical and syntactic resources of languages vary. For example, it appears that the vernacular concept for ‘*tavan*’ activated a vernacular understanding for the mathematical concept ‘exponential’. B1’s gesture suggests a failure to comprehend the term ‘*tavan*’ as being used in a mathematical sense whose meaning involves the notion of exponential growth and sometimes exponential decrease if it is a negative power.

One of the last remarks I would like to make in this section is to stress the particular ‘standpoint’ (Beattie, 2004) gesticulators take towards describing the same event in gestures in different languages. For example, in the analysis of cross-linguistic variation of ‘power’ versus ‘*tavan*’ reflected through gestures, what I also find of particular interest is not only how the variation of gestures could reveal the gesticulator’s mental image about the event they are describing in different languages, but how the gestural representation reveal the particular point of view that B1 or T1 has taken towards it. Both B1 and T2 had the choice of depicting the event from the point of view of the agent (the observer) or the mathematical concept ‘exponential’ (character) itself. In performing the gesture for ‘power’ it is clear that both B1 and T2 pointed to the top right hand side, indexing the notation (or the position of ‘power’ which is placed at the top right hand corner of the base, three to the power of five3^5).

It is clear that they were both *seeing* the event from the viewpoint of the agent because otherwise the manner of their pointing gesture would not have taken the form of diagonal top right hand corner. Interestingly enough, the image that was depicted through the gestural representation of ‘*tavan*’ was from the point of view of the concept itself. Although the concept revealed the vernacular aspect, hence no mathematical meaning was attached, but it illustrates how B1 and T2 were taking the viewpoint of the vernacular power and their gesticulation served to depict aspects of potency and strength. Therefore not only can two different languages generate two different gestural representations of the same mathematical concept, but two different points of view in depicting the same event; that is the 'agent' versus the 'concept' itself.

9.4. Conclusion

As stated earlier, I have built upon an already existing framework (McNeill, 1992) to recognise different forms and functions of gestures that carry mathematical meanings in a British-Iranian complementary school. By incorporating McNeill's (1992) gesture type category as a framework, it appears that beat gestures were not employed by the interlocutors during their mathematical talk. Based on forms and functions of those gestures that were produced by teachers and students, I offered two different gesture type categories to encapsulate gestures that conveyed mathematical meaning. Those were conceptual-based gestures (gestures that conveyed aspects of the concept) and linguistic-based gestures (gestures that reveal aspects of the terminology e.g. phonetic).

In addition, while I offered examples supporting this symbiotic and the embodied nature of mathematical thinking as speakers are engaged in speaking mathematically, I have also provided evidence which suggests how different languages (due to their apparent semantic and phonological properties) may generate two different gestural representations of the same mathematical concept (as the case for '*tavan*' and 'power'; 'isosceles' and '*motasavi-al-saghain*'). The gestural representation of the mathematical concept 'exponential' and 'isosceles triangles' varied between speakers of different languages because the lexical and syntactic resources of languages varied. The results show that different languages reveal aspects of everyday understanding of the mathematical concept 'exponential' and 'isosceles triangles' which are reflected through the students' and the teachers' gestures. As I stated earlier, I consider gestures to be important aspect of communication not only as they are communicational actions but because they show an avenue to the teachers' (and students') mathematics knowing when they are engaged in explanation.

10. Chapter ten - Conclusion

This chapter retraces the origin of my study and summarises my findings. I will also discuss issues to do with limitations of this study and further research that might be relevant as a consequence of my findings and reflection on this thesis.

10.1 Retracing the research design and research question

In this section I would like to highlight and place an emphasis again on the shift that has occurred in my research design and my research questions. My initial research interest was to scrutinise and describe the relation between learning mathematics in a multi-lingual context (complementary school) versus monolingual contexts (mainstream schools), and in terms of a research questions, my interest fell in the following area:

1) What prompts code-switching in a bilingual (English-Farsi) mathematics classroom in the UK? And
2) Does a complementary school 'complement' the work that goes on in mainstream school? And if so, how?

The answer to these questions required comparing and contrasting the teaching and learning that took place in a multilingual context followed by how mathematics is taught in a monolingual setting. As I have stated in the methodology chapter, due to the fact that I did not gain access to every research participants' mainstream school for the purpose of follow up, I was left with such a small and insufficient amount of data (from the mainstream schools) to compare and contrast between the two institutional educations. I decided to change my original focus of the study which resulted in different research question. I set out to address the following research question:

1. What is the pedagogic nature of gesture (forms and functions) and other body-based resources in a British-Iranian complementary mathematics classroom in the UK?

In the next section, I will address my finding to my research question.

10.2 Summarise of the contribution

In this section, by summarising the main points of analysis, I aim to show how gestures were used as a resource for teaching mathematics in a bilingual setting. Furtheremore, I will highlight how mathematical knowledge was expressed flexibly through a combination use of verbal and nonverbal resources for meaning making such as code-switching and bodily communication. I will conclude this chapter by rasing awareness to how gestures offered a different perspective on how certain mathematical properties can be seen differently across languages.

10.2.1 Mathematical knowledge expressed flexibly in languages and gestures

Aspects of this PhD study investigated the learners' linguistic practices during a bilingual mathematics classroom in an Iranian complementary school in the UK. Most often bilingual learners and the classroom teachers were engaged in participating through flexible bilingual practices (code-switching). Teachers provided opportunities by means of bilingual instruction to encourage bilingual learners to draw upon their linguistic repertoire. One of the most recognisable patterns of interaction was of code-switching. Code-switching in this particular complementary school bilingual mathematics classroom was welcomed; it was an integral part of the bilingual interaction between teacher-students and students-students. When learners had difficulties expressing and conveying their mathematical knowledge in one language, they often code-switched to their most dominant language, or even gestured to manage the problem of 'lexical access and retrieval' (Polinsky, 2008). It is worth noting that such discursive practices and switches within a classroom show the dynamicity and fluidity of a bilingual mathematics classroom. Learners' and teacher's code-switching served as a powerful mechanism to construct meaning and mediate understanding. Teachers constantly code-switched to include a wider range of audience; hence the instructional information that was given would have been recognised by a larger number of students. Furthermore the multilingual orientation of this complementary school mathematics classroom developed a bilingual pedagogy whereby multilingual mathematics teachers provided an opportunity to develop different pedagogic possibilities for British-Iranian learners. A bilingual pedagogy that provided a space for bilingual learners to incorporate not only their languages (as they were encouraged to draw on their linguistic repertoires flexibly) but incorporated aspects of histories and experiences into formal schooling, whereby classroom teachers reinforced the content by providing culturally relevant exemplifications in both languages.

It is interesting to note that moving across and between languages was not the only resource that was used to convey mathematical information. Gestures and other spontaneous production of figurative language were an integrated part of communication. Bilingual teachers very often conveyed aspect of their instructional information through a flexible use of verbal and nonverbal resources. I will now turn to focus on summarising the the pedagogic nature of gestures (forms and functions) and other body-based resources which conveyed aspects of mathematical meaning in a British-Iranian complementary mathematics classroom in the UK.

In the classrooms I observed, other non-verbal resources played a major role in conveying the instructional information. First I would like to recap about their role and their communicative function that provide an adjunct to the spoken discourse and then to show how the gestural forms and their functions differed depending on which language they were produced in. Later I would like to highlight the cross linguistic variation of the mathematical concepts ‘power’ and ‘isosceles triangles’ reflected through gestures.

I primarily focused on the gestures that conveyed mathematical meanings. I came to the realisation that there were two types of gestures that had different forms and functions in conveying aspects of mathematical concepts. Gestures whose particular form displayed a close relationship to the meaning of the accompanying speech were called ‘conceptual-based gestures’. A conceptual-based gesture would reveal the semantic content of a word, a sentence or an idea. For example within a mathematical context, a conceptual-based gesture would imply when someone uses gestures to simulate actions that present a conceptual understanding of a mathematical concept e.g. parallel or perpendicular (see Figure 10 and Figure 14 on pages 129 and 137 respectively). Conceptual-based gestures could also play a role to emphasise/convey the verbal content which can help in teaching the concept by providing a visual counterpart. Most often conceptual-based gestures and their accompanying verbal resources appeared to be in a symbiotic relation, but it was also possible for a concept to be presented through conceptual-gestures only.

I have called the second type of gestures ‘linguistic-based gestures’. These were actions that are influenced by the phonic of the speech-accompanying gesture and are not semantically affiliated with the on-going flow of utterance. Therefore this type of gesture may convey aspects of the prosody of the message but not the concept. An example of linguistic-based gesture is the gestural representation for the mathematical concept ‘isosceles’ (e.g. see Figure 33: The gestural representation for isosceles triangle performed by T2 on page 212).

It is worth considering that both types of gestures facilitated teachers’ language production as they were conveying aspects of the instructional information. Gestures either conveyed aspects of the meaning through the images created or emphasised the terminology. For example consider the gestural representation for the mathematical concept ‘addition’ and how T1’s gesticulation complements his instructional information conveyed in the speech.

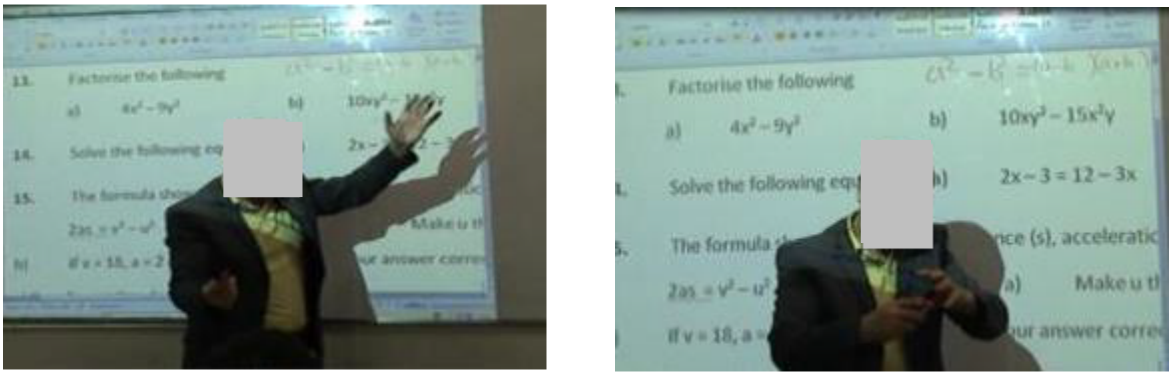


Figure 46: ‘jam’ {addition}

Both the speech and gesture refer to the same communicative event but they both present somewhat different aspects of it. The speech indicates the *concept* whereas the gesture signifies its *process*. Although speech and gesture encode meaning differently and present it in different ways, the two modalities form a single integrated unit; even the movement timing appears to be coordinated between the two modalities. In the end the meaning that emerges from the two modalities is clearer that either modality on its own.

Finally, I would like to re-emphasis the ways in which gestures were produced varied across languages and their realisation was also influenced by the linguistic, social and cultural factors. I have shown that the gestural representation of the mathematical concepts ‘power’ and ‘isosceles triangles’ varied between Farsi and English speakers because of the different lexical and syntactic resources of the languages used.

As a final point, different languages activated different conceptual understanding of the mathematical concept ‘power’ and ‘isosceles triangles’ which was reflected through the students’ and the teachers’ gestures.

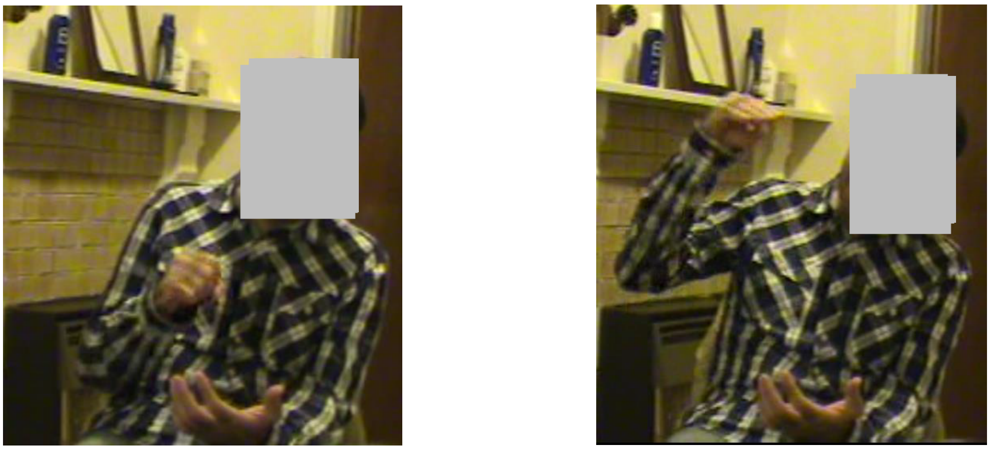


Figure 47: The gestural representation for ‘tavan’ and power

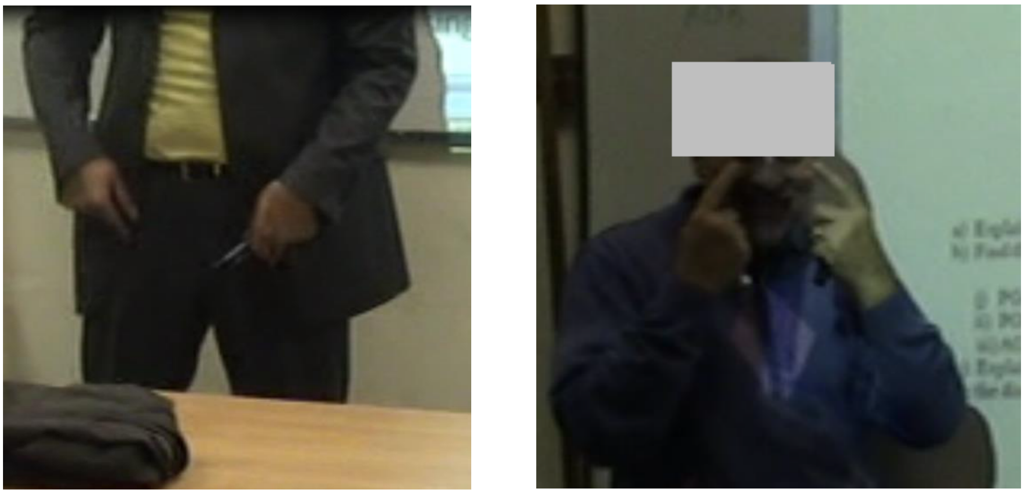


Figure 48: Cross-linguistic variation of the mathematical concept ‘isosceles triangles’ reflected through T1’s gestures

What I would also like to re-iterate was my methodological approach in understanding bilingual mathematics lesson. By developing a multi-modal approach to transcribing data I wanted to show a range of embodied and multi-semiotic resources that were presented within a bilingual mathematics classroom. In other words, not only I encounterd verbal, vocal and visual elements of communication, but showed how these modalities are in support of one another.

10.3 Limitation

Some of the limitations of this study were made due to certain methodological factors. They were of data collection and data presentation (transcription). I shall refer to each of these points in the subsequent paragraphs.

Throughout the data collection journey, I encountered two concerns: the first was to do with the data that was obtained from a single lens of a recording camera and second point is to do with nature of the classroom culture and the overlapping talk between the students. In practice, video recordings were generated through a single lens of a camera which did not capture the whole classroom interaction. The data that is obtained from a single camera has a single focus of attention whereas people are capable of attending to multiple aspects of a complex setting (Pimm, 1993). Therefore my recording method in the classroom did not pick up all the verbal and nonverbal communication: some missed data may have been potentially relevant in shaping my analysis. The second point to do with data collection was the fact that often interlocutors did not take turns smoothly in conversations. This made the actual realisation of the classroom discourse difficult to discern when multiple discourses were taking place at the same time.

One of the other major limitations of this study was to do with the transcription of the data. I was unsure as to how to incorporate: constant code-switching between two national languages (Farsi and English), prosodic and paralinguistic elements of speech, visual modes of communication (e.g. gestures), and also mathematical symbolism, algebraic notations and diagrammatic representations in a transcription. However, I partially managed to overcome this problem by incorporating multi-modal transcription. There still existed a big concern in transcription where I wanted to incorporate fluid and dynamic events that occurred in a three dimensional space onto a two dimensional fixed and static plane using pen and paper.

10.4 Policy, practice and further research design

Gestures have had a long curious relationship with meaning making. Whenever there is communication, there is gesticulation. The communicative notion of gestures have been long established and utilized in many different social contexts. For example, during the 1970s and 1980s, gestures were used in courts to detect deception. Gestures were later studied in courtship and dating, where they were first used as a visible indicator of liking or attraction. Only in the last decade or so have researchers focused on the communicative nature/function of gestures in educational contexts. For example, within a bilingual mathematics classroom, teachers' gestures together with its accompanying speech give an index to the richness of the instruction beyond words. Furthermore, research has shown that often information that is expressed in gesture cannot be found in speech. Research across multiple disciplines from linguistic to psychology to neuroscience, supports that gesture not only is an integral part of communication but have a special symbiotic and semantic connection to the words they accompany. It is important to note that the definition of gestures go beyond movements of the hands, posture and torso. Further attention needs to be paid to the micro-expression that interlocutors produce while conveying mathematical meanings. For example, when a teacher expresses a small number in words, s/he often squint his/her eyes whereas an eyebrow rise would accompany a big quantity. Micro-expression gestures, due to their small size, are not easily discerned and described. My interest lies in raising awareness for understanding the inter-relation of resources of representation and forms of knowledge. Therefore, there is a need not only to incorporate appropriate and precise visual methods to capture micro-expressions but also to be able to represent the harmony between verbal, vocal and visual forms of representations that occur spontaneously in a three dimensional space onto a two dimensional fixed and static transcription.

10.4.1 Recommendations and implications in education

This thesis primarily focused on gestures as a resource for teaching mathematics in a bilingual setting. In a sense I primarily focused on the pedagogic notion of gestures, that is, gestures that conveyed mathematical meaning in this bilingual institutional education. What I would like to address is a twofold:

1. The pedagogic nature of gestures is not merely limited to bilingual settings but also transferable to other institutional education, for example the mainstream education.
2. The role of gesture that convey other meaning than just mathematical ones.

As I have acknowledged previously, this thesis does not minimize the effectiveness of other gestures that were made during communication. Certain gestures were harder to *measure* while others were easier to discern and describe. For example, the notion of proximity, of how people regulate themselves in space. Girls maintained a closer proximity with a more turning angle to one another as they were discussing ideas/tasks. In contrast, boys maintained a further personal distance, less turning angle and less eye contact with one another. Other gestures were used for behavior management, disciplinary remarks and crowd control. For example, in a noisy classroom with several students talking off task, a teacher could retain complete control of the lesson without saying a single word. A teacher could bring his both hands up to the chest level (palm up) and hold for two seconds. This ‘unusual’ gesture very often resulted in students looking up to the teacher’s hands and stop talking. This subtle gesture was used and worked as a powerful nonverbal expression instead of saying “Oi, eyes front and stop talking!”.

One of the recommendations and implications for policy and practice in education is by incorporating *gestures studies* in teacher education courses in order to raise awareness to the communicative function of gestures. That is, not only to consider the pedagogic effects of gestures in teaching and learning but also how it can be used for disciplinary remarks.

10.5 Suggestions for future research

In the next three paragraphs, I would like to address three different but related suggestions for further research that I think would be a way forward after this thesis.

One of the main key concepts in this thesis was the pedagogic nature of gestures in communication in a particular British-Iranian complementary school bilingual mathematics classroom. I have shown how gestures play a role in communication; in particular in facilitating the speakers' language production. What I did not demonstrate was how learners extracted the information contained in gestures and in the speech channel. Therefore what I illustrated in this thesis was the possible communicational function of gestures rather than possible pedagogic functions of gestures. It would be interesting to demonstrate not only how students gleaned the information emerging from gestures in teacher's talk but to what extent gestures assisted in promoting learners' comprehension in a bilingual mathematics context.

Based on the findings of this study, I have offered a new perspective on the *cross-linguistic* variation of gestures of particular mathematical concepts and I have illustrated how the gestural representation for 'power' and 'isosceles triangle' conveyed different meanings. What I also found of particular interest was the *cross-cultural* variation of gestures of a particular mathematical concept. That is, two different cultures speaking the same language! For example, by exemplifying the gestural representation of a particular mathematical concept that gesticulators produce in two monoglossic societies: Anglophones in the UK and in the United States. Similarly a study that would examine the gestural representation of a particular mathematical concept between Farsi speakers in Iran and in Tajikistan.

One of the other foci of my research was on the development of linguistic and cultural awareness, and how they played a role in the effectiveness of interactional communication. By considering gestures as social and cultural constructs that played a role in interaction, it would be interesting to develop a nonverbal library of mathematics registers. A nonverbal dictionary of mathematics register that not only helped us to understand how mathematical concepts were depicted in gesture but also enacted.

Appendices

Appendix A: Students' profiles

B1

B1 and his family came to the UK in 2004 (when he was 10 years of age). B1 and his family went back to Iran in 2009 and stayed there for one year when he was 15 years of age. He went to an Iranian medium school for that period of a year. B1 found it difficult to adapt to the Iranian culture while he was in Iran. He felt that people and relatives around him asked him questions that were too personal or inappropriate for example, 'how much money do you get every month?'

In 2010 B1 and his father came back to the UK and started year 11 in his mainstream school. He is currently in year 11 and has moved up sets from set 4 to set 1 in mathematics.

Once B1 made a comparison in education he had experienced in the UK and in Iran. B1 started off by saying "Here in the UK I only go to school 5 days a week whereas in Iran its 6 days a week and we do not have any bank holidays or 13 weeks holiday or anything like that" (fieldnotes at complementary school, 09/01/2011). He also added that "schools in Iran are longer than in here [England] and break times are not as long". B1 continued by saying, "the kind of things [mathematics] that I do here [England] in year 11 are the sort of things that a 12 year old in Iran is expected to do ... In mathematics I think I am 3 or 4 years ahead of my other class mates". Moreover, his father's view about the educational differences in the two countries is very much in line with his son's perception. His father opines that "the kind of mathematics that my son does in England is very easy and relaxed in comparison to what he has done in Iran" (interview with B1's father, 21/05/2011, my translation).

Next year, B1 wants to take A-level mathematics as well as Further Mathematics because "everything somehow is maths related" (fieldnotes at complementary school, 19/09/ 2010). He is planning to go to university in the near future to pursue mathematics further. He shows a positive attitude towards understanding and learning mathematics and there is a great motivation from his family. His Dad encourages him to do whatever will make him happy in the future "and if he is happy to do mathematics as a university degree, I am willing to help him" (interview with B1's father, 21/05/2011, my translation).

The reason that B1 has gone to an Iranian complementary school on the weekend is twofold:

1) The Iranian complementary school often provides one-to-one help (if needed) with subjects or concepts that are unfamiliar to B1 like statistics.
2) B1's father as a parent cannot supply enough help with his son's mathematics at home because of the amount of hours that he puts into his job. However, "by sending him [B1] to that Iranian complementary school in which he is really happy with, it seems that this complementary school has helped him enormously to be in the top set in year 11" (interview with B1's father, 21/05/2011, my translation).

B1's favourite topic in mathematics is "anything and everything other that statistics and probability" (video recording at mainstream school, 10/02/2011) and the topic he dislikes is statistics and probability. With these topics he sometimes does not understand the question, and that makes him feel less able, "or perhaps because I have never done statistics and particularly probability before when I was in Iran" (video recording at mainstream school, 10/02/2011). Moreover, he adds in statistics "there is too much writing, it is too much English involved" (Interview with B1, 27/02/2011).

I have made fieldnotes, interviews and also video recorded B1 in his complementary school and his mainstream school. I have interviewed his father, and his mathematics teachers in both settings. I have also obtained further secondary data by collecting copies of B1's class work. Having access to the two institutions has enabled me to have two different perspectives on what assists or hinders B1 when he learns mathematics either monolingually or bilingually.

B1 uses Farsi regularly and on frequent intervals at home and he has exposure to Iranian television (Iranian satellite TV programs). Furthermore, B1 has access to other Farsi resources such as Persian websites on the internet, local newspapers/magazines and Farsi radio. He wishes to maintain his high level of proficiency in both languages, in both oral and written forms, academic and conversational in Farsi and in English so that it would enable him to communicate with a wider range of people, from different communities and diverse social class.

B1 is communicative competent in both Farsi and in English. His most dominant language is Farsi and he always converses in Farsi at home, with his extended family (over the phone or Skype) and in the Iranian community in the UK. Whilst he was in Iran for a period of a year, B1 had the opportunity to learn and develop using the formal version of Farsi in an academic setting among other native speakers of Farsi in Iran. Therefore, B1 not only can comprehend formal Farsi, but he can produce the formal form both in the form of verbal and written. B1's English language acquisition began in a classroom setting in Iran and then in the UK's mainstream school.

Boy 2

B2 was in Year 5 when his family came to the UK. He and his parents went back to Iran for some family reasons after a year of their arrival to the UK. They stayed in Iran for approximately 6 or 7 months and then they came back to the UK again and have stayed since. B2 is currently in the top set in English, mathematics and science.

He wants to be an architect and he knows very well that he needs mathematics for his career development. He knows very well that being able to speak two or more languages, will increase the chances of job opportunities in two or more different societies. B2’s parents are more concerns about B2 to enter an accredited medical school in the UK, and engineering comes second in their view. B2 wants to build his mathematical knowledge even stronger so that he could increase his chances of becoming who he wants to be. For this reason, he has taken extra mathematics lessons after school on Tuesdays and Thursdays at his mainstream school. In his complementary school, B2 also attends extra science lessons on Saturdays and extra mathematics lessons on Sundays.

B2 finds the complementary school sometimes really helpful and that is when the complementary school complements the work that goes on in his mainstream school. “Although I learn new things at the complementary school but sometimes complementary schools are not as helpful because sometimes we do a topic in the [mainstream] school and I come here [complementary school] and we do something totally different. It is hard for me to keep my mind off from that one [a topic in the mainstream school] and focus on the other [another topic in the complementary school]” (Interview with B2, 27/02/2011).

I have made fieldnotes, interviews and video recorded B2 in his complementary school. I have also interviewed T1 and T2 with regard to B2’s attitude to work in his complementary school. I did not get access to his mainstream school.

B2 just like B1 uses Farsi on a regular and frequent interval at home, with his extended family and in Iranian communities of practice in the UK. In addition, he reads local news papers in Farsi and watches his favourite TV series on the Iranian satellite TV.

B2 is communicative competent in both Farsi and in English. He understands Ajeri and speaks a little Azeri. His most dominant language is Farsi and he alternates between his languages depending on the topic at home. B2’s English language acquisition began at in UK in an after school classroom. He has developed most of his language skills within the first three years of his arrival to the UK.

Boy 3

B3 came to the UK with his parents 4 years ago. At that time he started year 7 and now he is currently in year 10. In his mainstream school, he is in the top set in mathematics only. He has a positive attitude in the learning of mathematics but he still does not know what is it that he wants to do in the future.

His parents, just like any other typical Iranian families are very concerns for their child to become a Doctor, enginee, or to enter a law school and they are less concern about B3's developing or sustaining his heritage language skills. B3 however, is not as proficient as he would like to be in English language and sometimes he struggles with word problems questions, such as those in 'statistics and probability'.

I have made fieldnotes and video recordings of B3 in his complementary school. I did not get access to his mainstream school to compare his learning in an "English-zone only" (Cummins, 2005:590).

B3's favourite topic in mathematics is algebra. He always likes to find the unknown factor in mathematics. He also added, "I like to play around with numbers and symbols, . . . [to] move them around and change their signs. . . something that most people get it wrong" (fieldnotes at complementary school, 19/09/2010, my translation). He does not really have a topic in mathematics which he dislikes.

B3's most dominant language is Farsi which he had learned it at a very early age at home. B3 has acquired English language in a team in his UK mainstream school that supports language support for EAL learners. He is competent in Kurdish and Turkish too. His English proficiency enables him to communicate and get his ideas across but sometimes he struggles to communicate effectively because of the language barrier. In all of B3's languages, his oral skills have much further developed than his formal written skills. At home he speaks Farsi with his parents and sister but to his other relatives, for example, his grandparents and other relatives, only speaks it minimally and instead he draws upon his other linguistic resources such as Kurdish and Turkish. It is apparent that the positive attitude of relatives (mostly from Iran) and extended family has played a major role in maintaining not only the heritage language (Farsi) but also in recognising the heritage Persian community which is reflected through its heritage speakers.

Boy 4

B4 arrived in the UK approximately 6 years ago. Unlike others, his attitude in the learning of mathematics is not positive. In some ways, he does not want to be in the complementary school because "Sundays are days to rest . . . I could be out here on the field playing football than being in here learning maths, . . . more and more maths that I do not ever need to know" (fieldnotes at complementary school, 03/10/2010). B4 does not know what he wants to be in the future but "definitely not a maths related one" (fieldnotes at complementary school, 27/02/2011). The only reason as to why he comes to this complementary school on the weekends is because of the parental aspirations so that he could learn better and do better in his exams. He opines that "there is like a family pressure on me to be here on Sundays. It is what they [parents] want and not what I want" (fieldnotes at complementary school, 09/01/2011). B4 is coming from a different social cultural background to most of the other students. He is an Armenian-Azerbaijani who shares different cultural perspectives, way of thinking and behaving at his mathematics lessons which he shares with other learners in the complementary school.

He is also known as the "Fonejacker" as he can produce variations of accents in each language. He tells a lot of funny jokes (in different dialects and languages about different nationalities) in lessons, in the playground, etc. By doing so, on a much bigger scale, he wants to connect himself with other Persian heritage students. He likes football and during break times he encourages the other students in the complementary school to play football. During most of the mathematics lessons, T1 and T2's make a lot of disciplinary comments which put B4 at the centre of attention. Unlike the other boys, B4 does not take an active part in the bilingual mathematics discussions. In general, he does not like mathematics and does not have a favourite topic in mathematics either.

Fieldnotes during participant observation were taken in B4's complementary school as well as interactional video recorded data. I did not get access to B4's mainstream school.

His most dominant language is Azerbaijani language. He is pretty competent in Armenian, Turkish and English. He is proficient enough in Farsi and often encounters difficulties expressing his thoughts and ideas. B4 holds a common view that different languages provide different job opportunities. He speaks a variety of languages depending on the topics, different social domain and with different interlocutors.

B4 only encounters Farsi at the complementary school. He acquired Farsi which was his forth language in a primary classroom setting in Iran and similarly his competency in English began in a classroom in the UK. Therefore B4 does not draw upon Farsi and English as much as Azerbaijani, Armenian and Turkish at home. He only speak Farsi and English occasionally at home with his family members. According to B4, his oral proficiency is much better that his written proficiency ten times. Unlike B1, B2 and B3, B4's Farsi acquisition occurred at a different time span of his life and in a different setting through a different source. His Farsi literacy skills were completed at a later age than those for B1, B2 and B3, whose level of language competency and literacy was more or less 'fixed' at a relatively young age.

Boy 5

B5 is a second generation British born Iranian. His mother is English. He is currently in year 10 and is in the top set in mathematics, science and English. He has recently joined the complementary school (in January 2011) and his main purpose in joining this school was to be able to write the community language, Farsi. Later on, he chose to do mathematics on Sundays so that he could do mathematics in Farsi too.

Like all second generation immigrant groups (Sohn & Merrill, 2008), second generation heritage bilingual learners tend to lose competency in their heritage language through successive generations. B5 is proficient enough in Farsi but sometimes he struggles to express his ideas if he is about to say something technical or a new concept in mathematics that he is unfamiliar with. However, his passive vocabulary knowledge allows him to understand any monolingual statements or monolingual discourse in Farsi, even if he is unable replying using the same sociolinguistic register. Mostly he substitutes a conversational style of communication for a technical one, and by doing so, he "make[s] meanings, transmit[s] information, and perform[s] identities using the linguistic signs at [his] disposal to connect with [his] audience in the community" (Creese & Blackledge, 2010:109).

I have made fieldnotes and video recordings of B2 in his complementary school. I was unable to get access to his mainstream school.

B5's favourite topic in mathematics is "everything, I like mathematics in general and that's the very reason why I am in here, because I chose to be in here on a Sunday. . . I want to learn mathematics in a different language . . . so that I can talk to my dad about it" (fieldnotes at complementary school, 27/02/2011). He does not really have any particular topic in mathematics which he dislikes but he hates some of the prepositions that are used extensively in the Farsi mathematics register, "those which stand for 'multiplication' and 'division', are so confusing". The preposition for multiplication and division in Farsi is 'dár' and 'bár' respectively (fieldnotes at complementary school, 27/02/2011). Moreover, B5 is always concern with technical words that have more than one meaning in mathematics such as 'moâdele' which means both 'inequality' and the verb 'equals to'.

B5's exposure to Farsi was mostly oral, therefore he has better oral skills (speaking/understanding) than written skills (writing/reading) in Farsi. He has been exposed to Farsi at home with the encouragement of his father. He would like to focus more on the written and formal form of Farsi so that would enable him not only to communicate with his father but also to be able to write letters to them and to be able to write Farsi orthography on Facebook and other media sources. Moreover he wants to learn Farsi so that he could talk about the Persian culture. In my fieldnotes, it is apparent that his positive attitude to learn the heritage language is to know about the culture that B5 inherits which exhibits a different perspective on the role on learning the community language. He has expressed different motivations for learning the community language and the role of his extended family was not the main factor his motivations as such.

B5's most dominant language is English. B5 knows an extensive vocabulary in Farsi, but unfortunately his linguistic range is limited to conversational, social and interactional domains at home only. Therefore he has difficulty talking about abstract topics in mathematics, using technical mathematical vocabulary, formal and polite terms in Farsi. He has however, a positive language in improving Farsi mathematics register and he comprehends and perceives well in Farsi. B5 acquired Farsi as a heritage language at home with his father. His Farsi literacy skills are however, not fully completed at this stage.

B5 understands and speaks a low level of Azeri. At home he speaks English most of the time, but sometimes Farsi and occasionally Azeri with his father. He cannot read or write in Azeri.

Boy 6

B6 is another British born Iranian. His father is an Iranian and his mother is from Russia. He is in year 7 and the only reason why he comes to this complementary school on the weekends is because of the family pressure and parental aspirations. B6's parents want him to improve his school subjects in the complementary school so he could apply and get into a 'King Edward' school (personal communication, June 2011).

Last year he was attending the complementary school for the purpose of learning how to write the community language. He has also attended a Russian complementary school for the same purpose, to be able to write in Russian. There seems to be a parental pressure on him to learn the two languages (Farsi and Russian) fluently as he often says "my parents want me to read and write in Farsi and Russian" (fieldnotes, 04/09/2010).

Fieldnotes and video recordings were taken in B6's complementary school. On one occasion I asked B6's father with regard to his intentions of sending his son to this complementary school. He said "as a family we are living in a new home country ... England is not my birth country nor English is my first language. Me and his Russian mother are unfamiliar with this culture which makes it even harder to raise and teach our child to familiarise himself in this culture ... This is one of the reasons why we sent our child to this school not only to learn his academic subjects but also so he can associate with those who have been living here longer and are more or less familiar with the system".

His mathematical ability is average and he is in set three in year 7. B6 often contributes to the class discussion in his complementary mathematics lessons and other science lessons. In general, he does what he has been told to do and he has never forgotten to complete and bring his homework.

B6 has access to media (such as Russian and Iranian satellite TV programs) and internet sources. His oral skills in his recognised languages outperform the written counterpart. His most dominant language is English. He is more or less equally competent in Russian and Farsi. At home he draws upon Farsi (with his father) and Russian (with his mother) interchangeably and sometimes English with friends who do not know either of the languages. His Farsi and Russian first began at home through the interactional exposure with his Russian mother and his Iranian father. All of B6's linguistic resources were developed in parallel with each other at a same time.

Boy 7

B7 is a British born Iranian. He was in year 7 when he first came to the complementary school in September 2010. He was put into the Iranian complementary school because “I was too noisy at home and my parents sent me here to learn some manners” (fieldnotes at complementary school, 12/09/2010). In his mainstream school, he is in the lowest set in mathematics.

He was in the complementary school for a period of a term and towards the end of the term (December 2010) he was suspended from the complementary school because of his aggressive behaviour towards other students. According to his 13 year old sister (G3) his parents sent B7 to the complementary school “so that he could learn how to write Farsi . . . [and] to learn other school subjects in Farsi” (fieldnotes, 28/08/2010, my translation).

B7’s parents like him to be a solicitor primarily because he talks a lot and most often he tends to outsmart other students in his age group during conversations.

B7 does not have a favourite topic in mathematics other than “counting money”. Other that this, he hates mathematics and school in general. A common question that he raised many times was: “What is the point of waking up at 7 in the morning everyday to get ready to go to school when I can be in bed at that time?” (fieldnotes at mainstream school, 15/02/2011). His attitude towards school (both complementary and mainstream school) was negative and he had no motivation to learn because “I do not need to study because when I am older I can work in my Dad’s shop and all I should know is how to count money” (fieldnotes at mainstream school, 09/02/2011). B7 and his sister (G3) are coming from different societal conditions and upper economic status. This is also apparent through the way they dress up (e.g. expensive cloth marks) and the cars they get picked up by after school.

I have only made fieldnotes in B7’s complementary and mainstream school. Having access to two institutions enabled me to compare and to be able to draw up themes across his profile. Teachers’ comments in both settings with regard to B7’s progress were helpful and supported my class observations.

B7’s most dominant language is English. He can only speak very basic conversations in Farsi and he can only write limited sentences in Farsi. At home he speaks English with his sisters, on Facebook and online games. He only speaks Farsi with his parents. B7’s Farsi’s linguistic repertoire is limited to informal-conversational form and he struggles to communicate and expresses his ideas in doing mathematics. He has a negative attitude towards Farsi and he does not want his friends at his mainstream school to know that he has an Iranian heritage. Moreover, as a respondent to the question ‘where do you come from?’, he shies away from that question or completely ignores that he has been asked a question by his peers in his mainstream school. He has shown a significant degree of discontent with regard to his heritage as a British born Iranian in his mainstream school but in the complementary school, he called himself as a Persian, and not an Iranian. B7 constantly adapt, construct and perform his social identity based on a specific community that he associates himself with (Heath, 1983; Erickson & Schultz, 1982).

Girl 1

G1 and her family came to the UK approximately 10 years ago when she was 4 years of age. She is currently in Year 9 and has been in the top set in mathematics for the last couple of years. She is in the top set in English and science too.

G1's education has been always through the medium of English in her mainstream school. She likes the complementary school to the point where she commits herself to go every Sunday, "It's so nice, it's just like going to church every Sunday" (fieldnotes at complementary school, 12/09/10). What she particularly likes about this complementary school is the bilingual nature of interaction during the classroom discussion because "using one language throughout the whole lesson can sometimes get boring" (fieldnotes at complementary school, 24/10/2010, my translation). G1 employs a metaphoric image to express her views on the nature of bilingual classrooms "it is like if it rains all day, it would seem too long and depressing but if it rains and then the sun comes out, it would not seem to be that long but exciting; it becomes short and meaningful . . . this is what I like about this Iranian school, so you can change your languages whenever you feel like it" (fieldnotes at complementary school, 24/10/2010, my translation).

She has a very positive attitude towards mathematics in general and she is planning to be a mathematics teacher in future. However, her parents want her to be a surgeon "even though they [my parents] know I do not like blood . . . I'm scared of a needle let alone a gamma knife" (fieldnotes at complementary school, 06/02/2011). She then shows her awareness of her parental expectation by saying "every Asian parent wants their children to be a 'doctor' [in an indian accent] or an engineer so that they can boast about: 'oh my son is a doctor' . . . perhaps I can understand why they want me to be a surgeon" (fieldnotes at complementary school, 06/02/2011).

G1 had first attended this complementary school 6 years ago when she had learned how to read and write in Farsi. Her parents sometimes provide her with help with her school subjects but if she does not understand, she discusses her questions bilingually in the complementary school.

Just like B1's opinion, G1's favourite topic in mathematics is 'anything and everything other that statistics and probability' and she hates probability "because of the language that is used in it" is too complex. G1 is "proficient enough in English language but sometimes she struggles with word problems questions, such as those in statistics and probability" (mainstream teacher 2, 26/04/2011).

I have made fieldnotes in her both complementary and mainstream mathematics lessons. Further secondary data was obtained by collecting copies of G1's work including any comments that she had made on her worksheets. I also collected copies of notebooks, coursework, and sample sheets of the teaching materials used by the teacher in her mainstream school. Interactional video recordings were captured during her discussions in the complementary school. I have interviewed her mathematics teachers in her mainstream school.

G1 has access to few Iranian satellite TV programs and she often watches Farsi language TV programs for Persian heritage learners. She actively listens and takes part in local Farsi radio quizzes. Just like B1, she likes to maintain high proficiency in both her language so that it could maximises the number of people that she could get to know of in a near future. . . . the other issue that must be pointed out is the attitude of relatives, extended family, and Persian community towards the Persian language skills of heritage speakers. She wants to be socially active primarily because "not many girls in Iran can be active as I can be here [in the UK] because the way females are viewed in Iran . . . and the sort of restrictions that are in place on Muslim girls". Because of certain religious limitations that have been enforced by the by social and cultural norms of gender rules on females in Iran, G1 want to show that Iranian heritage females outside Iran are just as capable as men are and they can be as active and attentive in different social domains.

G1 is a fluent speaker in both English and Farsi and she knows when and where to apply the formal form in both languages. She speaks Farsi effortlessly with a Farsi speaker at home and English with an English speaker outside home. According to her, "at the age of 13, she had completely developed a full level language competency and literacy in both languages". Her first exposure to Farsi was at home in Iran and her first exposure to English was both in primary classrooms and at home, with the help of her mother.

G2

G2 and her family came to England when she was 5 years of age. She is currently in Year 8 and in the top set in mathematics, English and science. Her parents registered her for this complementary school for one prime reason, and that is to learn how to read and write in Farsi. Occasionally she attended other school subject like mathematics and science in her complementary school. G2 parents expect her to be a medical doctor in the future.

G2 and G1 most of the time tend to work collaboratively with one another because:

- they knew each other from the previous years and
- at points, they were the only girls in their mathematics classrooms. So by sitting together, they would have felt more comfortable (Fieldnotes at complementary school, 06/02/2011).

Most often students who are in Year 7 and Year 8 are separated from those who are in Years 9, 10 and 11. G2 who is in Year 8 sometimes refuses to go to the class which is for Y7s and Y8s. Instead she wants to be in the other (9, 10 and 11) group. Sometimes she complains about the difficulty of the lesson (when she is in the higher age class) and if she goes to the lower age class, she complains about how easy and pointless the lessons are. She has never participated actively in any of the mathematics classroom discussions because they are either ‘too easy’ (“so what is the point”) or ‘too difficult’ (“I cannot understand it”). Her attendance is approximately 50% and more often than not she turns up late to her mathematics lessons.

She always talks in English when she complains to teachers or even when she is communicating with her friends on the playground.

I have made fieldnotes, and video recordings of G2 in her complementary school. I did not get access to her mainstream school.

G2’s most dominant language is English. G2 is not communicative competent enough in Farsi and sometimes she misunderstands certain cultural idioms/proverbs/expressions that have been said in the lessons. She does not know how to construct formal sentences in Farsi nor can she comprehend well it been spoken by others. She knows how to read and write in Farsi and she can understand a little Azeri. At home G2 speaks English with her parents most of the time but her parents sometimes talk to her in Farsi. She hears Azeri when her parents are talking to their family back home on the telephone or when they have visitors and guests. G2's English language acquisition began in an institutional setting in the UK. It seems that her parental and heritage language (Farsi) has almost lost its' application (Filmore, 1991), in a wider society in UK.

G3

G3 was two years old when her parents migrated to the UK. She is now in Year 9 and is in the top set in Mathematics, science and English. She first joined the complementary school 6/7 years ago so that she could learn to read and write the community language. She then rejoined this complementary school in September 2010 so that it could help her with her advance level sciences and mathematics. G3 and her brother (B7) left the complementary school in December 2010.

G3 wants to be a doctor in the future that is why she is attending advance level sciences and mathematics at the complementary school. G3 knows very well that achieving high results in mathematics can get her a step closer to her goal, which is to get on a medical course. The way G3's perceives her future-self as a medical doctor is also partially because her parents are pushing her to enter a medical school. However, G3's parents are less concerned with respect to sustaining her heritage language skills.

Most often in the mathematics classroom, she drew across her linguistic resources to think and to contribute to the class discussions bilingually. She often worked with B1 and B2 during class activities. Algebra was her strongest area in mathematics and she enjoyed doing algebra.

Because she was in the complementary school for only a period of a term and I did not have the permission to have any data in the format of video recordings, my primarily source of data collection was fieldnotes. Furthermore, I did not get access to her mathematics classroom in her mainstream school.

G3 is linguistic proficient in Farsi and English, but her written proficiency in Farsi is slightly weaker than the oral counterpart. She could draw across any of her languages to discuss and explore her thoughts. Both Farsi and English are her most dominant languages and she does not speak any other languages apart from these two. At home G3 speaks Farsi with her parents but English with her elder sister and little brother, in particular when they are fighting or discussing fashion. Her Farsi and English language acquisition began both at home and in an institutional setting in the UK at the same time. She has developed both 'native like fluency' in both languages at a same age.

Appendix B: Summary of students' biographies

Research participants	Year	Mathematics set	Number of languages	Citizenship	Future career	Favoriate topic in mathematics	Dislike topic in mathematics
B1	11	1	2	British citizen	Mathematics related	everything	statistics
B2	10	1	3	British citizen	Architect	Algebra	Statistics
B3	10	1	4	Iranian citizen	Not sure	Algebra	Nothing
B4	10	Lowest set	5	British citizen	Not maths related	Nothing	Everything
B5	10	1	3	British citizen	xxx	Everything	Farsi prepositions in mathematics
B6	7	3	3	British citizen	xxx	xxx	xxx

B7	7	Lowest set	2	British citizen	Family business	“counting money”	“hates mathematics”
G1	9	1	2	British citizen	Mathematics teacher	Everything	Statistics
G2	8	1	3	British citizen	xxx	xxx	xxx
G3	9	1	2	British citizen	Doctor	algebra	xxx

Appendix C 1

Informed Consent Form for Students

Dear Student,

My name is Danyal Farsani and I am doing a PhD in Mathematics Education. I am carrying out a research project about mathematics in your weekend school. I am asking permission for your voluntary participation in this study.

What interests me in learning mathematics is when students and teachers use more than one language in a mathematics classroom. For example, the use of English and Farsi in your weekend school. I am also interested in the use of language in learning mathematics within your normal Monday to Friday school.

My role as a researcher is to carry out observations in the school where you are studying both at the weekends and, at a later stage, at your weekday school. At a later date, I would like to make audio/video recordings so that I can analyse the classroom talk within the lesson. There is a chance that I may interview you in order to know your opinion about the learning of mathematics using two (or more) different languages. There will be no right or wrong answers in the interviews and no names will be mentioned in my reporting of this project. So no one can identify you as individual. Last but not least, your participation in my study is voluntary and, of course, you have the right to withdraw from the study at any stage.

Please note that some relevant clips from the recorded video maybe used in my thesis for clarification purposes. If you agree to be part of this study, please complete and return the attached form. If you have any further questions, please do not hesitate to contact me via T1.

Thank you for your time

<u>Danyal Farsani</u>

Reply slip

Please, could you complete this reply slip and send it back to T1.

Yes, I am willing to take part in this research study into the teaching and learning of mathematics in classes where students and teacher/s use two or more languages. I understand that this may involve me in both my weekend and weekday schools. I have read the details above and I understand that any information gathered will be used only for the purpose of the research study.

Name of student:____

Signed: _____________________________________

Date: _______________________________________

Thank you for your time

Appendix C2

Informed Consent Form for Parents of the Students

Dear Parent/Carer

My name is Danyal Farsani and I am a doctoral researcher in the School of Education, University of Birmingham. I have a first degree in mathematics and I am interested in doing research in mathematics education. I write to request your permission for your child to be included in this study.

I am a Persian/English speaker and my research is about the teaching and learning of mathematics in classes where students and teachers draw upon more than one language. My focus will be on Persian-speaking students of Iranian origin. This is why I would like to invite you and your children to participate in my research.

I want to carry out observations in the school where your child is studying at the weekends. At a later stage, I would also like to go into the mathematics lessons of your child's weekday school. Your child's participation is voluntary and both you and your child has the right to withdraw from the study at any stage. At a later date, I would like to make audio/video recordings in order to analyse the classroom talk. I may also want to interview your child and ask about his/her opinion about learning mathematics using different languages. Any writings which come from this study will not use your child's name, nor will the name of the school be used.

If you agree for your child to be part of the study, please complete and return the attached form by [date]. If you have any further questions, please do not hesitate to contact me, by my email address:

Thank you for your time

Danyal Farsani

Reply slip

Please, could you complete this reply slip and send it back to T1.

Yes, I am willing for my children to take part in my research study into the teaching and learning of mathematics in classes where students and teachers draw up on more than one language. I have read the details above and I understand that any information gathered will be used only for the purpose of the research study.

Name of child:____

Name of Parent/Carer:_______________________________________

Signed: ____________________________________

Date: ______________________________________

Thank you for your time

عزیز پدر و مادر / مراقب

من یک پژوهشگر دکتری در دانشکده آموزش و پرورش، دانشگاه بیرمنگام است. من یک درجه اول در ریاضیات و من علاقه نوشتن به شما اجازه برای فرزند شما را در این مطالعه.Iمند به انجام پژوهش در آموزش ریاضیات است.

من فارسی / انگلیسی صحبت می کنند و پژوهش در مورد آموزش و یادگیری ریاضی در کلاس که در آن دانش آموزان و معلمان به بیش از یک زبان در قرعه کشی. تمرکز من بر روی دانش آموزان فارسی زبان ایرانی تبار است. این است که چرا من می خواهم به شما و فرزندان خود را دعوت به شرکت در تحقیق من.

من می خواهم به انجام مشاهدات در مدرسه ای که فرزند شما در تعطیلات آخر هفته مطالعه. در مرحله بعد، من هم می خواهم برای رفتن به درس ریاضیات از مدرسه هفته فرزند شما است. مشارکت کودک شما داوطلبانه است و هم شما و هم کودک شما حق خروج از مطالعه در هر مرحله است. در تاریخ بعد، من می خواهم به ضبط شده صوتی / تصویری به منظور تجزیه و تحلیل بحث کلاس درس. من هم ممکن است کودک شما برای مصاحبه و در مورد / نظر خود را در مورد یادگیری ریاضیات با استفاده از زبان های مختلف خود را بپرسید. هر نوشته که از این مطالعه آمده است نام فرزند خود را استفاده نمی کند، و نه خواهد شد به نام مدرسه مورد استفاده قرار گیرد.

اگر شما برای فرزند شما توافق می کنید که بخشی از این مطالعه، لطفا کامل و به شکل متصل شده توسط [تاریخ] بازگشت. اگر شما هر گونه سوال بیشتر، لطفا دریغ نکنید برای تماس با من با آدرس ایمیل من: dxf950@bham.ac.uk و یا با من تماس بگیرد در 078 2877 2869.

تشکر از شما برای زمان

T1.لطفا، می تواند این پاسخ لغزش را تکمیل و ارسال آن به

بله، من حاضر برای کودکان من برای شرکت در مطالعه، پژوهش من به آموزش و یادگیری ریاضی در کلاس که در آن دانش آموزان و معلمان قرعه کشی در بیش از یک زبان است. من جزئیات بالا را خوانده و من درک می کنم که هر گونه اطلاعات جمع آوری خواهد شد تنها به منظور مطالعه پژوهشی استفاده می شود.

امضاء

Appendic C3

Informed Consent Form for Complementary School Staff

Institute: School of Education, University of Birmingham

Researcher: Danyal Farsani

I am a doctoral researcher in the School of Education, University of Birmingham and I am supported by a STEM studentship (Science, Technology and Mathematics).

Focus of this research: My research is about the teaching and learning of mathematics in multilingual classes in two distinct settings namely complementary schools and mainstream classrooms. My focus will be on bilingual, Persian-speaking students of Iranian origin. This is why I would like to invite you to participate in my research.

Reasons for doing this research: I have a first degree in mathematics and I am interested in mathematics education. Because I am a Persian/English bilingual, I am interested in the experiences that bilingual learners have in learning mathematics in different kinds of settings: in their local school in the UK, where only English is used and in other settings, such as their home or a local complementary school where they can draw on more than one language when talking about mathematics with their teacher or with other students or when solving mathematical problems. There is a growing tradition of research on mathematics in multilingual contexts, and I hope to contribute to the body of research by working with learners from Persian-speaking families within the West Midlands area. I am interested in two particular groups of bilingual Persian-speaking learners:

Firstly, I am interested in students whose families have recently moved to the UK and who are still at an early stage of learning English. The second group that will be included in my study is learners of Iranian origin who have grown up in the UK and who speak both Persian and English. Working with bilingual learners in these two group, I will address the following questions:

1. What prompts code-switching when English as an Additional Language (EAL) students are working towards a mathematical problem in the medium of English and how can this practice be imbedded in learning of mathematics?

2. What and how do EAL students learn from their bilingual interactions with their bilingual peers in learning environments other than the mainstream mathematics classroom e.g. home or complementary school?
3. What do Persian and English bilingual students think about learning mathematics in complementary schools where code-switching is regularly used, and mainstream classrooms, where the lessons are taught primarily in English only?

I hope with your participation in my research, we can learn more about these important topics.

What your participation would involve: There is nothing that I want you to do for me other than what you normally do in your classroom. I would like to observe some of your classes, offering help if needed. At a later date, I would like to make audio/video recordings, getting your advice beforehand about how I should do this. I will transcribe and analyse the audio/video recordings. I would also like to interview you about the complementary school and audio/video-record the interview. At the end of the research I will do a summary of my findings for you.

Right of refusal to participate and withdrawal: I will make sure that I gain permission from Parents/Carers for any students who will become part of the study. Should you agree to participate now, you still have the right to withdraw in the future and any data I have gained from you will not be used in any writings following the date of that withdrawal. Your participation is voluntary in my study.

Confidentiality: What will happen to any information that you provide: I will analyze the data that I have gathered with you (e.g. audio/video recording, field notes). When the tape recording is transcribed all the real names will be replaced with pseudonyms to ensure complete anonymity. Please note that some relevant clips from the recorded video maybe used in my thesis for clarification purposes. Once I have finished my research, I will destroy the tape recordings but until I finish my thesis, I will store your information in a secure place. Pseudonyms will be used in my thesis, including the name of the school.

Contacting me: If you have any further questions, please do not hesitate to contact me, by my email address: [redacted]

Thank you for your time

Reply slip

Please, could you complete this reply slip and send it back to T1 by [date]

Yes, I am willing to take part in this research study on the teaching and learning of mathematics in classes where students and teacher/s draw up on more than one language. I have read the details above and I understand that any information I give will remain confidential, and be used only for the purpose of the research study.

Name:____

Signed: ______________________________________

Date: ____

Thank you for your time

Appendic C4

Informed Consent Form for Mainstream School Students

Dear Students

My name is Danyal Farsani and I am doing a PhD in mathematics education at university of Birmingham. I have a first degree in mathematics and I am interested in doing research in your mathematics classrooms.

The purpose of this letter is to ask you for your permission to carry out observations in your mathematics classrooms. At a later date, I may wish to make audio/video recordings so as to observe the classroom talk and my focus will not be on you but it will be on the follow up of those students whose progress I have been following elsewhere and who have already given their consent. No names will be mentioned in this study so that no one can identify you as an individual.

If you agree to be audio/video recorded in your ordinary mathematics lesson where the focus of attention is not on you, please complete and return the attached form by date [...]. If you have any further [redacted]

Thank you for your time

<u>Danyal Farsani</u>

<u>Reply slip</u>

Please, could you complete this reply slip and send it back to Mr/Mrs by date [...].

Yes, I give permission to be audio/video recorded in my mathematics lesson where the focus of attention is not on me. I have read the details above and I understand that any information gathered will be used only for the purpose of the research study.

Name of Student:____

Signed: ______________________________________

Date: ____

Thank you for your time

Appendic C5

Informed Consent Form for Mainstream School Staff

Institution: School of Education, University of Birmingham

Researcher: Danyal Farsani

I am a doctoral researcher in the School of Education, University of Birmingham and I am interested on the teaching and learning of mathematics in classes where students and teacher/s draw up on more than one language.

Focus of this research: My research is about the teaching and learning of mathematics in multilingual classes in two distinct settings namely complementary schools and mainstream classrooms. My focus will be on bilingual, Persian/English speaking students whose progress I have been following and who are already been aware of my research interest.

Reasons for doing this research: I have a first degree in mathematics and I am interested in mathematics education. Because I am a Persian/English bilingual, I am interested in the experiences that bilingual learners have in learning mathematics in different kinds of settings: in their local school in the UK, where only English is used and in other settings, such as their home or a local complementary school where they can draw on more than one language when talking about mathematics with their teacher or with other students or when solving mathematical problems. I wish to explore is that of comparing their bilingual learning experiences in local complementary schools, where both Persian and English are used, and their experiences of learning solely through the medium of English in mainstream classrooms.

Through my research, I hope to raise the awareness of mainstream teachers (in the school(s) where I am working but also in other schools and local authorities) about the type of knowledge about mathematics that newly arrived Persian-Speakers EAL students from Iran are bringing to their learning of mathematics through the medium of English in the UK. I may use my data and the video transcripts to present at professional research conferences and in the writing of journal articles. Pseudonyms will be used in my thesis, including the name of the school.

What your participation would involve: Beyond allowing me to observe your classrooms, take notes and interview you, I would like you to continue with your normal classroom practice. At a later date, I would like to make audio/video recordings, getting your advice beforehand about how I should do this. I will transcribe and analyse the audio/video recordings. I would also like to interview you about the Persian/English learners and their participation in the classroom and audio/video-record the interview.

Right of refusal to participate and withdrawal: I will make sure that I gain permission from Parents/Carers for any students who will become part of the audio/video recordings. In an event that some parents or students do not consent to be videoed, I would ask that those students be placed in a position in the classroom which will not be within the video camera view during those lessons which are being recorded.

Should you agree to participate now, you still have the right to withdraw in the future and any data I have gained from you will not be used in any writings following the date of that withdrawal. Your participation is voluntary in my study.

Confidentiality: What will happen to any information that you provide: I will analyze the data that I have gathered with you (e.g. audio/video recording, field notes). When the tape recording is transcribed all the real names will be replaced with pseudonyms to ensure complete anonymity. Once I have finished my research, the video recording will be preserved for a period of ten years. The recordings will be stored in a secure place and only I and my supervisors will have access to them.

Thank you for your time

Reply slip

Yes, I am willing to take part in this research study on the teaching and learning of mathematics in classes where students and teacher/s draw up on more than one language. I have read the details above and I understand that any information I give will remain confidential, and be used only for the purpose of the research study.

Name: ____

Signed: _____________________________________

Date: _______________________________________

Thank you for your time

Appendix D

'Just Under'

During one of my classroom observation, I became aware of a possible trigger to confusion among the interlocutors. This trigger was primarily a linguistic one where the speaker B1, used the term 'just under' to refer to 'slower, just less than' and B5 interpreted the word in the context to mean 'faster'.

Before the start of the lesson, B1 was describing as to why he wants to buy a motorbike and not a car. He was describing about the nature of a sport motorbike and a BMW M3 and their corresponding 0-60 mph acceleration.

B1: a BMW M3 does 0-60 in like 5 seconds. A sport bike does 0-60 in just under half of what a M3 does, like in 2.7 seconds.

B5: That's not under, that's over. . . cause if 2.5 is half of what a M3 does, therefore 2.7 is just over and 2.3 is just under.

B1: No, I mean just less than, just slower than half.

What B5 had interpreted 'just under' in the context that could refer to the numerical value 2.3 (as it is under 2.5 which is the half) therefore faster. In contrast 2.7 could refer to 'just over' therefore just slower which is true. B5's most dminant language is English. He was confident enough and aware of this subtle linguistic 'mix-up' that 'under' or 'slower' in timing means faster.

What B1 had in mind was to express that a sport bike does 0-60 just a fraction of a second slower to half of what a M3 does. He therefore used the term 'just under' to refer to 'less', 'fewer', 'slower' which in reality is a representation for being faster and not slower.

Slow	fast
Over	under

Appendix E

Transcription of an interactional classroom recording

T2: If you get a taxi from xxx company, they use the following rule to work out How much does a taxi fare cost. Multiply the distance in Kilometre by two pounds fifty and then add one hundred and eighty pound to the answer.

B5: Is it two point five x plus one point eight?

T2: Yes, yeah. I haven’t read the question yet hehe. So using C to stand for the cost in pound and D to stand for distance in kilometre, express the rule for calculating this equation. So C is the cost is equal to two point, two pound fifty times what

B4: one eighty

T2: D, the distant plus one point eighty of a pound. *part B mige chi? Mige,* (what does part B says? It says} use your equation to work out the cost of a journey of one point eight kilometre. If d is one point six of a kilometre what is the cost?

Bx: four

T2: So c is equal to point five times one point six plus one point eight.

B5: five point eight

T2: *jaanam?* {sorry my dear?}

B5: five point eight

T2: five point eight. So five pound and eighty pence. *in ghesmate aval, ghesmate dovom mige chi? Mige,* {this is the first part. What does the second part say? It says} What is the length of a journey which costs ten pounds fifty five pence? *Khob, man in formula-ro tabdil mikonam be yek doone formula-ee ke subject-*

esh chi bayad bashe? {Ok, I will change the subject of this formula to a new formula. What should be the subject of the new formula?}

Bx: Ten pounds

T2: *Subject-esh bayad beshe?* {what should the subject be?} D, ok. So I will write two fifty, two point five times D is equal one pound eighty or basically C plus one pound eighty. One pound and eighty, what did I do? Moved this bit to the other side. Or D is equal to C plus one pound eight divided by two pound fifty. Now I change the subject of the formula from C to D. Ok, but we know in this case the cost is ten pound

B1: fifty five

T2: fifty five pence, do distance D is equal ten fifty five minus one point eight divided by two point five. What is the answer?

B5: three point five

T2: hmmm?

B5: three point five

T2: three point five of a kilometres. Ok, it says Joe's Taxis use this formula which is C is equal one point seven D plus two point one. So what is the difference between these two? What is the big different?

Bx: y intercept

T2: yes the y intercept but in this case the C intercept. but basically, *vaghti inaro ba ham compare mikonim ke common fare-eshoon, Joe's taxi is more expensive*. {when you compare them, you will realise that Joe's taxi is more expensive} If you get to Joe's taxi, you will get charged more than 'Car's R Us'. But in a long distant you pay less money. Ok, does everyone understand what I mean?

B5: Can you solve it with a line graph?

T2: yes, you can use a line graph and you will see one of them will look like that and the other one is like that . Ok and then you can figure out which one is more expensive for which journey. For example, for a very short journey, for one kilometre, which one is cheaper?

B1: ‘Cars R Us’

T2: ‘Cars R Us’ is cheaper. In a longer journey, hundred kilometre Joe’s taxi is cheaper. Now the second part, part two says find the distance where both these companies charge the same. Ok, how do we do this?

B5: line graph

T2: yes, one of the is line graph, but that is too long. Xxx assuming this C is equal to that C, what can I do? We can write C is equal to C. Now ‘Cars R Us’ C is equal to D plus one point eight. This is for ‘Cars R Us’ ok. The Joe’s taxi is the second one, one point seven D plus two point one. What does that D give us? Xxx so more that D over there, move this to the other side so two point five D takeaway one point seven D is equal to two point one takeaway one point eight. So what is the answer for this?

Bs: xxx zero point two five.

T2: ok, zero point eight D over nought point three. Ok, so D is equal to nought point three over nought point eight. How can we calculate this guys? Without a calculator.

Bx: Move them from decimal

T2: Fraction, so a big fraction line that is three over ten, eight over ten. Ok, how can I simplify this one? Very quickly?

B3: three times ten, eight times ten.

T2: Simplify these two because that is multiplied by and that is divide. So that is three over eight of a what? Of a kilometre. At three over eight f a kilometre, there two companies charge the same money. *Ok hame bacheha ino motavaje shodan?* {has everyone understood?} is it clear for everybody? Yeah, or you ok with this? Hello, are you fine with this?

BS/Gs: yes

T2: So which taxi do we use to travel long distant?

Bs: xxx

T2: Joe's taxi because we pay less in a long distance. *Khob, soaale badi fekr bekonid bacheha dar barash* {Ok, next question, guys think about it}. We have two equations and we want to draw the graph for it.

B5: can't we just sketch it for

T2: no xxx

B5: xxx you can sketch it, you don't need to draw the table.

T2: what do we do? Yes, basically we know that in some form y is equalled to x squared or something like that. And the other line should be like that because of negative, because of the gradient, but we have to do it properly, we have to show the value to the xxx when these sort of questions come to the exam, you have to do it properly. Xxx and now the question says draw the graph xx so that is two takeaway one takeaway x and that is x squared

B4: *oon mishe emm* {that will be emm}

T2: that one is nine four one zero one four nine

B4: *oon yeki mishe five* {the other one will be five}

T2: five

B4: four

T2: four

B4: three, three, two one zero

Bs: xxx

T2: xxx ok, I just need two of these lines to draw this line so that is zero and one and two, one point is there, one, two, three and minus one. This is our straight line, ok. The next one is when I have one, zero is zero, minus one

Gs: xxx

T2: so guys, the second one is xxx infinity and goes up there. g*hesmate aval* {first part} draw the graph which we just did there. *hameye in shomareha ro bacheha mikhastim* {we needed all these numbers} do we need all of these numbers?

B1: *na* {no}

T2: no, not really. We need at least four of them or three of them. Use your graph to find the solution of a simultaneous equation which you have there. where is the answer for these simultaneous equations? If I had the y equal to x squared and y is equalled to takeaway x what is the answer in the graph for these two? If you have to show them, what is it?

B5: its a y, its like cross xxx

T2: its a cross-section between, these two points *bacheha vaghti ke to ta, masalan age man yek doone graph dashte basham* {guys, its like when I have two graphs} something like that and the other graph is something like that if I have this one then it is like y is equalled to minus three x cubed plus or takeaway five, the other one is equalled y is minus or plus x takeaway three. *Ok, vaghti in dotaro ba ham hal mikonam* {when I solve it} what is the answer for this equation?

B5: xxx

T2: these two, when they hit each other, *khob pas inro mitoonid baraye hameye chizha estefadeh bekonid. Inro hal bekonid bacheha, chetori ino mitonid hal bekonid in simultaneous equation ro?* {so that we could use this for everything. Now solve it, how can you solve this simultaneous equation guys?} how did we solve it? *Ba che raveshi chiz mikonim?* {what strategies are we employing to solve it?} *which ravesh behtare?* {which method is the best?} substitution is the best and easiest method to solve this. Ok, how do we substitute? Can we just cross them over?

B2: No

T2: no, because this one doesn't have a x squared. So I will just substitute.

B2: *K ro as koja aavordin? Neveshtin two minus k.* {where is k coming from? You have written two minus k}

T2: That is x

B2: *na, bala bala* {no, up up}

T2: that supposed to be x.

B2: aha

T2: two take away x is equal to x squared or x square plus x takeaway two is equalled to zero. What are the two numbers when multiplied to each other

B2: two and one

T2: minus two and when added to each other it becomes one? It is two and one, but which one is the positive and which one is the negative?

Bs: xxx

T2: two is positive. *Hamishe sign-e bozorge sign-e middle number-e* {always the sign of the bigger number is carried from the same sign of the middle number} *in hamishe sign-e inro dare* {this always have the sign of this one}

B3: xxx

T2: *jaan?* {Sorry dear?}

B3: mage one nist dotashoon? {isn't it one?}

T2: two times minus one is? Minus two. Two takeaway one is?

B3: one

T2: one. Ok, got it, *khob, pas answer man hast* {ok, so my answer is} x plus two is equal to zero and x takeaway one is equalled to zero. *Baraye chi do ta answer daram?* {why do I have two answers?} why do I have two answers?

B1: chonke do tashoonan {because there are two of them}

T2: because that is a simultaneous equation xxx so x is equal to minus two xxx and x is equal to plus one. *Hala miam che kar mikonam, miam x haro mizaroom tooye yekdoone as ina.* {now I am gonna substitute my values back to the original question.} now this time I have x one is equalled to minus two. So y is equal takeaway takeaway two the answer is? Four, ok. Xxx so minus two and four. Which one is this?

B1: *avalie* {first one}

T2: minus two and minus four. *Soaale dovom*, {second question} x two is equal to xxx so the next one is plus one and

B2: minus one

T2: one, that is B.

B5: *oon avalie chera shod two minus minus two* {how come the first one became two minus minus two?}

T2: because that is two takeaway x. What is x? Minus two, so two takeaway takeaway two which becomes plus two. Because takeaway times takeaway is?

B2: takeaway, plus, minus

T2: Plus. We have an inequality and we want to show the answer. *Paak konameshoon inaro?* {shall I clear the board?} has everyone wrote it down? Fair enough.

B2: *shomareye haftim alan?* {Are we on number seven now?}

T2: Jaan? {sorry dear}

B2: *shomareye haftim?* {Are we on number seven?}

T2: *shomareye bistim* {we are on number twenty} actually. *Khob, bacheha say bekonid in inequalitiharo hal bekonid* {Ok, try to solve these inequalities guys}

B1: I've done it.

T2: you've done it? It is not that difficult, is it?

B1: no

T2: *khob, in inequalitimoon chi mige? Mige* {ok, what does our inequality says? It says} three x takeaway one is larger than eight. Then what do we do? The same thing we usually do for inequality equations. So three x is still larger than eight plus one, which is nine. Then x is still larger than nine divided by three which is? Three, so x is larger than three. *Khob, man ino tooye formula bezarim,* {Ok, if we put this back to the formula} assuming this is three, *x hamoon mikhaim larger than three bashe yani* {we want our x to be larger than three, that means} from there to the plus infinity, but because it is less than and not equal to, *in dayere-ee ke mikeshim chie?* {what happens to the circle that we draw on here?}

B2: *shadding toosh nist* {it won't be shaded inside it}

T2: *empty-e* {it is empty} *bacheha, inro hal bekonid baraye man* {guys, solve this for me} one takeaway three is larger than eight. What is the answer for this?

B3: *mishe hamin* {that's the same answer}

T2: *na na* {no, no} try to do it. So what I am gonna write down is minus three x is larger than what?

B5: *mishe, oonvari mire khatesh* {that is, the line will go to the other side}.

Bs: eight, seven

T2: minus three x is larger than seven. Now what do I want to do? I want to multiply, multiply, this is very important guys, this is really really important, multiply or divide the inequality by a negative value. When we do that the direction of the inequality

Bs: changes

T2: changes. This is really important, you have to remember this. So this is, this is not equal to x. This is wrong ok, this is wrong. When you do that, its x less than seven divided by minus three. *Baraye chi less than shod?* {why did it become less than?} Why did we change the direction of the inequality?

B1: because we took the negative numbers to the other side

T2: Yeah, so basically we multiply or divide an inequality by a negative number. This is important guys

B3: *khob se ro chera nayovordi ke, ke seven over three baashe?* {So how come you didn't do anything to the three, so it would have become seven over three?}

T2: *Jaan?* {sorry dear?}

B3: *chera seven over three nabaashe?* {why can't it be seven over three?}

T2: seven over three? That's seven over three.

B3: *minus three-e* {that is minus three}

T2: *basickly mishe chi, oonke shoma migin mishe man inro inja barat hal mikonam. Shoma oon chizi ke migi* {basically that is, what you are saying is, I will solve it in here. What you are saying} is minus x is still larger than seven over three. Ok, minus x is larger than seven over three.

B3: ahhh

T2: what is x? X is smaller than seven over three. So x is smaller than minus seven over three. *Doroste* {that is right} if we divide the whole thing by positive three, xxx

B3: nemishe minus three x-o masalan hamero divided by x konim? {can't we divide minus three x by x?}

T2: that does not help us, does it? Xxx then you have to reverse the whole thing to get back to where you already are. That wouldn't help us at all. *Ghesmate badi bacheha, badiro shoma hal bekonid.* {next part guys, you should solve the next part} ok, next one, I want to see everybody does the next one. *Agha B3, sooale bad ro anjam bede.* {Mr B3, do the next question} soaale badi bacheha, mikhaim in inequality ro anjam bedim {next question guys, we want to do/solve the next inequality} come on G2 *baraye chi anjam nemidi?* {why don't you do the question?}

G2: I haven't got that sheet.

T2: why didn't you tell me in the first stance?

G2: *yadam raft* {I forgot}

T2: *to mitooni ino anjam bedi G1?* {can you do/solve this one G1?}

B2: *to mitooni, are to mitooni* {you can, yes you can do it G1}

Bs: hehe

T2: ok, this time we have one over x is larger than one over two. What do we do here?

B1: *Times mikonim* {we times them}

T2: We times them. That's is

B5: cross-multiplication

T2: *khob, migim chi* {So, we say} two is larger than x, x is less than 2.

B3: *chera oonvari nist?* {how come it is not that way?}

T2: *x mire tooye numerator,* {x goes to the numerator} and one goes to the denominator xxx and when we draw our graph, x has to be less than two. *Age x less than or equal bood,* {if x was less than or equal} it becomes shaded. *Soaale badi bacheha,* {next question guys} we want to find the value of x.

B4: *khob midoonam, gofte avalesh, midooni, mige ke to, eigh-ta triangle dare* {Ok I know it. It says first, you know, its saying that you, it has eight triangles}

T2: *in soaalo bacheha deghat bekonid* {please pay attention to this question guys} this is a very simple question but interesting when we want to solve it. Basically, what sort of pentagon is it? Irregular, irregular pentagon. So

B2: *x do-e dige?* {is x two?}

T2: jaan? {sorry dear?}

B2: *x do mishe* {x is two}

B5: is it three hundred and sixty inside it?

T2: *hmmm, khob, mikhahim bebinim ke* {hmmm, well, we want to see} what are the sum of angles inside a pentagon?

B4: five times hundred and eighty

T2: yeah?

B3: *mibinim chand ta triangle-e toosh* {we should see how many triangles are inside it}

T2: so what I am gonna do, I am gonna find the centre

Bs: xxx

T2: how many triangle in there can you see?

B1: *panj-ta* {five of them}

B2: *se-ta* {three of them}

T2: so it is five times hundred and eighty

B3: *poonsad-o* {five hundred and}

T2: takeaway what? Bebinid, inharam maa darim hesab mikonim dige {look at here, we have already taken these into account}

Bs: three sixty

T2: takeaway three sixty xxx so the answer is five hundred and what?

Bx: fourty

T2: five hundred and forty. *Khob, hala miam migam* {Ok, now I say} that bit

B1: divided by eight

T2: is x, two x, two x, and two x. So two x plus two x plus two x plus x plus x five forty. Eight x five forty divided by eight, the answer is?

Bx: sixty seven point five

T2: sixty seven point five of a degree is the value of x.

B1: *tamoom shod dige?* {is that it then?/is it finished?}

Bs: xxx

T2: each internal angle of a regular pentagon is five times the size of the exterior angle. So calculate the size of the exterior angle. So we have like something like this, if this bit is x, this is five x. We want to find the x.

B4: *oon aval onesho dar miarim, em, midooni mishe five times sixty-seven and a half va* {that first, we work out, emm, you know, the, that part which will be five times sixty-seven and half and}

Bs: xxx

Gs: xxx

T2: that’s a regular pentagon xxx so x plus five x is equal to *chi bacheha?* {What is guys?}

B1: hundred and eighty

T2: hundred and eighty, no because that bit is hundred and eighty degrees.

B3: *hundred and eighty-ro divided by six mikoni?* {Do you divide hundred and eighty by six?}

B2: *na* {no}

B5: six x

T2: the answer is ninety, *doroste?* {is that correct?}

B3: *chera divided by two? Divided by six bayad mikardi.* {How come it is divided by two? You should have divided it by six}

T2: xxx if you divided by six, that is thirty degrees. So basically this is thirty and this is one hundred and fifty degrees. So we did that.

B3: *oon ham hal kardim, E-ro ham hal kardim* {we’ve done that, and also we have done E too}

T2: How many sides does a polygon have?

B2: *panje* {five}

B3: *yek, do, se, char panje* shish haft hast{one, two, three, four, five six seven eight}

B4: we know you can count

Bs: hehehe

T2: you have to, *bayad neshoon bedi* {you have to show your rationale}

B2: Tork-e khare {you dyslexic Turk}

Bs: xxx

T2: what sort of polygon is this *bacheha?* {guys?}

B1: regular

T2: regular. So if I just draw this, ok, and for example, this is the centre of this polygon, I draw a circle ok, what size is that bit? one hundred and fifty. *Pas inside-esh mishe cheghadr?* {so, what is the inside value?} hundred and fifty divided by two?

B2: seventy-five

Bs: xxx

T2: seventy-five. So what sort of triangle is this triangle *bacheha?* {guys}

Bs: isosceles

T2: isosceles triangle, ok, that is?

B1: seventy-five

B2: seventy-five

T2: seventy-five, so this bit is?

B1: eighty *na bebakhshi* {no sorry}

B3: thirty

B2: thirty

T2: So, I am gonna call that one y, so y is equal hundred and eighty takeaway two times seventy-five.

B2: *two times seventy-five mishe sado panjah* {two times seventy-five becomes hundred and fifty}

T2: yeah, y is thirty degrees. Ok, so this is thirty degrees. Khob, in pentagone ma hamintori bayad edame pida bekone dige {Ok, so our pentagon xxx}

Bx: xxx

350 T2: *so migim chi*, *migim* {so, what do we say, we say} how many thirty degrees are there in three hundred and sixty?

B2: *hashta* {eight}

B3: *mishe davazdahta* {there are twelve of them}

T2: three hundred and sixty divided by thirty

355 B2: Its eight

T2: Twelve. *Migim twelve triangle tooye ein shapemoon hastan*. {So there are twelve triangle in this shape}. So this shape has how many sides?

B3: Eight

T2: Twelve sides

360 Bs: hehehe Tork-e dige {he is a Turk, what can be done}

T2: we have like five triangles here and each triangle has a side so that is twelve. *Khob? Bacheha khaste nabashid* {Ok? Guys well done}

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