Finite Groups of Small Genus

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Abstract

For a finite group G, the Hurwitz space $\mathcal{H}^{in}_{r,g}(G)$ is the space of genus g covers of the Riemann sphere with r branch points and the monodromy group G. Let $\mathcal{E}_r(G) = \{(x_1, ..., x_r) : G = \langle x_1, ..., x_r \rangle, \prod_{i=1}^{r} x_i = 1, x_i \in G^{\#}, i = 1, ..., r\}$. The connected components of $\mathcal{H}^{in}_{r,g}(G)$ are in bijection with braid orbits on $\mathcal{E}_r(G)$.

In this thesis we enumerate the connected components of $\mathcal{H}^{in}_{r,g}(G)$ in the cases where $g \leq 2$ and G is a primitive affine group. Our approach uses a combination of theoretical and computational tools. To handle the most computationally challenging cases we develop a new algorithm which we call the Projection-Fiber algorithm.

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Chapter 1

Introduction

Suppose that R is a compact connected Riemann surface of genus g and that

$$\mu \colon R \longrightarrow \mathbb{P}^1 \tag{1.1}$$

is a meromorphic function where $\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$ is the Riemann sphere. For every meromorphic function, there is a number n such that the fiber $\mu^{-1}(p)$ is of size n for all but finitely many points $p \in \mathbb{P}^1$. The number n is called the degree of μ. The function field $C(R)$ is a finite algebraic extension of degree n of $C(\mathbb{P}^1) = \mathbb{C}(z)$. The points b where $|\mu^{-1}(b)| < n$ are called the branch points of μ. Let $B \leq \mathbb{P}^1$ be the set of branch points of μ. It is well known that B is a finite set. So one can label the points in B by $\{b_1, ..., b_r\}$. For any $p \in \mathbb{P}^1 \setminus B$, the fundamental group $\pi_1(\mathbb{P}^1 \setminus B, p)$ acts on $\mu^{-1}(p)$ via path lifting (see Section 2.2). Thus, one obtains a group homomorphism $\rho \colon \pi_1(\mathbb{P}^1 \setminus B, p) \to S_n$. The image of ρ is called the monodromy group of μ and denoted by $Mon(R, \mu)$. It is unique up to conjugacy in S_n. The monodromy group $Mon(R, \mu)$ is the Galois group associated to the Galois closure of the extension $C(R)/C(\mathbb{P}^1)$. The homomorphism is not unique, but certain features are. If R is connected, then $Mon(R, \mu)$ is a transitive subgroup of S_n (see Proposition 2.13). Furthermore, $\pi_1(\mathbb{P}^1 \setminus B, p)$ is generated by all homotopy classes of loops γ_i winding once around the point b_i. The loops γ_i can be chosen so that the generators γ_i satisfy the only relation

$$\gamma_1 \cdot \gamma_2 \cdot ... \cdot \gamma_r = 1. \tag{1.2}$$

Applying ρ to the canonical generators of $\pi_1(\mathbb{P}^1 \setminus B, p)$ gives the generators of a product one generating tuple in $Mon(R, \mu)$. Simplifying notation, we set $x_i = \rho(\gamma_i)$, $1 \leq i \leq r$ and $G = Mon(R, \mu)$. The following statements are true:

$$G = \langle x_1, x_2, ..., x_r \rangle \tag{1.3}$$

$$\prod_{i=1}^{r} x_i = 1,\ x_i \in G^{\#},\ i = 1, ..., r. \tag{1.4}$$

$$\sum_{i=1}^{r} ind\, x_i = 2(n + g - 1), \tag{1.5}$$

where $ind\, x_i$ is the minimal number of transpositions needed to express x_i as a product. Equation (1.5) is known as the Riemann-Hurwitz formula. It gives a numerical relation among g, n and r. Let C_i be the conjugacy class of x_i. Then the multi-set of non trivial conjugacy classes $C = \{C_1, ..., C_r\}$ in G is called the ramification type of the cover μ. While x_i is not uniquely determined by R and μ, the class C_i is. This non-uniqueness will be a very interesting fact that allows us to discuss braid actions.

In light of the above, we say that a transitive subgroup $G \leq S_n$ is a **genus g group** if there exist $x_1, ..., x_r \in G$ satisfying (1.3), (1.4) and (1.5) above, and we call $(x_1, ..., x_r)$ a **genus g system** of G. If the action of G on $\{1, ..., n\}$ is primitive, we call G a **primitive genus g group** and $(x_1, ..., x_r)$ a **primitive genus g system**.

A natural question is which possible groups G can occur for a fixed genus g? For instance, the map $\mu_p : \mathbb{P}^1 \to \mathbb{P}^1$ defined by $\mu_{p,a}(z) = (z-a)^p$ is ramified at a and ∞ with $G \cong \mathbb{Z}_p$. This is precisely what the conjecture made by Guralnick and Thompson from 1990, in [9]. In their paper, they conjectured that the set

$$\mathcal{E}^*(g) = (\bigcup_{(R,\mu)} cf(Mon(R,\mu))) \setminus \{A_n, C_p : \ n > 4, p \text{ a prime } \},$$

is finite for all natural number g, where $cf(G)$ denotes the isomorphism classes of composition factors of G. In 2001, the conjecture was proven by Frohardt and Magaard [7]. The proof of the conjecture suggests that we can actually determine the finite sets $\mathcal{E}^*(0), \mathcal{E}^*(1)$ and $\mathcal{E}^*(2)$ explicitly. Let $\mathcal{E}^{**}(g) = \bigcup_{(R,\mu)} cf(Mon(R,\mu))$, it is well known that for all R, all primes p and all $n > 4$, $C_p \in \mathcal{E}^{**}(g)$ and $A_n \in \mathcal{E}^{**}(g)$. Indeed, for each G which is either a C_p or an A_n, there is a cover $\psi\colon \mathbb{P}^1 \to \mathbb{P}^1$ depending on G such that $Mon(\mathbb{P}^1, \psi) \cong G$ (see [17, p. 3]).

The Fitting subgroup $F(G)$ is the largest nilpotent normal subgroup of G. A component of G is a subnormal subgroup which is perfect and simple module its center. The components of G generate $E(G)$. Two subgroups $F(G)$ and $E(G)$ of a group G generate the generalized Fitting subgroup $F^*(G)$ [12, p. 104].

If μ is an indecomposable function, then μ cannot be written as a composition of two non-linear functions. Indecomposable functions correspond to minimal extensions of $\mathbb{C}(z)$ which correspond to maximal subgroups of the monodromy group G via the Galois correspondence. Thus the action of G on cosets of the subgroup which fix $C(R)$ is primitive. The structure of a primitive group is made explicit by Aschbacher and O'Non-Scott Theorem [2].

Theorem 1.1. *(Aschbacher and O'Non-Scott) Suppose that G is a finite group and M is* *a maximal subgroup of G such that*

$$\bigcap_{g\in G} M^g = 1.$$

Let S be a minimal normal subgroup of G, let L be a minimal normal subgroup of S, and let $\Delta = \{L = L_1, L_2, ..., L_m\}$ be the set of the G-conjugates of L. Then L is simple, $S = \langle L_1, ..., L_m\rangle$, $G = MS$. Furthermore, either

(A) *L is of prime order p; or*

L is a non-abelian simple group and one of the following hold:

(B) *$F^*(G) = S \times R$, where $S \cong R$ and $M \cap S = 1$;*

(C1) *$F^*(G) = S$ and $M \cap S = 1$;*

(C2) *$F^*(G) = S$ and $M \cap S \neq 1 = M \cap L$;*

(C3) *$F^*(G) = S$ and $M \cap S = M_1 \times M_2 \times \cdots \times M_t$, where $M_i = M \cap L_i$, $1 \leq i \leq t$.*

Shih [19], and Guralnick and Thompson [9] respectively showed that the groups occurring in the conclusion of (B) and (C1) do not possess primitive genus zero systems. In case (C2) Aschbacher [3], showed that $F^*(G) = A_5 \times A_5$ in all genus zero examples. In case (C3), L_i is of Lie type of rank one and all elements of $\mathcal{E}^*(0)$ and $\mathcal{E}^*(1)$ were determined by Frohardt, Guralnick and Magaard [6]. Furthermore, they showed $t \leq 2$. In [5] they showed that if $t = 1$, L_i is classical and L_i/M_i is a point action, then $n = [L_i, M_i] \leq 10,000$. That result together with results of Aschbacher, Guralnick and Magaard [1] show that if $t = 1$ and L_i is classical, then $[L_i, M_i] \leq 10,000$. In the case (A), where $F^*(G)$ is abelian and which we refer to as the affine case, Guralnick and Thompson [9], showed that there are only finitely many simple groups occurring as composition factors of a primitive genus zero group. As well, Neubauer [17], showed that there are only finitely many simple groups occurring as composition factors of a primitive genus zero or one group. Finally, Magaard, Shpectorov, and Wang [14], produced a complete list of affine primitive genus zero groups. We are interested in the case where $F^*(G)$ is abelian and G has genus one or two. This case was first considered by Neubauer in his Ph.D. thesis [17]. He classified primitive genus one systems of affine type up to signature. Our goal is to classify primitive genus one and two systems of affine type up to diagonal conjugation by $Inn(G)$ and braid action (see below).

We now describe the work carried out in this thesis.

Chapter 2 is divided into four sections. In Sections 1 and 2 we will give some basic definitions and results in covering space theory. As well, we introduce the monodromy group of a covering map between topological spaces and define a meromorphic function between Riemann surfaces. In Section 3, we describe the connection between the homotopy classes of the fundamental group and the conjugacy classes of elements in G. Finally, we introduce the Riemann Existence Theorem, which tells us the converse of the above statements; that is, for every transitive subgroup G of S_n satisfying (1.3), (1.4) and (1.5) above, and for each discrete subset B of $\mathbb{P}^1$, there is a cover (1.1).

In Section 4 we will build the Hurwitz space $\mathcal{H}_r^A(G)$ as a space of A-equivalence classes of G-covers of the Riemann sphere $\mathbb{P}^1$ with r branched points, where A denotes a subgroup of the automorphism of G. In particular, if $A = Inn(G)$, then the Hurwitz space $\mathcal{H}_r^A(G)$ is denoted by $\mathcal{H}_r^{in}(G)$. Fixing the ramification type C, we are interested in a subset $\mathcal{H}_r^{in}(C)$ of $\mathcal{H}_r^{in}(G)$. It consists of all $[B, \phi]$ where $\phi : \pi_1(\mathbb{P}^1 \setminus B, p) \to G$ and $\phi(\sum_{b_i}) \in C_i$ for $i = 1, ..., r$ (Here $\sum_{b_i}$ is the conjugacy class of $\pi_1(\mathbb{P}^1 \setminus B, p)$) [20, p. 195]. Next we introduce the Nielsen classes in G as follows.

$$\mathcal{N}(C) = \{(x_1, ..., x_r) : G = \langle x_1, ..., x_r \rangle, \prod_{i=1}^r x_i = 1, \exists \sigma \in S_n \text{ such that } x_i \in C_{i\sigma} \text{ for all } i\}.$$

The braid group B_r on r strands is generated by $r-1$ generators (see [11, p. 1]). The braid $\sigma_i \in B_r$ acts on generating tuples $x = (x_1, ..., x_r)$ of a finite group G with $\prod_{i=1}^r x_i = 1$ as follows:

$$\sigma_i : (x_1, ..., x_i, x_{i+1}, ..., x_r) \to (x_1, ..., x_{i+1}, x_{i+1}^{-1} x_i x_{i+1}, ..., x_r) \quad (1.6)$$

for $i = 1, ..., r-1$. If P is a partition of $\{1, ..., r\}$ with stabilizer S_P when S_P is a subgroup of S_r, then the fiber of S_P under the homomorphism $f\colon B_r \to S_r$ is called a parabolic subgroup of B_r. It is denoted by B_P. The parabolic subgroup B_P preserves the order of conjugacy classes in C. We will see that the connected components of $\mathcal{H}_r^{in}(C)$ are parameterized by the B_P-orbits on the $\mathcal{N}(C)$. So computing the connected components of $\mathcal{H}_r^{in}(C)$ is equivalent to computing the B_P-orbits on Nielsen classes $\mathcal{N}(C)$. The latter is equivalent to classify primitive genus g systems up to diagonal conjugation and braid action. We achieve this classification with the aid of the computer algebra system GAP and the **MAPCLASS** package.

Chapter 3 consists of three sections. Section 1 is devoted to describing affine primitive permutation groups, as well as proving some basic results about them. In Section 2 we will find all possible ramification types that fit the Riemann-Hurwitz formula for $AGL(e, p)$ where $g = 1$ or 2. The degrees p^e are given in the main theorems of Neubauer's thesis [17, p. 6]. They serve as a starting point for our work.

Theorem 1.2. *If G is a primitive genus one group of affine type, then one of the following holds:*

(1) $G'' = 1$ *and* $p^e \leq 21$.

(2) $p = 2$ *and* $2 \leq e \leq 8$.

(3) $p = 3$ *and* $2 \leq e \leq 4$.

(4) $p = 5$ *or* 7 *and* $2 \leq e \leq 3$.

(5) $p = 11$ *and* $e = 2$.

Theorem 1.3. *If G is a primitive genus two group of affine type, then one of the following* *holds:*

(1) $G'' = 1$ *and* $p^e \leq 41$.

(2) $p = 2$ *and* $2 \leq e \leq 10$.

(3) $p = 3$ *and* $2 \leq e \leq 6$.

(4) $p = 5$ *and* $2 \leq e \leq 4$.

(5) $p = 7$ *and* $2 \leq e \leq 3$.

(6) $p = 11, 13, 17$ *and* $e = 2$.

In light of Theorem 1.2 (Theorem 1.3), we have exactly 703 (2144) primitive groups of affine type to deal with.

At the end of this chapter, we apply a series of filters to reduce the set of possible ramification types. For example, to show that a particular type cannot generate the group G, we can use Scott's theorem and its corollaries. Furthermore, for small groups, we can compute suitable class structure constants to rule out various potential ramification types. In this case, we show that there are no primitive genus two systems of affine type of degrees $5^4, 7^3, 11^2, 13^2, 3^5$ and 3^6. In Section 3 we will order and label the conjugacy classes of G according to certain rules to distinguish classes represented by elements of equal order.

Chapter 4 is organized as follows. In Sections 1,2 and 3 we introduce some results and methods to eliminate the remaining possible ramification types which are not generating types. We show that there are no primitive genus one systems of affine type of degree 2^8, by using the projection algorithm, and the translation algorithm [7]. Also, we use both algorithms to show that there are no primitive genus two systems of affine type of degree 2^9 and 2^{10}. Furthermore, we find the primitive genus two systems of affine type of degree

2^8 but one can show that $AGL(8,2)$ does not possess primitive genus two systems by using the same arguments as above. In Section 4 we use only the **MAPCLASS** package to compute the generating braid orbits for the given group and the given type.

Chapter 5 contains four sections. In section 1 we prove some results related to a new algorithm. In Section 2 we introduce a new algorithm, which we call the projection-fiber algorithm to deal with a tuple of type C of big length. More precisely we use the projection-fiber algorithm for those types where one cannot compute braid orbits directly by using the MAPCLASS package. The basic idea is to reduce the calculation to two smaller calculations. To do this, let G be an affine primitive group with point stabilizer M. We let $\pi\colon G \to M$ be the natural projection defined by $(x)\pi = m$ for all $x \in G$. Let $\hat{\pi}\colon G^r \to M^r$ be the mapping defined by $(x_1, ..., x_r)\hat{\pi} = ((x_1)\pi, ..., (x_r)\pi)$. The mapping $\hat{\pi}$ sends the tuple t of type C in G onto the tuple $(t)\hat{\pi}$ of type $\bar{C}$ in M. Now we can compute the generating braid orbits O for $\bar{C}$ in M. We pull back a representative tuple from O to G. In this way, we can compute the number of generating braid orbits in G. Finally, we write the function **LiftingQuotientOrbit** for our algorithm. Section 3 is devoted to illustrating the application of the projection-fiber algorithm in some examples.

Chapter 6 contains a summary of our results and the future work.

Our main results, Theorem 1.4 and Theorem 1.5 give the complete classification of primitive genus one and two systems of affine type when $G'' \neq 1$.

Appendix A contains tables representing the results of our computation of primitive genus one systems in affine groups satisfying Theorem 1.2, (2)-(5).

Theorem 1.4. *Up to isomorphism, there exist exactly 85 affine primitive genus one groups that satisfy Theorem 1.2, (2)-(5). The corresponding primitive genus one groups are enumerated in Tables A.2-16.*

The groups which satisfy (1) in Theorem 1.2 are well known [17].

Appendix B contains tables representing the results of our computation of primitive genus two systems in affine groups satisfying Theorem 1.3, (2)-(6).

Theorem 1.5. *Up to isomorphism, there exist exactly 95 affine primitive genus two groups that satisfy Theorem 1.3, (2)-(6). The corresponding primitive genus two groups are enumerated in Tables B.1-25.*

The computations show that there are exactly 6191 (25149) braid orbits of primitive genus one (two) groups of affine type with $G'' \neq 1$. The degrees, and the numbers of the branch points are given in Table 6.1 (Table 6.2).

The other main result is Theorem 1.6, which gives the complete classification of primitive genus two systems of affine type when $G'' = 1$.

Theorem 1.6. *Up to isomorphism, there exist exactly 14 affine primitive genus two groups that satisfy Theorem 1.3, (1). The corresponding primitive genus two groups are enumerated in Table B.26.*

Finally, the next theorems are essentially Theorem 1.2 and Theorem 1.3, except for the improved bounds for e for some primes.

Theorem 1.7. *If G is a primitive genus one group of affine type, then one of the following holds:*

(1) $G'' = 1$ *and* $p^e \leq 16$.

(2) $p = 2$ *and* $2 \leq e \leq 7$.

(3) $p = 3$ *and* $2 \leq e \leq 4$.

(4) $p = 5$ *or* 7 *and* $2 \leq e \leq 3$.

(5) $p = 11$ *and* $e = 2$.

Theorem 1.8. *If G is a primitive genus two group of affine type, then one of the following holds:*

(1) $G'' = 1$ *and* $p^e \leq 25$.

(2) $p = 2$ *and* $2 \leq e \leq 8$.

(3) $p = 3$ *and* $2 \leq e \leq 4$.

(4) $p = 5$ *and* $2 \leq e \leq 3$.

(5) $p = 7$ *or* 17 *and* $e = 2$.

Chapter 2

Background

The aim of this chapter is to communicate the background necessary for the study of the classification of primitive genus g systems for a finite group G up to diagonal conjugation by $Inn(G)$ and braiding. In particular, we shall see that this classification is achieved by computing braid orbits on Nielsen classes.

2.1 Fundamental Groups and Covering Spaces

In this section, we first review some basic properties for the fundamental group and covering spaces. We will investigate the relationship between them.

Definition 2.1. *Let R be a topological space.*

*(i) A **path** $\gamma : [0,1] \to R$ is a continuous map . We call $\gamma(0)$ and $\gamma(1)$ the initial point and the endpoint of γ, respectively. We denote by $C(R, p, q)$ the set of all paths γ in R with initial point p and endpoint q.*

*(ii) The path γ is a **loop** if the initial point and endpoint are equal.*

*(iii) γ is the **constant path** if $\gamma(t) = p$ for all $t \in [0,1]$.*

*(iv) Two paths γ_1 and γ_2 in $C(R, p, q)$ are **homotopic** if there is a continuous map*

$g : [0,1] \times [0,1] \to R$ such that

$$g(0,t) = \gamma_1(t), g(1,t) = \gamma_2(t) \text{ for all } t,$$

$$g(s,0) = p, g(s,1) = q \text{ for all } s.$$

*We call g a **homotopy** between γ_1 and γ_2.*

*(v) The **product** of paths γ_1 and γ_2 with $\gamma_1(1) = \gamma_2(0)$ is defined by $\gamma_1\gamma_2 : [0,1] \to R$,*

$$\gamma_1\gamma_2(t) = \begin{cases} \gamma_1(2t) & if\ t \in [0, \frac{1}{2}] \\ \gamma_2(2t-1) & if\ t \in [\frac{1}{2}, 1]. \end{cases}$$

Remark 2.2. *Indeed, homotopy is an equivalence relation on the set of all loops based at p. The equivalence class of γ under this relation denoted by $[\gamma]$. It is called the homotopy class of γ.*

Definition 2.3. *(1) The **product** of homotopy classes is defined by $[\gamma_1][\gamma_2] = [\gamma_1\gamma_2]$.*

*(2) The **inverse** of the path γ is γ^{-1} with $\gamma^{-1}(t) = \gamma(1-t)$ for all $t \in [0,1]$. Note that $\gamma^{-1}\gamma$ and $\gamma\gamma^{-1}$ are homotopic to a constant path.*

Theorem 2.4. *Let p be a point in a topological space R. Then the set of homotopy classes of loops based at p is a group with respect to the product $[\gamma_1][\gamma_2] = [\gamma_1\gamma_2]$.*

Proof. For a proof see [10, p. 26].

This group is called the **fundamental group** of R with base point p and denoted by $\pi_1(R,p)$.

Definition 2.5. *(i) A topological space R is called **locally path connected** if every point has a path connected neighborhood.*

*(ii) A topological space R is called **path connected** if for every points $p, q \in R$, there*

exists a path γ satisfying $\gamma(0) = p$ and $\gamma(1) = q$.

If R is a path connected, then we emphasize the fact that the fundamental group $\pi_1(R, p)$ is independent of the choice of the base point p. This means that given any points p, q in R, there is an isomorphism between $\pi_1(R, p)$ and $\pi_1(R, q)$. We illustrate this relation in such a way. We can find a path α form p to q, with inverse α^{-1} from q back to p.

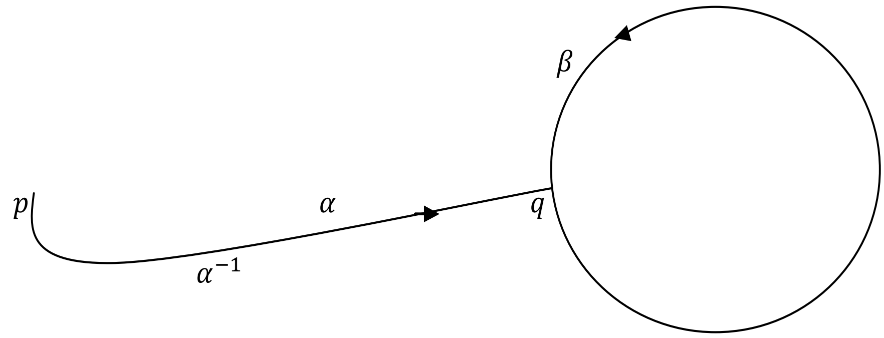

Figure 2.1: An isomorphism of fundamental groups

From Figure 2.1. It is clear that we have two choices, either $(\alpha\beta)\alpha^{-1}$ or $\alpha(\beta\alpha^{-1})$. These two choices are homotopic. So we can define an isomorphism $f \colon \pi_1(R, p) \to \pi_1(R, q)$ by $f(\beta) = [\alpha\beta\alpha^{-1}]$ [10, p. 28]. Furthermore, we call such space is **simply connected** if its fundamental group is trivial.

We would like to mention coverings are considered between general spaces, but we will presume that all spaces are Hausdorff, path connected and locally path connected. This presumption guarantees the basic properties of coverings hold.

Definition 2.6. *A topological space R is called **Hausdorff** if for any two distinct points x and y, there are disjoint open neighborhoods of x and y.*

Definition 2.7. *A surjective continuous map μ between two topological spaces R and T is called a **covering** if for every point $p \in T$ there is a connected neighborhood $V \subseteq T$ such that $\mu^{-1}(V)$ consists of a disjoint collection of open subsets $U_i \subseteq R$, and $\mu\,|_{U_i} \colon U_i \to V$, is a homeomorphism to each of these sets. We call (R, μ) a covering space of T.*

Example 2.8. Let $k \in \mathbb{N} \setminus \{0\}$ and define $\mu \colon \mathbb{C} \setminus \{0\} \to \mathbb{C} \setminus \{0\}$ by $\mu(z) = z^k$. We want to show that μ is a covering. Let $a \in \mathbb{C} \setminus \{0\}$ and pick $b \in \mathbb{C} \setminus \{0\}$ with $\mu(b) = a$. Since μ is a local homeomorphism, there are open neighbourhoods V_b and U_a such that $\mu/V_b \to U_a$ is a homeomorphism. So $\mu^{-1}(U_a) = V_b \cup wV_b \cup ... \cup w^{k-1}V_b$ where $w = e^{\frac{2\pi i}{k}}$ is a k-th root of unity. It is obvious that the sets $V_j = w^j V_b$ for $j = 0, ..., k-1$ are pairwise disjoint and each $\mu/V_j \to U_a$ is a homeomorphism. Hence μ is a covering map.

Definition 2.9. *Let R_1, R_2 and T be topological spaces.*

*(i) Two coverings $\mu_1 \colon R_1 \to T$ and $\mu_2 \colon R_2 \to T$ are **equivalent** if there is a homeomorphism $h \colon R_1 \to R_2$ such that $\mu_2 \circ h = \mu_1$.*

*(ii) A map $\mu \colon R \to T$ is called a **ramified** covering of T if there is a discrete subset B of T such that the restriction map $\bar{\mu} \colon R \setminus \mu^{-1}(B) \to T \setminus B$ of μ is a covering.*

*(iii) A covering space (R, μ) of T is called a **universal covering space** if R is simply connected. Moreover, it is unique up to isomorphism.*

*(iv) A map $\mu \colon R \to T$ is called a **proper** if the inverse image of any compact subset of T is compact.*

*(v) Let (R, μ) be a covering space of T and γ be a path in T. A **lift** of γ in R is a path $\tilde{\gamma}$ in R such that $\mu \circ \tilde{\gamma} = \gamma$.*

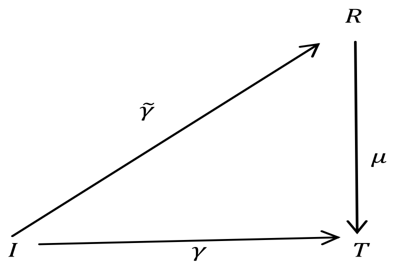

Figure 2.2: The path lifting

Proposition 2.10. *Let (R, μ) be a covering space of T.*

(1) *Let γ be a path in T with initial point p. For each $b \in \mu^{-1}(p)$, there exists a unique lift $\tilde{\gamma}$ with the initial point b.*

(2) *Let $\gamma \in C(T, p, q)$. For each $b \in \mu^{-1}(p)$, let γb be the endpoint of the lift of γ with initial point b. Then the map $b \to \gamma b$ is a bijection between $\mu^{-1}(p)$ and $\mu^{-1}(q)$.*

Proof. For a proof see [20, p. 64 and p. 68].

Part (2) of Proposition 2.10 implies that all fibers of μ have the same cardinality, called the degree of μ. It can be infinite, but in this thesis, we study coverings of a finite degree. To guarantee that $\mu^{-1}(p)$ is finite we need to assume that the topological spaces R and T are compact.

Theorem 2.11. *Let (R, μ) be a covering space of T, $p \in R$ and $q = \mu(p)$. Then the induced map $\mu_* \colon \pi_1(R, p) \to \pi_1(T, q)$ is a monomorphism.*

Proof. For a proof see [10, p. 34].

The fundamental group has an important application, namely there is a one to one correspondence between conjugacy classes of subgroups of $\pi_1(T, p)$ and the set of isomorphism classes of connected covering spaces (R, μ) of T as follows.

Theorem 2.12. *Let T and R be connected spaces and $q \in T$ be a base point. Then there is a one to one correspondence between equivalence classes of coverings $\mu \colon R \to T$ and conjugacy classes of subgroups $\pi_1(T, q)$.*

Proof. For a proof see [4, p. 47].

2.2 Monodromy Groups and Riemann Surfaces

In this section, we introduce the concept of the monodromy group of a covering map and explain how the covering map leads to the Galois covering. At the end of the section, we shall discuss a Riemann surface and meromorphic functions.

Suppose that μ: $R \to T$ is a covering map, the fiber $\mu^{-1}(p)$ is of size n and label the points in $\mu^{-1}(p)$ by $\{y_1, ..., y_n\}$. In light of Proposition 2.10, especially part (1) it follows that every loop γ in $\pi_1(T, p)$ based at p can be lifted to n paths $\tilde{\gamma}_1, ..., \tilde{\gamma}_n$, where $\tilde{\gamma}_i$ is the unique lift of γ, which begins at y_i; that is, $\tilde{\gamma}_i(0) = y_i$ for each i. Consider the endpoints $\tilde{\gamma}_i(1)$ which must be in $\mu^{-1}(p)$, since γ is a loop. So each $\tilde{\gamma}_i(1)$ is an y_j for some j. Denote $\tilde{\gamma}_i(1)$ by $y_{\sigma(i)}$. Then σ is a permutation of the indices $\{1, ..., n\}$ of $\pi_1(T, p)$ on $\mu^{-1}(p)$, and it only depends on the homotopy class $[\gamma]$. This is known as the **monodromy action**. It gives a group homomorphism $\rho : \pi_1(T, p) \to S_n$, and the image of this homomorphism is called the **monodromy group** of μ and denoted by $Mon(R, \mu)$ [16, p. 86].

Proposition 2.13. *Let $\rho : \pi_1(T, p) \to S_n$ be the monodromy representation of a covering map $\mu : R \to T$ of degree n. Then $Mon(R, \mu)$ is a transitive subgroup of S_n.*

Proof. Fix two indices i and j, and consider the two points y_i and y_j in $\mu^{-1}(p)$. Since R is connected, we may find a path $\tilde{\gamma}$ on R starting at y_i and ending at y_j. Let $\gamma = \mu \circ \tilde{\gamma}$ be the image of $\tilde{\gamma}$ in T. Note that γ is a loop in T based at p, since both y_i and y_j map to p under μ. Then by construction we have that $\rho([\gamma])$ is a permutation which sends i to j. □

Lemma 2.14. *Let $\mu : R \to T$ be a covering and $\mu^{-1}(p) = \{y_1, ..., y_n\}$. Then*

$$Mon(R, \mu) \cong \pi_1(T, p)/\bigcap_i Stab(y_i) \tag{2.1}$$

Proof. Recall that the group homomorphism $\rho : \pi_1(T, p) \to S_n$ is given by the monodromy group action of $\pi_1(T, p)$ on $\mu^{-1}(p)$. For any $y_i \neq y_j$ there exists $\rho(\gamma)$ in $Mon(R, \mu)$ such that $\rho(\gamma)y_i = y_j$ because the action is transitive. Since $Stab(y_i) = \{\gamma \in \pi_1(T, p) : \rho(\gamma)y_i = y_i\}$, then $Ker\,\rho = \bigcap_i Stab(y_i)$. Hence $Mon(R, \mu) = im\,\rho \cong \pi_1(T, p)/Ker\,\rho = \pi_1(T, p)/\bigcap_i Stab(y_i)$. □

Definition 2.15. *A homeomorphism $\alpha\colon R \to R$ is called a* ***deck transformation*** *with respect to a covering (R, μ) if $\mu\alpha = \mu$. By $Deck(\mu)$ we denote the set of all deck transformations of μ. This is a group under composition.*

It is clear that the group $Deck(\mu)$ acts on each fiber $\mu^{-1}(p)$, $p \in T$. If we pick an element $a \in \mu^{-1}(p)$, then the image of a under α is still in the same fiber $\mu^{-1}(p)$, for each $\alpha \in Deck(\mu)$. On the other hand, the monodromy action of the fundamental group $\pi_1(T, p)$ also acts on $\mu^{-1}(p)$. The following result tells us these two actions commute.

Proposition 2.16. *Let (R, μ) be a covering space of T. The action of $Deck(\mu)$ on $\mu^{-1}(p)$ commutes with the monodromy action of $\pi(T, p)$.*

Proof. For a proof see [20, p. 68].

Lemma 2.17. *Let $\mu : R \to T$ be a covering. For any q in T and any $p \in \mu^{-1}(q)$, the*

$$Deck(\mu) \cong Norm(\mu_*(\pi_1(R, p)))/\mu_*(\pi_1(R, p)) \tag{2.2}$$

where $Norm(\mu_*(\pi_1(R, p)))$ denotes the normalizer of the subgroup $\mu_*(\pi_1(R, p))$ in $\pi_1(T, q)$.

Proof. For a proof see [15, p. 134].

Lemma 2.18. *Let $\mu : R \to T$ be a finite covering. Then the equality $Deck(\mu) = Mon(R, \mu)$ holds if and only if $\mu_*(\pi_1(R, p))$ is a normal subgroup of $\pi_1(T, q)$.*

Proof. This follows from Lemma 2.14 and Lemma 2.17. □

Definition 2.19. *If (R, μ) is a covering space of T and $Deck(\mu)$ acts transitively on some fiber $\mu^{-1}(p)$, $p \in T$, then we say that a covering map is a* ***Galois covering****.*

Corollary 2.20. *Let (R, μ) be a universal Galois covering of T. Then $Mon(R, \mu) \cong \pi_1(T, q)$.*

Proof. This follows from Lemma 2.17 and 2.18. □

The "only if "part in Lemma 2.18 is equivalent to say that a covering map $\mu\colon R \to T$ is Galois covering. For the rest of our work, we can deal with the monodromy group of the cover μ instead of the deck transformation group by using Lemma 2.18.

Proposition 2.21. *Let $\mu\colon R \to T$ be a Galois covering and G be the monodromy group. Let $a \in R$ and $p = \mu(a)$. There is a unique surjective homomorphism $\phi_a\colon \pi_1(T,p) \to G$ such that $\phi_a([\gamma])$ maps $[\gamma]a$ to a, for each $[\gamma] \in \pi_1(T,p)$(Recall that $[\gamma]a$ is the endpoint of the lift of γ with initial point a).*

Proof. For a proof see [20, p. 69].

In the above proposition if the group G has trivial center, then for any two points $a, \bar{a}$ in the fiber $\mu^{-1}(p)$, we have $\phi_a = \phi_{\bar{a}}$ if and only if $a = \bar{a}$. Hence, the pair (p, ϕ_a) represent the covering $\mu\colon R \to T$. This property allows us to build the Hurwitz space for our study in Section 2.4.

We conclude this section by defining a Riemann surface and a meromorphic function and discuss some of their properties. Our approach uses topology to define a Riemann surface.

Definition 2.22. *(i) A **complex chart** on a topological space R is a homeomorphism h from an open set U in R onto an open set V in the complex plane. We say that two complex charts $h_1 : U_1 \to V_1$ and $h_2 : U_2 \to V_2$ are **compatible** if either $U_1 \cap U_2 = \phi$ or $h_2 \circ h_1^{-1}\colon h_1(U_1 \cap U_2) \to h_2(U_1 \cap U_2)$ is holomorphic.*

*(ii) A collection $\mathbb{A} = \{h_\alpha\colon U_\alpha \to V_\alpha\}$ of pairwise compatible complex charts whose domains cover R is called a **complex atlas**.*

(iii) Two complex atlases are equivalent if every chart of one is compatible with every chart of the other.

*(iv) An equivalence class of atlases on a topological space R is called a **complex structure** on R.*

We now have enough material to give the definition of a Riemann surface.

Definition 2.23. *A connected Hausdorff space R together with a complex structure is* *called a* ***Riemann surface****.*

Example 2.24. Consider the Riemann sphere $\mathbb{P}^1$. As a set this is $\mathbb{C}$ with an extra point, ∞. We make $\mathbb{P}^1$ to a Riemann surface with atlas of two charts $U_1 = \mathbb{C}$ and $U_2 = \mathbb{P}^1 \setminus \{0\}$. Let $h_1 \colon U_1 \to \mathbb{C}$, $h_1(z) = z$ and $h_2 \colon U_2 \to \mathbb{C}$,

$$h_2(z) = \begin{cases} \frac{1}{z} & if\ z \neq \infty \\ 0 & if\ z = \infty. \end{cases}$$

We see that $U_1 \cap U_2 = \mathbb{C} \setminus \{0\}$ and the map $h_2 \circ h_1^{-1}$ is equal to $\frac{1}{z}$ which is holomorphic on $\mathbb{C} \setminus \{0\}$.

Definition 2.25. *Let R and T be Riemann surfaces. A map $\mu : R \to T$ is called* holomorphic *at $p \in R$ if there exist complex charts $h_1 \colon U_1 \to V_1$ on R and $h_2 \colon U_2 \to V_2$ on T, with $\mu(p) \in U_2$, such that the composition $h_2 \circ \mu \circ h_1^{-1}$ is holomorphic at $h_1(p)$. We say that μ is holomorphic if it is holomorphic on all of R.*

Proposition 2.26. *Let R and T be Riemann surfaces and let $\mu : R \to T$ be a non constant holomorphic map defined at $r \in R$. Then there is a unique positive integer n which satisfies the following properties: for every complex chart $h_2 \colon U_2 \to V_2$ on T centered at $\mu(r)$, there exists a complex chart, $h_1 \colon U_1 \to V_1$ on R, cantered at r such that $h_2(\mu(h_1^{-1}(z))) = z^n$.*

Proof. For a proof see [16, p. 44].

Definition 2.27. *(i) The multiplicity of μ at r, denoted by $mult_r(\mu)$, is the unique integer n in Proposition 2.26.*

(ii) A point $r \in R$ is a ramification point of μ if $mult_r(\mu) \geq 2$.

(iii) The image of a ramification point r in T is called a ***branch point*** *of μ.*

Proposition 2.28. *Let R and T be Riemann surfaces and let $\mu : R \to T$ be a non-constant holomorphic map then for every $t \in T$, the fiber $\mu^{-1}(t)$ is a discrete subset of*

R. In particular, if R and T are compact, then $\mu^{-1}(t)$ is a nonempty finite set for every $t \in T$.

Proof. For a proof see [16, p. 41].

For a Riemann surface R, a **meromorphic** function on R is a non-constant holomorphic map μ from R to the Riemann sphere $\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$. For every meromorphic function, there is a number n, such that the fiber $\mu^{-1}(p)$, has size n for all but finitely many points $p \in \mathbb{P}^1$. The number n is called the **degree** of μ.

Definition 2.29. *A branched cover of the Riemann sphere $\mathbb{P}^1$ is a pair (R, μ) where R is a compact Riemann surface and $\mu \colon R \to \mathbb{P}^1$ is a meromorphic function.*

2.3 Riemann's Existence Theorem

In this section, we describe finite Galois coverings of the punctured Riemann sphere $\mathbb{P}^1$ (that is the Riemann sphere with a finite number of points removed) with monodromy group G and discuss the relationship between the elements of the monodromy group G and the elements in the fundamental group of $\mathbb{P}^1 \setminus B$. Finally, we turn our attention to Riemann's Existence Theorem.

First, we are going to consider coverings of the disc without a center. More precisely, we study the behavior close to a ramification point.

Lemma 2.30. *Let $\mathbb{K}(r) = \{z \in \mathbb{C} : 0 < |z| < r\}$ for any $r > 0$. Let $n \in \mathbb{N}$. The map $f_n \colon \mathbb{K}(r^{\frac{1}{n}}) \to \mathbb{K}(r)$, $z \to z^n$ is Galois and its monodromy group G is cyclic of order n.*

Proof. For a proof see [20, p. 70].

Remark 2.31. *(1) In Lemma 2.30 if $n > 1$ the map f_n is not one to one in any neigbourhood of 0. We have to remove 0 from the disc.*

(2) Each $\mathbb{K}(r)$ is homeomorphic to $\mathbb{K}(1)$ and we can study $\mathbb{K}(1)$ instead of $\mathbb{K}(r)$.

Lemma 2.32. *Let F be connected and $f \colon F \to \mathbb{K}(r)$ be a Galois covering of degree n.*

Then

(1) *f is equivalent to f_n.*

(2) *The group $G = Deck(f)$ is cyclic of order n and it is generated by x_F.*

Proof. For a proof see [20, p. 71].

Here, we remove finitely many points from R to obtain an unramified Galois covering of $\mathbb{P}^1 \setminus B$. Assume that $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ is a finite Galois covering. Let b be a branch point. Since μ is onto there exists $r \in R$ such that $\mu(r) = b$. There are neighborhoods U_r and V_b of r and b respectively, which are homeomorphic to the unit disc $\mathbb{K}(1)$. Lemma 2.32 tells us μ behaves like the map f around b.

For the rest of this thesis $B = \{b_1, ..., b_r\}$ denotes a finite subset of $\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$. We define the open disc $D_s(p)$, around $p \in \mathbb{P}^1$ with radius $s > 0$ as follows:

$$D_s(p) = \begin{cases} \{z \in \mathbb{C} : |z - p| < s\} & if\ p \in \mathbb{C} \\ \{z \in \mathbb{C} : |z| > s^{-1} \cup \infty\} & if\ p = \infty. \end{cases}$$

The open sets $D_s(p)$ are a basis for the topology of $\mathbb{P}^1$.

Proposition 2.33. *Let $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ be a finite Galois covering with monodromy group G and $D^* = D_s(b) \setminus \{b\}$ for some fixed $b \in B$. Then*

(1) *The group G permutes the components of $\mu^{-1}(D^*)$ transitively.*

(2) *If F is one of the components of $\mu^{-1}(D^*)$, and G_F is the stabilizer of F in G, then G_F is cyclic.*

Proof. For a proof see [20, p. 72].

We call x_F the distinguished generator if G_F generated by x_F. Let b be a branch point. We denote by C_b the conjugacy class in G containing x_F.

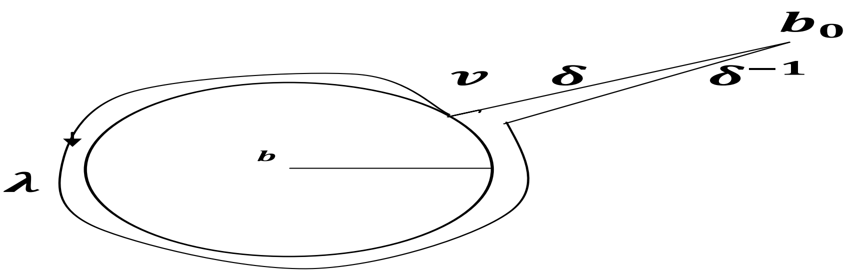

Figure 2.3: The conjugacy class in the group $\pi(\mathbb{P}^1 \setminus B, b_0)$

Figure 2.3 describes the conjugacy class corresponding to a branch point $b \in B$. It is clear that the path $\gamma = \delta^{-1}\lambda\delta$ starting from b_0 to point v on the boundary of $D_s(b)$ and winding once counterclockwise around $D_s(b)$ by λ. These elements $[\gamma]$ run through a conjugacy class of $\pi(\mathbb{P}^1 \setminus B, b_0)$ when δ and v vary. We shall use the notation $\sum_b$ to denote this conjugacy class.

For the rest of this section, all discs $D_s(b)$ are assumed to be sufficiently small, so that $D_s(b) \cap B = \{b\}$ and $\gamma = \delta^{-1}\lambda\delta$ represents elements $[\gamma]$ in $\pi(\mathbb{P}^1 \setminus B, b_0)$, where λ and δ are chosen in the special way as above.

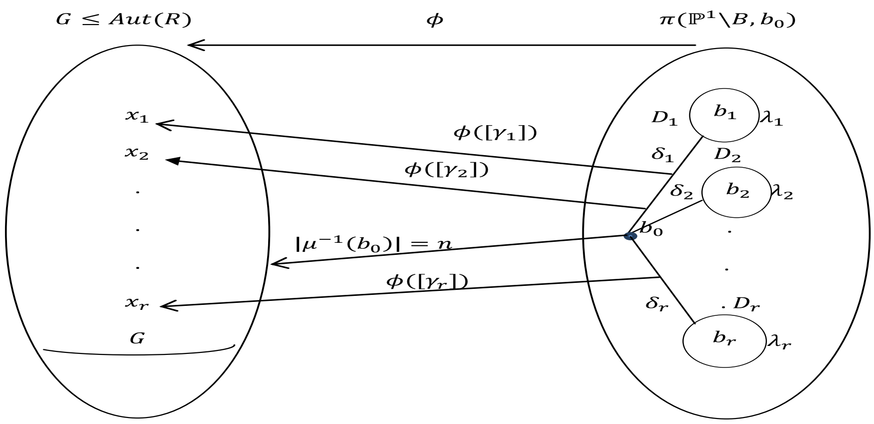

Figure 2.4: Elements in $\pi(\mathbb{P}^1 \setminus B, b_0)$ correspond to conjugacy classes of elements in G

In terms of Figure 2.3, the fundamental group $\pi(\mathbb{P}^1 \setminus B, b_0)$ of the punctured sphere is generated by the homotopy classes $[\gamma_i]$ which are pictured as in Figure 2.4. In addition,

Figure 2.4 shows that these homotopy classes correspond to conjugacy classes of elements in G.

The next proposition describes the correspondence between the homotopy classes in the fundamental group and conjugacy classes of elements in G.

Proposition 2.34. *Assume that $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ is a finite Galois covering. Fix some $b \in B$. Let $f\colon D^* \to \mathbb{K}(r)$ be the homeomorphism defined by*

$$f(z) = \begin{cases} \frac{1}{z} & if\ b = \infty \\ z - b & if\ b \neq \infty. \end{cases}$$

Let $b^* \in D^*$, $\bar{b} = f(b^*)$ and $\lambda(t) = f^{-1}(\bar{b}e^{2\pi it})$, a loop in D^* based at b^*. Let $c \in \bar{R}$ and $b_0 = \mu(c)$. Let δ be a path in $\mathbb{P}^1 \setminus B$ joining b^* and b_0. Then $\gamma = \delta^{-1}\lambda\delta$ is a loop in $\mathbb{P}^1 \setminus B$ based at b_0 and $\phi_c : \pi_1(\mathbb{P}^1 \setminus B, b_0) \to G$ sends $[\gamma]$ to an element of C_b.

Proof. For a proof see [20, p. 73].

Definition 2.35. *The surjective homomorphism $\phi\colon \pi_1(\mathbb{P}^1 \setminus B, b_0) \to G$ is called admissible if $\phi(\sum_b)$ is non-trivial for each $b \in B$.*

Definition 2.36. *The multi-set of non trivial conjugacy classes $C = \{C_{b_1}, ..., C_{b_r}\}$ in G is called the ramification type (or simply type) of the cover μ and we use the abbreviation (RT) for the ramification type.*

Lemma 2.37. *Let $\{D_1, ..., D_r\}$ be a set of distinct discs and d be a fixed point in $\mathbb{P}^1 \setminus \{\infty\}$. Let y_i be a point on the boundary of D_i satisfying the following condition. Let λ_i be a loop based on y_i going along the boundary of D_i and δ_i be the ray from y_i passing through d to ∞. The set of homotopy classes $\{[\gamma_1], ..., [\gamma_r]\}$ are disjoint pairwise. Then $\pi_1(\mathbb{P}^1 \setminus B, \infty)$ generated by $\{[\gamma_1], ..., [\gamma_r]\}$.*

Proof. For a proof see [20, p. 76].

The image of the set $\{[\gamma_1], ..., [\gamma_r]\}$ of distinguished generators of $\pi_1(\mathbb{P}^1 \setminus B, \infty)$ under the homomorphism ϕ, gives a set of distinguished generators $\{x_1, ..., x_r\}$ of G. We call $\{x_1, ..., x_r\}$ a generating tuple of G.

Remark 2.38. *We emphasize the fact that the existence of the generating tuple $x_1, x_2, ..., x_r$ is independent of the labeling of the branch points. For instance, $x_1, x_3, x_3^{-1}x_2x_3, x_4, ..., x_r$ is also a generating tuple of G with product 1. However the classes C_2 and C_3 are switched.*

Proposition 2.39. *Let $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ be a finite Galois covering and $D = D_s(b)$ be a disc where $b \in B$ and $D^* = D \setminus \{b\}$. The fiber $\mu^{-1}(D^*)$ is the union of the set of components F. For any small disc $\bar{D}$ inside D^*, there is a one to one correspondence between the component F and the component $\bar{F}$ inside $\mu^{-1}(\bar{D})$, given by inclusion. That is for each F there is exactly one $\bar{F}$ with $\bar{F} \subseteq F$ and vice versa. We call F a **circular component** of level s over b.*

Proof. For a proof see [20, p. 72].

Definition 2.40. *Let $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ be a finite Galois covering. We say that $s > 0$ is sufficiently small if $D_s(b) \cap B = \{p\}$ for all $b \in B$. Now fix $b \in B$. We define a relation on the set of circular components over b (of sufficiently small level): $F \cong \hat{F}$ if $F \subset \hat{F}$ or $\hat{F} \subset F$. By the above proposition this is an equivalence relation. The equivalence classes are called the **ideal points** over b.*

We now reverse the procedure, starting with an unramified Galois covering $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ of given type C by adding finitely many ideal points to $\bar{R}$, then we obtain a compact Riemann surface R. In other words, the Galois covering $\mu\colon \bar{R} \to \mathbb{P}^1 \setminus B$ can be extended to a branched cover $\mu\colon R \to \mathbb{P}^1$. See [20, p. 84-89] for details.

Finally, we can state the analytic version of the Riemann Existence Theorem as follows:

Theorem 2.41. *Let B be a discrete subset of the Riemann sphere $\mathbb{P}^1$. Given $n > 1$ and a transitive permutation representation $\rho : \pi_1(\mathbb{P}^1 \setminus B, b_0) \to S_n$, there is a Riemann surface R and a proper holomorphic map $\mu : R \to \mathbb{P}^1$ which realises ρ as its monodromy* *homomorphism. Morover R and μ are unique up to equivalence.*

Proof. For a proof see [4, p. 45].

The theory of permutation groups can be interpreted into the geometric problem via Theorem 2.41 and converse is true. The genus of R can be calculated by the Riemann-Hurwitz formula. We can state it as follows.

Theorem 2.42. *(Riemann-Hurwitz Formula) Let R be a Riemann surface of genus g and $\mu\colon R \to \mathbb{P}^1$ be a meromorphic function of degree n. Then*

$$2(n+g-1) = \sum_{r\in R}[mult_r(\mu) - 1]. \tag{2.3}$$

Proof. For a proof see [16, p. 52].

For all but the the ramification points of μ the values of $mult_r(\mu)$ will be equal to 1, so the summation need only be taken over the ramification points.

The following definition gives a method for computing the index of a permutation.

Definition 2.43. *Let G be a group acting on a finite set Ω. If $x \in G$, then the index of x is defined by $ind\, x = |\Omega| - orb\, x$ where $orb\, x$ is the number of orbits of x on Ω.*

For a branch point $b_i \in \mathbb{P}^1$ the homotopy class of loop around b_i is $[\gamma_i]$. Then the monodromy representation $\rho : \pi_1(\mathbb{P}^1 \setminus B, p) \to S_n$ gives the permutation $x_i = \rho([\gamma_i])$. If its ramification index is $mult_{r_i}(\mu)$, we have

$$\sum_{r_{ij}\in\mu^{-1}(b_i)} [mult_{r_{ij}}(\mu) - 1] = ind\, x_i. \tag{2.4}$$

This relationship arises as the cycles in the cycle decomposition of x_i correspond to the points in $\mu^{-1}(b_i)$. Therefore we may modify Equation 2.5 to give:

$$2(n+g-1) = \sum_{i=1}^{r} ind\, x_i. \tag{2.5}$$

2.4 Hurwitz Spaces and the Braid Action on Nielsen Classes

As usual $Inn(G)$ and $Aut(G)$ denote the inner-automorphism and automorphism groups of a group G respectively. $Z(G)$ denotes the center of G and A denotes a subgroup of $Aut(G)$. We denote by O_r, the space of subsets of $\mathbb{C}$ of cardinality r. Recall that Proposition 2.21 allows us to rebuild the Riemann surface R as a set of all pairs (p, ϕ_a).

Definition 2.44. *Let $B \in O_r$ and $\phi : \pi_1(\mathbb{P}^1 \setminus B, \infty) \to G$ be admissible. Then we say that two pairs (B, ϕ) and $(B^{'}, \phi^{'})$ are* A*-equivalent if and only if $B = B^{'}$ and $\phi^{'} = a \circ \phi$ for some $a \in \mathrm{A}$.*

Let $[B, \phi]_{\mathrm{A}}$ denote the A-equivalence class of (B, ϕ). The set of equivalence classes $[B, \phi]_A$ is denoted by $\mathcal{H}_r^{\mathrm{A}}(G)$ and is called the Hurwitz space of G-covers.

To define the topology of the Hurwitz space $\mathcal{H}_r^{\mathrm{A}}(G)$, we assume that $B = \{b_1, ..., b_r\} \in O_r$ and $D_1, ..., D_r$ be distinct discs of $b_1, ..., b_r$. A neighborhood of $[B, \phi]_{\mathrm{A}}$ is the set of all $[B^{'}, \phi^{'}]_{\mathrm{A}}$ where $B^{'} = \{b_1^{'}, ..., b_r^{'}\}$ such that $b_i^{'} \in D_i$ for $i = 1, ..., r$ and $\phi^{'}$ is the composition of ϕ with the canonical isomorphisms

$$\pi_1(\mathbb{P}^1 \setminus B^{'}, \infty) \to \pi_1(\mathbb{P}^1 \setminus (D_1 \cup D_2 \cup ... \cup D_r), \infty) \longrightarrow \pi_1(\mathbb{P}^1 \setminus B, \infty).$$

This gives a topology on $\mathcal{H}_r^{\mathrm{A}}(G)$. In particular if $\mathrm{A} = Inn(G)$, then the Hurwitz space $\mathcal{H}_r^{\mathrm{A}}(G)$ is denoted by $\mathcal{H}_r^{in}(G)$.

The next result will show the investigation.

Lemma 2.45. *The map $\Psi_{\mathrm{A}} : \mathcal{H}_r^{\mathrm{A}}(G) \longrightarrow O_r$, $\Psi_{\mathrm{A}}([B, \phi])) = B$ is covering.*

Proof. For a proof see [20, p. 184].

The topology on the Hurwitz space $\mathcal{H}_r^{\mathrm{A}}(G)$ is completely determined by the action of the fundamental group $\pi_1(O_r, B_0)$ where $B_0 = \{b_1, ..., b_r\}$ is the base point in O_r via path lifting (see Proposition 2.10). To be precise, to describe this action in more detail, we need a parametrization on $\Psi_{\mathrm{A}}^{-1}(B_0)$. The fiber $\Psi_{\mathrm{A}}^{-1}(B_0) = \{[B_0, \phi]_{\mathrm{A}} : \phi \colon \pi_1(\mathbb{P}^1 \setminus B_0, \infty) \longrightarrow G$ is admissible $\}$. This ϕ gives a product one generating tuple $(x_1, ..., x_r)$ of G.

Define $\mathcal{E}_r(G) = \{(x_1, ..., x_r) : G = \langle x_1, ..., x_r\rangle, \prod_{i=1}^{r} x_i = 1, x_i \in G^{\#}, i = 1, ..., r\}$. The group A acts on $\mathcal{E}_r(G)$ via sending $(x_1, ..., x_r)$ to $(x_1^a, ..., x_r^a)$, for $a \in \mathrm{A}$. Let $\mathcal{E}_r^{\mathrm{A}}(G) = \mathcal{E}_r(G)/\mathrm{A}$ to be the set of A-orbits. In particular, if $\mathrm{A} = Inn(G)$, then we have $Inn(G) \cong G/Z(G)$. Therefore $\mathcal{E}_r^{in}(G)$ is the set of G-orbits.

We see that each tuple in $\mathcal{E}_r^{\mathrm{A}}(G)$ corresponds to an admissible cover $\phi : \pi_1(\mathbb{P}^1 \backslash B, b_0) \longrightarrow G$ which also corresponds to a branched cover $\mu : R \longrightarrow \mathbb{P}^1$ via Theorem 2.41.

Lemma 2.46. *We obtain a bijection* $\Psi_{\mathrm{A}}^{-1}(B_0) \longrightarrow \mathcal{E}_r^{\mathrm{A}}(G)$ *by sending* $[B_0, \phi]_{\mathrm{A}}$ *to the generators* $(x_1, ..., x_r)$ *where* $x_i = \phi([\gamma_i])$ *for* $i = 1, ..., r$.

Proof. For a proof see [20, p. 194].

Let C be a fixed ramification type in G, then the subset $\mathcal{H}_r^{\mathrm{A}}(C)$ of $\mathcal{H}_r^{\mathrm{A}}(G)$ consists of all $[B, \phi]_{\mathrm{A}}$ with $B = \{b_1, ..., b_r\}$, $\phi \colon \pi_1(\mathbb{P}^1 \setminus B, \infty) \longrightarrow G$ and $\phi(\sum_{b_i}) \in C_i$ for $i = 1, ..., r$. It is a union of connected components in $\mathcal{H}_r^{A}(G)$.

Next we discuss the connection between braid orbits and connected components of the Hurwitz space. Here we give an algebraic definition of the braid group B_r, for a positive integer r.

Definition 2.47. *For* $r \geq 2$*, the Artin braid group* B_r *is generated by* $r-1$ *generators* $\sigma_1, \sigma_2, ..., \sigma_{r-1}$ *that satisfy the following relations:*

$$\sigma_i \sigma_j = \sigma_j \sigma_i \tag{2.6}$$

for all $i, j = 1, 2, ..., r-1$ with $|i-j| \geq 2$, and

$$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \tag{2.7}$$

for $i = 1, 2, ..., r-2$. These relations are known as the braid relations.

Lemma 2.48. *Given a group homomorphism $f \colon B_r \to G$, then the elements $f(\sigma_1), f(\sigma_2)$, ..., $f(\sigma_{r-1})$ of a group G satisfy the braid relations.*

Proof. For a proof see [11, p. 2].

Lemma 2.49. *If $x_1, x_2, ..., x_{r-1}$ are elements of a group G satisfying the braid relations then there is a unique group homomorphism $f \colon B_r \to G$ such that $x_i = f(\sigma_i)$ for all $i = 1, 2, ..., r-1$.*

Proof. For a proof see [11, p. 2].

We apply Lemma 2.49 to the symmetric group $G = S_r$. Consider the simple transpositions $s_1, ..., s_{r-1} \in S_r$, where s_i transposes i and $i+1$ and fixes all the other elements of $\{1, ..., r\}$. Note that the simple transpositions satisfy the braid relations and generate S_r. This transforms the Artin presentation of the braid group into the Coxeter presentation of the symmetric group:

$$S_r = \left\langle s_1, \ldots, s_{r-1} | s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}, s_i s_j = s_j s_i \text{ for } |i-j| \geq 2, s_i^2 = 1 \right\rangle.$$

The kernel of the homomorphism f (see Lemma 2.49) is called the **pure braid group** and denoted by B_r^P. The pure braid group is generated by the pure braid elements as follows:

$$\sigma_{ij} = \sigma_{j-1} \sigma_{j-2} ... \sigma_{i+1} \sigma_i^2 \sigma_{i+1}^{-1} ... \sigma_{j-2}^{-1} \sigma_{j-1}^{-1}$$

$$= \sigma_i^{-1} ... \sigma_{j-2}^{-1} \sigma_{j-1}^2 \sigma_{j-2} ... \sigma_i.$$

The braids $\{\sigma_{ij}\}_{i,j}$ are conjugate to each other in B_r [11, p. 19].

The braid σ_i acts on generating tuples $x = (x_1, ..., x_r)$ of a finite group G with $\prod_{i=1}^{r} x_i = 1$ as follows:

$$\sigma_i : (x_1, ..., x_i, x_{i+1}, ..., x_r) \to (x_1, ..., x_{i+1}, x_{i+1}^{-1} x_i x_{i+1}, ..., x_r) \tag{2.8}$$

for $i = 1, ..., r-1$.

Lemma 2.46 tells us, the action of B_r on $\Psi_{\mathrm{A}}^{-1}(B_0)$ yields an action on $\mathcal{E}_r^A(G)$. That is, the braid group B_r acts on $\mathcal{E}_r^A(G)$ via formula (2.8). Further, the braid action commutes with the action of $Aut(G)(\ Inn(G))$ on tuples.

The **braid orbit** of x is the smallest set of tuples which contains x and is closed under the operations (2.8).

Let $C = \{C_1, ..., C_r\}$ be the ramification type of $x = (x_1, ..., x_r)$ where $C_i = \{x_i^G\}$ is the conjugacy class of G containing x_i, $1 \leq i \leq r$. Then each braid σ permutes the classes in C, inducing the permutation $f(\sigma_i)$ on the indices. We can assume without loss of generality that the elements in the type C are ordered in such a way that if $C_i = C_j$, then for all k such that $i < k < j$, we have

$$C_k = C_i. \tag{2.9}$$

We now introduce the **Nielsen classes** as follows. For a ramification type $C = \{C_1, ..., C_r\}$, $\mathcal{N}(C) = \{(x_1, ..., x_r) : G = \langle x_1, ..., x_r \rangle, \prod_{i=1}^{r} x_i = 1, \exists \sigma \in S_n \text{ such that } x_i \in C_{i\sigma} \text{ for all } i\}$.

The group A also acts on $\mathcal{N}(C)$ via sending $(x_1, ..., x_r)$ to $(x_1^a, ..., x_r^a)$, for $a \in$ A. Let $\mathcal{N}^A(C) = \mathcal{N}(C)/A$. The Nielsen class $\mathcal{N}^A(C)$ in $\mathcal{E}_r^A(G)$ is a union of braid orbits. If Ψ_A in Lemma 2.45 restricts to a covering map $\mathcal{H} \to O_r$, then Lemma 2.46 implies that the fiber $\Psi_A^{-1}(B_0)$ corresponds to the set $\mathcal{N}^A(C)$. This yields a one to one correspondence between the connected components of $\mathcal{H}_r^A(C)$ and the braid orbits on $\mathcal{N}^A(C)$. In particular, if

$A = Inn(G)$, then the connected components of $\mathcal{H}_r^{in}(C)$ are in a one to one correspondence with braid orbits on $\mathcal{N}(C)$ [20, p. 196].

Let G be an affine primitive group with point stabilizer M. Since $\phi\colon \pi_1(\mathbb{P}^1 \setminus B, b) \to G$ is a surjective homomorphism and $\pi\colon G \to M$ is the natural projection, then $\eta := \phi \circ \pi\colon \pi_1(\mathbb{P}^1 \setminus B, b) \to M$ is also a surjective homomorphism which sends the canonical generators of $\pi_1(\mathbb{P}^1 \setminus B, b)$ to generators of M, say $(\lambda_i)\eta = m_i$. Similarly, we can define $\mathcal{N}^{in}(\bar{C})$ and $\mathcal{H}_r^{in}(\bar{C})$ in M where $\bar{C} = (C)\pi$. Clearly, the size of $\mathcal{N}(\bar{C})$ is less than the size of $\mathcal{N}(C)$. This fact useful for our computation.

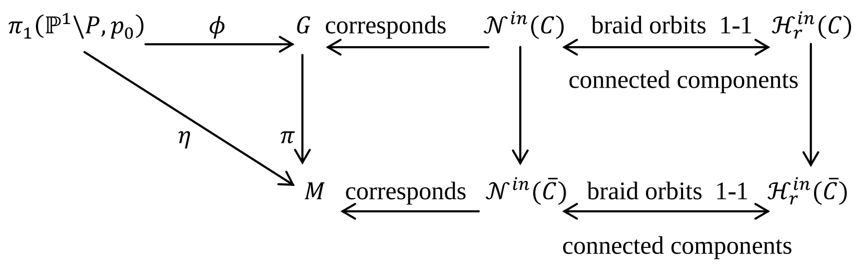

Figure 2.5: Nielsen Classes of G and M

Definition 2.50. *Two generating tuples are braid equivalent if they lie in the same orbit under the group generated by diagonal conjugation and the braid action.*

Theorem 2.51. *Two generating tuples are braid equivalent if and only if their corresponding covers are equivalent.*

Proof. For a proof see [20, p. 195].

Definition 2.52. *A partition P of the set $X = \{1, ..., r\}$, is a division of X into parts $P_1, P_2, ..., P_r$ such that $\bigcup_{i=1}^{r} P_i = X$ and $P_i \cap P_j = \emptyset$ if $i \neq j$.*

For the rest of this thesis, we let $P_1 = \{1, ..., n_1\}$, $P_2 = \{n_1 + 1, ..., n_2\}$,...,$P_r = \{n_k + 1, ..., r\}$.

We let the same elements in C are adjacent and from a part in C. If we indicate one class in each part, these different elements in C are in the same order of character table of G.

Definition 2.53. *If P is a partition of $\{1, ..., r\}$ with stabilizer S_P when S_P is a subgroup of S_r, then the fiber of S_P is called a* ***parabolic subgroup*** *of B_r and it is denoted by B_P.*

That is, B_P is generated by the set of those σ_{ij}'s with i and j lying in different parts of P, together with those σ_i's such that i and $i+1$ lie in the same part of P [14].

If $\{C_1, ..., C_r\}$ are ordered as (2.9) and P is the corresponding partition, then the parabolic subgroup B_P of B_r preserves the order of conjugacy classes, and B_P-orbits may be shorter than the B_r-orbits as much as by the factor of $[S_r : S_P]$, which switching to the B_P-orbits may be useful for our computations.

The following result is well-known (see[8]).

Proposition 2.54. *There is a one-to-one correspondence between connected components of $\mathcal{H}_r^{in}(C)$ and B_P-orbits on the Nielsen classes $\mathcal{N}(C)$.*

Hurwitz spaces can be interpreted to interesting things about:

(1) Components of moduli space of curves $\mathcal{M}_g$ where $\mathcal{M}_g$ denote the moduli space of a compact Riemann surfaces of a fixed genus $g \geq 0$.

(2) Equivalence class of ramified covers of the Riemann sphere.

Here our goal is to study the Hurwitz space $\mathcal{H}_r^{in}(G)$. In particular, we focus on the subset $\mathcal{H}_r^{in}(C)$ of $\mathcal{H}_r^{in}(G)$. To find this, one needs to find its connected components. Proposition 2.54 tells us the connected components of $\mathcal{H}_r^{in}(C)$ are in one to one correspondence with B_P-orbits on the Nielsen classes $\mathcal{N}(C)$. Finally, the **MAPCLASS** package of James, Magaard, Shpectorov and Volklein is designed to perform braid orbit computations for a given group and given type.

Chapter 3

Possible Ramification Types of Affine Primitive Groups

We begin this chapter by recalling a formal definition of genus g systems as follows. Suppose G acts transitively and faithfully on a finite G-set Ω where $|\Omega| = n$. **A genus** g **system** in G is a generating tuple $(x_1, ..., x_r)$ of elements of G such that $\prod_{i=1}^{r} x_i = 1,\ x_i \in G^{\#},\ i = 1, ..., r$ and $\sum_{i=1}^{r} ind\, x_i = 2(n + g - 1)$ where $ind\, x = n - orb\, x$ and $orb\, x$ is the number of orbits of x in G on Ω. We say that G **is a genus** g **group** if G possesses a genus g system for some transitive G-set Ω. Furthermore if G acts primitively on Ω, then we call this a **primitive genus** g **system**. We say that G **is a primitive genus** g **group** if G possesses a primitive genus g system.

3.1 Affine Groups

In this section, we discuss the structure of affine primitive permutation groups. Throughout this thesis, let $V = \mathbb{F}_p^e$ and G be a permutation group. The socle of G, denoted by $Soc(G)$, is generated by its minimal normal subgroups. The **general linear group** $GL(n, \mathbb{F})$ is the group of all invertible matrices of degree n with entries in a field $\mathbb{F}$.

The affine group, denoted by $AGL(e,p)$, is the set of all maps $f_{u,A} : V \to V$ such that $(v)f_{u,A} = vA + u$ for some $u \in V$ and $A \in GL(e,p)$. In particular, if $A = I_e$, then the set of the corresponding maps forms a subgroup $H \cong V$ which is normal in $AGL(e,p)$, H is regular on V and $H = Soc(AGL(e,p))$. The stabilizer of 0_V in $AGL(e,p)$ is a subgroup $GL(e,p)$ which acts irreducibly on V. Therefore, $AGL(e,p) \cong V : GL(e,p)$. We say that a primitive permutation group G is **affine** if $G \leq AGL(e,p)$ and $Soc(G) \cong V$. As we know, all primitive permutation groups of degrees up to 2500 are stored in GAP. We can extract them by using GAP function **AllPrimitiveGroups(DegreeOperation,p^e)**. Furthermore, we need to check the order of the **Socle** of those primitive groups to know which of them are affine.

For a given degree, we introduce the function **AffinePrimitiveGroups** to extract all affine primitive groups. It is of the form

$$AffinePrimitiveGroups(degree).$$

We use the following example to show the computation of all affine primitive groups of degree 8 by using the above function.

Example 3.1. `gap> AffinePrimitiveGroups(8);`

`[ AGL(1, 8), AGammaL(1, 8), ASL(3, 2) ]`

Remark 3.2. *The following table shows that the number of all affine primitive permutation groups of degree p^e where p and e satisfy the conditions in Theorems 1.2 and 1.3.*

Table 3.1: The Number of Affine Primitive Permutation Groups

p^e	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	3^2	3^3	3^4	3^5	3^6	5^2	5^3	5^4	7^2
#	3	20	3	64	3	238	36	105	7	11	145	34	471	22	33	647	33
p^e	7^3	11^2	13^2	17^2													
#	76	48	68	80													

Definition 3.3. *Let V be a vector space of dimension e over a field $\mathbb{F}_p$ where $p \neq 2$. A*

reflection is a diagonalizable linear transformation $\phi\colon V \to V$ having eigenvalues 1 and -1, such that eigenvalue 1 has multiplicity $e-1$.

Definition 3.4. *Let V be a vector space over a field $\mathbb{F}$. $End_{\mathbb{F}}(V)$ denotes the set of all linear transformations of V into itself. For $v \in V$ and $x \in End_{\mathbb{F}}(V)$, $[x, v] = vx - v$ is the commutator of v with x and $C_V(x) = \{v \in V : vx = v\}$ is the centralizer of x in V.*

Remark 3.5. *The eigenspace corresponding to the eigenvalue 1 is $C_V(x)$, the centralizer of x, and the 1-dimensional eigenspace corresponding to the eigenvalue -1 is $[x, V]$, the commutator space.*

Lemma 3.6. *If x is an involution in $GL(n, p)$ and $|[x, V]| = p$, where p is an odd prime, then x is a reflection.*

Proof. Let V be an e-dimensional vector space over $\mathbb{F}_p$ and $p \neq 2$. Let $\phi_x\colon V \to V$ be defined by $(v)\phi_x = [x, v] = vx - v$. Let $v_1, v_2 \in V$ and $\lambda \in \mathbb{F}_p$, then $(v_1 + v_2)\phi_x = [x, v_1 + v_2] = (v_1 + v_2)x - (v_1 + v_2) = v_1x - v_1 + v_2x - v_2 = (v_1)\phi_x + (v_2)\phi_x$. And also $(\lambda v)\phi_x = [x, \lambda v_1] = (\lambda v_1)x - (\lambda v_1) = \lambda(v_1x - v_1) = \lambda(v_1)\phi_x$. Therefore ϕ_x is linear. Since $im\, \phi_x = \{(v)\phi_x |\, v \in V\} = \{[x, v]|\, v \in V\} = [x, V]$. Thus, $im\, \phi_x = [x, V]$, and $\dim[x, V] = 1$ by hypothesis. Furthermore,

$$
\begin{aligned}
\operatorname{Ker} \phi_x &= \{v \in V |\ (v)\phi_x = 0\} \\
&= \{v \in V |\ [x, v] = 0\} \\
&= \{v \in V |\ vx - v = 0\} \\
&= \{v \in V |\ vx = x\} \\
&= C_V(x).
\end{aligned}
$$

So by the Rank-Nullity theorem,

$$\dim(V) = \dim(im\,\phi_x) + \dim(\mathrm{Ker}\,\phi_x) = \dim[x,V] + \dim C_V(x),$$

we obtain $\dim(C_V(x)) = e - 1$. It remains to show that $[x,V]$ has eigenvalue -1. Let $u \in [x,V]$. Then $u = vx - v$ and $ux = vx^2 - vx = v - vx = -u$, as required. □

Definition 3.7. *An invertible linear transformation $x : V \to V$ is called a transvection if $[x,V]$ is 1-dimensional and $[x,V] \subseteq C_V(x)$.*

Lemma 3.8. *If x is an involution in $GL(n,2)$ and $|[x,V]| = 2$, then x is a transvection.*

Proof. Let $u \in [x,V]$. Then $u = vx + v$ and $ux = vx^2 + vx = v + vx = u$. Then $ux = u$ implies $u \in C_V(x)$. Also, by hypothesis $\dim[x,V] = 1$ and the result follows. □

3.2 Numerical Conditions on the Ramification Type

The main goal of this section is to determine all possible ramification types for affine primitive permutation groups of genus one and two. To achieve this, we can use the following results.

- Theorems 1.2 and 1.3
- Riemann-Hurwitz formula (RHF for short).

Recall that the permutation index of $x \in S_n$ is defined by $ind\,x = n - orb\,(x)$ where $orb\,(x)$ is the number of orbits of x on n points and the RHF is

$$2(n + g - 1) = \sum_{i=1}^{r} ind\,x_i \tag{3.1}$$

where $ind\,x_i$ is the permutation index of x_i, n is the degree and g is the genus.

For the rest of this chapter, we focus on the groups $G = AGL(e, p)$ where p and e, satisfy the conditions of Theorems 1.2 and 1.3. Then we can use the RHF to determine all possible ramification types. Clearly, the left side of the RHF can be easily evaluated. For the right side, we first need to find all the conjugacy classes of G, and then we must compute the permutation index on $n = p^e$ points of each class representative as shown below.

We define an **acceptable partition** as an r-tuple $(i_1, ..., i_r)$ of non-negative integers, that are all values of the permutation index on the classes of G, such that $\sum_{s=1}^{r} i_s = 2(n+g-1)$.

All acceptable partitions for given G, n, g can be easily computed in GAP. Here is a sample code, for $n = 8, G = AGL(3, 2), g = 1$ and $r = 3$.

```
gap> k:=PrimitiveGroup(8,3);
ASL(3, 2)
gap> Con:=List(ConjugacyClasses(k),Representative);;
gap> Ind:=List([2..Length(Con)],i->[i,8-Length(Orbits(Group(Con[i]),[1..8]))]);
[ [ 2, 4 ], [ 3, 2 ], [ 4, 4 ], [ 5, 6 ], [ 6, 4 ],
  [ 7, 6 ], [ 8, 4 ], [ 9, 6 ], [ 10, 6 ], [ 11, 6 ] ]
gap> Indices:=List(Ind,x->x[2]);
[ 4, 2, 4, 6, 4, 6, 4, 6, 6, 6 ]
gap> acceptablepartitions:=RestrictedPartitions(16,Set(Indices),3);
[ [ 6, 6, 4 ] ]
```

The above computations show that there may be a repetition of indices. We use the GAP function **UnorderedTuple** in order to discover all acceptable partitions which include the repetition of indices.

For instance, in view of the preceding computation, the acceptable partition [6,6,4] is a possible genus one system for $AGL(3, 2)$, we have five different conjugacy classes with same index 6 and four different conjugacy classes with same index 4. The following computation shows how the function **UnorderedTuple** works.

```
gap> a:=acceptablepartitions[1];
[ 6, 6, 4 ]
gap> b:=Elements(a);
[ 4, 6 ]
gap>  T:=[];
[  ]
gap>  for j in [1..Length(b)] do
>      m:=Length(Filtered(a,x->x=b[j]));
>      c:=List(Filtered(Ind,x->x[2]=b[j]),x->x[1]);
>      Add(T,UnorderedTuples(c,m));
>      od;
gap> T;
[ [ [ 2 ], [ 4 ], [ 6 ], [ 8 ] ], [ [ 5, 5 ], [ 5, 7 ], [ 5, 9 ],
[ 5, 10 ], [ 5, 11 ], [ 7, 7 ], [ 7, 9 ], [ 7, 10 ], [ 7, 11 ],
[ 9, 9 ],[ 9, 10 ], [ 9, 11 ], [ 10, 10 ], [ 10, 11 ], [ 11, 11 ] ] ]
```

To put them together, we need to use the GAP codes such as **Cartesian** and **Concatenation**. In this way, we can successfully find all possible ramification types that satisfy the RHF.

Remark 3.9. *The following tables show that there are a large number of possible ramification types that fit the RHF for $AGL(e, p)$ where p and e satisfy the conditions in Theorems 1.2 and 1.3.*

Table 3.2: The Number of Possible RTs which Fit the RHF for Genus One

AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)
220	891	2150	4597	8712	14202
AGL(2,3)	AGL(3,3)	AGL(4,3)	AGL(2,5)	AGL(3,5)	-
76	293	864	127	287	-
AGL(2,7)	AGL(3,7)	AGL(2,11)	-	-	-
126	181	32	-	-	-

Table 3.3: The Number of Possible RTs which Fit the RHF for Genus Two

AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)	AGL(9,2)	AGL(10,2)
335	1250	1602	2755	1448	2453	623	755
AGL(2,3)	AGL(3,3)	AGL(4,3)	AGL(5,3)	AGL(6,3)	AGL(2,5)	AGL(3,5)	AGL(4,5)
111	286	385	310	243	157	291	194
AGL(2,7)	AGL(3,7)	AGL(2,11)	AGL(2,13)	AGL(2,17)	-	-	-
149	82	48	40	12	-	-	-

The next goal is to eliminate those ramification types which are not generating types. To do this, we apply a series of filters to each ramification type. We would like to present the main filters which will be used to eliminate possible ramification types.

- Scott's theorem and its corollaries.
- Structure constant.

A ramification type C is a **generating type** if there is at least one generating tuple of type C.

Recall that for a group G acting on a vector space V over field $\mathbb{F}$ for $x \in G$, set $[x, V]$ as a commutator space.

Theorem 3.10. *(Scott) Let G be a finite group which acts faithfully and irreducibly on a vector space V over a field $\mathbb{F}$ and let $G = \langle x_1, ..., x_r \rangle$ with $\prod_{i=1}^{r} x_i = 1$. Then*

$$\sum_{i=1}^{r} \dim[x_i, V] \geq 2 \dim V. \tag{3.2}$$

Proof. For a proof see [18].

Definition 3.11. *Let G be a group acting on Ω. For $x \in G$ define $Fix\, x = \{\omega \in \Omega : \omega x = \omega\}$ and $f(x) = |Fix\, x|$.*

Corollary 3.12. *If* $G = \langle x_1, ..., x_r \rangle$ *is an affine primitive group and* $\prod_{i=1}^{r} x_i = 1$ *if* $f(x_i) > 0$ *for all* $i \in \{1, ..., r\}$*, then*

$$\sum_{i=1}^{r} \dim[x_i, V] > 2 \dim V. \tag{3.3}$$

Proof. For a proof see [17, p. 23].

Scott's theorem and its corollary below give a sufficient condition, in terms of dimensions of commutator spaces, to eliminate a ramification type from further consideration. That is, any ramification type, which does not satisfy formulas (3.2) or (3.3), is not a generating type because G does not act irreducibly on V.

We use the following lemma to compute the dimension of commutator space in GAP.

Lemma 3.13. *Let* x *be an element in* G *and* $C_V(x)$ *be the centralizer of* x *in* V*. Then*

$$\dim[x, V] = \dim V - \dim C_V(x).$$

Proof. It is obvious.

Recall that we are considering $G = V : GL(e, p)$, where V is an elementary abelian p-subgroup of G with $|V| = p^e$. The vector space V has dimension e and the subspace $C_V(x)$, where $x \in G$, has dimension e_0 and $0 \leq e_0 \leq e$. We find e_0, by using the GAP function **Factors**. Let us continuous in the previous example to compute the dimensions of commutator spaces for the tuple "t" as follows.

```
gap> t:=[Con[3],Con[6],Con[8],Con[11]];
[ (5,6)(7,8), (3,4)(5,7,6,8), (3,5,7)(4,6,8), (2,3,5,6,8,4,7) ]
gap> V:=NormalSubgroups(k)[2];;
gap> C:=[];; b:=List(t,x->Size(Centralizer(V,x)));
[ 4, 2, 2, 1 ]
```

```
gap> for j in [1..Length(b)] do
>  if b[j]<>1 then
>  Append(C,[3-Length(Factors(b[j]))]);
>  else
>  Append(C,[3]);
>  fi;
>  od;
gap> C;
[ 1, 2, 2, 3 ]
```

Let us see the effect of Scott's theorem on ramification types to reduce the number of all possible ramification types. In particular, the following tables present the number of all possible ramification types which satisfy Scott's theorem for $AGL(e, p)$.

Table 3.4: The Number of Possible RTs which Pass Scott's Theorem for Genus One

AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)
137	459	368	370	168	102
AGL(2,3)	AGL(3,3)	AGL(4,3)	-	AGL(2,5)	AGL(3,5)
64	166	290	-	115	139
AGL(2,7)	AGL(3,7)	AGL(2,11)	-	-	-
110	52	249	-	-	-

Table 3.5: The Number of Possible RTs which Pass Scott's Theorem for Genus Two

AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)	AGL(9,2)	AGL(10,2)
262	788	563	799	103	357	34	106
AGL(2,3)	AGL(3,3)	AGL(4,3)	AGL(5,3)	AGL(6,3)	AGL(2,5)	AGL(3,5)	AGL(4,5)
103	249	263	112	43	139	288	158
AGL(2,7)	AGL(3,7)	AGL(2,11)	AGL(2,13)	AGL(2,17)	-	-	-
145	76	48	40	12	-	-	-

As well, we can apply the following GAP function **MovedPoints** to check which elements of G has fixed points. Let us keep the computation in the previous example, the parameter "F" as presented below is the set of the number of fixed points for t.

```
gap> F:=List(t,x->8-Length(MovedPoints(x)));
[ 4, 2, 2, 1 ]
```

We present the number of possible ramification types for $AGL(e,p)$, where p and e take values in Theorems 1.2 and 1.3, which pass the RHF, Scott's theorem and Corollary 3.12 in Table 3.6.

Table 3.6: The Number of Possible RTs which Pass Corollary 3.12 for Genus One and Two

genus	AGL(2,17)	AGL(2,13)	AGL(2,11)	AGL(2,7)	AGL(3,7)	AGL(2,5)	AGL(3,5)	AGL(4,5)
2	10	40	44	115	18	120	104	28
1	-	-	16	90	20	103	91	-
genus	AGL(2,3)	AGL(3,3)	AGL(4,3)	AGL(3,5)	AGL(6,3)	-	-	-
2	103	239	212	84	38	-	-	-
1	64	118	172	-	-	-	-	-
genus	AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)	AGL(9,2)	AGL(10,2)
2	262	788	401	493	55	232	16	76
1	137	423	192	197	71	52	-	-

The following lemma is well known for the representation theory of finite groups (see [13, p. 158]).

Lemma 3.14. *Let $x_i \in G$ and let C_i be the conjugacy class of x_i in G for $i = 1, ..., r$. Let $Irr(G)$ be the set of irreducible characters of G. Let $T(x_1, ..., x_r)$ be the set of tuples $(x_1, x_2, ..., x_r)$ with $x_i \in C_i$ and $\prod_{i=1}^{r} x_i = 1$. Then $T(x_1, ..., x_r) = \frac{\prod_{i=1}^{r} |C_i|}{|G|} \sum_{\chi \in Irr(G)} \frac{\prod_{i=1}^{r} \chi(x_i)}{\chi(1)^{r-2}}$*

Note that $|T(x_1, ..., x_r)|$ = structure constant $(C_1, ..., C_{r-1}, C_r^{-1})$. Next we are going to compute the structure constant (StC for short) for possible ramification types of small groups, which also eliminate many possible ramification types, given in the following table. Thus, there are no primitive genus two systems of degrees 169, 121 and 343.

Table 3.7: The Number of Possible RTs which Pass StC for Small Groups

genus	AGL(2,17)	AGL(2,13)	AGL(2,11)	AGL(2,7)	AGL(3,7)	AGL(2,5)	AGL(3,5)	AGL(4,5)
2	2	0	0	6	0	54	21	28
1	-	-	4	10	1	10	7	-
genus	AGL(2,3)	AGL(3,3)	AGL(4,3)	AGL(3,5)	AGL(6,3)	-	-	-
2	84	140	50	84	38	-	-	-
1	47	86	71	-	-	-	-	-
genus	AGL(3,2)	AGL(4,2)	AGL(5,2)	AGL(6,2)	AGL(7,2)	AGL(8,2)	AGL(9,2)	AGL(10,2)
2	258	754	395	493	55	232	16	76
1	132	390	186	197	71	52	-	-

We finish this section with the following corollary. As usual $G = \langle x_1, ..., x_r \rangle$ with $\prod_{i=1}^{r} x_i = 1$, $x_i = m_i v_i$, $m_i \in M$ and $v_i \in V$.

Corollary 3.15. *If* $M = \langle m_1, ..., m_r \rangle, \prod_{i=1}^{r} m_i = 1$, *acts irreducibly on* V, *let* $J \subset \{1, ..., r\}$ *with* $|J|$ *even. Then*

$$\sum_{i \in J} \dim[-m_i, V] + \sum_{i \in J^c} \dim[x_i, V] \geq 2 \dim V. \tag{3.4}$$

Proof. For a proof see [17, p. 21].

The main purpose of the Corollary 3.15 is to show that M does not act irreducibly on V, by forcing down the dimension of certain commutator spaces. When applying the Lemma 3.15 we need to remember that only an even number of generators multiplied by a $-I$ matrix. This necessary so as preserve the condition $\prod_{i=1}^{r} x_i = 1$.

The following example shows the application of the above corollary for a particular element in the group.

Example 3.16. Let x be an element of order 6 in $AGL(3, 7)$ with $\operatorname{ind} x = 275$, $f(x) > 0$ and $dim[x, V] = 3$. So $x^g = m = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ for some $g \in G$, we see that

$-m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ has order six, but $2 = \bar{e} = dim[-m, V]$.

The application of Corollary 3.15 shows that the number of possible ramification types, which are present in Table 3.7 drops down to two, one and two respectively, for the groups $AGL(4, 5)$, $AGL(5, 3)$ and $AGL(6, 3)$. The remaining possible ramification types are length three. These types can be eliminated by computing the conjugacy class of the first element. We see that the multiplication of each element in the first conjugacy class with the second conjugacy class representative is not the inverse of the third conjugacy class representative. Therefore, there are no primitive genus two systems of degrees 625, 243 and 729.

The application of Corollary 3.15 is not useful when the characteristic of the field equal to two because $-1 = 1$. In other words, the dimensions of commutator spaces become stable.

Remark 3.17. *We can represent the matrix $-I$ over a field $GF(p)^e$ as a permutation element by using GAP code such as* ***Permutation(***$-I$***,AsList(***$GF(p)^e$***))***.

3.3 Ordering of Conjugacy Classes

We are going to order and label conjugacy classes according to certain rules to make a distinction between elements that have the same order. This has to do with the fact that the ordering of the conjugacy classes in a group may not be the same for two different runs of GAP. For better consistency, we need to establish a canonical order of classes similar to the ATLAS notation. It works as follows.

Let G be an affine primitive permutation group and let $C_1, ..., C_m$ be the conjugacy classes of G. Assume that $x_i \in C_i$ has order d_i for $i = 1, ..., m$.

- We order that d_i, for $i = 1, ..., m$ form a non-decreasing sequence.
- If $d_t, ..., d_s$ are equal where $t, s < m$, then we compute $|C_G(x_i)|$ for $i = 1, ..., s$ and order them to form a non-increasing sequence.
- If equality holds among some terms in sequence of $|C_G(x_i)|$, then we compute the permutation indices in the natural action (that is, G acts on p^e points) for them and order them to form a non-decreasing sequence.
- If equality holds among some terms in sequence of indices, then we take power

functions for them such as $C_i^d := \{x^d | x \in C_i\}$, where d is some order of an element in G. Next we are going to check that they are conjugate to different elements in G or not.

According to the above rules, we obtain a sequence. We label the corresponding conjugacy class of the first term in sequence with a given order d_i by d_iA, the corresponding conjugacy class of the second term in sequence by d_iB and so on.

Finally, we introduce the function **Ordering** which has the form

$$Ordering(group, degree).$$

It arranges conjugacy classes according to the above rules.

For more detail, we will give the following example to explain the application of the above rules.

Example 3.18. In $G = AGL(3,3)$, we look at elements of order six, of which G has eight different conjugacy classes. We denote them by $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ and C_8. We first compute the centralizer size and permutation index of each of them, and the results appear in Table 3.8.

Table 3.8: Ordering and Labeling Conjugacy Classes

Conjugacy classes	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Centralizer Size	36	36	108	18	18	108	18	144
indices	17	18	19	21	21	21	22	22
After ordering								
ordered	C_8	C_3	C_6	C_1	C_2	C_4	C_5	C_7
size of centralizer	144	108	108	36	36	18	18	18
indices	22	19	21	17	18	21	21	22
power function	-	-	-	-	-	C_4^3	C_5^3	-
Types	6A	6B	6C	6D	6E	6F	6G	6H

In fact, $6A$ represents C_8 because it has a biggest centralizer size. However, the conjugacy class C_3 has the same centralizer size as the conjugacy class C_6, but the indices are different and so $6B$ must be represented C_3, because the index of C_3 is less than the index of C_6 and so on. For the conjugacy classes C_4 and C_5, we use a different technique such as; we take the third power of the conjugacy classes of C_4 and C_5, which are conjugate to the conjugacy classes of $2A$ and $2B$ respectively. Therefore, C_4 represents $6F$ and C_5 represents $6G$.

Remark 3.19. *(i) In the above example, if we take the second power of the conjugacy classes of C_4 and C_3, which are conjugate to the conjugacy classes $3D$ and $3E$ respectively. We obtain the same result.*

(ii) The ambiguities that we have are the following. Sometimes when we take the powers of conjugacy classes are not useful, because they conjugate to the same element in a group. Furthermore, if the conjugacy classes are inverse of each other, and so we cannot make distinctions among them. If the character table exists for the group, then we can make a distinction between them just by looking it.

Chapter 4

The Investigation of Possible Ramification Types

Those ramification types which survive the series of filters namely the Riemann-Hurwitz formula, Scott's theorem and Corollary 3.12 (in some cases structure constant) may contain the generating types for a given g and p^e in $AGL(e, p)$. For fixed degree and ramification type, it may happen that there exist non isomorphic affine primitive permutation groups of this prescribed type. For instance, we have the type $(2B, 2B, 2B, 2B, 2D, 5A)$ in $2^4.PSL(4, 2)$ when $g = 2$. The corresponding tuples of this type generate three groups that are isomorphic to $2^4.PSL(4, 2)$, $2^4.S(6)$ and $2^4.S(5)$ this will be explained in Section 4.4.

The first three sections are devoted to checking whether or not the possible ramification types are generating types for given g and p^e in all affine primitive permutation subgroups of $AGL(e, p)$. In the final section, we compute braid orbits on the Nielsen classes for most generating types of these subgroups by using **MAPCLASS** package.

Now we go through the remaining possible ramification types to check whether or not they are generating types. For this purpose, we develop some methods.

4.1 GeneratingTypes

The main idea of the final step of Wang's function **GeneratingTypes** came from [20]. Originally, it works by using the character table of the group G to count the number of genus g systems. We modify this function for our situation. The main change is that it does not compute the character table for big groups. This computation may be problematic in GAP for these groups. We only need to check the existence of genus g systems of G before computing braid orbits. Wang gave a description of his function in his Ph.D. thesis [8]. Let us give a brief description to explain how it works.

Let $C = \{C_1, ..., C_r\}$ be a ramification type. We build all subgroups of G generated by tuples $(x_1, ..., x_r) \in C_1 \times C_2 \times ... \times C_r$ with $\prod_{i=1}^{r} x_i = 1$. We compute all pairs (T_j, c_j) (up to G-conjugacy) consisting a subgroup T_j of G generated by a tuple $(x_1, x_2, ..., x_j)$ with $x_1 \cdots x_j = c_j$ and $x_i \in C_i$ for $1 \leq j \leq r$.

We first pick element x_1 and continue adding generators $x_k \in C_k$ in every cycle of the routine to the tuple $(x_1, ...)$ until we reach $(x_1, ..., x_{r-1})$ then we construct the subgroup generated by $\langle x_1, ..., x_r \rangle$. If the subgroup is new, then we add it to the list. Finally, we check the subgroup generated by $\langle x_1, ..., x_{r-1} \rangle$ corresponding to the inverse of an element of last conjugacy class. If this is G, then we say that C is a generating type.

The function **Generating Types** first computes all possible ramification types that satisfy the RHF, Scott's theorem and Corollary 3.12 for an affine primitive group, then input them into the final step to check if there exists a generating tuple of G.

The function **Generating Types** takes three inputs; which are the group, the degree of the action and the fixed genus. When it finishes, the output of it is a file which has the tuples of generating types, the name of the group and its conjugacy class representatives. In this method, we pick the specific group and discard all the ramification types which do not generate the group. For a given p^e and g, we find all generating types for all affine primitive subgroups of $AGL(e,p)$, except the groups $AGL(8,2), AGL(9,2)$ and $AGL(10,2)$. For instance; we have 64 subgroups of $AGL(6,2)$ and 493 possible ramification types for primitive genus two systems and 197 possible ramification types for primitive genus one systems. As displayed in Table 4.1, there are only 29 subgroups of $AGL(6,2)$ possessing primitive genus two systems and 24 subgroups of $AGL(6,2)$ possessing primitive genus one systems. To check each of these subgroups we need a few minutes. However, for the group $AGL(6,2)$ it takes a bit longer.

Table 4.1: Affine Primitive Groups of Degree 64 of Genera One and Two

g	1		2	
Up to isom. Group,	# of ramification type	# of generating type	# of ramification type	# of generating type
$2^6 : D_{14}$	0	0	21	21
$2^6 : 7 : 6$	0	0	7	4
$2^6 : 3^2 : Sym(3)$	0	0	10	2
$2^6 : 3^2 : Sym(3)$	0	0	8	2
$2^6 : 3^2 : D_{12}$	5	1	0	0
$2^6 : (3^2 : 3) : 4$	4	4	14	4
$2^6 : (3 \wr Sym(3))$	0	0	28	2
$2^6 : (3^2 : 3) : D_8$	230	3	0	0
$2^6 : 3^3 : D_{12}$	8	1	44	3
$2^6 : 3^3 : Alt(4)$	4	4	0	0
$2^6 : (3^2 : 3) : SD_{16}$	9	2	0	0
$2^6 : (GL(2,2) \wr Alt(3))$	0	0	42	2
$2^6 : 3^3 : Sym(4)$	10	6	0	0
$2^6 : 3^3 : Sym(4)$	4	2	50	3
$2^6 : (3^2 : 3) : Q_8 : 3$	0	0	33	2
$2^6 : (GL(2,2) \wr Sym(3))$	22	2	89	2
$2^6 : (3^2 : 3) : Q_8 : Sym(3)$	13	2	34	2
$AGL(6,2)$	197	21	493	102
$2^6 : (GL(3,2) \wr 2)$	0	0	43	11
$A\Gamma(3,4)$	0	0	59	2
$A\Sigma L(3,4)$	26	2	35	3
$ASL(3,4)$	0	0	45	6
$2^6 : 3.Alt(6)$	7	1	28	4
$2^6 : (S3 \times GL(3,2))$	0	0	30	3
$2^6 : (3 \times GL(3,2))$	6	1	0	0
$2^6 : Sp(6,2)$	123	2	248	14
$2^6 : GO-(6,2)$	49	16	119	19
$2^6 : O-(6,2)$	17	6	82	18
$2^6 : Sym(8)$	54	8	77	13
$2^6 : Alt(8)$	32	6	50	14
$2^6 : Sym(7)$	11	9	19	10
$2^6 : Alt(7)$	3	2	9	3
$2^6 : \Sigma U(3,3)$	19	1	30	3
$2^6 : SU(3,3)$	10	2	21	3
$2^6 : PGL(2,7)$	13	8	13	4

It may be possible that generating types exist for primitive genus one and two systems for the groups $AGL(8,2)$, $AGL(9,2)$ and $AGL(10,2)$. We will need to introduce another method to find generating types of these groups.

Definition 4.1. *Let H and K be subgroups of a group G and let $x \in G$. Then the set $HxK = \{hxk | h \in H, k \in K\}$ is called a double coset with respect to H and K.*

The next lemma is often useful to find braid orbits for a tuple of length three because the double coset HxK is the union of the orbit of Hx under right multiplication by K. In particular, for the groups $AGL(8,2), AGL(9,2)$ and $AGL(10,2)$, we can use Lemma 4.2 to check whether or not the ramification types of length three are generating types.

Lemma 4.2. *Let (C_1, C_2, C_3) be a generating type. Let $C_G(c_i)$ be the centralizer of $c_i \in G$, $i = 1, 2$. Then any generating tuple of this type up to conjugation in G is in the form $\langle c_1, c_2^k, (c_1 c_2^k)^{-1} \rangle$ where $c_1 \in C_1$, $c_2 \in C_2$ and k is in the double coset of $C_G(c_1)$ and $C_G(c_2)$ in G and $(c_1 c_2^k)^{-1} \in C_3$.*

Proof. For a proof see [8].

Remark 4.3. *(About Lemma 4.2)*

- *Without loss of generality, we fixed one entry to be the class representative c_1.*
- *Sometime the double coset computation fails because either subgroups being maximal of big indices or the total number of double cosets being prohibitive. In this case, we can take k as random elements in a group.*

4.2 Projection Algorithm

Recall that G is an affine primitive group with point stabilizer M. Let $\pi\colon G \to M$ be the natural projection defined by $(x)\pi = m$ for all $x \in G$. Let $\hat{\pi}\colon G^r \to M^r$ be the mapping defined by $(x_1, ..., x_r)\hat{\pi} = ((x_1)\pi, ..., (x_r)\pi)$. Our idea is to send the tuple t of type C in G via $\hat{\pi}$ to the tuple $\bar{t}$ of type $\bar{C}$ in M. This idea is quite useful because the size of M is less than the size of G. We can use this idea to check whether or not the tuple t generates

G. To do this, we need to verify that the tuple $\bar{t}$ generates M. If $\bar{t}$ does not generate M, then the tuple t cannot generate G.

For a group G, we introduce the function **QFind3Tuple**. The idea of this function comes from combining the projection algorithm and Lemma 4.2. Let us briefly describe how it works. Let $t = (x_1, x_2, x_3)$ be a tuple of type (C_1, C_2, C_3) in G. Then the mapping $\hat{\pi}$ sends the tuple t onto the tuple (m_1, m_2, m_3) of type $\bar{C} = (\bar{C_1}, \bar{C_2}, \bar{C_3})$ in M.

Next we apply Lemma 4.2 to the type $\bar{C}$ in M. As we mentioned in the previous page this lemma can be used to compute orbits for the type $\bar{C}$ in M via double cosets. For more details see codes in Appendix C.

The function **QFind3Tuple** takes three inputs; which are the tuple, the group and the degree of the action. When it finishes, the output of it is an empty list if the type does not generate M otherwise a list of tuples (which are the number of orbits of length one).

4.3 The Groups AGL(8,2), AGL(9,2) and AGL(10,2)

The group $AGL(10, 2)$ has order 375,234,700,595,146,883,504,949,480,652,800. The group $AGL(9, 2)$ has order 358,201,502,736,997,192,984,166,400, and the group $AGL(8, 2)$ has order 1,369,104,324,918,194,995,200. As we see the orders of these groups are too big to put in the function **GeneratingTypes**. However, the conjugacy class representatives of these groups have big permutation indices, we can see that most of the ramification types have length three except for the group $AGL(8, 2)$ when $g = 2$. The group $AGL(8, 2)$ has 232 ramification types. Only 12 of them have length four and the others have length three. For these groups, we will use the following results to check whether or not the ramification types are generating types.

Lemma 4.4. *Let $G = \langle x_1, x_2, x_3\rangle$ with $x_1 \cdot x_2 \cdot x_3 = 1$. Assume that the element x_2 has* *order n. Then the tuple*

$$t = (x_1, x_2x_1x_2^{-1}, x_2^2x_1(x_2^2)^{-1}, ..., x_2^{n-1}x_1x_2, x_3^n)$$

generate a normal subgroup V of G, and its product is also 1.

Proof. For a proof see [7].

Lemma 4.5. *Let $G = \langle x_1, x_2, x_3, x_4 \rangle$ with $x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 1$. Assume that the element x_3 has order n. Then the tuple*

$$t = (x_1, x_2, x_3x_1x_2x_3^{-1}, x_3^2x_1x_2(x_3^2)^{-1}, ..., x_3^{n-1}x_1x_2x_3, x_4^n)$$

generate a normal subgroup V of G, and its product is also 1.

Proof. For a proof see [7].

The main idea of the above lemmas is to translate a tuple of length four or three into longer one by removing one conjugacy class representative x_i whose commutator space $[x_i, V]$ has big dimension. If the tuple t does not satisfy the formula (3.2), then the original tuple of length four or three will not be a generating tuple of G because it does not act irreducibly on V.

4.3.1 Genus One and Two Systems for AGL(8,2)

Recall that we have 52 ramification types for primitive genus one systems for $AGL(8, 2)$ see Table 3.7. We represent them in the following tables.

Table 4.2: Part1: Possible Signatures for $AGL(8, 2)$

tuple	signature	# of ramification type	$dim[x_1, V]$	$dim[x_2, V]$	$dim[x_3, V]$	
(x_1, x_2, x_3)	(2,5,d)	8	1	8	7,8	$d \in \{24, 42\}$
(x_1, x_2, x_3)	(2,6,d)	10	1	8	7,8	$d \in \{15, 28, 30\}$
(x_1, x_2, x_3)	(2,3,d)	7	2	8	7,8	$d \in \{28, 30\}$

All signatures from Table 4.2 can be ruled out by applying Lemma 4.4 as follows: by removing the second conjugacy class representative. For instance, we take the tuple (x_1, x_2, x_3) with $x_1^2 = x_2^5 = x_3^{24} = 1$ and translate it to the tuple $(x_1, x_2^{-1}x_1x_2, x_2^{-2}x_1x_2^2, x_2^{-3}x_1x_2^3, x_2^{-4}x_1x_2^4, x_3^5)$. The sum of the dimensions of the commutator spaces of the tuple is equal to 13. So it is less than $16 = 2\dim V$, these tuples do not generate. The remaining cases present in Table 4.3.

Table 4.3: Part2: Possible Signatures for $AGL(8, 2)$

tuple	signature	# of ramification type	
(x_1, x_2, x_3)	(2,4,d)	4	$d \in \{5, 6, 12, 15\}$
(x_1, x_2, x_3)	(2,6,d)	9	$d \in \{5, 6, 8, 9, 12\}$
(x_1, x_2, x_3)	(2,3,d)	9	$d \in \{8, 12, 15\}$
(x_1, x_2, x_3)	(4,3,6)	1	
(x_1, x_2, x_3)	(3,3,4)	4	

We use the function **QFind3Tuple** to eliminate 25 of the ramification types in Table 4.3. Hence we are left with just two types, which can be represented by $(3D, 3D, 4X)$ and $(3D, 3D, 4S)$. The mapping $\hat{\pi}$ sends these two types onto two types in $GL(8, 2)$, which are $(3D, 3D, 4E)$ and $(3D, 3D, 4G)$. We compute the structure constants and observe that $\frac{StC}{|GL(8,2)|} < 1$. This means that these two types cannot generate $GL(8, 2)$. Hence the original types cannot generate $AGL(8, 2)$. Thus $AGL(8, 2)$ possesses no primitive genus one systems. Similarly, $AGL(8, 2)$ possesses no primitive genus two systems by the same arguments plus Lemma 4.5.

4.3.2 Genus Two Systems for AGL(9,2)

Recall that we have 16 ramification types for $AGL(9, 2)$ see Table 3.7. We represent them in the following table.

Table 4.4: Possible Signatures for $AGL(9, 2)$

tuple	signature	# of ramification type	$dim[x_1, V]$	$dim[x_2, V]$	$dim[x_3, V]$
(x_1, x_2, x_3)	(2,7,10)	8	1	9	8
(x_1, x_2, x_3)	(2,3,20)	5	2	8	8
(x_1, x_2, x_3)	(2,5,6)	3	2	8	8

The first two signatures can be ruled out by applying Lemma 4.4. However, the third signature cannot be eliminated by Lemma 4.4. We use the function **QFind3Tuple** to eliminate that one. Therefore, we have successfully eliminated all the signatures in Table 4.4. Therefore, $AGL(9,2)$ possesses no primitive genus two systems.

4.3.3 Genus Two Systems for AGL(10,2)

Recall that we have 76 ramification types for $AGL(10,2)$ see Table 3.7. We are able to rule out 73 of them by using Lemma 4.4. The three remaining types have the signature (4,3,4). The mapping $\hat{\pi}$ sends these three types onto three types in $GL(10,2)$, and they have the same signature. These three types can be ruled out by the computing structure constant. However, it is not a straightforward exercise to calculate the structure constant because the character table does not exist in GAP. By using the GAP program (see Appendix C), we can check that $x \cdot y \neq z^{-1}$, for all triples (x,y,z) of type $(4,3,4)$ in $GL(10,2)$. The key point of this program is to compute the conjugacy class of the second element by random elements in $GL(10,2)$. Therefore, $AGL(10,2)$ possesses no primitive genus two systems.

4.4 Explicit Braid Computations

In the previous sections, we found all generating types for each affine primitive group of genus one and two. It is now time to compute braid orbits on the Nielsen classes. Let t be a tuple of G with ramification type C. In principle, the computation of the braid orbit of t is straightforward. We simply apply the following functions which are contained in the **MAPCLASS** package to compute braid orbits on Nielsen classes. The main functions are the following:

4.4.1 GeneratingMCOrbits

This function computes only generating braid orbits of the given tuple and the group. The function takes three inputs: the group, the genus and the tuple where tuple is a tuple of conjugacy class representatives.

GeneratingMCOrbits(group,genus,tuple)

The output of this function is many things such as size of the centralizer etc. But we need the number of braid orbits and the length of orbits. Recall that we study the Hurwitz space $\mathcal{H}_r^{in}(G)$ which is the space of G-covers of base space $\mathbb{P}^1$.

Example 4.6. For the group $k := ASL(2,4)$ and $g = 2$, there is only one possible generating type of length five, namely $(2B, 2B, 2B, 2B, 3A)$. The

```
tuple:=[ (2,11)(3,15)(4,5)(6,10)(7,14)(9,16), (2,11)(3,15)(4,5)(6,10)
(7,14)(9,16), (2,11)(3,15)(4,5)(6,10)(7,14)(9,16), (2,11)(3,15)(4,5)
(6,10)(7,14)(9,16), (2,3,4)(5,14,10)(6,16,11)(7,15,9)(8,13,12) ]
```

is of type $(2B, 2B, 2B, 2B, 3A)$.

```
gap> orbits:=GeneratingMCOrbits(k,0,tuple);;
gap> Length(last);
gap> 3
gap> orb:=orbits[1];;
gap> Length(orb.TableTuple);
gap> 1080
```

The first orbit is of length 1080 and by inspection, we see that the others are as well.

4.4.2 AllMCOrbits

This function computes all braid orbits of the given group and the type, both generating and non generating. Once the program completes its work it is easy to select those orbits that are generating. However, the time required by this program may be significantly higher as it may be looking for a long time for the last small non generating orbits. The function takes three inputs: the group, the genus and the tuple.

AllMCOrbits(group,genus,tuple)

The output of this function is many things such as size of the centralizer, etc. But we need the number of braid orbits and the length of orbits.

Definition 4.7. *For a given ramification type, if there exist at least one braid orbit among all braid orbits such that a tuple from it generates an affine primitive subgroup of G, then the ramification type is a generating type.*

For a given p and e, we first find all braid orbits (including the non-generating ones) in $G = AGL(e, p)$. For each orbit O we consider the group $H = H_O$ generated by a tuple from O. This group is defined uniquely up to conjugation in G. We need to see whether H is affine and primitive. In particular, we will just show how to do one particular type.

The following example illustrates the above function.

Example 4.8. Let $G := 2^4.PSL(4, 2)$ and $g = 2$. The

```
tuple:=[ (2,7)(3,6)(9,16)(12,13), (2,7)(3,6)(9,16)(12,13),
(2,7)(3,6)(9,16)(12,13), (2,7)(3,6)(9,16)(12,13), (3,11)(4,12)(5,14)
(6,13)(7,8)(15,16), (2,7,13,5,16)(3,4,6,10,14)(8,11,9,12,15) ]
```

is of type $(2B, 2B, 2B, 2B, 2D, 5A)$. Now we find all braid orbits.

```
gap> G:=PrimitiveGroup(16,11);
```

```
2^4.PSL(4, 2)
gap> O:=AllMCOrbits(G,0,tuple);;
gap>Length(O);
7
```

Thus, we obtain seven braid orbits. For each braid orbit O, we consider the group generated by a tuple from O. We need to check whether the group is affine and primitive.

```
gap> for i in [1..Length(O)] do
> pp:=Group(O[i].TupleTable[1].tuple);
> if IsPrimitive(pp)=true and Size(Socle(pp))=16 then
> Print(i);
> Print("\n");
> fi; od;
1
6
7
```

We showed the first tuple from orbits one, six and seven generate affine primitive groups and the first tuple from other orbits do not. It is easy to check that these affine primitive groups obtained here match the groups in the following list.

```
gap> ll:=AllPrimitiveGroups(DegreeOperation,16);
[ 2^4:5, 2^4:D(2*5), AGL(1, 16), (A(4) x A(4)):2, (2^4:5).4,
AGL(1, 16):2, 2^4.S(3) x S(3), 2^4.3^2:4, AGammaL(1, 16), (S(4) x S(4)):2,
2^4.PSL(4, 2), AGammaL(2, 4), ASL(2, 4):2, AGL(2, 4), ASL(2, 4),2^4.S(6),
2^4.A(6), 2^4:S(5), 2^4:A(5), 2^4.A(7), A(16), S(16) ]
gap> pp1:=Group(O[6].TupleTable[1].tuple);;
gap> for i in [1..Length(ll)] do
```

```
> if Size(pp1)=Size(ll[i]) then
> Print(i);
> Print("\n");
> fi; od;
18
gap> IsomorphicSubgroups(pp1,ll[18]);
[ [ (1,4,16,3,8,14,7,6,10,5,2,12)(9,13,15,11), (3,16,9)(4,15,10)(7,12,13)
(8,11,14) ] -> [ (1,2,4,7,16,6,9,10,12,15,8,14)(3,5,11,13), (3,14,16)
(4,13,15)(5,12,10)(6,11,9) ] ]
```

Thus, the first tuple from orbit six $O[6]$, generates the affine primitive group which is isomorphic to $2^4.S(5)$ and $M \cong S(5)$.

```
gap> pp2:=Group(O[7].TupleTable[1].tuple);;
gap> for i in [1..Length(ll)] do
> if Size(pp2)=Size(ll[i]) then
> Print(i);
> Print("\n");
> fi;od;
16
gap> IsomorphicSubgroups(pp2,ll[16]);
[ [ (2,5)(3,15)(4,11)(7,16)(8,12)(9,14), (1,14,2,13)(3,9,7,15,6,11,4,10,
8,16,5,12) ] ->[ (2,8)(3,12)(4,13)(5,14)(6,11)(9,15), (1,15,11,10,7,4,2,
16,12,9,8,3)(5,14,6,13) ] ]
```

Also, the first tuple from orbit seven $O[7]$, generates the affine primitive group which is isomorphic to $2^4.S(6)$ and $M \cong S(6)$.

```
gap> pp3:=Group(O[1].TupleTable[1].tuple);;
```

```
gap> for i in [1..Length(ll)] do
> if Size(pp3)=Size(ll[i]) then
> Print(i);
> Print("\n");
> fi;od;
11
gap> pp3=ll[11];
true
```

So, the first tuple from orbit one $O[1]$, generates the affine primitive group which is isomorphic to $2^4.PSL(4,2)$ and $M \cong PSL(4,2)$. They appear in the tables in the appendix B, with the type $(2B, 2B, 2B, 2B, 2E, 5A)$ for $2^4.S(6)$, the type $(2C, 2C, 2C, 2C, 2D, 5A)$ for $2^4.S(5)$ and the type $(2B, 2B, 2B, 2B, 2D, 5A)$ for $2^4.PSL(4,2)$.

Chapter 5

Projection-Fiber Algorithm

In Section 4.4, we computed braid orbits on Nielsen classes for a vast majority of generating types of length $r > 3$ by using the **MAPCLASS** package. For the remaining types, we cannot use the **MAPCLASS** package directly because either the computations take too long, or they require a lot of memory. To deal with these types, we introduce a new algorithm, which we call the projection-fiber algorithm.

Throughout this chapter, we assume that $G = MV$, M is the point stabilizer in G, V is a regular normal subgroup of G and $M \cap V = 1$. We let $\pi\colon G \to M$ be the natural projection defined by $(x)\pi = m$ for all $x = mv$ in G. Note that since G is primitive, V is an elementary abelian p-group, M acts irreducibly on V. Let $|V| = p^e$.

5.1 Some Basic Results

Before describing our algorithm, we need to prove some results related to it.

Lemma 5.1. *Let* $\hat{\pi} : G^r \to M^r$ *be the mapping defined by* $(x_1, x_2, ..., x_r)\hat{\pi} = ((x_1)\pi, ..., (x_r)\pi)$. *Then* $\hat{\pi}$ *commutes with the braid action.*

Proof. Let $(x_1, ..., x_r)$ be a tuple in G^r and σ_i be one of the standard generator of B_r.

Let $(x_i)\pi = m_i$ for all i. Then we have

$$\begin{aligned}((x_1, x_2, ..., x_r)\hat{\pi})\sigma_i &= ((x_1)\pi, ..., (x_r)\pi)\sigma_i \\ &= (m_1, ..., m_r)\sigma_i \\ &= (m_1, ..., m_{i+1}, m_{i+1}^{-1}m_i m_{i+1}, ..., m_r) \\ &= ((x_1)\pi, ..., (x_{i+1})\pi, (x_{i+1}^{-1}x_i x_{i+1})\pi, ..., (x_r)\pi) \\ &= (x_1, ..., x_{i+1}, x_{i+1}^{-1}x_i x_{i+1}, ..., x_r)\hat{\pi} \\ &= ((x_1, x_2, ..., x_r)\sigma_i)\hat{\pi}.\end{aligned}$$

Hence $\hat{\pi}\sigma_i = \sigma_i\hat{\pi}$, as wanted. □

The next lemma tells us that if we have two tuples which are not braid equivalent in M, then their pullbacks under $\hat{\pi}$ are not braid equivalent in G.

Lemma 5.2. *If two tuples are not braid equivalent in M, then their pullbacks under $\hat{\pi}$ are not braid equivalent in G, either.*

Proof. Suppose that t and t' are braid equivalent in G. So $t' = t\sigma$ for some $\sigma \in B_r$ then $(t')\hat{\pi} = (t\sigma)\hat{\pi} = ((t)\hat{\pi})\sigma$ by Lemma 5.1. Therefore $(t')\hat{\pi}$ and $(t)\hat{\pi}$ are braid equivalent, and the proof is now complete. □

Lemma 5.3. *Suppose $m = (m_1, ..., m_r)$ is a generating tuple in M, and $x = (x_1, ..., x_r)$ is an arbitrary lift of m into G. Then either $G = \langle x_1, ..., x_r\rangle$ or $\langle x_1, ..., x_r\rangle$ is a complement to V in G.*

Proof. It is clear that $G_0 = \langle x_1, ..., x_r\rangle$ is a subgroup of G and that $G = G_0V$. Let $H = V \cap G_0$. Then H is normal in G_0, since V is normal in G. It is also normal in V because V is abelian. It follows that H is normal in G and thus either $H = 1$ or $H = V$, as claimed. □

Lemma 5.4. *Let $1 \neq m \in M$ and $C_x = \{x^G\}$ be a conjugacy class. If $\pi\colon G \to M$ is the natural projection and $(x)\pi = m$, then*

(a) $|(m)\pi^{-1}| = |V|$.

(b) $|(m)\pi^{-1} \cap C_x|$ *is a multiple of* $[V : C_V(x)]$.

Proof. (a) The fibers of π are cosets of the kernel, which is V. We know that $|xV| = |V| = |Vx|$. So $|(m)\pi^{-1}| = |V|$.

(b) If $v \in V$, then $k = [x, v] = x^{-1}v^{-1}xv = (v^{-1})^x v \in V$, because the latter is normal in G. For $y \in (m)\pi^{-1} \cap C_x$ and $v \in V$, $y^v = v^{-1}yv = y[y, v]$. Hence $(y^v)\pi = (y[y, v])\pi = (y)\pi([y, v])\pi = (y)\pi.1 = m$. That is, $(m)\pi^{-1} \cap C_x$, is invariant under conjugation by V. By Orbit-Stabilizer Theorem, we have $|y^V| = [V : C_V(y)]$ and this is the true for all $y \in C_x$. Thus $|(m)\pi^{-1} \cap C_x|$ is a multiple of $[V : C_V(y)]$. □

5.2 Description of Algorithm

We are now ready to describe our algorithm. Note that if t is a tuple of the generating type $C = (C_1, ..., C_r)$ in G, then $(t)\hat{\pi}$ is a tuple of type $\bar{C} = (\bar{C}_1, ..., \bar{C}_r)$ in M. We can compute the generating braid orbits of type $\bar{C}$ in M by using the function **GeneratingMCOrbits**. Let these orbits be denoted by $O_j(M, \bar{C})$, $j = 1, 2, ..., l$. For each generating braid orbit $O = O_j(M, \bar{C})$, we now try to find the generating braid orbits of type C in G, which project onto O. To do this, select a representative tuple $\bar{t} = (\bar{t}_1, ..., \bar{t}_r)$ from O and then compute the fibers $L_i = \pi^{-1}(\bar{t}_i)$, $i = 1, ..., r$. We compute $U_i = L_i \cap C_i$, and we let $U = U_1 \times U_2 \times ... \times U_r$. Once this is done, we collect those tuples in U, which satisfy $\prod_{i=1}^{r} u_i = 1$ and $G = \langle u_1, ..., u_r \rangle$, in a list which is denoted by $\mathcal{F}(\bar{t})$. So

$$\mathcal{F}(\bar{t}) = \{(u_1, ..., u_r) \in U : G = \langle u_1, ..., u_r \rangle \text{ and } \prod_{i=1}^{r} u_i = 1\} = U \cap \mathcal{N}(\mathcal{C}).$$

It follows from the above definition that $\mathcal{F}(\bar{t}) \subseteq \mathcal{N}(\mathcal{C})$ see page 27 for definition. Note that every element of the fiber $\mathcal{F}(\bar{t})$ is mapped to $\bar{t}$ via $\hat{\pi}$. It may be that $\mathcal{F}(\bar{t})$ is empty.

In view of Lemma 5.3, this means that every tuple from U generates a complement to V. Hence we focus on the case where $\mathcal{F}(\bar{t})$ is non-empty.

Clearly, V acts on $\mathcal{F}(\bar{t})$ via sending $(u_1, ..., u_r)$ to $(u_1^v, ..., u_r^v)$ for $v \in V$.

Claim 5.5. *$|\mathcal{F}(\bar{t})|$ is a multiple of $|V|$.*

Proof. To prove this, we need to show that the action of V on $\mathcal{F}(\bar{t})$ is semi-regular. Let $u \in \mathcal{F}(\bar{t})$ and $v \in V$. Then $u^v = (u_1^v, ..., u_r^v) = u$ if and only if $u_1^v = u_1, ..., u_r^v = u_r$. Since $u_i^{-1}u_i^v = 1$, we obtain $(v^{-1})^{u_i} = v^{-1}$. Thus for every i, $v \in C_V(u_i)$ and hence $v \in C_V(G)$ because $G = \langle u_1, ..., u_r \rangle$. Since $C_V(G) = 1$, we obtain $v = 1$ and the claim holds.

As V acts faithfully via diagonal conjugation on $\mathcal{F}(\bar{t})$, we can deal with V-orbits instead of the individual generating tuples. Let $\mathcal{F}^{(V)}(\bar{t})$ be the set of V-orbits and let q be the number of these orbits.

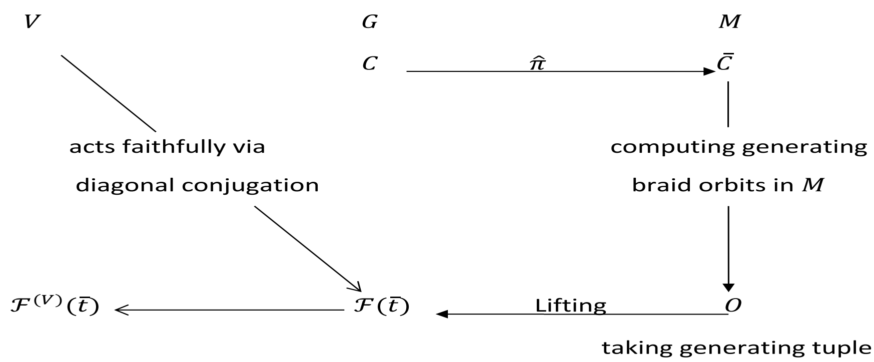

Figure 5.1: Projection-Fiber Algorithm

All the above computations are done by the function **LiftingQuotientOrbit**. It is of the form:

$$LiftingQuotientOrbit(group, degree, tuple).$$

When it terminates, it saves a file that contains the name of the group, the tuple and the list which, for each orbit $O_j(M, \bar{C})$, contains a record containing j, length of the orbit $O_j(M, \bar{C})$, the list $\mathcal{F}(\bar{t})$ for a representative tuple $\bar{t} \in O_j(M, \bar{C})$, and the number q of

V-orbits.

Let $\mathcal{S}_{\bar{t}} = \{\sigma \in B_r : \bar{t}\sigma = \bar{t}\}$. Clearly, $\mathcal{S}_{\bar{t}}$ acts on $\mathcal{F}(\bar{t})$.

Lemma 5.6. *The action of V on $\mathcal{F}(\bar{t})$ commutes with the action of $S_{\bar{t}}$.*

Proof. Let $(x_1, ..., x_r) \in \mathcal{F}(\bar{t})$. For each $v \in V$, we want to show that conjugation by v commutes with the action of every generators $\sigma_i \in S_{\bar{t}}$. We have

$$\begin{aligned}
((x_1, x_2, ..., x_r)\sigma_i)^v &= (x_1, ..., x_{i+1}, x_{i+1}^{-1}x_i x_{i+1}, ..., x_r)^v \\
&= (x_1^v, ..., x_{i+1}^v, (x_{i+1}^{-1})^v x_i^v x_{i+1}^v, ..., x_r^v) \\
&= (x_1^v, ..., x_{i+1}^v, (x_{i+1}^v)^{-1} x_i^v x_{i+1}^v, ..., x_r^v) \\
&= (x_1^v, ..., x_i^v, x_{i+1}^v, ..., x_r^v)\sigma_i \\
&= ((x_1, ..., x_i, x_{i+1}, ..., x_r)^v)\sigma_i.
\end{aligned}$$

Hence the lemma holds. □

The action of $\mathcal{S}_{\bar{t}}$ on $\mathcal{F}(\bar{t})$ commutes with the action of V, hence gives an action of $\mathcal{S}_{\bar{t}}$ on the set $\mathcal{F}^{(V)}(\bar{t})$ of V-orbits on $\mathcal{F}(\bar{t})$.

Lemma 5.7. *The $\mathcal{S}_{\bar{t}}$-orbits on $\mathcal{F}^{(V)}(\bar{t})$ are in a one-to-one correspondence with the braid orbits of $\mathcal{N}(\mathcal{C})$ which project onto the braid orbit O on $\mathcal{N}(\bar{\mathcal{C}})$.*

Proof. Since C is a generating type of G thus $\mathcal{N}(C) \neq \emptyset$ and it is the disjoint union of braid orbits, which project onto the braid orbit O on $\mathcal{N}(\bar{\mathcal{C}})$. It is clear that $\mathcal{F}^{(V)}(\bar{t}) \neq \emptyset$, where $\bar{t} \in O$. Also, we have that $\mathcal{F}^{(V)}(\bar{t}) \subseteq \mathcal{N}(C) = \bigcup_{i=1}^m B_i$. For each i, it is clear that $S_i := \mathcal{F}^{(V)}(\bar{t}) \cap B_i \neq \emptyset$. Next we want to prove that $\mathcal{S}_{\bar{t}}$ is transitive on S_i for each i. Let $t_1, t_2 \in S_i$. Then there exists an element $\sigma \in B_r$ such that $t_1\sigma = t_2$. Note that $t_1\sigma\hat{\pi} = t_2\hat{\pi} = \bar{t}$ and $t_1\hat{\pi}\sigma = \bar{t}\sigma$. From Lemma 5.1, it follows that $\sigma \in \mathcal{S}_{\bar{t}}$. □

In particular if $q = 1$, then the number of generaing B_r-orbits of type C in G is equal to the number of generaing B_r-orbits of type $\bar{C}$ in M. They have the same lengths.

The major step is leading toward the algorithm when $q > 1$. One crucial, but necessary step, we need to check whether or not the V-orbits in $\mathcal{F}^{(V)}(\bar{t})$ are transitive. This is achieved by doing the following steps:

Step1:- Compute the stabilizer $\mathcal{S}_{\bar{t}}$ in the braid group of the tuple $\bar{t}$.

Step2:- Compute the action of $\mathcal{S}_{\bar{t}}$ on $\mathcal{F}^{(V)}(\bar{t})$.

Step3:- Read off the result that is looking at the orbits.

Note that in our examples, the computations show that q is at most seven.

5.3 Applications

In this section we study some concrete examples to illustrate the application of the projection-fiber algorithm. Throughout this section, we use the notation (mA^n) instead of $(\underbrace{mA, mA, ..., mA}_{n-times})$ where mA is a conjugacy class in G.

Example 5.8. Let $C = (2B^2, 3A, 14A)$ be the type in $G = 2^4 : PSL(3, 2)$. The type C can be represented by the tuple $t := [(2, 7)(3, 6)(9, 16)(12, 13), (2, 7)(3, 6)(9, 16)(12, 13), (2, 10, 9)(3, 15, 13)(4, 8, 5)(6, 11, 16)(7, 14, 12), (1, 2, 14, 5, 15, 11, 6, 3, 4, 16, 7, 13, 9, 8)(10, 12)]$.

```
gap> k:=PrimitiveGroup(16,11);

2^4:PSL(4, 2)
```

First, we use the function **LiftingQuotientOrbit** which gives the following data

```
gap> LL:=LiftingQuotientOrbit(k,16,t);

Total Number of Tuples: 141120

Orbit1:

Length=7

Generating Tuple  =[( 3,10)( 4, 9)( 7,14)( 8,13),( 1, 9)( 2,10)( 5,13)
```

```
( 6,14),( 1,5,12)( 2, 3,13)( 4,11, 7)( 6,14, 8)( 9,15,10),
( 1,12, 8, 6,13,10,15)( 2, 5, 4,14, 7,11, 9) ]
[ [ rec( LargestLength := 7, F:= [ [ (5,7)(6,8)(13,15)(14,16),(2,14)
(3,15)(6,10)(7,11), (1,9,3)(2,8,13)(4,16,7)(5,12,6)(10,14,15),
(1,13,6,12,7,14,8,16,4,11,5,10,3,9)(2,15) ],...],
numberofquotientorbit:= 1, q := 2 ) ] ]
```

From the above, we obtain one braid orbit O of length seven in $M := PSL(4,2)$. Note that the parameter F as presented above is the set $\mathcal{F}(\bar{t})$ and $|F| = 32$. The full tuples in F are given in Appendix C. We denote by t_i, the ith tuple in F. Since $q = 2$, the extra calculations are needed. Second, we compute the stabilizer in the braid group of the first tuple in O and denoted by $\bar{t}$, because we pull back the first tuple in O to G. To do this, we need to extract the braid action on O from the MAPCLASS package.

```
gap> a:=O[1].ActionOnOrbit;
[ [ 2, 5, 6, 7, 1, 4, 3 ], [ 3, 4, 1, 7, 6, 5, 2 ], [ 4, 6, 5, 1, 7, 2, 3 ],
[ 2, 5, 4, 3, 1, 7, 6 ] ]
gap> x:=PermList(a[1]);
(1,2,5)(3,6,4,7)
gap> y:=PermList(a[2]);
(1,3)(2,4,7)(5,6)
gap> z:=PermList(a[3]);
(1,4)(2,6)(3,5,7)
gap> w:=PermList(a[4]);
(1,2,5)(3,4)(6,7)
gap> h:=O[1].OurAction;
[[ f1*f2*f1^-1, f1, f3, f4 ],[ f1, f3^-1*f2*f3, f3^-1*f2^-1*f3*f2*f3, f4 ],
```

```
[f1,f4^-1*f2*f4,f4^-1*f2^-1*f4*f2*f3*f2^-1*f4^-1*f2*f4,f4^-1*f2^-1*f4*f2*f4],
[f1,f2, f4^-1*f3*f4, f4^-1*f3^-1*f4*f3*f4 ]]
a:=h[1];;
b:=h[2];;
c:=h[3];;
d:=h[4];;
```

Straightforward computations show that $\mathcal{S}_{\bar{t}} = \mathcal{S}_{\bar{t}}|_{B_M} = \langle b^2, c^2, c^3a\rangle$, where $B_M :=$ image of B_4 in the action on the orbit of M-tuples of length seven. Finally, we work out the action of $\mathcal{S}_{\bar{t}}$ on $F^{(V)}$. As we mentioned before $q = 2$, this means that we have two V-orbits denoted by $F^{(V)}(1) = [t_1, t_3, t_5, t_7, t_9, t_{11}, t_{13}, t_{15}, t_{18}, t_{20}, t_{22}, t_{24}, t_{26}, t_{28}, t_{30}, t_{32}]$ and $F^{(V)}(2) = F^{(V)} \setminus F^{(V)}(1)$. The computation shows that $\mathcal{S}_{\bar{t}}$ acts transitively on $F^{(V)}$. For instance, if we take the representative tuples $t_1 \in F^{(V)}(1)$ and $t_{10} \in F^{(V)}(2)$, then there exists $b^2 \in \mathcal{S}_{\bar{t}}$ such that $t_1b^2 = t_{10}$. Thus, we obtain one $\mathcal{S}_{\bar{t}}$-orbit of length two on $F^{(V)}$. From Lemma 5.7, we obtain one generating braid orbit of type C in G and the length of this orbit is $7 \times 2 = 14$.

Example 5.9. Let t be a tuple of type $C = (3A^4)$ in $ASL(3, 2)$. The mapping $\hat{\pi}$ sends the tuple t onto a tuple $(t)\pi$ of type $\bar{C} = (3A^4)$. We can compute the generating braid orbit for $\bar{C}$ in $SL(3, 2)$ by using the function **LiftingQuotientOrbit**. We obtain two generating braid orbits $O_1(M, \bar{C})$ and $O_2(M, \bar{C})$ of lengths 90 and 144 respectively. We pull back one tuple from $O_i(M, \bar{C})$ to $ASL(3, 2)$. As V acts faithfully via diagonal conjugation on the lifting generating tuples we obtain two V-orbits. The same argument used in Example 5.8 shows that lifting one tuple from $O_1(M, \bar{C})$ gives one orbit of length 90×2. Lifting one tuple from $O_2(M, \bar{C})$ gives two orbits with the same length 144.

The previous examples are too simple, but they are good examples to explain our algorithm. The purpose of our algorithm is to deal with large braid orbits. So we can give two more examples.

Example 5.10. The group $G = 2^4 : PSL(4,2)$ acts faithfully on the 16 points of $\mathbb{F}_2^4$. We have only one generating type of length seven, which corresponds to the ramification type $C = (2B^5, 2D^2)$ and the structure constant (StC) for C is 1,137,259,549,440. The ratio $\frac{StC}{|G|} = \frac{1137259549440}{322560} \approx 3525730$ is an estimate of the sum of the lengths of all Generating Mapping Class Orbits. The direct computation of the braid orbits of this type is impossible because it takes too long. So we use the projection-fiber algorithm by applying the mapping $\hat{\pi}$ to C gives the ramification type $\bar{C} = (2A^5, 2B^2)$ in $PSL(4,2)$. The structure constant of $\bar{C}$ is 29,632,277,430. The ratio $\frac{StC}{|PSL(4,2)|} = \frac{29632277430}{20160} \approx 1469855$ is an estimate of the sum of the lengths of all Generating Mapping Class Orbits. We first compute the braid orbits O of type $\bar{C}$ in $SL(4,2)$. Second; we lift one tuple from O to G. The computations are done by the function **LiftingQuotientOrbit**. Note that on the screen appears the following:

```
Total Number of Tuples: 18192384000
Orbit1:
Length=902400
Generating Tuple  =[ ( 1, 7)( 3, 5)( 9,15)(11,13), ( 2,15)( 4,13)( 5,12)
( 7,10),( 1, 2)( 5, 6)( 9,10)(13,14), ( 1, 9)( 2,10)( 5,13)( 6,14), ( 2,13)
( 4,15)( 5,10)( 7,12), ( 1, 8)( 2, 3)( 4, 5)( 6, 7)(10,14)(11,15), ( 1, 8)
( 2, 5)( 3, 6)( 4, 7)(10,12)(13,15) ]
[ [ rec( LargestLength := 902400, numberofquotientorbit:= 1, q := 1 ) ] ]
```

Note that the size of $PSL(4,2)$ divides the total number of tuples, that is $\frac{18192384000}{20160} = 902400$. Since $q = 1$, we obtain one braid orbit of type C in G.

Example 5.11. Similarly, for the group $G = ASL(3,2)$. There is only one generating type of length nine, which is $C = (2B^9)$ and the structure constant for it is 310,088,862,720.

The ratio $\frac{StC}{|G|} = \frac{310088862720}{1344} = 230720880$ is an estimate of the sum of the lengths of all Generating Mapping Class Orbits. The direct computation of the braid orbits of this type is impossible because the estimated number of orbits is outside the countable range of Generating Mapping Class Orbits. We have the same steps as above. The only difference is that $q = 7$. Note that the size of $SL(3,2)$ divides the total number of tuples, that is $\frac{4511324160}{168}$ =26853120. Therefore, we obtain one braid orbit for C in G.

Table 5.1: Summary of Examples

Group	Ramification Type	# of orbits	the Largest length of orbit	Size of constant structure	Time spent
$PSL(4,2)$	$(2A^5, 2B^2)$	1	902400	29,632,277,430	1781 minutes
$2^4.PSL(4,2)$	$(2B^5, 2D^2)$	1	902400	1,137,259,549,440	1782 minutes
$SL(3,2)$	$(2A^9)$	1	26853120	4,732,175,280	64800 minutes
$ASL(3,2)$	$(2B^9)$	1	7×26853120	310,088,862,720	65160 minutes

We end this section with another useful property of our algorithm, which is a reduction of time of braid orbit computations in the whole group G. It follows from the fact that different types in G could merge in M. For more detail, we present the following example.

Example 5.12. Let t_i, for $i = 1,2,3$, be tuples of generating types $(2D^2, 4F, 4G)$, $(2D^2, 4F, 4H)$ and $(2D, 2E, 4F^2)$ respectively in $ASL(5,2)$. We send tuples t_i via $\hat{\pi}$ onto tuples $(t_i)\hat{\pi}$ of the type $\bar{C} = (2B^2, 4B^2)$ in $SL(5,2)$. We first compute generating braid orbits O for $\bar{C}$ in $SL(5,2)$. It gives one orbit of length 624. When we pull back a representative tuple from O to $ASL(5,2)$, we obtain one orbit for each of these three types in $ASL(5,2)$ and the length of each of them is 624, 1248 and 624 respectively. Thus, we compute three different types of $ASL(5,2)$ in a single fast computation which takes one day. However, the direct computations of the braid orbits of these three types take three days.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, we enumerated the connected components of $\mathcal{H}^{in}_{r,g}(G)$ in the cases where $g = 1, 2$ and G is an affine primitive group with $G'' \neq 1$. The total numbers of connected components of $\mathcal{H}^{in}_{r,g}(G)$ are summarized in Tables 6.1 and 6.2.

Table 6.1: Affine Primitive Genus One Systems: Number of Components

Degree	#Group Iso types	# RTs	# comp's $r = 3$	# comp's $r = 4$	# comp's $r = 5$	# comp's $r = 6$	# comp's $r = 7$	# comp's $r = 8$	# comp's total
128	1	2	2	-	-	-	-	-	2
64	24	114	738	19	-	-	-	-	757
32	1	131	2247	30	3	-	-	-	2280
16	18	599	2150	273	94	18	1	-	2544
8	2	134	68	71	28	13	4	1	185
81	14	37	71	4	-	-	-	-	75
27	7	53	163	14	2	-	-	-	179
9	5	49	34	25	9	3	-	-	71
125	2	5	24	-	-	-	-	-	24
25	5	17	34	1	-	-	-	-	35
343	1	12	12	-	-	-	-	-	12
49	4	15	14	1	-	-	-	-	15
121	1	4	20	-	-	-	-	-	20
Totals	85	1172	5577	438	136	34	5	1	6191

Table 6.2: Affine Primitive Genus Two Systems: Number of Components

Degree	#Group Iso types	# RTs	# comp's $r=3$	# comp's $r=4$	# comp's $r=5$	# comp's $r=6$	# comp's $r=7$	# comp's $r=8$	# comp's $r=9$	# comp's total
256	4	10	98	-	-	-	-	-	-	98
128	1	3	100	-	-	-	-	-	-	100
64	29	285	3072	35	-	-	-	-	-	3107
32	1	363	13418	86	7	-	-	-	-	13511
16	16	1159	6091	614	274	76	14	1	-	7070
8	3	272	76	143	74	34	14	4	1	346
81	13	29	131		-	-	-	-		131
27	7	100	438	21	2	-	-	-		461
9	4	96	26	75	19	3	-	-		194
125	2	6	12	-	-	-	-	-		12
25	13	90	118	28	4	-	-	-		150
49	1	1	24		-	-	-	-		24
289	1	2	16	-	-	-	-	-		16
Totals	95	2417	23620	1002	380	113	28	5	1	25149

To achieve this, we needed two types of calculations.

Type 1 We needed to check whether or not an affine primitive group possessed primitive genus g systems.

Type 2 We needed to compute braid orbits of the affine primitive group that possessed primitive genus g systems.

In most cases, both calculations were pretty straightforward except for some groups.

In type 1, for a given p^e and g, we used the function **GeneratingType** to check whether or not the affine primitive group possessed primitive genus g systems. The function worked for all affine primitive groups except the groups $AGL(8,2)$, $AGL(9,2)$ and $AGL(10,2)$ because they are too big. We used some other arguments such as the projection algorithm, and the translation algorithm in [7] to check them.

In type 2, using the MAPCLASS package, we computed braid orbits for all affine primitive groups and types except some types in the groups $2^4.PSL(4,2)$, $ASL(5,2)$ and $AGL(6,2)$. We introduced the projection-fiber algorithm to compute braid orbits of those types with the aid MAPCLASS package. We verified the algorithm by comparing results in M with explicit results computed in G for some cases.

6.2 Future work

The algorithm introduced in this thesis is targeted to handle large braid orbits. Note that our algorithm can only be used when a group is a semidirect product. Furthermore, another algorithm exists for computing large braid orbits in [8]. Our idea is to combine both algorithms together as follows. The mapping $\hat{\pi}$ sends the tuple t of type C in G onto the tuple $(t)\hat{\pi}$ of type $\bar{C}$ in M. We can use the matching algorithm in [8] to compute generating braid orbits O for $(t)\pi$ of type $\bar{C}$ in M. Next, we pull back a representative tuple from O to G. It seems this idea saves a lot of time.

Appendix A
Genus One Covers

Note that N.O means number of orbits, L.O means largest length of the orbit and GOS means Genus one System.

Table A.1: Symbols

Symbols	Description
S_n	Symmetric Group
A_n	Alternating Group
C_n	Cyclic Group of order n
$AGL(n, p)$	Affine general linear Group
$ASL(n, p)$	Affine special linear Group
Q_n	quaternion group
$D(2 * n)$	dihedral group
Sp	Symplectic group
SU	Special unitary group
GO	GeneralOrthogonalGroup
PGL	ProjectiveSpecialLinearGroup
$A\Gamma L$	Affine semilinear group

Table A.2: GOSs for Affine Primitive Groups of Degree 121

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(2, 11) : 2$	(2B,3A,10A)	5	1	(2B,3A,10B)	5	1
	(2B,3A,10C)	5	1	(2B,3A,10D)	5	1

Table A.3: GOSs for Affine Primitive Groups of Degree 49 and 343

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$7^2 : S_3$	(2A,3A,14A)	1	1	(2A,3A,14B)	1	1
	(2A,3A,14C)	1	1	(2A,3A,14D)	1	1
	(2A,3A,14E)	1	1	(2A,3A,14F)	1	1
$7^2 : 3 \times D(2*6)$	(2A,6C,6H)	1	1	(2A,6D,6I)	1	1
	(2B,6C,6G)	1	1	(2B,6D,6F)	1	1
$7^2 : D(2*6)$	(2A,2B,2C,3A)	1	24			
$7^2 : 3 \times (Q_8 : 3)$	(3D,3F,6C)	1	1	(3C,3F,6E)	1	1
	(3B,3G,6D)	1	1	(3A,3G,6F)	1	1
$7^3 : PSL(2,7)$	(2A,3A,7U)	1	1	(2A,3A,7V)	1	1
	(2A,3A,7K)	1	1	(2A,3A,7L)	1	1
	(2A,3A,7M)	1	1	(2A,3A,7N)	1	1
	(2A,3A,7O)	1	1	(2A,3A,7P)	1	1
	(2A,3A,7Q)	1	1	(2A,3A,7R)	1	1
	(2A,3A,7S)	1	1	(2A,3A,7T)	1	1

Table A.4: GOSs for Affine Primitive Groups of Degree 25 and 125

group	ramification type	N.O	L.O	ramification type	N.O	L.O
AGL(2,5)	(2B,4F,24A)	1	1	(2B,4F,24B)	1	1
	(2B,4E,24C)	1	1	(2B,4E,24D)	1	1
	(3A,4E,4F)	6	1			
ASL(2,5)	(3A,4A,5C)	2	1	(3A,4A,5B)	2	1
$5^2 : 4 \times D(2*3)$	(2B,4C,12B)	1	1	(2B,4D,12A)	1	1
	(2A,4C,12A)	1	1	(2A,4D,12B)	1	1
$5^2 : Q_8 : 3$	(3A,3B,4A)	12	1			
$5^2 : D(2*4)$	(2A,4A,10C)	1	1	(2A,4A,10D)	1	1
	(2B,4A,10A)	1	1	(2B,4A,10B)	1	1
	(2A,2B,2C,4A)	1	12			
$5^3 : S(5)$	(2A,4A,5G)	1	1	(2A,4A,5I)	1	1
	(2A,4A,5H)	1	1	(2A,4A,5J)	1	1
$5^3 : S(5)$	(2B,4A,6A)	20	1			

Table A.5: GOSs for Affine Primitive Groups of Degree 81

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$3^4 : Sp(4,3) : 2$	(2C,6N,5A)	14	1			
$3^4 : (GL(1,3) \wr A(4))$	(2D,6H,6N)	1	1	(2D,6G,6M)	1	1
$3^4 : (Q_8 : 2)S(3)$	(2A,2C,2D,3F)	1	12	(2A,2C,2D,3G)	1	12
$3^4 : (2 \times S(5))$	(2C,6H,6N)	1	1			
$3^4(2 \times A(6).2)$	(2C,4C,8B)	2	1	(2C,4C,8D)	2	1
	(2C,4B,8A)	2	1	(2C,4B,8C)	2	1
$3^4 : (Q_8 : 3) : 2$	(2A,3J,12B)	1	1	(2A,3J,12C)	1	1
	(2A,3I,12A)	1	1	(2A,3I,12D)	1	1
	(2A,3H,12A)	1	1	(2A,3H,12D)	1	1
	(2A,3G,12B)	1	1	(2A,3G,12C)	1	1
$3^4 : 2.A(5)$	(2C,3F,10A)	1	1	(2B,3F,10B)	1	1
	(2B,2C,2C,3F)	1	18			
$3^4(2^{(3+4)}) : 4$	(2D,4E,8A)	1	1	(2D,4E,8B)	1	1
	(2D,4D,8C)	1	1	(2D,4D,8D)	1	1
$3^4 : S(6)$	(2C,5A,6I)	6	1			
$3^4 : 2.A(5) : 2$	(2B,5A,6C)	1	1	(2B,5A,6B)	1	1
$3^4 : (2^3 : A(4)) : S(3)$	(2G,6O,6U)	3	1	(2G,6N,6V)	3	1
$3^4 : (2^3 : 2^2) : 3^2 : D(2*4)$	(2A,6N,8D)	1	1	(2A,6N,8E)	1	1
$3^4 : (2^3 : 2^2) : (3^2 : 4)$	(2C,4E,8C)	4	1	(2C,4E,8D)	4	1
	(2C,4D,8A)	4	1	(2C,4D,8B)	4	1
$3^4 : Q_8 : S(4)$	(2C,2D,2D,3F)	1	24			

Table A.6: GOSs for Affine Primitive Groups of Degree 27

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(3,3)$	(3E,6D,6F)	8	1	(3E,6D,6G)	8	1
	(3E,4B,4B)	32	1	(4B,6E,6D)	8	1
	(4B,4A,6D)	12	1			
	(2C,6D,26B)	1	1	(2C,6D,26C)	1	1
	(2C,6D,26D)	1	1	(2C,6D,26A)	1	1
	(2B,6F,13B)	1	1	(2B,6F,13C)	1	1
	(2B,6F,13D)	1	1	(2B,6F,13A)	1	1
	(2B,6G,13B)	1	1	(2B,6G,13C)	1	1
	(2B,6G,13D)	1	1	(2B,6G,13A)	1	1
	(2B,2B,4B,6D)	1	144	(2B,2B,2C,13A)	1	13
	(2B,2B,2C,13B)	1	13	(2B,2B,2C,13C)	1	13
	(2B,2B,2C,13D)	1	13	(2B,2C,3E,6D)	1	144
$ASL(3,3)$	(2A,6B,13A)	2	1	(2A,6B,13B)	2	1
	(2A,6B,13C)	2	1	(2A,6B,13D)	2	1
	(2A,4A,13A)	2	1	(2A,4A,13B)	2	1
	(2A,4A,13C)	2	1	(2A,4A,13D)	2	1
	(3F,3F,8A)	16	1	(3F,3F,8B)	16	1
	(3F,3F,6D)	8	1	(3F,3F,6C)	8	1
$3^3 : S_4 \times 2$	(2A,2E,2E,6H)	1	12	(2B,2A,2E,9A)	1	3
	(2B,2E,2C,6H)	1	12	(2B,2B,2E,12A)	1	4
	(2B,2B,2C,2E,2E)	1	48	(2B,2A,2E,2E,2E)	1	48
	(2A,6A,12A)	1	1			
$3^3 : S_4$	(2A,2A,2B,9A)	1	3	(2A,2A,2B,9B)	1	3
	(3E,4A,4A)	8	1			
$3^3 : 2.A_4$	(2A,2C,3D,3E)	1	4	(2A,6E,9A)	1	1
	(2A,6E,9B)	1	1	(2B,6E,6F)	4	1
$3^3 : A_4 \times 2$	(2B,2B,2B,4A)	1	32	(2B,4A,9A)	1	1
	(2B,4A,9B)	1	1			
$3^3 : A(4)$	(3E,3F,6A)	2	1			

Table A.7: GOSs for Affine Primitive Groups of Degree 9

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(2,3)$	(3B,8A,8A)	1	1	(3B,8B,8B)	1	1
	(3B,6B,8A)	1	1	(3B,6B,8B)	1	1
	(2B,2B,3C,3C)	1	24	(2B,2B,3C,4A)	1	36
	(2B,2B,3C,6A)	1	36	(2B,2B,4A,6A)	1	48
	(2B,2B,6A,6A)	1	48	(2B,2A,3B,8A)	1	4
	(2B,2A,3B,8B)	1	4	(2B,3B,3B,6B)	1	12
	(2B,3B,3B,8A)	1	8	(2B,3B,3B,8B)	1	8
	(2B,2B,3B,3B,3B)	1	120	(2B,2B,2A,3B,3B)	1	48
	(2B,2B,2B,2B,3C)	1	648	(2B,2B,2B,2B,4A)	1	768
	(2B,2B,2B,2B,6A)	1	864	(2B,2B,2B,2B,2B,2B)	1	15360
$ASL(2,3)$	(2A,3C,3C,3D)	1	3	(3C,3C,3C,4A)	1	4
	(2A,3B,3C,4A)	2	2	(3C,3B,3B,3D)	1	3
	(3B,3C,3C,6B)	1	2	(2A,3B,3B,3E)	1	3
	(3B,3C,3C,3E)	1	3	(3C,3B,3B,6A)	1	2
	(3B,3B,3B,4A)	1	4	(3E,3D,4A)	4	1
	(3D,3D,6B)	2	1	(3E,4A,6B)	3	1
	(6B,6B,6B)	4	1	(3E,3E,6A)	2	1
	(3D,4A,6A)	3	1	(4A,6B,6A)	4	1
	(6A,6A,6A)	4	1			
AΓ L(1,9)	(2B,2B,4B,4B)	1	32			
$3^2 : D(2*4)$	(2B,2B,4A,4A)	1	8	(2A,2B,3B,4A)	1	2
	(2A,2B,3A,4A)	1	2	(2A,2A,4A,4A)	1	8
	(2A,2B,2B,2B,4A)	1	16	(2A,2A,2B,2B,3B)	1	4
	(2A,2A,2B,2B,3A)	1	4	(2A,2A,2A,2B,4A)	1	16
	(2A,2A,2B,2B,2B,2B)	1	32	(2A,2A,2A,2A,2B,2B)	1	32
AGL(1,9)	(2A,8B,8A)	1	1	(2A,8C,8D)	1	1
$3^2 : Q_8$	(4A,4B,4C)	4	1			
$3^2 : 4$	(3A,4A,4B)	1	1	(3B,4A,4B)	1	1

Table A.8: Part1: GOSs for Affine Primitive Groups of Degree 8

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(1,8)$	(2A,7D,7F)	1	1	(2A,7B,7C)	1	1
	(2A,7A,7E)	1	1			
$A\Gamma L(1,8)$	(3B,6B,6B)	2	1	(3B,6A,7B)	2	1
	(3B,6A,7A)	2	1	(3A,7B,7B)	2	1
	(3A,6B,7B)	2	1	(3A,6B,7A)	2	1
	(2A,3B,3B,3B)	1	14	(3A,3A,3B,3B)	1	48
	(2A,3A,3A,3A)	1	14			
$ASL(3,2)$	(3A,7B,7B)	2	1	(3A,7A,7B)	2	1
	(3A,7A,7A)	2	1	(3A,6A,7B)	2	1
	(3A,6A,7A)	2	1	(3A,4C,7B)	2	1
	(3A,4C,7A)	2	1	(3A,4C,6A)	4	1
	(3A,4A,7A)	2	1	(3A,4A,7B)	2	1
	(4B,7A,7B)	6	1	(4B,6A,7B)	4	1
	(4B,6A,7A)	4	1	(4B,6A,6A)	2	1
	(4B,4C,7A)	2	1	(4B,4C,7B)	2	1
	(4B,4C,6A)	2	1	(4B,4C,4C)	4	1
	(4B,4A,7A)	2	1	(4B,4A,7B)	2	1
	(2C,7B,7B)	1	1	(2C,7A,7A)	1	1
	(2C,6A,7B)	1	1	(2C,6A,7A)	1	1
	(3A,3A,3A,3A)	3	180	(3A,3A,3A,4B)	2	384
	(3A,3A,4B,4B)	4	288	(3A,4B,4B,4B)	2	216
	(4B,4B,4B,4B)	3	168	(2C,4B,4B,4B)	1	72
	(2C,3A,4B,4B)	1	132	(2C,3A,3A,4B)	1	168
	(2C,3A,3A,3A)	1	120	(2C,2C,3A,4B)	1	24
	(2C,2C,4B,4B)	1	24	(2B,3A,3A,7B)	1	168
	(2B,3A,3A,7A)	1	168	(2B,3A,3A,6A)	1	240
	(2B,3A,3A,4C)	1	132	(2B,3A,3A,4A)	1	60
	(2B,3A,4B,7A)	1	126	(2B,3A,3A,7B)	1	126
	(2B,3A,4B,6A)	1	168	(2B,3A,3A,4C)	1	132
	(2B,3A,4B,4A)	1	48	(2B,4B,4B,7B)	1	105
	(2B,4B,4B,7A)	1	105	(2B,4B,4B,6A)	1	132
	(2B,4B,4B,4A)	1	72	(2B,4B,4B,4B)	1	48
	(2B,2C,3A,7A)	1	42	(2B,2C,3A,7B)	1	42
	(2B,2C,3A,6A)	1	30	(2B,2C,3A,4C)	1	24
	(2B,2C,4B,4C)	1	24	(2B,2C,4B,7B)	1	28
	(2B,2C,4B,7A)	1	28	(2B,2C,4B,6A)	1	24
	(2B,2C,2C,7A)	1	7	(2B,2C,2C,7A)	1	7
	(2B,2B,7B,7B)	2	14	(2B,2B,7A,7A)	2	14
	(2B,2B,7A,7B)	1	42	(2B,2B,6A,7B)	1	42
	(2B,2B,7A,7A)	1	42	(2B,2B,6A,6A)	1	30
	(2B,2B,4C,7B)	1	28	(2B,2B,4C,7A)	1	28
	(2B,2B,4C,6A)	1	24	(2B,2B,4C,4C)	1	24
	(2B,2A,3A,7A)	1	7	(2B,2A,3A,7B)	1	7
	(2B,2A,4B,7A)	1	7	(2B,2A,4B,7B)	1	7
	(2A,3A,3A,4B)	2	14	(2A,3A,4B,4B)	1	14
	(2A,4B,4B,4B)	2	14	(2B,2B,4A,7B)	1	14
	(2B,2B,3A,3A,3A)	1	7812	(2B,2B,4A,7A)	1	14
	(2B,2B,3A,3A,4C)	1	5868	(2B,2B,3A,4B,4B)	1	4374
	(2B,2B,4B,4B,4B)	1	3564	(2B,2B,2C,3A,3A)	1	1728
	(2B,2B,2C,3A,4B)	1	1296	(2B,2B,2C,4B,4B)	1	912
	(2B,2B,2C,2C,3A)	1	216	(2B,2B,2C,2C,4B)	1	192
	(2B,2B,2B,3A,7A)	1	1323	(2B,2B,2B,3A,7B)	1	1323
	(2B,2B,2B,3A,6A)	1	1728	(2B,2B,2B,3A,4A)	1	1296
	(2B,2B,2B,3A,4B)	1	432	(2B,2B,2B,4B,7A)	1	1029
	(2B,2B,2B,4B,7B)	1	1029	(2B,2B,2B,4B,6A)	1	1296
	(2B,2B,2B,4B,4C)	1	912	(2B,2B,2B,4B,4A)	1	384
	(2B,2B,2B,2C,7A)	1	294	(2B,2B,2B,2C,7B)	1	294
	(2B,2B,2B,2C,6A)	1	294	(2B,2B,2B,2C,4C)	1	294
	(2B,2B,2B,2A,7A)	1	49	(2B,2B,2B,2A,7B)	1	49
	(2B,2B,2A,3A,3A)	1	210	(2B,2B,2A,3A,4B)	1	168
	(2B,2B,2A,4B,4B)	1	168	(2B,2B,2B,2B,3A,3A)	1	60426

Table A.9: Part2: GOSs for Affine Primitive Groups of Degree 8

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(3,2)$	(2B,2B,2B,2B,2A,3A)	1	1512	(2B,2B,2B,2B,2A,4B)	1	1344
	(2B,2B,2B,2B,3A,4B)	1	45360	(2B,2B,2B,2B,4B,4B)	1	34992
	(2B,2B,2B,2B,2C,3A)	1	12960	(2B,2B,2B,2B,2C,4B)	1	9600
	(2B,2B,2B,2B,2C,2C)	1	1680	(2B,2B,2B,2B,2B,7A)	1	10290
	(2B,2B,2B,2B,2B,6A)	1	12960	(2B,2B,2B,2B,2B,7B)	1	10290
	(2B,2B,2B,2B,2B,4A)	1	3360	(2B,2B,2B,2B,2B,4C)	1	9600
	(2B,2B,2B,2B,2B,2B,3A)	1	466560	(2B,2B,2B,2B,2B,2B,4B)	1	354240
	(2B,2B,2B,2B,2B,2B,2C)	1	97920	(2B,2B,2B,2B,2B,2B,2A)	1	11760
	(2B,2B,2B,2B,2B,2B,2B,2B)	1	3623760			

Table A.10: Part1: GOSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4 : 5$	(2C,5B,5C)	1	1	(2C,5A,5D)	1	1
	(2B,5B,5C)	1	1	(2B,5A,5D)	1	1
	(2A,5B,5C)	1	1	(2A,5A,5D)	1	1
$2^4 : D(2*4)$	(2D,2D,2C,5B)	1	5	(2D,2D,2C,5A)	1	5
	(2D,2D,2B,5B)	1	5	(2D,2D,2B,5A)	1	5
	(2D,2D,2A,5B)	1	5	(2D,2D,2A,5A)	1	5
	(2D,2D,2D,2D)	1	60	(2D,2D,2D,2D,2B)	1	60
	(2D,2D,2D,2D,2A)	1	60			
$2^4 : (A(4) \times A(4)) : 2$	(3B,6D,6D)	1	1	(3B,4A,6C)	1	1
	(3A,6C,6C)	1	1	(3A,4A,6D)	1	1
	(2C,2C,3B,6B)	1	2	(2A,2A,3A,6A)	1	2
$(2^4 : 5).4$	(4C,4C,4A)	1	1	(4C,4C,4D)	4	1
	(4B,4C,5A)	3	1	(4B,4B,4A)	1	1
	(2C,2C,4B,4C)	1	18	(4B,4B,4D)	4	1
$AGL(1,16) : 2$	(2B,6B,15A)	1	1	(2B,6B,15B)	1	1
	(2B,6A,15C)	1	1	(2B,6A,15D)	1	1
$2^4 : S(3) \times S(3)$	(2D,2E,3A,6C)	1	6	(2D,2E,3A,4A)	1	3
	(2C,2E,3A,6B)	1	6	(2C,2E,3A,4B)	1	3
	(2C,2E,3A,4D)	1	2	(2C,2E,3A,4C)	1	1
	(2D,2D,2E,2E,3A)	1	27	(2C,2C,2E,2E,3A)	1	27
	(2C,2C,2D,2D,3A)	1	3			
$2^4.3^2 : 4$	(4C,4C,4D)	4	1	(4C,4C,4A)	2	1
	(4B,4C,6A)	3	1	(4B,4B,4D)	4	1
	(4B,4B,4A)	2	1	(3A,4C,8A)	1	1
	(3A,4B,8B)	1	1	(2C,2C,4B,4C)	3	24
$A\Gamma L(1,16)$	(4C,4C,6A)	3	2	(4B,4B,6A)	3	2
$S(4) \times S(4) : 2$	(4F,6B,6C)	6	1	(4F,4E,6B)	3	1
	(2C,2F,4F,4F)	1	24	(2C,2C,4F,8A)	1	8
	(2C,2C,6C,6C)	1	12	(2C,2C,4C,6C)	1	6
	(2C,2E,4F,6C)	1	27	(2C,2E,4F,4E)	1	12
	(2C,2D,4F,4D)	1	8	(2C,2D,4F,4C)	1	4
	(2C,2D,4F,6A)	1	6	(2C,2D,6B,6C)	1	12
	(2C,2D,4E,6B)	1	6	(2C,2D,3A,8A)	1	2
	(2D,2E,4F,6B)	1	27	(2D,2D,4F,4F)	1	24
	(2D,2D,6B,6B)	1	12	(2D,2F,3A,4F)	1	6
	(2C,2C,2D,2F,4F)	1	48	(2C,2C,2D,2E,6C)	1	54
	(2C,2C,2D,2D,4B)	1	16	(2C,2C,2D,2E,6D)	1	24
	(2C,2C,2D,2D,4C)	1	8	(2C,2C,2D,2D,6A)	1	12
	(2C,2D,2E,2E,4F)	1	120	(2C,2D,2D,2E,6B)	1	54
	(2C,2D,2D,2D,4F)	1	48	(2C,2D,2D,2F,3A)	1	12
	(2C,2C,2C,2D,8A)	1	16	(2C,2C,2C,2D,2D,2F)	1	96
	(2C,2C,2D,2D,2E,2E)	1	240	(2C,2C,2D,2D,2D,2D)	1	96
$A\Gamma L(2,4)$	(4C,4C,6A)	6	1	(2C,6C,15A)	3	1
	(2C,6C,15B)	3	1	(3B,6C,6C)	6	1
	(2C,4C,8A)	2	1	(2C,2C,4C,4C)	1	192
	(2C,2C,3B,5A)	1	30	(2C,3B,3B,4C)	1	32
	(2B,2C,2C,15A)	1	15	(2B,2C,2C,15A)	1	15
	(2B,2C,3B,6C)	1	42	(2B,2B,2C,2C,3B)	1	288
$ASL(2,4) : 2$	(2C,6A,8A)	2	1	(3A,4D,6A)	12	1
	(4D,4D,5A)	2	1	(2B,5A,8A)	2	1
	(2C,2C,4D,4D)	1	32	(2B,2C,2C,8A)	2	8
	(2B,2C,4D,3A)	2	30	(2B,2B,4D,4D)	1	16

Table A.11: Part2: GOSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(2,4)$	(2B,6B,15C)	1	1	(2B,6B,15B)	1	1
	(2B,6A,15D)	1	1	(2B,6A,15A)	1	1
	(3D,6C,5B)	1	1	(3D,6C,5A)	1	1
	(3D,3E,15C)	1	1	(3D,3E,15B)	1	1
	(3D,6A,5B)	1	1	(3D,6A,5A)	1	1
	(3C,6D,5B)	1	1	(3C,6D,5A)	1	1
	(3C,3E,15D)	1	1	(3C,3E,15A)	1	1
	(3C,6B,5B)	1	1	(3C,6B,5A)	1	1
	(2B,2B,3D,6C)	1	18	(2B,2B,3D,6A)	1	18
	(2B,2B,3C,6D)	1	18	(2B,2B,3D,6B)	1	18
	(2B,3C,3D,3E)	1	24			
$ASL(2,4)$	(3A,3A,5B)	12	1	(3A,3A,5A)	12	1
	(2B,2B,3A,3A)	3	72			
$2^4.A(6)$	(3B,4C,5B)	12	1	(3B,4C,5A)	12	1
	(3B,4C,6A)	12	1	(4C,4C,5B)	8	1
	(4C,4C,5A)	8	1	(2B,5B,8A)	4	1
	(3A,5A,5B)	2	1	(3A,3B,8A)	4	1
	(2B,2B,4C,3B)	2	144	(2B,2B,4C,4C)	3	80
	(2B,2B,2B,8A)	4	24	(2B,2B,3A,5B)	1	60
	(2B,2B,3A,5B)	1	60	(2B,2B,3A,3B)	1	36
	(2B,2B,2B,2B,3A)	1	1728			
$2^4:S(5)$	(5A,6C,6C)	3	1	(4B,6C,6C)	1	1
	(4D,6C,6C)	2	1	(6C,6C,6A)	1	1
	(6C,6C,6B)	2	1	(4E,5A,6C)	6	1
	(4E,4B,6C)	1	1	(4E,4D,6C)	2	1
	(4E,6A,6C)	2	1	(4E,6B,6C)	4	1
	(4E,4E,5A)	3	1	(4E,4E,4B)	1	1
	(4E,4E,6B)	4	1	(3A,6C,12A)	1	1
	(3A,6C,8A)	2	1	(3A,4E,12A)	2	1
	(3A,4E,8A)	1	1	(2D,2D,6C,6C)	1	36
	(2D,2D,4E,6C)	1	54	(2D,2D,4E,4E)	1	48
	(2D,2E,3A,6C)	1	21	(2D,2E,3A,4E)	1	30
	(2C,2D,6C,4E)	1	30	(2C,2D,6C,4B)	1	6
	(2C,2D,6C,4D)	1	12	(2C,2D,6C,6A)	1	7
	(2C,2D,6C,6B)	1	14	(2C,2D,4E,5A)	1	45
	(2C,2D,4E,4C)	1	8	(2C,2D,4E,4D)	1	16
	(2C,2D,4E,6A)	1	10	(2C,2D,4E,6B)	1	20
	(2C,2D,3A,12A)	1	7	(2C,2D,3A,8A)	1	18
	(2C,2E,6C,6C)	1	18	(2C,2E,4E,6C)	1	24
	(2C,2E,4E,4E)	1	24	(2C,2E,3A,5A)	1	15
	(2C,2C,5A,5A)	1	30	(2C,2C,6C,12A)	1	6
	(2C,2C,6C,8A)	1	8	(2C,2C,4E,12A)	1	8
	(2C,2C,4E,8A)	1	8	(2C,2C,4B,5A)	1	5
	(2C,2C,4D,5A)	1	10	(2C,2C,6A,5A)	1	5
	(2C,2C,6B,5A)	1	10	(2C,2D,2D,2D,6C)	1	270
	(2C,2D,2D,2D,4E)	1	408	(2C,2D,2D,2E,3A)	1	144
	(2C,2C,2D,2D,5A)	1	225	(2C,2C,2D,2D,4B)	1	40
	(2C,2C,2D,2D,4D)	1	80	(2C,2C,2D,2D,6A)	1	48
	(2C,2C,2D,2D,6B)	1	96	(2C,2C,2D,2E,6C)	1	108
	(2C,2C,2D,2E,4E)	1	144	(2C,2C,2C,2D,12A)	1	36
	(2C,2C,2C,2D,8A)	1	48	(2C,2C,2C,2E,5A)	1	75
	(2C,2C,2C,2D,2D,2E)	1	730	(2C,2C,2D,2D,2D,2D)	1	2016
$2^4:A(5)$	(3A,5B,5B)	3	1	(3A,5A,5B)	3	1
	(3A,5A,5A)	3	1	(3A,5B,6C)	1	1
	(3A,5B,6A)	1	1	(3A,5B,6B)	1	1
	(3A,5A,6B)	1	1	(3A,5B,6C)	1	1
	(3A,5A,6C)	1	1	(3A,4A,5A)	1	1
	(3A,4A,5B)	1	1	(3A,4B,5A)	1	1
	(2C,2C,3A,5B)	1	45	(3A,4B,5B)	1	1
	(2C,2C,3A,5A)	1	45	(2C,2C,3A,6A)	1	18
	(2C,2C,3A,6B)	1	18	(2C,2C,3A,6C)	1	18
	(2C,2C,3A,4A)	1	18	(2C,2C,3A,4B)	1	36
	(2C,2C,2C,2C,3A)	1	810			

Table A.12: Part3: GOSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4.A(7)$	(3A,6B,6A)	6	1	(3A,6B,6B)	6	1
	(3A,3B,14B)	2	1	(3A,3B,14A)	2	1
	(3A,6B,7B)	3	1	(3A,6B,7A)	3	1
	(3A,5A,6A)	6	1	(3A,5A,6B)	18	1
	(3A,5A,7A)	8	1	(3A,4A,7B)	8	1
	(3A,5A,5A)	9	2	(3A,4B,14B)	6	1
	(3A,4B,14A)	6	1	(3A,4B,8A)	8	1
	(3B,4B,7A)	6	1	(3B,4B,7B)	6	1
	(4B,4B,6A)	24	1	(4B,4B,6B)	36	1
	(4B,4B,7B)	24	1	(4B,4B,7A)	24	1
	(4B,4B,5A)	32	2	(2B,6B,14B)	4	1
	(2B,6B,14A)	4	1	(2B,7B,14B)	2	1
	(2B,7B,14A)	1	1	(2B,7B,8A)	2	1
	(2B,7A,14B)	1	1	(2B,7A,14A)	2	1
	(2B,7A,8A)	2	1	(2B,5A,14B)	4	1
	(2B,2B,3A,6A)	1	132	(2B,5A,14A)	4	1
	(2B,2B,3A,6B)	1	270	(2B,2B,3A,7B)	1	126
	(2B,2B,3A,7A)	1	126	(2B,2B,3A,5A)	1	450
	(2B,2B,4B,4B)	3	504	(2B,2B,2B,14B)	2	42
	(2B,2B,2B,14A)	2	42	(2B,3A,3A,3B)	1	186
	(2B,3A,3A,4B)	1	576	(2B,2B,2B,2B,3A)	1	9720
$2^4.S(6)$	(4E,4F,6C)	8	1	(3B,6B,6C)	12	1
	(4F ,5A,6B)	8	1	(4E,6B,6C)	6	1
	(5A,6B,6B)	14	1	(2C,5A,12A)	2	1
	(2E,6C,6C)	6	1	(2D,5A,6C)	6	1
	(2E,6C,12A)	6	1	(2B,2B,2B,2B,2E,3B)	1	540
	(2C,2C,4F,6B)	1	12	(2C,2C,6B,6B)	1	24
	(2C,2C,3A,5A)	1	10	(2C,2D,3A,3B)	1	12
	(2E,2C,4E,4F)	1	40	(2E,2C,3B,6B)	2	27
	(2E,2C,4E,6B)	1	42	(2E,2C,3A,6C)	1	24
	(2E,2C,2D,5A)	1	30	(2E,2E,4F,4F)	1	32
	(2E,2E,4F,6B)	1	120	(2E,2E,6B,6B)	2	144
	(2E,2E,2C,12A)	2	12	(2E,2E,2D,6C)	1	72
	(2B,2C,3B,5A)	2	15	(2B,2C,3B,6A)	2	6
	(2B,2C,4F,6C)	1	18	(2B,2C,4E,5A)	1	20
	(2B,2C,6B,6C)	1	18	(2B,2E,3B,6C)	2	36
	(2B,2E,4F,5A)	1	40	(2B,2E,4E,6C)	1	42
	(2B,2E,6B,5A)	1	70	(2B,3A,3B,4F)	1	14
	(2B,3A,3B,6B)	1	18	(2B,2D,3B,4E)	1	24
	(2B,2B,6C,6C)	1	24	(2B,2B,5A,5A)	1	20
	(2B,2B,3B,8A)	1	8	(2E,2E,2C,2C,3A)	1	108
	(2E,2E,2E,2C,2D)	1	360	(2B,2E,2C,2C,4F)	1	80
	(2B,2E,2E,2C,3B)	2	171	(2B,2E,2C,2C,6B)	1	108
	(2B,2E,2E,2C,4E)	1	224	(2B,2E,2C,2E,4F)	1	624
	(2B,2E,2E,2E,6B)	1	972	(2B,2B,2C,2C,5A)	1	50
	(2B,2B,2C,2D,3B)	1	54	(2B,2B,2E,2C,6C)	1	108
	(2B,2B,2E,2E,5A)	1	350	(2B,2B,2E,3A,3B)	1	96
	(2B,2B,2B,3B,4F)	1	72	(2B,2B,2B,3B,6B)	1	108
	(2B,2B,2E,2E,2C,2C)	1	568	(2B,2B,2E,2E,2E,2E)	1	5040
$2^4.PSL(4,2)$	(6B,6B,7B)	64	1	(6B,6B,7A)	64	1
	(6B,6B,6C)	126	1	(6B,6B,6A)	30	1
	(6B,6B,5A)	114	1	(6B,6B,4E)	24	1
	(4F,6B,7B)	52	1	(4F,6B,7A)	52	1
	(4F,6B,6C)	90	1	(4F,6B,6A)	30	1
	(4F,6B,5A)	42	1	(4F,6B,4E)	36	1
	(4F,4F,7B)	18	1	(4F,4F,7A)	18	1
	(4F,4F,4E)	24	1	(4D,6B,7B)	18	1
	(4D,6B,7A)	18	1	(4D,6B,6C)	24	1
	(4D,6B,5A)	36	1	(4D,4D,6C)	4	1
	(4D,4D,5A)	6	1	(4C,6C,7B)	15	1
	(4C,6C,7A)	15	1	(4C,6C,6C)	36	1
	(4C,6B,15B)	12	1	(4C,6B,15A)	12	1
	(4C,6B,14B)	18	1	(4C,6B,14A)	18	1
	(4C,6B,12A)	12	1	(4C,6B,8A)	18	1

Table A.13: Part4: GOSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(4C,5A,6C)	24	1	(4C,5A,7B)	12	1
	(4C,5A,7A)	12	1	(4C,5A,5A)	12	1
	(4C,6A,6A)	6	1	(4C,6A,5A)	12	1
	(4C,4F,15B)	12	1	(4C,4F,15A)	12	1
	(4C,4F,14B)	14	1	(4C,4F,14A)	14	1
	(4C,4F,12A)	18	1	(4C,4F,8A)	12	1
	(4C,4E,6C)	8	1	(4C,4E,5A)	12	1
	(4C,4D,15B)	3	1	(4C,4D,15A)	3	1
	(3A,4C,15B)	3	1	(3A,4C,15A)	2	1
	(3A,4C,14B)	2	1	(3A,4C,14A)	2	1
	(3B,6C,7B)	11	1	(3B,6C,7A)	11	1
	(3B,6B,15B)	14	1	(3B,6B,15A)	14	1
	(3B,6B,14B)	16	1	(3B,6B,14A)	16	1
	(3B,6B,12A)	10	1	(3B,6B,8A)	10	1
	(3B,5A,7B)	3	1	(3B,5A,7A)	3	1
	(3B,4F,15B)	8	1	(3B,4F,15A)	8	1
	(3B,4F,14B)	6	1	(3B,4F,14A)	6	1
	(3B,4F,12A)	20	1	(3B,4E,6C)	8	1
	(3B,4E,5A)	4	1	(3B,4D,15B)	4	1
	(4F,4D,6C)	24	1	(3B,4D,15A)	4	1
	(4F,4D,7A)	14	1	(4F,4D,7B)	14	1
	(4F,4D,5A)	12	1			
	(2D,7B,15B)	2	1	(2D,7A,14B)	3	1
	(2D,7B,15A)	2	1	(2D,7B,14A)	3	1
	(2D,7B,12A)	2	1	(2D,7A,15B)	2	1
	(2D,7A,15A)	2	1	(2D,7A,12A)	2	1
	(2D,4E,15B)	2	1	(2D,4E,15A)	2	1
	(2C,6C,7B)	3	1	(2C,6C,7A)	3	1
	(2C,6B,15B)	3	1	(2C,6B,15A)	3	1
	(2C,5A,7B)	3	1	(2C,5A,7A)	3	1
	(2C,4F,15B)	3	1	(2C,4F,15A)	3	1
	(2B,15B,15B)	1	1	(2B,15A,15A)	1	1
	(2B,14B,15B)	1	1	(2B,14B,15A)	1	1
	(2B,14A,15B)	1	1	(2B,14A,15A)	1	1
	(2B,12A,15B)	1	1	(2B,12A,15A)	1	1
	(2B,8A,15B)	1	1	(2B,8A,15A)	1	1
	(3A,6B,7A)	6	1	(3A,6B,7B)	6	1
	(3A,4D,7A)	2	1	(3A,4D,7B)	2	1
$2^4.PSL(4,2)$	(2B,4C,6B,6B)	1	2700	(2B,4C,3A,6B)	1	234
	(2B,4C,4D,6A)	1	660	(2B,3A,4C,4D)	1	48
	(2B,4C,4C,6C)	1	654	(2B,4C,4C,5A)	1	450
	(2B,4C,4F,6B)	1	2154	(2B,3A,4C,4F)	1	252
	(2B,4C,4F,4D)	1	528	(2B,4C,4F,4F)	1	1692
	(2B,2C,6B,6B)	1	288	(2B,2C,4C,6C)	1	90
	(2B,2C,4C,5A)	1	90	(2B,2C,4F,6B)	1	360
	(2B,2C,4F,4F)	1	288	(2B,2B,6C,6C)	1	216
	(2B,2B,6C,7B)	1	126	(2B,2B,6C,7A)	1	126
	(2B,2B,6B,14B)	1	140	(2B,2B,6B,14A)	1	140
	(2B,2B,6B,15B)	1	120	(2B,2B,6B,15A)	1	120
	(2B,2B,6B,12A)	1	96	(2B,2B,6B,8A)	1	120
	(2B,2B,5A,7B)	1	105	(2B,2B,5A,7A)	1	105
	(2B,2B,5A,6C)	1	210	(2B,2B,5A,5A)	1	150
	(2B,2B,3A,14B)	1	14	(2B,2B,3A,14A)	1	14
	(2B,2B,3A,15B)	1	15	(2B,2B,3A,15A)	1	15
	(2B,2B,4D,15B)	1	30	(2B,2B,4D,15A)	1	30
	(2B,2B,4E,6C)	1	60	(2B,2B,4E,5A)	1	60
	(2B,2B,6A,6C)	1	72	(2B,2B,5A,6A)	1	60
	(2B,2B,4E,14B)	1	112	(2B,2B,4E,14A)	1	112
	(2B,2B,4E,15B)	1	90	(2B,2B,4E,15A)	1	90
	(2B,2B,4E,12A)	1	120	(2B,2B,4E,8A)	1	96
	(2B,3B,6B,6B)	1	2448	(2B,3B,3A,6B)	1	216
	(2B,3B,4D,6C)	1	552	(2B,3B,3A,4D)	1	60
	(2B,3B,4C,6C)	1	510	(2B,3B,3A,4C)	1	450
	(2B,2C,3B,6C)	1	72	(2B,2C,3B,3A)	1	60

Table A.14: Part5: GOSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2B,3B,3B,6C)	1	456	(2B,3B,3B,3A)	1	370
	(2B,3B,4F,6B)	1	1942	(2B,3B,3A,4F)	1	198
	(2B,3B,4F,4D)	1	456	(2B,3B,4F,4F)	1	1442
	(2B,2D,6B,6C)	1	756	(2B,2D,6B,6A)	1	198
	(2B,2D,6B,7B)	1	406	(2B,2D,6B,7A)	1	406
	(2B,2D,4E,6B)	1	168	(2B,2D,5A,6B)	1	660
	(2B,2D,3A,7B)	1	42	(2B,2D,3A,7A)	1	42
	(2B,2D,4D,6C)	1	156	(2B,2D,4D,5A)	1	180
	(2B,2D,4D,7B)	1	98	(2B,2D,4D,7A)	1	98
	(2B,2D,4C,14B)	1	98	(2B,2D,4C,14A)	1	98
	(2B,2D,4C,15B)	1	75	(2B,2D,4C,15A)	1	75
	(2B,2D,4C,12A)	1	84	(2B,2D,4C,8A)	1	96
	(2B,2D,2C,14B)	1	15	(2B,2D,2C,14A)	1	15
	(2B,2D,3B,14B)	1	84	(2B,2D,3B,14A)	1	84
	(2B,2D,3B,15B)	1	75	(2B,2D,3B,15A)	1	75
	(2B,2D,3B,12A)	1	66	(2B,2D,3B,8A)	1	60
	(2B,2D,4F,6C)	1	540	(2B,2D,4F,6A)	1	180
	(2B,2D,4F,7B)	1	322	(2B,2D,4F,7A)	1	322
	(2B,2D,4F,5A)	1	360	(2B,2D,4F,4E)	1	192
	(2B,2D,4C,6C)	1	252	(2B,2D,4E,4C)	1	288
	(2D,4C,4C,6B)	1	1896	(2D,3A,4C,4C)	1	264
	(2D,4C,4C,4D)	1	424	(2D,4C,4C,4E)	1	1680
	(2D,2C,4C,6B)	1	252	(2D,2C,4C,4F)	1	288
	(2D,3B,4C,6B)	1	1896	(2D,3B,3A,4C)	1	138
	(2D,3B,4C,4D)	1	444	(2D,3B,4C,4F)	1	1320
	(2D,2B,2C,6B)	1	198	(2D,2B,2C,4F)	1	180
	(2D,3B,3B,6B)	1	1560	(2D,3B,3B,4D)	1	296
	(2D,3B,3B,4F)	1	648	(2D,2D,6B,6B)	2	2088
	(2D,2D,4D,6B)	1	672	(2D,2D,4D,4D)	1	88
	(2D,2D,4C,6B)	1	618	(2D,2D,4C,6A)	1	168
	(2D,2D,4C,7B)	1	308	(2D,2D,4C,7A)	1	308
	(2D,2D,4C,5A)	1	330	(2D,2D,4C,4E)	1	176
	(2D,2D,2C,7B)	1	42	(2D,2D,2C,7A)	1	42
	(2D,2D,3B,7B)	1	154	(2D,2D,3B,7A)	1	154
	(2D,2D,3B,4E)	1	112	(2D,2D,4E,6C)	1	1656
	(2D,2D,4E,4D)	1	384	(2B,2D,2D,2D,4C)	1	10944
$2^4.PSL(4,2)$	(2B,2B,2B,6B,6B)	1	23112	(2B,2B,2B,3A,6B)	1	1944
	(2B,2B,2B,4D,6B)	1	5184	(2B,2B,2B,3A,4D)	1	432
	(2B,2B,2B,4C,6C)	1	4698	(2B,2B,2B,4C,5A)	1	4050
	(2B,2B,2B,2C,6C)	1	648	(2B,2B,2B,2C,5A)	1	600
	(2B,2B,2B,3B,6C)	1	4212	(2B,2B,2B,3B,5A)	1	3600
	(2B,2B,2B,4F,6B)	1	18594	(2B,2B,2B,3A,4F)	1	1944
	(2B,2B,2B,4F,4D)	1	4320	(2B,2B,2B,4F,4F)	1	14208
	(2B,2B,2B,2D,14B)	1	784	(2B,2B,2B,2D,14A)	1	784
	(2B,2B,2B,2D,12A)	1	648	(2B,2B,2B,2D,8A)	1	672
	(2B,2B,2B,2D,15B)	1	675	(2B,2B,2B,2D,15A)	1	675
	(2B,2B,2D,4C,6B)	1	16776	(2B,2B,2D,3A,4C)	1	1584
	(2B,2B,2D,4C,4D)	1	3888	(2B,2B,2D,4C,4F)	1	13344
	(2B,2B,2D,2C,6B)	1	1944	(2B,2B,2D,2C,4F)	1	2016
	(2B,2B,2D,3B,6B)	1	15084	(2B,2B,2D,3B,3A)	1	1140
	(2B,2B,2D,3B,4D)	1	3264	(2B,2B,2D,3B,4F)	1	11784
	(2B,2B,2D,2D,6C)	1	4536	(2B,2B,2D,2D,6A)	1	1296
	(2B,2B,2D,2D,7B)	1	2548	(2B,2B,2D,2D,7A)	1	2548
	(2B,2B,2D,2D,5A)	1	3900	(2B,2B,2D,2D,4E)	1	1152
	(2B,2D,2D,4C,4C)	1	12120	(2B,2D,2D,2C,4C)	1	1728
	(2B,2D,2D,3B,4C)	1	11544	(2B,2D,2D,2C,3B)	1	1296
	(2B,2D,2D,3B,3B)	1	9804	(2B,2D,2D,2D,6B)	1	17064
	(2B,2D,2D,2D,4D)	1	3840	(2B,2D,2D,2D,4F)	1	11232
	Projection-fiber Algorithm					
	(2B,2B,2B,2B,2B,6C)	1	38889	(2B,2B,2B,2B,2B,5A)	1	33750
	(2B,2B,2B,2B,2D,6B)	1	142560	(2B,2B,2B,2B,2D,3A)	1	12960
	(2B,2B,2B,2B,2D,4D)	1	31488	(2B,2B,2B,2B,2D,4F)	1	113280
	(2B,2B,2B,2D,2D,4C)	1	106176	(2B,2B,2B,2D,2D,2C)	1	12960
	(2B,2B,2B,2D,2D,3B)	1	94824	(2B,2B,2D,2D,2D,2D)	1	104976
				(2B,2B,2B,2B,2B,2D,2D)	1	902400

Table A.15: Part1: GOSs for Affine Primitive Groups of Degree 32

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(5,2)$	(4F,6C,6F)	84	1	(4F,4J,12B)	72	1
	(4F,4J,8B)	72	1	(4F,4J,6E)	78	1
	(4F,4J,5A)	78	1	(4F,4I,6F)	78	1
	(4F,4F,21B)	10	1	(4F,4F,21A)	10	1
	(4F,4F,12C)	12	1	(4F,4F,10A)	18	1
	(4F,4F,8C)	24	1	(3B,4F,14D)	10	1
	(3B,4F,14C)	10	1	(3B,4F,6F)	48	1
	(3B,4B,10A)	6	1	(3B,4B,8C)	4	1
	(3B,6C,12B)	84	1	(3B,6C,8B)	90	1
	(3B,6C,6E)	144	1	(3B,6C,6D)	12	1
	(3B,6C,5A)	120	1	(3B,4J,4J)	18	1
	(3B,4I,12B)	78	1	(3B,4I,8B)	72	1
	(3B,4I,6E)	90	1	(3B,4I,6D)	12	1
	(3B,4I,5A)	90	1	(3B,4D,8B)	12	1
	(3B,4D,5A)	18	1	(3B,3B,12B)	48	1
	(3B,3B,8B)	48	1	(3B,3B,6E)	48	1
	(3B,3B,5A)	36	1	(3A,3B,10A)	2	1
	(3A,3B,8C)	2	1	(2E,12B,12B)	16	1
	(2E,8B,12B)	20	1	(2E,8B,8B)	16	1
	(2E,6E,12B)	16	1	(2E,6E,8B)	16	1
	(2E,6E,6E)	22	1	(2E,6C,21B)	2	1
	(2E,6C,21A)	2	1	(2E,5A,12B)	30	1
	(2E,5A,8B)	12	1	(2E,5A,6E)	18	1
	(2E,5A,5A)	14	1	(2E,4I,21B)	2	1
	(2E,4F,31A)	1	1	(2E,4F,31B)	1	1
	(2E,4F,31C)	1	1	(2E,4F,31D)	1	1
	(2E,4F,31E)	1	1	(2E,4F,31F)	1	1
	(2D,8B,8C)	20	1	(2E,3B,15B)	1	1
	(2E,3B,15A)	1	1	(2D,12B,21B)	13	1
	(2D,12B,21A)	13	1	(2D,12B,12C)	16	1
	(2D,10A,12B)	20	1	(2D,8C,12B)	20	1
	(2D,8B,21A)	13	1	(2D,8B,21B)	13	1
	(2D,8B,12C)	16	1	(2D,8B,10A)	20	1
	(2D,6F,14D)	13	1	(2D,6F,14C)	13	1
	(2D,6F,6F)	22	1	(2D,6E,21B)	18	1
	(2D,6E,8C)	16	1	(2D,6E,21A)	18	1
	(2D,6E,12C)	16	1	(2D,6E,10A)	18	1
	(2D,6D,21A)	2	1	(2D,6D,21B)	2	1
	(2D,5A,21A)	18	1	(2D,5A,21B)	18	1
	(2E,4I,21A)	2	1	(2D,5A,12C)	20	1
	(2D,5A,10A)	14	1	(2D,5A,8C)	12	1
	(2D,4J,31A)	4	1	(2B,4J,31B)	4	1
	(2D,4J,31C)	4	1	(2B,4J,31D)	4	1
	(2D,4J,31E)	4	1	(2B,4J,31F)	4	1
	(2B,6F,31A)	1	1	(2B,6F,31B)	1	1
	(2B,6F,31C)	1	1	(2B,6F,31D)	1	1
	(2B,6F,31E)	1	1	(2B,6F,31F)	1	1
	(2B,10A,21B)	1	1	(2B,10A,21A)	1	1
	(2B,8C,21B)	1	1	(2B,8C,21A)	1	1

Table A.16: Part2: GOSs for Affine Primitive Groups of Degree 32

Projection-fiber Algorithm						
group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(5,2)$	(2B,3B,4F,4F)	1	432	(2B,2B,3B,8C)	1	24
	(2B,2E,3B,4I)	1	84	(2B,2B,3B,10A)	1	30
	(2D,2E,3B,6C)	1	78			
	(2B,2D,4F,6F)	1	588	(2B,2D,2E,21A)	1	14
	(2B,2D,2E,12B)	1	588	(2B,2D,2E,21B)	1	14
	(2B,2D,2E,6D)	1	78	(2B,2D,2E,8B)	1	600
	(2B,2D,2E,6E)	1	780	(2B,2D,2E,5A)	1	780
	(2D,2E,3A,3B)	1	46	(2D,2E,3B,4B)	1	88
	(2D,2E,4F,4F)	1	624	(2D,2D,4F,4J)	1	3360
	(2D,2D,2E,12B)	1	720	(2D,2D,2E,8B)	1	672
	(2D,2D,2E,6E)	1	720	(2D,2D,2E,5A)	1	680
	(2D,2D,2D,21A)	1	630	(2D,2D,2D,21B)	1	630
	(2D,2D,2D,12C)	1	720	(2D,2D,2D,8C)	1	672
	(2D,2D,2D,10A)	1	680	(2D,2D,3B,4I)	1	3720
	(2D,2D,3B,6C)	1	4368	(2D,2D,3B,4D)	1	264
	(2D,2D,3B,3B)	1	1680	(2B,2B,2D,2E,3B)	1	528
	(2B,2D,2D,2D,3B)	1	30024	(2D,2D,2D,2D,2E)	1	31744

Table A.17: GOSs for Affine Primitive Groups of Degree 64 and 128

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^6 : 3^2 : D_{12}$	(2F,2G,2H,3D)	1	9			
$2^6 : (3^2 : 3) : 4$	(2D,4F,12A)	1	1	(2D,4F,12B)	1	1
	(2D,4E,12C)	1	1	(2D,4E,12D)	1	1
$2^6 : (3^2 : 3) : D(2*4)$	(2D,2F,2G,4G)	1	12			
	(2G,4E,12C)	1	1	(2G,4E,12D)	1	1
$2^6 : 3^3 : D(2*6)$	(2G,6I,6K)	3	1			
$2^6 : 3^3 : A(4)$	(2E,3F,9C)	3	1	(2E,3F,9D)	3	1
	(2E,3E,9A)	3	1	(2E,3E,9B)	3	1
$2^6 : (3^2 : 3) : SD_{16}$	(2E,4H,8A)	3	1	(2E,4H,8B)	4	1
$2^6 : 3^3 : S(4)$	(2F,4G,9A)	3	1	(2F,4G,9B)	3	1
	(2F,3D,24A)	1	1	(2F,3D,24B)	1	1
	(2F,2F,2F,4G)	1	96	(2F,2F,2F,3D)	1	72
$2^6 : 3^2 : S(4)$	(2D,4H,9A)	1	1	(2D,4H,9B)	1	1
$2^6 : (GL(2,2) \wr S(3))$	(2L,4R,12I)	4	1	(2E,2L,2L,4S)	1	16
$2^6 : (3^2 : 3) : Q_8 : S(3)$	(2E,6C,8C)	1	1	(2E,6C,8D)	1	1
$AGL(6,2)$	(2F,6J,8D)	48	1	(2F,6J,6J)	192	1
	(2F,4O,15D)	7	1	(2F,4O,15E)	7	1
	(2F,4O,14H)	10	1	(2F,4O,14G)	10	1
	(2F,4K,15D)	1	1	(2F,4K,15E)	1	1
	(2F,4H,21A)	1	1	(2F,4H,21B)	1	1
	(2F,3C,42A)	2	1	(2F,3C,42B)	2	1
	(2F,3C,21A)	4	1	(2F,3C,21B)	4	1
	(2D,8D,7E)	16	1	(2D,6J,7E)	82	1
	(2D,6F,14H)	4	1	(2D,6F,14G)	4	1
	(2B,7E,15D)	1	1	(2B,7E,15E)	1	1
	(3C,4O,6F)	92	1			
$A\Sigma L(3,4)$	(2C,4D,14A)	1	1	(2C,4D,14B)	1	1
$2^6 : 3.A(6)$	(3C,3D,4D)	6	1			
$2^6 : (3 \times GL(3,2))$	(3C,3E,4D)	4	1			
$2^6 : Sp(6,2)$	(2H,6H,7A)	14	1	(2H,6I,7A)	42	1
$2^6 : GO-(6,2)$	(2I,6G,8F)	4	1	(2I,4R,12G)	4	1
	(2I,4R,10B)	5	1	(2I,4R,8E)	6	1
	(2I,4P,10B)	3	1	(2I,4O,12I)	4	1
	(2I,4O,9A)	6	1	(2I,4L,12I)	2	1
	(2I,4L,9A)	3	1	(2C,8F,12I)	1	1
	(2C,8F,9A)	1	1	(2I,2I,2I,10B)	1	192
	(2I,2I,2I,4L)	1	96	(2F,2I,2I,4R)	1	112
	(2F,2I,2I,4P)	1	60	(2C,2I,2I,8F)	1	32
$2^6 : O-(6,2)$	(2E,4H,12F)	3	1	(2E,4H,12E)	3	1
	(2E,4H,9A)	3	1	(2E,4H,9B)	3	1
	(2E,4F,9A)	3	1	(2E,4F,9B)	3	1
$2^6 : S(8)$	(2J,6G,7A)	6	1	(2J,4S,12F)	4	1
	(2J,4S,10B)	6	1	(2J,4R,8E)	6	1
	(2J,4O,15A)	4	1	(2J,2J,2J,4O)	1	192
	(2F,2J,2J,4S)	1	48	(2D,2J,2J,4R)	1	88
$2^6 : A(8)$	(2G,4G,15A)	1	1	(2G,4G,15B)	1	1
	(2G,4H,15A)	1	1	(2G,4H,15B)	1	1
	(2C,4J,15A)	1	1	(2C,4J,15B)	1	1
$2^6 : S(7)$	(2H,4N,12F)	2	1	(2H,4N,10B)	3	1
	(2H,4M,12G)	4	1	(2H,3B,12I)	2	1
	(2H,3B,12H)	1	1	(2D,7A,12B)	1	1
	(2H,2H,2I,3B)	1	66	(2H,2H,2H,4L)	1	48
	(2F,2H,2H,4N)	1	48			
$2^6 : A(7)$	(2E,4E,7A)	6	1	(2E,4E,7B)	6	1
$2^6 : \Sigma U(3,3)$	(2D,6A,6C)	6	1			
$2^6 : SU(3,3)$	(2B,6A,7A)	2	1	(2B,6A,7B)	2	1
$2^6 : PGL(2,7)$	(2G,4I,8A)	2	1	(2G,4I,8B)	2	1
	(2G,4G,8A)	1	1	(2G,4G,8B)	1	1
	(2G,4G,12A)	1	1	(2G,2G,2F,3A)	1	30
	(2D,2G,2G,4I)	1	24	(2D,2G,2G,4G)	1	12
$ASL(7,2)$	(2F,3B,14K)	1	1	(2F,3B,14L)	1	1

Appendix B
Genus Two Covers

Note that N.O means number of orbits, L.O means largest length of the orbit and GTS means Genus two System.

Table B.1: GTSs for Affine Primitive Groups of Degree 289 and 49

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$17^2 : Q_8 : S(3)$	(2A,3A,8A)	8	1	(2A,3A,8B)	8	1
$7^2 : Q_8 : 3$	(3A,3B,4A)	24	1			

Table B.2: GTSs for Affine Primitive Groups of Degree 25 and 125

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(2,5)$	(4C,4E,6A)	1	1	(4D,4F,6A)	1	1
	(4C,4E,5B)	1	1	(4D,4F,5B)	1	1
	(2B,2B,4C,4E)	1	32	(2B,2B,4D,4F)	1	32
$ASL(2,5):2$	(2B,6A,20A)	1	1	(2B,6A,20B)	1	1
	(2B,6A,20C)	1	1	(2B,6A,20D)	1	1
	(2B,5D,20A)	1	1	(2B,5D,20B)	1	1
	(2B,5E,20C)	1	1	(2B,5E,20D)	1	1
	(2B,5D,12A)	1	1	(2B,5D,12B)	1	1
	(2B,5E,12A)	1	1	(2B,5E,12B)	1	1
	(2B,10B,20A)	1	1	(2B,10B,20B)	1	1
	(2B,10A,20C)	1	1	(2B,10A,20D)	1	1
	(2B,10A,12A)	1	1	(2B,10A,12B)	1	1
	(2B,10B,12A)	1	1	(2B,10B,12B)	1	1
	(2B,2B,2B,12A)	1	72	(2B,2B,2B,12B)	1	72
	(2B,2B,2B,20A)	1	40	(2B,2B,2B,20B)	1	40
	(2B,2B,2B,20C)	1	40	(2B,2B,2B,20D)	1	40
	(2B,2B,3A,5B)	1	30	(2B,2B,3A,5C)	1	30
	(2B,2B,3A,3A)	1	216			
$ASL(2,5)$	(3A,5B,6A)	2	1	(3A,5C,6A)	2	1
	(3A,3A,10A)	12	1	(3A,3A,10B)	12	1
	(3A,5B,10B)	2	1	(3A,5C,10A)	2	1
	(3A,5C,5D)	1	1	(3A,5B,5F)	1	1
	(3A,5C,5E)	1	1	(3A,5B,5G)	1	1
$5^2:Q_+(2,5)$	(2B,2B,4B,4F)	1	2	(2B,2B,4A,4E)	1	2
$5^2:(Q_8:3)'4$	(3A,4C,8B)	6	1	(3A,4D,8A)	6	1
$5^2:(Q_8:3)'2$	(2B,6A,12A)	1	1	(2B,6A,12B)	1	1
	(2B,6B,12C)	1	1	(2B,6B,12D)	1	1
	(2B,2B,3A,3B)	1	36			
$5^2:Q_8:3$	(3A,3A,6A)	12	1	(3B,3B,6B)	12	1
$5^2:8:2$	(2A,8A,8C)	1	1	(2A,8B,8D)	1	1
$5^2:Q_{12}$	(3A,4A,4B)	12	1			
$5^2:D(2*6)$	(2B,6A,10A)	1	1	(2B,6A,10C)	1	1
	(2A,6A,10B)	1	1	(2A,6A,10D)	1	1
	(2A,2A,2B,10B)	1	12	(2A,2A,2B,10D)	1	12
	(2A,2B,2B,10A)	1	12	(2A,2B,2B,10C)	1	12
	(2A,2A,2C,6A)	1	12	(2B,2B,2C,6A)	1	12
	(2A,2A,2A,2B,2C)	1	48	(2A,2B,2B,2B,2C)	1	48
$5^2:D(2*4)$	(2A,2B,2B,10C)	1	2	(2A,2B,2B,10D)	1	2
	(2A,2A,2B,10A)	1	2	(2A,2A,2B,10B)	1	2
	(2A,2A,2B,2B,2C)	1	24			
$5^2:S_3$	(2A,2A,2A,10A)	1	4	(2A,2A,2A,10B)	1	4
	(2A,2A,2A,10C)	1	4	(2A,2A,2A,10D)	1	4
	(2A,2A,3A,3A)	1	48			
$5^2:3$	(3A,3B,5A)	1	1	(3A,3B,5B)	1	1
	(3A,3B,5C)	1	1	(3A,3B,5D)	1	1
	(3A,3B,5D)	1	1	(3A,3B,5E)	1	1
	(3A,3B,5F)	1	1	(3A,3B,5G)	1	1
$5^3:4^2:3$	(3A,3B,4C)	4	1	(3A,3B,4D)	4	1
$5^3:2\times A(5)$	(2A,5I,6A)	1	1	(2A,5J,6A)	1	1
	(2A,5K,6A)	1	1	(2A,5M,6A)	1	1

Table B.3: GTSs for Affine Primitive Groups of Degree 81

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(4,3)$	(2D,6P,8I)	2	1	(2D,6P,8J)	2	1
	(2B,5A,24I)	8	1	(2B,5A,24J)	8	1
$ASL(4,3)$	(2B,5A,9B)	8	1	(2B,5A,9C)	24	1
	(2B,5A,5A)	32	1			
$3^4 : Sp(4,3)$	(2B,5A,9A)	1	1	(2B,5A,9B)	1	1
$3^4 : (GL(1,3) \wr S(4))$	(2B,6T,8A)	1	1			
$3^4 : 4.S(5)$	(2D,6C,6K)	2	1	(2D,6D,6J)	2	1
$3^4 : S(5)$	(2A,5A,12A)	1	1	(2A,5A,12B)	1	1
$3^4 : SA_{16} : 2$	(2B,4E,8B)	2	1	(2B,4F,8A)	2	1
$3^4 : (2 \times Q_8) : 6$	(2C,6F,6L)	1	1	(2C,6E,6K)	1	1
	(2C,6D,6I)	1	1	(2C,6C,6J)	1	1
$3^4 : ((2 \times Q_8) : 2) : 5$	(2B,5A,5C)	4	1	(2B,5B,5D)	4	1
$3^4 : 2.A(5)$	(2B,5A,6G)	4	1	(2B,5B,6G)	4	1
$3^4 : A(5)$	(2A,5A,5B)	6	1			
$3^4(2^3 : 2^2) : 3^2 : D(2*4)$	(2D,6J,8D)	1	1	(2D,6J,8E)	1	1
$3^4 : Q_8^2 : S(3)^2$	(2E,6O,6T)	6	1	(2E,6P,6S)	6	1

Table B.4: GTSs for Affine Primitive Groups of Degree 27

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(3,3)$	(3E,6B,8A)	1	1	(3E,6B,8B)	1	1
	(3E,4B,8A)	9	1	(3E,4B,8B)	9	1
	(3E,4B,6G)	24	1	(3E,4B,6F)	24	1
	(4B,4B,4A)	24	1	(4B,4B,6E)	48	1
	(6D,6E,6F)	8	1	(6D,6E,6G)	12	1
	(6D,6D,6H)	8	1	(6D,6D,8C)	8	1
	(6D,6D,8D)	8	1	(6D,4D,6F)	8	1
	(6D,4A,6G)	8	1			
	(3B,4B,26B)	1	1	(3B,4B,26C)	1	1
	(3B,4B,26D)	1	1	(3B,4B,26A)	1	1
	(3B,4B,24B)	1	1	(3B,4B,24A)	1	1
	(2C,4B,26A)	4	1	(2C,4B,26B)	4	1
	(2C,4B,26C)	4	1	(2C,4B,26D)	4	1
	(2B,6H,26A)	1	1	(2B,6H,26B)	1	1
	(2B,6H,26C)	1	1	(2B,6H,26D)	1	1
	(2B,8D,26A)	1	1	(2B,8D,26B)	1	1
	(2B,8D,26C)	1	1	(2B,8D,26D)	1	1
	(2B,8D,24A)	1	1	(2B,8C,24B)	1	1
	(2B,8C,26A)	1	1	(2B,8C,26B)	1	1
	(2B,8C,26C)	1	1	(2B,8C,26D)	1	1
	(2B,3B,3E,6B)	1	12	(2B,3B,3E,4B)	1	140
	(2B,2C,3E,4B)	1	480	(2B,2C,6D,6E)	1	132
	(2B,2B,3E,6H)	1	144	(2B,2C,4A,6D)	1	144
	(2B,2B,3E,8C)	1	144	(2B,2B,3E,8D)	1	144
	(2B,2B,6D,6F)	1	144	(2B,2B,6D,6G)	1	132
	(2B,2B,4B,4B)	1	496	(2B,2B,2B,2C,6D)	1	2160
ASL(3,3)	(3F,3F,13A)	8	1	(3F,3F,13B)	8	1
	(3F,3F,13C)	8	1	(3F,3F,13D)	8	1
	(3F,6B,6C)	12	1	(3F,6B,6D)	12	1
	(3F,6B,8A)	18	1	(3F,6B,8B)	18	1
	(3F,3E,8A)	6	1	(3F,3E,8B)	6	1
	(3F,3D,8A)	1	1	(3F,3D,8B)	1	1
	(3F,3C,8A)	1	1	(3F,3C,8B)	1	1
	(3F,4A,8A)	18	1	(3F,4A,8B)	18	1
	(3F,4A,6C)	8	1	(3F,4A,6D)	8	1
	(2A,8A,8B)	8	1	(3F,3F,12A)	12	1
	(2A,2A,3F,3F)	1	864	(2A,3B,3F,3F)	1	256
$3^3 : S_4 \times 2$	(2A,2E,3D,4B)	1	12	(2B,2D,3D,4A)	1	4
	(2B,2A,3D,6H)	1	3	(2B,2B,2A,2D,3D)	1	12
$3^3 : S_4$	(2A,2A,4A,4A)	1	32	(2A,2A,3E,6F)	1	6
	(2A,2B,3E,4A)	1	24	(3E,4A,6B)	1	1
	(3E,4A,6E)	2	1	(3E,4A,6A)	1	1
	(3E,4A,6C)	2	1	(3E,4A,6D)	2	1
$3^3 : A_4$	(2A,2A,3F,3E)	1	24	(3E,3E,9A)	1	1
	(3E,3E,9C)	1	1	(3F,3F,9B)	1	1
	(3F,3F,9D)	1	1			
$3^3 : A_4 \times 2$	(2B,2B,3D,3D)	1	48	(3D,4A,6B)	2	1
$3^3 : 13.3$	(3D,3D,9C)	4	1	(3D,3D,9D)	4	1
	(3C,3C,9A)	4	1	(3C,3C,9B)	4	1
	(3C,3D,13A)	2	1	(3C,3D,13B)	2	1
	(3C,3D,13C)	2	1	(3C,3D,13D)	2	1

Table B.5: GTSs for Affine Primitive Groups of Degree 9

group	ramification type	N.O	L.O	ramification type	N.O	L.O
AGL(2,3)	(3C,8A,8A)	4	1	(3C,8B,8B)	4	1
	(3C,8B,6B)	2	1	(3C,8A,6B)	2	1
	(6A,8A,8B)	1	1	(6A,6B,8A)	1	1
	(6A,6B,8B)	1	1	(2B,3B,3C,8A)	1	24
	(2B,3B,4A,6B)	1	36	(2B,3B,3C,8B)	1	24
	(2B,3B,4A,8A)	1	24	(2B,3B,4A,8B)	1	24
	(2B,3B,3C,6B)	1	24	(2B,3B,3C,8A)	1	24
	(2B,3B,3C,8B)	1	24	(2B,3B,6A,6B)	1	36
	(2B,3B,6A,8A)	1	32	(2B,3B,6A,8B)	1	32
	(2B,2A,3C,8A)	1	9	(2B,2A,3C,8B)	1	9
	(2B,2A,6A,8A)	1	13	(2B,2A,6A,8B)	1	13
	(2B,2B,8B,8B)	2	16	(2B,2B,8A,8B)	2	32
	(2B,2B,8A,8A)	2	16	(2B,2B,6B,8A)	1	48
	(2B,2B,6B,8B)	1	48	(2B,2B,3B,3B,6A)	1	480
	(2B,2B,3B,3B,3C)	1	360	(2B,2B,3B,3B,3C)	1	360
	(2B,2B,3B,3B,4A)	1	360	(2B,2B,2A,3B,6A)	1	156
	(2B,2B,2A,3B,3C)	1	108	(2B,2B,2A,3B,4A)	1	156
	(2B,2B,2B,3B,8A)	1	576	(2B,2B,2B,3B,8B)	1	576
	(2B,2B,2B,2A,8A)	1	208	(2B,2B,2B,2A,8B)	1	208
	(2B,2B,2B,3B,6B)	1	648	(2B,2B,2B,2B,3B,3B)	1	8640
	(2B,2B,2B,2B,2A,3B)	1	2808			
ASL(2,3)	(3C,2A,3D,3D)	1	6	(3C,2A,4A,3E)	1	9
	(3C,2A,6B,4A)	3	6	(3C,2A,6A,6A))	3	6
	(3B,2A,3E,3E)	1	6	(3B,2A,4A,3D)	1	9
	(3B,2A,6B,6B)	3	6	(3B,2A,6A,4A)	1	6
	(3C,3C,3E,3E)	1	6	(3C,3C,4A,3D)	1	12
	(3C,3C,6B,3E)	1	6	(3C,3C,6B,6B)	2	6
	(3C,3C,6A,3A)	1	4	(3C,3C,6A,4A)	3	8
	(3B,3C,3D,3E)	1	6	(3B,3C,4A,3A)	1	4
	(3B,3C,6B,3D)	1	6	(3B,3C,4A,4A)	3	12
	(3B,3C,6A,3E)	1	6	(3B,3C,6A,6B)	3	6
	(3B,3B,3D,3D)	1	6	(3B,3B,4A,3E)	1	12
	(3B,3B,6B,3A)	1	4	(3B,3B,6B,4A)	3	8
	(3B,3B,6A,3D)	1	6	(3B,3B,6A,6A)	2	6
	(2A,3B,3B,3C,3C)	2	4	(3B,3B,3B,3B,3C)	1	9
	(3B,3C,3C,3C,3C)	1	9			
$A\Gamma L(1,8)$	(2A,2B,4B,8A)	1	13	(2A,2B,4B,8B)	1	13
	(2B,2B,8A,8A)	1	8	(2B,2B,8A,8B)	1	8
	(2B,2B,8B,8B)	1	8	(4B,6A,8A)	3	1
	(4B,6A,8B)	3	1			
$3^2 : D(2*4)$	(4A,6A,6B)	4	1	(2B,2B,6A,6A)	1	4
	(2B,2C,4A,6A)	1	9	(2A,2B,6A,6B)	1	4
	(2A,2C,4A,6B)	1	40	(2A,2A,6B,6B)	1	4
	(2A,2B,2B,2C,6A)	1	18	(2A,2B,2C,2C,4A)	1	40
	(2A,2A,2B,2B,2C,2C)	1	80	(2A,2A,2B,2C,6B)	1	18
$AGL(1,9)$	(4A,8A,8A)	1	1	(4A,8B,8D)	1	1
	(4A,8C,8C)	1	1	(4B,8A,8C)	1	1
	(4B,8B,8B)	1	1	(4B,8D,8D)	1	1
	(3A,8A,8B)	1	1	(3A,8C,8D)	1	1
$3^2 : 4$	(2A,2A,4A,4B)	3	8			

Table B.6: Part1: GTSs for Affine Primitive Groups of Degree 8

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(1,8)$	(7E,7F,7F)	1	1	(7C,7F,7F)	1	1
	(7C,7C,7D)	1	1	(7B,7D,7E)	1	1
	(7B,7B,7F)	1	1	(7A,7D,7D)	1	1
	(7A,7C,7F)	1	1	(7A,7A,7B)	1	1
$A\Gamma L(1,8)$	(6B,6B,6B)	2	1	(6A,6A,6A)	2	1
	(6A,6B,7A)	2	1	(6A,6B,7B)	2	1
	(2A,3B,3B,6B)	1	14	(3B,3B,3B,7B)	1	49
	(3B,3B,3B,7A)	1	49	(2A,3A,3B,7B)	1	14
	(2A,3A,3A,6A)	1	14	(2A,3A,3B,7A)	1	14
	(3A,3B,3B,6A)	1	64	(3A,3A,3B,6B)	1	64
	(3A,3A,3A,7A)	1	49	(3A,3A,3A,7B)	1	49
ASL(3,2)	(7B,7B,7B)	6	1	(7A,7A,7A)	6	1
	(6A,7B,7B)	4	1	(6A,7A,7B)	4	1
	(6A,7A,7A)	4	1	(6A,6A,7B)	4	1
	(6A,6A,7A)	4	1	(4C,7A,7B)	6	1
	(4C,6A,7B)	4	1	(4C,6A,7A)	4	1
	(4C,6A,6A)	4	1	(4C,4C,7B)	1	1
	(4C,4C,7A)	1	1	(4C,4C,6A)	1	2
	(4A,7B,7B)	4	1	(4A,7A,7A)	4	1
	(4A,6A,7A)	2	1	(4A,6A,7B)	2	1
	(4A,4C,7A)	2	1	(4A,4C,7B)	2	1
	(3A,3A,3A,7A)	4	252	(3A,3A,3A,7B)	4	252
	(3A,3A,3A,6A)	4	288	(3A,3A,3A,4C)	2	384
	(3A,3A,3A,4A)	1	480	(3A,3A,4B,7A)	2	392
	(3A,3A,4B,7B)	2	392	(3A,3A,4B,6A)	2	512
	(3A,3A,4B,4C)	3	384	(3A,3A,4B,4A)	1	336
	(3A,4B,4B,7A)	2	294	(3A,4B,4B,7B)	2	294
	(3A,4B,4B,6A)	3	384	(3A,4B,4B,4C)	2	288
	(3A,4B,4B,4A)	1	264	(4B,4B,4B,7A)	2	196
	(4B,4B,4B,6A)	2	288	(4B,4B,4B,7B)	2	196
	(4B,4B,4B,6B)	2	288	(4B,4B,4B,4A)	1	144
	(2C,3A,3A,7A)	1	168	(2C,3A,3A,7B)	1	168
	(2C,3A,3A,6A)	1	240	(2C,3A,3A,4C)	1	84
	(2C,3A,3A,4A)	1	60	(2C,3A,4B,7A)	1	168
	(2C,3A,4B,7B)	1	168	(2C,3A,4B,6A)	1	168
	(2C,3A,4B,4C)	1	132	(2C,3A,4B,4A)	1	48
	(2C,4B,4B,7A)	1	140	(2C,4B,4B,7B)	1	140
	(2C,4B,4B,6A)	1	132	(2C,4B,4B,4C)	1	72
	(2C,4B,4B,4A)	1	48	(2C,2C,3A,7A)	1	21
	(2C,2C,3A,7B)	1	21	(2C,2C,3A,6A)	1	30
	(2C,2C,4B,7A)	1	28	(2C,2C,4B,7B)	1	28
	(2C,2C,4B,6A)	1	24	(2C,2C,4B,4C)	1	24
	(2C,3A,7A,7A)	1	168	(2C,3A,7A,7B)	1	168
	(2C,3A,7B,7B)	1	168	(2C,3A,6A,7A)	1	224
	(2C,3A,6A,7B)	1	224	(2C,3A,6A,6A)	1	240
	(2C,3A,4C,7A)	1	168	(2C,3A,4C,7B)	1	168
	(2C,3A,4C,6A)	1	168	(2C,3A,4C,4C)	1	132
	(2B,3A,4A,7A)	1	84	(2B,3A,4A,7B)	1	84
	(2B,3A,4A,6A)	1	60	(2B,3A,4A,4C)	1	48
	(2B,4B,7A,7A)	1	168	(2B,4B,7B,7B)	1	168
	(2B,4B,7A,7B)	1	70	(2B,4B,6A,7A)	1	168
	(2B,4B,6A,7B)	1	168	(2B,4B,6A,6A)	1	168
	(2B,4B,4C,7A)	1	140	(2B,4B,4C,7B)	1	140
	(2B,4B,4C,6A)	1	132	(2B,4B,4C,4C)	1	72
	(2B,4B,4A,7A)	1	56	(2B,4B,4A,7B)	1	56
	(2B,4B,4A,6A)	1	48	(2B,4B,4A,4C)	1	48
	(2B,2C,7A,7A)	1	28	(2B,2C,7B,7B)	1	28
	(2B,2C,7A,7B)	1	56	(2B,2C,6A,7A)	1	28
	(2B,2C,6A,6A)	1	30	(2B,2C,6A,7B)	1	28
	(2B,2C,4C,7A)	1	28	(2B,2C,4C,7A)	1	28
	(2B,2C,4C,6A)	1	24	(2B,2C,4C,4C)	1	24
	(2B,2A,7A,7A)	1	14	(2B,2A,7B,7B)	1	14
	(2B,2A,6A,7A)	1	7	(2B,2A,6A,7B)	1	7

Table B.7: Part2: GTSs for Affine Primitive Groups of Degree 8

group	ramification type	N.O	L.O	ramification type	N.O	L.O
ASL(3,2)	(2B,2A,4C,7A)	1	7	(2B,2A,4C,7B)	1	7
	(2A,3A,3A,7A)	1	14	(2A,3A,3A,7B)	1	14
	(2A,3A,3A,4C)	2	14	(2A,3A,4B,7B)	2	14
	(2A,3A,3A,6A)	4	7	(2A,3A,4B,7A)	2	14
	(2A,3A,3A,4C)	2	7	(2A,4B,4B,7A)	1	14
	(2A,4B,4B,6A)	1	4	(2A,4B,4B,7B)	1	14
	(2A,4B,4B,4C)	2	14	(2A,2C,3A,7A)	1	7
	(2A,2C,4B,7A)	1	7	(2A,2C,3A,7B)	1	7
	(2A,2C,4B,7B)	1	7	(2B,2C,4B,7A)	1	14
	(2B,3A,3A,3A,3A)	1	50400	(2B,2C,4B,7B)	1	14
	(2B,3A,3A,3A,4B)	1	37632	(2B,3A,3A,4B,4B)	1	28392
	(2B,3A,4B,4B,4B)	1	21168	(2B,4B,4B,4B,4B)	1	14784
	(2B,2C,3A,3A,4B)	1	7824	(2B,2C,3A,3A,3A)	1	10416
	(2B,2C,3A,4B,4B)	1	5832	(2B,2C,4B,4B,4B)	1	4752
	(2B,2C,2C,3A,3A)	1	1728	(2B,2C,2C,3A,4B)	1	1296
	(2B,2C,2C,4B,4B)	1	912	(2B,2C,2C,2C,3A)	1	216
	(2B,2C,2C,2C,4B)	1	192	(2B,2B,3A,3A,7A)	1	8232
	(2B,2B,3A,3A,6A)	1	10416	(2B,2B,3A,3A,7B)	1	8232
	(2B,2B,3A,3A,4C)	1	7824	(2B,2B,3A,3A,4A)	1	3465
	(2B,2B,3A,4B,7A)	1	6174	(2B,2B,3A,4B,7B)	1	6174
	(2B,2B,3A,4B,6A)	1	7824	(2B,2B,3A,4B,4C)	1	5832
	(2B,2B,4B,4B,7A)	1	4459	(2B,2B,3A,4B,4A)	1	2592
	(2B,2B,4B,4B,7B)	1	4459	(2B,2B,4B,4B,6A)	1	5832
	(2B,2B,4B,4B,4C)	1	4752	(2B,2B,4B,4B,4A)	1	1824
	(2B,2B,2C,3A,7A)	1	1764	(2B,2B,2C,3A,7B)	1	1764
	(2B,2B,2C,3A,6A)	1	1728	(2B,2B,2C,3A,4C)	1	1296
	(2B,2B,2C,3A,4A)	1	432	(2B,2B,2C,4B,7A)	1	1372
	(2B,2B,2C,4B,6A)	1	1296	(2B,2B,2C,4B,7B)	1	1372
	(2B,2B,2C,4B,4C)	1	912	(2B,2B,2C,4B,4A)	1	384
	(2B,2B,2C,2C,7A)	1	294	(2B,2B,2C,2C,7B)	1	294
	(2B,2B,2C,2C,6A)	1	216	(2B,2B,2C,2C,4C)	1	192
	(2B,2B,2B,7A,7A)	1	1519	(2B,2B,2B,7B,7B)	1	1519
	(2B,2B,2B,7A,7B)	1	1176	(2B,2B,2B,6A,7A)	1	1764
	(2B,2B,2B,6A,6A)	1	1728	(2B,2B,2B,6A,7B)	1	1764
	(2B,2B,2B,4C,7B)	1	1372	(2B,2B,2B,4C,7A)	1	1372
	(2B,2B,2B,4C,6A)	1	1296	(2B,2B,2B,4C,4C)	1	912
	(2B,2B,2B,4A,4C)	1	384	(2B,2B,2B,4A,7B)	1	588
	(2B,2B,2B,4A,6A)	1	432	(2B,2B,2B,4A,7A)	1	588
	(2B,2B,2A,3A,7A)	1	294	(2B,2B,2A,3A,7B)	1	294
	(2B,2B,2A,3A,6A)	1	210	(2B,2B,2A,3A,4C)	1	168
	(2B,2B,2A,4B,7A)	1	196	(2B,2B,2A,4B,7B)	1	196
	(2B,2B,2A,4B,6A)	1	168	(2B,2B,2A,4B,4C)	1	168
	(2B,2B,2A,2C,7A)	1	49	(2B,2B,2A,2C,7B)	1	49
	(2B,2A,3A,3A,3A)	1	1680	(2B,2A,3A,3A,4B)	1	1176
	(2B,2A,3A,4B,4B)	1	924	(2B,2A,4B,4B,4B)	1	504
	(2B,2A,2C,3A,3A)	1	210	(2B,2A,2C,3A,4B)	1	168
	(2B,2B,2B,3A,3A,3A)	1	394632	(2B,2A,2C,4B,4B)	1	168
	(2B,2B,2B,3A,3A,4B)	1	295596	(2B,2B,2B,3A,4B,4B)	1	221886
	(2B,2B,2B,4B,4B,4B)	1	162204	(2B,2B,2B,2C,3A,3A)	1	80568
	(2B,2B,2B,2C,3A,4B)	1	60480	(2B,2B,2B,2C,4B,4B)	1	46656
	(2B,2B,2B,2C,2C,3A)	1	12690	(2B,2B,2B,2C,2C,4B)	1	9600
	(2B,2B,2B,2C,2C,2C)	1	1680	(2B,2B,2B,2B,3A,7B)	1	64827
	(2B,2B,2B,2B,3A,6A)	1	80568	(2B,2B,2B,2B,3A,7A)	1	64827
	(2B,2B,2B,2B,3A,4C)	1	60480	(2B,2B,2B,2B,3A,4A)	1	25920
	(2B,2B,2B,2B,4B,7A)	1	48020	(2B,2B,2B,2B,4B,7B)	1	48020
	(2B,2B,2B,2B,4B,6A)	1	60480	(2B,2B,2B,2B,4B,4C)	1	46656
	(2B,2B,2B,2B,4B,4A)	1	19200	(2B,2B,2B,2B,2C,7A)	1	13720
	(2B,2B,2B,2B,2C,6A)	1	12960	(2B,2B,2B,2B,2C,7B)	1	13720
	(2B,2B,2B,2B,2C,4C)	1	9600	(2B,2B,2B,2B,2C,4A)	1	3360
	(2B,2B,2B,2B,2A,7A)	1	2058	(2B,2B,2B,2B,2A,7B)	1	2058
	(2B,2B,2B,2B,2A,6A)	1	1512	(2B,2B,2B,2B,2A,4C)	1	1344
	(2B,2B,2B,2A,3A,3A)	1	12096	(2B,2B,2B,2A,3A,4B)	1	9072
	(2B,2B,2B,2A,2C,3A)	1	1512	(2B,2B,2B,2A,4B,4B)	1	6384
	(2B,2B,2B,2A,2C,4B)	1	1344	(2B,2B,2B,2B,2B,2A,2C)	1	11760

Table B.8: Part3: GTSs for Affine Primitive Groups of Degree 8

group	ramification type	N.O	L.O	ramification type	N.O	L.O
ASL(3,2)	(2B,2B,2B,2B,2B,2A,4B)	1	67200	(2B,2B,2B,2B,2B,2A,3A)	1	90720
	(2B,2B,2B,2B,2B,2B,7A)	1	504210	(2B,2B,2B,2B,2B,2B,7B)	1	504210
	(2B,2B,2B,2B,2B,2B,6A)	1	622080	(2B,2B,2B,2B,2B,2B,4C)	1	472320
	(2B,2B,2B,2B,2B,2C,2C)	1	97920	(2B,2B,2B,2B,2B,2B,4A)	1	195840
	(2B,2B,2B,2B,2B,2C,4B)	1	472320	(2B,2B,2B,2B,2B,2C,3A)	1	622080
	(2B,2B,2B,2B,2B,4B,4B)	1	1722000	(2B,2B,2B,2B,2B,3A,4B)	1	2312912
	(2B,2B,2B,2B,2B,3A,3A)	1	3084480	(2B,2B,2B,2B,2B,2B,2B,2A)	1	685440
	(2B,2B,2B,2B,2B,2B,2B,2C)	1	4831679	(2B,2B,2B,2B,2B,2B,2B,4B)	1	18029760
	(2B,2B,2B,2B,2B,2B,2B,3A)	1	24086160	(2B,2B,2B,2B,2B,2B,2B,2B,2B)	1	187971840

Table B.9: Part1: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$(A(4) \times A(4)) : 2$	(3E,6C,6D)	5	1	(2C,2A,3B,6C)	1	2
	(2C,2B,3B,6C)	1	3	(2C,3B,3B,6D)	1	2
	(2C,2A,3A,6D)	1	2	(2A,2B,3A,6D)	1	3
	(2C,3A,3B,4A)	1	2	(2C,3A,3A,6C)	1	2
	(2C,2C,3D,6B)	1	4	(2C,2C,3C,6A)	1	4
$(2^4 : 3).4$	(2C,2B,4C,4C)	1	10	(2C,2B,4B,4B)	1	10
	(2C,2A,4C,4C)	1	1	(2C,2A,4B,4B)	1	1
	(2C,8B,8B)	5	1	(2C,8A,8A)	5	1
$2^4 : S(3) \times S(3)$	(2D,2E,3C,6C)	1	15	(2C,2E,3B,6B)	1	60
	(2D,2D,2E,2E,3C)	1	15	(2C,2D,2E,3A,3A)	1	12
	(2C,2D,2E,2A,3A)	1	60	(2C,2D,2E,2B,3A)	1	6
	(2C,2C,2E,2E,3B)	1	9			
$2^4.3^2 : 4$	(2C,8B,8B)	2	1	(2C,8A,8A)	2	1
	(3B,4C,8A)	4	1	(3B,4B,8B)	4	1
	(2C,3A,4C,4C)	1	27	(2C,3A,4B,4B)	1	27
	(2C,2A,4C,4C)	2	6	(2C,2A,4B,4B)	2	6
	(2C,2B,4C,4C)	2	9	(2C,2B,4B,4B)	2	9
$A\Gamma L(1, 16)$	(4B,4C,15A)	3	1	(4B,4C,15B)	3	1
$S(4) \times S(4) : 2$	(4F,4A,6C)	1	1	(4F,4B,6C)	3	1
	(2F,6C,8A)	3	1	(2D,8A,12A)	1	1
	(2C,2D,4A,6C)	1	2	(2C,2D,4C,6C)	1	6
	(2C,2D,3B,8A)	1	8	(2C,3A,4F,6C)	1	12
	(2C,3A,4F,4E)	1	6	(2C,2A,4F,6C)	1	6
	(2C,2A,4F,6C)	1	9	(2D,2F,2F,6C)	1	18
	(2D,2F,3B,4F)	1	24	(2D,2E,4F,4A)	1	4
	(2D,2E,4F,4B)	1	12	(2D,2E,2F,8A)	1	12
	(2D,2D,4A,6B)	1	2	(2D,2D,4C,6B)	1	6
	(2D,2D,2F,12A)	1	6	(2D,3A,4F,6B)	1	12
	(2D,2A,4F,6B)	1	6	(2D,2B,4F,6B)	1	9
	(2C,4F,4F,6B)	1	54			
	(2D,2D,2E,2F,2F)	1	72	(2C,2D,2D,2F,3B)	1	48
	(2C,2D,2D,2E,4A)	1	8	(2C,2D,2D,2E,4B)	1	24
	(2C,2D,2D,3A,6B)	1	24	(2C,2D,2D,2A,6B)	1	12
	(2C,2C,2C,4F,6C)	1	108	(2C,2D,2D,2B,6B)	1	18
	(2C,2C,2C,4F,4E)	1	48	(2C,2C,2E,4F,4F)	1	240
	(2C,2C,2D,4F,6B)	1	108	(2C,2C,2D,3A,6C)	1	24
	(2C,2C,2D,2A,6C)	1	12	(2C,2C,2D,3A,4E)	1	12
	(2C,2C,2D,2B,6C)	1	18	(2C,2D,2E,3A,4F)	1	54
	(2C,2D,2E,2A,4F)	1	24	(2C,2D,2E,2B,4F)	1	36
	(2C,2C,2C,2C,2D,6C)	1	216	(2C,2C,2C,2C,2D,4E)	1	96
	(2C,2C,2C,2D,2E,4F)	1	480	(2C,2C,2C,2D,2D,6B)	1	216
	(2C,2C,2D,2D,2E,3A)	1	108	(2C,2C,2D,2D,2E,2A)	1	48
	(2C,2C,2D,2D,2E,2B)	1	72	(2C,2C,2C,2C,2D,2D,2E)	1	960
$A\Gamma L(2, 4)$	(4C,4C,15A)	4	1	(4C,4C,15B)	4	1
	(4C,6A,6C)	24	1	(4C,6B,6C)	14	1
	(2C,8A,15A)	2	1	(2C,8A,15B)	2	1
	(3B,6C,8A)	8	1	(2C,2C,4C,6C)	1	448
	(2C,2C,3B,15A)	1	30	(2C,2C,3B,15B)	1	30
	(2C,3B,3C,4C)	1	112	(2C,3B,3B,6C)	1	120
	(2B,2C,4C,6B)	1	112	(2B,2C,4C,6A)	1	120
	(2B,2C,3B,8A)	1	40	(2B,3B,4C,4C)	1	198
	(2B,2C,2C,2C,4C)	1	2688	(2B,2C,2C,3B,3B)	1	696
$ASL(2, 4) : 2$	(4D,5A,6A)	12	1	(3A,4D,8A)	4	1
	(4D,4C,6A)	2	1	(4D,4A,6A)	4	1
	(4D,4B,5A)	6	1	(3A,6A,6A)	8	2
	(2C,2C,4D,6A)	2	54	(2C,2C,4D,4B)	2	24
	(2B,2C,3A,6A)	2	56	(2B,2C,4D,5A)	2	45
	(2B,2C,3A,4C)	2	8	(2B,2C,4D,4A)	1	32
	(2B,2B,3A,5A)	3	30	(2B,2B,4D,6A)	2	24
	(2B,2C,2C,2C,4D)	2	408	(2B,2B,2C,2C,3A)	2	384
	(2B,2B,2B,2C,4D)	2	144			

Table B.10: Part2: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$AGL(2,4)$	(3D,5B,15C)	1	1	(3D,5B,15B)	1	1
	(3D,5A,15C)	1	1	(3D,5A,15B)	1	1
	(3D,6D,15D)	1	1	(3D,6D,15A)	1	1
	(3D,4A,15C)	1	1	(3D,4A,15B)	1	1
	(3D,6B,15D)	1	1	(3D,6B,15A)	1	1
	(3C,5B,15D)	1	1	(3C,5B,15A)	1	1
	(3C,5A,15D)	1	1	(3C,5A,15A)	1	1
	(3C,6C,15C)	1	1	(3C,6C,15B)	1	1
	(3C,4A,15D)	1	1	(3C,4A,15A)	1	1
	(3C,6A,15C)	1	1	(3C,6A,15B)	1	1
	(3C,15B,15D)	1	1	(3C,15A,15C)	1	1
	(2B,2B,3D,15C)	1	15	(2B,2B,3D,15B)	1	15
	(2B,2B,3C,15D)	1	15	(2B,2B,3D,15A)	1	15
	(2B,3D,3D,6D)	1	18	(2B,3D,3D,6B)	1	24
	(2B,3C,3D,5B)	1	20	(2B,3C,3D,5A)	1	20
	(2B,3C,3D,4A)	1	18	(2B,3C,3C,6C)	1	18
	(2B,3C,3C,6A)	1	24	(3D,3D,3D,3E)	1	18
	(3C,3C,3C,3E)	1	18	(2B,2B,2B,3C,3D)	1	360
$ASL(2,4)$	(3A,5B,5B)	12	1	(3A,5A,5B)	12	1
	(3A,5A,5A)	1	1	(3A,4C,5B)	4	1
	(3A,4C,5A)	4	1	(3A,4B,5B)	4	1
	(3A,4B,5A)	4	1	(3A,4A,5B)	4	1
	(3A,4A,5A)	4	1	(2B,2B,3A,5B)	3	60
	(2B,2B,3A,5A)	3	60	(2B,2B,3A,4C)	2	36
	(2B,2B,3A,4B)	2	36	(2B,2B,3A,4A)	2	36
	(2B,2B,2B,2B,3A)	3	1080			
$2^4.A(6)$	(3B,5A,5B)	14	1	(3B,6A,5B)	12	1
	(3B,5A,6A)	12	1	(4C,5B,5B)	24	1
	(4C,5A,5B)	24	1	(4C,5A,5A)	24	1
	(4C,4C,8A)	8	1	(4C,4B,5B)	4	1
	(4C,4B,5A)	4	1	(4C,4A,5B)	8	1
	(4C,6A,5A)	12	1	(4C,4A,5A)	8	1
	(4C,6A,5B)	12	1	(3A,5B,8A)	4	1
	(3A,5A,8A)	4	1	(2B,2B,3B,5B)	2	240
	(2B,2B,3B,5A)	2	240	(2B,2B,3B,6A)	2	108
	(2B,2B,4C,5B)	6	120	(2B,2B,4C,5A)	6	120
	(2B,2B,4C,4B)	4	24	(2B,2B,4C,4A)	2	96
	(2B,2B,4C,6A)	6	48	(2B,2B,3A,8A)	2	48
	(2B,3A,3B,3B)	2	160	(2B,3A,3B,4C)	2	174
	(2B,3A,4C,4C)	1	192	(2B,3A,3A,5B)	1	40
	(3A,3A,3A,3B)	2	12	(2B,3A,3A,5A)	1	40
	(2B,2B,2B,2B,3B)	2	6912	(2B,2B,2B,2B,4C)	6	2880
	(2B,2B,2B,3A,3A)	1	1440			
$2^4 : S(5)$	(4C,5A,6C)	3	1	(4A,5A,6C)	1	1
	(4E,4C,5A)	3	1	(4E,4A,5A)	2	1
	(2D,12A,12A)	1	1	(2D,8A,12A)	1	1
	(2E,5A,12A)	3	1	(2E,5A,8A)	4	1
	(2D,2D,4C,6C)	1	18	(2D,2D,4A,6C)	1	6
	(2D,2D,4E,4C)	1	24	(2D,2D,4E,4A)	1	8
	(2D,2D,2E,12A)	1	18	(2D,2D,2E,8A)	1	24
	(2D,3A,6C,6C)	1	48	(2D,3A,4E,6C)	1	78
	(2D,2E,2E,5A)	1	45	(2D,2B,6C,6C)	1	10
	(2D,2B,4E,6C)	1	10	(2D,2A,6C,6C)	1	5
	(2D,2A,4E,6C)	1	5	(2E,3A,3A,6C)	1	30
	(2C,6C,6C,6C)	1	30	(2E,3A,3A,4E)	1	24
	(2C,4E,6C,6C)	1	48	(2C,4E,4E,6C)	1	78
	(2C,4E,4E,4E)	1	96	(2C,2D,4C,5A)	1	15
	(2C,2D,4A,5A)	1	5	(2C,3A,5A,6C)	1	45
	(2C,3A,6C,4B)	1	7	(2C,3A,5A,4D)	1	14
	(2C,3A,6C,6A)	1	10	(2C,3A,5A,6B)	1	20
	(2C,3A,3A,12A)	1	10	(2C,3A,4E,5A)	1	60
	(2C,3A,4E,4B)	1	13	(2C,3A,4E,4D)	1	20
	(2C,3A,4E,6A)	1	8	(2C,3A,4E,6B)	1	16
	(2C,3A,3A,8A)	1	8	(2C,2B,5A,6C)	1	10

Table B.11: Part3: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4 : S(5)$	(2C,2A,5A,6C)	1	5	(2C,2B,5A,4E)	1	10
	(2C,2A,5A,4E)	1	5	(2D,2D,2D,2E,2E)	1	360
	(2C,2D,2D,2D,4B)	1	120	(2C,2D,2D,2D,4A)	1	40
	(2C,2D,2D,3A,6C)	1	369	(2C,2D,2D,3A,4E)	1	540
	(2C,2D,2D,2B,6C)	1	60	(2C,2D,2D,2B,4E)	1	80
	(2C,2D,2D,2A,6C)	1	30	(2C,2D,2D,2A,4E)	1	40
	(2C,2D,2E,3A,3A)	1	174	(2C,2C,2D,6C,6C)	1	246
	(2C,2C,2D,4E,6C)	1	372	(2C,2C,2D,4E,4E)	1	528
	(2C,2C,2D,3A,5A)	1	300	(2C,2C,2D,3A,4B)	1	48
	(2C,2C,2D,3A,4D)	1	96	(2C,2C,2D,3A,6A)	1	58
	(2C,2C,2D,3A,6B)	1	116	(2C,2C,2D,2B,5A)	1	50
	(2C,2C,2D,2A,5A)	1	25	(2C,2C,2E,3A,6C)	1	135
	(2C,2C,2E,3A,4E)	1	144	(2C,2C,2C,5A,6C)	1	225
	(2C,2C,2C,4B,6C)	1	36	(2C,2C,2C,4D,6C)	1	72
	(2C,2C,2C,6A,6C)	1	45	(2C,2C,2C,6B,6C)	1	90
	(2C,2C,2C,4E,5A)	1	300	(2C,2C,2C,4E,4B)	1	48
	(2C,2C,2C,4E,4D)	1	96	(2C,2C,2C,4E,6A)	1	48
	(2C,2C,2C,4E,6B)	1	96	(2C,2C,2C,3A,12A)	1	45
	(2C,2C,2C,3A,8A)	1	48	(2C,2C,2C,2D,2D,6C)	1	1836
	(2C,2C,2C,2D,2D,4E)	1	2688	(2C,2C,2C,2D,2E,3A)	1	864
	(2C,2C,2D,2D,2D,3A)	1	2700	(2C,2C,2D,2D,2D,2B)	1	400
	(2C,2C,2C,2C,2D,5A)	1	1500	(2C,2C,2D,2D,2D,2A)	1	200
	(2C,2C,2C,2C,2D,4B)	1	240	(2C,2C,2C,2C,2D,4D)	1	317
	(2C,2C,2C,2C,2D,6A)	1	288	(2C,2C,2C,2C,2D,6B)	1	576
	(2C,2C,2C,2C,2E,6C)	1	648	(2C,2C,2C,2C,2E,4E)	1	768
	(2C,2C,2C,2C,2C,12A)	1	216	(2C,2C,2C,2C,2C,8A)	1	256
	(2C,2C,2C,2C,2D,2D,2D)	1	13440	(2C,2C,2C,2C,2D,2D,2E)	1	4320
$2^4 : A(5)$	(2C,3A,3A,5B)	1	60	(2C,3A,3A,5A)	1	60
	(2C,3A,3A,6A)	1	24	(2C,3A,3A,6B)	1	24
	(2C,3A,3A,6C)	1	24	(2C,3A,3A,4A)	1	18
	(2C,3A,3A,4B)	1	36	(2C,2B,3A,5B)	1	10
	(2C,2B,3A,5B)	1	10	(2C,2A,3A,5A)	1	5
	(2C,2A,3A,5B)	1	5	(2C,2C,2C,3A,3A)	1	1080
	(2C,2C,2C,2B,3A)	1	180	(2C,2C,2C,2A,3A)	1	90
$2^4.A(7)$	(3A,6B,14B)	6	1	(3A,6B,14A)	6	1
	(3A,6B,8A)	12	1	(3A,7B,14B)	2	1
	(3A,7B,14A)	6	1	(3A,7B,8A)	6	1
	(3A,7A,14B)	6	1	(3A,7A,14A)	2	1
	(3A,7A,8A)	6	1	(3A,5A,14B)	16	1
	(3A,5A,14A)	16	1	(3A,5A,8A)	20	1
	(3A,6A,7B)	6	1	(3A,6B,7B)	7	1
	(3A,7B,7B)	8	1	(3A,6A,7A)	6	1
	(3A,6B,7A)	7	1	(3A,7B,7A)	9	1
	(3B,7A,7A)	4	1	(3B,5A,7B)	7	1
	(3B,5A,7A)	7	1	(4B,6B,6A)	36	1
	(4B,6B,6B)	28	1	(3B,4B,14B)	8	1
	(3B,4B,14A)	8	1	(3B,4B,6A)	18	1
	(3B,4B,6B)	24	1	(3B,4B,8A)	12	1
	(4B,6A,7A)	18	1	(4B,6B,7A)	24	1
	(4B,7A,7B)	8	1	(4B,7A,7A)	12	1
	(4B,5A,6A)	60	1	(4B,5A,6B)	56	1
	(4B,5A,7B)	54	1	(4B,5A,7A)	54	1
	(4B,5A,5A)	56	1	(4B,4B,14B)	44	1
	(4B,4B,14A)	44	1	(4B,4A,7B)	6	1
	(4B,4A,7A)	6	1	(2B,14B,14B)	1	1
	(2B,14A,14B)	2	1	(2B,8A,14B)	2	1
	(2B,8A,14A)	2	1	(3A,3A,3A,3B)	3	378
	(3A,3A,3A,4B)	2	816	(2B,3A,3A,6A)	1	312
	(2B,3A,3A,6B)	1	712	(2B,3A,3A,7B)	1	364
	(2B,3A,3A,7A)	1	364	(2B,3A,3A,5A)	1	1620
	(2B,3A,3A,4B)	1	132	(2B,3A,3B,4B)	1	882
	(2B,3A,4B,4B)	1	4752	(2B,2B,3A,14B)	1	252
	(2B,2B,3A,14A)	1	252	(2B,2B,3A,8A)	1	384
	(2B,2B,3B,7B)	1	196	(2B,2B,3B,7A)	1	196

Table B.12: Part4: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4.A(7)$	(2B,2B,4B,6A)	3	384	(2B,2B,4B,6B)	3	532
	(2B,2B,4B,7A)	3	378	(2B,2B,4B,7B)	3	378
	(2B,2B,4B,5A)	3	840	(2B,2B,2B,3A,3A)	1	32264
	(2B,2B,2B,2B,4B)	3	20160			
$2^4.S(6)$	(4F,5A,6C)	24	1	(3B,4F,12A)	4	1
	(4F,4F,8A)	4	1	(4F,6A,6C)	6	1
	(4E,6C,6C)	12	1	(4E,4F,8B)	4	1
	(4E,4F,12A)	8	1	(5A,6B,6C)	48	1
	(3B,6B,8B)	4	1	(3B,6B,12A)	12	1
	(4F,6B,8A)	8	1	(4E,6B,8B)	8	1
	(4E,6B,12A)	8	1	(6B,6B,8A)	8	1
	(4C,5A,6B)	6	1	(4B,6B,6C)	6	1
	(4D,6B,6C)	12	1	(6A,6B,6C)	12	1
	(3A,6C,8B)	2	1	(3A,6C,12A)	4	1
	(2C,2C,4E,5A)	2	30	(2C,2C,4E,6A)	2	12
	(2C,2C,6B,6C)	2	36	(2C,2C,3A,8A)	1	8
	(2C,3A,3B,4F)	2	4	(2C,3A,4E,4F)	1	42
	(2C,3A,3B,6B)	2	42	(2C,3A,3B,4E)	1	36
	(2C,3A,3A,6C)	2	12	(2C,2D,4F,4F)	1	36
	(2C,2D,4E,4E)	1	36	(2C,2D,4F,6B)	1	54
	(2C,2D,6B,6B)	1	60	(2C,2D,3A,5A)	1	30
	(2E,2C,4F,5A)	2	60	(2E,2C,4F,6A)	2	18
	(2E,2C,4E,6C)	2	75	(2E,2C,6B,5A)	2	132
	(2E,2C,4B,6B)	2	12	(2E,2C,4D,6B)	1	48
	(2E,2C,6A,6B)	2	36	(2E,2C,4F,6A)	2	18
	(2E,2C,3A,8B)	1	12	(2E,2C,3A,12A)	2	12
	(2E,2E,4F,6C)	2	180	(2E,2E,6B,6C)	2	324
	(2E,2E,4C,6B)	1	72	(2E,3A,4F,4F)	1	48
	(2E,3A,4F,6B)	1	96	(2E,3A,6B,6B)	1	144
	(2E,2D,3B,4F)	1	71	(2E,2D,4E,4E)	1	72
	(2E,2D,3B,6B)	1	144	(2E,2D,4E,6B)	1	150
	(2E,2D,3A,6C)	1	54	(2B,3B,3B,4F)	2	64
	(2B,4F,4F,4F)	1	20	(2B,3B,4E,4F)	2	63
	(2B,4E,4E,4F)	1	84	(2B,3B,3B,6B)	3	73
	(2B,4F,4F,6B)	1	64	(2B,3B,4E,6B)	2	69
	(2B,4E,4E,6B)	1	80	(2B,4F,6B,6B)	1	104
	(2B,6B,6B,6B)	1	168	(2B,2C,6C,6C)	2	30
	(2B,2C,5A,5A)	2	60	(2B,2C,4F,8A)	1	12
	(2B,2C,4F,12A)	2	9	(2B,2C,4E,8A)	1	12
	(2B,2C,6B,8B)	1	18	(2B,2C,6B,12A)	2	10
	(2B,2C,4B,5A)	2	5	(2B,2C,4D,5A)	1	20
	(2B,2C,5A,6A)	2	15	(2B,2E,5A,6C)	2	135
	(2B,2E,3B,8B)	1	24	(2B,2E,3B,12A)	2	24
	(2B,2E,4F,8A)	1	32	(2B,2E,4E,8B)	1	32
	(2B,2E,4E,12A)	2	25	(2B,2E,6B,8A)	1	50
	(2B,2E,4C,5A)	1	30	(2B,2E,4B,6C)	2	12
	(2B,2E,4D,6C)	1	48	(2B,2E,6A,6C)	2	27
	(2B,3A,3B,6C)	2	42	(2B,3A,3B,4C)	1	12
	(2B,3A,4F,5A)	1	20	(2B,3A,4E,6C)	1	54
	(2B,3A,5A,6B)	1	50	(2B,2D,3B,5A)	1	60
	(2B,2D,4F,6C)	1	36	(2B,2D,4E,5A)	1	60
	(2B,2D,6B,6C)	1	72	(2B,2B,6C,8B)	1	12
	(2B,2B,6C.12A)	2	12	(2B,2B,5A,8A)	1	20
	(2C,2C,2C,2D,3A)	1	54	(2E,2C,2C,2C,6B)	2	162
	(2E,2C,2C,3A,3A)	1	120	(2E,2E,2C,2C,4E)	2	384
	(2E,2E,2C,2D,3A)	1	342	(2E,2E,2E,2C,4F)	2	936
	(2E,2E,2E,2C,6B)	2	1782	(2B,2C,2C,2C,5A)	2	75
	(2B,2C,2C,2C,6A)	2	75	(2B,2C,2C,3A,4F)	1	96
	(2B,2C,2C,3A,6B)	1	96	(2B,2C,2C,2D,4E)	1	96
	(2B,2E,2C,2C,6C)	2	180	(2B,2E,2C,3A,3B)	2	207
	(2B,2E,2C,2D,4F)	1	240	(2B,2E,2C,3A,4E)	1	240
	(2B,2E,2C,2D,6B)	1	360	(2B,2E,2E,2C,5A)	2	750
	(2B,2E,2E,2C,4B)	2	60	(2B,2E,2E,2C,4D)	1	240
	(2B,2E,2E,2C,6A)	2	171	(2B,2E,2E,2E,6C)	2	1782

Table B.13: Part5: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4.S(6)$	(2B,2E,2E,2E,4C)	1	360	(2B,2E,2E,3A,4F)	1	480
	(2B,2E,2E,3A,6B)	1	768	(2B,2E,2E,2D,3B)	1	648
	(2B,2E,2E,2D,4E)	1	768	(2B,2B,2C,3B,4F)	2	144
	(2B,2B,2C,4E,4F)	1	200	(2B,2B,2C,3B,6B)	2	180
	(2B,2B,2C,4E,6B)	1	210	(2B,2B,2C,2C,8A)	1	32
	(2B,2B,2C,3A,6C)	1	126	(2B,2B,2C,2D,5A)	1	150
	(2B,2B,2E,3B,3B)	2	480	(2B,2B,2E,4F,4F)	1	316
	(2B,2B,2E,4E,4F)	2	432	(2B,2B,2E,4E,4E)	1	508
	(2B,2B,2E,4F,6B)	1	546	(2B,2B,2E,6B,6B)	1	852
	(2B,2B,2E,2C,8B)	1	80	(2B,2B,2E,2C,12A)	2	60
	(2B,2B,2E,2E,8A)	1	256	(2B,2B,2E,3A,5A)	1	250
	(2B,2B,2E,2D,6C)	1	324	(2B,2B,3A,3A,3B)	1	78
	(2B,2B,2B,3B,6C)	2	216	(2B,2B,2B,3B,4C)	1	54
	(2B,2B,2B,4F,5A)	1	150	(2B,2B,2B,4E,6C)	1	270
	(2B,2B,2B,5A,6B)	1	300	(2B,2B,2B,2B,2E,2C,2C)	1	3168
	(2B,2E,2E,2C,2C,2C)	2	936	(2B,2E,2E,2E,2E,2C)	1	9936
	(2B,2B,2C,2C,2C,2D)	1	240	(2B,2B,2E,2C,2C,3A)	1	594
	(2B,2B,2E,2E,2C,2D)	1	1872	(2B,2B,2E,2E,2E,3A)	1	4050
	(2B,2B,2B,2C,2C,4F)	1	480	(2B,2B,2B,2C,2C,6B)	1	540
	(2B,2B,2B,2E,2C,3B)	2	1053	(2B,2B,2B,2E,2C,4E)	1	1272
	(2B,2B,2B,2E,2E,4F)	1	2832	(2B,2B,2B,2E,2E,6B)	1	4428
	(2B,2B,2B,2B,2C,6C)	1	648	(2B,2B,2B,2B,2E,5A)	1	1500
	(2B,2B,2B,2B,3A,3B)	1	432			
	(2B,2B,2B,2B,2E,2E,2E)	1	23328	(2B,2B,2B,2B,2B,2B,3B)	1	2430
$2^4.PSL(4,2)$	(6B,6C,6C)	168	1	(6B,6C,7B)	144	1
	(6B,7B,7B)	64	1	(6B,6C,7A)	144	1
	(6B,7A,7B)	64	1	(6B,7A,7A)	64	1
	(6B,6B,14B)	128	1	(6B,6B,14A)	128	1
	(6B,6B,12A)	144	1	(6B,6B,8A)	104	1
	(6B,6B,15B)	102	1	(6B,6B,15A)	102	1
	(5A,6B,6C)	84	1	(5A,5A,6B)	28	1
	(5A,6B,7B)	96	1	(5A,6B,7A)	96	1
	(4E,6B,6C)	48	1	(4E,5A,6B)	72	1
	(4E,6B,7B)	36	1	(4E,6B,7A)	36	1
	(6A,6B,6C)	72	1	(5A,6A,6B)	60	1
	(6A,6B,7B)	48	1	(6A,6B,7A)	48	1
	(4B,6B,7B)	6	1	(4B,6B,7A)	6	1
	(3A,7B,7B)	3	1	(3A,7A,7A)	3	1
	(3A,6B,14B)	8	1	(3A,6B,14A)	8	1
	(3A,6B,15B)	14	1	(3A,6B,15A)	14	1
	(3A,4E,7B)	4	1	(3A,4E,7A)	4	1
	(4D,6C,6C)	48	1	(4D,6C,7B)	30	1
	(4D,7B,7B)	16	1	(4D,6C,7A)	30	1
	(4D,7A,7B)	14	1	(4D,7A,7A)	16	1
	(4D,6B,14B)	18	1	(4D,6B,14A)	18	1
	(4D,6B,15B)	24	1	(4D,6B,15A)	24	1

Table B.14: Part6: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4.PSL(4,2)$	(4D,6B,12A)	12	1	(4D,6B,8A)	18	1
	(4D,5A,6C)	32	1	(4D,5A,7B)	24	1
	(4D,5A,7A)	24	1	(4D,5A,5A)	16	1
	(3A,4D,14B)	2	1	(3A,4D,14A)	2	1
	(3A,4D,7B)	4	1	(3A,4D,7A)	4	1
	(4D,4D,7B)	3	1	(4D,4D,7A)	3	1
	(4D,4E,6C)	8	1	(4D,4E,5A)	12	1
	(4D,6A,6C)	12	1	(4D,5A,6A)	6	1
	(4D,6C,14B)	30	1	(4D,6C,14A)	30	1
	(4D,6C,7B)	21	1	(4D,6C,7A)	21	1
	(4D,6C,12A)	24	1	(4D,6C,8A)	24	1
	(4C,7B,14B)	14	1	(4D,7B,14A)	14	1
	(4C,7B,15B)	12	1	(4D,7B,15A)	12	1
	(4C,7B,12A)	18	1	(4D,7B,8A)	14	1
	(4C,7A,14B)	14	1	(4D,7A,14A)	14	1
	(4C,7A,15B)	12	1	(4D,7A,15A)	12	1
	(4C,7A,12A)	18	1	(4D,7A,8A)	14	1
	(4C,5A,14B)	24	1	(4D,5A,14A)	24	1
	(4C,5A,15B)	24	1	(4D,5A,15A)	24	1
	(4C,5A,12A)	36	1	(4D,5A,8A)	12	1
	(4C,4E,15A)	6	1	(4D,4E,15B)	6	1
	(4C,6A,15A)	12	1	(4D,6A,15B)	12	1
	(4C,4B,15A)	3	1	(4D,4B,15B)	3	1
	(2C,6C,15A)	6	1	(2C,6C,15B)	6	1
	(2C,6C,14A)	3	1	(2C,6C,14B)	3	1
	(2C,7B,15A)	3	1	(2C,7B,15B)	3	1
	(2C,7A,15A)	3	1	(2C,7A,15B)	3	1
	(2C,5A,14A)	3	1	(2C,5A,14B)	3	1
	(2C,5A,15A)	6	1	(2C,5A,15B)	6	1
	(3B,6C,14A)	22	1	(3B,6C,14B)	22	1
	(3B,6C,15A)	21	1	(3B,6C,15B)	21	1
	(3B,6C,12A)	24	1	(3B,7B,14B)	12	1
	(3B,7B,12A)	16	1	(3B,7B,14A)	12	1
	(3B,7B,15A)	11	1	(3B,7B,15A)	11	1
	(3B,7B,8A)	6	1	(3B,7A,14B)	12	1
	(3B,7A,12A)	16	1	(3B,7A,14A)	12	1
	(3B,7A,15B)	11	1	(3B,7A,15A)	11	1
	(3B,7A,8A)	6	1	(3B,5A,14A)	6	1
	(3B,5A,12A)	20	1	(3B,5A,14B)	6	1
	(3B,5A,15A)	9	1	(3B,5A,15B)	9	1
	(3B,4E,15A)	8	1	(3B,4E,15B)	8	1
	(3B,6A,15A)	6	1	(3B,6A,15B)	6	1
	(4F,7B,6C)	66	1	(4F,7B,7B)	36	1
	(4F,7A,6C)	66	1	(4F,7A,7B)	32	1
	(4F,7A,7A)	36	1	(4F,4E,15B)	8	1
	(4F,6B,14A)	104	1	(4F,6B,15B)	104	1
	(4F,6B,15A)	96	1	(4F,6B,15B)	96	1
	(4F,6B,12A)	104	1	(4F,6B,8A)	64	1
	(4F,5A,7A)	18	1	(4F,5A,7B)	18	1
	(4F,4D,14A)	14	1	(4F,4D,14B)	14	1
	(4F,4D,15A)	24	1	(4F,4D,15B)	24	1
	(4F,4D,12A)	18	1	(4F,4D,8A)	12	1
	(4F,4E,7A)	28	1	(4F,4E,7B)	28	1
	(4F,4E,6C)	48	1	(4F,4E,5A)	12	1
	(4F,6A,7A)	18	1	(4F,6A,7B)	18	1
	(4F,4F,14A)	36	1	(4F,4F,14B)	36	1
	(4F,4F,15A)	45	1	(4F,4F,15B)	45	1
	(4F,4F,12A)	64	1	(2D,14A,14B)	3	1
	(2D,12A,14A)	2	1	(2D,12A,14B)	2	1
	(2D,15B,14A)	4	1	(2D,15A,14B)	4	1
	(2D,15B,12A)	6	1	(2D,15B,15B)	3	1
	(2D,15A,14A)	4	1	(2D,15A,14B)	4	1
	(2D,15A,15B)	6	1	(2D,15A,15A)	3	1
	(2D,15A,12A)	6	1			

Table B.15: Part7: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(4C,4C,4C,6B)	1	12744	(3A,4C,4C,4C)	1	1296
	(4C,4C,4C,4D)	5	1368	(4C,4C,4C,4F)	1	9216
	(2C,4C,4C,6B)	1	1980	(2C,3A,4C,4C)	1	144
	(2C,4C,4C,4F)	1	1584	(2B,6B,6B,6B)	1	25572
	(2B,3A,6B,6B)	1	2016	(2B,4D,6B,6B)	1	5400
	(2B,3A,4D,6B)	1	312	(2B,4D,4D,6B)	1	660
	(2B,3A,4D,4D)	1	48	(2B,4C,6B,6C)	1	5220
	(2B,4C,6B,7B)	1	2562	(2B,4C,6B,7A)	1	2562
	(2B,4C,6B,5A)	1	4230	(2B,4C,6B,4E)	1	1320
	(2B,4C,6B,6A)	1	1656	(2B,4C,6B,4B)	1	252
	(2B,3A,4C,6C)	1	420	(2B,3A,4C,5A)	1	420
	(2B,3A,4C,7A)	1	315	(2B,3A,4C,7B)	1	315
	(2B,3A,4C,4E)	1	96	(2B,3A,4C,6A)	1	180
	(2B,4C,4D,6C)	1	1128	(2B,4C,4D,5A)	1	900
	(2B,4C,4D,7A)	1	574	(2B,4C,4D,7B)	1	574
	(2B,4C,4C,14B)	1	574	(2B,4C,4C,14A)	1	574
	(2B,4C,4C,15B)	1	495	(2B,4C,4C,15A)	1	495
	(2B,4C,4C,12A)	1	660	(2B,4C,4C,8A)	1	528
	(2B,4C,4F,6C)	1	3996	(2B,4C,4F,5A)	1	3510
	(2B,4C,4F,7A)	1	1918	(2B,4C,4F,7B)	1	1918
$2^4.PSL(4,2)$	(2B,4C,4F,6A)	1	1368	(2B,4C,4F,4B)	1	288
	(2B,2C,6B,6C)	1	648	(2B,2C,6B,5A)	1	720
	(2B,2C,6B,7B)	1	420	(2B,2C,6B,7A)	1	420
	(2B,2C,3A,7B)	1	42	(2B,2C,3A,7A)	1	42
	(2B,2C,4D,6C)	1	90	(2B,2C,4D,5A)	1	90
	(2B,2C,4C,15A)	1	90	(2B,2C,4C,15B)	1	90
	(2B,2C,4F,6C)	1	576	(2B,2C,4F,5A)	1	480
	(2B,2C,4F,7B)	1	336	(2B,2C,4F,7A)	1	336
	(2B,2B,6C,14B)	1	252	(2B,2B,6C,14A)	1	252
	(2B,2B,6C,15B)	1	210	(2B,2B,6C,15A)	1	210
	(2B,2B,6C,12A)	1	216	(2B,2B,6C,8A)	1	192
	(2B,2B,7B,14B)	2	112	(2B,2B,7B,14A)	2	112
	(2B,2B,7B,15B)	1	105	(2B,2B,7B,15A)	1	105
	(2B,2B,7B,12A)	1	140	(2B,2B,7B,8A)	1	112
	(2B,2B,7A,14B)	2	112	(2B,2B,7A,14A)	2	112
	(2B,2B,7A,15B)	1	105	(2B,2B,7A,15A)	1	105
	(2B,2B,7A,12A)	1	140	(2B,2B,7A,8A)	1	112
	(2B,2B,5A,14A)	1	210	(2B,2B,5A,14B)	1	210
	(2B,2B,5A,15A)	1	165	(2B,2B,5A,15B)	1	165
	(2B,2B,5A,12A)	1	240	(2B,2B,5A,8A)	1	160
	(2B,2B,4E,15A)	1	60	(2B,2B,4E,15B)	1	60
	(2B,2B,6A,15A)	1	75	(2B,2B,6A,15B)	1	75
	(2B,2B,4B,15A)	1	15	(2B,2B,4B,15B)	1	15
	(2B,3B,6B,6C)	1	4590	(2B,3B,6B,5A)	1	3930
	(2B,3B,6B,7A)	1	2296	(2B,3B,6B,7B)	1	2296
	(2B,3B,6B,4E)	1	1104	(2B,3B,6B,6A)	1	1380
	(2B,3B,6B,4B)	1	198	(2B,3B,3A,6C)	1	252
	(2B,3B,3A,7A)	1	294	(2B,3B,3A,7B)	1	294
	(2B,3B,3A,5A)	1	280	(2B,3B,3A,6A)	1	144
	(2B,3B,3A,4E)	1	120	(2B,3B,4D,6C)	1	1020
	(2B,3B,4D,7A)	1	504	(2B,3B,4D,7B)	1	504
	(2B,3B,4D,5A)	1	900	(2B,3B,4C,14B)	1	504
	(2B,3B,4C,12A)	1	504	(2B,3B,4C,14A)	1	504
	(2B,3B,4C,15A)	1	552	(2B,3B,4C,15B)	1	552
	(2B,2C,3B,15A)	1	75	(2B,2C,3B,15B)	1	75
	(2B,3B,3B,14B)	1	448	(2B,3B,3B,14A)	1	448
	(2B,3B,3B,15B)	1	390	(2B,3B,3B,15A)	1	390
	(2B,3B,3B,12A)	1	460	(2B,3B,3B,8A)	1	376
	(2B,3B,4F,6C)	1	3564	(2B,3B,4F,5A)	1	2730
	(2B,3B,4F,7A)	1	1708	(2B,3B,4F,7B)	1	1708
	(2B,3B,4F,4E)	1	912	(2B,3B,4F,6A)	1	1128
	(2B,3B,4F,4B)	1	180	(2B,3B,4C,8A)	1	456
	(2B,4F,6B,6B)	1	19740	(2B,3A,4F,6B)	1	1470

Table B.16: Part8: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2B,4F,4D,6B)	1	4308	(2B,3A,4D,4F)	1	840
	(2B,4F,4D,4D)	1	528	(2B,4F,4F,6B)	1	15180
	(2B,4F,4F,4D)	1	3384	(2B,4F,4F,4F)	1	11208
	(2B,3A,4F,4F)	1	840	(2B,2D,6C,6C)	1	1008
	(2B,2D,7B,6C)	1	756	(2B,2D,7B,7B)	1	378
	(2B,2D,7A,6C)	1	756	(2B,2D,7A,7B)	1	364
	(2B,2D,7A,7A)	1	378			
	(2B,2D,6B,14B)	1	812	(2B,2D,6B,14A)	1	812
	(2B,2D,6B,15B)	1	705	(2B,2D,6B,15A)	1	705
	(2B,2D,6B,12A)	1	828	(2B,2D,6B,8A)	1	624
	(2B,2D,5A,6C)	1	630	(2B,2D,5A,5A)	1	350
	(2B,2D,5A,7A)	1	630	(2B,2D,5A,7B)	1	630
	(2B,2D,3A,14B)	1	64	(2B,2D,3A,14A)	1	64
	(2B,2D,3A,15B)	1	70	(2B,2D,3A,15A)	1	70
	(2B,2D,4D,14B)	1	98	(2B,2D,4D,14A)	1	98
	(2B,2D,4D,15B)	1	150	(2B,2D,4D,15A)	1	150
	(2B,2D,4D,12A)	1	84	(2B,2D,4D,8A)	1	96
	(2B,2D,4E,6C)	1	312	(2B,2D,4E,5A)	1	360
	(2B,2D,4E,7B)	1	196	(2B,2D,4E,7A)	1	196
	(2B,2D,6A,6C)	1	432	(2B,2D,6A,5A)	1	360
	(2B,2D,6A,7B)	1	252	(2B,2D,6A,7A)	1	252
	(2B,2D,4F,14B)	1	644	(2B,2D,4F,14A)	1	644
$2^4.PSL(4,2)$	(2B,2D,4F,15B)	1	585	(2B,2D,4F,15A)	1	585
	(2B,2D,4F,12A)	1	624	(2B,2D,4F,8A)	1	432
	(2B,2D,4B,7B)	1	42	(2B,2D,4B,7A)	1	42
	(3B,4C,4C,6B)	1	10848	(3B,4C,4C,3A)	1	1416
	(3B,4C,4C,4D)	1	2304	(3B,4C,4C,4F)	1	8640
	(2C,3B,4C,6B)	1	1656	(2C,3A,3B,4C)	1	180
	(3B,3B,4C,6B)	1	9984	(3B,3B,3A,4C)	1	1116
	(3B,3B,4C,4D)	4	1632	(3B,3B,4C,4F)	1	7296
	(2C,3B,4C,4F)	1	1368	(2C,3B,3B,6B)	1	1380
	(2C,3B,3B,3A)	1	144	(2C,3B,3B,4F)	1	1128
	(3B,3B,3B,6B)	2	4320	(3B,3B,3B,3A)	1	432
	(3B,3B,3B,4D)	1	1824	(3B,3B,3B,4F)	1	5136
	(2D,4C,6B,6B)	1	18216	(2D,4C,3A,6B)	1	1722
	(2D,4C,4D,6B)	1	3672	(2D,3A,4C,4D)	1	352
	(2D,4C,4D,4D)	1	424	(2D,4C,4C,6C)	1	3768
	(2D,4C,4C,7A)	1	1806	(2D,4C,4C,7B)	1	1806
	(2D,4C,4C,5A)	1	3600	(2D,4C,4C,4E)	1	848
	(2D,4C,4C,6A)	1	1332	(2D,4C,4C,4B)	1	264
	(2D,4C,4F,6B)	1	15660	(2D,4C,4F,3A)	1	924
	(2D,4C,4F,4D)	1	3360	(2D,4C,4F,4F)	1	10728
	(2D,2C,6B,6B)	1	2484	(2D,2C,4D,6B)	1	252
	(2D,2C,4C,6C)	1	648	(2D,2C,4C,5A)	1	540
	(2D,2C,4C,7A)	1	294	(2D,2C,4C,7B)	1	294
	(2D,2C,4F,6B)	1	1872	(2D,2C,4C,4D)	1	288
	(2D,2C,4F,4F)	1	1296	(2D,3D,6B,6B)	1	17172
	(2D,3B,3A,6B)	1	966	(2D,3B,4D,6B)	1	3792
	(2D,3B,3A,3A)	1	184	(2D,3B,4D,4D)	1	444
	(2D,3B,4C,6C)	1	3414	(2D,3B,4C,5A)	1	2460
	(2D,3B,4C,7A)	1	1596	(2D,3B,4C,7B)	1	1596
	(2D,3B,4C,4E)	1	888	(2D,3B,4C,6A)	1	888
	(2D,3B,4C,4B)	1	168	(2D,3B,2C,6C)	1	432
	(2D,3B,2C,7B)	1	252	(2D,3B,2C,7A)	1	252
	(2D,3B,2C,5A)	1	360	(2D,3B,3B,6C)	1	2142
	(2D,3B,3B,7B)	1	1036	(2D,3B,3B,7A)	1	1036
	(2D,3B,3B,5A)	1	1020	(2D,3B,3B,4E)	1	592
	(2D,3B,3B,6A)	1	540	(2D,3B,4F,6B)	1	12456
	(2D,3B,4F,4D)	1	2640	(2D,3B,4F,4F)	1	5310
	(2D,2D,6B,6C)	1	3528	(2D,2D,6B,5A)	1	1680
	(2D,2D,6B,7B)	1	2772	(2D,2D,6B,7A)	1	2772
	(2D,2D,6B,4E)	1	1344	(2D,2D,6B,6A)	1	1296
	(2D,2D,4D,6C)	1	824	(2D,2D,4D,5A)	1	440
	(2D,2D,4D,7B)	1	616	(2D,2D,4D,7A)	1	616

Table B.17: Part9: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2D,2D,4D,4E)	1	176	(2D,2D,4D,6A)	1	168
	(2D,2D,4C,14B)	1	616	(2D,2D,4C,14A)	1	616
	(2D,2D,4C,15B)	1	630	(2D,2D,4C,15A)	1	630
	(2D,2D,4C,12A)	1	672	(2D,2D,4C,8A)	1	384
	(2D,2D,2C,14B)	1	42	(2D,2D,2C,14A)	1	42
	(2D,2D,2C,15B)	1	90	(2D,2D,2C,15A)	1	90
	(2D,2D,3B,14B)	1	308	(2D,2D,3B,14A)	1	308
	(2D,2D,3B,15B)	1	345	(2D,2D,3B,15A)	1	345
	(2D,2D,3B,12A)	1	432	(2D,2D,4F,4E)	1	768
$2^4.PSL(4,2)$	(2D,2D,4F,7A)	1	1092	(2D,2D,4F,7B)	1	1092
Projection-fiber Algorithm						
	(2B,2B,4C,4C,6B)	1	105672	(2B,2B,4C,4C,3A)	1	12312
	(2B,2B,4C,4C,4D)	1	23136	(2B,2B,4C,4C,4F)	1	81312
	(2B,2B,2C,4C,6B)	1	15552	(2B,2B,2C,4C,3A)	1	1296
	(2B,2B,2C,4C,4F)	1	12960	(2B,2B,2B,6B,6C)	1	44712
	(2B,2B,2B,6B,7B)	1	21756	(2B,2B,2B,6B,7A)	1	21756
	(2B,2B,2B,6B,5A)	1	37800	(2B,2B,2B,6B,4E)	1	10368
	(2B,2B,2B,6B,6A)	1	13014	(2B,2B,2B,6B,4B)	1	1944
	(2B,2B,2B,3A,6C)	1	3024	(2B,2B,2B,3A,5A)	1	3150
	(2B,2B,2B,3A,7B)	1	2646	(2B,2B,2B,3A,7A)	1	2646
	(2B,2B,2B,3A,4E)	1	864	(2B,2B,2B,3A,6A)	1	1296
	(2B,2B,2B,4D,6C)	1	9396	(2B,2B,2B,4D,5A)	1	8100
	(2B,2B,2B,4D,7B)	1	4704	(2B,2B,2B,4D,7A)	1	4704
	(2B,2B,2B,4C,14B)	1	4704	(2B,2B,2B,4C,14A)	1	4704
	(2B,2B,2B,4C,15B)	1	4050	(2B,2B,2B,4C,15A)	1	4050
	(2B,2B,2B,4C,12A)	1	5184	(2B,2B,2B,4C,8A)	1	4320
	(2B,2B,2B,2C,15B)	1	675	(2B,2B,2B,2C,15A)	1	675
	(2B,2B,2B,3B,14B)	1	4116	(2B,2B,2B,3B,14A)	1	4116
	(2B,2B,2B,3B,15B)	1	3600	(2B,2B,2B,3B,15A)	1	3600
	(2B,2B,2B,3B,12A)	1	4338	(2B,2B,2B,3B,8A)	1	3600
	(2B,2B,2B,4F,6C)	1	34668	(2B,2B,2B,4F,5A)	1	28500
	(2B,2B,2B,4F,7B)	1	16464	(2B,2B,2B,4F,7A)	1	16464
	(2B,2B,2B,4F,4E)	1	8640	(2B,2B,2B,4F,6A)	1	10800
	(2B,2B,2B,4F,4B)	1	2016	(2B,2B,3B,4C,6B)	1	94320
	(2B,2B,3B,4C,3A)	1	11196	(2B,2B,3B,4C,4D)	1	20064
	(2B,2B,3B,4C,4F)	1	71760	(2B,2B,3B,2C,6B)	1	13014
	(2B,2B,3B,2C,3A)	1	1296	(2B,2B,3B,2C,4F)	1	10800
	(2B,2B,3B,3B,6B)	1	83808	(2B,2B,3B,3B,3A)	1	10152
	(2B,2B,3B,3B,4D)	1	17472	(2B,2B,3B,3B,4F)	1	63664
	(2B,2B,2D,6B,6B)	1	162692	(2B,2B,2D,3A,6B)	1	12096
	(2B,2B,2D,4D,6B)	1	33552	(2B,2B,2D,4D,3A)	1	2112
	(2B,2B,2D,4D,4D)	1	3888	(2B,2B,2D,4C,6C)	1	32400
	(2B,2B,2D,4C,7B)	1	15484	(2B,2B,2D,4C,7A)	1	15484
	(2B,2B,2D,4C,5A)	1	27000	(2B,2B,2D,4C,4E)	1	7776
	(2B,2B,2D,4C,6A)	1	9792	(2B,2B,2D,4C,4B)	1	1728
	(2B,2B,2D,2C,6C)	1	3888	(2B,2B,2D,2C,5A)	1	3900
	(2B,2B,2D,2C,7B)	1	2352	(2B,2B,2D,2C,7A)	1	2352
	(2B,2B,2D,3B,6C)	1	28728	(2B,2B,2D,3B,5A)	1	24000
	(2B,2B,2D,3B,7B)	1	13720	(2B,2B,2D,3B,7A)	1	13720
	(2B,2B,2D,3B,4E)	1	6528	(2B,2B,2D,3B,6A)	1	8136
	(2B,2B,2D,3B,4B)	1	1296	(2B,2B,2D,4B,6B)	1	126792
	(2B,2B,2D,4B,3A)	1	8904	(2B,2B,2D,4B,4D)	1	26688
	(2B,2B,2D,4B,4F)	1	96768	(2B,2B,2D,2D,12A)	1	4896
	(2B,2B,2D,2D,14B)	1	5096	(2B,2B,2D,2D,14A)	1	5096
	(2B,2B,2D,2D,15B)	1	4500	(2B,2B,2D,2D,15A)	1	4500
	(2B,2B,2D,2D,8A)	1	3840	(2B,2D,4C,4C,4C)	1	79228
	(2B,2D,2C,4C,4C)	1	11664	(2B,2D,3B,4C,4C)	1	67788
	(2B,2D,3B,2C,4C)	1	9792	(2B,2D,3B,3B,4C)	1	60744
	(2B,2D,3B,3B,2C)	1	8136	(2B,2D,3B,3B,3B)	1	53124
	(2B,2D,2D,4C,6B)	1	119232	(2B,2D,2D,4C,3A)	1	9912
	(2B,2D,2D,4C,4D)	1	24240	(2B,2D,2D,4C,4F)	1	97056
	(2B,2D,2D,2C,6B)	1	14688	(2B,2D,2D,2C,4D)	1	1728
	(2B,2D,2D,2C,4F)	1	11520	(2B,2D,2D,3B,6B)	1	108090

Table B.18: Part10: GTSs for Affine Primitive Groups of Degree 16

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^4.PSL(4,2)$	(2B,2D,2D,3B,3A)	1	6552	(2B,2D,2D,3B,4D)	1	23088
	(2B,2D,2D,3B,4F)	1	81432	(2B,2D,2D,2D,6C)	1	21672
	(2B,2D,2D,2D,7B)	1	17934	(2B,2D,2D,2D,7A)	1	17934
	(2B,2D,2D,2D,5A)	1	13650	(2B,2D,2D,2D,4E)	1	7680
	(2B,2D,2D,2D,6A)	1	9072	(2D,2D,2D,4C,4C)	1	98640
	(2D,2D,2D,2C,4C)	1	11520	(2D,2D,2D,3B,4C)	1	75168
	(2D,2D,2D,3B,2C)	1	9072	(2D,2D,2D,3B,3B)	1	41148
	(2D,2D,2D,2D,6B)	1	66528	(2D,2D,2D,2D,4D)	1	14592
	(2B,2B,2B,2B,4C,6B)	1	898560	(2B,2B,2B,2B,4C,3A)	1	103680
	(2B,2B,2B,2B,4C,4D)	1	191616	(2B,2B,2B,2B,4C,4F)	1	691584
	(2B,2B,2B,2B,2C,6B)	1	123120	(2B,2B,2B,2B,2C,3A)	1	10692
	(2B,2B,2B,2B,2C,4F)	1	103680	(2B,2B,2B,2B,2B,12A)	1	41040
	(2B,2B,2B,2B,2B,14B)	1	38416	(2B,2B,2B,2B,2B,14A)	1	38416
	(2B,2B,2B,2B,2B,15B)	1	33750	(2B,2B,2B,2B,2B,15A)	1	33750
	(2B,2B,2B,2B,2B,8A)	1	34560	(2B,2B,2B,2B,3B,6B)	1	799200
	(2B,2B,2B,2B,3B,3A)	1	94282	(2B,2B,2B,2B,3B,4D)	1	165888
	(2B,2B,2B,2B,3B,4F)	1	611136	(2B,2B,2B,2B,2D,6C)	1	276048
	(2B,2B,2B,2B,2D,7B)	1	131712	(2B,2B,2B,2B,2D,7A)	1	131712
	(2B,2B,2B,2B,2D,5A)	1	232500	(2B,2B,2B,2B,2D,4E)	1	62976
	(2B,2B,2B,2B,2D,6A)	1	79056	(2B,2B,2B,2B,2D,4B)	1	12960
	(2B,2B,2B,2D,4C,4C)	1	658704	(2B,2B,2B,2D,2C,4C)	1	94464
	(2B,2B,2B,2D,3B,4C)	1	584064	(2B,2B,2B,2D,3B,2C)	1	79056
	(2B,2B,2B,2D,3B,3B)	1	516546	(2B,2B,2B,2D,2D,6B)	1	1036800
	(2B,2B,2B,2D,2D,3A)	1	73710	(2B,2B,2B,2D,2D,4D)	1	212352
	(2B,2B,2B,2D,2D,4F)	1	808128	(2B,2B,2D,2D,2D,4C)	1	774144
	(2B,2B,2D,2D,2D,2C)	1	91008	(2B,2B,2D,2D,2D,3B)	1	691200
	(2B,2B,2D,2D,2D,2D)	1	470400	(2B,2B,2B,2B,2B,2B,6B)	1	7620480
	(2B,2B,2B,2B,2B,2B,3A)	1	874800	(2B,2B,2B,2B,2B,2B,4D)	1	1589760
	(2B,2B,2B,2B,2B,2B,4F)	1	5875200	(2B,2B,2B,2B,2B,2D,4C)	1	5625600
	(2B,2B,2B,2B,2B,2D,2C)	1	766080	(2B,2B,2B,2B,2B,2D,3B)	1	4983120
	(2B,2B,2B,2B,2D,2D,2D)	1	6659136	(2B,2B,2B,2B,2B,2B,2B,2D)	1	48041280

Table B.19: Part1: GTSs for Affine Primitive Groups of Degree 32

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(5,2)$	(6C,6C,6F)	552	1	(6C,6C,6B)	12	1
	(4J,6C,12B)	456	1	(4J,6C,8B)	384	1
	(4J,6C,6E)	444	1	(4J,6C,5A)	408	1
	(4I,6C,6F)	444	1	(4I,6C,6B)	12	1
	(4I,4J,12B)	360	1	(4I,4J,8B)	312	1
	(4I,4J,6E)	360	1	(4I,4J,5A)	306	1
	(4F,12B,12B)	360	1	(4F,8B,12B)	288	1
	(4F,8B,8B)	240	1	(4F,6E,12B)	372	1
	(4F,6E,8B)	300	1	(4F,6E,6E)	384	1
	(4F,6D,12B)	90	1	(4F,6D,8B)	96	1
	(4F,6D,6E)	84	1	(4F,6C,21B)	93	1
	(4F,6C,21A)	93	1	(4F,6C,12C)	90	1
	(4F,6C,10A)	96	1	(4F,6C,8C)	96	1
	(4F,5A,12B)	336	1	(4F,5A,8B)	264	1
	(4F,5A,6E)	342	1	(4F,5A,6D)	96	1
	(4F,5A,5A)	264	1	(4F,4J,14D)	84	1
	(4F,4J,14C)	84	1	(4F,4J,6F)	78	1
	(4F,4I,21B)	72	1	(4F,4I,21A)	72	1
	(4F,4I,12C)	72	1	(4F,4I,10A)	78	1
	(4F,4I,8C)	72	1	(4F,4H,12B)	24	1
	(4F,4H,8B)	48	1	(4F,4H,6E)	24	1
	(4F,4H,5A)	36	1	(4F,4G,12B)	12	1
	(4F,4G,8B)	24	1	(4F,4G,6E)	12	1
	(4F,4F,31A)	17	1	(4F,4F,31B)	17	1
	(4F,4F,31C)	17	1	(4F,4F,31D)	17	1
	(4F,4F,31E)	17	1	(4F,4F,31F)	17	1
	(4F,4F,30B)	18	1	(4F,4F,30A)	18	1
	(4F,4F,28B)	20	1	(4F,4F,28A)	20	1
	(4F,4D,21B)	6	1	(4F,4D,21A)	6	1
	(3B,4F,21B)	48	1	(3B,4F,21A)	48	1
	(3B,4F,15B)	18	1	(3B,4F,15A)	18	1
	(3B,4F,14B)	6	1	(3B,4F,14A)	6	1
	(3B,4F,12C)	24	1	(3B,4F,10A)	36	1
	(3B,4F,8C)	24	1	(4B,6F,12B)	78	1
	(4B,6F,8B)	68	1	(4B,6B,8B)	4	1
	(4B,6E,6F)	72	1	(4B,5A,6F)	74	1
	(4B,5A,6B)	6	1	(4B,4J,21B)	18	1
	(4B,4J,21A)	18	1	(4F,4G,5A)	18	1
	(3B,4B,31A)	74	1	(3B,4B,31B)	74	1
	(3B,4B,31C)	74	1	(3B,4B,31D)	74	1
	(3B,4B,31E)	74	1	(3B,4B,31F)	74	1
	(3B,4B,30B)	3	1	(3B,4B,30A)	3	1
	(3B,4B,28B)	1	1	(3B,4B,28A)	1	1
	(3B,6C,14D)	93	1	(3B,6C,14C)	93	1
	(3B,6C,6F)	288	1	(3B,4J,12B)	156	1
	(3B,4J,8B)	144	1	(3B,4J,7B)	18	1
	(3B,4J,6E)	180	1	(3B,4J,7A)	18	1
	(3B,4J,6D)	12	1	(3B,4J,5A)	180	1
	(3B,4I,14D)	72	1	(3B,4I,14C)	72	1
	(3B,4I,6F)	180	1	(3B,4D,14D)	3	1
	(3B,4D,14C)	3	1	(3B,3B,14C)	54	1
	(3B,3B,14D)	54	1	(3B,3B,6F)	64	1
	(3A,6F,12B)	52	1	(3A,6F,8B)	54	1
	(3A,6B,8B)	2	1	(3A,6F,6E)	58	1
	(3A,5A,6F)	60	1	(3A,5A,6B)	2	1
	(3A,4J,21B)	16	1	(3A,4J,21A)	16	1
	(2E,5A,6F)	18	1	(3A,3B,30B)	4	1
	(3A,3B,31A)	3	1	(3A,3B,31B)	4	1
	(3A,3B,31C)	3	1	(3A,3B,31D)	4	1
	(3A,3B,31E)	3	1	(3A,3B,31F)	4	1
	(3A,3B,30A)	4	1	(3A,3B,28B)	2	1
	(3A,3B,28A)	2	1	(2E,6F,12B)	16	1
	(2E,12B,14D)	13	1	(2E,12B,14C)	13	1
	(2E,8B,14D)	16	1	(2E,8B,14C)	16	1

Table B.20: Part2: GTSs for Affine Primitive Groups of Degree 32

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2E,8B,6F)	16	1	(2E,6E,14C)	13	1
	(2E,6E,14D)	13	1	(2E,6E,6F)	22	1
	(2E,6C,31A)	5	1	(2E,6C,31B)	5	1
	(2E,6C,31C)	5	1	(2E,6C,31D)	5	1
	(2E,6C,31E)	5	1	(2E,6C,31F)	5	1
	(2E,5A,14D)	16	1	(2E,5A,14C)	16	1
	(2E,4J,21B)	2	1	(2E,4J,21A)	2	1
	(2E,4I,31A)	4	1	(2E,4I,31B)	4	1
	(2E,4I,31C)	4	1	(2E,4I,31D)	4	1
	(2E,4I,31E)	4	1	(2E,4I,31F)	4	1
	(2E,3B,31A)	2	1	(2E,3B,31B)	2	1
	(2E,3B,31C)	2	1	(2E,3B,31D)	2	1
	(2E,3B,31E)	2	1	(2E,3B,31F)	2	1
	(2E,3B,30B)	1	1	(2E,3B,30A)	1	1
	(2D,14D,21B)	12	1	(2D,14D,21A)	12	1
$ASL(5,2)$	(2D,14D,12C)	13	1	(2D,14D,10A)	16	1
	(2D,14D,8C)	16	1	(2D,14C,21A)	12	1
	(2D,14C,21B)	12	1	(2D,14C,12C)	13	1
	(2D,14C,10A)	16	1	(2D,14C,8C)	16	1
	(2D,12B,28B)	16	1	(2D,12B,30B)	16	1
	(2D,12B,30A)	16	1	(2D,12B,28A)	13	1
	(2D,12B,31A)	17	1	(2D,12B,31B)	17	1
	(2D,12B,31C)	17	1	(2D,12B,31D)	17	1
	(2D,12B,31E)	17	1	(2D,12B,31F)	17	1
	(2D,8B,31A)	13	1	(2D,8B,31B)	13	1
	(2D,8B,31C)	13	1	(2D,8B,31D)	13	1
	(2D,8B,31E)	13	1	(2D,8B,31F)	13	1
	(2D,8B,30A)	16	1	(2D,8B,30B)	16	1
	(2D,8B,28A)	16	1	(2D,8B,28B)	16	1
	(2D,6F,21A)	36	1	(2D,6E,21B)	36	1
	(2D,6F,15A)	17	1	(2D,6F,15B)	17	1
	(2D,6F,12C)	16	1	(2D,6F,10A)	18	1
	(2D,6F,8C)	16	1	(2D,6B,15B)	1	1
	(2D,6E,31A)	17	1	(2D,6E,31B)	17	1
	(2D,6E,31C)	17	1	(2D,6E,31E)	17	1
	(2D,6E,31E)	17	1	(2D,6E,31F)	17	1
	(2D,6E,30A)	17	1	(2D,6E,30B)	17	1
	(2D,6E,28A)	13	1	(2D,6E,28B)	13	1
	(2D,6D,31A)	36	1	(2D,6D,31B)	36	1
	(2D,6D,31C)	36	1	(2D,6D,31D)	36	1
	(2D,6D,31E)	36	1	(2D,6D,31F)	36	1
	(2D,6B,15A)	1	1			
	(2D,5A,31A)	14	1	(2D,5A,31B)	14	1
	(2D,5A,31C)	14	1	(2D,5A,31D)	14	1
	(2D,5A,31E)	14	1	(2D,5A,31F)	14	1
	(2D,5A,30A)	18	1	(2D,5A,30B)	18	1
	(2D,5A,28A)	16	1	(2D,5A,28B)	16	1
	(2D,4H,31A)	2	1	(2D,4H,31B)	2	1
	(2D,4H,31C)	2	1	(2D,4H,31D)	2	1
	(2D,4H,31E)	2	1	(2D,4H,31F)	2	1
	(2D,4G,31A)	1	1	(2D,4G,31B)	1	1
	(2D,4G,31C)	1	1	(2D,4G,31D)	1	1
	(2D,4G,31E)	1	1	(2D,4G,31F)	1	1
	(2B,21B,31A)	1	1	(2B,21B,31B)	1	1
	(2B,21B,31C)	1	1	(2B,21B,31D)	1	1
	(2B,21B,31E)	1	1	(2B,21B,31F)	1	1
	(2B,21A,31A)	1	1	(2B,21A,31B)	1	1
	(2B,21A,31C)	1	1	(2B,21A,31D)	1	1
	(2B,21A,31E)	1	1	(2B,21A,31F)	1	1
	(2B,21A,30A)	1	1	(2B,21A,30B)	1	1
	(2B,21A,28B)	1	1	(2B,21B,28A)	1	1
	(2B,21B,30A)	1	1	(2B,21B,30B)	1	1

Table B.21: Part3: GTSs for Affine Primitive Groups of Degree 32

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2B,12C,31A)	1	1	(2B,12C,31B)	1	1
	(2B,12C,31C)	1	1	(2B,12C,31D)	1	1
	(2B,12C,31E)	1	1	(2B,12C,31F)	1	1
	(2B,10A,31A)	1	1	(2B,10A,31B)	1	1
$ASL(5,2)$	(2B,10A,31C)	1	1	(2B,10A,31D)	1	1
	(2B,10A,31E)	1	1	(2B,10A,31F)	1	1
	(2B,8C,31A)	1	1	(2B,8C,31B)	1	1
	(2B,8C,31C)	1	1	(2B,8C,31D)	1	1
	(2B,8C,31E)	1	1	(2B,8C,31F)	1	1
	Projection-fiber Algorithm					
	(2B,3A,3B,4J)	1	552	(2B,3B,4B,4J)	1	660
	(2B,4F,4F,4J)	1	3456	(2B,3B,4F,4I)	1	3192
	(2B,3B,4F,6C)	1	3888	(2B,3B,4F,4D)	1	198
	(2B,3B,4F,3B)	1	2088	(2B,2B,4J,21A)	1	140
	(2B,2B,12C,6F)	1	576	(2B,2B,4J,21B)	1	140
	(2B,2B,8B,6F)	1	552	(2B,2B,8B,6B)	1	24
	(2B,2B,6E,6F)	1	600	(2B,2B,3B,31F)	1	31
	(2B,2B,3B,31E)	1	31	(2B,2B,3B,31D)	1	31
	(2B,2B,3B,31C)	1	31	(2B,2B,3B,31B)	1	31
	(2B,2B,3B,31A)	1	31	(2B,2E,3B,4J)	1	84
	(2B,2B,3B,28A)	1	14	(2B,2B,3B,28B)	1	14
	(2B,2B,3B,30A)	1	30	(2B,2B,3B,30B)	1	30
	(2B,2B,5A,6F)	1	600	(2B,2B,5A,6B)	1	30
	(2B,2E,4F,12C)	1	600	(2B,2E,4F,8B)	1	672
	(2B,2E,4F,6E)	1	588	(2B,2E,4F,5A)	1	660
	(2B,2D,4J,12C)	1	2976	(2B,2D,4J,8B)	1	2592
	(2B,2D,4J,6E)	1	2952	(2B,2D,4J,5A)	1	2700
	(2B,2D,4I,6F)	1	2952	(2B,2D,4I,6B)	1	84
	(2B,2D,6C,6F)	1	3624	(2B,2D,6C,6B)	1	78
	(2B,2D,4F,21A)	1	588	(2B,2D,4F,21B)	1	588
	(2B,2D,4F,12D)	1	600	(2B,2D,4F,8C)	1	672
	(2B,2D,4F,10A)	1	660	(2B,2D,3B,14D)	1	588
	(2B,2D,2E,31F)	1	31	(2B,2D,2E,31E)	1	31
	(2B,2D,2E,31D)	1	31	(2B,2D,2E,31C)	1	31
	(2B,2D,2E,31B)	1	31	(2B,2D,2E,31A)	1	31
	(2B,2D,3B,14C)	1	588	(2B,2D,3B,6F)	1	1560
	(2D,3A,3B,4F)	1	2520	(2D,3B,4B,4F)	1	3186
	(2D,4F,4F,4F)	1	16752	(2D,2E,4F,4I)	1	3360
	(2D,2E,4F,6C)	1	4008	(2D,2E,4F,3B)	1	1248
	(2D,2D,4I,4J)	2	12384	(2D,2D,6C,4J)	1	19152
	(2D,2D,3A,6F)	1	2328	(2D,2D,3A,6B)	1	46
	(2D,2D,4B,6F)	1	3072	(2D,2D,4B,6B)	1	88
	(2D,2D,4F,12C)	1	16224	(2D,2D,3B,4J)	1	7440
	(2D,2D,4F,6D)	1	4008	(2D,2D,4F,8B)	1	13632
	(2D,2D,4F,4G)	1	624	(2D,2D,4F,4H)	1	1248
	(2D,2D,4F,6E)	1	17352	(2D,2D,4F,5A)	1	15360
	(2D,2D,2D,31F)	1	682	(2D,2D,2D,31E)	1	682
	(2D,2D,2D,31D)	1	682	(2D,2D,2D,31C)	1	682
	(2D,2D,2D,31B)	1	682	(2D,2D,2D,31A)	1	682
	(2D,2D,2D,28B)	1	588	(2D,2D,2D,28A)	1	588
	(2D,2D,2D,30B)	1	720	(2D,2D,2D,30A)	1	720
	(2D,2D,2E,14D)	1	588	(2D,2D,2E,14C)	1	588
	(2D,2D,2E,6F)	1	720	(2D,2D,2D,2D,4F)	1	776448
	(2B,2D,2D,2E,4F)	1	27641	(2B,2D,2D,2D,4J)	1	131904
	(2B,2B,2D,4F,3B)	1	25704	(2B,2B,2D,2D,6F)	1	24336
	(2B,2B,2D,2D,6B)	1	528	(2B,2B,2B,3B,4J)	1	5184

Table B.22: Part1: GTSs for Affine Primitive Groups of Degree 64

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^6 : D(2*7)$	(2I,4G,7A)	1	1	(2I,4G,7B)	1	1
	(2I,4G,7C)	1	1	(2I,4F,7A)	1	1
	(2I,4F,7B)	1	1	(2I,4F,7C)	1	1
	(2I,4E,7A)	1	1	(2I,4E,7B)	1	1
	(2I,4E,7C)	1	1	(2I,4D,7A)	1	1
	(2I,4D,7B)	1	1	(2I,4D,7C)	1	1
	(2I,4C,7A)	1	1	(2I,4C,7B)	1	1
	(2I,4C,7C)	1	1	(2I,4B,7A)	1	1
	(2I,4C,7B)	1	1	(2I,4C,7C)	1	1
	(2I,4B,7A)	1	1	(2I,4B,7B)	1	1
	(2I,4B,7C)	1	1	(2I,4A,7A)	1	1
	(2I,4A,7B)	1	1	(2I,4A,7C)	1	1
$2^6 : 7 : 6$	(2E,6C,6A)	2	1	(2E,6C,6F)	1	1
	(2E,6D,6E)	2	1	(2E,6D,6B)	1	1
$2^6 : 3^2 : S(3)$	(2E,2F,2G,6E)	1	12	(2E,2F,2G,6F)	1	12
$2^6 : 3^2 : S(3)$	(2E,6C,6E)	3	1	(2E,6D,6F)	3	1
$2^6 : (3^2 : 3) : 4$	(2E,4B,9B)	1	1	(2E,4B,9A)	1	1
	(2E,4A,9D)	1	1	(2E,4A,9C)	1	1
	(2H,4B,4E)	1	1	(2H,4A,4F)	1	1
	(2I,4B,4E)	1	1	(2I,4A,4F)	1	1
$2^6 : (3 \wr S(3))$	(2D,6Q,9B)	1	1	(2D,6O,9A)	1	1
$2^6 : (3^3 : D(2*6))$	(2G,6K,6H)	3	1	(2G,6C,12B)	1	1
	(2D,2G,2G,6I)	1	12			
$2^6 : (GL(2,2) \wr A(3))$	(2B,9B,12J)	1	1	(2B,9A,12I)	1	1
$2^6 : 3^3 : S(4)$	(3E,4G,4H)	8	1	(2F,4G,9A)	3	1
	(2F,4G,9B)	3	1			
$2^6 : (3^2 : 3) : Q_8 : 3$	(2F,4C,6P)	1	1	(2E,4C,6Q)	1	1
$2^6 : (GL(2,2) \wr S(3))$	(2L,2L,2J,4R)	1	48	(2B,2L,2L,12I)	1	12
$2^6 : (3^2 : 3) : Q_8 : S(3)$	(2E,6I,8A)	1	1	(2E,6I,8B)	1	1
$2^6 : (GL(3,2) \wr 2)$	(3B,4S,4O)	14	1	(2H,6J,8C)	10	1
	(2H,4U,14H)	3	1	(2H,4U,14G)	3	1
	(2G,6J,12D)	4	1	(2G,4S,28H)	1	1
	(2G,6S,28G)	1	1	(2G,4S,28F)	1	1
	(2C,8C,14H)	1	1	(2G,4S,28E)	1	1
	(2C,8C,14G)	1	1			
$A\Gamma L(3,4)$	(2D,6D,14A)	3	1	(2D,6D,14B)	3	1
$A\Sigma L(3,4)$	(2B,6C,8C)	8	1	(2B,4G,14A)	1	1
	(2B,4G,14B)	1	1			
$ASL(3,4)$	(3C,4D,4F)	8	1	(3C,4D,4E)	8	1
	(3C,4C,4G)	8	1	(3C,4C,4F)	8	1
	(3C,4B,4G)	8	1	(3C,4B,4E)	8	1
$2^6 : 3.A(6)$	(2C,4D,15A)	1	1	(2C,4D,15B)	1	1
	(2C,4D,15C)	1	1	(2C,4D,15D)	1	1
$2^6 : (6 \times GL(3,2))$	(3C,4G,4J)	2	1	(2F,4G,21B)	1	1
	(2F,4G,21A)	1	1			
$2^6 : \Sigma U(3,3)$	(2D,6A,8B)	2	1	(2D,4J,7A)	6	1
	(2B,2D,2D,6A)	1	30			
$2^6 : SU(3,3)$	(3B,3B,6A)	16	1	(2B,6A,8C)	2	1
	(2B,6A,8D)	2	1			
$2^6 : PGL(2,7)$	(2D,6B,8D)	1	1	(2D,6B,8C)	1	1
	(2G,2G,2E,4E)	1	12	(2G,2G,2F,4E)	1	24
$2^6 : Sp(6,2)$	(2H,6I,12F)	2	1	(2H,6I,10B)	10	1
	(2H,6I,8)	6	1	(2H,6I,6J)	12	1
	(2H,6H,12J)	14	1	(2H,6H,12I)	12	1
	(2H,6H,9A)	18	1	(2H,6H,8H)	4	1
	(2H,6H,8G)	8	1	(2H,4R,7A)	20	1
	(2E,6I,9A)	6	1	(2E,6I,8H)	2	1
	(2H,2H,2H,6H)	1	1296	(2E,2H,2H,6I)	1	138

Table B.23: Part2: GTSs for Affine Primitive Groups of Degree 64

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$2^6 : GO-(6,2)$	(2I,6J,12G)	3	1	(2I,6J,10B)	3	1
	(2I,6I,12I)	6	1	(2I,6I,10C)	4	1
	(2I,6I,10A)	2	1	(2I,6I,9A)	9	1
	(2I,4Q,24A)	4	1	(2I,4Q,20A)	5	1
	(2I,6H,20A)	1	1	(2H,6I,8E)	6	1
	(2H,6I,10B)	6	1	(2F,6L,8F)	4	1
	(2F,6I,20A)	2	1	(2I,2I,2I,6I)	1	270
	(2I,2I,2H,4Q)	1	360	(2I,2I,2H,4H)	1	60
	(2F,2I,2I,6I)	1	72	(2F,2I,2H,6I)	1	72
	(2C,2C,4P,6J)	1	3			
$2^6 : O-(6,2)$	(2F,5A,6G)	3	1	(2F,5A,6F)	3	1
	(2F,4G,12F)	9	1	(2F,4G,12E)	9	1
	(2F,5A,9A)	9	1	(2F,5A,9B)	9	1
	(2F,4D,9A)	3	1	(2F,4D,9B)	3	1
	(2E,6G,10C)	1	1	(2F,6G,10B)	1	1
	(2E,6G,10A)	1	1	(2F,6G,9A)	1	1
	(2E,6F,10C)	1	1	(2F,6F,10B)	1	1
	(2E,6F,10A)	1	1	(2F,6F,9A)	3	1
	(2E,2E,2F,6G)	1	27	(2E,2E,2F,6F)	1	27
$2^6 : S(8)$	(2J,6K,6L)	6	1	(2J,6G,15A)	4	1
	(2J,4N,24A)	2	1	(2J,4N,20A)	3	1
	(2I,6G,8E)	6	1	(2I,6G,10B)	2	1
	(2D,6G,20A)	1	1			
	(2J,2J,2J,6G)	1	180	(2J,2J,2I,4N)	1	96
	(2J,2J,2H,4Q)	1	132	(2F,2J,2H,6G)	1	20
	(2D,2J,2I,6G)	1	30	(2C,2J,2H,7A)	1	14
$2^6 : A(8)$	(2H,4E,15A)	3	1	(2H,4E,15B)	3	1
	(2G,6J,7B)	2	1	(2G,6J,7A)	2	1
	(2G,6H,7A)	2	1	(2G,6H,7B)	2	1
	(2E,6F,7A)	1	1	(2E,6F,7B)	1	1
	(2F,6F,7A)	1	1	(2F,6F,7B)	1	1
	(2F,4I,15A)	1	1	(2F,4I,15B)	1	1
	(2E,4I,15A)	1	1	(2E,4I,15B)	1	1
$2^6 : S(7)$	(2H,6I,7A)	6	1	(2H,6G,7A)	3	1
	(2H,6F,12G)	1	1	(2H,4M,12I)	2	1
	(2H,4M,12H)	3	1	(2D,7A,12I)	1	1
	(2D,7A,12H)	1	1	(2H,2H,2I,4M)	1	144
	(2D,2H,2I,7A)	1	42	(2D,2H,2H,12G)	1	12
$AGL(6,2)$	(4O,4O,6F)	48	1	(4K,4O,6F)	144	1
	(4G,4H,7E)	28	1	(3C,4G,7E)	84	1
	(3C,6E,6F)	64	1	(3C,4O,4P)	32	1
	(3C,4H,6J)	72	1	(3C,3C,8D)	64	1
	(3C,3C,6J)	352	1	(3C,3C,6G)	18	1
	(2F,7E,12D)	54	1	(2F,8D,12G)	36	1
	(2F,8D,10B)	96	1	(2F,8D,8F)	48	1
	(2F,7E,8C)	48	1	(2F,6J,12G)	226	1
	(2F,6J,12E)	56	1	(2F,6J,10B)	202	1
	(2F,6J,8F)	426	1	(2F,6J,6I)	102	1
	(2F,6H,7E)	34	1	(2F,6F,28D)	8	1
	(2F,6F,21D)	14	1	(2F,6F,28C)	8	1
	(2F,6F,21C)	14	1	(2F,6E,15E)	4	1
	(2F,6E,15D)	4	1	(2F,6E,14H)	10	1
	(2F,6E,14H)	10	1	(2F,4Q,7E)	30	1
	(2F,4O,42A)	7	1	(2F,4O,42B)	7	1
	(2F,4O,28E)	13	1	(2F,4O,28F)	13	1
	(2F,4O,21A)	7	1	(2F,4O,21B)	7	1
	(2F,4M,14I)	6	1	(2F,4M,14H)	6	1
	(2F,4K,28F)	2	1	(2F,4K,28E)	2	1
	(2F,4K,21A)	3	1	(2F,4K,21B)	3	1
	(2F,4H,63A)	1	1	(2F,4H,63B)	1	1
	(2F,4H,63C)	1	1	(2F,4H,63D)	1	1
	(2F,4H,63E)	1	1	(2F,4H,63F)	1	1
	(2F,3C,63A)	2	1	(2F,3C,63B)	2	1
	(2F,3C,63C)	2	1	(2F,3C,63D)	2	1

Table B.24: Part3: GTSs for Affine Primitive Groups of Degree 64

group	ramification type	N.O	L.O	ramification type	N.O	L.O
	(2F,3C,63E)	2	1	(2F,3C,63F)	2	1
	(2F,3C,62A)	2	1	(2F,3C,62B)	2	1
	(2F,3C,62C)	2	1	(2F,3C,62D)	2	1
	(2F,3C,62E)	2	1	(2F,3C,62F)	2	1
	(2F,3C,60B)	1	1	(2D,7E,12G)	70	1
	(2D,7E,12E)	12	1	(2D,7E,10B)	66	1
	(2D,7E,8F)	58	1	(2D,8D,15C)	20	1
	(2D,8D,12H)	16	1	(2D,8D,9A)	20	1
	(2D,6J,15C)	76	1	(2D,6J,12H)	78	1
	(2D,6J,9A)	78	1	(2D,6J,8G)	58	1
	(2D,6I,7E)	30	1	(2D,6G,28E)	2	1
	(2D,6F,60F)	2	1	(2D,6G,28D)	2	1
$AGL(6,2)$	(2D,6F,60G)	2	1	(2D,6F,21A)	6	1
	(2D,4P,15E)	3	1	(2D,6F,21B)	6	1
	(2D,4P,15D)	3	1	(2B,14I,15C)	1	1
	(2B,12H,14I)	1	1	(2B,14H,15C)	1	1
	(2B,12H,14H)	1	1	(2B,9A,15E)	1	1
	(2B,9A,15D)	1	1	(2B,9A,14I)	1	1
	(2B,9A,14H)	1	1	(2B,8G,15E)	1	1
	(2B,8G,15D)	1	1	(2B,7E,21A)	1	1
	(2B,7E,21B)	1	1	(2F,3C,60A)	1	1
	Projection-fiber Algorithm					
	(2D,2F,2F,8D)	1	1632	(2D,2F,2F,6J)	1	7032
	(2D,2D,2F,7E)	1	2100	(2B,2F,3C,6F)	1	444
	(2B,2F,2F,14E)	1	70	(2B,2F,2F,14D)	1	70
	(2B,2F,2F,15E)	1	30	(2B,2F,2F,15D)	1	30

Table B.25: GTSs for Affine Primitive Groups of Degree 128 and 256

group	ramification type	N.O	L.O	ramification type	N.O	L.O
$ASL(7,2)$	(2F,3B,14M)	2	1	(2F,3B,14N)	2	1
	(2F,4T,8N)	96	1			
$2^8:2^3:2^2:3$	(2E,3E,9C)	3	1	(2E,3E,9D)	3	1
$2^8.A(10)$	(2E,3A,9B)	28	1	(2E,3A,9A)	28	1
$2^8:PSL(2,8)$	(2E,3A,9A)	4	1	(2E,3A,9B)	4	1
	(2E,3A,9C)	4	1			
$2^8:PSL(2,17)$	(2C,3A,9A)	8	1	(2C,3A,9B)	8	1
	(2C,3A,9C)	8	1			

Table B.26: GTSs for Affine Primitive Groups with $G'' = 1$

degree p^e	group G	ramification type
4	A_4	$(2A,3A,3A,3B,3B),(2A,2A,3B,3B,3B),(2A,2A,3A,3A,3A)$ (2A,2A,2A,3A,3B) (3A,3B,3B,3B,3B),(3A,3A,3A,3A,3B)
8	$C_2^3.C_7$	(7E,7F,7F),(7C,7F,7F),(7C,7C,7D),(7B,7D,7E),(7B,7B,7F),(7A,7D,7D),(7A,7C,7F), (7A,7A,7B)
3	S_3	(2A,2A,2A,2A,2A,2A,2A,2A),(2A,2A,2A,2A,2A,2A,3A),(2A,2A,2A,2A,3A,3A), (2A,2A,3A,3A,3A)
9	$C_3^2.C_4$	(2A,2A,4A,4B)
	$C_3^2.C_8$	(4A,8A,8A),(4A,8B,8D),(4A,8C,8C),(4B,8A,8C),(4B,8B,8B), (4B,8D,8D),(3A,8A,8B),(3A,8C,8D)
5	$C_5.C_4$	(2A,4B,4B,5A),(2A,4A,4A,5A),(4B,4B,4B,4B),(4A,4A,4B,4B), (4A,4A,4A,4A),(2A,2A,2A,4B,4B),(2A,2A,2A,4A,4A)
	$C_5.C_2$	(2A,2A,5A,5A),(2A,2A,5B,5B),(2A,2A,5A,5B), (2A,2A,2A,2A,5B),(2A,2A,2A,2A,5B),(2A,2A,2A,2A,2A,2A)
25	$C_5^2.C_3$	(3A,3B,5A), (3A,3B,5B),(3A,3B,5C),(3A,3B,5D),(3A,3B,5D),(3A,3B,5E),(3A,3B,5F),(3A,3B,5G)
7	$C_7.C_6$	(2A,2A,6A,6B),(2A,3B,3B,6B),(2A,3A,3A,6B),(6A,6B,7A)
	$C_7.C_3$	(3A,3A,3B,3B)
11	$C_{11}.C_5$	(5C,5C,5D),(5B,5D,5D),(5A,5B,5B),(5A,5A,5C)
13	$C_{13}.C_3$	(3A,3B,13A),(3A,3B,13B),(3A,3B,13C), (3A,3B,13D)
	$C_{13}.C_6$	(3A,6B,6B),(3B,6A,6A),(2A,2A,3A,3B)
17	$C_{17}.C_8$	(2A,8B,8D),(2A,8A,8C)

Appendix C
Program Code

Note that some of the programs can be found in [8].

```
# LoadPackage
LoadPackage ("mapclass");
# Generate ramification types that fit the Riemann Hurwitz number
# Assign the group g first, then fix the list of conjugacy class representatives
CC:=[];
ct:=[];
CheckingTheGroup:=function(group)
 CC:=List(ConjugacyClasses(group),Representative);
 ct:=CharacterTable(group);
end;

# The index of a permutation
PermIndex:=function(perm,degree)
 return degree-Length(Orbits(Group(perm),[1..degree]));
end;

# The indices of conjugacy class representatives for a group
FindInd:=function(group,degree)
 local i,IndexSet,t,Ind;
IndexSet:=[];
 for i in [2..Length(CC)] do
t:=CC[i];
  Ind:=PermIndex(t,degree);
  Append(IndexSet,[rec(pos:=i,index:=Ind , Ord:=Order(t),
 size:=Size(Centralizer(k,t)))]);
 od;
return IndexSet;
end;
```

```
# Find all possible ramification types for a group with fixed degree and genus
Dim:=[];
RamiTypes:=function(group,degree,genus)
 local h,bb,dim,n,RH,IndexSet,Indices,PossibleCombinations,
 i,a,b,Temp,j,k,c,RamificationTypes;
 dim:=Length(Factors(degree));
 n:=Filtered(NormalSubgroups(group),x->Size(x)=degree)[1];
 RamificationTypes:=[];
 RH:=2*(degree+genus-1);
 IndexSet:=FindInd(group,degree);
 Indices:=List(IndexSet,x->x.index);
 PossibleCombinations:=RestrictedPartitions(RH,Elements(Indices));
 for i in [1..Length(PossibleCombinations)] do
  Temp:=[];
    a:=PossibleCombinations[i];
    b:=Elements(a);
    for j in [1..Length(b)] do
    k:=Length(Filtered(a,x->x=b[j]));
    c:=List(Filtered(IndexSet,x->x.index=b[j]),x->x.pos);
    Append(Temp,[UnorderedTuples(c,k)]);
    od;
    Temp:=Cartesian(Temp);
    for j in [1..Length(Temp)] do
    Append(RamificationTypes,[Concatenation(Temp[j])]);
    od;
    od;
 for i in [1..Length(RamificationTypes)] do
    a:=RamificationTypes[i];
    b:=List(a,x->Size(Centralizer(n,CC[x])));
    c:=[];
        for j in [1..Length(b)] do
    if b[j]<> 1 then
    Append(c,[dim-Length(FactorsInt(b[j]))]) ;
    else
    Append(c,[dim]);
    fi;
         od;
bb:=List(a,x->CC[x]);
h:=List(bb,x->degree-Length(MovedPoints(x)));
if Sum(c)<2*dim then
   Unbind(RamificationTypes[i]);
elif Sum(c)=2*dim and not 0 in h then
   Unbind(RamificationTypes[i]);
```

```
# elif ClassStructureCharTable(ct,RamificationTypes[i])=0 then
# Unbind(RamificationTypes[i]);
else
   Append(Dim,[c]);
  fi;
od;
 return Elements(RamificationTypes);
end;

AddOneGenerator:=function(SubgroupList,NewGen,group)
 local NewSubgroupList,a,b,x,Cgx,gg,c,t,y,j,r,h,flag,hh;
 NewSubgroupList:=[];
 for a in [1..Length(CC)] do
  NewSubgroupList[a]:=[];
 od;

 for b in [1..Length(CC)] do
  x:=CC[b];
  Cgx:=Centralizer(group,x);
  for gg in SubgroupList[b] do
   for c in DoubleCosetRepsAndSizes(group,Centralizer(group,NewGen),
                                    Normalizer(Cgx,gg)) do
    t:=NewGen^(c[1]);
    y:=x*t;

    j:=1;
    while j < Length(CC)+1 do
     r:=RepresentativeAction(group,y,CC[j]);
     if not r=fail then
      break;
     else
      j:=j+1;
     fi;
    od;
    h:=Group(Concatenation(GeneratorsOfGroup(gg),[t]))^r;
    flag:=0;
    for hh in NewSubgroupList[j] do
     if IsConjugate(Centralizer(group,CC[j]),hh,h) then
      flag:=1;
      break;
     fi;
    od;
    if flag=0 then
```

```
     Add(NewSubgroupList[j],h);
    fi;
   od;
  od;
 od;
  return NewSubgroupList;
end;

AddOneGenerator1:=function(SubgroupList,NewGen,group,p)
 local NewSubgroupList,a,b,x,Cgx,gg,c,t,y,r,h,stop;
 stop:=0;
 for b in [1..Length(CC)] do
  x:=CC[b];
  Cgx:=Centralizer(group,x);
  for gg in SubgroupList[b] do
   for c in DoubleCosetRepsAndSizes(group,Centralizer(group,NewGen),
                                    Normalizer(Cgx,gg)) do
    t:=NewGen^(c[1]);
    y:=x*t;
    r:=RepresentativeAction(group,y,CC[p]);
    if r<>fail then
     h:=Group(Concatenation(GeneratorsOfGroup(gg),[t]))^r;
     if h=group then
      stop:=1;
      break;
     fi;
    fi;
   od;
  od;
 od;
  return stop;
end;

# Find generating types
GeneratingType:=function(group,degree,genus)
 local RamificationTypes,GeneratingTypes,i,SubgroupList,
       ClassRepTuple,NewGen,m,j,k,n,a,p;
GeneratingTypes:=[];
 RamificationTypes:=RamiTypes(group,degree,genus);
 for i in [1..Length(RamificationTypes)] do
  Print("\r","Checking the ramification type ",i," with ",
        Length(RamificationTypes)-i," remaining ","\c");
  SubgroupList:=[];
```

```
  p:=0;
  ClassRepTuple:=List(RamificationTypes[i],x->CC[x]);
  for k in [1..Length(CC)] do
   SubgroupList[k]:=[];
   if IsConjugate(group,CC[k]^-1,
         ClassRepTuple[Length(ClassRepTuple)]) then
     p:=k;
   fi;
  od;

  SubgroupList[RamificationTypes[i][1]]:=[Group(ClassRepTuple[1])];
  for j in [2..Length(ClassRepTuple)-1] do
   NewGen:=ClassRepTuple[j];
   if j=Length(ClassRepTuple)-1 then
    m:=AddOneGenerator1(SubgroupList,NewGen,group,p);
    if m=1 then
     Append(GeneratingTypes,[RamificationTypes[i]]);
    fi;
   else
    m:=AddOneGenerator(SubgroupList,NewGen,group);
   fi;
   SubgroupList:=m;
  od;
 od;
 n:=Concatenation("CasesFor","degree",String(degree),"group",
    String(Position(AllPrimitiveGroups(DegreeOperation,degree),k)),
    "genus",String(genus));
 if Length(GeneratingTypes)<>0 then
  AppendTo(n,"GeneratingTypes:=",GeneratingTypes,";\n");
  AppendTo(n,"group:=",group,";\n");
  AppendTo(n,"CC:=",CC,";\n");
 fi;
 Print("\n");
 return GeneratingTypes;
end;

# Lifting Generating braid orbits from the quotient to the whole group.
LiftingQuotientorbit:=function(group,degree,tuple)
local GeneratingTypes,kk,Qorbits,gg,g,genus,q,f,l,n,e,nn,phi,s,ss,OO,orbits,
xx,i,j,A,LL,PP,a,b,c,d,h,m,z,zz,qz;

Qorbits:=[];
kk:=Stabilizer(group,degree);;
```

```
gg:=GeneratorsOfGroup(group);;
q:=Socle(group);;
nn:=List(gg,x->x*RepresentativeAction(q,degree^x,degree));;
phi:=GroupHomomorphismByImages(group,kk,gg,nn);;
s:=tuple;
ss:=List(s,x->x^phi);
orbits:=GeneratingMCOrbits(kk,0,ss);;
for j in [1..Length(orbits)] do
xx:=orbits[j].TupleTable[1].tuple;;
LL:=[];
for i in [1..Length(xx)] do
A:=Elements(PreImages(phi,xx[i]));;
A:=Filtered(A,x->IsConjugate(group,x,s[i]));
Add(LL,A);
od;
PP:=[];
if Length(LL)=3 then
for a in LL[1] do
for b in LL[2] do
c:=(a*b)^-1;
if c in LL[3] and Subgroup(group,[a,b])=group then
Add(PP,[a,b,c]);
fi;
od;
od;
if Size(Center(kk))=1 then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e]; end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e]; end);;
l:=Length(OO);
fi;
fi;
if Length(LL)=4 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
d:=(a*b*c)^-1;
if d in LL[4] and Subgroup(group,[a,b,c])=group then
Add(PP,[a,b,c,d]);
```

```
fi;
od;
od;
od;
if Size(Center(kk))=1 then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e]; end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e]; end);;
l:=Length(OO);
fi;
fi;
if Length(LL)=5 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
for d in LL[4] do
e:=(a*b*c*d)^-1;
if e in LL[5] and Subgroup(group,[a,b,c,d])=group then
Add(PP,[a,b,c,d,e]);
fi;
od;
od;
od;
od;
if Size(Center(kk))=1  then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e];end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e];end);;
l:=Length(OO);
fi;
fi;
if Length(LL)=6 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
```

```
for d in LL[4] do
for e in LL[5] do
f:=(a*b*c*d*e)^-1;
if f in LL[6] and Subgroup(group,[a,b,c,d,e])=group then
Add(PP,[a,b,c,d,e,f]);
fi;
od;
od;
od;
od;
od;
if Size(Center(kk))=1  then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e,
p[6]^e];end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e,
p[6]^e];end);;
l:=Length(OO);
fi;
fi;
if Length(LL)=7 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
for d in LL[4] do
for e in LL[5] do
for f in LL[6] do
g:=(a*b*c*d*e*f)^-1;
if g in LL[7] and Subgroup(group,[a,b,c,d,e,f])=group then
Add(PP,[a,b,c,d,e,f,g]);
fi;
od;
od;
od;
od;
od;
od;
if Size(Center(kk))=1 then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
```

```
,p[6]^e,p[7]^e];end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
,p[6]^e,p[7]^e];end);;
l:=Length(OO);
fi;
fi;
if Length(LL)=8 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
for d in LL[4] do
for e in LL[5] do
for f in LL[6] do
for g in LL[7] do
h:=(a*b*c*d*e*f*g)^-1;
if h in LL[8] and Subgroup(group,[a,b,c,d,e,f,g])=group then
Add(PP,[a,b,c,d,e,f,g,h]);
fi;
od;
od;
od;
od;
od;
od;
od;
if Size(Center(kk))=1 then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
,p[6]^e,p[7]^e,p[8]^e];end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
,p[6]^e,p[7]^e,p[8]^e];end);;
l:=Length(OO);
fi;
fi;
```

```
if Length(LL)=9 then
for a in LL[1] do
for b in LL[2] do
for c in LL[3] do
for d in LL[4] do
for e in LL[5] do
for f in LL[6] do
for g in LL[7] do
for h in LL[8] do
m:=(a*b*c*d*e*f*g*h)^-1;
if m in LL[9] and Subgroup(group,[a,b,c,d,e,f,g,h])=group then
Add(PP,[a,b,c,d,e,f,g,h,m]);
fi;
od;
od;
od;
od;
od;
od;
od;
od;
if Size(Center(kk))=1 then
OO:=Orbits(q,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
,p[6]^e,p[7]^e,p[8]^e,p[9]^e];end);;
l:=Length(OO);
else
zz:=Center(kk).1;
z:=Random(PreImages(phi,zz));
qz:=Subgroup(group,Concatenation(GeneratorsOfGroup(q),[z]));
OO:=Orbits(qz,PP,function(p,e) return [p[1]^e,p[2]^e,p[3]^e,p[4]^e,p[5]^e
,p[6]^e,p[7]^e,p[8]^e,p[9]^e];end);;
l:=Length(OO);
fi;
fi;
if l<>1 then
Add(Qorbits,[rec(numberofquotientorbit:=j,
LargestLength:=Length(orbits[j].TupleTable), q:=l, List:=PP )]);
else
Add(Qorbits,[rec(numberofquotientorbit:=j,
LargestLength:=Length(orbits[j].TupleTable), q:=l)]);
fi;
od;
 n:=Concatenation("CasesFor","degree",String(degree),"group");
```

```
 if Length(Qorbits)<>0 then
  AppendTo(n,"group:=",group,";\n");
  AppendTo(n,"GT:=",GT,";\n");
  AppendTo(n,"Lifting orbits:=",Qorbits,";\n");
 fi;
 Print("\n");
return Qorbits;
end;

# Checking the ramification type
 Find3Tuple:=function(RamificationType,group)
 local GeneratingTuples,ClassRepTuple,g1,g2,g3,gg,Cgx,c,t,y,r,h,i,TT,flag;
 ClassRepTuple:=List(RamificationType,x->CC[x]);
 GeneratingTuples:=[];
 g1:=ClassRepTuple[1];
 g2:=ClassRepTuple[2];
 g3:=ClassRepTuple[3];
 gg:=Group(g1);
 Cgx:=Centralizer(group,g1);
 for c in DoubleCosetRepsAndSizes(group,Centralizer(group,g2),Cgx) do
  t:=g2^(c[1]);
  y:=g1*t;
  r:=RepresentativeAction(group,y,Inverse(g3));
  if r<>fail then
   h:=Group(Concatenation(GeneratorsOfGroup(gg),[t]));
   if h=group then
    TT:=[g1,t,Inverse(y)];
    flag:=0;
    for i in [1..Length(GeneratingTuples)] do
     if RepresentativeAction(group,TT,GeneratingTuples[i],OnTuples)<>fail then
      flag:=1;
      break;
     fi;
    od;
    if flag=0 then
     Append(GeneratingTuples,[TT]);
    fi;
   fi;
  fi;
 od;
  return GeneratingTuples;
end;
```

```
# Checking the ramification type in the quotient group.
QFind3Tuple:=function(tuple,group,degree)
local GeneratingTuples,ClassRepTuple,g1,g2,g3,gg
,Cgx,c,t,y,r,h,i,TT,flag,Cl,gp,kk,q,nn,phi;

kk:=Stabilizer(group,degree);;
gp:=GeneratorsOfGroup(group);;
q:=Socle(group);;
nn:=List(gp,x->x*RepresentativeAction(q,degree^x,degree));;
phi:=GroupHomomorphismByImages(group,kk,gp,nn);;

ClassRepTuple:=List(tuple,x->x^phi);
 GeneratingTuples:=[];

 g1:=ClassRepTuple[1];
 g2:=ClassRepTuple[2];
 g3:=ClassRepTuple[3];

 gg:=Group(g1);
 Cgx:=Centralizer(kk,g1);
 for c in DoubleCosetRepsAndSizes(kk,Centralizer(kk,g2),Cgx) do
  t:=g2^(c[1]);
  y:=g1*t;
  r:=RepresentativeAction(kk,y,Inverse(g3));
  if r<>fail then
   h:=Group(Concatenation(GeneratorsOfGroup(gg),[t]));
   if h=kk then
    TT:=[g1,t,Inverse(y)];
    flag:=0;
    for i in [1..Length(GeneratingTuples)] do
     if RepresentativeAction(kk,TT,GeneratingTuples[i],OnTuples)<>fail then
      flag:=1;
      break;
     fi;
    od;
    if flag=0 then
     Append(GeneratingTuples,[TT]);
    fi;
   fi;
  fi;
 od;
  return GeneratingTuples;
end;
```

```
# Find affine primitive groups of degree p^e.
AffinePrimitiveGroups:=function(degree)
local
GroupL,GroupLL,k,i;
GroupL:=[];
GroupLL:=AllPrimitiveGroups(DegreeOperation,degree);;
for i in [1..Length(GroupLL)] do
k:=GroupLL[i];
if Size(Socle(k))=degree  then
Append(GroupL, [k]);
   fi;
 od;
  return GroupL;
end;

# Find affine primitive group of degree p^e.
AffinePrimitiveGroup:=function(degree,pos)
local
GroupL,position;
GroupL:=[];
position:=AffinePrimitiveGroups(degree);
if pos>Length(position) then
Print("There are only ", Length(position) ," affine primitive groups");
else
GroupL:=position[pos];
fi;
  return GroupL;
end;

# Ordering the conjugacy class representives.
Ordering:=function(group,degree)
 local i,j,IndexSet,x,y,oo,c,d,a,Label,LL,zz;
 IndexSet:=[];
Label:=FindInd(group,degree);
 for i in [1..Length(Label)] do
 for j in [i..Length(Label)] do
if i<>j then
if Label[i].Ord> Label[j].Ord then
zz:= Label[i];
Label[i]:= Label[j];
Label[j]:=zz;
fi;
```

```
if Label[i].Ord = Label[j].Ord  and Label[i].size < Label[j].size then
zz:= Label[i];
Label[i]:= Label[j];
Label[j]:=zz;
fi;
if Label[i].Ord = Label[j].Ord  and Label[i].size = Label[j].size
and Label[i].index > Label[j].index then
zz:= Label[i];
Label[i]:= Label[j];
Label[j]:=zz;
fi;
if Label[i].Ord = Label[j].Ord  and Label[i].size = Label[j].size
and Label[i].index = Label[j].index then
oo:=Set(IndexSet,x->x.Ord);
x:=Label[i].con;
y:=Label[j].con;
for c in [2..Length(CC)-1] do
for d in [c+1..Length(CC)] do
for a in oo do
if IsConjugate(group,x^a,CC[c])=true and
IsConjugate(group,y^a,CC[d])=true then
if Size(Centralizer(group,CC[c])) <
Size(Centralizer(group,CC[d])) then
zz:= Label[i];
Label[i]:= Label[j];
Label[j]:=zz;
else
break;
fi;
if Size(Centralizer(group,CC[c]))=Size(Centralizer(group,CC[d]))
 and PermIndex(CC[c],degree) > PermIndex(CC[d],degree) then
zz:= Label[i];
Label[i]:= Label[j];
Label[j]:=zz;
else
break;
fi;
else
break;
fi;
od;
od;
od;
```

```
fi;
fi;
od;
  Add(IndexSet, Label[i]);
od;
 LL:=[];
 for i in [1..Length(IndexSet)] do
 x:=IndexSet[i].pos;
 y:=IndexSet[i].Ord;
 Append(LL,[rec(pos:=x,order:=y)]);
 od;
 return LL;
end;

# Computing a conjugacy class by random conjugation and use it
to check a triple (aa,bb,cc).
h:=Centralizer(group,aa);;
Size(h);
Index(group,Centralizer(group,bb));
orbs:=[];
orbs:=List(orbs,x->[x,Order(aa*x)]);;
 for i in [1..1000] do
 x:=bb^Random(group);
 o:=Order(aa*x);;
 new:=true;
 for i in [1..Length(orbs)] do
 if orbs[i][2]=o then
 if RepresentativeAction(h,orbs[i][1],x)<>fail then
 new:=false;
 break;
 fi;
 fi;
 od;
 if new then
 Add(orbs,[x,o]);
 fi;
 od;
sum:=0;
for i in [1..Length(orbs)] do
sum:=sum+Index(h,Centralizer(h,orbs[i][1]));
od;
sum;
# We stop if
```

```
sum=Index(group,Centralizer(group,bb))
goodorbs:=Filtered(orbs,x->RepresentativeAction(group,(aa*x[1])^-1,cc)<>fail);;
Length(goodorbs);
GT:=Filtered(goodorbs,x->Size(group)=Size(Group(aa,x[1])));;
Length(GT);

# Application of Lemma 3.13.
Minusidentity:=function(group,degree,genus,p,e)
local m,q,I,C,n,ll,U,i,j,B,P,PP,RamificationTypes,A;
n:=Filtered(NormalSubgroups(group),x->Size(x)=degree)[1];
RamificationTypes:=RamiTypes(group,degree,genus);
if p=3 and e=3 then
m:=[[1,0,0],[0,1,0],[0,0,1]]*Z(p);;
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=3 and e=4 then
m:=[[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]]*Z(p);;
q:=Permutation(m,AsList(GF(p)^e));
 fi;
if p=3 and e=5 then
m:=[[1,0,0,0,0],[0,1,0,0,0],[0,0,1,0,0],[0,0,0,1,0],[0,0,0,0,1]]*Z(p);;
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=3 and e=6 then
m:=[[1,0,0,0,0,0],[0,1,0,0,0,0],[0,0,1,0,0,0],
[0,0,0,1,0,0],[0,0,0,0,1,0],[0,0,0,0,0,1]]*Z(p);
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=5 and e=3 then
m:=[[1,0,0],[0,1,0],[0,0,1]]*Z(p)^2;;
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=5 and e=4 then
m:=[[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]]*Z(p)^2;;
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=7 and e=3 then
m:=[[1,0,0],[0,1,0],[0,0,1]]*Z(p)^3;;
q:=Permutation(m,AsList(GF(p)^e));
fi;
if p=7 and e=4 then
m:=[[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]]*Z(p)^3;;
q:=Permutation(m,AsList(GF(p)^e));
```

```
fi;
P:=[];
 for i in [1..Length(RamificationTypes)] do
 U:=List(RamificationTypes[i],x->CC[x]);;
 ll:=[U[1]*q,U[2]*q,U[3]];;
 B:=List(ll,x->Size(Centralizer(n,x)));
 A:=[];
 for j in [1..Length(B)] do
 if B[j]<> 1 then
 Append(A,[e-Length(Factors(B[j]))]);
 else
 Append(A,[e]);
 fi;
 od;
if Sum(A)>=2*e then
Add(P,RamificationTypes[i]);
fi;
od;
 PP:=[];
 for i in [1..Length(P)] do
 U:=List(P[i],x->CC[x]);;
 ll:=[U[1]*q,U[2],U[3]*q];;
 B:=List(ll,x->Size(Centralizer(n,x)));
 A:=[];
 for j in [1..Length(B)] do
 if B[j]<> 1 then
 Append(A,[e-Length(Factors(B[j]))]);
 else
 Append(A,[e]);
 fi;
 od;
if Sum(A)>=2*e then
Add(PP,P[i]);
fi;
od;
 C:=[];
 for i in [1..Length(PP)] do
 U:=List(PP[i],x->CC[x]);;
 ll:=[U[1],U[2]*q,U[3]*q];;
 B:=List(ll,x->Size(Centralizer(n,x)));
 A:=[];
 for j in [1..Length(B)] do
 if B[j]<> 1 then
```

```
 Append(A,[e-Length(Factors(B[j]))]);
 else
 Append(A,[e]);
 fi;
 od;
if Sum(A)>=2*e then
Add(C,PP[i]);
fi;
od;
return C;
end;
# List F.
[ [(5,7)(6,8)(13,15)(14,16),(2,14)(3,15)(6,10)(7,11),(1,9,3)(2,8,13)(4,16,7)
(5,12,6)(10,14,15), (1,13,6,12,7,14,8,16,4,11,5,10,3,9)(2,15) ],
[ (5,7)(6,8)(13,15)(14,16),(2,14)(3,15)(6,10)(7,11),(1,10,13)(2,7,3)(4,15,9)
(5,11,12)(8,14,16), (1,15,4,9,3,11,7,16,2,13,8,14,6,10)(5,12) ],
[(5,7)(6,8)(13,15)(14,16), (2,14)(3,15)(6,10)(7,11), (1,11,9)(2,6,7)(4,14,13)
(5,10,16)(8,15,12), (1,9,5,14,4,15,6,16,8,12,3,13,2,11)(7,10) ],
[ (5,7)(6,8)(13,15)(14,16),(2,14)(3,15)(6,10)(7,11), (1,12,7)(2,5,9)(3,4,13)
(6,8,16)(10,15,11), (1,11,3,15,8,10,5,16,6,14,2,9,7,12)(4,13) ],
[(5,7)(6,8)(13,15)(14,16), (2,14)(3,15)(6,10)(7,11), (1,13,6)(2,4,12)(3,5,16)
(7,8,9)(11,15,14), (1,10,8,11,2,12,4,16,7,9,6,15,5,13)(3,14) ],
[(5,7)(6,8)(13,15)(14,16),(2,14)(3,15)(6,10)(7,11),(1,14,12)(2,3,6)(4,11,16)
(5,15,13)(8,10,9), (1,12,2,10,6,13,3,16,5,15,7,11,4,14)(8,9) ],
[(5,7)(6,8)(13,15)(14,16), (2,14)(3,15)(6,10)(7,11),(1,15,16)(3,7,6)(4,10,12)
(5,14,9)(8,11,13), (1,14,7,13,5,9,2,16,3,10,4,12,8,15)(6,11) ],
[(5,7)(6,8)(13,15)(14,16),(2,14)(3,15)(6,10)(7,11),(1,16,2)(3,8,12)(4,9,6)
(5,13,7)(10,11,14), (1,16)(2,14,5,11,8,13,7,15,3,12,6,9,4,10) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (2,16,15)(3,9,11)(4,8,5)
(6,13,10)(7,12,14), (1,15,4,9,3,11,7,16,2,13,8,14,6,10)(5,12) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,2,15)(3,10,5)(4,7,11)
(6,14,8)(9,12,13), (1,13,6,12,7,14,8,16,4,11,5,10,3,9)(2,15) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,3,11)(2,14,5)(4,6,15)
(7,10,8)(12,16,13), (1,11,3,15,8,10,5,16,6,14,2,9,7,12)(4,13) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,4,5)(2,13,11)(3,12,15)
(6,16,14)(7,9,10), (1,9,5,14,4,15,6,16,8,12,3,13,2,11)(7,10) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12),(1,5,8)(2,12,10)(3,13,14)
(6,9,15)(7,16,11), (1,12,2,10,6,13,3,16,5,15,7,11,4,14)(8,9) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,6,10)(2,11,8)(3,14,4)
(5,7,15)(9,16,12), (1,10,8,11,2,12,4,16,7,9,6,15,5,13)(3,14) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,7,14)(2,10,4)(3,15,8)
(5,6,11)(9,13,16), (1,16)(2,14,5,11,8,13,7,15,3,12,6,9,4,10) ],
[ (5,7)(6,8)(13,15)(14,16), (1,13)(4,16)(5,9)(8,12), (1,8,4)(2,9,14)(3,16,10)
```

```
(6,12,11)(7,13,15), (1,14,7,13,5,9,2,16,3,10,4,12,8,15)(6,11) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,9,3)(2,8,13)(4,16,7)
(5,12,6)(10,14,15), (1,15,4,9,3,11,7,16,2,13,8,14,6,10)(5,12) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,10,13)(2,7,3)(4,15,9)
(5,11,12)(8,14,16), (1,13,6,12,7,14,8,16,4,11,5,10,3,9)(2,15) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,11,9)(2,6,7)(4,14,13)
(5,10,16)(8,15,12), (1,11,3,15,8,10,5,16,6,14,2,9,7,12)(4,13) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,12,7)(2,5,9)(3,4,13)
(6,8,16)(10,15,11), (1,9,5,14,4,15,6,16,8,12,3,13,2,11)(7,10) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,13,6)(2,4,12)(3,5,16)
(7,8,9)(11,15,14), (1,12,2,10,6,13,3,16,5,15,7,11,4,14)(8,9) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11),(1,14,12)(2,3,6)(4,11,16)
(5,15,13)(8,10,9), (1,10,8,11,2,12,4,16,7,9,6,15,5,13)(3,14) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,15,16)(3,7,6)(4,10,12)
(5,14,9)(8,11,13), (1,16)(2,14,5,11,8,13,7,15,3,12,6,9,4,10) ],
[ (1,3)(2,4)(9,11)(10,12), (2,14)(3,15)(6,10)(7,11), (1,16,2)(3,8,12)(4,9,6)
(5,13,7)(10,11,14), (1,14,7,13,5,9,2,16,3,10,4,12,8,15)(6,11) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (2,16,15)(3,9,11)(4,8,5)
(6,13,10)(7,12,14), (1,13,6,12,7,14,8,16,4,11,5,10,3,9)(2,15) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,2,15)(3,10,5)(4,7,11)
(6,14,8)(9,12,13), (1,15,4,9,3,11,7,16,2,13,8,14,6,10)(5,12) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,3,11)(2,14,5)(4,6,15)
(7,10,8)(12,16,13), (1,9,5,14,4,15,6,16,8,12,3,13,2,11)(7,10) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,4,5)(2,13,11)(3,12,15)
(6,16,14)(7,9,10), (1,11,3,15,8,10,5,16,6,14,2,9,7,12)(4,13) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,5,8)(2,12,10)(3,13,14)
(6,9,15)(7,16,11), (1,10,8,11,2,12,4,16,7,9,6,15,5,13)(3,14) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,6,10)(2,11,8)(3,14,4)
(5,7,15)(9,16,12), (1,12,2,10,6,13,3,16,5,15,7,11,4,14)(8,9) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,7,14)(2,10,4)(3,15,8)
(5,6,11)(9,13,16), (1,14,7,13,5,9,2,16,3,10,4,12,8,15)(6,11) ],
[ (1,3)(2,4)(9,11)(10,12), (1,13)(4,16)(5,9)(8,12), (1,8,4)(2,9,14)(3,16,10)
(6,12,11)(7,13,15), (1,16)(2,14,5,11,8,13,7,15,3,12,6,9,4,10) ] ]
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